CALCULATION OF THE EFFICIENCY OF THE CURRENT SHIMS USED IN PS189*

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1 Introduction

A radiofrequency spectrometer has been built for PS189 experiment[1] which requires a very homogeneous magnetic field[2, 3, 4]. The magnet is described in Ref.[5], but here is the calculation of the efficiency of the electric shims used to correct the main field B and its radial and axial gradients. The results of the present calculation are reported in Ref.[5], and this note must be considered as an appendix to Ref.[5]. All notations are indeed identical, and not given again.
The goal is to calculate the magnitude of B_z, $\partial B_z/\partial R$, and $\partial B_z/\partial z$ as produced by the different printed circuits installed on the 2 pole faces of the magnet. Two different approaches have been used. First, it has been assumed that the loops which are longer in the azimuthal direction than in the radial one could be approximated by 2 infinite parallel wires in azimuthal direction. This approach allows to establish analytic relations easy to use. They give results in good agreement with our measurements. However, in order to check the validity of this approach and to better understand why it worked so well though our loops were not so similar to infinite wires, we finally also did a calculation taking into account the exact dimensions of the loops. These two approaches will be now developed in chapters **2** and **3** respectively, but the approximation by infinite wires is very generally good enough.

2 Calculation of the magnetic induction produced by infinite wires

2.1 Magnetic induction produced by an infinite straight wire near a pole face

The magnetic induction produced at a point M(R, z) by a single wire parallel to x axis, at position (y_0, h) with current intensity I (Fig. 1a) is :

$$b = \frac{\mu_0 I}{2\pi[(h-z)^2 + (R-y_0)^2]^{1/2}}$$

with $\mu_0 = 4\pi 10^{-7}$ MKSA
The axial component b_z is given by

$$b_z = \frac{\mu_0 I}{2\pi} \frac{(R-y_0)}{(h-z)^2 + (R-y_0)^2}$$

Following Neyret and Parain[6], we may use the image method to take into account the presence of the two pole pieces : the medium is then treated as homogeneous air, but the single wire is replaced by an infinite set of wires including itself plus its images given by the 2 pole faces (Fig. 1b).

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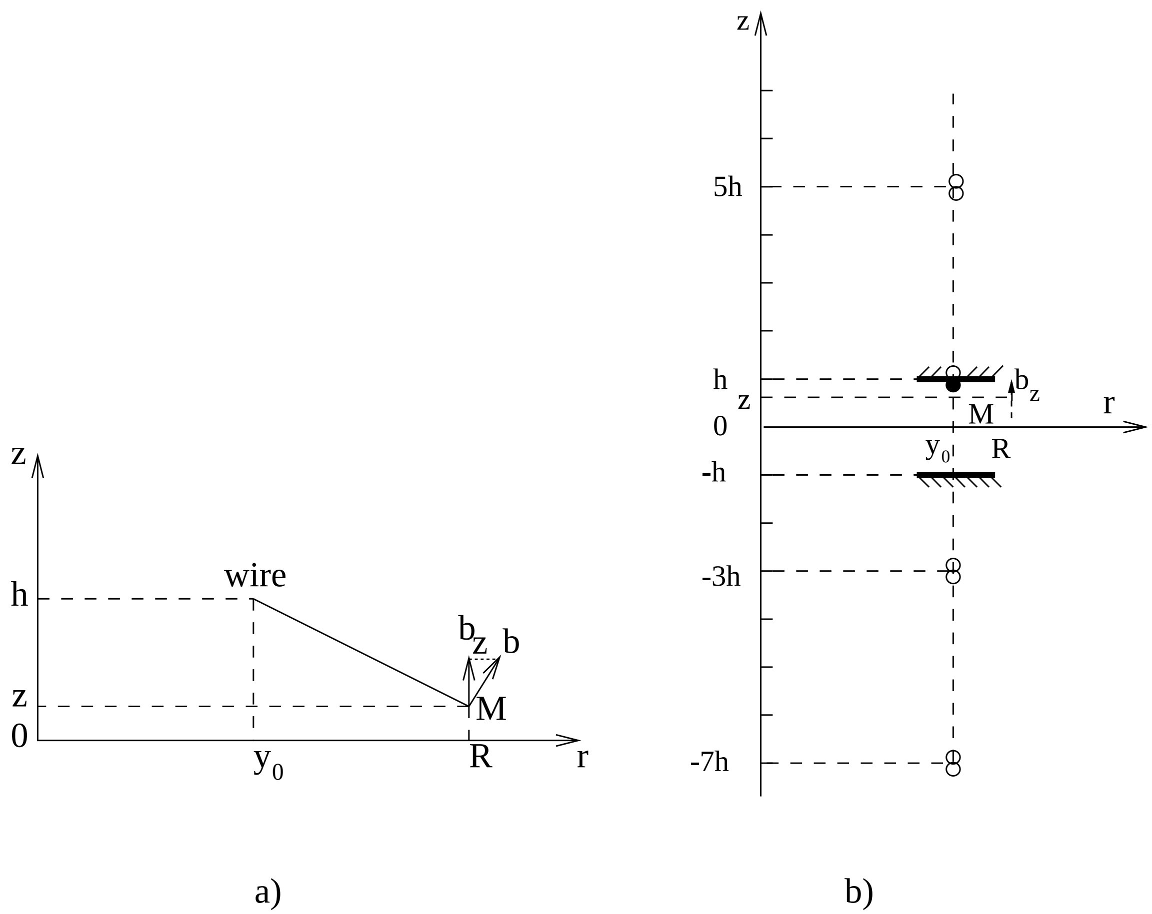

Figure 1: Magnetic induction created in M(R, z) by a wire(y_0, h) with current I
a) in air b) in presence of pole pieces

We make the approximation that the wire and first image very near the pole face are exactly at the same ordinate h, so that

$$b_z = 2 \sum_{n=-\infty}^{+\infty} \frac{\mu_0 I}{2\pi} \frac{(R - y_0)}{(h - z + 4nh)^2 + (R - y_0)^2}$$

Defining $\lambda = (R - y_0)/4h$, and $\mu = (h - z)/4h$, one finds

$$b_z = 2 \frac{\mu_0 I}{8\pi h} \sum_{n=-\infty}^{+\infty} \frac{\lambda}{(\mu + n)^2 + \lambda^2}$$

which, according to Neyret and Parain[6], is identical to

$$b_z = \frac{\mu_0 I}{4\pi h} \frac{\pi \sinh\ 2\pi\lambda}{\cosh\ 2\pi\lambda - \cos\ 2\pi\mu}$$

$$b_z = \frac{\mu_0 I}{4h} \frac{\sinh\ \pi(R - y_0)/2h}{\cosh\ \pi(R - y_0)/2h\ -\ \sin\ \pi z/2h}$$

From this formula, we may deduce the partial derivatives of b_z relative to R and z :

$$\frac{\partial b_z}{\partial R} = \frac{\mu_0 I}{4h} \frac{\pi}{2h} \frac{1\ -\ \cosh\ \pi(R - y_0)/2h\ \ \sin\ \pi z/2h}{[\cosh\ \pi(R - y_0)/2h\ -\ \sin\ \pi z/2h]^2}$$

$$\frac{\partial b_z}{\partial z} = \frac{\mu_0 I}{4h} \frac{\pi}{2h} \frac{\sinh\ \pi(R - y_0)/2h\ \ \cos\ \pi z/2h}{[\cosh\ \pi(R - y_0)/2h\ -\ \sin\ \pi z/2h]^2}$$

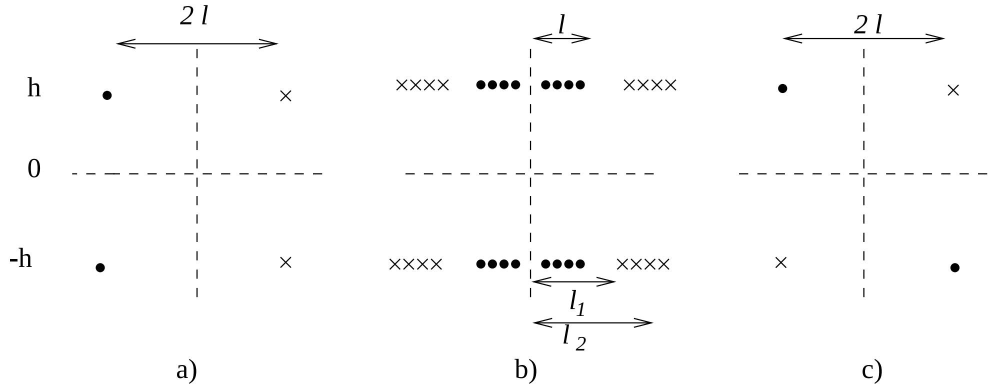

Figure 2: Combinations of wire positions and current directions used to correct
a) the main field b) the radial gradient c) the axial gradient

We may now restrict ourselves to the values of b_z, $\partial b_z/\partial R$, and $\partial b_z/\partial z$ for $R = R_0$ in the median plane ($z = 0$). For $y_0 = R_0 - l$, one gets :

$$\begin{aligned} b_z &= \frac{\mu_0 I}{4h} \tanh\ \pi l/2h \\ \frac{\partial b_z}{\partial R} &= \frac{\pi \mu_0 I}{8h^2} \frac{1}{\cosh^2 \pi l/2h} \\ \frac{\partial b_z}{\partial z} &= \frac{\pi \mu_0 I}{8h^2} \frac{\sinh\ \pi l/2h}{\cosh^2 \pi l/2h} \end{aligned}$$

2.2 Main field correction

Using 2 loops with currents in the same directions provides 4 wires as shown in Fig. 2a. Their effects cancel for the two gradients, but add up for b_z. The resulting effect is indeed 4 times that of a single wire :

$$b_z = \frac{\mu_0 I}{h} \tanh\ \pi l/2h$$

Numerical values are h=.07 m and l = .115 m which give :

$$b_z/I \simeq \mu_0/h = .18\ 10^{-4}\ \mathrm{TA}^{-1} = .18\ \mathrm{GA}^{-1}$$

our experimental value is .17 GA^{-1}.
It must be noticed that applying the Ampere theorem[7] on a closed Γ contour through the magnet and through the 2 correcting loops will give $\int_\Gamma \vec{H}.\vec{dl} = 2I$, which leads to $b_z = \mu_0 I/h$ if it is assumed that the iron permeability is infinite and that the magnetic field is constant inside the magnet gap. This last assumption looks reasonable since the adopted configuration is cancelling the radial and axial gradients at first order. This means that the approximation of the loop by 2 infinite wires must be good in the case of the main field correction.

2.3 Radial gradient correction

Using sheets of 8 parallel wires separated by small intervals p, with identical current I in the same direction on both pole faces, provides a pattern as shown in Fig. 2b. The effects of the 2 sheets cancel for b_z and for the axial gradient, while they add up for the radial gradient. Following again Neyret and Parain[6], we may sum the effects of the 2×8 central wires : we replace I by Idl/p and integrate over the radial extension of the 2 sheets from $-l$ to $+l$:

$$\frac{\partial b_z}{\partial R} = \frac{2}{p}\frac{\pi\mu_0 I}{8h^2}\int_{-l}^{+l}\frac{dl}{\cosh^2 \pi l/2h}$$
$$\frac{\partial b_z}{\partial R} = \frac{\mu_0 I}{hp}\tanh\ \pi l/2h$$

The external wires necessary to close the current loops are expected to have a small influence. However, their effect may be calculated the same way by integrating $\partial b_z/\partial R$ from l_1 to l_2, which gives :

$$\frac{\partial b_z}{\partial R} = \frac{\mu_0 I}{2hp}(\tanh\ \pi l_1/2h\ -\ \tanh\ \pi l_2/2h)$$

The total effect is thus :

$$\frac{\partial b_z}{\partial R} = \frac{\mu_0 I}{2hp}(2\tanh\ \pi l/2h + \tanh\ \pi l_1/2h\ -\ \tanh\ \pi l_2/2h)$$

Numerical values are h=.07 m, l=.055 m, l_1=.07 m, l_2=.115 m and p=2l/7=.11/7. The effect of loop closing is only 4% and we find

$$\frac{1}{I}\frac{\partial b_z}{\partial R} = 9.2\ 10^{-4}\ \mathrm{Tm^{-1}A^{-1}} = .092\ \mathrm{Gcm^{-1}A^{-1}}$$

Our experimental value is .095 $\mathrm{Gcm^{-1}A^{-1}}$.

2.4 Axial gradient correction

Using 2 loops with currents in opposite directions provides 4 wires as shown in Fig. 2c. Their effects cancel for b_z and for the radial gradient, but add up for the axial one. The resulting effect is again 4 times that of a single wire :

$$\frac{\partial b_z}{\partial z} = \frac{\pi\mu_0 I}{2h^2}\frac{\sinh\ \pi l/2h}{\cosh^2 \pi l/2h}$$

Numerical values are h = .07 m and l = .075 which give :

$$\frac{1}{I}\frac{\partial b_z}{\partial z} = 1.4\ 10^{-4}\ \mathrm{Tm^{-1}A^{-1}} = .014\ \mathrm{Gcm^{-1}A^{-1}}$$

Our experimental value is .013 G cm^{-1} A^{-1}.

3 Magnetic induction produced by rectangular loops

3.1 Magnetic induction produced by a straight wire of finite length

The magnetic induction produced by a straight wire of finite length AB with current intensity I, at a point M (Fig. 3a), is given by Ref. [7].

$$b = \frac{\mu_0 I}{4\pi a}(\sin\theta_2 - \sin\theta_1)$$

For a wire parallel to x axis, at coordinates (y_0, z_0), running from $-x_0$ to $+x_0$, (Fig. 3b), the magnetic induction at M(x, y, z) is

$$b = \frac{-\mu_0 I}{4\pi[(y_0 - y)^2 + (z_0 - z)^2]^{1/2}} \times K$$

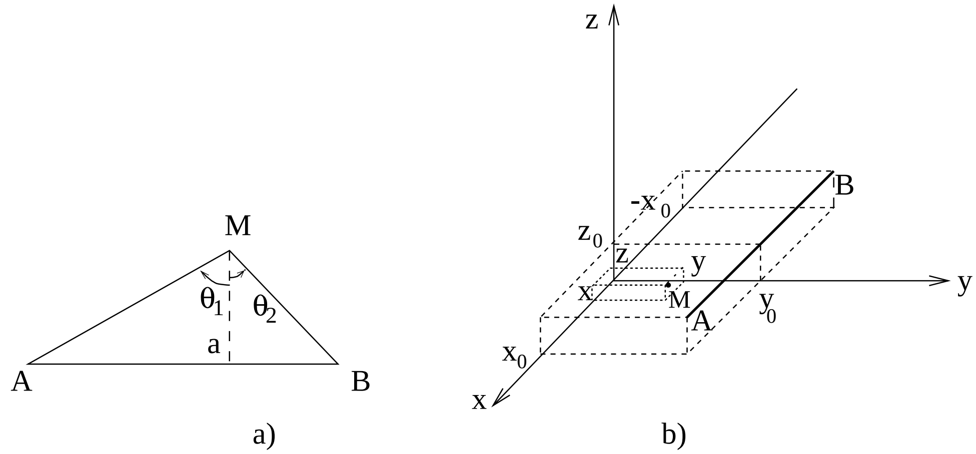

Figure 3: Magnetic induction created in M(x, y, z) by a wire (y_0, z_0) running from $-x_0$ to $+x_0$ with current I

with

$$K = \frac{x_0 - x}{[(x_0 - x)^2 + (y_0 - y)^2 + (z_0 - z)^2]^{1/2}} + \frac{x_0 + x}{[(x_0 + x)^2 + (y_0 - y)^2 + (z_0 - z)^2]^{1/2}}$$

Its z component is

$$b_z = \frac{\mu_0 I}{4\pi} \frac{y_0 - y}{(y_0 - y)^2 + (z_0 - z)^2} \times K$$

For an infinite wire, $x_0 \to \infty$, and $K \to 1$. K thus describes the effect of the finite length which decreases the field.
For wires parallel to y axis, we have just to exchange $(x_0 - x)$ and $(y_0 - y)$. In a loop, the effects of the wires parallel to x and y axis add up.
Then we shall have to add up the contributions of the images by setting

$$z_0 = h_n = (-1)^{n+1}(2n - 1)h$$

so that b_z is twice the infinite sum from n=1 to ∞ of the corresponding b_z values.

3.2 Main field correction

At M(0,0,0), for the 2 loops, we finally get :

$$b_{0z} = \sum_{n=1}^{\infty} \frac{4\mu_0 I}{\pi} \left(\frac{y_0}{y_0^2 + h_n^2} \times \frac{x_0}{(x_0^2 + y_0^2 + h_n^2)^{1/2}} + \frac{x_0}{x_0^2 + h_n^2} \times \frac{y_0}{(x_0^2 + y_0^2 + h_n^2)^{1/2}} \right)$$

$$b_{0z} = \frac{4\mu_0 I x_0 y_0}{\pi} \sum_{n=1}^{\infty} \frac{1}{(x_0^2 + y_{0i}^2 + h_n^2)^{1/2}} \left(\frac{1}{y_{0i}^2 + h_n^2} + \frac{1}{x_0^2 + h_n^2} \right)$$

In the case of the magnet for PS189, the loops are not exactly rectangular, but are certainly very near rectangular loops with $x_0 = \pi R_0/8 = 0.195$ m, $y_0 = 0.115$ m, and $h = 0.07$ m.
The result of the computation is $b_z = .18$ GA^{-1}.
As expected in §**2.2**, this result confirms the validity of the infinite wires approximation in this case. Even for a nearly square loop, the difference is only a few percent. However, if l becomes small, one has to take into account the hyperbolic tangent term, and not to use the formula deduced from the Ampere theorem.

3.3 Radial gradient correction

3.3.1 azimuthal wires

For wires in azimuthal direction, at M(0,y,0), we have

$$b_z(y) = \frac{\mu_0 I}{2\pi}\left(\frac{y_0 - y}{(y_0 - y)^2 + z_0^2} \times \frac{x_0}{[x_0^2 + (y_0 - y)^2 + z_0^2]^{1/2}}\right)$$

First, we derive $b_z(y)$ relative to y, and take its value at M(0,0,0,). Then, we have to sum from i=1 to 4 the effects of the 4 wires at positions

$$y_{0i} = (2i - 1)p/2$$

and sum from n=1 to ∞ the effects of the images at positions h_n. Taking into account the fact that we have 4 times 4 wires and double images, we get :

$$\frac{\partial b_z}{\partial y} = \frac{4\mu_0 I x_0}{\pi}\sum_{n=1}^{\infty}\sum_{i=1}^{4}\frac{1}{(y_{0i}^2 + h_n^2)(x_0^2 + y_{0i}^2 + h_n^2)^{1/2}}\left(-1 + \frac{2y_{0i}^2}{(y_{0i}^2 + h_n^2)} + \frac{y_{0i}^2}{x_0^2 + y_{0i}^2 + h_n^2}\right)$$

The azimuthal loop-closing wires produce a radial gradient in opposite direction which may be calculated using the same formula, but with y'_{0i} instead of y_{0i} :

$$y'_{0i} = l_2 - (i - 1)p$$

where l_2 is defined as in §**2.3**. It thus must be subtracted.

3.3.2 radial wires

The effect of the radial wires must be added up to that of the central sheet. The radial wires extend from y_{0i} to y'_{0i}. For one wire, we get :

$$\begin{aligned} b_z(y) &= \frac{\mu_0 I}{4\pi}\frac{x_0}{x_0^2 + z_0^2}\left(\frac{y'_{0i} - y}{(x_0^2 + (y'_{0i} - y)^2 + z_0^2)^{1/2}} - \frac{y_{0i} - y}{(x_0^2 + (y_{0i} - y)^2 + z_0^2)^{1/2}}\right) \\ \frac{\partial b_z}{\partial y} &= \frac{\mu_o I x_0}{4\pi}\left(\frac{1}{(x_0^2 + y_{0i}^2 + z_0^2)^{3/2}} - \frac{1}{(x_0^2 + {y'_{0i}}^2 + z_0^2)^{3/2}}\right) \end{aligned}$$

This value must be summed up the same way as done for the azimuthal wires :

$$\frac{\partial b_z}{\partial y} = \frac{4\mu_o I x_0}{\pi}\sum_{n=1}^{\infty}\sum_{i=1}^{4}\left(\frac{1}{(x_0^2 + y_{0i}^2 + h_n^2)^{3/2}} - \frac{1}{(x_0^2 + {y'_{0i}}^2 + h_n^2)^{3/2}}\right)$$

3.3.3 total radial gradient

For $x_0 = \pi R_0/8 = .195$ m, $h = .07$ m, $l = .055$ m, and $l_2 = .115$ m , the computation gives :

$$\frac{1}{I}\frac{\partial b_z}{R} = .088 \text{ Gcm}^{-1}\text{A}^{-1}$$

The 4% difference with the analytic formula in §**2.3** is not due to the infinite wires approximation but to the calculation of the contributions of the different wires of the sheet which have been assimilated to a homogeneous distribution in §**2.3**.

3.4 Axial gradient correction

For wires in azimuthal direction, at M(0,0,z), we have

$$b_z(z) = \frac{\mu_0 I}{2\pi}\left(\frac{y_0}{y_0^2+(z_0-z)^2} \times \frac{x_0}{(x_0^2+y_0^2+(z_0-z)^2)^{1/2}}\right)$$

First, we derive $b_z(z)$ relative to z, and take its value at M(0,0,0,). Then, we sum up the contributions of the different images :

$$\frac{\partial b_z}{\partial z} = \frac{4\mu_0 I x_0 y_0}{\pi}\sum_{n=1}^{\infty}\frac{h_n}{(y_0^2+h_n^2)(x_0^2+y_0^2+h_n^2)^{1/2}}\left(\frac{2}{y_0^2+h_n^2}+\frac{1}{x_0^2+y_0^2+h_n^2}\right)$$

The contributions of the radial wires must be added up. It is obtained easily by exchanging x_0 and y_0 :

$$\frac{\partial b_z}{\partial z} = \frac{4\mu_0 I x_0 y_0}{\pi}\sum_{n=1}^{\infty}\frac{h_n}{(x_0^2+h_n^2)(x_0^2+y_0^2+h_n^2)^{1/2}}\left(\frac{2}{x_0^2+h_n^2}+\frac{1}{x_0^2+y_0^2+h_n^2}\right)$$

For $x_0 = \pi R_0/8 = .195$ m, $h = .07$ m, and $y_0 = .075$ m, the computation gives :

$$\frac{1}{I}\frac{\partial b_z}{\partial z} = .014 \text{ Gcm}^{-1}\text{A}^{-1}$$

This value is in good agreement with the one calculated by using the analytical formula.

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