Single particle behaviour at (and away from) stability

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Abstract. Short of proposing a full microscopic mass formula, we outline the steps that should make it possible. The construction rests on defining the nuclear monopole Hamiltonian, H_m that has to be extracted phenomenologically because of the bad saturation properties of the realistic forces. We propose a preliminary form of H_m that makes clear the origin of shell effects ,and with only three paraters, reproduces the known spectra of particle and hole states on doubly magic cores to within 300 keV. Predictions are made for the yet unobserved levels around ^{132}Sn, and those associated to ^{22}O, ^{34,42}Si, ^{68,78}Ni, ^{100}Sn and for the particle-hole gaps in these nuclei. Finally we analyse the single particle gaps for the N=50 and 82 isotones, to conclude that the major closures are likely to persist away from stability.

Text-books on many body theory usually start from the assumption that the Hamiltonian may be split as $H = H_0 + (H - H_0)$, where H_0, is some "conveniently" defined single-particle field. There a difficulty here: H is two-body, and it is not clear how to extract from it a one body part. The mathematically correct solution is to extract all quadratic (two body) forms in the scalar products $a_r^+ \cdot a_s$. The resulting object, the monopole Hamiltonian H_m is closed under Hartree Fock (HF) variation (i.e., unitary transformations of the a operators).The correct H_0 is then the diagonal part of H_m

$$H_m^d = \frac{1}{2}\frac{\hbar\omega}{\hbar\omega_0}\sum_{k,l} V_{kl} m_k (m_l - \delta_{kl}), \tag{1}$$

where $V_{kl} = \sum_J (2J+1) V_{klkl}^J / \sum_J (2J+1)$ is the centroid calculated at $\hbar\omega_0$, and m_k is either the number of neutrons n_k, or protons z_k in orbit k. The expectation value of H_m^d for any state is the average energy of the configuration to which it belongs (a configuration is a set of states with fixed m for each orbit). In particular H_m^d reproduces the exact energy of closed shells (cs) and single particle (or hole) states built on them ($(cs) \pm 1$), since for this set ($cs \pm 1$) each configuration contains a single member. To arrive at this form we assume that a nucleus of $A = N + Z$ particles, and isospin $T = (N - Z)/2$, has the correct radius. In other words, we assume that HF variation of H_m with respect to $\hbar\omega$ yields a minimum at the observed radius $< r^2 > \approx 0.9A^{2/3}(1 - (2T/A)^2)$ [1], so that [2]

$$\hbar\omega = \frac{34.6A^{1/3}}{< r^2 >} \approx 40/\rho, \quad \rho = (A^{1/3}(1 - (2T/A)^2). \tag{2}$$

As we know that a typical matrix element goes as [4,5]

$$V(\omega)_{klmn} \cong \frac{\omega}{\omega_0} V(\omega_0)_{klmn}, \tag{3}$$

and then eq. (1) follows after a bit of work.

Unfortunately, H_m, as extracted from potentials describing the NN scattering data does not have the correct properties. Recent *exact* results for $A \leq 8$ [3] indicate that a potential describing *perfectly* the NN phase-shifts, supplemented by a three body term, produces excellent spectroscopy, but still has problems with the absolute bindings, the symmetry energy and the spin-orbit splitting, terms essentially given by H_m. Very much the same happens in shell model calculations, where the damage due to bad saturation increases with particle number. The problem cannot be solved at present, but it can be bypassed cleanly by the separation $H = H_m + H_M$. All the trouble comes from H_m and can be phenomenologically corrected. Then, the multipole term H_M produces detailed agreement with the observables *provided it is derived from potentials consistent with the NN phase-shifts* [4,6,7,5]. The necessary condition that H_m must satisfy, is to reproduce properly the $(cs) \pm 1$ excitation spectra.

To understand the crucial importance of the quadratic effects in H_m, let us consider two shells h, p of degeneracies D_p, D_h on top of a core. Then

$$H_m = \varepsilon_p m_p + \varepsilon_h m_h + \frac{1}{2} V_{pp} m_p(m_p - 1) + \frac{1}{2} V_{hh} m_h(m_h - 1) + V_{ph} m_p m_h. \tag{4}$$

To make things more tranparent, let us introduce the operators

$$\Gamma_{ph} = \frac{m_p D_h - m_h D_p}{D_p + D_h}, \tag{5}$$

which produce unit splittings betwen orbits p, h: Then, we can recast eq. (4 as

$$H_m = \varepsilon_0 + \frac{1}{2} m(m-1)W_0 + \Gamma_{ph}[\varepsilon_1 + (m-1)W_1] + \Gamma_{ph}^2 W_2. \tag{6}$$

Take now a core of ^{12}C, call the $p_{1/2}$ orbit on top of it h, and the next two, $s_{1/2}$ and $d_{5/2}$, p. When 4 particles are allowed in this space, we have a simple model for ^{16}O, where one can infer from binding energies that the lowest ph states should come at around the value of the "gap" i.e., BE(^{17}O) +BE(^{15}O)$-$2BE(^{16}O)=11.5 MeV. Experimentally, there is a doublet near 6 MeV, consisting of a 3^- ph state and a 0^+ state of well established 4p-4h nature. The symmetry energy (i.e. a $T(T+1)$ term) brings easily down the 3^- from 11.5 to 6 Mev. What about the 0^+: If we neglect the quadratic term in Γ^2_{ph} in eq. (6, it would come at about $4\times 11.5=46$ Mev. By including it, and borrowing numbers from the calculation that first gave a consistent picture of the low lying states in the region [8], the average energies for $kp-kh$ excitations are roughly those of Fig. (1).

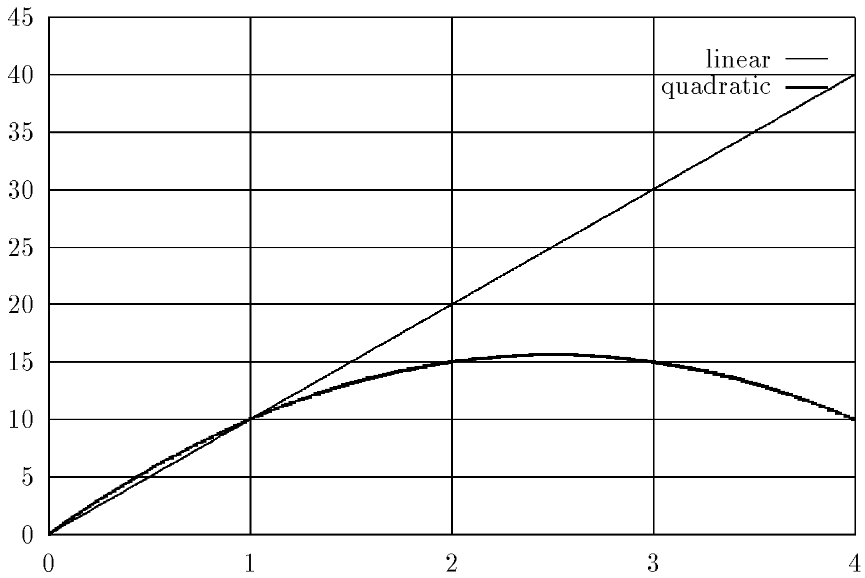

FIGURE 1. kp-kh excitation energies for two shells

Let us describe briefly the indispensable ingredients of the general H_m, bearing in mind this simple example that suggests we separate "constant" (bulk terms in total m) from linear an quadratic contributions. We shall certainly need the kinetic energy ho, compensated by an attraction which we take to go as the leading monopole term in the decomposition of the interaction (gm) [5,9]. The combination responsible for shell effects (p is the principal quantum number, an overall factor $\rho\equiv\omega/\omega_0$ is assumed),

$$gm-4ho=(\sum_p\frac{n_p+z_p}{\sqrt{(p+2)(p+1)}})^2-\sum_p\frac{n_p+z_p}{(p+2)(p+1)}-4(p+3/2)(n_p+z_p),\quad (7)$$

has no bulk or suface contributions (we ignore them in this talk).

Next we need linear operators that we take to be either of "$l{\cdot}s$" type (i.e.,they split spin-orbit (so) partners $l_<$, $l_>$), or "$l{\cdot}l$" type (i.e., they split the l centroids). Finally, we should decide on the quadratic operators, which produce basically modulations of the above ones. As the data demand a special treatment for the so partners with $l=p$, we introduce the following sets of orbits with degeneracies D_x:

$$
\begin{array}{rcl}
p, & \text{the full shell,} & D_p = (p+1)(p+2) \\
p_>, & \text{largest orbit,} & D_{p_>} = 2(p+1) \\
p_<, & \text{its } l\cdot s \text{ partner,} & D_{p_<} = 2p \\
q, & p_> + p_<, & D_q = 2(2p+1) \\
r, & p - q, & D_r = p(p-1) \\
R, & p - p_>, & D_R = p(p+1),
\end{array}
$$

define the following set of operators, all simple variants of $l \cdot s$, and $l \cdot l$, (for the r-orbits we use $m_l = m_{l_<} + m_{l_>}$):

$$a_p = \frac{p}{p+2}\Gamma_{p>p<}, \quad b_p = \Gamma_{rq}, \quad s_p = \Gamma_{p>R}, \quad v_p = \Gamma_{p<r} \tag{8a}$$

$$d_p = \sum_{l\neq p} \frac{l}{p+2}\Gamma_{l>l<}, \quad e_p = \sum_{l\neq p} \frac{[2l(l+1)-(p-2)(p+1)]\, m_l}{2D_r} \tag{8b}$$

and select the ones we shall need

$$\mathcal{O} = \sum_p \mathcal{O}_p, \qquad ff'g\mathcal{O} = \sum_{[pp'][xx']} \frac{b_p^x \mathcal{O}_{p'}^{x'}}{(D_p D_{p'})^{1/4}}, \quad x, x' \equiv n \text{ or } z \tag{9}$$

where $\mathcal{O} = a, b, v, d, e$, the denominators ensure splittings of order unity, and the sums are restricted so that

$$
\begin{aligned}
ff'g\mathcal{O} &= ffi\mathcal{O}, \text{ for } x = x',\ p = p' \\
ff'g\mathcal{O} &= ffc\mathcal{O}, \text{ for } x = x',\ p \neq p' \\
ff'g\mathcal{O} &= zni\mathcal{O}, \text{ for } x \neq x',\ p = p' \\
ff'g\mathcal{O} &= znc\mathcal{O}, \text{ for } x \neq x',\ p \neq p'
\end{aligned}
$$

The operators $\Gamma(ij)$ can act in either the same or different fluids (ff or nz) and either the same or different major shells (i or c). Eqs. (9) achieve maximum symmetry.

The quadratic operators in eq. (9) have the property of producing negligible splittings in hole states for HO closures. There is an exception that the data seem to demand. We call it simply ffi because it is the only representative of its class that will appear:

$$ffi = \sum_{xp} \frac{m_{p_>}^x}{D_{p_>}} \left(s_p^x - \frac{D_p - m_p^x}{D_p - 1} \right). \tag{10}$$

The guiding idea in writing these forms is that in the harmonic oscillator closures (HO), only the linear terms act. Since the major EI (extruder intruder) closures at N,Z=6(sic), 14, 28, etc. differ from HO ones only by the presence of a $p_>$ shell—whatever difference there is in the $(cs)\pm 1$ spectra with respect to the linear contributions must be due to a quadratic in which one of the factors contains a $p_>$ operator. (The "driver' b in the equation above.)

It proves convenient to regroup terms, and we have adopted the following, minimal, combination (using H'_m as a reminder that bulk terms have been omitted):

$$-\rho H'_m = c_1(d + znce + znie) + c_2(znib + ffi) +$$
$$c_3\{W - 4K + 2[a + b(1 - 3/\rho)] + 2zncv + ffcv\}, \qquad (11)$$

which yields (c_1, c_2, c_3)=(12.90,4.75,8.03) with a deviation (rmsd) of 282 KeV, in a fit to the experimental excitations adopted in table I. The parameters change to (12.91,4.45,8.17) with rmsd=244 KeV, if only the $A > 60$ data are kept, and to (14.71,5.18,8.03) with rmsd=280 KeV for $A < 60$. These last parameters when used for $A > 60$ yield rmsd=324 KeV, and no important differences with the numbers in Table I, indicating a remarkable stability of the fit with respect to changes in input data.

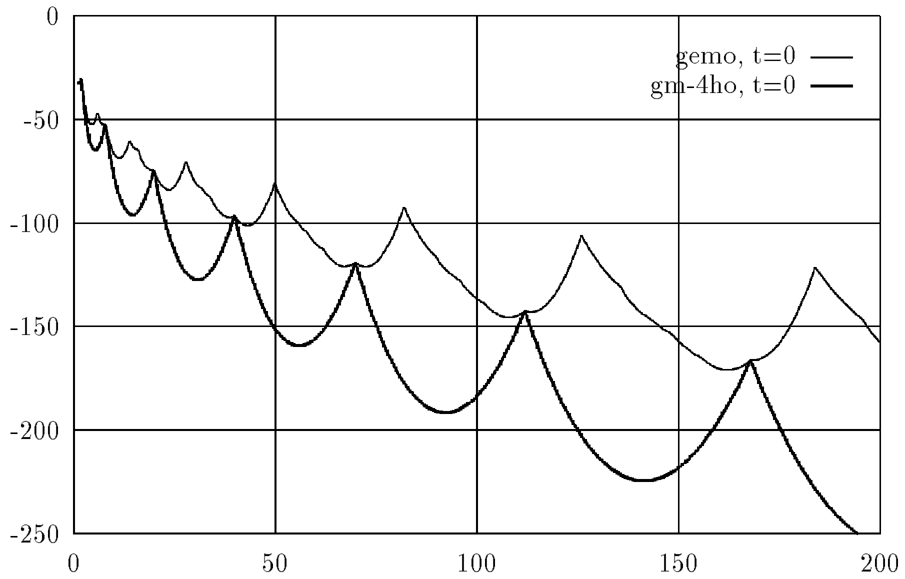

FIGURE 2. Shell formation mechanisms

In Fig 2 we can follow the energy for $T = 0$ states as a function of N. The $gm - 4ho$ in eq. (7) yields the shell effects associated with the HO closures. When all the contributions are added ($gemo$), the HO closures go (except at the very beginning) and are replaced by the EI ones.

Energies of the of the $(cs) \pm 1$ states around existing and possible new closures can be retrieved from ref. [10]. Because of lack of space we shall not comment on them, except to say that the agreement with observation is good enough to give plausibility to the predictions.

To close, the last figure compares the predictions for the single particle gaps around N=50, with the data from [11]. We need here the symmetry energy

$-32[4T(T+1)](1-1.6/\rho)/A$ taken from [9], and a perturbative estimate of multipole correlations, which amounts to a constant shift of about 1 MeV. However we have not estimated the correct occupancies of the proton orbits, which will smooth the residual subshell effects. The agreement is quite satisfactory, and there no sign of shell erosion in going away from stability. Very much the same happens for the closures at N,Z=82 and Z=50.

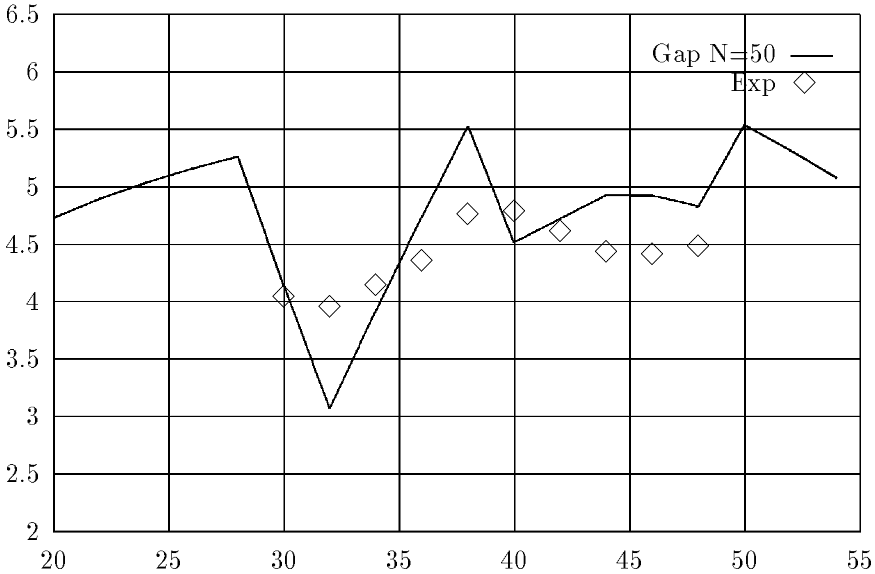

FIGURE 3. The single particle gaps for N=50

REFERENCES

1. J. Duflo Nuc. Phys. **A576**, 29 (1994)
2. A. Bohr and B. Mottelson, *Nuclear Structure* vol I (Benjamin, Reading, 1964).
3. See S. Pieper contribution to ENAM98 (this volume)
4. A. Abzouzi, E. Caurier et A. P. Zuker, Phys. Rev. Lett. **66**, 1134 (1991).
5. M. Dufour and A.P. Zuker, Phys. Rev. C **54**, 1641 (1996)
6. G. Martínez-Pinedo, A.P. Zuker, A. Poves and E. Caurier, Phys. Rev. C **55**, 187 (1997)
7. A.P. Zuker, in Contemporary Shell Model, edited by X.W. Pan (Springer Lecture Notes 482, 1997)
8. A. P. Zuker, B. Buck and J. B. Mc Grory Phys. Rev. Lett. **21**, 39 (1968).
9. J. Duflo and A.P. Zuker, Phys. Rev. C **52**, R23 (1995)
10. J. Duflo and A.P. Zuker, http://csnwww.in2p3.fr/amdc/ (theory file du_zu_ph.ps)
11. G. Audi and A.H. Wapstra, Nuc. Phys. **A565**, 1 (1993)