

Generalized Reich-Moore *R*-matrix Approximation

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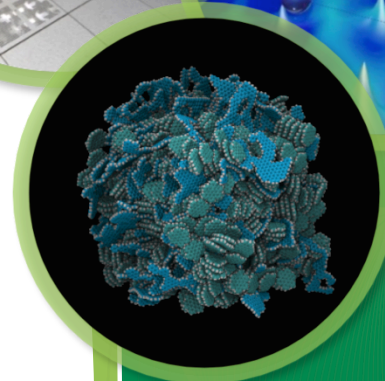
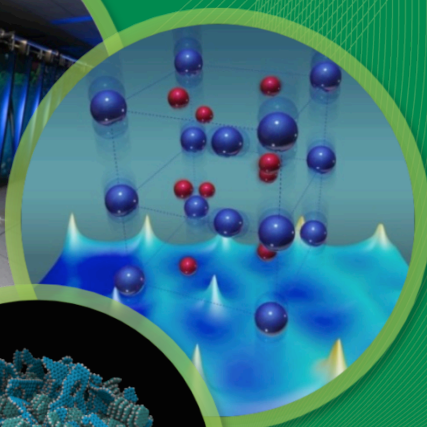
Andrew Holcomb, ORNL

Pablo Ducru, MIT

Marco Pigni, ORNL

Dorothea Wiarda, ORNL

Consultant's Meeting on
R-Matrix Codes for Charged-Particle Reactions
in the Resolved Resonance Region,
IAEA Headquarters,
Vienna, Austria
December 5-7, 2016



Talk outline

- History of Reich-Moore (RM) approximation
- Motivation for generalized RM
 - Considerations of Unitarity, Brune alternative R-matrix parameters
- Derivation
 - like RM but retain level-level interference via γ -ray channels
- Implications
 - Total capture cross section fitted by fewer parameters than full R-matrix
 - Manifest Unitarity
 - Froehner's prescription
 - Insight about statistics of level-level interference via γ -channels
- SAMMY 8.1

RM History and use

Reich and Moore, Phys. Rev. 111, 929 (1958)

- Reich-Moore divides full R -matrix into particle and γ -ray blocks
 - Formal expressions derived for reduced R -matrix of particle channels
 - The effect of γ -ray channels on particle-channels approx. by a diagonal
 - Level-level interference among γ -ray channels neglected

$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c} \gamma_{\lambda c'}}{E_{\lambda} - E - i\Gamma_{\lambda\gamma}/2} \quad (c, c' \notin \gamma), \quad (194)$$

corresponding SLBW level matrix. Otherwise it is so exact that although the reduced collision matrix cannot be unitary - because of transitions into eliminated channels - the overall collision matrix can still be considered as unitary, i. e. as conserving probability flux, so that the capture cross section may alternatively be obtained as the difference

$$\sigma_{c\gamma} = \sigma_c - \sum_{c' \notin \gamma} \sigma_{cc'}, \quad (196)$$

with σ_c calculated from the reduced collision matrix element U_{cc} according to Eq. 153.

$$\sigma_c \equiv \sum_{c'} \sigma_{cc'} = 2\pi\lambda_c^2 g_c (1 - \text{Re } U_{cc}), \quad (153)$$

Motivation for generalized Reich-Moore

- Conversion of RM R-matrix parameters
 - Between formal and alternative (a.k.a. physical) ones (C. Brune, 2002)
- Investigate whether Reich-Moore (RM) approx. is unitary
 - It may appear not to be as particle-channel R-matrix is complex (not real)
 - RM derivation was revisited to investigate unitarity
 - In this process a generalization of RM was found
 - This generalization is manifestly unitary
 - Corollary: Conventional RM is also unitary
 - It provides basis for Fritz Froehner's prescription used by SAMMY
- Other potential benefits Generalized RMA (gRMA)
 - May provide better fits to total capture (and other cross sections)
 - Because gRMA reproduces total cross section formally
 - May shed light on resonant-interference effects neglected by conv. RM
 - Including statistical properties of capture widths

Derivation uses full R-matrix via level-matrix A

$$A^{-1} = e - E\mathbf{1} + \gamma(L - B)\gamma^T$$

- Separate channel space into particle and γ -ray channels:

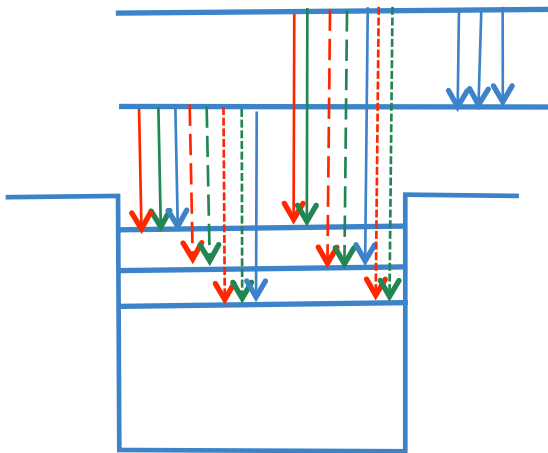
$$\gamma \equiv (\gamma_c, \gamma_\gamma)$$

$$L - B \equiv \begin{pmatrix} L_c - B_c & \mathbf{0} \\ \mathbf{0} & L_\gamma - B_\gamma \end{pmatrix}$$

$$A^{-1} = e - E\mathbf{1} + \gamma_\gamma(L_\gamma - B_\gamma)\gamma_\gamma^T + \gamma_c(L_c - B_c)\gamma_c^T$$

Full R-matrix g-ray channels

- γ -ray channels
 - defined by EM multipolarity, helicity, and final state quantum numbers
 - Selection rules based on f.s. quantum numbers, γ -ray multipolarity
 - Electric: E1, E2, E3, ...
 - Magnetic: M1, M2, M3, ...
- Level-level interference takes place via identical γ -ray channels
 - Use conventional approximation S-B=0
 - Or use Brune alternative R-matrix parameters for which S-B=0 always



$$i\gamma_{\gamma} P_{\gamma}^{1/2} P_{\gamma}^{1/2} \gamma_{\gamma}^T$$

Generalized RM Derivation

- Consider capture-width matrix Γ_γ inside the level matrix \mathbf{A}

$\gamma_\gamma = N_\lambda \times N_\gamma$ matrix of physical capture channel widths
 $\gamma_{gRM} = N_\lambda \times N_\lambda$ matrix of gRMA capture channel widths

$$\begin{array}{c}
 \left[\begin{array}{c} N_\lambda \\ \hline \gamma_\gamma \mathbf{P}_\gamma^{1/2} \\ \hline N_\gamma \end{array} \right] \times \left[\begin{array}{c} \mathbf{P}_\gamma^{1/2} \gamma_\gamma \\ \hline N_\lambda \end{array} \right] = \left[\begin{array}{c} \Gamma_\gamma / 2 \end{array} \right] = \left[\begin{array}{c} \gamma_{gRM} \end{array} \right] \times \left[\begin{array}{c} \gamma_{gRM} \end{array} \right]
 \end{array}$$

$N_\lambda \ll N_\gamma$

$$\begin{array}{c}
 \gamma_\gamma (\mathbf{L}_\gamma - \mathbf{B}_\gamma) \gamma_\gamma^T \\
 i \gamma_\gamma \mathbf{P}_\gamma^{1/2} \mathbf{P}_\gamma^{1/2} \gamma_\gamma^T
 \end{array}$$

- Since *total capture* cross section depends on Γ_γ , it could be fit equally well by N_λ as it could by all N_γ capture channels
 - True for *total* capture only (individual γ -channels require full R-matrix)

Total capture of gRMA equals that of full R-matrix

- Working with alternative R-matrix parameters since $S(E)-B=0$

$$A^{-1} = e - E1 + \gamma_{\gamma}(L_{\gamma} - B_{\gamma})\gamma_{\gamma}^T + \gamma_c(L_c - B_c)\gamma_c^T$$

$$A^{-1} \approx e - E1 + i\gamma_{\gamma}P_{\gamma}^{1/2}P_{\gamma}^{1/2}\gamma_{\gamma}^T + \gamma_c(L_c - B_c)\gamma_c^T$$

$$= e - E1 + i\Gamma^{(\gamma)}/2 + \gamma_c(L_c - B_c)\gamma_c^T$$

- The total capture cross section is proportional to:

$$\begin{aligned} U_{c\gamma}U_{c\gamma}^{*T} &= \Omega_c P_c^{1/2} \gamma_c^T A \gamma_{\gamma} P_{\gamma}^{1/2} \Omega_{\gamma} \Omega_{\gamma}^* P_{\gamma}^{*1/2} \gamma_{\gamma} A \gamma_c^T P_c^{*1/2} \Omega_c^* \\ &= \Omega_c P_c^{1/2} \gamma_c^T A (\gamma_{\gamma} P_{\gamma}^{1/2} P_{\gamma}^{1/2} \gamma_{\gamma}^T) A \gamma_c^T P_c^{*1/2} \end{aligned} \quad (14)$$

$$= \Omega_c P_c^{1/2} \gamma_c^T A (\Gamma^{(\gamma)}/2) A \gamma_c^T P_c^{*1/2} \Omega_c^* \quad (15)$$

→ since $\Gamma^{(\gamma)}$ is the same for full R-matrix and gRMA

Full R-matrix vs. conventional RM: total capture

- full R-matrix equivalent to gRMA; 2-level SAMMY example:

Table II B2.1. Parameter values used to illustrate Reich-Moore vs. full R-matrix calculations

	λ	Energy (MeV)	$\bar{\Gamma}_{\lambda\gamma}$ (eV)	$\Gamma_{\lambda n}$ (eV)	Sign $\times \Gamma_{\lambda\gamma}$ (eV) ^a
Reich Moore	1	1.0	1.0	10000.	
	2	1.1	1.1	11000.	
Pseudo-full R-matrix # 1	1	1.0	10^{-8}	10000.	1.0
	2	1.1	10^{-8}	11000.	1.1
Pseudo-full R-matrix # 2	1	1.0	10^{-8}	10000.	1.0
	2	1.1	10^{-8}	11000.	-1.1

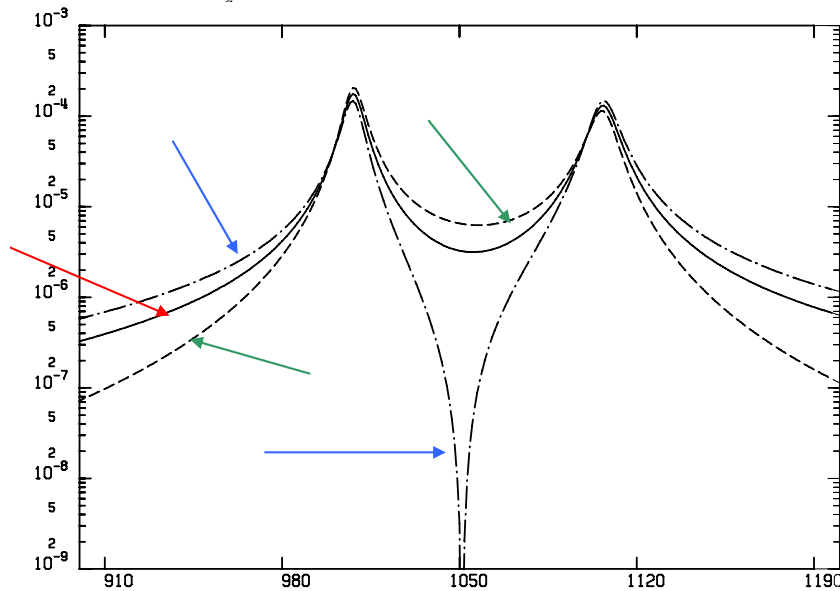
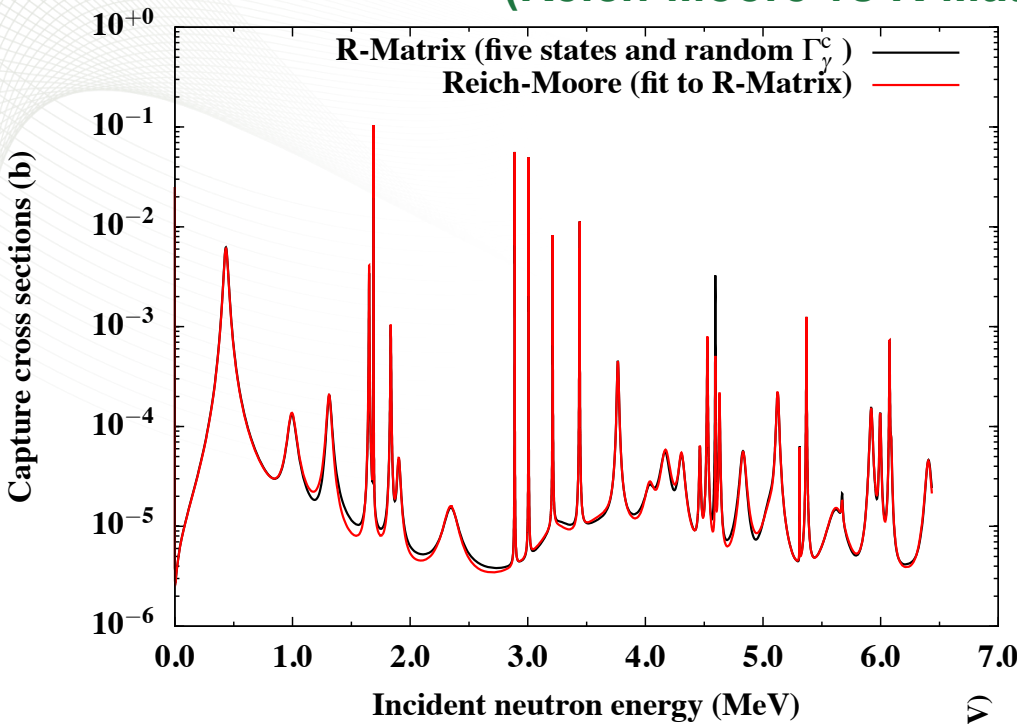


Figure II B2.1. Reich-Moore approximation vs. full R-matrix for artificial example of test case tr110.

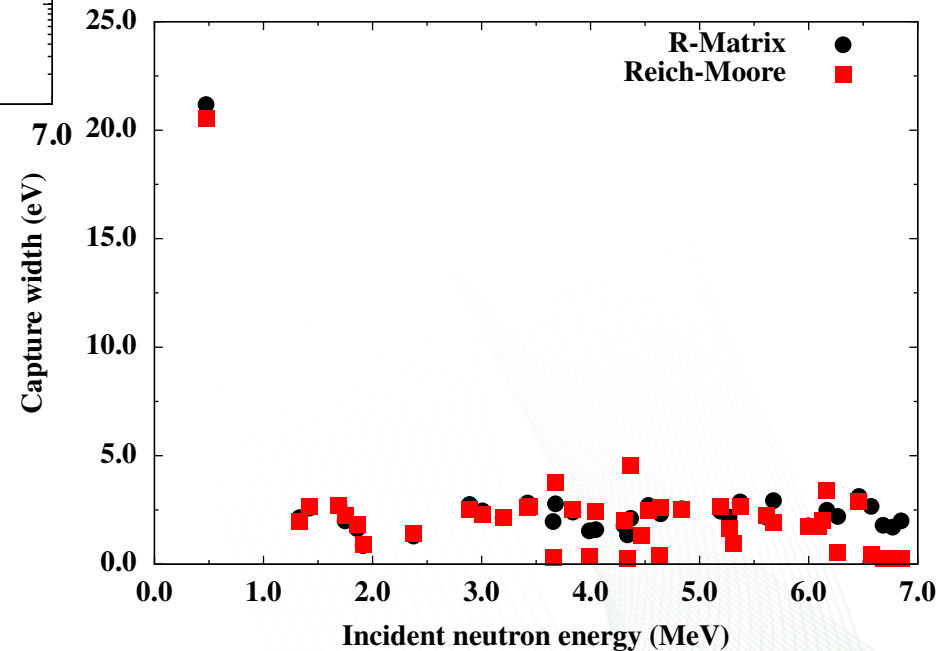
- Solid line = Reich Moore
- Dot-dash = full R-matrix # 1
- Dash = full R-matrix # 2

^{16}O Full R-matrix vs. conventional RM toy case (Reich-Moore vs R-Matrix calculations)



Good agreement between Reich-Moore and R-Matrix capture widths

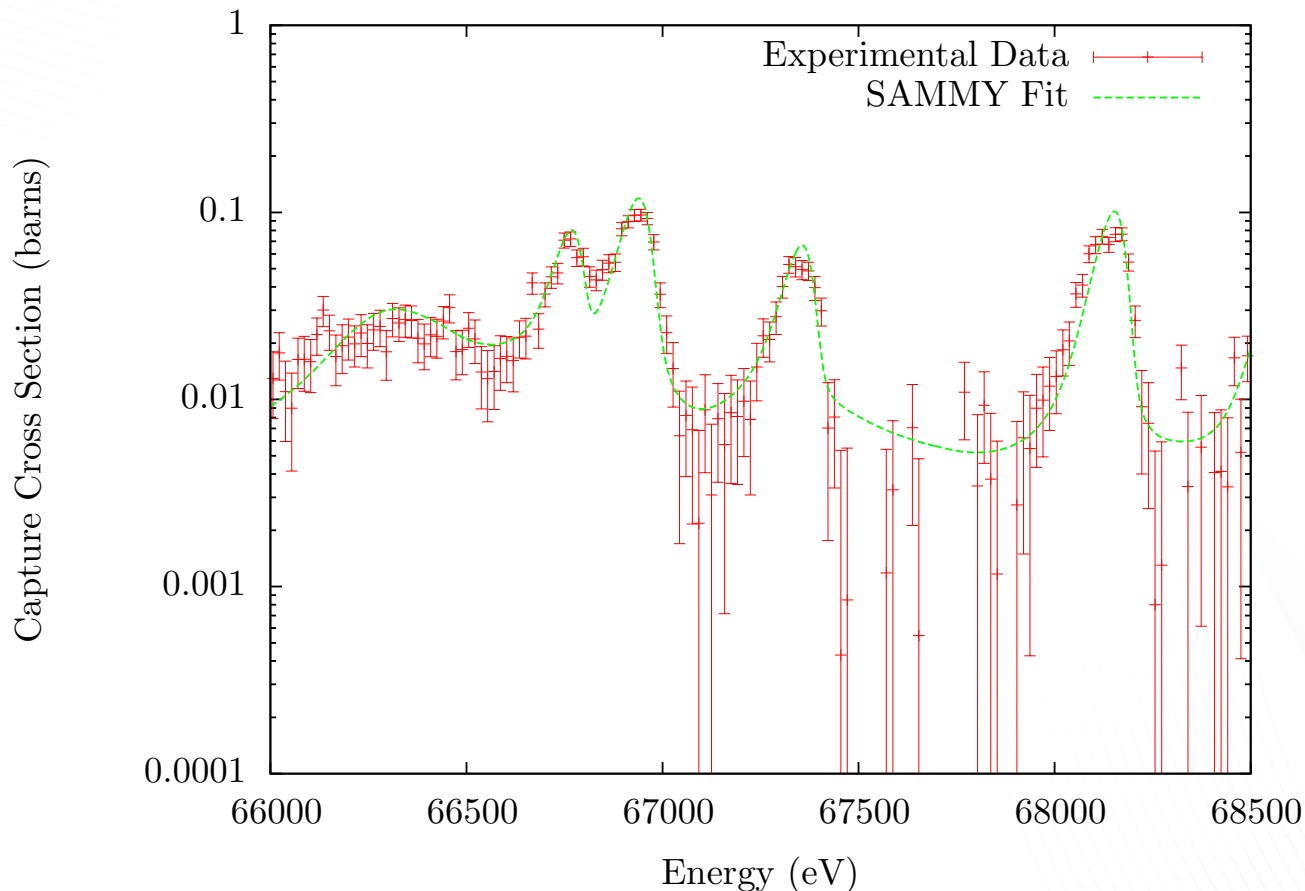
$$\Gamma_{\mu\mu} = 2 \sum_c \gamma_{\mu c} \gamma_{\mu c}$$



- R-matrix case (five states for ^{17}O) generated by randomly sampling capture widths (in black)
- The R-matrix capture cross sections were used to obtain capture widths in Reich-Moore approximation and related capture cross sections (in red)
- The fit of the capture cross sections did not impact other reaction channels

Cu(n,γ) evaluation

- Note: other ways to improve the fit besides resorting to gRMA
 - $^{63,65}\text{Cu}$ evaluation by V. Sobes (ORNL)



Tungsten ^{183}W evaluation

- By M. Pigni (ORNL)
- Slight room for improvement:

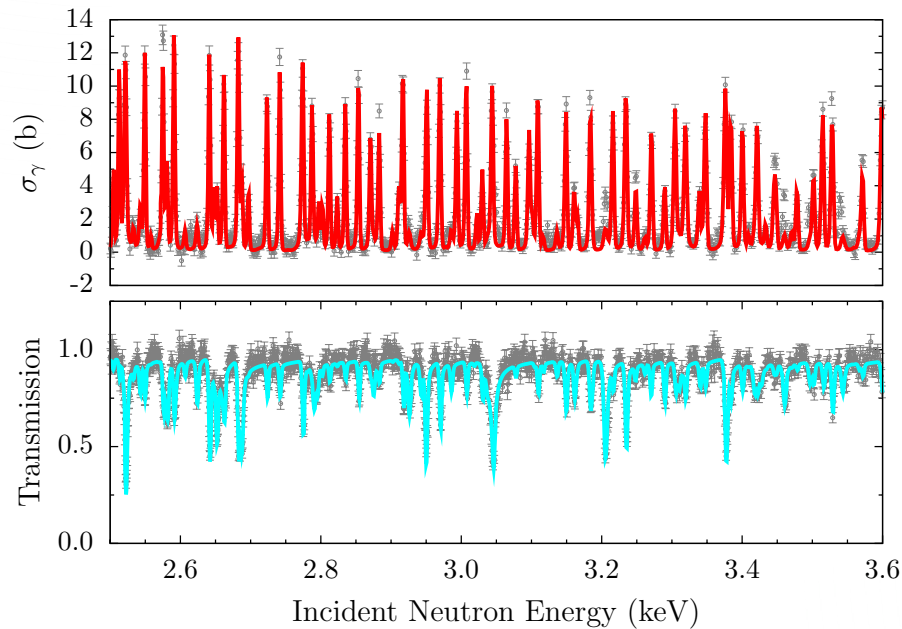


FIG. 1. Neutron capture cross sections (top) and transmission data (bottom) of ^{183}W in the energy range of 2.5-3.6 keV. The solid red lines calculated by the resonance parameters are compared with the experimental data.

Implications: Unitarity

- Generalized RM R -matrix is manifestly unitary
 - its R -matrix is real, its resonance energies are real
 - A reduced number (N_λ) of “resonant” γ -ray channels is retained
 - R -matrix particle-channel sub-matrix alone is not unitary
- Insight: γ -ray channels retained in gRMA make R -matrix unitary
- Consequently, conventional RMA is unitary
- gRMA thus provides foundation for Froehner’s prescription
 - Total cross section is computed assuming a unitary S- or U-matrix
 - Capture cross section is then a deviation of particle R -matrix from unitarity
 - Total capture computed as difference between total and particle channels
 - Implemented in SAMMY

Implications: Statistics

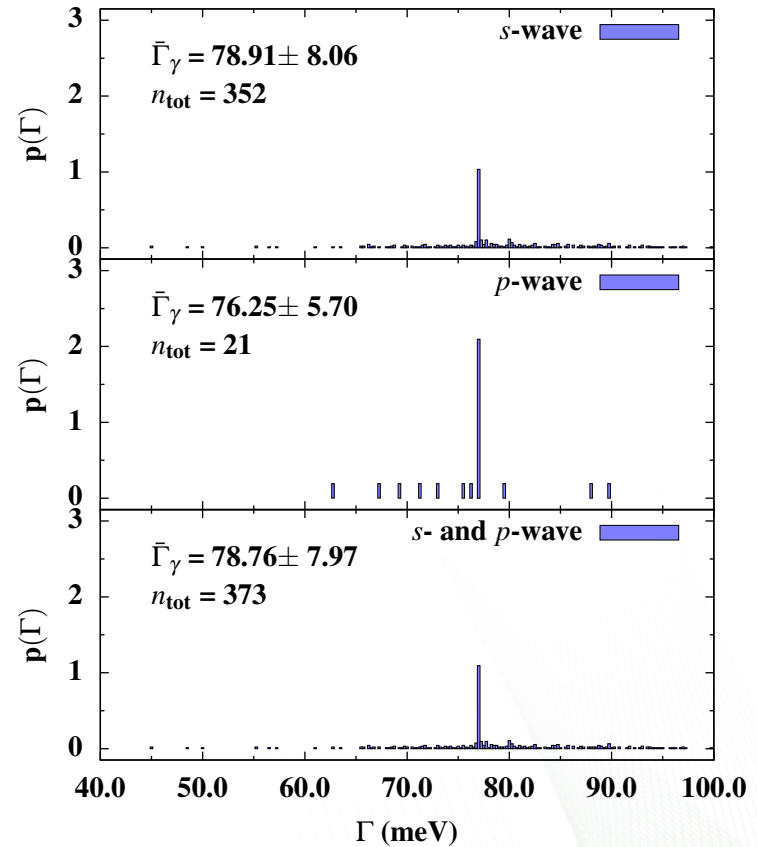
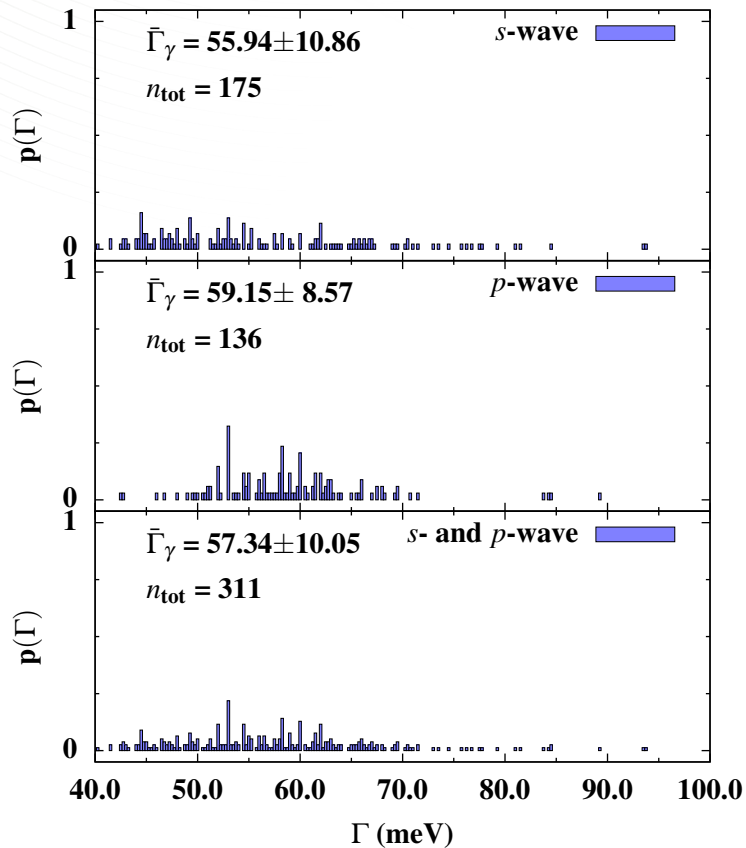
- Related to Wishart matrices
 - Marginal distribution of diagonal and off-diagonal elements
 - Correlations may affect marginal distrib.'s
- Diagonal elements
 - Related to conventional RM capture widths
 - Approximately χ^2 -distribution with N_γ degrees of freedom (DOF)
 - Dominant γ -rays may reduce the effective number of DOF
 - P. Koehler finds DOF is smaller than expected in data
- Off-diagonal elements
 - “XY” distribution with mean 0 and variance $\frac{1}{2}$ that of the diagonal
- → Variance of RM widths estimates neglected off-diagonal
 - May be useful to estimate isotopes to re-evaluate.

Statistics of conventional RM widths on $^{182,183}\text{W}$

- Their variance is approx. twice that of (neglected) off-diagonal:

^{182}W (Leal)

^{183}W (Pigni)



Implications: Brune's alternative R-matrix

- Brune alternative R-matrix parameterization:
 - Independent of boundary conditions thanks to
 - $S_c(E)-B_c=0$ energy-dependent boundary conditions
 - The real part of Kapur-Peierls $L_c(E)-B_c=S_c(E)+iP_c(E)-B_c=0$
 - Alternative resonance energies aligned with resonant peaks
 - Intuitive parameterization
 - C. Brune Phys. Rev. **C66** 044611 (2002)
- Generalized RM capture widths
 - Brune transformation matrices not affected by γ -channels, if $S_\gamma(E)=B_\gamma=0$
 - Unambiguous transformation to/from Brune alternative capture widths
 - Also true for conventional RM widths (diagonal only)
 - But these will not remain diagonal when transformed

Alternative R-matrix parameterization

- C. Brune transform

C. R. BRUNE PHYSICAL REVIEW C **66**, 044611 (2002)

$$U_{c'c} = \Omega_{c'} \Omega_c [\delta_{c'c} + 2i(P_{c'} P_c)^{1/2} \gamma_{c'}^T A \gamma_c]$$

$$A^{-1} = e - E\mathbf{1} - \gamma(L - B)\gamma^T \quad e_{\lambda\mu} = E_\lambda \delta_{\lambda\mu}$$

- Transform from Formal to alternative parameters:

- a non-linear eigenvalue problem:

$$\mathcal{E} = e - \sum_c \gamma_c \gamma_c^T (S_c - B_c)$$

$$\mathcal{E} a_i = \tilde{E}_i a_i$$

- All widths in generalized Reich-Moore approximation transform as:

$$\tilde{\gamma}_{ic} = a_i^T \gamma_c$$

- This shows that exact Brune transform is possible in gRM, not in RM

- Alternative A- and R-matrix derived by C. Brune (2002)

Generalized RM (gRM): Conclusions and Outlook

- gRM provides a better way to fit *total* capture cross sections
 - It is analytically equivalent to full R-matrix when $\mathbf{S}_\gamma - \mathbf{B}_\gamma = \mathbf{0}$
 - It will likely improve fitting of total and other partial cross sections too
- gRM is manifestly unitary because its R-matrix is *real*
 - Justifies Froehner's prescription for unitary conventional RM in SAMMY
- gRM alternative resonance parameters are unambiguous
 - Carl Brune (2002) transform can be applied exactly when $\mathbf{S}_\gamma - \mathbf{B}_\gamma = \mathbf{0}$
- gRM yields insight on distributions of capture matrix elements
 - Deviations from statistics of Wishart matrices may reveal doorways
 - Variance of gRM capture matrix diagonal is 2x that of off-diagonal el.'s
 - Elements near the diagonal (e.g. tri-diagonal) may be sufficient for fitting
- Identify evaluations that may benefit from gRMA
 - Implement it and apply it in a future publication

SAMMY 8.1 coming soon to RSICC.ornl.gov

- Includes SAMINT program designed to adjust nuclear data parameters based on integral data: <http://www.osti.gov/scitech/biblio/1185560/>
 - In multi-group or continuous-energy framework
 - Informs the evaluation of resolved resonance parameters
 - Leverages ORNL expertise in sensitivity studies of IBE's and applications
 - SCALE modules TSUNAMI and TSURFER
- New detector resolution functions based on MCNP simulations
 - In collaboration with Y. Danon (RPI)
- SAMMY CI and SQA
 - Cmake/ctest build and test on Linux/Mac/Windows, gfortran/ifort
 - Helped identify and resolve several minor problems
- SAMMY code modernization on the way:
 - SAMRML already rewritten into C++
- Many thanks for feedback and inquires from SAMMY users!

Backup SAMMY slides

History

- Developed by Dr. Nancy Larson since 1970's through 2008
- Includes SAMMY + 25 auxiliary codes
 - e.g. SAMRML
- Architecture
 - Large Fortran (77) container array for memory management
- 185 multi-step test cases + 10 tutorial examples
- Comprehensive Documentation:
 - <http://info.ornl.gov/sites/publications/files/Pub13056.pdf>
- Employed for resolved resonance evaluations in ENDF
- Distributed via RSICC <https://rsicc.ornl.gov/>
 - SAMMY 8.1 is forthcoming

Capabilities

- Multi-level Multi-channel R-matrix code
- Bayesian fitting of R-matrix resonance parameters (RP)
 - a.k.a. Generalized Least Squares
 - yields covariance matrix of RP
- Data reduction:
 - Experimental Facility Resolution functions: ORELA, RPI, GELINA
 - Normalization, background
- Detector resolution functions
 - Configurable for variety of detectors
- Doppler broadening
 - Solbrig's kernel, Leal-Hwang method
- Multiple scattering effects, and other target effects
- Charged projectiles (p , α)
- Unresolved Resonance Range (FITACS by F. Froehner)

Capabilities: R-matrix *and* Reich-Moore Approx.

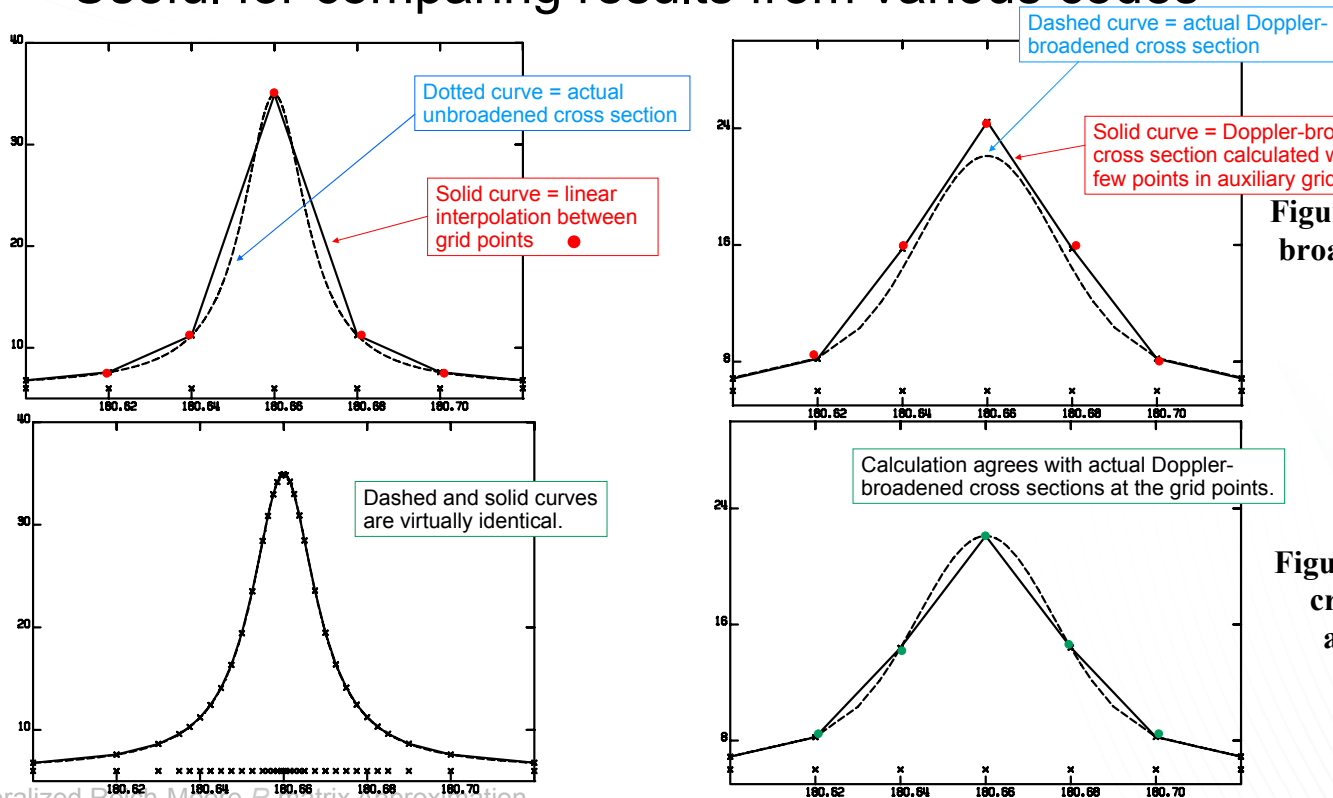
- Reich-Moore approximation (RMA)
 - For channels approximated statistically via Random Phase Hypothesis
 - Applied mostly for capture channels
 - penetrability $P=1$ for capture in RMA (and in exact R-matrix below)
- R-matrix w/o RMA
 - Make capture channels as reaction channels in SAMMY input files
 - Marco Pigni's talk:
 - Quantify accuracy of RMA relative to R-matrix on ^{16}O using 5 γ -rays/level

Capabilities: Resolution broadening

- Experimental Resolution broadening
 - Convolution of 4 components:
 - The electron burst
 - a square function of time
 - Neutron sources:
 - tantalum target
 - water moderator
 - Neutron detectors
 - model NE-110 recoil proton detector
 - lithium glass detector
 - Time-of-flight channel width
 - a square function of time
- Doppler broadening (DB)
 - numerical convolution of cross sections by Solbrig kernel over E
 - Double-differential c.s. DB-ed approximately by SAMMY now
 - our group developed and published an exact method
 - Leal-Hwang: efficient, used by SCALE

Capabilities: configurable energy mesh

- Auxiliary energy mesh for computations
 - Includes data energy points and additional points
 - especially at and near resonance peaks to trace their shapes completely
 - Needed because resonance are narrower at $T=0$ K before Doppler broadening to room temperature for data
 - Useful for comparing results from various codes



Figures from SAMMY Users' Guide:

Figure III A1.2. Incorrect Doppler-broadened cross section calculated with too few points in the auxiliary grid.

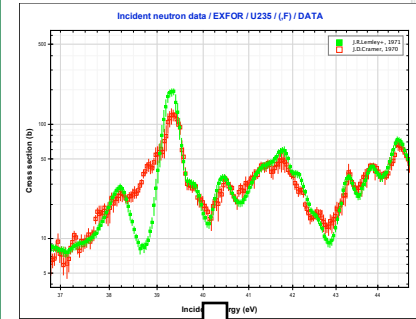
Figure III A1.4. Doppler-broadened cross section calculated with an adequate number of points in the auxiliary grid.

SAMINT: Nuclear Data Adjustment Based on Integral Benchmark Experiments (IBE)

- SAMINT is an auxiliary program designed to allow SAMMY to adjust nuclear data parameters based on integral data.
- Enables coupling of differential and integral data evaluation in a continuous-energy framework
- Informs the evaluation of resolved resonance parameters
 - Based on performance in simulations of IBE's
- Leverages RNSD (ORNL) expertise in sensitivity studies of nuclear IBE's and applications
 - SCALE modules TSUNAMI and TSURFER

Integral Experiments to Aid Nuclear Data Evaluation

- SAMINT can be used to extract information from integral benchmarks to aid the nuclear data evaluation process.
- Near the end of the evaluation based on differential experimental data, integral data can be used to:
 - Resolve remaining ambiguity between differential data sets
 - Guide the evaluator to troublesome energy regions
 - Inform the evaluator of the most important nuclear data parameters to integral benchmark calculations
 - Improve the nuclear data covariance matrix evaluation



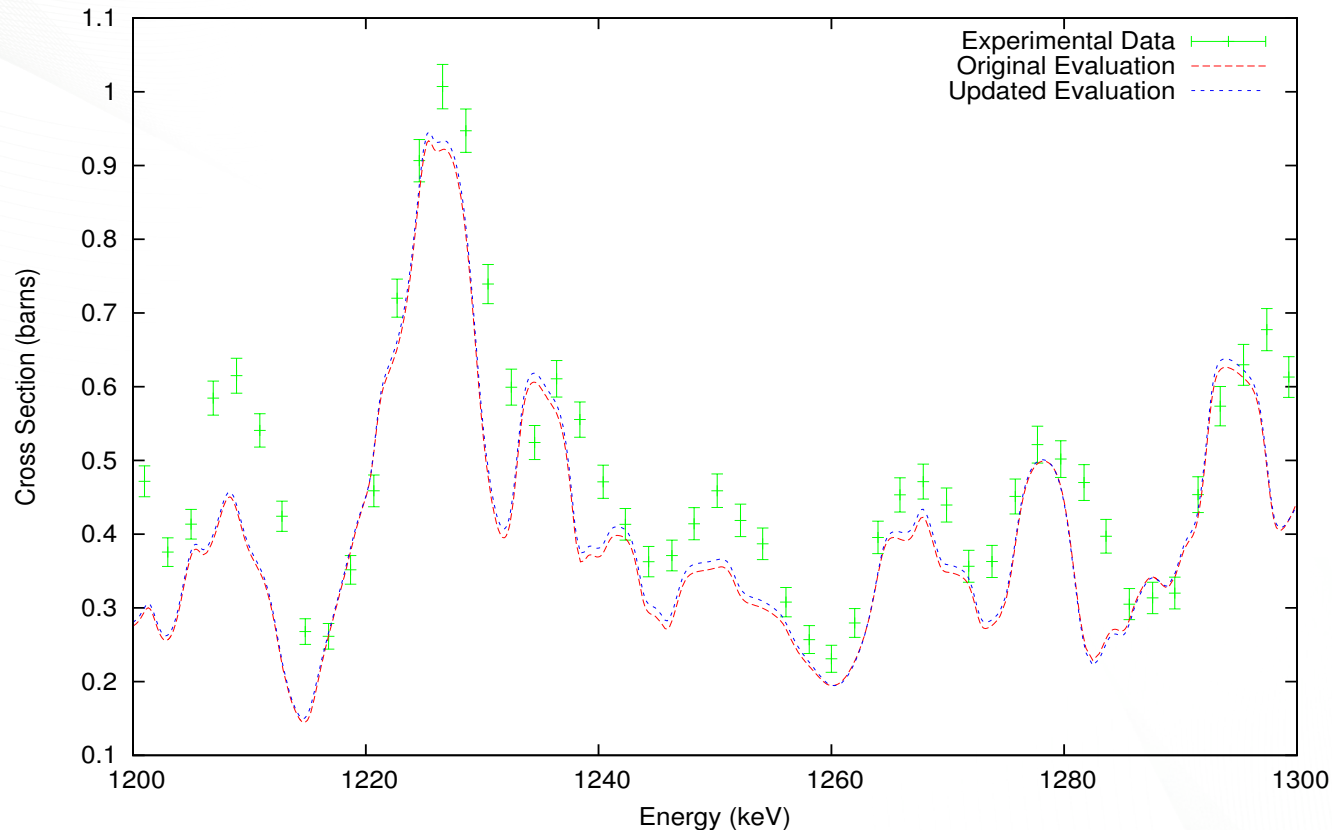
SAMMY



ENDF

Cross Section Changes: Finer Scale than Differential Experimental Data

Inelastic Cross Section



Inelastic cross section of ^{56}Fe before ($\chi^2 = 23.6023$) and after ($\chi^2 = 22.9036$) the adjustment based on integral experimental data plotted on top of differential experimental data of Perey, presented with one standard deviation error bars.

Connections to AMPX and SCALE

- AMPX: data processing suite for SCALE data libraries
 - Dorothea Wiarda, Andrew Holcomb, Michael Dunn (ORNL)
 - Shipped with SCALE 6.2 via <https://rsicc.ornl.gov>
- SCALE: nuclear modeling suite for design, safety, licensing
 - <http://scale.ornl.gov> Brad Rearden (ORNL) Manager
- SAMMY modernization follows AMPX and SCALE
 - Mercurial version control system
 - Cmake automated build and ctest automated testing
 - Fogbugz Bug tracking system
- SAMMY Module SAMRML “shared” with AMPX:
 - Stripped-down SAMMY for computing resonant cross sections
 - no parameter fitting, no Doppler or resolution broadening
 - also used in data processing codes: AMPX, NJOY, PREPRO
 - Modernized into C++ by Andrew Holcomb
- Provides a framework for modernizing SAMMY

Modernization: SQA

- Version control of source code and test cases using Mercurial
- Bug tracking and workflow
- Cmake: auto configuration build (make -j)
- 25 executables built automatically on several platforms
- Ctest: auto testing tool
 - Test whether result are within a prescribed tolerance ($1E-4$)
 - SAMMY files tested: LST, PAR, LPT...
 - Makes it much easier to notice discrepancies.
 - 178 test cases from SAMMY 8.0.0 and
 - 4 new test cases for SAMINT
 - 1 new test case for RPI Lithium Glass detector resolution function
 - (All test cases include subcases.)

Supported Platforms and Compilers

- Mac: gfortran
- Linux: gfortran, ifort
- Windows: ifort
 - Revealed few remaining issues that were corrected

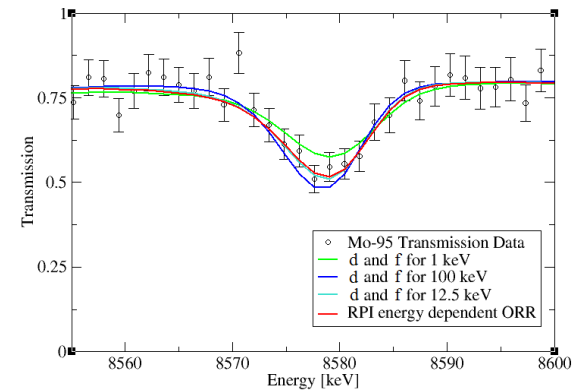
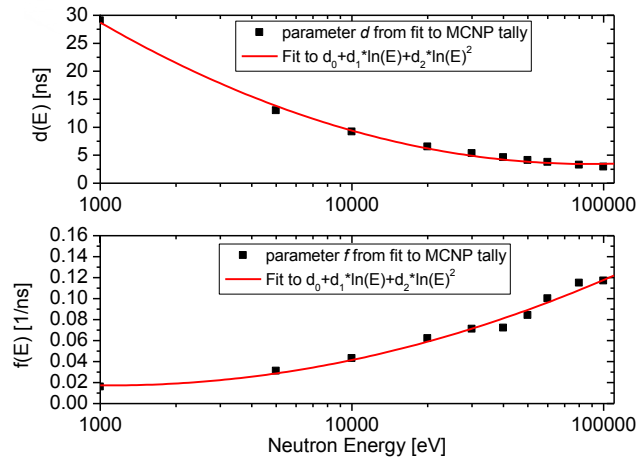
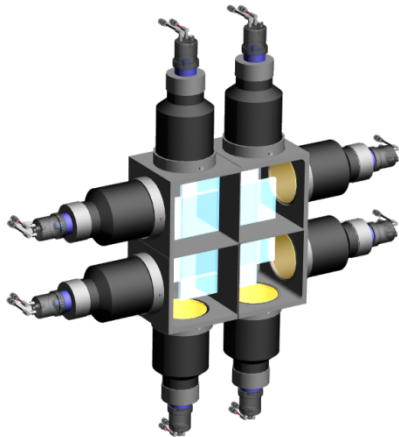
Modernization cont.'d

- Proposed SAMMY re-organization
 - Delineate modules that compute cross sections from those that fit resonance parameters to the data
 - i.e. keep SAMRML a standalone module called by a fitting program
 - Fitting method could remain Bayesian or Generalized Least Squares

⁶Li-glass Neutron Detector Array MELINDA

– Improved parameterization

- Based on MCNP simulations by Amanda Youmans (RPI)



Comparing the original SAMMY detector resolution function to the modified function with Mo-95 data

Before:

$$I(l) = \begin{cases} Dg & \text{for } 0 < t < d \\ D e^{-f(t-d)} & \text{for } d < t \end{cases}$$

After:

$$d(E) = d_0 + \ln(E) * (d_1 + d_2 * \ln(E))$$

$$f(E) = f_0 + \ln(E) * (f_1 + f_2 * \ln(E))$$

Liquid Scintillator Detector liquid CH_{1.212}

– Based on MCNP simulations by Amanda Youmans (RPI)

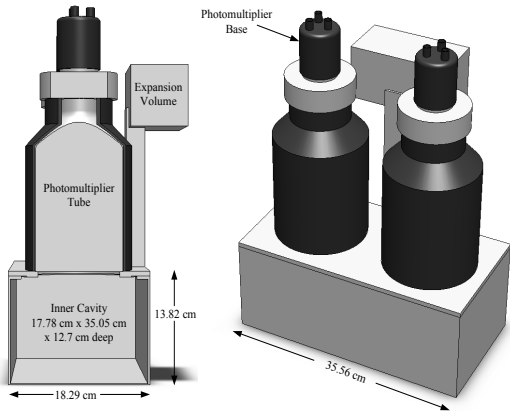
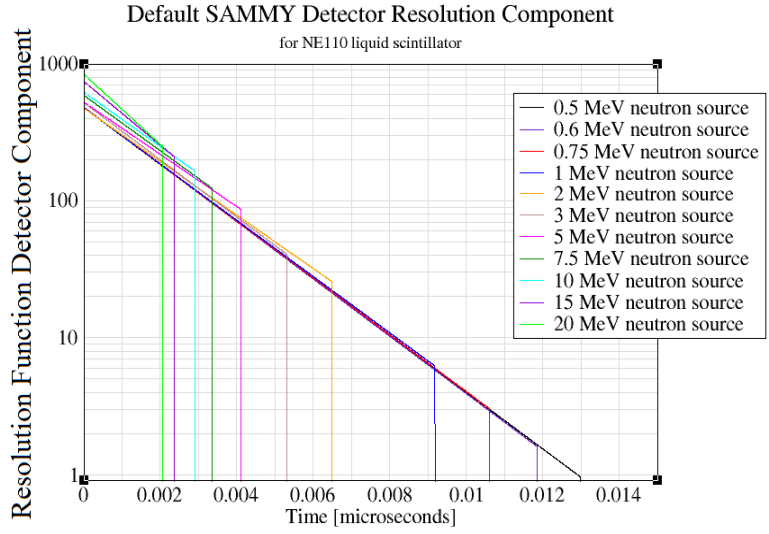
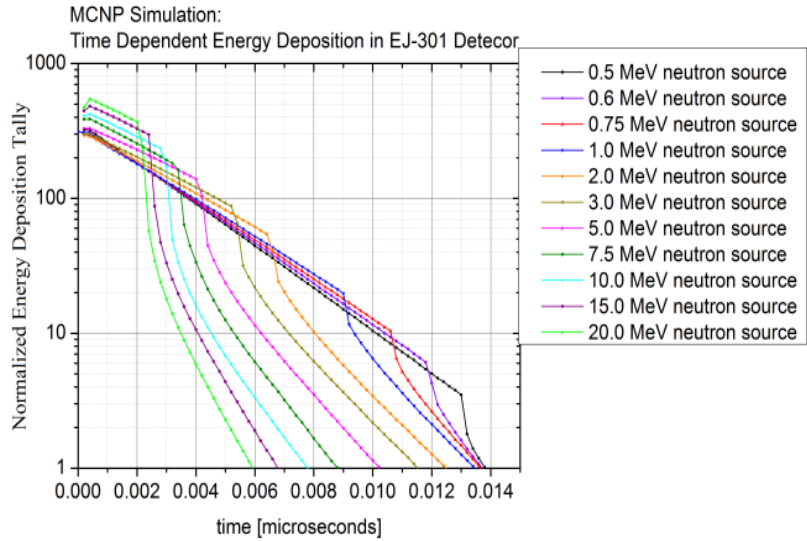


Table IV: Cross Sections for EJ-301 SAMMY Input

Energy [eV]	Cross section [b]
500,000	7.967348
600,000	6.963397
750,000	5.958675
1,000,000	4.953201
2,000,000	3.101132
3,000,000	2.329274
5,000,000	1.683733
7,500,000	1.554593
10,000,000	1.172541
15,000,000	0.877705
20,000,000	0.663136



Minor improvements in SAMMY 8.1

- Updated physical constants
 - Consistent with SAMRML
 - SAMMY and SAMRML yield identical results now
- Corrected a misplaced index causing incorrect matrix multiplication for non-diagonal data covariance matrix
 - (uncovered and corrected by Vlad Sobes)
- Several other minor bug fixes
 - Revealed by compiler or platform idiosyncrasies

Recent Developments

- Convert SAMMY resonance parameters to formal R-matrix
 - SAMMY evaluations set $S_c(E)-B_c=0$ boundary condition (b.c.)
 - Advantage: resonance energies coincide with resonance peaks
 - Disadvantage: slight deviations from formal R-matrix
 - SAMMY does support formal R-matrix $B_c=-1$ b.c.
 - Andrew Holcomb programmed conversion to/from formal R-matrix
 - Converted ^{16}O $S_c(E)-B_c=0$ into formal parameters for Marco Pigni's talk
 - and is extending it to $L_c(E)-B_c=0$ in complex plane
- S-matrix pole representation of R-matrix
 - via e.g. Brune transform of R-matrix param.'s for $L_c(E)-B_c=0$ b.c.
 - Useful for on-the-fly Doppler broadening in neutron transport app.'s
 - Developed by Hwang (ANL) and Fritz Froehner (INR, Karlsruhe)
 - Ongoing collaboration Vlad Sobes and N.E. at M.I.T. (Pablo Ducru)
 - Talk by Vlad Sobes this Friday
 - Use Nicolas Michel's complex Coulomb w.f. library (CPC, 176 (2007) 232)

Conclusions and outlook

- ORNL has enlarged SAMMY user and developer expertise
 - various RRR evaluations ongoing and upcoming
 - Marco Pigni, Vlad Sobes, Luiz Leal
 - Klaus Guber collecting data at GELINA
 - SAMMY modernization continuing in a modern SQA framework
 - code sharing with AMPX modules
 - e.g. SAMRML modernized into C++ by Andrew Holcomb
 - Interaction with other R-matrix and data evaluation codes is encouraged.