



Technical Meeting on the Nuclear Data Processing  
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# **GRUCON Code Package Capacities in Creating of ACE Data Files for Monte Carlo Transport Calculations**

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**Appendix B. ACE files with Angular Distribution Reconstructed from Resonance Parameters**

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# Plan of Inter-Comparison Exercise

1. Prepare working ENDF and ACE working libraries from 235U, 238U, 239Pu files, loaded from IAEA-CIELO Web page and processed by PREPRO-2015, NJOY-2016 and GRUCON codes;
2. Compare reconstructed from resonance parameters and Doppler broadened cross sections
3. Compare the effective cross sections derived from the Probability Tables (PT) and Subgroup Parameters (SP), at infinite and 1 barn dilutions, using ACE\_UNPACK, ACE\_PLOT /V.Sinitsa/, PLOTTAB, PLOTSAVE /D.Cullen/;
4. Run MCNP code for benchmarks, sensitive to shielding in the URR, using identical inputs data but different ACE files for the 235U, 238U, 239Pu nuclides, taking the rest from the ENDF/B-VII.1;
5. Inter-compare Monte Carlo simulation responses for the cases with/without self-shielding;
6. Compare calculated and experimental values.

# **1. Working Data Libraries**

## Data libraries, involved in inter-comparison procedure, contain:

### ACE-1 library, with

- **U-235** and **U-238** (26Sep2017) **CIELO/ACE** files downloaded from the Web page <https://www-nds.iaea.org/CIELO/>
- **Pu-239 ACE** file prepared from **Pu-239 CIELO/ENDF** file (original name is Pu239e80b4\_5\_corDN, as submitted by D. Neudecker on 22 February) through **NJOY-2016** ( date of installation 20Sep2017)
- **ACE** files for **56** materials, involved in benchmarks calculations (including thermal polyethylene H(CH<sub>2</sub> ) ); they were obtained from the **ENDF/B-VII.1** data by **NJOY-2016** processing code

### ACE-2 library, with

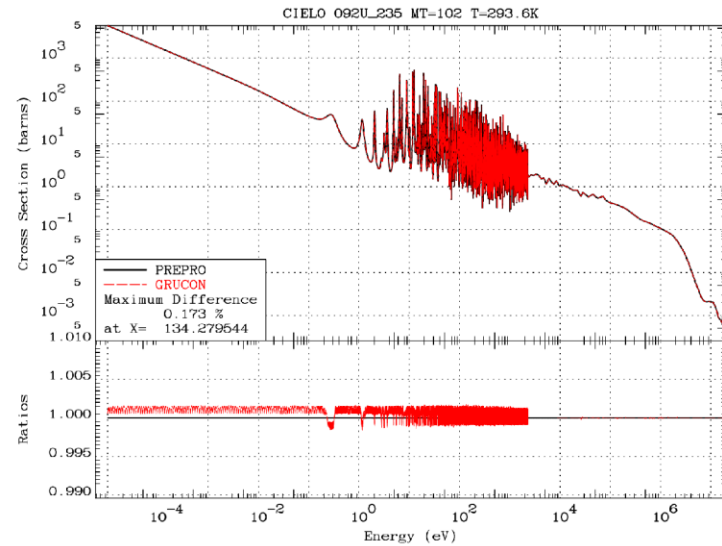
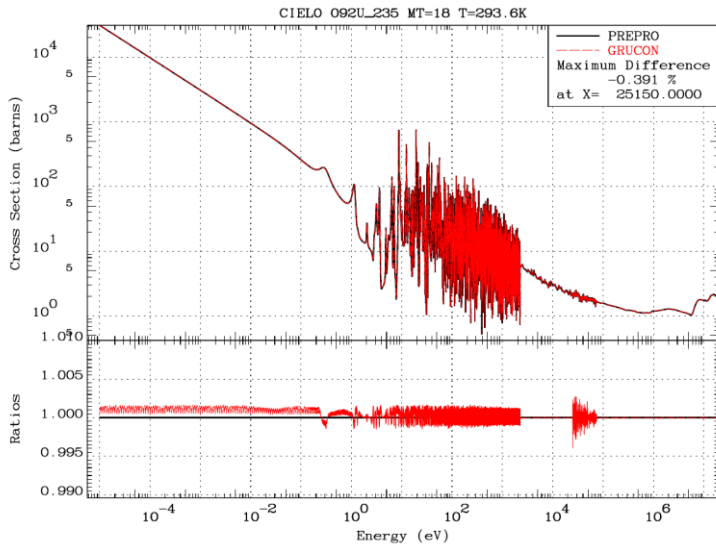
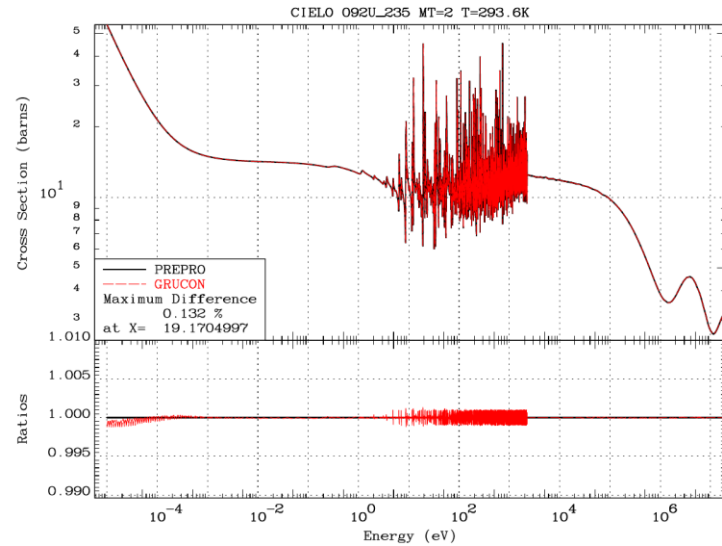
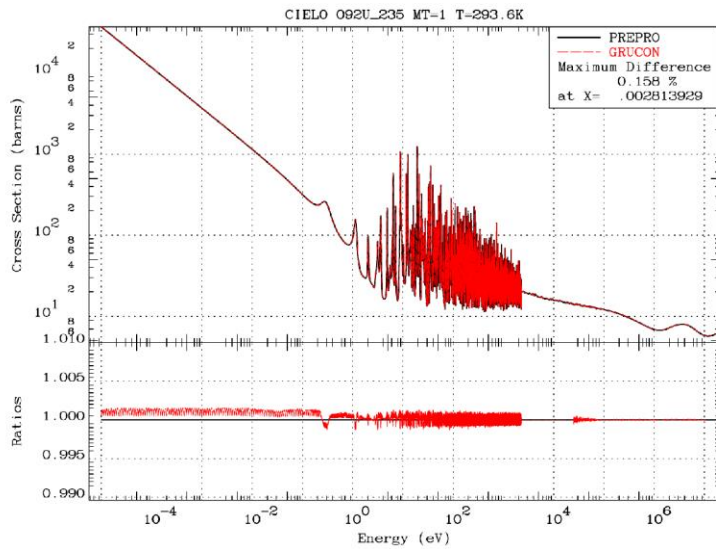
- **56 ACE** files, prepared from the **ENDF/B-VII.1** data by **NJOY-2016** processing code system
- **ACE** files for **U-235,U-238** and **Pu-239** prepared from **CIELO/ENDF** files by **GRUCON** code package

**ENDF-PREPRO** and **ENDF-GRUCON** auxiliary libraries with point-wise cross sections, reconstruction from resonance parameters and Doppler broadened by **PREPRO-2015** and **GRUCON** processing codes, for **U-235,U-238** and **Pu-239 CIELO/ENDF** Files.

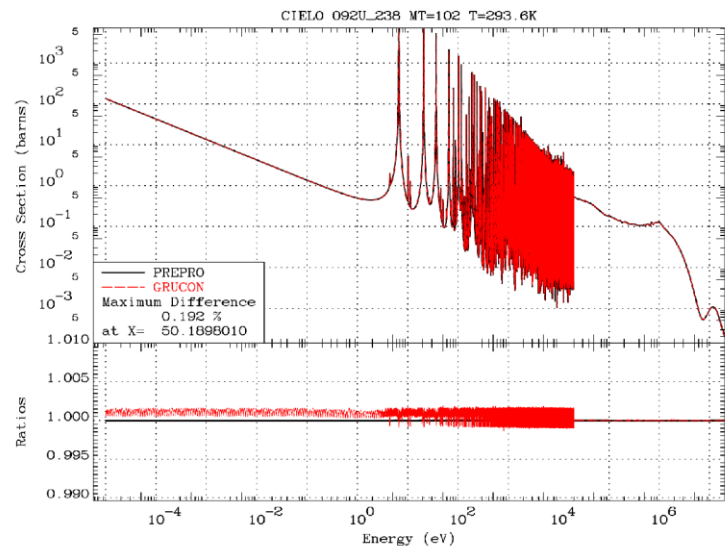
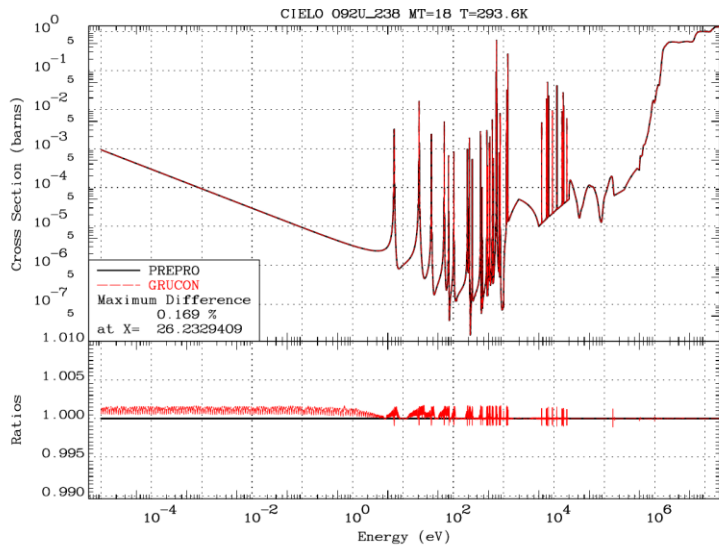
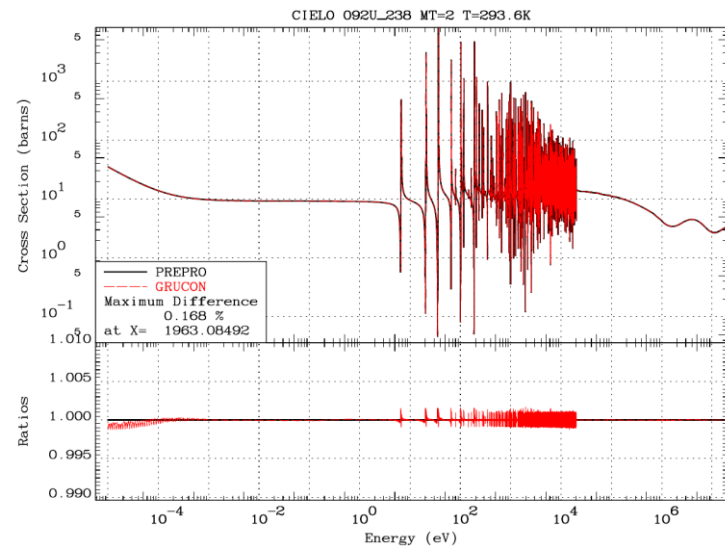
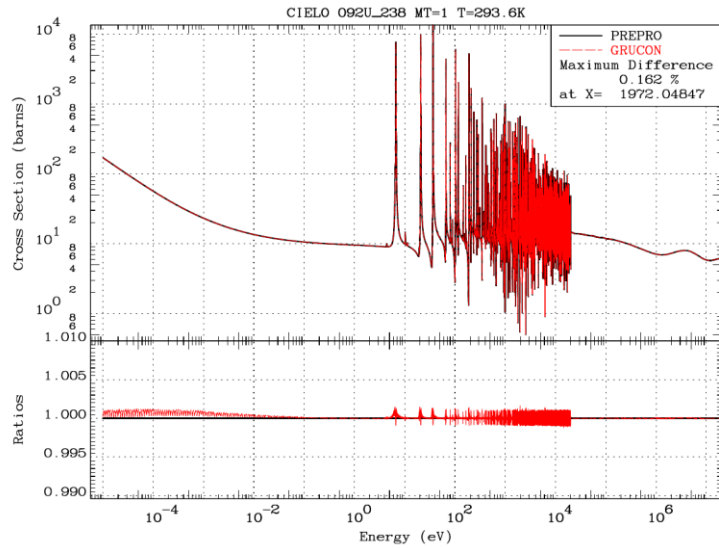
Tolerance parameter: **EPS=0.001**

## **2. Comparison of the Reconstructed from Resonance Parameters and Doppler Broadened Cross Sections**

# CIELO/ENDF/U-235 : GRUCON/PREPRO comparison

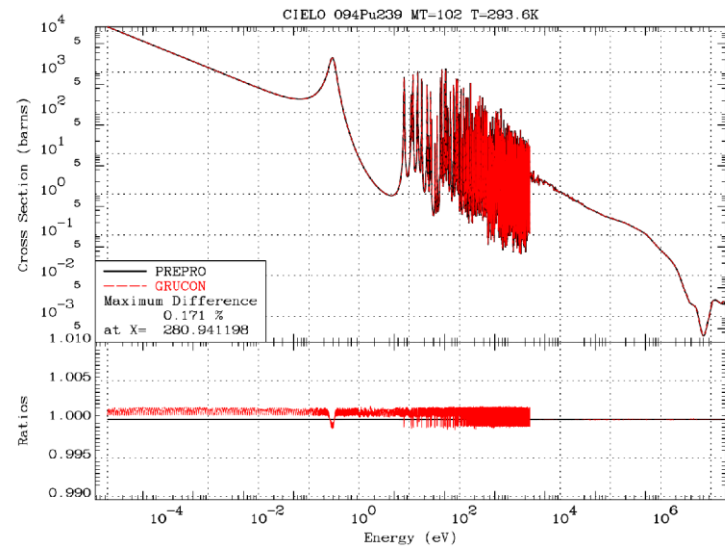
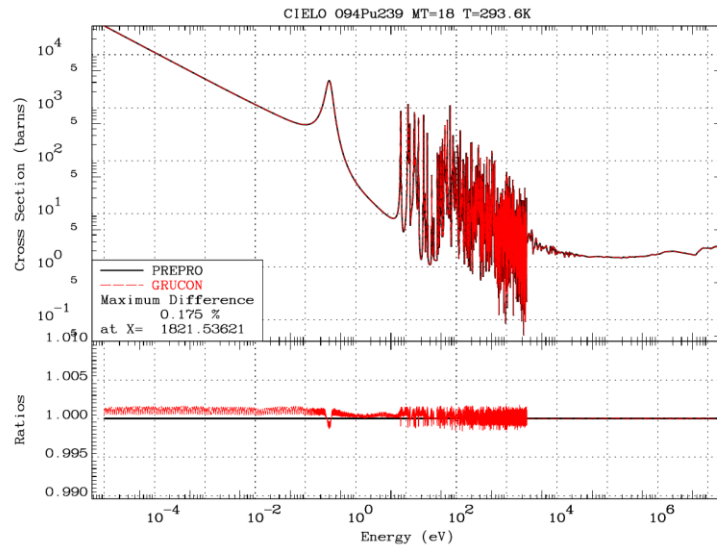
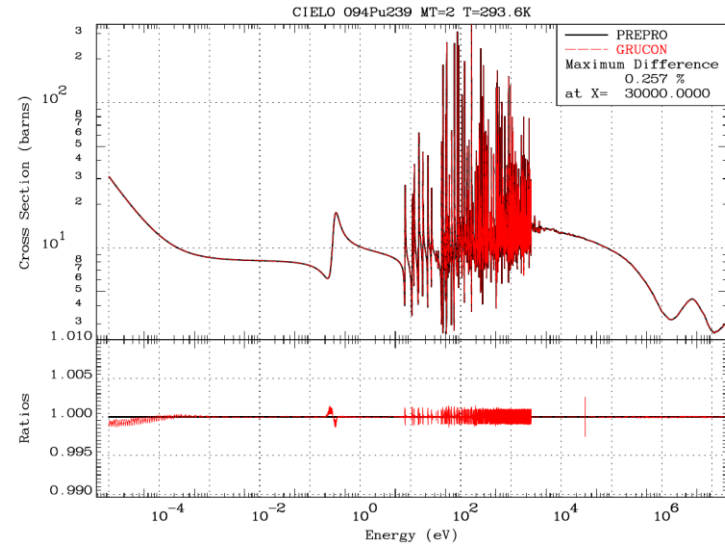
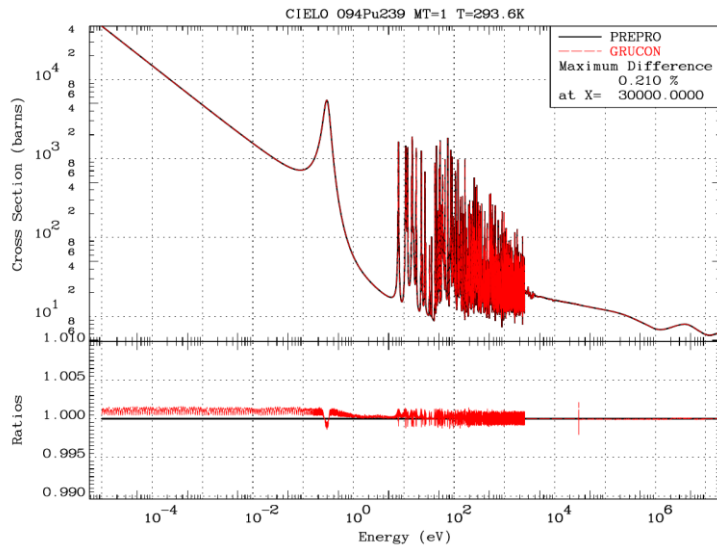


# CIELO/ENDF/U-238 : GRUCON/PREPRO comparison





# CIELO/ENDF/Pu-239 : GRUCON/PREPRO comparison

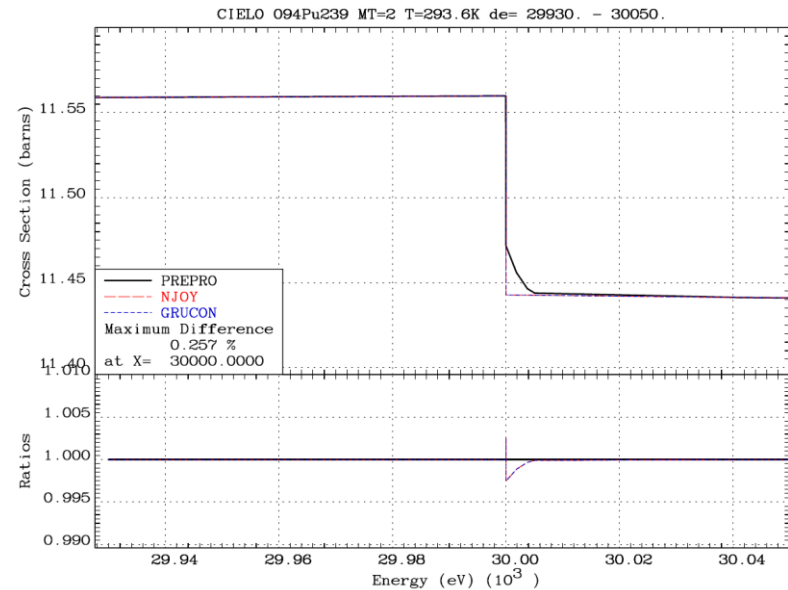
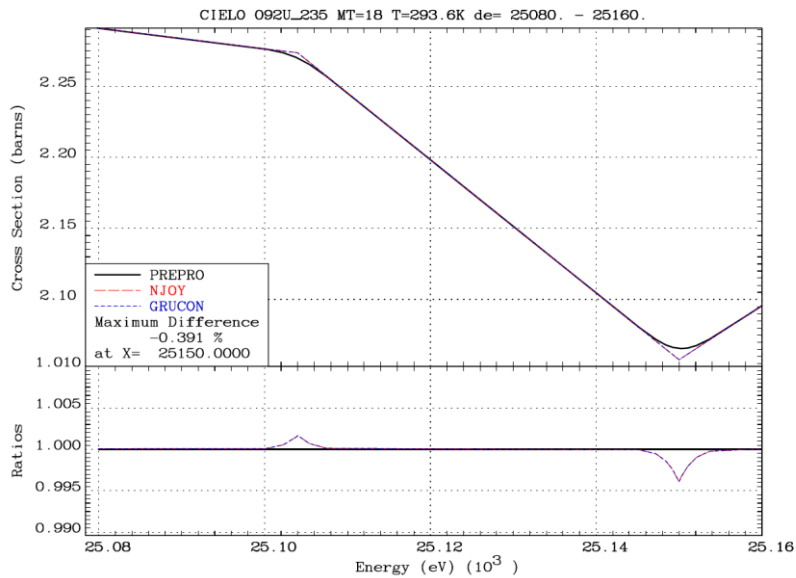


Obtained Difference <0.2% , with exceptions:

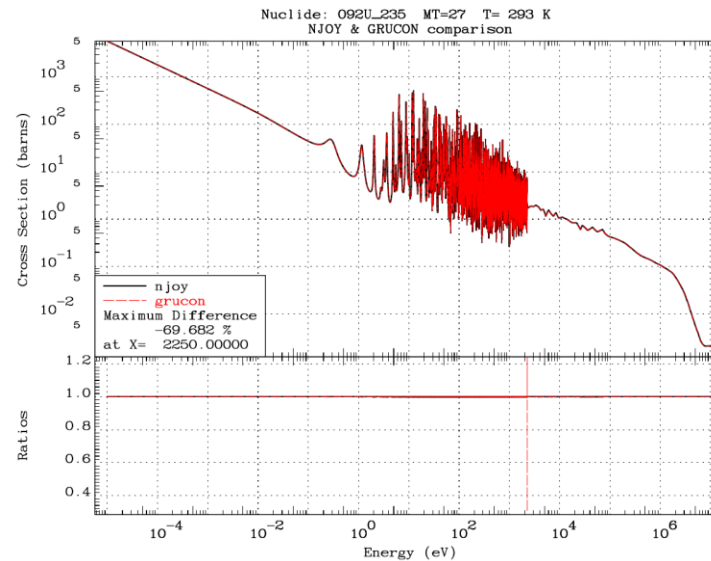
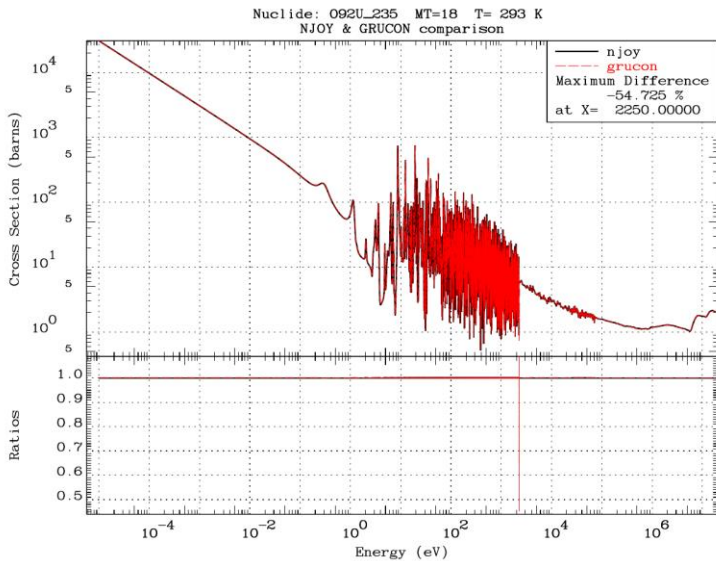
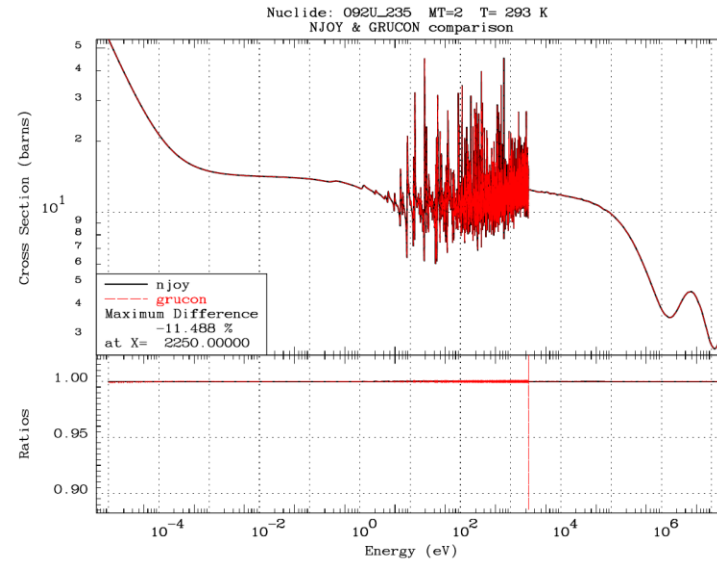
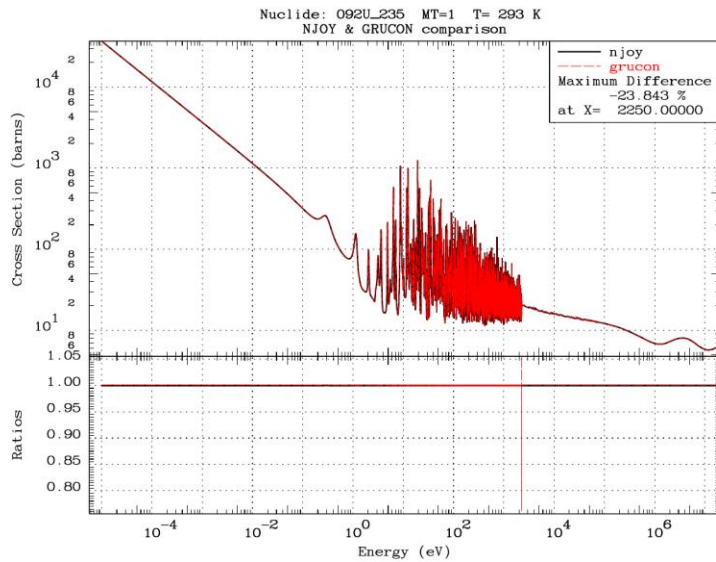
U-235 Fission – 0.39%

Pu-239 Elastic – 0.26%

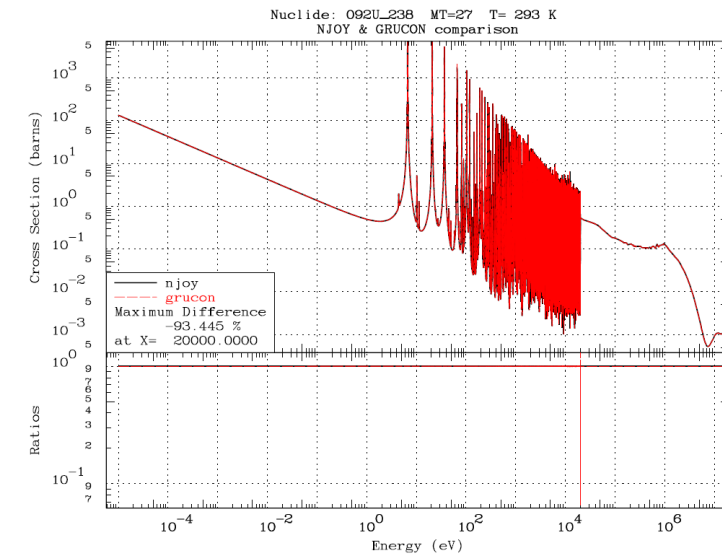
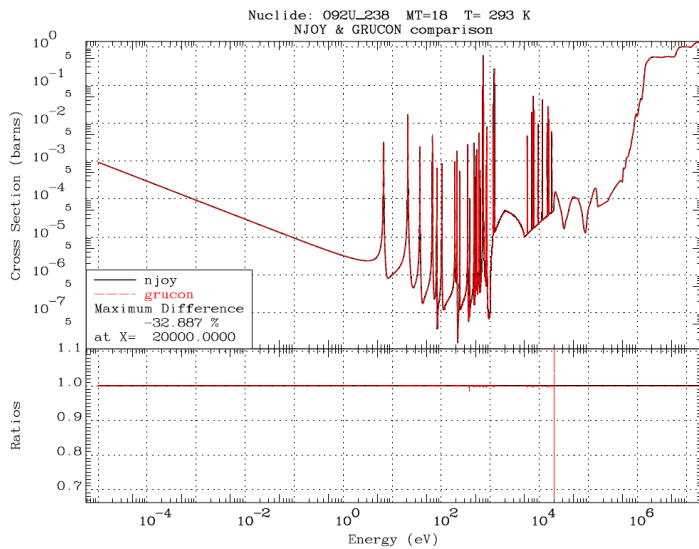
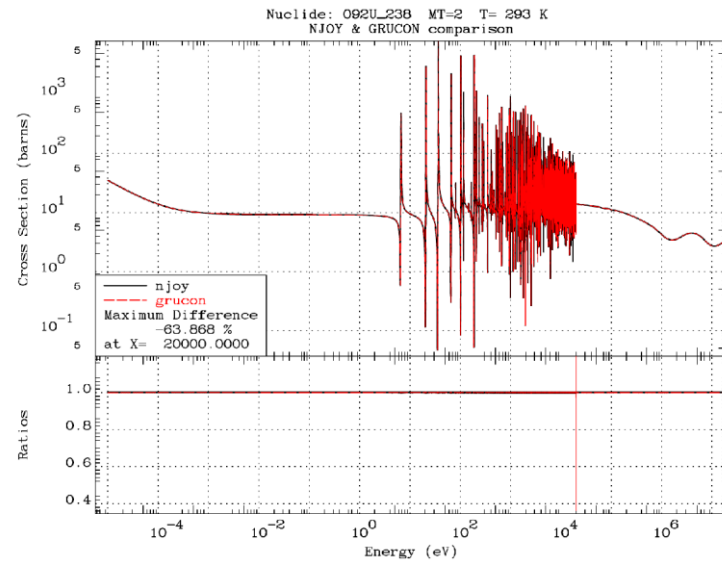
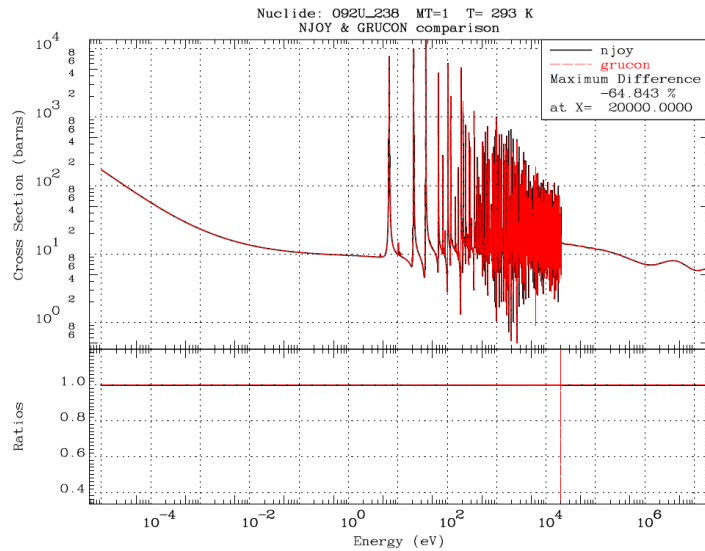
Explanation: GRUCON ( and NJOY) don't make Doppler broadening above URR (25keV for U-235 and 30keV for Pu239) , but PREPRO do it:



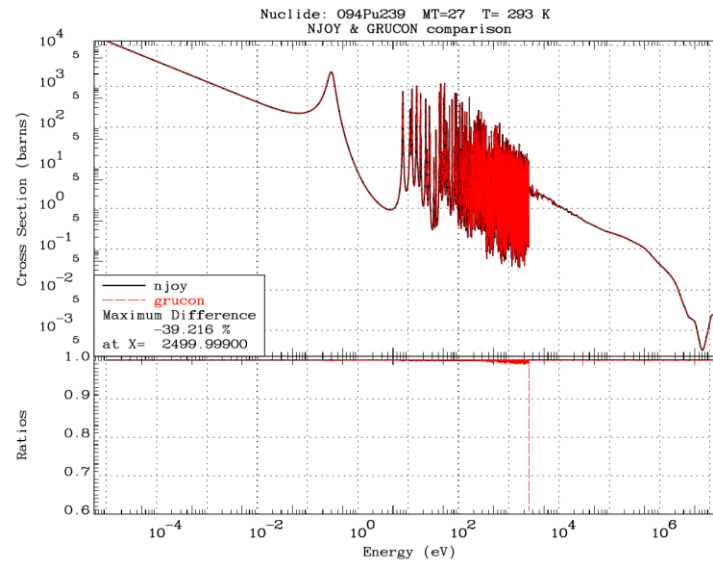
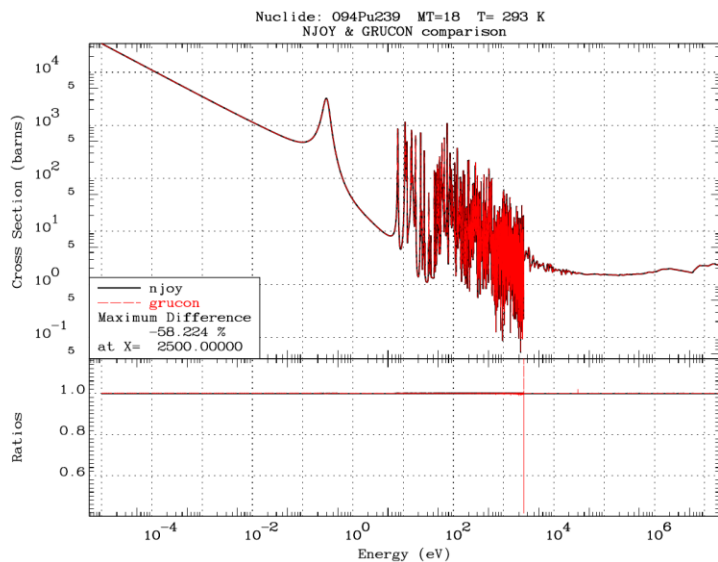
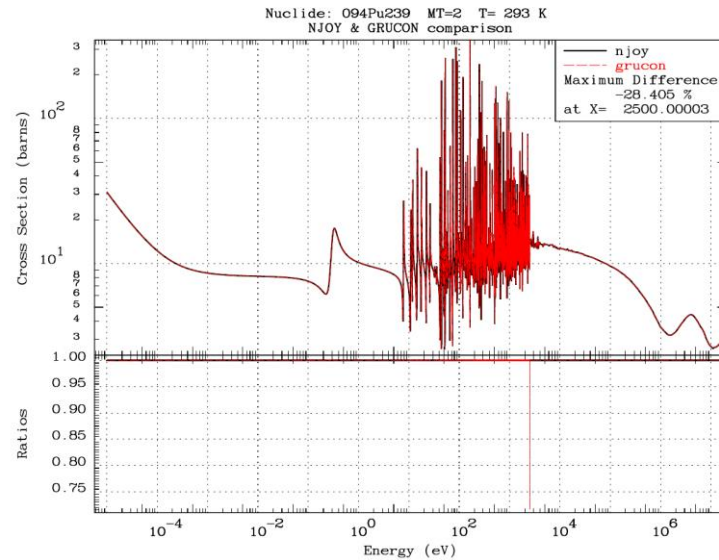
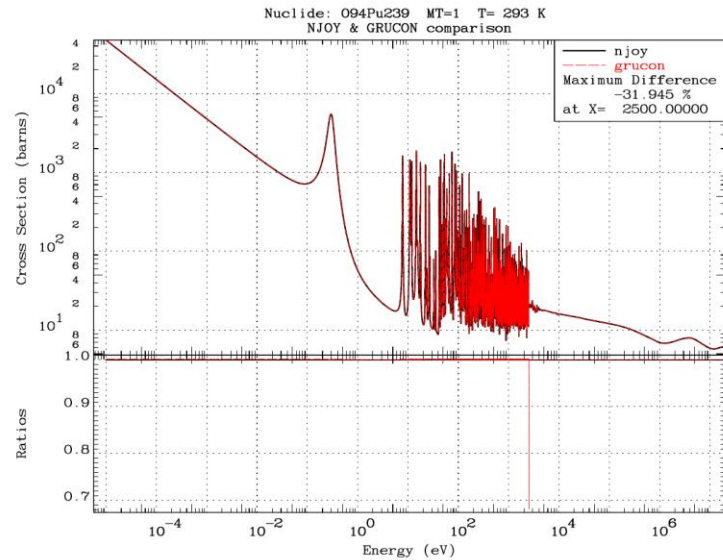
# CIELO/ACE/U-235 : GRUCON/NJOY comparison



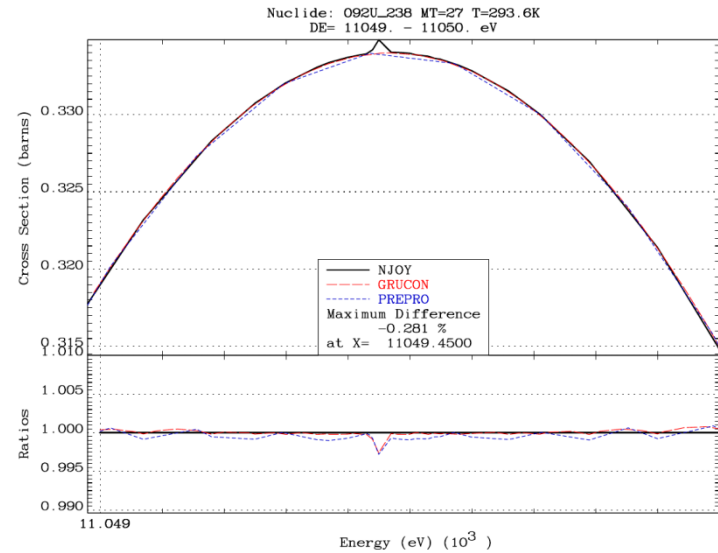
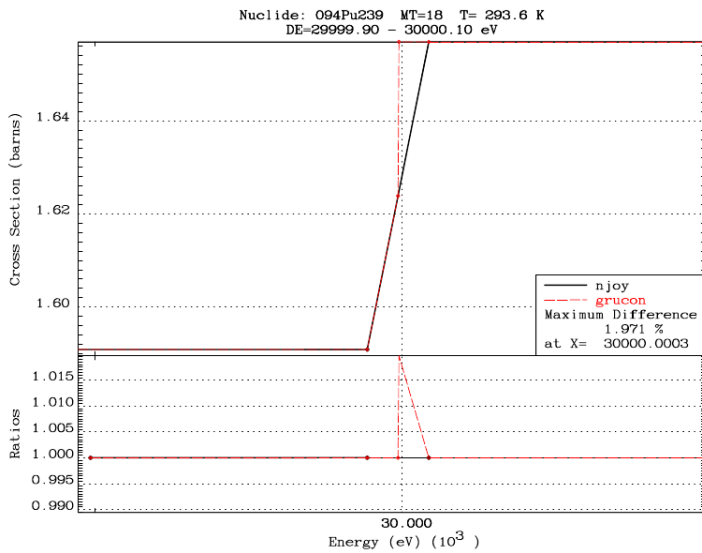
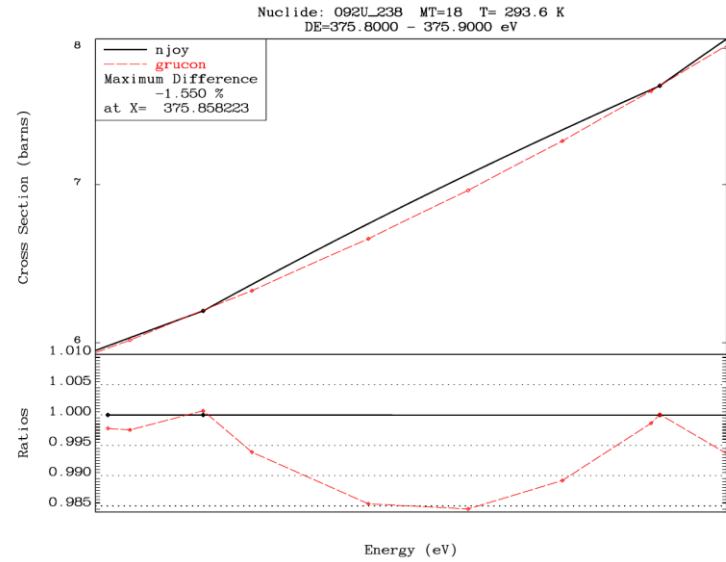
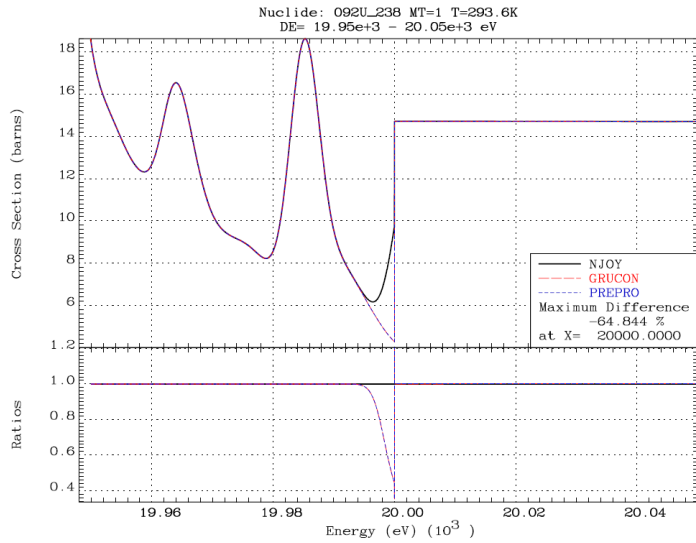
# CIELO/ACE/U-238 : GRUCON/NJOY comparison



# CIELO/ENDF/ACE/Pu-239: GRUCON/NJOY comparison



# CIELO/ACE/U-238 : some differences



# Summary

Comparison of cross sections, reconstructed and Doppler broadened through NJOY, PREPRO and GRUCON codes, reveal no serious discrepancies.

ACE files for U-235 and U-238, prepared in CIELO project, sometimes have lack of points in the RR energy range.

The absorption cross section in U238 CIELO ACE file shows irregularity, connected, probably with loss of accuracy.

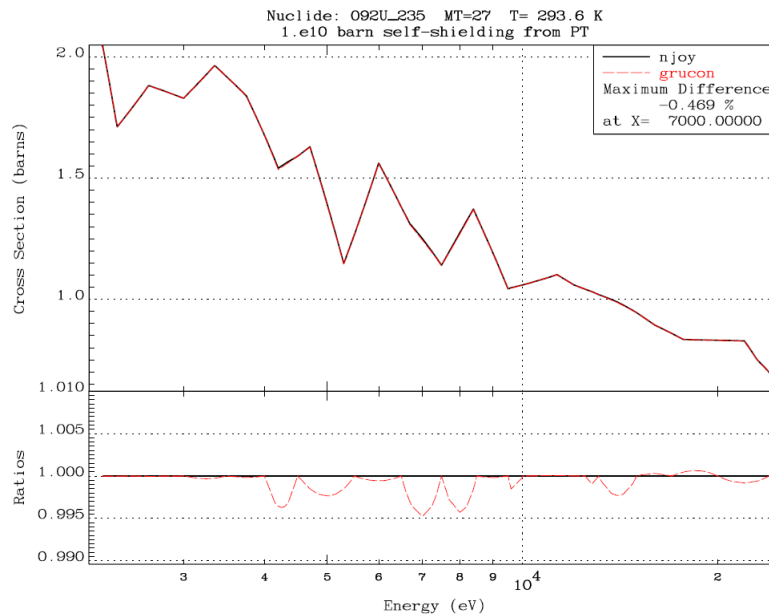
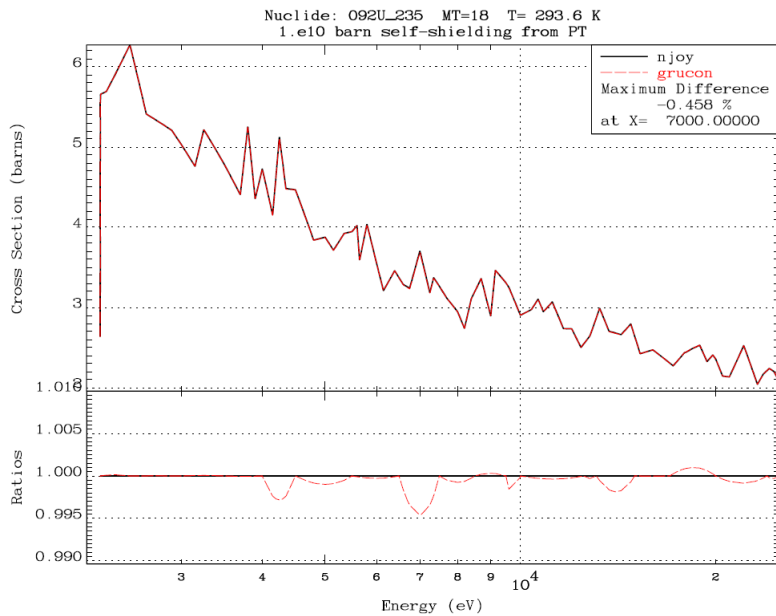
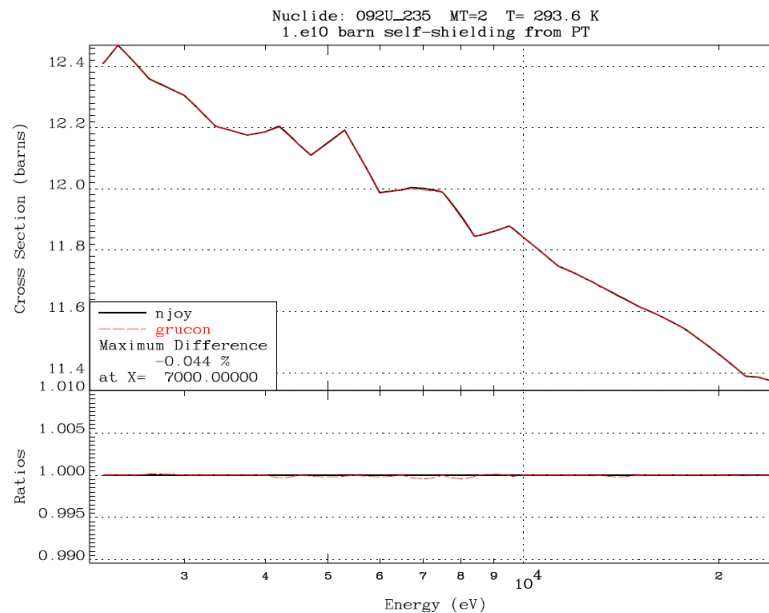
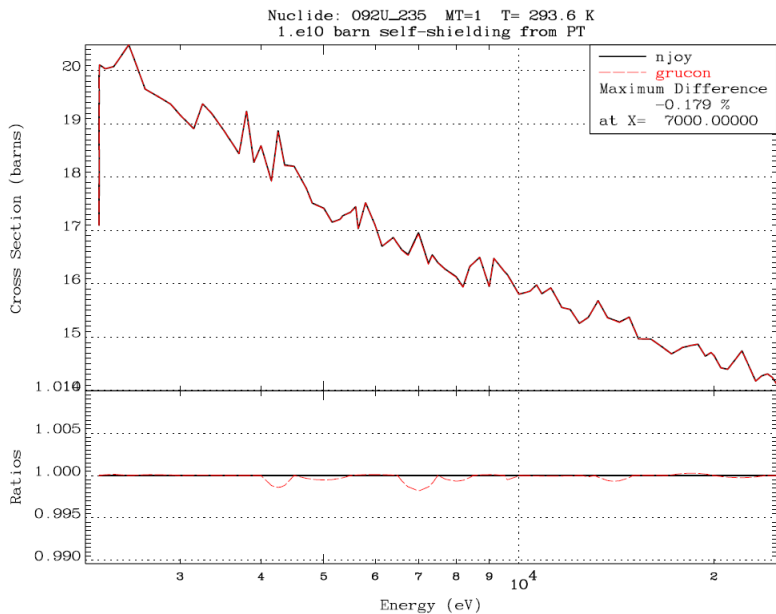
Differences, found in detailed cross sections, can not have essential influence on results of benchmark calculation.

### **3. Comparison of Cross Sections Derived from Probability Tables and Subgroup Parameters (ACE/LUNR Record)**

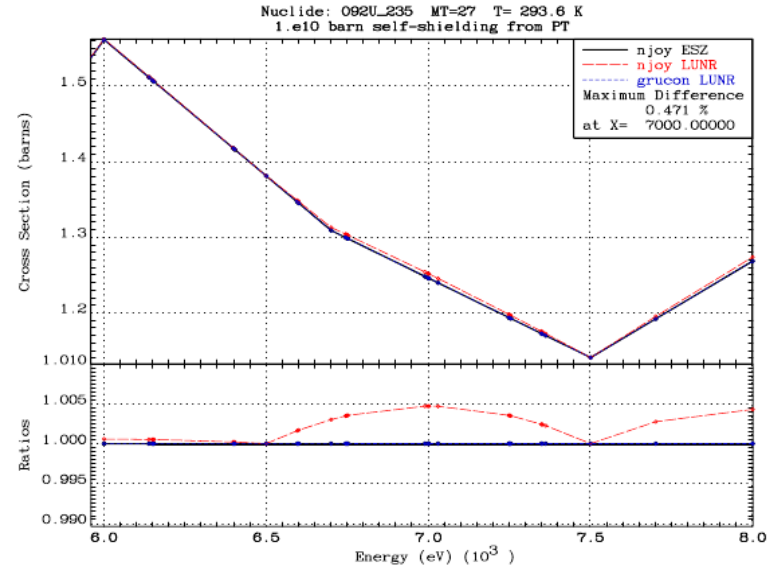
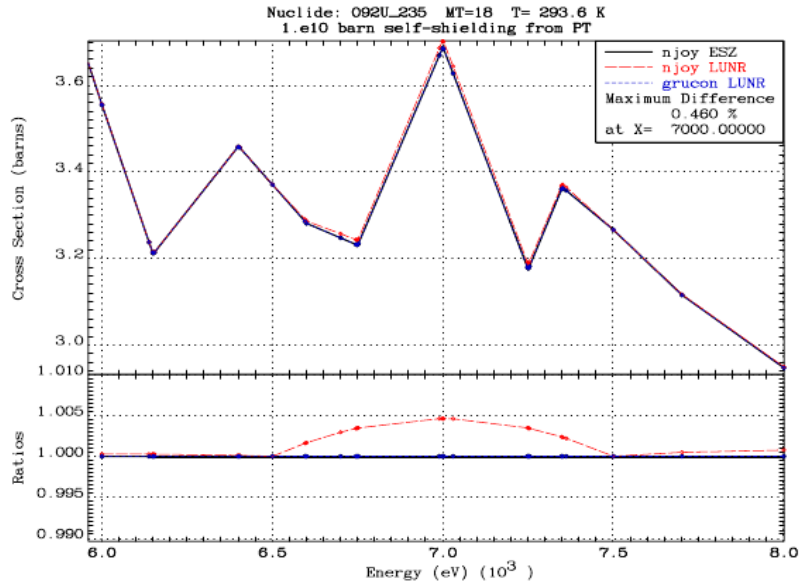


## **3.1 Comparison of Unshielded Cross Sections Derived from ACE/LUNR Record**

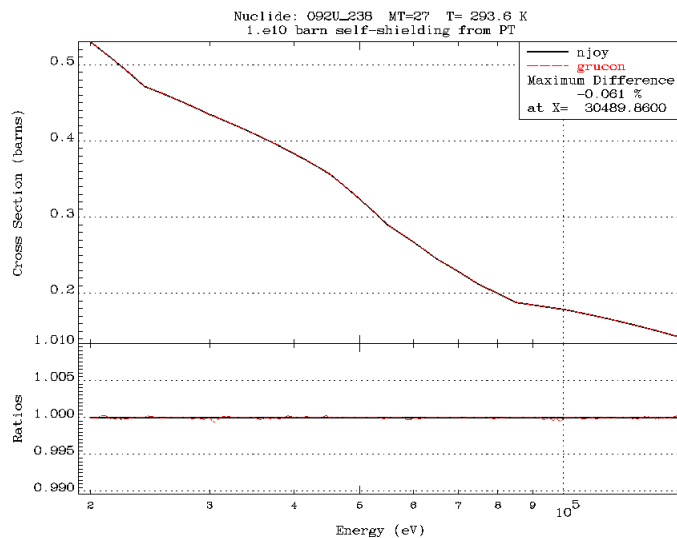
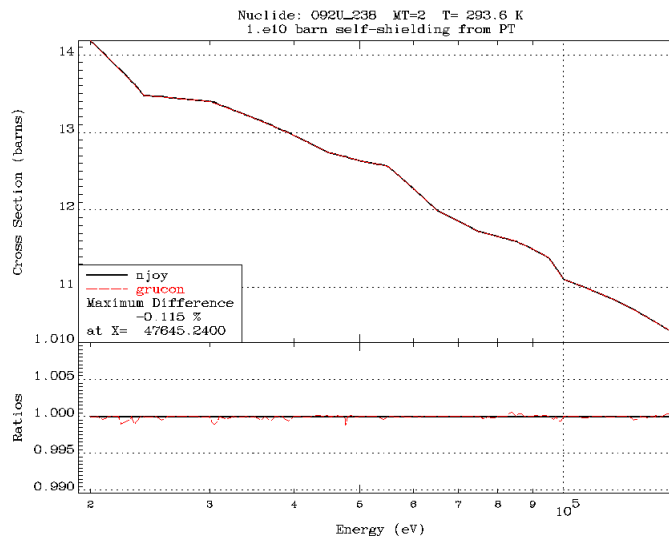
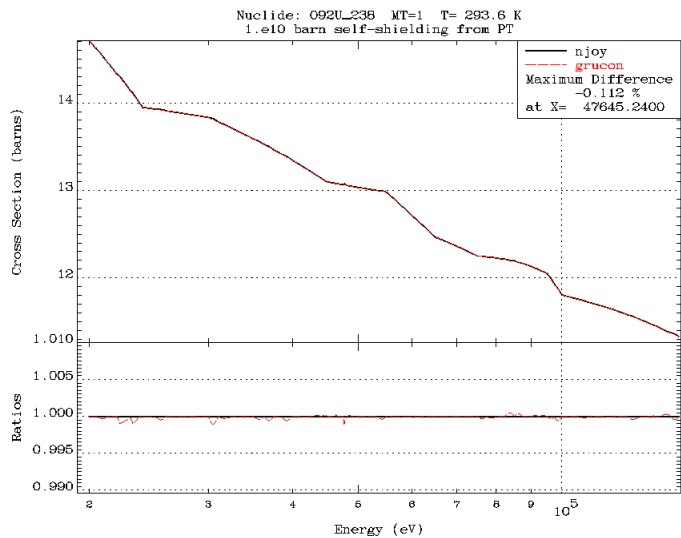
# CIELO/ACE/U-235: GRUCON/NJOY comparison



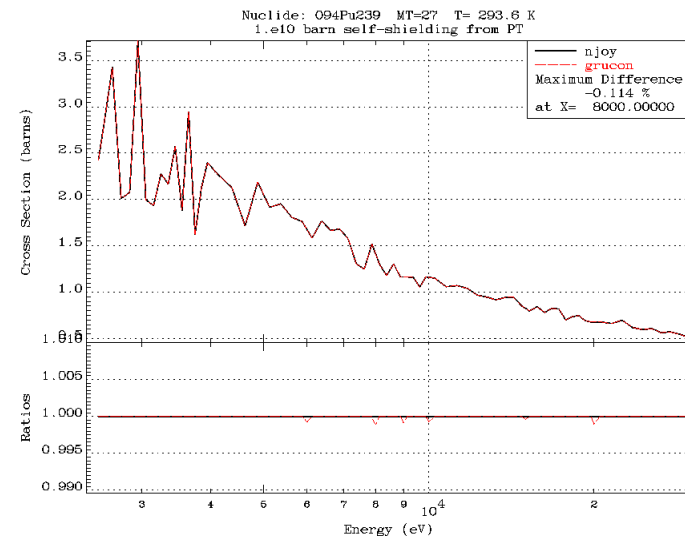
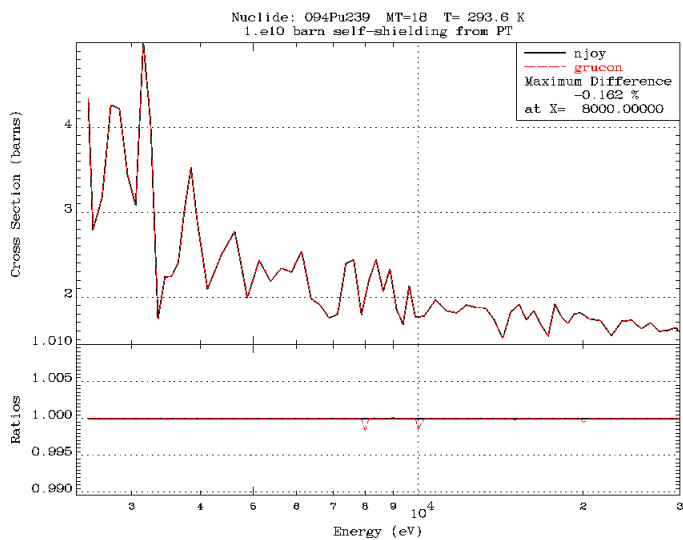
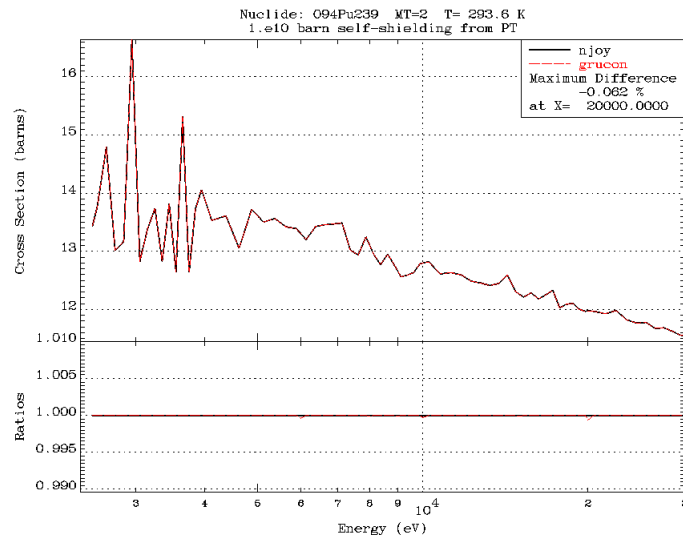
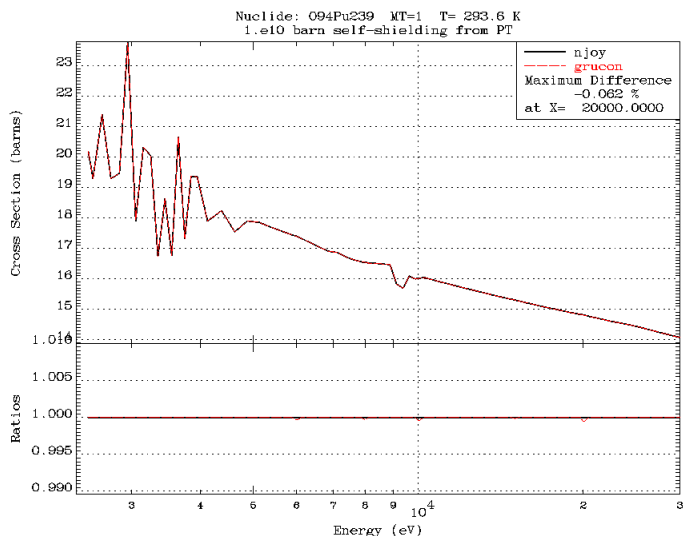
# CIELO/ACE/U-235: Comparison of Cross Sections from ACE/ESZ and ACE/LUNR Records



# CIELO/ACE/U-238: GRUCON/NJOY comparison

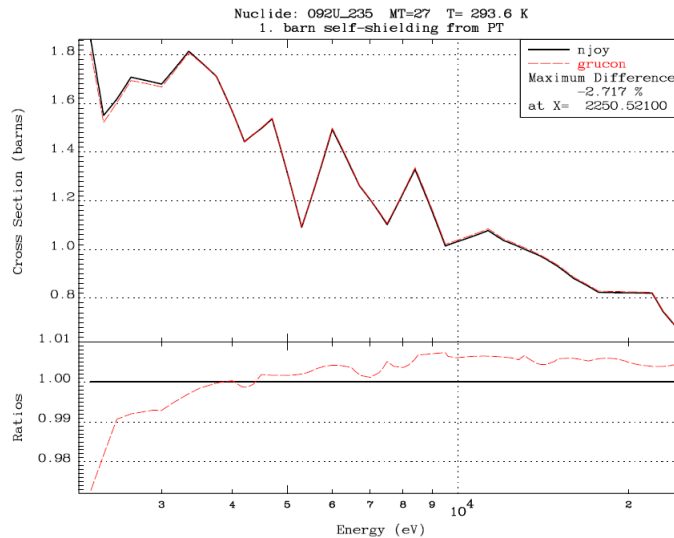
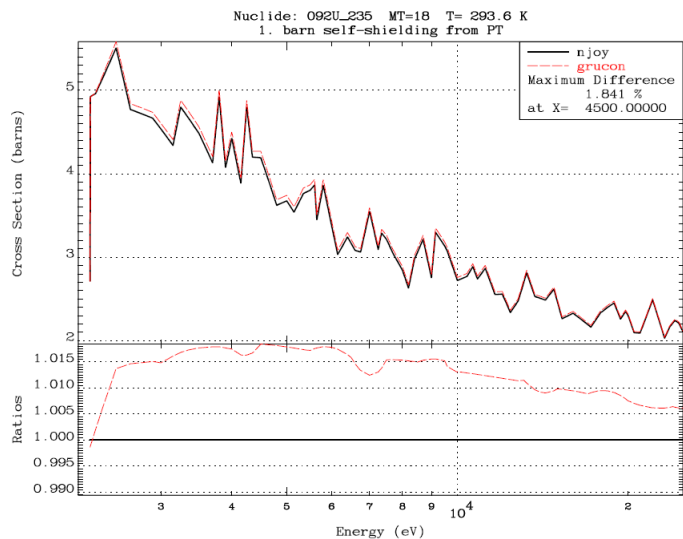
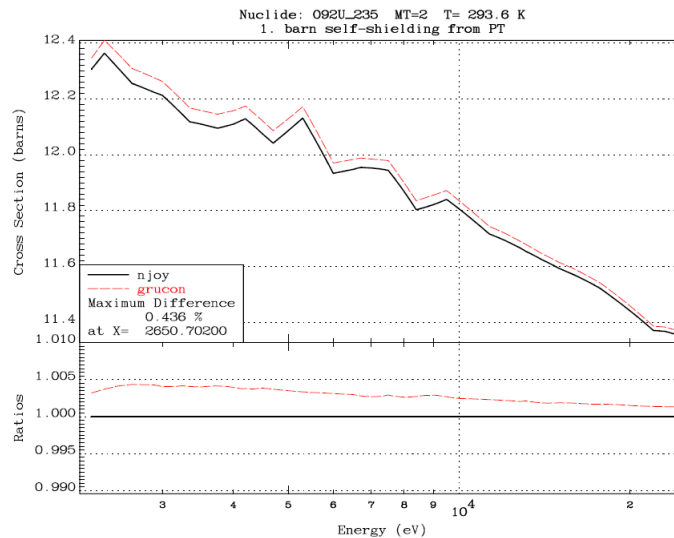
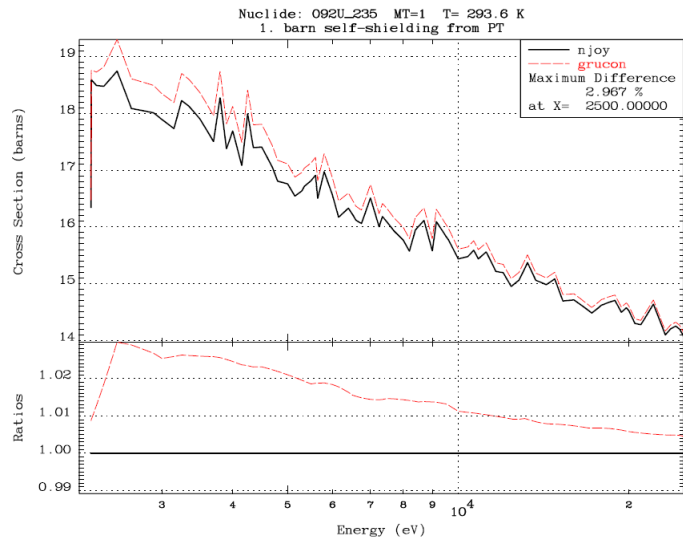


# CIELO/ENDF/ACE/Pu-239: GRUCON/NJOY comparison

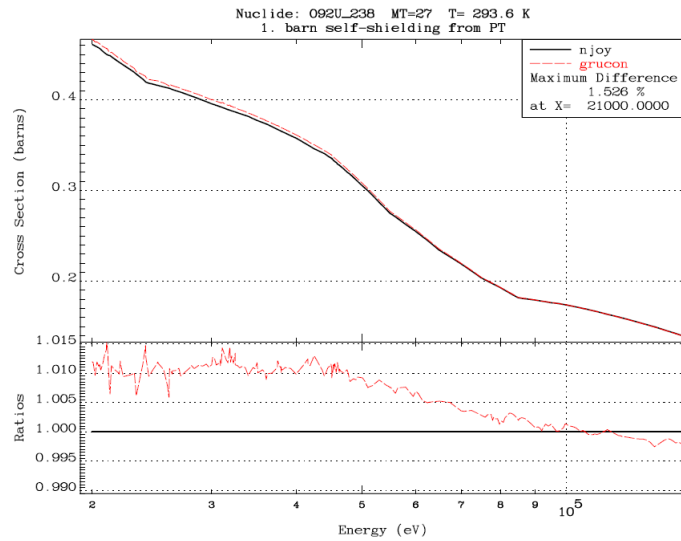
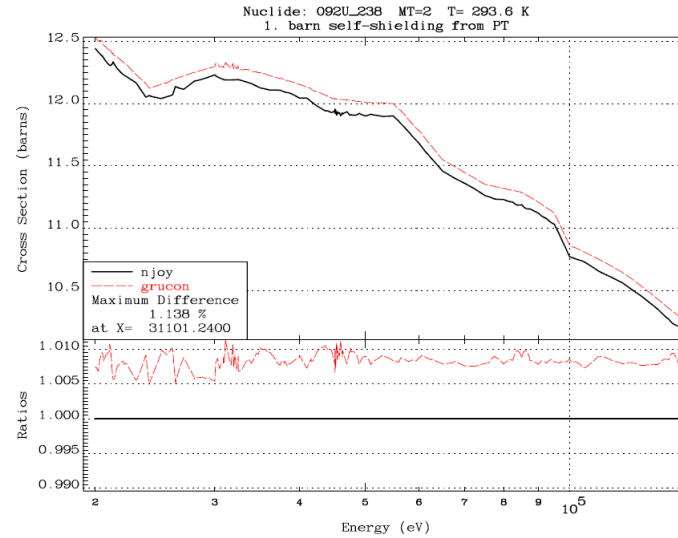
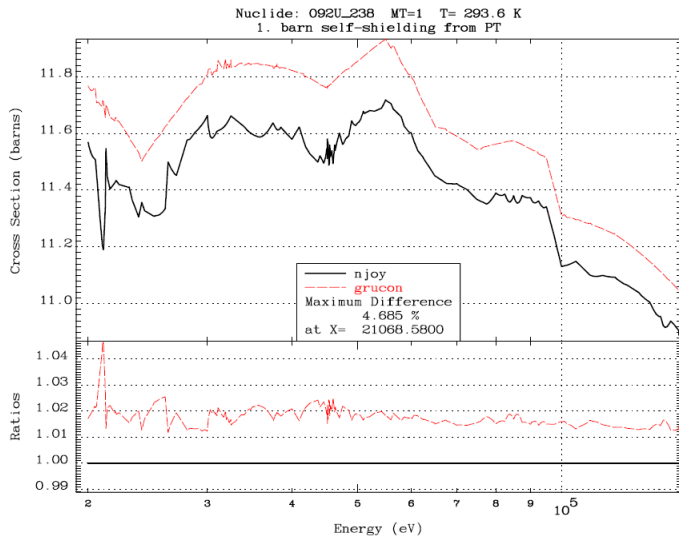


## **3.2 Comparison of Shielded (1b) Cross Sections Derived from ACE/LUNR Record**

# CIELO/ACE/U-235: GRUCON/NJOY comparison

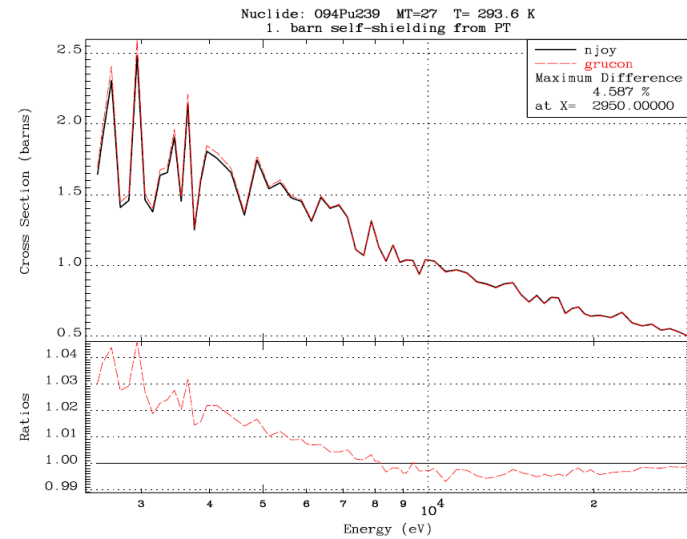
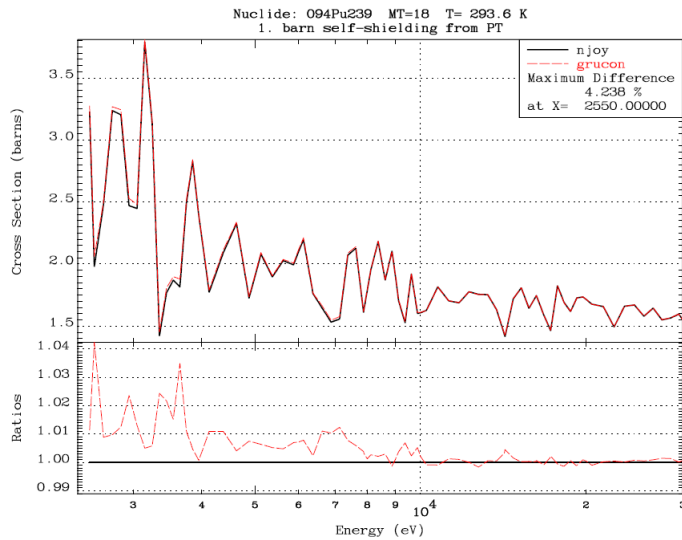
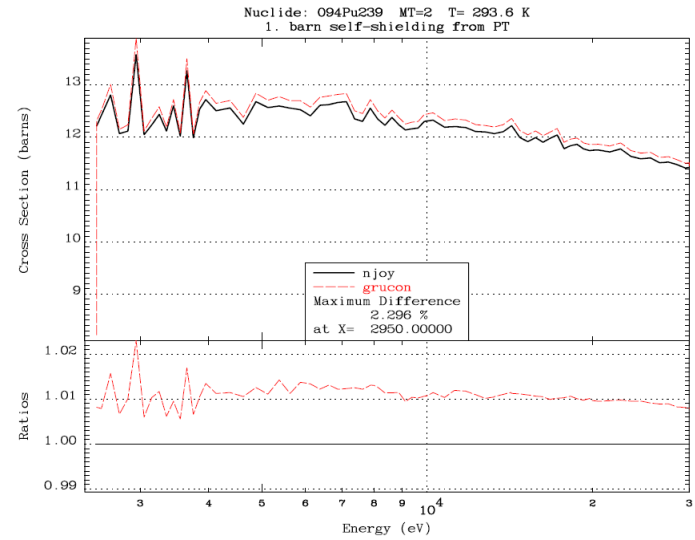
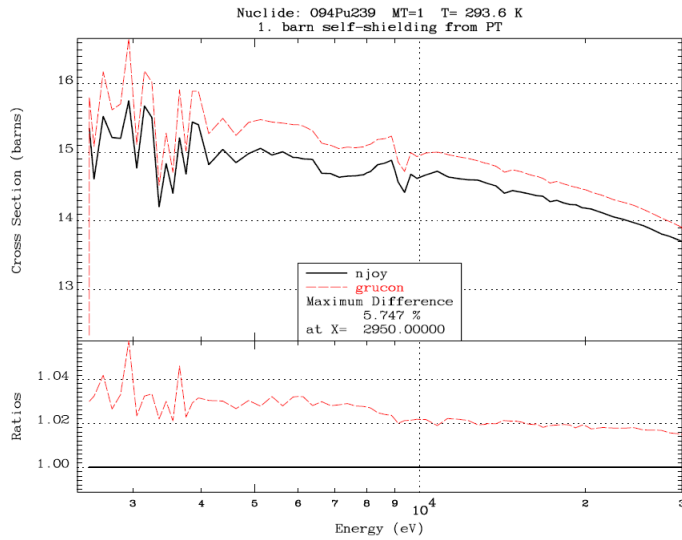


# CIELO/ACE/U-238: GRUCON/NJOY comparison





# CIELO/ENDF/ACE/Pu-239: GRUCON/NJOY comparison



# Summary

Infinitely diluted cross sections, derived from Probability Tables (NJOY) and Subgroup Parameters (GRUCON) reveal no essential disagreement. A small difference ( $\sim 0.47\%$ ), obtained in fission and absorption cross sections of U-235, is due to interpolation error the interpolation error of ACE/LUNR table, prepared by NJOY.

Effective cross sections at 1 barn dilution, in contrary, demonstrate essential differences (up to  $\sim 6\%$  in case of total cross section of Pu-239) , and require additional consideration.

## **4. Keff Calculations**

# NJOY/MCNP

N	Benchmark	Experiment	Error	Keff (PT)	Error	Keff (NOPT)	Error
0	scherzo -5.56	1.000	0.002	1.00367	0.00005	0.99466	0.00005
1	heu-met-fast-001	1.000	0.001	1.00026	0.00008	1.00012	0.00008
2	heu-met-fast-002	1.000	0.003	0.99988	0.00009	0.99976	0.00009
3	heu-met-fast-003_01	1.000	0.005	0.99322	0.00009	0.99362	0.00009
4	heu-met-fast-003_02	1.000	0.005	0.99239	0.00009	0.99291	0.00009
5	heu-met-fast-003_03	1.000	0.005	0.99718	0.00009	0.99762	0.00009
6	heu-met-fast-003_10	1.000	0.005	1.00453	0.00008	1.00455	0.00009
7	heu-met-fast-003_11	1.000	0.005	1.00923	0.00009	1.00916	0.00010
8	heu-met-fast-014	0.9989	0.0017	0.99549	0.00020	0.99632	0.00009
9	heu-met-fast-032_01	1.0000	0.0016	1.00212	0.00021	1.00282	0.00009
10	heu-met-fast-032_02	1.0000	0.0027	1.00250	0.00020	1.00337	0.00020
11	heu-met-fast-032_03	1.0000	0.0017	0.99837	0.00019	0.99889	0.00020
12	heu-met-fast-032_04	1.0000	0.0017	1.00003	0.00019	1.00029	0.00019
13	ieu-met-fast-007	1.0045	0.0007	1.00397	0.00008	1.00091	0.00007
14	ieu-met-fast-007d	1.0045	0.0007	1.00403	0.00008	1.00102	0.00007
15	ieu-met-fast-10	0.9954	0.0024	0.99575	0.00015	0.99213	0.00015
16	ieu-met-fast-013	0.9941	0.0023	0.99632	0.00016	0.99414	0.00016
17	ieu-met-fast-014-2	0.9958	0.0022	0.99668	0.00016	0.99519	0.00016
18	mix-misc-fast-001_01	0.9736	0.0071	0.96937	0.00012	0.96120	0.00012
19	mix-misc-fast-001_02	1.0050	0.0057	0.99717	0.00013	0.98888	0.00012
20	mix-misc-fast-001_03	0.9959	0.0059	0.98945	0.00014	0.98088	0.00013
21	mix-misc-fast-001_09	1.0188	0.0072	1.01146	0.00014	1.00711	0.00014
22	mix-misc-fast-001_10	0.9732	0.0064	0.97405	0.00013	0.97006	0.00013
23	mix-misc-fast-001_11	1.0153	0.0074	1.01514	0.00015	1.01368	0.00015
24	ieu-met-fast-022_1	1.00077	0.00134	1.00595	0.00008	1.00512	0.00008
25	ieu-met-fast-022_2	0.99325	0.00110	0.99696	0.00009	0.99664	0.00009
26	ieu-met-fast-022_3	0.98748	0.00110	0.98639	0.00011	0.98607	0.00011
27	ieu-met-fast-022_4	0.98629	0.00107	0.99041	0.00011	0.98996	0.00011
28	ieu-met-fast-022_5	0.99775	0.00123	1.00172	0.00011	1.00121	0.00011
29	ieu-met-fast-022_6	1.00121	0.00172	1.00330	0.00010	1.00258	0.00010
30	ieu-met-fast-022_7	0.99758	0.00126	1.00395	0.00011	1.00376	0.00011
31	ieu-met-fast-012	1.00070	0.0027	1.00255	0.00016	1.00030	0.00016
32	ieu-comp-fast-004	0.9978	0.0015	0.99846	0.00017	0.99758	0.00017

# GRUCON/MCNP

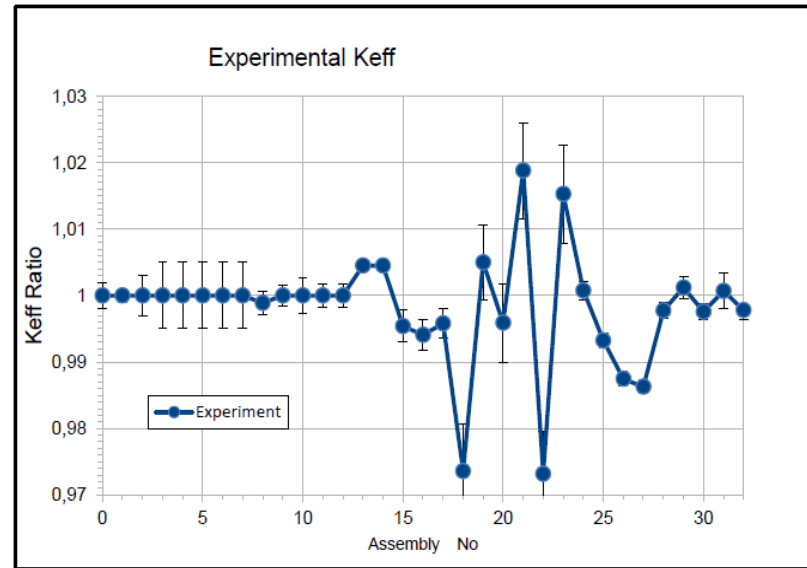
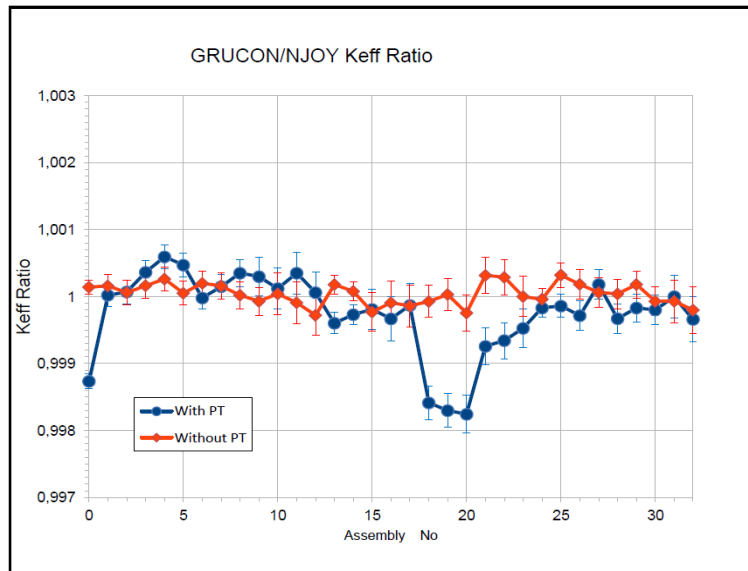
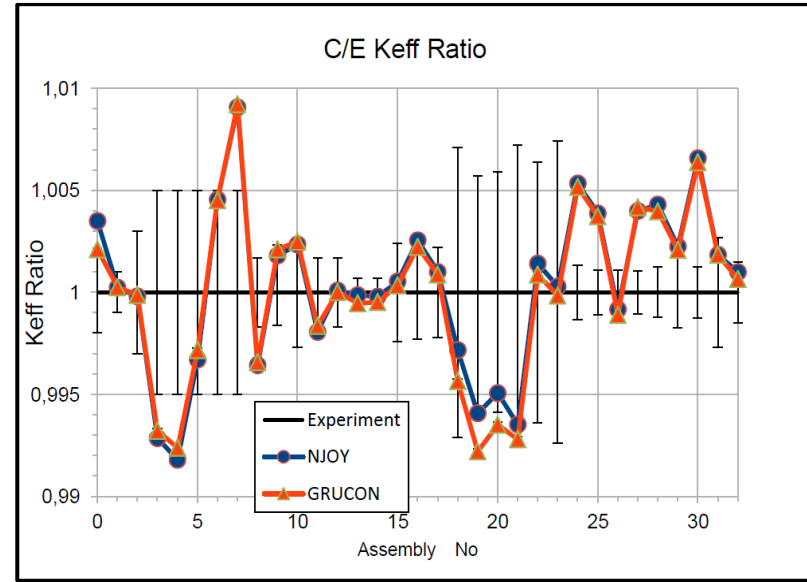
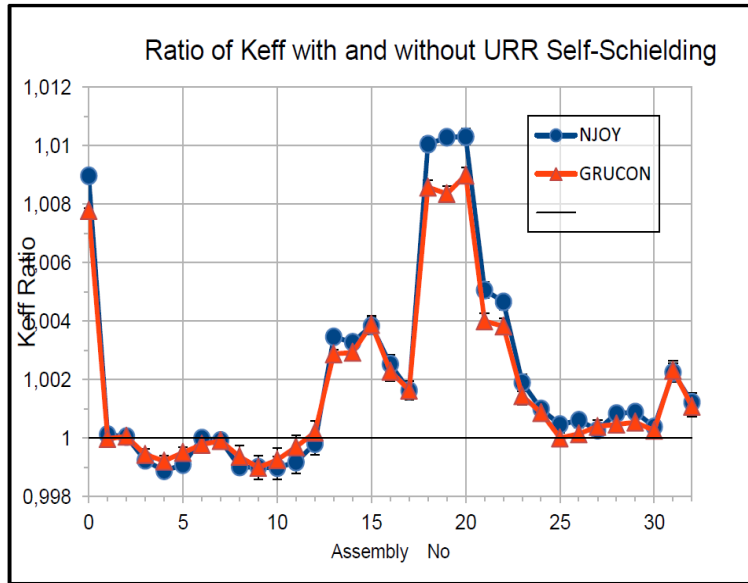
N	Benchmark	Experiment	Error	Keff (SP)	Error	Keff (NOSP)	Error
0	scherso -5.56	1.000	0.002	1.00240	0.00006	0.99480	0.00005
1	heu-met-fast-001	1.000	0.001	1.00026	0.00008	1.00028	0.00009
2	heu-met-fast-002	1.000	0.003	0.99988	0.00009	0.99982	0.00009
3	heu-met-fast-003_01	1.000	0.005	0.99322	0.00009	0.99378	0.00009
4	heu-met-fast-003_02	1.000	0.005	0.99239	0.00009	0.99317	0.00009
5	heu-met-fast-003_03	1.000	0.005	0.99718	0.00009	0.99767	0.00009
6	heu-met-fast-003_10	1.000	0.005	1.00453	0.00008	1.00475	0.00009
7	heu-met-fast-003_11	1.000	0.005	1.00923	0.00009	1.00932	0.00009
8	heu-met-fast-014	0.9989	0.0017	0.99549	0.0002	0.99612	0.00019
9	heu-met-fast-032_01	1.0000	0.0016	1.00212	0.00021	1.00313	0.0002
10	heu-met-fast-032_02	1.0000	0.0027	1.00232	0.00019	1.00325	0.0002
11	heu-met-fast-032_03	1.0000	0.0017	0.99847	0.0002	0.99867	0.0002
12	heu-met-fast-032_04	1.0000	0.0017	0.99983	0.00019	0.99983	0.00019
13	ieu-met-fast-007	1.0045	0.0007	1.00360	0.00008	1.00109	0.00007
14	ieu-met-fast-007d	1.0045	0.0007	1.00361	0.00008	1.00110	0.00007
15	ieu-met-fast-10	0.9954	0.0024	0.99490	0.00015	0.99190	0.00014
16	ieu-met-fast-013	0.9941	0.0023	0.99600	0.00017	0.99405	0.00016
17	ieu-met-fast-014-2	0.9958	0.0022	0.99661	0.00015	0.99505	0.00015
18	mix-misc-fast-001_01	0.9736	0.0071	0.96932	0.00013	0.96113	0.00012
19	mix-misc-fast-001_02	1.0050	0.0057	0.99735	0.00012	0.98891	0.00012
20	mix-misc-fast-001_03	0.9959	0.0059	0.98925	0.00014	0.98064	0.00014
21	mix-misc-fast-001_09	1.0188	0.0072	1.01146	0.00014	1.00743	0.00013
22	mix-misc-fast-001_10	0.9732	0.0064	0.97394	0.00014	0.97034	0.00013
23	mix-misc-fast-001_11	1.0153	0.0074	1.01512	0.00015	1.01368	0.00015
24	ieu-met-fast-022_1	1.00077	0.00134	1.00575	0.00008	1.00508	0.00008
25	ieu-met-fast-022_2	0.99325	0.00110	0.99637	0.00009	0.99696	0.00009
26	ieu-met-fast-022_3	0.98748	0.00110	0.98582	0.00011	0.98625	0.00011
27	ieu-met-fast-022_4	0.98629	0.00107	0.98971	0.00011	0.99002	0.00011
28	ieu-met-fast-022_5	0.99775	0.00123	1.00122	0.00010	1.00125	0.00011
29	ieu-met-fast-022_6	1.00121	0.00172	1.00206	0.00010	1.00276	0.0001
30	ieu-met-fast-022_7	0.99758	0.00126	1.00328	0.00011	1.00369	0.00011
31	ieu-met-fast-012	1.00070	0.0027	1.00165	0.00016	1.00023	0.00016
32	ieu-comp-fast-004	0.9978	0.0015	0.99796	0.00017	0.99738	0.00018

**SCHERZO-556 - imaged critical infinity composition  
of 5.56% <sup>235</sup>U and 94.44% <sup>238</sup>U.**

**Experiments in Pure Uranium Lattices with Unit Kinf Assemblies  
SNEAK-8/8Z; UK 1 and UK 5 in ERMINE and HARMONIE,  
Compiled by P. Chaudat, M. Darrouzet, E. A. Fischer, Rep.  
KFK 1865, CEA-R-4552 (1974)**

	<b>Experiment</b>	<b>NJOY (PT)</b>	<b>GRUCON (PT)</b>	<b>NJOY (C/E-1)%</b>	<b>GRUCON (C/E-1)%</b>
<b>Kinf</b>	<b>1.000 ± 0.2%</b>	<b>1.00367 (5)</b>	<b>1.00240(6)</b>	<b>0.367</b>	<b>0.240</b>
<b>F238/F235</b>	<b>0.0227 ± 1.3%</b>	<b>0.022122</b>	<b>0.022163</b>	<b>-2.54</b>	<b>-2.37</b>
<b>C238/F235</b>	<b>0.1154 ± 1.5%</b>	<b>0.113749</b>	<b>0.114105</b>	<b>-1.43</b>	<b>-1.12</b>

# Comparison of Keff Calculation Results



# Summary

Differences in of self-shielded cross sections obtained from Probability Tables (NJOY) and Subgroup Parameters (GRUCON) are manifested in the  $K_{eff}$  lowering on  $\sim 200$  pcm in cases when self-shielding effect in the URR plays an appreciable role ( $\sim 1\%$  in SCERZO-5.56 and MIX-MISC-FAST benchmarks).



# **Semi-analytical approach to calculation of Cross Section Moments in the URR (U/D-F,U/D-S modules)**

Sinitsa V.V., "Calculation of Self-Shielding Factors for Cross  
Sections in the Unresolved Resonance Region Using the  
GRUCON Applied Code Package,"  
INDC(CCP)-228/GV, Translated by the IAEA, November 1984

# Averaging on Resonance Location

Resonance, closest to given E point:  $|E_0 - E| \leq \frac{D}{2}$

$$\sigma(E, E_0) = \frac{4\pi}{k^2} g \frac{\Gamma_{0n}}{\Gamma_0} \frac{\cos 2\varphi - \left(2 \frac{E_0 - E}{\Gamma_0}\right) \sin 2\varphi}{1 + \left(2 \frac{E_0 - E}{\Gamma_0}\right)^2} + \sigma^B(E, E_0) + \sigma_p$$

$$\sigma_x(E, E_0) = \frac{4\pi}{k^2} g \frac{\Gamma_{0n}\Gamma_{0x}}{\Gamma_0^2} \frac{1}{1 + \left(2 \frac{E_0 - E}{\Gamma_0}\right)^2} + \sigma_x^B(E, E_0)$$

Contribution from other resonances:

$$\sigma^B(E, E_0) = \frac{4\pi}{k^2} g \sum_{\lambda \neq 0} \frac{\Gamma_{\lambda n}}{\Gamma_\lambda} \frac{\cos 2\varphi - \left(2 \frac{E_0 - E + \lambda D}{\Gamma_0}\right) \sin 2\varphi}{1 + \left(2 \frac{E_0 - E + \lambda D}{\Gamma_\lambda}\right)^2}$$

$$\sigma_x^B(E, E_0) = \frac{4\pi}{k^2} g \sum_{\lambda \neq 0} \frac{\Gamma_{\lambda n}\Gamma_{\lambda x}}{\Gamma_\lambda^2} \frac{1}{1 + \left(2 \frac{E_0 - E + \lambda D}{\Gamma_\lambda}\right)^2}$$

$$\Gamma_\lambda = \sum_x \Gamma_{\lambda x} ;$$

# Averaging on Resonance Location (continuation)

Setting  $\Gamma_{\lambda,x} = \Gamma_x, \quad \lambda \neq 0$

and averaging  $\langle (\dots) \rangle \equiv \frac{1}{D} \int_{E-\frac{D}{2}}^{E+\frac{D}{2}} dE_0 (\dots),$

we finally obtain for contribution of all resonances, except the closest one (with  $\lambda=0$ )

$$\langle \sigma^B (E) \rangle = \frac{4\pi}{k^2} g \frac{\Gamma_n}{D} \left( \frac{\pi}{2} - \operatorname{arctg} \frac{D}{\Gamma} \right) \cos 2 \varphi$$

$$\langle \sigma_x^B (E) \rangle = \frac{4\pi}{k^2} g \frac{\Gamma_n \Gamma_x}{\Gamma D} \left( \frac{\pi}{2} - \operatorname{arctg} \frac{D}{\Gamma} \right)$$

# Averaging on Parameter Distributions

For any  $x$  -reaction: 
$$\sigma_x \sim \frac{\Gamma_n \Gamma_x}{(\Gamma_n + \Gamma_x + \dots + \Gamma_z)^2} \sim \frac{x}{(x + \gamma)^2}$$

where  $x = \frac{\Gamma_x}{\bar{\Gamma}_x}$ , and  $\gamma$  includes all other reaction widths:  $\gamma = \frac{1}{\bar{\Gamma}_x} \sum_{y \neq x} \Gamma_y$

If  $x$  is distributed as

$$P_\nu(x) = \frac{\nu}{2\Gamma(\frac{\nu}{2})} \left(\frac{x\nu}{2}\right)^{\frac{\nu}{2}-1} e^{-\frac{x\nu}{2}}$$

the  $M_n$  moments can be expressed as

$$M_n = \int_0^\infty (x + \gamma)^n P(x) dx = \gamma^n \left(\frac{\nu\gamma}{2}\right)^{\frac{\nu}{2}} U\left(\frac{\nu}{2}, \frac{\nu}{2} + n + 1; \frac{\nu\gamma}{2}\right)$$

where  $U(a, b; z)$  - hyper geometrical function, defined from recurrent relation:

$$(b - a - 1)U(a, b - 1; z) + (1 - b - z)U(a, b; z) + zU(a, b + 1; z) = 0$$

with initial values:  $U(a, a + 1; z) = z^{-a}$

$$U(a, a; z) = e^z \Gamma(1 - a, z)$$

Setting  $\bar{\gamma}_x = \frac{1}{\bar{\Gamma}_x} \sum_{y \neq x} \bar{\Gamma}_y$ , the set of quadrature parameters  $\{a_i, x_i\}_{i=1}^{i=N}$

for  $x$  -reaction can be found as solution of equation

$$M_n = \sum_{i=1}^{i=2N} a_i (x_i + \bar{\gamma}_x)^n, \quad L - N \leq n \leq L + N - 1$$

# Averaging on Parameter Distributions (continuation)

The moments for each spin group , averaged on parameter distributions, are summation by quadrature formulas

$$\overline{\langle M_n (\sigma_x(E; T), \sigma(E; T)) \rangle} \cong \sum_{i_1}^{N_1} \dots \sum_{i_m}^{N_m} a_{i_1} \times \dots \times a_{i_m} \langle M_n (\sigma_x(E; T), \sigma(E; T) | x_{i_1}, \dots, x_{i_m}) \rangle$$

of the moments, averaged numerically on resonance location

$$\langle M(\sigma_x(E; T), \sigma(E; T) | x_1, \dots, x_m) \rangle = \frac{1}{D} \int_{E-\frac{D}{2}}^{E+\frac{D}{2}} dE_0 \langle M(\sigma_x(E, E_0; T), \sigma(E, E_0; T) | x_1, \dots, x_m) \rangle$$

for cross sections, defined by formulas

$$\sigma(E, E_0; T) = \frac{4\pi}{k^2} g \frac{\Gamma_{0n}}{\Gamma_0} (\Psi(x, \xi) \cos 2\varphi + X(x, \xi) \sin 2\varphi) + \overline{\langle \sigma^B(E) \rangle} + \sigma_p$$

$$\sigma_x(E, E_0; T) = \frac{4\pi}{k^2} g \frac{\Gamma_{0n} \Gamma_{0x}}{\Gamma_0^2} \Psi(x, \xi) + \overline{\langle \sigma_x^B(E) \rangle}$$

$$x = 2 \frac{E - E_0}{\Gamma_0}; \quad \xi = \frac{\Gamma_0}{\Delta}; \quad \Delta = 2 \sqrt{\frac{k_B T E}{M/m_n}}$$

# Moments convolution (F/C-F module)

Badikov S.A., Gai E.V., Rabotnov N.S., Sinitsa V.V.,  
“Use of Padé Approximation to calculate subgroup constants  
and to Include the Doppler Effect in Resonance Analysis of  
Neutron Cross Sections,”  
Soviet Atomic Energy, Volume 60, Issue 1 (1986) pp 35-43

Rineiski A.A., Sinitsa V.V.,  
“Extended Probability Tables for Approximation Multigroup  
Cross –Sections,”  
M&C, Gatlinburg, Tennessee, April 6-11,2003

# Type of Moments

## Rational Moments ( shielded cross sections)

$$S_n(\sigma | \sigma_0) = \overline{\langle (\sigma + \sigma_0)^n \rangle}$$

$$S_n(\sigma_x, \sigma | \sigma_0) = \overline{\langle \sigma_x (\sigma + \sigma_0)^n \rangle}$$

## Exponential Moments (transmission functions)

$$T_n(\sigma | t_0) = \overline{\langle e^{-\sigma t_0 n} \rangle}$$

$$T_n(\sigma_x, \sigma | t_0) = \overline{\langle \sigma_x e^{-\sigma t_0 n} \rangle}$$

### Allowable moment's parameters:

(a)  $\sigma_0 = \text{const}$ ,  $L - N \leq n \leq L + N - 1$

(b)  $n = -1$  (fixed), set of values  $\{\sigma_{0j}\}_{j=1}^{j=M}$

(c)  $t_0 = \text{const}$ ,  $0 \leq n \leq 2N - 1$

(d)  $n = 1$  (fixed)  $\{t_{0i}\}_{i=1}^M$

### Approximations:

( Gauss )

( Padé-II )

( Gauss )

(no approximation)

# Rational Moment Approximations

## Gauss quadrature approach

$$\sigma_0 = \text{const}, \quad L - N \leq n \leq L + N - 1$$

$$\langle (\sigma + \sigma_0)^n \rangle = \sum_{i=1}^N a_i (\sigma_i + \sigma_0)^n$$

$$\langle \sigma_x (\sigma + \sigma_0)^n \rangle = \sum_{i=1}^N (a\sigma_x)_i (\sigma_i^{(x)} + \sigma_0)^n$$

## Padé-II approximation approach

$$n = -1 \text{ (fixed)} \quad \{\sigma_{0j}\}_{j=1}^{j=M \geq 2N}$$

$$\langle 1/(\sigma + \sigma_{0j}) \rangle \cong \sum_{i=1}^N a_i / (\sigma_i + \sigma_{0j})$$

$$\langle \frac{\sigma_x}{\sigma + \sigma_{0j}} \rangle \cong \sum_{i=1}^N (a\sigma_x)_i / (\sigma_i^{(x)} + \sigma_{0j})$$



# Rational Moments Convolution

M – number of components

$$S_n(\sigma|\sigma_0) = \sum_{\vec{i}} a_{\vec{i}} S_n(\sigma_{\vec{i}}|\sigma_0) ; \quad a_{\vec{i}} \equiv \prod_{m=1}^M a_{i_m}$$

$$\sigma_{\vec{i}} \equiv \sum_{m=1}^M \sigma_{i_m}$$

$$\vec{i} \equiv (i_1, i_2, \dots, i_M)$$

$$i_m = 1, \dots, N_m$$

$$S_n(\sigma_x, \sigma|\sigma_0) = \sum_{m=1}^M \sum_{\vec{i}} a_{\vec{i}}^{(m)} S_n(\sigma_{\vec{i}}^{(m)}|\sigma_0) ; \quad a_{\vec{i}}^{(m)} \equiv (a\sigma_x)_{i_m} \prod_{k \neq m} a_{i_k}$$

$$\sigma_{\vec{i}}^{(m)} \equiv \sigma_{i_m}^{(x)} + \sum_{k \neq m} \sigma_{i_k}$$

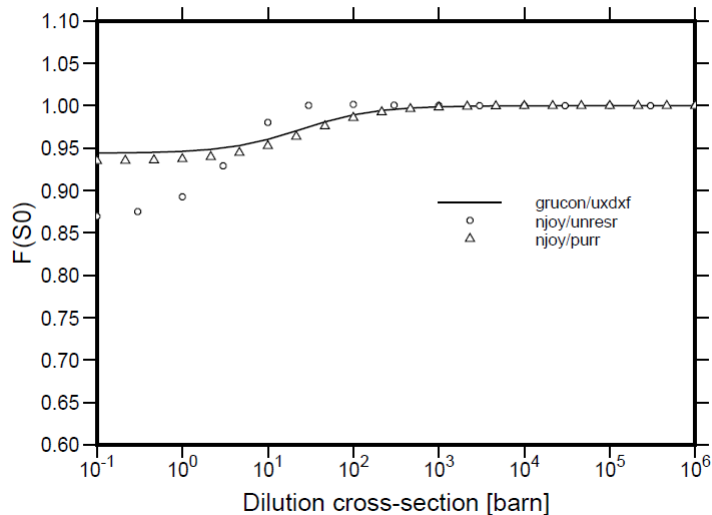
# Exponential Moments Convolution

$$T_n(\sigma|t) = \prod_{m=1}^M T_n(\sigma^{(m)}|t)$$

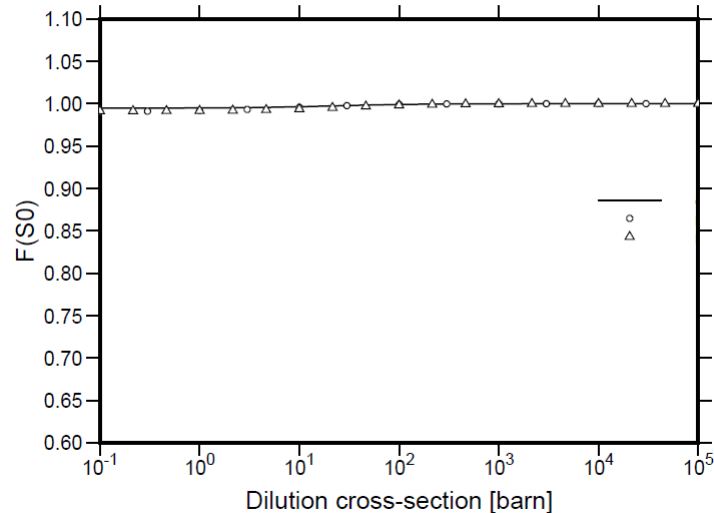
$$T_n(\sigma_x, \sigma|t) = \sum_{m=1}^M \left\{ T_n(\sigma_x^{(m)}, \sigma^{(m)}|t) \prod_{k \neq m} T_n(\sigma^{(k)}|t) \right\}$$

# U-235 URR SS Factors: GRUCON/NJOY comparison

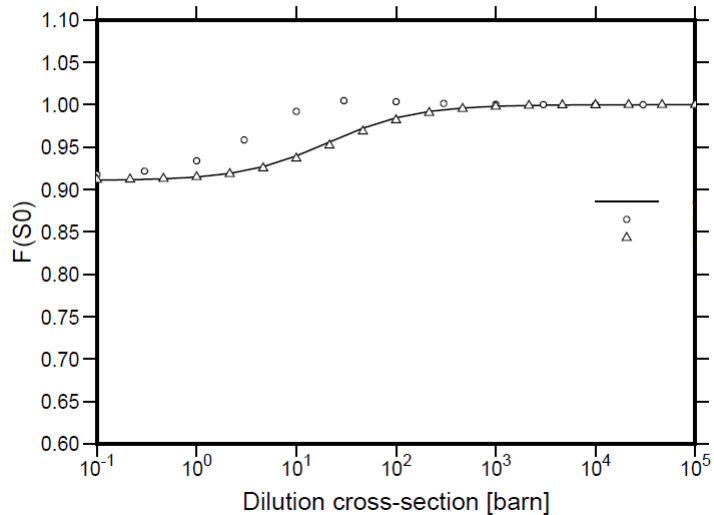
092-U -235 NG= 0 EG= 2.250+3 MT= 1 T= 293.6K  
Self-shielding factor



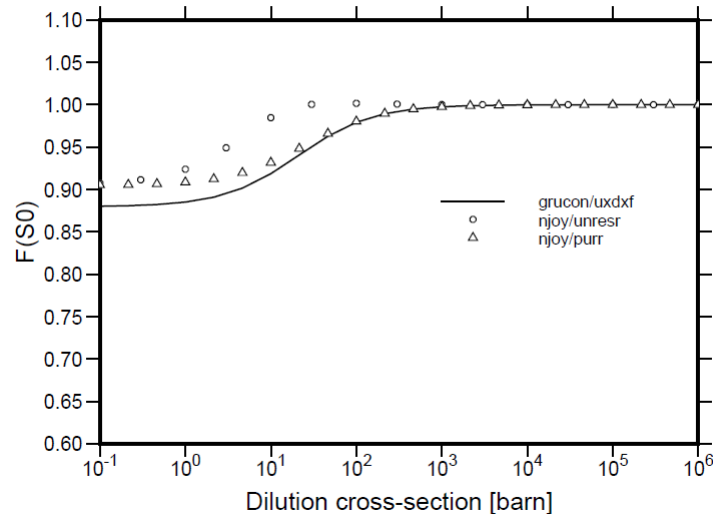
092-U -235 NG= 0 EG= 2.250+3 MT= 2 T= 293.6K  
Self-shielding factor



092-U -235 NG= 0 EG= 2.250+3 MT= 18 T= 293.6K  
Self-shielding factor

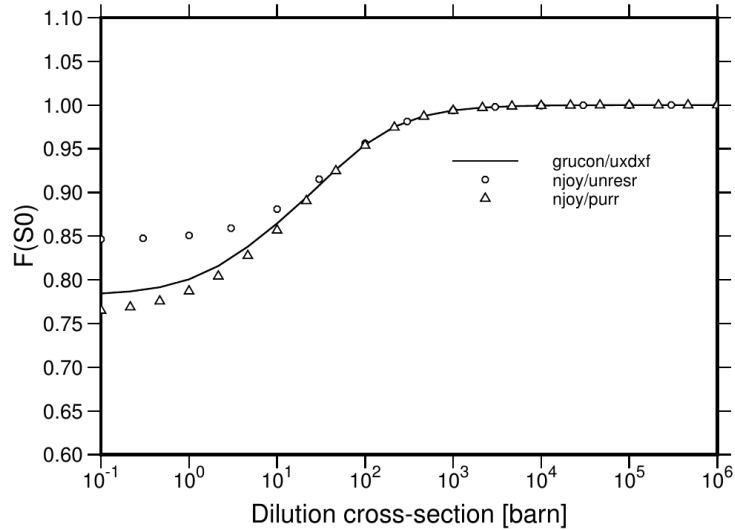


092-U -235 NG= 0 EG= 2.250+3 MT=102 T= 293.6K  
Self-shielding factor

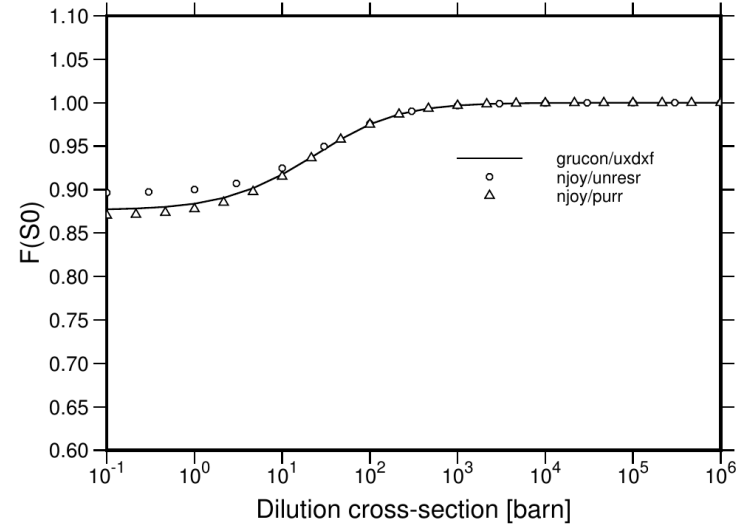


# U-238 URR SS Factors: GRUCON/NJOY comparison

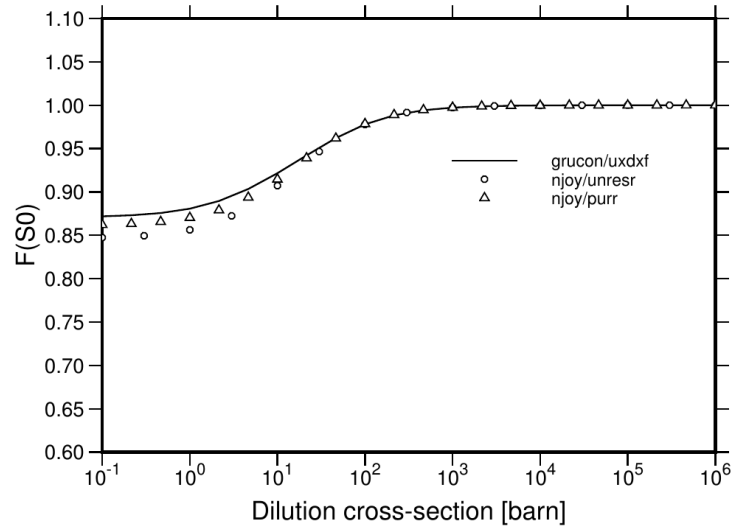
092-U -238 NG= 0 EG= 2.000+4 MT= 1 T= 293.6K  
Self-shielding factor



092-U -238 NG= 0 EG= 2.000+4 MT= 2 T= 293.6K  
Self-shielding factor

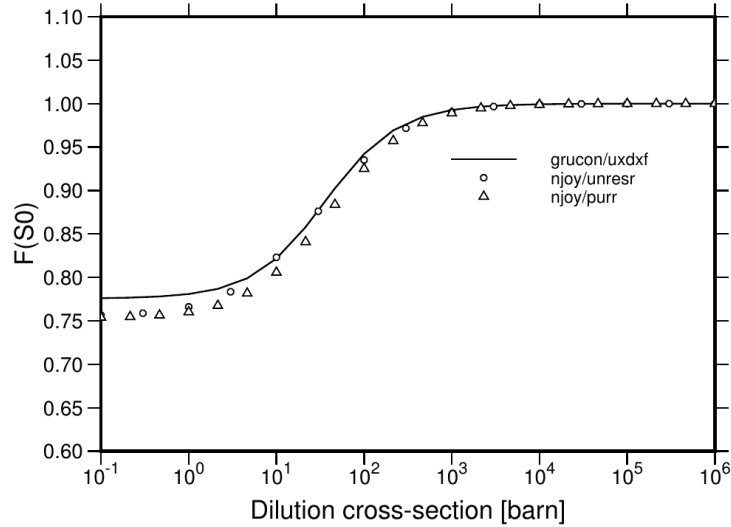


092-U -238 NG= 0 EG= 2.000+4 MT=102 T= 293.6K  
Self-shielding factor

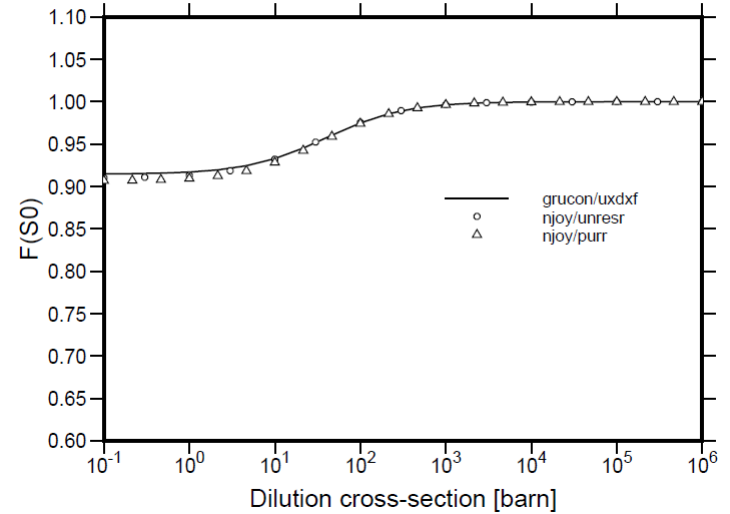


# Pu-239 URR SS Factors: GRUCON/NJOY comparison

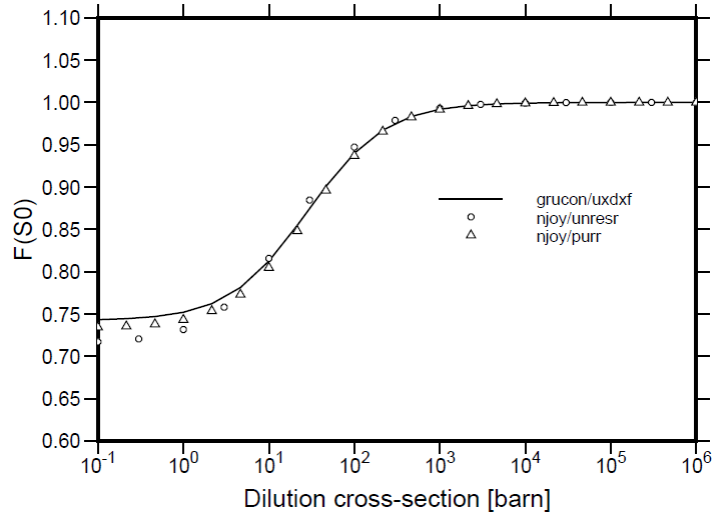
094-Pu-239 NG= 0 EG= 2.500+3 MT= 1 T= 293.6K  
Self-shielding factor



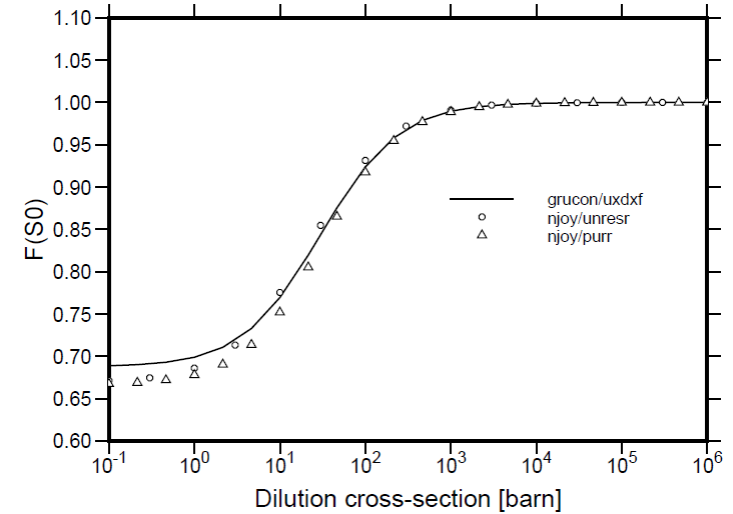
094-Pu-239 NG= 0 EG= 2.500+3 MT= 2 T= 293.6K  
Self-shielding factor



094-Pu-239 NG= 0 EG= 2.500+3 MT= 18 T= 293.6K  
Self-shielding factor



094-Pu-239 NG= 0 EG= 2.500+3 MT=102 T= 293.6K  
Self-shielding factor



**Probability Tables, Correlation  
Matrices and Subgroup Parameters  
(F/E-P, S/P-PN, PN/P-S, PN/PN-PC,  
PN/D-PC, P/C-P, P/D-F modules)**

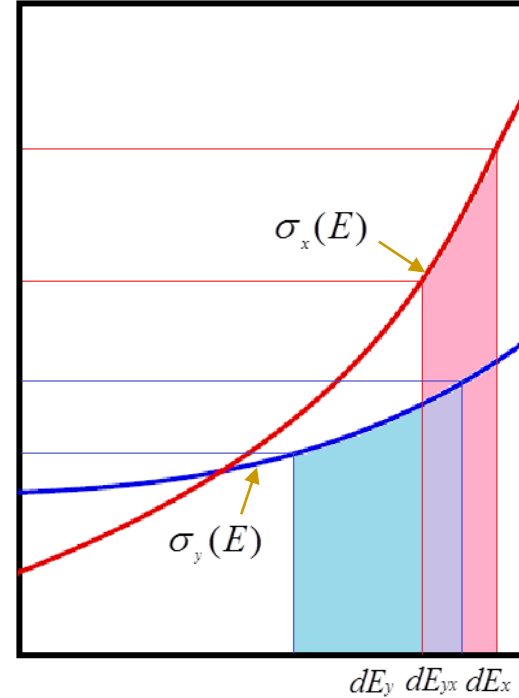
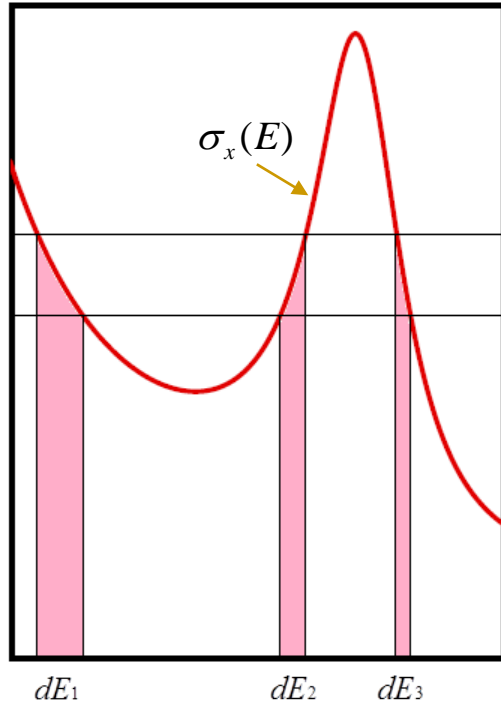
# Probability Tables

# Correlation Matrices

“x”, “y” = {reaction, temperature, nuclide}

$$P(\sigma_x) = \sum_i \left| \frac{dE_i}{d\sigma_x} \right|$$

$$P(\sigma_y | \sigma_x)$$



## Structural Parameters

$$P(\sigma_x) \sim \sum_{i=1}^{N_x} a_i^{(x)} \delta(\sigma_x - \sigma_i^{(x)})$$

$$P(\sigma_y | \sigma_x) \sim \sum_{j=1}^{N_y} \sum_{i=1}^{N_x} \omega_{j,i}^{(y,x)} a_i^{(x)} \delta(\sigma_x - \sigma_i^{(x)})$$

$$\sum_{i=1}^{N_x} a_i^{(x)} = 1, \quad a_i^{(x)} > 0$$

$$\sigma_{\min}^{(x)} \leq \sigma_i^{(x)} \leq \sigma_{\max}^{(x)}$$

## Physical constraints

$$\sum_{j=1}^{N_y} \sum_{i=1}^{N_x} \omega_{j,i}^{(y,x)} = 1, \quad \omega_{j,i}^{(y,x)} \geq 0$$

# Calculation of Subgroup Parameters

( Moments approximation with balance and normalization constraints )

Gauss approach

$$\langle (\sigma + \sigma_0)^n \rangle = \sum_{i=1}^N a_i (\sigma_i + \sigma_0)^n$$

$$\langle \sigma_x (\sigma + \sigma_0)^n \rangle = \sum_{i=1}^N a_i \sigma_{xi} (\sigma_i + \sigma_0)^n$$

$$\sum_{i=1}^N a_i = 1$$

$$\sum_x \sigma_{xi} = \sigma_i$$

*Padé-II* approximation

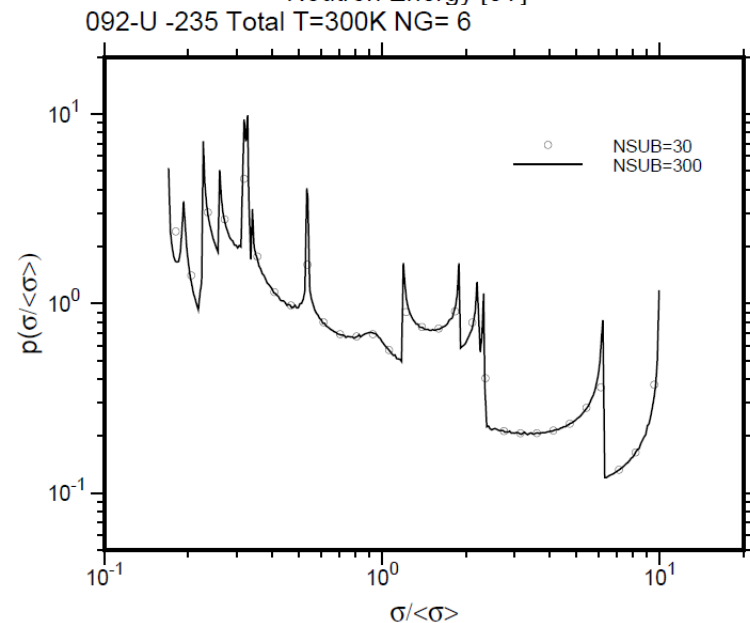
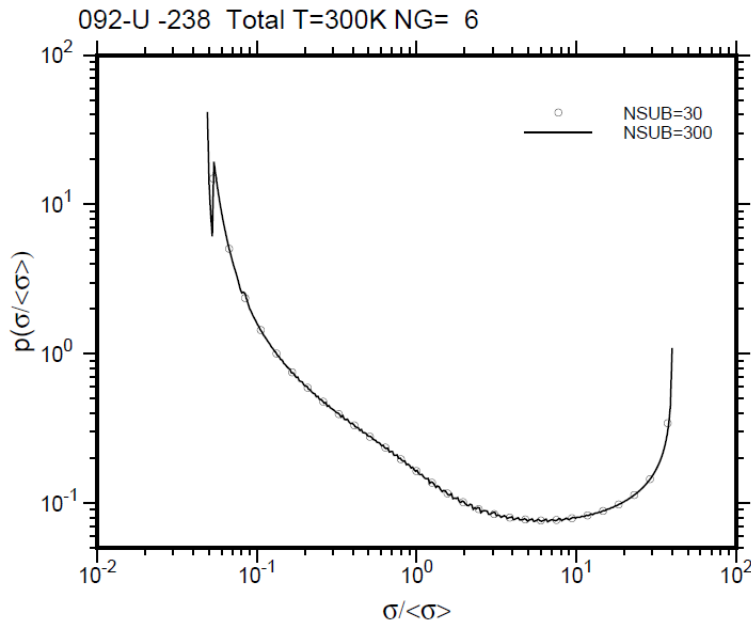
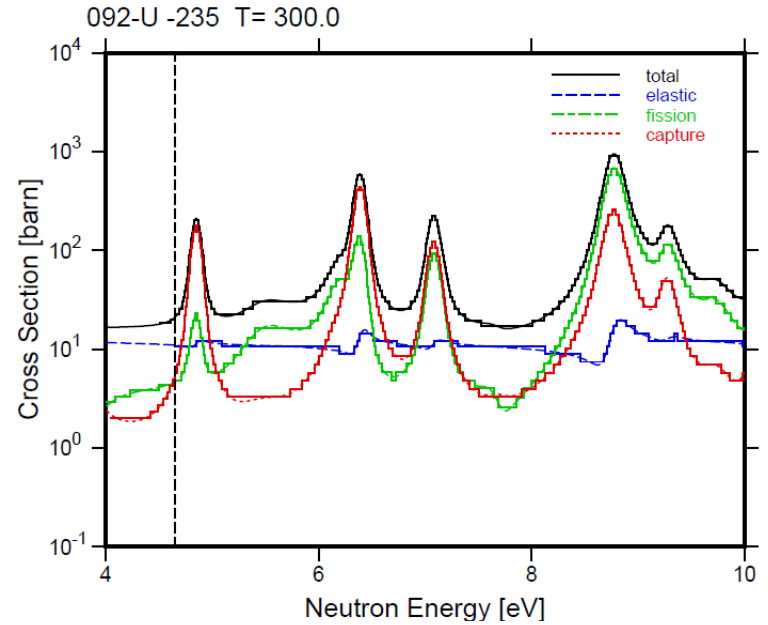
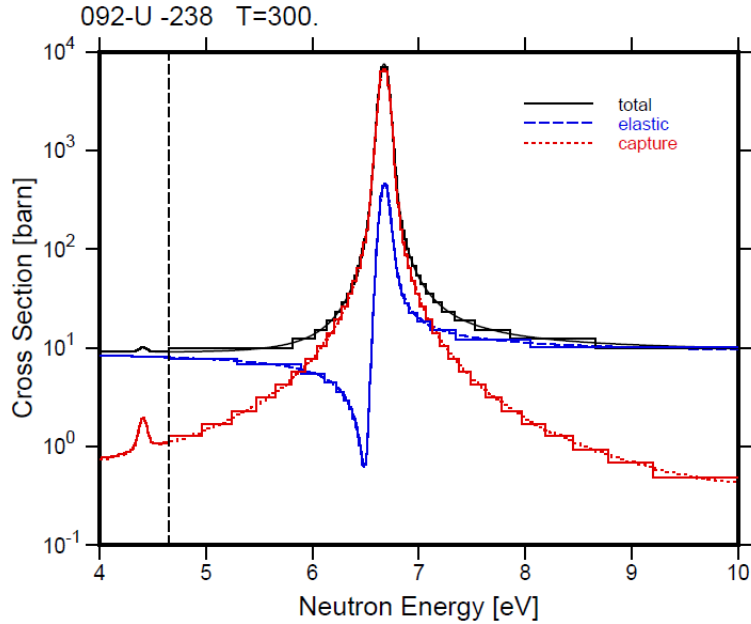
$$\langle 1/(\sigma + \sigma_{0j}) \rangle \cong \sum_{i=1}^N a_i / (\sigma_i + \sigma_{0j})$$

$$\langle \frac{\sigma_x}{\sigma + \sigma_{0j}} \rangle \cong \sum_{i=1}^N a_i \sigma_{xi} / (\sigma_i + \sigma_{0j})$$

$$\sum_{i=1}^N a_i \sigma_i = \langle \sigma \rangle$$

$$\sum_{i=1}^N a_i \sigma_{xi} = \langle \sigma_x \rangle$$

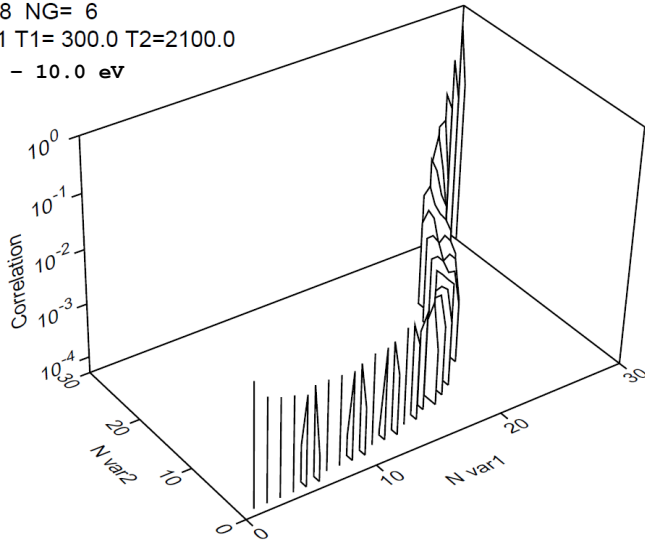
# Calculation of Probability Tables



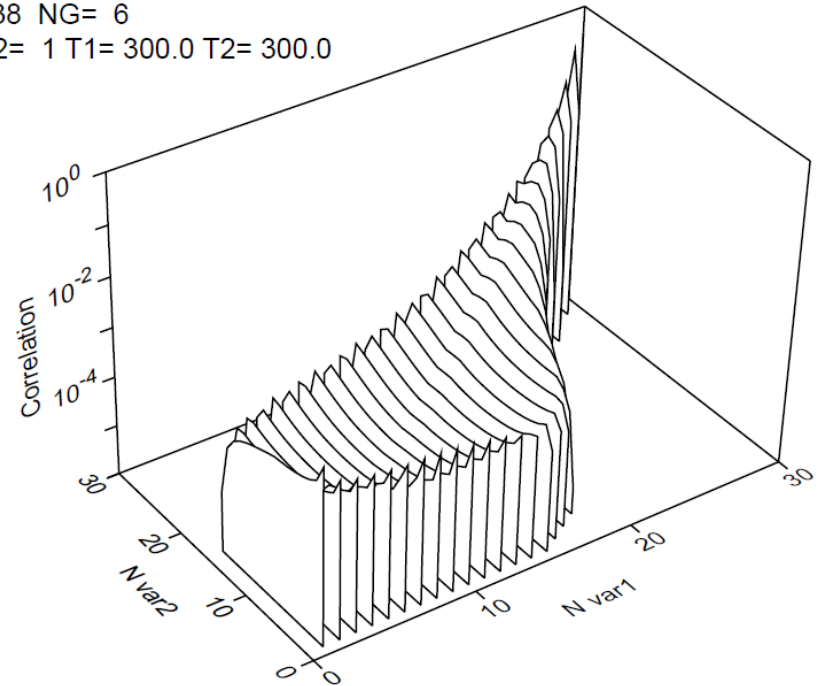


# Correlation Matrices

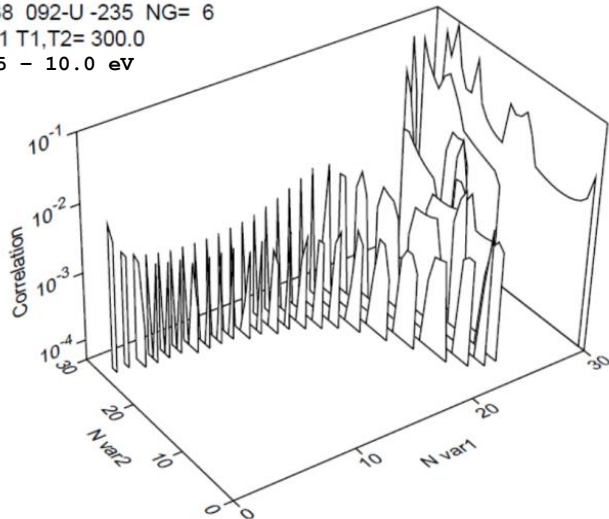
092-U -238 NG= 6  
Is1=Is2= 1 T1= 300.0 T2=2100.0  
EG= 4.65 - 10.0 eV



092-U -238 NG= 6  
Is1= 1 Is2= 1 T1= 300.0 T2= 300.0

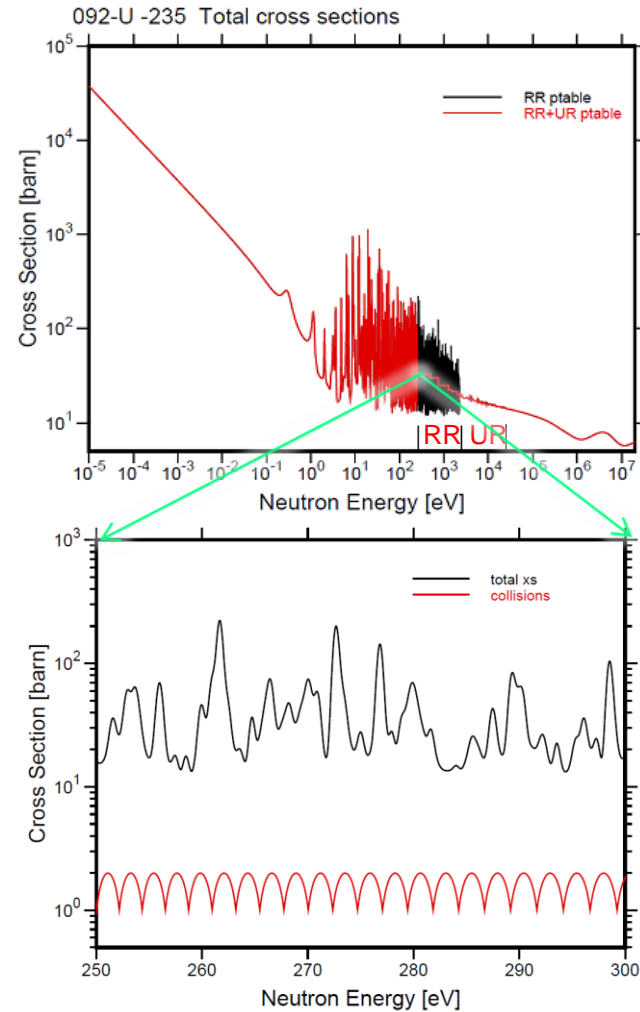
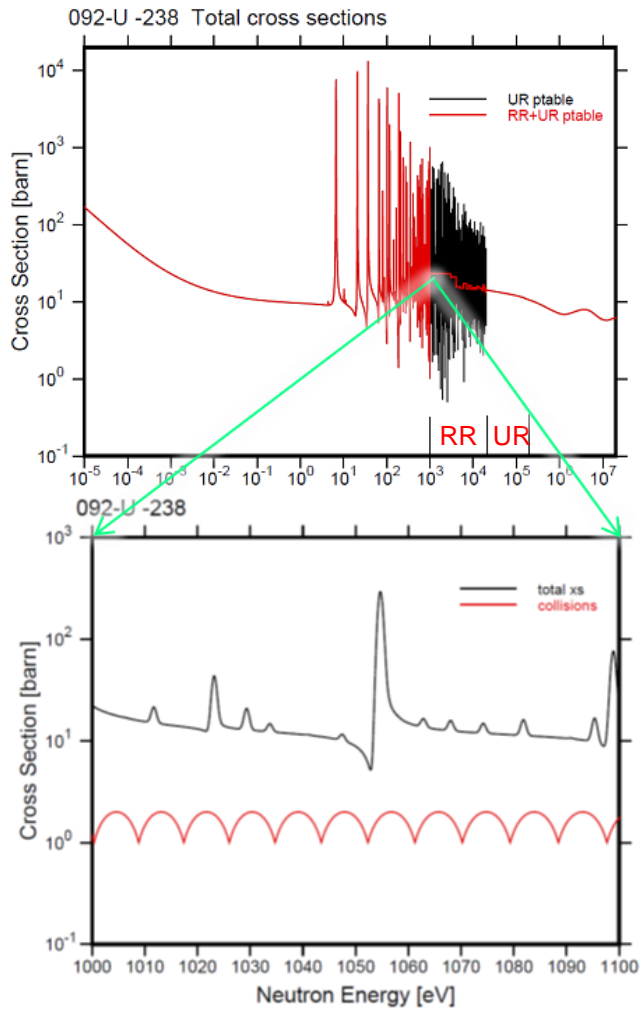


092-U -238 092-U -235 NG= 6  
Is1=Is2= 1 T1,T2= 300.0  
EG= 4.65 - 10.0 eV



# **Appendix A. ACE files with Combined Probability Tables and Subgroup Parameters**

# ACE files with Combined Probability Tables

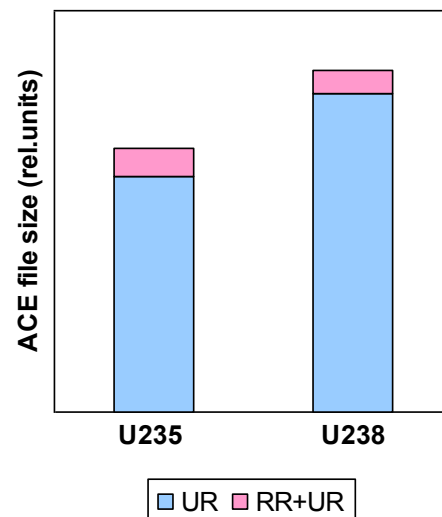


## Case A: subgroup parameters in URR

## Case B: subgroup parameters in RRR & URR

### Benchmark:

SCHERZO-556 - imaged critical infinity  
composition of 5.56%  $^{235}\text{U}$  and 94.44%  $^{238}\text{U}$



	Experiment	A: UR	$(C_A/E-1)\%$	B: RR+UR	$(C_B/C_A-1)\%$
Kinf	1.000 ± 0.2%	1.0008±(1)	0.08	1.0009±(1)	0.01
F238/F235	0.0227 ± 1.3%	0.02234±(1)	-1.60	0.02233±(1)	-0.04
C238/F235	0.1154 ± 1.5%	0.11552±(3)	0.10	0.11552±(3)	0.00

**Appendix B. ACE Files with Angular Distributions  
Reconstructed from Resonance Parameters  
(R/T-S, S/L-A, A/E-A modules)**

## Blatt-Biedenharn formalism:

$$d\sigma_{\alpha\alpha'} / d\Omega = \frac{\lambda_\alpha^2}{(2i+1)(2I+1)} \sum_{s,s'} \sum_{L=0}^{\infty} B_L(\alpha s, \alpha' s') P_L(\cos \theta)$$

$$B_L(\alpha s, \alpha' s') = \frac{(-1)^{s-s'}}{4} \sum_{J_1, J_2} \sum_{l_1, l_2} \sum_{l'_1, l'_2} \bar{Z}(l_1 J_1 l_2 J_2 | s L) \bar{Z}(l'_1 J'_1 l'_2 J'_2 | s' L)$$

$$\times \left( \delta_{\alpha\alpha'} \delta_{l_1 l'_1} \delta_{ss'} - U_{\alpha l_1 s, \alpha' l'_1 s'}^{J_1} \right) \left( \delta_{\alpha\alpha'} \delta_{l_2 l'_2} \delta_{ss'} - U_{\alpha l_2 s, \alpha' l'_2 s'}^{J_2} \right)$$

$$\bar{Z}(l_1 J_1 l_2 J_2 | s L) = \sqrt{(2l_1+1)(2l_2+1)(2J_1+1)(2J_2+1)} (l_1 l_2 00 | L 0) W(l_1 J_1 l_2 J_2 | s L)$$

Where  $\bar{Z}$  is a Blatt-Biedenharn coefficients,

$W(l_1 J_1 l_2 J_2 | s L)$  is a Racah coefficients,

$(l_1 l_2 00 | L 0)$  is a Clebsh-Gordan coefficients of angular momentum coupling,

and  $U_{\alpha l_1 s, \alpha' l'_1 s'}^{J_1}$  is a collision function.

# Collision Matrix Calculation

**Breight-Wigner approximation:**

$$U_{cc'} = e^{-i(\varphi+\varphi')} \left( \delta_{cc'} + i \sum_{\lambda} \frac{\Gamma_{\lambda c}^{\frac{1}{2}} \omega_{\lambda cc'} \Gamma_{\lambda c'}^{\frac{1}{2}}}{E_{\lambda} + \Delta_{\lambda} - E - \frac{i\Gamma_{\lambda}}{2}} \right)$$

with

$\omega_{\lambda cc'} = 1$  (SLBW case) or

$$\omega_{\lambda cc'} = 1 + \sum_{\mu \neq \lambda} \left( \frac{\gamma_{\mu c}}{\gamma_{\lambda c}} + \frac{\gamma_{\mu c'}}{\gamma_{\lambda c'}} \right) \frac{\sum_{c''} \gamma_{\lambda c''} L_{c''}^0 \gamma_{\mu c''}}{E_{\mu} + \Delta_{\mu} - E_{\lambda} - \Delta_{\lambda} - i(\Gamma_{\mu} - \Gamma_{\lambda})/2} \quad (\text{MLBW case})$$

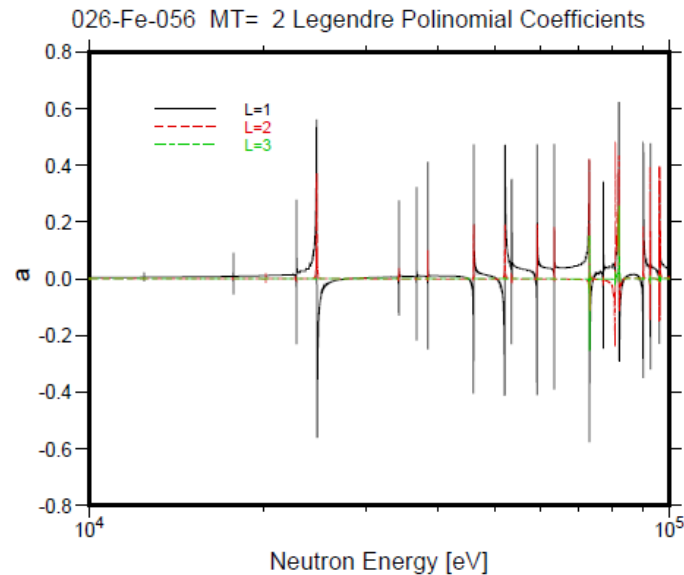
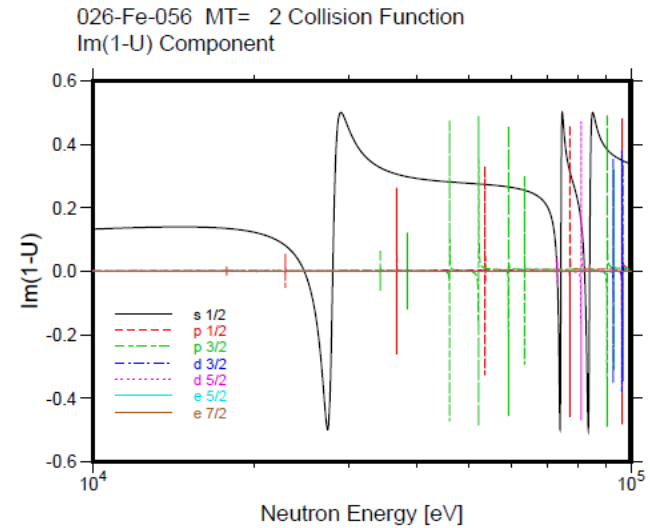
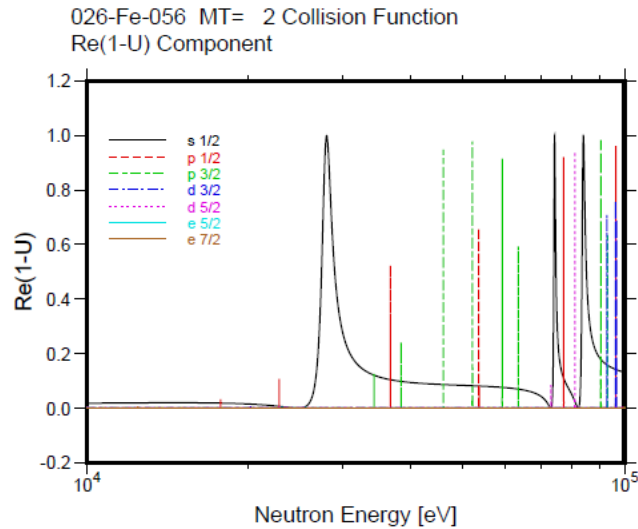
**Reich-Moore approximation:**

$$U_l = e^{-i2\varphi_l} \left( 1 + \frac{2iR_l P_l}{1 - R_l L_l^0} \right)$$

with  $R_l$  defined as

$$R_l \equiv \sum_{\lambda} \frac{\gamma_{\lambda l}^2}{E_{\lambda} - E - \frac{i\Gamma_{\lambda}}{2}}$$

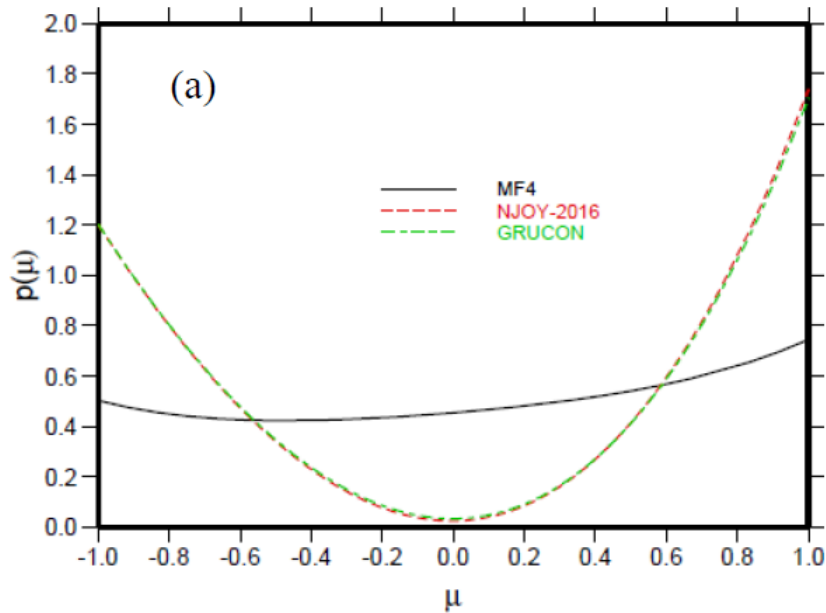
# Collision function and Legendre polynomial coefficients



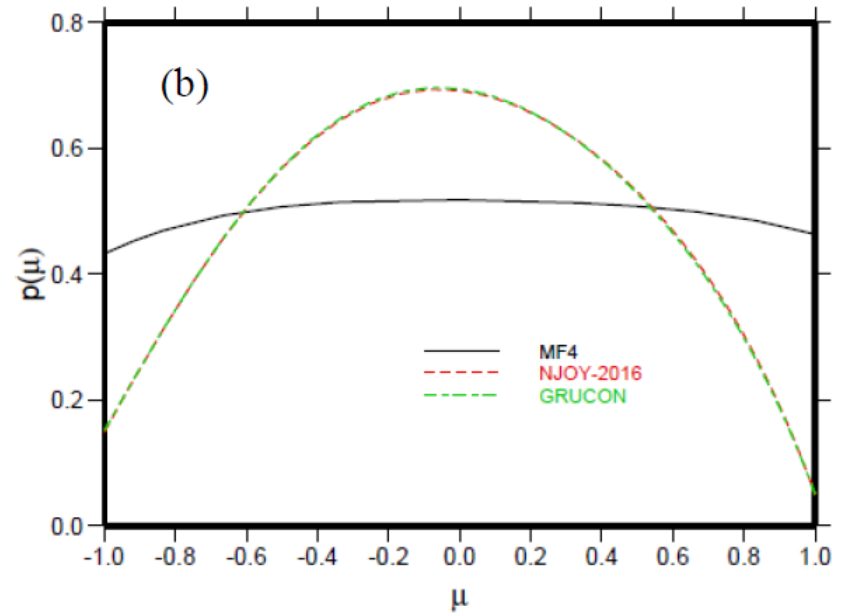


# Comparison with NJOY and File Data

026-Fe-056 MT= 2 En= 3.000000+5  
Angular Distribution

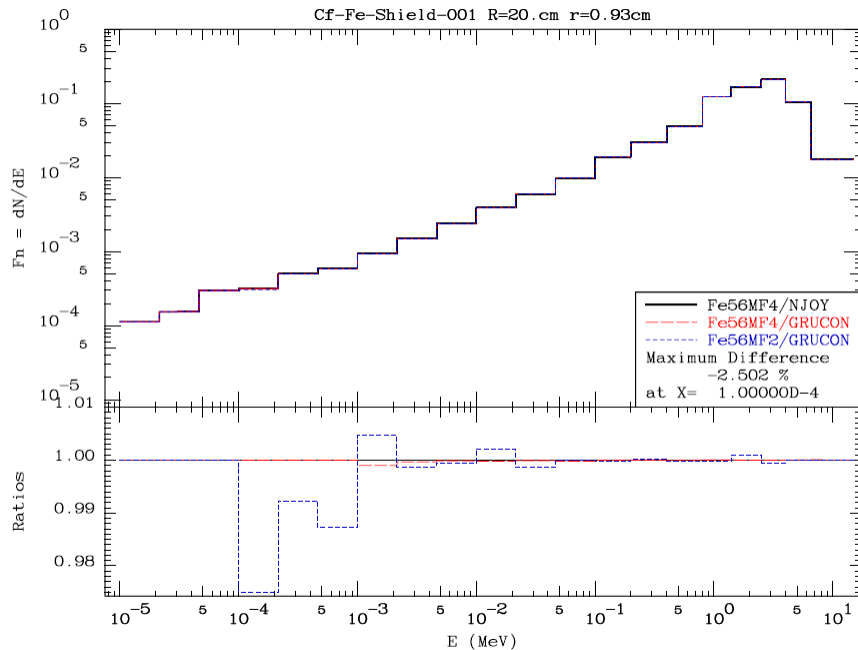


026-Fe-056 MT= 51 En= 2.000000+6  
Angular Distribution

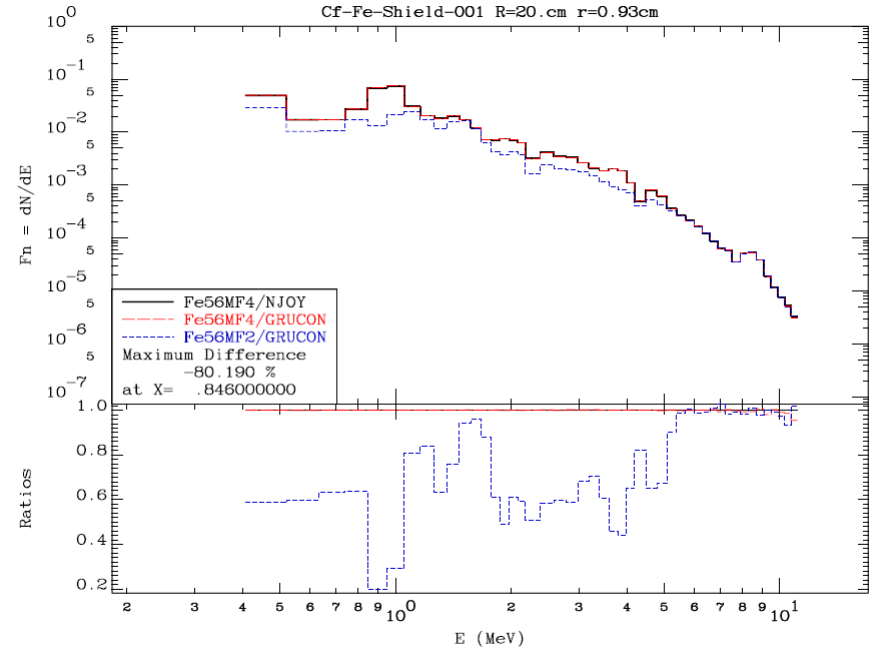


# Leakage from Iron Shell with Cf-252 Source (R=20cm r=0.93cm)

## Neutrons



## Photons



**MF4 – angular distributions are taken from ENDF file**

**MF2 – angular distributions are restored from resonance parameters**

**Appendix C. Thermal ACE Files  
with Resonance Scattering Data  
(TH/-DS, S/T-DS module)**

# Neutron Scattering on Free Nuclei

Free gas approximation ( Wigner-Wilkins - scattering cross section is constant ) :

$$\sigma_s^T(E \rightarrow E', \vec{\Omega} \rightarrow \vec{\Omega}') = \frac{\sigma_s}{v} \int_{all V: v_r > 0} v_r P(v, V \rightarrow E', \vec{\Omega}') M^T(V) dV$$

Scattering on resonances ( Ouisloumen - Sanchez ) :

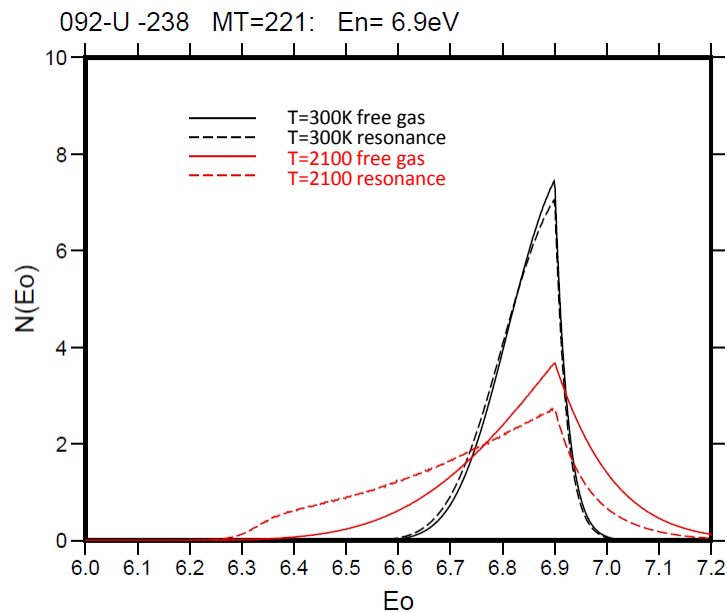
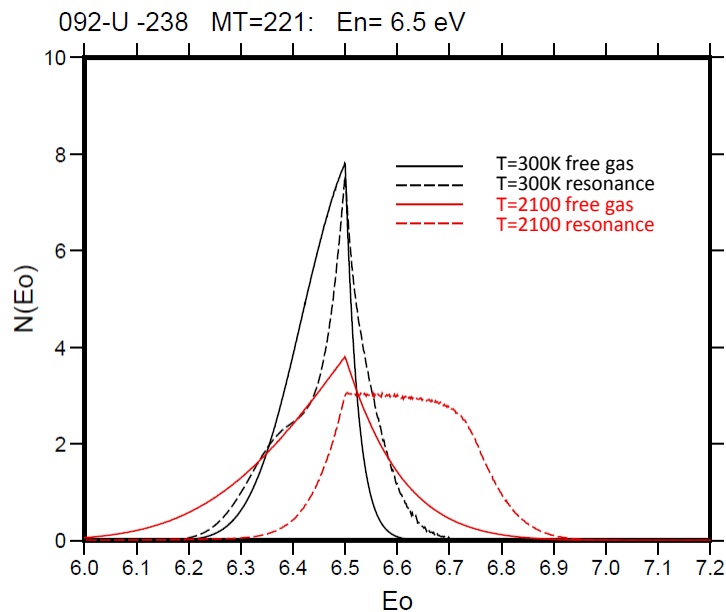
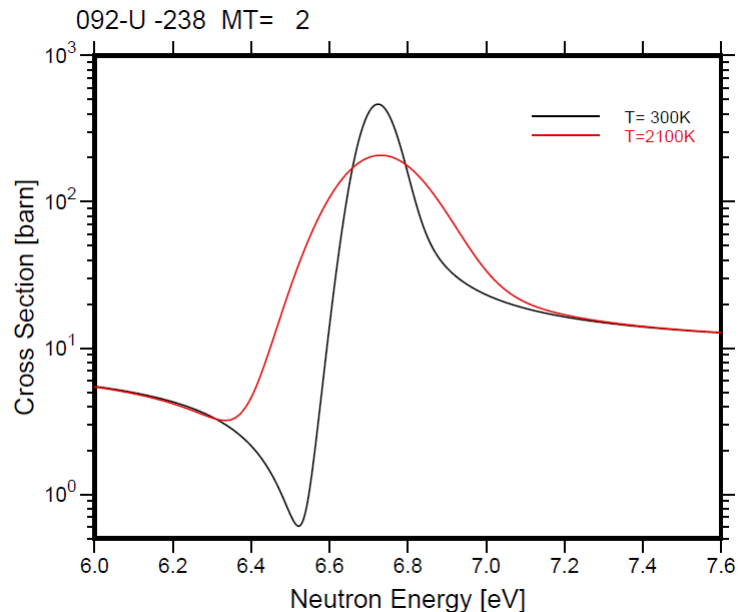
$$\sigma_s^T(E \rightarrow E', \vec{\Omega} \rightarrow \vec{\Omega}') = \frac{1}{v} \int_{all V: v_r > 0} v_r \sigma_s(v_r, 0) P(v, V \rightarrow E', \vec{\Omega}') M^T(V) dV$$

Where

$$P(V, T) = \frac{4}{\sqrt{\pi}} \beta^{3/2} V^2 e^{-\beta V^2} \quad - \quad \text{Maxwell-Boltzmann distribution}$$

# Scattering on the U-238 Resonance

Neutron spectra on the left ( $E_n=6.5\text{eV}$ ) and the right ( $E_n=6.9\text{eV}$ ) wings of U-238 resonance, calculated in free gas and resonance scattering models



# ACE Files for Thermal Neutrons with Resonance Scattering Data

## Mosteller Benchmark Specification

**Task** – calculate Doppler coefficient of reactivity

$$DC = \frac{\Delta\rho_{Dop}}{\Delta T_{Fuel}}$$

where  $\Delta\rho_{Dop} = \frac{k_{HFP} - k_{HZP}}{k_{HFP} \times k_{HZP}}$  - Doppler defect

$\Delta T_{Fuel}$  - difference of fuel temperatures for conditions:

HZP – hot zero power (600K)

HFP – hot full power (900K)

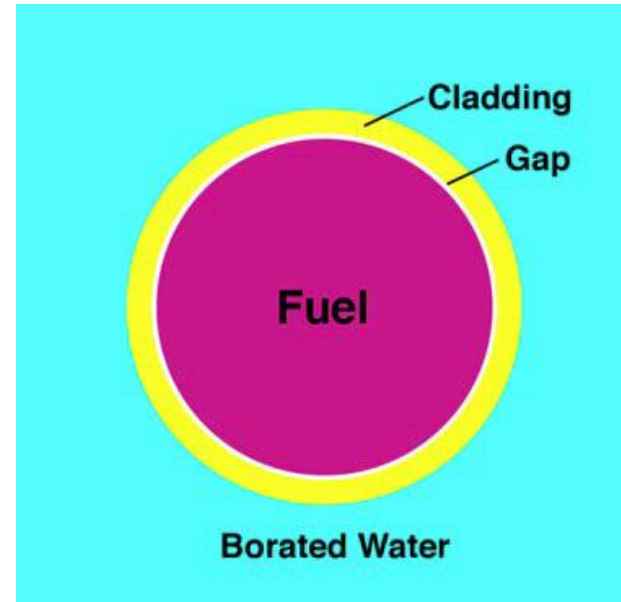
### Subsets of benchmarks:

- UO<sub>2</sub>, fuel, ranging from normal to 5 wt. % enriched uranium
- Reactor–Recycled MOX (UO<sub>2</sub>+PuO<sub>2</sub>), PuO<sub>2</sub> from 1 wt. % to 8 wt. %
- Weapons-Grade MOX (UO<sub>2</sub>+PuO<sub>2</sub>), PuO<sub>2</sub> from 1 wt. % to 6 wt. %

**Moderator:** Water + B (1400 ppm) , T=600K

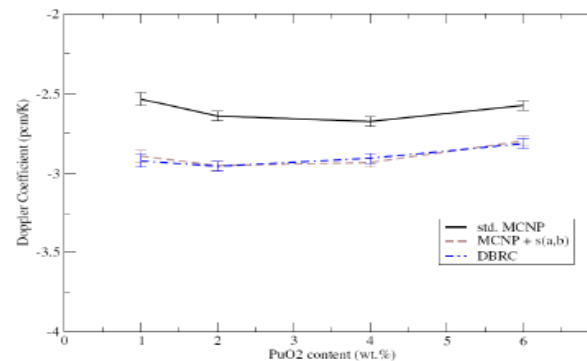
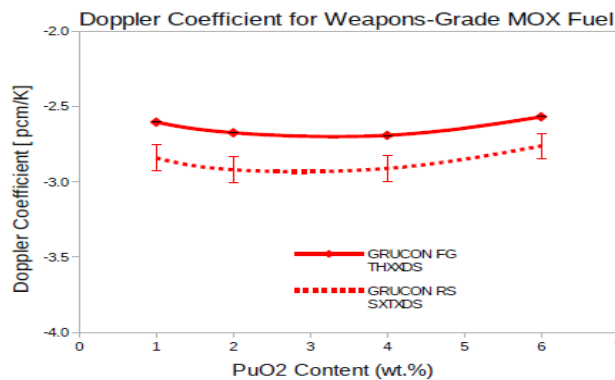
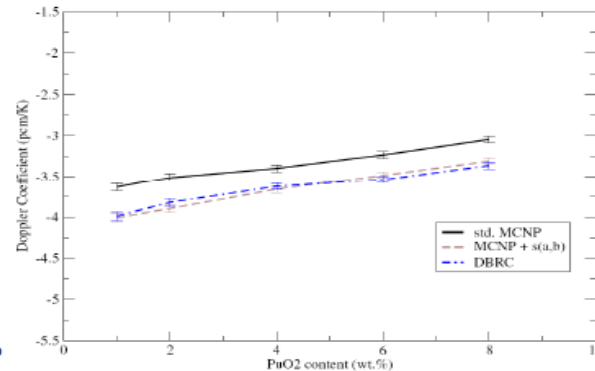
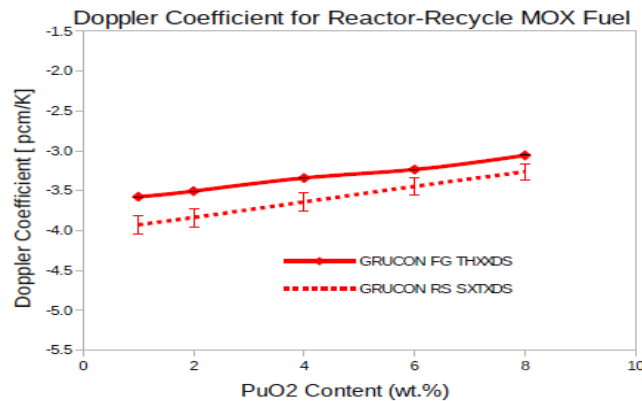
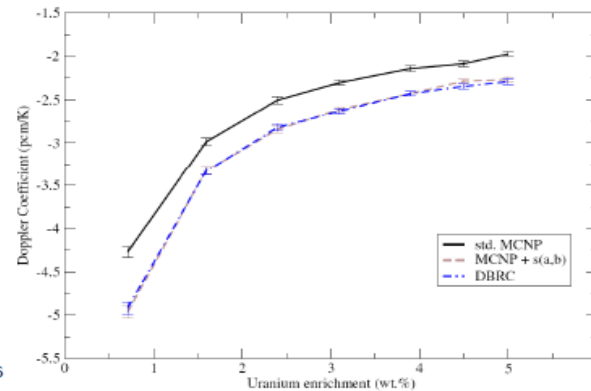
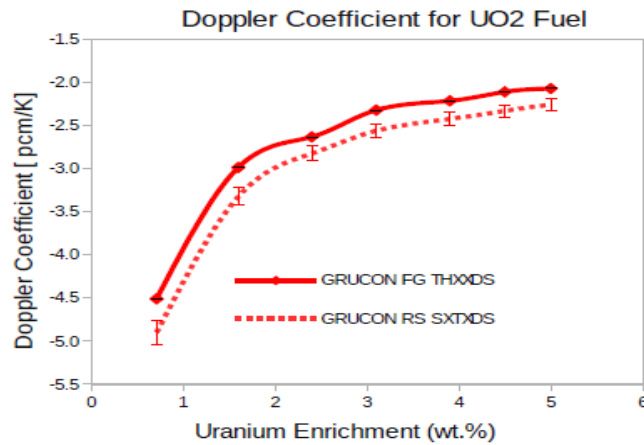
**Cladding:** Zr, T=600K

## Schematic of the Geometry



# Resonance Scattering Effect

B.Becker, R.Dagan,  
C.H.M.Broeders, G.Lohnert,  
"An Alternative Stochastic  
Doppler Broadening  
Algorithm", M&C, Saratoga  
Springs, New York, May 3-7,  
2009



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**Thank You for Attention!**