**Nuclear Data Section**

**International Atomic Energy Agency**

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**Memo CP-D/1129**

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**To:** Distribution

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**Subject: Compilation of parameter covariances and sensitivity coefficients**

When the number of the data points is huge (e.g., 1000) but their uncertainties originate from counts and a few (e.g., 10) parameters, submission of the uncertainties in the counts and covariances of the parameters may significantly reduce the size of the data for our exchange.

Suppose the quantity z at n incident energies is expressed by the number of counts xi (i=1,n), and parameters yi (i=1,m):

zi = zi (xi, y1, y2, …, ym) (1)

Its absolute covariance Cov (zi,zj) is expressed in terms of Δxi and Δyp (p=1,m) by

Cov (zi,zj) = δij (∂zi/∂xi Δxi)2 + Σp,q (∂zi/∂yp) Cov (yp,yq) (∂zi/∂yq), (2)

where δij is Kronecker delta and q=1,m. This demonstrates that the n×n covariances can be expressed by (∂zi/∂xi) Δxi, (∂zi/∂yp) and Cov (yp,yq) (i=1,n; p,q=1,m). In this expression, we need to store the following number of elements:

* (∂zi/∂xi) Δxi n elements (statistical uncertainty in zi)
* ∂zi/∂yp n×m elements (sensitivities)
* Cov (yp,yq) m×m elements (symmetric matrix)

The total number of elements is n + n×m + m×(m+1)/2. If there are 1000 incident energies and 10 parameters, then the total number of elements required to express the 1000×1000 covariances is reduced from 1000×(1000+1)/2=500500 to 1000 + 10000 + 55 = 11055!

In EXFOR, we can store the ∂zi/∂xi Δxi under ERR-S and ∂zi/∂yp under MISC1, MISC2, …, MISCm, while Cov (yp,yq) can be stored as free text under COVARIANCE. If there are more than one counts (e.g., foreground counts and background counts), ERR-1, ERR-2 etc. with the correlation property flag U may be used instead of ERR-S.

When the z is expressed as a product of m+1 parameters like

zi = xi y1 y2 … ym (3)

and Cov(yp,yq) are diagonal, (2) is simplified to

cov (zi,zj) = δij (δxi)2 + Σp (δyp)2 (4)

where cov(zi,zj)=Cov(zi,zj)/(zi zj), δxi = Δxi/xi and δyp = Δyp/yp. This becomes the **quadrature sum of the partial uncertainties** when i=j.

**Distribution:**

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***Example*** (EXFOR 14836.002 from J.M.Brown+,J,NSE,198,1155,2024)

Transmission at 7030 time-of-flight T(t) is modelled by (1) 2 counts for sample-in Cs(t) and sample-out Co(t) as well as 10 parameters a, b, ks, ko, B0s, B0o and αi (i=1,4):

T(t) = [α1 C(E) – α2 ks a exp(-bt) – B0s] / [α3 C(E) – α4 ko a exp(-bt) – B0o]

The DATA section contains T, ΔtotT (total uncertainty), ΔstatT (statistical uncertainty), and partial derivatives of T in terms of the 10 parameters received from the author in the following form (numbers received from the authors are rounded to four digits after the decimal point for readability):

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| E | T | ΔtotT | ΔstatT | ∂T/∂a | ∂T/∂b | ∂T/∂ks | ∂T/∂ko | ∂T/∂B0s | ∂T/∂B0o | ∂T/∂α1 | ∂T/∂α2 | ∂T/∂α3 | ∂T/∂α4 |
| eV | no dim. | no dim. | no dim. | ? | ? | ? | ? | ? | ? | ? | ? | ? | ? |
| E | DATA | ERR-T | ERR-S | MISC1 | MISC2 | MISC3 | MISC4 | MISC5 | MISC6 | MISC7 | MISC8 | MISC9 | MISC10 |
| EV | NO-DIM | NO-DIM | NO-DIM | ? | ? | ? | ? | ? | ? | ? | ? | ? | ? |
| 150.0312 | 0.5749 | 0.1941 | 0.0772 | 0.0000 | 2.9851 | -0.8054 | 0.4588 | -0.0093 | 0.0053 | 2.4388 | -1.3313 | -1.6710 | 0.8636 |
| 150.1353 | 0.6403 | 0.1291 | 0.0656 | 0.0000 | 1.8091 | -0.6246 | 0.3979 | -0.0072 | 0.0046 | 2.0866 | -1.0325 | -1.5922 | 0.7490 |
| 150.2394 | 0.8181 | 0.0972 | 0.0803 | 0.0000 | 0.4951 | -0.6755 | 0.5521 | -0.0078 | 0.0064 | 2.3848 | -1.1167 | -2.1422 | 1.0391 |
| 150.3437 | 0.7549 | 0.0964 | 0.0700 | 0.0000 | 0.9084 | -0.6053 | 0.4560 | -0.0070 | 0.0052 | 2.1578 | -1.0006 | -1.8476 | 0.8583 |
| … | … | … | … | … | … | … | … | … | … | … | … | … | … |

with the following definitions of ERR-T and ERR-S as well as MISC1 to MISC10:

MISC-COL (MISC1) Derivative of T w.r.t a

(MISC2) Derivative of T w.r.t b

(MISC3) Derivative of T w.r.t ks

(MISC4) Derivative of T w.r.t ko

(MISC5) Derivative of T w.r.t B0s

(MISC6) Derivative of T w.r.t B0o

(MISC7) Derivative of T w.r.t alpha1

(MISC8) Derivative of T w.r.t alpha2

(MISC9) Derivative of T w.r.t alpha3

(MISC10) Derivative of T w.r.t alpha4

ERR-ANALYS (ERR-T) Total uncertainty

(ERR-S) Statistical uncertainty

The parameter covariances may be given with short description on how to construct the full covariances ((numbers received from the authors are rounded to four digits after the decimal point for readability):

COVARIANCE Full covariance can be derived from the statistical

uncertainty (ERR-S), sensitivities to the 10

parameters (MISC1 to MISC10) and the following

parameter covariances (a1=alpha1 etc.):

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uncertainty a b ks ko B0s B0o a1 a2 a3 a4

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a 3.8349E+02 1

b 6.0052E-02 0.9931 1

ks 2.4131E-02 0 0 1

ko 4.3340E-02 0 0 0 1

B0s 8.2000E-01 0 0 0 0 1

B0o 1.0000E+00 0 0 0 0 0 1

a1 1.0000E-02 0 0 0 0 0 0 1

a2 1.0000E-02 0 0 0 0 0 0 0 1

a3 1.0000E-02 0 0 0 0 0 0 0 0 1

a4 1.0000E-02 0 0 0 0 0 0 0 0 0 1

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