WP2009-06

Uncertainty propagation in TOF-data

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Compilation of covariance matrix for TOF-data (transmission, reaction yield) into EXFOR is proposed by Geel group. In usual expression, covariance of TOF data is very huge (n^2, n is the number of TOF channel, ~10^4). By using the expression adopted in the AGS format, the size is reduced into $(k+1)n$ (k is the number of systematic error sources, "+1" is from uncorrelated uncertainty) and covariance in this expression can be tabulated in k+1 columns and n lines. So this covariance expression is friendly for the EXFOR format.

NDS analyzed the AGS format and summarized some technical questions in compilation in Memo CP-D/564). Technical questions:

(1) Can we code time-of-flight instead of incident energy as an independent variable?

(2) Can we code negative value as partial uncertainty?

(3) Can we code Cholesky decomposition of covariance matrix as partial uncertainty?

A sample data set prepared in EXFOR template by Geel group is in Appendix 1 of Memo CP-D/564. Corresponding coding sample by NDS is in Appendix 2 of the memo. A short note of the quantities given in the AGS format is summarized in Appendix 3 of this memo.

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Memo CP-D/564 (Rev.)

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To: Distribution
From: N. Otsuka

Subject: **Compilation of covariance matrix given in the AGS format**

Compilation of experimental covariance matrix for transmission and reaction yield will be proposed in the NRDC 2009 meeting by Drs. Stefan Kopecky and Peter Schillebeeckx (EC-JRC-IRMM, Geel). By using their expression of covariance matrix - AGS (Analysis of Geel Spectra) - total size covariance matrix is reduced from n^2 to $n(k+1)$, where n is the total number of TOF (time of flight) channel (~10^4). A note for this formalism in Appendix 3 might be helpful for discussion.

They propose compilation of matrices D and S defined in Eqs.10 and 11 in the note as a function of TOF. I prepared a coding sample (Appendix 2) according to their template for EXFOR compilation (Appendix 1). Some possible technical questions are summarized below:

- Can we add TOF (in second) as an independent variable instead of incident energy?

- Can we allow negative values under `ERR-`*n*? Sensitivity coefficient in Eq. 12 ($\partial\sigma_i / \partial x^l$) can be negative.

- We do not need new headings for uncertainties in the coding sample because the size of covariance matrices of systematic effect sources (defined in matrices W in Eq. 7) is 1 x 1 for all 4 sources, and the matrix S then gives usual partial uncertainty (See Eq.12). But the situation is not simple if the matrix is larger than 1 x 1.

  Example of 2 x 2 matrix:
  If the background correction is given in linear function like $B = a + b*TOF$, W^l_{qr} is a 2 x 2 symmetric matrices (q, r = 1 or 2) for each i, and we will compile 2 elements of S^l_{ip} (p = 1 or 2) under two columns in EXFOR. They are however not usual partial uncertainties `ERR-`*n*.

Appendix 1: EXFOR template from Geel

1. EXPERIMENT DESCRIPTION

Main Reference		1
Facility	GELINA	2,3
Neutron production		4
Primary neutron production target	Uranium	
Time resolution primary beam (ns)	1 ns	
Moderator material	H_2O	
Surface Dimensions (mm x mm or diameter in mm)	2 containers 100 x 100 mm	
Thickness (mm)	40 mm	
Experimental details		
Measurement type	Transmission	
Flight path length (m) (moderator – target (detector): face to face distance)	26.444 +/- 0.006 m	
Angle (with respect to cp beam)	99 deg	
Beam dimensions (mm x mm or diameter in mm)	Diameter 40 mm	
Detector		
Type	Scintillator	
Material	Li-glass (NE 905)	
Surface Dimensions (mm x mm or diameter in mm)	120 mm diameter	
Thickness (mm)	12.7 mm	
Sample		
Type (metal, powder)	Metal	
Chemical composition	Cd	
Atomic abundance of main element Weight	100 % 2.0993 +/- 0.0001 g	
Weight per unit area (g/cm^2)	0.04172 g/cm2	
Geometry		
Surface dimensions (mm x mm or diameter in mm)	Diameter 80.04 +/- 0.03 mm	
Thickness (mm)	0.050 mm (nominal)	
Thickness of main element (at/b)	2.2352 10^{-4} at/b	
Containment description	No container	
Uncertainties (at 1 sigma level)		
Normalization (relative)		
Other sources creating correlated uncertainty components (see data format)	Dead Time correction sample in [1] Background correction sample in parameter one [2] Dead Time correction sample out [3] Background correction sample out parameter one [4]	

References

[1] S. Kopecky et al., submitted to NIMB
[2] J.M. Salome and R. Cools, Nucl. Instrum. Methods Phys. Res. A **179** (1981), 13
[3] D. Tronc, J.M. Salome, K.H. Böckhoff, Nucl. Instrum. Methods Phys. Res. A **228** (1985), 217
[4] M. Flaska et al. Nucl. Instrum. Methods Phys. Res. A **531** (2004), 392

2. DATA FORMAT

According to AGS format

Time-of-Flight [ns]		Observable	Uncertainty					
Low	High	Transmission	Total	Uncorrelated	Correlated Components			
			σ	σ^2	[1]	[2]	[3]	[4]
3.1188E+06	3.1444E+06	9.1309E-01	5.2664E-03	2.7570E-05	6.5601E-06	-1.0072E-04	-7.4295E-06	9.3673E-05
3.1444E+06	3.1700E+06	9.0347E-01	5.2812E-03	2.7719E-05	6.3922E-06	-1.0226E-04	-7.2767E-06	9.4143E-05
3.1700E+06	3.1956E+06	9.0134E-01	5.2989E-03	2.7902E-05	6.3386E-06	-1.0281E-04	-7.2228E-06	9.4387E-05
3.1956E+06	3.2212E+06	8.8804E-01	5.2316E-03	2.7195E-05	6.2074E-06	-1.0196E-04	-7.1256E-06	9.2229E-05
3.2212E+06	3.2468E+06	8.8347E-01	5.2576E-03	2.7461E-05	6.1074E-06	-1.0321E-04	-7.0280E-06	9.2872E-05
…	…	…	…	…	…	…	…	…
6.5116E+06	6.6140E+06	5.1049E-01	1.7897E-03	3.0786E-06	2.7113E-06	-5.1153E-05	-4.0677E-06	2.6553E-05
6.6140E+06	6.7164E+06	5.1780E-01	1.8092E-03	3.1479E-06	2.7654E-06	-5.0203E-05	-4.1206E-06	2.6437E-05
6.7164E+06	6.8188E+06	5.2519E-01	1.8266E-03	3.2107E-06	2.8231E-06	-4.9156E-05	-4.1785E-06	2.6257E-05
6.8188E+06	6.9212E+06	5.3394E-01	1.8541E-03	3.3090E-06	2.8821E-06	-4.8495E-05	-4.2320E-06	2.6329E-05
6.9212E+06	7.0236E+06	5.4117E-01	1.8762E-03	3.3897E-06	2.9320E-06	-4.7730E-05	-4.2778E-06	2.6267E-05
7.0236E+06	7.0246E+06	5.6613E-01	1.9202E-02	3.6855E-04	3.0999E-06	-4.8282E-05	-4.4270E-06	2.7783E-05

Appendix 2: Coding sample

```
ENTRY            00001   20090507                                  0000100000001
SUBENT        00001001   20090507                                  0000100100001
BIB                 14         66                                  0000100100002
TITLE      The total cross section and resonance parameters for    0000100100003
           the 0.178 eV resonance of 113Cd                         0000100100004
AUTHOR     (S.Kopecky, I.Ivanov, M.Moxon, P.Schillebeeckx,         0000100100005
           P.Siegler, I.Sirakov)                                   0000100100006
INSTITUTE  (2ZZZGEL)                                               0000100100007
REFERENCE  (J,NIM/B,,,2009) Submitted to NIM/B                     0000100100008
           #doi: 10.1016/j.nimb.2009.04.010                        0000100100009
REL-REF    (N,,J,NIM/A,179,13,1981)                                0000100100010
            J.M.Salome+, Details of GELINA facility                0000100100011
           (N,,J,NIM/A,228,217,1985)                               0000100100012
            D.Tronc+, Details of GELINA facility                   0000100100013
           (N,,J,NIM/A,531,392,2004)                               0000100100014
            M.Flaska+, Details of neutron production               0000100100015
FACILITY   (LINAC,2ZZZGEL) GELINA                                  0000100100016
INC-SOURCE Diameter of neutron beam = 40 mm                        0000100100017
           (PHOTO) (g,n) on uranium target                         0000100100018
                   (2 containers, 100 x 100 mm, 40 mm thick)       0000100100019
           (THCOL) Water moderator                                 0000100100020
METHOD     (TRN,TOF) Flight path (moderator - target (detector)    0000100100021
                     face to face distance) = 26.444 +/- 0.006 m   0000100100022
DETECTOR   (SCIN,GLASD) Li-glass scintillator (NE 905)             0000100100023
                        120 mm diameter, 12.7 mm thick             0000100100024
SAMPLE     Physical type:             Metal                        0000100100025
           Chemical composition:      Element (Cd)                 0000100100026
           Purity of main element:    100%                         0000100100027
           Weight per area:           0.04172 g/cm2                0000100100028
           Diameter:                  80.04 +/- 0.03 mm            0000100100029
           Thickness:                 0.050 mm (nominal)           0000100100030
           Thickness of main element: 2.2352E-04 atom/b            0000100100031
CORRECTION Data are corrected for                                  0000100100032
              - dead time at sample-in detector   (din);           0000100100033
              - background at sample-in detecto   (Bin);           0000100100034
              - dead time at sample-out detector  (dout);          0000100100035
              - background at sample-out detector (Bout),          0000100100036
           as follows:                                             0000100100037
             T=(din*Cin-Bin)/(dout*Cout-Bout)                      0000100100038
           , where                                                 0000100100039
              T:    Transmission;                                  0000100100040
              Cin:  Count at sample-in detector;                   0000100100041
              Cout: Count at sample-out detector.                  0000100100042
ERR-ANALYS (ERR-T) Total uncertainty                               0000100100043
           (ERR-S) Sum of uncorrelated uncertainties               0000100100044
                   due to counting statistics                      0000100100045
                   (sample-in and sample-out)                      0000100100046
           (ERR-1) Uncertainty due to dead-time correction         0000100100047
```

```
                (sample-in)                                       0000100100048
          (ERR-2) Uncertainty due to background correction        0000100100049
                (sample-in)                                       0000100100050
          (ERR-3) Uncertainty due to dead-time correction         0000100100051
                (sample-out)                                      0000100100052
          (ERR-4) Uncertainty due to background correction        0000100100053
                     (sample-out)                                 0000100100054
COVARIANCE If i=j,                                                0000100100055
           cov(T_i,T_j) = (ERR-S)i**2 +(ERR-1)i**2+(ERR-2)i**2    0000100100056
                                     +(ERR-3)i**2+(ERR-4)i**2     0000100100057
          ,else                                                   0000100100058
           cov(T_i,T_j) = (ERR-1)i*(ERR-1)j                       0000100100059
                        + (ERR-2)i*(ERR-2)j                       0000100100060
                        + (ERR-3)i*(ERR-3)j                       0000100100061
                        + (ERR-4)i*(ERR-4)j                       0000100100062
          .                                                       0000100100063
HISTORY   (20090403R) On. Received by e-mail from S.Kopecky       0000100100064
          (20090423C) On.                                         0000100100065
          (20090507A) On. ERR-ANALYSIS updated,                   0000100100066
                          CORRECTION and COVARIANCE added.        0000100100067
          (20090520A) On. TITLE added, AUTHOR corrected           0000100100068
ENDBIB              66          0                                 0000100100069
NOCOMMON             0          0                                 0000100100070
ENDSUBENT           69          0                                 0000100199999
SUBENT        00001002   20090507                                 0000100200001
BIB                  3          3                                 0000100200002
REACTION  (48-CD-0(N,TOT),TRN)                                    0000100200003
STATUS    (TABLE) Data on MS word file                            0000100200004
ENDBIB               3          0                                 0000100200006
COMMON               2          3                                 0000100200007
TOF-L      TOF-L-ERR                                              0000100200008
M          M                                                      0000100200009
26.444     0.006                                                  0000100200010
ENDCOMMON            3          0                                 0000100200011
DATA                 9        101                                 0000100200012
TOF-MIN   TOF-MAX   DATA       ERR-T      ERR-S      ERR-1        0000100200013
ERR-2     ERR-3     ERR-4                                         0000100200014
NS        NS        NO-DIM     NO-DIM     NO-DIM     NO-DIM       0000100200015
NO-DIM    NO-DIM    NO-DIM                                        0000100200016
 3.1188E+06 3.1444E+06 9.1309E-01 5.2664E-03 5.2507E-03 6.5601E-060000100200017
-1.0072E-04-7.4295E-06 9.3673E-05                                 0000100200018
 3.1444E+06 3.1700E+06 9.0347E-01 5.2812E-03 5.2649E-03 6.3922E-060000100200019
-1.0226E-04-7.2767E-06 9.4143E-05                                 0000100200020
 3.1700E+06 3.1956E+06 9.0134E-01 5.2989E-03 5.2822E-03 6.3386E-060000100200021
-1.0281E-04-7.2228E-06 9.4387E-05                                 0000100200022
 3.1956E+06 3.2212E+06 8.8804E-01 5.2316E-03 5.2149E-03 6.2074E-060000100200023
-1.0196E-04-7.1256E-06 9.2229E-05                                 0000100200024
…
 6.7164E+06 6.8188E+06 5.2519E-01 1.8266E-03 1.7918E-03 2.8231E-060000100200211
-4.9156E-05-4.1785E-06 2.6257E-05                                 0000100200212
 6.8188E+06 6.9212E+06 5.3394E-01 1.8541E-03 1.8191E-03 2.8821E-060000100200213
-4.8495E-05-4.2320E-06 2.6329E-05                                 0000100200214
 6.9212E+06 7.0236E+06 5.4117E-01 1.8762E-03 1.8411E-03 2.9320E-060000100200215
-4.7730E-05-4.2778E-06 2.6267E-05                                 0000100200216
 7.0236E+06 7.0246E+06 5.6613E-01 1.9202E-02 1.9198E-02 3.0999E-060000100200217
-4.8282E-05-4.4270E-06 2.7783E-05                                 0000100200218
ENDDATA            206          0                                 0000100200219
ENDSUBENT          218          0                                 0000100299999
ENDENTRY             2          0                                 0000199999999
ENDTRANS             1          0                                 Z999999999999
```

Appendix 3: Quantitites given in the AGS format

Note: A compact expression of experimental covariance

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This is a note for compact expression of experimental covariance (AGS format)[1, 2].

Formalism

Let us consider a quantity at i-th incident energy bin σ_i ($1 \leq i \leq n$) as a function of M uncorrelated quantities y_i^m ($1 \leq m \leq M$) and $K \times L$ correlated quantities x_r^l ($1 \leq l \leq L$, $1 \leq r \leq K_l$, $K = K_1 + K_2 + ... + K_L$):

$$\sigma_i \equiv \sigma(\{y_i^m\}, \{x_r^l\}). \tag{1}$$

The total uncertainty of σ_i is decomposed to uncorrelated and correlated terms

$$
\begin{aligned}
(\Delta\sigma_i)^2_{\mathrm{tot}} &= \sum_{m=1}^{M} (\Delta\sigma_i)^2_m + \sum_{l=1}^{L} (\Delta\sigma_i)^2_l &(2)\\
&= \sum_{m=1}^{M} \left(\frac{\partial\sigma_i}{\partial y_i^m}\Delta y_i^m\right)^2 + \sum_{l=1}^{L}\sum_{q,r=1}^{K_l} \left(\frac{\partial\sigma_i}{\partial x_q^l}\Delta x_q^l\right)\left(\frac{\partial\sigma_i}{\partial x_r^l}\Delta x_r^l\right) c_{qr}^l &(3)
\end{aligned}
$$

where c_{qr}^l is the correlation matrix of the quantity x^l. We assume that there is no correlation between x_q^k and x_r^l if $k \neq l$.

The covariance matrix of σ_i is then

$$
\begin{aligned}
V_{ij} &= \sum_{m=1}^{M} (\Delta\sigma_i)_m (\Delta\sigma_j)_m \delta_{ij} + \sum_{l=1}^{L} (\Delta\sigma_i)_l (\Delta\sigma_j)_l C_{ij}^l &(4)\\
&= \sum_{m=1}^{M} \frac{\partial\sigma_i}{\partial y_i^m}\frac{\partial\sigma_j}{\partial y_j^m} U_{ij}^m + \sum_{l=1}^{L}\sum_{q,r=1}^{K_l} \frac{\partial\sigma_i}{\partial x_q^l}\frac{\partial\sigma_j}{\partial x_r^l} W_{qr}^l \quad (V, U \in R^{n\times n}, W \in R^{K_l \times K_l}) &(5)
\end{aligned}
$$

, where two covariance matrices

$$
\begin{aligned}
U_{ij}^m &\equiv (\Delta y_i^m)^2 \delta_{ij}, &(6)\\
W_{qr}^l &\equiv \Delta x_q^l \Delta x_r^l c_{qr}^l &(7)
\end{aligned}
$$

are introduced (δ_{ij}: Kronecker's delta).

The matrix W^l is positive definite and symmetric and, therefore, can be decomposed to an upper triangle matrix R ($R \in R^{K_l \times K_l}$) and its transposed matrix R^T as $W^l = R^l R^{l^T}$ (Cholesky decomposition). Then

$$
\begin{aligned}
V_{ij} &= \sum_{m=1}^{M} \frac{\partial\sigma_i}{\partial y_i^m}\frac{\partial\sigma_j}{\partial y_j^m} U_{ij}^m + \sum_{l=1}^{L}\sum_{p,q,r=1}^{K_l} \frac{\partial\sigma_i}{\partial x_q^l}\frac{\partial\sigma_j}{\partial x_r^l} R_{qp}^l R_{pr}^l &(8)\\
&= D_{ij} + \sum_{l=1}^{L}\sum_{p=1}^{K_l} S_{ip}^l S_{jp}^l &(9)
\end{aligned}
$$

, where

$$D_{ij} \equiv \sum_{m=1}^{M} \frac{\partial\sigma_i}{\partial y_i^m}\frac{\partial\sigma_j}{\partial y_j^m}U_{ij}^m = \delta_{ij}\sum_{m=1}^{M}(\Delta\sigma_i)_m^2 \quad (D \in R^{n\times n}\text{ ,diagonal}), \tag{10}$$

$$S_{ip}^l \equiv \sum_{q=1}^{K_l}\frac{\partial\sigma_i}{\partial x_q^l}R_{qp}^l \quad (S^l \in R^{n\times K_l}) \tag{11}$$

. The covariance matrix V is now expressed by the n elements of the diagonal matrix D and $n\times K$ elements of the matrices S^l $(1 \le l \le L)$. This reduces total number of elements from $n\times n$ to $n\times(1+K)$

If K_l=1 for all l, $W_{qr}^l \to W^l = (\Delta x^l)^2$, $c_{qr}^l \to c^l = 1$, $R_{qp}^l \to R^l = \sqrt{W^l} = \Delta x^l$, and

$$S_{ip}^l \to S_i^l = \frac{\partial\sigma^i}{\partial x^l}\Delta x^l \equiv (\Delta\sigma_i)_l\,, \tag{12}$$

which sigature is positive or negative. Therefore the covariance matrix and its diagonal component are

$$V_{ij} = \delta_{ij}\sum_{m=1}^{M}(\Delta\sigma_i)_m^2 + \sum_{l=1}^{L}(\Delta\sigma_i)_l(\Delta\sigma_j)_l \tag{13}$$

$$(\Delta\sigma)_{\text{tot}}^2 = V_{ii} = \sum_{m=1}^{M}(\Delta\sigma_i)_m^2 + \sum_{l=1}^{L}(\Delta\sigma_i)_l^2 \tag{14}$$

EXFOR

We need one column for D_{ii} $(1 \le i \le n)$ and $K = K_1 + ... + K_L$ columns for S_{ip}^l $(1 \le l \le L, 1 \le p \le K_l, 1 \le i \le n)$.

If $K_l = 1$ for all L and the incident energy E_i is known, we can use the current EXFOR format:

`EN`	`ERR-S`	`ERR-1`	...	`ERR-L`
	D_{ii}	S_i^1	...	S_i^L
E_1	$(\Delta\sigma_1)_0$	$(\Delta\sigma_1)_1$	...	$(\Delta\sigma_1)_L$
E_2	$(\Delta\sigma_2)_0$	$(\Delta\sigma_2)_1$	...	$(\Delta\sigma_2)_L$
...	...	...	...	...
E_i	$(\Delta\sigma_i)_0$	$(\Delta\sigma_i)_1$	...	$(\Delta\sigma_i)_L$
...	...	...	...	...
E_n	$(\Delta\sigma_n)_0$	$(\Delta\sigma_n)_1$	...	$(\Delta\sigma_n)_L$

, where we allow to use the heading `ERR-S` for total uncorrelated uncertainty

$$(\Delta\sigma_i)_0^2 = \sum_{m=1}^{M}(\Delta\sigma_i)_m^2 \tag{15}$$

.

Example - Transmission

Transmission T (, `TRN` in EXFOR) is related to total cross section σ_T as follows:

$$T(E) = \exp\left(-L\rho\sigma_T(E)\right) \tag{16}$$

(L: target thickness, ρ: target density), and expressed in terms of the 6 observables

- C_i^{in} : Count at sample-in detector (uncorrelated)

- C_i^{out}: Count at sample-out detector (uncorrelated)
- d^{in} : Dead time correction for sample-in detector (correlated)
- d^{out} : Dead time correction for sample-out detector (correlated)
- B^{in} : Background correction for sample-in detector (correlated)
- B^{out} : Background correction for sample-out detector (correlated)

as follows:

$$T_i = T_i(C_i^{\text{in}}, C_i^{\text{out}}, d^{\text{in}}, d^{\text{out}}, B^{\text{in}}, B^{\text{out}}) = \frac{d^{\text{in}} C_i^{\text{in}} - B^{\text{in}}}{d^{\text{out}} C_i^{\text{out}} - B^{\text{out}}} \tag{17}$$

This is the case of $M = 2$, $L = 4$ and $K_l = 1$ ($1 \leq l \leq 4$) in the formalism. Covariance matrices for uncorrelated quantities are

$$U_{ij}^{\text{Cin}} = \left(\Delta C_i^{\text{in}}\right)^2 \delta_{ij}, U_{ij}^{\text{Cout}} = \left(\Delta C_i^{\text{out}}\right)^2 \delta_{ij}, \tag{18}$$

while covariance matrices for correlated quantities and these Cholesky decompositions are

$$W^{\text{din}} = \left(\Delta d_i^{\text{in}}\right)^2, W^{\text{dout}} = \left(\Delta d_i^{\text{out}}\right)^2, W^{\text{Bin}} = \left(\Delta B_i^{\text{in}}\right)^2, W^{\text{Bout}} = \left(\Delta B_i^{\text{out}}\right)^2 \tag{19}$$

and

$$R^{\text{din}} = \Delta d^{\text{in}}, R^{\text{dout}} = \Delta d^{\text{out}}, R^{\text{Bin}} = \Delta B^{\text{in}}, R^{\text{Bout}} = \Delta B^{\text{out}}. \tag{20}$$

Therefore the covariance matrix V is expressed by n elements of the matrix D

$$\begin{aligned} D_{ij} &= \frac{\partial T_i}{\partial C_i^{\text{in}}} \frac{\partial T_j}{\partial C_j^{\text{in}}} U_{ij}^{\text{Cin}} + \frac{\partial T_i}{\partial C_i^{\text{out}}} \frac{\partial T_j}{\partial C_j^{\text{out}}} U_{ij}^{\text{Cout}} & (21) \\ &= (\Delta T_i)^2_{\text{Cin}} \delta_{ij} + (\Delta T_i)^2_{\text{Cout}} \delta_{ij} & (22) \end{aligned}$$

and $n \times 4$ elements of S matrices:

$$\begin{aligned} S_i^{\text{din}} &= \frac{\partial T_i}{\partial d^{\text{in}}} R^{\text{din}} = \frac{\partial T_i}{\partial d^{\text{in}}} \Delta d^{\text{in}} \equiv (\Delta T_i)_{\text{din}} & (23) \\ S_i^{\text{dout}} &= \frac{\partial T_i}{\partial d^{\text{out}}} R^{\text{dout}} = \frac{\partial T_i}{\partial d^{\text{out}}} \Delta d^{\text{out}} \equiv (\Delta T_i)_{\text{dout}} & (24) \\ S_i^{\text{Bin}} &= \frac{\partial T_i}{\partial B^{\text{in}}} R^{\text{Bin}} = \frac{\partial T_i}{\partial B^{\text{in}}} \Delta B^{\text{in}} \equiv (\Delta T_i)_{\text{Bin}} & (25) \\ S_i^{\text{Bout}} &= \frac{\partial T_i}{\partial B^{\text{out}}} R^{\text{Bout}} = \frac{\partial T_i}{\partial B^{\text{out}}} \Delta B^{\text{out}} \equiv (\Delta T_i)_{\text{Bout}} & (26) \end{aligned}$$

Covariance matrix can be reconstructed from these elements as follows:

$$\begin{aligned} V_{ij} &= D_{ij} S_i^{\text{din}} S_j^{\text{din}} + S_i^{\text{dout}} S_j^{\text{dout}} + S_i^{\text{Bin}} S_j^{\text{Bin}} + S_i^{\text{Bout}} S_j^{\text{Bout}} & (27) \\ &= (\Delta T_i)^2_{\text{Cin}} \delta_{ij} + (\Delta T_i)^2_{\text{Cout}} \delta_{ij} \\ &+ (\Delta T_i)_{\text{din}} (\Delta T_j)_{\text{din}} + (\Delta T_i)_{\text{dout}} (\Delta T_j)_{\text{dout}} + (\Delta T_i)_{\text{Bin}} (\Delta T_j)_{\text{Bin}} + (\Delta T_i)_{\text{Bout}} (\Delta T_j)_{\text{Bout}} & (28) \end{aligned}$$

. The diagonal element V_{ii} gives the total uncertainty of T^i

$$(\Delta T_i)^2_{\text{tot}} = (\Delta T_i)^2_{\text{Cin}} + (\Delta T_i)^2_{\text{Cout}} + (\Delta T_i)^2_{\text{din}} + (\Delta T_i)^2_{\text{dout}} + (\Delta T_i)^2_{\text{Bin}} + (\Delta T_i)^2_{\text{Bout}} \tag{29}$$

This can can be compiled as follows

EN	ERR-S	ERR-1	...	ERR-4
E_1	$(\Delta T_1)_{\rm C}$	$(\Delta T_1)_{\rm din}$	...	$(\Delta T_1)_{\rm Bout}$
E_2	$(\Delta T_2)_{\rm C}$	$(\Delta T_2)_{\rm din}$	...	$(\Delta T_2)_{\rm Bout}$
...	...	...	...	...
E_i	$(\Delta T_i)_{\rm C}$	$(\Delta T_i)_{\rm din}$	...	$(\Delta T_i)_{\rm Bout}$
...	...	...	...	...
E_n	$(\Delta T_n)_{\rm C}$	$(\Delta T_n)_{\rm din}$	...	$(\Delta T_n)_{\rm Bout}$

, where

$$(\Delta T_i)^2_{\rm C} = (\Delta T_i)^2_{\rm Cin} + (\Delta T_i)^2_{\rm Cout} \tag{30}$$

References

[1] C.Bastian, Proceedings of the International Symposium on Nuclear Data Evaluation Methodology, Upton, US, 1992, p.642 (1993).

[2] C.Bastian, Proceedings of the International Conference on Neutron Research and Industry, Crete, Greece, 1996, p.611 (1997).