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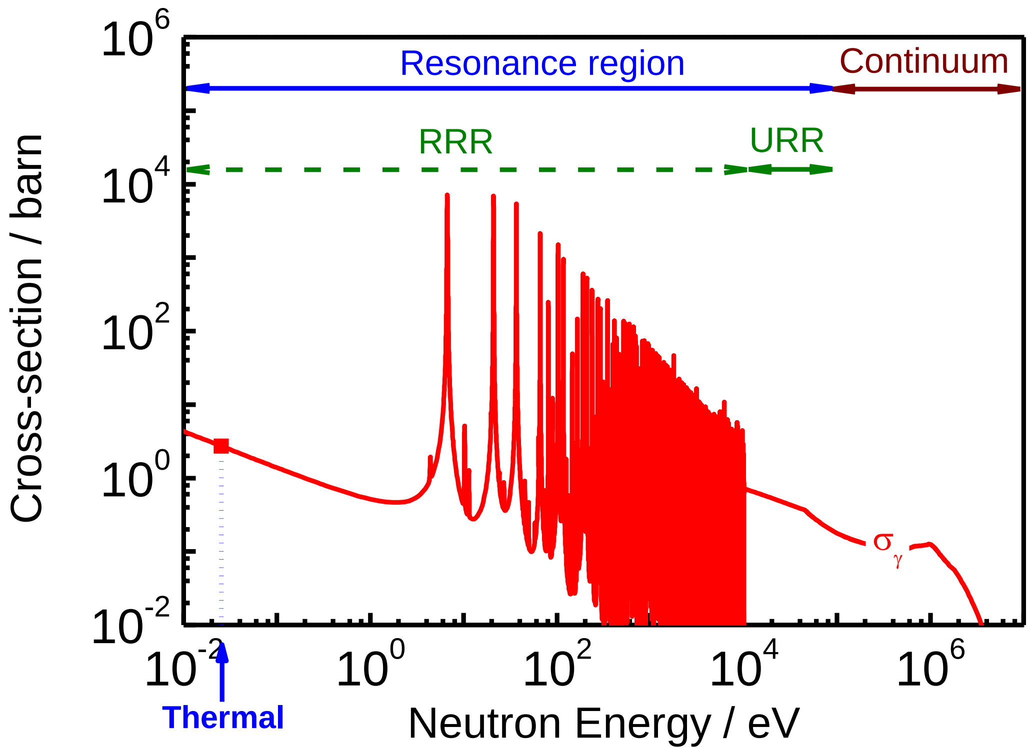
Derivation of resonance parameters from time-of-flight measurements

Stefan Kopecky

European Commission

Neutron induced reaction cross sections

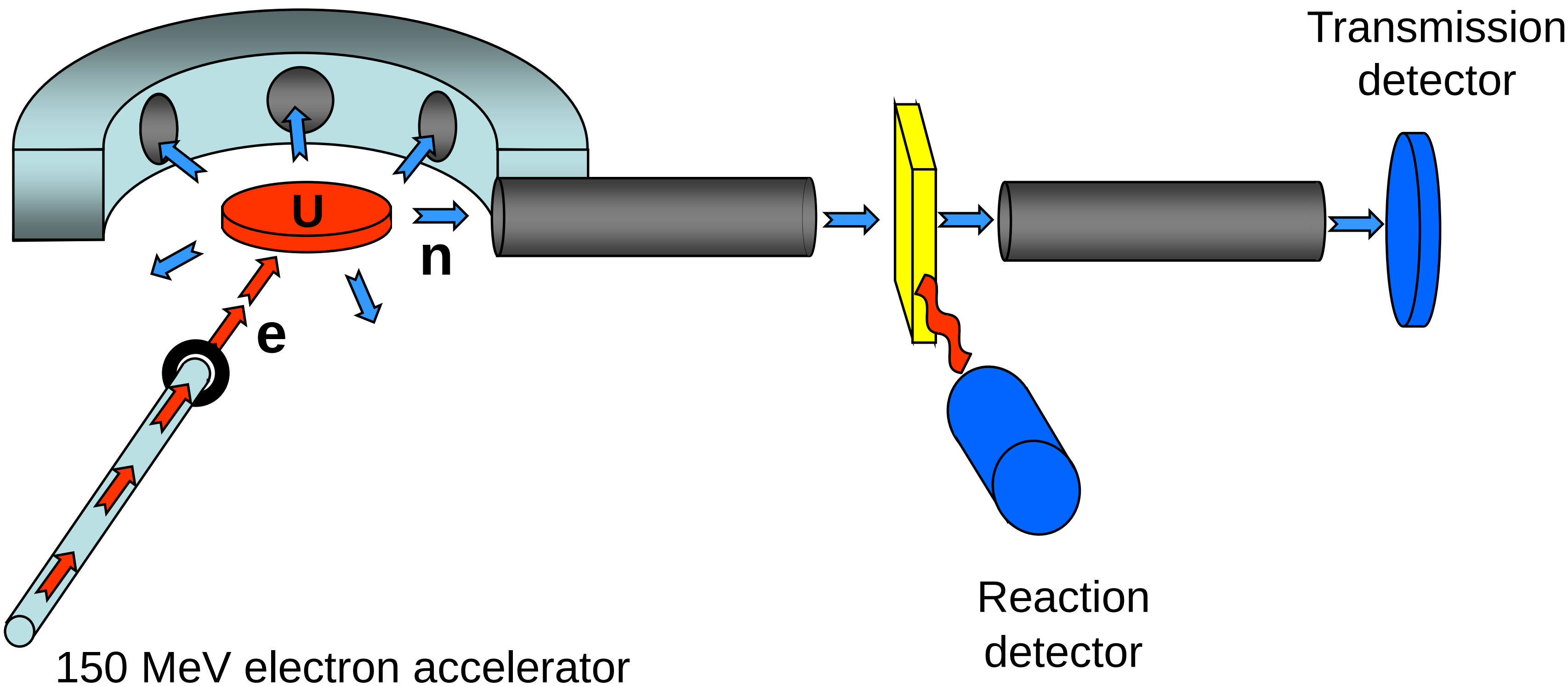


Cross sections cannot be predicted by nuclear theory from first principles

⇒ parameterized by nuclear reaction models

⇒ different models in different energy regions

Time-of-flight measurement



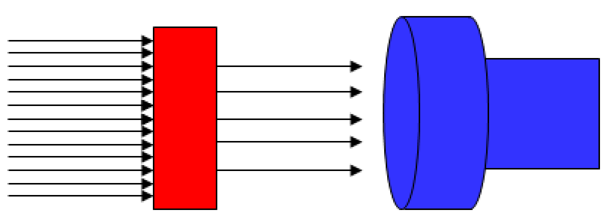
Cross section measurements : transmission + reaction

Transmission

$$T = e^{-n\sigma_{tot}}$$

T : transmission
Fraction of the neutron beam traversing the sample without any interaction

$$T_{exp} = \frac{C_{in}}{C_{out}}$$



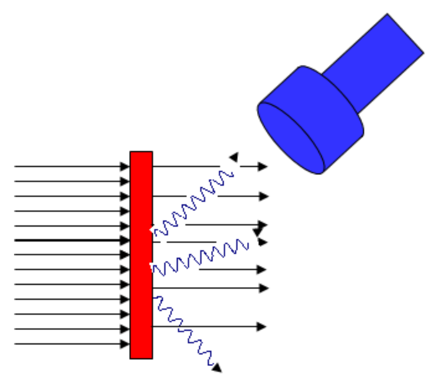
Reaction

$$Y_r \cong (1 - e^{-n\sigma_{tot}})\frac{\sigma_r}{\sigma_{tot}} + \ldots$$

Y_r : reaction yield
Fraction of the neutron beam creating a (n,r) reaction in the sample

Only for thin samples : $Y_r \approx n\,\sigma_r$

e.g. (n,γ)



Cross section measurements : transmission + reaction

Transmission

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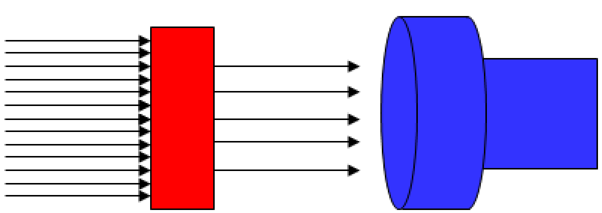
T : transmission
Fraction of the neutron beam traversing the sample without any interaction

$$T_{exp} = \frac{C_{in}}{C_{out}}$$

Absolute measurement



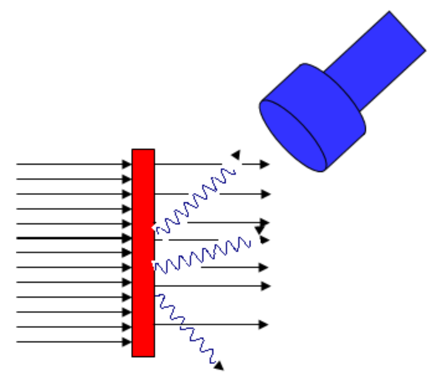
Reaction

$$Y_r \cong (1 - e^{-n\sigma_{tot}})\frac{\sigma_r}{\sigma_{tot}} + \ldots$$

Y_r : reaction yield
Fraction of the neutron beam creating a (n,r) reaction in the sample

$$Y_{r,exp} = \frac{C_r}{\varepsilon_r\, \Omega\, P_r\, A_r\, \varphi} = K_r \frac{C_r}{\varphi}$$

e.g. (n,γ)



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Experimental observables (resonance region)

$$T_{exp} = K_T \frac{C_{in} - B_{in}}{C_{out} - B_{out}}$$ $$\frac{u_T}{T_{exp}} \approx 0.3\%$$

$$Y_{exp,\gamma} = K_\gamma \frac{C_{in} - B_{in}}{C_\varphi - B_\varphi} Y_\varphi$$ $$\frac{u_{Y_\gamma}}{Y_{exp,\gamma}} \approx 2.0\%$$ (without fission)

$$Y_{exp,f} = K_f \frac{C_{in} - B_{in}}{C_\varphi - B_\varphi} Y_\varphi$$ $$\frac{u_{Y_f}}{Y_{exp,f}} \approx 2.0\%$$

Methodologies to determine (Z_{exp},$V_{Z_{exp}}$) and report them in EXFOR are well established

Nuclear Data Sheets 113 (2012) 3054 - 3100

Becker et al., "AGS-concept", J. of Instrumentation 7 (2012) P11002

$\Rightarrow$ EXFOR Consultants' Meeting, 8 to 10 October 2013, IAEA, INDC(NDS)-0647

Reaction model parameters + covariance (θ, V_θ)

(θ, V_θ) ← **Model $H_M(\theta,\kappa)$**
Reaction + experiment ← $(Z_{exp}, V_Z, \kappa, V_\kappa)$

(θ, V_θ) determined by :

- Experimental data and parameters $(Z_{exp}, V_Z, \kappa, V_\kappa)$
- Model H_M : reaction model (θ) + experiment (κ)
  - Nuclear reaction model $F_M(\theta)$
  - Model to account for experimental parameters κ (i.e. resonance region)
- **Methods or procedures to estimate model parameters (θ, V_θ) from $(Z_{exp}, V_Z, \kappa, V_\kappa)$**

Reaction model parameters + covariance (θ, V_θ)

(θ, V_θ) ← **Model $H_M(\theta,\kappa)$**
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- **Methods or procedures to estimate model parameters (θ, V_θ) from $(Z_{exp}, V_Z, \kappa, V_\kappa)$**

Model for experimental parameters

$$T_{\exp} = \int R(t,E)T(E)dE = \int R(t,E)e^{-n\sigma(E)}dE$$

$$\frac{\ln\left(T_{\exp}\right)}{n} \neq \int R(t,E)\sigma(E)dE$$

$$Y_{\exp} = K_{\exp} \cdot \int R(t,E)Y_{Mod}(E)dE$$

$$Y_{Mod}(E) = \varepsilon_c Y_c(E) + \varepsilon_{ns}(E)Y_s$$

$$Y_c(E) = F_0 Y_0(E) + F_m Y_m(E)$$

Experimental broadening

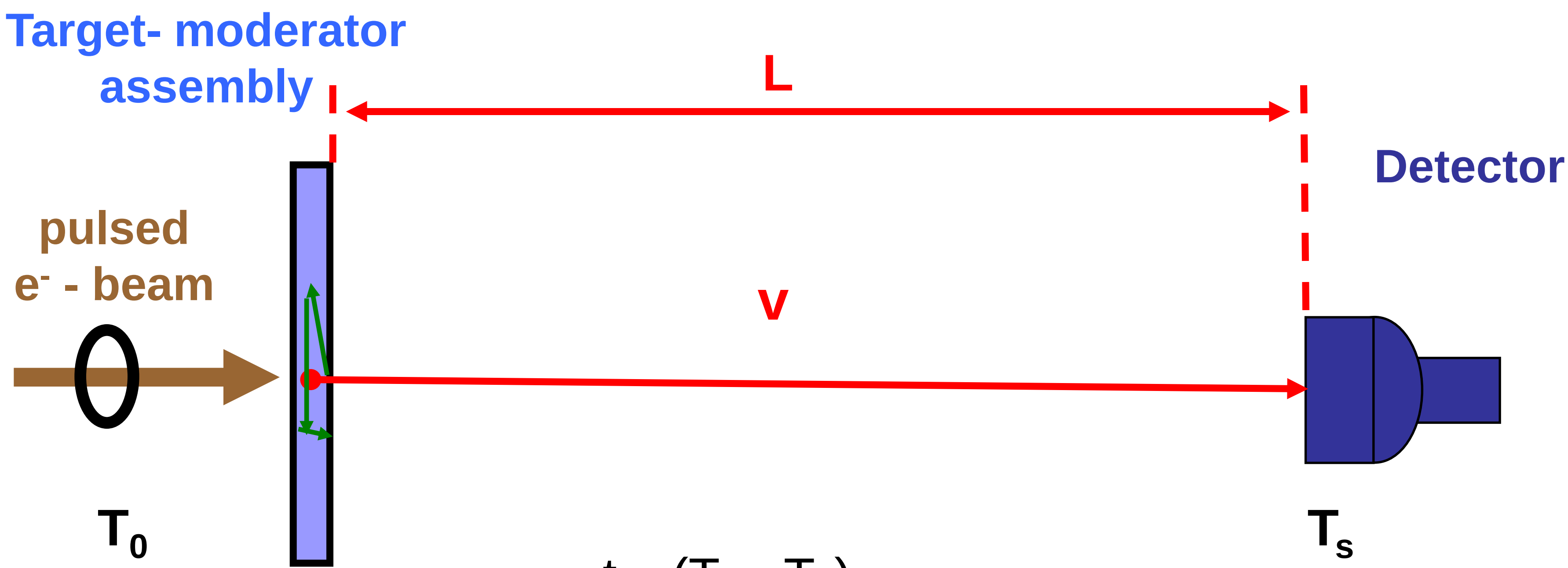
Sample inhomogeneity

Multiple Scattering

Gamma attenuation

Neutron sensitivity

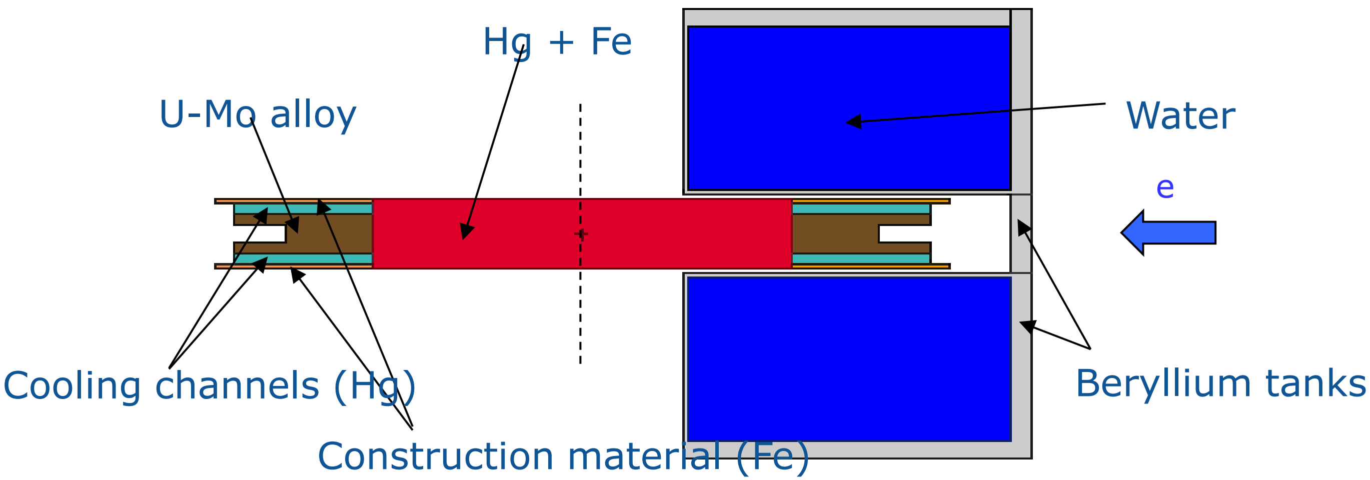
TOF -> Energy

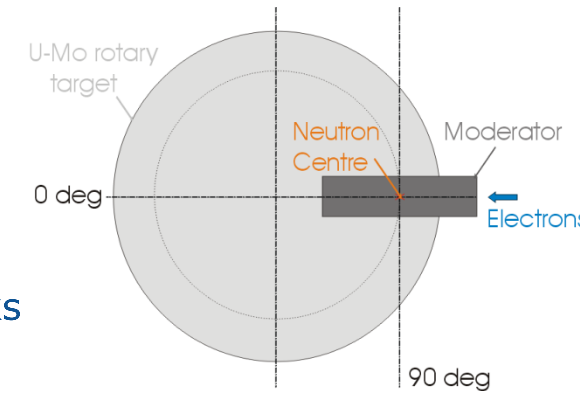


$$t = (T_s - T_0)$$

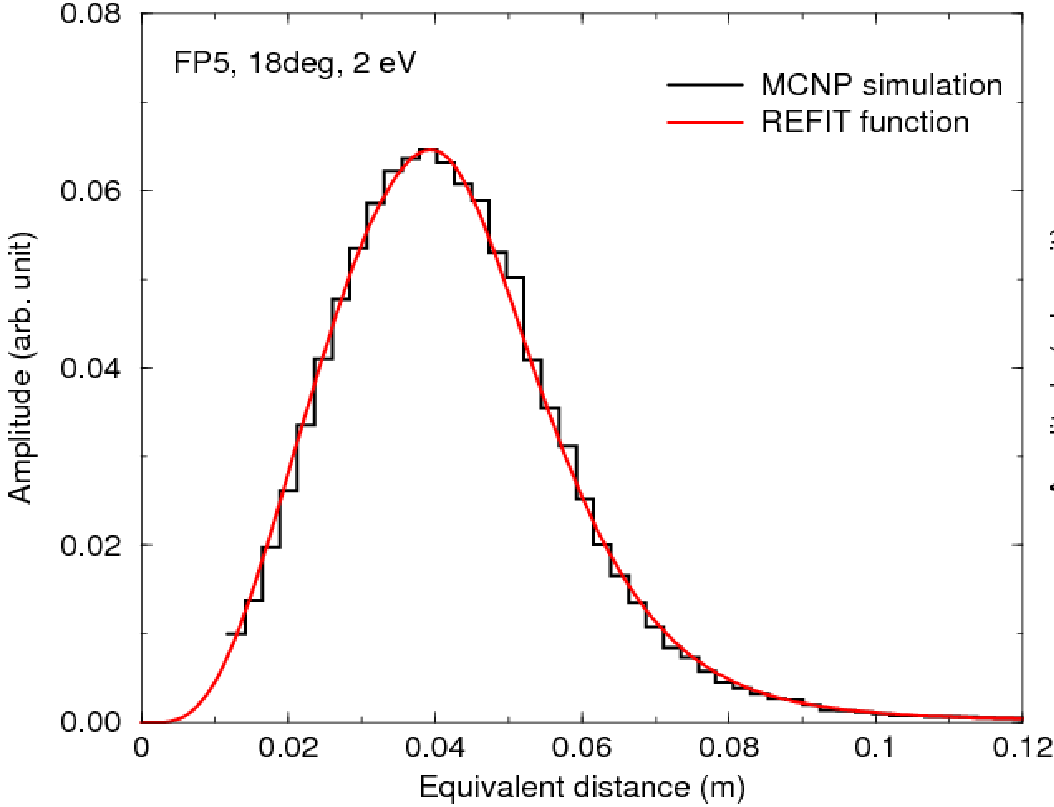
$$v = \frac{L}{t} \quad \Rightarrow \quad E = \frac{1}{2}mv^2$$

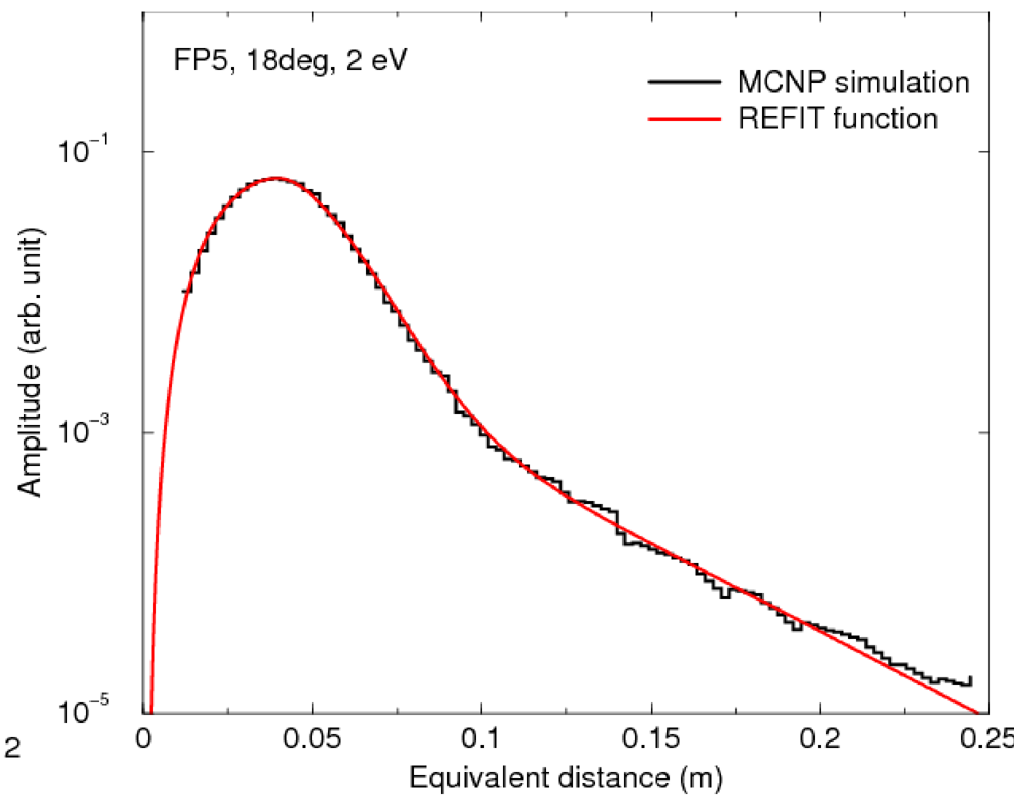
GELINA Target-moderator assembly



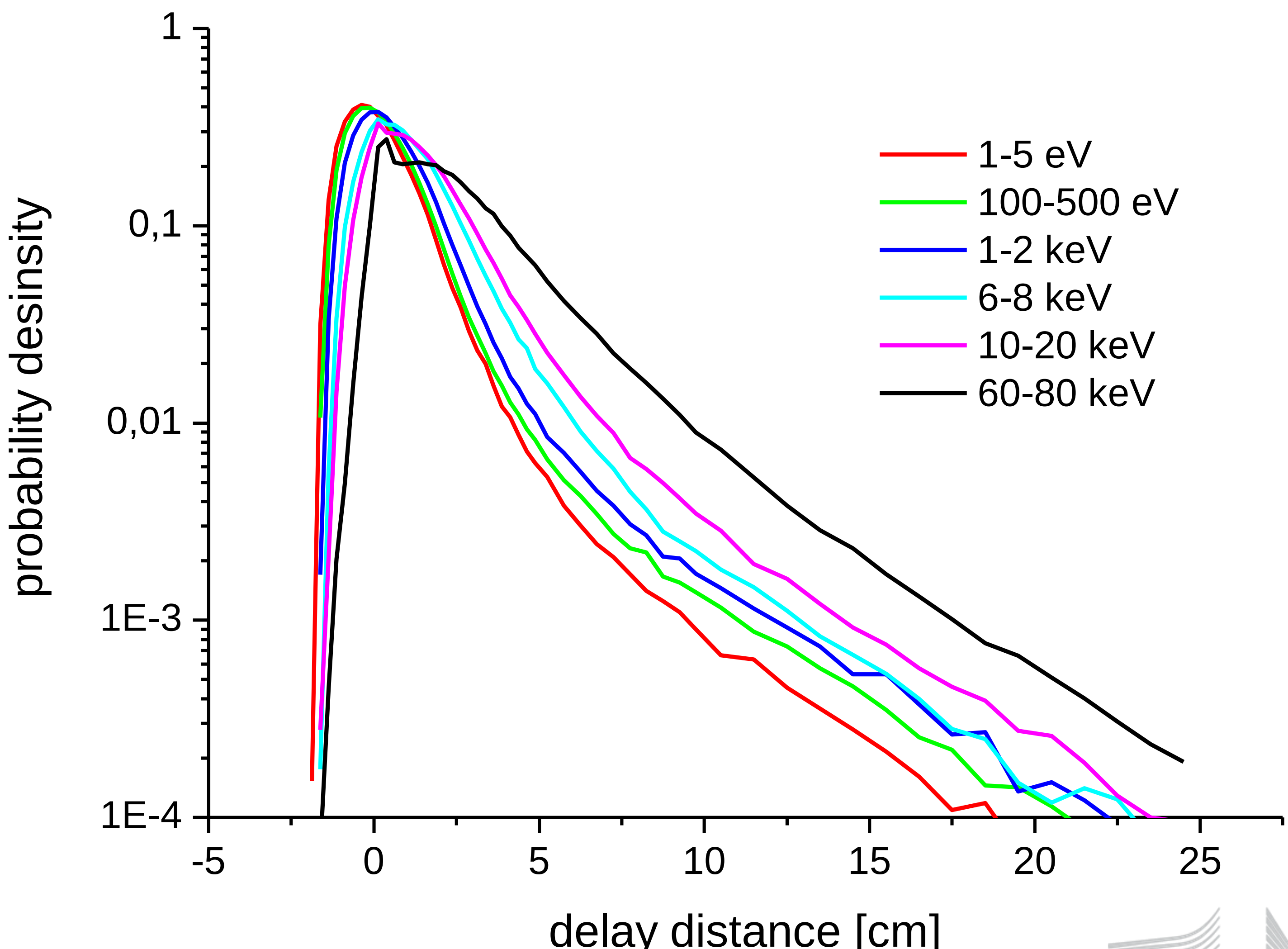


GELINA Response function





GELINA Response function



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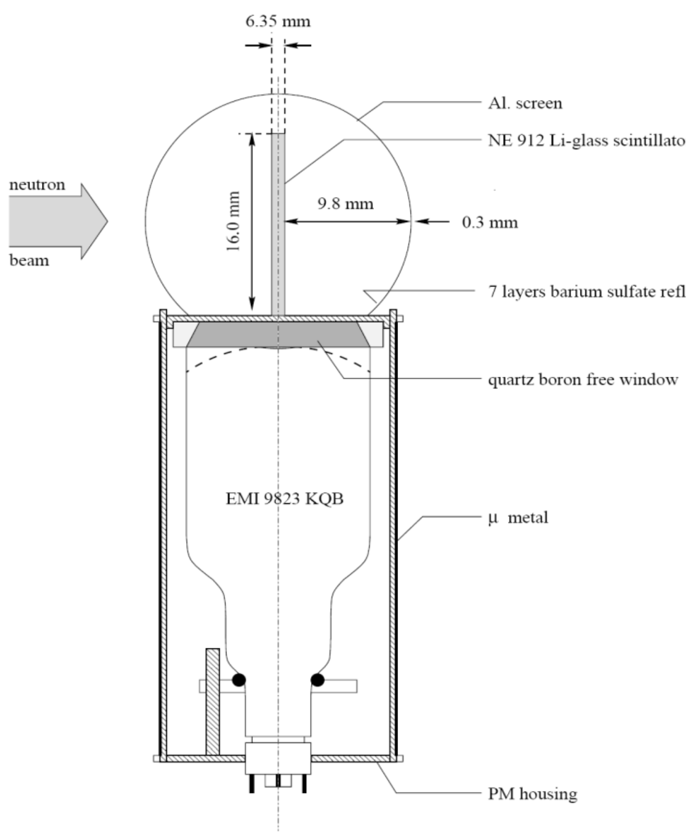
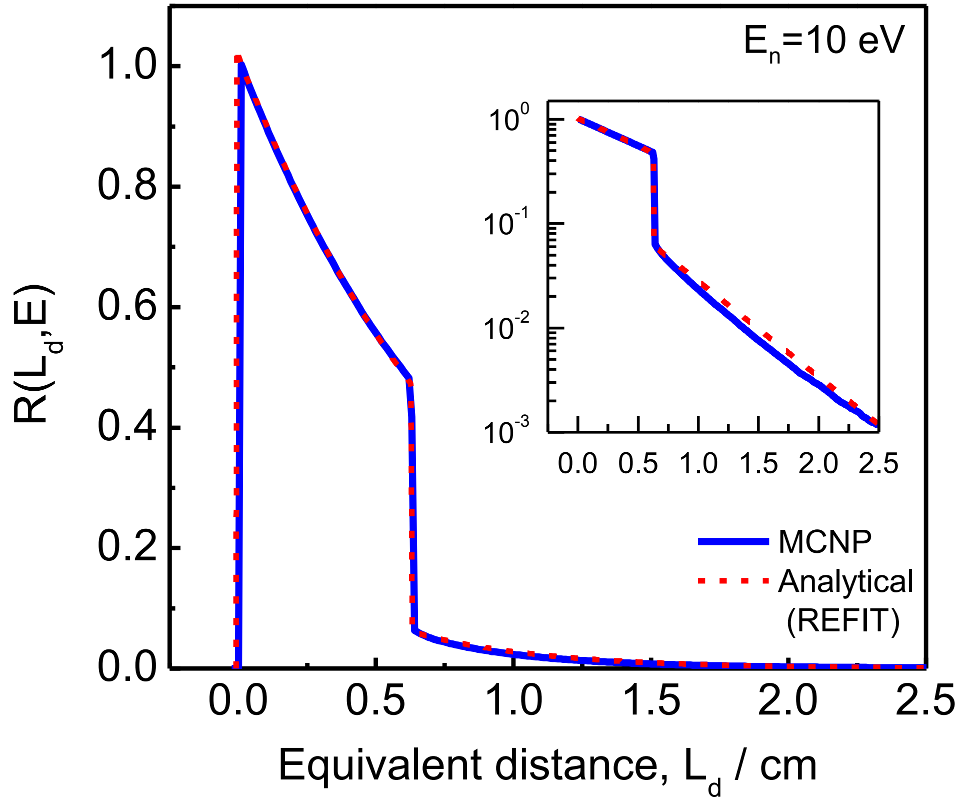
Detector Response

^{6}Li(n,t)α Scintillator + Photomultiplier



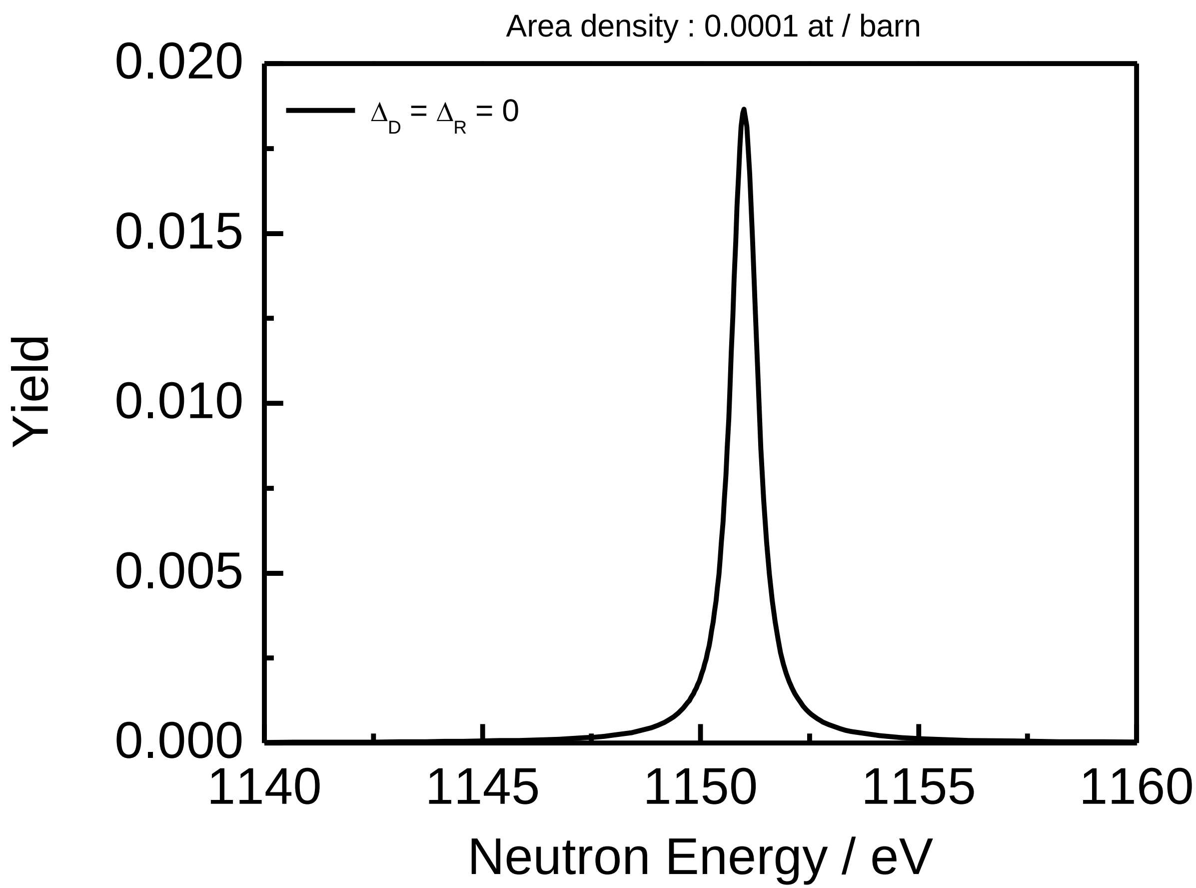
$R(t_d,E_n)$



Theoretical yield

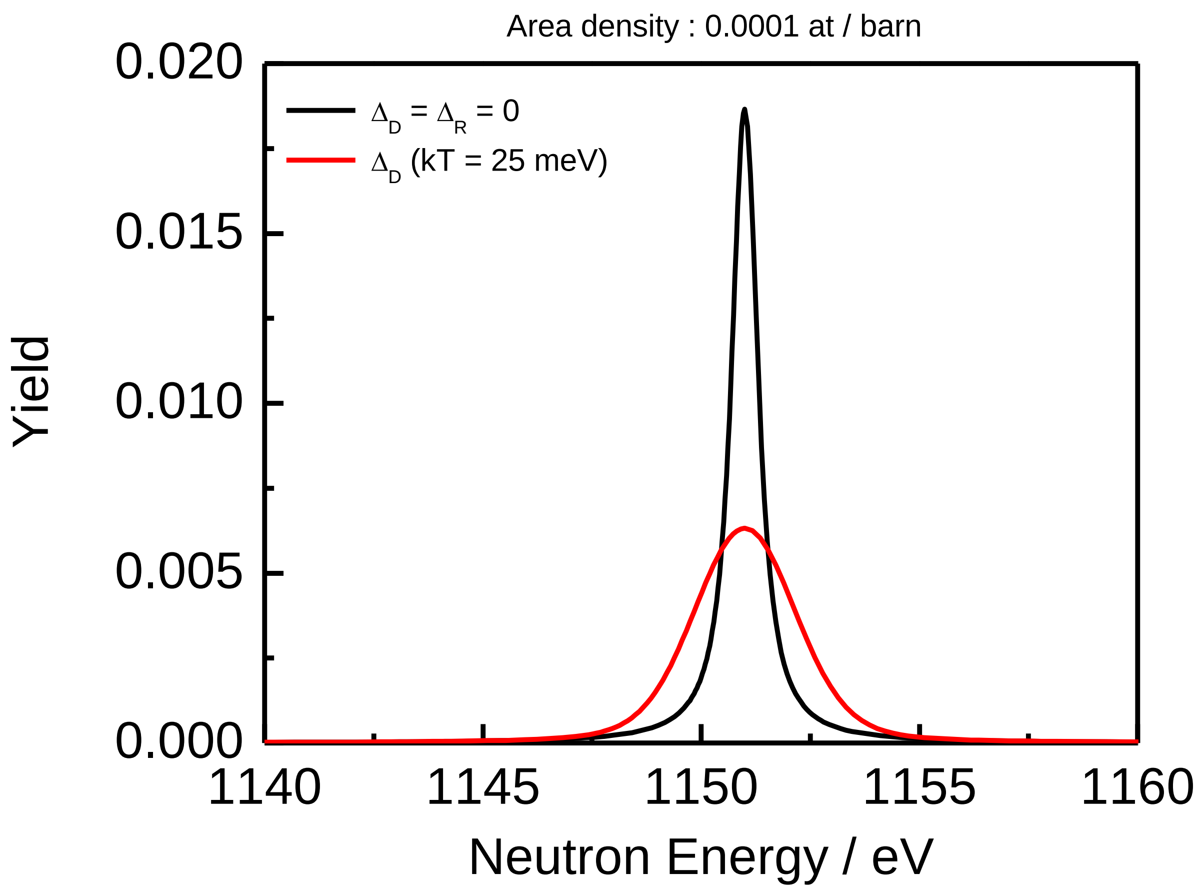
^{56}Fe 1.15 keV resonance

$$Y_\gamma \cong \frac{\sigma_\gamma}{\sigma_{tot}}(1-e^{-n\sigma_{tot}})+\ldots$$



+ Doppler

^{65}Fe 1.15 keV resonance



$$Y_\gamma \cong \frac{\bar{\sigma}_\gamma}{\bar{\sigma}_{tot}}\left(1 - e^{-n\bar{\sigma}_{tot}}\right) + \ldots$$

$$\bar{\sigma}(E) = \int dE' S(E') \sigma(E - E')$$

$$\Delta_D = \sqrt{\frac{4 E k_B T}{m_X / m_n}}$$

$$FWHM = 2\sqrt{\ln 2}\, \Delta_D$$

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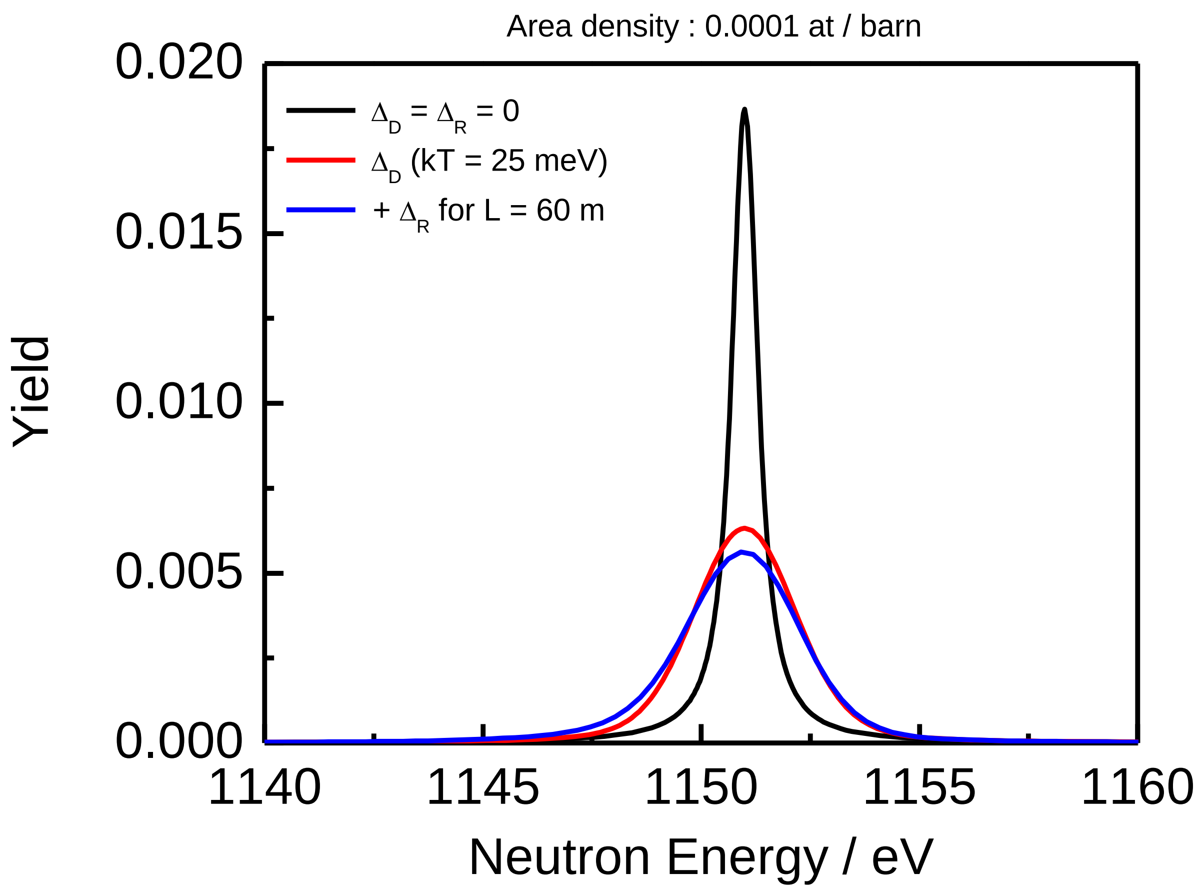
+Doppler + Response

^{65}Fe 1.15 keV resonance



$$Y_\gamma \cong \frac{\bar{\sigma}_\gamma}{\bar{\sigma}_{tot}}\left(1 - e^{-n\bar{\sigma}_{tot}}\right) + \ldots$$

$$\bar{\sigma}(E) = \int dE' S(E')\sigma(E - E')$$

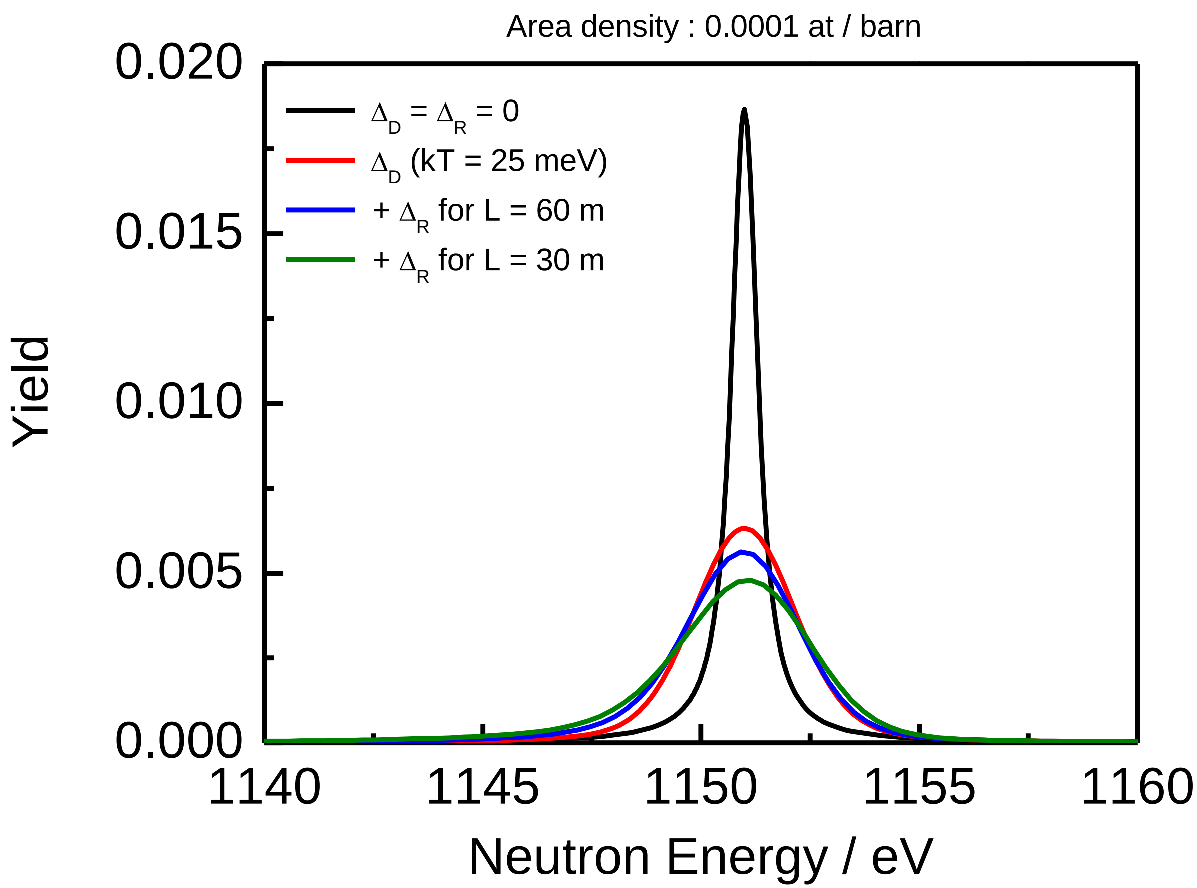
$$Y_{exp} = \frac{\int R(t,E) Y_\gamma(E) dE}{\int R(t,E) dE}$$

L = 60 m

$$\frac{\Delta E}{E} = 2\sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}$$

+Doppler + Response

^{65}Fe 1.15 keV resonance



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$$Y_{exp} = \frac{\int R(t,E) Y_\gamma(E) dE}{\int R(t,E) dE}$$

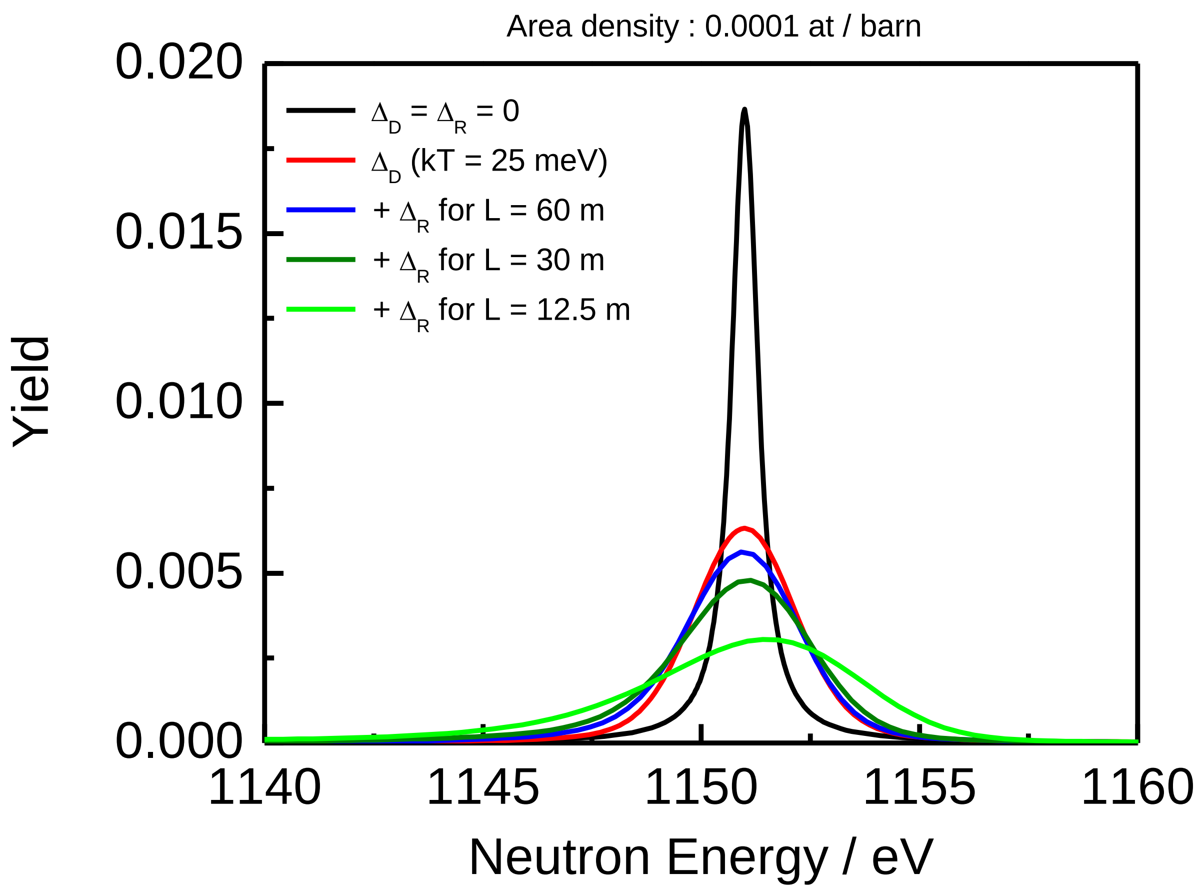
L = 60 m

L = 30 m

$$\frac{\Delta E}{E} = 2\sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}$$

+Doppler + Response

^{65}Fe 1.15 keV resonance



$$Y_\gamma \cong \frac{\bar{\sigma}_\gamma}{\bar{\sigma}_{tot}}\left(1 - e^{-n\bar{\sigma}_{tot}}\right) + \ldots$$

$$\bar{\sigma}(E) = \int dE' S(E')\sigma(E - E')$$

$$Y_{exp} = \frac{\int R(t,E) Y_\gamma(E) dE}{\int R(t,E) dE}$$

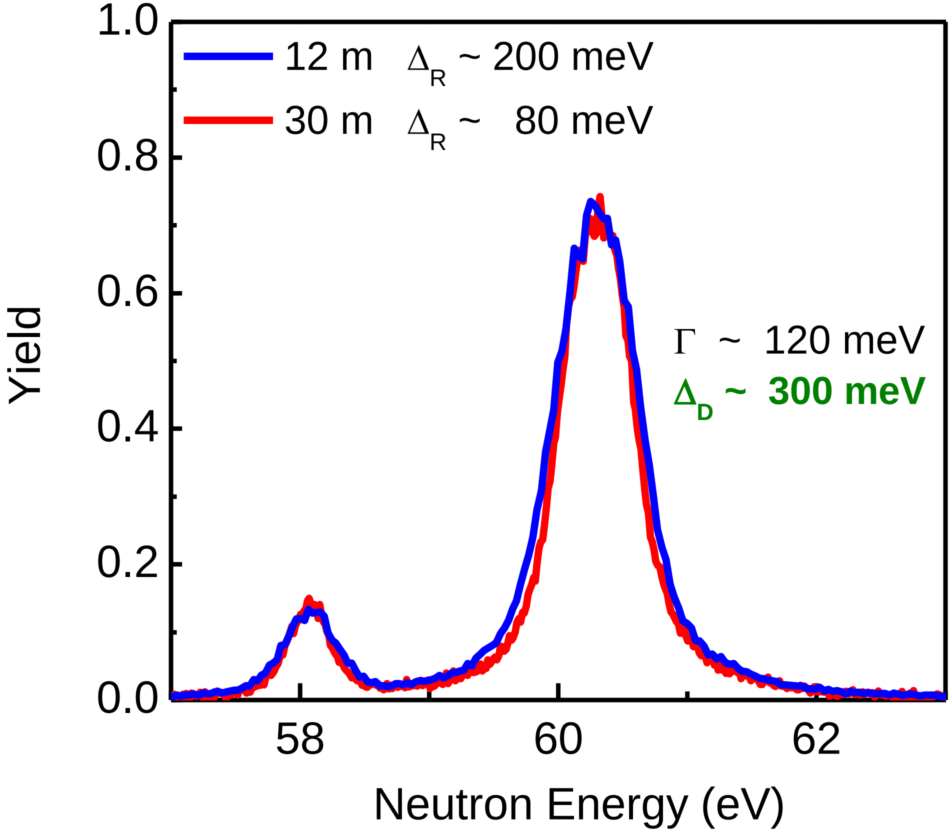
L = 60 m
L = 30 m
L = 12.5 m

$$\frac{\Delta E}{E} = 2\sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}$$

^{197}Au(n,γ) at L=12m and 30 m

$$\frac{\Delta E}{E} = 2\sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}$$

$$\Delta_D = \sqrt{\frac{4 E k_B T}{m_X / m_n}} \qquad \mathrm{FWHM} = 2\sqrt{\ln 2}\,\Delta_D$$



^{197}Au(n,γ) at L=12m and 30 m

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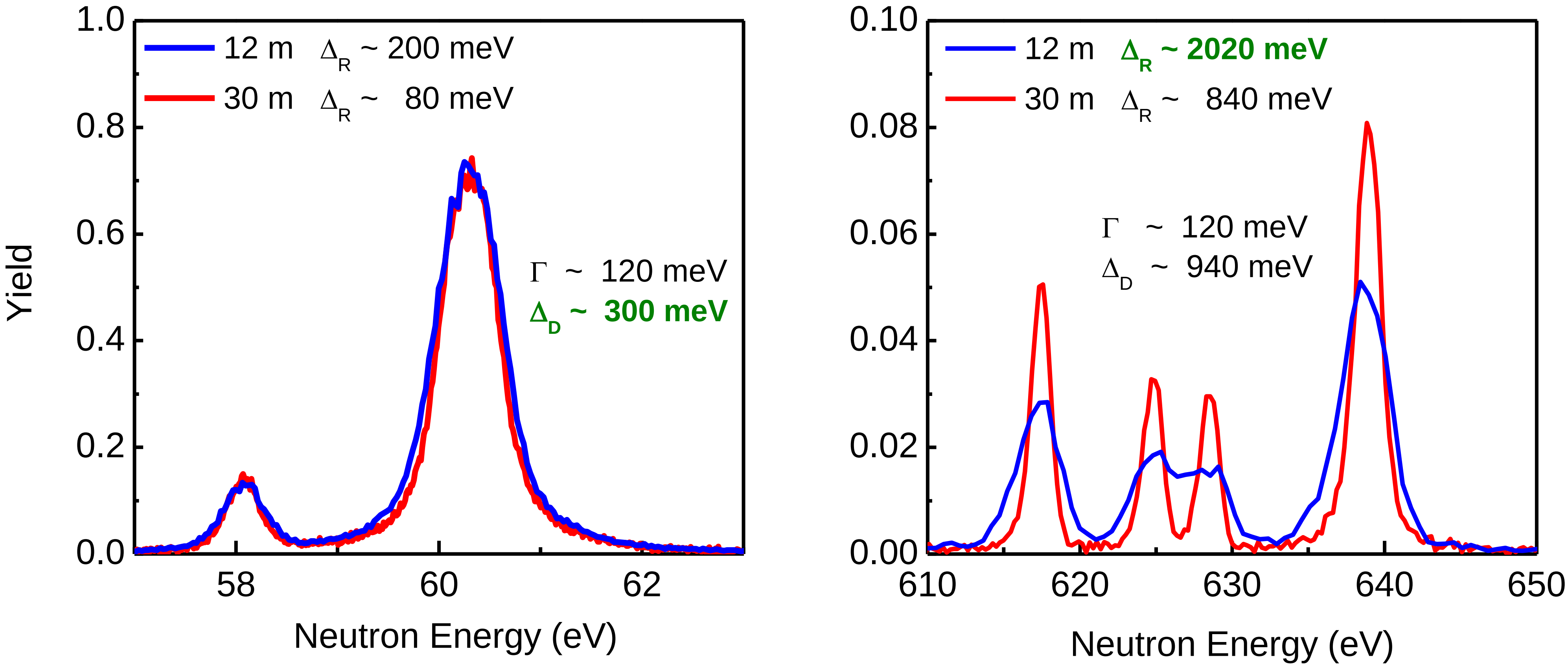


$^{56}Fe(n,\gamma)$ at L= 30 m

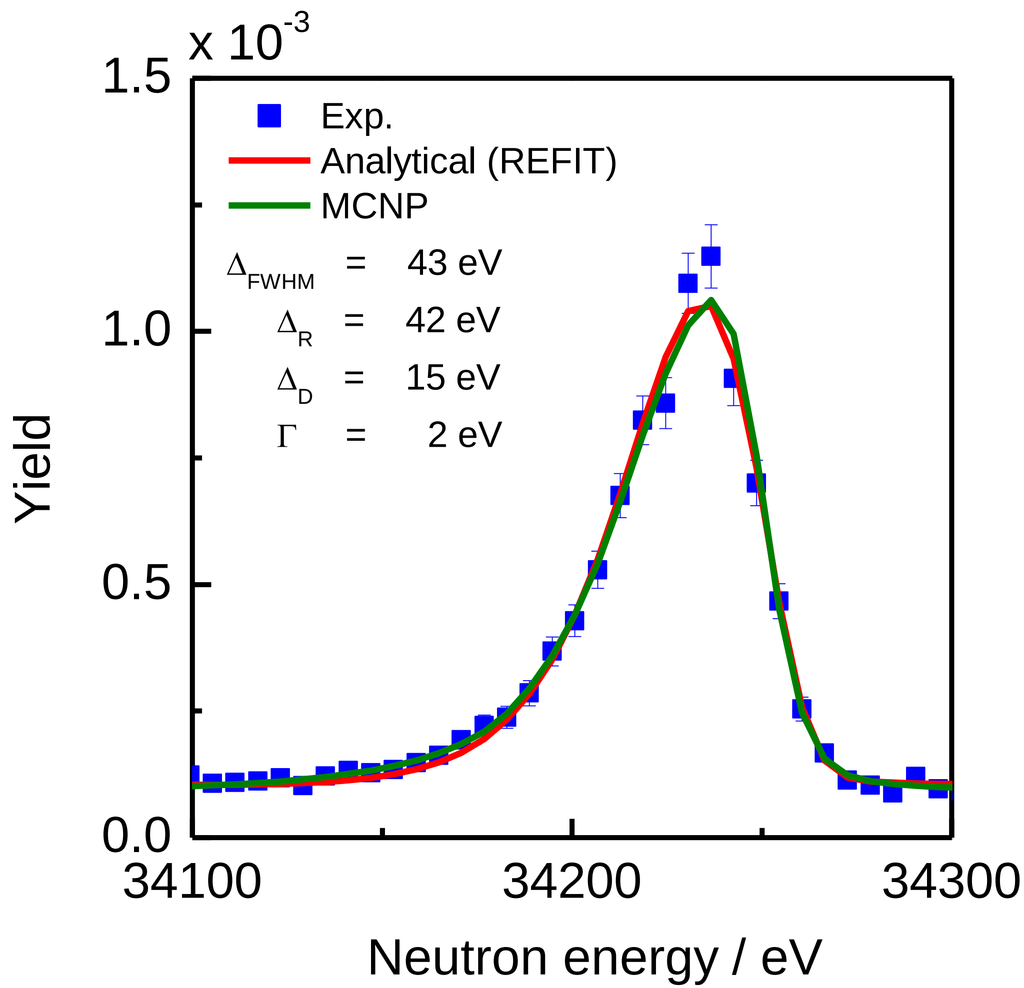
$$\Delta_{FWHM} = \sqrt{\Gamma^2 + \Delta_D{}^2 + \Delta_R{}^2}$$

dominated by Δ_R or Δ_D

with

- **Δ_R Experimental resolution**
- **Δ_D Doppler broadening**
- **Γ Total resonance width**

⇒ effective experimental observable is resonance area



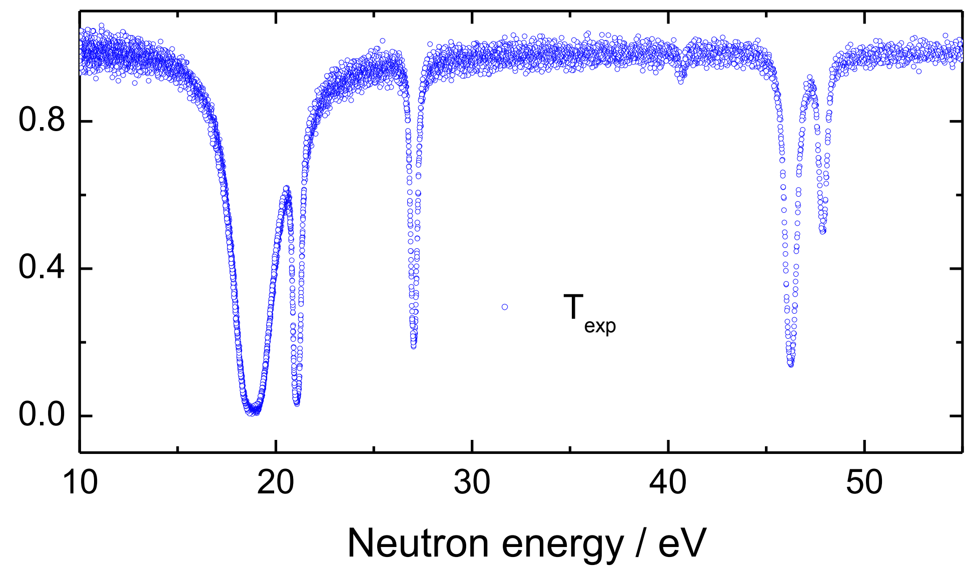
Sample inhomogeneity

Declared : $n_W = (1.084 \pm 0.014)\ 10^{-3}$ at/b

Heterogeneous sample:

$$\bar{T} = \int T(n')p(n')dn' = \int e^{-n'\sigma_{tot}} p(n')dn'$$

^{nat}W-powder mixed with ^{nat}S-powder
(80 cm diameter, 14 g ^{nat}W, 3.5 g ^{nat}S)



Transmission measurements

- a 25 m station of GELINA

- ^{6}Li detector

Sample inhomogeneity

Declared : $n_W = (1.084 \pm 0.014)\ 10^{-3}$ at/b

T_M (hom.) : $n_W = (0.939 \pm 0.003)\ 10^{-3}$ at/b

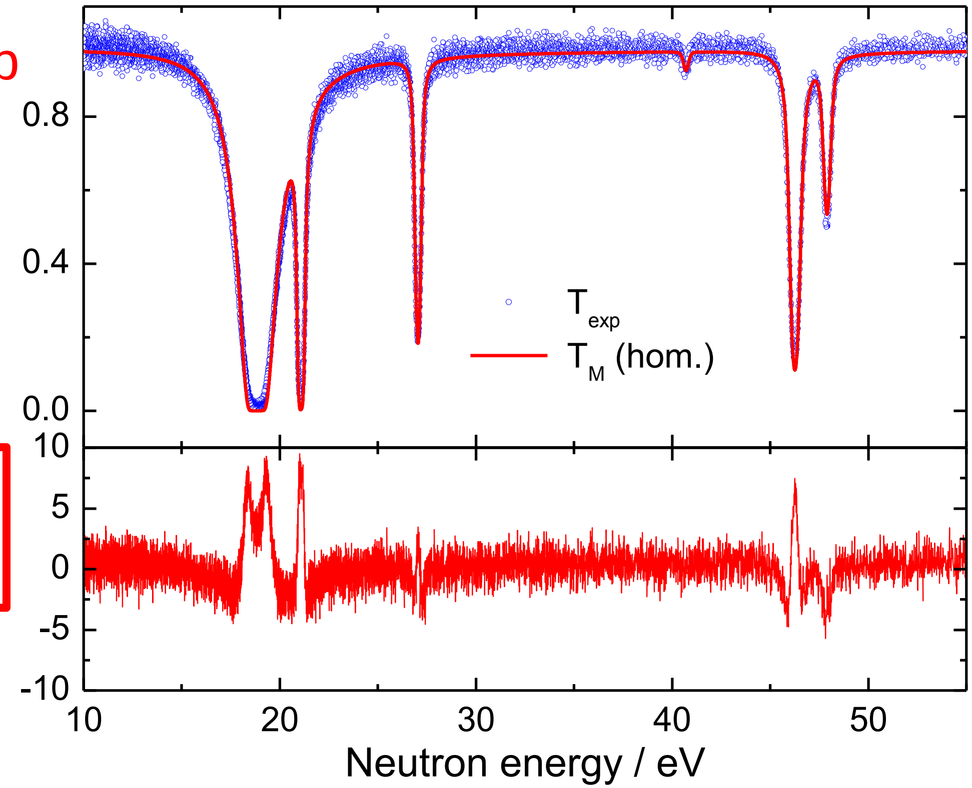
Heterogeneous sample:

$$\bar{T} = \int T(n')p(n')dn' = \int e^{-n'\sigma_{tot}} p(n')dn'$$

$$\neq$$

$$T(\bar{n}) = e^{-\bar{n}\sigma_{tot}}$$

^{nat}W-powder mixed with ^{nat}S-powder (80 cm diameter, 14 g ^{nat}W, 3.5 g ^{nat}S)



T_M: REFIT + JEFF 3.2

Sample inhomogeneity

Declared : $n_W = (1.084 \pm 0.014)\ 10^{-3}$ at/b

T_M (hom.) : $n_W = (0.939 \pm 0.003)\ 10^{-3}$ at/b

T_M (inhom.) : $n_W = (1.096 \pm 0.003)\ 10^{-3}$ at/b

$$\bar{T} = \int T(n')p(n')dn' = \int e^{-n'\sigma_{tot}} p(n')dn'$$

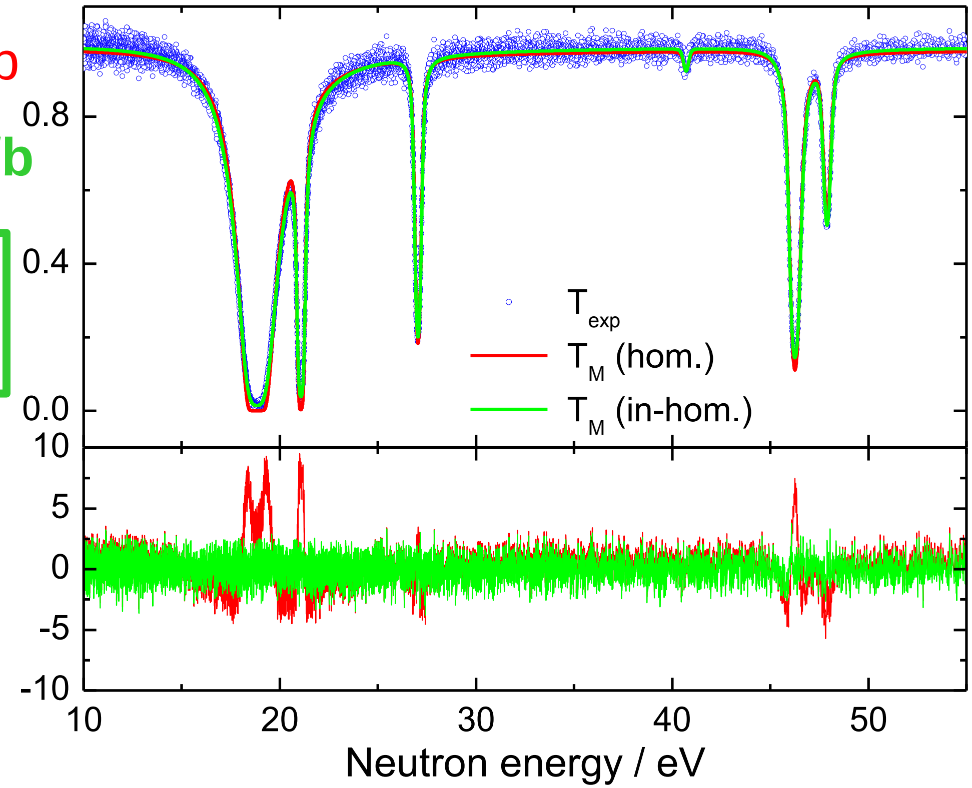
LP Model

Levermore, Pomraning et al., J. Math. Phys. 27, 2526, 1986

Implemented in REFIT

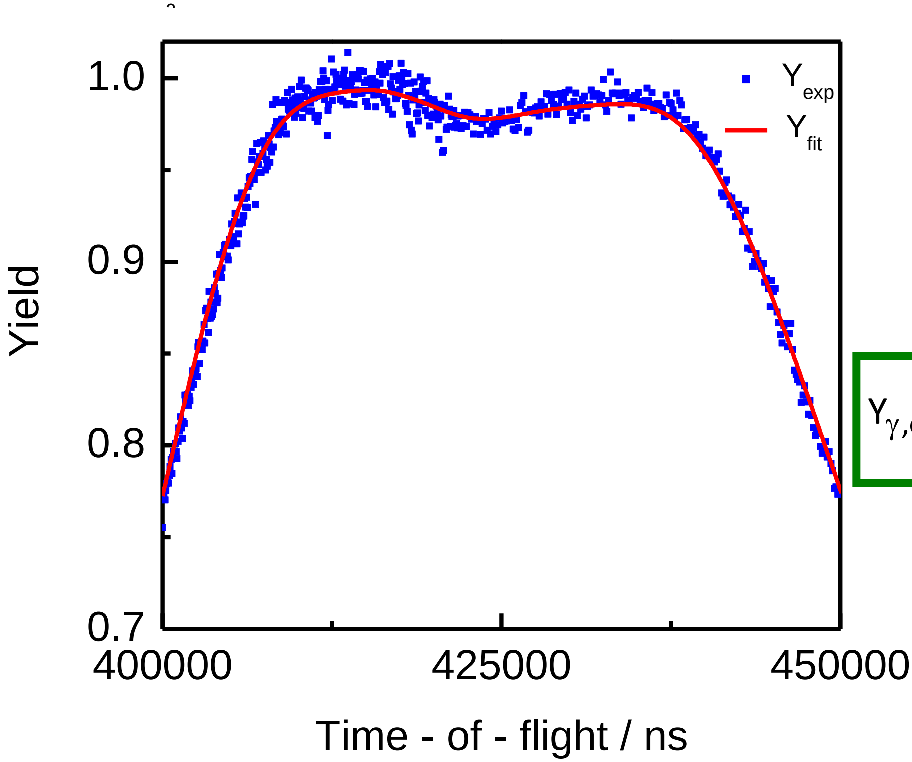
Becker et al., Eur. Phys. J. Plus 129 (2014) 58 - 9

^{nat}W-powder mixed with ^{nat}S-powder
(80 cm diameter, 14 g ^{nat}W, 3.5 g ^{nat}S)

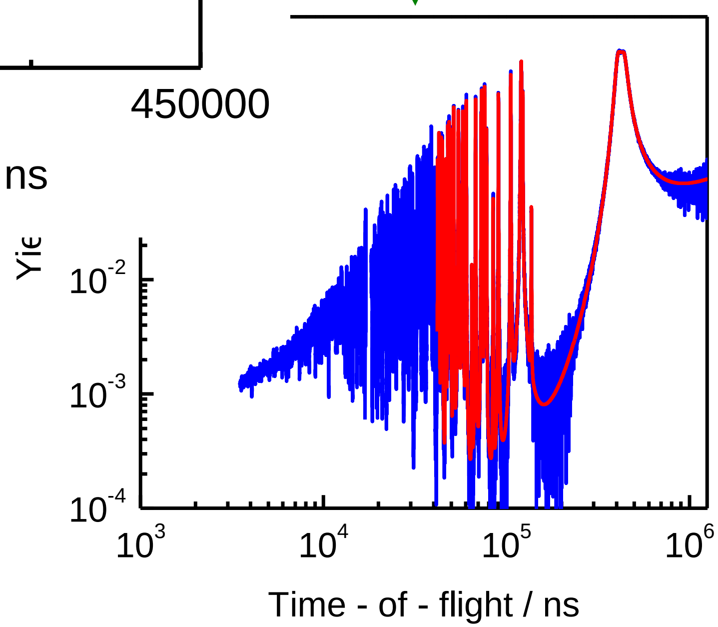


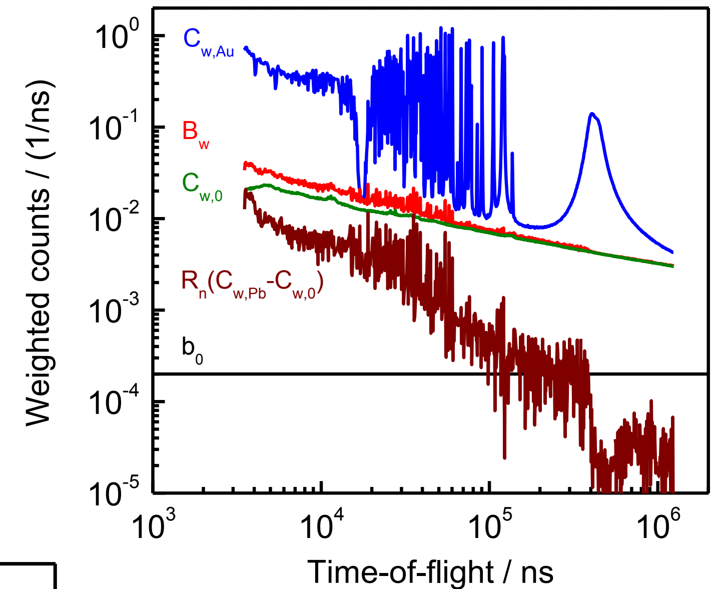
T_M: REFIT + JEFF 3.2

Normalization at saturated resonances



$$Y_{\gamma,exp} = N_C \frac{C_w - B_w}{C_\varphi - B_\varphi} Y_\varphi$$





Saturated resonance at 4.9 eV with $\Gamma_n << \Gamma_\gamma$

$\Rightarrow$ no reference cross section except for shape of $^{10}B(n,\alpha)$

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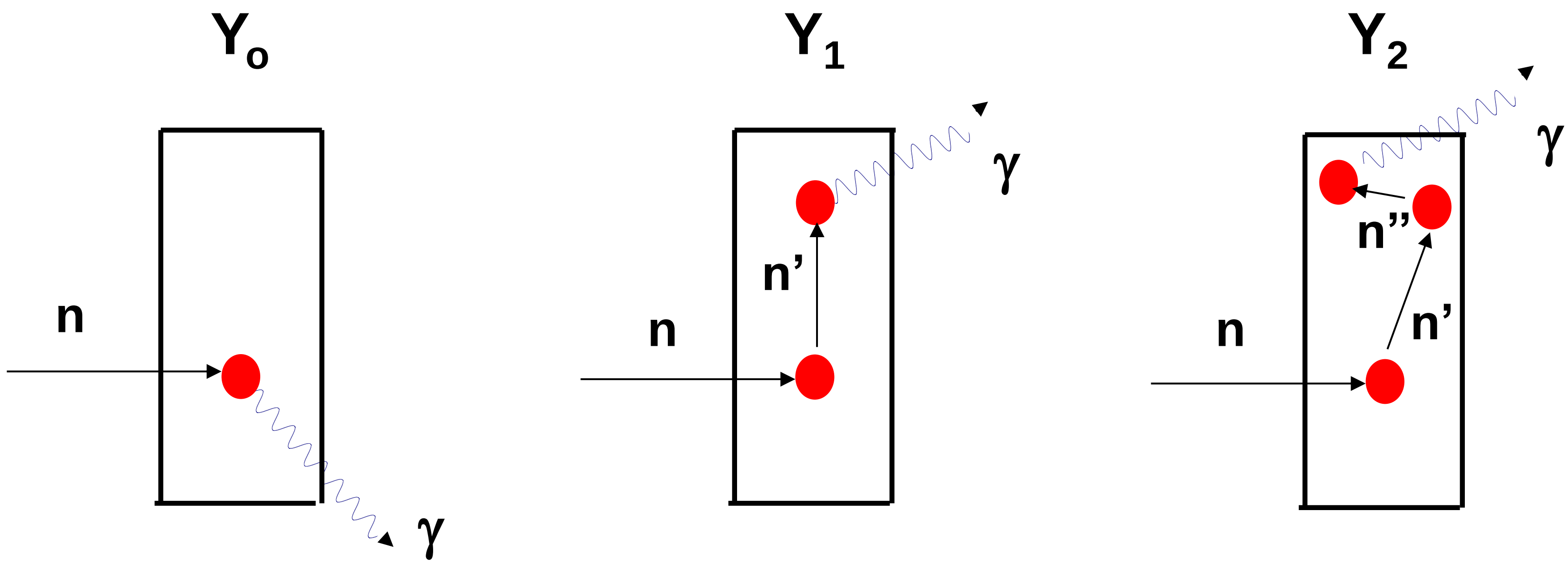
Multiple Scattering

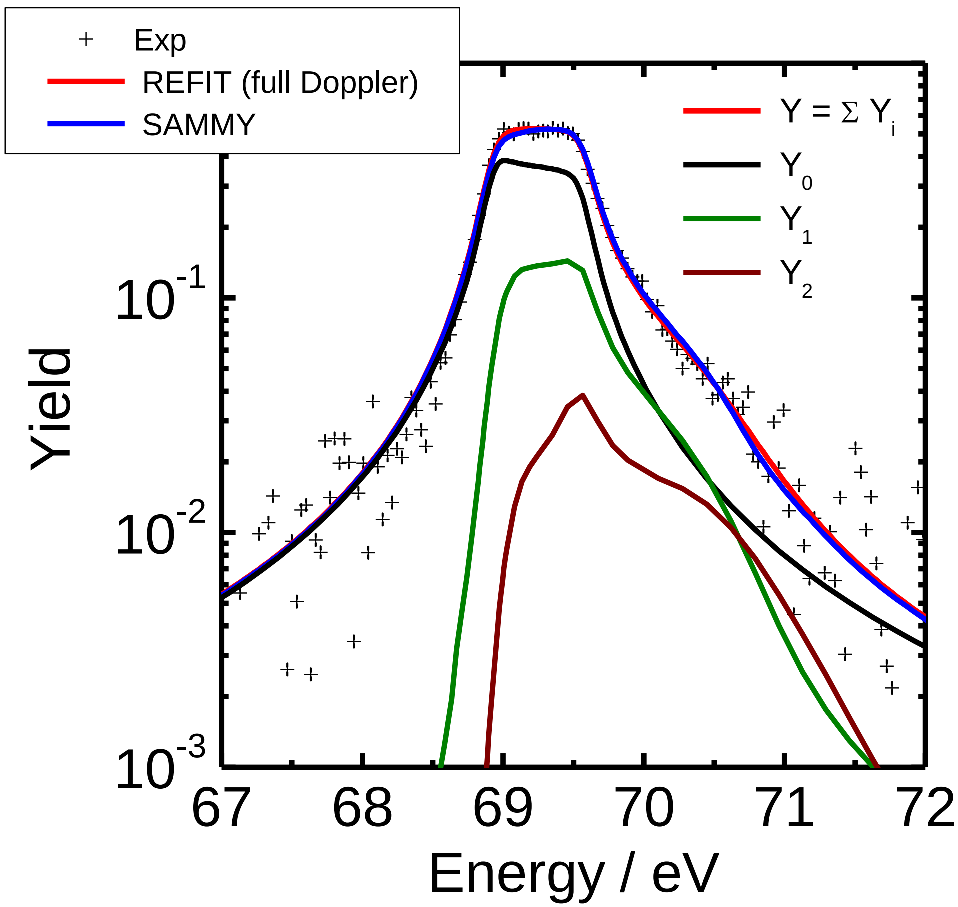
$$Y_r = Y_0 + Y_1 + Y_2 + \ldots = Y_0 + Y_m$$



$$Y_0 = (1 - e^{-n\,\bar{\sigma}_{tot}})\,\frac{\bar{\sigma}_\gamma}{\bar{\sigma}_{tot}}$$

Multiple Scattering

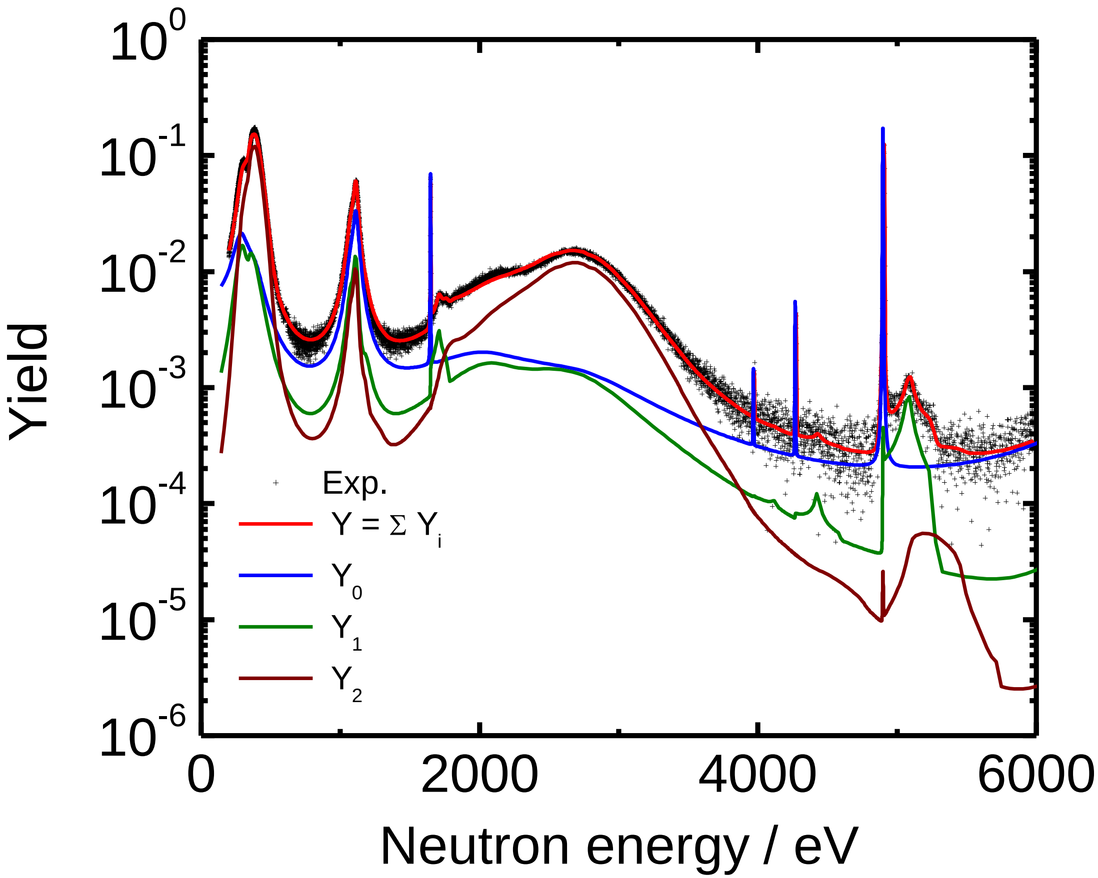
69 eV resonance in $^{232}Th+n$



Th metal disc
80 mm diameter
1 mm thick

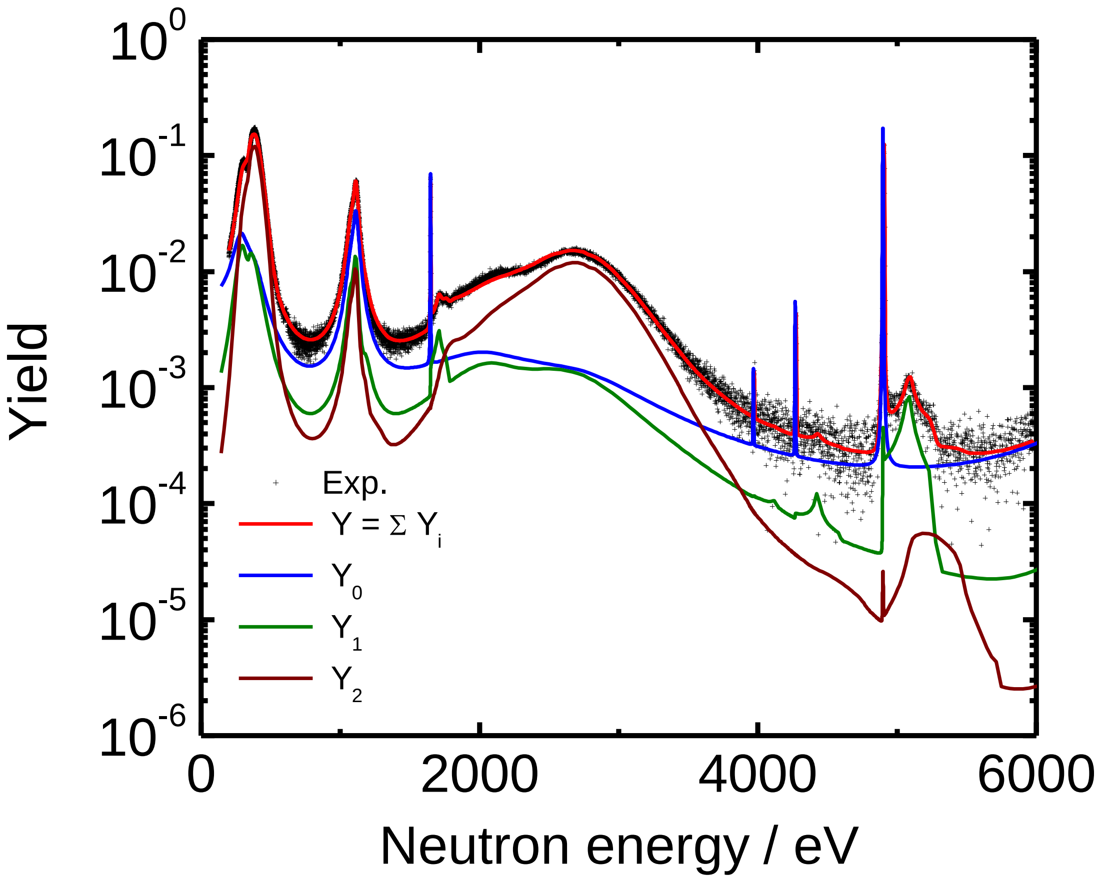
Multiple Scattering

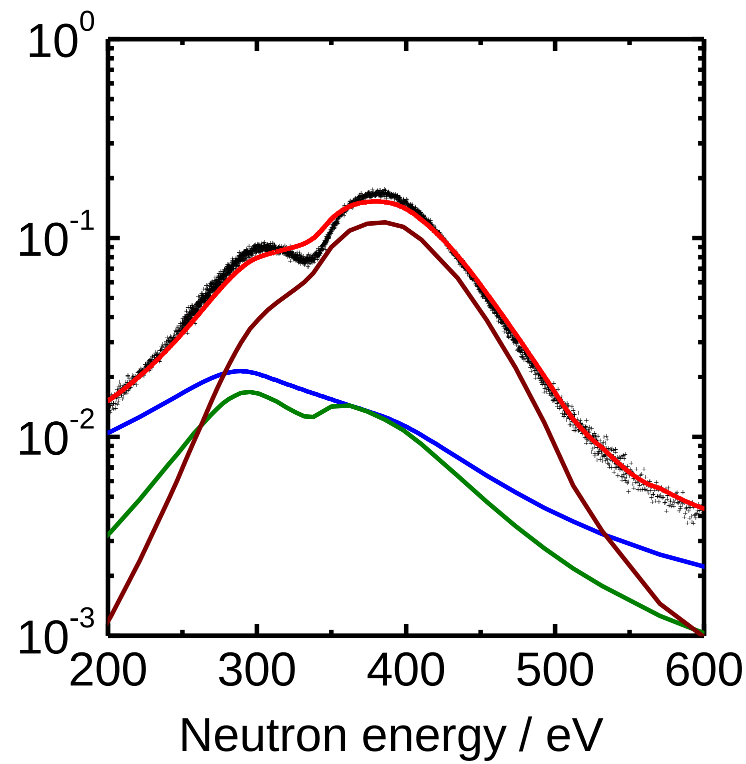
^{55}Mn + n: Sputtering target , 77 mm diameter and 3 mm thick



Multiple Scattering

^{55}Mn + n: Sputtering target , 77 mm diameter and 3 mm thick

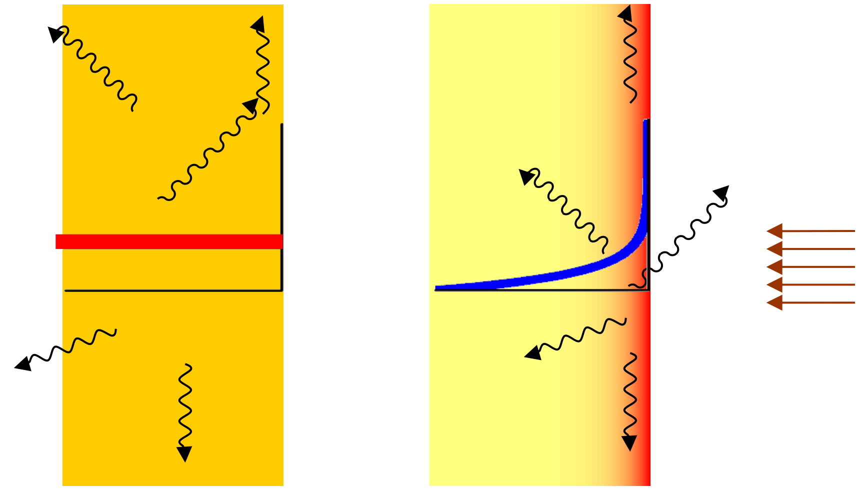




Gamma Attenuation

γ- ray attenuation depends on resonance strength

Each resonance requires special WF



WR
Weak
Resonance

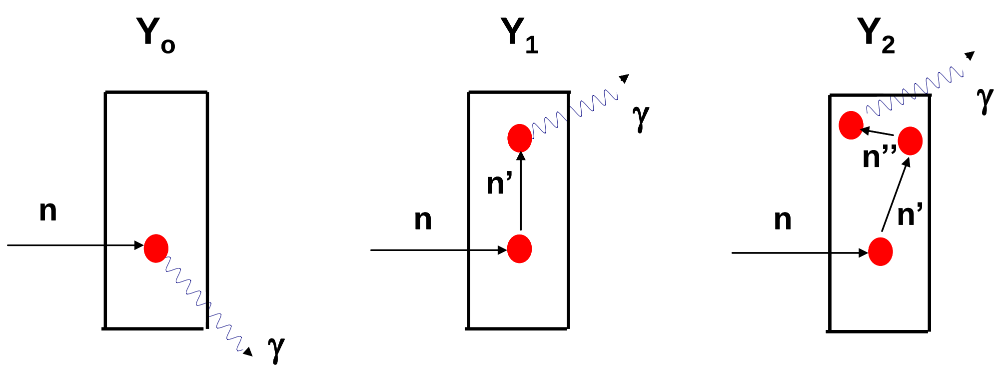
SR
Strong
Resonance

γ - ray attenuation depends on $n\sigma_{tot}$

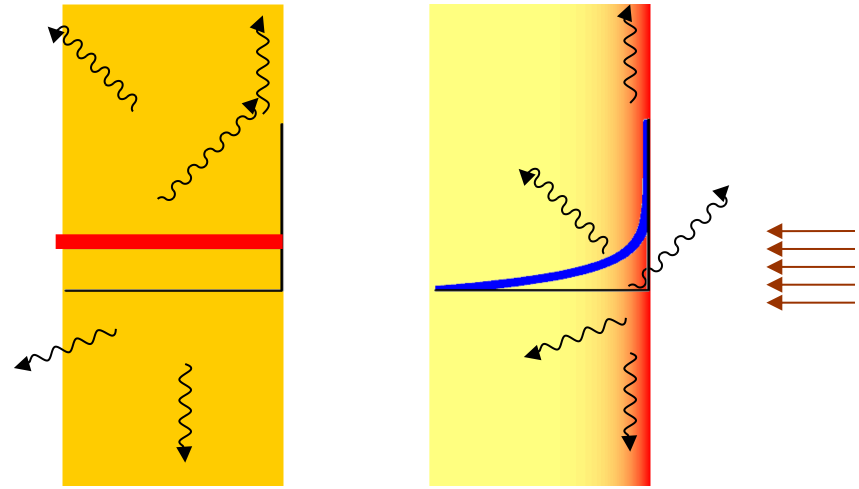
γ- ray attenuation depends on resonance strength

Each resonance requires special WF

+ each component Y_o, Y_1, Y_2



⇒ in practice not possible



WR
Weak
Resonance

SR
Strong
Resonance

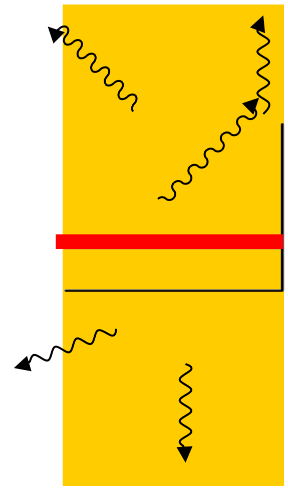
Schillebeeckx et al., Nuclear Data Sheets, December 2012

Gamma Attenuation

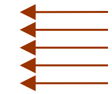
Procedure:

(1) Analyse experimental data for WR

i.e. supposing homogeneous distribution of γ-rays



WR
Weak
Resonance



Gamma Attenuation

Procedure:

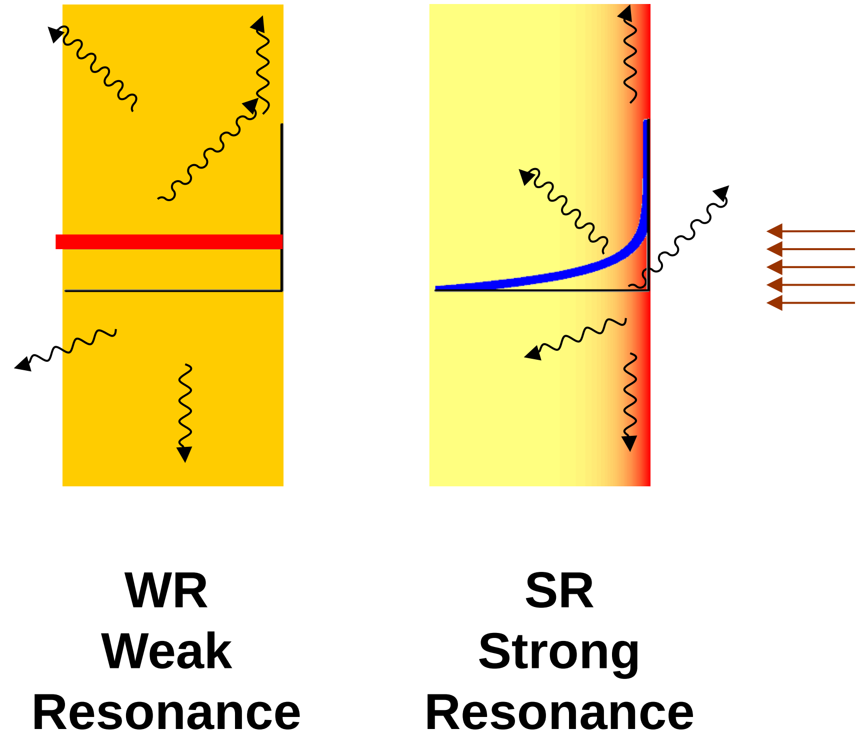
(1) Analyse experimental data for WR

i.e. supposing homogeneous distribution of γ-rays

(2) Correction factor on calculated yield mostly based on calculations

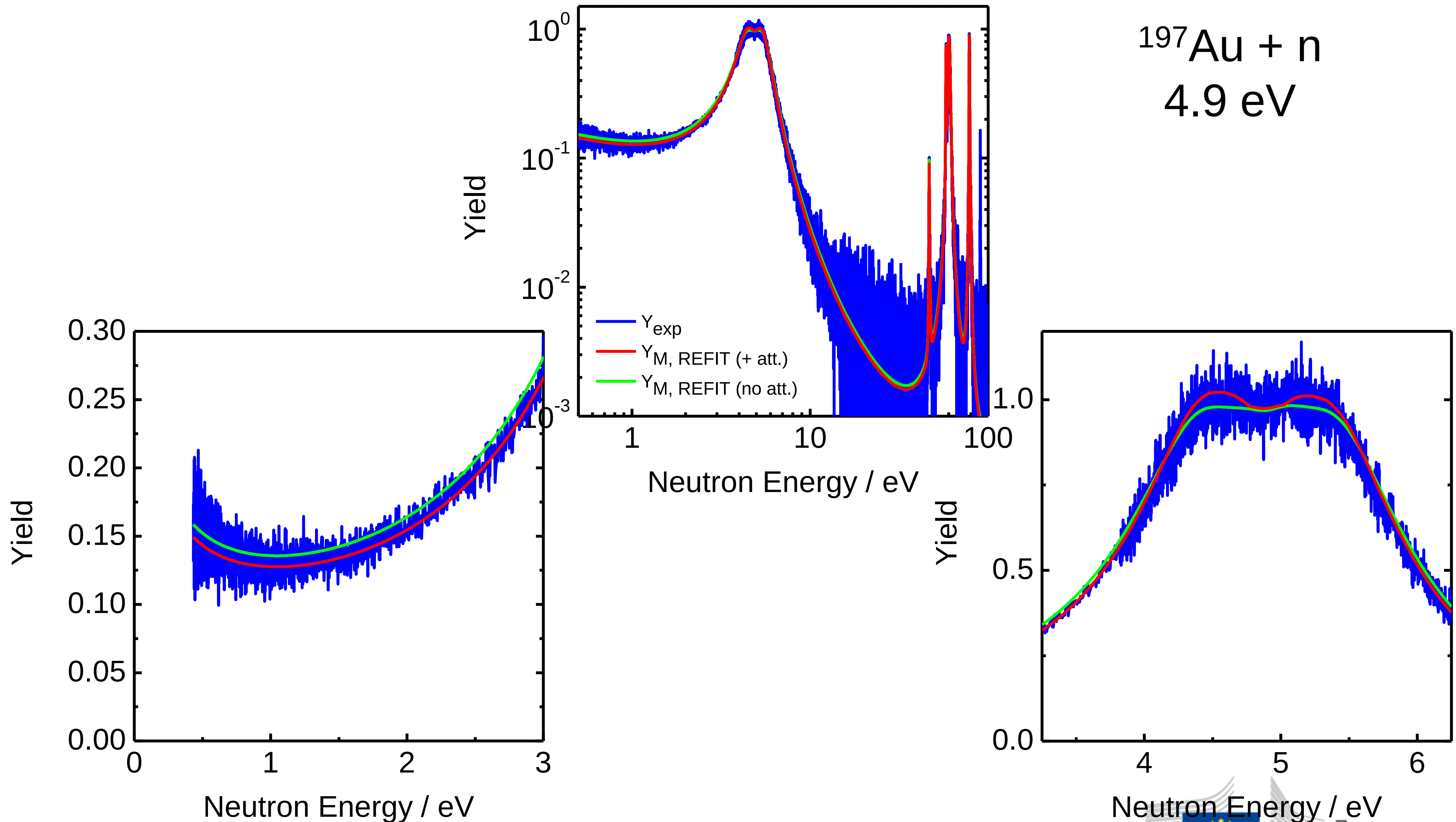
$$F_0(n\sigma_{tot}) \qquad F_m(n\sigma_{tot}) \approx 1$$

$$Y_c = F_0 \, Y_0(E_n) + F_m \, Y_m(E_n)$$

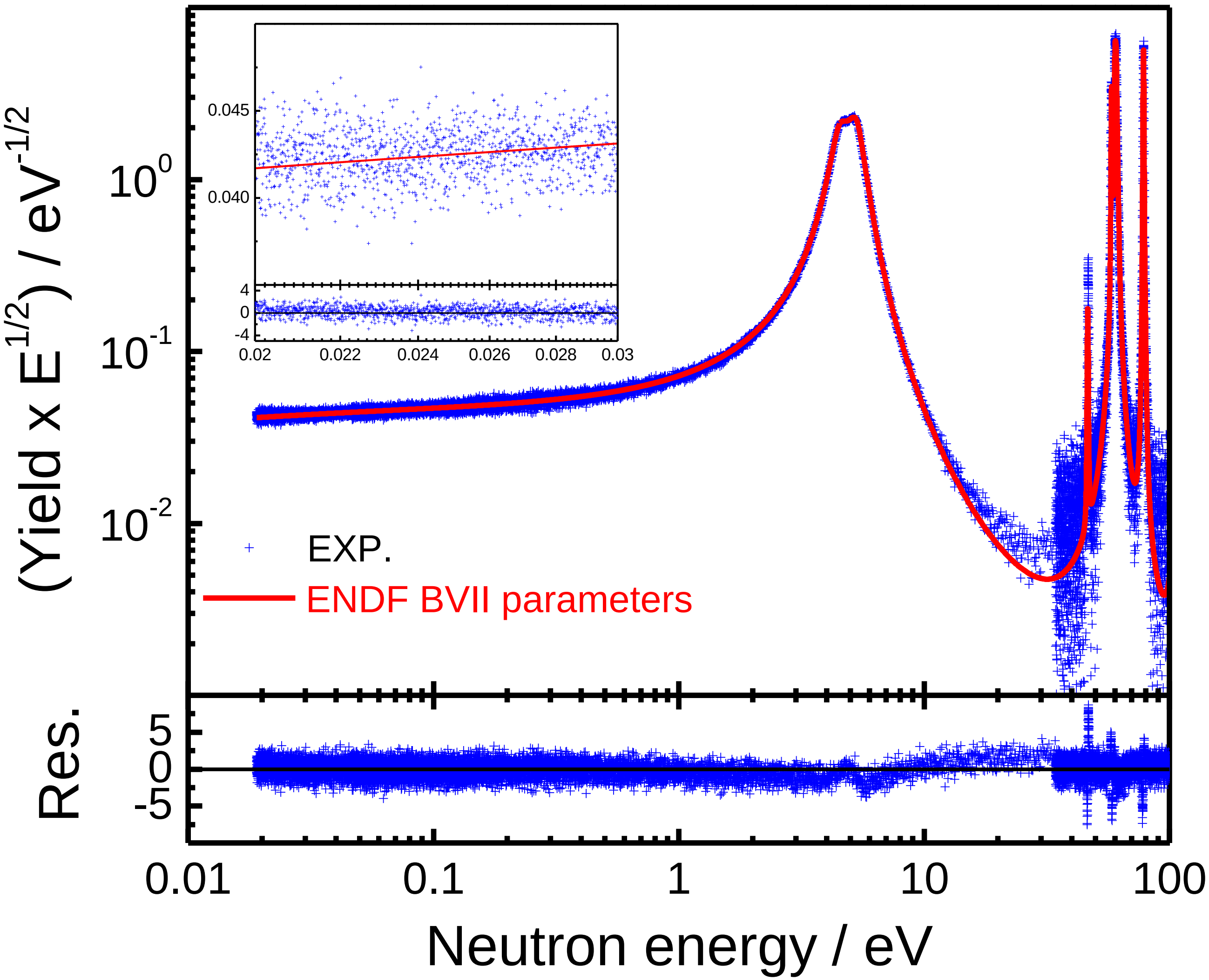


Schillebeeckx et al., Nuclear Data Sheets, December 2012

Gamma Attenuation



Gamma Attenuation



Standard :

From transmission

$\sigma(n_{th}, \gamma) = (98.7 \pm 0.1)$ b

GELINA :

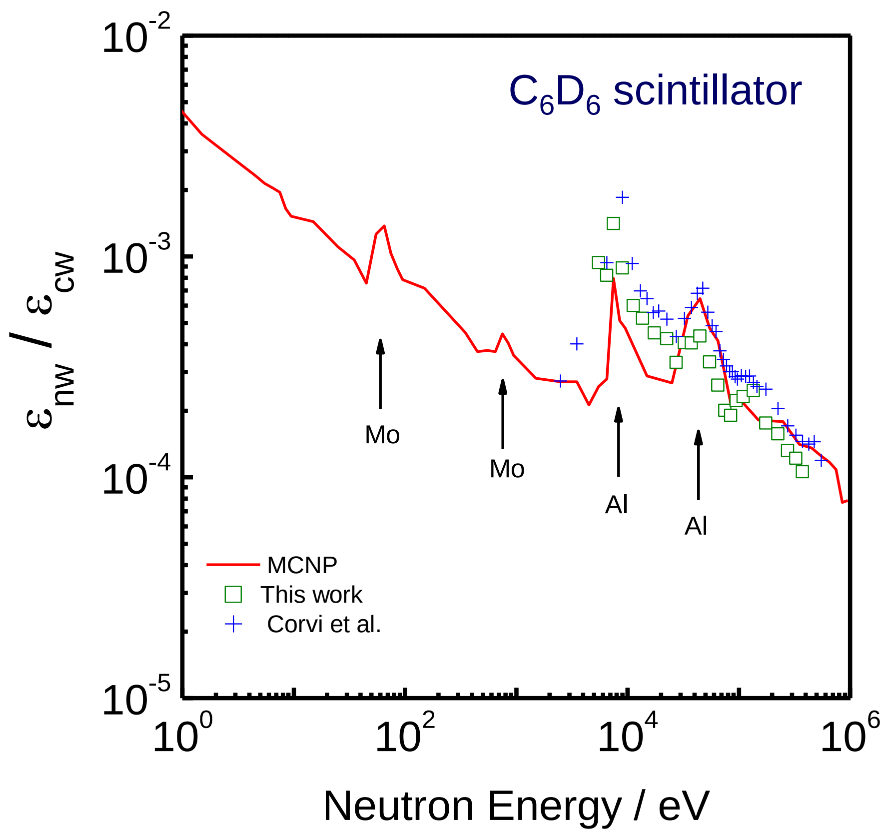
From capture

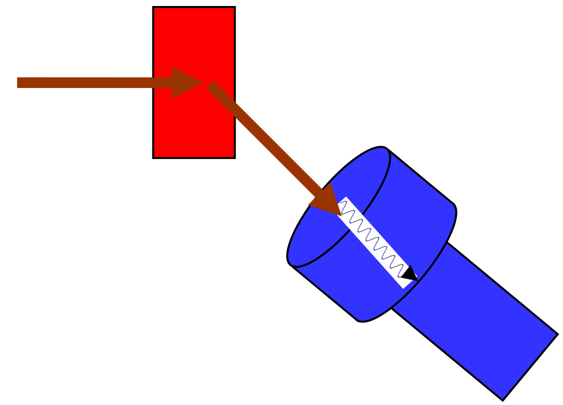
$\sigma(n_{th}, \gamma) = (99.0 \pm 1.0)$ b

Schillebeeckx et al., JKPS 59 (2011) 1563

Neutron Sensitivity

Neutron scattered by the sample creates a capture event in the detector





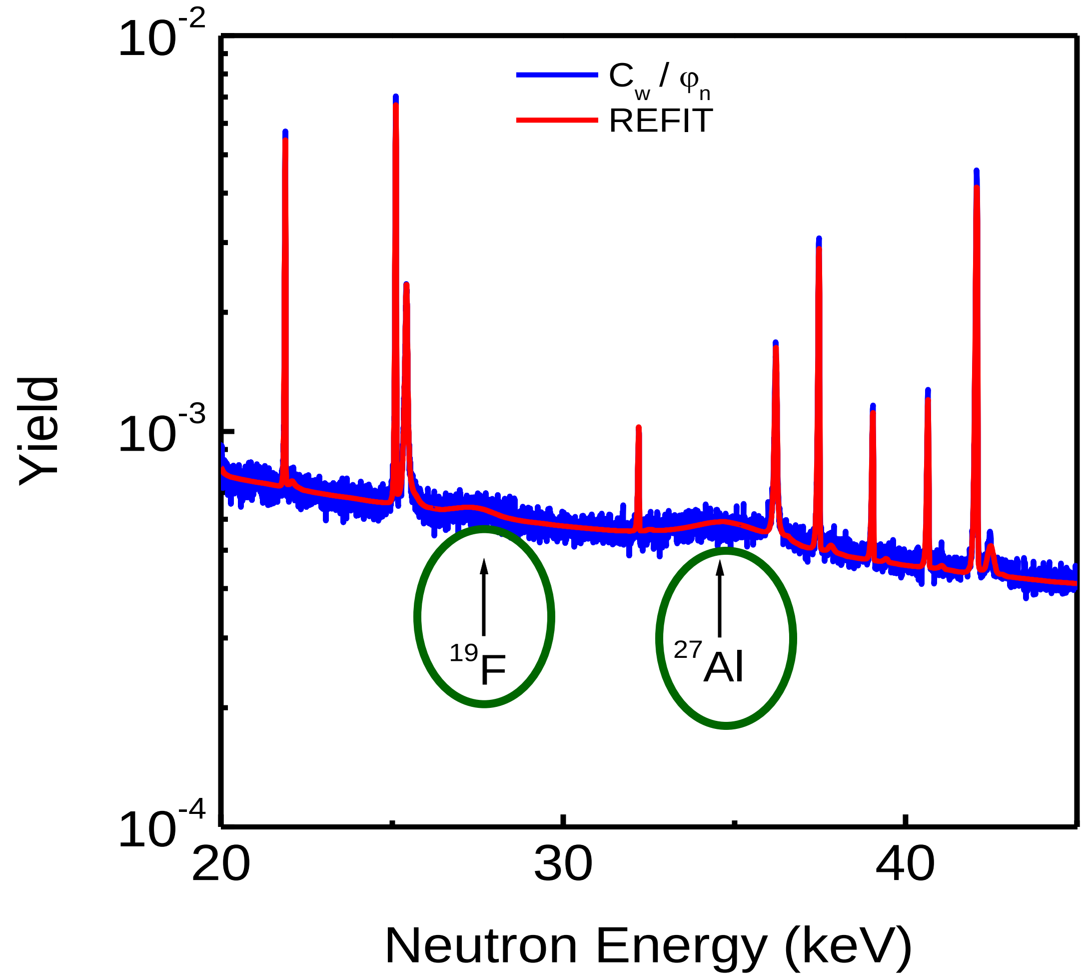
- **Mo from μ-metal**
- **F from teflon in C_6D_6**
- **Al from**
  - **Sample holder**
  - **Detector structure**

Neutron Sensitivity

REFIT : correction for NS

$$Y_M = \varepsilon_c\ Y_c + K_{ns}\ Y_n$$

$$Y_c = F_0\ Y_0(E_n) + F_m\ Y_m(E_n)$$



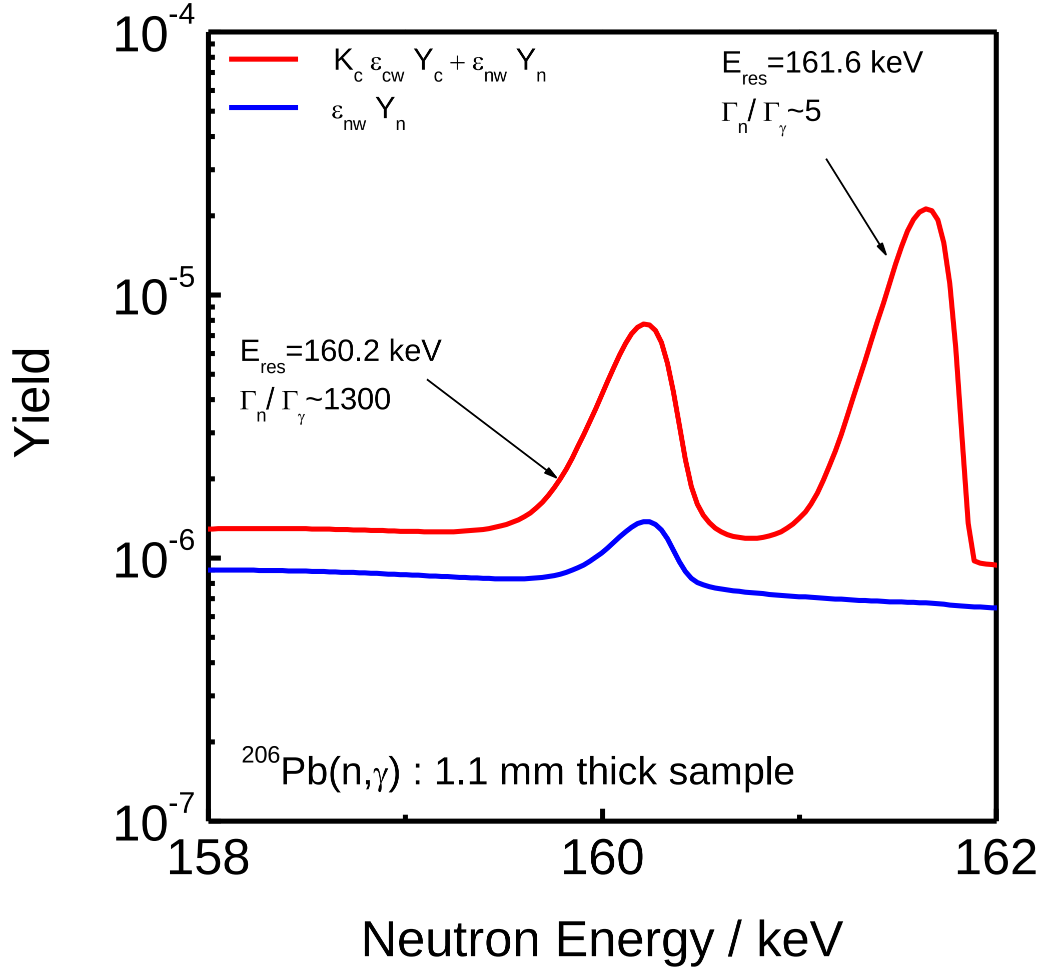
$^{206}Pb(n,\gamma)$

REFIT : correction for NS

$$Y_M = \varepsilon_c\, Y_c + K_{ns}\, Y_n$$

$$Y_c = F_0\, Y_0(E_n) + F_m\, Y_m(E_n)$$

includes impact of multiple interaction events



Borella et al., NIM A 577 (2007) 626

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Conclusions

- Modelling of experimental effects
    - Important for reliable determination of nuclear parameters

- The question "How good is the experimental model" remains unanswered

- Uncertainty of nuclear parameters are determined in an interplay between (experimental) model and experimental uncertainties

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