SOME EXAMPLES OF UNCERTAINTY ANALYSIS IN ACTIVATION MEASUREMENTS

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Outlook

- Importance of the correct uncertainty estimation
- Uncertainty analysis of a neutron induced activation measurement at single energy point
- Energy-energy correlations
- Uncertainties of interfering reaction cross sections deduced from measurements using samples with different isotopic compositions
- Reaction-reaction correlations

Importance of the correct uncertainty estimation

"A primary benefit of improved nuclear data lies in the perspective of cost reductions in developing and operating nuclear reactors; with precise nuclear data, nuclear systems can be designed to reach high efficiencies whilst maintaining adequate safety standards in a cost-effective manner.

Nuclear data are an integral part of reactor codes. Such codes are used by many in the research, development and plant operations fields, often with little awareness of the impact of nuclear data on the final results. That impacts are large. It is manifestly demonstrated by sensitivity studies that relate the uncertainties of calculated system parameters to the uncertainties of the underlying nuclear data. " [1]

Table 2 . Summary of the SG26 Highest Priority Target Accuracies for Fast Reactors

		Energy Range	**Current Accuracy (%)**	**Target Accuracy (%)**
U238	σ_{inel}	6.07 ÷ 0.498 MeV	10 ÷ 20	2 ÷ 3
	σ_{capt}	24.8 ÷ 2.04 keV	3 ÷ 9	1.5 ÷ 2
Pu241	σ_{fiss}	1.35 MeV ÷ 454 eV	8 ÷ 20	2 ÷ 3 (SFR,GFR,LFR) 5 ÷ 8 (ABTR,EFR)
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[1] EUR 23977 EN : Nuclear data for sustainable nuclear energy
Coordinated action on nuclear data for industrial development in Europe CANDIDE

Uncertainty analysis of a neutron induced activation measurement at single energy point I

$$\sigma_x = \sigma_{st} \frac{A_x}{A_{st}} \frac{[I\epsilon Fn\Phi_0]_{st}}{[I\epsilon Fn\Phi_0]_x} \frac{C_{flux,x}}{C_{flux,st}} \frac{C_{low,x}}{C_{low,st}} \frac{C_{cions,x}}{C_{coins,st}}$$

Uncorrelated: σ_{st}, A_x, A_{st}, I_x, n_x, n_{st}

Correlated: *F*, $\varepsilon_x/\varepsilon_{st}$, *some correction factors*

$$F = \frac{\lambda}{exp(-\lambda t_c)\left(1 - exp(-\lambda t_e)\right)\left(1 - exp(-\lambda t_m)\right)}$$

$$F = \frac{1}{F\left(T_{1/2}\right)F\left(T_{1/2}, t_{irr}\right)F\left(T_{1/2}, t_c\right)F\left(T_{1/2}, t_m\right)}$$

$$X = F\left(z_1(m), z_2(m), \ldots, z_p(m)\right)$$

$$S^2(x) = \sum_{i=1}^{p}\left[\left(\frac{\partial F}{\partial z_k}\right)\Big|_{z_k}\right]^2 S^2(z_k) + 2\sum_{k=1}^{p}\sum_{j<k}^{p}\left(\frac{\partial F}{\partial z_k}\right)\Big|_{z_k}\left(\frac{\partial F}{\partial z_j}\right)\Big|_{z_j} S\left(z_k, z_j\right)$$

$$z_{k,j} = f_{k,j}(m)$$

$$S\left(z_k, z_j\right) = \left(\frac{\partial z_k}{\partial m}\right)\Big|_m \left(\frac{\partial z_j}{m}\right)\Big|_m S^2(m)$$

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Uncertainty analysis of a neutron induced activation measurement at single energy point II

Covariance matrix of detector efficiency

$$\epsilon_i = aE_i^b$$

$$< \delta\epsilon_i\delta\epsilon_j > = < \delta a\delta a > + b^2(lnE_i)(lnE_j) < \delta b\delta b > + (b(\ln(E_i) + b(\ln(E_j)) < \delta a\delta b >$$

Table 3 Uncertainties of interpolated values in detection efficiency

No.	Radio-nuclide	E_γ (MeV)	$\varepsilon(E_\gamma)$	Std. dev. (%)	Correlation matrix ($\times 100$)										
1	^{51}Ti	0.320	9.45 *E*-2	2.28	100										
2	^{115m}In	0.336	8.96 *E*-2	2.23	100	100									
3	^{196}Au	0.356	8.43 *E*-2	2.18	100	100	100								
4	^{113m}In	0.392	7.61 *E*-2	2.08	100	100	100	100							
5	^{58}Co	0.811	3.51 *E*-2	1.42	96	96	96	97	100						
6	^{54}Mn	0.835	3.40 *E*-2	1.40	95	95	96	96	100	100					
7	^{27}Mg	0.844	3.36 *E*-2	1.39	95	95	96	96	100	100	100				
8	^{56}Mn	0.847	3.35 *E*-2	1.39	95	95	96	96	100	100	100	100			
9	^{64}Cu	1.346	2.04 *E*-2	1.07	80	80	81	82	94	95	95	95	100		
10	^{24}Na	1.369	2.01 *E*-2	1.06	79	80	80	82	94	94	94	94	100	100	
11	^{32}P	β		2.0	0	0	0	0	0	0	0	0	0	0	100

W. Mannhart, "A Small Guide to Generating Covariance of Experimental Data", INDC(NDS)-0588

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Uncertainty analysis of a neutron induced activation measurement at single energy point III

Non-linear function

Nonlinear case should be carefully applied. It is applicable only at certain conditions. The interconnection of the expectation value of the function <*f*> and the expectation values of the various variables $<x_i>$ requires the knowledge of the joint probability density function.

$$<x_i>=\mu_i \qquad f(x_i) \cong f(\mu_i) + \sum_i \left.\frac{\partial f}{\partial x_i}\right|_\mu (x_i - \mu_i)$$

$$< \delta f \delta f >= \sum_i < \delta x_i \delta x_i > + \sum_i \sum_j b_i b_j < \delta x_i \delta x_j >$$

The criterion for applicability of the formula is:

$$< dx_i dx_i > \ll \mu_i^2$$

If the explicit expression of the derivatives cannot be obtained in analytical form the approximate value of the derivatives can be calculated numerically using the expression:

$$\frac{\partial f}{\partial x_i} \approx \frac{f(x_i + \Delta x_i) - f(x_i - \Delta x_i)}{2 \Delta x_i}$$

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Uncertainty analysis of a neutron induced activation measurement at single energy point V

$$\sigma_x = \sigma_{st} \frac{S_x}{S_{st}} \frac{[I\epsilon F n \Phi_0]_{st}}{[I\epsilon F n \Phi_0]_x} \frac{C_{flux,x}}{C_{flux,st}} \frac{C_{low,x}}{C_{low,st}} \frac{C_{cions,x}}{C_{coins,st}}$$

Table 4.2: Principal sources of uncertainty and their estimated magnitudes in %.

	Neutron energy (MeV)									
	8.34	9.15	13.33	16.1	17.16	17.9	19.36	19.95	20.61	
σ_{Al}	1.9	1.9	1.6	2	2	2.2	3.1	4.1	5.4	
S_{Am}	5.0	4.0	2.5	2.1	1.5	1.3	6.3	1.4	5.7	
S_{Al}	1.0	1.0	1.0	1.0	1.0	0.7	2.0	1.0	1.6	
I_{Am}	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	
n_{Al}	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	
n_{Am}	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	
$\epsilon_{\mathrm{Al}}/\epsilon_{\mathrm{Am}}$	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	
$(f_\Sigma f_r)_{\mathrm{Am}}$	0.9	0.6	0.4	0.6	0.6	0.7	0.6	0.6	0.6	0. 6
$\frac{C_{\mathrm{low,Am}}}{C_{\mathrm{low,Al}}}$			0.3	0.3	0.3	0.3	1.3	1.4	1.4	

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Energy-energy correlations I

$$\sigma_x = \sigma_{st}\,\frac{S_x}{S_{st}}\,\frac{[I\epsilon Fn\Phi_0]_{st}}{[I\epsilon Fn\Phi_0]_x}\,\frac{C_{flux,x}}{C_{flux,st}}\,\frac{C_{low,x}}{C_{low,st}}\,\frac{C_{cions,x}}{C_{coins,st}}$$

Fully correlated: I_x, F_x, $\varepsilon_x/\varepsilon_{st}$, I_{st}, F_{st},

Partially correlated terms: σ_{st}, (n_x, n_{st})

Uncorrelated S_x, S_{st} *(only diagonal elements)*

If we use functional standard deviations (Δx/<x>) and functional covariance (V_{xy}/(<x><y>)) the covariance matrix can be presented as $C=\Sigma A_i$ with A_i relative covariance matrices corresponding to different terms needed to calculate the cross sections.

$$\begin{array}{r}13.8\\15.6\\17.6\\18\\19.3\end{array}\begin{bmatrix}5^2+1^2 & & & & \\ & 3^2+1^2 & & & \\ & & 8^2+1^2 & & \\ & & & 3^2+1^2 & \\ & & & & 3^2+1^2\end{bmatrix} + \begin{array}{r}13.8\\15.6\\17.6\\18\\19.3\end{array}\begin{bmatrix}3.7^2 & & & & \\ 3.7*4 & 4^2 & & & \\ 3.7*4 & 4*4 & 4^2 & & \\ 3.7*3.4 & 4*3.4 & 4*3.4 & 3.4^2 & \\ 3.7*3.3 & 4*3.3 & 4*3.3 & 3.4*3.3 & 3.3^2\end{bmatrix} +$$

$$+ \begin{array}{r}13.8\\15.6\\17.6\\18\\19.3\end{array}\begin{bmatrix}0.07^2 & & & & \\ & 0.07^2 & & & \\ 1*0.07*0.07 & & 0.07^2 & & \\ & & & 0.07^2 & \\ & & & & 0.1^2\end{bmatrix} + \cdots =$$

Energy-energy correlations II

$$
= \begin{array}{r}
13.8 \\ 15.6 \\ 17.6 \\ 18 \\ 19.3
\end{array}
\begin{bmatrix}
53.5449 & & & & \\
26.282 & 41.2949 & & & \\
26.3829 & 27.77 & 96.2949 & & \\
24.1868 & 25.418 & 26.65 & 37.6949 & \\
24.0456 & 25.172 & 26.97 & 24.99 & 42.29
\end{bmatrix}
$$

En	Cross section	Rel. Std.Dev.	Correlation matrix x 100				
13.8	xxx	7.32	100				
15.6	xxx	6.43	56	100			
17.6	xxx	9.81	37	44	100		
18	xxx	6.14	54	64	44	100	
19.3	xxx	6.50	51	60	42	63	100

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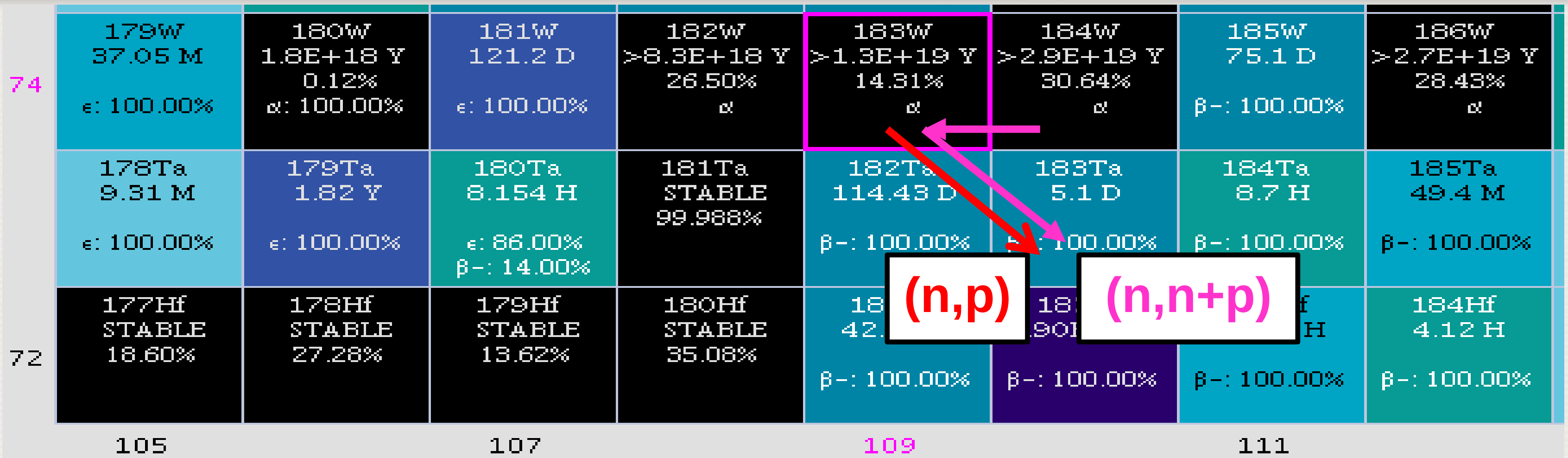
Uncertainties of interfering reaction cross sections deduced from measurements using samples with different isotopic compositions



$$RR_{183W\_np} = \frac{N_{184,es}RR_{ns} - N_{184,ns}RR_{es}}{N_{183,ns}N_{184es} - N_{184,ns}N_{183,es}}$$

$$RR_{184W\_nx} = \frac{N_{183,ns}RR_{es} - N_{183,es}RR_{ns}}{N_{183,ns}N_{184es} - N_{184,ns}N_{183,es}}$$

$$RR_{183W\_np} = \mathrm{a}_1 RR_{ns} + \mathrm{a}_2 RR_{es}$$

$$RR_{184W\_nx} = \mathrm{a}_3 RR_{es} + \mathrm{a}_4 RR_{ns}$$

$$P_1 = (x_1, x_2),\ P_2 = (x_1, x_2)$$

$$\begin{bmatrix} u_{p_1}^2 & \\ cov(p_1, p_2) & u_{p_2}^2 \end{bmatrix} = \begin{bmatrix} \frac{\partial p_1}{\partial x_1} & \frac{\partial p_1}{\partial x_2} \\ \frac{\partial p_2}{\partial x_1} & \frac{\partial p_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} {u_{x1}}^2 & \\ cov(x1, x2) & {u_{x2}}^2 \end{bmatrix} \begin{bmatrix} \frac{\partial p_1}{\partial x_1} & \frac{\partial p_2}{\partial x_1} \\ \frac{\partial p_1}{\partial x_2} & \frac{\partial p_2}{\partial x_2} \end{bmatrix}$$

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Reaction-reaction correlations

$\sigma_2[^{183}W(n,p)^{183}Ta] / \sigma_1[^{27}Al(n,\alpha)]$ $\sigma_3[^{184}W(n,x)]^{183}Ta / \sigma_1[^{27}Al(n,\alpha)]$

$$\sigma_i/\sigma_j=P_i/P_j$$

$$P_i = \frac{A_i}{\epsilon_i}\frac{1}{N_i}\sum_l k_i^l$$

$$R_{12}=P_2/P_1 \text{ and } R_{13}=P_3/P_1$$

$$<\delta R_{12}\delta R_{12}> =<\delta P_2\delta P_2>+<\delta P_1\delta P_1>-2<\delta P_1\delta P_2>$$

$$<\delta R_{13}\delta R_{13}> =<\delta P_3\delta P_3>+<\delta P_1\delta P_1>-2<\delta P_1\delta P_3>$$

$$<\delta R_{12}\delta R_{13}> =<\delta P_1\delta P_1>+<\delta P_2\delta P_3>-<\delta P_1\delta P_3>-< \delta P_2\delta P_1>$$

$$<\delta R_{13}\delta R_{12}> =<\delta P_1\delta P_1>+<\delta P_3\delta P_2>-<\delta P_3\delta P_1>-< \delta P_1\delta P_2>$$

$$\begin{bmatrix} < \delta R_{12}\delta R_{12} > & < \delta R_{12}\delta R_{13} > \\ < \delta R_{13}\delta R_{12} > & < \delta R_{13}\delta R_{13} > \end{bmatrix}$$

$$<\delta P_i\delta P_j> = <\delta A\delta A_j> + <\delta\varepsilon_i\delta\varepsilon_j> + <\delta N_i\delta N_j> +\Sigma <\delta k_i\delta k_j>$$