Workshop on EXFOR Compilation, 2013, Vienna

Covariance analysis of the measured ^{40}Ca(n, tot) cross sections up to 20 MeV

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Background —— Why covariance?

- **Covariance:** $Cov(X,Y) = \iint (X - \langle X \rangle)(Y - \langle Y \rangle) f(X,Y)\, dXdY$
- **The requirements are growing**
- **Covariance data in evaluation libraries**

Library	Numbers of nuclei with covariance
ENDF/B-VII.1	189
JENDL-4.0	95
JEFF-3.1	36

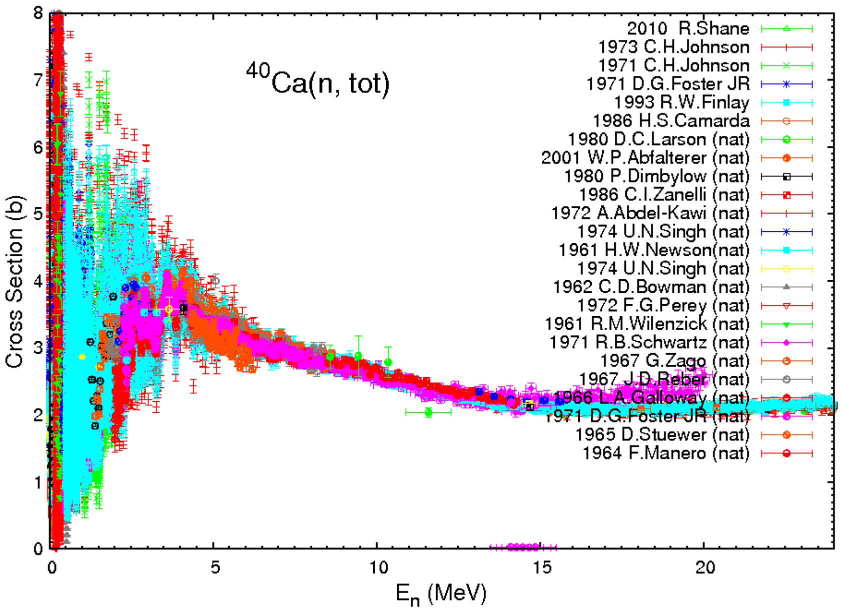
- **The main research methods:**
  - **Deterministic method**
  - **Monte Carlo**

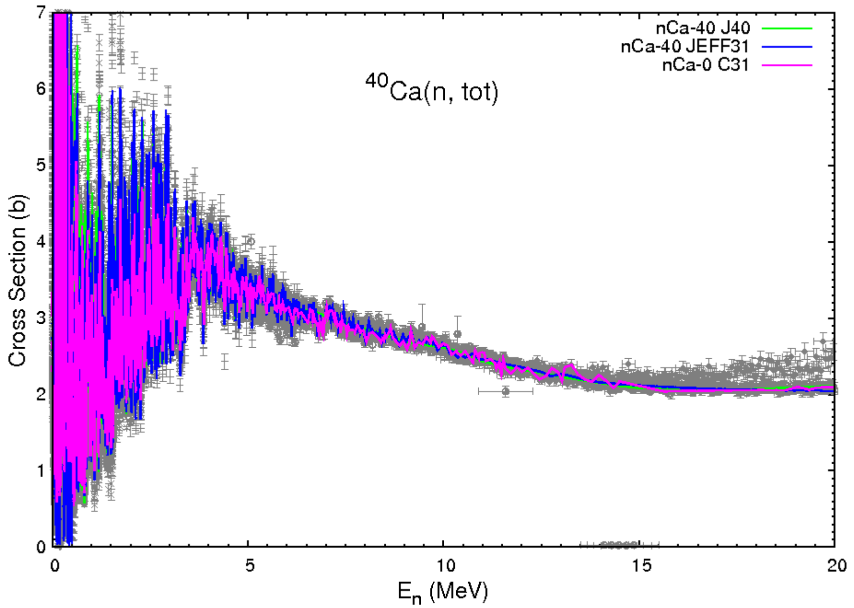
Background ——Why ^{40}Ca(n, tot) cross sections?

- **Calcium is widely existed in environment**
- **^{40}Ca is a typical structural nuclide**
- **(n, tot) cross section is the basic quantity of the neutron induced nuclei interactions**
- **Characters of ^{40}Ca(n, tot) cross section**
  - **representative structures**
  - **obvious regional features**
  - **abundant experimental data**
  - **visual divergences on experimental data**

Background —— Existing data

Reaction	Year	Sets	En_min(MeV)	En_max(MeV)	Points
^{40}Ca(n, tot)	1967 ~ 2010	11	0.04	600	10093
^{nat}Ca(n, tot)	1949 ~ 2001	43	2.11e-9	559	17859





All the evaluations have no covariance.

Background —— Experimental Principle

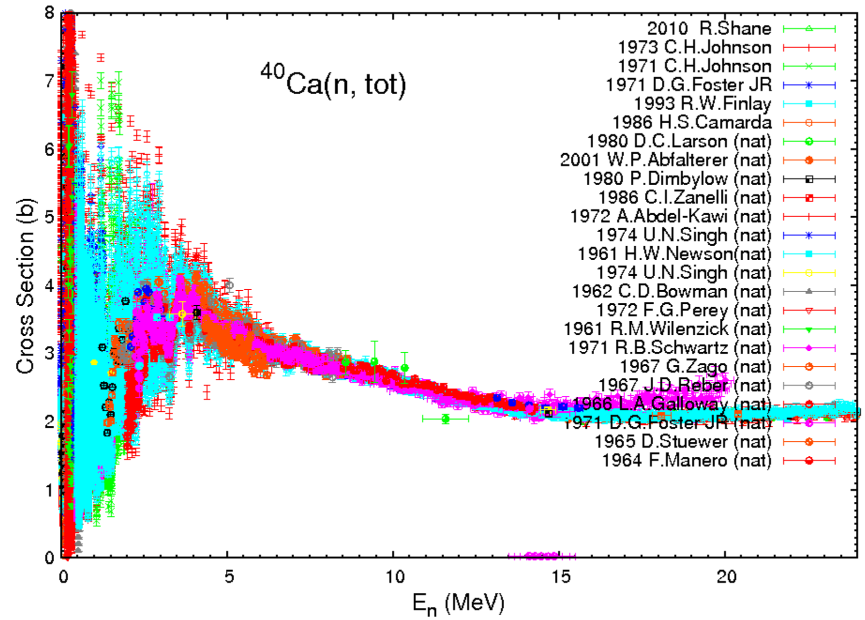
- Experimental principle:

$$\sigma_T = -\frac{1}{nl}\ln\frac{R_i - B_i}{R_o - B_o}$$

- Experimental methods:
  - TRN (transmission method) / TRN+ TOF (time-of-flight)
- Sources of errors:
  - statistics,
  - background,
  - neutron flux monitoring and normalizing,
  - inscattering correction,
  - counting rate loss correction,
  - sample shape and impurities, …

Background —— Problems facing

- **How to evaluate both the cross section and its covariance self-consistently?**
  - **to distinguish which measurements are more reliable**
  - **to re-estimate errors those are not given very clear**
  - **to recommend credible values**
  - **to give appropriate error bars**
  - **to give appropriate associations, avoiding "little errors, big correlations"**



The energy range is divided to two parts, called structural and smooth regions, respectively.

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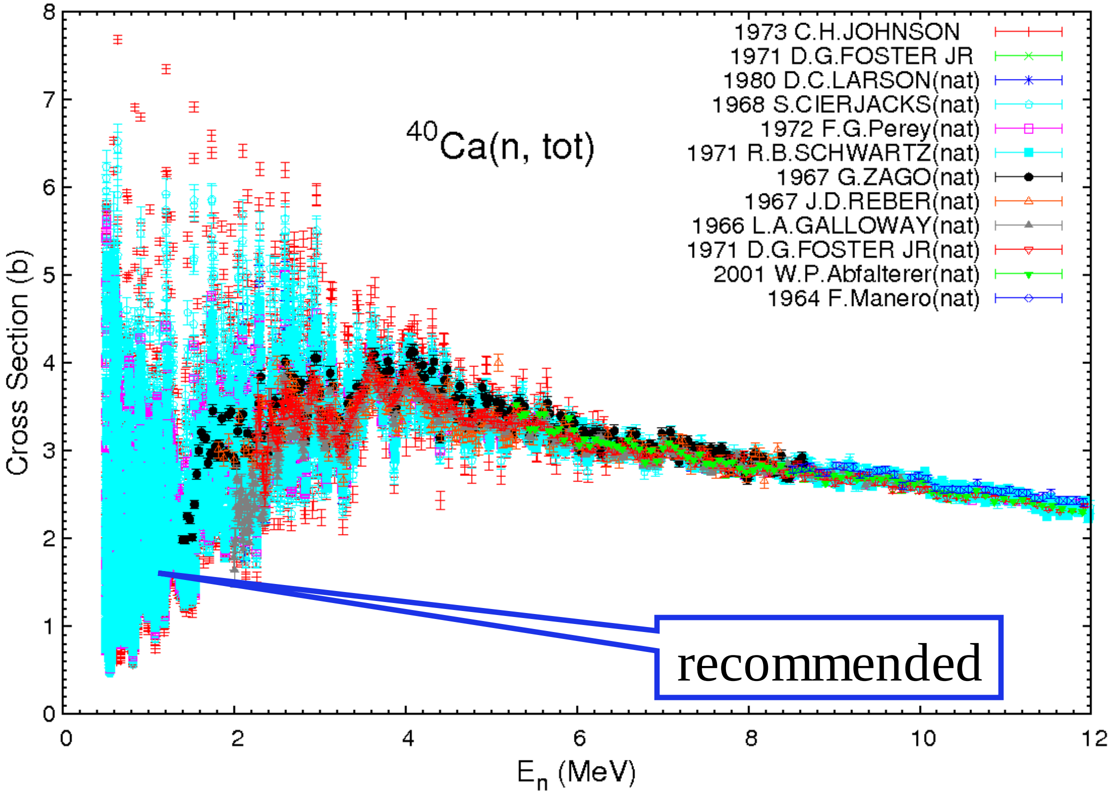
Evaluations of Exp. Data —— structural region

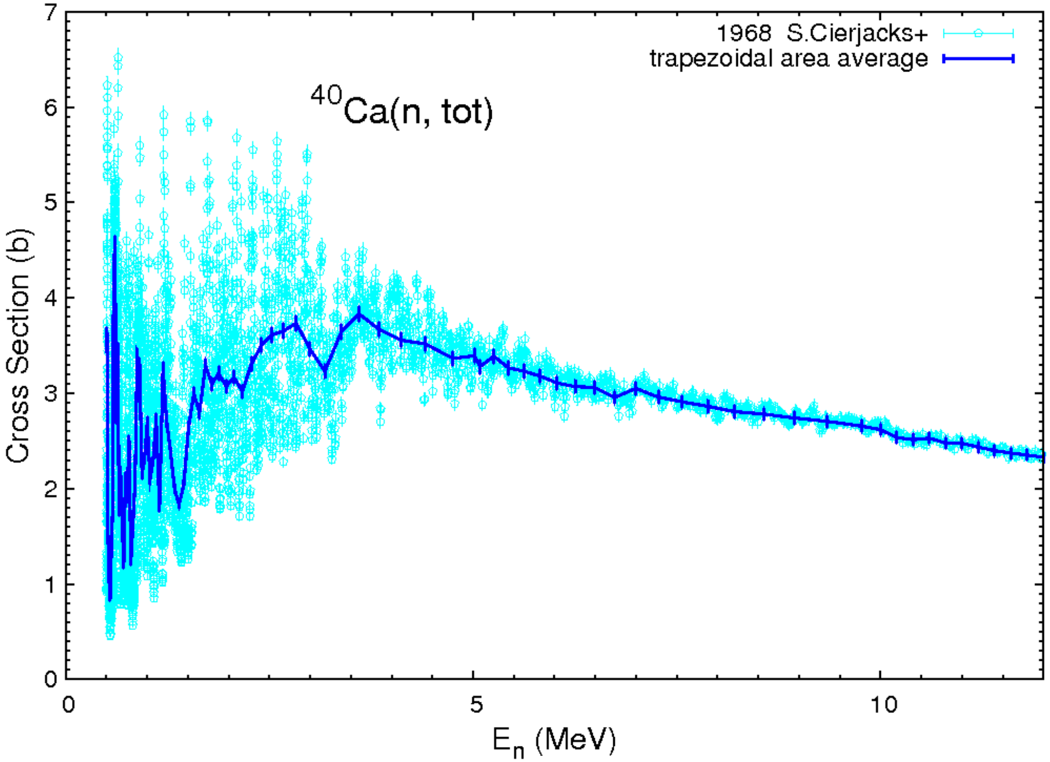


- sets of experiments with different peaks and valleys
- Processing: recommending one set of experimental data as evaluated total CS
- Recommend qualifications:
  - wide energy covering,
  - good energy resolution,
  - obvious peak and valley positions,
  - clear error analysis, …
- Data recommended
  - 1968 S. Cierjacks+ (for ^{nat}Ca)

Evaluations of Exp. Data —— structural region

- **EXFOR information: Energy Region**
  - **Year: 1968**
  - **Author: S. Cierjacks, P. Forti, and D. Kopsch, et. al.**
  - **Energy Region: 0.5 ~ 30 MeV**
  - **Points: 5113**
  - **Method: TOF (57.5m)**
  - **Inc-Source : (EVAP) U(d,nx)-reaction with a broad neutron spectrum in the energy region from 0.3 to 30 MeV**
  - **Detector: (SCIN) liquid scintillator (Ne-213), 9 cm diam., 1 cm thick viewed by an xp-1040 photo tube**
  - **Err-Analys: .Total error given is an upper limit (3.00 percent).**
    .The statistical error runs from better than 1 percent
    to about 3 percent in most of the channels.

Evaluations of Exp. Data —— structural region



Error estimate needs experts' help.

- Data processing:
  - recommended the cross section values
  - re-estimated the errors:

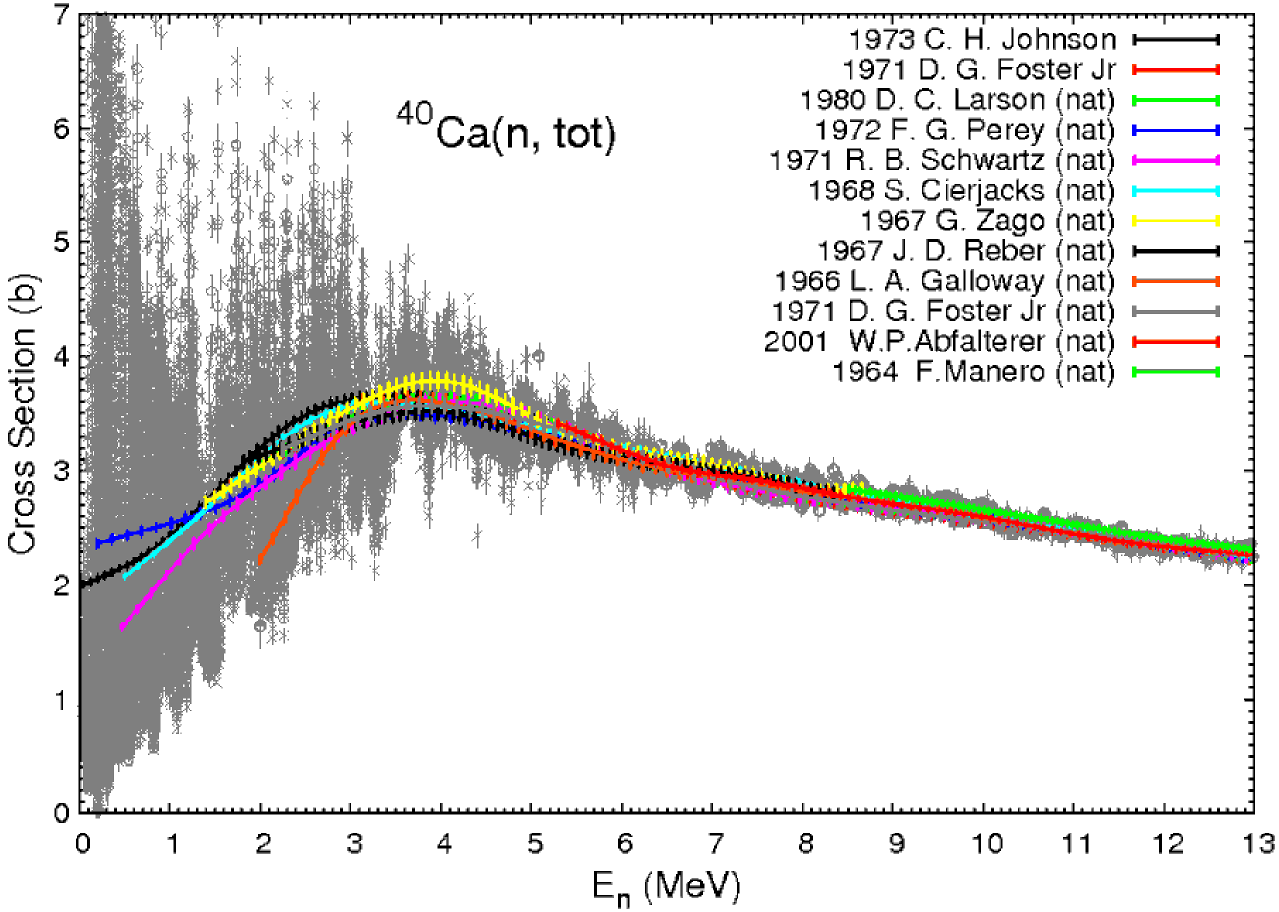
    systematic error: 1%

    statistic error: 1~2.8%

    total error: 1.4~3%
  - mergered the data for covariance energy points (4000+ points are too much), holding the main structures

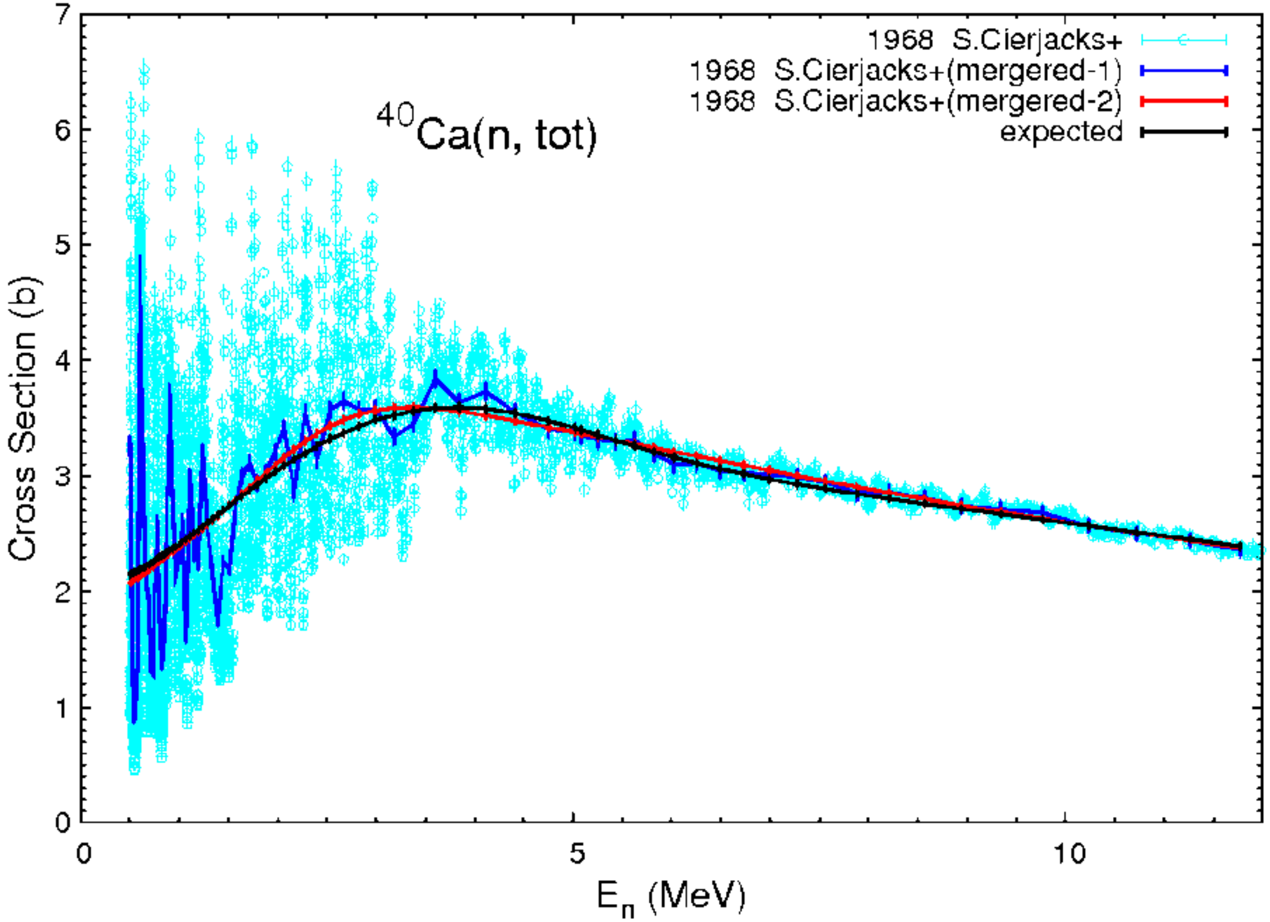
Evaluations of Exp. Data —— structural region

- To avoid systematic deviation of the recommendation , other experiments are considered
- All the measurements are smoothed , systematic errors maintained



Evaluations of Exp. Data —— structural region

- The smoothed recommendation is compared with the expected mean value



Evaluations of Exp. Data —— structural region

- **Negligent error** :
  - The errors can't be discovered and avoided by single experimenter, performing in sets measure system divergences, unknowable reasons,.
  - Different from statistics and systematic errors.
- **Negligent error calculation**

for two measurements, $x \pm \delta_x, \quad y \pm \delta_y,$

let $g = y - x, \quad \delta_g = (\delta_x^2 + \delta_y^2)^{1/2}$

Introduce λ

$$p(\bar{g}|\lambda) = \frac{\lambda^{1/2}}{\delta_g} \exp\left[-\frac{\lambda}{2}(\bar{g}/\delta_g)^2\right]$$

$$\bar{\lambda} = \begin{cases} 1/[(g/\delta_g)^2 - 1) & \text{for } |g| \geqslant \delta_g \\ \infty & \text{for } |g| \leqslant \delta_g \end{cases}$$

$$\bar{g} = \begin{cases} g(1 - \delta_g^2/g^2) & \text{for } |g| \geqslant \delta_g \\ 0 & \text{for } |g| \leqslant \delta_g \end{cases}$$

$$\bar{\delta}_g = \begin{cases} \delta_g(1 - \delta_g^2/g^2)^{1/2} & \text{for } |g| \geqslant \delta_g \\ 0 & \text{for } |g| \leqslant \delta_g \end{cases}$$

negligent error

Ref 1:
Y. CHAO, "A New Approach to the Adjustment of Group Cross Section Fitting Integral Measurements", Nuclear Science and Engineering, 72, 1 (1979).

Evaluations of Exp. Data —— structural region

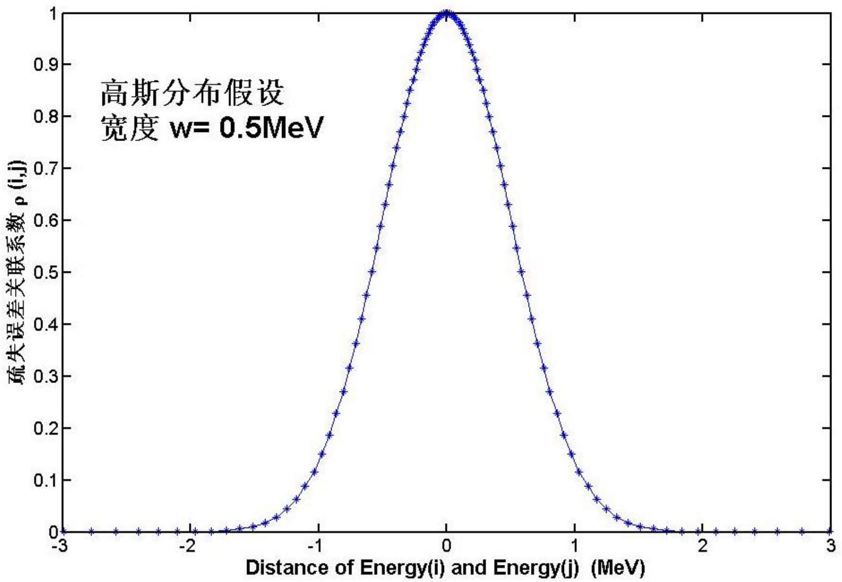
- **Negligent error**:
  - Defined as middle-range correlation, the correlative coefficient depends on the distance between two points. Gauss factor is adopt.

$$Err_{neg} = \rho_{ij} \bullet \Delta\sigma_i \bullet \Delta\sigma_j$$

- **errors formation**:

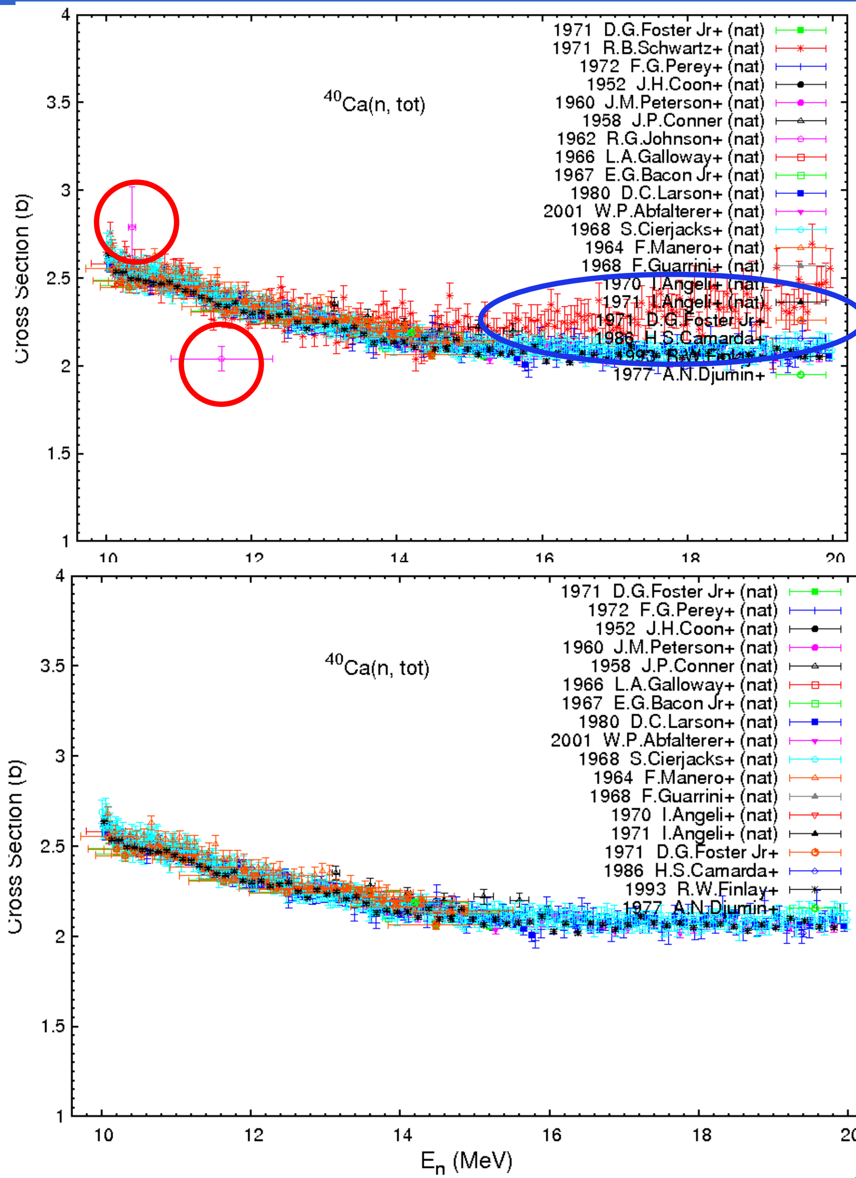
$$Err_{tot}^2 = Err_{sys}^2 + Err_{sta}^2 + Err_{neg}^2$$

- Err_{sys} long distance association
- Err_{neg} middle distance association
- Err_{sta} no association



Evaluations of Exp. Data —— smooth region

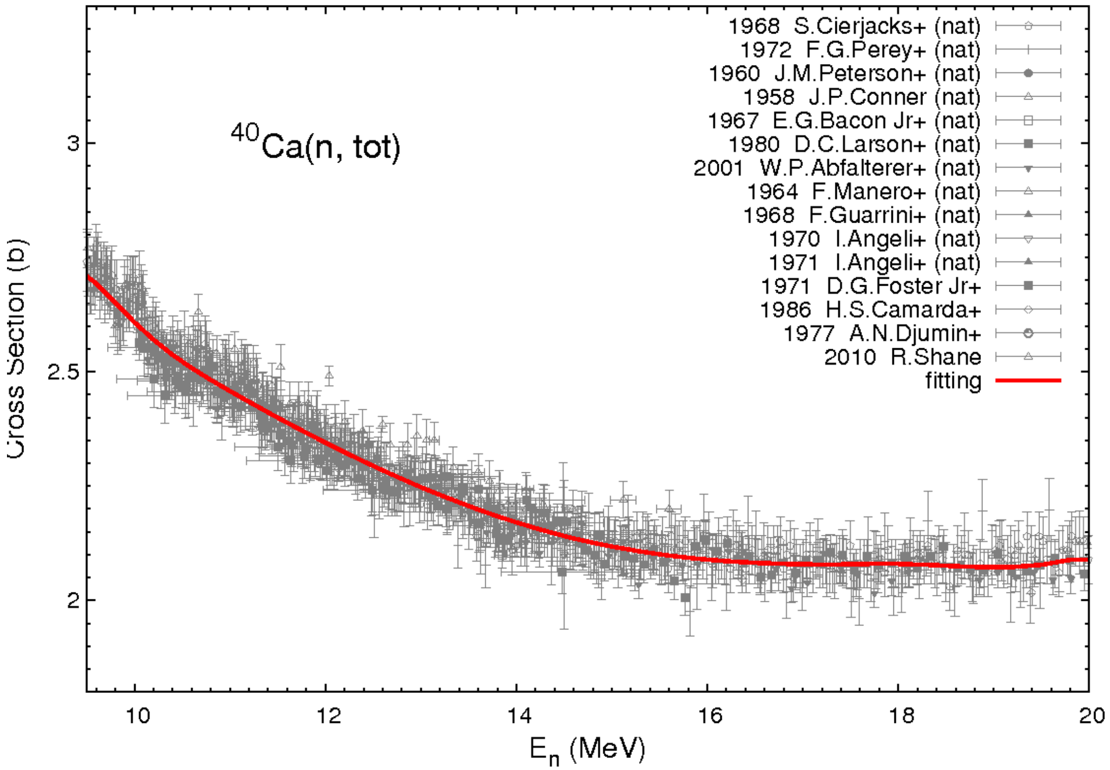
- 20 sets measurements
- rejected those depart obviously from others (2 sets)
- Analyzed errors for each set of data carefully
- Error analysis principles:
  - ➢ subjecting to original reference
  - ➢ Judging from experi-mental conditions



Evaluations of Exp. Data —— smooth region

Year	Author	Reaction	Points	Err_t(%)	Err_sta(%)	Err_sys(%)	Remarks
1972	F.G.Perey	nCa-0	156/3501	1.99-3.24	0.97-2.73	1.74	background0.1%， dead time1%， flux normalizing 1%， others1%
1952	J.H.Coon	nCa-0	1/1	1.83	1	1.53	single point, rejected
1960	J.M.Peterson	nCa-0	1/4	1.71	0.82	1.5	Original err_sys 0.5%， 1.5% estimated
1958	J.P.Conner	nCa-0	6/6	1.70-1.82	0.89-1.1	1.45	no error information, 1.45% estimated
1967	E.G.Bacon Jr	nCa-0	3/7	1.38-1.72	0.95-1.4	1	few points, no error information, rejected
1980	D.C.Larson	nCa-0	106/685	1.46-1.96	1.07-1.68	1	no error information, 1% estimated
2001	W.P.Abfalterer	nCa-0	69/467	1.25-1.6	0.75-1.21	1	no error information, 1% estimated
1968	S.Cierjacks	nCa-0	90/5113	1.38-1.45	0.95-1.05	1	Same processing as structural region
1964	F.Manero	nCa-0	45/61	1.5-2.0	1-1.66	1.12	Background, inscattering correction, and others
1968	F.Guarrini	nCa-0	1/1	1.7	1	1.38	Background, inscattering correction, and others
1970	I.Angeli	nCa-0	1/1	1.63	1.43	1	no error information, 1% estimated
1971	I.Angeli	nCa-0	1/1	1.89	1.60	1	no error information, 1% estimated
1971	D.G.Foster Jr	nCa-40	34/244	2.12-2.47	0.692-1.45	2	no error information, 1% estimated
1986	H.S.Camarda	nCa-40	5/17	1.15-1.40	0.57-0.98	1	Flux normalizing, background, sample thickness
1977	A.N.Djumin	nCa-40	1/1	1.57	0.5	1.5	single point, rejected
2010	R.Shane	nCa-40	7/69	1.31-1.37	0.85-0.93	1	no error information, 1% estimated

Evaluations of Exp. Data —— smooth region



- Values recommended:
  - fitting curve obtained from spline fit code SPCC
- Err analysis:
  - In our opinion,

  Recommended systematic errors shouldn't less than any single Err_sys!
  - About weighted Err_sys adopt.
  - No negligent errors.

Ref 2:

- **Liu T, Zhou H. The Spline Fitting for Multi-Sets of Correlative Data. Communication of Nuclear Data Progress. 1994, 11: 116.**

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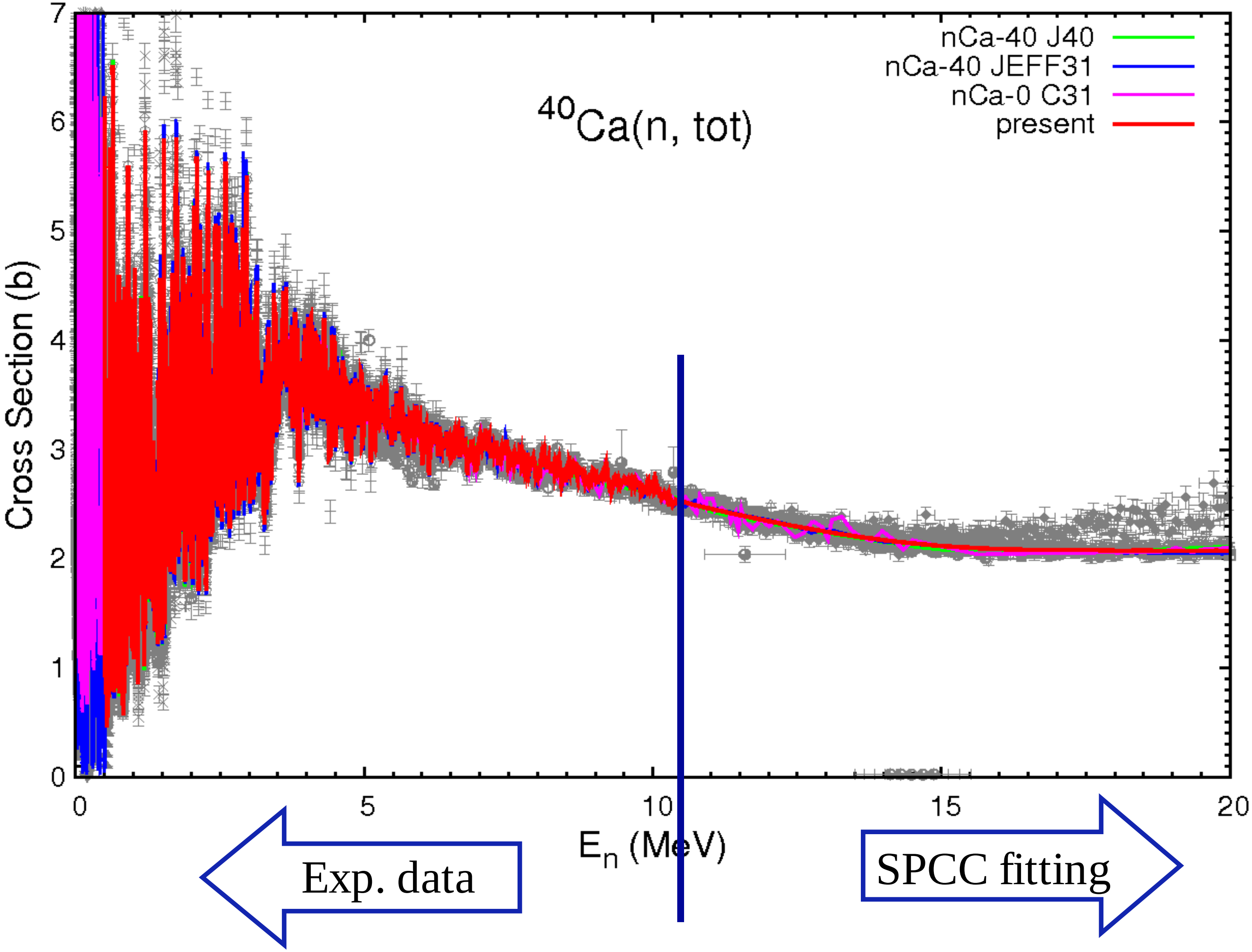
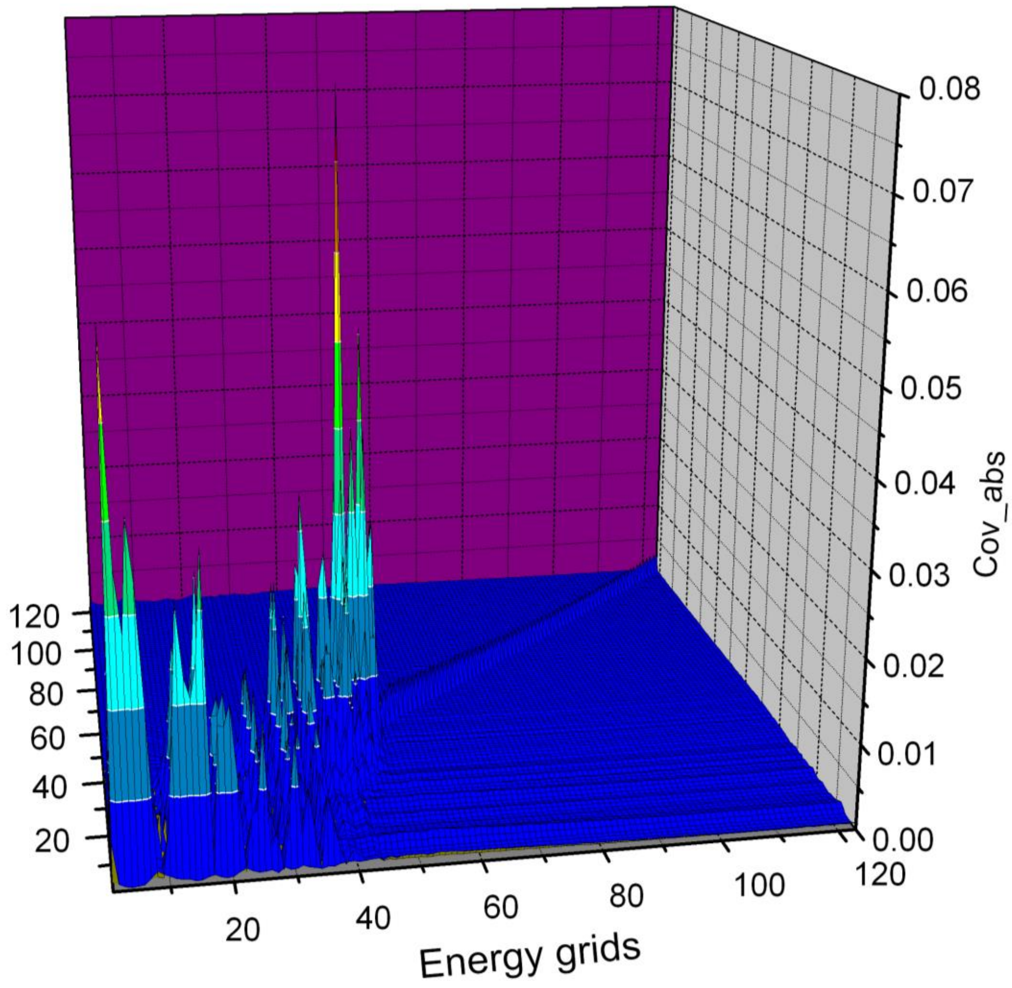
Recommendations and Covariance



Recommendations and Covariance

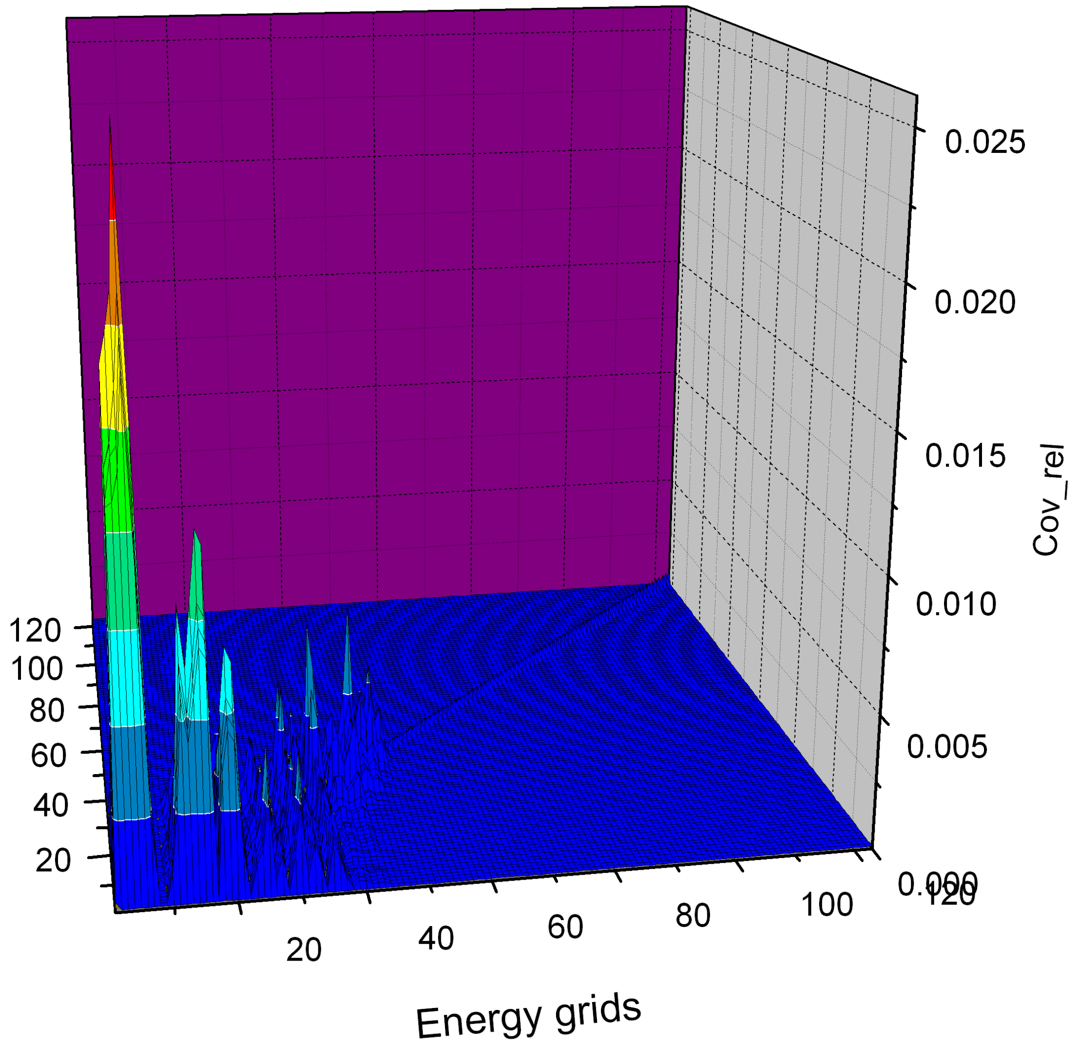
- Absolute covariance matrix



- **121 points, 0.1~10 MeV 76 points, 10~20 MeV 45 points.**

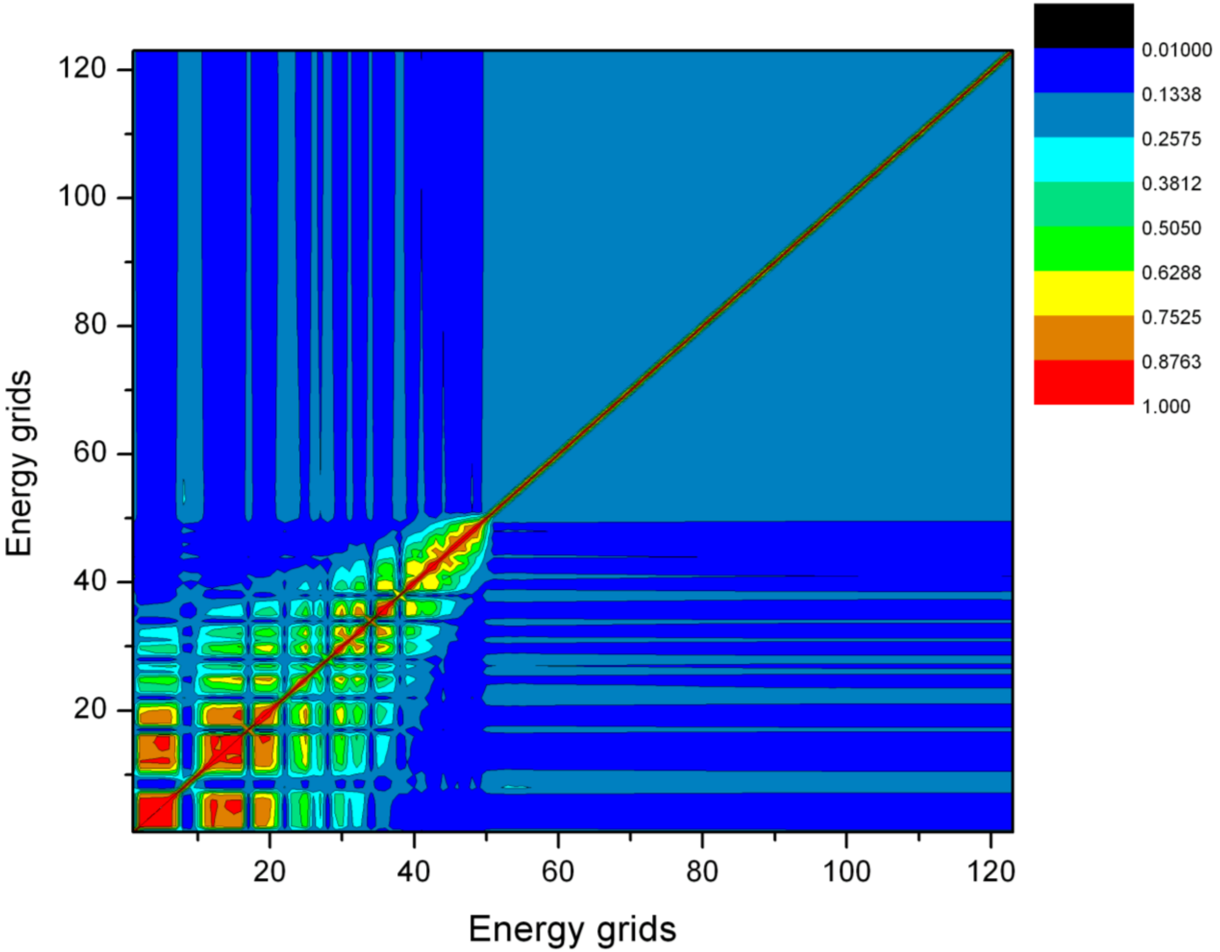
Recommendations and Covariance

- Relative covariance matrix



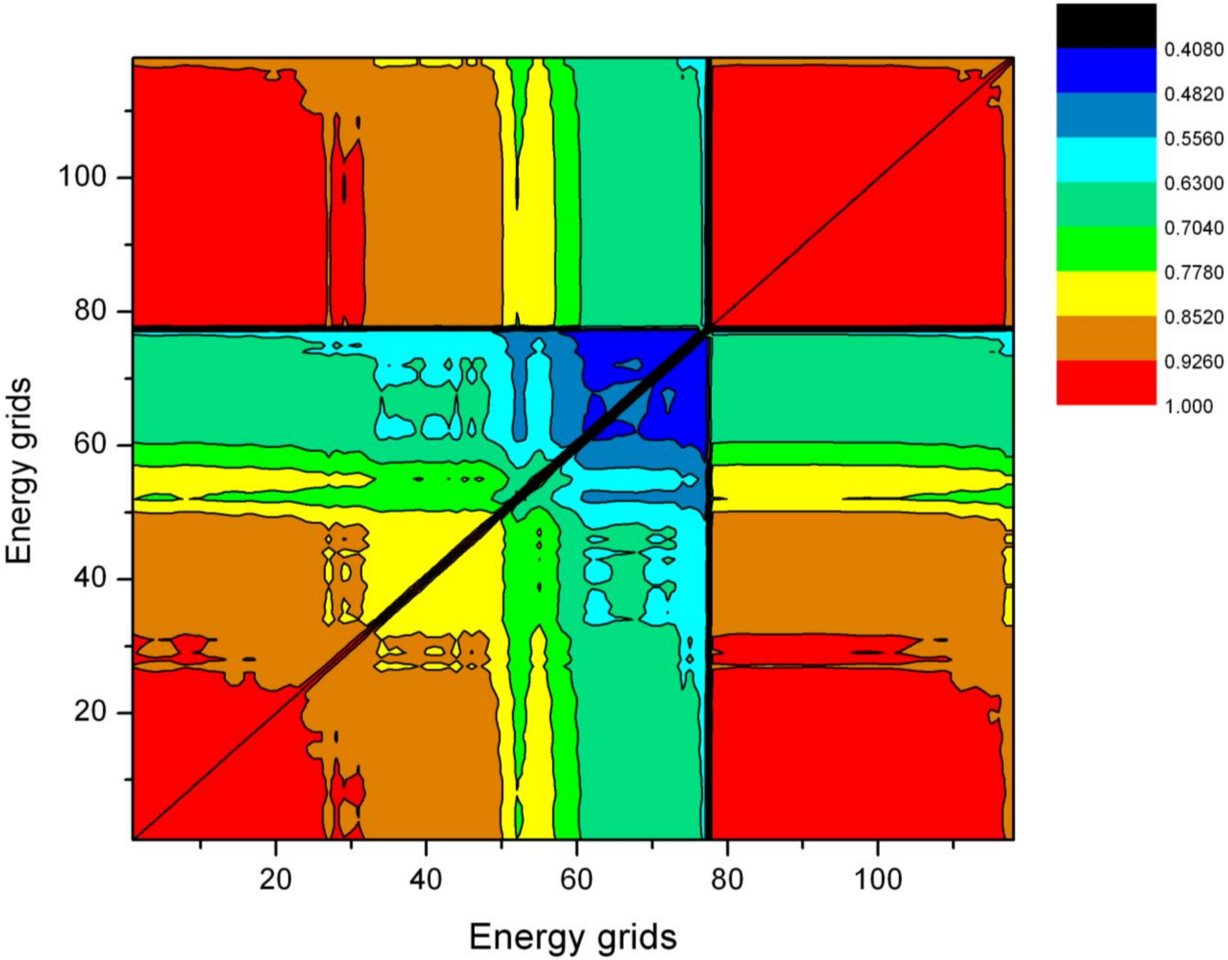
Recommendations and Covariance

- Covariance matrix of correlation coefficient



Addendum

- Over-calculated associations.



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Summary

- Both the evaluated value and the covariance information are given based on experimental data.
- Provide one method to evaluate total cross sections of structural nuclei.
- Submit an idea to evaluate sets and numerous measurements with obvious divergences, considering the negligent error.
- Physical give associations in the smooth region, avoiding unreasonable systematical error and over-calculated associations.

Thank you！

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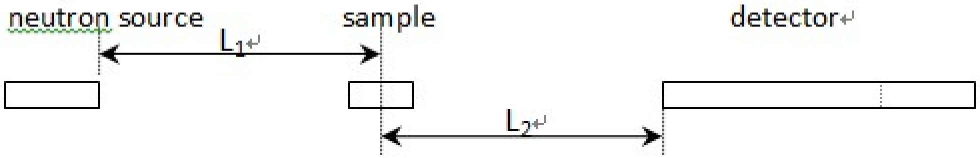
- **Experimental principle:**

$$\sigma_T = -\frac{1}{nl}\ln\frac{R_i - B_i}{R_o - B_o}$$



- **n = the number of atoms per unit volume**
- **l = the sample length**
- **R_i, R_0 = the counts within and without sample in a given time bin per beam monitor count**
- **B_i, B_0 = the background counts within and without sample per beam monitor count**

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❖ 结构区主要实验数据

0	1973	C.H.JOHNSON	Ca-40	En = 4.0103E-02~6.3951E+00 MeV
1	1971	D.G.FOSTER JR	Ca-40	En = 2.2510E+00~1.2949E+01 MeV
2	1980	D.C.LARSON	Ca-0	En = 1.9974E+00~1.3099E+01 MeV
3	1972	F.G.Perey	Ca-0	En = 2.0002E-01~1.3047E+01 MeV
4	1971	R.B.SCHWARTZ	Ca-0	En = 4.7670E-01~1.3070E+01 MeV
5	1968	S.CIERJACKS	Ca-0	En = 5.0016E-01~1.3018E+01 MeV
6	1967	G.ZAGO	Ca-0	En = 1.4010E+00~8.6500E+00 MeV
7	1967	J.D.REBER	Ca-0	En = 1.8180E+00~8.3430E+00 MeV
8	1966	L.A.GALLOWAY	Ca-0	En = 1.9960E+00~1.0060E+01 MeV
9	1971	D.G.FOSTER JR	Ca-0	En = 2.2510E+00~1.3112E+01 MeV
10	2001	W.P.Abfalterer	Ca-0	En = 5.2930E+00~1.3020E+01 MeV
11	1964	F.Manero+	Ca-0	En = 8.5200E+00~1.3060E+01 MeV

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- **疏失误差计算：**

对两个测量值，x± δ_x， y± δ_y，令 g=y−x， $\delta_g = (\delta_x^2 + \delta_y^2)^{1/2}$

（g是疏失误差（认为是统计变量）的一个样本，）则

$$p(g|\bar{g}) = \frac{1}{\delta_g} \exp\left[-\frac{1}{2\delta_g^2}(g-\bar{g})^2\right]$$

并引进疏失误差**不存在可信程度因子λ**（λ=∞表示疏失误差不存在），

$$p(\bar{g}|\lambda) = \frac{\lambda^{1/2}}{\delta_g} \exp\left[-\frac{\lambda}{2}(\bar{g}/\delta_g)^2\right]$$

由此进行统计分析，给出了以下则最可几的λ、g、δ_g为：

参考文献：

- 赵永安 (NSE,72,1-8(1979))；
- 刘廷进 (系统疏失误差c，内部交流)：

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$$\bar{\lambda} = \begin{cases} 1/[(g/\delta_g)^2 - 1) & \text{for } |g| \geqslant \delta_g \\ \infty & \text{for } |g| \leqslant \delta_g \end{cases}$$

$$\bar{g} = \begin{cases} g(1 - \delta_g^2/g^2) & \text{for } |g| \geqslant \delta_g \\ 0 & \text{for } |g| \leqslant \delta_g \end{cases}$$

$$\bar{\delta}_g = \begin{cases} \delta_g(1 - \delta_g^2/g^2)^{1/2} & \text{for } |g| \geqslant \delta_g \\ 0 & \text{for } |g| \leqslant \delta_g \end{cases}$$

上述各式的物理意义：

1） 当 $\bar{g} \leqslant \delta_g$ 时，疏失误差不存在；

2） 疏失误差 $\bar{g}$ 始终小于两个值差的绝对值 $|g|$；

3） 疏失误差 $\bar{g}$ 的大小决定于 δ_g/g，即两个量的误差越大、差别越小，疏失误差 $\bar{g}$ 越小，当 δ_g=g时，疏失误差 $\bar{g}$=0