Description of the fission probability with the GEF code *

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1 Introduction

The description of fission observables above the threshold of multi-chance fission requires the knowledge of the competition between fission, neutron emission and gamma decay as a function of excitation energy and angular momentum of the compound nucleus, because they determine the relative weights of the different chances. Entrance-channel-specific pre-compound processes must eventually be considered in addition. They are not included in the present study. Since the GEF code aims for modelling the fission process in a global way without being locally adjusted to experimental data of specific systems, global descriptions of the relevant decay widths are required. This ensures that the GEF code can predict fission observables for systems for which no experimental data are available. However, this also means that specific nuclear-structure effects can only be considered in an approximate way.

2 Formulation of the fission probability

The fission probability is calculated as

$$P_f = \Gamma_f/(\Gamma_f + \Gamma_n + \Gamma_\gamma). \tag{1}$$

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The gamma-decay width is calculated by the global formula

$$\Gamma_\gamma = 0.62410^{-9} \cdot A_{CN}^{1.6} \cdot T_i^5 MeV \qquad (2)$$

proposed by Ignatyuk [1]. A_{CN} is the mass number and T_i is the temperature of the compound nucleus with energy E_i.

The neutron-decay width is calculated by the global formula

$$\Gamma_n = 0.13 \cdot (A_{CN} - 1)^{2/3} \cdot T_n^2 / exp(< S_n > /T_n) \qquad (3)$$

proposed in ref. [2], which is valid for an exponential neutron-energy spectrum. S_n is the neutron separation energy, T_n is the maximum temperature of the daughter nucleus at the energy $E_i- < S_n >$. This expression was multiplied by

$$1 - exp(-(E_i- < S_n >)/(1.6 \cdot T_n)) \qquad (4)$$

in order to approximately adapt to the Maxwellian shape of the neutron-energy spectrum. The use of $< S_n >= S_{2n}/2$ is another way to consider the shift of the level density by Δ and 2Δ in odd-mass and even-even nuclei, respectively, with respect to odd-odd nuclei. Γ_n is set to zero at energies below the neutron separation energy S_n.

The calculation of the fission-decay width is based on the following equations proposed in ref. [3] with a few extensions:

$$\Gamma_f = F_{rot} \cdot T_f / (G \cdot exp(B_m/T_f)). \qquad (5)$$

B_m is the maximum value of the inner fission barrier B_A and the outer barrier B_B, T_f is the temperature of the compound nucleus at the barrier B_m. $F_{rot} = exp((I_{rms}/15)^2)$ considers the influence of the root-mean square value I_{rms} of the angular-momentum distribution of the compound nucleus.

$$G = G_A \cdot exp((B_A - B_{max})/T_f) + G_B \cdot exp((B_B - B_{max})/T_f) \qquad (6)$$

whereby G_A and G_B consider the collective enhancement of the level densities on top of the inner barrier(assuming triaxial shapes) and the outer barrier (assuming mass-asymmetric shapes) and of tunneling through the corresponding barrier:

$$G_A = F_A \cdot 0.14/\sqrt{\pi/2}, \qquad (7)$$

$$F_A = 1/(1 + exp(-(E - B_A)/T_{equi}), \qquad (8)$$

$$G_B = F_B/2, \qquad (9)$$

and

$$F_B = 1/(1 + exp(-(E - B_B)/T_{equi}). \tag{10}$$

T_{equi} is related to the values of $\hbar\omega_A$ and $\hbar\omega_B$ at the inner and outer barriers by $T_{equi} = \hbar\omega/2\pi$, assuming $\hbar\omega_A = \hbar\omega_B = 0.9$ MeV.

In order to account for the low level density above B_m at energies below the pairing gap 2Δ in even-even nuclei, the value of Γ_f was multiplied at energies in the vicinity of the barrier B_m by a reduction factor that was deduced from the average behaviour of measured fission probabilities. The function is shown in figure 1.

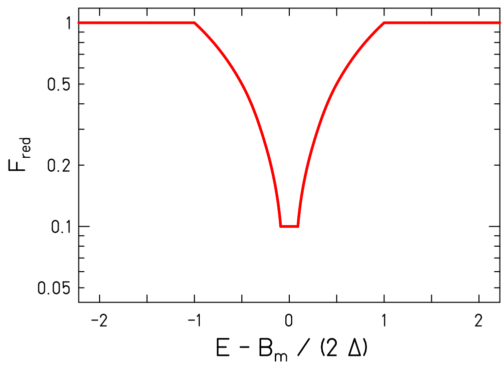

Figure 1: Adapted reduction of the fission-decay width around the fission barrier for even-even nuclei

The collective-enhancement factors at the inner and outer barrier with respect to the daughter nucleus after neutron decay that is assumed to have a quadrupole shape (the inverse of $0.14/\sqrt{\pi/2}$ and 0.5, respectively) are assumed to fade out at higher energies, where the shape of the fissioning nucleus at scission becomes mass symmetric. They are multiplied by the attenuation factor:

$$F_{att} = exp(0.05(E - B_A))/(1/G_A + exp(0.05(E - B_A))) \tag{11}$$

for the inner barrier and an analogue factor for the outer barrier.

The temperature values were determined as the inverse logarithmic derivative of the nuclear level density with respect to excitation energy. The nuclear level density both in the ground-state minimum and at the fission barrier was modelled by the constant-temperature description of v. Egidy and Bucurescu [4] at low energies. The level density was smoothly joined at higher energies with the modified Fermi-gas description of Ignatyuk et al. [5, 1] for the nuclear-state density:

$$\omega \propto \frac{\sqrt{\pi}}{12\tilde{a}^{1/4}U^{5/4}} exp(2\sqrt{\tilde{a}U}) \tag{12}$$

with $U = E + E_{cond} + \delta U(1 - exp(-\gamma E))$, $\gamma = 0.55$ and the asymptotic level-density parameter $\tilde{a} = 0.078A + 0.115A^{2/3}$. The shift parameter $E_{cond} = 2$ MeV $-n\Delta_0$, $\Delta_0 = 12/\sqrt{A}$ with $n = 0, 1, 2$, for odd-odd, odd-A and even-even nuclei, respectively, as proposed in ref. [6]. δU is the ground-state shell correction. Because the level density in the low-energy range is described by the constant-temperature formula, a constant spin-cutoff parameter was used. The matching energy is determined from the matching condition (continuous level-density values and derivatives of the constant-temperature and the Fermi-gas part). Values slightly below 10 MeV are obtained. The matching condition also determines a scaling factor for the Fermi-gas part. It is related with the collective enhancement of the level density.

The fission barriers were modelled on the basis of the Thomas-Fermi fission barriers of Myers and Swiatecki [7], using the topographical theorem of the same authors [8] to account for the contribution of the ground-state shell effect. Adjustments to measured barrier values [9] were applied. Details are described elsewhere.

3 Comparison with experimental data

Figures 2 to 9 show a survey on measured fission probabilities in comparison with the results of the GEF code. The data are taken from the following publications: refs. [10, 11, 12, 13, 14, 15] and references cited therein. Some of the figures show the data from different reactions with different symbols. (See the original publications for details.)

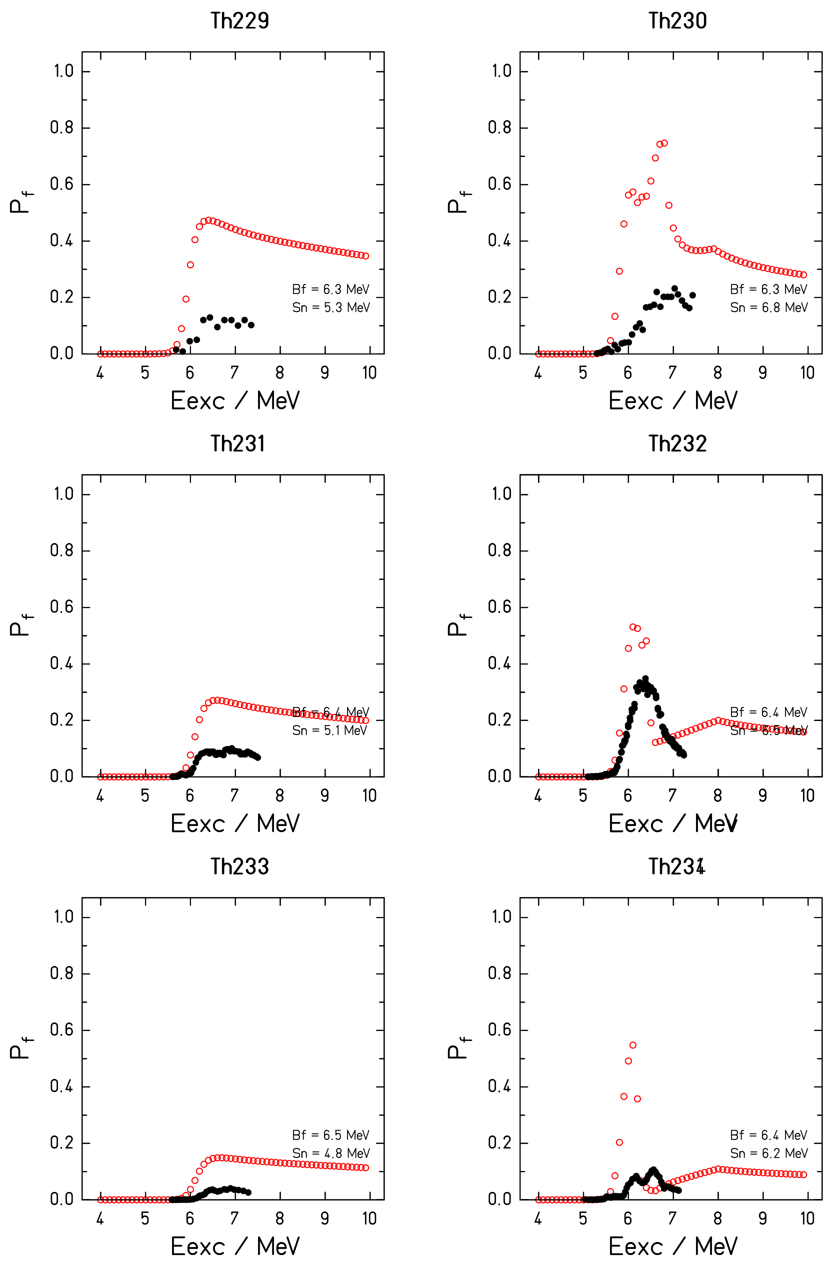

Figure 2: Comparison of the measured fission probabilities (black symbols) with calculations with the GEF code (red symbols). The fission barrier and the neutron separation energy used in the calculations are listed.

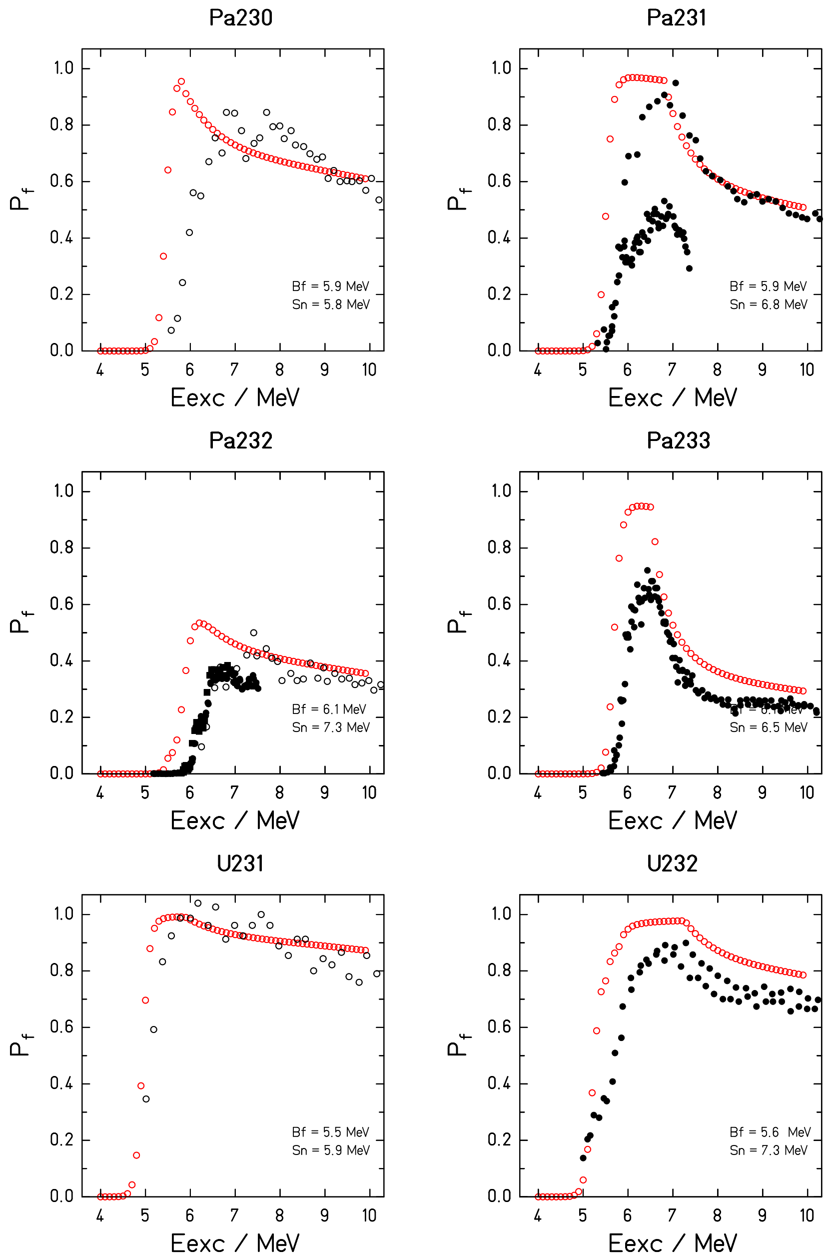

Figure 3: Comparison of the measured fission probabilities (black symbols) with calculations with the GEF code (red symbols).

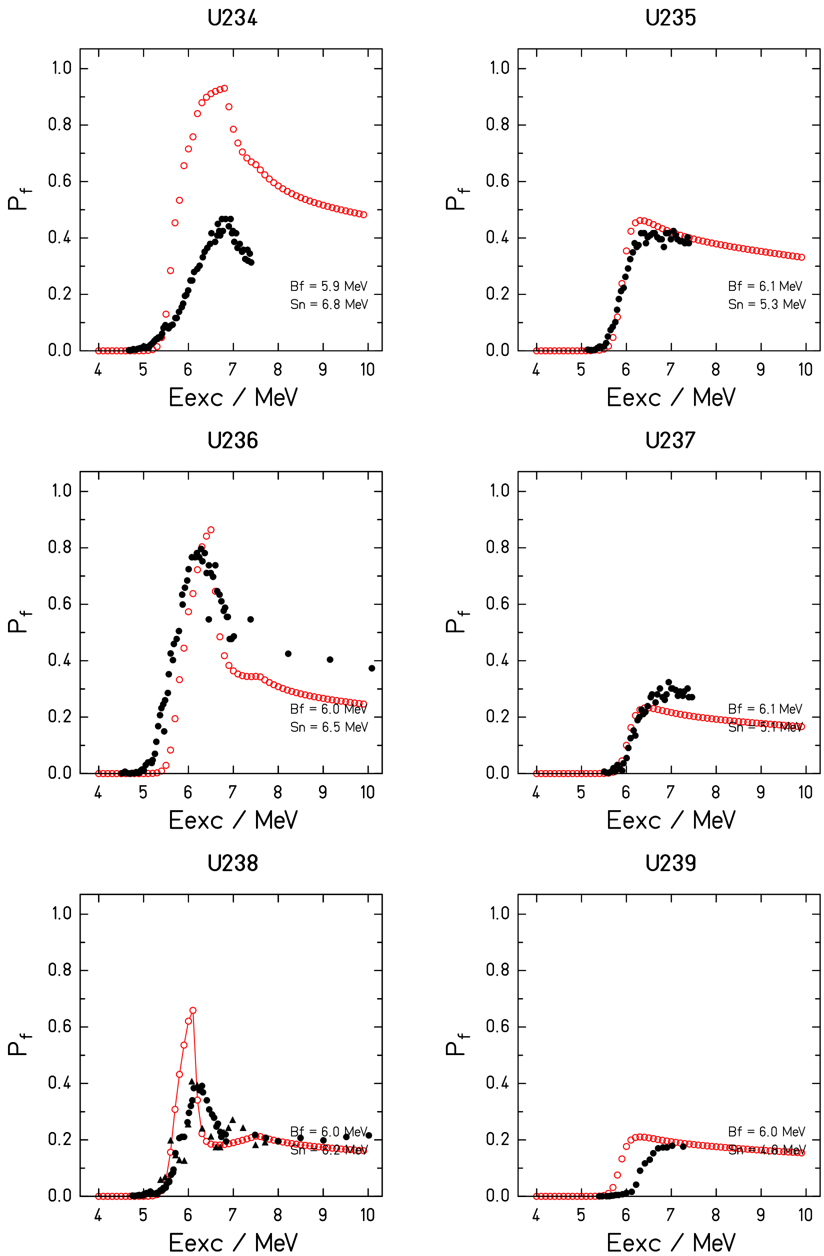

Figure 4: Comparison of the measured fission probabilities (black symbols) with calculations with the GEF code (red symbols).

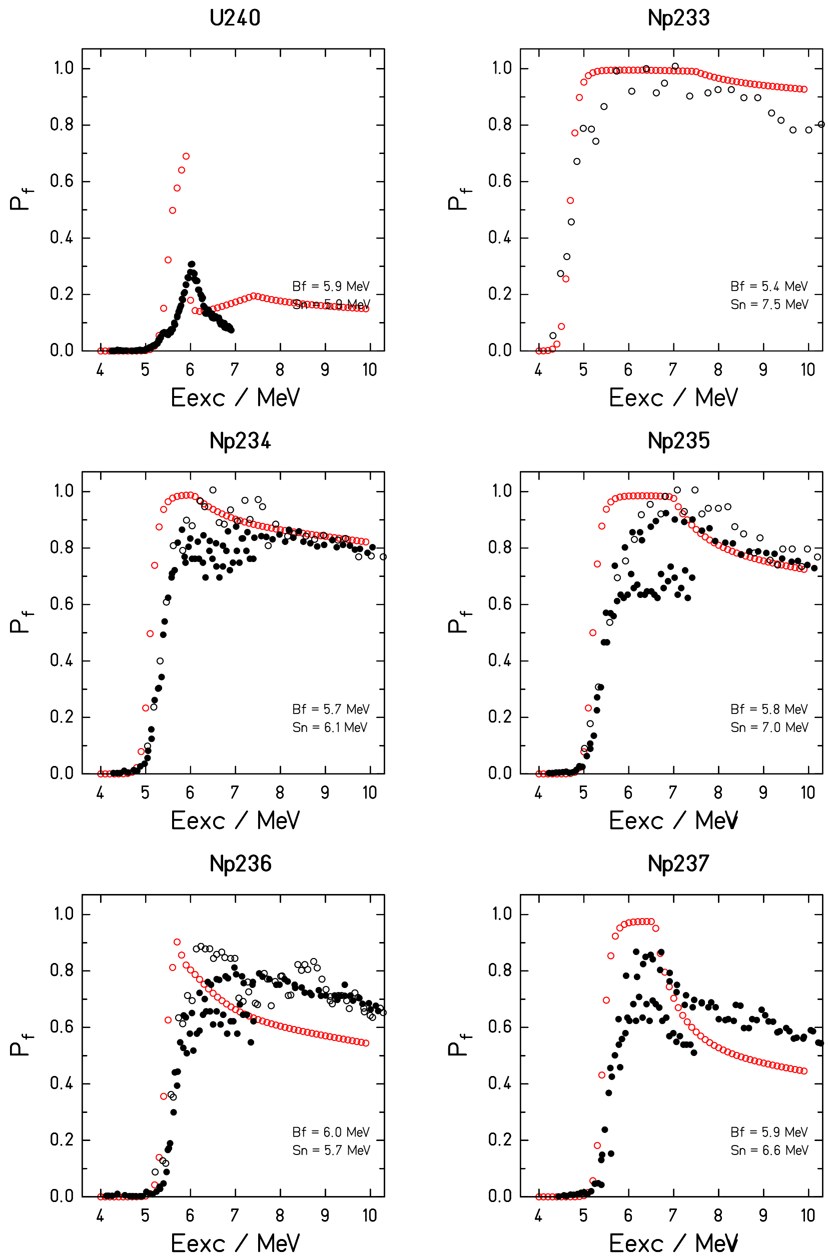

Figure 5: Comparison of the measured fission probabilities (black symbols) with calculations with the GEF code (red symbols).

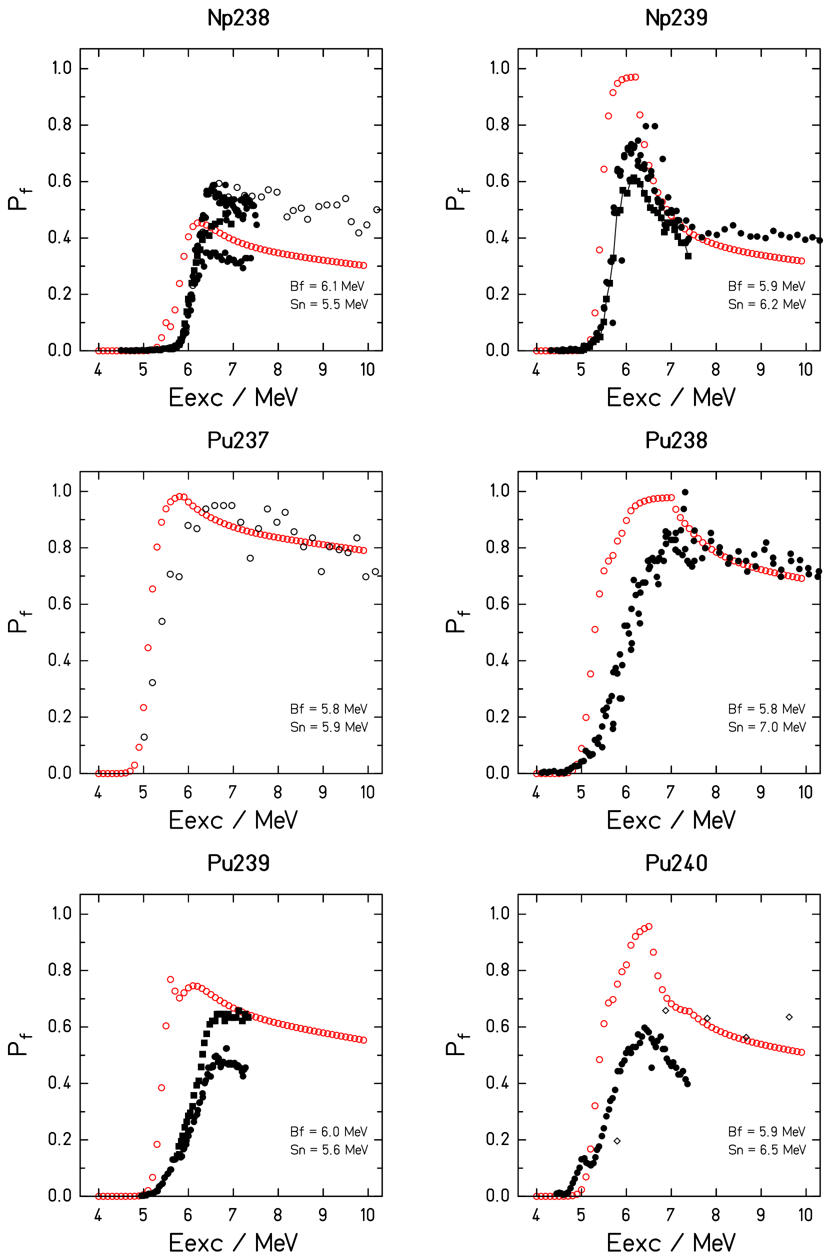

Figure 6: Comparison of the measured fission probabilities (black symbols) with calculations with the GEF code (red symbols).

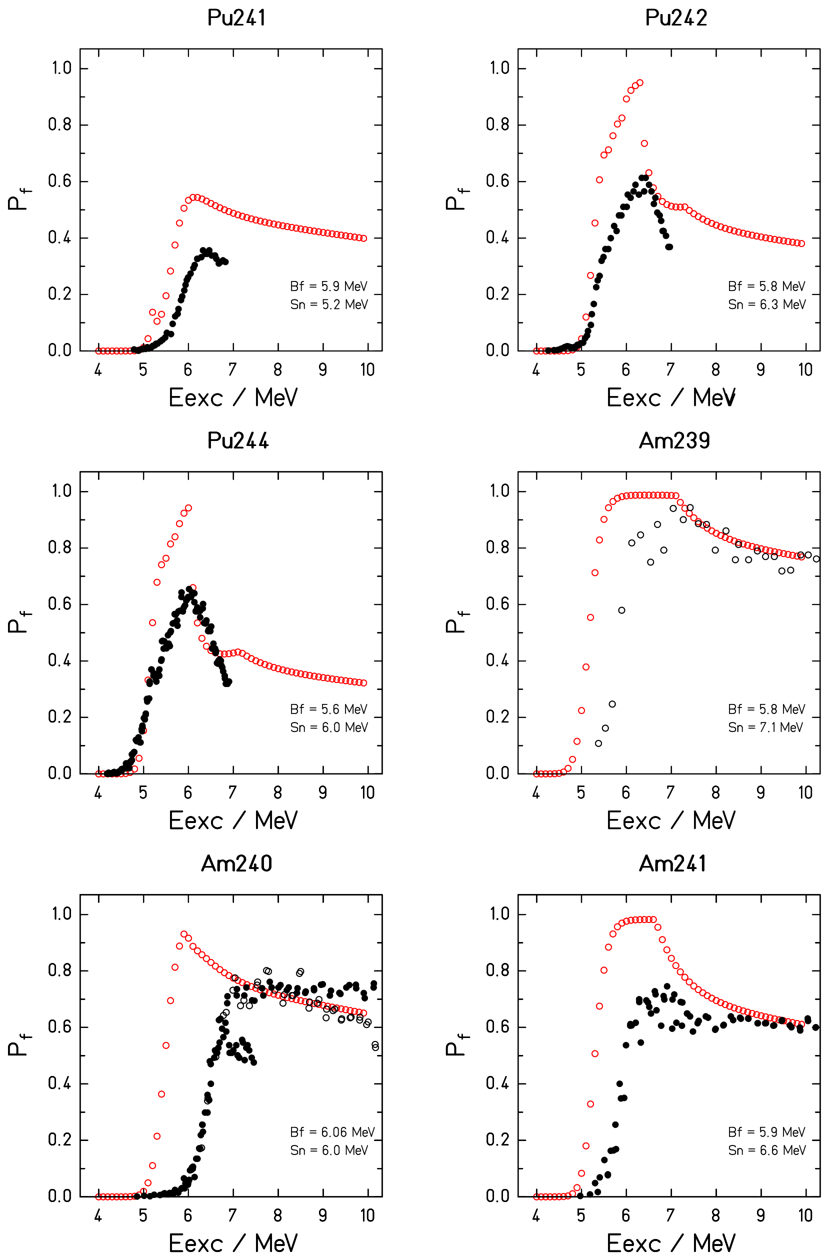

Figure 7: Comparison of the measured fission probabilities (black symbols) with calculations with the GEF code (red symbols).

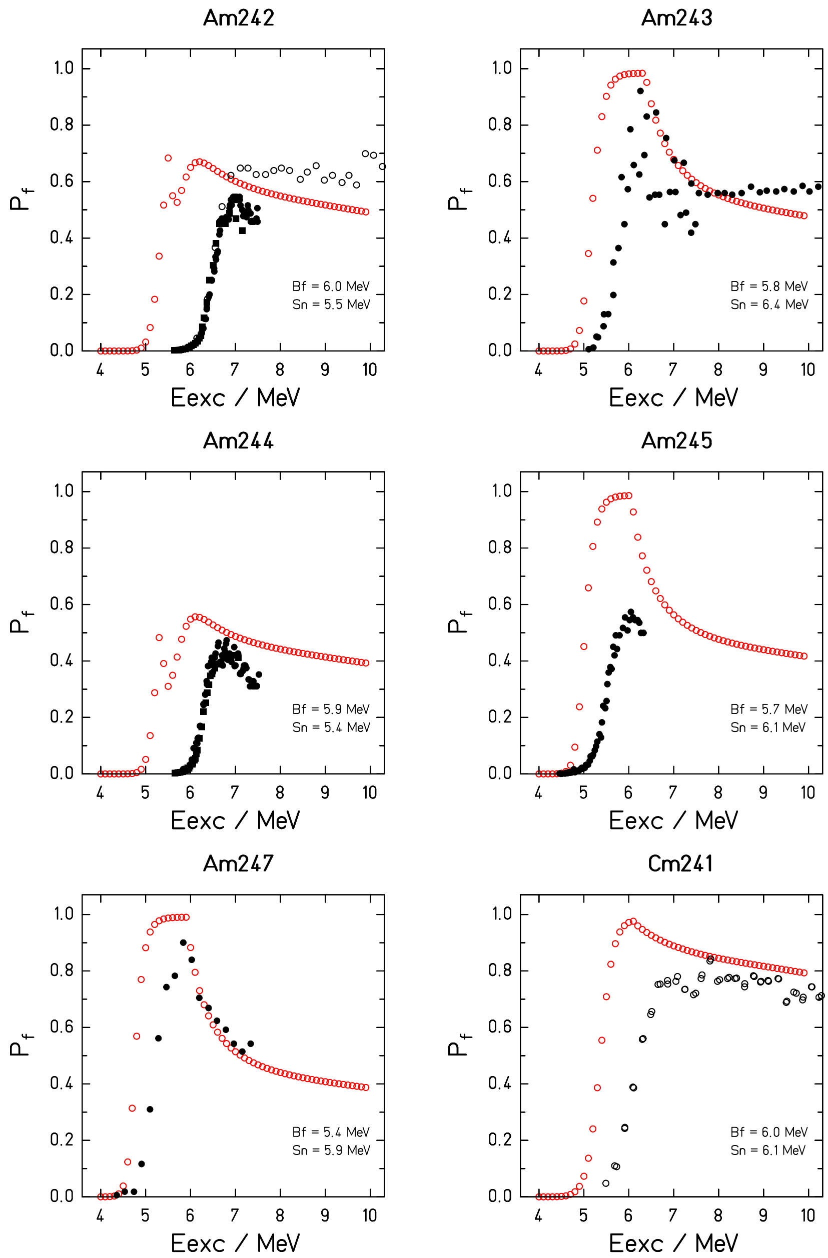

Figure 8: Comparison of the measured fission probabilities (black symbols) with calculations with the GEF code (red symbols).

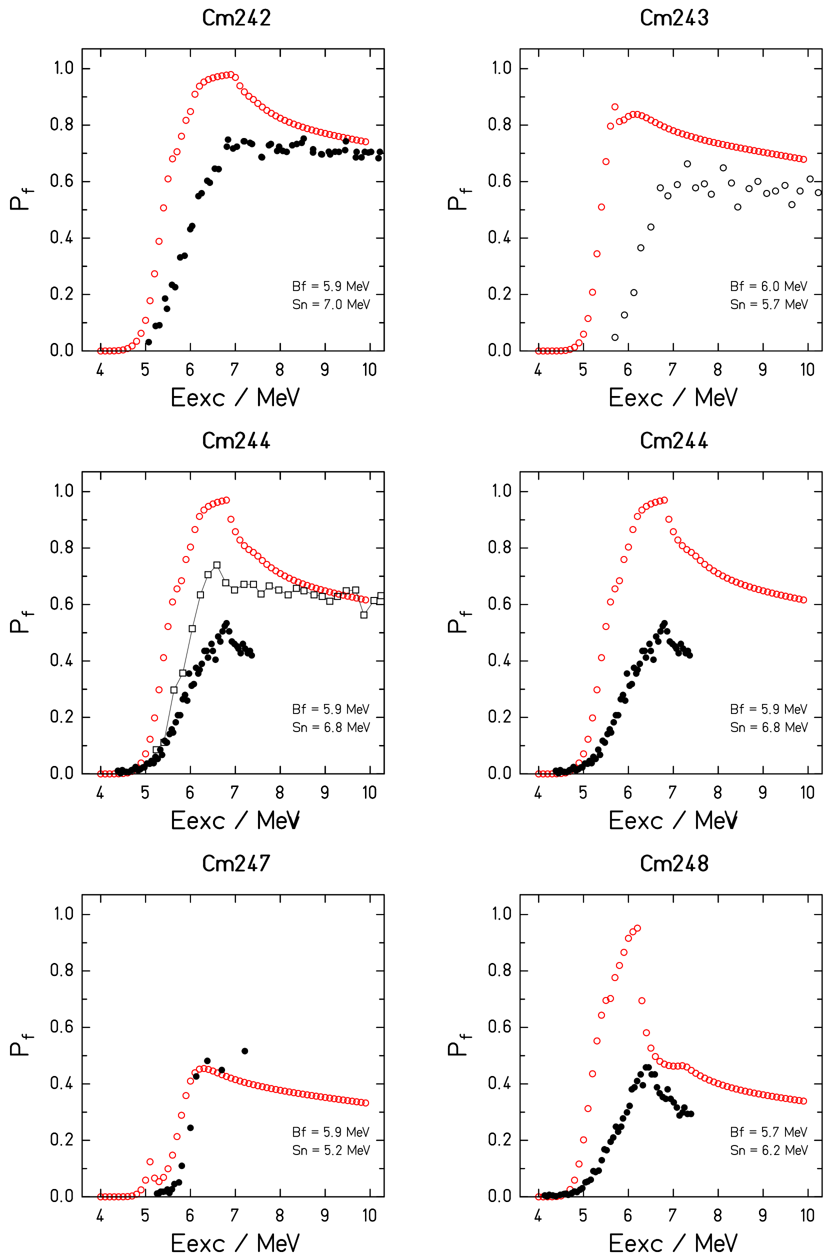

Figure 9: Comparison of the measured fission probabilities (black symbols) with calculations with the GEF code (red symbols).

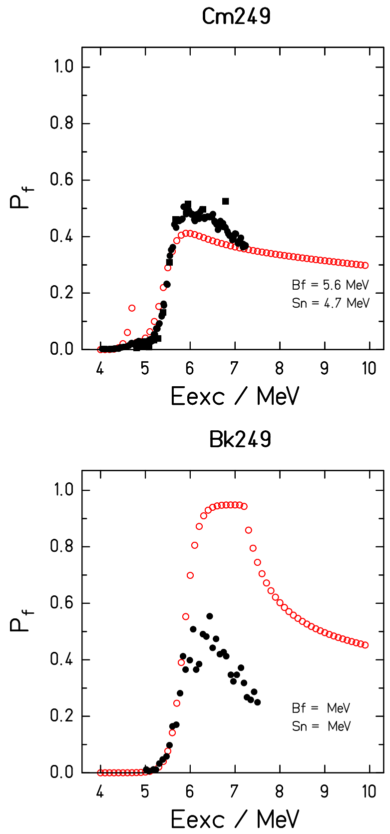

Figure 10: Comparison of the measured fission probabilities (black symbols) with calculations with the GEF code (red symbols).

4 Discussion

The absolute values and the energy dependence of the fission probabilities of most systems reaching from Pa to Cm are rather well reproduced by the GEF code. However, some more or less drastic deviations are found. One may distinguish two kinds of problems: (i) In several cases, the measured fission probabilities are considerably lower than the calculated ones, while the threshold and the energy dependence are rather similar. The most pronounced cases are almost all thorium isotopes, ^{234}U, ^{241}Pu, ^{244}Cm, and ^{249}Bk. (ii) In many cases, the variation and the absolute value of the fission probability in the vicinity of the threshold is not correctly reproduced by the calculation.

A possible key to the first problem may be seen in the figures for ^{231}Pa, ^{235}Np, ^{239}Pu, ^{240}Pu, and ^{244}Cm, where different sets of measured data exist. In all these cases, one of the data sets gives appreciably higher values than the other one, and the higher values agree rather well with the model calculations. For fission probabilities obtained with transfer reactions, there may be a background originating from reactions on target contaminants (e.g. oxygen) or from other parasitic reactions like the breakup of the projectile (deuteron-breakup in particular). This may explain the differences encountered between the different groups of experimental data. Thus, the first problem might have its origin in the experimental data at least in some of the cases.

The second problem must be attributed to the shortcoming of the model due to its global description. Specific structural effects at low excitation energies, either at the fission barrier or in the daughter nucleus after neutron evaporation, are not properly considered. The problem is most severe for even-even fissioning systems, but there are cases, where this description works rather well, see e.g. ^{236}U. Moreover, some of the fission barriers might deviate from the global description used in the code. Whenever experimental fission probabilities are available, these can be used to improve the model calculations. However, the kind of disagreement seen in the figures gives a realistic impression about the quality of the predictions of the model for cases, where no experimental data exist.

5 Conclusion

A global description of the fission probability of the actinides has been derived which reproduces the experimental data rather well. Discrepancies in the absolute values over the whole energy range might be caused by a background contribution due to the presence of light target contaminants in the experiment. The global description of the nuclear level densities near the ground state and near the fission threshold used in the code can only give a rather crude approximation of the behaviour of the fission probability near the fission threshold. This explains the discrepancies in the fission probabilities near the fission threshold found for several systems. The energy-dependent fission probabilities are important to calculate the relative weights of the different fission chances at higher energies.

Acknowledgement

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