APPENDIX E

Kinematic Formulas

The notation used to describe two-body kinematics is shown in Fig. E-1. The scattering treatment used for neutrons can be generalized to two-particle reactions by allowing the emitted particle mass to be different from the incident particle mass (*i.e.*, $m_3 \neq m_1$). Applying conservation of mass, energy, and momentum gives the following non-relativistic kinematic equations:

$$A = \frac{m_2}{m_1}, \tag{E.1}$$

$$A' = \frac{m_3}{m_1}, \tag{E.2}$$

$$\beta = \left(\frac{A\left(A+1-A'\right)}{A'} \left[1 + \frac{1+A}{A} \frac{Q}{E_1} \right] \right)^{1/2} \tag{E.3}$$

$$\gamma = \frac{A'}{A+1-A'} \beta, \tag{E.4}$$

$$\frac{\varepsilon_3}{\varepsilon_1} = \frac{A'}{A^2} \beta^2, \tag{E.5}$$

$$\varepsilon_1 = \left(\frac{A}{A+1} \right)^2 E_1, \tag{E.6}$$

$$\mu_3 = \mu, \tag{E.7}$$

$$\frac{\varepsilon_4}{\varepsilon_1} = \frac{A'}{A+1-A'} \frac{\varepsilon_3}{\varepsilon_1}, \tag{E.8}$$

$$\mu_4 = -\mu, \tag{E.9}$$

$$\frac{E_3}{E_1} = \frac{A'}{(1+A)^2} \left(\beta^2 + 1 + 2\beta\mu \right), \tag{E.10}$$

$$\omega_3 = \frac{1+\beta\mu}{\sqrt{\beta^2+1+2\beta\mu}}, \tag{E.11}$$

$$\frac{E_4}{E_1} = \frac{A+1-A'}{(1+A)^2}\left(\gamma^2+1-2\gamma\mu\right), \tag{E.12}$$

$$\omega_4 = \frac{1-\gamma\mu}{\sqrt{\gamma^2+1-2\gamma\mu}}. \tag{E.13}$$

If the incident and scattered particles are the same, A′=1, and these formulas reduce to the familiar set used for neutron scattering. The elastic reaction corresponds to A′=1 and Q=O.

According to these formulas, the secondary energy distribution of the emitted particle and the complete energy-angle distribution of the recoil nucleus are completely determined by A, A′, Q, and the angular distribution for the emitted particle.

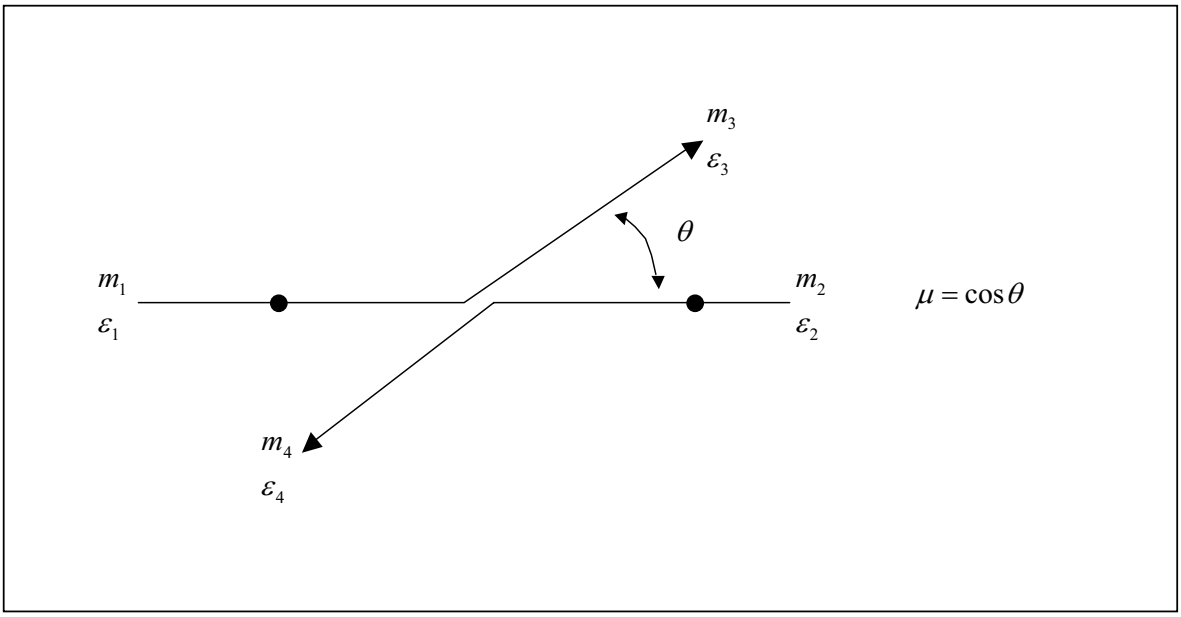

Figure E.1
Kinematics Variables for Two-Particle Reactions
Center-of-Mass System

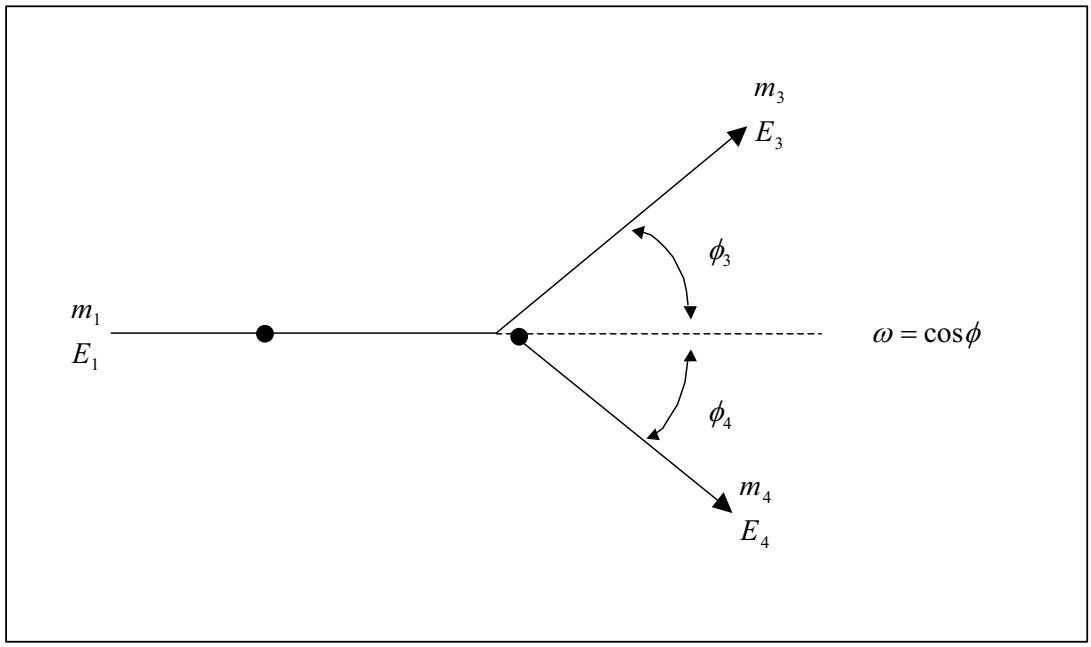

Figure E.2
Kinematics Variables for Two-Particle Reactions
Laboratory System