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The Statistical Estimation of Thermal-Neutron Cross Sections

J. L. Cook and A. L. Wall

AAEC Research Establishment, Private Mail Bag, Sutherland, NSW, Australia

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The thermal radiative capture cross sections of 87 nuclides were computed using a Monte Carlo selection of reduced neutron widths, and the assumption that distant resonance levels determine the cross section.

Histograms of possible cross-section values were prepared for each nuclide, and the 87 samples analyzed to find the overall accuracy of estimation. The results indicated a fluctuation of 0.4 ± 0.6 for the logarithm of the ratio of experiment to the calculated mean cross section.

Tables of results for means and standard deviations are given together with the results of Keane's summation formula. The possible use of this technique in estimating unknown cross sections is discussed.

I. INTRODUCTION

From the time when reactor physics was in its early infancy to the present day, reactor designers and engineers have been asking the nuclear physicists if it is at all possible to calculate unmeasured neutron cross sections. Invariably, the answer has been that average cross sections of intermediate and heavy nuclei can be estimated in the keV-MeV range, but at low energies, where resonances are well resolved, the statistical fluctuations in resonance locations and strengths make such predictions impossibly inaccurate. However, a quantitative assessment of these errors has not been undertaken.

We have attempted here to synthesize thermal cross sections by using statistical resonance parameters along with a quantitative estimate of the errors involved. We report the results of Monte Carlo calculations on 87 nuclides whose thermal-neutron cross sections are known. A mean cross section and standard deviation were computed for each nuclide, assuming that the reduced neutron widths obey the Porter-Thomas distribution law.[1] The overall accuracy with which thermal-neutron cross sections can, in general, be predicted was determined. The Monte Carlo method was used because it is a rigorous technique if all relevant information is available and allows the checking of analytical approximations against exact answers. In this problem, no exact analytical solution exists.

In particular, the technique can be applied to the estimation of unmeasured cross sections for relatively short-lived fission products.

II. RESONANCE THEORY

The basic assumption underlying our calculations is that the thermal-neutron cross section for the (n,γ) and (n,f) reactions can be expressed as the contribution from the sum over distant Breit-Wigner levels[2] as follows:

Each level contributes an amount

$$\sigma_{n,\gamma}(E) = 4\pi\lambda\!\!\!^{-2}\ \frac{\Gamma_n\,\Gamma_\gamma g_J}{\Gamma^2}\cdot\frac{1}{1+X^2}\ , \qquad (1)$$

where

$\lambda\!\!\!^{-}$ = the wavelength of the neutron in cm

$\Gamma_n = \Gamma_n^0\sqrt{E}$ = the neutron width

Γ_n^0 = the reduced, s-wave neutron width

[1] C. E. PORTER and R. G. THOMAS, *Phys. Rev.*, 104, 483 (1956).

[2] G. BREIT and E. WIGNER, *Phys. Rev.*, 49, 519 (1936).

Γ_γ = the radiation width

$\Gamma = \Gamma_n + \Gamma_\gamma + \Gamma_f$

Γ_f = the fission width

$X = \frac{2}{\Gamma}(E - E_r)$

E = the neutron energy

E_r = the energy at exact resonance.

The reduced neutron widths Γ_n^0 fluctuate over wide ranges of values and, according to Porter and Thomas, obey a chi-squared distribution with one degree of freedom

$$P(y) = \exp(-y/2)/\sqrt{2\pi y}, \tag{2}$$

where

$y = \Gamma_n^0/\bar{\Gamma}_n^0$

$\bar{\Gamma}_n^0$ = the average reduced width.

Summing over all levels, one would obtain a thermal cross section of

$$\sigma_{n,\gamma}(E) = \frac{4\pi\lambda\!\!\!^{-2}\,\Gamma_\gamma\,\bar{\Gamma}_n^0\sqrt{E}}{\Gamma^2}\sum_{i=-\infty}^{\infty}\frac{y_i}{1+X_i^2}, \tag{3}$$

where we have neglected variations in Γ_n which affect Γ.

If we further assume that levels are equally spaced, then

$$E_{ri} = E_{r0} + N\bar{D}, \tag{4}$$

where

$\bar{D}$ = the average level spacing

N = an integer.

Keane[3] introduced the assumption that all levels have the same average Γ_n^0, and was able to sum Eq. (3) to an analytic form, using a Poisson summation rule, obtaining

$$\bar{\sigma}_{n,\gamma}(E) = 4\pi\lambda\!\!\!^{-2}\,\frac{\bar{\Gamma}_n^0\,\Gamma_\gamma\sqrt{E}}{\bar{\Gamma}^2}\,\frac{a}{2}\,\frac{\sinh a}{\cosh(a) - \cos(a\alpha)}, \tag{5}$$

where

$$\alpha = \frac{2(E_{r0} - E)}{\bar{\Gamma}},$$

$\bar{\Gamma} = \Gamma_\gamma + \bar{\Gamma}_f + \bar{\Gamma}_n^0\sqrt{E}$, and

$$a = \frac{\pi\bar{\Gamma}}{\bar{D}}.$$

To estimate the thermal cross section, taking account of neutron width fluctuations, we used

$$\sigma_{n,\gamma}(E) = \frac{4\pi\lambda\!\!\!^{-2}\,\Gamma_\gamma\,\bar{\Gamma}_n}{\bar{\Gamma}^2}\sum_{i=-N}^{N}\frac{X_i}{1+(\alpha+i\beta)^2}, \tag{6}$$

where

$$\beta = \frac{2\bar{D}}{\bar{\Gamma}}.$$

III. CALCULATIONS AND RESULTS

The standard random-number routine from the IBM-7040 systems library at the AAEC was used to select random X_i values from a Porter-Thomas distribution. The thermal-neutron cross section was computed 50 times using the summation formula of Eq. (6) with $N = 10$. A histogram of probability values per interval in σ was computed, while the mean and standard deviations were also determined by the usual statistical methods. This operation was repeated for each of the 87 nuclides and an overall estimate was obtained for the quantity

$$z = \log\frac{\sigma(\text{experiment})}{\sigma(\text{theory})}. \tag{7}$$

The parameter N was varied from 2 to 100 to find the optimum value which gave less than 5% error in the summation, yet presented a reasonable time for computation. A value of $N = 10$ was eventually chosen from test cases in which the error in summation rarely exceeded 2%. To secure reliable statistics, the value of 50 was chosen as the number of trials for each nuclide. With 20 trials or less, the standard deviation appeared to fluctuate more than 10%, but with 50, the maximum fluctuation found was 5%. With much greater than 50 trials the computation time became an obstacle.

The influence of irregularities in level spacings can be taken into account properly only by simultaneous sampling of widths from a Porter-Thomas and spacings from a Wigner distribution while recomputing the cross section at least 500 times to ensure good statistics. This posed a considerable problem in computing time, and intuitive arguments suggest that the mean values already obtained would be affected little, though the standard deviations would certainly increase.

It was assumed that only the lowest resonance energy was known. Without this assumption, the resultant thermal cross section could vary over five orders of magnitude with an appropriate probability distribution, because a resonance is equally probable at all energies, if the location of other resonances has not been fixed. Should the energy of one resonance become fixed, the span of values for its neighbors immediately becomes narrower, because the other resonances will be distributed according to the Wigner distribution

[3] A. KEANE, "An Estimate of the Decrease in the Effective Resonance Integral due to Resonance Overlap," AAEC/TM296, Australian Atomic Energy Commission (1965).

relative to the first. Nevertheless, there will still be a finite probability that a resonance could occur right at 0.0253 eV, or at a maximum distance from this energy which leads to a very high or abnormally low cross section. Such alternatives do not exist if one assumes that resonances are equally spaced.

We therefore justify our assumption of equal increments between levels by surmising that this is the most probable many-resonance configuration and intend to inquire further into this feature of level statistics.

Figure 1 is a typical histogram obtained for ^{232}Th. Note the long exponential tail for large values of σ, and the prominent peak. This shape is characteristic of all nuclides. The mean value of 9.5 b is quite close to the experimental value of 7.4 ± 1 b. Keane's formula gives 5.4 b, again a quite reasonable estimate, though the standard deviation is 12 b, which clearly indicates that the only certain result of such calculations is a probable upper limit.

Tables I and II give the results for the 87 nuclides. The mean cross section, the standard deviation, the value from Keane's formula, and the quantities z are shown. The z values fluctuated between 1 and -1 showing that in nearly every case the experimental result was reproduced to within one order of magnitude. The average value of z was found to be -0.40, which implies that the most probable experimental value lies at about 0.4 of the calculated mean. The standard deviation in z was about 0.63, indicating that the actual value is such that

$$0.093 \times \sigma(\text{calculated mean}) < \sigma(\text{experiment}) < 1.7 \times \sigma(\text{calculated mean}), \quad (8)$$

to within a confidence interval of 67%.

The frequency distribution of z is illustrated by Fig. 2. Rather than appearing as a normal distribution, it is biased toward the positive values

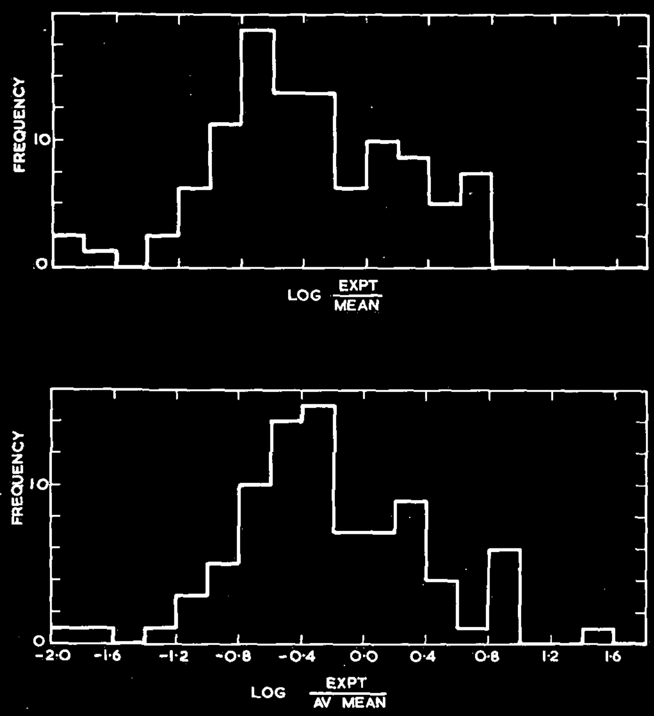

Fig. 2. Distribution of z values.

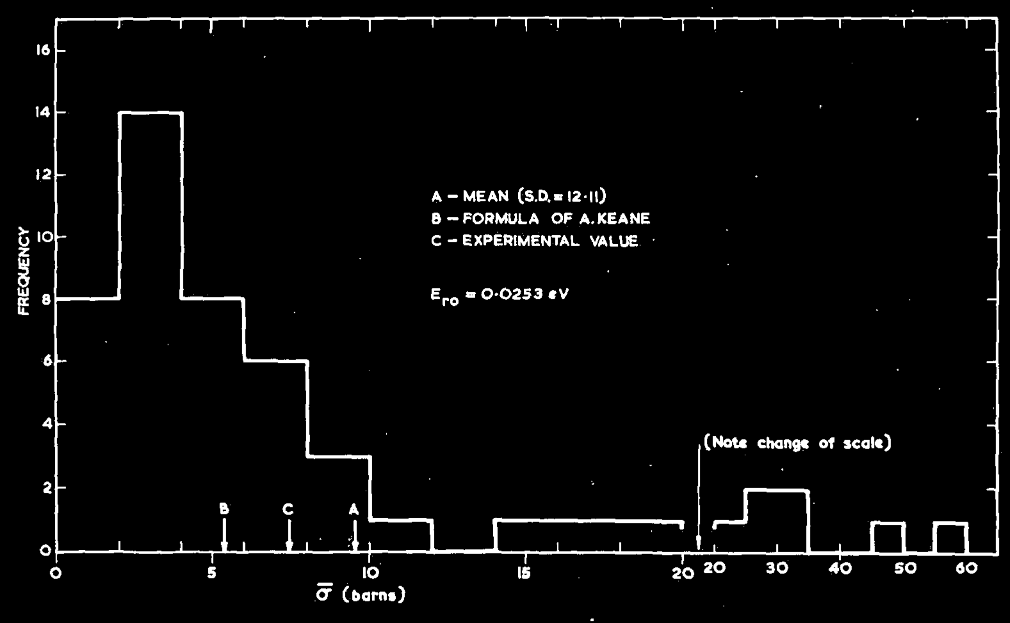

Fig. 1. Frequency distribution of σ for ^{232}Th.

TABLE I

Calculated Thermal Cross Sections: Sampled Neutron Widths

Nuclide	Experiment,b	Mean,b	Standard Deviation,b	log (expt/mean)	No. of Resonances	Source	
						Data	σ expt
^{45}Sc	25. ± 2.	13.64	21.53	0.2631	15	a	a,b
^{50}V	80. ± 60.	321.3	678.9	-0.6038	16	a	a
^{50}Cr	15.9 ± 1.6	2.966	2.679	0.7292	4	a	c
^{52}Cr	0.76 ± 0.06	1.337	1.933	-0.2453	14	a	d
^{55}Mn	13.3 ± 0.1	83.28	133.1	-0.7966	42	a	a,b
^{54}Fe	2.8 ± 0.4	0.7937	0.9720	0.5475	21	a	c
^{56}Fe	2.7 ± 0.2	10.94	17.59	-0.6077	23	a	d
^{57}Fe	2.5 ± 0.2	9.894	12.44	-0.5974	7	a	d
^{59}Co	37.2 ± 0.6	301.4	576.7	-0.9086	62	a	a,b
^{63}Cu	4.51 ± 0.23	3.818	4.658	0.0723	30	a	c
^{65}Cu	2.2 ± 0.2	15.44	24.44	-0.8462	18	a	d
^{68}Zn	1.095 ± 0.11	8.383	14.53	-0.8840	2	a	a
^{69}Ga	2.1 ± 0.2	12.96	19.87	-0.7904	8	a	d
^{71}Ga	5.15 ± 1.0	88.81	91.10	-1.2297	4	a	a
^{75}As	4.3 ± 0.2	61.10	78.83	-1.1526	112	a	e
^{74}Se	50. ± 7.	232.3	459.3	-0.6671	2	a	d
^{76}Se	22. ± 1.	3.562	7.117	0.7907	11	a	a
^{77}Se	42.0 ± 4.0	142.7	227.8	-0.5312	10	a	e
^{80}Se	0.61 ± 0.06	1.252	1.623	-0.3123	9	f	e
^{79}Br	10.9 ± 0.6	89.80	83.99	-0.9159	7	a	a
^{85}Rb	0.91 ± 0.08	4.630	6.336	-0.7065	9	a	e
^{87}Rb	0.12 ± 0.03	6.725	9.788	-1.7485	8	a	e
^{84}Sr	1.05 ± 0.17	9.748	13.86	-0.9677	8	a	a
^{86}Sr	0.8 ± 0.1	1.221	1.911	-0.1836	8	a	a
^{88}Sr	0.005 ± 0.001	0.02695	0.04463	-0.7316	19	a	e
^{90}Zr	0.10 ± 0.07	0.3881	0.6115	-0.5889	13	f	e
^{91}Zr	1.58 ± 0.12	6.434	7.421	-0.6098	13	a	e
^{92}Zr	0.25 ± 0.12	0.2207	0.2424	0.0541	8	a	e
^{93}Zr	1.1 ± 0.4	6.220	10.53	-0.7524	-	g	e
^{94}Zr	0.075 ± 0.008	16.23	29.47	-2.3352	17	a	a
^{96}Zr	0.05 ± 0.01	4.262	8.828	-1.9306	18	a	a
^{95}Mo	14.5 ± 0.5	11.56	18.52	0.0984	-	g	a
^{97}Mo	2.2 ± 0.7	4.405	5.540	-0.3015	9	a	e
^{98}Mo	0.15 ± 0.2	0.2425	0.3214	-0.2086	6	a	a
^{100}Mo	0.5 ± 0.5	4.101	7.361	-0.9139	4	a	e
^{99}Tc	22. ± 3.	96.09	121.7	-0.6403	-	g	a
^{101}Ru	3.1 ± 0.9	11.66	15.08	-0.5753	11	a	a
^{102}Ru	1.44 ± 0.16	138.2	217.6	-1.9821	-	g	e
^{103}Rh	150. ± 5.	1041.	1708.	-0.8414	47	a	a,b
^{105}Pd	11.0 ± 6.0	223.9	417.5	-1.3087	26	a	e
^{106}Pd	0.292 ± 0.029	0.04839	0.0218	0.7806	-	g	a
^{108}Pd	12.2 ± 0.2	9.519	12.90	0.1078	-	g	a
^{109}Ag	91. ± 3.	112.4	218.8	-0.0917	-	g	a
^{113}Cd	(2 ± 0.03) × 10^4	4.391 × 10^4	5.306 × 10^4	-0.3415	9	a	e
^{115}In	199. ± 8.	135.9	231.2	0.1674	11	a	a
^{127}I	6.2 ± 0.2	33.25	35.49	-0.7294	-	g	a,b
^{129}I	28. ± 3.	26.04	29.35	0.0315	5	a	a
^{131}Xe	110. ± 20.	24.39	29.53	0.6542	-	g	a
^{135}Xe	(3.6 ± 0.4) × 10^6	3.939 × 10^7	6.405 × 10^7	-1.0391	-	g	a
^{133}Cs	31.6 ± 1.7	87.20	148.0	-0.4408	123	a	a,b
^{138}Ba	0.35 ± 0.15	0.2050	0.3014	0.2323	23	a	a
^{139}La	8.2 ± 0.8	3.604	7.717	0.3570	-	g	a
^{141}Pr	12. ± 3.	16.80	27.68	-0.1461	-	g	a,b
^{143}Nd	335. ± 10.	195.9	199.1	0.2330	7	a	a
^{145}Nd	52. ± 2.	454.7	725.5	-0.9417	-	g	a

TABLE I (Continued)

Nuclide	Experiment, b	Mean, b	Standard Deviation, b	log (expt/mean)	No. of Resonances	Source: Data	Source: σ expt
^{148}Nd	2.9 ± 0.5	12.89	18.61	-0.6479	-	g	a
^{150}Nd	3.0 ± 1.5	8.926	13.20	-0.4735	-	g	e
^{147}Pm	235. ± 24.	1176.	1564.	-0.6993	-	g	e
^{147}Sm	87. ± 60.	184.3	305.9	-0.3260	-	g	e
^{148}Sm	9.0 ± 9.0	4.916	8.366	0.2626	-	g	e
^{149}Sm	(4.08 ± .09) × 10^4	1.114 × 10^5	1.996 × 10^5	-0.4362	-	g	e
^{150}Sm	97.0	25.69	37.39	0.5770	-	g	e
^{151}Sm	(1.5 ± .04) × 10^4	9.264 × 10^3	1.486 × 10^4	0.2093	-	g	e
^{152}Sm	216 ± 6.	75.62	123.3	0.4558	-	g	e
^{154}Sm	5.5 ± 1.1	19.02	37.56	-0.5388	-	g	e
^{151}Eu	(7.8 ± 0.2) × 10^3	3.404 × 10^3	3.941 × 10^3	0.3601	-	g	c
^{153}Eu	450. ± 20.	1.517 × 10^3	2.013 × 10^3	-0.5278	-	g	e
^{155}Eu	(1.4 ± .4) × 10^4	2.005 × 10^5	3.536 × 10^5	-1.1560	-	g	e
^{155}Gd	(5.8 ± .3) × 10^4	1.556 × 10^5	2.277 × 10^5	-0.4286	-	g	e
^{157}Gd	(2.42 ± .04) × 10^5	4.703 × 10^5	8.992 × 10^5	-0.2886	-	g	e
^{158}Gd	3.9 ± 0.4	21.85	32.46	-0.7484	-	g	e
^{176}Lu	(4.0 ± .8) × 10^3	6.594 × 10^3	8.411 × 10^3	-0.2171	-	g	d
^{186}W	35. ± 3.	30.57	43.84	0.0588	-	g	d
^{232}Th	7.4 ± 0.1	9.509	12.11	-0.1089	257	a	a
^{231}Pa	200. ± 10.	3002.	5434.	-1.1764	21	a	a
^{233}Pa	43. ± 5.	22.75	40.35	0.2765	15	a	a
^{232}U	78. ± 4.	13.26	18.30	0.7696	14	a	a
^{233}U	49. ± 6.	624.5	1191.	-1.1053	46	a	a
^{234}U	90.	19.61	34.08	0.6618	20	a	a,h
^{235}U	101. ± 2.	557.8	934.6	-0.7422	221	a	a
^{236}U	6. ± 1.	38.55	61.84	-0.8079	14	a	a
^{238}U	2.73 ± .04	7.676	12.16	-0.4490	148	a	a
^{237}Np	170. ± 5.	310.9	450.4	-0.2622	65	a	d,h
^{239}Pu	273.9	256.2	544.7	0.0290	-	g	a
^{241}Pu	425. ± 40.	955.1	1502.	-0.3517	31	a	a
^{241}Am	622. ± 35.	684.5	856.9	-0.0416	42	a	a
^{243}Am	180. ± 20.	309.3	231.8	-0.2351	11	a	a

a. From Stehn et al.[6]
b. Natural parameters used.
c. From Hughes et al.[5]
d. From Hughes and Schwartz.[4]
e. From England.[7]
f. From Stehn et al.,[6] modified.
g. From Ref. 8.
h. Estimated.

for z. This is produced by the long exponential tail in the distribution of σ's, which allows for an appreciable probability of overestimating the answer.

The experimental thermal-neutron cross sections were taken from BNL-325[4-6] and England,[7] while resonance parameters came from BNL-325. The direct arithmetic average of spacings and widths was used where parameters were available.[8] In cases where no resonances have been resolved, the level spacing was computed from an improved version of Gilbert and Camerons'[9] free-gas formula (Cook et al.[10]). Neutron widths were then estimated from the nuclear systematics of the s-wave strength functions, taken from the CINDA

[4] D. J. HUGHES and R. B. SCHWARTZ, "Neutron Cross Sections," BNL-325, Second ed., Brookhaven National Laboratory (1958).

[5] D. J. HUGHES, B. A. MAGURNO, and M. K. BRUSSEL, "Neutron Cross Sections," BNL-325, Suppl., 1, Second ed., Brookhaven National Laboratory (1960).

[6] J. R. STEHN, M. D. GOLDBERG, R. WEINER-CHASMAN, S. F. MUGHABGHAB, B. A. MAGURNO, and V. M. MAY, "Neutron Cross Sections," BNL-325, Suppl. 2, Vol. III, Brookhaven National Laboratory (1965).

[7] T. R. ENGLAND, "Time-Dependent Fission Product Thermal and Resonance Absorption Cross Sections," WAPD-TM-333 Addendum No. 1., Bettis Atomic Power Laboratory, Westinghouse Electric Corporation (1965).

[8] Unpublished data of the Australian Atomic Energy Commission.

[9] A. GILBERT and A. G. W. CAMERON, *Can. J. Phys.*, **43**, 1446 (1965).

[10] J. L. COOK, H. FERGUSON, and A. MUSGROVE, *Aust. J. Phys.*, **20**, 5 (1967), to be published.

TABLE II

Calculated Thermal Cross Sections: Keane's Formula

Nuclide	Experiment,b	Av mean,b	log (expt/av mean)
^{45}Sc	25. ± 2.	15.32	0.2126
^{50}V	80. ± 60.	209.9	-0.4189
^{50}Cr	15.9 ± 1.6	4.932	0.5084
^{52}Cr	0.76 ± 0.06	0.9003	-0.0736
^{55}Mn	13.3 ± 0.1	45.67	-0.5357
^{54}Fe	2.8 ± 0.4	0.3719	0.8767
^{56}Fe	2.7 ± 0.2	9.836	-0.5615
^{57}Fe	2.5 ± 0.2	7.676	-0.4872
^{59}Co	37.2 ± 0.6	177.4	-0.6695
^{63}Cu	4.51 ± 0.23	3.944	0.0582
^{65}Cu	2.2 ± 0.2	7.312	-0.5216
^{68}Zn	1.095 ± 0.11	5.686	-0.7154
^{69}Ga	2.1 ± 0.2	8.402	-0.6022
^{71}Ga	5.15 ± 1.0	67.27	-1.1090
^{75}As	4.3 ± 0.2	39.86	-0.9671
^{74}Se	50. ± 7.	160.5	-0.5065
^{76}Se	22. ± 1.	2.784	0.8978
^{77}Se	42.0 ± 4.0	77.90	-0.2683
^{80}Se	0.61 ± 0.06	0.8814	-0.1598
^{79}Br	10.9 ± 0.6	58.91	-0.7328
^{85}Rb	0.91 ± 0.08	3.879	-0.6297
^{87}Rb	0.12 ± 0.03	6.076	-1.7044
^{84}Sr	1.05 ± 0.17	8.086	-0.8865
^{86}Sr	0.8 ± 0.1	1.207	-0.1786
^{88}Sr	0.005 ± 0.001	0.01533	-0.4866
^{90}Zr	0.10 ± 0.07	0.2251	-0.3524
^{91}Zr	1.58 ± 0.12	3.414	-0.3346
^{92}Zr	0.25 ± 0.12	0.1936	0.1110
^{93}Zr	1.1 ± 0.4	3.499	-0.5026
^{94}Zr	0.075 ± 0.008	9.779	-2.1152
^{95}Zr	0.05 ± 0.01	3.694	-1.8685
^{95}Mo	14.5 ± 0.5	8.192	0.2480
^{97}Mo	2.2 ± 0.7	2.914	-0.1221
^{98}Mo	0.15 ± 0.2	0.1544	-0.0126
^{100}Mo	0.5 ± 0.5	3.407	-0.8334
^{99}Tc	22. ± 3.	88.52	-0.6046
^{101}Ru	3.1 ± 0.9	6.553	-0.3251
^{102}Ru	1.44 ± 0.16	154.0	-2.0292
^{103}Rh	150. ± 5.	621.6	-0.6174
^{105}Pd	11.0 ± 6.0	202.0	-1.2640
^{106}Pd	0.292 ± 0.029	0.01039	1.4488
^{108}Pd	12.2 ± 0.2	5.531	0.3436
^{109}Ag	91. ± 3.	92.93	-0.0091
^{113}Cd	(2 ± 0.03) × 10^4	5.262 × 10^4	-0.4201
^{115}In	199. ± 8.	120.3	0.2203
^{127}I	6.2 ± 0.2	23.63	-0.5811
^{129}I	28. ± 3.	19.30	0.1616
^{131}Xe	110. ± 20.	14.79	0.8714
^{135}Xe	(3.6 ± 0.4) × 10^6	4.611 × 10^7	-1.1075
^{133}Cs	31.6 ± 1.7	64.03	-0.3067
^{138}Ba	0.35 ± 0.15	0.1430	0.3887
^{139}La	8.2 ± 0.8	3.063	0.4277
^{141}Pr	12. ± 3.	10.71	0.0494
^{143}Nd	355. ± 10.	159.3	0.3228
^{145}Nd	52. ± 2.	266.4	-0.7095

TABLE II (Continued)

Nuclide	Experiment,b	Au mean,b	log (expt/av mean)
^{148}Nd	2.9 ± 0.5	7.96	-0.4385
^{150}Nd	3.0 ± 1.5	6.504	-0.3361
^{147}Pm	235. ± 24.	1097.	-0.6691
^{147}Sm	87. ± 60.	144.2	-0.2194
^{148}Sm	9.0 ± 9.0	3.723	0.3834
^{149}Sm	(4.08 ± 0.09) × 10^4	6.985 × 10^4	-0.2335
^{150}Sm	97.	14.80	0.8165
^{151}Sm	(1.5 ± .04) × 10^4	5.896 × 10^3	0.4055
^{152}Sm	216. ± 6.	50.25	0.6333
^{154}Sm	5.5 ± 1.1	12.18	-0.3453
^{151}Eu	(7.8 ± 0.2) × 10^3	2.714 × 10^3	0.4585
^{153}Eu	450. ± 20.	1.349 × 10^3	-0.4768
^{155}Eu	(1.4 ± .4) × 10^4	1.213 × 10^5	-0.9377
^{155}Gd	(5.8 ± .3) × 10^4	1.387 × 10^5	-0.3786
^{157}Gd	(2.42 ± .04) × 10^5	4.144 × 10^5	-0.2336
^{158}Gd	3.9 ± 0.4	15.79	-0.6073
^{176}Lu	(4.0 ± .8) × 10^3	9.520 × 10^3	-0.3766
^{186}W	35. ± 3.	19.55	0.2529
^{232}Th	7.4 ± 0.1	5.369	0.1393
^{231}Pa	200. ± 10.	1900.	-0.9777
^{233}Pa	43. ± 5.	18.08	0.3763
^{232}U	78. ± 4.	9.567	0.9113
^{233}U	49. ± 6.	525.6	-1.0305
^{234}U	90.	13.49	0.8242
^{235}U	101. ± 2.	356.6	-0.5479
^{236}U	6. ± 1.	18.61	-0.4916
^{238}U	2.73 ± .04	6.019	-0.3434
^{237}Np	170. ± 5.	254.2	-0.1747
^{239}Pu	273.9	232.1	0.0719
^{241}Pu	425. ± 40.	919.3	-0.3351
^{241}Am	622. ± 35.	445.4	0.1450
^{243}Am	180. ± 20.	309.0	-0.2347

compilation,[11] while radiation widths were also obtained by interpolation through the periodic table. The position of the lowest energy resonance was initially assigned at $\bar{D}/2$. For reactor physics cross-section calculations, this assignment was later varied until the correct thermal cross section was attained. In this way, estimates of unmeasured resonance integrals can also be calculated.

One source of error in the calculations is that we assumed averaged parameters for even-odd and odd-even nuclei were the same in each spin state, the level sequences of which are randomly located. Although this is not a bad approximation, one should properly specify two low-lying resonances, one from each state to fix the relative sequence, then compute the contribution from each

[11] "CINDA - An Index to the Literature on Microscopic Neutron Data," EANDC 46 "U" NYO-GEN-72-27, New York Operations Office, USAEC (1965).

spin state separately. Our approximation amounts to replacing the double sequence by an average single sequence, and we do not expect the error incurred by this assumption to be comparable with that produced by neutron-width uncertainties. In a preliminary survey of this kind, it was felt to be a justifiable approximation.

IV. CONCLUSION

The statistical distribution of reduced neutron widths was taken into account in predicting the possible range of values for the thermal-neutron cross section. In taking a sample of 87 nuclides it was found that the fluctuation in the order of magnitude was 0.4 ± 0.6 for the logarithm of the ratio of experiment to the calculated mean. This result allowed us to estimate unmeasured thermal-neutron cross sections to within one order of magnitude.

There is an additional statistical uncertainty we must take into account if estimates are to be made of cross sections where no resonances are available. This is the so far unknown probability distribution for the location of the lowest energy resonance. We are investigating the 87 nuclides listed in this paper to search for correlations that may give us this law. The general conclusions of this investigation are not expected to be altered appreciably by inclusion of such a distribution, though standard deviations may increase.

The result may be applied to the resonance overlap theory developed by Keane to give estimates of errors in overlap corrections. It could also be used to estimate unmeasured fission-product cross sections.