

**A review of the Total Neutron Cross Section of Carbon
up to 2 MeV**

K. Nishimura, S. Igarasi, S. Tanaka and T. Fuketa

**Japan Atomic Energy Research Institute
Tokai-mura, Ibaraki-ken, Japan**

May 1969

000301



**IAEA
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ABSTRACT

A review of the total neutron cross section $\sigma_{nT}(E)$ for carbon up to 2 MeV has been made by analyzing the numerical data available in SCISRS and some other reports. Those numerical data which are reported as preliminary or obsolete by the authors are not included. No special weight is given to any of the data points adopted in the analysis. A polynomial expression for $\sigma_{nT}(E)$ is derived by the least-squares method as follows:

$$\sigma_{nT}(E) = 4.729 - 2.968E + 0.551E^2 + 0.413E^3 - 0.166E^4,$$

where $\sigma_{nT}(E)$ is in barns and E in MeV. There is a systematic deviation of about 3.5 % in the cross section at 350 keV between the two groups of experimental data sets, one obtained by the time-of-flight and the other by the direct-current-beam method.

1. Introduction

In May 1965, international collaboration on "compilation and evaluation of data related to nuclear energy standards" was discussed at the subcommittee of EANDC held in Washington, D.C., and the nuclear data on the following reactions $H(n,n)$, ${}^3\text{He}(n,p)$, ${}^6\text{Li}(n,\alpha)$, ${}^{10}\text{B}(n,\alpha)$, $\text{C}(\text{total})$, $\text{Au}(n,\gamma)$, $\text{Pb}(\text{total})$, ${}^{235}\text{U}(n,f)$, ${}^{239}\text{Pu}(n,f)$ and $\bar{\nu}$ of ${}^{252}\text{Cf}$ were selected as the standards. Two years later, an IAEA panel on nuclear standards was held at Brussels, where problems and progress associated in the relevant branch of the field of nuclear data were reviewed and discussed. As mentioned in the report¹⁾ of the panel, these "nuclear standards" pertain to reactions basic to absolute flux determination and reactions suitable as secondary standards.

Among the above standard nuclear cross sections, the total neutron cross section of carbon is a useful standard for scattering cross section measurements in the energy region below 2.0 MeV. The main reasons for this are (1) the major mode of the reaction is elastic scattering, the only competing process to scattering being the (n,γ) reaction which has a very small cross section, (2) the angular distribution of scattered neutrons is believed to be almost isotropic below about 1.0 MeV^{2)*} and the angular variation in the energy of scattered neutrons is not large as compared with the $H(n,n)$ reaction, (3) the total cross section shows a monotonic shape with no resonance structure below 2.0 MeV, and (4) procurement of high purity sample is easy.

In the evaluation work of KFK-120/Part 1³⁾, the overall accuracy of the recommended σ_{nT} values below 1.4 MeV is estimated to be between

* According to the tabulation of $\overline{\cos \theta}(E)$ in KFK 750, anisotropy begins at about 50 keV.

± 5 and 10% . This evaluation revealed that existing data on microscopic cross sections of carbon at that time were not good enough for practical use as a standard. It is thus very desirable to re-evaluate the microscopic cross-section data on σ_{nT} including a number of recent measurements and to improve the reliability of the recommended values of σ_{nT} .

The present report reviews the total neutron cross section $\sigma_{nT}(E)$ of carbon up to 2 MeV as a first step of evaluation. The numerical data stored in SCISRS and available in published reports are analyzed by least-squares method. An empirical formula for $\sigma_{nT}(E)$ of carbon is deduced by 4th-order polynomial fitting. The study was initiated by a suggestion made by Dr. R. F. Taschek, LASL, U.S.A. and Dr. J. Spaepen, BCMN, Belgium at the 4th meeting of the INDSWG (International Nuclear Data Scientific Working Group) of the IAEA, and has been performed as one of the programs of the Japanese Nuclear Data Committee.

2. Compilation and Representation of the Data

1) Data compilation

The evaluation of nuclear data starts with extensive collection of the experimental data. We requested and received a copy of the relevant part of the reference list and numerical data in SCISRS from CCDN, Saclay, France. In December 1967, the number of references stored in SCISRS under the quantity "SIGMA TOTAL" of carbon was 59 over the entire neutron energy region. In addition to SCISRS, 18 references containing numerical data which were not available in SCISRS, were surveyed by ourselves. In CINDA (October 1967) there were 115 references for the total cross section on carbon. Some of these were repetitions of other reports in their contents and some others gave no data; the number of references useful for numerical work was actually not so many as would have appeared from a glance at the CINDA index.

So far as the energy region from 1 eV to 2 MeV is concerned, there are 17 references ⁴⁾⁻²⁰⁾ among the above 59 references in SCISRS, and 5 references ²¹⁾⁻²⁵⁾ among those 18 references which are not available in SCISRS. Numerical data in these 22 references are collected and assigned for the object of present study.

A curve of the total neutron cross section of carbon is given in BNL-325, 2nd edition ²⁶⁾ in the energy range 0 ~ 1.4 MeV. Three ²⁷⁾²⁸⁾²⁹⁾ of those data sets plotted on this graph are not stored in SCISRS and are not used in the present review. Concerning about the data of Wilenzick et al. ²⁷⁾ taken with time-of-flight (TOF) method in the energy region from 185 keV to 700 keV, we were informed by one of the authors, K. K. Seth, that there was uncertainty of about 10 % in the determination of backgrounds in time spectra. He recommended the use of another data taken with direct-current-beam (DCB) method by Seth et al. ¹¹⁾

2) Data representation

The total number of data points collected for the present review amounts to almost 2,700. The collection contains the following: 87 points from Harwell ⁹⁾, 109 points from ANL (Hibdon) ¹⁰⁾, 684 points from Duke ¹¹⁾, 931 points from RPI ¹⁹⁾, 122 points from Wisconsin ²⁰⁾, and 546 points from ANL (Whalen) ²²⁾. These data are plotted in figure 1 by using a Calcomp plotter.

Experimental methods of measuring the total neutron cross section can be classified into time-of-flight (TOF) and direct-current-beam (DCB) methods. The marks $\bigcirc$ and X in figure 1 correspond to the data obtained by the TOF and DCB methods, respectively. The DCB and TOF data above 1 keV are plotted separately in the upper and lower part, respectively, of figure 1(a).

In order to clarify the dense parts of the data points in figure 1 and figure 1(a), an enlarged linear scale of neutron energy is adopted in figure 1(b) in the energy range higher than 100 keV. These data are also classified according to the method of measurements.

3) Procedure and Results

Some data points deviate anomalously from the majority at near-by energies. To exclude such data points from the object of evaluation, those data which have a deviation larger than $\sqrt{10} \varepsilon$ are discarded. Here, ε is the square root of the mean square deviation of the data from a cross section curve obtained by the least-squares fit. Thus, a part or all of the data of the references (10), (16), (17), and (23) are excluded and are not plotted in figure 1. The number of these rejected data points is 24 out of total number of data points about 2,700. These data, however, are plotted in figure 1(b).

There are several papers⁹⁾¹¹⁾²⁸⁾ which contain a second or fourth order polynomial fit to the experimental data reported therein. The monotonic shape of the experimental data plotted in figure 1 indicates the possibility of a representation by a low-order polynomial. In the present work a fourth order polynomial is chosen:

$$\sigma_{nT}(E) = a_0 + a_1E + a_2E^2 + a_3E^3 + a_4E^4 \quad (1).$$

The coefficients of a_i ($i=0,1,2,3,4$) of equation (1) are deduced from fitting the cross-section data shown in figure 1 by the least-squares method. The calculation was carried out using an IBM 7090. All the numerical data adopted in figure 1 were dealt with on an equal basis and no special weight was given to any of the data points. The empirical formula of the cross-section curve thus obtained is

$$\sigma_{nT}(E) = 4.729 - 2.968E + 0.551E^2 + 0.413E^3 - 0.166E^4 \quad (2),$$

where $\sigma_{nT}(E)$ is in barns and E in MeV.. The result of the formula is shown by the solid curve in figures 1 and 1(b).

The values of the variances $(\Delta a_i)^2$ of the coefficients a_i are calculated at a confidence level of 95 % from the relation $\Delta a_i = 1.96 \varepsilon_i$, where ε_i are given as $\varepsilon_i^2 = A_{ii} \varepsilon^2$.

The coefficients A_{ii} are the diagonal elements of variance matrix A.

The matrix A is equal to the inversion of a matrix B, whose elements

consists of the coefficients in the normal equation. The values of Δa_i

corresponding to a confidence of 95 % are obtained as follows:

$$\Delta a_0 = 0.0127$$

$$\Delta a_1 = 0.124$$

$$\Delta a_2 = 0.322$$

$$\Delta a_3 = 0.279$$

$$\Delta a_4 = 0.0716$$

Since the values of a_i are correlated with each other, they should not be modified even if they are within the range of $a_i \pm \Delta a_i$.

The variance $(\Delta \sigma)^2$ is given as a function of energy E by use of the variance matrix A and the standard deviation ε as follows:

$$(\Delta \sigma)^2 = (1, E, E^2, E^3, E^4) \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{pmatrix} \begin{pmatrix} 1 \\ E \\ E^2 \\ E^3 \\ E^4 \end{pmatrix} (1.96 \varepsilon)^2 \quad (3).$$

The numerical values of $\sigma_{nT}(E)$ and the square roots of its variances $\Delta \sigma$ at a confidence level of 95 % are listed in table 1, and shown in figure 3.

At thermal and epithermal energies, values of the total neutron cross section having high accuracy are reported in several references⁴⁾³⁰⁾³¹⁾³²⁾. These data are not used in the above least-squares fit nor shown in figure 1, but will be discussed later in relation with the cross-section curves below 1 keV.

4. Discussion

The relatively small values of the error $\Delta\sigma$ shown in table 1 and figure 2 merely come from statistical treatment of a large number of data points. The individual measurements are not uniformly distributed over the energy range of the above least-squares fit, and there may exist appreciable systematic errors in some measurements. Actual reliability of the curve may be different at different energies beyond the estimate of the variances $\Delta\sigma$ by equation (3) mentioned above.

In order to check possible differences between the sets of data taken with different experimental methods, all the numerical values were classified according to the technique of measurement, i.e., the data taken by the time-of-flight (TOF) method and those by the direct-current-beam (DCB) method. Each method has specific characteristics in the total cross-section measurement. The TOF method is of great advantage in determining the whole shape of the cross-section curve over a large range of energy. The DCB method, on the other hand, is suitable for the determination of the absolute cross-section values with high accuracy at a limited number of energy points.

Since monochromatic neutrons obtained by use of the DCB method are not intense below a few keV, it is reasonable for the DCB data to set the lowest energy limit at 1 keV. Thus, the DCB data were plotted for the energy region above 1 keV in figure 1(a) upper. The TOF data were divided into two regions, i.e., from 1 keV to 2 MeV (figure 1(a) lower) and the region below 1 keV (figure 2), for the sake of comparison with the DCB data.

The same procedure of polynomial fitting by the least-squares method as before was applied for both the DCB data (figure 1(a) upper) and TOF data (figure 1(a) lower), separately. The results are shown by the solid curves in figure 1(a); and the respective formulas are as follows:

$$\sigma_{nT}(E) = 4.841 - 3.792E + 2.333E^2 - 0.976E^3 + 0.184E^4 \quad (4)$$

for the curve in figure 1(a) upper (DCB), and

$$\sigma_{nT}(E) = 4.740 - 4.030E + 2.934E^2 - 1.306E^3 + 0.234E^4 \quad (5)$$

for the curve in figure 1(a) lower (TOF, above 1 keV).

For the TOF data below 1 keV (figure 2), first order polynomial fitting by least-squares was applied. The result is shown by the solid curve in figure 2, which is represented by the following formula:

$$\sigma_{nT}(E) = 4.637 + 0.003E \quad (6),$$

where E is given in keV.

1) E_n above 1 keV

Confidence bands of the cross-section curves from the equations (4) and (5) are calculated at a level of 95 %, and are shown in figure 4. They do not overlap below $E_n = 950$ keV and have a discrepancy of 0.136 barns (about 3.5 %) at $E_n = 350$ keV. In other words, if one took the TOF data as true values of the cross section, almost all of the DCB data would have to be rejected even at the confidence level of 95 %.

As seen in figures 1, 1(a), and 1(b), the DCB data concentrate around 100 to 650 keV, and the data of Duke¹¹⁾ and Argonne¹⁰⁾²²⁾ form the majority. On the other hand, the TOF data are mostly distributed around 0.8 to 2 MeV, and the data of Harwell⁹⁾ form the majority below 0.8 MeV. The discrepancy between the two confidence bands in figure 4 should be resolved by further measurements and a critical analysis below 950 keV.

2) E_n below 1 keV

At thermal and epithermal energies, absolute values of the total neutron cross sections having high accuracy are reported in several articles. They are 4.66 ± 0.03 barns (graphite) and 4.74 ± 0.06 barns (diamond dust) at $E_n = 1.44$ eV⁴⁾; 4.77 ± 0.05 barns at $E_n = 0.025$ eV³⁰⁾; 4.743 ± 0.02 barns at

$E_n = 33.9 \text{ eV}^{31)}$; $4.7264 \pm 0.0024 \text{ barns at } E_n = 61.1 \text{ eV}^{31)}$, and $4.7534 \pm 0.0045 \text{ barns in the energy range } 0.3 \text{ to } 400 \text{ eV}^{32)}$, respectively. These data points and the curves of equations (2) and (6) are shown in figure 5.

Recently, a TOF measurement³³⁾ has been made at Harwell from 100 eV to 100 keV. By fitting their own data to 1 % over the energy range measured, the following equation is reported:

$$\sigma_{nT}(E) = 4.767 - 4.00 E \quad (7),$$

where E is given in MeV. These data points are not included in the present analysis, but equation (7) is shown in figure 5 for comparison.

Looking at figure 5, representation of data points by the empirical equation (2) at thermal and epithermal energy region is better than that by equations (6) and (7), although they are in reasonable agreement with that of equation (2) within the accuracy of one percent. The deviation of about 0.1 barns at thermal energy between the data points in figure 5 and the curve of equation (6) is significant.

The cross-section values obtained by the TOF method so far are lower than those obtained by the DCB method as shown in figure 4. The recent TOF measurement at Harwell, however, indicate the higher values of cross section as seen by equation (7) in figure 5. The data of previous TOF measurements seem to indicate a systematic tendency of having low values of cross section.

In the low energy region, below 1 keV, therefore, further measurements with good accuracy are highly desirable in order to settle the above discrepancy.

In the present work, as mentioned above, all the numerical data which satisfy the criterion of $\sqrt{10} \%$ are equally weighted, and no specific consideration is given to any individual set of data. This treatment has been adopted as a first step of an evaluation, because about half of the data sets are provided with no discussion of errors and, furthermore, no effort

has been made to estimate the systematic errors in different sets of data measured at different laboratories. This treatment reported here results in weighting each set of data with a weight which is dependent upon the number of its data points. In so far as this method of treatment is used, any new experiment, which has a relatively small number of data points, would not affect the present empirical formula substantially, even if the new experiment had a very high accuracy. Thus, in the next step of the evaluation, accuracies of individual sets of data should be investigated in order to find proper weights.

The authors are indebted to Dr. T. Momota for his suggestions and interest to this work. They would like to express their gratitude to Dr. K. Tsukada and Professor R. Nakasima for helpful and stimulating discussions. The authors are also grateful to Drs. K. Parker, J. J. Schmidt, S. Schwarz, L. Stewart and R. F. Taschek for careful reading and useful comments to a preliminary draft of the manuscript.

Table 1. Cross-section values $\sigma_{nT}(E)$ are calculated from the equation (2), which is derived from fitting the cross-section data shown in figure 1 by the least-squares method. The errors $\Delta\sigma$, square roots of variances, of the $\sigma_{nT}(E)$ are deduced at a confidence level of 95 %.

E(MeV)	$\sigma_{nT}(E)$ (barns)	$\Delta\sigma$ (barns)
Energy	Cross sections	Errors
0.000	4.729	$\pm$ 0.013
0.050	4.582	0.010
0.100	4.438	0.009
0.150	4.298	0.009
0.200	4.161	0.010
0.250	4.027	0.010
0.300	3.898	0.010
0.350	3.773	0.009
0.400	3.652	0.009
0.450	3.536	0.008
0.500	3.424	0.007
0.550	3.317	0.007
0.600	3.214	0.007
0.650	3.116	0.007
0.700	3.023	0.008
0.750	2.935	0.009
0.800	2.851	0.010
0.850	2.771	0.010
0.900	2.696	0.011
0.950	2.626	0.011
1.000	2.559	0.012
1.050	2.496	0.012
1.100	2.438	0.013
1.150	2.382	0.013
1.200	2.330	0.014
1.250	2.281	0.014
1.300	2.235	0.015
1.350	2.191	0.016
1.400	2.149	0.017
1.450	2.109	0.018
1.500	2.070	0.019
1.550	2.032	0.020
1.600	1.995	0.021
1.650	1.957	0.021
1.700	1.918	0.022
1.750	1.879	0.022
1.800	1.838	0.024
1.850	1.795	0.028
1.900	1.748	0.033
1.950	1.699	0.041
2.000	1.645	0.052

Figure Captions

Figure 1. The total neutron cross-section data of Carbon from 1 ev up to 2 MeV used in the present analysis. The time-of-flight (TOF) and direct-current-beam (DCB) data are indicated by $\odot$ and X, respectively. The solid curve presents equation (2), which is an empirical formula deduced by the least-squares method from fitting all the data shown in the figure.

Figure 1(a). Classified plotting of the cross-section data from 1 keV to 2 MeV in accordance with the method of DCB and TOF. The solid curves in the upper (DCB) and lower (TOF) figures present equations (4) and (5), respectively.

Figure 1(b). Plotting of the dense portions of figure 1(a) with an enlarged linear^a scale of neutron energy. Several data points anomalously deviated from majority at near-by energies are not plotted in figure 1(a), but in figure 1(b). The solid curves represent equation (2).

Figure 2. The total neutron cross-section data of Carbon from 1 ev to 1 keV, obtained by the TOF method. The solid curve presents equation (6), which is derived by the least-squares analysis from fitting all the data shown in the figure.

Figure 3. The total neutron cross-section curve calculated from equation (2) and its confidence band obtained at confidence level of 95 % from equation (3).

Figure 4. Confidence bands of equations (4) and (5). The bands are calculated at confidence level of 95 %.

Figure 5. The total neutron cross-section measurements of Carbon with high precision in the energy region from thermal to 1 keV are compared with the curves of equation (2), (6) and (7). The experimental data $\odot$ are taken from reference (4), $\frac{1}{2}$ from reference (7), and $\blacksquare$ from reference (31), the errors

being of the order of the size of the square. The horizontally long rectangle indicates energy range and error limit of the data from reference (32).

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