AAEC/TM536

INDC(AUL)-9/G
383

AAEC/TM536

# AUSTRALIAN ATOMIC ENERGY COMMISSION
## RESEARCH ESTABLISHMENT
## LUCAS HEIGHTS

**THE CALCULATION OF NEUTRON CROSS SECTIONS AT ENERGIES BETWEEN 0.5 MeV AND 15 MeV**

by

**W.K. BERTRAM**

April 1970

IAEA
NUCLEAR DATA SECTION
MASTER COPY

AUSTRALIAN ATOMIC ENERGY COMMISSION
RESEARCH ESTABLISHMENT
LUCAS HEIGHTS

# THE CALCULATION OF NEUTRON CROSS SECTIONS AT ENERGIES BETWEEN 0.5 MeV AND 15 MeV

by

W. K. BERTRAM

## ABSTRACT

The optical model is used to calculate the elastic, non-elastic, and total cross sections for scattering of neutrons with energies between 0.5 and 15 MeV, by nuclei in the mass range $65 \leqslant A \leqslant 160$. A simple method for determining the compound elastic scattering cross section is given.

## CONTENTS

## 1. INTRODUCTION

The available experimental data on neutron cross sections of fission fragments, that is, nuclei in the mass range $60 \lesssim A \lesssim 160$, is almost entirely confined to total cross sections. Measurements of the elastic and non-elastic cross sections are very limited. Consequently when these cross sections are required for reactor calculations one must rely almost entirely on evaluated data.

To test the possibility of producing reliable evaluated data for these nuclei, we have compared the elastic, non-elastic and total cross sections, calculated for neutron energies between 0.5 and 15 MeV using the optical model, with experimental results. The optical potential used in these calculations was the local equivalent of the non-local potential of Perey and Buck (1962). This potential is known to give a good description of elastic as well as inelastic scattering over a wide range of incident neutron energies (Wilmore and Hodgson 1964, Wilmore 1964, Bertram 1970).

From the optical model, only the shape elastic $\sigma_{se}$ and the absorption $\sigma_a$ cross sections can be calculated directly. At high energies, greater than about 5 MeV, $\sigma_{se}$ and $\sigma_a$ are equal to the elastic and non-elastic cross sections. However below 5 MeV, compound elastic scattering becomes very important and must be taken into account.

In this report it is shown that the compound elastic scattering cross section can be calculated from the optical model absorption cross section using a method based on simple statistical considerations.

## 2. METHOD OF CALCULATION

The shape elastic $\sigma_{se}$, absorption $\sigma_a$, and total $\sigma_t$, cross sections were calculated using the optical model code COMPOST (Bertram 1969 a). The optical potential $\mathcal{U}$ was taken to be of the form

$$\mathcal{U} = \mathcal{U}_L + \mathcal{U}_S \ . \tag{1}$$

$\mathcal{U}_L$ is a local, energy dependent potential obtained from the non-local potential $\mathcal{U}_{NL}$ of Perey and Buck (1962) by numerically solving the equation (Wilmore 1964):

$$\mathcal{U}_L \exp\left\{\frac{m\alpha^2}{2\hbar^2}(E - \mathcal{U}_L)\right\} = \mathcal{U}_{NL} \quad , \tag{2}$$

in which E is the energy of the incident neutron, m the reduced mass of the system and $\alpha$ a constant parameter. $\mathcal{U}_{NL}$ was assumed to be of the form:

$$\mathcal{U}_{NL} = V_o\, f(r) + i\, W_o\, g(r) \ , \tag{3}$$

with

$$f(r) = V_o \{1 + \exp[(r - R_1)/d_1]\}^{-1} \quad , \text{ and} \tag{4}$$

$$g(r) = 4\, W_o \exp[(r-R_2)/d_2]\, \{1 + \exp[(r-R_2)/d_2]\}^{-2} \ . \tag{5}$$

The spin-orbit interaction $\mathcal{U}_S$ was taken as

$$\mathcal{U}_S = V_{so}\left(\frac{\hbar}{\mu c}\right)^2 \sigma . L\, \frac{1}{r} \left| \frac{d\, f(r)}{dr} \right| \quad , \tag{6}$$

where $\mu$ is the $\pi$-meson mass.

The parameters in Equations 4 – 6 remained fixed throughout our calculations, their values being those obtained by Perey and Buck from scattering of 7 MeV and 14.5 MeV neutrons by lead.

$V_c = 71$ MeV, $W_c = 15$ MeV, $V_{so} = 7$ MeV, $R_1 = R_2 = 1.22A^{1/3}$F,

$d_1 = 0.65$F , $d_2 = 0.47$F and $a = 0.85$F .

## 3. COMPOUND ELASTIC SCATTERING

When the energy of the incident neutrons is sufficiently high, greater than about 5 MeV for the nuclei we are considering, the elastic and non-elastic cross sections are just the shape elastic and absorption cross sections of the optical model. Below 5 MeV the absorption cross section as obtained from the optical model contains the compound elastic scattering cross section as well as the non-elastic cross section.

$$\sigma_a = \sigma_{ne} + \sigma_{ce} \quad . \tag{7}$$

The compound elastic scattering cross section can be calculated from the statistical theory of Hauser and Feshbach (1952). However this requires knowledge of the energies, spins and parities of all energy levels of the target nuclei up to 5 MeV excitation energy. This infomation is usually not available. In addition, for the heavier nuclei in the mass range we are considering, the Hauser-Feshbach theory does not give the correct behaviour of the inelastic scattering cross section, and therefore also does not give the correct behaviour of the compound elastic scattering cross section, near the inelastic scattering threshold (Bertram 1970).

The compound elastic scattering cross section can be calculated using a much simpler theory. As in the Hauser-Feshbach theory, we assume that the only processes contributing to $\sigma_a$ are those involving neutron emission. In other words $\sigma_{ne}$ represents only inelastic scattering. This is a good approximation because, of the other possible processes, only the $(n,\gamma)$ reaction has any significance between 0.5 and 5 MeV. In most cases the $(n,\gamma)$ reaction contributes less than 1 percent to the non-elastic cross sections in this energy range.

We now assume that the compound elastic scattering cross section is inversely proportional to the number of open exit channels, that is,

$$\sigma_{ce} = N(E)^{-1} \sigma_a(E) \quad . \tag{8}$$

$N(E)$ is the number of exit channels which in this case is just the number of energy levels that can be excited by the incoming neutron.

The number of levels with excitation energies up to $\epsilon$ is approximately given by Gilbert and Cameron (1965) as,

$$N(E) \approx e^{a(\epsilon - \epsilon_0)} \quad , \tag{9}$$

where $a^{-1}$ is the nuclear temperature.

When the incident energy E is less than the inelastic threshold energy, that is, when

$$E < E_1 \quad ,$$

where $E_1$ is the energy of the first excited state of the nucleus,

$$\sigma_{ce} = \sigma_a$$

and

$$\sigma_{ne} = 0 \quad .$$

Hence $\quad N(E) = 1 \quad$ for $\quad E < E_1 \quad . \qquad (10)$

The parameter $\epsilon_0$ in Equation 9 must therefore be the energy of the first excited state of the nucleus. The compound elastic and the non-elastic cross section can then be written:

$$\sigma_{ce} = e^{-a(E-E_1)}\,\sigma_a(E) \tag{11}$$

$$\sigma_{ne} = \left[1 - e^{-a(E-E_1)}\right]\sigma_a(E) \quad . \tag{12}$$

For several nuclei in the mass range $65 \lesssim A \lesssim 160$ the values of the parameter a were determined from experimental data on inelastic scattering and non-elastic cross sections (Figure 1). The results can be represented approximately by the equation:

$$a = 0.70 + 0.0017A \quad , \tag{13}$$

which leads to nuclear temperatures much higher than those tabulated, for example, by Gilbert and Cameron, and which yield inelastic scattering cross sections rising too rapidly near threshold. Similar discrepancies have also been found when (n,2n) excitation functions, calculated using tabulated level density parameters, were compared with experiment (Bertram 1969b). Inelastic scattering cross sections for heavy nuclei also increase too rapidly near threshold when calculated using the Hauser-Feshbach theory with or without fluctuation corrections (Bertram 1970).

## 4. RESULTS

The calculated elastic, non-elastic and total cross sections for Se, Mo, Ag, Sn, Ba and Sm were compared with experimental data compiled by Howerton (1958) and Goldberg et al. (1966). Available data on total cross sections of these nuclei is quite extensive, but very little is available for elastic and non-elastic cross sections.

The calculated total cross sections were found to be in very good agreement with experiment (Figures 2 – 7). At energies above 5 MeV the calculated elastic and non-elastic cross sections of Ag (Figure 4) and Sn (Figure 5), for which some experimental results exist, also agree well with experiment.

Compound elastic scattering cross sections were calculated from Equations 11 and 13 using the values of the parameters a and $E_1$ as follows:

| Nucleus | a($MeV^{-1}$) | $E_0$(MeV) |
|---|---|---|
| Se | 0.83 | 0.65 |
| Mo | 0.86 | 0.70 |
| Ag | 0.88 | 0.32 |
| Sn | 0.90 | 1.20 |
| Be | 0.93 | 1.40 |
| Sm | 0.95 | 0.13 |

When compound elastic scattering is taken into account the calculated non-elastic cross sections for incident energies below 5 MeV give a reasonable fit to the experimental results. The results for elastic scattering are also good except for Ag (Figure 4) where the calculated cross section is too large, and Sm (Figure 7) where the experimental cross section varies rapidly with energy, probably owing to resonance effects.

## 5. CONCLUSIONS

The results of our calculations show that the optical model, with Perey and Buck's potential, can be used to predict with good accuracy neutron cross sections of fission products for incident neutron energies from 0.5 to 15 MeV. From the little available experimental data it appears that our procedure for calculating the compound elastic scattering cross sections gives reasonable results.

## 6. REFERENCES


Bertram, W.K. (1970). – A.A.E.C. report in preparation.

Bertram, W.K. (1969 a). – AAEC/TM524.

Bertram, W.K. (1969 b). – AAEC/TM522.

Gilbert, A. and Cameron, A.G.W. (1965). – Can. J. Phys. 43: 1446.

Goldberg, M.D. et al. (1966). – BNL–325 (2nd edition, supplement 2).

Hauser, W., and Feshbach, H. (1952). – Phys. Rev. 87: 366.

Howerton, R. J. (1958). – UCRL–5226.

Perey, F.G. and Buck, B. (1962). – Nucl. Phys. 32: 353.

Wilmore, D. (1964). – AERE–R4649.

Wilmore, D. and Hodgson, P.E. (1964). – Nucl. Phys. 55: 672.

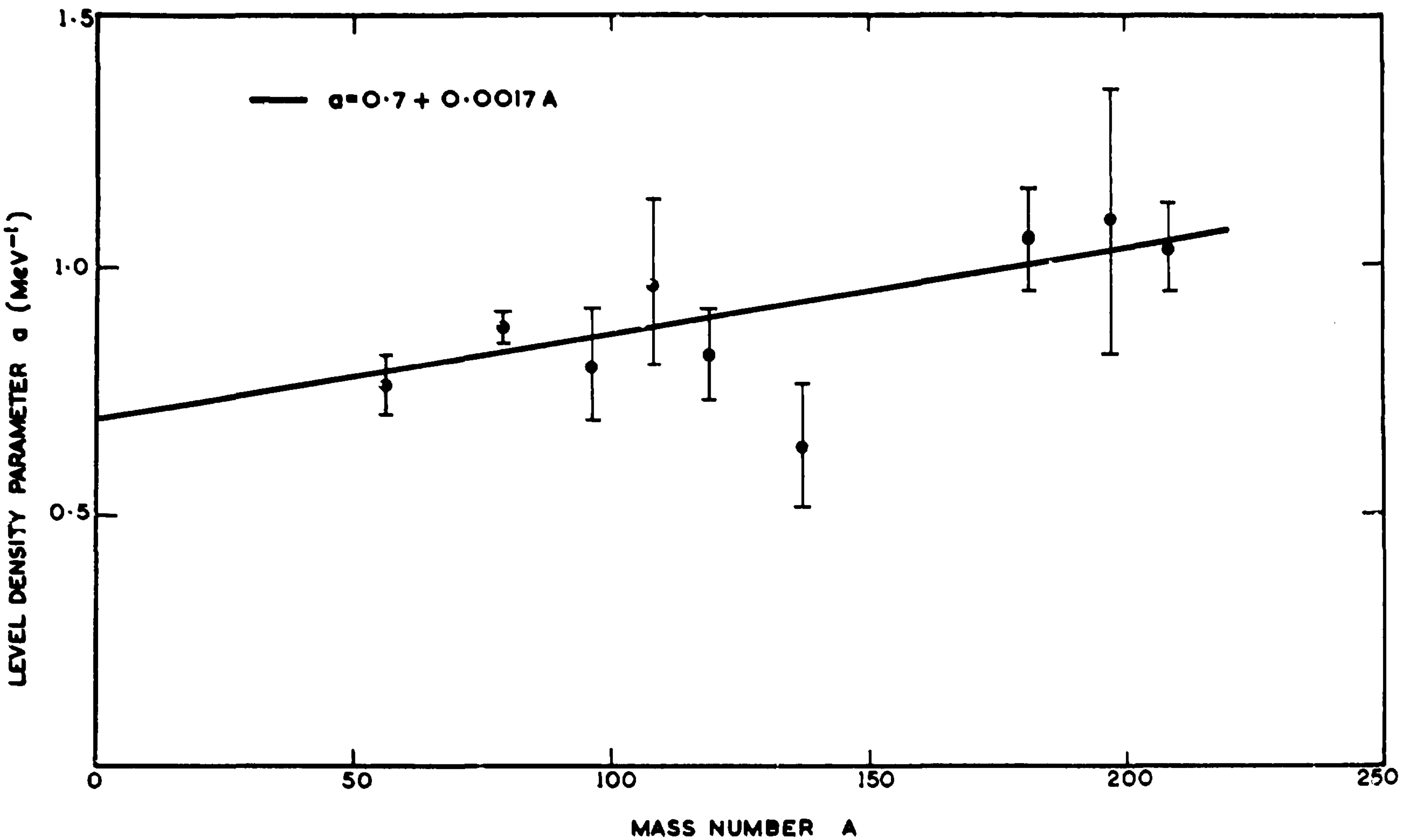


FIGURE 1. LEVEL DENSITY PARAMETER a AS A FUNCTION OF MASS NUMBER A

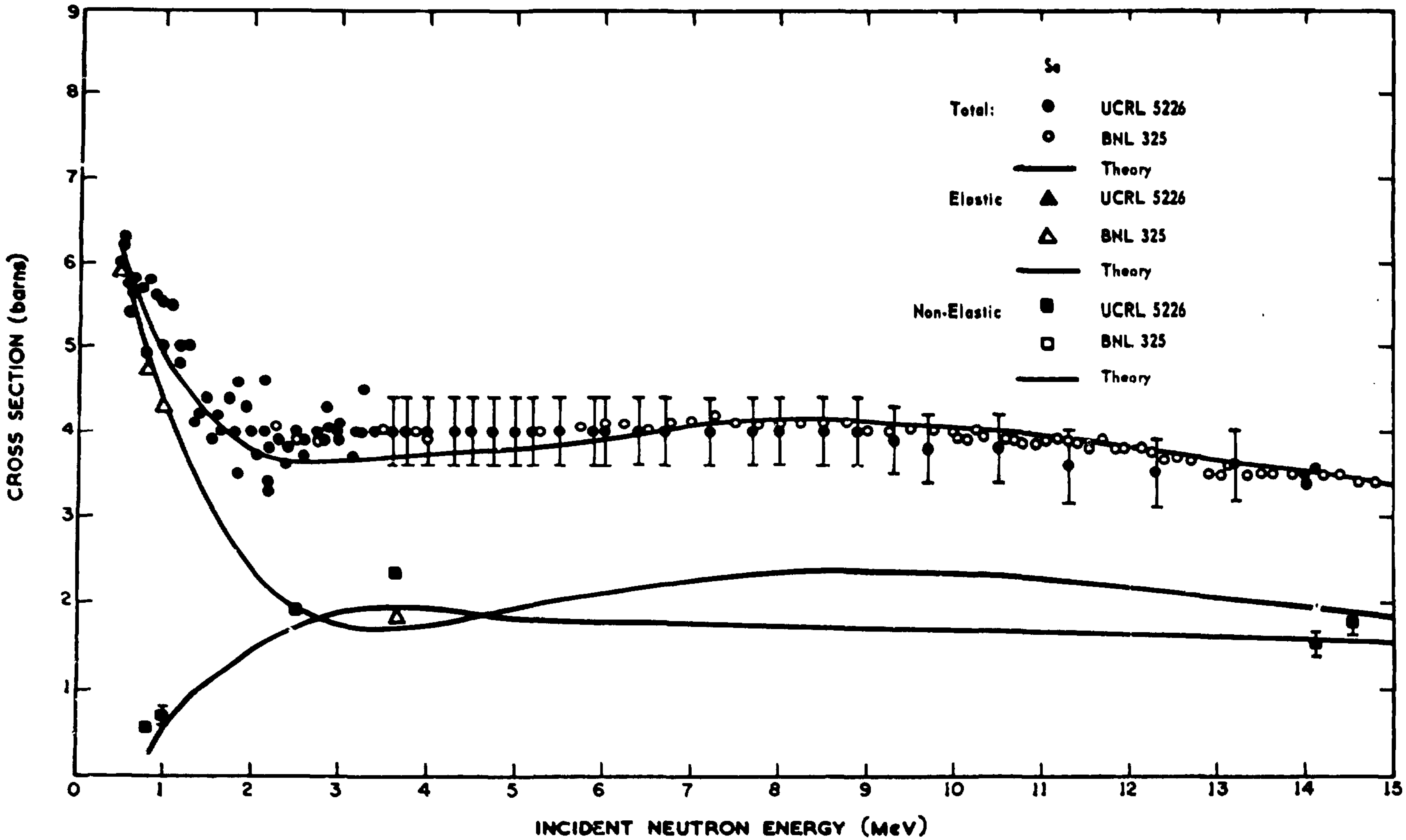


FIGURE 2. COMPARISON OF THEORETICAL NEUTRON CROSS SECTIONS OF Se WITH EXPERIMENT

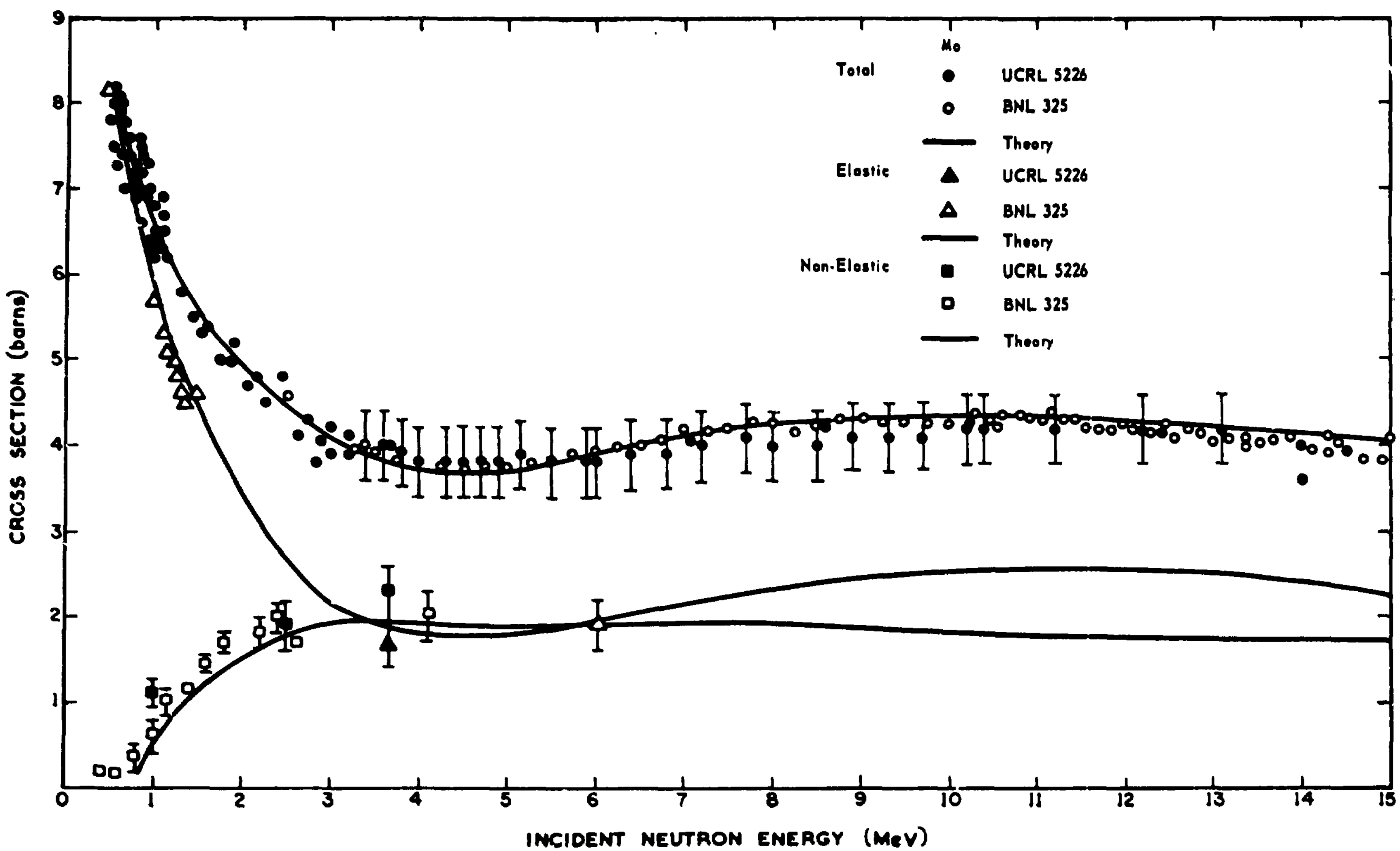


FIGURE 3. COMPARISON OF THEORETICAL NEUTRON CROSS SECTIONS OF Mo WITH EXPERIMENT

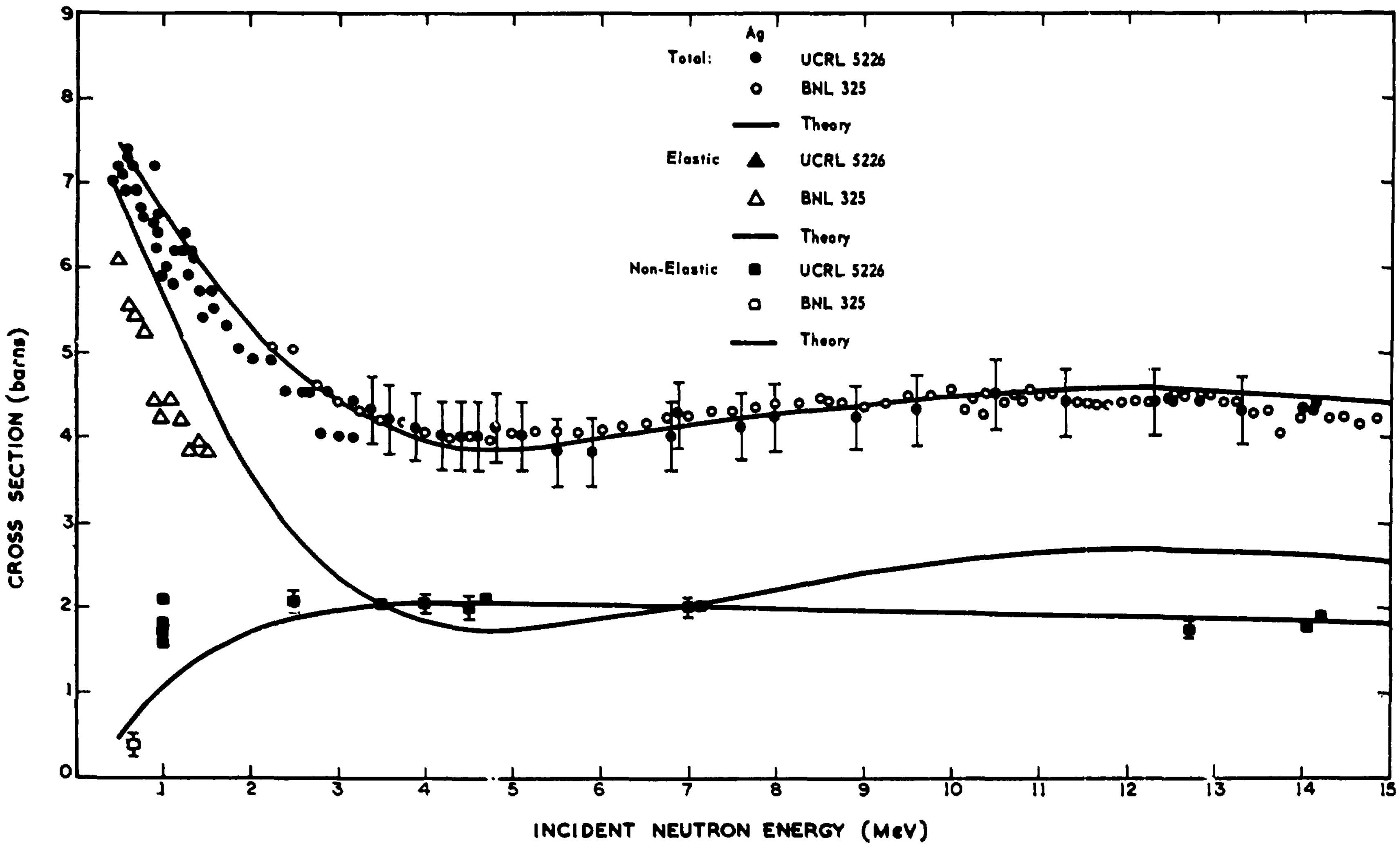


FIGURE 4. COMPARISON OF THEORETICAL NEUTRON CROSS SECTIONS OF Ag WITH EXPERIMENT

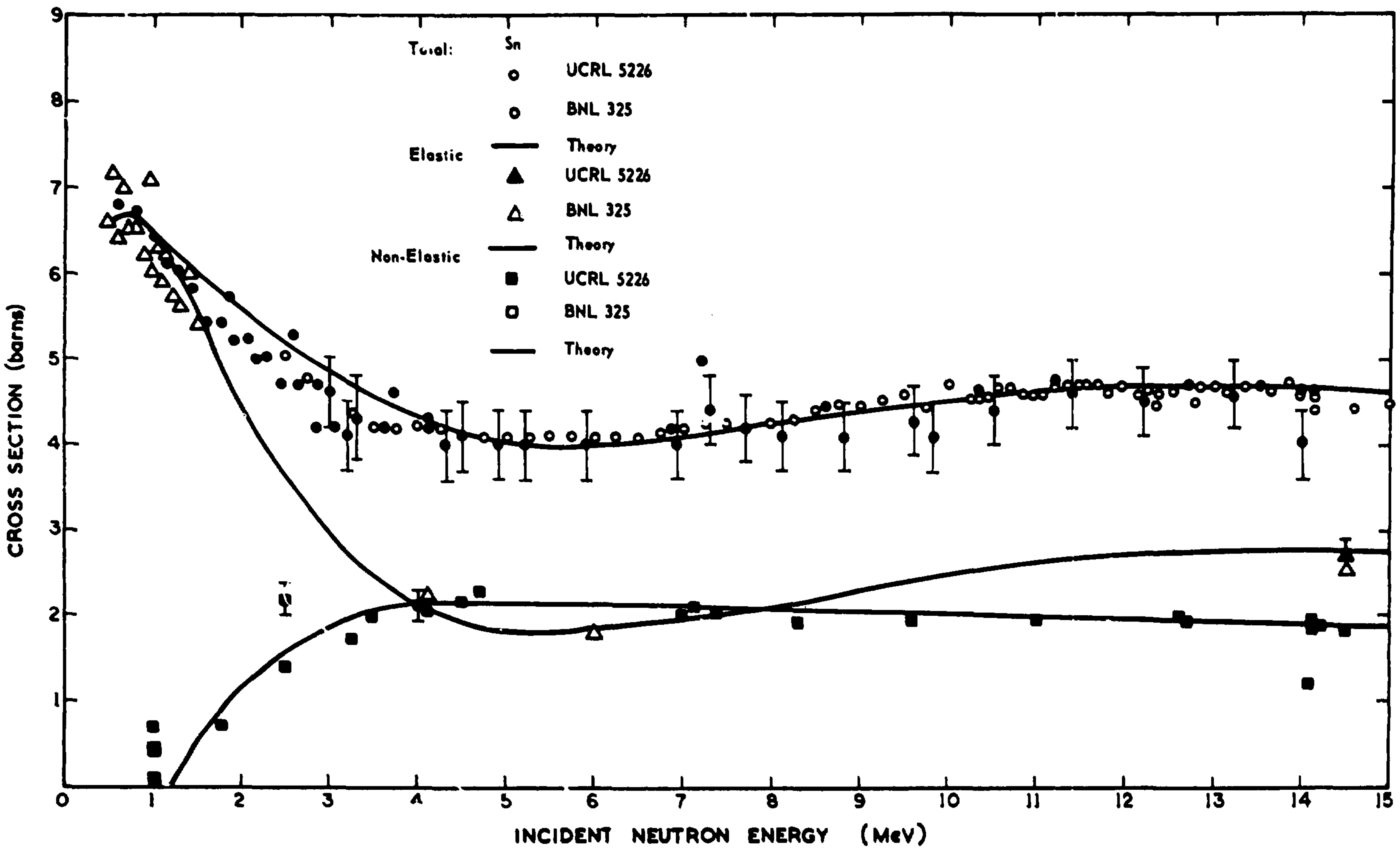


FIGURE 5. COMPARISON OF THEORETICAL NEUTRON CROSS SECTIONS OF Sn WITH EXPERIMENT

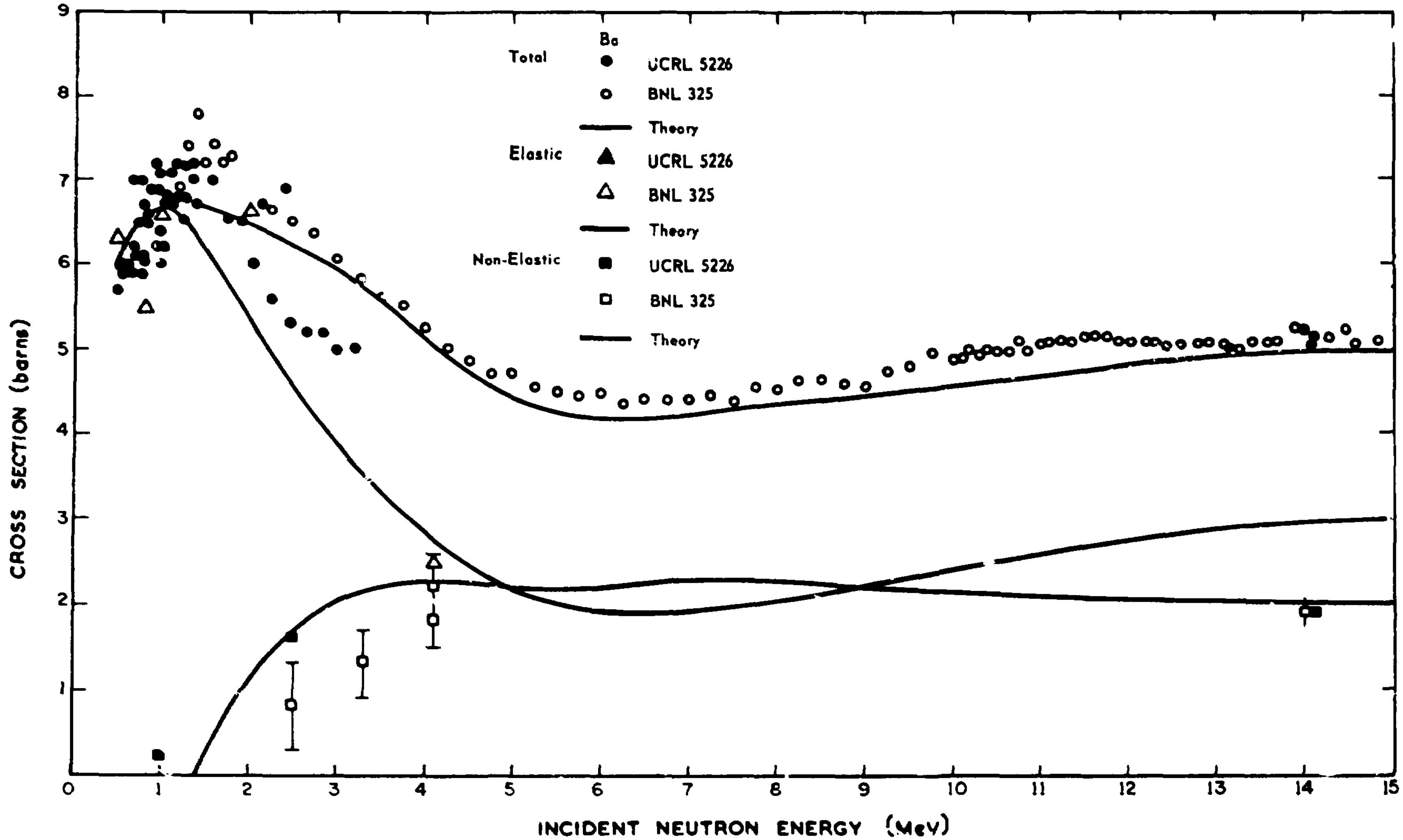


FIGURE 6. COMPARISON OF THEORETICAL NEUTRON CROSS SECTIONS OF Ba WITH EXPERIMENT

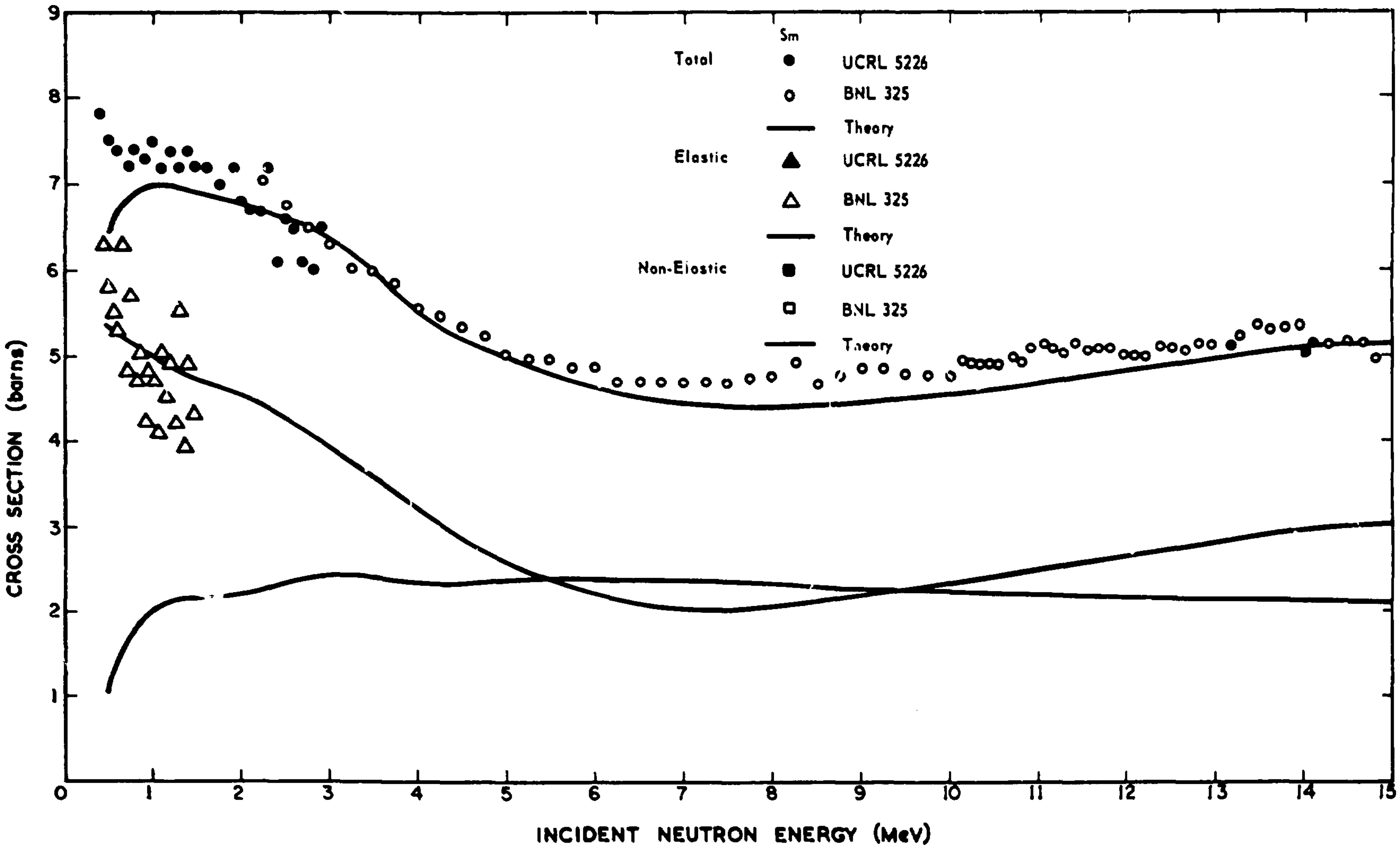


FIGURE 7. COMPARISON OF THEORETICAL NEUTRON CROSS SECTIONS OF Sm WITH EXPERIMENT