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RESEARCH ESTABLISHMENT

LUCAS HEIGHTS

THE CALCULATION OF (n,2n) CROSS SECTIONS USING THE HAUSER-FESHBACH THEORY

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ABSTRACT

The statistical theory of Hauser and Feshbach is used to calculate the (n,2n) cross sections of ^{63}Cu, ^{197}Au, ^{203}Tl, ^{232}Th and ^{238}U, for incident neutron energies below the (n,3n) threshold. Results are compared with experiment and with the results from the evaporation model.

CONTENTS

1. INTRODUCTION

In a previous paper (Bertram 1969a) the evaporation model was used to calculate excitation functions of (n,2n) reactions. It was found that when the level density parameters used were consistent with resonance data and emission spectra measurements, the calculated (n,2n) cross sections of heavy nuclei increased too rapidly with energy near the threshold. The evaporation model contains assumptions which may not be correct, and which may be the cause of the discrepancies between theory and experiment. The aim of the present work was to see if these difficulties could be overcome by using a more sophisticated theory.

The (n,2n) cross sections for ^{63}Cu, ^{197}Sn, ^{203}Tl, ^{232}Th and ^{238}U were calculated using the Hauser-Feshbach theory (Hauser and Feshbach 1952) with transmission coefficients obtained from the optical model. The potential used in these calculations was the local equivalent of the non-local potential of Perey and Buck (1962). Previously Towle and Owens (1967) found that this method gave good fits to their measurements of the (n,n') emission spectra of ^{56}Fe and ^{60}Ni at 7 MeV incident energy. Similar calculations at lower energies and calculations of the inelastic scattering cross sections of other nuclei have also been found to give good agreement with experiment (Bertram - to be published). Optical model calculations of elastic scattering as well as non-elastic and total cross sections for a wide range of different nuclei have shown that the Perey and Buck potential can be used for energies up to at least 15 MeV (Wilmore 1964, Bertram 1970).

2. THEORY

The (n,2n) process may be regarded as inelastic scattering in which the final nucleus is left excited to such a degree that a second neutron can be emitted. In the Hauser-Feshbach theory, the cross section for inelastic scattering, where the outgoing neutrons have energies between E_f and E_f+dE_f, is given (Hauser and Feshbach 1952, Towle and Owens 1967) as

$$\sigma(E_n,E_f)\,dE_f = \frac{\pi}{2(2I_o+1)k^2}\sum_{\ell j} T_\ell^{\,j}(E_n)\sum_J \frac{2J+1}{D_{\ell J}}\sum_{I_f\pi_f j'\ell'} T_{\ell'}^{\,j'}(E_f)\,\rho(I_f,\pi_f,\mathcal{E}_f)\,d\mathcal{E}_f \qquad .....(1)$$

with

$$D_{\ell J} = \sum_{p\ell'' j''} T_{\ell''}^{\,j''}(E_p) + \sum_{I_p\pi_p j''\ell''}\int_{\mathcal{E}_o}^{\mathcal{E}_n} T_{\ell''}^{\,j''}(E_p)\,\rho(I_p,\pi_p,\mathcal{E}_p)\,d\mathcal{E}_p \quad , \qquad .....(2)$$

where $\mathcal{E}_f = E_n - E_f$ and $\mathcal{E}_p = E_n - E_p$

$T_\ell^{\,j}(E_p)$ are the transmission coefficients and $\rho(I,\pi,\mathcal{E})$ is the density of levels with spin I and parity π. The angular momentum quantum numbers satisfy

$$\begin{aligned}
j &= \ell \pm \tfrac{1}{2} \quad \text{(similarly for } j' \text{ and } j''\text{)} \\
|I_o - j| &\leq J \leq (I_o + j) \\
|J - I_f| &\leq j' \leq (J + I_f) \\
|J - I_p| &\leq j'' \leq (J + I_p) \\
(-1)^{\ell}\pi_o &= (-1)^{\ell'}\pi_f \\
(-1)^{\ell}\pi_o &= (-1)^{\ell''}\pi_p
\end{aligned}$$

In Equations 1 and 2 the energy spectrum of the nucleus has been broken up into two parts. The first consists of individual levels with excitation energies up to $\mathcal{E} = \mathcal{E}_o$. In the second part, for energies greater than $\mathcal{E}_o$, the levels are assumed to form a continuum with an appropriate level density.

When the excitation energy of the final nucleus is greater than the binding energy $\mathcal{E}_A$ of the last neutron, a second neutron can be emitted. It is assumed that the final nucleus always decays via neutron emission when this is energetically possible. The (n,2n) cross section can then be written as

$$\sigma_{n,2n} = \int_0^{E_n - E_A} \sigma(E_n, E_f)\, dE_f \qquad \text{.....(3)}$$

3. METHOD OF CALCULATION

The transmission coefficients in Equation 1 were calculated using the optical model code COMPOST (Bertram 1969b) with the local equivalent of the non-local potential of Perey and Buck. The level density formula was the one used by Towle and Owens for their analysis of inelastic scattering data.

$$\rho(I,\pi,\mathcal{E}) = \frac{(2I+1)\exp(2\sqrt{\alpha u})}{1.796\ \alpha A U^2}, \qquad \text{.....(4)}$$

where
$$\begin{aligned}
U &= \mathcal{E} - P(Z) - P(N), \\
\alpha &= (0.00917S + 0.142)A, \quad \text{and} \\
S &= S(N) + S(Z).
\end{aligned}$$

P(Z), P(N) are the pairing energies and S(Z) and S(N) the shell corrections, with values taken from the tabulations of Gilbert and Cameron (1965). Equation 1 does not take into account reactions which involve charged particle and gamma ray emission so that the inelastic scattering cross section is normalised to the optical model absorption cross section σ_a.

$$\int_0^{E_n} \sigma(E_n,E_f)\, dE_f = \sigma_a\,(E_n) \quad , \qquad .....(5)$$

At the high incident energies required for the (n,2n) reaction to take place, this is usually not a good approximation. Instead Equation 5 should be written as

$$\int_0^{E_n} \sigma(E_n,E_f) \quad = \sigma_{nM}\,(E_n) \quad , \qquad .....(6)$$

where

$$\sigma_{n,M} = \sigma_{n,n'} + \sigma_{n,2n} + \sigma_{n,3n} + \ldots.. \quad ,$$

so that the (n,2n) cross sections given by Equation 3 must be multiplied by a factor $P(E_n)$ given by

$$P(E_n) = \frac{\sigma_{n,M}(E_n)}{\sigma_a\,(E_n)} \qquad .....(7)$$

As in the evaporation model this factor is assumed to be independent of energy and its magnitude is simply obtained by renormalising the calculated cross section to experimental results at a particular energy.

4. RESULTS AND DISCUSSION

The results of our calculations are summarised in Figures 1 to 5. Curves marked <u>a</u> are evaporation model calculations (Bertram 1969a) with values of the level density parameter α given by Gilbert and Cameron (1965).

The results of the Hauser-Feshbach calculation without normalisation of the cross sections (curves <u>b</u>) are much too large in all cases, and only for ^{197}Au (Figure 2) are the calculated cross sections anywhere near the experimental values. When these cross sections are normalised (curves <u>c</u>) they are found to be almost identical with the results obtained from the much simpler evaporation model. Therefore the use of the Hauser-Feshbach theory does not result in more accurate (n,2n) cross sections. The problem of why a level density formula, which is consistent with both resonance data and emission spectra for such reactions as (n,n'), (n,α), (n,p) and (p,α), does not give the correct (n,2n) excitation functions still remains unsolved. Büttner et al. (1964) used what was essentially a modified evaporation model to calculate (n,2n) cross sections with gamma ray emission as a competing process. They found that gamma ray competition reduced the (n,2n) cross sections near threshold quite appreciably. This could possibly explain the discrepancies between our results and experiment. However, the results of Büttner et al. are not as meaningful as they might seem, since the calculation of the effect of gamma ray emission requires knowledge of the total radiation width, Γ_γ.

The procedure adopted by Büttner et al. for determining Γ_γ is not very reliable and can result in values for Γ_γ which are wrong by a factor of more than 3 (Lynn 1968, Starfelt 1964). Until a better method for calculating radiation widths is found it will be very difficult to obtain an accurate estimate of the effect of gamma ray emission on (n,2n) cross sections.

5. ACKNOWLEDGEMENTS

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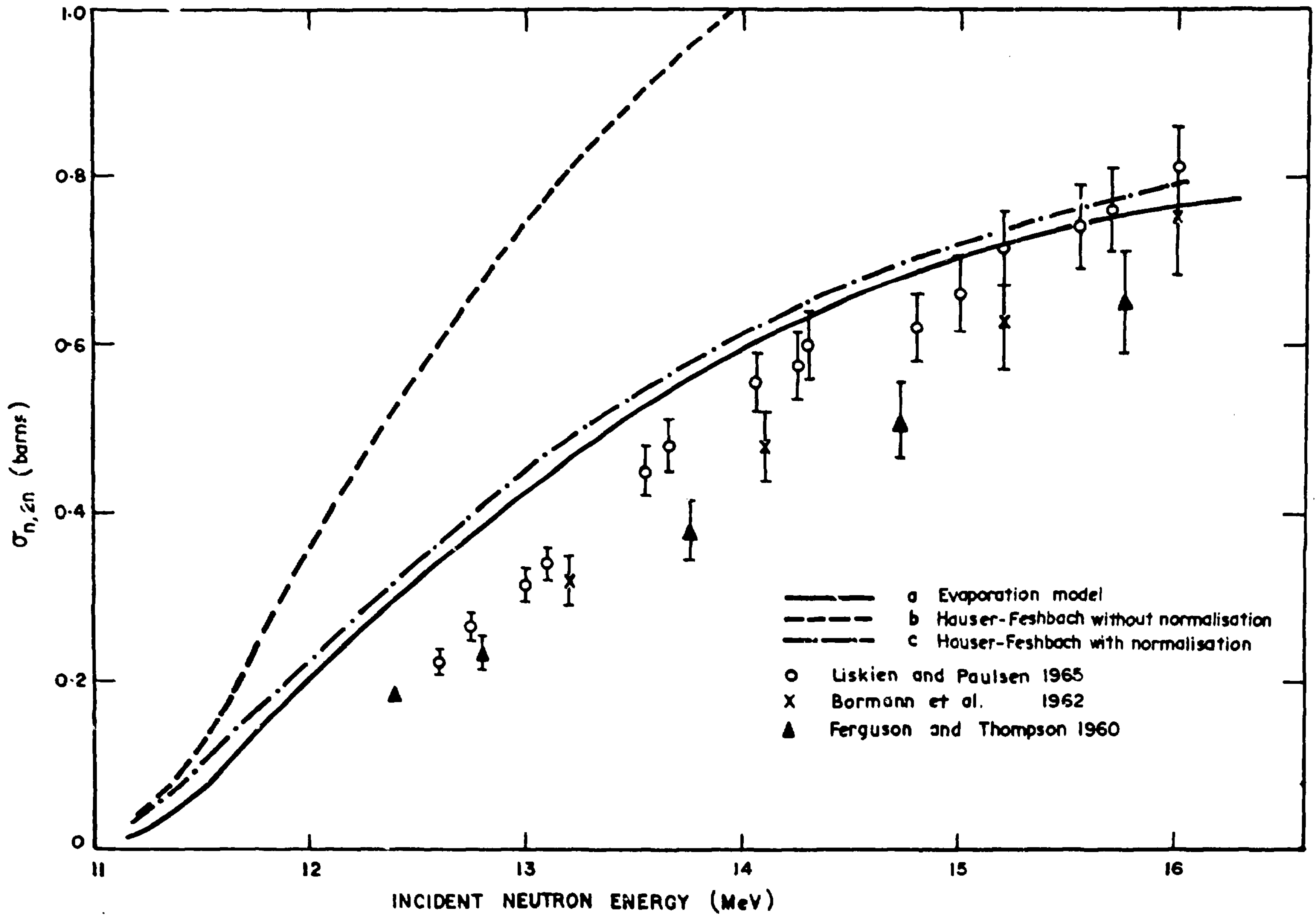

FIGURE 1. (n,2n) CROSS SECTIONS FOR ^{63}Cu - COMPARISON OF RESULTS FROM HAUSER-FESHBACH THEORY, EVAPORATION MODEL AND EXPERIMENT

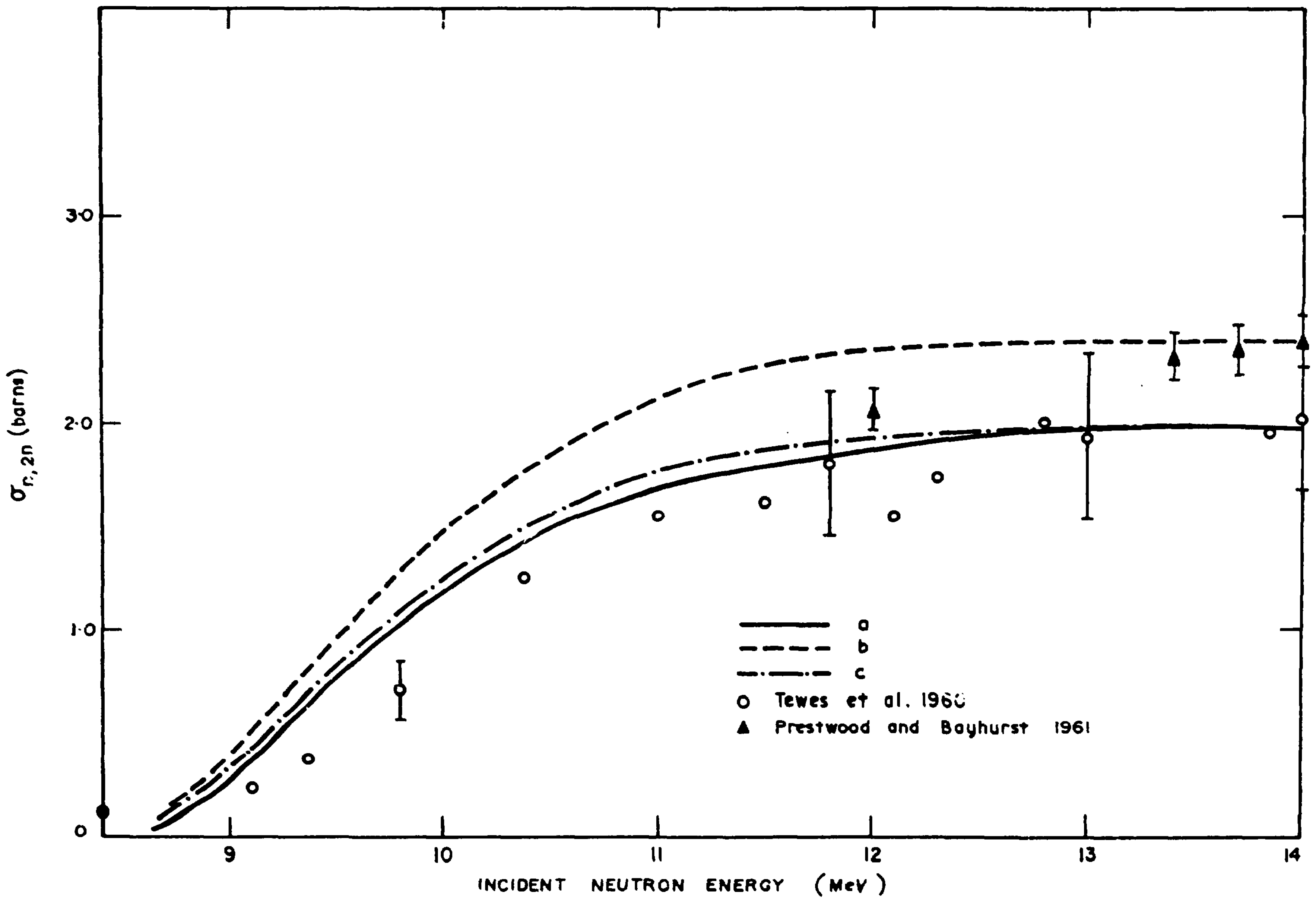

FIGURE 2. (n,2n) CROSS SECTIONS FOR ^{197}Au

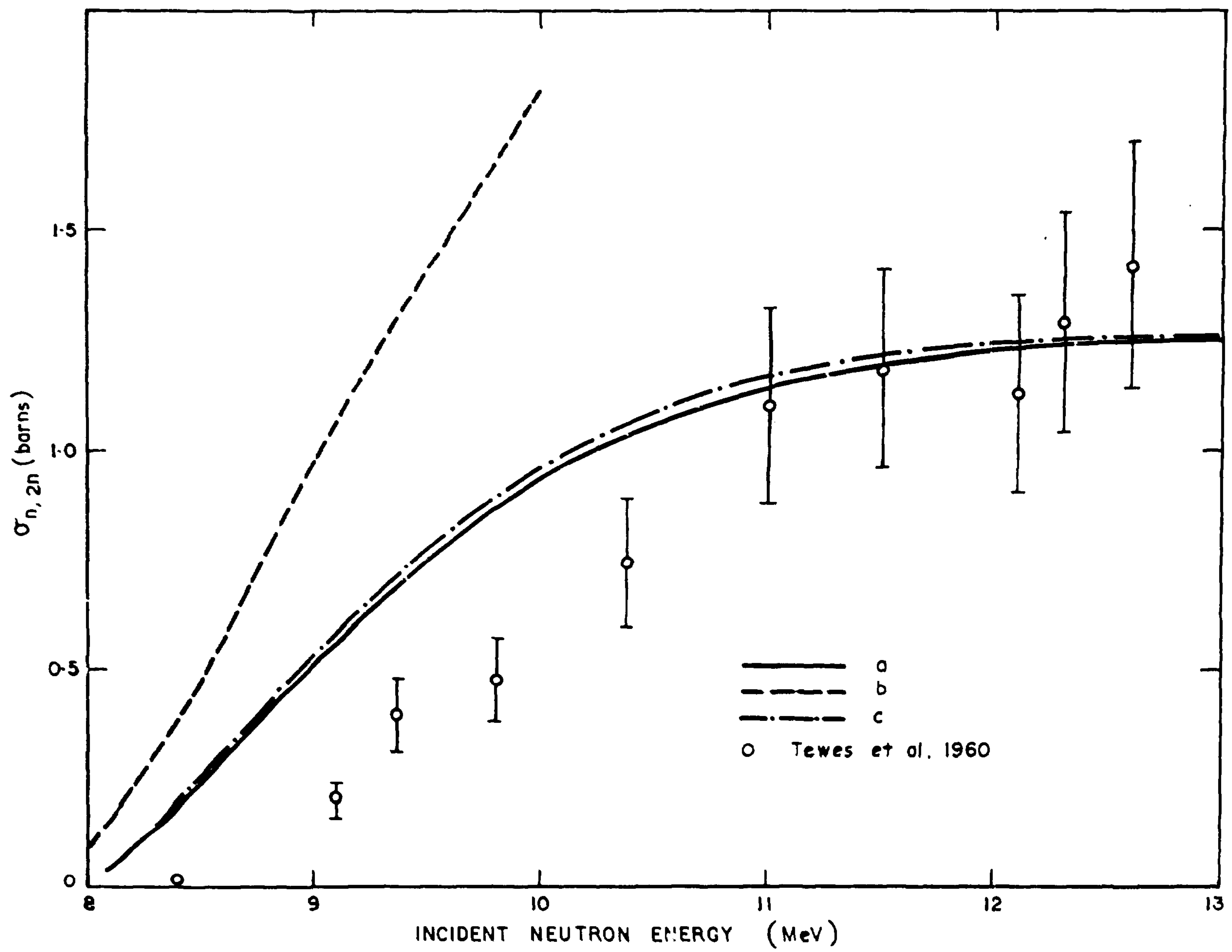

FIGURE 3. (n,2n) CROSS SECTIONS FOR ^{203}Tl

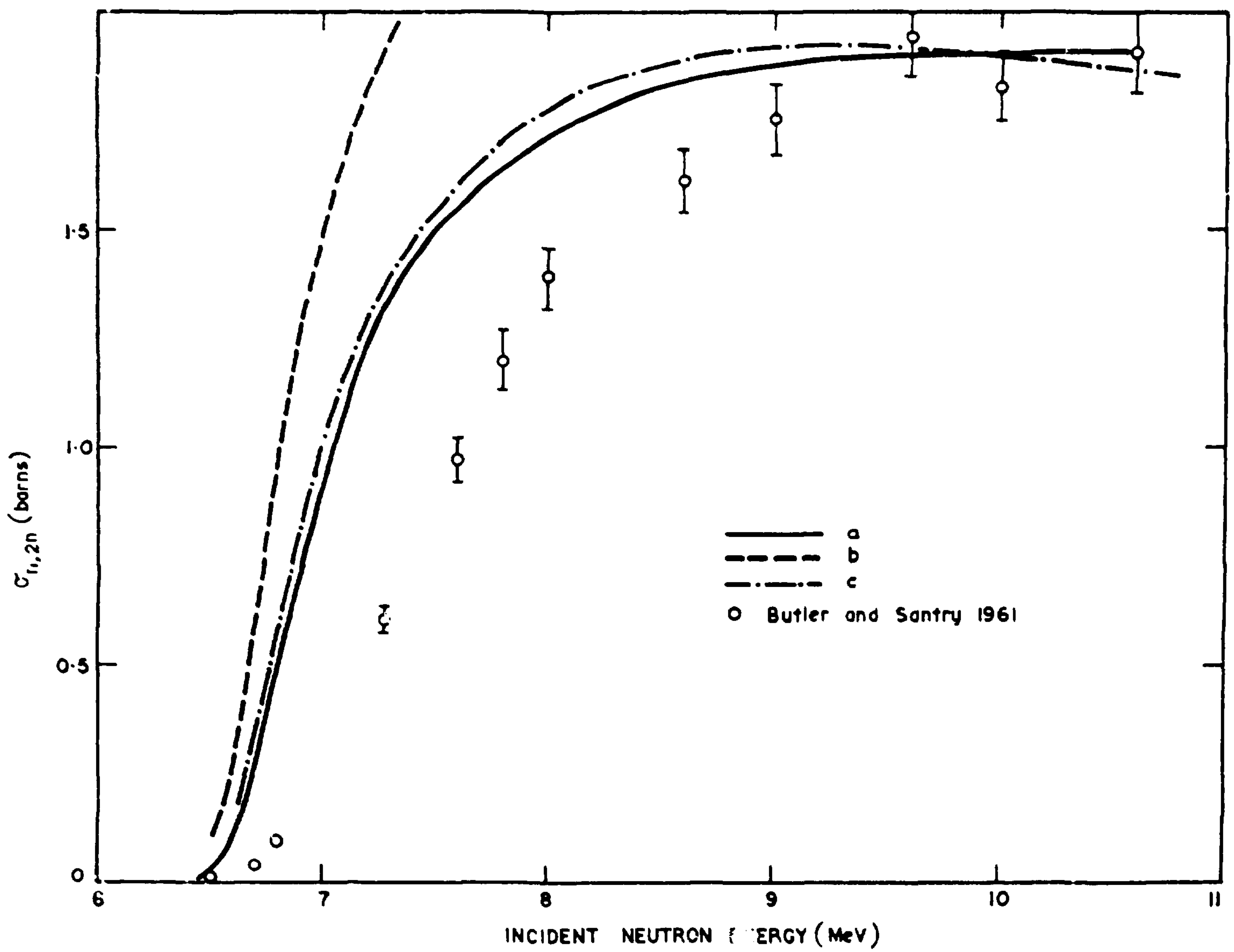

FIGURE 4. (n,2n) CROSS SECTIONS FOR ^{232}Th

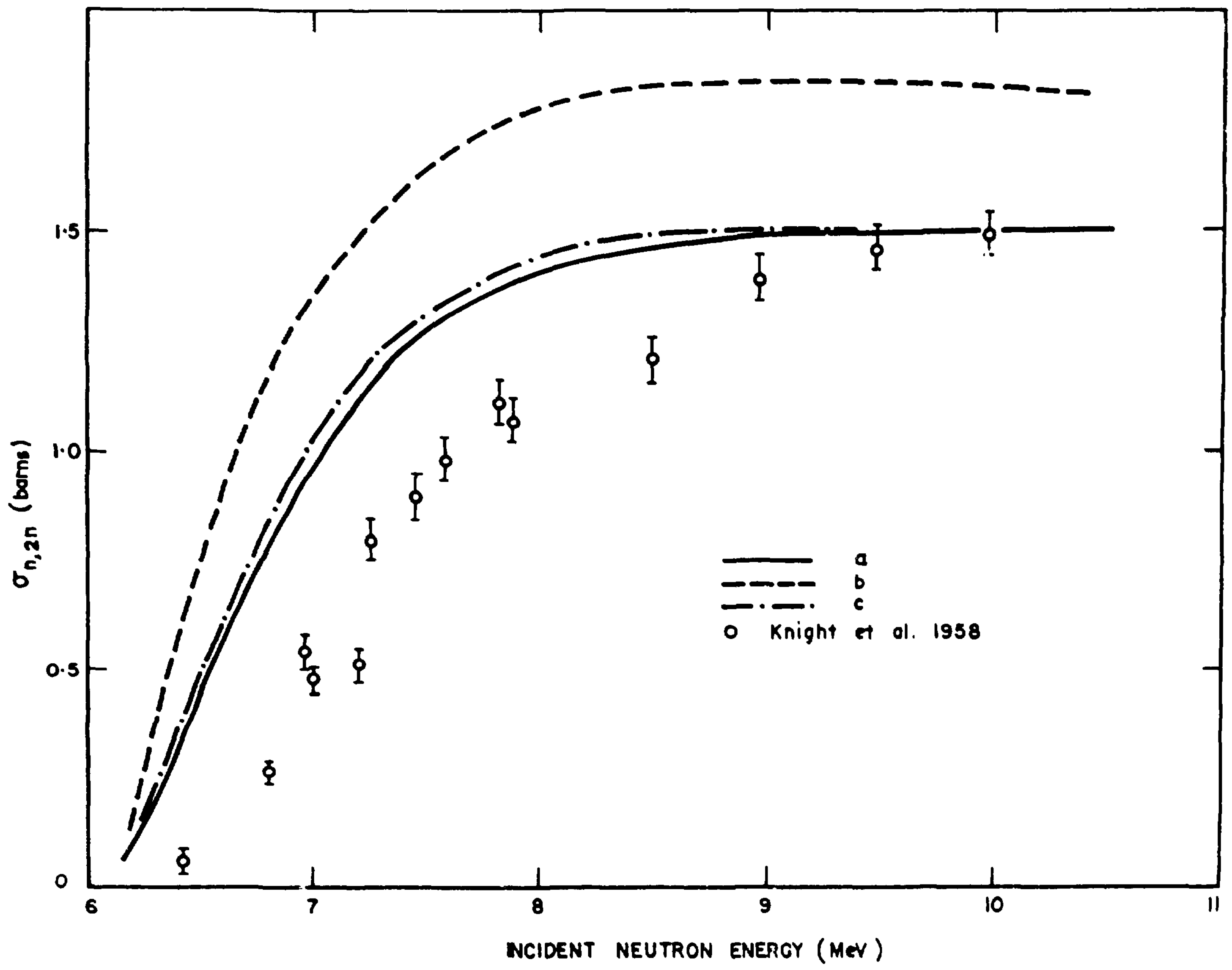

FIGURE 5. (n,2n) CROSS SECTIONS FOR ^{238}U