# AUSTRALIAN ATOMIC ENERGY COMMISSION RESEARCH ESTABLISHMENT <br> LUCAS HEIGHTS 

## meV NEUTRON RESONANCE CAPTURE IN BARIUM -135


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* Oak Ridge National Laboratory, Oak Ridge, Tennessee, USA Research sponsored in part by the USAEC under contract to Union Carbide Corporation


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keV NEUTRON RESONANCE CAPTURE IN BARIUM-135
by

A.R. deL. MUSGROVE<br>B.J. ALLEN<br>R.L. MACKLIN*


#### Abstract

Tine neutron capture cross section of ${ }^{133}$ Ba has ween measured with high resolution at the Oak Ridge Linear Accelerator in the energy range 3 to 100 keV. From over ninety observed resonances in the 3 to 6 keV energy range, the average resonance parameters obtained were: $\left\langle\Gamma_{\gamma}\right\rangle=150 \pm 20 \mathrm{meV}$; (D) = $39.3: 4 \mathrm{eV}$ and $10^{4} \mathrm{~S}_{1}=0.8 \pm 0.2$. The quoted radiation width and p-wave strength function also have a normalisation error of $\pm 20$ per cent. The method of separation of $s$ - and p-wave populations by statistical methods is described.


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BARIUM 135; CAPTURE; CROSS SECTIONS; ENERGY LEVELS; KEV RANGE 01-10; KEV RANGE 10-100; MONTE CARLO METHOD; NEUTRCN REACTIONS; P WAVES; RESONANCE; S WAVES

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Table 1 Final Resonance Parameters for ${ }^{135} \mathrm{Ea}$


## 1. INTRODUCTION

The neutron casture cross sections of separated isotopes of barium ( ${ }^{134}$, $135,136,137,138$ ba) were measured at the Oak Ridey Electron Linear

Accelerator (ORELA) facility; the data obtained are being analysed in detail at Lucas Heights to yield information on the s- and p-wave neutron resonance parameters and on the neutron capture cross sections in the region 3 to 100 kev. Preliminary rasults of this analysis were reported recently at the Soviet National Conference on Neutron Physics (Allen et al. 1973a), and the earlier results are now superseded. The present report gives details of the analysis of the ${ }^{135} \mathrm{Ba}$ capture data with the techniques used to discriminate between s- and p-wave levels. The resuiting s-wave levels have average level spacing in excellent agreement with that found at low energy by other experimenters. The most probable p-wave neutron strength function is deduced from the neutron widths extracted from the assumed p-wave levels.

## 2. EXPERIMENTAL DETAILS

The dstails of the experimental arrangement and equipment have already been extensively documented (Macklin 1971, Macklin \& Allen 1971, Allen et al. 1973b). In brief, this measurement was performed at the 40 m station of ORELA. The capture $\gamma$-rays were collacted by a total energy detector with. efficiency independent of the nature of the $\gamma$-ray spectrum. A calculated pulse height weighting scheme ensures that, on average, the detector response is proportional to the total energy of the ever.t (i.e. the binding energy plus the centre of mass neutron bombarding energy).

For each event detected, a weight (GWT) - actually an energy weight is obtained from the calculated table and is then added into the appropriate time channel file along with its variance. Finally, the number of capture events at energy $E$ is obtained from:

$$
N_{c}(E)=\frac{\text { GWT }(E)}{E_{B}+E A /(A+1)},
$$

- 

where $E$ is the newiron kinetic energy and GWT ( F ) is the cumulated product of weight and total energy at the time channel corresponding to $E$. The denominator is the total energy of the capture events in this channel.

The experiment described here was performed earlier than the lead runs already published (Allen et al. 1973b). In that paper, a description of the thin $(0.5 \mathrm{~mm}){ }^{6} \mathrm{Li}$ glass neutron flux monitor is given. The glass is inserted in the neutron heam at 39 m and provides, via the ${ }^{6} \mathrm{Li}(\mathrm{n}, \alpha)$ cross section (Uttley et al. 1971), a diiect concemporary measure of the incident neutron
flux. For early measurements such as the one here descrihed, a time-gated communal fission detector measures the time integrated neutron yield at the source (assumed to be proportional to the integrated flux at 40 m ).

After installation of the ${ }^{6} \mathrm{Li}$ glass, the ${ }^{6} \mathrm{Li}(\mathrm{n}, \alpha)$ yield was measured (alonc with the number of fission counts), and the barium results can be normalised to the ${ }^{6} \mathrm{Li}(n, \alpha)$ cross section by using the ratio of the two fission counts. It is believed that the normalisation, relying on the fission monitor alone, is no more than 30 per cent accurate over long time periods. Fortunately for the ${ }^{135} \mathrm{Ba}$ run, a secondary standard is available. Two runs on gold were performed directly after the barium-135 run and, when these runs are normalised to the ${ }^{6}$ Li yield via the fission counter, the cross section in the keV region can be compared with previous measurements. For the gold runs we obtain at 30 keV :

Run $1 \quad A u \sigma($ exp $) / \sigma($ standard $)=1.04 \pm 30$ per cent
Rum $2 \quad$ Au $\sigma(\exp ) / \sigma($ standard $)=0.96 \pm 30$ per cent
wherethe 30 pir cent error is the expected long time-scale normalisation error.
From thes e gold results and also from the lead results (Allen et al. 1973b) it is claimed that, for short time periods, the normalisation achieved with the fission monitor is reproducible to within about 10 per cent.

Sumarising the above argument then, the normalisation to the much later ${ }^{6} \mathrm{Li}(\mathrm{n}, a)$ yield is expected to be accurate to about 30 per cent. However, the results could have been normalised to the subsequent gold runs with an error of about 10 per cent. Taking a pessimistic view, a 20 per cent error is assumed in the normalisation to the ${ }^{6} \mathrm{Li}(\mathrm{n}, \alpha)$ cross section, which is believed to be known to better than 2 per cent below 100 keV .

The target consisted of an enriched (93.6 per cent) sample of ${ }^{135} \mathrm{Ba} \mathrm{CO}_{3}$ containing $14,27 \mathrm{~g}$ of ${ }^{135} \mathrm{Ba}$. The target thickness was 0.0061 atoms/barn with linear dimensions $2.61 \times 2.61 \times 0.84 \mathrm{~cm}$.

The accelerator operated with a pulse width of 5 ns giving $26.8 \times 10^{8}$ bursts during the run, while $1.06 \times 10^{6}$ fissions were counted on the fission monitor.

## 3. DATA REDUCTION

At the completion of the capture measuremert (about one day's running time) the data are dumped on tape for further analysis. Routine dead time and time independent backgzound corrections are made and the GWT data, originally in time channels, are transformé to an energy scale. The data are then converted to a capture yield (in mb) via the fission counter and later ${ }^{6}$ Li monitor yields as previously described. The data still contain a
time dependent background and have not been corr, ted for self shielding and multiple scattering. The ${ }^{135}$ Ba capture yield data at this stage are displayed in Figure 1 for the energy region 3 to 90 keV . Also indicated is the background (assumed linear in each region) along with its assumed error.

## 4. ANALYSIS

4.1 Area Analysis Using Monte Carlo Method

A modified version of the RPI Monte Carlo code (Sullivan et al. 1969, plus later errata) is used extensively in the analysis of capture experiments. The code uses initial guesses for the resonance neutron and radiative widths for up to ten resonances at a time, and calculates the multiple scattering component of the capture yield. This component is subtracted from the experimental yield and the :ode then performs an iterative area fit to each resonance which is assumed to sit on a linear background. Fo:- ${ }^{135} \mathrm{Ba}$ between 3 to 6 keV we examined 100 cilannel segments ( 100 eV ) of the data in turn which contain, on average, about three resonances to be fittea. The calculated primary capture yield at each resonance is convoluted with the resolution function, and the shape fit to the data is examined visually (at the end of the iteration, the capture areas should be equal). It is claimed that the FWHM of the resolution function is known very accurately as a function of energy by Gaussian width fitting of many large unresolved (i.e. with $\Gamma<$ resolution width; resonances in this nucleus. The Doppler broadening width is, of course, added in quadrature to the resolution width

Experience with other isotopes has shown that resonances with widths down to about 0.5 resolution widths can be resolved. That is, as well as an area fit, a shape fit to the resonance can also be performed tc extract $\Gamma_{n}$. For resonances with widths smaller than this limit, no detectable EreitWigner tail can be discerned, nor the consequent reduction in resonance peak height.

For ${ }^{135} \mathrm{Ba}$, most resonances fall into the latter category and only one resonance can be resolved out of the 90 observed.

The quant ity obtained from the area fit is the ideailiscd thin sample capture area:

$$
A_{Y}=2 \pi^{2} \hbar^{2} g \Gamma_{n} \Gamma_{Y} / \Gamma
$$

and it is assumed that the reader is familiar with the conventional meanings of these terms. The cunu..t of self shielding occuraing due to finite thickness of the target is calculated in the programe and, for tine largest resonances in ${ }^{135} \mathrm{Ba}$, this effect reduces the capture yield by a factor of
about 0.7 to 0.8 of the thin target $y$ :eld. This self shielding effect predominates over the (negligible) multiple scattering correction for this nucleus at the en rgies considered here. For the smaller resonances the self shielding correcion rare.' y exceeds 5 per cent.

### 4.2 The Resonance Parameter Kernel

The resonance parameter kernel of $A_{\gamma}$ is also of interest. This quantity is denoted by:

$$
K=g \Gamma_{n} \Gamma_{\gamma} / \Gamma(e V)
$$

When the programme is iterating to determine $A_{\gamma}$, it alters either $\Gamma_{n}$ or $\Gamma_{\gamma}$, whichever is the smaller. Therefore, for small resonances wiere $\Gamma_{n}<\Gamma_{\gamma} a$ value for ${ }^{r}{ }_{\gamma}$ is put into the programe which will not be altered by the fitting procedure. For very small resonances, $K \simeq g \Gamma_{n}$ and this is the output quantity.

For the larger resonances having $\Gamma_{n}>\Gamma_{\gamma}$, a nice separation of $k$ into two groups corresponding to the two $g$ values for s-neutrons is expected. For ${ }^{135} \mathrm{Ba}$, which has target spin of $3 / 2^{+}$, the $g$ values are sufficiently different ( 0.625 and 0.375 ) to make $J$ assignments with some confidence. Of course, it must be assumed that $\Gamma_{Y}$ is independent of $J$ in order to do this.

More qualitatively, from the previously published low erergy information on this nucleus (Van der Vyver \& Pattenden 1971, Alves et al. 1968) may be taken estimates of the average resonance parameters as follows: $S_{0} \simeq 1.0 \times$ $i 0^{-4},(\mathrm{D}) \simeq 40 \mathrm{eV}$ and $\left(\Gamma_{\gamma}\right) \simeq 100 \mathrm{meV}$. A rough estimate of the average s-wave neutron width at 3 keV based on these parameters is $0.4 \mathrm{eV}, \mathrm{o}=$ about 4 times the radiation width. From Porter-Thomas statistics, some 60 per cenc of the s-wave levels wijl have neutron widths larger than their radiative widchs. On the other hand, the resolution FWHM at 3 keV is 5 eV increasing to 10 eV at 6 keV . Thus, it is extremely unlikely that any resonances will be resolved. In fact, the $4,629 \mathrm{eV}$ resonance is just resolved with $\Gamma_{\mathrm{n}}=6.0$ $e V$, but for no other large s-wave level can a reasonable estimate of $\Gamma_{n}$ be made.

### 4.3 Example of Analysis on the 3,199 eV Resonance

In figure 2 is given a typical example of a detailed Monte Carlo area analysis performed on the $3,199 \mathrm{eV}$ resonance. The resonance $\Gamma_{n}$ has been varied from 0.2 eV to 2.0 eV . At $\Gamma_{\mathrm{n}}=2.0 \mathrm{eV}$, the Breit-Wigner tail of the resonance and the compensating drop in the peak height makes the visually inspected shape fit to the resonance discernibly worse than with $\Gamma_{n}=1.0 \mathrm{eV}$.

For all $\Gamma_{n}$ values $\leqslant 1.0 \mathrm{eV}$, the resonance shape is fitted very well. From Figure $2(a)$ it can be seen that $\Gamma_{\gamma}$ remains close to 150 meV for $\Gamma_{n}$ values in the range 0.8 to 0.4 eV before beginning to increase sharply as $\Gamma_{n}$ approaches $\Gamma_{\gamma}$.

The behaviour of the self shielaing factor as a function of $\Gamma_{n}$, shown in Figure $2(b)$, is typical of the large s-wave levels and the effect which this produces on the true resonance capture area is displayed in Figure 2(c). To obtain the resonance parameters for Table 1 , a number of different approaches are possible. For example, the expectation value of the unknown capture area is presumabl ${ }_{i j}$ an average over the Porter-Thomas $\Gamma_{n}$ distribution. To perform such an average, a value for $\left\langle\Gamma_{n}\right\rangle$ would be required, but alsc, while such an approach may be sound mathematically, it is readily seen in Figure $2(a)$ that the average would actually weight highly a region (with $\Gamma_{n}$ close to 0.2 ) which may be judged to be physically unlikely. That is, for this region the raaiation widths are much larger than the mean radiation width obtained for this nucleus. In the event, the radiative width for this resonance is estimated to be $\Gamma_{\gamma} \gtrsim 150 \mathrm{meV}$ and a somewhat arbitrary value has been taken for $\Gamma_{n}$ of 0.4 eV to obtain the capture area indicated in Figure 2 (c) with the error associated with this estimate. It is felt that this procedure can at least be more readily corrected if further measuremerts are made on this nucleus.

For the analysis of the $3,199 \mathrm{eV}$ resonance, it was assumed $J=2$ and hence $g=0.625$. Had $J=1$ been assumed for this resonance, the curve in Figure $2(a)$ woild be shifted considerably to the right (i.e. to much higher $r_{\gamma}$ values). The lower value was judged tc be more likely and this level was assigned to the $J=2$ sequence on that basis.

### 4.4 Separation of $g$ Values

In the course of the present analysis, the higher $g$ value nas been acarmed whenever $k \geqslant 0.055 \mathrm{eV}$ and the appropriate $J$ value has been given in Table 1. In the range 3 to $5 \mathrm{kev}, 15$ levels have been assigned as $J=2$ and 8 as $J=1$ in satisfactory agreement with a $2 J+1$ level density law. Of course, as shown in Figure $2(c)$, the resonance area and hence $k$ varies with $r_{n}$. Borderline cases therefore require a physical assessment of which $J$ value seems more likely, and to this extent, may be regarded as uncertain. To further amplify this problem, reference is made to the group of levels near 5.1 keV . These six large resonances look identical in the capture cross section and would, on the basis of their apparent $k$ values, be all assigned to tr. $J=1$ sequence. Clearly this is not a physically possible interpretation, bearing in mind the Wigner distribution's injunction against small spacings of levels of the same
spin sequ $n$ nce. The first level of this group was analysed as $J=1$ and the remainder have been analysed assuming $g=0.5$ and are not assigned to either sequence.

### 4.5 Average Parameters

Although each particular resonance $A_{\gamma}$ suffers from the uncertainty in $\Gamma_{n}$, the average cross sections as finally presented in Figure 9 contain contributions from a number of large s-wave levels (indicated by * in Table 1) and the authors are confident that the error on tre final average cross section, due to the lack of knowledge of any particular resonance $\Gamma_{n}$, is small compared with the overall normalisation error. A similar observation applies to the final best estinate for the average radiative width for this nucleus. From the eight large resonances, between 3 and $4 \mathrm{keV}\left\langle\Gamma_{\gamma}\right\rangle$ can be estimated to be 150 meV with an estimated error of $\pm 20 \mathrm{meV}$. The resolved resonance at $4,629 \mathrm{eV}$ has $\Gamma_{\gamma}=135 \pm 15 \mathrm{meV}$ which agrees with this value within the errors.

Clearly, the resonance parameters presented in Table 1 are not the result of a 'once through' analysis. Above has been indicated the sort of analysis required to obtain an estimate for ( $\Gamma_{\gamma}$ ) for this nucleus from those resonances having the largest capture areas (and presumably $\Gamma_{n}>\Gamma_{\gamma}$ ). In this energy region, and with a believable value for the p-wave neutron strength function, it can be calculated that these are (almost) certainly s-wave levels. From the earlier rough calculation, about 60 per cent of the s-wave levels are expected to be found at this stage, however all indications show that many p-wave levels have also been detected amongst the remaining small levels. For example, far too many small values of $g \Gamma_{n}$ are obtained than would be expected from Porter-Thomas statistics. Also, an examination of the distribution of level spacings comparel with the expected wigner distribution (for two s-wave leve? sequences) shows too many small spacings. The detectability limit in the energy range covered increases approximatcely $a s E^{2}$, which means that for the small p-wave levels (with capture arca $\alpha_{\mathrm{g}} \mathrm{I}_{\mathrm{n}}$ ), the best chance of observing them is at low energy (i.e. near 3 keV ). Also, at low energy it is more easy from a calculation of the relative probabilities to decide if a level is more likely to be s-wave or p-wave.
4.6 Bayes' Theorem Analysis

In this section thi workings of such a calculation, usually tarmed a
'Bayes' theorem analysis', are examined.
A resonance occurs at energy $E$ with carture kernel $g \Gamma_{n} \Gamma_{\gamma} / \Gamma$ in the range $k \pm \Delta r$. Assuming the level to have been formed by either $s$ - or p-wave inter-
action gives 6 possible ( $\ell, J$ ) sequences (mutually exclusive) to which the level can belong. Estimates are required of $S_{0}$ and $S_{1}$, the $s-$ and p-wave neutron strength functions, ( $D$ ), the s-wave level spacing and, $\Gamma_{\gamma}$, the average radiation width. The average reduced neutron widths can be calculated for each of the $6(\ell, J)$ sequences:

$$
\left\langle\Gamma_{n J}^{\ell}\right\rangle=S_{\ell} \frac{\langle D\rangle}{g} \varepsilon_{I J}^{\ell}
$$

where $\varepsilon_{I J}^{\ell}$ is the number of channel spin contributions (either 1 or 2). Now for each sequence the quantity $k \pm \Delta k$ is transformed to a corresponding $\Gamma_{n J} \pm$ $\Delta \Gamma_{n}$, and this is further reduced to the reduced neutron width $\Gamma_{n J}^{\ell} \pm \Delta \Gamma_{n}^{\ell}$ by dividing by the $\ell$-wave penetration factor and $\sqrt{E}$. If $\varepsilon_{I J}^{\ell}=1$, the neutron widths come from a Porter-Thomas distribution, while when $\varepsilon_{I J}^{\ell}=2$, which occurs for two of the p-wave sequences, the neutron widths come from the $\chi_{2}^{2}$ or negative exponential distribution. The probabilit: of finding $\Gamma_{n J}^{\ell}$ in the specified error range is now readily calculated an. the result of this is further multiplied by the a priori density factor $\rho_{J}=2 \mathrm{~J}+1$. The probabilities so calculated are finally normalised such that the sum is unity and the probability that the level in question is either s-wave or p-wave is found. In Figure 3 this calculation is illustrated with the final best estimate parameters for $E=3 \mathrm{keV}$ and $\mathrm{E}=6 \mathrm{keV}$. For each value of k are shown the calculated probabilities for a level to belong to each of the six ( $\ell, J$ ) sequences. The $s / p$ wave boandary is shown as a heavy line. Utilising these figures, it is seen that the group of four levels designated pwave near 3.1 keV have $\mathrm{k} \leqslant 0.007$ which, by reference to Figure 3 , gives $\leq 15$ per cent probability that these are s-wave. This is about the level at which the 'clear cut' decision to call these p-wave resonances operates. Others who use a similar technique to separate small p-wave levels (e.g. Bollinger \& Thomas 1968, Thomas et al. 1972, Liou et al. 1972a) are able to achieve much greater confidence levels for their separations since they operate at lower energies.
4.7 Statistical Tests
In addition to separating a p-wave component in the manner described above, further checks have been made which can be applied to the remnant swave population. Statistical tests which can be applied to a single sequence (i.e. one $\ell, J$ value) have been reported by Liou et $a l$. '1972b) and are used extensively by the Columbia group in their analyses. In ${ }^{135} \mathrm{Ba}$ there are two s-wave sequences, and there is the further problem caused by missing levels from the main sequence. In the circumstances, the s-wave level spacing data
aretested against the expected $2 \mathrm{~J}+1$ weighted Wigner distribution for two sequences and, since many of the neutron widths for these levels are unknown, tests can only be made to ensure that the number of small s-wave levels is approximately correct, assuming the value $S_{o}=1.0 \times 10^{-4}$ recommended by Van der Vyver \& Pattenden (1971).

## 5. RESULTS

Finally, the stairsase plot of s-wave levels is obtained (Figure 4). An average spacing of 41 eV is indicated frcm a straight line fit to the lower portion of this staircase, and the distribution of spacings about this average is given in Figure 5 compared with the expected distribution. This plot shows that 4 to 5 small spacings are missing and a similar number appear to be missing from the staircase plot between about 4 and 6 keV . It would be a mistake to assume that these missing levels are simply misassigned p-wave levels, although some misassignments have undoubtedly occurred in this analysis. crerlapping of 5 -wave levels can occur, although the only likely candidate seems to be the level at $5,978 \mathrm{eV}$ which has a very large value of k . The most probable reason for missing s-wave levels is simply that the detectability limit increases faster than the resonance area for the smallest s-wave levels and, therefore, at 6 keV somr s-wave levels are below the limit of detectability.

Using the low energy levels found by Van der Vyver \& Pattenden (1971) the level spacing has also been evaluated for this nucleus. Their speculative 26.0 eV resonance was omitted from this evaluation shown in Figure 6 , and the best estimate for the level spacing in ${ }^{135} \mathrm{Ba}$ was found to be $39.3 \pm 3.0 \mathrm{eV}$.

In Figure 7, the quantity $k$ is plotted for all resonances observed versus neutron energy. The transverse line marks the bourdary, calculated by the Bayes' theorem analysis, at which a level has equal a priori probability of being s-wave or p-wave. This boundary has been calculated with the final best set of average parameters. The position of this boundary is quite a sensitive function of the parameters, in particular, the p-wave strength function and the average radiation width.

Figure 8 gives the cumulative sum of $g r_{n}^{1}$ versus neutron energy for this nucleus. A straight line fit to the slope here has produced a strength function estimate of $\sim 0.6 \times 10^{-4}$; however, taking into account the large number of p-wave le;els missed (only 25 versus 68 s-wave levels are seen), a value $S_{1}=0.8 \pm 0.2 \times 10^{-4}$ is estimated for this nusleus.

Finally, the capture cross section in the energy range 3 to 90 keV has been obtained. From 3 to 6 keV the cross section is almost ertirely subscribed
by the observed resonances, but above 6 keV the cross section has been obtained by integrating the capture yield in each region, subtracting a linear background islown in Figure l) and then making an average self shielding and multiple scattering correction using the approximation of Macklin (!964). In this process, the background subtraction is tine major source of error as shown in Figure 9. The normalisation error has not been included. Also shown in Figure 9 is the calculated statistical model cross section using the best set of average resonance parameters. The fit is good and tends to lenu some support to the g -w.we strength function, which was derived as described from resolved resonances near 3 kev yet still fits the cruss sec:ion in regions where the $p$-wave coritribution exceeds the s-wave contribution.
6. CONCLUSICN

The measured radiative width is larger than that found by early workers at low energy (Van der Vyver \& Pattenden lo71, Alves et al. 1968). They report values around $1 C O$ meV and this may point to a normalisation error in the present results. On the other hand, the weiy:.:ニ入 rontribution of the ${ }^{135} \mathrm{Ba}$ capture cross section to the measured natural element cross section at an energy of $61 \pm 5 \mathrm{mb}$ (Macklin et al. 1963) is $20 \pm 7 \mathrm{mb}$. This is by far the major contribution to the natural element cross section and can scarcely be reduced. The preliminary results for the other sotopes to date give a calculated elemental cross section of only $38 \pm 10 \mathrm{mb}$.

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TABLE 1
FINAL RESONANCE PARAMETERS FOR ${ }^{135} \mathrm{Ba}$

| Resonance | Energy | $g \Gamma_{n} \Gamma_{\gamma} / \Gamma$ <br> eV | Area BeV | $\begin{gathered} \text { Statistical } \\ \text { Error } \\ \text { in Area } \\ \% \end{gathered}$ |  | $\ell$ | J | $\begin{gathered} g \Gamma_{\mathrm{n}} \\ \mathrm{ev} \end{gathered}$ | $\Gamma_{\gamma}$ <br> meV | $\begin{aligned} & g r_{n}^{0} \\ & \mathbf{e V} \end{aligned}$ | $g \Gamma_{n}^{1}$ <br> eV |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 3.059 | 0.72 | 97.6 | 22 | * | 0 | 2 | (0.3) | $\geq 187$ |  |  |
| 2 | 3.098 | 0.005 | 6.1 | 30 |  | 1 |  | 0.005 | (150) |  | 0.012 |
| 3 | 3.108 | 0.005 | 6.3 | 30 |  | 1 |  | 0.005 | (150) |  | 0.012 |
| 4 | 3.120 | 0.007 | 8.9 | 30 |  | 1 |  | 0.007 | (150) |  | 0.019 |
| 5 | 3.147 | 0.004 | 5.4 | 30 |  | 1 |  | 0.004 | (150) |  | 0.010 |
| 6 | 3.199 | 0.068 | 88.7 | 22 | * | 0 | 2 | (0.4) | $\geqslant 150$ |  |  |
| 7 | 3.231 | 0.013 | 16.9 | 19 |  | 0 |  | 0.016 | (150) | 0.000 |  |
| 8 | 3.277 | 0.028 | 35.7 | 8 |  | 0 |  | 0.045 | (150) | 0.001 |  |
| 9 | 3.288 | 0.010 | 12.8 | 30 |  | 1 |  | 0.011 | (150) |  | 0.026 |
| 10 | 3.305 | 0.006 | 7.9 | 25 |  | 1 |  | 0.007 | (150) |  | 0.015 |
| 11 | 3.321 | 0.003 | 3.5 | 30 |  | 1 |  | 0.003 | (150) |  | 0.006 |
| 12 | 3.340 | 0.009 | 11.5 | 20 |  | 1 |  | 0.011 | (150) |  | 0.024 |
| 13 | 3.358 | 0.019 | 23.9 | 16 |  | 0 |  | 0.026 | (150) | 0.000 |  |
| 14 | 3.402 | 0.024 | 29.8 | 1 C |  | 0 |  | 0.036 | (150) | 0.001 |  |
| 15 | 3.419 | 0.064 | 77.6 | 25 | * | 0 | 2 | (0.5) | $\geqslant 120$ |  |  |
| 16 | 3.432 | 0.015 | 17.6 | 16 |  | 0 |  | C. 018 | (150) | 0.000 |  |
| 17 | 3.464 | 0.014 | 16.5 | 15 |  | 0 |  | 0.016 | (150) | 0.000 |  |
| 18 | 3.481 | 0.058 | 69.7 | 26 | * | 0 | 2 | (0.5) | $\geqslant 116$ | 0.000 |  |
| 19 | 3.507 | 0.024 | 28.7 | 9 |  | 0 |  | 0.036 | (150) | 0.001 |  |
| 20 | 3.588 | 0.030 | 34.3 | 13 |  | 0 |  | 0.049 | (150) | 0.001 |  |
| 21 | 3.605 | 0.052 | 60.2 | 17 | * | 0 | 1 | (0.8) | $\geqslant 170$ |  |  |
| 22 | 3.626 | 0.010 | 12.0 | 25 |  | 1 |  | 0.012 | (150) |  | 0.024 |
| 23 | 3.639 | 0.017 | 19.6 | 15 |  | 0 |  | 0.022 | (150) | 0.000 |  |
| 24 | 3.649 | 0.027 | 30.9 | 15 |  | 0 |  | 0.042 | (150) | 0.001 |  |
| 25 | 3.675 | 0.036 | 40.9 | 6 |  | 0 |  | 0.067 | (150) | 0.001 |  |
| 26 | 3.710 | 0.030 | 33.4 | 8 |  | 0 |  | 0.049 | (15.) | 0.001 |  |
| 27 | 3.754 | 0.046 | 50.6 | 16 | * | 0 | 1 | (0.8) | $\geq 145$ |  |  |
| 28 | 3.785 | 0.008 | 9.3 | 50 |  | 1 |  | 0.009 | (150) |  | 0.017 |
| 29 | 3.818 | 0.019 | 20.1 | 20 |  | 0 |  | 0.024 | (150) | 0.000 |  |
| 30 | 3.826 | 0.024 | 25.9 | 20 |  | 0 |  | 0.035 | (150) | 0.001 |  |
| 31 | 3.837 | 0.010 | 10.5 | 50 |  | 1 |  | 0.011 | (150) |  | 0.020 |
| 32 | 3.959 | 0.049 | 51.7 | 16 | * | 0 | 1 | (0.8) | $\geq 157$ |  |  |
| 33 | 3.987 | 0.072 | 75.2 | 20 | * | 0 | 2 | (0.6) | $\geq 143$ |  |  |
| 34 | 4.022 | 0.013 | 13.2 | 19 |  | 1 |  | 0.015 | (150) |  | 0.025 |
| 35 | 4.638 | 0.021 | 21.2 | 19 |  | 0 |  | 0.026 | (150) | 0.000 |  |
| 36 | 4.075 | 0.098 | 99.6 | 19 | * | 0 | 2 | (0.6) | $>210$ |  |  |
| 37 | 4.100 | 0.044 | 44.8 | 15 | * | 0 | 1 | (0.6) | $\geq 146$ |  |  |
| 38 | 4.174 | 0.034 | 33.5 | 16 | * | 0 | 1 | (0.6) | $\geqslant 106$ |  |  |

TABLE 1 (continued)

| Resonance | Energy kev | $\begin{gathered} g \Gamma_{n} \Gamma_{\gamma} / \Gamma \\ e v \end{gathered}$ | Area BeV | Statistical Error in Area - |  | $\ell$ | J | $\mathbf{g} \boldsymbol{\Gamma}_{\mathrm{n}}$ <br> eV | $\sum_{\mathrm{meV}}^{r_{\mathrm{r}}}$ | $\begin{aligned} & \mathbf{g} \Gamma_{n}^{0} \\ & \mathrm{ev} \end{aligned}$ | $g \Gamma_{n}^{1}$ $e V$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 39 | 4.221 | 0.064 | 62.4 | 20 | * | 0 | 2 | (0.6) | $\geq 122$ |  |  |
| 40 | 4.245 | 0.080 | 78.2 | 19 | * | 0 | 2 | (0.6) | $\geqslant 161$ |  |  |
| 41 | 4.269 | 0.010 | 9.4 | 40 |  | 1 |  | 0.011 | (150) |  | 0.017 |
| 42 | 4.329 | 0.084 | 80.9 | 18 | * | 0 | 2 | (1.5) | $\geqslant 148$ |  |  |
| 43 | 4.337 | 0.027 | 25.8 | 19 |  | 0 |  | 0.043 | (150) | 0.001 |  |
| 44 | 4.361 | 0.010 | 9.6 | 19 |  | 1 |  | 0.011 | (150) |  | 0.017 |
| 15 | 4.411 | 0.055 | 51.5 | 19 | * | 0 | (2) | (0.3) | $\geq 124$ |  |  |
| 46 | 4.443 | 0.112 | 205.0 | 17 | * | 0 | 2 | (2.0) | $\geq 198$ |  |  |
| 47 | 4.464 | 0.009 | 8.1 | 19 |  | 1 |  | 0.009 | (150) |  | 0.014 |
| 48 | 4.523 | 0.014 | 13.0 | 19 |  | 1 |  | 0.017 | (150) |  | 0.024 |
| 49 | 4.568 | 0.036 | 32.8 | 12 |  | 0 |  | 0.066 | (150) | 0.001 |  |
| 50 | 4.629 | 0.083 | 74.1 | 10 | R | 0 | 2 | $r_{n}=6.0 \pm 1$ | 135 |  |  |
| 51 | 4.677 | 0.022 | 19.9 | 10 |  | 0 |  | 0.031 | (150) | 0.000 |  |
| 52 | 4.689 | 0.045 | 39.8 | 14 | * | 0 | 1 | (0.4) | $>170$ |  |  |
| 53 | 4.699 | 0.038 | 33.4 | 14 | * | 0 | 1 | (0.6) | $>120$ |  |  |
| 54 | 4.723 | 0.024 | 20.8 | 13 |  | 0 |  | 0.033 | (150) | 0.000 |  |
| 55 | 4.803 | 0.075 | 65.0 | 18 | * | 0 | 2 | (0.7) | $>147$ |  |  |
| 56 | 4.829 | 0.006 | 5.1 | 19 |  | 1 |  | 0.006 | (150) |  | 0.008 |
| 57 | 4.850 | 0.010 | 8.5 | 19 |  | 1 |  | 0.011 | (150) |  | 0.015 |
| 58 | 4.865 | 0.063 | 54.1 | 19 | * | 0 | 2 | (0.7) | $\geq 120$ |  |  |
| 59 | 4.908 | 0.030 | 25.1 | 16 |  | 0 |  | 0.047 | (150) | 0.001 |  |
| 60 | 4.930 | 0.017 | 14.6 | 19 |  | 1 |  | 0.020 | (150) |  | 0.026 |
| 61 | 4.943 | 0.041 | 34.3 | 17 | * | 0 | 1 | (0.3) | $>172$ |  |  |
| 52 | 4.965 | 0.068 | 57.2 | 19 | * | 0 | 2 | (0.4) | $\geq 166$ |  |  |
| 63 | 5.024 | 0.050 | 41.3 | 19 | * | 0 |  | (0.3) | $>139$ |  |  |
| 64 | 5.064 | 0.026 | 21.3 | 15 |  | 0 |  | 0.038 | (150) | 0.001 |  |
| 65 | 5.076 | 0.046 | 36.3 | 18 | * | 0 | 1 | (0.7) | $\geq 150$ |  |  |
| 66 | 5.100 | 0.043 | 35.0 | 18 | * | 0 |  | (0.7) | $\geq 93$ |  |  |
| 67 | 5.113 | 0.042 | 34.2 | 18 | * | 0 |  | (0.7) | $>90$ |  |  |
| 68 | 5.150 | 0.050 | 40.1 | 21 | * | 0 |  | (0.7) | $\geq 110$ |  |  |
| 69 | 5.163 | 0.047 | 38.0 | 25 | * | 0 |  | (0.7) | $>104$ |  |  |
| 70 | 5.172 | 0.048 | 38.2 | 25 | * | 0 |  | (0.7) | $>104$ |  |  |
| 71 | 5.275 | 0.033 | 26.3 | 1.3 |  | 0 |  | 0.057 | (150) | 0.001 |  |
| 72 | 5.342 | 0.021 | 16.2 | 15 |  | 0 |  | 0.028 | (150) | 0.000 |  |
| 73 | 5.357 | 0.065 | 50.3 | 17 | * | 0 | 2 | (0.7) | $>122$ |  |  |
| 74 | 5.414 | 0.053 | 40.4 | 14 | * | 0 | 1 | (0.7) | $\geq 175$ |  |  |
| 75 | 5.448 | 0.067 | 51.0 | 19 | * | 0 | 2 | (0.7) | $\geq 128$ |  |  |
| 76 | 5.500 | 0.061 | 45.8 | 19 | * | 0 | 2 | (0.7) | $>114$ |  |  |
| 77 | 5.525 | 0.025 | 19.1 | 20 |  | 1 |  | 0.033 | (150) |  | 0.036 |
| 78 | 5.552 | 0.039 | 29.3 | 15 |  | 0 |  | 0.077 | (150) | 0.001 |  |

TARLE 1 (continued)

| Resonance | Energy | $\begin{gathered} \mathrm{gr}_{\mathrm{ri}^{2} \mathrm{r}^{\prime} / \Gamma} \\ \mathrm{eV} \end{gathered}$ | Area BeV | $\begin{array}{\|c\|} \text { Statistical } \\ \text { Error } \\ \text { in Area } \\ \hline \end{array}$ |  | $\ell$ | $J$ | $\begin{gathered} \mathrm{g} \Gamma_{n} \\ \mathrm{ev} \\ \hline \end{gathered}$ | $\mathrm{r}_{\boldsymbol{\gamma}}$ | $\begin{aligned} & \mathrm{gr} \\ & \mathrm{n} \\ & \mathrm{eV} \end{aligned}$ | $\begin{gathered} \mathrm{g} \Gamma_{n}^{1} \\ \mathrm{eV} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 79 | 5.570 | 0.022 | 16.4 | 19 |  | 1 |  | 0.029 | (150) |  | 0.030 |
| 80 | 5.593 | 0.069 | 50.9 | 19 | * | 0 | 2 | (0.7) | $\geq 130$ |  |  |
| 81 | 5.620 | 0.046 | 35.2 | 18 | * | 0 | 1 | (0.7) | $\geq 150$ |  |  |
| 82 | 5.659 | 0.020 | 14.9 | 20 |  | 1 |  | 0.025 | (150) |  | 0.026 |
| 93 | 5.686 | 0.024 | 17.7 | 20 |  | 0 |  | 0.035 | (150) | 0.000 |  |
| 84 | 5.718 | 0.015 | 10.9 | 20 |  | 1 |  | 0.018 | (150) |  | 0.018 |
| 85 | 5.756 | 0.034 | 24.4 | 13 |  | 0 |  | 0.058 | (150) | 0.001 |  |
| 86 | 5.781 | 0.026 | 18.8 | 13 |  | 0 |  | 0.039 | (150) | 0.001 |  |
| 87 | 5.809 | 0.023 | 16.5 | 15 |  | 0 |  | 0.033 | (150) | 0.000 |  |
| 88 | 5.842 | 0.014 | 10.2 | 20 |  | 1 |  | 0.017 | (150) |  | 0.028 |
| 89 | 5.871 | 0.029 | 20.4 | 20 |  | 0 |  | 0.047 | (150) | 0.001 |  |
| 90 | 5.888 | 0.100 | 70.4 | 19 | * | 0 | 2 | (0.75) | 200 |  |  |
| 91 | 5.905 | 0.070 | 49.1 | 23 | * | 0 | 2 | (0.7) | 140 |  |  |
| 92 | 5.941 | 0.045 | 31.5 | 14 | * | 0 | 1 | (0.7) | 150 |  |  |
| 93 | 5.978 | 0.172 | 119.4 | 14 | * | 0 |  | comple | peak |  |  |

* Large s-wave levels analysed as for 3,199 eV resonance (Figure 2).
() indicates assumed value.


FIGURE I(a) ${ }^{135}$ Ba (CAPTURE YIELD DATA 3.5 keV



FIGURE $1(b)^{135}$ Ba CAPTURE YIELD DATA 5.8 kcV




FIGURE ((d) ${ }^{135}$ Ba CAPTURE YIELD DATA 12.25 keV



ENERGY (keV)
FIGURE I(e) ${ }^{135}$ [ja CAPTURE YIELD DATA 25.50 keV




THE AREA BELOW THE HEAVY LINE REFERS TO E-WAVE
LEVELS WITH $\ell, J=01$ AND 02 AS SHOWN. THE
PARAMETERS USED IN THE CALCULATION ARE SHOWN
FIGURE 3. A bayes' theorem analysis for the probability that resonance with g $\Gamma_{n} \Gamma^{\prime} / \Gamma=k$ beloncis to each of the b possible e, I SfQuences in ${ }^{135}$ ba


FIGURE 4. STAIRCASE PLOT OF NUMBER OF LEVELS UP TJ ENERGY E


FIGURE 5. ${ }^{135}$ Ba s-WAVE LEVELS COMPARED WITH A $2 \mathrm{~J}+1$ WEIGHTED WIGNER DISTRIBUTION


FIGURE 6. AAEC LEVEL DATA COMPARED WITH LOW ENERGY DATA


AAEC $\ell$-WAVE ASSIGNMENTS ARE SHOWN WITH THE BOUNDARY CALCULATED FROM BAYES' TIEOREM AT WHICH A LEVEL HAS EQUAL PROBABILITY FOR BEING s-WAVE OR p-WAVE

FIGURE 7. Values of $g \Gamma_{n} F_{\gamma} / \mathcal{F}$ FOR all Levels detected versus E


FIGURE 8. CUMULATIVE SUM OF $g \Gamma_{n}^{1}$ VERSUS E. THE SLOPE OF THE STRAIGHT LINE GIVES A p-WAVE STRENGTH FUNCTION OF $0.6 \times 10^{-4}$


FIGURE 9. EXPERIMENTAL AND CALCULATED CROSS SECTIONS FOR ${ }^{135} \mathrm{Ba}$

