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by
C.H. WESTCOTT

## ABSTRACT

At this time, when the $2200 \mathrm{~m} / \mathrm{sec}$ values of the neutron cross sections and other constants of fissile nuclei are again under review, it is important to have up-to-date values of the $g$-factors for these nuclei and particularly to have reliable estimates of their accuracy. This is because the g-factors which arise in interpreting measurements of data made in Maxwellian neutron spectra, or reactor spectra approximating thereto, affect many of the values which form the basis of a regression analysis, and the relative weights given to different values depend directly on accuracy estimates. The present report describes a detailed study of the possible changes in the shapes of curves of variation of the cross sections with neutron energy, based on the measured values used to fix these curves, and complements a study on a more theoretical basis by E. Vogt et al (unpublished). It is concluded that the best estimates of the accuracy of the room-temperature g-factors (quoted as standard deviations) are as follows:

|  | $\underline{U-233}$ | $\underline{U-235}$ | $\underline{\text { Pu }-239}$ |
| :--- | :--- | :--- | :--- |
| 9 (abs) | $\pm 0.12 \%$ | $\pm 0.09 \%$ | $\pm 0.285 \%$ |
| $G$ (fiss) | $\pm 0.20 \%$ | $\pm 0.155 \%$ | $\pm 0.285 \%$ |
| $g$ (eta) | $\pm 0.16 \%$ | $\pm 0.174 \%$ | $\pm 0.325 \%$ |

A less complete study of this problem for Pu-241 is also described.

Applied Mathematics Branch Chalk River Nuclear Laboratories

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\text { April, } 1969
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Etude de 1a précision des facteurs-g pour les spectres maxwelliens
à la température ambiante des isotopes $U$ et $P u$
par C.H. Westcott

## Résumé

A 1'heure où les valeurs de $2200 \mathrm{~m} / \mathrm{sec}$ des sections efficaces de neutrons et autres constantes des noyaux fissiles sont de nouveau passées en revue, il est important d'avoir des valeurs à jour pour les facteurs-g de ces noyaux et de pouvoir fiablement évaluer leur précision. La raison en est que les facteurs-g (dont il est question lorsqu'on interprète les mesures effectuees dans les spectres neutroniques maxwelliens ou dans les spectres de reacteurs qui s'en rapprochent) influencent un grand nombre de valeurs qui forment la base d'une analyse de régression et les poids relatifs donnés aux différentes valeurs dépendent directement de la précision des estimations. Ce rapport décrit une étude détaillée sur les changements pouvant se produire dans les formes des courbes de variation des sections efficaces avec l'énergie neutronique. Cette étude repose sur les valeurs mesurées employees pour fixer ces courbes et elle complète un travail plus thêorique effectué par E. Vogt et ses collaborateurs (non publié). La conclusion est que les meilleures estimations de la précision des facteurs-g à la température de la pièce (cités comme déviations types) sont les suivantes:

|  | $\underline{U-233}$ | $\underline{U-235}$ | $\underline{\text { Pu-239 }}$ |
| :--- | :--- | :--- | :--- |
| g (abs) | $\pm 0.12 \%$ | $\pm 0.09 \%$ | $\pm 0.285 \%$ |
| g (fiss) | $\pm 0.20 \%$ | $\pm 0.155 \%$ | $\pm 0.285 \%$ |
| g (eta) | $\pm 0.16 \%$ | $\pm 0.174 \%$ | $\pm 0.325 \%$ |

Il.est également question d'une étude moin: complète de ce problème pour $1^{\prime}$ isotope $\mathrm{Pu}-241$.

L'Energie Atomique du Canada, Limitée<br>Chalk River, Ontario

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ERRATA (In Text)
p. 13, second line of section 5, for "adjust read "adjust".
p. A.III.l, bottom line for contribution read contributions.
p. A.V.2, top line for ("normal" read ("normal").
p. A.V.12, line 24 , for If was read It was.

## ERRATA (Cont'd)

## (In Figures, Unclear "Case" Codes)

p. 61 Fig. $16 A$ is case $B V$, Fig. $16 B$ is case $B C$.
p. 64-66 Fig. 19A, 20A, 2lA are case $B,(19 B, 20 B$ and $21 B$ are R).
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p. 70 Fig. $25 A, B$ and $C$ are case $A, 25 D$ and $E$ are $A+$.
p. 71 Fig. $26 A$ and 26 F are $H D, 26 \mathrm{C}$ is HD+; Fig. 26 B and G are $N J, 26 D$ is $N J+$ and Fig. $26 E$ is $N$.

## 1. INTRODUCTION

Earlier compilations (from this laboratory) (AECL-407, 607 and 1101) of the $g$-factors which express (in terms of the $2200 \mathrm{~m} / \mathrm{sec}$ values) the effective cross sections for Maxwellian neutron spectra, or (together with an s-factor) for spectra of well-moderated thermal reactors, gave values for many isotopes and for temperatures up to $600^{\circ} \mathrm{C}$ or $1500^{\circ} \mathrm{C}$. The present study concentrates on establishing the accuracy of the $g$-factors for $U-233$, U-235, Pu-239 and (less exhaustively) Pu-241, primarily for a $20^{\circ} \mathrm{C}$ neutron spectrum, and with less precision for somewhat higher temperatures. This question of the accuracy of " $g$ " has never been adequately studied, and especially since the rate of improvement of the experimental data on which the $g$-factors were based has slowed down of recent years, it appeared preferable to study accuracies than to recalculate $g$-factors for a wider set of reactions or a wider range of temperature. This study has been done concurrently with a study under IAEA auspices for a revision of the $1965 *$ survey of $2200 \mathrm{~m} / \mathrm{sec}$ constants (cross sections and, e.g. $\bar{u}, \eta, \alpha$ ) for the same isotopes. For the regression-type analysis used in this work, knowledge of the accuracy of g-values is essential. The lower accuracy of $\mathrm{Pu}-241$ data generally, and the recent g-factor reevaluation for Pu-241 by Lemmel and Westcott. (Journal of Nuclear Energy, Vol. 21 p. 417, 1967), together relieve us of the necessity of studying this isotope in the same depth as the others. In all the isotopes studied it is found that the largest uncertainty in the low-temperature g-factors derives from

[^0]the fact that the experimental data is scarce or inaccurate for low neutron energies, i.e., in general below about 10 to 20 milli-ev.

The g-factor is defined exactly as in AECL-110l, so that $g-1$ measures the departure of the variation of the cross-section with energy from the $1 / \mathrm{v}-1$ aw; consequently it is convenient for both $\sigma_{a}$ and $\sigma_{f}$ to work in terms of the variation of $\sigma \sqrt{E}$ with energy. Notation otherwise follows normal practice; note that $\sigma_{\gamma}=\sigma_{a}-\sigma_{f}$, while $\alpha$ is defined as $\sigma_{\gamma} / \sigma_{f}$ so that $1+\alpha=\sigma_{a} / \sigma_{f}$. Therefore, writing $x=\sigma_{\sqrt{E}}$ and $M(E)$ for the Maxwellian density (not flux) distribution function* for temperature $T$, we have since $\int_{0}^{\infty} M(E) d E=1$, $g=\left(1 / x_{0}\right) \int_{0}^{\infty} x \cdot M(E) d E$, which for a given reaction is only a function of $T$. We should note that since $g$ depends on $x / x_{0}$, a renormalization of $\sigma$ does not in itself change the $g$-factor. Also, if the $1 / v-1$ aw applies, so that $\sigma \propto E^{-\frac{1}{2}}$, $g \equiv 1$; further $T_{0}$ is the absolute (Kelvin) temperature equivalent to $20.4_{4}{ }^{\circ} \mathrm{C}$. The quantity $g_{\eta}$ is defined, as usual** as equal to $g_{f} / g_{a}$, and $g_{1+\alpha}=1 / g_{\eta}$. The quantity obtained by

* This requires that $M(E)=2 \sqrt{E / \pi}(k T)^{-3 / 2} \cdot e^{-E / k T}$; we also write $x_{0}=\sigma_{2200} \sqrt{k T_{0}}$ where $k T_{0}$ is the energy of a $2200 \mathrm{~m} / \mathrm{sec}$ neutron ( 0.025298 eV ).
** Although $\eta=\nu /(1+\alpha)$ and $\nu$ is constant in the thermal region, so that we can define a $g(1+\alpha)$ equal to the reciprocal of $g_{\eta}$, this is not in fact equivalent to $g(1+\alpha)=I /\left(I+\alpha_{0}\right) \int_{0}^{\infty}[I+\alpha(E)] M(E) d E$ where $\int_{0}^{\infty} M(E) d E=1$. Instead, in defining $g_{\eta}$ we in fact average over the rate of absorption of neutrons and not just over the Maxwellian spectrum, so that $g_{\eta}=\left(1+\alpha_{0}\right) \times$ $\int_{0}^{\infty} \frac{\sigma_{a} \sqrt{ } E}{1+\alpha} M(E) d E /\left[\int_{0}^{\infty} \sigma_{a} \sqrt{ } E M(E) d E\right]$ which is readily seen to be equal to $g_{f} / g_{a}$ since $\sigma_{f}=\sigma_{a} /(I+\alpha)$.
writing $\mathrm{x}=\alpha$ in the above definition of "g" is not a normally used quantity, although it may be applicable in special cases.

2. SOURCE OF EXPERIMENTAL DATA AND OUTLINE OF METHODS

Most of the experimental data is now available in SCISRS* and a magnetic tape version of this for the isotopes concerned was obtained from the National Neutron CrossSection Centre at Brookhaven; this was supplemented by some additional data which was added to the tape in a similar format. This procedure differs from that used in the 1960 and earlier compilations (AECL-407, -670, and -1101) where the shapes of the curves were taken from the BNL-325 compilation, and only amended where new data had become available or inconsistencies were found (e.g. between $\sigma_{f}, \sigma_{a}$ and $\alpha$ ). Since our interest was confined to "thermal" values, only data below 0.4 eV were retrieved from the SCISRS tape. The abbreviations in use in SCISRS for identifying data sets are those used here in tables and curves wherever abbreviations are necessary.

Unfortunately we do not have available a uniform and reliable indicator of the true accuracy of each datum stored in SCISRS - even when errors are quoted (which is for a small fraction only of the data) it is usually unclear whether systematic (e.g., normalization), statisiical, or a combination of both errors is entered. For our purpose we only wish to establish curve shapes, so that we would be interested in systematic errors, if they were known to exist, only if their origin could be identified in such a

[^1]way that corrections for them could be made. We have therefore been led to attach "weights" (i.e. accuracy criteria) to the various sets of data from the internal evidence, i.e., from the "spread" of the points themselves, and generally we have done this for each data set from averages taken over several discrete ranges of energy.

In view of this limitation of the data it has not been possible to avoid the arbitrariness of making judgments based on familiarity with the technology used in measuring this type of data; a logical self-consistency was not always possible. Thus "best" curves through a collection of several sets of data for a given quantity were chosen "by eye", and then described mathematically - "spreads" or mean square residuals from these lines were used as a basis for weights. Sometimes the "eye-ball" curves were modified as a result of a previous "adjust" operation, giving the work a partly iterative character--there could also be other changes (e.g. data renormalization) as the work proceeded. Subsequently an "adjust" operation yielded curves for $\sigma_{f}, \sigma_{a}$ and $\alpha$ which were "consistent" (i.e. $\sigma_{a}=\sigma_{f}(1+\alpha)$ for each energy) to do this required a knowledge of the relative 'accuracy' of the previously-chosen $\sigma_{a}, \sigma_{f}$ and $\alpha$ curves stated as a weight per unit energy interval derived from the number and weight of the data points on which each curve was based. This was necessary since each of the original curves had been drawn using data for one quantity considered separately. At a later stage, weights were attached to the individual data sets (generally for a number of energy ranges) and a least-squares method used to fit an analytic function of $E$ to the points. By varying such things as the order of the polynomial in this function end the range over which the fit
is made a number of different fitted curves is produced. By examining these curves, it is hoped to learn how far the curve shapes would be expected to be able to vary while still giving a fit which is not unreasonable in view of what is known about the physics of neutron-nuclei interactions.

It then appears, although some elements of arbitrariness have to be admitted, that a reasonably representative set of fits has been obtained; applying some "factor of safety" (discussed below) we can use the range of variation of the g-factors corresponding to these sets to estimate standard deviations for these g-factors, i.e. how much variation in $g$-factors is likely in view of all the available experimental data. This was also confirmed in some cases by using different (alternative) "best" curves estimated by eye, applying the "adjust" procedure described above and noting how far the g-factors for alternative adjusted curves differ from one another (without any polynomial fitting). For U-235 and Pu-239, where alternative "best" curves were suggested by the data, this additional method tends to give changes in $g$ of the order of the standard deviation derived from fitting but if it gives smaller changes, it can do little to confirm the errors estimated from the fitting procedure. With this introduction, a detailed account of the work follows.
3. PRESENTATION AND ASSESSMENT OF EXPERIMENTAL DATA

As a first step, all data are plotted using the $G-20$ computer. Since the points are numerous and small differences are to be studied, an open scale (displaced zero) plot has been adopted--thus for $U-235$ ( $\sigma_{a}$ and $\sigma_{f}$ ) the whole (10-inch) range of ordinates represents only about a $10 \%$ range of $\sigma \sqrt{E}$. The "zero displacement" (quantity subtracted from $\sigma \sqrt{E}$ for
plotting) may be a function of energy and those used are given in Appendix I. This is purely a matter of convenience since all data points and curve-fitting recipes are similarly treated, but the use of an open scale of this type greatly aids the assessment of the differences between different data sets. Note also, that the choice of $\sigma \sqrt{E}$ rather than $\sigma$ as the quantity plotted in itself allows a smaller (and more nearly constant) zero displacement to be used. Some sets of points were immediately seen to have a very large spread or "scatter"--these are usually from the older measurements, and in general these data were rejected. In a few cases the spreads were not unacceptably large but the shapes of curve indicated were so different from those of other data for the same quantity that their rejection for this reason alone seemed indicated. It is true that a very numerous set of points having too large a spread could be treated by averaging several successive data, so as to reduce the spread, but in most of the unacceptable cases the original spread was so large that this approach was of little value. A list, with comments, of data sets not used for these reasons is given in Appendix IV.

In a few cases where the SCISRS files indieated the statistical accuracy of each point, and in a few other important cases cited below where this information was obtained from the literature, these errors were used rather than values based on the internal evidence (spread of the points) which was the general basis for the "weights". Curves are then drawn by eye so as to give a good representation of the trends indicated by the points, most weight being given to the sets of points exhibiting the smallest spreads. Clearly this is a somewhat arbitrary process, partly superseded by later stages of this study by
the polynomial "fits", but this "eye-balling" forms an unavoidable first step, and for producing "best" curves it seems surprisingly good in the light of the subsequent more sophisticated procedures. Analytical descriptions are given to these smooth "eye-balled" curves; this is done piecewise, using for each of a reasonable number of ranges of energy a polynomial in $E$, with or without a Breit-Wigner term added. The resulting analytical "recipes" (v. Appendix I), produced for $\sigma_{f}, \sigma_{a}$ and $\alpha$, are then plotted automatically with the SCISRS data to confirm the correctness of the steps so far. At this stage the computer also forms the sums of squares of the deviations of the points of each set from the "eye-ball" curves, the sums being taken over selected ranges of energy, corresponding to the energy regions for which the "weights" are calculated (v. Appendix II).

The weights given are the reciprocals of the mean square deviation of the points from the "eye-ball" or "consensus" curve expressed* in (per cent) ${ }^{2}$, i.e., $10^{-4} \bar{x}^{2} / \overline{\Delta x^{2}}$. For the "adjust" procedure, what is required is the weight per unit energy interval (not per point) summed for all the data sets contributing to a given "consensus" curve. For this we arbitrarily take the sum of the products of weight per point times the number of points per $12 \frac{1}{2} \mathrm{milli}-e V\left(\approx \frac{1}{2} k T_{0}\right)$ energy interral; since only relative weights are needed this convention is acceptable. The results are given in Table A.II. 2 ; from the Appendix it will also be seen that the adjust weights have been made continuous, whereas the weights of data sets for "fitting" may jump suddenly from one value to another at the ends of the energy range. For "adjust" to give smooth curves, this continuity feature is necessary. * It is important that for the $\alpha$ data the percentages are taken in terms of $1+\alpha$, not $\alpha$, since we later use "adjust" based on $\sigma_{f}(1+\alpha) / \sigma_{a}=1$.
4. PROBLEMS IN ASSIGNING "WEIGHTS"

There are several problems in defining weights from these sums of squared residuals for the points in a given set of data; these are of two types, viz., problems of philosophy and the difficulties due to small-sample statistics or special problems. We consider the latter first. In Appendix I are explained the cases where alternative "eyeball" lines seemed desirable, respectively fitting different groups of sets of measurements of the same quantity, i.e., when the consensus indicated by several sets of measurements differed from that which would fit other sets. These differences are unexplained but could be due to systematic errors affecting some classes of experiment--they are an unsatisfactory feature of the data especially for $\sigma_{f}(U-235)$ and $\sigma_{a}(P u-239)$, and give rise to a considerable difficulty in pursuing the present study.

Another systematic effect may arise from the arbitrariness of the eye-ball curves--the weights of sets of points lying neax the curve concerned would be much greater than of a set of points following a parallel line (especially if the spread of the points is less than the displacement of the line); renormalization is clearly the answer in such a case, and this has been done whenever it seemed needed. Also, to help meet objections of this type, some of the eyeball curves were readjusted and the changes in the resulting weights or fitted curves observed as a measure of what changes were possible; see also Appendix IIC for the procedure in the difficult cases of $\sigma_{ \pm}(U-235)$ and $\sigma_{a}(P u-239)$. The problems of small sample statistics also arose, but mostly these affected the weights significantly only when a few points fell in the energy range concerned; for more numerous sets of points
the weight seemed to depend relatively weakly on the choice of "eye-ball" curves for any changes likely to be made in practice, except for the cases, already mentioned, discussed in Appendix IIC. But there seemed to be no need, judging from the effects found in fits for different weights, to know weights generally to better than $\pm 25 \%$ (or up to $\pm 40 \%$ for less important curves), so that the accuracies achieved seem satisfactory.

There is another possible logical problem involved in the procedure described above for obtaining "weights", viz.. that the "eye-ball" curves for $\sigma_{a}, \sigma_{f}$ and $\alpha$ are determined separately and are not in fact mutually consistent. It should be noted that the "weights" attached t:o data sets are used in two processes. For the second procedure ("fits") the results depend on the relative weights of individual sets of data, while in the first procedure ("adjust"), all that is requirea is the total relative weight (per unit length of curve) attached to the sum total of $\sigma_{a}$ (and $\sigma_{T}$ ) measurements, of $\sigma_{f}$ and of $\alpha$ measurement. At least for "adjust", it seems clear that a weighting depending on the measured data for each quantity separately is the correct quantity to use, so that "spreads" from the separate unadjusted curves are appropriate.

Initially, the same basis was used in estimating weights to be used in the "fitting" procedure, but here the underlying philosophy is less clear. As a general proposition, and for completely arbitrary choices of "recipes" if the data points lay completely randomly on the graphs, it cannot be denied that there would be a definite risk of getting into tautological argument by first choosing weights for sets of data from their mean square deviations from an arbitrarily-
chosen "recipe" and then using a least squares procedure to determine the "best" curve through the points; obviously for simply random data the result would be likely to be that the original recipe would be more or less well reproduced as the "best" fitted curve. The onus is therefore on us to show that this is not the case for our study.

There seems to be no simple fundamental answer to this point; but as practical answers, several points can be made. First, a few fits have been made on the simplest assumption (e.g., giving all data points equal weights), and the results so obtained lay quite as close as could be expected to the curves obtained by our much more sophisticated procedure. And renormalization was always used to remove the worst effects of one set of data lying systematically "off" the recipe or consensus curves.

The point is rather that the data certainly cannot be described as filling the whole area of the graphs uniformly. Giving a uniform and equal weight to 311 data points-certainly an extreme assumption--was found not to displace the fitted curves, to any serious extent. Indeed, the whole process may be regarded as in a general way iterative, starting from this point (see discussion in Appendix II). Using a first (rather approximate) set of weights in "adjust", the shape changes called for by the consistency requirement may be seen, and if they represent definite trends, might cause us to make a revision in the "eye-ball" recipes used; weights for "fitting" are not needed until a later stage and therefore these may be deduced from the deviations from improved "recipes". Although the present report presents only final "recipes" and "weights", and the stages of the present study are not detailed, a process having a considerable similarity
to that just described formed part of the evolution of both the "old" to "new" sets of weights of which mention is made in Appendix IIC. It must be admitted, however, that limitations of time for this study rendered an evolutionary approach necessary--if the work were now repeated a more logiral procedure, including allowing at least some normalization factors to be fitted also, could be adopted. However, it would appear that this would not appreciably change the results obtained, since the main limitations are all in the data available.

To summarize, there appears to be no practical alternative to the use of the "spread" of the data points as a measure of their accuracy, with all the small-sample statistical problems that this theoretically involves. However, and especially for U-233 where non-statistical divergent trends have not been identified, it has seemed desirable to apply "factors of safety" to our deduced errors to allow for this difficulty. This is in addition to using a $x^{2}$ test as a measure of statistical goodness, and multiplying our computed error estimates by a mean $X$. The place where the worst effects of the paucity of experimental data shows up is always for low energies, below about 0.02 eV , and this, while true for all isotopes, stands out most for U-233. In the descriptions which follow, therefore, U-235 and Pu-239, which have special problems, are discussed first, and $\mathrm{U}-233$, where the problem is mainly a low-energy indeterminacy, is discussed later in the light of the lowenergy problems of the other isotopes.

There then remains the question of whether weights should be determined from departures from "adjusted" curves, instead of setting weights before adjusting (in the sense that
$\sigma_{f}(l+\alpha)$ should $\left.=\sigma_{a}\right)$. This could relatively easily have been done using the preliminary version of the "adjust" results as the base curves for deducing the mean squares residuals from which final "weights" for eack data set could have been estimated, i.e., by extending the iterative process already described (note: the procedure for "adjust" itself requires the weights to be already known).

This question was examined most closely for U-233, where the difficulties due to divergert recipes did not arise, although a trial derivation of weights from "consistent" curves was in fact made for each of the three isotopes. For this the mean square deviations of each set of points from one or more sets of curves which the polynomial fit gave was used; these curves were automatically consistent, and in fact curves were chosen from the fitted sets which were closely of the same shape as the final version of the "adjust" curves. The results showed that except for the "difficult" cases of $\sigma_{f}(U-235)$ and $\sigma_{a}(P u-239)$, the "weights" would indeed vary within a range of the order of $\pm 30 \%$ wide, though on the average the changes might be only some $\pm 10 \%$. Few systematic trends were evident, but it was only for the cases where alternative weights are given in Table AII.l that the alternative recipes gave weights differing by factors of l.5, 2 or 3 or more.

Taking therefore the $U-233$ case, we see from Figure 6 that the largest difference (in the important region below $0.1 \mathrm{eV})$ between the original recipes and the results from "adjust" occurs for fission. This corresponds to the statement in Appendix $I(C .3)$, that for this isotope $\sigma_{f}$ is the least well determined quantity. However, comparing the points of Figure 1 with the curve of Figure 6 , we see that
if an "adjusted" curve were used for deriving the mean square deviation, the result would be a considerable further downweighting of all the fission data. While renormalization could largely correct this, a large renormalization would be involved; while this is an unwelcome situation, we feel that we should give the $\sigma_{f}$ measurements in this case the "benefit of the doubt" and retain the weights obtained in the original process of using unadjusted recipes. Since this situation may apply elsewhere also, the same has been done for the other isotopes.
5. THE "ADJUST" PROCEDURE

So far the $\sigma_{f}, \sigma_{a}$ and $a$ data and the corresponding recipes have been dealt with separately. The "adjust procedure is used to generate consistent curves for absorption, fission and $1+\alpha$. The adjustment is done on a point-forpoint least squares basis, with as inputs to the computer the three $\left(\sigma_{\mathrm{a}} \sqrt{E}, \sigma_{\mathrm{f}} \sqrt{E}\right.$ and $\left.l+\alpha\right)$ recipes and relative weights for each quantity. These are weights-per-unit energy range as explained above, generated by summing for all the data contributing to each quantity. However, the weights are estimated for fairly broad energy ranges, but in order to avoid producing discontinuities in the adjusted curves, it is necessary to make the weights continuous functions of the energy variable. Thus, as the Table A.II. 2 (cf Appendix II.B) shows, one may use a constant weight for $0>E>0.02 \mathrm{eV}$, and from say 0.03 eV onwards, but for the transition region (in this case from 0.02 to 0.03 eV ) a linear variation of the weight is assumed. This is in fact also physically reasonable, although it is impracticable to derive directly from the data a continuously varying weight.

Adjustments in the sense of reducing the weights are made whenever systematic diserspancies appear, or when the indicated weight approaches or exceeds that corresponding tc a $0.1 \%$ a.. "cy, which is taken as the limit believed attainable . present measuring techniques.

The results of some typical "adjust" procedures are shown in Figures $5 b$ and 6 to 9, where insets show how the low energy ends of the curves vary if alternative recipes (taken from Appendix I) are chosen for U-235 and Pu-239.

In the "adjust" procedure also the g-factors corresponding to the input recipes are calculated, as well as those given by the resulting adjusted curves; in general these are obtained for $20.44^{\circ} \mathrm{C}$ temperature, and for higher temperatures up to 140 or $180^{\circ} \mathrm{C}$ in intervals of $20^{\circ} \mathrm{C}$ or $40^{\circ} \mathrm{C}$. Generally, the "adjust" procedure is truncated at $E=0.2 \mathrm{eV}$, and between this energy and 0.4 eV , where the recipes terminate, the integral is calculated from the unadjusted recipes. Up to $140^{\circ} \mathrm{C}$ the errors caused by this procedure are quite small; however, if the temperature is above $150^{\circ} \mathrm{C}$ this may cease to be a good approximation. For Pu-239 both the unadjusted and the adjusted recipes contain the dominant term due to the 0.3 eV resonance, only the small excess being adjusted, so that the error likely due to not doing the adjustment above 0.2 eV remains small. Since the main aim of the present work was to obtain g-factors and estimates of their accuracy for the purpose of interpreting measurements in terms of $2200 \mathrm{~m} / \mathrm{sec}$ values, and also because the experimental data generally become less accurate for the energy range 0.2 to 1 eV , no attempt to calculate g-factors for higher temperature has been made.

The g-factors resulting from the "adjust" procedure for the most important of the alternative recipes of Appendix I are given in Table l. The correspondence of some of these cases to the graphs of figs. 5b to 9 will be obvious. In the table for each isotope the first combination of recipes is that preferred; the temperature-increments are given only for these recipes. Other increments can be inferred from the $\Delta(R-S H)$, $\Delta(R-L O)$ and $\Delta(R-R S)$ entries. The temperature effects and the "spreads" of the $g^{\prime} s$ will be further discussed in section 9 below; the cases involving MLA or $H I$ are of minor importance but are included to show what the effects would be.
6. PROCEDURES FOR POLYNOMIAL FITTING TO THE DATA POINTS

The recipes so far used ("eye-ball" curves) are defined piecewise; it appears also worth while to attempt to produce single recipes to fit the data in one range (say for the whole range 0 to 0.2 eV$)$. Each quantity would still be expressed as a power series in $E_{o}$ using a least squares procedure to determine its coefficients; if a Breit-Wigner term is also needed it is chosen at the outset and its coefficients are not fitted. Discussion of an unpublished MS by E. Vogt et al. was helpful in this connection.

It is not a priori clear over what range of energies such a polynomial, or what order of polynomial, will give a good fit. For example, the large resonance for Pu-239 at 0.3 eV might have rendered a least squares polynomial fitting for this element difficult, or limited the energy range over which the fitting would be satisfactory; in fact it was found that the
$\underline{\text { g-factors from } \frac{\text { TABLE } I}{\text { "adjust" }} \text { for } 20.4_{4}{ }^{\circ} \mathrm{C}}$

| Isotope code | Absorption |  | Fission |  | $\operatorname{Eta}(1 / 1+\alpha)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | orig. | adja. | orig. | adjd. | orig. | adjd. |
| U-233 |  |  |  |  |  |  |
| RST/R/R | 0.9960 | 0.99615 | 0.99645 | 0.9963 | 1.0004 | 1.00015 |
| $\Delta$ for $T$ | -30 | -28 | -28 | -355 | +2 | -65 |
| $\underline{\mathrm{U}-235}$ |  |  |  |  |  |  |
| R/RSH/R | 0.9790 | 0.97905 | 0.9763 | 0.9769 | 0.99725 | 0.9978 |
| $\Delta$ for $T$ | -237 | -238 | -2395 | -256 | -245 | -20 |
| $\mathrm{R} / \mathrm{R} / \mathrm{R}$ | 0.9790 | 0.9793 | 0.9770 | 0.9771 | 0.99795 | 0.9977 |
| $\mathrm{R} / \mathrm{SH} / \mathrm{R}$ | 0.9790 | 0.9788 | 0.9756 | 0.9767 | 0.99655 | 0.9979 |
| $\Delta(\mathrm{R}-\mathrm{SH})$ at $20^{\circ}$ | - | +5 | +14 | +4 | - | -2 |
| $\Delta(\mathrm{R}-\mathrm{SH})$ at $140^{\circ}$ | - | +25 | -7 | -8 | -- | -11 |
| R/MLA/R | 0.9790 | 0.9797 | 0.97815 | 0.9776 | 0.99915 | 0.99785 |
| Pu-239 |  |  |  |  |  |  |
| LOR/R/RRS | 1.0736 | 1.0757 | 1.05445 | 1.0523 | 0.9822 | 0.9783 |
| $\triangle$ for $T$ | +1511 | +1533 | +1131 | +11165 | -288 | -311 |
| LO/R/RRS | 1.07325 | 1.07535 | 1.05445 | 1.05165 | 0.9825 | 0.9780 |
| R/R/RRS | 1.07455 | 1.07675 | 1.05445 | 1.0543 | 0.9813 | 0.97915 |
| $\Delta(\mathrm{R}-\mathrm{LO})$ at $20^{\circ}$ | +13 | +14 | - | +265 | -12 | +115 |
| $\Delta(\mathrm{R}-\mathrm{LO})$ at $140^{\circ}$ | -35 | -25 | - | +5 | +275 | +24 |
| HI/R/RRS | 1.0725 | 1.07555 | 1.05445 | 1.0565 | 0.9832 | 0.9823 |
| $\Delta(\mathrm{R}-\mathrm{RS})$ at $20^{\circ}$ | - | +5 | - | -16 | - | -195 |

Pu-239
(A final "5" digit indicates next place $=4,5$ or 6).
$\Delta$ for $T$ is increment for a $120^{\circ} \mathrm{C}$ temperature rise (to $140.4_{4}{ }^{\circ} \mathrm{C}$ ).
Note that (cf App. I) $\Delta(L O R-L O)=\frac{1}{4} \Delta(R-L O)$; RSH is halfway between $R$ and $S H$ and RRS between $R$ and RS.
inclusion of a suitable pre-set Breit-Wigner term avoided any such trouble. We therefore approached the problem with an open mind as to the upper limit of fitting and as to the polynomial order. As a code in the tables below we write $E_{m} ; N$ (case), to denote a particular fit, the "case" code being as in Appendix II, thus $0.1 ; 5(0)$ means case "O", fit for $0<E<0.1 \mathrm{eV}$ with 5 terms (a power series in $E$ up to and including $E^{4}$ ) for both $\sigma_{a} \sqrt{E}$ and $\sigma_{f} \sqrt{E}$.

We therefore have the possibility of repeating the fitting process many times, say for $E_{\max } 0.10,0.15$ and 0.20 eV ( 0.12 eV was also used) and for series of 4, 5, 6, 7 or 8 terms; thus, instead of one "adjust" result, we might have, say 10-15 different fits. There were also the alternative choices of weights (cases), given in Appendix II, giving a total of up to 50-70 fits, as against only about 5 alternatives in "adjust". It was hoped that the variety so obtained would enable us to find limits on what variations of $g$-factors were possible or acceptable, and this indeed seems to have been achieved. From the considerations of the unpublished MS (E. Vogt et al) already mentioned, it would seem that the type of effect (e.g. due to a small resonance whose existence had not been suspected) which would make a loworder polynomial an unlikely representation of the data is indeed likely to occur only rarely on the basis of nuclear systematics.

A large series of polynomial fits have therefore been undertaken for $U-233, U-235$ and Pu-239. Some of these exhibited certain peculiar features to be discussed below, but in general it may be said that this approach appears to have yielded a representative set of reasonably "good" fits, from the spread of which the accuracy of $g$ can be estimated; there were some
"unacceptable" fits which serve well to indicate the limitations inherent in the data. The lowest order fit which seemed reasonable was a cubic, and fits of up to 7 th order ( $N=8$ ) were undertaken; the upper limit of order which gives reasonable fits is discussed later. For now we note only that generally a lower order fit suffices for the narrower ( 0 to 0.10 or 0.12 eV ) ranges of fitting, than is desirable for a wider range, e.g. 0 to 0.2 eV . The question of low order fits merits separate consideration. For a fit to $\sigma \sqrt{E}$ vs $E$, a one-term fit ( $\sigma \sqrt{E}=$ Const) is a trivial case for which $g \equiv 1$ identically. A linear (2-term) fit permits $g$ to take values other than $l$, but is not physically very useful, especially if fitting over a range $6 k T_{0}-8 k T_{o}$ wide, since points at the end of the range tend to determine the slope and hence the quantity $g-l$, whereas $g$ should be mainly determined by points in the range from $\frac{1}{2} k T$ to $2 k T$, where the Maxwellian spectrum is relatively intense. For reasons of this type, quadratic fits (3 terms) have not been employed in this work; while cubic (4-term) fits over the range $0-0.2 \mathrm{eV}$ have been included, these have mainly served to demonstrate the relative irvariance of the g-factors so deduced in spite of changes of the weights or other assumptions. Examination of typical fitted curves, or of the position of data points for, e.g. U-235 (cf figs. l3-18) will demonstrate that if we restrict ourselves to a parabolic or cubic fit over the whole range, the (negative) slope of the curve near energy $k T_{\rho}$ is almost entirely dictated by the rise of the consensus curve above 0.07 eV , for both fission and absorption, notwithstanding the spread of the data (especially for fission) at low energies*. Similarly, the statistical "standard" error for $g$ which the fitting calculations indicate is found to be unduly small for

* For an actual fit restricted to cubic form see Fig. l2A for U-233.
$0-0.2$ eV cubic-iype fits. Clearly this does not mean that the $g$-factor obtained is correspondingly accurate, since it is due to a limitation implicit in the choice of this type of fit; this is confirmed by noting that if either the range is reduced (to 0.12 or 0.10 eV ) or the order of fit is increased (to say 6 or 7) the "error" figure rises to a value which varies little with the choice of order or range of fit.

One other quite striking general conclusion from an examination of the fitted curves is that the experimental data are all relatively scarce and inaccurate at energies below about 0.02 eV , and that therefore many of the fitted curves tend to exhibit quite considerable variation from one another and departures from the "eye-ball" curves as the $E=0$ axis is approached. The extreme case of this occurred for one set of weights for $U-233$ (for an $0.1 ; 8$ ( 0 ) fit) see fig.l2c; for this case the $\sigma_{a}$ and $\sigma_{f}$ lines crossed, which would imply negative $\sigma_{\gamma}$ or $\alpha$ (this is clearly physically impossible, but this condition was not implicit in the fit procedure). Lesser variations of curve shape at low energies are quite frequent, and it appears that the largest uncertainty in the $20^{\circ} \mathrm{C}$ g-factors is usually that due to the uncertainty in $\sigma \sqrt{E}$ (or $\alpha$ ) in this range--in some cases the variations start only below 0.01 or 0.015 eV but often they persist up to 0.025 eV .
7. DISCUSSION OF RESULTS OF FITTING: $\mathrm{U}-235$ and Pu-239

We discuss these two isotopes together since for each the main problems are connected with the alternative recipes (App. I) and the consequent options in choosing weights (App. II) for $\sigma_{f}$ (U-235) and $\sigma_{a}(P u-239)$. The "adjust" operation did not for
either isotope show that either alternative was unreasonable (though it did indicate some slight trend favouring LO for Pu-239 over R); the data for both isotopes show a general scarcity and decrease of accuracy at low energies. For U-235 the $\sigma_{a}$ data are better than the $\sigma_{f}$ and $\alpha$ data in this region, but for Pu-239 it is in $\sigma_{a}$ that the recipe options appear, so that no quantity is well determined near $\mathrm{E}=0$ for this isotope. The general accuracy of all data is also lower for Pu-239 than for $U-235$. In contrast to U-233 (see below), the "old" sets of weights did not give rise to specific difficulties (cf App. II C), except that there were early problems (before "fliers" were excluded) for Pu-239, where also the nearness of the large 0.3 eV resonance might have been expected to give rise to problems for a wide ( 0 to 0.2 eV ) range of fit, but difficulties for this case were in fact not clearly seen in the final fits.

When the spread of the $g$-factors from fitting (tables 3 and 4) are examined, trends are seen, as expected, depending on which option for weights is used, but for the most affected quantities (those for which the alternate recipes occurred, viz. $\sigma_{f}$ for U-235 and $\sigma_{a}$ for Pu-239) only about $50 \%$ of the total "spread" appears to be a systemative $\Delta g$ correlated with option; the "spreads" found within the individual options (being the other 50\%) are of about equal size. For the other quantities ( $\sigma_{a}$ or $\alpha$ of $U-235 ; ~ \sigma_{f}$ or $\alpha$ of Pu-239) the "spreads" found for individual options are greater than the option-correlated deviations. There is, of course, a similar trend in the fitted curves, that cases based on "SH" weights tend to reproduce "SH" recipes, and similarly.

The behaviour of the $\chi^{2}$ (as depending on weight-option) are also interesting. We may expect that a weighting option using
a "whichever is best" basis may lead to a higher $x^{2}$, since data near one of the alternative recipes may have a rather high weight--at the same time other data near the other recipe can also have high weights, but no fit can lie close to them both in the "discrepant" region. For all "O" sets of weights this basis was used, and for these, and case "B" of Pu-239 (also tending to a "whichever is best" basis) $\chi^{2}$ will be seen (in Tables 2-4) to be noticeably higher than for other cases. Naturally the magnitude of the difference depends on the data in question--for U-235 both "BC" and "BV" are compromises with some best-weight tendency-in fact the use of authors' claimed weights for "BV" makes $\chi^{2}$ higher for this than for "BC"--but there is also in "MR" a "whichever is best" element in respect of the alternative recipes $R$ and MLA, all of which is reflected in the trends of $X^{2}$. One cannot push this point too far, however, in view of the sometimes rather arbitrary treatment in estimating data-accuracies.
8. DISCUSSION OF RESULTS OF FITTING FOR U-233 AND GENERAI POINTS
Table 2 gives the numerical results from fitting for $U-233$. For this isotope both $\sigma_{f}$ and $\alpha$ data were of poor accuracy, at low energies especially, but no alternative recipes (App. I) were needed. In the following discussion it is the problem of the shape of the curves at low-energy which predominates; this is really a general problem for all three isotopes, as will be seen from Figs. 10-24. The discussion of App.II.C. 3 refers particularly to the use of a fictitious datum for $\alpha$ at $E=0$ to stabilize the curve-shapes at low energy; for the later sets of weights this "pseudo-point" appeared unnecessary. Comparing the computed errors of the $g^{\prime}$ s in Table 2 for cases 2 and 3 (the latter has the

TABLE 2
$\mathrm{U}^{233}$

| FIT | $x^{2}$ | $\hat{S}_{\text {a }}$ | $\sigma_{a}\left(E_{0}\right)$ | 9 a | $\hat{\sigma}_{E}$ | $\sigma_{f}\left(E_{0}\right)$ | $\mathrm{g}_{\mathrm{f}}$ | ${ }_{9}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1:4 (0+) | 1.18 | $572.8=.4$ | $574.6=.4$ | . $99672 \pm 55$ | $523.9=.5$ | $525.0=.6$ | $.99775=78$ | $1.00103=65$ |
| 0.1:4 (0) | 1.21 |  |  | . $99730=54$ |  |  | $.99841=94$ | 1.00121 $\pm 91$ |
| 0.1;4 (1) | 1.02 | $572.2=.3$ | $574.2=.3$ | $.99643=44$ | $523.1=.6$ | $524.6=.5$ | . $99734=88$ | $1.00091=82$ |
| 0.1:4 (2) | 1.02 | $572.2=.3$ | $574.2=.3$ | $.99644=44$ | $522.9 \div .6$ | $524.5 \pm .5$ | $.99715 \pm 86$ | $1.00071=80$ |
| 0.1:4 (3) | 1.02 | $572.2=.3$ | $574.2=.3$ | . $99643=54$ | $523.0=.5$ | $524.5 \pm .5$ | $.99720=75$ | $1.00077=66$ |
| 0.1; $5(0+)$ | 1.18 | $573.0=.4$ | $574.3=.5$ | $.99778=80$ | $524.1=.5$ | $524.6=.7$ | $.99893=105$ | $1.00115=76$ |
| 0.1:5 (0) | 1.20 |  |  | $.99845=79$ |  |  | $1.00094=164$ | $1.00265=162$ |
| 0.1:5 (1) | 1.03 | $572.2=.3$ | $574.2=.4$ | . $99645=61$ | $523.1 \pm .7$ | $524.0=.6$ | . $99741=156$ | $1.00097=150$ |
| 0.1:5 (2) | 2.03 | $572.2 \pm .3$ | $574.2=.4$ | . $99646=61$ | $522.9 \pm .7$ | $524.5 \pm .6$ | $.99681=142$ | $1.00085 \pm 133$ |
| 0.1;5 (3) | 1.03 | $572.2=.3$ | $574.2=.4$ | . $99645=61$ | $523.0 \pm .5$ | $524.5=.6$ | $.99714=101$ | $1.00068 \pm 84$ |
| 0.1;6 (0+) | 1.18 | $573.0=.4$ | $574.2 \pm .5$ | . $99785=81$ | $524.1=.6$ | $524.6=.7$ | $.99912=109$ | $1.00128=81$ |
| 0.1:6 (0) | 1.20 |  |  | $.99860=79$ |  |  | $1.00238=204$ | $1.00401=213$ |
| 0.1;6 (1) | 1.04 | $572.2=.3$ | $574.2=.4$ | . $99644=61$ | $523.0 \pm .8$ | $524.5=.6$ | . $99718=161$ | $1.00075 \pm 199$ |
| 0.1;6 (2) | 1.03 | $572.2=.3$ | $574.2 \pm .4$ | $.99644=61$ | $522.7 \pm .7$ | $524.6=.6$ | $.99631=101$ | . $99986=152$ |
| 0.1:6 (3) | 1.03 | $572.2=.3$ | $574.2 \pm .4$ | . $99644=61$ | $523.0=.5$ | $524.5=.6$ | $.99713=211$ | $1.00070 \pm 85$ |
| 0.1:7 (0+) | 1.18 | $573.1=.4$ | $574.1=.5$ | . $99819 \pm 88$ | $524.1=.6$ | $524.6=.7$ | . $99916=133$ | $1.00097 \pm 113$ |
| 0.1;7 (0) | 1.19 |  |  | . $99912=90$ |  |  | $1.00566=250$ | $1.00692=252$ |
| 0.1;7 (1) | 1.03 | $572.3=.4$ | $574.1=.4$ | . $99682=71$ | $524.2=1.1$ | $524.4=.7$ | . $99964=253$ | $1.00284 \pm 246$ |
| 0.1:7 (2) | 1.04 | $572.3=.4$ | $574.1=.4$ | $.99681=71$ | $522.9 \pm .7$ | $524.6=.6$ | . $99669=164$ | $.99988=154$ |
| 0.1:7 (3) | 1.04 | $572.3=.4$ | $574.1=.4$ | $.99681=71$ | $523.0=.5$ | $524.6=.6$ | , $99683=122$ | $1.00002=107$ |
| 0.1:8 (0+) | 1.18 | $572.9=.7$ | $574.2=.5$ | . $99771=159$ | $525.2=1.0$ | $524.0=.8$ | $1.00212=245$ | 1. $000441=215$ |
| 0.1:8 (0) | 1.19 |  |  | . $99932 \pm 110$ |  |  | $1.00476=300$ | $1.00785 \pm 301$ |
| 0.1;8 (1) | 1.04 | $572.6=.6$ | $574.0=.4$ | . $99746=135$ | $524.4 \pm 1.3$ | $524.3=.7$ | $1.00015=273$ | $1.00259 \pm 251$ |
| 0.1;8 (2) | 1.04 | $572.6=.6$ | $574.1=.4$ | . $99746=135$ | $523.7=.9$ | $524.3=.7$ | $.99881=227$ | $1.00135=199$ |
| 0.1:8 (3) | 1.04 | $572.6=.6$ | $574.1=.4$ | $.99747=135$ | $523.8=.9$ | $524.3=.7$ | $.99906=225$ | $1.00160 \pm 197$ |
| 0.15:4 (0+) | 1.19 | $572.4 \pm .4$ | $574.8 \pm .4$ | . $99596=29$ | $523.2=.5$ | $526.0=.6$ | . $99461=41$ | . $99865=35$ |
| 0.15:4 (0) | 1.21 |  |  | . $99656=29$ |  |  | . $99481=42$ | . $99829=37$ |
| 0.15;4 (1) | 1.09 | $571.8 \pm .3$ | $574.4 \pm .3$ | . $99556=24$ | $521.8 \pm .5$ | $524.9=.5$ | . $99419=37$ | . 99863 = 32 |
| 0.15;4 (2) | 1.09 | $571.8 \pm .3$ | $574.4 \pm .3$ | . $99556=24$ | $521.8=.5$ | $524.9=.5$ | $.99419=37$ | . $99862 \pm 32$ |
| 0.15:4 (3) | 1.09 | $572.0=.3$ | $574.4 \pm .3$ | $.99590=24$ | $522.1=.5$ | $525.1=.5$ | $.99431=36$ | $.99840 \pm 31$ |
| 0.15:5 ${ }^{(0+)}$ | 1.16 | $572.8=.4$ | $574.6 \pm .4$ | $.99690=58$ | $523.6=.5$ | $525.1=.6$ | . $99707=80$ | $1.00019=63$ |
| 0.15:5 (0) | 1.19 |  |  | . $99764=56$ |  |  | $.99767=99$ | $1.00010 \pm 91$ |
| 0.15:5 (1) | 1.08 | $572.0 \pm .3$ | $574.2 \pm .3$ | . $99607=46$ | $522.7=.6$ | $524.5=.5$ | . $99645=90$ | $1.00038=83$ |
| 0.15:5 (2) | 1.08 | $572.0=.3$ | $574.2=.3$ | . $99608 \pm 47$ | $522.6=.6$ | $524.5=.5$ | . $99627 \pm 88$ | $2.00018 \pm 80$ |
| 0.15:5 (3) | 1.08 | $572.0=.3$ | $574.2 \pm .3$ | . $99642 \pm 47$ | $522.6=.5$ | $524.5 \pm .5$ | $.99640=76$ | . $99998 \pm 65$ |
| 0.15:6 (0+) | 1.15 | $573.1=.4$ | $574.2 \pm .5$ | . $99798=77$ | $523.8=.5$ | $524.5=.7$ | $.99867 \pm 102$ | $1.00009 \pm 73$ |
| 0.15;6 (0) | 1.17 |  |  | $.99878=78$ |  |  | $1.00137 \pm 157$ | $1.00269=149$ |
| 0.15;6 (1) | 1.08 | $572.2 \pm .3$ | $574.1=.4$ | . $99668=59$ | $523.2 \pm .7$ | $524.2=.6$ | . $99806=146$ | $1.00138=140$ |
| 0.15;6 (2) | 1.08 | $572.2=.3$ | $574.1=.4$ | . $99671=59$ | $522.8=.6$ | $524.2=.6$ | $.99733 \pm 134$ | $1.00062=126$ |
| 0.15;6 (3) | 1.08 | $572.2 \pm .3$ | $574.1 \pm .4$ | .99671 $=59$ | $522.7=.5$ | $524.3 \pm .6$ | . $99710 \pm 97$ | $1.00038 \pm 81$ |

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| FIT | $x^{2}$ | $\hat{\sigma}_{a}$ | $\sigma_{a}\left(E_{0}\right)$ | ${ }^{9}$ | $\hat{\sigma}_{\mathbf{f}}$ | $\sigma_{f}\left(E_{0}\right)$ | $\mathrm{g}_{\mathrm{E}}$ | $\mathrm{S}_{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.15;7 (0+) | 1.16 | $573.1=.4$ | $574.3 \pm .5$ | . $99788=80$ | $523.9 \pm .6$ | $524.6=.7$ | . $99865=104$ | $1.00077=74$ |
| 0.15;7 (0) | 1.175 |  |  | $.99879 \pm 80$ |  |  | $1.00187=201$ | $1.00318=194$ |
| 0.15;7 (1) | 1.07 | $572.3 \pm .3$ | $574.3 \pm .4$ | $.99653=60$ | $522.8=.8$ | $524.5=.6$ | $.99679=190$ | $1.00026=185$ |
| 0.15:7 (2) | 1.07 | $572.3=.3$ | $574.3 \pm .4$ | . $99654 \pm 60$ | $522.5 \pm .7$ | $524.6=.6$ | $.99601=156$ | . $99947 \pm 148$ |
| 0.15:7 (3) | 1.07 | $572.3=.3$ | $574.3=.4$ | $.99653 \pm 60$ | $522.8=.5$ | $524.5=.6$ | $.99680=98$ | $1.00027=81$ |
| 0.15;8 (0+) | 1.16 | $572.9=.4$ | $574.3=.5$ | $.99761=84$ | $523.9 \pm .6$ | $524.5=.7$ | . $99870=119$ | $1.00109=97$ |
| 0.15;8 (0) | 1.175 |  |  | $.99856=83$ |  |  | $1.00292=234$ | $1.00447 \pm 227$ |
| 0.15:8 (1) | 1.08 | $572.2 \pm .4$ | $574.3=.4$ | $.99642=66$ | $523.0 \pm 1.0$ | $524.5 \pm .7$ | . $99729=225$ | $1.00087 \pm 219$ |
| 0.15:8 (2) | 1.08 | $572.2=.4$ | $574.3=.4$ | $.99642 \pm 66$ | $522.5=.7$ | $524.6=.6$ | $.99592=161$ | $.99950 \pm 152$ |
| 0.15:8 (3) | 1.08 | $572.2 \pm .4$ | $574.3 \pm .4$ | . $99642 \pm 66$ | $522.7=.5$ | $524.5 \pm .6$ | $.99655 \pm 109$ | $1.00013=94$ |
| 0.2:4 ${ }^{(0+1)}$ | 1.24 | $572.1 \pm .4$ | $574.8 \pm .4$ | $.99539=19$ | $522.7 \pm .5$ | $526.3 \pm .6$ | . $99325=26$ | . $99785 \pm 22$ |
| 0.2:4 (0) | 1.21 | $505.5 \pm .6$ | $573.5=.4$ | $.99586=19$ | $401.4=1.4$ | $524.6 \pm .6$ | . $99370=26$ | . $99781=22$ |
| 0.2:4 (1) | 1.10 | $571.8=.3$ | $574.4=.3$ | $.99541=17$ | $521.4 \pm .5$ | $524.9 \pm .5$ | . $99331 \pm 23$ | . $99788 \pm 19$ |
| 0.2:4 (2) | 1.10 | $571.8=.3$ | $574.4 \pm .3$ | $.99541=16$ | $521.4=.5$ | $524.9 \pm .5$ | . $99331=23$ | . $99788 \pm 19$ |
| 0.2;4 (3) | 1.11 | $571.8=.3$ | $574.4=.3$ | $.99541 \pm 17$ | $521.7=.5$ | $525.2=.5$ | $.99327=23$ | $.99784 \pm 19$ |
| 0.2;5 (0) | 1.22 | $572.5 \pm .4$ | $574.8 \pm .4$ | $.99596=40$ | $523.2=.5$ | $525.8 \pm .6$ | $.99517=56$ | . $99921=47$ |
| 0.2;5 (0) | 1.24 |  |  | . $99670=40$ |  |  | $.99528=62$ | . $99855=55$ |
| 0.2:5 (1) | 1.09 | $572.0=.3$ | $574.3=.3$ | $.99602=34$ | $522.0=.5$ | $524.8=.5$ | . $99465=56$ | . $99862 \pm 50$ |
| 0.2;5 (2) | 1.09 | $572.0=.3$ | $574.3=.3$ | . $99603=34$ | $521.0=.5$ | $524.8=.5$ | $.99461=56$ | $.99858 \pm 49$ |
| 0.2;5 (3) | 1.09 | $572.0=.3$ | $574.3 \pm .3$ | $.99601=34$ | $522.3=.5$ | $524.0 \pm .5$ | $.99488=52$ | $.99887=45$ |
| 0.2;6 (0+) | 1.17 | $573.1=.4$ | $574.3 \pm .4$ | . $99795=64$ | $523.8=.5$ | $524.6 \pm .6$ | $.99850=87$ | $1.00055=67$ |
| 0.2;6 (0) | 1.19 |  |  | $.99872=64$ |  |  | $.99984 \pm 115$ | $1.00109=107$ |
| 0.2;6 (1) | 1.07 | $572.3 \pm .3$ | $574.0 \pm .4$ | . $99700 \pm 51$ | $523.1=.6$ | $524.2 \pm .6$ | $.99801=107$ | $1.00101 \pm 99$ |
| 0.2;6 (2) | 1.07 | $572.3 \pm .3$ | $574.0 \pm .4$ | $.99702=51$ | $522.9 \pm .6$ | $524.1 \pm .6$ | $.99704 \div 102$ | $1.00062 \pm 94$ |
| 0.2:6 (3) | 1.07 | $572.3=.3$ | $574.0 \pm .4$ | $.99703=51$ | $522.8 \pm .5$ | $524.1=.6$ | $.99746 \pm 84$ | $1.00043=71$ |
| 0.2:7 $(0+)$ | 1.17 | $573.0=.4$ | $574.4 \pm .5$ | . $99760=77$ | $523.7 \pm .5$ | $524.7=.7$ | . $99811 \pm 101$ | $1.00052=72$ |
| 0.2:7 (0) | 1.19 |  |  | $.99845=77$ |  |  | $1.00056 \pm 165$ | $1.00209=156$ |
| 0.2:7 (1) | 1.06 | $572.2=.3$ | $574.3=.4$ | $.99642=59$ | $522.9 \pm .7$ | $524.4 \pm .6$ | . $99710 \pm 154$ | $1.00069=148$ |
| 0.2;7 (2) | 1.06 | $572.2=.3$ | $574.3 \pm .4$ | . $99644=59$ | $522.6=.7$ | $524.5=.6$ | . $99642 \pm 138$ | $.99998=130$ |
| 0.2:7 (3) | 1.06 | $572.2=.3$ | $574.3=.4$ | . $99643 \pm 59$ | $522.7 \pm .5$ | $524.4=.6$ | $.99657=96$ | $1.00014=80$ |
| 0.2:8 (0+) | 1.18 | $573.0=.4$ | $574.3 \pm .5$ | . $99783 \pm 79$ | $523.7 \pm .6$ | $-74.6=.7$ | . $99838=104$ | $1.00055=74$ |
| 0.2;8 (0) | 1.18 |  |  | . $99887 \pm 79$ |  |  | $1.00267=202$ | $1.00392=194$ |
| 0.2;8 (1) | 1.065 | $572.2=.3$ | $574.2 \pm .4$ | . $99644 \pm 59$ | $523.1=.8$ | $524.3 \pm .6$ | . $99771=191$ | $1.00128 \pm 186$ |
| 0.2;8 (2) | 1.07 | $572.2 \pm .3$ | $574.2 \pm .4$ | $.99646 \pm 59$ | $522.6=.7$ | $524.5 \pm .6$ | $.99645=156$ | . $99999 \pm 148$ |
| 0.2;8 (3) | 1.07 | $572.2=.3$ | $574.2=.4$ | $.99645 \pm 59$ | $522.7 \pm .5$ | $524.5=.6$ | $.99660=97$ | $1.00015 \pm 80$ |

TABLE 3
$\underline{\mathrm{U}}^{235}$

| FIT |  | $x^{2}$ | $\hat{\tilde{\sigma}}_{a}$ |  | $\sigma_{a}\left(E_{0}\right)$ |  | $\mathrm{g}_{\mathrm{a}}$ | $\hat{\hat{\sigma}}_{\mathrm{f}}$ | $\sigma_{\mathrm{f}}\left(E_{0}\right)$ |  | $\mathrm{g}_{\mathrm{E}}$ | $9_{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1;4 | (0) | 1.123 | 663.2 | $\pm .35$ | 677.5 | $\pm .4$ | $.97890 \pm 45$ | $567.2 \pm .4$ | 580.9 | $\pm .6$ | . $97632 \pm 69$ | . $99737 \pm 81$ |
| 0.1;4 | (BC) | 1.083 | 663.0 | $\pm .25$ | 677.2 | $=.3$ | $.97893 \pm 30$ | $567.3 \pm .35$ | 580.6 | $\pm .5$ | . $97701 \pm 59$ | . $99804 \pm 63$ |
| 0.1:4 | (BV) | 1.107 | 662.9 | $\pm .25$ | 677.2 | $\pm .3$ | . $97887 \pm 30$ | $567.0 \pm .35$ | 580.6 | $=.5$ | . $97662 \pm 59$ | .99771 $\pm 63$ |
| 0.1;4 | (MR) | 1.015 | 663.1 | $=.25$ | 677.3 | $\pm .3$ | $.97906 \pm 30$ | $568.1 \pm .4$ | 580.8 | $\pm .5$ | . $97827 \pm 66$ | $.99919 \pm 69$ |
| 0.1:4 | (SH) | 1.086 | 652.8 | $\pm .25$ | 677.1 | $=.3$ | . $97884 \pm 30$ | $566.4 \pm .35$ | 580.0 | $\pm .5$ | . $97667 \pm 58$ | $.99778=63$ |
| 0.1;5 | (0) | 1.091 | 663.2 | $\pm .3_{5}$ | 677.4 | $\pm .5$ | . $97904 \pm 64$ | 567.2 2 . 4 | 582.1 | $\pm .7$ | . $97438 \pm 86$ | . $99523 \pm 106$ |
| 0.1:5 | (BC) | 1.075 | 663.0 | $\pm .25$ | 677.2 | $\pm .3$ | $.97904 \pm 40$ | $567.3 \pm .35$ | 581.3 | $\pm .55$ | $.97591 \pm 72$ | $.99680 \pm 79$ |
| 0.1:5 | (BV) | 1.095 | 662.9 | $\pm .25$ | 677.2 | $\pm .3$ | . $97895 \pm 40$ | $567.0 \pm .35$ | 581.4 | $\pm .5 \mathrm{~s}$ | . $97530 \pm 72$ | $.99626 \pm 78$ |
| 0.1:5 | (MR) | 1.016 | 663.1 | $\pm .25$ | 677.2 | $\pm .3$ | . $97925 \pm 40$ | $568.1 \pm .4$ | 580.9 | $\pm .6$ | $.97792 \pm 87$ | $.99864 \pm 90$ |
| 0.1;5 | (SH) | 1.074 | 662.8 | $\pm .25$ | 677.1 | $\pm .3$ | . $97892 \pm 40$ | $566.5 \pm .35$ | 580.8 | $\pm .55$ | . $97533 \pm 73$ | $.99634 \pm 79$ |
| 0.1:6 | (0) | 1.055 | 663.2 | $\pm .35$ | 677.2 | $\pm .5$ | . $97936 \pm 67$ | $567.4 \pm .4$ | 581.9 | $\pm .7$ | . $97502 \pm 88$ | . $99557 \pm 109$ |
| 0,1;6 | (BC) | 1.058 | 663.0 | $\pm .25$ | 677.2 | $\pm .3$ | . $97906 \pm 40$ | $567.4 \pm .35$ | 581.2 | $\pm .5 \mathrm{~s}$ | .97631 $\pm 73$ | $.99719 \pm 79$ |
| 0.1:6 | (BV) | 1.073 | 662.9 | $\pm .25$ | 677.2 | $\pm .3$ | . $97897 \pm 40$ | $567.2 \pm .35$ | 581.3 | $\pm .55$ | $.97571 \pm 73$ | $.99667 \pm 79$ |
| 0.1:6 | (MR) | 1.009 | 663.1 | $\pm .25$ | 677.2 | $\pm .3$ | $.97923 \pm 40$ | $568.1 \pm .4$ | 580.9 | $\pm .6$ | . $97786 \pm 87$ | . $99860 \pm 90$ |
| 0.1;6 | (SH) | 1.055 | 662.9 | $\pm .25$ | 677.1 | $\pm .3$ | $.97892 \pm 40$ | $566.6 \pm .35$ | 580.9 | $\pm .6$ | $.97550 \pm 73$ | $.99651 \pm 80$ |
| 0.15;4 | (0) | 1.130 | 663.0 | $\pm .3$ | 677.6 | $\pm .4$ | $.97848 \pm 25$ | $567.1 \pm .4$ | 580.3 | $\pm .5$ | . $97740 \pm 28$ | $.99890 \pm 36$ |
| 0.15;4 | (BC) | 1.073 | 662.8 | $\pm .25$ | 677.3 | $\pm .2_{5}$ | $.97859 \pm 20$ | $567.2 \pm .35$ | 580.3 | $\pm .4$ | $.97750 \pm 27$ | . $99889 \pm 32$ |
| 0.15:4 | (BV) | 1.095 | 662.8 | $\pm .25$ | 677.3 | $\pm .3$ | . $97857 \pm 20$ | $567.1 \pm .35$ | 580.1 | $\pm .4$ | . $97743=27$ | . $99884 \pm 32$ |
| 0.15;4 | (MR) | 1.111 | 663.0 | $\pm .25$ | 677.4 | $\pm .3$ | . $97865 \pm 21$ | $568.0 \pm .35$ | 580.9 | $\pm .4$ | .97785 $\pm 27$ | . $99918 \pm 32$ |
| 0.15; 4 | (SH) | 1.077 | 662.7 | $\pm .25$ | 677.2 | $\pm .25$ | $.97856 \pm 20$ | $566.4 \pm .35$ | 579.5 | $\pm .4$ | . $97751 \pm 26$ | . $99892 \pm 32$ |
| 0.15;5 | (0) | 1.127 | 663.0 | $\pm .35$ | 677.6 | $\pm .4$ | . $97845 \pm 47$ | $567.1 \pm .4$ | 580.9 | $\pm .6$ | . $97628 \pm 64$ | $.99778 \pm 77$ |
| 0.15; 5 | (BC) | 1.073 | 662.8 | $\pm .25$ | 677.3 | $\pm .3$ | . $97869 \pm 32$ | $567.2 \pm .35$ | 580.6 | $\pm .5$ | $.97684 \pm 56$ | $.99812 \pm 62$ |
| 0.15;5 | (BV) | 1.093 | 662.8 | $\pm .25$ | 677.3 | $\pm .3$ | $.97864 \pm 32$ | $567.0 \pm .35$ | 580.6 | $\pm .5$ | $.97649 \pm 56$ | $.99780 \pm 62$ |
| 0.15; 5 | (MR) | 1.113 | 663.0 | $\pm .25$ | 677.3 | $\pm .3$ | . $97883 \pm 32$ | $568.1 \pm .4$ | 580.9 | $\pm .5$ | . $97794 \pm 63$ | $.99909 \pm 67$ |
| 0.15:5 | (SH) | 1.075 | 662.7 | $\pm .25$ | 677.2 | $\pm .3$ | $.97861 \pm 32$ | $565.4 \pm .35$ | 580.0 | $\pm .5$ | $.97653 \pm 55$ | . $99788 \pm 61$ |
| 0.15; 6 | (0) | 1.106 | 663.1 | $\pm .35$ | 677.3 | $\pm .5$ | $.97903 \pm 62$ | $567.1 \pm .4$ | 581.9 | $\pm .65$ | . $97463 \pm 81$ | $.99551 \pm 100$ |
| 0.15;6 | (BC) | 1.069 | 562.9 | $\pm .25$ | 677.1 | $\pm .3$ | $.97891 \pm 38$ | $567.2 \pm .35$ | 581.1 | $\pm .5$ | . $97601 \pm 69$ | . $99704 \pm 76$ |
| 0.15:6 | (BV) | 1.086 | 662.8 | $\pm .25$ | 677.1 | $\pm .3$ | . $97884 \pm 38$ | $567.0 \pm .3_{5}$ | 581.2 | $\pm .5$ | $.97544 \pm 69$ | $.99652 \pm 76$ |
| 0.15:6 | (MR) | 1.113 | 663.0 | $\pm .25$ | 677.1 | $\pm .3$ | $.97913 \pm 39$ | $569.1 \pm .4$ | 580.9 | $\pm .55$ | . $97787 \pm 83$ | $.99871 \pm 86$ |
| 0.15;6 | (SH) | 1.068 | 662.7 | $\pm .25$ | 677.1 | $\pm .3$ | . $97881 \pm 38$ | $566.4 \pm .35$ | 580.6 | $\pm .55$ | $.97550 \pm 70$ | $.99662 \pm 76$ |
| 0.15:7 | (0) | 1.093 | 663.1 | $\pm .35$ | 677.2 | $\pm .5$ | $.97920 \pm 66$ | $567.2 \pm .4$ | 582.2 | $\pm .7$ | . $97433 \pm 82$ | . $99502 \pm 104$ |
| 0.15;7 | (BC) | 1.061 | 662.9 | $\pm .25$ | 677.2 | $\pm .3$ | . $97889 \pm 39$ | $567.3 \pm .35$ | 581.3 | $\pm .55$ | . $97578 \pm 70$ | $.99682 \pm 76$ |
| 0.15;7 | (BV) | 1.075 | 662.8 | $\pm .25$ | 677.2 | $\pm .3$ | $.97882 \pm 39$ | $567.1 \pm .35$ | 581.5 | $\pm .55$ | $.97520 \pm 69$ | . $99630 \pm 76$ |
| 0.15;7 | (MR) | 1.114 | 663.0 | $\pm .25$ | 677.2 | $\pm .3$ | . $97911 \pm 39$ | $568.0 \pm .4$ | 581.0 | $\pm .6$ | . $97762 \pm 85$ | . $99848 \pm 88$ |
| 0.15;7 | (SH) | 1.058 | 662.7 | $\pm .25$ | 677.1 | $\pm .3$ | $.97878 \pm 39$ | $566.5 \pm .4$ | 580.9 | $\pm .55$ | $.97513 \pm 71$ | $.99627 \pm 77$ |


| FIT |  | $x^{2}$ | $\hat{\sigma}_{a}$ |  | $\sigma_{a}\left(E_{0}\right)$ |  | $\mathrm{ga}_{\mathrm{a}}$ | $\hat{\sigma}_{\text {f }}$ |  | $\sigma_{f}\left(E_{0}\right)$ |  | $g_{E}$ | ${ }_{7} 7$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.15; 8 |  | 1.062 | 663.1 | $\pm .35$ | 677.2 | . 5 | $.97926 \pm 66$ | 567.4 | $\pm .4$ | 581.4 | $\pm .7$ | . $97590 \pm 91$ | . $99657 \pm 111$ |
| 0.2:4 | (0) | 1.155 | 663.0 | $\pm .3$ | 677.6 | $\pm .4$ | . $97856 \pm 16$ | 567.1 | $\pm .4$ | 580.3 | $\pm .5$ | . $97723 \pm 16$ | . $99864 \pm 21$ |
| 0.2;4 | (BC) | 1.049 | 662.8 | $\pm .25$ | 677.3 | $\pm .25$ | . $97865 \pm 16$ | 567.2 | $\pm .35$ | 580.4 | $\pm .4$ | . $97729 \pm 15$ | . $99861 \pm 20$ |
| 0.2:4 | (BV) | 1.068 | 662.8 | $\pm .25$ | 677.3 | $\pm .3$ | . $97864 \pm 16$ | 567.0 | $\pm .35$ | 580.2 | $\pm .4$ | . $97731 \pm 15$ | . $99863 \pm 20$ |
| 0.2;4 | (MR) | 1.091 | 662.9 | $\pm .25$ | 677.4 | $\pm .3$ | . $97867 \pm 16$ | 567.9 | $\pm .4$ | 581.1 | $\pm .4$ | . $97727 \pm 15$ | . $99857 \pm 20$ |
| 0.2;4 | (SH) | 1.050 | 662.7 | $\pm .25$ | 677.2 | $\pm .25$ | $.97865 \pm 16$ | 566.4 | $\pm .35$ | 579.5 | $\pm .4$ | . $97743 \pm 15$ | . $99876 \pm 20$ |
| 0.2;5 | (0) | 1.158 | 663.0 | $\pm .35$ | 677.6 | $\pm .4$ | $.97847=33$ | 567.1 | $\pm .4$ | 580.2 | $\pm .5$ | . $97756 \pm 40$ | . $99907 \pm 50$ |
| 0.2;5 | (BC) | 1.051 | 662.8 | $\pm .25$ | 677.3 | $\pm .3$ | . $97867 \pm 26$ | 567.3 | $\pm .35$ | 580.2 | $\pm .4$ | . $97760 \pm 37$ | . $99891 \pm 43$ |
| 0.2;5 | (BV) | 1.071 | 662.8 | $\pm .25$ | 677.3 | $\pm .3$ | . $97864 \pm 26$ | 567.1 | $\pm .35$ | 580.1 | $\pm .4$ | . $97744 \pm 37$ | . $99877 \pm 43$ |
| 0.2;5 | (MR) | 1.084 | 663.0 | $\pm .25$ | 677.4 | $\pm .3$ | $.97878=26$ | 568.1 | $\pm .4$ | 580.4 | $\pm .45$ | . $97832 \pm 39$ | . $99952 \pm 44$ |
| 0.2;5 | (SH) | 1.053 | 662.7 | $\pm .25$ | 677.2 | $\pm .3$ | $.97862 \pm 26$ | 566.4 | $\pm .4$ | 579.5 | $\pm .4$ | . $97748 \pm 36$ | . $99883 \pm 42$ |
| 0.2;6 | (0) | 1.142 | 663.0 | $\pm .35$ | 677.6 | $\pm .4$ | $.97849 \pm 51$ | 567.1 | $\pm .4$ | 581.2 | $\pm .6$ | $.97570 \pm 69$ | . $99715 \pm 83$ |
| 0.2;6 | (BC) | 2.048 | 662.8 | $\pm .25$ | 677.3 | $\pm .3$ | . $97859 \pm 34$ | 567.2 | $\pm .35$ | 580.8 | $\pm .5$ | . $97661 \pm 60$ | . $99797 \pm 66$ |
| 0.2;6 | (BV) | 1.065 | 662.8 | $\pm .25$ | 677.3 | $\pm .3$ | . $97854 \pm 34$ | 567.0 | $\pm .35$ | 580.8 | $\pm .5$ | . $97619 \pm 60$ | . $99760 \pm 66$ |
| 0.2:6 | (MR) | 1.086 | 663.0 | $\pm .25$ | 677.4 | $\pm .3$ | $.97877 \pm 35$ | 568.1 | $\pm .4$ | 580.9 | $\pm .5$ | $.97797 \pm 68$ | . $99918 \pm 72$ |
| 0.2;6 | (SH) | 1.047 | 662.7 | $\pm .25$ | 677.3 | $\pm .3$ | $.97851 \pm 34$ | 566.4 | $\pm .35$ | 580.1 | $\pm .5$ | $.97624 \pm 60$ | . $99768 \pm 65$ |
| 0.2;7 | (0) | 1.136 | 663.1 | $\pm .35$ | 677.4 | $\pm .45$ | . $97884 \pm 62$ | 567.1 | $\pm .4$ | 581.8 | $\pm .65$ | . $97474 \pm 81$ | . $99581 \pm 100$ |
| 0.2;7 | (BC) | 1.042 | 662.8 | $\pm .25$ | 677.1 | $\pm .3$ | . $97887 \pm 38$ | 567.2 | $\pm .3 \mathrm{~s}$ | 581.2 | $\pm .5$ | . $97594 \pm 59$ | . $99701 \pm 75$ |
| 0.2;7 | (BV) | 1.057 | 662.8 | $\pm .25$ | 677.1 | $\pm .3$ | $.97880 \pm 38$ | 567.0 | $\pm .4$ | 581.3 | $\pm .5$ | . $97537 \pm 69$ | . $99649 \pm 75$ |
| 0.2;7 | (MR) | 1.084 | 662.0 | $\pm .25$ | 677.2 | $\pm .3{ }^{\prime}$ | . $97909 \pm 39$ | 568.1 | $\pm .4$ | 581.0 | $\pm .55$ | . $97776 \pm 82$ | . $99865 \pm 86$ |
| 0.2:7 | (SH) | 1.040 | 662.7 | $\pm .25$ | 677.1 | $\pm .3$ | . $97877 \pm 38$ | 566.4 | $\pm .4$ | 580.6 | $\pm .55$ | . $97542=70$ | $.99658 \pm 76$ |
| 0.2;8 | (0) | 1.109 | 663.1 | $\pm .3_{5}$ | 677.2 | $\pm .5$ | . $97928 \pm 66$ | 567.3 | $\pm .4$ | 582.2 | $\pm .5_{5}$ | . $97439 \pm 81$ | $.99500 \pm 103$ |
| 0.2;8 | (BV) | 1.048 | 662.8 | $\pm .25$ | 677.2 | $\pm .3$ | . $97880 \pm 38$ | 567.1 | $\pm .35$ | 581.5 | $\pm .5$ | . $97520 \pm 69$ | . $99632 \pm 75$ |
| 0.2:8 | (MR) | 1.084 | 663.0 | $\pm .25$ | 677.2 | $\pm .3$ | . $97907 \pm 39$ | 568.0 | $\pm .4$ | 581.1 | $\pm .5 \mathrm{~s}$ | . $97756 \pm 69$ | . $99845 \pm 87$ |
| 0.2;8 | (SH) | 1.032 | 662.7 | $\pm .2 \mathrm{~s}$ | 677.10 | $\pm .3$ | . $97876 \pm 38$ | 566.5 | $\pm .3_{5}$ | 580.9 | $\pm .55$ | $.97513 \pm 70$ | $.99630 \pm 76$ |

## TABLE 4 <br> $\mathrm{Pu}^{239}$

| FIT |  | $x^{3}$ | $\stackrel{\wedge}{\sigma}_{a}$ | $\sigma_{a}\left(E_{0}\right)$ | 9 | $\hat{\sigma}_{f}$ | $0_{f}\left(E_{0}\right)$ | $9_{f}$ | $\mathrm{g}_{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1:4 | (0) | L. 37 | 1088.2 $=$ | $1012.7 \pm 1.3$ | $1.07454 \pm 100$ | $780.3 \pm$ | $739.7=1.6$ | $1.05487 \pm 168$ | $.98169 \pm 166$ |
| 0.1:4 | (B) | 1.22 | 1087.2 $\pm .9$ | $1011.1 \pm 1.4$ | $1.07528 \pm 103$ | $778.8 \pm 1.3$ | $738.5 \pm 1.3$ | $1.05452 \pm 142$ | $.98069 \pm 145$ |
| 0.1;4 | (R) | 1.02 | $1096.5 \pm 1.4$ | $1015.1 \pm 1.7$ | $1.08019 \pm 132$ | $782.4 \pm 1.3$ | $740.8 \pm 1.4$ | $1.05619 \pm 144$ | . $97778 \pm 152$ |
| 0.1:4 | (L) | 1.07 | $1086.1 \pm .8$ | $1009.7 \pm 1.4$ | $1.07571 \pm 100$ | $778.3 \pm 1.3$ | $738.0 \pm 1.3$ | $1.05466 \pm 142$ | . $98042 \pm 144$ |
| 0.1; 5 | (0) | 1.35 | $1088.7 \pm$ | $1014.6 \pm 1.4$ | $1.07303 \pm 106$ | $779.0=$ | $741.8=1.9$ | $1.05146 \pm 278$ | . $97989 \pm 260$ |
| 0.1:5 | (B) | 1.19 | $1087.6 \pm .9$ | $1012.9 \pm 1.4$ | $1.07371 \pm 109$ | $777.7 \pm 1.4$ | $740.9 \pm 1.5$ | $1.04964 \pm 239$ | . $97758 \pm 223$ |
| 0.1:5 | (R) | 1.01 | $1096.4 \pm 1.4$ | $1017.5 \pm 1.9$ | $1.07754 \pm 173$ | $781.2 \pm 1.5$ | $743.2 \pm 1.6$ | $1.05103 \pm 247$ | . $9754 \pm 230$ |
| 0.1;5 | (L) | 1.05 | $1086.5 \pm .8$ | $1011.4 \pm 1.4$ | $1.07427=107$ | $777.3 \pm 1.4$ | $740.3=1.5$ | $1.04997 \pm 239$ | $.97738 \pm 222$ |
| 0.12:4 | (0) | 1.34 | $1088.5 \pm .8$ | $1012.3 \pm 1.3$ | $1.07532=92$ | $779.9=1.4$ | $739.6=1.5$ | $1.05445 \pm 127$ | . $98059 \pm 130$ |
| 0.12; 4 | (B) | 1.21 | $1087.4 \pm .9$ | $1010.6 \pm 1.3$ | $1.07599 \pm 94$ | $778.7 \pm 1.2$ | $738.2 \pm 1.3$ | $1.05482 \pm 106$ | . $98032 \pm 115$ |
| 0.12; 4 | (R) | 1.03 | $1096.6 \pm 1.4$ | $1015.7 \pm 1.6$ | $1.07966=113$ | $782.3 \pm 1.3$ | $740.8 \pm 1.3$ | $1.05591 \pm 108$ | . $97800 \pm 121$ |
| 0.12:4 | (L) | 1.07 | $1086.4 \pm .84$ | $1009.5 \pm 1.3$ | $1.07619 \pm 91$ | $778.3 \pm 1.2$ | $737.8 \pm 1.3$ | $1.05491 \pm 106$ | . $98023 \pm 114$ |
| 0.12:5 | (0) | 1.33 | $1088.7 \pm .8$ | $1014.2 \pm 1.4$ | $1.07348 \pm 106$ | $780.1 \pm 1.5$ | $740.5 \pm 1.8$ | $1.05340 \pm 239$ | . $98129 \pm 227$ |
| 0.12:5 | (B) | 1.20 | $1087.7 \pm .9$ | $1012.4 \pm 1.4$ | $1.07431 \pm 109$ | $778.2 \pm 1.4$ | $739.7=1.4$ | $1.05202 \pm 205$ | . $97926 \pm 196$ |
| 0.12;5 | (R) | 1.03 | $1096.6 \pm 1.4$ | $1015.8 \pm 1.8$ | $1.07949 \pm 163$ | $781.5=1.4$ | $741.5 \pm 1.5$ | $1.05402 \pm 210$ | . $97640 \pm 206$ |
| 0.12;5 | (L) | 1.06 | $1086.6 \pm .8$ | $1010.9 \pm 1.4$ | $1.07487=107$ | $777.7 \pm 1.4$ | $739.1 \pm 1.4$ | $1.05226 \pm 205$ | . $97897 \pm 196$ |
| 0.12;6 | (0) | 1.32 | 1088.9 $\pm .8$ | $1014.2=1.4$ | $1.07366 \pm 109$ | $779.1 \pm 1.6$ | $742.4=2.0$ | $1.04945 \pm 310$ | . $97745 \pm 288$ |
| 0.12;6 | (B) | 1.18 | 1087.9 $\pm .9$ | $1012.7 \pm 1.4$ | $1.07426 \pm 111$ | $776.7 \pm 1.5$ | $741.5 \pm 1.6$ | $1.04741 \pm 272$ | . $97500 \pm 249$ |
| 0.12;6 | (R) | 1.01 | $1096.9 \pm 1.4$ | $1018.0=2.0$ | $1.07754 \pm 176$ | $780.2 \pm 1.6$ | $744.2 \pm 1.7$ | $1.04832 \pm 281$ | . $97288 \pm 255$ |
| 0.12:6 | (L) | 1.05 | $1086.8 \pm .9$ | $1011.2=1.4$ | $1.07473 \pm 108$ | $776.2 \pm 1.5$ | $740.8 \pm 1.6$ | $1.04778=272$ | . $97492 \pm 250$ |
| 0.12:7 | (0) | 1.32 | $1089.0=0.9$ | $1015.1=1.6$ | $1.07280=134$ | $778.8 \pm 1.7$ | $742.8 \pm 2.0$ | $1.04852 \pm 337$ | $.97736 \pm 311$ |
| 0.15; 4 | (0) | 1.30 | 1088.6 亡 . 8 | $1011.4 \pm 1.2$ | $1.07631 \pm 81$ | $779.0=1.3$ | $740.0 \pm 1.5$ | $1.05265 \pm 92$ | . $97802 \pm 97$ |
| 0.15:4 | (B) | 1.26 | $1087.6 \pm .9$ | $1009.9 \pm 1.2$ | $1.07684 \pm 80$ | $777.7 \pm 1.1$ | $738.3=1.2$ | $1.05345 \pm 75$ | . $97828 \pm 83$ |
| 0.15;4 | (R) | 1.09 | $1096.4 \pm 1.4$ | $1016.3 \pm 1.5$ | $1.07874 \pm 89$ | $781.4 \pm 1.2$ | $741.3 \pm 1.3$ | $1.05408 \pm 76$ | . $97714 \pm 85$ |
| 0.15;4 | (L) | 1.13 | $1086.5 \pm .9$ | $1008.9 \pm 1.2$ | $1.07695 \pm 78$ | $777.3=1.2$ | $737.8 \pm 1.2$ | $1.05352 \pm 75$ | . $97824 \pm 82$ |
| 0.15;5 | (0) | 1.29 | $1088.6 \pm .9$ | $1013.4 \pm 1.4$ | $1.07439 \pm 103$ | $780.0 \pm 1.4$ | $739.8 \pm 1.6$ | $1.05431 \pm 185$ | . $98130 \pm 179$ |
| 0.15;5 | (B) | 1.25 | $1087.7 \pm .9$ | $1011.4 \pm 1.4$ | $1.07539 \pm 105$ | $778.4 \pm 1.3$ | $738.5 \pm 1.3$ | $1.05375 \pm 158$ | . $97988 \pm 156$ |
| 0.15:5 | (R) | 1.09 | $1096.8 \pm 1.4$ | $1014.9 \pm 1.7$ | $1.08074=143$ | $781.9=1.4$ | $740.5 \pm 1.4$ | $1.05587 \pm 162$ | . $97698 \pm 164$ |
| 0.15; 5 | (L) | 1.13 | $1086.6 \pm .9$ | $1010.1 \pm 1.4$ | $1.07579 \pm 103$ | $778.9 \pm 1.3$ | $738.1 \pm 1.3$ | $1.05397 \pm 158$ | . $97974 \pm 156$ |

(Cont'd)


1
N
(All errors in $g^{\prime} s$ are $\times 10^{-5}$ )
$\alpha=0.095$ (at zero energy) pseudo-point, weight 3.0 units) we see that for the average fit the $g_{a}$ accuracy is not improved, while the error of $g_{\alpha}$ is reduced by about $25 \%$ and $g_{f}$ almost as much. This is a modest change for the effect of a point placed where there are no other data. The case $3 g^{\prime} s$ are therefore included in the estimate of "spreads", but (because cases l, 2 and 3 all have a considerable similarity) the older 0 and $0+$ cases are retained but treated separately, as giving definitely less reliable fits.

In fact, the main problem for $U-233$, and one also applying to both other isotopes, is deciding, for those curves behaving anamalously near $E=0$, at what point we must draw the line and declare a fit "unacceptable", and so exclude the corresponding g-factors from account when compiling the total "spread" for each type of $g$. In this context, acceptability is judged from the reasonableness of the shapes in view of what is known of the theory of neutron-nuclear interactions. Although it is known that multi-level curves may vary in shape much more than single level Breit-Wigners, the smallest likely level-widths (set by $\Gamma_{\gamma}$ as a lower limj.t, see BNL-325, 1965, Vol. III) for these heavy nuclei makes sudden departures below 10-15 milli-eV from the trends established above this energy rather unlikely, but it is still a matter of judgment based on familiarity with both theory and the available measured data. In Appendix $V$ some aspects of this are discussed further.

Tables 2, 3 and 4 show the $\hat{\sigma}, \sigma_{2} \sigma_{0}$ and g-factors obtained from the fitting procedure for $U-233, U-235$ and Pu-239 respectively, together with the standard deviations calculated as explained in Appendix III, i.e., from the accuracy assigned to each input datum (multiplying by $x$ may give a rough estimate based on the
average spreads of the points, as m_ationed in the middle of section 5 above). However, the "spreads" of the g-values themselves appear to be a better indication of the overall accuracy, though this should be no less than the mean $X$ times the computed "errors". This is because the computations assume that a given order of polynomial plus the chosen Breit-wigner terms can be expected to give a good fit to the $\sigma \sqrt{E}$ concerned, and although mathematically convenient, this assumption has no physical basis. Inspection of the selection of fitted curves shown in figs. 10 - 24 may support the feeling that we have achieved an adequate number of fits representative of various possible fluctuations but it would be quite difficult to prove this. The use of a "factor of safety", applied to errors deduced from "spreads", has already been mentioned--this factor will be largest (in fact almost $\times 2$ ) for $U-233$, where the "cases" used (except 0 and $0+$ ) are rather similar, and least for Pu-239 where the source of the divergent trends of the recipes seems clear and no awkward decisions between claimed accuracies and observed spreads within a single data set arose to complicate the assessment.
9. THE "SPREADS" OF g-FACTORS OBTAINED

We have already noted that for low-order wide-range fits the choice of fit in itself reduces the calculated errors and makes the fits for all "cases" similar; this is clear from Tables 2-4. The trends represented by the systematic differences between $G$ 's for the cases ("SH" and "MR") based mainly on SH (in contrast to $R$ ) recipes for $U-235$, and simi. $\because . i y$ for $R, L$ and $B$ cases
for Pu-239, are also seen in this table, and the curve-shape differences (see figures cited above) also show the characteristics indicated in App. II.C.

The fact that for $g_{f}(U-235)$ and $g_{a}(P u-239)$ the systematic difference (due to the recipe differences (R-SH and R-LO) respectively) is of the same order as the "spread" of $g$ 's (for acceptable curve shapes) for either group taken separately was mentioned above (section 7) and is in fact derived from the values* in Tables 3 and 4; the only slightly smaller systematic differences for $g_{T}(U-235)$ and the definitely smaller ones for $g_{f}$ and $g_{.}(P u-239)$ are also seen from this table. These facts are relevant to the interpretation of the observed spread of the g-values in terms of a standard deviation (the former being generally 3 to $3 \frac{1}{2}$ times the latter, see Appendix $\mathrm{V}(\mathrm{A})$ ). Also in Appendix V is a summary of which curve shapes were felt to be unacceptable (some "borderline" cases also exist). The spreads shown in Table 5 are those given by the $g$ 's of tables $2-4$ and it will be seen that the mean of the spreads with and without borderline cases is that adopted. Table 5 also shows the spread of the $20.44^{\circ} \mathrm{C}$ g's deduced from "adjust", from Table 1 ; the additional spread which would be given by the (subsidiary) option between $R$ and $R S$ has been included, but MLA is regarded as something between borderline and unacceptable (HI has already been regarded as rejected-it is in Table 1 only to show what effect it would produce). Both these "adjust" spreads and the spreads from "fitting" are for $20.44^{\circ} \mathrm{C}$ Maxwellian spectra--in App. V(C) the question of widening the "spreads" (because of an accidental cancellation at * See, for example, the differences between cases MR and SH of Table 3 and cases $R$ and $L$ of Table 4.

|  | $\mathrm{g}_{\mathrm{abs}}$ | $\mathrm{g}_{\text {fiss }}$ | $\mathrm{g}_{\text {eta }}$ |
| :---: | :---: | :---: | :---: |
| U-233 |  |  |  |
| A | 0.99615 | 0.9963 | 1.00015 |
| F | $0.99703]-0.99549$ | $0.99764]$ - 0.99419 | $1.00138-[0.99840$ |
| FB | $0.99747]-0.99549$ | $0.99806]-0.99419$ | $1.00138-10.99784$ |
| $\Delta\left(x 10^{5}\right)$ | 176 | 365 | 326 |

U-235


Pu-239


Deduced Standard Deviations and Factors of Safety

|  | $\frac{U-233}{}$ | $\frac{U-235}{}$ | $\frac{\text { Pu-239 }}{1.1}$ |
| :--- | :---: | :---: | :---: |
| Safety Factor | $1.65(a b s, 2.3)$ | $1.25(\mathrm{abs}, 3.75)$ | 0.285 |
| Std. Dev. \%, abs | 0.12 | 0.09 | 0.285 |
| Std. Dev. \%, fiss | 0.20 | 0.155 | 0.325 |

For detailed explanation and acceptability criteria (also of asterisk *) see App. V. B. 4
$A=$ from "adjust" procedure, $F=$ from fitting procedure; (B) denotes borderline acceptability. $\Delta=$ "spread", followed (in brackets) by $\Delta T$ if applicable.
$20^{\circ} \mathrm{C}$ of the effects of some of the differences between alternative recipes) is discussed, and the final "effective" spreads chosen, taking this into account, are also listed in Table 5, together with the factor of safety (see section 8 above) and the final standard deviation. The "independent" errors (v. App. V(D)), needed for the $2200 \mathrm{~m} / \mathrm{sec}$ regression analysis, are listed, with the recommended g-values and the standard errors from Table 5 in Table 7 below.
10. STUDY OF Pu-241

So far pu-241 has not been considered in detail. The $\alpha$ data (apart from Maxwellian averages) consist only of two points, so that only the ratio of the two is significant for our work. Consequently the "adjust" procedure is inapplicable and the fitting procedure depends mainly on the data for $\sigma_{a}$ and $\sigma_{f}$. To obtain the weights for the $\sigma_{f}$ points an "eye-ball" fit was used, as for the other isotopes, while for $\sigma_{a}$ the most important data were known to be of similar quality (the CR 64C and MTR 6lI values both used a BNL-type chopper and the same oxide sample material) so that the relative weights were estimated and an "absolute" weighting system derived from these aided by some mean-square deviation calculations. In fact, the accuracy for $\sigma_{a}$ was quite high and that for $\sigma_{f}$ relatively low, and the relative weights of total and fission cross-section data could be varied considerably without changing the fits (had there been no $\alpha$ data at all the fits would have been completely independent). The only alternative "cases" (i.e., sets of weights) used were obtained by varying the weights for some special sets of data (see App, V.F),
and for some fits by adding a "fictitious" point for $\alpha$ at 0.005 eV energy. This point was introduced, as discussed below and in the Appendix, to provide some way of including the qualitative knowledge which nuclear systematics provides as to what kinds of behaviour of $\sigma \sqrt{E}$ seem quite unlikely. The data from Harwell (Raffle, AERE-R 2998, and James (1965)) and Hanford (the HAN 58D set, with points in the energy range 2.5-4.7 milli eV), for all of which the authors' accuracy claims were known, all required a weight function varying with energy, or from point to point. The problems raised by these data sets is discussed fully in App. V.F--for James the fact that SCISRS contained a very large number of data, each of very low accuracy, introduced problems, while for HAN 58D the normalization accuracy of the data set required examination. Unfortunately this means that all the low-energy $\sigma_{f}$ data are in some way problematical. The low energy $\sigma_{a}$ data (especially the MTR 68L set) appeared very good in comparison, although the steep rise at low energies of $\sigma_{a} . / \mathrm{E}$ is also not so easily explained in terms of systematics (an unexpectedly narrow negative-energy resonance with a very small $\Gamma_{f} / \Gamma_{\gamma}$ would be needed to explain tinis, since $\sigma_{f} \sqrt{E}$ does not show a similar rise).

The problem which arises is seen from Figs. 25-26; the $\sigma_{a} \sqrt{E}$ rise near $E=0$ seems well-determined but is increasingly steep as $E=0$ is approached, and many of the fits (at least without the "fictitious" $\alpha$-dacum) show $\sigma_{f} / E$ falling in the $\mathrm{E}<0.015$ (approx.) eV region where the data become sparse. The only two $\alpha$-points indicate only a small negative slope and a "flat" curve seems reasonable, but the fits rise steeply as $\alpha \rightarrow 0$. It is difficult to see what these trends can
represent in terms of the theory of neutron resonances; while multi-level analysis may yield curve-shapes (for $\sigma / \mathrm{E}$ ) rather different from those expected from single-level theory, it seems unlikely that a definite rise of $\sigma_{a} \sqrt{E}$ for energies below 25-40 milli-eV would be accompanied by an even more sudden (concentrated below 0.02 eV ) fall off of $\sigma_{f} \sqrt{\mathrm{E}}$. But since the data below 0.015 eV are so poor, we have preferred to assume that $\sigma_{f} \sqrt{E}$ should be at least flat, if it does not rise slightly as $\mathrm{E} \rightarrow 0$, and have performed fits to study what special assumptions would be needed to ensure this.

In the final set of fits (specified in the Appendix) are included cases with and without the James data (where the $\chi^{2}$ shows that the "with" set give these points too much weight), with and without the downweighting applied to HAN 58D data, and with and without the single "fictitious" $\alpha$ datum. Table 6 gives the values of g-factors corresponding to the cases which were done and sample graphs are given in Fig. 25-26. To illustrate the changes due to different choices of weights, only curves for $0.1 ; 5$ fits are shown--considered as a set, the 0.l;5 are all "reasonable" fits, and more extreme differences (or, e.g. wiggliness) will occur for other indices and ranges of fit. On the question of which curve-shapes could be treated as acceptable, only the $\sigma_{f} /$ curves are discussed below; as App. V makes clear all absorption, but none of the $\alpha$ curves, were "acceptable" if we apply nuclear systematics criteria. For the rest, the following general statements can be made: With neither HAN 58D nor James data included (case "N") we get typically a highly variable fit, with most influence from the extra $\alpha$ datum; practically all fits without this datum give $\sigma_{f}$ curve shapes rejected on systematics ground (with it, most are OK).

## TABLE 6

## g-factors for Pu-241 Fits

|  |  | $x^{2}$ | $\mathrm{g}_{\mathrm{a}}$ | $\mathrm{g}_{\mathrm{f}}$ | $\mathrm{g}_{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1; 4 | A | 1.478 | $1.03738 \pm 69$ | $1.04836 \pm 232$ | $1.01058 \pm 233$ |
|  | A+ | 1. 485 | $1.03733 \pm 69$ | $1.05005 \pm 226$ | $1.01227 \pm 228$ |
|  | HD | 1.474 | $1.03739 \pm 69$ | $1.04651 \pm 246$ | $1.00879 \pm 246$ |
|  | HD+ | 1.483 | $1.03733 \pm 69$ | $1.04889 \pm 237$ | $1.01114 \pm 237$ |
|  | NJ | 1.158 | $1.03738 \pm 69$ | $1.05077 \pm 251$ | $1.01290 \pm 250$ |
|  | NJ+ | 1.169 | $1.03733 \pm 69$ | $1.05274 \pm 241$ | $1.01485 \pm 241$ |
| 0.1:5 | A | 1.480 | $1.03768 \pm 86$ | $1.04862 \pm 383$ | $1.01054 \pm 376$ |
|  | A+ | 1.486 | $1.03762 \pm 86$ | $1.05244 \pm 364$ | $1.01428 \pm 359$ |
|  | HD | 1.475 | $1.03768 \pm 86$ | $1.04310 \pm 442$ | $1.00523 \pm 432$ |
|  | HD+ | 1.484 | $1.03761 \pm 86$ | $1.04984 \pm 405$ | $1.01179 \pm 397$ |
|  | NJ | 1.158 | $1.03771 \pm 86$ | $1.05545 \pm 413$ | $1.01709 \pm 405$ |
|  | NJ+ | 1.166 | $1.03765 \pm 86$ | $1.05903 \pm 389$ | $1.02060 \pm 382$ |
|  | N | 1.164 | $1.03771 \pm 86$ | $1.04296 \pm 638$ | $1.00506 \pm 620$ |
|  | $\mathrm{N}+$ | 1.179 | $1.03765 \pm 85$ | $1.05680 \pm 507$ | $1.01846 \pm 493$ |
| 0.1;6 | A | 1.480 | $1.03762 \pm 86$ | $1.05117 \pm 431$ | $1.01306 \pm 422$ |
|  | A+ | 1.485 | $1.03757 \pm 85$ | $1.05556 \pm 411$ | $1.01734 \pm 402$ |
|  | HD | 1.477 | $1.03760 \pm 86$ | $1.04401 \pm 547$ | $1.00617 \pm 532$ |
|  | HD+ | 1.485 | $1.03756 \pm 86$ | $1.05334 \pm 483$ | $1.01522 \pm 470$ |
|  | NJ | 1.159 | $1.03765 \pm 87$ | $1.05789 \pm 455$ | $1.01950 \pm 445$ |
|  | NJ+ | 1.165 | $1.03761 \pm 85$ | $1.06177 \pm 429$ | $1.02329 \pm 419$ |
|  | N | 1.166 | $1.03764 \pm 86$ | $1.03993 \pm 920$ | $1.00221 \pm 889$ |
|  | N+ | 1.179 | $1.03761 \pm 86$ | $1.06173 \pm 638$ | $1.02325 \pm 617$ |
| 0.14;4 | HD | 1.425 | $1.03644 \pm 50$ | $1.04735 \pm 134$ | $1.01054 \pm 138$ |
|  | NJ | 1.120 | $1.03643 \pm 50$ | $1.04818 \pm 238$ | $1.01134 \pm 141$ |
| 0.14;5 | A | 1.426 | $1.03779 \pm 77$ | $1.04902 \pm 266$ | $1.01083 \pm 266$ |
|  | A+ | 1.432 | $1.03773 \pm 77$ | $1.05114 \pm 258$ | $1.01293 \pm 258$ |
|  | HD | 1.423 | $1.03779 \pm 77$ | $1.04662 \pm 287$ | $1.00851 \pm 286$ |
|  | HD+ | 1.431 | $1.03772 \pm 77$ | $1.04974 \pm 274$ | $1.01158 \pm 273$ |
|  | NJ | 1.114 | $1.03779 \pm 77$ | $1.05219 \pm 287$ | $1.01387 \pm 286$ |
|  | NJ+ | 1.123 | $1.03774 \pm 77$ | $1.05454 \pm 272$ | 1.01620 \# 274 |
|  | N | 1.118 | $1.03782 \pm 77$ | $1.04636 \pm 363$ | $1.00824 \pm 357$ |
|  | N+ | 1.132 | $1.03773 \pm 77$ | $1.05244 \pm 320$ | $1.01417 \pm 317$ |
| 0.14;6 | A | 1.428 | $1.03775 \pm 84$ | $1.04779 \pm 378$ | $1.00968 \pm 372$ |
|  | A+ | 1.434 | $1.03768 \pm 84$ | $1.05168 \pm 359$ | $1.01349 \pm 354$ |
|  | HD | 1.424 | $1.03774 \pm 84$ | $1.04194 \pm 440$ | $1.00404 \pm 430$ |
|  | HD+ | 1.432 | $1,03767 \pm 84$ | $1.04896 \pm 403$ | $1.01088 \pm 395$ |
|  | NJ | 1.116 | $1.03777 \pm 85$ | $1.05437 \pm 405$ | $1.01600 \pm 398$ |
|  | NJ+ | 1.123 | $1.03772 \pm 85$ | $1.05800 \pm 381$ | $1.01955 \pm 374$ |
|  | N | 1.119 | $1.03778 \pm 84$ | $1.04072 \pm 640$ | $1.00284 \pm 622$ |

(Cont'a)

|  |  | $\chi^{2}$ | $\mathrm{ga}_{\text {a }}$ | $g_{\text {E }}$ | $g_{\eta}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.14;7 | A | 1.427 | $1.03766 \pm 86$ | $1.05143 \pm 425$ | $1.01327 \pm 416$ |
|  | A+ | 1.432 | $1.03761 \pm 86$ | $1.05566 \pm 399$ | $1.01740 \pm 391$ |
|  | HD | 1.424 | $1.03765 \pm 81$ | $1.04494 \pm 530$ | $1.00703 \pm 515$ |
|  | HD+ | 1.431 | $1.03760 \pm 88$ | $1.05369 \pm 437$ | $1.01551 \pm 465$ |
|  | NJ | 1.113 | $1.03769 \pm 79$ | $1.05807 \pm 469$ | $1.01964 \pm 457$ |
|  | $\mathrm{NJ}+$ | 1.120 | $1.03765 \pm 85$ | $1.06183 \pm 413$ | $1.02330 \pm 404$ |
|  | N | 1. 121 | $1.03768 \pm 86$ | $1.04308 \pm 881$ | $1.00521 \pm 852$ |
|  | N+ | 1.131 | $1.03765 \pm 88$ | $1.06240 \pm 608$ | $1.02385 \pm 589$ |
| 0.14;8 | A | 1.428 | $1.03715 \pm 254$ | $1.05162 \pm 384$ | $1.01395 \pm 444$ |
|  | HD | 1.426 | $1.03714 \pm 104$ | $1.04492 \pm 529$ | $1.00751 \pm 502$ |
|  | HD+ | 1.432 | $1.03708 \pm 207$ | $1.05449 \pm 816$ | $1.01678 \pm 810$ |
|  | NJ | 1.115 | $1.03720 \pm 239$ | $1.05808 \pm 119$ | $1.02013 \pm 199$ |
|  | NJ+ | 1.121 | $1.03714 \pm 119$ | $1.06179 \pm 488$ | $1.02377 \pm 479$ |
|  | N | 1.121 | $1.03717=225$ | $1.03634 \pm 1006$ | $0.99920 \pm 993$ |
| 0.18; 5 | A | 1.462 | $1.03702 \pm 65$ | $1.04654 \pm 199$ | $1.00918 \pm 201$ |
|  | A+ | 1. 469 | $1.03697 \pm 65$ | $1.04790 \pm 194$ | $1.01054 \pm 197$ |
|  | HD | 1.459 | $1.03703 \pm 65$ | $1.04505 \pm 208$ | $1.00773 \pm 210$ |
|  | HD+ | 1.467 | $1.03697 \pm 65$ | $1.04691 \pm 202$ | $1.00958 \pm 204$ |
|  | NJ | 1.127 | $1.03702 \pm 65$ | $1.04864 \pm 213$ | $1.01120 \pm 215$ |
|  | N | 1.128 | $1.03705=65$ | $1.04499 \pm 243$ | $1.00766 \pm 243$ |
|  | $\mathrm{N}+$ | 1.145 | $1.03698 \pm 65$ | $1.04844 \pm 226$ | $1.01106 \pm 226$ |
| 0.18;6 | A | 1.461 | $1.03799 \pm 81$ | $1.04930=316$ | $1.01090 \pm 314$ |
|  | A+ | 1. 467 | $1.03792 \pm 81$ | $1.05210 \pm 303$ | $1.01366 \pm 301$ |
|  | HD | 1.458 | $1.03799 \pm 81$ | $1.04583 \pm 353$ | $1.00756 \pm 348$ |
|  | HD+ | 1.465 | $1.03792 \pm 81$ | $1.05030 \pm 331$ | $1.01193 \pm 327$ |
|  | NJ | 1.121 | $1.03800 \pm 81$ | $1.05401 \pm 343$ | $1.01543 \pm 339$ |
|  | NJ+ | 1.129 | $1.03794 \pm 81$ | $1.05692 \pm 325$ | $1.01828 \pm 322$ |
|  | N | 1.126 | $1.03802 \pm 81$ | $1.04585 \pm 475$ | $1.00754 \pm 464$ |
|  | N+ | 1.139 | $1.03794 \pm 81$ | $1.05480 \pm 401$ | $1.01625 \pm 393$ |
| 0.18;7 | A | 1.462 | $1.03783 \pm 80$ | $1.04963 \pm 405$ | $1.01137 \pm 396$ |
|  | A+ | 1.468 | $1.03777 \pm 84$ | $1.05361 \pm 373$ | $1.01526 \pm 367$ |
|  | HD | 1.459 | $1.03782 \pm 80$ | $1.04362 \pm 464$ | $1.00558 \pm 452$ |
|  | HD+ | 1.466 | $1.03776 \pm 85$ | $1.05110 \pm 436$ | $1.01285 \pm 426$ |
|  | NJ | 1.122 | $1.03785 \pm 78$ | $1.05618 \pm 426$ | $1.01766 \pm 416$ |
|  | NJ+ | 1.129 | $1.03780 \pm 85$ | $1.05982 \pm 399$ | $1.02121 \pm 391$ |
|  | N | 1.127 | $1.03785 \pm 83$ | $1.04245 \pm 715$ | $1.00443 \pm 693$ |
|  | N+ | 1.139 | $1.03780 \pm 86$ | $1.05829 \pm 539$ | $1.01974 \pm 524$ |
| 0.18;8 | A | 1.463 | $1.03758 \pm 85$ | $1.05000 \pm 313$ | $1.01197 \pm 288$ |
|  | A+ | 1.468 | $1.03753 \pm 77$ | $1.05450 \pm 349$ | 1. $01636 \pm 342$ |
|  | HD | 1.460 | $1.03757 \pm 144$ | $1.04186 \pm 453$ | $1.00413 \pm 456$ |
|  | HD+ | 1.467 | $1.03752 \pm 92$ | $1.05179 \pm 397$ | $1.01376 \pm 390$ |
|  | NJ | 1.123 | $1.03761 \pm 71$ | $1.05689 \pm 360$ | $1.01858 \pm 351$ |
|  | $\mathrm{NJ}+$ | 1.129 | $1.03757 \pm 80$ | $1.06077 \pm 354$ | $1.02236 \pm 347$ |
|  | N | 1.127 | $1.03760 \pm 148$ | $1.03655 \pm 836$ | $0.99899 \pm 816$ |
|  | N+ | 1.140 | $1.03757 \pm 43$ | $1.06007 \pm 493$ | $1.02169 \pm 469$ |

(All erxors in $g$ 's are $\times 10^{-5}$ )

With both these data sets at full weight, or with only James data excluded (cases A or NJ) most fits are acceptable and the $\alpha$ (extra) datum of much less importance.

If HAN 58D is downweighted but James retained (case HD) the sets with the fictitious $\alpha$ point are reasonably good (in $\sigma_{f}$ curve shape) but most fits without it are rejects or at best borderline.

In Table 7 the attempt is made, in these difficult circumstances, to give ranges of $g$-factors selected from these fits either (A) applying the rejection criteria based on systematics as described above, or (B) being much more lenient concerning the shapes of the $\sigma_{f} \sqrt{E}$ curves. It is seen that this choice has a considerable effect on the "accuracy" which one could claim for the $g_{f}$ and $g_{\eta}$ factors.

As a result, it is felt desirable to present this situation without a definitive conclusion, except insofar as it is necessary to choose g-factor "best values" and accuracies for the IAEA $2200 \mathrm{~m} / \mathrm{s}$ cross section evaluation. It does not appear likely that a further theoretical examination of the Pu-241 data will clarify the situation, though comments from measurers of data will be sought when this report has been circulated. However, the only way to resolve this question appears to be to obtain better $\sigma_{f}$ or $\alpha$ data, as a function of neutron energy below about 0.02 eV , and it is hoped that this will be found to give values whose interpretation in terms of resonance theory of neutron-nucleus interactions is clear. In the meantime for the $2200 \mathrm{~m} / \mathrm{sec}$ evaluation the "A" values of Table 7 will be used.

## TABLE 7

## Ranges of g -factors for Pu-241

|  | $\mathrm{g}_{\text {ajos }}$ | $\underline{g}_{\text {fiss }}$ | $\mathrm{g}_{\text {eta }}$ |
| :---: | :---: | :---: | :---: |
| Max-Min (A) | 1.03801-1.03697 | 1.05800-1.04494 | 1.01964-1.00703 |
| Max-Min (B) | 1.03643 | 1.05982-1.04401 | $1.02121-1.00617$ |
| Max-Min (C) | 1.03802-1.03643 | 1.00007-1.04072 | 1.02169-1.00284 |
| Range ( $A+B$ mean) | 131 | 1444 | 1388 |
| Range (C) | 159 | 1935 | 1885 |
| Std. Dev'n (A/B) | $\pm .00043$ | $\pm .00481$ | $\pm .00463$ |
| \% | say 0.1 | 0.5 | 0.5 |
| Std. Dev'n (C) | $\pm .00053$ | $\pm .00645$ | $\pm .00628$ |
| \% | * | 0.7 | 0.7 |

Notes: All ranges are in units of $10^{-5}$; $A=$ acceptable, $B=$ borderline (both using $\sigma_{f}$ shapes); $C=$ assuming wider limits (not systematics)

* If $\sigma_{a} \sqrt{E}$ low-energy measurement assumed not to have systematic error, $\pm 0.125 \%$, say, but if most recent measurements could be systematically high, then say $\pm 0.2 \%$ or even more.

From the foregoing it appears that the limits within which the curve shapes and g-factors are expected to lie, based on the existing experimental data, have been adequately explored, except that for $P u-241$ the exploration extends only as far as seems justified in view of the limitations of the data. It seems less likely that more refined resonance theory, applied to these problems, would appreciably improve the situation.

It is, however, more difficult to define a "best" curve for $\sigma_{a} \sqrt{E}, \sigma_{f} \sqrt{E}$, or $\alpha$ as a function of energy. An attempt to estimate the actual values of $g_{a}, g_{f}$ and $g_{\eta}$ has been made, but the skewness of the distribution of some of our $g^{\prime}$ s obtained by fitting has not made this easy--it is, for example, unlikely that the centre of our "acceptable" range of $g^{\prime} s$ will generally be the most probable value. By examining the histograms (cf. App. V(E)) and curve shapes, and values from the "adjust" procedure, the "best" values given in Table 8 were obtained, but it is not claimed that an accuracy of better than perhaps $15-25 \%$ of the standard deviation is meaningful, so that in presenting these values only three decimal places have been generally given, and Pu-241 is only included in a tentative sense. Also given in the tables are the standard deviations of the $g^{\prime}$ s (corresponaing to the percentage errors of Table 5) as well as the (percentage) errors of three "independent" quantities which (with the relation $g_{\eta}=g_{f} / g_{a}$ ) together reproduce the total accuracies (cf. App. V.D.; these quantities are usually needed for least squares treatments of $g$-factors, as in the IAEA study already cited.)

## Estimated "Best" Values of $\mathrm{g}\left(20.4_{4}{ }^{\circ} \mathrm{C}\right)$ and Their Accuracy

|  | $\underline{\mathrm{g}} \mathrm{abs}$ | $\underline{g}_{\text {fiss }}$ | $g_{\text {eta }}$ |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { U-233 } \\ & \quad \text { (Indpt. errors) } \end{aligned}$ | $\begin{gathered} 0.966 \pm .0012 \\ (0.12 \%) \end{gathered}$ | $\underset{(\infty)}{0.966 \pm .0020}$ | $\begin{gathered} 1.000 \pm .0016 \\ (0.16 \%) \end{gathered}$ |
| $\begin{aligned} & \text { U-235 } \\ & \quad \text { (Indpt. errors) } \end{aligned}$ | $\begin{gathered} 0.979 \pm .0009 \\ (0.092 \%) \end{gathered}$ | $\begin{gathered} 0.977 \pm .00151 \\ (0.164 \%) \end{gathered}$ | $\begin{gathered} 0.998_{5} \pm .00174 \\ (0.459 \%) \end{gathered}$ |
| $\begin{aligned} & \text { Pu-239 } \\ & \quad \text { (Indpt. errors) } \end{aligned}$ | $\begin{gathered} 1.076 \pm .00307 \\ (0.285 \%) \end{gathered}$ | $\begin{gathered} 1.052 \pm .00300 \\ (0.285 \%) \end{gathered}$ | $\begin{gathered} 0.978 \pm .00318 \\ (0.325 \%) \end{gathered}$ |
| $\begin{aligned} & \text { Pu-241 (tentative) } \\ & \quad \text { (Choice of errors) } \end{aligned}$ | $\begin{gathered} 1.037_{5} \\ \pm 0.1 \text { or } 0.2 \% \end{gathered}$ | $\begin{gathered} 1.050_{5} \\ \pm 0.5 \text { or } 0.7 \% \end{gathered}$ | $\begin{aligned} & 1.012_{5} \\ & \pm 0.5 \text { or } 0.7 \% \end{aligned}$ |

# EREX E U M <br> A STUDY OF THE ACCURACY OF g-FAC'IORS FOR ROOM-TEMPERATURE MAXWELLIAN SPECIRA FOR U AND PU ISOTOFES 

by
C.H. WESTCOTT

An unfortunate error occured in setting up Table 8 (p.40), where the errors in parentheses for Pu-239 (indpt. errors) are shown incorrectly. These values should be
(indpt. errors)
$( \pm 0.331 \%)$

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> September 1969

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The next problem is two-fold; to define "best" curves (as a function of energy) and to deduce how the g-factors will behave at temperatures above $20.4{ }^{\circ} \mathrm{C}$ (there is seldom interest in practice in lower temperatures). Now if the g's are exactly known for all temperatures we can in theory define (apart from an arbitrary multiplier) the curve shapes, and vice versa, but if the g's are only known for one temperature, or a narrow range of temperatures, several different curves may be equally possible. Similarly, should a regression analysis (e.g., the IAEA study cited) show that $g$ should be raised by, say, $0.3 \%$, there is no unambiguous curve adjustment which can be justified as corresponding to this change. It may be possible in favourable cases where two clearly-differing alternative "recipes" have been identified (in our case $R$ or LO for Pu-239 $\sigma_{a}$ or $R$ or $S H$ for $U-235 \sigma_{f}$ ) to select a linear combination of the two in chosen proportions to duplicate quite closely the required g-factors, and suggest curves (from linear combinations of "adjusted" recipes) which correspond to these g's, but there is still no proof that such curves will be "correct" even if all three g's are reproduced. And in general no adjustment based on one recipe difference can reproduce the three g's exactly, since there are two degress of freedom involved in such a process. Nevertheless, in the actual case this single adjustment may be the best procedure available, and is likely to produce curve-shapes which are not unreasonable in the light of all available information, even if the g's are not exactly correct. We must emphasize, however, that no such recipe can give us any proof of the correctness of the resulting
curve-shapes. A similar problem arises if we find, on performing a regression analysis for all data relevant to $2200 \mathrm{~m} / \mathrm{sec}$ constants (as in the concurrent IAEA study cited above) that the g-factors here proposed should be adjusted by up to 1 or 2 standard deviations to give a "best fit"--there is also here no unambiguous curve-shape adjustment which can be shown to correspond to the new g-factor values, or to be indicated by the totality of the information available.

A further related point is the temperature-variation of the g-factors for $\mathrm{U}-233, \mathrm{U}-235$ or Pu-239. As indicated in Table l, the increments in the g's for an increment of, say, 120 centigrade degrees above $20.4_{4}{ }^{\circ} \mathrm{C}$, varies somewhat according to which "adjust" recipe is used. The "linear combination" suggested above may indicate, for any chosen $g\left(20.4_{4}{ }^{\circ} \mathrm{C}\right)$ 's, how the $g$ 's are likely to change with $T$, but this deduction also is not unambiguous--fortunately in practice these $\Delta g$ changes are relatively similar for the different recipes, and the problem is seldom serious even though in principle ambiguity must arise (for Pu-239, the worst case, other uncertainties in the actual neutron spectrum may help to reduce the importance of this effect). The values which are finally obtained (for temperatures up to $160^{\circ} \mathrm{C}$ ) are given in App. V.G.

Nevertheless, in spite of these reservations of principle, it seems useful to make available to the data centres values of $\sigma_{a}(E), \sigma_{f}(E)$ and $\alpha(E)$ which are consistent with the present work; these will be sent initially to the Brookhaven NNCSC in a format of the ENDF/B type. A transition region near 0.2 eV will be needed in which our functions can be joined to the present evaluated data files, and
indications for this will also be sent to Brookhaven. We therefore conclude with Table 8 (giving recommended g-values) and by arranging to communicate to the data centres typical curve-shapes at low energy, although with the reservations stated, whose importance must not be overlooked. It is also hoped that it will be clear that these are secondary to our main purpose, which was to study what accuracy it would seem reasonable to attribute to g-factors. Moreover for any reliable estimates of $g^{\prime}$ s at higher temperatures than about $130^{\circ} \mathrm{C}$, a further study concentrating more on data for neutron energies above 0.1 eV would be necessary.

## 12. ACKNOWLEDGMENTS

This work has been undertaken concurrently with a review of the $2200 \mathrm{~m} / \mathrm{sec}$ constants on behalf of the IAEA and an indebtedness to those who supplied data or comments thereon for the IAEA study, must also be expressed in connection with this report. Discussions of more particular relevance to g-factor accuracy problems with prof. E. Vogt, of the University of British Columbia, and Mr. G.C. Hanna are also acknowledged.

The contributions of Dr. D. McPherson and the staff of the Chalk River computation centre were much more direct, and some of the basic procedures adopted were chosen, and the early stages of the work programmed by Dr. McPherson; Miss C. Sprague also greatly assisted in arranging computer operations and graphical and tabular results and without their cooperation the present work might not have been possible.

## NOTES ON FIGURES $1-12$

Figs. l-4 and 5(a) siow the data together with "recipes" ("eye-ball" curves drawn through sets of data.)

Fig. 1 (U-233) has absorption (upper curve) and fission (lower) data.

Figs. 2 and 3 show three alternative U-235 (fission) recipes. The data which fit the "SH" curve best are (with tinat curve) drawn 3 inches low.

Fig. 4 also has those data fitting "LO" best drawn 3 inches low.
Figs. 5b and 6-9 are curves of "adjust" procedure--solid curves are the adjusted curves, dotted are the original recipes. The recipes used (in order, "abs, fiss, alpha") are indicated after the title.

Note, e.g., on Fig. 7(a) the "kink" near 0.02 eV due to change of weight in this region--this is acceptably reduced for the R.RSH.R or R.SH.R (cf. Fig. 8(a)) fits.

Figs. l0-l2 are U-233 fitted curves (title still names three recipes, but the solid line is only the one--or two--recipes concerned). The dotted lines are the polynomial fits obtained.

Fig. 10 shows a good (0.15;5) fit, and a "wiggly" one (0.l;8).
Fig. 11 illustrates "even-odd" effects, for fits 0.2; 6 and 0.2;7.
Fig. $12(\mathrm{a})$ and (b) is an "over-rigid" fit (low-order, "limited to cubic") as discussed on p.l9; the others (c)(d) are an "old" case giving unacceptable fits $\left(\sigma_{\gamma}\right.$ becomes negative as $\left.E \rightarrow 0\right)$.

## SUMMARY OF U-233 FITS SHOWN IN GRAPHS

Fits rejected are those of Fig. $10(c, d), 12(a, b)$ and $12(c, d)--$ the last an extremely bad case. Fig. ll(a,b) is only moderately good but is deemed acceptable. The other two, lO(a,b) and ll(c,d), are good fits.

## NOTES ON FIGURES 13-26

U-235 Fits (Figs. 13-18). Full line is recipe (which is shown in title). Dotted line is fitted curve.

Figs. 13-14 are one case in full (higher $E$ range on smaller scale).

Figs. 15-16 and l7a are other cases but still for 0.15;5 fits, and illustrate $M R$ vs $S H$ differences, evc.
Figs. $17(\mathrm{~b}), 18(\mathrm{~d})$ and $18(\mathrm{a}, \mathrm{c})$ are fits which are too "wiggly". The upper end, $18(\mathrm{~b})$, of Fig. $17(\mathrm{~b})$ shows that wiggles are mainly at low $E$ end, but also occur at upper end of E-range. For this case $\sigma_{a}$ is well-behaved except just at upper end of range (like $\sigma_{f}$ there).

Pu-239 Fits (Figs. 19-24)
Figs. 19-20 have one full-size $\sigma_{a}$ and one full-size $\sigma_{f}$ curves. Others are cut down and therefore do not show all data points (top left abs., top left and bottom right, fission).
Figs. 19-21 show $B$ vs $R$ contrast in fits for $0.15 ; 6$.
Fig. 24 shows case $L$ (not exactly same order fit).

Pu-241 Fits (Figs. 25-26)
These are all 0.10;5 fits to illustrate difference between cases. Note the low tendency for $\sigma_{f} \sqrt{E}$ near $E=0$ for cases HD and N .

SUMMARY: The only curves corresponding to rejected cases are Fig. $17(\mathrm{~b})$ and all on Fig. 18 and for Pu-241 the cases of Figs. 26(a,f) and 26(e); all others were deemed acceptable, except the fig. $26(d)$ case which is taken as borderline for wiggliness.
N.B. No data points for HAR 65C $\sigma_{f}$ data appear on Figs. 25-26; they are so numerous that it ${ }^{\mathbf{f}}$ seemed better to omit them, and their "scatter" is very large.











$n$
0
1

















## A.I. 1 <br> APPENDIX I

A. NUMERICAL COEFFICIENTS OF "RECIPES" FOR $\sigma_{a} \sqrt{E}, \sigma_{f} /$ E AND $\alpha$

On the following sheets are listed the coefficients of the power series "recipes" (and Breit-Wigner term to be added) for absorption, fission and (l+a) for U-233, v-235 and Pu-239. The format of the listing is: (2 lines)

| Quantity | Code | $E_{\min }$ | $E_{\max }$ | $E_{r}$ | $W$ | $C_{r}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Card No. | $C_{0}$ | $C_{1}$ | $C_{2}$ | $C_{3}$ | $C_{4}$ | $C_{5}$ |

The code digit is 1 for absorption, 2 for fission, 4 for $1+\alpha . E_{\min } E_{\max }(b o t h$ in $e V)$ are energy range of validity of the recipe given by this pair of cards. $C_{0}, C_{1}, C_{2}$ etc., are coefficients of power series, where the units of energy are $0.1 \mathrm{eV} ; E_{r}$ and $W$ are the energy and $\Gamma^{2} / 4$ of the resonance and $C_{r}$ the numerator term (the units of $E_{r}$ and $\sqrt{\mathrm{W}}$ are also 0.1 eV , for convenience). The expressions to be evaluated are therefore (units for $\sigma \sqrt{ } \mathrm{E}$, barns $\sqrt{e v}$ ).

$$
\begin{aligned}
(\sigma \sqrt{E} \text { or } 1+\alpha)= & C_{0}+10 C_{1} E+100 C_{2} E^{2}+10^{3} C_{3} E^{3}+\ldots \\
& +C_{r}\left[\left(10 E-E_{r}\right)^{2} / W+1\right] \begin{cases}\text { between } E=E_{\min } \\
\text { and } & E=E_{\max }\end{cases}
\end{aligned}
$$

Following the listing (for each isotope) of the recipes recommended, are listed alternatives. Such listings are partial, it being assumed that in the energy ranges for which alternative are not given, the "recommended" recipes listed at first will apply. Comments on alternative recipes follow.

| URANIJM-233 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| absorption heciprs |  |  |  |  |  |  |
| C3ABS | 1 |  | . 04 |  |  |  |
| RST! | 41.94912 | -2.0335 | -1.05 | -. 017 |  |  |
| 23ays | 1.46763 | -.04 | -26.10553 |  |  |  |
|  | 90.46763 | A.3368\% | -26.60.553 | 25.9084 | -9.25925 |  |
| $23 A 8 S$ 9ST3 | $107.7106$ | -33.5375 | $14: 1475$ |  |  |  |
| 23185 | 1 | . 14 | . 185 |  |  |  |
| RST4 | 76.6177 | 24.15 | -20 | 6.6067 |  |  |
| 23AHS RST5 | $-44^{1}, 6693 k$ | $\begin{gathered} 934: 857 \\ \hline 1857 ? \end{gathered}$ | $\begin{gathered} -322 \\ -344: 026 \end{gathered}$ | 50.61 |  |  |
| $\begin{array}{r} 23 A H 5 \\ \text { RST6 } \end{array}$ | $97.2254$ | . 22 | . 4 |  |  |  |
| alternatives inifffrences small) |  |  |  |  |  |  |
| 23485 | 1 |  | . 04 |  |  |  |
|  | 91.61499 | $-2.0335$ | -2.1 | -. 035 |  |  |
| R2 | 48.6521 | 20.104 | -63.21106 | 51.816R | -18.51849 |  |
| $\begin{gathered} 23 A 85 \\ \text { SII } \end{gathered}$ | $92.24326$ | $\begin{gathered} .0 \\ -3.435 \end{gathered}$ | . 1 |  |  |  |
| -ISSION AECIFE |  |  |  |  |  |  |
| 23F1s | 2 | . 0 | . 12 | . 6 | . 02 | . 15 |
| R1 | 83.89307 | -. 145 | -1.125 | -2.15 | 1.25 |  |
| 23 FIS | 2 | . 12 | . 19 |  |  |  |
| R2 | -14.15961 | 293.48291 | -322.49512 | 148. ${ }^{\text {H }}$ | -24.0 |  |
| 23F15 | 2 | . 19 | . 28 |  |  |  |
| $\mathrm{RJ}^{\text {d }}$ | -112.33374 | 2.25.63086 | -82.65894 | 10.7501 |  |  |
| 23 F IS | 2 | . 28 | . 4 |  |  |  |
| R4 | -157.32397 | 246.0443 | -80.11938 | R. 61409 |  |  |
| ALPHa RECIPE |  |  |  |  |  |  |
| C3ALF | 4 | . 0 | .07 |  |  |  |
| CRALF | 4.095 | :005 | :0012 | -.01702 | . 072 |  |
| H2 | 1.16814 | -. 2184 | . 150 |  |  |  |
| 23ALF | 4 | . 09 | . 095 |  |  |  |
| R3 | 1.04395 | . 06 |  |  |  |  |
| 23ALF | 4 | . 095 | . 1 h |  |  |  |
| R4 | 1.01118 | . 1344 | -. 042 |  |  |  |
| 23 ALF | 4 | . 16 | . 24 |  |  |  |
| Hb | 1.00055 | .14769 | -. 04615 |  |  |  |
| 23 ALF | 4 | . 24 | . 4 |  |  |  |
| R6 | 3.2659 | -2.46979 | 1.01693 | -. 18272 | . 012 |  |

## NOTES ON RECIPE TABLES

Alternative recipes are partial; i.e., for ranges where no alternative recipe is given, the first-listed recipe applies. For alternatives listed as "differences small" or "either could fit data", the main recipe was a compromise between $R$ (used earlier) and a second recipe, but the change involved was of little significance.

The $\sigma_{f}(U-235)$ and $\sigma_{a}(p u-239)$ alternatives are discussed at length in the text and appendices, although MLA was a recipe suggested by some data sets carrying relatively low weight, and was less important than the others. The HI $\sigma_{a}(\mathrm{Pu}-239)$ recipe was used for early weight determination ( $v$, App II) but was aropped later when it was found that generally fits lay between $R$ and Lo ( $R$ was itself a compromise between LO and HI). The Einal LOR compromise was $75 \%$ LO, $25 \% \mathrm{R}$, other compromises were 50-50.

Recipes used in exploratory whry but not listed included two $\sigma_{f}(U-233)$ recipes rividally difeerent in shape near 0.06 eV , and early a(U-235) recipes, one actually flat below 0.1 eV , and other independent of energy from 0.09 down to about 0.03 eV , and then "dipping" sharply as the energy axis was approached.

The $L U \sigma_{a}(U-235)$ recipe was only used to test how much the $g$-factors depended on the curve shape above $\sim 3 \mathrm{kT}$.

URANIUM-235
ABSORPTION RECIDES


## HLUTONIUM-239

ABSORPTION RECIPES

| 49 abs | 1 |  | .06 | 2.975 | . 25 | 2850.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LOR1 | $73.4543^{\mu}$ | -13.9625 | -26.4375 | 16.943735 |  |  |
| 49 ABS | 1 | . 16 | . 18 | 2.975 | .25 | 2850.0 |
| LOR2 | 76.8999 | -33.5 | 6.?.5 |  |  |  |
| 49ABS <br> LOR3 | 36.850 ll | . 18 | . 4 | 2.975 | . 25 | 2850.0 |


| 49abs | 1 |  | . 06 | 2.975 | . 25 | 2850.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{H}_{1}$ | 79.1499 | -41 | 12.5 |  |  |  |
| $\begin{gathered} 49 \text { ABS } \\ \text { LOI } \end{gathered}$ | 71.556 | $-.00$ | $-40: 018634$ | $\begin{gathered} 2.975 \\ 22.65741 \end{gathered}$ | . 25 | 2856.0 |

aliernative useu for weights eearlyi

| 49ahs | 1 | - 0 | .0n | 2.915 | . 25 | 2850.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| HIL | 87.96997 | $-71.0$ | $3 \mathrm{P}, 0$ |  |  |  |
| +ISSION HECIPE. |  |  |  |  |  |  |
| $\begin{aligned} & 49 F \text { is } \\ & R 1 \end{aligned}$ | $\begin{gathered} 2 \\ 67.30005 \end{gathered}$ | -21.63 | . 4 | 2.915 | . 25 | 1700.0 |

alpha hecipes

| -9ALF | 4 | . 0 | . 04 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| RRS 1 | 1.3395 | . 08 | . 0975 |  |  |
| 49ALF |  | . 04 | $.0 \%$ |  |  |
| RRS2 | 1.35295 | -.0.044 | .6015 | -. 84 | . 525 |
| 49ALF | 4 | . 07 | .18 |  |  |
| RRS 3 | 1.22524 | . 3568 | -. 056 |  |  |
| 49ALF | 4 | . 18 | . 275 |  |  |
| RRS4 | . 49 | . 64444 | -. 14321 |  |  |
| 49ALF | 4. | .2750 | . 325 |  |  |
| RR55 | 3.15606 | -.93ña | .14321 |  |  |
| 49ALF | 4.6434 | . 325 | . 4 |  |  |


| alternatives (eithfy could fit datas |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 49 ALF | 4 |  | . 07 |  |  |
| R1 | 1.352 |  | . 195 |  |  |
| $\begin{gathered} \text { 49GLF } \\ \text { R51 } \end{gathered}$ | $\begin{aligned} & 4.327 \\ & 1.327 \end{aligned}$ | $\begin{aligned} & .0 \\ & .16 \end{aligned}$ | . 04 |  |  |
| $4.9 A L F$ RS 2 | 4.35384 |  | ¢.007 | -1.6H | 1.05 |

$$
\text { A.I. } 4
$$

B. RENORMALIZATION, DISPLACED ZERO GRAPHS AND $\sigma_{s}$ CORRECTION Several details should be explained briefly; as mentioned in Section 1, the $g$-factors depend on the shapes of the curves, so that renormalization is allowable*; these recipes are arbitrary to this extent but large renormalization changes are to be avoided, since then the "adjust" or "fitting" procedures may involve problems in resolving inconsistencies $\left(\sigma_{f}(I+\alpha) / \sigma_{a}\right.$ should equal unity). For the techniques commonly used for the measurement of $\sigma_{T}$, renormalization is somewhat dubious, but it may be appropriate if errors of sample assay are suspected--it is much less easy to adjust $\sigma_{T}$ data for other types of systematic experimental errors, such as an incorrect estimate of background. For $\sigma_{f}$ and $\eta$ (used to give plots of $\alpha$ ) measurements, renormalization is a more justifiable procedure, and this is also true for most techniques for measuring $\alpha$ directly. In general, however, only a few sets of data required renormalization to fall within the region of consensus, and only by factors slightly different from unity.

In obtaining data for $\sigma_{a}$ an arbitrary (constant) value of $\sigma_{S}$ was subtracted from the $\sigma_{T}$ values in SCISRS. These $\sigma_{S}$ values were initially, 13, 15 and 11 barns for $U-233, U-235$ and Pu-239 respectively, but, since their reassessment was part of the concurrent IAEA ( $2200 \mathrm{~m} / \mathrm{sec}$ ) study, provision was made for the "recipes" to be adjusted accordingly, by adding a term $\pm \mathrm{K} \sqrt{E}$ to the $\sigma_{\mathrm{a}} \sqrt{E}$ recipe.

Also, when only single data values appeared in SCISRS, they were usually ignored as giving no curve shape information, but if a special reason for their use existed, an "absolute"

* Normalization multipliers used appear in Appendix II (Table); here the absence of a number means ro renormalization; "x" with a number, indicates the multiplier used. A number followed by $" \div "$ indicates that $\eta$ (or, say, $\eta / \eta_{\text {thermal) }}$ is stored in SCISRS, so that $1+\alpha$ is obtained as a constant (that given) divided by the SCISRS datum (e.g. $1+\alpha=\nu / \eta$ ).

$$
\text { A.I. } 5
$$

value not otherwise normalized to our curves was down-weighted if necessary to allow for any possible normalization uncertainty in the rest of our data.

As explained at the beginning of Section 3 of the report, the absorption and fission graphs of this report are on an open scale with displaced zero. For U-233 the (constant) displacement indicated on the ordinate scales, and for a curve no displacement is used. The quantities subtracted from $\sigma \sqrt{E}$ before plotting are

U-235 (absorption) 100-200E ( $\mathrm{E}<0.12 \mathrm{eV}$ ), $53.29 / \Delta(\mathrm{E}>0.1 \mathrm{eV})$
U-235 (fission) 85-200E ( $\mathrm{E}<0.12 \mathrm{eV}$ ), 39.97/D (E>0.1 eV) Where $\Delta=$ Breit Wigner denominator $=1+(10 E-2.9)^{2} / 0.4556$ Pu-239 (absorption) 2850/[1 + (10E - 2.975) 2/0.25] Pu-239 (fission) $1700 /\left[1+(10 E-2.975)^{2} / 0.25\right]$

## C. DISCUSSION OF ALTERNATIVE RECIPES AND SPECIAL POINTS

Figures l-5 show, with the relevant data points, the lowenergy portions of the more important alternative recipes. In Figure 2, where the SH recipe and the points associated with it are displaced downwards by 3" in an attempt to separate the two groups, we see most of the data for U-235 fission and the two main ( R and SH ) recipes--note the open scale used. Figure 3, shows the alternative MLA recipe (ANL data shown on this as well as on Figure 2), but these are relatively low-weight data*. In Figure 4 the Pu-239 $\sigma_{a}$ points are in two groups, one fitting best to recipes HI or R--for the rest (also lowered 3" on the graph) curves $R$ or LO fit better: in Figure 5 ( $\alpha$ for Pu-239) it is seen that the data is scarce and different recipes are chosen to see what effects they have, rather than because the data indicate specific alternatives.

[^2]$$
\text { A.I. } 6
$$

## C.I Comments for U-235

Although generally the U-235 data are more accurate than for either U-233 or Pu-239, accuracy decreases above 0.1 eV and also at low energies $\alpha$ data become rather scarce. The two "single" points were added to the low-energy a region for this reason, downweighted as explained in "B" of this Appendix. For a also, the early recipes (flat and with a sharp dip near $E=0$ ), mentioned in text on tables, were drawn (before the "single" points were used) to see whether a dip such as the ANL (1958) data alone might indicate would, if adopted, change $g_{\alpha}$ appreciably; only about 0.08 - $0.1 \%$ change resulted, and since the "dip" trend was not a serious suggestion, only a suggestion put up to test the point, no further use was made of these recipes. For $\sigma_{a}(U-235)$ the low energy data are good, but a test (with recipe LU) was made of the influence of the exact curve above $\sim 3 \mathrm{kT}_{0}$. Below 0.6 eV LU and R coincide, while above 0.1 eV LU is some 0.8 barns$\sqrt{ } \mathrm{leV}$ below R ; the resulting $20^{\circ} \mathrm{C} g_{\mathrm{a}}$-factor was only $0.08 \%$ lower than for $R$, so that no further study of the effect of small recipe changes above $\sim 0.08 \mathrm{eV}$ seemed necessary.

Thus (MLA being a recipe of minor importance) only the SH vs R alternative recipes represent a serious difference, to be considered again in Appendix II and taken up in the body of this report. This is especially important since the Hanford and Columbia (Safford) measurements, which support SH, were careful measurements for which a high accuracy was claimed.

## C. 2 Comments for Pu-239

For Pu-239, $\alpha$ and $\sigma_{f}$ both had some uncertainties at low energies, but the main discrepancy was for $\sigma_{a}$. For $\sigma_{f}$ it was the old Sacl.ay data which were discrepant (by a large amount, enough to change $g_{f}$ by $\sim 2 \%$ ) but enquiries of Saclay led us to reject these data as unreliable. For $\alpha$ the $R$ recipe was the first choice, based

$$
\text { A.I. } 7
$$

on an examination of the log-log curves of BNL-325, but in fact on a linear scale $R$ and $R S$ are equally likely and RRS appears to be the recipe to be recommended. There is no serious discrepancy involved in this option; either recipe fits the data (Figure 5) equally well.

For $\sigma_{a}$ (Pu-239) Figure 4 shows that about half the data favour HI or R, and half LO; the "adjust" procedure indicated some further preference,but not a clear-cut one, for LO. Only if the rejected Saclay $\sigma_{f}$ data had been used as the basis of a recipe would HI have led to an "adjusted" a curve of the expected form. The discrepancy, now seen as one between the recipes $R$ and LO, is significant and is taken up in Appendix II and the body of the report.

## C. 3 Comments for U-233

Accuracies of the data for $U-233$ were only modest, especially for $\sigma_{f}$, and for both this and $\alpha$, low energy data were scanty; however, no diverqent recipes were indicated for U-233. The low-energy a point $C E$ Cocking was downweighted (on the grounds given in $B$ of this Appentli! lue to the $\alpha$ normalization uncertainty (the MTR 1966 data had had to be normalized upwards $0.3 \%$, and all other data downwards by $0.3 \%$, so a $\pm 0.3 \%$ uncertainty was taken), but in fact even wifh thly Hia point lies only about





 source of difficulty for $\mathrm{U}-233$ is the lack ok godd data near zero energy.

## A.II. 1

## APPENDIX II

## A. EXPERIMENTAL DATA USED AND WEIGHTS ASSIGNED

In the tables which follow are listed the data sets used (SCISRS codes and short-form references) with the weights as used for "fitting" to a polynomial function as described in the text. A number (with $x$ before or $\div$ after it) indicates a renormalization, or deduction of $1+\alpha$ from $\eta$, as explained in Appendix I. The weights are in units of (\%) ${ }^{-2}$, being generally derived as explained in the text from the mean square deviations.

One important point will be clear from the tables; weights have been assigned over certain discrete energy ranges. In fact, from a general examination of the data curves initially plotted, it was clear that measurements were more accurate (and probably easier) in some energy ranges than others. For most types of measurement it was obvious that from 0.025 eV to 0.08 or 0.1 eV a better accuracy (and more plentiful data) was obtained, than for lower or higher energies. The decrease in accuracy has often occurred rather suddenly near $0.08-0.12 \mathrm{eV}$, with a gradual worsening (to a degree varying from data set to data set) up to 0.2 eV . Below 0.025 eV , the loss of data accuracy, or of number of points, was even more marked, and the onset of this effect was usually quite sudden and near to 0.025 eV , although for $\sigma_{\mathrm{T}}$ it was generally more gradual, but none the less definite.

It appears then that experimental techniques begin to fail below 0.025 eV ; but the fact that experimenters have tended to concentrate their measurements in the energy range of $k T_{0}$ to a few $x \mathrm{kT}_{0}$, and have sometimes "pushed" their measurements down in energy so as just to include a $2200 \mathrm{~m} / \mathrm{sec}$ point, has probably also been operative. For these reasons, it was certainly convenient to assign "weights" (usually constant but sometimes varying with

$$
\text { A.II. } 2
$$

the form $\mathrm{a}+\mathrm{bE}$ ) separately for the ranges of energy $0-0.025 \mathrm{eV}$, 0.025 to $0.1 \mathrm{eV}, 0.1$ to 0.15 or 0.2 eV and if necessary 0.15 0.2 eV , with in special cases weight changes at other energy points. The format used in listing the weights is straightforward, footnotes being used whenever an energy boundary is other than at the energies just listed; a dash (-) indicates no weight assigned (equivalent to zero weight), often because there is no data in this range. An equals ( $\Rightarrow$ ) sign indicates that the weight shown for the next lower energy range continues into the range concerned. See summary at the end of this appendix.

We should also note that a few data points were rejected as "fliers"; this (see Appendix $V$ below) is indicated by a note "excluding" (i.e., giving zero weight) to certain short energy ranges, or sometimes by changing the lowest (or highest) limit of energy from (or to) which a weight is assigned. Usually these lay more than $\sqrt{10}$ (usually much more) standard deviations from the "consensus" or "eye-ball" curve, so that no precise rejection criterion was required; the cases were obvious ones and represented much less than $1 \%$ of the data used.

There were several alternative sets of weights for each isotope, as explained in the text. Since a preliminary use of the "adjust" procedure (using weights as explained in "B" below) served to improve the "eye-ball" recipes, the development of these weights had an iterative feature; the penultimate set of weights is indicated in the fits by code o (for "old")--these old values are not listed in this appendix--and the codes for more recent sets for the respective isotopes are as follows: U-233: " $\mathrm{O}+$ " old weights plus a pseudo-point (see C. 3 below)
"1" single Cocking point (v Appendix I) excluded
"2" Cocking point included, no pseudo-point
"3" adding a pseudo-point $\alpha=0.095$ at $E=0$, wt $=3.0$

## A.II. 3

U-235 "BC", "BV", "SH" and "MR" (v Appendix II C) Pu-239 "R", "LO" and "B" based on a preference for recipe R, for recipe LO, and for both treated equally (v Appendix I and II C below), respectively.
B. WEIGHTS USED FOR "ADJUST" PROCEDURE

In Table A II, 2 (last column of second page) are given the weights-per-unit-energy (in units of (\%) ${ }^{-2}$ per $\frac{1}{2} k T_{0}$, an arbitrary but convenient choice) for the sum of all available information for $\sigma_{a}, \sigma_{f}$ and $I+\alpha$ respectively of each isotope. only relative weights are needed (for these three quantities) and occasionally all three values have been (equally) normalized upwards as regions where all data deteriorate (e.g., near $\mathrm{E}=0$ ) are approached. These are made from the weights of Appendix II A by multiplying by the number of points of each set per $\frac{l_{2} k T o}{}$ energy range, and summing for all sets. As this is (see above) an iterative process, some of the "adjust" weights may have been deduced from an older set of separate-data weights, but the results are adequate for the purpose.

For the "adjust" procedure, the weight functions should be continuous. Trapezoidal (i.e., with alternately horizontal and sloping straight lines comprising the graph) functions are usually used, and in the tables are given $E_{a}, E_{b}, E_{C}, W_{f}, W_{g}$ where the significance is that from $E_{a}$ to $E_{b}$ the weight is constant and equals $W_{f}$, changing linearly from this value to $W_{g}$ from $E_{b}$ to $E_{C}$; dashes (-) indicate cases where one or other of these sections is omitted.
C. COMMENTS ON THE WEIGHTS AND SPECIAL PROBLEMS C. $1 \mathrm{U}-235$

For U-235 the divergent recipes (Appendix I above) for $\sigma_{f}$ corresponded to trends on which the alternative sets of weights

TABLE A.II. 1



## A.II. 6

are based. The "O" (old) case was in effect a "whichever-isbest" weight based on SH or R (MLA was not then in final form). In the newer cases not only was the $S H$ vs $R$ choice involved, but also whether the authors' claimed accuracy was accepted for the two $\sigma_{f}$ data sets (HAN 1957 and COL 1959) supporting the SH recipe, instead of using the (lower) weights given by the normal procedure based on the spread of the values. The authors' claims resulted in weights varying with energy (a form $a+b E$ over certain energy ranges was an adequate approximation), or a constant weight over this range (giving a similar total weight) could also be used. Both were larger than the (constant) weight which was derived from the spread of the points. The reasons for considering this special treatment for these data sets were that both were measurements carefully made when it was known that absolute values were in doubt, and for which the authors' claims for the accuracy of each datum were available (for coL in a publication for HAN from SCISRS--also private communication); they were also the data primarily causing the $R$ vs SH discrepancy, since all other (except MOL) values clustered around the $R$ recipe as a consensus curve.

The cases (other than " O ") used thus were:
"BC": based on a compromise (mean) of the weights
deduced from a spread from recipes $R$ and $S H$; COL and HAN with constant weights.
"BV": as BC above, but basing weights for COL and HAN $\sigma_{f}$ data sets on authors' claims (varying with energy, see above). "MR": based on spread from $R$ recipe for a compromise between $R$ and MLA if better); constant for COL and HAN. "SH": based on spread from SH recipe, with authors' claimed (variable with energy) weights for COL 1959 and HAN $1957 \sigma_{f}$ 's. Certainly using these sets, one expects the results of fitting using $S H$ to yield a curve nearer the $S H$ recipe, and using

## A.II. 7

MR the $R$ recipe. The old set of weights was not very different From BC, but the low-energy Safford and Cocking $\alpha$ points were excluded; by accident this one omission and the lack of weights for MOL data seemed to tend to cancel one another.
C. 2 Pu-239

As a point of interest, case "O" for Pu-239 did not exclude all the "fliers" excluded by the tabulated weights (it also gave some, but a small, weight to the Saclay $1955 \sigma_{f}$ data); in even earlier sets, with even more "fliers" in, the "fitting" procedure had failed to converge for ranges of fit up to 20.15 eV . This experience led us to look for "fliers" systematically, using the $\sqrt{10}$ standard deviation criterion mentioned above.

The other cases, corresponding to the alternative
recipes for $\sigma_{a}(v$ Appendix I) are:
B: weights using both recipes $R$ and $L O$ and using $a$ compromise which is close to the value for whichever recipe gives the higher weight.

R: a set based on deviations from recipe $R$ only.
L: a similar set based on recipe LO only.
There were no problems concerning $\sigma_{f}$, and the alternative $\alpha$ recipes for $P u-239$ did not justify special treatment (v. App. I).
C. $3 \underline{\mathrm{U}-233}$

The cases 1,2 and 3 differed only trivially as shown in "A" above. The main difficulty with U-233 was connected with the sparsity of data at low energies, and to meet this an additional "pseudo-point" (weight 7.1 units in case "O", a value chosen to equal one point from the best (MTR) $\alpha$ data set, but only 3.0 units in case 3) was added to the data, being $\alpha=0.095$ at $\mathrm{E}=0$.

The use of this point is discussed in the text; it may be that an accident when case " $O$ " was in use was really what led

## A.II. 8

to the use of such a point--though since the fits for U-233 all showed, to a greater degree than the other isotopes, the effects of a lack of good data below 0.02 eV , its use for this isotope was certainly instructive. The "accident" mentioned for case o--reinforced in its effects by the fact that at this date no "flier" exclusion had been made--was the use twice over of the 1957 Harwell $\sigma_{f}$ data, once as now used and also, with source 1955 Geneva Conference, the same data with a different normalization. When the identity of these two sets of values was discovered, so that the accidental duplication of their weight disappeared, the fits certainly improved and an extreme case like the 0.1;8 case "O" fit (see Table 2 in text for notation) where $\sigma_{\gamma}$ became negative for low energies, was not again encountered. Figure 12 includes a curve for this case, and Figure 10 the worst comparable curve obtained with the "new" weights.

## SUMMARY OF FORMAT OF TABLE A.II. 1

lst line: Code, Author or reference Normalization (if any)
2nd line: Code (if any) weights .....

$$
\begin{aligned}
& 0-0.025 \mathrm{eV} \text { 0.025-0.1 ev } 0.1-0.15 \mathrm{eV} 0.15-0.2 \mathrm{eV} \\
& \text { (unless modified by notes) }
\end{aligned}
$$

The entry "-" is equal to zero weight; "=" same as next lower energy range. "Exclusions" are also regions of zero weights, usually placed so as to include "fliers" to be rejected.

## A.III. 1

APPENDIX III
D. McPherson

## Least Squares Fitting Program

An option in the SCISRS retrieval program provided for least squares fitting of the quantities $\sigma_{a} \sqrt{E}, \sigma_{f} \sqrt{E}$ and $1+\alpha$ in the range $0<E \leq E{ }_{c}$ and the evaluation of the $g$-factors from the fitted quantities. The functional forms adopted were

$$
\begin{align*}
& \sigma_{a} \sqrt{E}=\left\{\sum_{i=1}^{N_{a}} c_{i} E^{i-1}\right\}_{E \leq E_{c}}+Q_{a}  \tag{la}\\
& \sigma_{f} \sqrt{E}=\left\{\sum_{i=N_{a}+1}^{N_{a}+N_{f}} c_{i} E^{i-N_{a}-1}\right\}_{E \leq E_{c}}+Q_{f}  \tag{1b}\\
& 1+\alpha=\sigma_{a} / \sigma_{f} \tag{lc}
\end{align*}
$$

where $C_{i}$ are the parameters to be determined by the fit
$N_{a}$ is the number of parameters used to describe the absorption curve
$N_{f}$ is the number of parameters used to describe fission curve

Qa is a Breit-Wigner resonance term plus, for $E>E_{C}$, a polynomial to describe absorption
$Q_{f}$ analogous to $Q_{a}$. but for fission.
The polynomials in $Q_{a}$ and $Q_{f}$ are used only for $E>E_{C}$, and are the appropriate "eye-balled" recipes described in Appendix $I$; the polynomial contribution to the $Q$ 's are
A.III. 2
not, of course, involved in the fit, but make a small contribution to the $g$-factors.

The quantities $N_{a}, N_{f}, E_{c}$ were specified for each fit, the weights to be used for the data appeared as part of the data set specification (see Table A.II.l).

Trial values of the $C_{i}$ were determined by fitting independently the linear systems la and 13 . The fit to the complete system of Eq. (l) was then obtained by iterating until the maximum change in any $C_{i}$ was less than 1 part in $10^{6}$. The output of the fitting program consisted of the set of $C_{i}{ }^{\prime} s$ and their variance-covariance matrix $V$.

The effective cross section in a Maxwellian spectrum of temperature $T=E_{o} / \mathrm{K}\left(=20.44^{\circ} \mathrm{C}\right)$ is defined by

$$
\hat{\sigma}=\frac{2}{\sqrt{\pi}} \frac{1}{E_{0}^{2}} \int_{0}^{\infty} E e^{-E / E_{O}} \sigma(E) d E
$$

In terms of the functions of Eq. (1),

$$
\begin{align*}
\hat{\sigma}_{a} & =\sum_{i=1}^{N_{a}} c_{i} I_{i}+R_{a} \\
\hat{\sigma}_{f} & =\sum_{i=N_{a}+1} c_{i} I_{i-N_{a}}+R_{f} \tag{2a}
\end{align*}
$$

where

$$
I_{i}=\frac{4}{\sqrt{\pi}} \frac{1}{E_{O}^{2}} \int_{0}^{\sqrt{E}} C E^{i} e^{-E / E_{O}} d(\sqrt{E})
$$

$$
\begin{aligned}
& \text { A.III. } 3 \\
&=\left(i-\frac{1}{2}\right) E_{O} I_{i-1}-\frac{2 e^{-E_{C} / E_{O}}}{E_{0} \sqrt{\pi}} E_{C}^{i-\frac{1}{2}}, \\
& I_{0}=\frac{2}{E_{0}^{3 / 2}} \operatorname{Erf}\left(\sqrt{E_{C} / E_{O}}\right)
\end{aligned}
$$

and

$$
R_{a}=\frac{2}{\sqrt{\pi}} \frac{1}{E_{0}^{2}} \int_{0}^{\infty} Q_{a}(E) \sqrt{E} e^{-E / E_{0}} d E
$$

with a similar expression for $R_{f}$.
The g-factors are defined by

$$
g_{a}=\hat{\sigma}_{a} / \sigma_{a}\left(E_{O}\right), \quad g_{f}=\hat{\sigma}_{f} / \sigma_{f}\left(E_{O}\right) \quad \text { and } \quad g_{\eta}=g_{f} / g_{a}
$$

The covariances of the effective cross sections and the g-factors are given in terms of the variance-covariance matrix (of the $c_{i}{ }^{\prime} s$ ) $v$ by:

$$
\begin{aligned}
\operatorname{var}\left(\hat{\sigma}_{a}\right)= & \sum_{i, j=1}^{N_{a}} I_{i} I_{j} V_{i j} \\
\operatorname{var}\left(\hat{\sigma}_{f}\right)= & \sum_{i, j=N_{a}+1}^{N_{a}+N_{f}} I_{i-N_{a}} I_{j-N_{a}} V_{i j}
\end{aligned}
$$

## A.III. 4

$$
\begin{aligned}
& \operatorname{var}\left(g_{a}\right)=\sum_{i, j=1}^{N_{a}} \frac{\partial g_{a}}{\partial C_{i}} \frac{\partial g_{a}}{\partial C_{j}} v_{i j} \\
& \operatorname{var}\left(g_{f}\right)=\sum_{i, j=N_{a}+1}^{N_{a}+N_{f}} \frac{\partial g_{f}}{\partial C_{i}} \frac{\partial g_{f}}{\partial C_{j}} v_{i j} \\
& \operatorname{var}\left(g_{\eta}\right)=\sum_{i, j=1}^{N_{a}+N_{f}} \frac{\partial g_{\eta}}{\partial C_{i}} \frac{\partial G_{\eta}}{\partial C_{j}} v_{i j}
\end{aligned}
$$

where the $\partial g / \partial c_{i}$ are obtained from the definition of the g-factors and Eq. (2).

As used in this study, $\mathrm{N}_{\mathrm{a}}=\mathrm{N}_{\mathrm{f}}$ except where otherwise stated. The computer output includes, in addition to values previously indicated, the values and standard deviations of $\hat{\sigma}, \sigma\left(E_{O}\right)$ and $g\left(20.44^{\circ} \mathrm{C}\right)$ for both absorption and fission, and $g_{\eta}$ and its standard deviation; it also includes $\chi^{2}$, defined by

$$
x^{2}=\frac{10^{4}}{M-\left(N_{a}^{+N_{f}}\right)} \sum_{j=1}^{M} \frac{w_{j}}{x_{j}^{2}}\left(x_{j}-x_{e}\right)^{2}
$$

Where $M$ is the number of data points, $x$ is the $j$-th datum $(\sigma \sqrt{E}$ or $1+a), x_{e}$ is the value calculated for the energy $E_{j}$ of the $j-t h$ datum, and $W_{j}$ is the weight (in our units, so that $10^{-4} x_{j}^{2} / W_{j}$ is the variance) of the j-th datum.

## A.IV. 1

## APPENDIX IV

Data in SCISRS Not Used

The following (SCISRS codes given) data were not used for reasons listed below, although more than one datum for them were in SCISRS.

## U-233

| TOT | HAR | 56 | Pattenden* | NFE | HAR | 59 G | R-2998 | $* * *$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| NFE | HAR | 59 G | R-2998 | $* * *$ | NF | HAR | 65 F | Brooks | $*$ |
| all | HAR | 66 H | Brooks | $*$ | TOT | CCP | 55 H | Geneva | $*$ |
| NF | CR | $51 A$ | CRGP-458 $* * *$ | TOT | HAR | 55 | Brooks | $*$ |  |
|  |  |  |  |  | TOT | HAR | 55 | Lynn | $*$ |
|  |  |  |  |  | NF | SAC | 55 | Auclair $*$ |  |

Pu-239

U-235

Pu-241


Others listed in Table A.II.l as "no weights" were initially given weights, which turned out to be low, and discarded after enquiries from authors or representatives of laboratories involved. Such enquiries were usually also made for data marked $*$ or $* *$ in above table. Note also that deduced values (e.g. $\sigma_{f}$ from $\alpha$ ) were not generally included, nor data sets which started near or above 0.2 eV .

## Code of Remarks

* Unacceptably large spread of data values.
* Unreliable shape--evidence of systematic errors--no satisfactory explanation available. (for NF HAN 58D cf App. V.F, first para.)
** Cld values--enquiries difficult or replies not encouraging.
** Spread of values not good, very small numbers of data, not worth inclusion in view of other data available.


## APPENDIX V

A. RELATION OF SPREAD TO STANDARD DEVIATION

Fundamentally the only reason for stating our results as standard deviations is because they are to be used in a least squares regression analysis--our real conclusions are the range within which it seems that a given g-factor is likely to vary. Curves which were obtained in our fits and which would correspond to $g^{\prime} s$ outside this range ( $g_{\text {min }}$ to $g_{\text {max }}$, say), have been rejected as physically implausible. We therefore choose as the standard deviation that for which the assumption of a normal distribution would lead to about a unit expectation for the number of fits giving $g>g_{\max }$, and also for $g<g_{m i n}$, from among the $N$ (say) acceptable fits obtained. Most of this expectation (for the normal distribution) would represent cases giving $g$ only slightly outside our range, with a similar liklihood of $g^{\prime} s$ just within our range occurring. The more normal approach of deducing a standard deviation from a sum of squares of deviations is more sensitive to the incorrect rejection or acceptance of extreme values, so in spite of its logjcality we have chosen not to use it for this work; we have really been searching as much for the limits of unacceptability as to produce a representative sample of fits of all types with the "correct" frequency of occurrence. However, for $\sigma_{f}(P u-239)$ we have also calculated the standard deviation of $g^{\prime} s$ from our sample in a conventional manner; the values obtained was $\pm 266 \times 10^{-5}$ in excellent agreement with the value (Table 5) obtained from the spread using a x 1.l factor of safety. Note: this was a case for which no rejection on account of curve shape was necessary.

We consider first a presumed Gaussian ("normal" distribution for $N=40$, say; beyond the $95 \%$ confidence limits at each end of the range (at $\pm 2 \sigma$, where $\sigma=$ standard deviation) ~ $\sim$ datum is expected; then $\sigma=$ one-quarter of the spread. A second distribution may have two components, one "normal" and the other a distribution where two discrete values (differing by $x$, say) are presumed equally probable, folded together. If $x=2 \sigma$, say, we can form the combined distribution of the average of two Gaussians centered on $\pm x$ from the centre of the whole--the $95 \%$ confidence limits are then at $\pm 2.644$ $\sigma(f o r \mathrm{x}=\sigma$ it would be $\pm 2.18 \sigma$ ) whereas the standard deviation is in fact $\sqrt{2 \sigma}$ (or $\sqrt{1.25 \sigma}$ for $x=\sigma$ ), so that the full width of the spread is 3.74 (or 3.90) times the standard deviation, instead of 4 times. This alternative corresponds for example, to the $\sigma_{T}(P u-239)$ LOR-R alternative folded with a Gaussian (cf. second paragraph of section 9 of text).

It seems in our cases that at most half of the total spread is from a two-discrete-value distribution, but the rest of the distribution may be a somewhat truncated normal distribution, so that we should probably reduce the factor (4 or 3.74 as above) by which we divide the "spread" to estimate the standard deviation, to a value of 3.33 ; this adjustment covers the truncation possibility and other contingencies.

The "factors of safety" mentioned in section 8 (and listed in Table 5) also contain some adjustments but are used with the 3.33 factor just quoted. For U-233, where the twovalue distribution effect is probably absent, 3.50 might have

$$
\text { A.V. } 3
$$

been more appropriate than 3.33, so the factors of safety have been adjusted downwards (from the original estimate of $\sqrt{3}$, or more for $g_{a b s}$ ) to the values shown.
B. REJECTION FOR CURVE SHAPE REASONS: SUMMARY

The general question of rejection has been dealt with in section 8; apart from unacceptable anomalous behaviour near $E=0$, curves may be rejected for general "wiggliness" if too high an order or fit gives too sinuous a curve. An extreme case is that of case "O" U-233 for the 0.1; 8 fit (fig.l2c), when $\sigma_{\gamma}$ becomes negative. Less serious cases (rejected or categorized as "borderline") may have faults as follows
$R$ rises suddenly (too much to be acceptable) near $E=0$
$F$ falls suddenly (too much to be acceptable) near $E=0$
w generally too wiggly

In what follows, $R B, F B$ and $W B$ denote borderline effects of a similar type; $a, f$ or $\alpha$ preceding these symbols denote which quaritities misbehave.
B. 1 For U-233

The cases "O" and "O+" are taken separately--all but the low-order fits show large or at least appreciable wiggliness; the ranges in Table 5 are taken for the remaining ("new") cases but 0.2;4 is rejected as the "too rigid fit"* and 0.1;7 and 0.1;8 fits are also rejected for wiggliness. The case "l" (single HAR 58 point omitted) also becomes borderline for many of the fits. This isotope is discussed again below. * In the sense described in section 6, p. 19-20 ("low order fits")

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\text { A.V. } 4
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## B. 2 For U-235

No fits for 0.1;7 or 0.1;8 and only case "O" for 0.15;8 was done, and this reduced the number of rejections for curve shape reasons. For the older weights (case "0") 0.1;6, 0.15;7 and 0.2;8 all gave fF, $\alpha$ R rejections with (especially for 0.1;6) also a general wiggliness and 0.15;8 case "0" was rejected even more definitely, while 0.1;5. 0.15;6 and 0.2;7 were all treated as borderline cases. By contrast for case "MR" all fits done were acceptable, with 0.l;6 being still acceptable but the nearest of the "MR" cases to the borderline (fFB, $\alpha R B$ ). The other cases were intermediate (see below). Since (apart from wiggliness for lowrange high-order fits) the most frequent fault was fF, $\alpha R$, it is perhaps not surprising that case $S H$, which gives the lowest fission curve for $E \approx 0$, should be more liable, and case MR less liable, to give curves rejected for these reasons. Physically, a quite sudden rise of $\alpha$ near $E=0$ (in the last $\approx 8 \mathrm{meV}$ ) seems unlikely, since for this isotope $\Gamma_{\gamma} \approx 40 \mathrm{meV}$, for even a small capture-only resonance would probably not give as sharp a rise as our fits indicate--this would still leave the fall in $\sigma_{f} \sqrt{E}$ to be explained. It is almost certain that accidental features in the few data for $\alpha$ and $\sigma_{E}$ at low energies cause the $f F, \alpha R$ tendency, and we feel that beyond a certain point this should be taken as a ground for rejection of the fit as physically unreasonable.

For case "SH", to complete the story in detail, 0.1;6, 0.15;7 and 0.2;8 are rejected, though on the basis of the fission curve shape alone the $0.2 ; 8$ case might be accepted but the $\alpha R$ fault is definitely bad--indeed 0.1;6 and 0.2;8
A.V. 5
are rejected also for $B V$ and the former for $B C$ (the latter was not done for this case); for 0.15;7 case BV and BC both give wiggly but acceptable fission curves, but BV gives an $\alpha R$ characteristic and is treated as rejected, while BC is deemed borderline since its $\alpha$ rise is less marked. The only other cases near the borderline are (for $\alpha \mathrm{RB}$ reasons) 0.2;7 "SH" and "BV" 0.15;6 "SH" and 0.1;5 "BV", but these are all treated as acceptable cases.
B. 3 For Pu-239

For $\mathrm{E}_{\max }=0.1 \mathrm{eV}$ only 4- and 5-term fits were done, but a set for $\mathrm{E}_{\text {max }}=0.12 \mathrm{eV}$ (up to 6 terms, or 7 for case "O") was included. The 0.12;7 "O" fit was rejected (reason code: $f F$, aWR) but all others were either borderline or acceptable. For case "0" the borderline fits were 0.12; 5 and 0.1;4 (reason codes: $\alpha F B$ ) but these were both close to acceptable and the "borderline" designation reflects as much a lack of confidence in the "old" weight set as any really dubious curve shapes. For the 0.2;8 fits, case "R" showed a borderline trend (fFB, $\alpha R B$ ) while for 0.15;7 fits a similar trend for case "R" was rather more marked, while cases "B" and "L" also showed this trend, in the latter case the fission curves being rather more, and the $\alpha$ curves less affected than for case "R". For 0.12;6 cases "R" and "L" showed a similar trend ( $\alpha$ curves being satisfactory for "L" and fission for "R") but the cases were deemed acceptable.

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## B. 4 General

In some cases odd/even effects were seen, where the behaviour at low energies depended on whether the order of fit was odd or even (perhaps due to data peculiarities at the high end of the range) but generally a good variation of curves was obtained. The number of acceptable fits for U-235 was about 45 (plus borderline), and for Pu-239 slightly more, so that statistically the sample seems adequate. For U-233 the new fits, for cases 2 and 3 only (case 1 being near borderline rather often) we have only 28 good fits, or say 38 with most case 1 fits included; these are rather uniform fits (within the spreads shown in Table 5) but as already explained are based on a narrow choice of weights. The rather large factor of safety (/3, corrected as explained in $A$ above to 1.64 ) used brings the overall claimed errors up to the point where the case "0" curve shapes correspond to g-values within the errors proposed in cases where the shapes are not unreasonable.

We also tend to use larger "factors of safety" when the computed error is small--this occurs for absorption of U-235 and U-233, being particularly marked for the former as seen in Table 5. In any case it is felt that a claim of much better accuracy than $0.1 \%$ for a $g$-factor cannot be sustained, and for U-233, where the causes of the larger spreads obtained using "old weights" is not fully understood, the larger factor of safety listed for absorption is felt to be appropriate. There was one other consideration for

## A.V. 7

U-235, which led to the factors of safety in Table 5 being increased a little over what would have been used otherwise; this was that in the course of preliminary work some cases had been fitted with somewhat different weights and a few of the curves which resulted had shapes which would be deemed acceptable, but gave g-factors outside the limits for fits in Table 5. These were for cases discarded or superseded, but it was nevertheless felt desirable to make a token upward adjustment to safety factors on this account.

A few other points re Table 5 can also be clarified here. Whexe two borderline cases are shown bracketed (with an asterisk*) the mean of the $t \cdot$, is used as a borderline figure, to be averaged with the "acceptable" limit shown in the line above. This has been done where the outer limit for the "borčerline" case is felt to be only marginally acceptable, even as borderline, and the next highest (or lowest) value for another borderline case in given also. Note that the limits from "adjust" are obtained (from those of Table 1) by adding for $\mathrm{Pu}^{239}$ at each end of the range a quantity $\frac{1}{2}|\Delta(R-R S)|$ to allow for the possible extra spread in adjust due to the $\alpha$ recipe alternatives.

Figures 10-24 have been chosen so as to include borderline and unacceptable cases as well as a set of acceptable fits for each isotope--these can be identified from the foregoing by the fitting "code" given for each graph; see also remarks on pages 45 and 46 .

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## C. TEMPERATURE EFFECTS ON g-FACTORS

Especially for differences in recipes like R-LO
icf. Fig. 4) there is the possibility of two effects cancelling when a room-temperature g-factor is computed. Thus the integrai of $\sigma \cdot \sqrt{E}$ times the Maxwellian function will be lower for Lo than for $R$, but so will $\sigma_{2200}$. Thus it may be that $g$ will be insensitive to such a recipe difference "by accident"; however at higher temperatures the cancellation may diminish. It is therefore dangerous to take the "spread" or variation of $g^{\prime} s$ at one temperature only. For U-233 any such effect can only be conjectured, since no clear alternative recipes were seen; for $\Delta(R S-R)$ for $P u-239$ all $\Delta g^{\prime} s$ diminish (arithmetically) with rising temperature, so the effect would be unimportant. But for $\Delta(R-I O)$ Pu-239 or $\Delta(R-S H)$ U-235 differences it is found that the corresponding $\Delta g^{\prime}$ s are only arithmetically decreasing (for positive $\Delta T$ ) for one of the three $g^{\prime}$ s for either isotope; for one other $g$ in each case $\Delta g$ increases without change of sign and for the third, 9 passes through a sign change and then increases with increasing T. For such Cases it is felt desirable to add the additional* $|\Delta g|$ to the existing spread of $g^{\prime} s$ calculated at $20^{\circ} \mathrm{C}$ to give the range used in Table 5 to calculate the total inaccuracy of $g$. For this purpose we use the changes in $g$ possible for temperatures up to $140^{\circ} \mathrm{C}$--this somewhat arbitrary figure is chosen to include the range of neutron temperatures which actually occur

* We take the excess over the $20^{\circ} \mathrm{C} \Delta \mathrm{g}$ when the sign does not change, and the $140^{\circ} \mathrm{C}|\Delta g|$ plus half of the $20^{\circ} \mathrm{C}|\Delta g|$ when it does.

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in measurements of the type used in the $2200 \mathrm{~m} / \mathrm{sec}$ study already mentioned.

## D. TOTAL ACCURACY AND INDEPENDENT ERRORS

For the least squares analysis of $2200 \mathrm{~m} / \mathrm{sec}$ constants the $g$-factor accuracies are required as independent errors, say $g_{a} \pm h_{a}, g_{f} \pm h_{f}$ and $g_{\eta} \pm h_{\eta}$, the total accuracy of $g_{f}$, say, being the direct accuracy taken together with an accuracy deduced from $g_{f}=g_{\eta} g_{a}$, and similarly for the other quantities. Writing for the $\left(\%^{2}\right)$ variances of the $g^{\prime} s, v_{a}, v_{f}$ and $v_{\eta}$, and the corresponding independent quantities $\mu_{a}, \mu_{f}$ and $\mu_{\eta}$ (where for example, $\mu_{f}=10^{4} h_{f}^{2} / g_{f}^{a}$, so that the total weights are $1 / \mathrm{V}$, $1 / v_{f}, 1 / v_{\eta}$ (and similarly for the independent weights), we see that the equations defining the total weight in terms of the independent accuracies are $1 / v_{f}=1 / \mu_{f}+\left(\mu_{a}+\mu_{\eta}\right)^{-1}$ and similarly. G.C. Hanna has shown that the solutions are of the form, writing $v=v_{a}+v_{f}+v_{\eta}$ and $t_{a}=v-2 v_{a}, t_{f}=v-2 v_{f}$, $t_{\eta}=v-2 v_{\eta}$,

$$
\begin{aligned}
& \mu_{a}=\frac{t_{f} t_{n}}{2}\left(\frac{1}{t_{a}}+\frac{1}{t_{f}}+\frac{1}{t_{\eta}}\right) \\
& \mu_{f}=\frac{t_{\eta} t_{a}}{2}\left(\frac{1}{t_{a}}+\frac{1}{t_{f}}+\frac{1}{t_{\eta}}\right) \\
& \mu_{\eta}=\frac{t_{a} t_{f}}{2}\left(\frac{1}{t_{a}}+\frac{1}{t_{f}}+\frac{1}{t_{\eta}}\right)
\end{aligned}
$$

Hence the $h^{\prime} s$ can be found using $\mu_{i}=10^{4} h_{i}{ }^{2} / g_{i}{ }^{2}$ (see above).

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## E. ADDITIONAL COMMENTS

It should be recorded that, as an additional aid in studying curve-shapes in "B" above, histograms were constructed giving the distribution of slopes of curves (expressed by $C_{1} / C_{o,}$ v. App. I) near the origin. These were found very helpful-oespecially in correlating the more extreme values ẇth particular cases or orders of fit--but it seems unnecessary to discuss this in detail. They also provided warning of any skewness of the distributions (giving a mid-range value,
 values) which were used in considering the "best value" problem for $g--$ see section 11 of this report.
F. SPECIAL QUESTIONS ARISING FOR Pu-241

As mentioned in the text above, the fission data for $P u-241$ are of low accuracy, being especially poor below 0.015 eV , while for $\alpha$ (or $\eta$ ) the only two points which exist (J.R. Smith and S.D. Reeder, NBS (Spec. Pub.) 299, p.590) were absolute m measurements, not intended to give information on the energy-variations. These have been downweighted (from $0.9 \%$ to $1.15 \%$ error in $\eta$, or ( $1+\alpha$ )) following our usual rules for using absolute values, due to having to regard all other curves as renormalizable in fitting curve shapes. The use of authors' claimed accuracies for the Seppi, Friesen and Leonard (HW-55879, p. 3) $\sigma_{f}$ data, without allowance for the $2 \%$ error of the $514-$ barn normalization (at $E=0.1 \mathrm{eV}$ ) point, also represents an over-weighting,
discussed further below, since it could be claimed that the total weight of all these points (correlated by the normalization) should correspond to not less than a $2 \%$ error, which would require a rather large downweighting factor. The Raffle data were also taken with the author's claimed accuracy, and the James (Nucl. Phys. 65, 353, 1965) data (given in SCISRS as almost 1000 points of individual accuracy varying from 8 to over $40 \%$ ) had also to be specially treated, to fit the computing code used.

As stated in the text, the main difficulty is that the a curve derived from separate $\sigma_{a}$ ard $\sigma_{f}$ fits rises steeply as E approaches zero, while the Smith and Reeder points and nuclear systematics, indicate a flat trend in $\alpha$ (since this is found in fact above about 0.035 eV , systematics suggest not too great a change in $\alpha$-slope should occur below ~0.02 eV).

In order to study the "spread" of g-factors, several different alternative bases for fitting were used. For a preliminary study the James data were excluded (see below, remarks re biased averaging), especially since these numerous data slowed down the computation process considerably--some fits were also made excluding also the seppi et al (HAN 58D) points, as representing an extreme form of "downweighting". In these exploratory runs the tendency of the $\sigma_{f} \sqrt{E}$ fitted curve to fall off sharply near $E=0$ was almost universal, and in order to combat this tendency (to see how much the g's would be changed), an arbitrary extra datum (for $\alpha$ at $E=0.005 \mathrm{eV}$ ) was added as a "fictitious" point to the input. Initially this extra point was given an error 5 times, and later 3.5 times, larger than the error of either Smith and

Reeder a point, since as low a weight as would suffice was felt to be desirable for this fictitious datum, in a desire to perturb the fits as little as possible (the weights were 0.03 and 0.06122 units) ; the value of $\alpha$ used for this point was $\alpha=0.380$, (a rough extrapolation to 0.005 eV from the two other $\alpha$ points). Although the use of a nearly-constant a curve would tend, with the actual $\sigma_{a}$ curve, to produce a definitely-rising $\sigma_{f} \sqrt{E}$ curve, the accuracies $( \pm 0.0795$ or $\pm 0.0557$ in $\alpha$, corresponding to about $\pm 135$ or $\pm 95$ barns in $\sigma_{f}$ at 0.005 eV ) were so low that the upward-pull on the fission curve was relatively weak. It did, however, act as a counterbalance to the existing tendency of $\sigma_{f^{2}} \sqrt{E}$ to fall. Since this tendency seems to be due not to adequate data but to fluctuations in what (relatively inaccurate) data there are, this idea of a "fictitious" point may not be unreasonable.

The problem which arose for the James data (cited above as "biased averaging"), and which was largely overcome by modifying the least squares code to use the percentage error specified in SCISRS for each point, was that because initially Eixed percentage accuracies (or \%-accuracies dependent only on $E$ ) were used, any points actually fluctuating downwards received a too high weight, while upward fluctuations gave a downweighting. If was near $E=0$ that, with $\sim 40 \%$ standard deviations for single data points, this became serious, and the "average" $\sigma \sqrt{E}$ deduced for the James data dropped off, exactly where other data tended to cause the curves to "dip" in an unacceptable (for systematics) way.

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Even with the modified computing code taking the accuracy of each point from SCISRS this effect seem not to have disappeared entirely.

In the later, definitive, curve-fitting studies for Pu-241, four options were used, each without and usually also with the extra 0.005 eV fictitious $\alpha$ point discussed above. The four options are (a " + " is added to the code if the extra $\alpha$ point is used).

A all data included at indicated weights (v. Table A.V.l)
HD HAN '58D data downweighted for uncertain normalization, James data at indicated weight
NJ James data excluded, HAN '58D with indicated weight
N Both James and HAN 58D data excluded, others with indicated weights.

In fact, when the James data were included (with errors from SCISRS) the result was a definite increase in $X^{2}$, indicating that the weight given James data was too large lby about 1.8 times). Without further computer code changes this was not correctable, so, remembering also that some biased averaging effect appeared to remain, the "with James" and "no James" fits were felt to be equally ike: ." =epresentations which should "bracket" the most acceptable choice. The HAN 58D downweighting for uncertain normalization (by a factor of 3.2) was also felt to be problematical, so that "A" and "HD" can also be considered as fits to be taken on an equal footing. Case N is included to show how much shapes can change if both these "special" sets of data are rejected, but in fact $\mathbb{N}+$ seems more reasonable than $N$ (in the absence of the HAN 58D data the $\sigma_{f} / E \operatorname{dip}$ near $E=0$ is accentuated).

## A.V. 14

TABLE A.V. 1
Pu-241 DATA : WEIGHTS IN (\%)
(In brackets are energy limits in eV, between are given weights in (\%) ${ }^{-2}$. The code ( $T$ ) indicates 0.025 eV , L indicates a linear variation of the weight with energy in the range indicated, $E$ "exclusion" (i.e., wt $=0$ ), ";" indicates discontinuity in weight, when linear variation in use, at end of range).

## Total Cross Sections

MTR 61I (0) 0.25(T,L, 0.04)1.0(0.1)0.5(0.159,E,0.160)0.5(0.2).
CR 64C (0.02)0.5(0.1)0.025(0.18) - craig et al
MTR 68L (0)0.5(0.1) - Young et al, Priv. Comm.

Fission Cross Sections
MTR 64F (0)0.0186(T)0.113(0.1)0.284(0.2) IDO-16995
CCP 55H (0.05)0.06(0.105)0.16(0.15)0.06(0.2)
CCP 55 (0) $0.015(T) 0.037(0.1)$
HAN 58C (0.02)0.165(0.0325)2.0(0.1)0.4(0.15)0.2(0.2) HW-63492
HAN 58D *(indicated) $0.004(0.0025, \tau, 0.0035) 0.094(0.0035, L, 0.00475) 0.315$
*(downw'td) 0.0013 (same) 0.030 (same) 0.1
HAR 59G $0.005(0.006, L, 0.0075) 0.015(0.0075, L, 0.09) 0.02 ;(0.09) 0.03(0.105)$
HAR 65C (James) - either \% errors as given in SCISRS or no weight*

* options - see mid-page A.V.13

Alpha Data
MTR 68 wt. 0.75 (fictitious point, if used, wt $=0.06122$ ).
N.B. For fuller references CINDA should be consulted (Above codes are from SCISRS)

## A.V. 15

No full list of rejections or rejection criteria are given; simply, the situation is that if an $\alpha$-curve shape (based on systematics) were used as a basis, essentially all fits would be rejects, since all give an abrupt rise near $E=0$. On the basis of systematics, indicating that $\sigma_{f} \sqrt{E}$ should not fall too sharply at low energies, a rejection criterion can be chosen (excluding about half of the fitted curves) which seems not unreasonable. The ranges shown in Table 7 for Pu-241 marked "A" are made on this basis--for "B" the rejection of $\sigma_{f} \sqrt{E}$ dips is restricted to one or two of the worst cases. Thus we see that whether or not we feel able to apply some limit based on the idea that a narrow low-energy captureonly resonance in Pu-24l is possible, but a sharp dip in $\sigma_{f} \sqrt{E}$ is not, may affect the accuracy attributable to the Pu-241 g's by a factor of almost l.5. The only real answer to this situation is more data--for either $\sigma_{f}$ or $\alpha$ as a function of energy below 0.02 eV--and preferably also more accurate data in this energy region.
G. TABLE OF g-VALUES FROM "ADJUST" FOR VARIOUS TEMPERATURES

To conclude this appendix we list the g-factors obtained from an "adjust" operation for temperatures up to $\sim 140^{\circ} \mathrm{C}$, for U-233, U-235 and Pu-239, choosing the recipes used to give $20^{\circ} \mathrm{C}$ g's close to those of Table 8 (actually a small constant increment has been added to all tabulated values to bring the $20.4_{4}{ }^{\circ} \mathrm{C}$ values exactly to agree with those given in that table).

Earlier it was mentioned that provisional values for
$\sigma_{S}$ (used in $\sigma_{a}=\sigma_{T}-\sigma_{S}$ ) were used, and these seemed adequate for all the earlier work on the "spread" (i.e., the accuracy) of $g$-factors.

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## TABLE A.V. 2

## Low-Temperature g-Factors for U-233 (Based on RST.R.R)

| $\mathrm{T}\left({ }^{\circ} \mathrm{C}\right)$ | $\mathrm{g}_{\mathrm{a}}$ | $\mathrm{g}_{f}$ | $\mathrm{~g}_{\eta}$ |
| ---: | :--- | :--- | :--- |
| $20.4_{4}$ | 0.9963 | $0.996 I$ | 0.9998 |
| $40.4_{4}$ | 0.9959 | $0.9955_{5}$ | 0.9997 |
| $60.4_{4}$ | 0.9955 s | 0.9951 | 0.9995 |
| $80.4_{4}$ | $0.9952_{5}$ | $0.9946_{5}$ | 0.9994 |
| $100.4_{4}$ | $0.9950_{5}$ | 0.9943 | 0.9992 |
| $120.4_{4}$ | 0.9949 | 0.9940 | 0.9991 |
| $140.4_{4}$ | 0.9948 | 0.9937 | 0.9989 |

## For U-235 (Based on R.RSH.R)

| $20.4_{4}$ | 0.9787 | 0.9772 | 0.9984 |
| ---: | :--- | :--- | :--- |
| $40.4_{4}$ | 0.97405 | 0.97235 | 0.9982 |
| $60.4_{4}$ | 0.96965 | 0.9671 | 0.99795 |
| $80.4_{4}$ | 0.9655 | 0.9627 | 0.9977 |
| $100.4_{4}$ | 0.96165 | 0.9586 | 0.9974 |
| $120.4_{4}$ | 0.9580 | 0.9546 | $0.9970_{5}$ |
| $140.4_{4}$ | 0.9546 | 0.9509 | 0.9967 |

For Pu-239 (Based on LR.R.R)

| $20.4_{4}$ | 1.0762 | 1.0522 | 0.9777 |
| ---: | :--- | :--- | :--- |
| $40.4_{4}$ | 1.0948 | 1.0654 | 0.9731 |
| $60.4_{4}$ | 1.1159 | 1.0805 | 0.9683 |
| $80.4_{4}$ | 1.1396 | $1.0977_{5}$ | $0.9632_{5}$ |
| $100.4_{4}$ | 1.1662 | 1.1172 | 0.9580 |
| $120.4_{4}$ | 1.1958 | 1.1391 | 0.9526 |
| $140.4_{4}$ | 1.22845 | $1.1634_{5}$ | 0.9471 |

N.B. For the table above the $\sigma_{a}$ recipes were altered to correspond to $\sigma_{s}$ values of 11,16 and 8 barns respectively for $\mathrm{U}-233, \mathrm{U}-235$ and Pu-239.



[^0]:    * Westcott, Ekberg, Hanna, Pattenden, Sanatani and Attree Atomic Energy Rev., Vol. 3, No. 2, p. 2, (1955)

[^1]:    * Denotes "Sigma Center Information Storage and Retrieval System", See Report BNL-883 (1964)

[^2]:    * The MOL authors used a linear fit vs time-of-flight, i.e. $\sigma=a+b E^{-\frac{1}{2}}$, and MLA is of somewhat similar shape, but there is no theoretical reason to expect a straict line on Mol coordinates. Note also on Figures 1-' som lata sets listed have points only at higher enery than the graphs show.

