International Atomic Energy Agency

INDC(CCP)-63/L

INDC INTERNATIONAL NUCLEAR DATA COMMITTEE

EVALUATING THE CROSS-SECTIONS OF THE (n,2n) AND (n,3n) REACTIONS FOR ^{239}Pu

E.Sh. Sukhovitskij, V.A. Kon'shin

Translation of a Reprint from

Vestsi Akad. Navuk BSSR, Ser. Fiz.-Energ. Navuk 3(1974)

Translated by the IAEA
February 1975

IAEA NUCLEAR DATA SECTION, KÄRNTNER RING 11, A-1010 VIENNA

75-0979 INDC(CCP)-63/L
Translated from Russian

EVALUATING THE CROSS-SECTIONS OF THE (n,2n) AND (n,3n) REACTIONS FOR ^{239}Pu

E.Sh. Sukhovitskij, V.A. Kon'shin

Translation of a Reprint from
Vestsi Akad. Navuk BSSR, Ser. Fiz.-Energ. Navuk 3(1974)

Translated by the IAEA
February 1975

EVALUATING THE CROSS-SECTIONS OF THE (n,2n) AND (n,3n) REACTIONS FOR ^{239}Pu

E.Sh. Sukhovitskij, V.A. Kon'shin

In compiling a comprehensive file of constants, the authors found that there is a complete lack of experimentally measured cross-section data for the (n,2n) and (n,3n) reactions. Preference could not be given to any of the existing models for calculating these cross-sections, since the results they give for practically identical premises differ by a factor of 3-4. We tried to establish a model from which the (n,2n) and (n,3n) cross-sections could be calculated using experimental data on the stages in the process.

We shall assume that for the energy range in which we are interested (5-15 MeV), the (n,2n) and (n,3n) reactions involve a compound nucleus. The cross-sections of these reactions can then be written in the form

$$\sigma_{n,2n} = \sum_{\alpha} \sigma_{1\alpha} P_{2\alpha} P_{4\alpha},$$

$$\sigma_{n,3n} = \sum_{\alpha} \sigma_{1\alpha} P_{2\alpha} P_{4\alpha} P_{3\alpha},$$

where $\sigma_{1\alpha}$ is the cross-section for the emission of a neutron in the α channel;

$P_{2\alpha}$, $P_{3\alpha}$ are the probabilities for the emission of a second and third neutron in the α channel;

$P_{4\alpha}$ is the probability that no further neutrons will be emitted by the nucleus.

In the calculations below, we shall make the following simplifications:

$$\sigma_{n,2n} = \overline{P}_{2\alpha} \overline{P}_{4\alpha} \sum_{\alpha} \sigma_{1\alpha},$$

$$\sigma_{n,3n} = \overline{P}_{2\alpha} \overline{P}_{3\alpha} \overline{P}_{4\alpha} \sum_{\alpha} \sigma_{1\alpha}.$$

It can easily be seen that

$$\sigma_{ne} = \sigma_{nf} + \sigma_{n\gamma} + \sum_{\alpha} \sigma_{1\alpha},$$

where σ_{nf} is the primary partial fission cross-section ($\sigma_{nF} = \sigma_{nf} + \sigma_{n,n'f} + \sigma_{n,2nf}$).

According to the statistical model of the nucleus, after emission of the first neutron the target nucleus remains in an excited state with energy E, which is distributed in accordance with

$$B(E) = \sqrt{E}\, \sigma_{n\gamma}(E)\, e^{\sqrt{4aE}} .$$

It can be assumed that the A nucleus is formed from the (A-1) nucleus by the capture of a neutron of energy $E-E_1$, where E_1 is the neutron binding energy in the A nucleus.

Since

$$\sigma_{n,n'} + \sigma_{n,2n} + \sigma_{n,3n} + \ldots = \sigma_{ne} P_{2\alpha}, \text{ TO}$$

$$P_{2\alpha} = \frac{\sigma_{n,n'} + \sigma_{n,2n} + \sigma_{n,3n} + \ldots}{\sigma_{ne}} = \left(\frac{\sigma_{ne} - \sigma_{nf} - \sigma_{n\gamma}}{\sigma_{ne}}\right)_{A-1, E-E_1} .$$

Hence

$$\overline{P}_{2\alpha} = \frac{\int\limits_{E_1}^{E_n} \left(\frac{\sigma_{ne} - \sigma_{nf} - \sigma_{n\gamma}}{\sigma_{ne}}\right)_{A-1, E-E_1} B_A(E)\, dE}{\int\limits_0^{E_n} B_A E\, (dE)} .$$

After emission of the first neutron, the mean excitation E_{n_1} of the nucleus is:

$$E_{n_1} = \frac{\int\limits_0^{E_n} E B_A(E)\, dE}{\int\limits_0^{E_n} B_A(E)\, dE} .$$

Using the above-described procedure we find the probability $\overline{P}_{3\alpha}$:

$$\overline{P}_{3\alpha} = \frac{\int\limits_{E_2}^{E_{n_1}-E_1} \left(\frac{\sigma_{ne} - \sigma_{n\gamma} - \sigma_{nf}}{\sigma_{ne}}\right)_{A-2, E-E_2} \cdot B_{A-1}(E)\, dE}{\int\limits_0^{E_{n_1}-E_1} B_{A-1}(E)\, dE} ,$$

where E_2 is the neutron binding energy in the (A-1) nucleus.

In the (n,2n) process, after the emission of two neutrons, the residual excitation should be eliminated by the emission of γ-rays, i.e.

$$\bar{P}_{4\alpha} = \frac{\int\limits_{0}^{E_{n_1}-E_1} \left(\frac{\sigma_{n\gamma}}{\sigma_{ne}}\right)_{A-2,E-E_2} B_{A-1}(E)\,dE}{\int\limits_{0}^{E_{n_1}-E_1} B_{A-1}(E)\,dE}.$$

Similar reasoning can be applied to other processes taking place in stages, i.e. the $\sigma_{n,n'f}$, $\sigma_{n,2nf}$ cross-sections can be calculated.

If we assume that in the energy range in which we are interested, $\sigma_{n\gamma}(E) \sim \frac{1}{\sqrt{E}}$, we get $B(E) = e^{\sqrt{4aE}}$. It follows from this that

$$\sigma_{n,2n}(E_n) = (\sigma_{ne} - \sigma_{nf} - \sigma_{n\gamma})_{A,E_n} \frac{\int\limits_{E_1}^{E_n} \left(\frac{\sigma_{ne} - \sigma_{nf} - \sigma_{n\gamma}}{\sigma_{ne}}\right)_{A-1,E-E_1} \cdot e^{\sqrt{4aE}}\,dE}{\int\limits_{0}^{E_n} e^{\sqrt{4aE}}\,dE} \times$$

$$\times \frac{\int\limits_{0}^{E_{n_1}-E_1} \left(\frac{\sigma_{n\gamma}}{\sigma_{ne}}\right)_{A-2,E-E_2} e^{\sqrt{4aE}}\,dE}{\int\limits_{0}^{E_{n_1}-E_1} e^{\sqrt{4aE}}\,dE};$$

$$\sigma_{n,3n}(E_n) = (\sigma_{ne} - \sigma_{nf} - \sigma_{n\gamma})_{A,E_n} \frac{\int\limits_{E_1}^{E_n} \left(\frac{\sigma_{ne} - \sigma_{nf} - \sigma_{n\gamma}}{\sigma_{ne}}\right)_{A-1,E-E_1} e^{\sqrt{4aE}}\,dE}{\int\limits_{0}^{E_n} e^{\sqrt{4aE}}\,dE} \times$$

$$\times \frac{\int\limits_{E_2}^{E_{n_1}-E_1} \left(\frac{\sigma_{ne} - \sigma_{n\gamma} - \sigma_{nf}}{\sigma_{ne}}\right)_{A-2,E-E_2} \cdot e^{\sqrt{4aE}}\,dE}{\int\limits_{0}^{E_{n_1}-E_1} e^{\sqrt{4aE}}\,dE} \times$$

$$\times \frac{\int\limits_{0}^{E_{n_2}-E_2} \left(\frac{\sigma_{n\gamma}}{\sigma_{ne}}\right)_{A-3,E-E_3} e^{\sqrt{4aE}}\,dE}{\int\limits_{0}^{E_{n_2}-E_2} e^{\sqrt{4aE}}\,dE},$$

where $E_{n_2} = \overline{E_{n_1} - E_1}$, as in the case of E_{n_1}.

If the energy dependences of the cross-section are sufficiently smooth, the formulae above can be simplified as follows:

$$\sigma_{n,2n}(E_n) = (\sigma_{ne} - \sigma_{nf} - \sigma_{n\gamma})_{A,E_n} \left(\frac{\sigma_{ne} - \sigma_{nf} - \sigma_{n\gamma}}{\sigma_{ne}} \right)_{A-1,E_{n_1}-E_1} \times$$

$$\times \frac{\int\limits_{E_1}^{E_n} e^{\sqrt{4aE}}\, dE}{\int\limits_{0}^{E_n} e^{\sqrt{4aE}}\, dE} \left(\frac{\sigma_{n\gamma}}{\sigma_{ne}} \right)_{A-2,E_{n_2}-E_2};$$

$$\sigma_{n,3n}(E_n) = (\sigma_{ne} - \sigma_{nf} - \sigma_{n\gamma})_{A,E_n} \cdot \left(\frac{\sigma_{ne} - \sigma_{nf} - \sigma_{n\gamma}}{\sigma_{ne}} \right)_{A-1,E_{n_1}-E_1} \times$$

$$\times \frac{\int\limits_{E_1}^{E_n} e^{\sqrt{4aE}}\, dE}{\int\limits_{0}^{E_n} e^{\sqrt{4aE}}\, dE} \left(\frac{\sigma_{ne} - \sigma_{nf} - \sigma_{n\gamma}}{\sigma_{ne}} \right)_{A-2,E_{n_2}-E_2} \frac{\int\limits_{E_2}^{E_{n_1}-E_1} e^{\sqrt{4aE}}\, dE}{\int\limits_{0}^{E_{n_1}} e^{\sqrt{4aE}}\, dE} \times$$

$$\times \left(\frac{\sigma_{n\gamma}}{\sigma_{ne}} \right)_{A-3,E_{n_3}-E_3}.$$

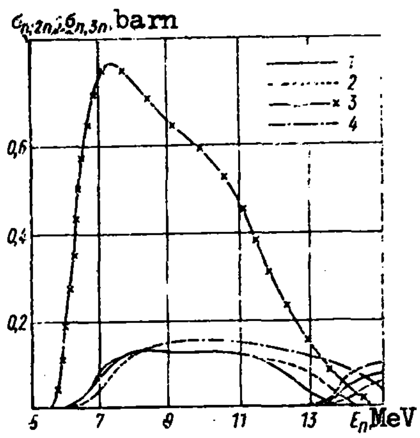
From what we have said above, it is easy to obtain an expression for the emitted neutron flux at each stage of the process

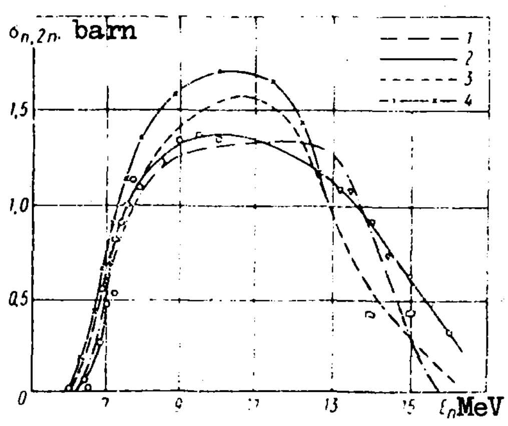
$$J_i(E) \sim \sqrt{E}\, e^{\sqrt{4a(E_{n_i} - E_i)}},$$

while the mean energies of the neutrons produced in the (n,2n) and (n,3n) reactions are equal to:

$$2T_1 = \sqrt{\frac{E_n}{a} - \frac{1}{a}},$$

$$2T_2 = \sqrt{\frac{E_n - 2T_1 - E_1}{a} - \frac{1}{a}}.$$





/For captions, see top of page 5/

Fig. 1. Cross-sections of the (n,2n) and (n,3n) reactions for ^{239}Pu.
1 - Our work; 2 - Calculation for a = 9.5 MeV; 3 - [6]; 4 - [8].

Fig. 2. Cross-sections of the (n,2n) reaction for ^{238}U. 1 - Our work; 2 - Curve based on experimental data; 3 - [7]; 4 - [7] renormalized data.

The formulae given above were used to calculate the (n,2n) and (n,3n) reaction cross-sections. The ^{239}Pu cross-sections needed for this were taken from Kon'shin [1], and the ^{238}Pu cross-sections from Dunford and Alter [2]. Instead of the ^{237}Pu and ^{236}Pu cross-sections, for which no data are available, use was made of the ^{239}Pu and ^{238}Pu cross-sections. The neutron binding energies were taken from Gorbachev [3]. The level density parameter was taken from Malyshev [4] and from Baba and Baba [5], and this value, 29 MeV^{-1}, is in our opinion the most likely one.

Figure 1 gives the cross-section for the (n,2n) and (n,3n) reactions obtained in our calculations. The $\sigma_{n,2n}$, $\sigma_{n,3n}$ cross-sections were calculated with a level density parameter a = 10 MeV^{-1}. This value of a was used in other cross-section evaluations, for instance those of Douglas and Barry [6] and of Pearlstein [7].

Figure 1 compares the results of our calculations with those of other authors. Our results are in good agreement with Prince's evaluation [8], but they differ greatly from the results of Douglas and Barry [6].

Our results are given in the table below.

Comparison of calculations based on our model with those based on the Pearlstein model [7] shows that the latter model does not take into account the competition of other processes at stages following the formation of the compound nucleus. In addition, in the Pearlstein model a contribution to $\sigma_{n,2n}$ and $\sigma_{n,3n}$ is made by inelastic scattering, and this was not taken into account. A comparison of the models for the case of the ^{238}U nucleus, for which experimental data are available (see Fig. 2), shows that the results given by the Pearlstein model are somewhat too high.

Evaluated values for the $\sigma_{n,2n}$, $\sigma_{n,3n}$ cross-sections

E, MeV	$\sigma_{n,2n}$, barn	$\sigma_{n,3n}$, barn	E, MeV	$\sigma_{n,2n}$, barn	$\sigma_{n,3n}$, barn
6	0,008	—	11,0	0,120	—
6,5	0,020	—	11,5	0,108	—
7,0	0,072	—	12,0	0,093	—
7,5	0,108	—	12,5	0,068	—
8,0	0,132	—	13,0	0,034	—
8,5	0,131	—	13,5	0,008	0,006
9,0	0,130	—	14,0	0,00	0,034
10,0	0,130	—	14,5	—	0,060
10,5	0,129	—	15,0	—	0,080

It will be noted that the calculations of Pearlstein [7], given in Fig. 2, have been renormalized to the present constants σ_{ne}, σ_{nf}.

The authors wish to express their gratitude to academician A.K. Krasin for supporting the work.

SUMMARY

A nuclear model for calculation of the (n,2n) and (n,3n) neutron cross-sections is proposed. The model uses experimental data on the process of neutron interaction with nuclei. The (n,2n) and (n,3n) cross-sections for both ^{238}U and ^{239}Pu are calculated and the results are compared with those of other evaluations.

REFERENCES

[1] KON'SHIN, V.A. et al., All-Union Conference on Neutron Physics, Kiev, May 1973.

[2] DUNFORD, C.L., ALTER, H., NAA-SR-12272, Physics, 1967.

[3] GORBACHEV, V.M., et al., The main characteristics of isotopes of heavy elements, Atomizdat 1970 (in Russian).

[4] MALYSHEV, A.V., Level density and structure of atomic nuclei, Atomizdat 1969 (in Russian).

[5] BABA, H., BABA, S., Report JAERI-1183, August 1969.

[6] DOUGLAS, A.C., BARRY, J.F., AWRE-O-79/64, 1964.

[7] PEARLSTEIN, S., Nucl. Sci. Eng., 23, 238, 1965.

[8] PRINCE, A., IAEA Conference on Nuclear Data for Reactors, Helsinki, June, 1970, 2, p. 825, 1970.