INDC(CCP)-77/U

International Atomic Energy Agency

INDC INTERNATIONAL NUCLEAR DATA COMMITTEE

# EVALUATION OF $^{240}$PU NEUTRON CROSS-SECTIONS IN THE UNRESOLVED RESONANCE REGION

G.V. Antsipov, V.A. Konshin, E.Sh. Sukhovitskij

Institute of Nuclear Power Engineering
Academy of Sciences of the Byelorussian SSR

January 1976

IAEA NUCLEAR DATA SECTION, KÄRNTNER RING 11, A-1010 VIENNA

INDC(CCP)-77/U

# EVALUATION OF $^{240}$PU NEUTRON CROSS-SECTIONS IN THE UNRESOLVED RESONANCE REGION

G.V. Antsipov, V.A. Konshin, E.Sh. Sukhovitskij

Institute of Nuclear Power Engineering
Academy of Sciences of the Byelorussian SSR

January 1976

75-4489
Translated from Russian

# EVALUATION OF $^{240}Pu$ NEUTRON CROSS-SECTIONS IN THE UNRESOLVED RESONANCE REGION

G.V. Antsipov, V.A. Kon'shin, E.Sh. Sukhovitskij
Institute of Nuclear Power Engineering,
Academy of Sciences of the Byelorussian SSR

## ABSTRACT

The mean resonance parameters for $^{240}Pu$ were evaluated using both available experimental data and theoretical concepts. The assumption of a double-humped fission barrier structure was used for accurate calculation of the fission widths and the width fluctuation factor. The parameters of a double-humped fission barrier were evaluated. The mean resonance parameters permit calculation of the cross-sections $\sigma_t$, $\sigma_{n\gamma}$, $\sigma_f$ and $\sigma_{nn'}$ in the 1-142 keV energy region.

The unresolved resonance region of the $^{240}Pu$ nucleus occupies the 1-142 keV energy range in this evaluation. On one side it is limited by the resolved resonance region, and at the upper end its limitation is due mainly to the fact that the strength function $S_2$ is not known, and to the accuracy with which the second-level excitation cross-section is taken into account (142 keV $2^+$). Because the energy region is so large, the S, p and d waves were included in the calculation in order to obtain a more accurate analysis, and the dependence of the level density on energy was also taken into account.

The average distance between the levels $\langle D\rangle_J$ of the spin J was determined, in the present work, from the average observed distances $\langle D\rangle_{exp}$ in the resolved resonance region, using an independent particle model (see Ref. [1]):

$$\langle D\rangle_J = \frac{24\sqrt{2}\, a^{\frac{1}{4}}\, U^{5/4}\, G^3 \cdot 10^6}{(2J+1)} \operatorname{Exp}\left[-2(aU)^{\frac{1}{2}} + \frac{(J^{\frac{1}{2}})}{2\sigma^2}\right] \qquad (1)$$

where G is the cut-off parameter defined by the expression $G^2 = 0.0889\,(aU)^{\frac{1}{2}}A^{2/3}$, and U is the excitation energy of the compound nucleus $U = B_n + E_n - \Delta$. $B_n$ is the energy of neutron detachment from the nucleus A + 1, equalling 5.241 MeV, and $\Delta$ is the proton pairing energy, equal to 0.39 MeV. The parameter $a$ was determined from the condition

$$\langle D\rangle_{exp} = \langle D\rangle_{\frac{1}{2}} = 13.5 \pm 0.5 \text{ eV} \qquad (2)$$

We obtained such a value of $\langle D\rangle_{exp}$ when we evaluated the resolved resonance parameters. From (2), the parameter $a$ was found to be 25.66 $MeV^{-1}$. Figure 1 shows $\langle D\rangle_J$ as a function of energy in the 1-142 keV energy region; the average distance between the levels is reduced by about ~ 20%.

The mean neutron widths $<\Gamma_n>_s$ can be represented in terms of the strength functions $S_e$ as

$$<\Gamma_n>_s = S_e <D>_J E^{\frac{1}{2}} P_e \nu_g \tag{3}$$

where $\nu_s$ is the number of degrees of freedom for a given S-state, defined by the number of possible channels, and $P_e$ is the penetration coefficient for the partial wave.

$$P_0 = 1, \; P_1 = \frac{(ka)^2}{1 + (ka)^2}, \; P_2 = \frac{(ka)^4}{g + 3(ka)^2 + (ka)^4} \tag{4}$$

Here k is the neutron wave number $k = 2.196771 \times 10^{-3} \left(\frac{AW}{AW + 1}\right)E^{\frac{1}{2}}$, where AW is the isotopic mass of $^{240}Pu$, equal to 237.992, and $a$ is the radius of the scattering channel $a = 0.123 \; (AW \times 1.008665)^{1/3} + 0.08$.

The mean inelastic widths were determined from the expression

$$<\Gamma_{n'}>_s = <D>_J \sum_{ge'} S_{e'} \, \varepsilon_g^{\frac{1}{2}} P_{e'} \, (\varepsilon_g) \, \nu_{Je'g} \tag{5}$$

where $\varepsilon_g = E - E_g$ is the neutron energy in the inelastic channel, e' the spin of the inelastically scattered neutron. Thus, one has to know the strength function in order to determine the neutron and inelastic widths. The strength function $S_o$ was taken from our evaluation in the resolved resonance region, and is $S_o = (1.1 \pm 0.16) \; 10^{-4} \; eV^{-\frac{1}{2}}$, and the value of $S_2$ was assumed to be $S_o$. Selection of the strength function $S_1$ was based on the condition of obtaining the best possible agreement of calculated and experimental data on $\sigma_t$, $\sigma_{n\gamma}$ throughout the energy region studied, i.e. $S_1 = (2.8 \pm 0.4) \; 10^{-4} \; eV^{-\frac{1}{2}}$.

In some experimental work, groups of powerful fission resonances have been found in the sub-barrier fission region of the $^{240}Pu$ nucleus. Such behaviour of the fission cross-section can be understood in terms of the double-humped fission barrier model described by Strutinskij and Bjørnholm ([2]). The existence of such a barrier has no practical effect on the fission width $<\Gamma_f>_s$, but it alters the fission width distribution. The penetration curves for fission envelop the resonances in the second well; by regarding these resonances as being of the Breit-Wigner type, one can obtain the following width distribution (cf. Ref. [3]):

$$\varphi(x)dx = \frac{dx}{\pi x}(x - x_{min})^{-\frac{1}{2}}(x_{max} - x)^{-\frac{1}{2}} \tag{6}$$

where $x = \frac{\Gamma_f}{<\Gamma_f>}$, $x_{max} = \frac{\Gamma_{f\ max}}{<\Gamma_f>}$, $x_{min} = \frac{\Gamma_{f\ min}}{<\Gamma_f>}$, and $<\Gamma_f> = \sqrt{\Gamma_{f\ min}\,\Gamma_{f\ max}}$.
We also think that, apart from the distribution (6), the widths $\Gamma_f$ as well as the widths $\Gamma_n$ and $\Gamma_{n'}$ also experience local fluctuations, described as usual by the $\chi^2$-distribution with the number of degrees of freedom equal to the number of open channels. The mean fission widths $<\Gamma_f>_s$ were determined by the expression

$$<\Gamma_f>_s = \sum_k \frac{<D>_s}{2\pi} P_{fsk}$$

where $P_{fsk}$ is the fission penetration factor of the k-channel of the S-state. Following Equation (6),

$$P_{f\ \substack{max \\ min}} = \frac{P_A\, P_B}{1 \mp \sqrt{(1-P_A)(1-P_B)}} \tag{7}$$

where $P_A$ and $P_B$ are the penetration factors of the first and second hump of the double-humped fission barrier. From Equation (6), it follows that

$$\begin{aligned} <\Gamma_f> = \Gamma_{f\ max}\,\Gamma_{f\ min} &= \frac{<D>}{2\pi}\,\frac{P_A\, P_B}{1 - (1-P_A)(1-P_B)} \\ X_{max} = \frac{1}{X_{min}} &= \frac{1 + \sqrt{(1-P_A)(1-P_B)}}{1 - \sqrt{(1-P_A)(1-P_B)}} \end{aligned} \tag{8}$$

The penetration factors $P_A$ and $P_B$ were calculated by the Hill-Wheeler formula:

$$P_f = \frac{1}{1 + \mathrm{Exp}\left[\frac{2\pi(E_f - E)}{\hbar\omega}\right]} \tag{9}$$

where $E_f$ is the height of the fission barrier, and $\hbar\omega$ is its curvature parameter. In the present work, the number of fission channels was taken to be $(2J + 1)$, and the parameters of barriers A and B were considered to be independent of spin and parity. On this assumption

$$<\Gamma_f>_s = \frac{<D>_J}{2\pi}(2J + 1)\ P(E_n, E_{fA}, \hbar\omega_A, E_{fB}, \hbar\omega_B) \tag{10}$$

The curvature parameter $\hbar\omega_A$ we evaluated as 1.00 ± 0.05 MeV [4, 5], $\hbar\omega_B$ as 0.55. Using $<\Gamma_{f\,max}>$ = 70 ± 30 MeV, $<\Gamma_{f\,min}>$ = 0.15 ± 0.04 MeV, $<\Gamma_f>$ = 3.34 ± 1 MeV (evaluated by us in the resolved resonance region), we obtained $E_{fA}$ = 1.028 MeV, $E_{fB}$ = 0.143 MeV, the energy being deduced from the neutron binding energy. At first we intended to make the barrier parameters more precise as a result of calculating $<\sigma_f>$, but the good agreement of the experimental and calculated data in the 1-500 keV energy range enabled us to leave them unchanged.

A mean radiation width of 30.7 MeV was used in the calculations. Hockenbury et al. [7] have shown that agreement with experiment can be improved by increasing $<\Gamma_\gamma>$ from 30 MeV to 33 MeV with an increase in energy from 6 keV to 30 keV; the same effect can be produced by reducing $<D>_J$, so we introduced a dependence of the average distance between the levels on energy in the unresolved resonance region.

The quality of the mean resonance parameters was checked by comparing experimental and calculated cross-sections. The total cross-section $<\sigma_t>$ was calculated according to the formula

$$\langle\sigma_t\rangle = \frac{4\pi}{k^2}\sum_\ell (2\ell+1)\sin^2\varphi_\ell + \frac{2\pi^2}{k^2}\sum(2\ell+1)\sqrt{E}\,S_\ell P_\ell - \frac{4\pi^2}{k^2}\sum(2\ell+1)E^{1/2}S_\ell P_\ell \sin^2\varphi_\ell$$

and for the phase shifts $\varphi_\ell$ the formulae

$$\pi R^2 = 9.2474\ \text{барн},\quad \varphi_0 = kR,\quad \varphi_1 = kR - \operatorname{arctg}(kR)$$
$$\varphi_2 = kR - \operatorname{arctg}\left(\frac{3(kR)}{3-(kR)^2}\right) \qquad (11)$$

were used.

The cross-sections of the reactions taking place across the compound nucleus $<\sigma_{nr}>$ were determined by the expression

$$\langle\sigma_{nr}\rangle = \frac{B}{E}\sum_s \frac{g_s}{\langle D\rangle_J}\left\langle\frac{\Gamma_{ns}\Gamma_{rs}}{\Gamma_{ts}}\right\rangle$$

where $g_s$ is a statistical factor equal to $\frac{(2J+1)}{(2I+1)2}$, B = 4.124226 x $10^6$ $\frac{\text{barn}}{\text{eV}}$, $\Gamma_{rs}$ is the partial width of the (n,r) reaction and $\Gamma_{ts}$ is the total width.

In order to calculate $\left\langle \frac{\Gamma_{ns}\,\Gamma_{rs}}{\Gamma_{ts}} \right\rangle$ from the values of $\langle \Gamma_{rs} \rangle$, we considered that the neutron and inelastic widths were subject to a $\chi^2$ distribution with the number of degrees of freedom given in Table 1.

Table 1

| $\ell$ | $J$ | $\pi$ | $\nu_n$ | $\nu_{n'}$ | $\nu_f$ | $\nu_\gamma$ |
|---|---|---|---|---|---|---|
| 0 | 1/2 | + | 1 | 2 | 1 | $\infty$ |
| 1 | 1/2 | - | 1 | 1 | 2 | |
| 1 | 3/2 | - | 1 | 2 | 4 | |
| 2 | 3/2 | + | 1 | 1 | 4 | |
| 2 | 5/2 | + | 1 | 1 | 6 | |

Furthermore, the widths $\Gamma_f$ were considered to be distributed according to distribution (6).

We averaged the values of $\frac{\Gamma_{ns}\,\Gamma_{rs}}{\Gamma_{ts}}$, which were obtained by random testing of equivalent distributions, the testing being continued until the calculation error due to the finiteness of the selection process was less than three per cent for each channel. The initial calculations of the cross-sections $\langle \sigma_t \rangle$ and $\langle \sigma_{n\gamma} \rangle$ were made with a view to determining the value of the strength function $S_1$. The results showed that in order to obtain agreement with the data of Smith et al. [8], $S_1$ must equal $\sim 2.65 \times 10^{-4}$ eV$^{-\frac{1}{2}}$. At the same time, calculations on $\sigma_{n\gamma}$ show that in order to achieve agreement with the data of Hockenbury et al. [7], $S_1$ must be $\sim 2.9 \times 10^{-4}$ eV$^{-\frac{1}{2}}$. We selected the value $S_1 = 2.8 \times 10^{-4}$ eV$^{-\frac{1}{2}}$. Figures 2, 3 and 4 show a comparison of the calculated and experimental data on $\langle \sigma_t \rangle$, $\langle \sigma_{n\gamma} \rangle$ and $\langle \sigma_f \rangle$. Agreement is good within the margin of error. $\langle \sigma_f \rangle$ coincides with the experimental results throughout the region in which the programme permitted calculation of the cross-section $\langle \sigma_f \rangle$ (up to 700 keV).

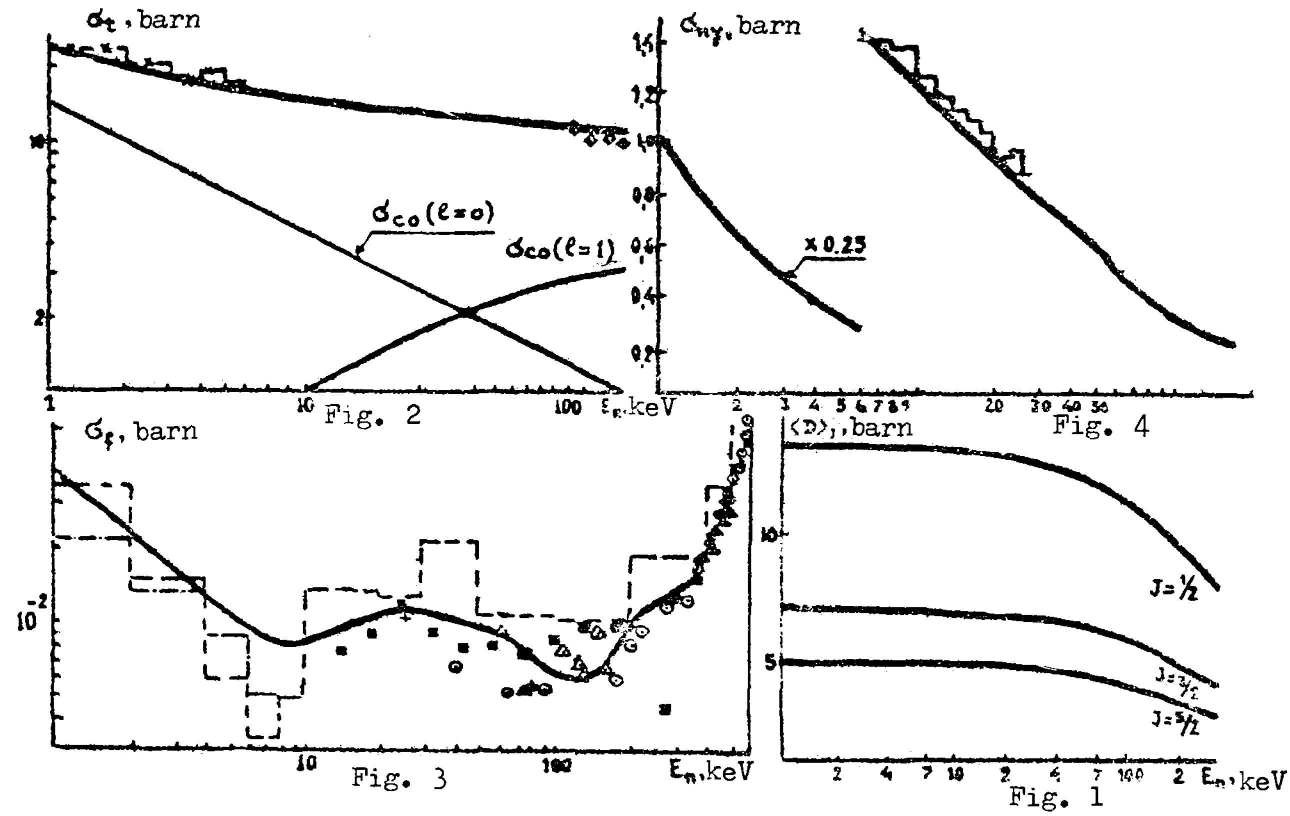


Fig. 1 — $\langle D \rangle_J$ as a function of energy

Figs 2, 3 and 4 — Comparison of calculated and experimental data on $\sigma_t$, $\sigma_f$ and $\sigma_{n\gamma}$

| | |
|---|---|
| ———— | Calculation from evaluated parameters; |
| x - x - x | Averaged data from Ref. [14]; |
| - - - - - | Averaged data from Ref. [9]; |
| - . - . - | Averaged data from Ref. [10]; |
| ⊓⊔⊓⊔ | Averaged data from Ref. [7]; |
| ◀▷ | Data from Ref. [8]; |
| ■ | Data from Ref. [11]; |
| Δ | Data from Ref. [12]; |
| ⊙ | Data from Ref. [13]. |

## REFERENCES

[1] MALYSHEV, A.V., Plotnost' urovnej i structura atomnogo yadra (Level density and atomic nucleus structure), Atomizdat, 1962.

[2] STRUTINSKIJ, V.M., BJØRNHOLM, S., Nuclear Structure: Dubna Symposium, 1968.

[3] GAJ, E.V., IGNATYUK, A.V., RABOTNOV, N.S., SMIRENKIN, G.N., Yad. Fiz. 10 (1969) 542.

[4] BACK, B.B., HANSEN, Ole, IAEA Symposium on Physics and Chemistry of Fission, Rochester, New York, V.1, p.3 (1973).

[5] BRITT, H.C. et al., Phys. Rev., C, 7 (1973) 801.

[6] TYAPIN, A.S., MARSHALKIN, V.E., Yad. Fiz. 18 (1973) 277.

[7] HOCKENBURY, R.W., MOYER, W.R. et al., Nucl. Sci. Eng. 49 (1972) 153.

[8] SMITH, A.B., LAMBROPOULOS, P. et al., Nucl. Sci. Eng. 47 (1972) 19.

[9] BYERS, D.H., DIVEN, B.C., SILBERT, M.G., LA-3568 (1966).

[10] MIGNECO, E., THEOBALD, G., Nucl. Phys., A112 (1968) 603.

[11] GILBOY, W.B., KNOLL, G.F., IAEA Symposium on Nuclear Data for Reactors, Paris, V.1, p.295 (1966).

[12] RUDDICK, P.R., WHITE, P.H., J. Nucl. Energy 18 (1964) 561.

[13] NESTEROV, V.G., SMIRENKIN, G.N., At. Ehnerg. 9 (1960) 16.

[14] KOLAR, W., BOECKHOFF, K.H., J. Nucl. Energy 22 (1968) 299.

Reproduced by the IAEA in Austria
January 1976
75-10062