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THE INFLUENCE OF DIFFERENT REPRESENTATIONS OF THE STATISTICAL PROPERTIES OF FISSION WIDTHS ON THE CALCULATION OF AVERAGE REACTION CROSS-SECTIONS

G.V. Antsipov, V.A. Kon'shin and V.M. Maslov

It is usual, when calculating average reaction cross-sections for fissionable nuclei in the unresolved resonance region, to use the Hauser-Feshbach formalism [1], which was modified by Lane and Lynn [2] in order to take into account fluctuations in partial widths, but which is correct in the absence of interference between resonances and of correlations of the widths for different processes. The expression for the average cross-section $\langle\sigma_{nx}\rangle_r$ of the (n,x) reaction and the state r of a compound nucleus characterized by particular values for J and for parity (π) is as follows:

$$\langle \sigma_{nx} \rangle_r = 2\pi^2\lambda^2 \frac{g_r}{\langle D \rangle_r} \frac{\langle \Gamma_n \rangle_r \langle \Gamma_x \rangle_r}{\langle \Gamma \rangle_r} S_{nxr}, \qquad (1)$$

where g_r is the statistical factor of the state r;

$\langle D\rangle_r$ is the average distance between levels of the compound nucleus;

$\langle\Gamma_n\rangle_r$ is the average neutron width;

$\langle\Gamma_x\rangle_r$ is the average width of the (n,x) reaction;

$\langle\Gamma\rangle_r$ is the average total width of the state r; and

S_{nxr} is a factor which takes into account the effect of fluctuations in partial widths:

$$S_{nxr} = \left\langle \frac{\Gamma_n \Gamma_x}{\Gamma} \right\rangle_r \Big/ \frac{\langle \Gamma_n \rangle_r \langle \Gamma_x \rangle_r}{\langle \Gamma \rangle_r} \qquad (2)$$

The averaging in Eq. (2) is performed in accordance with the generally accepted laws of width distribution. This is normally the Porter-Thomas distribution with ν degrees of freedom:

$$P(y)\,dy = \left(\frac{\nu y}{2}\right)^{(\nu-2)/2} \exp\left(-\frac{\nu y}{2}\right) \Gamma^{-1}\left(\frac{\nu}{2}\right) \frac{\nu}{2}\,dy, \qquad (3)$$

where $\Gamma(\nu/2)$ is the gamma function and $y = \Gamma_{xr}/\langle\Gamma_x\rangle_r$.

The number of degrees of freedom ν_{xr} is the same as the number of channels making a contribution to the width of the (n,x) reaction or as the effective quantity $\nu_{eff.xr}$ obtained from analysis of the experimental resonance widths $\Gamma_{\lambda xr}$:

$$\nu_{eff.xr} = 2\frac{\langle\Gamma_x\rangle_r^2}{\langle\Gamma_x^2\rangle_r - \langle\Gamma_x\rangle_r^2} \qquad (4)$$

or, what comes to the same thing, from the penetrabilities of the channels:

$$\nu_{eff.xr} = \frac{\left(\sum\limits_{k=1}^{\nu} P_{kxr}\right)^2}{\sum\limits_{k=1}^{\nu} P_{kxr}^2}. \qquad (5)$$

The first method of determining the number of degrees of freedom does not take into account the fact that channels may make different contributions to the average width. The second method takes this into account only in an approximate way. Clearly, the same value for $\nu_{eff.xr}$ can be obtained using different combinations of numbers of channel and their relative contributions.

A distribution which explicitly takes into account the presence of several channels with different relative contributions has been proposed by Shaker and Luk'yanov [3]. The quantity y can in this case be represented in the following way:

$$y = \frac{\Gamma_{xr}}{\langle\Gamma_x\rangle_r} = \sum_{k=1}^{\nu} \frac{\Gamma_{xrk}}{\langle\Gamma_x\rangle_{rk}} \frac{\langle\Gamma_x\rangle_{rk}}{\langle\Gamma_x\rangle_r} = \sum_{k=1}^{\nu} x_k a_k, \qquad (6)$$

where α_k is the contribution of the k-th channel to the average width. Assuming that the values x_k obey the distribution in expression (3) with $\nu = 1$, by the reduction of ν distributions we can obtain the following distribution law for y:

$$P(y, \alpha_1, \alpha_2, \ldots, \alpha_\nu)\, dy = \left[\frac{y^{\nu-2}}{(2\pi)^\nu \alpha_1\alpha_2 \ldots \alpha_\nu} \right]^{1/2} \exp\left(- \frac{y}{2\alpha_\nu}\right) \times$$
$$\times \int_0^1 z_1^{-1/2} \exp(- A_1 z_1 y)\, dz_1 \int_0^{1-z_1} z_2^{-1/2} \exp(- A_2 z_2 y)\, dz_2 \ldots \times$$
$$\times \int_0^{1-z_1-\ldots-z_{\nu-2}} [z_{\nu-1}(1 - z_1 - \ldots - z_{\nu-1})]^{-1/2} \exp(- A_{\nu-1} z_{\nu-1}\, y)\, dz_{\nu-1}\, dy, \qquad (7)$$

where $A_k = (\alpha_\nu - \alpha_k)/(2\alpha\nu\,\alpha_k)$.

As a rule, in calculating average cross-sections the distribution in expression (3) has a whole number of degrees of freedom, so that the possibility of a reaction occurring in different channels is not fully taken into account. In this connection it is worth considering how different methods of describing partial width distributions affect the average cross-sections for the case of several channels. It is convenient to do this on the basis of fissionable nuclei for which the contribution to the fission widths $\langle\Gamma_f\rangle_r$ of s-resonances generally gives a small number of channels (1-4). Let us confine our analysis to three processes: elastic scattering, radiative capture and fission. There is sufficient justification for considering the radiation width $\Gamma_{\gamma r}$ non-fluctuating, which corresponds to an infinite number of degrees of freedom $\nu_{\gamma r}$. When we use the law in expression (3) to describe the fluctuations of the widths, the factor S_{nxr} is written:

$$S_{nxr} = \langle \Gamma \rangle_r \int_0^\infty \frac{\left(1 + 2\,\frac{\delta_{nx}}{\nu_{nr}}\right) \exp(- \langle \Gamma_\gamma \rangle_r t)\, dt}{\left(1 + 2\frac{\langle \Gamma_n \rangle_r}{\nu_{nr}}\, t\right)^{1+\delta_{nx}+\frac{1}{2}\nu_{nr}} \left(1 + 2\, \frac{\langle \Gamma_f \rangle_r}{\nu_{fr}}\, t\right)^{\delta_{fx}+\frac{1}{2}\nu_{fr}}} \qquad (8)$$

If we use the distribution in Eq. (7), the expression for S_{nxr} in the case of two fission channels is:

$$S_{nxr} = \langle \Gamma \rangle_r \int_0^\infty \frac{\left(1 + 2\frac{\delta_{nx}}{\nu_{nr}}\right) \exp\left(-\langle \Gamma_\gamma \rangle_r t\right)}{\left(1 + 2\frac{\langle \Gamma_n \rangle_r}{\nu_{nr}} t\right)^{1+\delta_{nx}+\frac{1}{2}\nu_{nr}}} \times$$

$$\times \frac{\xi^{1/2}\left(\langle \Gamma_f \rangle_r t + \xi\right)^{\delta_{fx}} dt}{\left[\left(\left(\langle \Gamma_f \rangle_r t + \xi\right)^2 - \left(\frac{\alpha_1 - \alpha_2}{\xi}\right)^2\right]^{1/2+\delta_{fx}}}, \tag{9}$$

where $\xi = (4\alpha_1\alpha_2)^{-1}$.

For the case of three channels we can get:

$$S_{nxr} = \frac{\langle \Gamma \rangle_r}{2^{3/2}(\alpha_1\alpha_2\alpha_3)^{1/2}} \int_0^\infty \frac{\left(1 + 2\frac{\delta_{nx}}{\nu_{nr}}\right) \exp\left(-\langle \Gamma_\gamma \rangle_r t\right)}{\left(1 + 2\frac{\langle \Gamma_n \rangle_r}{\nu_{nr}} t\right)^{1+\delta_{nx}+\frac{1}{2}\nu_{nr}}} \times$$

$$\times \sum_{k=0}^\infty \frac{A_2^{2k}}{2^{4k+\delta_{fx}}} \frac{(2k+1)!}{(k!)^2} F^{1/2}\left(\frac{1}{2\alpha_1}\right) F^{2(k+1)-\delta_{fx}}\left[\frac{1}{4}\left(\frac{1}{\alpha_2} + \frac{1}{\alpha_3}\right)\right] \times$$

$$\times \left\{\frac{F\left(\frac{1}{2\alpha_2}\right)}{F\left[\frac{1}{4}\left(\frac{1}{\alpha_2} + \frac{1}{\alpha_3}\right)\right]} + 4k + 2\right\}^{\delta_{fx}} dt, \tag{10}$$

where $F^q(\delta) = \left(\langle \Gamma_f \rangle_r t + \delta\right)^{-q}$.

In the particular case where we have equal penetrabilities for two channels ($\alpha_1 = \alpha_2$), Eq. (10) becomes:

$$S_{nxr} = \frac{\langle \Gamma \rangle_r}{2^{1/2+\delta_{fx}}(2\alpha)\alpha_3^{1/2}} \int_0^\infty \frac{\left(1 + 2\frac{\delta_{nx}}{\nu_{nr}}\right) \exp\left(-\langle \Gamma_\gamma \rangle_r t\right)}{\left(1 + 2\frac{\langle \Gamma_n \rangle_r}{\nu_{nr}} t\right)^{1+\delta_{nx}+\frac{1}{2}\nu_{nr}}} \times$$

$$\times \frac{\left(3\langle \Gamma_f \rangle_r + \frac{1}{2\alpha} + \frac{1}{\alpha_3}\right) dt}{\left(\langle \Gamma_f \rangle_r t + \frac{1}{2\alpha}\right)^{1+\delta_{fx}} \left(\langle \Gamma_f \rangle_r t + \frac{1}{2\alpha_3}\right)^{1/2+\delta_{fx}}}, \tag{11}$$

where $2\alpha = \alpha_1 + \alpha_2$.

Let us now see how different methods of describing the fission width distributions affect the values of S_{nxr} and, accordingly, the average cross-sections $\langle\sigma_{nn}\rangle_r$, $\langle\sigma_{n\gamma}\rangle_r$ and $\langle\sigma_{nf}\rangle_r$ for ^{239}Pu.

In the calculations, the following average resonance parameters were used: force functions $S_o = 1.06 \times 10^{-4}$ and $S_1 = 2.5 \times 10^{-4}$; the average radiation width $\langle\Gamma_\gamma\rangle_{observed} = 0.043$ eV. The average distances $\langle D\rangle_r$ between levels of the compound nucleus were calculated with the Fermi-gas model using $\langle D\rangle_{observed} = 2.38$ eV, $B_n = 6.534$ MeV and $\Delta = 0.919$ MeV.

For the state 0^+, in accordance with Lynn's scheme of transitional states [4], there are two channels whose contributions to the average width $\langle\Gamma_f\rangle_{0^+}$ differ greatly [5]. An analysis of fission widths in the resolved resonance region produces the same results. Ribon and Le Coq [6] give the value $\nu_{eff.0^+} = 1.4$, which in the case of a contribution of two channels gives $\alpha_1 = 0.83$ and $\alpha_2 = 0.17$.

Figure 1 (graphs a–c) shows the dependences of the factors S_{nn0^+}, $S_{n\gamma0^+}$ and S_{nf0^+} on $|\alpha_1-\alpha_2|$ calculated from Eqs (8) and (9) for an energy of 0.1 keV. The upper and lower straight lines correspond to Eq. (3); $\nu = 1$ and $\nu = 2$, respectively. Curve 4 corresponds to Eq. (7) and curve 3 to Eq. (3), with $\nu_{fr} = \nu_{eff.fr}$ from Eq. (5), where $\nu_{eff.fr}$ is found from the relative contributions of channels. The calculations were performed on the assumption that the average fission width $\langle\Gamma_f\rangle_{0^+}$ remained constant and equal to 1.59 eV and that only the relative contributions of channels varied. The figure shows that differences in methods for describing the distributions of fission widths have considerable influence on S-factors. This is particularly noticeable for $S_{n\gamma0^+}$ and S_{nn0^+}. A comparison of curves 3 and 4 will show that values for S_{nx0^+} depend to different degrees on the ratio of the contributions of channels despite the agreement of $\nu_{eff.0^+}$ with $|\alpha_1-\alpha_2|$, while the differences in S_{nn0^+} and $S_{n\gamma0^+}$ reach approximately 18% and in S_{nf0^+} approximately 5%, where $|\alpha_1-\alpha_2| = 0.7$–$0.9$.

The calculations performed show (Fig. 1, graphs d–f) that with the rise in energy leading to a significant increase in $\langle\Gamma_n\rangle_{r0^+}/\langle\Gamma\rangle_{0^+}$, while $\langle\Gamma_\gamma\rangle_{0^+}$ and $\langle\Gamma_f\rangle_{0^+}$ vary little, the difference decreases between the results obtained with the traditional method (involving the use of ν_{eff}) of taking fluctuations of fission widths into account and those obtained with the method based on two-channel distribution. With an energy of 100 keV, the difference decreases by a factor of 2–3 for S_{nn0^+} and $S_{n\gamma0^+}$ and by a factor of 1.5–2 for S_{nf0^+}.

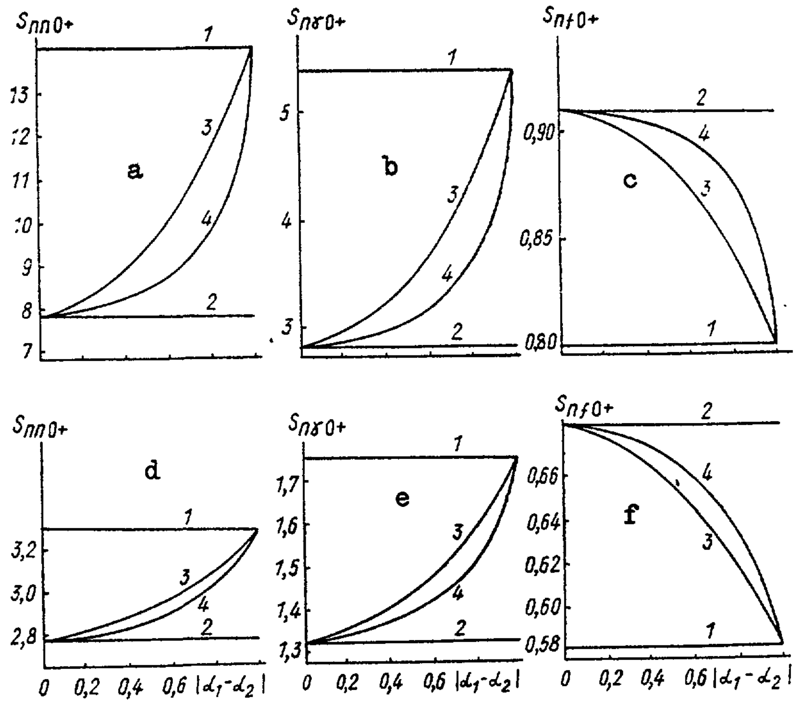
It should be pointed out that the minium error in values for S-factors in the transition from $\nu = 1$ to $\nu = 2$ occurs when $|\alpha_1 - \alpha_2| \simeq 0.9$ and when $\nu_{eff.} = 1.35$. This circumstance must be borne in mind when using Eq. (3) with a whole number of degrees of freedom. It should also be mentioned that the high values for S_{nn0^+} and $S_{n\gamma 0^+}$ are due to the strong fission competition; this is in line with the conclusions reached in Ref. [7].

Let us now examine the case of the contribution of three channels to the average width $\langle\Gamma_f\rangle_r$. In accordance with Lynn's scheme of transitional states, this occurs for the 1^- state of a compound nucleus. The contributions of two channels are taken to be equal to $(\alpha_1 = \alpha_2 = \alpha)$ and these will be considered, for example, to be completely open. The contribution of the third channel α_3 satisfies the condition $\alpha_3 \leqslant 0.5\ (\alpha_1 + \alpha_2)$. The average fission width $\langle\Gamma_f\rangle_{r^-} = 0.396$ eV at an energy of 0.1 keV. Fig. 2 shows that the differences noted above for the two-channel case decrease, although for S_{nn1^-} and $S_{\nu\gamma 1^-}$ they are still considerable. Where $E = 0.1$ keV (Fig. 2, graphs a-c), the divergences between curves 3 and 4 are approximately 8% for S_{nn1^-} and $S_{n\gamma 1^-}$ and approximately 0.5% for S_{nf1^-}. It decreases with the increase in competition of the neutron channel and at 100 keV (Fig. 2, graphs d-f) it does not exceed ~ 3% for S_{nn1^-} and $S_{n\gamma 1^-}$. However, where $\alpha_3 > 0.5\ (\alpha_1 + \alpha_2)$ and $\alpha_1 \sim \alpha_2$, the differences noted between these approaches should be expected to increase.

It follows from the above that, when calculating average reaction cross-sections of fissionable nculei in the unresolved resonance region, the generalized Porter-Thomas distribution must be used for describing fluctuations in fission widths with a small number of channels. The use of $\nu_{eff.fr}$ for describing Γ_{fr} fluctuations is justified only in the case of very small or very large differences between the relative contributions of channels, for which whole numbers for ν_{fr} can be used with equal justification.

[Figure caption]

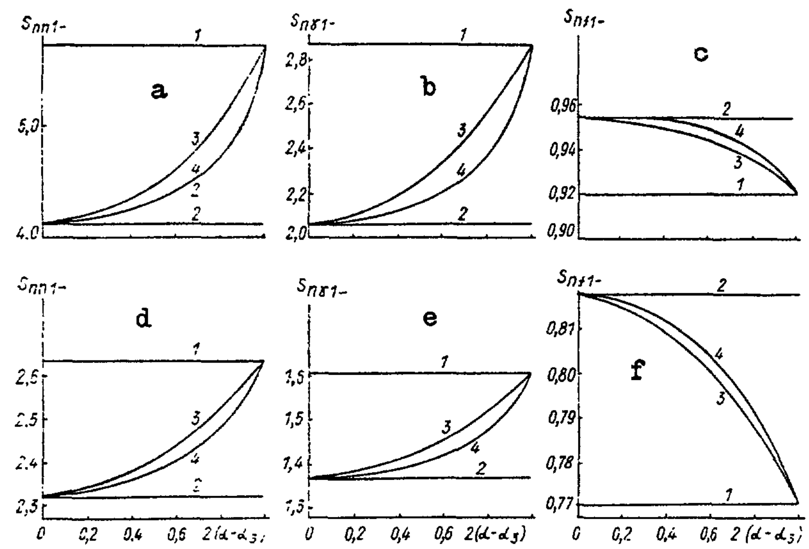
Fig. 1. Dependence of the factor S_{nx0t} on the difference in relative contributions of two fission channels for ^{239}Pu, where E = 0.1 keV (graphs a–c) and 100 keV (graphs d–f): graphs a and d show S_{nn0+}, graphs b and e show $S_{n\gamma 0+}$ and graphs c and f show S_{nf0+}. Curves 1–3 show the Porter–Thomas distribution; in curve 1 $\nu = 2$, in curve 2 [ν =]1*/, in curve 3 $\nu = \nu_{eff}$ and curve 4 shows the generalized Porter–Thomas distribution.



*/ Translator's note: The expression "ν=" has been omitted in the original for curve 2.

[Figure caption]

<u>Fig. 2</u>. Dependence of the factor $S_{nx1}-$ in the case of three fission channels with $2(\alpha-\alpha_3)$ for ^{239}Pu, where E = 0.1 keV (graphs a-c) and 100 keV (graphs d-f): graphs a and d show $S_{nn1}-$, graphs b and e show $S_{n\gamma 1}-$ and graphs c and f show $S_{nf1}-$. For explanation of curves see Fig. 1.



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