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SIMULTANEOUS MULTI-LEVEL ANALYSIS OF THE TOTAL AND  
FISSION CROSS-SECTIONS OF  $^{239}\text{Pu}$  UP TO 160 eV

V.V. Kolesov and A.A. Luk'yanov

Translation from Nuclear Constants 2(46) 3 (1982)

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ABSTRACT

In order to describe cross-sections a multi-level scheme based on S-matrix theory was used taking the Doppler effect and the resolution into account. A multi-level analysis of the total cross-section and fission cross-section of  $^{239}\text{Pu}$  was made by the least-squares method in the energy region up to 160 eV. The experimental cross-section data used have a good resolution. The multi-level parameters can be used to represent all the features of the detailed energy structure of experimental cross-sections where the regions of interference minima are of the greatest interest.

An exact knowledge of the detailed structure of the cross-section of fissionable nuclei in the resonance energy region is of great importance where practical applications are concerned. Although in recent years considerable progress has been made with measurements, the results of different experiments do not fully agree with each other. The problem of the evaluation of cross-sections and the comparison of different experimental results in a given range is therefore rather complex.

In many cases this problem is best solved by deriving an analytical representation of the cross-sections by introducing a given number of parameters obtained by analysing experiments. This parametrization makes it possible, for example, to solve the following problems:

- (1) Intercomparison of the results of different experiments with different resolutions and sample temperatures;
- (2) Reconstitution of the true cross-sections at different temperatures;

- (3) Comparatively easy calculation of functionals from cross-sections;
- (4) Representation of a large number of experimental data with relatively small number of parameters.

At present three basic assumptions have become the most widely used for parametrization and representation of the energy structure of cross-sections: the Brite-Wigner approximation, a scheme based on the R-matrix theory and a scheme based on the S-matrix theory.

The first approximation is easiest to apply but owing to significant interference effects between resonances for many elements, and in particular for  $^{239}\text{Pu}$ , it should really not be used. The main advantage of the R-matrix scheme is simplicity of the interpretation of the parameters obtained. Among its disadvantages the main one to be pointed out is the difficulty arising when taking the Doppler effect and resolution into account. This also makes the search for parameters extremely laborious. For fissionable nuclei it is probably better to use the S-matrix scheme. When so doing it is easy to take into account the Doppler effect and the instrumental resolution and it is possible to represent cross-sections with any number of reaction channels and any degree of interference between resonance levels. The representation is in practice just as convenient for use in various calculations as is the Brite-Wigner approximation.

It should be pointed out that most of the literature, Ref. [1] for example, and also many evaluated data libraries give only resonance parameters obtained with the Brite-Wigner approximation. This makes it necessary to use relatively complex sets of programmes for reconstructing the detailed shape of the cross-section.

Method of calculation. For representing cross-sections in the present paper use was made of the multi-level scheme based on the S-matrix theory. Taking into account the Doppler effect, in accordance with the data of Ref. [2], general cross-section equations are written in the form

$$\begin{aligned} \sigma_T &= \sigma_p + 0,65 \cdot 10^6 / \sqrt{E} \sum_{\lambda} 1/\nu_{\lambda} \left\{ G_{\lambda}^T \Psi \left[ (E - \mu_{\lambda})/\nu_{\lambda}; \nu_{\lambda}/\Delta_T \right] + H_{\lambda}^T \chi \left[ (E - \mu_{\lambda})/\nu_{\lambda}; \nu_{\lambda}/\Delta_T \right] \right\}; \\ \sigma_F &= 0,65 \cdot 10^6 / \sqrt{E} \sum_{\lambda} 1/\nu_{\lambda} \left\{ G_{\lambda}^F \Psi \left[ (E - \mu_{\lambda})/\nu_{\lambda}; \nu_{\lambda}/\Delta_T \right] + H_{\lambda}^F \chi \left[ (E - \mu_{\lambda})/\nu_{\lambda}; \nu_{\lambda}/\Delta_T \right] \right\}, \end{aligned}$$

where  $\sigma_p$  is the potential scattering,  $G_{\lambda}^T$ ,  $H_{\lambda}^T$ ,  $G_{\lambda}^F$ ,  $H_{\lambda}^F$  are parameters,  $\mu_{\lambda}$  is the

position of the resonance,  $v_\lambda$  is its width,  $\Delta_T = \sqrt{4 KTE/(A + 1)}$  is the Doppler width, and  $\Psi$  and  $\chi$  are symmetrical and asymmetrical Doppler functions containing the main energy dependence of the cross-section and taking the resonance form. The resolution of the instruments is taken into account here by replacing  $\Delta_T$  by  $\Delta = \sqrt{\Delta_T^2 + \Delta_R^2}$  where  $\Delta_R^2$  is the resolution dispersion function.

Analysis of experiments and results. For this analysis use was made of the following sets of experimental data:

For  $\sigma_T$  in the region 0.014-4.5 eV, with a resolution of 2.7-0.52  $\mu\text{s/m}$  [3] and also in the region above 4.45 eV, with a resolution of 0.018-0.001  $\mu\text{s/m}$  [4];

For  $\sigma_F$  in the region 0.019-2 eV, with a resolution of 2.5-0.8  $\mu\text{s/m}$  [5], in the region 2-4.5 eV, with a resolution of 0.2-0.03  $\mu\text{s/m}$  [6], in the region 4.5-37.5 eV, with a resolution of 0.025  $\mu\text{s/m}$  [7], and in the region above 37.5 eV with a resolution of 0.007-0.004  $\mu\text{s/m}$  [8].

Virtually all measurements were performed with the best resolution available at present for the particular energy region. In order to match the positions of resonances in the total and fission cross-sections, it proved necessary to shift the energy scales in the measurements taken from Refs [7,8] so as to bring them into line with the scale in Ref. [4]. The shift was made to obey the law  $E' = E - \alpha E + \beta$  where  $\alpha = 0.0047729$ ,  $\beta = 0.0152823$  for Ref. [7], and  $\alpha = 0$  and  $\beta = 0.033$  for Ref. [8]. Analysis of cross-sections was based on the least squares method using the program described in Ref. [9]. The potential scattering was taken to be equal to  $10.3 \text{ b}^{*/}$ . The resonance parameters obtained in this way are shown in the table, together with single-level parameters from Ref. [1]. In Figs 1-3 the reconstructed cross-sections and experimental data are compared.

The results of the multi-level parametrization illustrated the feasibility of simultaneously representing the total cross-section and the fission cross-section of  $^{239}\text{Pu}$  in the resonance region using a single consistent set of resonance parameters. The cross-sections plotted from the parameters found represented all the features of the detailed energy structure of the experimental cross-sections where the region of interference minima is of greatest interest.

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\*/ 1 barn =  $10^{-28} \text{ m}^2$

In the future it is planned to test the derived parameters, mainly  $H_{\lambda}^T$  and  $H_{\lambda}^F$ , against the measurements of neutron spectra and fission cross-sections in filtered beams for relatively thick samples [10].



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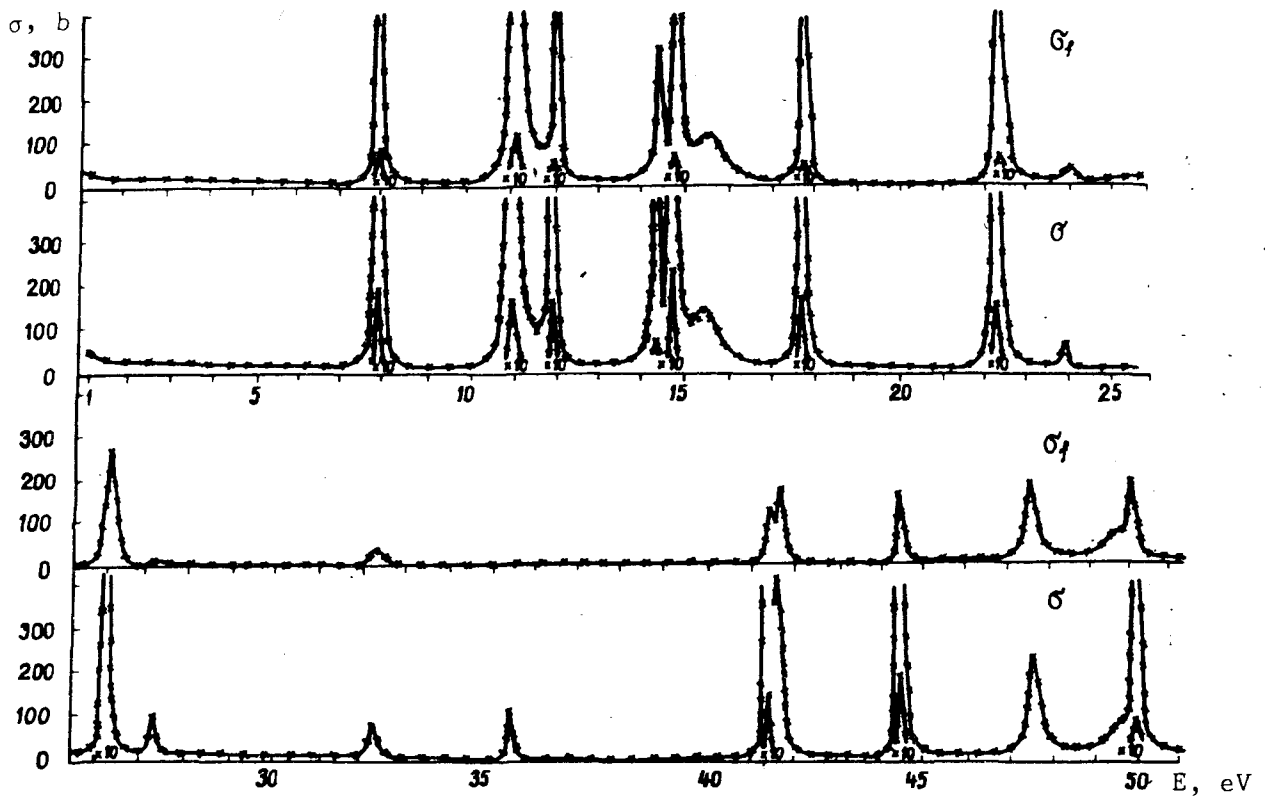


Fig. 1. Calculated and experimental cross-sections in the region 1-50 eV

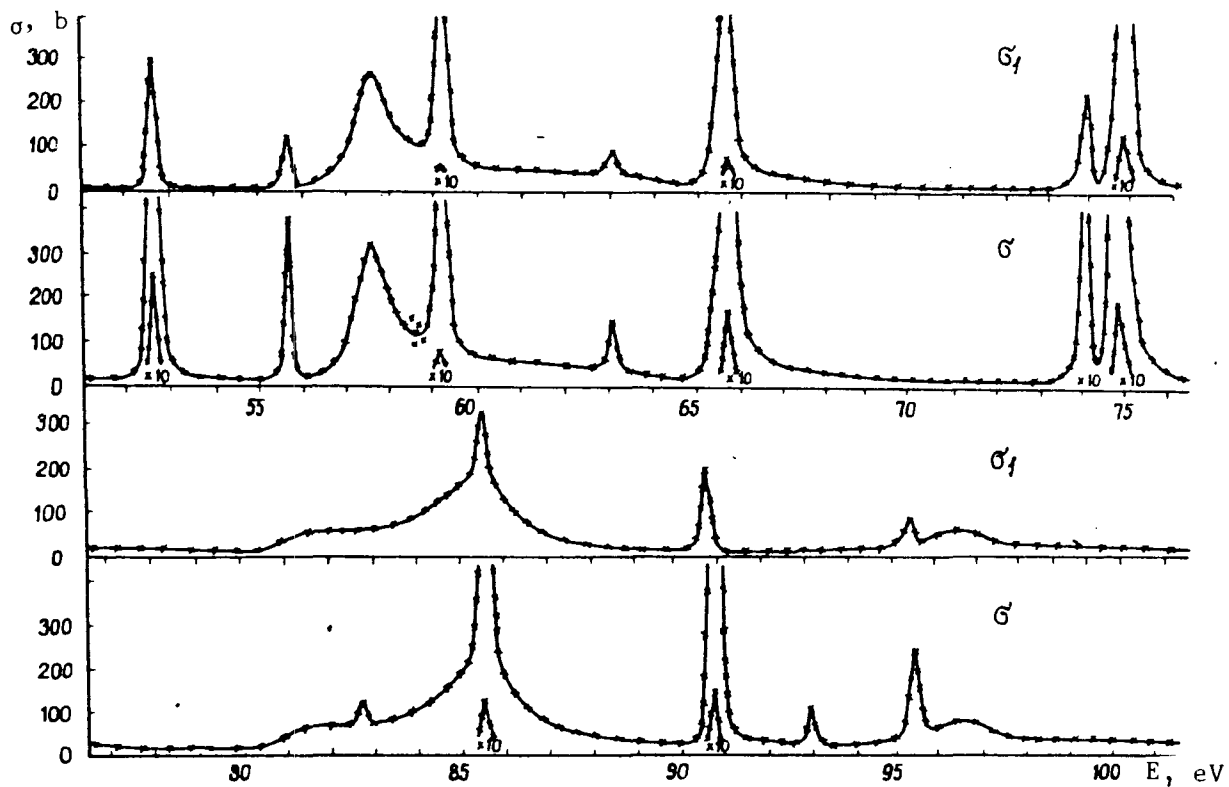


Fig. 2. Calculated and experimental cross-sections in the region 20-100 eV

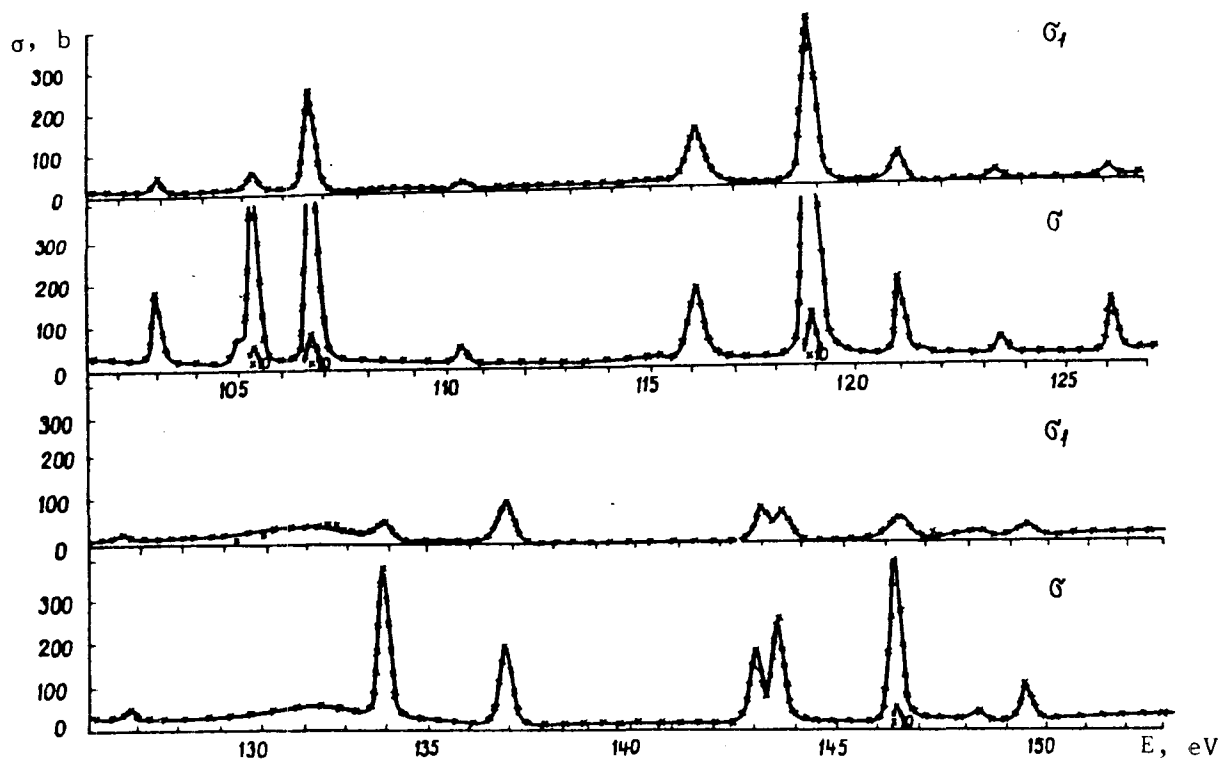


Fig. 3. Calculated and experimental cross-sections in the region 100-150 eV

Resonance parameters for  $^{239}\text{Pu}$  (single-level parameters of Ref. [1] are shown in brackets)

$\mu$ , eV	$\nu$ , eV	$G^T \cdot 10^4$ , eV <sup>1/2</sup>	$H^T \cdot 10^4$ , eV <sup>1/2</sup>	$G^F \cdot 10^4$ , eV <sup>1/2</sup>	$H^F \cdot 10^4$ , eV <sup>1/2</sup>	$\mu$ , eV	$\nu$ , eV	$G^T \cdot 10^4$ , eV <sup>1/2</sup>	$H^T \cdot 10^4$ , eV <sup>1/2</sup>	$G^F \cdot 10^4$ , eV <sup>1/2</sup>	$H^F \cdot 10^4$ , eV <sup>1/2</sup>
-0,26 (-1,81) (-0,08)	0,100 (1,650) (0,035)	0 (3,1998) (0,0518)	0,4274 - -	0 (2,8314) (0,0489)	0,3100 - -	15,42 (15,45)	0,405 (0,350)	2,8498 (2,3758)	-0,1814 -	2,6114 (2,1999)	-0,0598 -
0,30 (0,29)	0,047 (0,047)	2,9322 (2,2315)	0,0482 -	1,3247 (1,3534)	0,0341 -	17,63 (17,65)	0,038 (0,037)	6,4463 (5,8296)	0,0190 -	2,9145 (2,5307)	-0,1193 -
3,15 (5,89)	2,250 (1,651)	0,4700 (0,3870)	0,0830 -	0,4500 (0,3819)	-0,0532 -	22,24 (22,28)	0,052 (0,054)	8,2352 (7,8679)	0,2296 -	4,7341 (4,4822)	0,2243 -
7,81 (7,81)	0,043 (0,044)	4,1733 (4,1017)	-0,0922 -	2,3591 (2,2526)	0,0412 -	23,88 (23,92)	0,046 (0,035)	0,2852 (0,2610)	-0,0430 -	0,1806 (0,1490)	-0,0485 -
10,92 (10,92)	0,089 (0,100)	8,3851 (8,0416)	0,5114 -	6,4547 (6,2786)	0,7015 -	26,23 (26,22)	0,043 (0,042)	4,5331 (3,5111)	0,0262 -	2,2577 (1,9244)	-0,0651 -
11,49 (11,49)	0,026 (0,026)	0,2508 (0,2508)	-	0,0505 (0,0505)	-	27,24 (27,22)	0,025 (0,021)	0,3962 (0,4114)	0,0217 -	0,0558 (0,0586)	-0,0048 -
11,88 (11,88)	0,033 (0,038)	4,2233 (3,8830)	-0,3242 -	1,6253 (1,4645)	-0,2832 -	32,29 (32,29)	0,083 (0,076)	0,7340 (0,6596)	0,0207 -	0,5092 (0,4851)	0,0413 -
14,30 (14,30)	0,053 (0,051)	2,6068 (2,2851)	-0,3247 -	1,5852 (1,5072)	-0,3154 -	34,58 (34,58)	0,046 (0,046)	0,0311 (0,0311)	-	0,0168 (0,0168)	-
14,66 (14,67)	0,036 (0,035)	7,4337 (7,4113)	0,5076 -	3,2002 (3,0963)	0,3614 -	35,43 (35,48)	0,019 (0,024)	0,5888 (0,6861)	0,0006 -	0,0568 (0,0581)	0,0014 -
						41,38 (41,40)	0,023 (0,026)	8,9349 (9,8778)	0,3422 -	0,7149 (0,9455)	0,0664 -
						41,63 (41,64)	0,051 (0,052)	3,0294 (3,4550)	-0,1235 -	1,3963 (1,5393)	-0,0887 -

$\mu$ , eV	$\nu$ , eV	$G^T \cdot 10^4$ , eV <sup>1/2</sup>	$H^T \cdot 10^4$ , eV <sup>1/2</sup>	$G^F \cdot 10^4$ , eV <sup>1/2</sup>	$H^F \cdot 10^4$ , eV <sup>1/2</sup>	$\mu$ , eV	$\nu$ , eV	$G^T \cdot 10^4$ , eV <sup>1/2</sup>	$H^T \cdot 10^4$ , eV <sup>1/2</sup>	$G^F \cdot 10^4$ , eV <sup>1/2</sup>	$H^F \cdot 10^4$ , eV <sup>1/2</sup>
44,44 (44,46)	0,026 (0,029)	13,5942 (14,1207)	0,3824 -	1,2069 (1,3084)	-0,0266 -	74,03 (74,01)	0,039 (0,036)	5,7519 (5,7009)	-0,7261 -	2,6380 (2,5197)	-0,8253 -
47,56 (47,57)	0,141 (0,156)	3,7368 (4,0877)	0,2674 -	3,2502 (3,2136)	0,1571 -	74,90 (74,91)	0,093 (0,073)	34,4847 (36,3043)	2,3414 -	22,3753 (21,1433)	1,0277 -
49,65 (49,68)	0,367 (0,401)	2,4500 (2,8772)	0,1974 -	2,3500 (2,6869)	0,1177 -	78,94 (78,91)	0,063 (0,046)	0,1079 (0,2303)	0,0231 -	0,0022 (0,1219)	-0,0037 -
50,04 (50,05)	0,027 (0,029)	6,3568 (6,8441)	0,1099 -	1,3829 (1,5547)	-0,0474 -	81,13 (81,72)	0,835 (1,023)	2,5826 (4,7818)	4,0203 -	2,3074 (4,6613)	3,7796 -
52,54 (52,57)	0,029 (0,034)	20,2067 (19,8633)	0,7271 -	2,7112 (2,4681)	0,0467 -	82,66 (82,64)	0,024 (0,035)	0,5693 (0,8259)	0,0167 -	0,0338 (0,3456)	-0,0287 -
55,58 (55,60)	0,029 (0,029)	3,0807 (3,6128)	-0,0041 -	1,1610 (1,3211)	-0,1479 -	83,48 (83,48)	0,875 (1,3404)	-	-	(1,3057)	-
57,42 (57,41)	0,466 (0,255)	15,0716 (10,6615)	6,2855 -	13,7402 (9,3002)	5,2303 -	85,42 (85,28)	1,165 (1,049)	28,6130 (27,8232)	-3,4829 -	25,9597 (26,5710)	-4,7143 -
58,81 (58,81)	0,551 (0,551)	7,5472 (7,5472)	-	7,4624 (7,4624)	-	85,49 (85,44)	0,038 (0,037)	12,5233 (12,3335)	0,4069 -	2,3643 (2,8105)	-0,1821 -
59,16 (59,19)	0,069 (0,090)	9,1128 (10,5124)	0,8103 -	6,5670 (7,0514)	-0,249 -	90,72 (90,70)	0,030 (0,030)	17,6433 (17,8340)	0,8753 -	2,6941 (2,6829)	-0,0635 -
63,60 (60,91)	3,511 (3,399)	10,0000 (12,9075)	-11,9197 -	9,8000 (12,7871)	-10,9950 -	92,97 (92,92)	0,021 (0,029)	1,0150 (1,3749)	-0,0112 -	0,0930 (0,2013)	-0,0685 -
63,03 (63,05)	0,049 (0,076)	1,1107 (1,5228)	0,0007 -	0,7143 (1,0962)	-0,0189 -	95,37 (95,31)	0,039 (0,049)	2,9964 (3,2589)	0,2744 -	0,8838 (0,9632)	-0,0540 -
65,45 (65,33)	0,196 (0,046)	7,2213 (0,6373)	2,1266 -	5,3894 (0,3435)	3,1132 -	96,65 (96,44)	0,732 (0,850)	5,3368 (6,9614)	-1,1699 -	4,9017 (6,7293)	-1,0593 -
65,70 (65,68)	0,336 (0,068)	16,7377 (20,5695)	1,9750 -	5,9393 (11,0313)	-0,254 -	98,87 (100,20)	4,652 (3,001)	14,5392 (6,0530)	1,5010 -	14,0253 (5,9975)	-0,8727 -

$\mu$ , eV	$\nu$ , eV	$G^T \cdot 10^4$ , eV <sup>1/2</sup>	$H^T \cdot 10^4$ , eV <sup>1/2</sup>	$G^F \cdot 10^4$ , eV <sup>1/2</sup>	$H^F \cdot 10^4$ , eV <sup>1/2</sup>	$\mu$ , eV	$\nu$ , eV	$G^T \cdot 10^4$ , eV <sup>1/2</sup>	$H^T \cdot 10^4$ , eV <sup>1/2</sup>	$G^F \cdot 10^4$ , eV <sup>1/2</sup>	$H^F \cdot 10^4$ , eV <sup>1/2</sup>
103,01 (102,94)	0,025 (0,023)	2,3071 (2,5535)	0,1037 -	0,4703 (0,4906)	-0,0383 -	132,04 (131,69)	1,575 (1,900)	11,1373 (16,5727)	-1,0295 -	10,6627 (16,2195)	-2,3901 -
105,31 (105,25)	0,030 (0,024)	6,5878 (6,2304)	0,2973 -	0,6896 (0,7794)	-0,0168 -	133,61 (133,72)	0,022 (0,028)	6,4109 (6,4892)	0,1869 -	0,8250 (0,7599)	-0,0349 -
106,69 (106,62)	0,036 (0,038)	12,6625 (13,9489)	0,0342 -	4,6437 (4,7907)	-0,3053 -	135,23 136,79 (136,68)	7,506 0,050 (0,063)	10,6572 4,1424 (4,2112)	-6,7919 0,1111 -	6,8196 2,7971 (2,7774)	-1,6538 -0,2782 -
110,42 (110,33)	0,030 (0,022)	0,6378 (0,6807)	0,0206 -	0,2580 (0,2035)	-0,0395 -	139,21 (139,21)	0,00002 (0,161)	0,0444 (0,1363)	0,0303 -	0 (0,1183)	0,0061 -
113,96 (114,38)	0,919 (0,749)	0,2115 (0,7848)	0,2115 -	0,2100 (0,7613)	0,9800 -	142,96 (142,85)	0,041 (0,069)	3,7593 (4,2409)	-0,0849 -	2,4286 (2,4696)	-0,3129 -
115,28 (115,04)	0,086 (0,103)	0,1852 (0,3210)	-0,1980 -	0 (0,2560)	-0,1901 -	143,48 (143,40)	0,042 (0,042)	5,1322 (5,1962)	0,5257 -	2,2537 (1,8748)	-0,0155 -
116,06 (115,97)	0,122 (0,134)	4,8072 (5,4545)	0,5479 -	4,2813 (4,4244)	-0,0809 -	146,14 (147,37)	0,406 (0,501)	1,4373 (1,0995)	0,5271 -	1,2514 (1,0494)	-0,2300 -
118,84 (118,77)	0,041 (0,051)	22,3501 (25,7776)	0,9402 -	9,0941 (10,3383)	-0,4038 -	146,27 (146,18)	0,002 (0,035)	6,5670 (8,9219)	0,7200 -	1,0224 (1,5253)	0,0888 -
121,02 (120,93)	0,024 (0,039)	3,0593 (3,7027)	-0,0709 -	1,4482 (1,8166)	-0,2362 -	148,29 (148,14)	0,047 (0,075)	0,4334 (0,5148)	-0,0964 -	0,3032 (0,3583)	-0,3138 -
123,48 (123,38)	0,040 (0,032)	0,6926 (0,6318)	-0,0036 -	0,3583 (0,3783)	-0,1555 -	148,93 149,45 (149,35)	2,402 0,026 (0,059)	2,9578 1,6839 (1,9555)	-1,0449 0,1667 -	2,9360 0,5990 (0,8740)	3,0953 -0,2506 -
126,23 (126,14)	0,020 (0,046)	2,1035 (2,7451)	0,1266 -	0,4517 (0,5728)	-0,0093 -	156,22 157,08 (157,01)	0,162 0,342 (0,613)	0,1900 13,9589 (13,8339)	-0,3589 1,2623 -	0,1800 9,9080 (6,1005)	0,3213 -0,4117 -