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DISSOCIATION OF HYDROGEN IONS IN $H_2^+ + H_2^+$ AND $H_3^+ + H_3^+$ REACTIONS AT LOW ENERGIES

V.A. Belyaev, M.M. Dubrovin, L.I. Men'shikov
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DISSOCIATION OF HYDROGEN IONS IN $H_2^+ + H_2^+$ AND $H_3^+ + H_3^+$ REACTIONS AT LOW ENERGIES

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The dissociation cross-sections for H_2^+ and H_3^+ ions in $H_2^+ + H_2^+$ and $H_3^+ + H_3^+$ reactions are found at collision energies lower than 100 eV. It is shown that simultaneous dissociation of two colliding ions is the most likely process. Theoretical cross-section values agree with the experimental results.

INTRODUCTION

As the density and degree of ionization of a hydrogen plasma increase, elementary processes resulting from paired ion collisions may begin to play an important role. One of them - dissociation of molecular hydrogen ions in collisions with each other ($H_2^+ + H_2^+$) - has recently attracted attention in connection with the development of research into controlled thermonuclear fusion. To inject $H_2^+(D_2^+)$ ions into open traps, the ions must travel from a gas-discharge source over a distance of several metres as part of an intense beam. This process in a beam with low ion collision energy, may substantially reduce the number of ions. There are indirect indications that this process, occurring also at low energies in the gas-discharge source plasma, may affect the yield from the source of H^- ions [1, 2], used in some tokamak beam heating systems. Low energies may also turn out to be the most important energies for this process in the region near the wall in modern thermonuclear facilities. However, at present, it is not even possible to evaluate the contribution of this process to proton production from H_2^+ at low energies, since there have been no papers devoted to a study of it.

The merging overlapping beam technique [3-5] is a suitable method for studying low-energy processes experimentally, and can be used to measure the cross-sections of inelastic collisions of two ions in a wide range of energies up to a hundredth even a thousandth of an electron volt [6].

When the ions are completely identical, however, the difficult experimental task of spatially merging two ion beams with virtually identical parameters has to be solved in order to use that technique.

On the other hand, the difficulty of a theoretical examination is that accurate calculation of dissociation cross-sections for H_2^+ ions then involves solving the quantum-mechanical problem of the motion of six bodies (four protons and two electrons) and in the case of H_3^+ ions, eight bodies.

This paper is the first attempt to obtain dissociation cross-sections for hydrogen ions in collisions with each other at energies lower than 100 eV both theoretically and experimentally.

1. CALCULATION OF THE DISSOCIATION CROSS-SECTIONS OF H_2^+ IONS IN $H_2^+ + H_2^+$ REACTIONS AT COLLISION ENERGIES OF 10-100 eV

In view of the difficulty of quantum-mechanical calculations of the motion of six, or even eight, bodies, simple and fairly general methods of evaluating dissociation cross-sections in order of magnitude are particularly important. In our work, we have used the theory of sudden perturbations to determine the dissociation cross-sections of H_2^+ and H_3^+ ions at energies of $10 \lesssim E(\text{eV}) \lesssim 100$ with about 50% accuracy. Since in this energy range the ion approach velocity ($\lesssim 10^7$ cm/s) is small compared to the characteristic electron velocity in the H_2^+ ion ($\sim 2 \times 10^8$ cm/s), ion excitation and ionization is more unlikely than dissociation caused by their electrical interaction.

To evaluate the dissociation cross-sections of H_2^+ ions in

$$H_2^+ + H_2^+ \begin{cases} \nearrow H_2^+ + H + p \\ \searrow H + p + H + p \end{cases} \qquad (1)$$

reactions and in similar reactions involving H_3^+ ions we assume that an effective charge $q = \frac{1}{2}$ and $q = 1/3$, respectively, is concentrated near each proton in these ions (here and further on $\hbar = me = e = 1$). More accurate analysis by the variational method gives the value $q = 0.53$ for the H_2^+ ion, which does not differ significantly from $q = \frac{1}{2}$. Therefore, the interaction energy between protons in H_2^+ ions is

$$V_0 = q^2 \left(\frac{1}{|\vec{R}_1 - \vec{R}_3|} + \frac{1}{|\vec{R}_1 - \vec{R}_4|} + \frac{1}{|\vec{R}_2 - \vec{R}_3|} + \frac{1}{|\vec{R}_2 - \vec{R}_4|} \right), \qquad (2)$$

where $\vec{R}_{1,2}$ and $\vec{R}_{3,4}$ are the proton co-ordinates in the first and second H_2^+ ions, respectively.

Let us assume that two conditions are fulfilled:

$$\rho_0 \gg \frac{r_0}{2}, \tag{3}$$

$$\tau_{coll} \sim \frac{\rho_0}{v} \gg \frac{1}{\omega}, \tag{4}$$

where ρ_0 and v are the characteristic impact parameter and relative collisional velocity of H_2^+ ions; $\omega \approx 0.01$ a.u. is the vibrational frequency of H_2^+ ions; $r_0 = 2$ a.u. is the equilibrium distance between the protons in H_2^+ ions.

Condition (3) enables us to perform an expansion in equation (2) with respect to the small parameter $r_0/2\rho_0$. Condition (4) means that during the collision time τ_{coll}, the directions of the H_2^+ ion axes and the distances between protons making up the H_2^+ do not have time to alter. The theory of sudden perturbations can therefore be applied.

For the interaction energy, we obtain from Eqs (2) and (3)

$$V_0 = \frac{1}{R} + V(\vec{r}_1, \vec{r}_2, \vec{R}), \tag{5}$$

where

$$V(\vec{r}_1, \vec{r}_2, \vec{R}) = q^2 \frac{3n_i n_k - \delta_{ik}}{R^3} \left[(r_1)_i (r_1)_k + (r_2)_i (r_2)_k \right];$$

$\vec{R}$ is the vector uniting the centres of gravity of the H_2^+ ions; $\vec{r}_{1,2}$ are the vectors uniting protons in the first and second H_2^+ ions, respectively: $\vec{n} = \vec{R}/R$.

Since motion of the protons during the dissociation of H_2^+ ions is quasi-classical we can apply the laws of classical mechanics to describe it.

The classical Hamiltonian of the $H_2^+ + H_2^+$ system is

$$H = H_0(\vec{R}) + \frac{\vec{p}_1^{\,2}}{M_p} + \frac{\vec{p}_2^{\,2}}{M_p} + u(r_1) + u(r_2) + V(\vec{r}_1, \vec{r}_2, \vec{R}),$$

where $H_o(\vec{R}) = \frac{\vec{p}^2}{2M_p} + \frac{1}{R}$; $\vec{p} = M_p\vec{v}_{rel}$ is the relative ion pulse; $\vec{p}_1$, $\vec{p}_2$ are the relative proton pulses in the first and second H_2^+ ions, respectively; M_p is the proton mass; and $u(\vec{r})$ is the potential energy of the protons in the H_2^+ ion ground state.

If condition (4) is fulfilled, the following substitution is valid:

$$\frac{3n_i n_k - \delta_{ik}}{R^3} \longrightarrow \delta(t)\int_{-\infty}^{+\infty} dt\, \frac{3n_i n_k - \delta_{ik}}{R^3} ,$$

where the integral is performed over the trajectory of relative ion movement $\vec{R}(t)$. Below we prove that when conditions (3) and (4) are fulfilled, the trajectory is virtually a straight line, hence $\vec{R} = \vec{\rho} + \vec{v}t$, where ρ is the impact parameter. For straight-line trajectories we have

$$\frac{3n_i n_k - \delta_{ik}}{R^3} \longrightarrow \delta(t)\,\frac{2Q_{ik}}{v\rho^2} ,$$

where $Q_{ik} = 2S_i S_k + K_i K_k - \delta_{ik}$; $\vec{S} = \frac{\vec{\rho}}{\rho}$; $\vec{K} = \frac{\vec{v}}{v}$ $(\vec{K} \perp \vec{S})$.

Therefore in the approximation of the theory of sudden perturbations, the Hamiltonian of the $H_2^+ + H_2^+$ system equals

$$H = \frac{\vec{p}_1^{\,2}}{M_p} + \frac{\vec{p}_2^{\,2}}{M_p} + u(r_1) + u(r_2) + \frac{\delta(t)q^2}{v\rho^2}Q_{ik}\left[(r_1)_i(r_1)_k + (r_2)_i(r_2)_k\right].$$

From this Hamiltonian, we conclude that in the approximations (3) and (4), the proton movement in different ions takes place independently.

Equations for proton motion in the first ion are:

$$\dot{\vec{r}}_1 = \frac{2}{M_p}\vec{p}_1 ,$$

$$(\dot{p}_1)_i = -\nabla_i u(r_1) - \frac{2\delta(t)q^2}{v\rho^2} - Q_{ik}(r_1)_k$$

When $t = 0$, the pulse $\vec{p}_1$ at the fixed co-ordinate $\vec{r}_1$ instantly changes.

Let us assume that before collision, the H_2^+ ions were in ground vibrational states, i.e. when $t = -0 \vec{r}_1 = r_o \vec{i}_1$, $\dot{\vec{r}}_1 = 0$, where $\vec{i}_1$ is the unit vector along the axis of the first H_2^+ ion. Then, when $t = +0 \vec{r}_1 = r_o \vec{0}_1$

$$(P_1)_i = -\frac{2q^2 r_o}{v\rho^2} Q_{ix} (i_1)_x .$$

Since $u(r_o) = -D$, where $D = 2.7$ eV, is the H_2^+ dissociation energy, the energy of the first ion after collision is

$$E_1 = -D + \frac{\vec{P}_1^2}{M_p} = -D + \frac{4q^4 r_o^2}{M_p v^2 \rho^4} \left[Q_{ix} (i_1)_x \right]^2 =$$

$$= -D + \frac{4q^4 r_o^2}{M_p v^2 \rho^4} \left[1 - (\vec{k} \vec{i}_1)^2 \right] .$$

When $E_1 \geqslant 0$, the first ion dissociates, therefore the probability of its dissociation is

$$W_1 = \theta(E_1) = \theta(\rho_o^2 \sin \vartheta_1 - \rho^2) , \qquad (6)$$

where $\theta(x) = \begin{cases} 1 & \text{when } x > 0, \\ 0 & \text{when } x < 0; \end{cases}$ θ_1, is the angle between the vectors $\vec{i}_1$ and $\vec{k}$;

$$\rho_o^2 = \frac{2q^2 r_o}{v\sqrt{M_p D}} = \frac{r_o}{2v\sqrt{M_p D}} . \qquad (7)$$

By analogy with Eq. (6), the probability of dissociation of the second H_2^+ ion is:

$$W_2 = \theta(\rho_o^2 \sin \vartheta_2 - \rho^2) . \qquad (8)$$

Let us introduce the following notations for the cross-sections: $\sigma_{1\bar{2}}$ is the dissociation cross-section of the first ion provided that the second ion does not dissociate; σ_{12} is the dissociation cross-section of the two ions simultaneously.

Obviously,

$$\sigma_{1\bar{2}} = \sigma_{\bar{1}2} = 2\pi \left\langle \int_0^\infty W_1(1-W_2)\rho\, d\rho \right\rangle , \tag{9}$$

where the angular brackets indicate the average over the axes of the H_2^+

$$\langle \ldots \rangle = \frac{1}{4} \iint_0^\pi \sin\vartheta_1 \sin\vartheta_2 \, d\vartheta_1 d\vartheta_2 \ldots = \iint_0^{\frac{\pi}{2}} \sin\vartheta_1 \sin\vartheta_2 \, d\vartheta_1 d\vartheta_2 \ldots$$

From Eqs (6), (8) and (9) we obtain

$$\begin{aligned}
\sigma_{1\bar{2}} = \sigma_{\bar{1}2} &= 2\pi \left\langle \int_0^\infty \theta(\rho_0^2 \sin\vartheta_1 - \rho^2)\theta(\rho^2 - \rho_0^2 \sin\vartheta_2)\rho\, d\rho \right\rangle = \\
&= \pi\rho_0^2 \left\langle (\sin\vartheta_1 - \sin\vartheta_2)\theta(\sin\vartheta_1 - \sin\vartheta_2) \right\rangle = \\
&= \pi\rho_0^2 \iint_0^{\frac{\pi}{2}} \sin\vartheta_1 \sin\vartheta_2 (\sin\vartheta_1 - \sin\vartheta_2)\theta(\vartheta_1 - \vartheta_2) d\vartheta_1 d\vartheta_2 = \\
&= \pi\rho_0^2\left(\frac{\pi}{4} - \frac{2}{3}\right) \approx 0{,}37\rho_0^2
\end{aligned} \tag{10}$$

Similarly to this equation, we have

$$\sigma_{12} = \frac{2}{3}\pi\rho_0^2 \approx 2{,}1\rho_0^2 . \tag{11}$$

The dissociation cross-section of the first ion, regardless of the state of the second, is:

$$\sigma_1 = \sigma_{12} + \sigma_{1\bar{2}} = \frac{\pi^2}{4}\rho_0^2 \approx 2{,}47\rho_0^2 . \tag{12}$$

The dissociation cross-section of one of the ions

$$\sigma = \sigma_{1\bar{2}} + \sigma_{\bar{1}2} \approx 0{,}74\,\rho_0^2 \tag{13}$$

is 35% σ_{12}. Therefore, simultaneous dissociation of both colliding ions is the most likely process.

Substituting Eq. (7) into expressions (3) and (4), we obtain the following validity criteria for expressions (10)-(13):

$$10 \ll E(\mathrm{eV}) \ll 100\,, \tag{14}$$

where E is the total kinetic energy of colliding particles in the centre-of-mass system. The parameter $r_o/2\rho_o^+ \sim 0.5$, and hence the accuracy of Eqs (10) to (13), is not very great (~50%).

Substituting into Eqs (10)-(13) numerical values for parameters r_o and D for H_2^+ ions we obtain (cross-sections, cm^2):

$$\begin{aligned} \sigma_{12} &= \frac{6{,}9\cdot 10^{-16}}{\sqrt{E(\mathrm{eV})}}\,, \\ \sigma_{1} &= \frac{8{,}1\cdot 10^{-16}}{\sqrt{E(\mathrm{eV})}}\,, \\ \sigma_{\bar{1}2} = \sigma_{1\bar{2}} &= \frac{1{,}2\cdot 10^{-16}}{\sqrt{E(\mathrm{eV})}}\,, \\ \sigma = 2\sigma_{1\bar{2}} &= \frac{2{,}4\cdot 10^{-16}}{\sqrt{E(\mathrm{eV})}}\,. \end{aligned} \tag{15}$$

In the near threshold region $2.7 < E(\mathrm{eV}) < 10$, Eqs (15) are not valid for two reasons. Firstly, in this area the approximation of the theory of sudden perturbations is not valid since the protons have time to adjust to the change in electrostatic potential (2). Secondly, at energies $E \lesssim 20$ eV, Coulomb repulsion between the ions becomes significant. As a result of these two effects there is a reduction in the dissociation cross-sections at energies of $T \lesssim 20$ eV.

From Eq. (7) it follows that

$$\sigma_{\rm diss} \sim \frac{1}{\sqrt{D}} ,$$

hence the dissociation cross-section of the H_2^+ ion in an excited vibrational state is greater than that of the ion in the ground state. Suppose that H_2^+ ions are produced by ionization of hydrogen molecules due to electron impact. According to the Franck-Condon principle, the probability $W(\nu)$ of an H_2^+ ion being in a vibrational state with a quantum number ν is equal to the square of the overlap integral for the wave functions of the ν-th vibrational H_2^+ state and the ground vibrational H_2 state. To evaluate the influence of vibrational excitation of the ions on the dissociation cross-section in terms of vibrational H_2^+ and H_2 wave functions, let us use the wave functions of a harmonic oscillator. Then for the probabilities $W(\nu)$, we obtain the values: $W(0) = 0.098$; $W(1) = 0.292$; $W(2) = 0.350$; $W(3) = 0.204$; $W(4) = 0.053$ (probabilities $W(\nu \geqslant 5)$ are small). This calculation of vibrational ion excitation means that the dissociation cross-section increases by the multiple

$$\eta = \sqrt{D} \sum_{\nu} \frac{W(\nu)}{\sqrt{D - \omega\nu}} \approx 1{,}12 .$$

As evaluations show, in the range $10 < E(\mathrm{eV}) < 20$ Coulomb repulsion leads to a 50% reduction in dissociation cross-sections, as calculated from Eqs (15). Therefore, none of the three effects enumerated above move the calculated cross-sections beyond the limits of accuracy of Eqs (15) over the energy range of Eq. (14).

The angular and energy distribution of protons formed in reactions (1) is determined mainly by the Coulomb scattering of H_2^+ ions.

In the zero-order approximation in parameter $r_0/2\rho_0$ this scattering is defined by $1/R$ in Eq. (5). In this approximation, the H_2^+ ions scatter elastically and the angular distribution of protons may easily be obtained by the Rutherford formula. The proton distribution function through angles χ and with energies ε_p in the centre-of-mass system for $H_2^+ + H_2^+$ is

$$dW \sim \left[W_1(\rho) + W_2(\rho)\right] 2\pi\rho\,\delta(\varepsilon_p - \varepsilon_o)\,d\rho\,d\varepsilon_p \qquad (16)$$

where $W_{1,2}$ is determined by Eqs (6) and (8); $\varepsilon_o = 1/8M_p v^2$. Averaging over the orientations of the ion axes in Eq. (16), we obtain

$$dW = \frac{4\delta(\varepsilon_p - \varepsilon_o)}{a}\left\{\frac{\cos\frac{\chi}{2}}{\sin^3\frac{\chi}{2}} F\left[\frac{1}{a}\,\mathrm{ctg}^2\frac{\chi}{2}\right] + \frac{\sin\frac{\chi}{2}}{\cos^3\frac{\chi}{2}} F\left[\frac{1}{a}\,\mathrm{tg}^2\frac{\chi}{2}\right]\right\} d\varepsilon_p\,d\chi\,,$$

where χ is the angle of elastic scattering of H_2^+ ions in the centre-of-mass system ($0 \leqslant \chi \leqslant \pi$):; $a = M_p^2 v^4\ \rho_o^2$;

$$F(\chi) = \theta(1-\chi)\int_0^{\pi/2}\left[\frac{\pi}{2} - \mathrm{tg}\,\varphi\,\ln\left(\mathrm{ctg}\,\frac{\varphi}{2}\right)\right]\theta(\sin\varphi - \chi)\sin\varphi\,d\varphi\,. \qquad (17)$$

A plot of the function $F(\chi)$ is shown in Fig. 1. The W function is normalized to unity

$$\int_0^\infty d\varepsilon_p \int_0^\pi \frac{dW}{d\varepsilon_p\,d\chi}\,d\chi = 1\,,$$

i.e. has a meaning of probability.

The intensity of the proton flux is equal to the number of protons fixed per unit time per unit solid angle. Therefore the intensity is proportional to

$$J_{\text{цм}} = \int \frac{dW}{d\Omega_p\,dv_p}\,v_p\,dv_p = \frac{v}{8\pi a}\,G(\chi)\,, \qquad (18)$$

where $$G(\chi) = \frac{1}{\sin^4\frac{\chi}{2}} F\left(\frac{1}{a}\,\mathrm{ctg}^2\frac{\chi}{2}\right) + \frac{1}{\cos^4\frac{\chi}{2}} F\left(\frac{1}{a}\,\mathrm{tg}^2\frac{\chi}{2}\right).$$

The dependence $\mathcal{J}_{c.m.}(\chi)$ when the kinetic ion energy in the centre-of-mass system is $E = \frac{\mu v^2}{2} = \frac{M_p v^2}{2} = 30$ eV is shown in Fig. 2. The proton intensity near $\chi = 0$ and $\chi = \pi$ is almost zero since the values of χ correspond to the large impact parameters $\rho > \rho_o$ at which the ions do not dissociate.

Evaluations show that distortions in angular and energy proton distribution as a result of spread due to pulses occurring during dissociation of H_2^+ ions are insignificant and can be described by the small parameter $\xi \approx 0.04\sqrt{E\ (eV)}$.

2. DISSOCIATION OF H_2^+ IONS IN $H_3^+ + H_3^+$ REACTIONS

An H_3^+ ion may dissociate into two types of fragments: $H_2^+ + H$ or $H_2 + p$, where H_2 is either a hydrogen molecule, or H + H. The $H_2^+ + H$ system has 2.4 eV more energy than $H_2 + p$, and is obtained from $H_2 + p$ as a result of the non-resonance recharge reaction $H_2 + p \rightarrow H_2^+ + H - 2.4$ eV. Hence it is the $H_2 + p$ decay channel that corresponds to the ground electron term of the H_3^+ ion. Decay to $H_2^+ + H$ occurs only when the electrons in the H_3^+ ion are excited and is therefore rare.

The main reaction channels are

$$H_3^+ + H_3^+ \begin{cases} H_3^+ + H_2 + p \\ H_2 + p + H_2 + p \end{cases}$$

The H_3^+ ion is an equilateral triangle with sides b = 0.88 Å. The dissociation energy along the $H_2 + p$ channel is D = 4 eV.

The Lagrangian for the protons in the H_3^+ ion colliding with a rapidly moving point charge has the form

$$\mathcal{L} = \frac{1}{2} M_p \sum_n \dot{\vec{r}}_n^{\,2} - \sum_n U(\vec{r}_n) - \frac{1}{2} \sum_n (\vec{r}_n \nabla_R)(\vec{r}_n \nabla_R) \frac{1}{R} ,$$

where $\vec{R} = \vec{\rho} + \vec{v}t$; $\vec{r}_n (n = 1, 2, 3)$ are the co-ordinates of the protons in the H_3^+ ion, reckoned from the centre of gravity of the ion. In equilibrium $|\vec{r}_n| = r_o = b/\sqrt{3}$.

Writing out the equations of motion for the protons and applying the same reasoning as in the case of $H_2^+ + H_2^+$, we find the pulses imparted to the protons:

$$\vec{P}_n = \frac{2}{3v\rho^2}\left[\vec{r}_n - 2\vec{S}(\vec{S}\vec{r}_n) - \vec{K}(\vec{K}\vec{r}_n)\right].$$

The energy of the H_3^+ ion after collision is

$$E = -D + \frac{1}{2M_p}\sum_n (\vec{P}_n)^2 =$$

$$= -D + \frac{2}{9M_p v^2\rho^4}\left(\delta_{ik} - K_i K_k\right) Q_{ik} \quad ,$$

where $$Q_{ik} = \sum_n (r_n)_i (r_n)_k = \frac{3}{2} r_0^2 \left(\delta_{ik} - m_i m_k\right),$$

$\vec{m}$ is the normal to the plane of the H_3^+ ion. Hence we obtain:

$$E = -D + \frac{r_0^2}{3M_p v^2 \rho^4}\left(1 + \cos^2\vartheta\right),$$

where θ is the angle between $\vec{m}$ and $\vec{k}$. The probability of ion dissociation is

$$W = \theta(E) = \theta\left(\rho_0^2\sqrt{1+\cos^2\vartheta} - \rho^2\right),$$

where $$\rho_0^2 = \frac{r_0}{v\sqrt{3M_p D}}.$$

By analogy with Eqs (11) and (12) we obtain

$$\sigma_{12} = 2\pi\rho_0^2\left\{\frac{1}{\sqrt{2}} + \frac{1}{2}\ln(1+\sqrt{2}) - \frac{2\sqrt{2}}{3} + \frac{1}{3}\right\} \approx 3{,}36\,\rho_0^2,$$

$$\sigma_1 = \pi\rho_0^2\left\{\frac{1}{\sqrt{2}} + \frac{1}{2}\ln(1+\sqrt{2})\right\} \approx 3{,}59\,\rho_0^2,$$

$$\sigma_{1\bar{2}} = \sigma_{\bar{1}2} = \sigma_1 - \sigma_{12} \approx 0{,}23\,\rho_0^2.$$

Substituting the numerical values of parameters r_o and D for the H_3^+ ion into these equations, we obtain (cross-section cm^2; energy eV):

$$\sigma_{12} = \frac{4{,}8 \cdot 10^{-16}}{\sqrt{E}} ,$$

$$\sigma_{1} = \frac{5{,}1 \cdot 10^{-16}}{\sqrt{E}} ,$$

$$\sigma_{1\bar{2}} = \sigma_{\bar{1}2} = \frac{3{,}3 \cdot 10^{-17}}{\sqrt{E}} ,$$

where $E = \frac{\mu v_{rel}^2}{2}$, $\mu = 3/2\ M_p$. These equations have an accuracy of ~50% over the energy range $20 < E(eV) < 100$.

3. EXPERIMENTAL METHOD FOR STUDYING COLLISIONS OF IDENTICAL IONS

As has been said already, the technique of merged overlapping beams to study non-elastic processes resulting from the collision of two ions at low energies, causes considerable experimental difficulties when the colliding ions are identical. On the other hand, the identical nature of the ions means that the technique can be modified so that studying the results of their collisions at low energies can be made much easier. Ion beams travelling in the same direction do not merge spatially but cross in a small angle. That possible experimental solution to the technique of overlapping (non-merging) beams is given in Refs [3, 7, 8].

Let us examine two monokinetic beams crossing at angle ϕ. The relative collisional velocity of particles composing different beams with identical velocities v_o, in the case of a small angle is $v = \phi v_o$. Hence the collision energy, i.e. the particle energy in one beam in the co-ordinate system associated with the particles of the other beam, given equal mass, will be:

$$T = \varphi^2 E_o , \tag{19}$$

where E_o is the particle energy in the laboratory co-ordinate system. Taking ion beam particle energy of several keV as a convenient size for the purpose of the experiment, we find that to obtain collision energies of a few eV, the crossing angle must definitely be $\phi \approx 1 - 2^o$. To create such a small cross angle for identical ion beams is not much easier than spatially merging beams of almost identical parameters.

However, if the colliding ions are completely identical, one single beam may be used. To do this, it must be fixed so that the ion trajectories cross and hence collide in the region of the focal point. As a result new particles are formed in the beam and the process that takes place may be judged from the type and quantity of the particles. The maximum crossing angle for the trajectories under these conditions will equal the angle of divergence of the beam ϕ, and the minimum angle will be zero, which leads to collisions in the energy range 0 - T, in accordance with Eq. (19). That range may be reduced by placing a screen inside the beam in front of the focal point so that it interrupts the trajectories of some of the ions i.e. creates a hollow (split) beam. The dimensions and location of the trajectory chopper determine the range of crossing angles in the region of the beam focal point and therefore also the range of collision energies.

In order to determine the cross-sections of the non-elastic process in an experimental system of this type, let us examine two plane beams with particle velocities v_1 and v_2, intersecting at angle ϕ (see Fig. 3).

Let the collision of these particles be accompanied by an elementary process leading to the production of new "secondary" particles which can be separated and recorded. Then the rate of their production, equal to the number of effective collisions per unit of time (i.e. those which lead to the process we are considering), is

$$N_{eff} = N_1 \left[1 - \exp(-t/\tau)\right]$$

Here N_1 is the intensity of the first beam in particles/s; τ is the average lifetime of the first beam particles before their effective collision with particles of the second beam; t is the interaction time of particles in the first beam with particles in the second.

The experimental conditions were chosen so that the free path length of the particles was considerably greater than the characteristic dimension of the beam interaction region (the condition of single time collisions); therefore $t \ll \tau$ and $N_{eff} = N_1 t/\tau$. Since $t_1 = \ell_1/v_1 = \delta_2/v_1 \sin\phi$ and $\tau = (n\sigma v)^{-1}$, where n is the particle density in the second beam and equal to $N_2/h\delta_2 v_2$ (N_2 is the intensity of the second beam, δ_2 is the width, and h is the height of the beams), while v is the approach velocity of the particles, for N_{eff} we have

$$N_{eff} = N_1 N_2 \frac{\sigma}{h \sin\varphi} \cdot \frac{v}{v_1 v_2} \cdot$$

In the case of a split (hollow) beam $v_1 = v_2 = v_0$ and $\nu = \nu_0\sqrt{2(1 - \cos\phi)}$. Therefore

$$N_{eff} = N_1 N_2 \frac{\sigma}{h v_0} \chi(\varphi) \quad \text{where} \quad \chi(\varphi) = \frac{\sqrt{2(1-\cos\varphi)}}{\sin\varphi} .$$

Hence the cross-section of the process is

$$\sigma = \frac{N_{eff}}{N_1 N_2} v_0 h \frac{1}{\chi(\varphi)} . \qquad (20)$$

For $\phi \lesssim 10^{o}$ $\chi(\phi) = 1$ within the limits of 0.5%. Hence the uncertainty of the measured cross-section hardly depends at all on the shape and horizontal dimensions of the interaction region (given a constant particle density in the beam with respect to h) and is determined only by the energy spread in the beam, in accordance with Eq. (20): $\Delta\sigma/\sigma = \Delta E_o/2E_o = \Delta v_o/v_o$.

However the angular spread together with the energy spread of particles in the beam leads, in accordance with Eq. (19), to uncertainty in the collision energy: $\Delta T/T = \Delta E_o/E_o + 2\Delta\phi/\phi$.

4. EXPERIMENT

The method examined was used to measure the cross-section for the process of the formation of protons in paired H_2^+ ions collisions at energies of $T \approx 15$ eV.

Fig. 4 gives a schematic of the experimental facility. The H_2^+ ion beam passed through slot S_1, at the focal point of the magnet- monochromator and reached into the first vacuum chamber with a quadrupole lens system which projected the image of slot S_1 onto slot S_2 in the high-vacuum measuring chamber. The measuring chamber contained the splitting and beam focusing system SS, the energy analyser EA, and the detectors D_1 to D_5. The cross-pieces of the double slots S_3/S_4 and S_5/S_6 were used as choppers. The intensities of the split parts of the beam and the angle of intersection (from 2 to 4^{o}) behind slot S_5/S_6 varied by selecting the potentials at the focusing electrodes P_1 and P_2. The detectors D_4 and D_5 were used to monitor the location and shape of the area of interaction.

The energy analyser - a plane electrostatic mirror with an entry angle of 45^o and focal point near the plane of the plate [9] - meant that several detectors could be used simultaneously to record the various beam components. Secondary electron multipliers were used as the detectors. One of them (D_1) was fixed at the maximum distance from the entry slot of the analyser which the design permitted and was used to record the primary beam. The others (D_2 and D_3) could be moved along the analyser plate during measurement. One of the main advantages of such a detection system was that processes of the type

$$H_3^+ + H_3^+ \longrightarrow H_3^+ + H_2^+ + H_1^+ + e$$

with a signal feed into the coincidence circuit could be measured.

From the proton current recorded by one of the movable detectors we determine the cross-section of proton formation in the $H_2^+ + H_2^+$ reaction. At the same time the arrival of two protons at the detector was recorded as a single event, just as if it were one proton. As a result, the number of pulses from the detector was equivalent to the number of effective (causing dissociation) collisions (N_{eff} in Eq. (20)), and so it was possible to calculate the total cross-section $\sigma_{exp.} = \sigma_{1\bar{2}} + \sigma_{\bar{1}2} + \sigma_{12}$ (see Eq. (15)).

Measurements were made at an H_2^+ ion energy $E_o = 3$ keV with an energy spread in the beam of $\Delta E_o/E_o \approx 0.01$, and at an angle of divergence (average crossing angle of the parts of the split beam) $\phi = 4^o$ with an uncertainty of $\Delta\phi/\phi \approx 0.125$. This corresponded to the collision energy $T = 14.6 \pm 1.9$ eV.

The presence of residual gas in the region where parts of the split beam crossed caused a proton noise current as a result of dissociation of H_2^+ ions in the residual gas, recorded by the detector together with the effect current. Since the noise current was greater than the effect current by two orders of magnitude, it took a long period of continuous measurement to determine the degree of statistical error in the number of effective collisions, and therefore the calculation error, in accordance with Eq. (20), of the cross-section σ_{exp}.

From the measurements it was found that $\sigma_{exp} = (0.87 \pm 0.51) \times 10^{-16} cm^2$. The corresponding value of $\sigma_{1\bar{2}} + \sigma_{\bar{1}2} + \sigma_{12}$, calculated from Eq. (15), for kinetic energy in the centre-of-mass system E = 7.3 eV (which corresponds to T = 14.6 eV), was 3.4×10^{-16} cm^2. Taking into account the accuracy of the calculations (about 50%) and measurements, and also the fact that the measurements were made for energy values outside the validity criterion (14) for the final theoretical expressions, and furthermore lie in the energy field where an increase in the cross-sections calculated from Eq. (15) is to be expected, we can consider that the theoretical and experimental results do not contradict each other.

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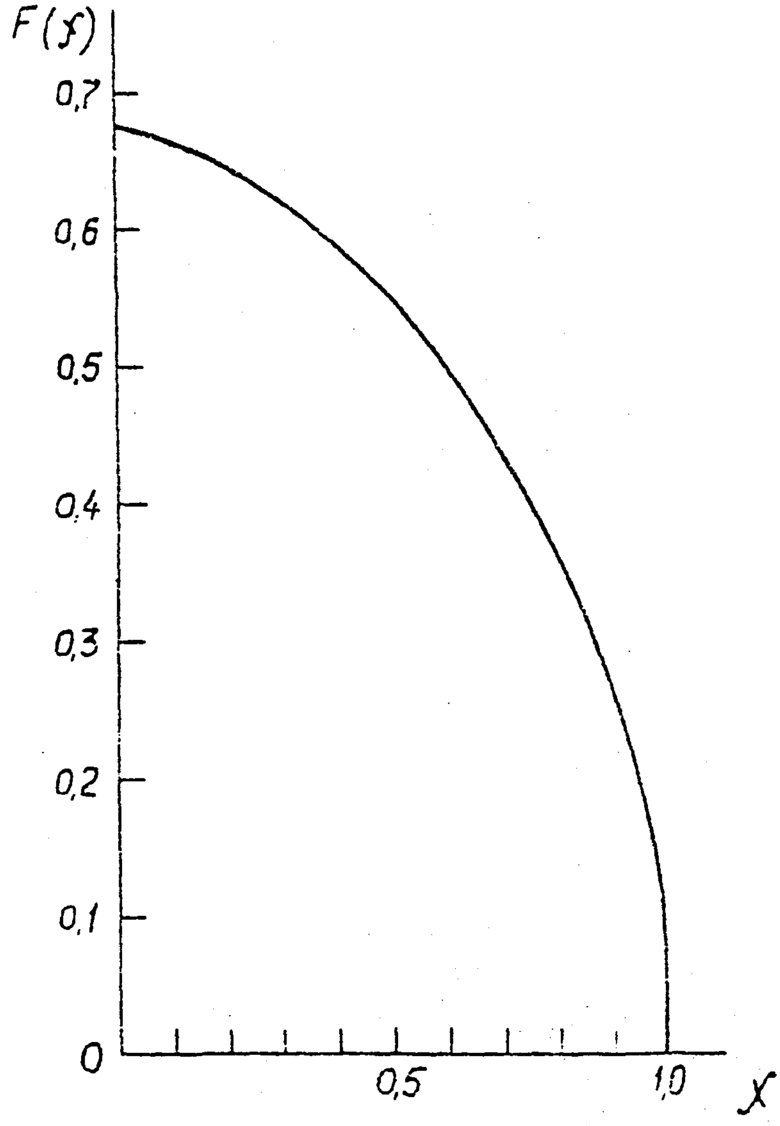

Fig. 1. Plot of the $F(\chi)$ function corresponding to Eq. (17).

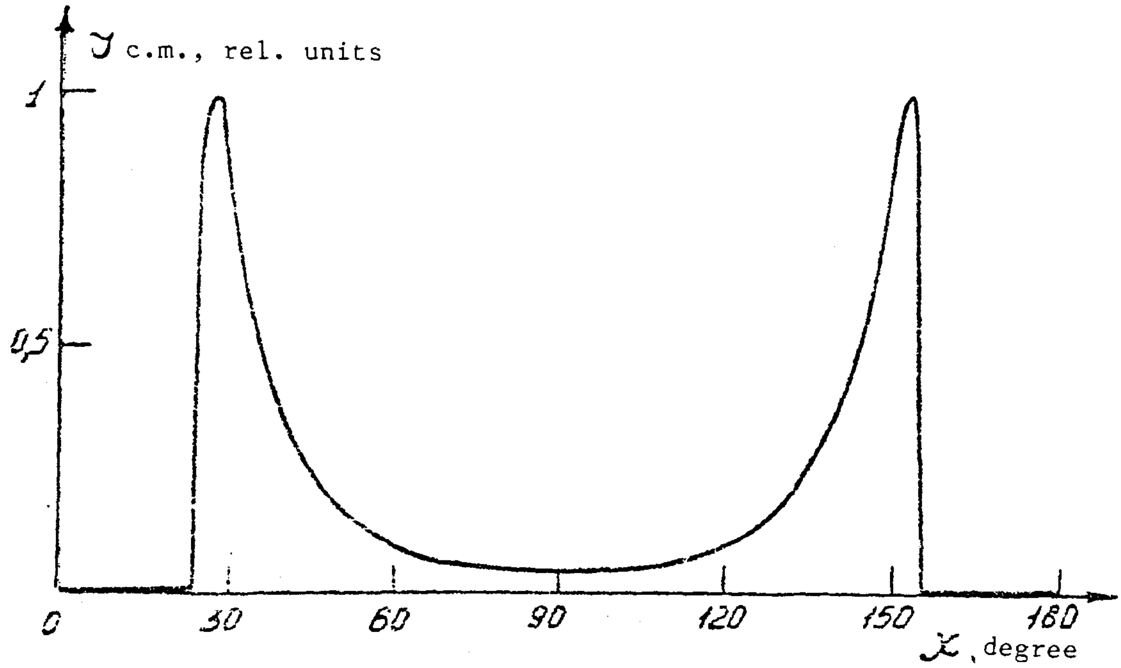

Fig. 2. Intensity $\mathcal{J}$ c.m. of the secondary proton flux occurring in H_2^+ ion collisions with kinetic energy in the centre-of-mass system E = 30 eV as a function of the angle of departure χ in accordance with Eq. (18).

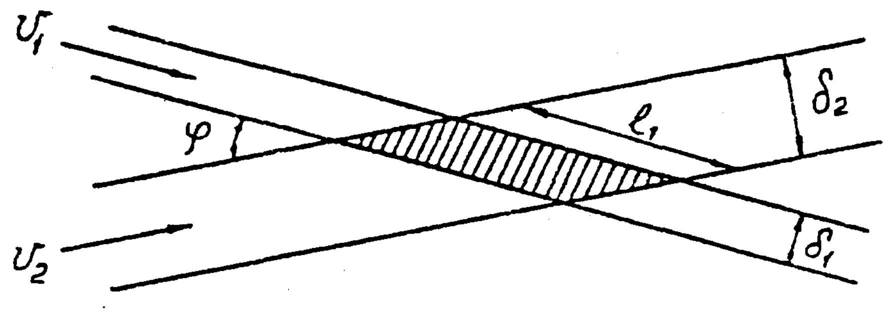

Fig. 3. Derivation of Eq. (20)

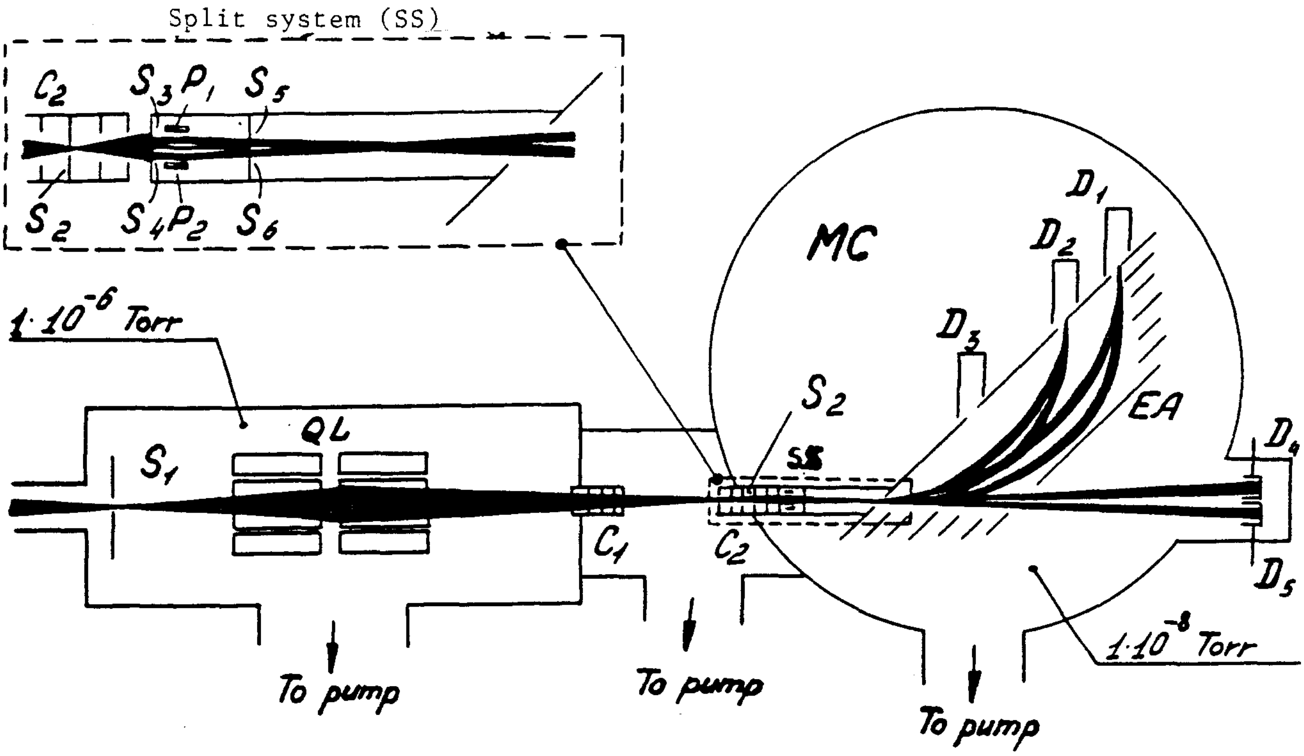

Fig. 4. Experimental facility layout: S_1 is the slot at the magnet-monochromator focal point; QL is the system of quadrupole lenses; S_2 is the slot at the focal point of the system of quadrupole lenses; SS is the splitting and focusing system; EA is the energy analyser; D_1 to D_5 are the detectors; S_3/S_4 and S_5/S_6 are the double slots; P_1 and P_2 are the convergence electrodes; C_1 and C_2 are the vacuum resistors; MC is the high-vacuum measuring chamber.