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# EVALUATION OF $^{232}$Th NEUTRON DATA IN THE UNRESOLVED RESONANCE REGION

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## ABSTRACT

The data on the total, radiative-capture and elastic and inelastic scattering cross-sections for $^{232}$Th in the 1-1000 keV neutron region are analysed within the framework of the Hauser-Feshbach-Moldauer statistical model. The average resonance parameters - neutron and radiative strength functions - have been obtained. The evaluated accuracies of the average radiative-capture cross-sections for $^{232}$Th in the 1-300 keV neutron region were $\pm$ 3-5%. The results of the study can be used for compilation of evaluated data files for $^{232}$Th.

The purpose of the present study was to obtain reliable evaluated data for $^{232}$Th in the unresolved resonance region. The interest in $^{232}$Th is due, firstly, to its use in the U-Th fuel cycle and, secondly, to the similarity of the properties of its nucleus to those of the $^{238}$Th nucleus, which plays an important part in fast reactor physics.

The study deals chiefly with the evaluation of the radiative-capture cross-section. The quantities evaluated directly were the average resonance parameters - neutron and radiative strength functions for s-, p- and d-waves. This approach not only gives evaluations of the average cross-sections but can also be used to apply the results to other functionals of cross-sections. For example, for reactor physics calculations it is very important to determine cross-sections which take into account the self-shielding effect: $\bar{\sigma}_{s \cdot sh} = \bar{\sigma},f$ where $\bar{\sigma}$ is the ordinary cross-section and f the so-called resonance self-shielding factor for the cross-section, which is dependent on the temperature and composition of the medium. If the average cross-section $\bar{\sigma}$ can be evaluated from an analysis of experimental data, the resonance self-shielding factor f can often be determined by calculation alone. In the unresolved resonance region these factors can be calculated conveniently from the average resonance parameters.

Analysing the experimental data which have become available since 1971, i.e. in the last 10 years, we note that for the radiative-capture cross-section the data show good mutual agreement to within ± 10%. According to the list of nuclear data requirements in WRENDA-81/82, the accuracy required for the radiative-capture cross-section in the 1-1000 keV neutron region for fast reactors is ± 3%. There is hope that these requirements can be satisfied to a large extent by further improving the evaluation procedure. In order to enhance the reliability of evaluation, we describe here at the same time data on the total, radiative-capture and elastic and inelastic scattering cross-sections, using for the purpose the Hauser-Feshbach-Moldauer model. The data were analysed in the 1-1000 keV neutron energy region.

## EVALUATION METHOD

The neutron cross-sections and average resonance parameters for $^{232}Th$ in the unresolved resonance region were evaluated by the maximum likelihood method. It is known from the literature on the subject that evaluations obtained by this method are unbiased, consistent and effective and show minimum dispersion [1].

The method is essentially the following. Let $\bar{\sigma}_0$ be the vector for the experimental results and $\bar{\sigma}_1$ the vector for the evaluations calculated by the given theoretical model having a set of parameters $\bar{p}$ and whose a priori evaluations are $\bar{p}_0$ : $\bar{\sigma}_1 = \bar{\sigma}(\bar{p}_0)$. Let us further make the valid assumption that the errors in the experimental results $\bar{\sigma}_0$ and a priori evaluations $\bar{p}_0$ are normally distributed and that the covariance matrices $V_0$ and $W_0$ of these distributions are known. It is assumed that the only reason for differences between the experimental ( $\bar{\sigma}_0$ ) and calculated ($\bar{\sigma}_1$ ) results is that the experimental data $\bar{\sigma}_0$ and model parameters $\bar{p}_0$ contain non-systematic random errors. Then the most probable evaluations for parameters $\bar{p}'$ are evaluations which minimize the quadratic form

$$s^2(\bar{p}) = \left[\bar{\sigma}_0 - \bar{\sigma}(\bar{p})\right]^T V_0^{-1} \left[\bar{\sigma}_0 - \bar{\sigma}(\bar{p})\right] + (\bar{p} - \bar{p}_0)^T W^{-1} (\bar{p} - \bar{p}_0). \tag{1}$$

The most probable model paramters $\bar{p}'$ are determined from the condition $\partial s^2/\partial p_k = 0$ on the assumption of a linear dependence of the cross-sections on parameters:

$$\bar{\sigma}(\bar{p}') = \bar{\sigma}(\bar{p}_0)\left(1 + H \frac{\Delta \bar{p}}{p}\right), \tag{2}$$

where H is the matrix for sensitivities of the calculated cross-sections $\bar{\sigma}(\bar{p}_0)$ to the model parameters:

$$H = \| h_{ik} \| = \left\| \frac{\partial \sigma_i / \sigma_{ik}}{\partial p_k / p_k} \right\|_{\bar{p} = \bar{p}_0} . \tag{2A}$$

Here the covariance matrix for the errors in the most probable evaluations of parameters $\bar{p}'$ is given by the formula:

$$W = (W_0^{-1} + H^T V^{-1} H)^{-1} . \tag{3}$$

Within the framework of linear hypothesis (2) we can obtain an evaluation for the covariance matrix of the errors in the cross-sections calculated from the parameters $\bar{p}'$ obtained:

$$V = HWH^T . \tag{4}$$

Two criteria were used to verify the statistical consistency of the experimental data analysed.

Criterion 1. The minimum value $s^2_{min}$ of quadratic form (1), being a function of random quantities, is itself a random quantity and its distribution, in the case of validity of linear hypotheses (2), coincides with a $\chi^2$ - distribution having N degrees of freedom, where N is the number of experimental points. The mathematical expectation of $s^2_{min}$ is equal to N and the dispersion is 2N. Thus, the value of $s^2_{min}$ serves as an integral statistical criterion for agreement of experimental data among themselves and with the adopted theoretical model.

Criterion 2. *If the hypothesis of normal distribution of the random* quantities $\bar{\sigma}_0$ and $\bar{p}_0$ is valid and the matrices $V_0$ and $W_0$ are true covariance matrices for this distribution, the components of difference vector $\bar{\sigma}_0 - \bar{\sigma}(\bar{p}')$ will also be distributed normally with the covariance matrix:

$$U = V + H\tilde{W}H^T, \tag{4A}$$

where $\tilde{W}$ is the true covariance matrix of parameters $\bar{p}'$. If linear hypothesis (2) is not satisfied, evaluation (3) is not necessarily a good approximation of $\tilde{W}$. It transpires, however, that for the evaluation of matrix U the inaccuracy of matrix $\tilde{W}$ is not substantial since in our case $|V| >> |HWH^T|$ , and in a first approximation we can consider that $U \approx V$.

Representing $V = Y^T Y$, where Y is the upper triangular matrix, we write down the quadratic form of difference vector $\bar{\sigma}_0 - \bar{\sigma}(\bar{p}')$ as:

$$\theta = \left[\bar{\sigma}_0 - \bar{\sigma}(\bar{p}')\right]^T Y^T Y \left[\bar{\sigma}_0 - \bar{\sigma}(\bar{p}')\right] = \bar{Z}^T \bar{Z} \ . \tag{5}$$

The components of vector $\bar{Z}$ (provided that the experimental data do not contradict each other or the theoretical model) will have a normal distribution with zero average and unit dispersions. The Kolmogorov criterion [1] can be used to verify the hypothesis of normal distribution for the components of vector $\bar{Z}$.

The latter criterion is obviously stronger than the $s^2_{min}$ criterion. For a high value of N, criterion 1 permits detection of only very rough divergences. Criterion 2 can be used to detect a wider class of divergences in the experimental data and to consider the data from each experiment and even for each point in relation to the whole set of data, thereby detecting and eliminating contradictory results.

CALCULATION MODEL

The average cross-sections were calculated within the framework of the statistical theory of nuclear cross-sections by the Hauser-Feshbach-Moldauer formulae [2, 3]. It is assumed that the interaction of the neutron with the target nucleus occurs through the formation of a compound nucleus, which decays subsequently by one of the possible reaction channels: $(n,\gamma)$, $(n,n)$, $(n,n')$ etc., the method of decay of the nucleus being independent of the method of its formation.

Let $\ell$ be the incident neutron angular moment, let the states of the compound nucleus be characterized by the total moment J and parity $(-I)^{\ell}$ and let the spin of the nucleus be I. Then the average cross-section $\bar{\sigma}_{nx}$ of the reaction (n,x) will then be

$$\bar{\sigma}_{nx} = \frac{\pi}{k^2}\sum_{\ell J}\frac{(2J+1)}{2(2I+1)}\,\frac{\varepsilon^{0}_{\ell\ell J}\,T^{\ell J}_{n}T^{\ell J}_{x}}{\sum_{c}T^{\ell J}_{c}}\,S^{\ell J}_{nx}\ , \qquad (6)$$

where k is the wave number $(k=2{,}196771\frac{A}{A+1{,}00866}\sqrt{E\ \mathrm{MeV}})$, $\varepsilon^{0}_{\ell\ell J}$ is the degree of degeneracy of state $\ell J$; $T^{\ell J}_{n}$ are the transmission coefficients $(T^{\ell J}_{n}\leq I)$; $\sum_{c}$ means summation over all possible reaction channels, and $S^{\ell J}_{nx}$ is a factor which takes account of neutron width fluctuations.

The total cross-section is calculated by the equation

$$\bar{\sigma}_{t} = \frac{\pi}{k^2}\sum_{\ell}(2\ell+1)(T^{\ell}_{n}\cos 2\varphi_{\ell} + 4\sin^2\varphi_{\ell})\,, \qquad (7)$$

where $\varphi_{\ell}$ is phase shift.

It is assumed that the transmission coefficients $T^{\ell J}_{n}$ do not depend on total moment J, i.e. $T^{\ell J}_{n}=T^{\ell}_{n}$. The transmission coefficients $T^{\ell}_{n}$ were calculated by the relationship

$$\Gamma_n^{\ell} = \frac{2\pi S_n^{\ell} \vartheta_{\ell} \sqrt{E}}{\left(1 + \frac{\pi}{2} S_n^{\ell} \vartheta_{\ell} \sqrt{E}\right)^2}, \tag{8}$$

where $S_n^{\ell}$ is the neutron strength function and the multiplier $\vartheta_{\ell}\sqrt{E}$ takes into account the energy dependence of it. By definition strength function $S_n^{\ell}$ is

$$S_n^{\ell} = \frac{\overline{g\Gamma_{n0}^{\ell}}}{(2\ell+1)\bar{D}_{\ell}}, \tag{8A}$$

where g is a statistical factor, $\Gamma_{n0}$ the reduced neutron width and $\bar{D}$ the average distance between the levels of the compound nucleus. For s-neutrons with $\ell=0$: $\bar{D}_{\ell=0} = \bar{D}_{obs}$ and for neutrons with $\ell \neq 0$ it is usually assumed that $(2\ell+1)\bar{D}_{\ell} = \bar{D}_{obs}$.

The values of $\varphi_{\ell}$ in expression (7) and $\vartheta_{\ell}$ in expression (8) were calculated in the "black nucleus" model by the equations

$$\vartheta_0 = 1; \quad \vartheta_1 = \frac{x^2}{1+x^2}; \quad \vartheta_2 = \frac{x^4}{9+3x^2+x^4}, \tag{8B}$$

where $x = ka_{\ell}$, $k$ being the wave number and $a_{\ell}$ the radius of the nucleus (in fermi units $10^{-13}$ cm). The value of $a_{\ell}$ was taken as equal to $1.23\ A^{1/3} + 0.8$.

$$\varphi_0 = \rho_0; \quad \varphi_1 = \rho_1 - \mathrm{arctg}\,\rho_1; \quad \varphi_2 = \rho_2 - \mathrm{arctg}\frac{3\rho_2}{3-\rho_2}, \tag{8C}$$

where $\rho_{\ell} = kR'_{\ell}$ and $R'_{\ell}$ is the effective potential scattering radius.

The quantity $R'_0$ (or simply R') determines the potential scattering cross-section at low energies, i.e.

$$\sigma_{pot} = 4\pi (R')^2. \tag{8D}$$

As was shown in Refs [4-6], for example, potential scattering radius $R'_\ell$ and radius of the nucleus $a_\ell$ are connected by the relationship

$$R'_\ell = a_\ell(1 - R^\infty_\ell) , \tag{9}$$

where $R^\infty_\ell$ is a parameter which takes into account the influence of all resonances.

To allow for competition from the radiative channel and to calculate the neutron radiative-capture cross-section, we need to know coefficients $T^{\ell J}_\gamma$ and $T^{\ell J}_{\gamma c}$ which are determined in the following manner

$$T^{\ell J}_\gamma = \frac{2\pi \bar{\Gamma}^{\ell J}_\gamma}{\bar{D}_{\ell J}} ; \qquad T^{\ell J}_{\gamma c} = \frac{2\pi \bar{\Gamma}^{\ell J}_{\gamma c}}{\bar{D}_{\ell J}} , \tag{9A}$$

where $\bar{\Gamma}_\gamma$ is the total radiative width determining the decay probability of the state $\ell J$ for any radiation channel, and $\bar{\Gamma}_{\gamma c}$ the width corresponding to the neutron radiative-capture channel $^{N}_{Z}A + n \longrightarrow {}^{N+1}_{Z}A + \gamma$ .

On the assumption that the main type of transition from the highly-excited states of the nucleus are electric dipole gamma transitions, the energy-spin dependence of the widths $\bar{\Gamma}_\gamma$ and $\bar{\Gamma}_{\gamma c}$ takes the form [7]:

$$\bar{\Gamma}^{\ell J}_\gamma(U) = \frac{\text{const}}{\rho(U,J)} \sum_{i=|J-1|}^{J+1} \int_0^U \varepsilon^3_\gamma f(\varepsilon_\gamma)\rho(U-\varepsilon_\gamma, i)d\varepsilon_\gamma ; \tag{10}$$

$$\bar{\Gamma}^{\ell J}_{\gamma c}(U) = \frac{\text{const}}{\rho(U,J)} \sum_{i=|J-1|}^{J+1} \int_{U-Bn}^{U} \varepsilon^3_\gamma f(\varepsilon_\gamma)\rho(U-\varepsilon_\gamma, i)d\varepsilon_\gamma , \tag{11}$$

where $U = Bn + E - \Delta$; $\Delta$ is the correction for nucleon pairing in nuclei with even Z and (or) N [8]. The factor $f(\epsilon_\gamma)$ takes into account the energy dependence of the mean square of the matrix element for dipole gamma transitions and is usually chosen in the form of a Lorentz dependence approximating the photoabsorption cross-section in the neighbourhood of the giant dipole resonance

$$f(\varepsilon_\gamma) = \frac{\varepsilon_\gamma \Gamma_1^2}{(\varepsilon_\gamma^2 - E_1^2)^2 + \varepsilon_\gamma^2 \Gamma_1^2} \quad , \tag{11A}$$

where $\Gamma_1 \approx 5$ MeV is the giant resonance width and $E_1 \approx 80\ A^{-1/3}$ MeV the position of the resonance.

The absolute values of radiation widths $\bar{\Gamma}_\gamma$ and $\bar{\Gamma}_{\gamma c}$ in expressions (10) and (11) are normalized to the average value of radiative width $\bar{\Gamma}_{\gamma obs}$ (Bn) and resonance density $\rho_{obs} = \bar{D}^{-1}_{obs}$.

The level density of excited states $\rho(U, J)$ was calculated in the Fermi gas model with allowance for collective phenomena in the highly excited nucleus [9, 10]:

$$\rho(U,J) = k_{rot}(U) k_{vibr}(U) \rho_{F.g.}(U,J) ;$$
$$\rho_{F.g.}(U,J) \approx \frac{2J+1}{U^2} \exp\left[ 2\sqrt{aU} - \frac{J(J+1)}{2\sigma^2} \right] \quad , \tag{11B}$$

where $\sigma^2 = 0{,}146\sqrt{aU}\,A^{2/3}$ the spin dependence parameter. Correction factors $k_{rot}$ and $k_{vibr}$ in the excitation energy region (for phenomenological analysis of experimental data) take the following form

$$k_{vibr}(U) \approx \exp\left[0{,}25\,U^{2/3}\right] ; \tag{11C}$$

$k_{rot}$ (U) = 1 (for spherical nuclei) or $k_{rot}$ (U) $= \mathcal{F}_\perp t$ (for deformed nuclei), where $t = [U/\alpha]^{1/2}$ is the temperature of the excited nucleus and $\mathcal{F}_\perp$ the moment of inertia relative to the direction perpendicular to the axis of symmetry. The level density parameter $\alpha$, with allowance for shell effects, has the following dependence on excitation energy [9]:

$$a(U,Z,A) = \tilde{a}(A)\left[1 + f(U)\delta W(Z,A)/U\right] , \tag{11D}$$

where $\tilde{a}$ is the asymptotic value of the level density parameter at high excitation energies [9]: $a(A) \approx 0{,}093A$; $f(U) = 1 - \exp(-\gamma U)$ is the dimensionless universal function of the energy dependence of parameter $a$ ($\gamma = 0{,}064$) and $\delta W$

the shell correction in the nucleus binding energy. Transmission coefficients $T^{\ell J}_{in,k}$ in expression (6), which correspond to the channel of neutron inelastic scattering with excitation of a level of energy $E_k$, are determined by the relation

$$T^{\ell J}_{in,k} = \sum_{\ell'} \varepsilon^{k}_{\ell\ell' J} \, T^{\ell'}_{n}(E - E_k), \qquad (12)$$

where $\varepsilon^{k}_{\ell\ell' J}$ is the number of values of spins $j_k$ satisfying the rule of vector summation $\vec{I}+\vec{j}=\vec{J}=\vec{I}_k+\vec{j}_k$ and the law of conservation of parity $(-1)^{\ell}\Pi_0=(-1)^{\ell'}\Pi_k$ where $\Pi_o$ is the parity of the target nucleus in the ground state and $\Pi_k$ and $I_k$ are respectively, the parity and spin of the k-th excited level of the target nucleus.

In Eq. (12) it is assumed that the transmission coefficients for the excited state of the nucleus are determined, just as in the case of the ground state, by expression (8). However, the difference between the elastic and inelastic scattering channels in the calculation of transmission coefficients lies only in the energy. Relationship (12) is valid for neutron energies $E \geqslant E_k(A+1)/A$.

The fluctuation factor $S^{\ell J}_{nx}$ in Eq. (6) is determined as

$$S^{\ell J}_{nx} = \frac{\overline{\Gamma^{\ell J}_{n}\Gamma^{\ell J}_{x}}}{\Gamma^{\ell J}} \bigg/ \frac{\bar{\Gamma}^{\ell J}_{n}\bar{\Gamma}^{\ell J}_{x}}{\bar{\Gamma}^{\ell J}}, \qquad (12A)$$

where the averaging makes allowance for neutron width distribution in accordance with the Porter-Thomas law. However, integration over distributions demands considerable computer time. It was shown in Ref. [11] that, in the case of average cross-sections, the computation difficulties can be overcome successfully with the help of generalized Gauss quadrature formulae. The expressions for the fluctuation factors $S^{\ell J}_{nj}$, $S^{\ell J}_{nn'}$ and $S^{\ell J}_{nn}$ are

$$S_{n\gamma}^{\ell J} = (1+\alpha+\beta)\sum_{i=1}^{N^{(\nu)}}\sum_{j=1}^{N^{(\mu)}}\frac{a_i^{(\nu)}a_j^{(\mu)}x_i^{(\nu)}}{1+\alpha x_i^{(\nu)}+\beta x_j^{(\mu)}} ;$$

$$S_{nn'}^{\ell J} = (1+\alpha+\beta)\sum_{i=1}^{N^{(\nu)}}\sum_{j=1}^{N^{(\mu)}}\frac{a_i^{(\nu)}a_j^{(\mu)}x_i^{(\nu)}x_j^{(\mu)}}{1+\alpha x_i^{(\nu)}+\beta x_j^{(\mu)}} ; \qquad (12B)$$

$$S_{nn}^{\ell J} = (1+\alpha+\beta)\sum_{i=1}^{N^{(\nu)}}\sum_{j=1}^{N^{(\mu)}}\frac{a_i^{(\nu)}a_j^{(\mu)}(x_i^{(\nu)})^2}{1+\alpha x_i^{(\nu)}+\beta x_j^{(\mu)}} ,$$

where $\alpha = T_n^{\ell J}/T_\gamma^{\ell J}$ ; $\beta = \sum_k T_{in,k}^{\ell J}/T_\gamma^{\ell J}$, $\alpha_i$ and $x_i$ are, respectively, the weights and nodes of the quadrature equations (given in Ref. [11]), $\nu$ is the effective number of degrees of freedom for the input channel: $\nu = \varepsilon_{\ell\ell J}^0$ and $\mu$ the effective number of degrees of freedom for the inelastic scattering channel:

$$\mu^{\ell J} = \left(\sum_{\ell'}\varepsilon_{\ell\ell' J}^k T_n^{\ell'}\right)^2 \Big/ \sum_{\ell'}\varepsilon_{\ell\ell' J}^k (T_n^{\ell'})^2 . \qquad (12C)$$

The scheme of the low-lying levels of the $^{232}$Th nucleus for calculation of the inelastic scattering channel is taken from Ref. [12]; fifteen levels were taken into account.

CALCULATIONS BY THE OPTICAL MODEL

Before starting the analysis, we carried out calculations by the coupled channel method, using the CCROT program [13], in order to verify that the calculation of the transmission coefficients $T_n^\ell$ and $T_{in,k}^{\ell J}$ as determined by (8) and (12) was correct and to find the value of parameter $R_\ell^\infty$ from Eq. (9) and its energy dependence.

The non-spherical potential parameters and those for deformation were taken from Ref. [14]:

$$V_R = 46{,}2 - 0{,}3\,E;\quad r_R = 1{,}26;\quad a_R = 0{,}63;$$
$$V_S = 3{,}6 + 0{,}4\,E;\quad r_S = 1{,}26;\quad a_S = 0{,}52;$$
$$V_{S_0} = 6{,}2;\quad r_{S_0} = 1{,}12;\quad a_{S_0} = 0{,}47. \tag{12D}$$
$$\beta_2 = 0{,}19;\quad \beta_4 = 0{,}071;$$

Figure 1 shows a comparison of the transmission coefficients $T_n^\ell$ obtained by the optical model and by means of Eq. (8) in the "black nucleus" model. The results of calculating $T_n^0$ and $T_n^1$ agree with each other to within 2-5%, except in the neutron region above 500 keV, where the divergence is of the order of 10%. For $T_n^2$ the calculation results for the "black nucleus" model in the neutron region up to 500 keV are 10% lower than for the optical model and in the region above 500 keV they are 20-40% lower so that the contribution of the d-wave to the radiative-capture cross-section may be underestimated by 10-20%. The parameter $R_\ell^\infty$ and its energy dependence were evaluated from the results of CCROT program calculation by the coupled channel method. In the 1-1000 keV neutron energy region the value of parameter $R_\ell^\infty$ will fluctuate within $\ell=0$, $R_0^\infty=-(0{,}05-0{,}20)$; $\ell=1$, $R_1^\infty=+(0{,}10-0{,}19)$; $\ell=2$, $R_2^\infty=-(0{,}02-0{,}10)$. The derived dependences of parameter $R_\ell^\infty$ agree with the data of Ref. [6] for the $^{238}U$ nucleus with similar properties.

The CCROT program was used also to evaluate the contribution made by the direct interaction processes. In the energy region under consideration the direct processes are most substantial in the case of neutron inelastic scattering with excitation of the first two low-lying levels $2^+$ (49.4 keV) and $4^+$ (162.1 keV). The contribution of the direct process is shown in Fig. 2.

## EXPERIMENTAL DATA

Over the 1-1000 keV neutron region under consideration, there are experimental data available on average cross-sections - total cross-section (results of treatment of transmission functions), radiative-capture cross-section, and the elastic scattering cross-section and inelastic scattering cross-section for discrete levels. All these data were taken into account (Figs 2-6).

In the case of the radiative-capture cross-section, we considered data taken from 13 authors [15-27]; these can be tentatively divided into two groups: (1) the data of Refs [15-20] obtained up to 1971 (see Fig. 3) and (2) the data of Refs [21-27] obtained after 1971 (see Fig. 4) and published between 1976 and 1981.

The data of the first group [15-20] are mainly activation measurements. The error assigned by the authors is ~ 10-12%, although the disagreement between the different data considerably exceeds the stated accuracies (see Fig. 3). Hence the data of Refs [15-20] were weighted with a factor of 0.7 in the subsequent analysis.

For total cross-section we considered the data given in Refs [28-34]. It will be seen from Fig. 5, where a comparison is made, that the data of the different authors agree satisfactorily with each other to within ± 5% except those of Tabony and co-workers [28], which are systematically lower than all others by 6-12% for a 4-6% accuracy stated by the authors. The data of Ref. [28] were therefore rejected and not considered in the subsequent analysis.

Only a small amount of data is available on scattering cross-sections (see Figs 2 and 6) in the 1-1000 keV region considered. The elastic scattering cross-section data are from earlier studies [35-37]; they were obtained at a low level of accuracy ~ ± 15-20% and virtually all lie in the region above 300 keV.

As for inelastic scattering cross-sections, we took into account the experimental data of two studies [35] and [38] on the excitation cross-sections for the discrete levels $2^+$ (49.3 keV) and $4^+$ (162.1 keV).

## EVALUATION RESULTS

At the first stage we evaluated the average resonance parameters and cross-sections of $^{232}$Th by the method of greatest probability in the 1-300 keV neutron region, where the direct processes make a small contribution and can be neglected, i.e. it can be assumed that all the reactions considered take place through compound nucleus formation.

In selecting a priori evaluations of parameters $\bar{p}_o$ we considered the existing results of measurements, calculations and evaluations of the average resonance parameters of $^{232}$Th given in Tables 1 and 2. From analysis of the data, we took as a priori evaluations of parameters $\bar{p}_o$ the evaluations from BNL-325 [4] for the neutron strength functions and the potential scattering radius R'. The value of 17.0 eV obtained in Ref. [43] was taken as the average distance between levels $\bar{D}_{obs}$. For the average radiative width $\bar{\Gamma}_{\gamma obs}$ the result of Ref. [41] 21.2 MeV was taken as the a priori evaluation. The corresponding value of the radiative strength function is $S_\gamma = (1,25 \pm 0,08) \cdot 10^{-3}$. For the p-neutron scattering radius we took the value $R_1' = 7.5$ fm corresponding to a value of parameter $R_1^\infty = +0.10$. In the case of d-neutrons, their small contribution to the total cross-section at these energies is such that we assumed $R_2' = R_1'$. The radiative strength function in all calculations was assumed to be independent of the incident-neutron orbital moment $\ell$.

In the 1-300 keV region the following experimental data were considered: total cross-section - 107 points [29-34], radiative-capture cross-section - 154 points [15-26] and scattering cross-sections - 5 points [35-38].

Table 3 gives a number of versions of the description of the experimental data and the criterion $S^2_{min}/N$. All the versions describe the total cross-section equally well. Calculation on the basis of the parameters from BNL-325 (version 1) also describes the radiative-capture cross-section fairly well - to within $\pm$ 10% - but gives a low value for the inelastic scattering cross-section and a 3-5% lower value for the total cross-section in the neutron energy region below 10 keV.

Fitting only to the experimental data for the radiative-capture cross-section or with inclusion of the total cross-section data (versions 2 and 3, respectively) yields somewhat different values of the strength functions and slightly changes the picture for description of the radiative-capture cross-section without substantially altering the description of the inelastic scattering cross-section - at 300 keV, for example, the value of the cross-section is about 40% lower. It should be noted that in versions 2 and 3 it was not required to distinguish the neutron strength functions of the s- and d-waves; in the description of experimental data they were taken to be identical: $S_2 = S_o$. This is due to the fact that for $^{232}$Th it is difficult to extract information on the d-wave from the total and radiative cross-section data since its contribution to the cross-sections at these energies is still small. Additional information on the d-wave is contained in the inelastic scattering cross-section data.

Version 4 takes into account the experimental data on cross-sections of all types of reaction in the 1-300 keV region - total cross-section, radiative-capture cross-section and scattering cross-sections. One degree of freedom was added to the d-wave neutron strength function for a better description of the inelastic scattering cross-section data. The obtained value of strength function $S_2$ was close to $S_1$ ($S_1$ = 1.81 x $10^{-4}$, $S_2$ = 1.71 x $10^{-4}$) but in this case the value of parameter $R_l'$ had to be slightly reduced from 7.5 to 7.0 fm, whic in agreement with the evaluations of change in parameter $R_1^{\infty}$ obtained above in the optical model calculations: $R_1^{\infty}$ = +(0.10-0.19). The description of the inelastic scattering cross-section data was improved in version 4.

In variant 5 we used the experimental data on inelastic scattering cross-section weighted by a factor of two. The derived value of strength function $S_2$ exceeds the value of $S_1$ by 10% and is evidently within the accuracy of its determination. The curve showing the inelastic scatteriong cross-section (see Fig. 2) is closer to the latest data obtained by McMurray [38].

An attempt was made to describe the entire set of experimental data available in the region up to 1000 keV (version 6) by the greatest probability method. This did not substantially modify the description of the total and radiative-capture cross-sections but increased the inelastic scattering cross-section (by about 10%). The strength functions varied by about 7-10% in comparison with the preceding version. As a result of the analysis by the greatest probability method, the evaluation corresponding to version 5 was taken for the average cross-sections and average resonance parameters of $^{232}$Th (see Table 3).

In Figs 2-5 we have compared the experimental data and the various evaluations (present work, ENDF/B-14 and JENDL-1) for the average cross-sections of $^{232}$Th - total, radiative-capture and elastic and inelastic scattering.

For the radiative-capture cross-section (see Fig. 4) the adopted evaluation in the region below 100 keV follows the data of Kobayashi and co-workers [26] and Macklin and Winters [22]. In the region above 100 keV the evaluation adopted follows the data of Lindner [21], Macklin and Winters [22], and Poenitz and Smith [25]; below 10 keV it coincides with the ENDF/B-IV data. Above 10 keV the ENDF/B-IV and JENDL-1 evaluations lie above the adopted evaluation by 10-30% on an average. The JENDL-1 evaluation in the entire energy region, except 40-70 keV, gives about 30% higher value for the cross-section. For the total cross-section and the elastic scattering cross-section (Figs 5 and 6) the adopted evaluation in the entire neutron energy region follows the available experimental data and agrees with the JENDL-1 evaluation (the divergence does not exceed 3-4%). For the inelastic scattering cross-section (see Fig. 2) the adopted evaluation lies between the ENDF/B-IV and JENDL-1 although it is closer to the former. The calculation of the excitation functions for the low-lying discrete levels agrees quite satisfactorily with the available experimental data. Tables 1 and 2 show a comparison of the existing experimental data and evaluations of the average resonance parameters of $^{232}Th$. The value of radiative width $\overline{\Gamma}_{\gamma}$ obtained in the present study agrees with the data of Garg [39] and Rahn and co-workers [41]. The obtained values of strength functions $S_o$ and $S_1$ show satisfactory agreement with the available experimental data [40-43] and with the results of other evaluations [4, 44-47].

ANALYSIS OF THE STATISTICAL CONSISTENCY OF DATA

Our evaluation is based on the hypothesis that random values of experimental results $\overline{\sigma}_o$ and parameters $\overline{p}_o$ and their erros have a normal distribution. It is assumed that $V_o$ and $W_o$ are true covariance matrices for this distribution. Will the results of evaluation not be inconsistent with the adopted hypothesis? For the adopted version 5, $S^2_{min}/N = 1.05$. This value lies within the limits of one standard deviation, indicating the consistency of the data. Let us apply the more differential criterion 2. Figure 7 shows the normal distribution function of the standardized value $u=(z-\xi)/\sigma$, where z is a component of vector $\overline{Z}$ in expression (5), $\xi$ (equal to o) the centre of distribution and $\sigma=\sqrt{S^2_{min}/N}$. In accordance with the Kolmogorov criterion, the measure of disagreement between the theoretical and statistical distributions will be the maximum value of the modulus of difference of their distribution functions: $D = \max|\phi_{st}(u) - \phi_t(u)|$. This value has a fairly simple law of distribution [1]. In our case, $D\sqrt{N} \approx 0{,}7$. According to the Kolmogorov criterion, the probability that, for purely random reasons, the maximum disagreement between $\phi_{st}(u)$ and $\phi_t(u)$ will be smaller than that observed is 0.71. This is a quite high probability.

Thus, on the basis of this analysis we can conclude that the experimental data and the evaluation results are statistically consistent and that the latter results are reliable.

## EVALUATION OF THE ACCURACY OF THE RESULTS

The accuracy of the results obtained for the average resonance parameters and cross-sections can be determined by using the greatest probability method for analysis of data. Apart from evaluated values, Table 1 gives the errors of strength functions obtained by Eq. (3). The evaluated errors should be considered together with their correlation matrix, which is given in Table 4. The errors of the calculated values of radiative-capture cross-sections obtained within the framework of the linear hypothesis (2) by Eq. (4) are given in Table 5. The derived evaluation of the errors of the average radiative-capture cross-sections of $^{232}$Th makes no allowance for statistical fluctuations in the resonance parameters averaged over energy intervals and this fact should be taken into account when evaluating the errors in the group-averaged cross-sections.

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Table 1

Experimental data on the average resonance parameters of $^{232}Th$

| Data | $D_{obs}$, eV | $\bar{\Gamma}_{\gamma}$, MeV | $\bar{\Gamma}^{0}_{n}$, MeV | $S_0 \cdot 10^4$ | $S_1 \cdot 10^4$ | $R'_1$, fm | $R_0^{\infty}$ | |
|---|---|---|---|---|---|---|---|---|
| Garg [39] | I7,5±0,7 | I9,0 | | 0,69±0,07 | | | | |
| Uttley [40] | | | | | I,64±0,24 | | -0,I78±0,0I5 | +0, |
| Rahn [41] | I6,7±0,2 | 2I,2±0,3 | I,40±0,I5 | 0,84±0,08 | | 9,I±0,3 | | |
| Camarda [42] | | | | | I,5±0,4 | 9,72±0,30 | -0,I3±0,03 | +0,I±0,I |
| Corvi [43] | I7,0±I,0 | | I,49±0,26 | | 2,0±0,5 | | | |
| Kobayashi [31] | | | | | | 9,65±0,25 | | |

Table 2

Results of calculations and evaluations
of the average resonance parameters of $^{232}Th$

| Data | Energy region | $\bar{D}_{obs}$, eV | $\bar{\Gamma}_{\gamma}$, MeV | $S_0 \cdot 10^4$ | $S_1 \cdot 10^4$ | $S_2 \cdot 10^4$ | $R'$, fm |
|---|---|---|---|---|---|---|---|
| BNL-325 [4] | < 4 keV | | | 0,84±0,08 | I,6±0,2 | | 9,65±0,08 |
| ENDF/B-IV | 3,94-50 keV | I7,0 | 25,9 | 0,73 | I,20±I,4I | 0,73 | 8,9874 |
| Ref. [46] | <3 keV | I6,6±0,9 | 2I,0±0,8 | 0,856±0,09 | I,5±0,4 | | 9,72±0,3 |
| Derrien [44] | < 4 keV | I6,9 | 2I,45 | 0,89 | I,58 | | 9,65 |
| Keyworth [45] | < 4 keV | I6,4±I,0 | | 0,88±0,07 | I,64±0,50 | | |
| Macklin [47] | 2,6-I0 keV | I7,0 | 25,0±0,8 | 0,86±0,I0 | I,48±0,07 | I,I2±0,06 | |
| Present work | I-300 keV | I7,0 | 20,0±0,6 | 0,93±0,03 | I,82±0,05 | 2,00±0,I4 | 9,65 |

Table 3

Different versions and descriptions of experimental data (strength functions and average cross-sections)

| Data | | Version | | | | | |
|---|---|---|---|---|---|---|---|
| | | I | 2 | 3 | 4 | 5 | 6 |
| Scattering radius | $R'_0$ | 9,65 | 9,65 | 9,65 | 9,65 | 9,65 | 9,55 |
| | $R'_1$ | 7,5 | 7,5 | 7,5 | 7,0 | 7,0 | 6,7 |
| Strength functions | $S_0$ | 0,84 | 0,994 | 0,95 | 0,992 | 0,93 | 0,95 |
| | $S_1$ | I,60 | I,82 | I,80 | I,8I | I,82 | 2,0 |
| | $S_2$ | 0,84 | 0,994 | 0,95 | I,7I | 2,0 | 2,3 |
| | $S_\gamma$ | I2,0 | I2,9 | II,7 | II,7 | II,7 | II,2 |
| $\bar{\sigma}_{n\gamma}$ | I keV | 2,35 | 2,95 | 2,62 | 2,63 | 2,58 | 2,52 |
| | I0 keV | 0,732 | 0,793 | 0,746 | 0,752 | 0,750 | 0,753 |
| | 30 keV | 0,465 | 0,500 | 0,466 | 0,477 | 0,480 | 0,478 |
| | I00 keV | 0,203 | 0,2I5 | 0,I97 | 0,2I2 | 0,2I5 | 0,2I3 |
| | 300 keV | 0,I45 | 0,I52 | 0,I38 | 0,I42 | 0,I47 | 0,I44 |
| | 700 keV | 0,I78 | 0,I80 | 0,I65 | 0,I75 | 0,I78 | 0,I67 |
| | I000 keV | 0,I3I | 0,I38 | 0,I26 | 0,I38 | 0,I39 | 0,I38 |
| $\bar{\sigma}_{tot}$ | I0 keV | I5,2 | I5,9 | I5,7 | I5,9 | I5,6 | I5,6 |
| | 300 keV | 9,3I | 9,67 | 9,65 | 9,68 | 9,85 | 9,86 |
| | I000 keV | 6,45 | 7,02 | 7,02 | 6,92 | 7,05 | 7,05 |
| $\bar{\sigma}_{nn'}$ | I00 keV | 0,338 | 0,390 | 0,385 | 0,4I9 | 0,432 | 0,483 |
| | 300 keV | 0,67I | 0,768 | 0,753 | 0,9I2 | 0,956 | I,05 |
| | I000 keV | I,45 | I,66 | I,62 | I,93 | 2,03 | 2,2I |
| $\delta^2_{min}/N$ | | 2,5 | I,03 | I,5I | I,06 | I,05 | I,03 |

Table 4

Correlation matrix for the errors of the strength functions of $^{232}Th$

| $\rho$ | $S_0$ | $S_1$ | $S_2$ | $S_\gamma$ |
|---|---|---|---|---|
| $S_0$ | I | -0,4 | -0,I | -0,5 |
| $S_1$ | -0,4 | I | -0,4 | 0 |
| $S_2$ | -0,I | -0,4 | I | |
| $S_\gamma$ | -0,5 | 0,3 | 0,7 | |

Table 5

Errors in the radiative-capture cross-sections for $^{232}Th$ and their correlations

| E, keV | I | 40 | 200 | I000 | % |
|---|---|---|---|---|---|
| I | I | 0,98 | 0,94 | 0,90 | I,7 |
| 40 | 0,98 | I | 0,95 | 0,96 | 2,8 |
| 200 | 0,94 | 0,95 | I | 0,98 | 5,4 |
| I000 | 0,90 | 0,96 | 0,98 | I | 3,7 |

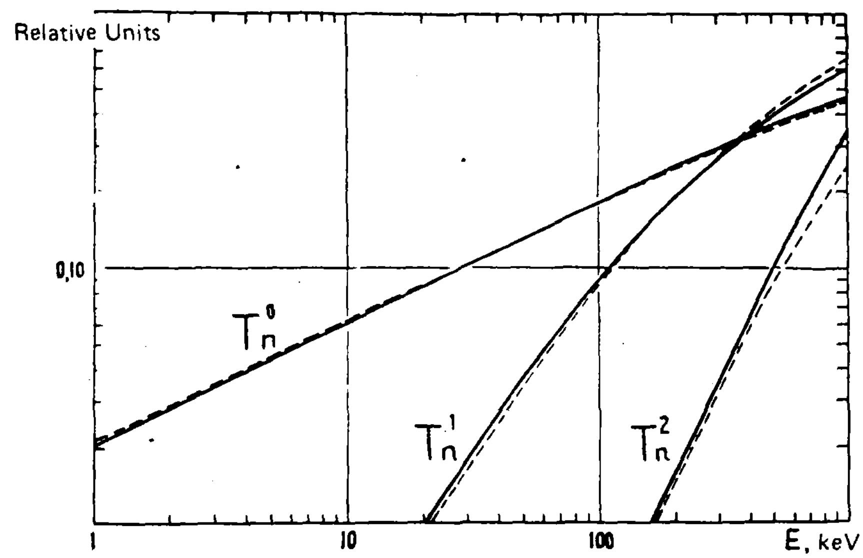


Fig. 1. Comparison of the calculated values of transmission coefficients $T_n^{\ell}$: —— optical model; - - - "black nucleus" model.

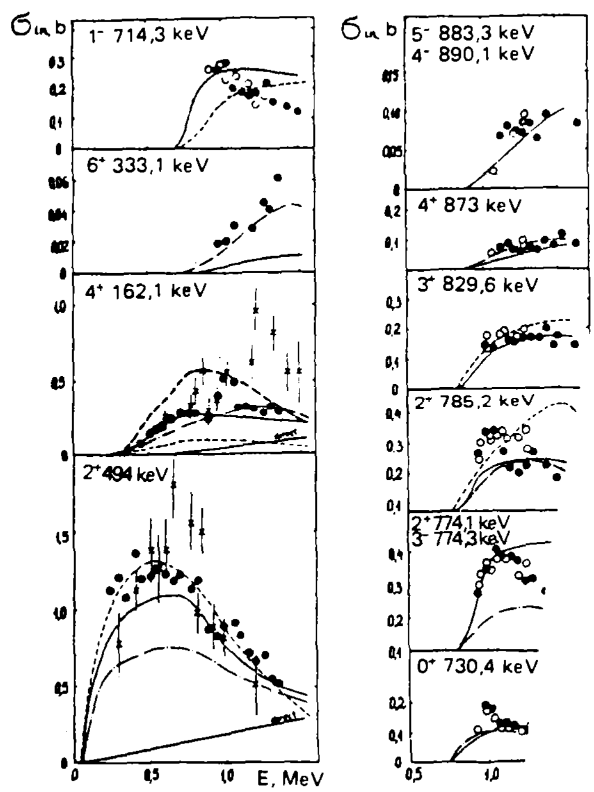


Fig. 2. Inelastic scattering cross-sections for discrete levels of $^{232}Th$: * Ref. [35], o • Ref. [38]. -.- JENDL-1; - - - ENDF/B-IV; —— present work.

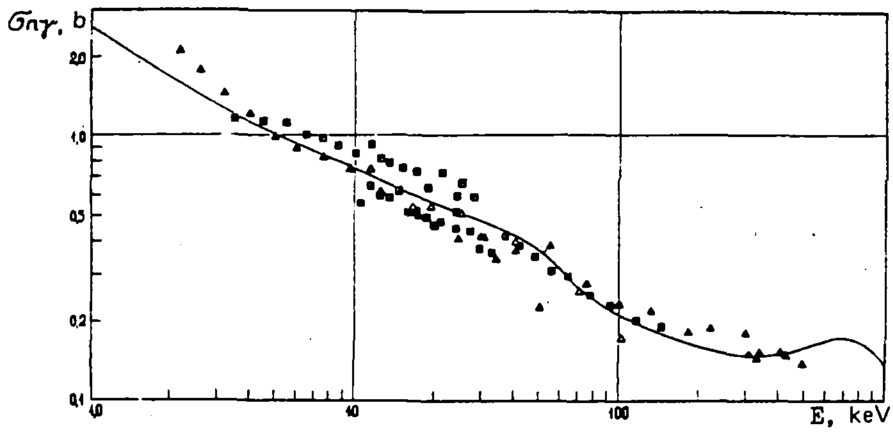


Fig. 3. Radiative-capture cross-section of $^{232}Th$. Comparison of old experimental data with the present evaluation: ◩ Ref. [15], ⊠ Ref. [16], ◪ Ref. [17], ▲ Ref. [18], ▲ Ref. [19], △ Ref. [20], ― present work.

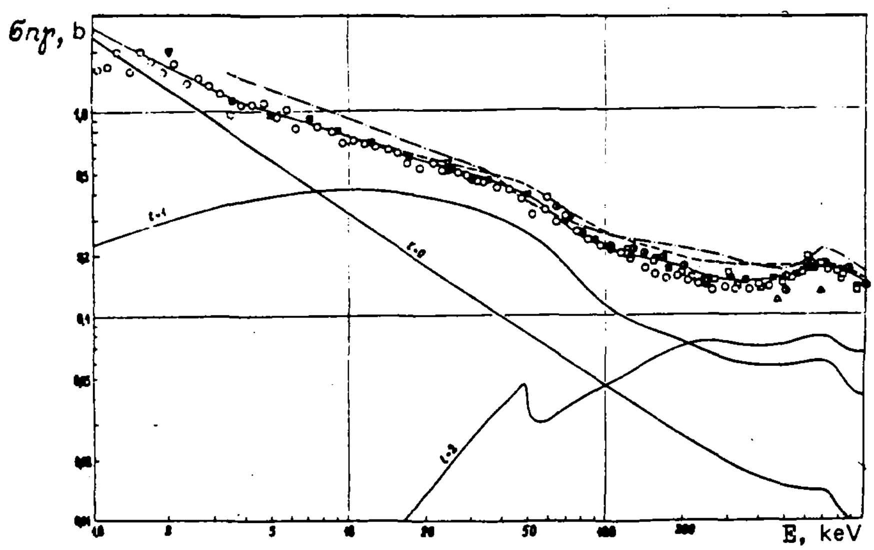


Fig. 4. Radiative-capture cross-section of $^{232}Th$. Comparison of the latest experimental data and different evaluations: □ Ref. [21], ■ Ref. [22], ▽ Ref. [23], ⊠ Ref. [24], ◐◑⊗ Ref. [25], ○ Ref. [26], △ Ref. [27]; -.- JENDL-1, - - - ENDF/B-IV; ― present work.

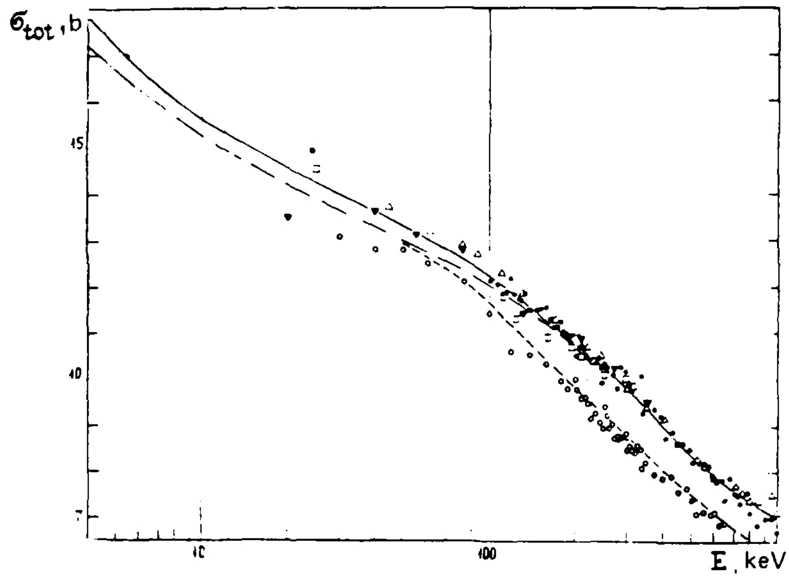


Fig. 5. Total cross-section of $^{232}Th$: ○ Ref. [28], □ Ref. [29], △ Ref. [30], ⊗ Ref. [31], ⊠ Ref. [32], ● Ref. [33], ▼ Ref. [34]; -.- JENDL-1; - - - ENDF/B-IV; ___ present work.

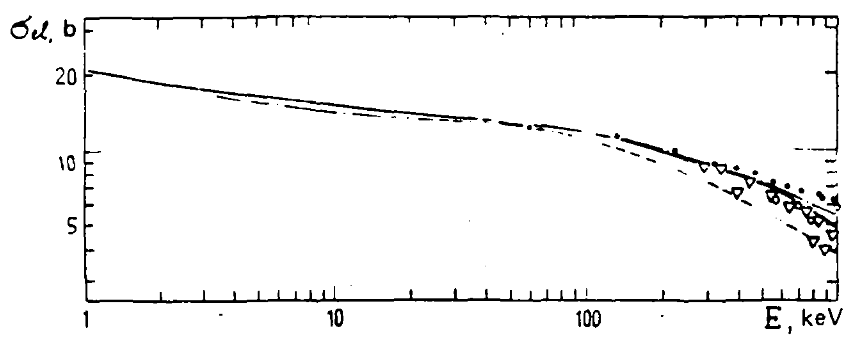


Fig. 6. Elastic scattering cross-section of $^{232}Th$: ○ Ref. [35], ● Ref. [36], ▽ Ref. [37], -.- JENDL-1; - - - ENDF/B-IV; ___ present work.

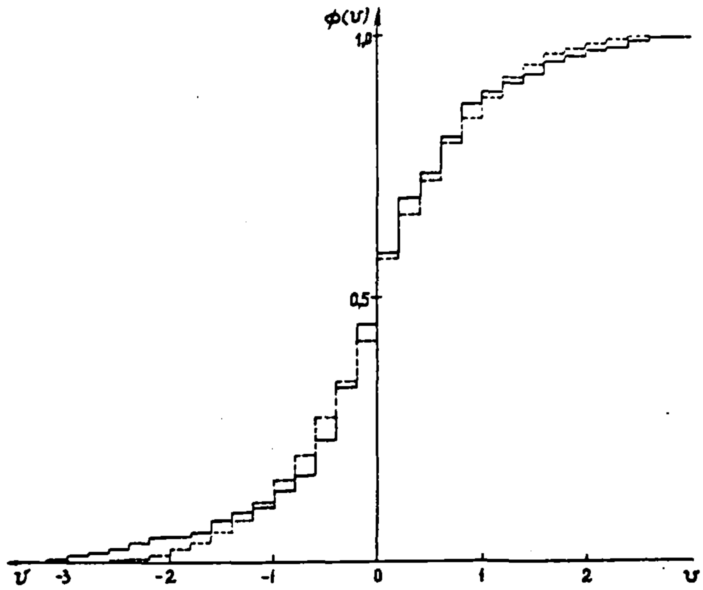


Fig. 7. The normal distribution function of the standardized value of differences $\bar{\sigma}_0 - \bar{\sigma}_1$ $(\bar{p}')$: - - - Theoretical distribution; ___ observed distribution.