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EVALUATION OF RESONANCE SELF-SHIELDING FACTORS FOR ^{238}U IN THE UNRESOLVED RESONANCE REGION

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ABSTRACT

On the basis of a theoretical model of identical equidistant resonances for the energy dependence of cross-sections in the unresolved resonance region, the authors have parametrized the values of the resonance self-shielding factors and their Doppler increments for ^{238}U. They have proposed a method by which the Doppler increments of the self-shielding factors can be calculated from simple analytical formulae by redetermination of the model parameters. Analysing the experimental data on direct and capture transmissions in the unresolved resonance region, they demonstrate the possibility of describing those data as a whole and of deriving from them the cross-section group functionals.

One of the most complex problems encountered in describing the process of the slowing down, absorption and transport of neutrons in media is to evaluate the effects of resonance self-shielding in the region of unresolved levels. Owing to lack of direct experimental information in this region, various approximations are used to plot the energy structure of cross-sections. The best known of these are the method of sub-groups [1] and, abroad, the use of probability tables [2]. In the sub-group method, which was developed by M.N. Nikolaev, any group-averaged functional of cross-section σ-$F(\sigma)$ (including group-averaged transmissions) is represented in the form $\langle F(\sigma)\rangle = \sum_k a_k F(\sigma_k)$; to find it, one needs to specify the sub-group paramaters $a_k \sigma_k$ [1,3]. The advantage of these methods lies in the ease of their practical use. For example, the sub-group method is used successfully in the practice of reactor calculations, while the question of the physical sense of the parameters remains open, their determination requiring special integral measurements, especially energy-averaged transmissions as a function of sample thickness [1-3]. In the present paper we propose a method of parametrizing these data and a relatively simple method for calculating, on the basis of the transmission parameters, the main cross-section functionals used in group calculations.

In order to describe the resonance structure of neutron cross-sections, we use the model of identical equidistant resonances [4], which takes into account the basic qualitative characteristics of the behaviour of actual cross-sections. In the formalism of the R-matrix theory for nuclei below the inelastic scattering threshold the total cross-section σ and the radiative capture cross-section σ_c can be represented by fairly simple formulae [5]:

$$\sigma(E) = \sigma_m + \sigma_0 (s\cos\varphi - \mathrm{tg}\, z \sin\varphi)^2/(s^2 + \mathrm{tg}^2 z) \; ; \tag{1}$$

$$\sigma_c(E) = \sigma_{0c} s^2 (\mathrm{tg}^2 z + 1)/(\mathrm{tg}^2 z + s^2), \tag{2}$$

where $z = \Pi E/D$, E is the neutron energy, D the distance between resonances, σ_m the cross-section at the resonance minimum, $\sigma_o = \sigma_M - \sigma_m$ characterizes the cross-section value at resonance maximum σ_M; $S \approx \Pi\Gamma/2D$ is the strength function and ϕ the interference phase of potential and resonance scattering. In the case of identical scattering with radiative capture and without the Doppler broadening of resonances, the parameters of formulae (1) and (2) can be calculated with the help of the evaluations [5,6]: $\sigma_0 = (4\pi/\kappa^2)(\Gamma_n/\Gamma)$, $\varphi = \kappa R$, $S = (\Pi/2)S_o\sqrt{E}$, where R is the optical radius of the nucleus; S_o the reduced strength function and Γ_n and Γ are the neutron and total widths. In a case where there are several scattering channels, formulae (1) and (2) describing the total and radiative capture cross-sections are also valid, but in this case the parameters σ_m, σ_o, S and ϕ will be based on the model and can be obtained from an analysis of experimental data on transmission or by fitting to the available values of cross-section moments or resonance self-shielding factors [7,8].

The averaging over energy of formulae (1) and (2) corresponds to averaging over the period $-\Pi/2 \le z \le \Pi/2$ and gives the well-known results for average cross-sections [9]:

$$\langle\sigma\rangle = \sigma_m + \sigma_0(\sin^2\varphi + s\cos^2\varphi)/(1+s) = \sigma_p + \sigma_0\, s\cos\varphi/(1+s) \; ; \tag{3}$$

$$\langle\sigma_c\rangle = \sigma_{0c} s \; , \tag{4}$$

where $\sigma_p = \sigma_m + \sigma_o \sin^2\phi$ is the potential cross-section.

Transmission with cross-section (2) will take the form [4,6].

$$T = \langle \exp(-n\sigma)\rangle = \frac{1}{\pi}\exp(-n\sigma_m)\int_{-\pi/2}^{+\pi/2} \exp\left\{-n\sigma_0 \frac{(s\cos\varphi - \mathrm{tg}\, z\sin\varphi)^2}{s^2 + \mathrm{tg}^2 z}\right\} dz \, . \tag{5}$$

The cross-section movements are accordingly determined as [6].

$$\langle \sigma^2 \rangle = \langle \sigma \rangle^2 + \left\{ s\sigma_0^2 \Big/ \left[2(1+s)^2 \right] \right\}; \tag{6}$$

$$\left\langle \frac{1}{\sigma} \right\rangle = \frac{1}{\sqrt{\sigma_m \sigma_M}} \, \frac{(s\sqrt{\sigma_m} + \sqrt{\sigma_M})(s\sqrt{\sigma_M} + \sqrt{\sigma_m})}{(s\sqrt{\sigma_m} + \sqrt{\sigma_M})^2 + \sigma_0 \sin^2\varphi (1-s^2)} \quad ; \tag{7}$$

$$\left\langle \frac{1}{\sigma^2} \right\rangle = \left\langle \frac{1}{\sigma} \right\rangle^2 + \frac{s\sigma_0^2}{2\left[\sigma_M \sigma_m\right]^{3/2} \left[(s\sqrt{\sigma_M} + \sqrt{\sigma_m})^2 + \sigma_0 \sin^2\varphi (1-s^2) \right]} \, , \tag{8}$$

where $\sigma_M = \sigma_o + \sigma_m$.

The values of cross-section moments (6) and (7) characterize the resonance self-shielding factors which, with allowance for the "dilution" cross-section σ_R, are determined as [3,5]:

$$f_t(\sigma_R) = 1/\langle \sigma \rangle \left[\left\langle \frac{1}{\sigma + \sigma_R} \right\rangle \Big/ \left\langle \frac{1}{(\sigma + \sigma_R)^2} \right\rangle - \sigma_R \right] \; ; \tag{9}$$

$$f_c(\sigma_R) = 1/\langle \sigma_c \rangle \left[\left\langle \frac{\sigma_c}{\sigma + \sigma_R} \right\rangle \Big/ \left\langle \frac{1}{\sigma + \sigma_R} \right\rangle \right] ; \tag{10}$$

$$f_e(\sigma_R) = 1/\langle \sigma_e \rangle \left[\left\langle \frac{\sigma_e}{\sigma + \sigma_R} \right\rangle \Big/ \left\langle \frac{1}{\sigma + \sigma_R} \right\rangle \right] , \tag{11}$$

where the subscript t refers to the total cross-section, e to scattering and c to absorption. The group cross-sections used in reactor calculations are obtained by multiplying the average cross-sections by the corresponding resonance self-shielding factors. Addition of the cross-section σ_R in formulae (9)-(11) is equivalent in the scheme considered to redetermining the value of the cross-section at the minimum $\sigma_m^* = \sigma_m + \sigma_R$. In this scheme, the group-averaged capture cross-section on filtered beams as a function of changes in filter thicknesses is found as

$$\langle \sigma_c \exp(-n\sigma) \rangle = \langle \sigma_c \rangle \exp\left\{ -n\left[\sigma_m + (\sigma_0/2) \right] \right\} I_0(n\sigma_0/2) \, , \tag{12}$$

where I_o is the Bessel function of the imaginary argument [4]. Here the effective resonance integral of absorption $\langle \sigma_c/\sigma \rangle$ will take the form

$$\langle \sigma_c/\sigma \rangle = \int_0^\infty \langle \sigma_c \exp(-n\sigma) \rangle dn = \langle \sigma_c \rangle \, 1/\sqrt{\sigma_m \sigma_M} \quad , \tag{13}$$

and the self-shielding factor f_c can be represented by the formula [5]

$$f_c = \frac{(s\sqrt{\sigma_M^*} + \sqrt{\sigma_m^*})^2 + \sigma_0 \sin^2\varphi(1-s^2)}{(s\sqrt{\sigma_M^*} + \sqrt{\sigma_m^*})(s\sqrt{\sigma_m^*} + \sqrt{\sigma_M^*})} , \tag{14}$$

where in $\sigma_m^* = \sigma_m + \sigma_R$, $\sigma_M^* = \sigma_m^* + \sigma_R$ the dilution cross-section σ_R has been taken into account.

On the basis of the definition of f_c and f_e in (10), (11), formulae (13), (14) and relation $\langle\sigma_c/\sigma\rangle + \langle\sigma_e/\sigma\rangle = 1$ we find

$$\langle 1/(\sigma + \sigma_R)\rangle = 1/(\langle\sigma_e\rangle f_e + \langle\sigma_c\rangle f_c + \sigma_R) = \left[1/\sqrt{\sigma_m^*(\sigma_m^* + \sigma_0^*)}\right] f_c \tag{15}$$

or for zero dilution ($\sigma_R = 0$)

$$1/\sqrt{\sigma_m \sigma_M} = \langle 1/\sigma\rangle f_c = \left[1/(\langle\sigma_c\rangle f_c + \langle\sigma_e\rangle f_e)\right] f_c \, . \tag{16}$$

Formula (14) for calculation of the capture resonance self-shielding factors describesthe absorption by one system of resonance levels (for example, with $\ell = 0$). In the case where several systems of resonance levels have to be taken into account and one of them is predominant, the capture self-shielding factors can be calculated as [5]:

$$f_c = f_c^* \left\{ \langle\langle\sigma_{cs}\rangle/\langle\sigma_c\rangle\rangle + \left[1 - \langle\langle\sigma_{cs}\rangle/\langle\sigma_c\rangle\rangle\right] \sqrt{\sigma_m^*/\langle\sigma^*\rangle f_e^*} \right\} , \tag{17}$$

where $\langle\sigma_{cs}\rangle/\langle\sigma_c\rangle$ characterizes the contribution of the S-wave (main system of resonance levels) to the average capture cross-section; f_c^* is calculated by formula (14); σ_m^*, f_e^*, $\langle\sigma^*\rangle$ are calculated with allowance for the dilution cross-section σ_R by redetermination of the parameter σ_m.

Consideration of the Doppler broadening of resonances

The influence of the Doppler broadening of resonances on the shape of the resonance curve (which can be neglected in the case of some structural elements) is very substantial in the case of heavy elements. At present, the classical scheme for describing such an effect is to use the Maxwellian distribution for the velocities of the nuclei in the target [9]:

$$f(E')dE' = (1/\sqrt{\pi}\,\Delta)\exp\left[-(E'-E)^2/\Delta^2\right]dE' , \tag{18}$$

where $\Delta = 2\sqrt{kTE}/(A+1)$ is the Doppler width, k the Boltzmann constant and T the absolute temperature. The use of the Maxwellian distribution to describe resonance shape gives us the well-known functions $\Psi(\zeta_\lambda, x)$, $X(\zeta_\lambda, x)$ [9]:

$$\left.\begin{aligned} \Psi(\xi_\lambda, x) &= \frac{\xi_\lambda}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp\left[-\xi_\lambda^2 (x-y)^2\right] dy}{1-y^2}, \\ X(\xi_\lambda, x) &= \frac{\xi_\lambda}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{y \exp\left[-\xi_\lambda^2 (x-y)^2\right]}{1+y^2} dy . \end{aligned}\right\} \tag{19}$$

Here $\zeta_\lambda = \Gamma_\lambda / 2\Delta$, Γ_λ being the natural resonance width. For example, in the case of the single-level Breit-Wigner formula for the total neutron-nucleus interaction cross-section which takes into account the interference of resonance and potential scattering ($\phi \neq 0$), we get

$$\sigma(E) = \sigma_p + \sigma_0 \left[\Psi(\xi_\lambda, x) \cos 2\varphi - X(\xi_\lambda, x) \sin 2\varphi\right], \tag{20}$$

where $x = 2(E_\lambda - E)/\Gamma_\lambda$.

Applying distribution (18) to cross-section (1) (which gives the Breit-Wigner formula for $S \ll 1$) we obtain

$$\sigma(E) = \sigma_m + \sigma_0 \sin^2\varphi + \frac{\sigma_0 s^2 \cos 2\varphi}{\sqrt{\pi}\,\Delta} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{\Delta^2}\right) \frac{dx}{s^2 + \operatorname{tg}^2\left[\pi(E+x)/D\right]} -$$

$$- \frac{\sigma_0 s \sin 2\varphi}{\sqrt{\pi}\,\Delta} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{\Delta^2}\right) \frac{dx \operatorname{tg}\left[\pi(E+x)/D\right]}{s^2 + \operatorname{tg}^2\left[\pi(E+x)/D\right]} . \tag{21}$$

As will be seen from formulae (19)-(21), the application of the Maxwellian distribution leads to integrals which are fairly complex for practical use, while if the model resonance parameters σ_m, ϕ, S, σ_o are used, the accuracy with which the Doppler broadening of resonances can be described using the functions $\Psi(\xi_\lambda, x)$, $X(\xi_\lambda, x)$ will not be very high. A more rational way of considering the Doppler broadening is to redetermine the parameters σ_m, ϕ, S, σ_o so that the temperature dependence of the resonance structure of cross-sections can be described more efficiently. Assuming that the temperature broadening does not affect $\langle\sigma\rangle$ (3) and $\sigma_p = \sigma_m + \sigma_o \sin^2\phi$, we come to the conclusion that only two of the four parameters need to be redetermined. There are several methods for such redetermination. For example, we can calculate the strength function as $\tilde{S} = \pi(\Gamma + \tilde{\Delta})/2D = s(\Gamma + \tilde{\Delta})/\Gamma$, where the relationship between $\tilde{\Delta}$ and Δ (Doppler width) is considered in Ref. [10]. With the norm $\Psi(\zeta_\lambda, x)$ remaining constant, we can redetermine the parameter $\sigma_0 : \tilde{\sigma}_0 = \bar{\sigma}_0 \Psi(\sqrt{2}\,\xi, 0)$ [4,11]. Given this redetermination, the parameter ϕ can be left unchanged. The accuracy of such redeterminations was studied in Refs [10,11]; in this case the Doppler changes in the effective resonance integral $\langle\sigma_c/\sigma\rangle$ are described satisfactorily, while the description of the Doppler increments of the self-shielding factors f_t and f_c is less satisfactory.

The calculation results show that, when the model parameters are used, a satisfactory agreement with the well-known values of the Doppler increments of the resonance self-shielding factors is achieved by a redetermination of the form

$$\tilde{\sigma}_0 = \sigma_0 \Psi(\xi,0) ; \quad (22)$$

$$\tilde{\sigma}_m = (\sigma_m + \sigma_0)\sin^4\varphi/(2\xi^2 + \sin^2\varphi). \quad (23)$$

Formula (22) reflects a reduction in the resonance maxima during the Doppler broadening (see expression (20)), while formula (23) approximately describes the temperature dependence of the cross-section σ_m at the minimum. For a specific case in practice, where we know the value of parameters σ_m, σ_o, S and also σ_p at a certain temperature (for example, 300K) formulae (22) and (23) will take the form

$$\tilde{\sigma}_0 = \sigma_0 \Psi(\xi_T,0)\Big/\left[\Psi(\xi,0)\right] ; \quad (24)$$

$$\tilde{\sigma}_m = \sigma_p - (\sigma_p - \sigma_0)\left\{16\xi^2\xi_T^2\Big/\left[4\xi^2\xi_T^2 + K(2\xi^2 - 2\xi_T^2)\right]\right\} . \quad (25)$$

Here $K = (\sigma_p - \sigma_m)/\sigma_0$; $\xi_T = \Gamma/2\Delta_T$ (Δ_T is the Doppler width at the temperature sought T; $\Gamma = 2DS/\pi$); $\xi = \Gamma/2\Delta$ (Δ is the Doppler width at the initial temperature, for example 300K). After the parameters $\tilde{\sigma}_m$ and $\tilde{\sigma}_o$ have been determined, $\tilde{S}$ and $\tilde{\phi}$ can be found from the conditions

$$\tilde{\varphi} = \arcsin(\sqrt{\sigma_p - \tilde{\sigma}_m}/\sigma_0) ; \quad (26)$$

$$\tilde{s} = (\langle\sigma\rangle - \sigma_p)\Big/(\tilde{\sigma}_0 \cos 2\tilde{\varphi} + \sigma_p - \langle\sigma\rangle) . \quad (27)$$

It is possible to redetermine parameters (25)-(27) both in the case of single-channel scattering with radiative capture, when parameters σ_o, ϕ, S are well known, and in the case of model parameters. Redetermining parameters σ_m, ϕ, S, σ_o for different temperatures, by formulae (9)-(11) we can then calculate the Doppler increments of the resonance self-shielding factors with an accuracy good enough for practical applications.

Parametrization of the values of self-shielding factors

The calculation of self-shielding factors, their Doppler increments and the transmissions for different thicknesses in the case of elastic scattering and radiative capture can be illustrated conveniently if we use the isotope ^{238}U as an example [12]. For this isotope the experimental data on transmission were analysed and the available values of the resonance self-shielding factors were parametrized; since this particular method involves averaging over a large number of resonances, the analysis was performed in groups 10-16 of the BNAB-78 26-group system of constants [3].

In order to parametrize resonance self-shielding factors with simultaneous solution of the corresponding equations, we determined the parameters σ_m, Φ, S, σ_o. In groups 10-14 we used the equations for $\langle\sigma\rangle$, $\langle 1/\sigma\rangle$, $\langle 1/\sigma^2\rangle$ (see formulae (3), (7) and (8)) and $\sigma_p = \sigma_m + \sigma_0 \sin^2\varphi$. For σ_p we took the value of 10.9 b recommended for the unresolved region [3,12]; $\langle 1/\sigma\rangle$ was calculated by formula (15) for zero dilution and $\langle 1/\sigma^2\rangle = (1/\langle\sigma\rangle)/[\langle\sigma\rangle f_t(0)]$. The values of f_c, f_e, f_t, $\langle\sigma_c\rangle$, $\langle\sigma_e\rangle$, $\langle\sigma\rangle$ were taken from Ref. [3].

The four equations with four unknown quantities were solved with the aid of the FUMILI and NEWTON programs from the BEhSM-6 computer library. The FUMILI program fitted the parameters by the least-square method [5-7], and the NEWTON program solved the four equations with four unknown quantities by the Newton method of solution of a system of transcendental equations [8,13]. The results of these programs were in agreement. As was shown by the calculation data, the value of σ_p in groups 15 and 16 was somewhat higher than 10.9 b; in these groups the contribution of the p-wave to the average capture cross-section was small [14,15]; as the fourth equation, instead of σ_p, we therefore used $\sqrt{\sigma_m(\sigma_m+\sigma_o)}$, which was calculated by formula (16) with the data of Ref. [3]. The set of four equations was solved with the two programs and, as in groups 10-14, the results tallied.

In Table 1 the numerator gives the values of parameters σ_m, Φ, S, σ_o obtained by solving the set of four equations in groups 10-16, and the moments of the cross-sections $\langle\sigma\rangle$, $\langle\sigma^2\rangle$, $\langle 1/\sigma\rangle$, $\langle 1/\sigma^2\rangle$ calculated from them as well as the self-shielding factors f_t, f_e and f_c for zero dilution σ_R, (formulae (3), (6)-(11)); the denominator contains the resonance parameters and group reactor functionals obtained from the analysis of data on direct and capture transmissions. Table 2 presents the dependence on the dilution cross-section σ_R of the resonance self-shielding factors f_t, f_e and f_c found by formulae (9) and (11) with the use of the derived parameters, in comparison with the data of Ref. [3]. In groups 13-16, f_c was calculated by means of formula (17), the contribution of the S-wave to the average capture cross-section being taken from Ref. [15]. In groups 10-12 the contribution of the resonance levels of the p- and d-waves to the capture

Table 1. Average resonance parameters, cross-section moments and self-shielding factors for groups 10-16.

Group No.	σ_m, σ	φ	s	σ_0, σ	$\langle\sigma\rangle$, σ	$\langle\sigma^2\rangle$, σ^2	$\langle 1/\sigma\rangle$, σ^{-1}	$\langle 1/\sigma^2\rangle$, σ^{-2}	f_c	f_t	f_e
I0	9,32 6,57	0,305 0,506	0,2I7 0,I77	I7,5I 24,36	I3,464 I4,24	203,6 240,6	0,08I4 0,0825	0,00707 0,0076I	0,777 0,85	0,855 0,702	0,9I7 0,85
II	8,I9 5,59	0,3II 0,4I7	0,I8I 0,073	28,76 42,26	I4,48 I4,47	263 266	0,082 0,082I	0,0075 0,0077	0,70I 0,744	0,755 0,734	0,849 0,846
I2	7,76 3,98	0,279 0,532	0,I65 0,0533	4I,52 34,5	I5,88 I3,7	356,5 2I6,5	0,08II 0,083I	0,0076 0,00807	0,63I 0,972	0,672 0,568	0,785 0,75
I3	5,I9 I,9I	0,28I 0,3I6	0,I47 0,0362	74,I3 II6	I8,95 I6,39	675,5 495,8	0,0885 0,09	0,0I045 0,0I3I	0,477 0,74I	0,449 0,665	0,60I 0,673
I4	3,75 4,75	0,I77 0,I63	0,0554 0,0548	229 229,5	22,I9 22,07	I795 I784	0,0973 0,0923	0,0II4 0,0III	0,3I8 0,325	0,33 0,377	0,474 0,500
I5	3,I5 4,93	0,0907 0,0859	0,0I08 0,0I47	I097 8II	23,72 22,48	695I 5206	0,0888 0,092	0,0I026 0,00993	0,I82 0,I7I	0,365 0,4I2	0,522 0,538
I6	5,6 6,32	0,0666 0,07I3	0,00636 0,0I29	I629 840	23,03 2I,02	8860 49I0	0,0786 0,09II	0,00687 0,00922	0,I33 0,I50	0,497 0,465	0,656 0,6I6

cross-section is large and approximations of formula (17) are not valid [5,14,15]. In these groups the capture resonance self-shielding factors were calculated by formula (14) with allowance for only one effective system of resonance levels; this evidently accounts for the disagreements with the data of Ref. [3] although the derived values lie within the existing scatter of evaluations of f_c in these groups [3,12].

For parameters σ_m, σ_o, S and Φ (see the numerator in Table 1) in groups 10-14 we carried out redetermination as a function of temperature using formulae (24)-(27), and calculated the Doppler increments in the resonance self-shielding factors for the total capture and scattering cross-sections; these are given in Table 3 for comparison with the data of Ref. [3]. The relative error in these data, apart from the values for f_c in groups 10-12, where the f_c values themselves differ, does not exceed 50% (mainly not more than 10-20%), i.e. it is of the same order as the accuracy of those data [3,12].

As can be seen from Table 1, the values of σ_m in groups 10-16 and σ_o in groups 10-14, which were obtained as a result of parametrization, agree satisfactorily with the minimum and maximum values of the total cross-sections in the respective groups [16]; here σ_o is described approximately by the dependence $\sigma_0=(4\pi/k^2)(\Gamma_n/\Gamma)\Psi(\xi,0)$ for the strongest resonances, while σ_m nowhere drops below the value $\sigma_m = 4\pi R^2(\Gamma_\gamma/\Gamma)$, where Γ_γ and Γ are the radiative and total neutron resonance widths in the group. Parameters S

Table 2. Resonance self-shielding factors for ^{238}U at 300K.

Group No.	f_t for σ_R, equal to					f_e for σ_R, equal to					f_c for σ_R, equal to				
	I0000	I000	I00	I0	0	I0000	I000	I00	I0	0	I0000	IU00	I00	I0	0
I0	-	0,997 (0,997)	0,973 (0,974)	0,90I (0,907)	0,855 (0,855)	-	0,998 (0,998)	0,987 (0,986)	0,947 (0,946)	0,9I7 (0,9I2)	-	0,996 (0,998)	0,962 (0,988)	0,85I (0,948)	0,777 (0,9I)
II	0,999 (0,999)	0,993 (0,99I)	0,944 (0,936)	0,82I (0,828)	0,755 (0,755)	-	0,996 (0,995)	0,972 (0,963)	0,898 (0,880)	0,849 (0,844)	0,999	0,992 (0,996)	0,937 (0,968)	0,78 (0,884)	0,70I (0,83)
I2	0,999 (0,998)	0,987 (0,978)	0,908 (0,882)	0,746 (0,756)	0,672 (0,672)	0,999 (0,999)	0,994 (0,989)	0,953 (0,93)	0,847 (0,832)	0,785 (0,78)	0,999 (0,999)	0,988 (0,990)	0,908 (0,929)	0,72 (0,795)	0,63I (0,7I3)
I3	0,997 (0,990)	0,968 (0,9I7)	0,805 (0,724)	0,56 (0,58I)	0,449 (0,447)	0,998 (0,995)	0,984 (0,955)	0,894 (0,8II)	0,706 (0,674)	0,6 (0,603)	0,985 (0,995)	0,965 (0,954)	0,833 (0,79I)	0,585 (0,592)	0,477 (0,50I)
I4	0,988 (0,977)	0,905 (0,846)	0,642 (0,609)	0,44 (0,477)	0,33 (0,384)	0,994 (0,988)	0,952 (0,9I3)	0,775 (0,7I8)	0,576 (0,579)	0,474 (0,479)	0,983 (0,988)	0,9I3 (0,903)	0,63I (0,625)	0,376 (0,377)	0,3I8 (0,285)
I5	0,952 (0,968)	0,756 (0,800)	0,564 (0,556)	0,464 (0,46I)	0,365 (0,365)	0,979 (0,968)	0,87I (0,89I)	0,695 (0,686)	0,588 (0,572)	0,522 (0,522)	0,946 (0,979)	0,703 (0,84)	0,330 (0,47I)	0,I83 (0,239)	0,I82 (0,I83)
I6	0,939 (0,957)	0,743 (0,777)	0,6 (0,596)	0,537 (0,537)	0,497 (0,497)	0,977 (0,986)	0,879 (0,889)	0,756 (0,746)	0,688 (0,68I)	0,656 (0,656)	0,929 (0,962)	0,626 (0,75)	0,269 (0,358)	0,I46 (0,I7I)	0,I33 (0,I33)

N.B. The data from Ref. [3] are given in brackets.

Table 3. Doppler increments in the resonance self-shielding factors for groups 10-14 for ^{238}U.

Group No.	Δ	Δf_t for σ_R, equal to				Δf_e for σ_R, equal to				Δf_c for σ_R, equal to			
		I000	I00	I0	0	I000	I00	I0	0	I000	I00	I0	0
I0	Δ_1	0,00I7 (0,00II)	0,0I36 (0,0082)	0,0442 (0,0275)	0,0585 (0,0456)	0,0008 (0,0005)	0,0066 (0,0043)	0,0252 (0,0I58)	0,0372 (0,0252)	0,002 (0,0005)	0,02 (0,0044)	0,072 (0,0I9I)	0,I02 (0,032I)
	Δ_2	0,0007 (0,0006)	0,006 (0,0057)	0,022 (0,0I7)	0,0302 (0,0264)	0,0003 (0,0003)	0,0027 (0,0025)	0,0II (0,0097)	0,0I67 (0,0I52)	0,00I4 (0,0002)	0,0I2 (0,002)	0,05 (0,0088)	0,0779 (0,0I49)
II	Δ_1	0,0036 (0,003)	0,027 (0,0I8)	0,074 (0,0395)	0,094 (0,0628)	0,00I8 (0,00I5)	0,0I39 (0,0I04)	0,0463 (0,0265)	0,066 (0,0373)	0,0037 (0,00I5)	0,0285 (0,0II5)	0,085 (0,039I)	0,I09 (0,0585)
	Δ_2	0,00I6 (0,00I6)	0,0I2 (0,0II)	0,036I (0,0272)	0,0436 (0,0374)	0,0007 (0,0008)	0,0057 (0,006I)	0,02 (0,0I76)	0,0278 (0,0243)	0,0022 (0,0007)	0,0I8 (0,0057)	0,067 (0,02I3)	0,094 (0,0327)
I2	Δ_1	0,006 (0,008I)	0,0383 (0,033I)	0,0757 (0,045I)	0,0845 (0,07I8)	0,0028 (0,0042)	0,020I (0,02I4)	0,053 (0,037)	0,068 (0,0457)	0,006 (0,0044)	0,0428 (0,0276)	0,I07 (0,0649)	0,I24 (0,0807)
	Δ_2	0,0029 (0,0045)	0,02I3 (0,0222)	0,053 (0,0356)	0,058I (0,0467)	0,00I4 (0,0023)	0,0I04 (0,0I36)	0,0324 (0,0272)	0,042I (0,0334)	0,0032 (0,0022)	0,0253 (0,0I58)	0,0837 (0,0433)	0,II2 (0,0573)
I3	Δ_1	0,0I4 (0,02)	0,067 (0,045)	0,086 (0,06I)	0,0987 (0,095)	0,0066 (0,0II)	0,039 (0,034)	0,074 (0,039)	0,092 (0,049)	0,0I2 (0,0I2)	0,0689 (0,054)	0,I25 (0,083)	0,I24 (0,084)
	Δ_2	0,007 (0,0I2)	0,044 (0,038)	0,052 (0,037)	0,0677 (0,055)	0,0034) (0,006)	0,024 (0,027)	0,0556 (0,036)	0,0646 (0,04I)	0,006 (0,007)	0,0428 (0,036)	0,II2 (0,07)	0,I34 (0,08)
I4	Δ_1	0,0342 (0,0404)	0,0652 (0,0408)	0,0479 (0,0257)	0,0744 (0,0853)	0,0I76 (0,0246)	0,0533 (0,04II)	0,0526 (0,0293)	0,0653 (0,0442)	0,033 (0,033)	0,I02 (0,083)	0,09II (0,077I)	0,0630 (0,064I)
	Δ_2	0,0203 (0,0285)	0,06I6 (0,0407)	0,0559 (0,0274)	0,0648 (0,I048)	0,0I02 (0,0I65)	0,0429 (0,0367)	0,0566 (0,0307)	0,0645 (0,04I8)	0,0I7 (0,020)	0,074 (0,065)	0,0879 (0,073I)	0,0755 (0,0697)

N.B. The data from Ref. [3] are given in brackets (Δ_1 = f(900K) - f(300K), Δ_2 = f(2100K) - f(900K).

and ϕ are sensitive to the Doppler broadening of resonances and are based on the model.

The parametrization method considered is clearly inapplicable to small S and ϕ and to large σ_o (which corresponds to groups 17-22) since for these values the error in calculating functions Ψ and X will be large and cross-section (1) is described by the approximate dependence $\sigma(E) \approx \sigma_m + \sigma_o S/(tg^2 Z + S^2)$. In this expression there is a very strong correlation between σ_o and S, and in groups 15 and 16 these parameters can no longer be regarded as independent.

Analysis of experimental data on transmission

With the help of the four paramters σ_m, Φ, S, σ_o we can calculate not only the self-shielding factors together with the Doppler increments but also the transmission data [5,12]. As an example we analysed the experimental data on the total and capture transmissions (formula (5) and (12)) in groups 10-16 for ^{238}U. In groups 10 and 11 we analysed the data contained in Ref. [17], and since transmissions are given only for small thicknesses, we also added the data of Ref. [18]. In groups 12-14 we analysed the data on direct and capture transmissions of Ref. [18]; in groups 14-16 we used the data from the ENDF/B library for calculation of the direct and capture transmissions by the URAN program [19].

The parameters σ_m, Φ, S, σ_o of formulae (5) and (12) were fitted to the respective experimental data with the help of the FUMILI library program on the BEhSM-6 computer by the least-square method. In the FUMILI program, a corresponding statistical weight can be assigned to them, depending on the existing accuracy of experimental data. A large error is typical of capture transmissions.

The results of fitting transmissions in groups 10-14 are given in Fig. 1, and those for capture transmissions in groups 10-16 in Fig. 2. The parameters σ_m, ϕ, S, σ_o obtained from analysis of the experimental data are given in Table 1 (denominator). From Figs 1 and 2 it will be seen that the data of Ref. [18] lie above the results of fitting transmissions for large thicknesses, and this may be due to the influence of secondary neutrons [20] and background. At the same time, the data on direct and capture transmissions contained in Refs [17,19] are described satisfactorily by formulae (5) and (12).

For the parameters σ_m, ϕ, S and σ_o obtained from analysis of experimental data by formulae (3), (5)-(7), (9)-(11) we calculated the values of $\langle\sigma\rangle$, $\langle\sigma^2\rangle$, $\langle 1/\sigma\rangle$, $\langle 1/\sigma^2\rangle$, f_t, f_e and f_c (see Table 1, denominator). In the case of parameters of groups 15 and 16, by means of redetermination on the basis of formulae (25)-(27) we calculated the Doppler increments of the resonance

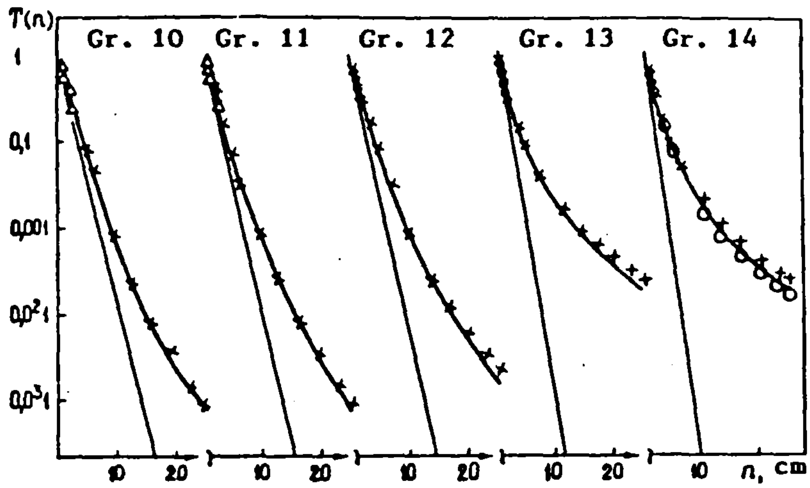

Fig. 1. Experimental and calculated values of transmissions T(n) as a function of thickness n in groups 10–14 for ^{238}U. x – data from Ref. [18]; Δ – data from Ref. [17]; o – data from Ref. [19]; —— – values calculated by formula (4); — – exp(-n<σ>).

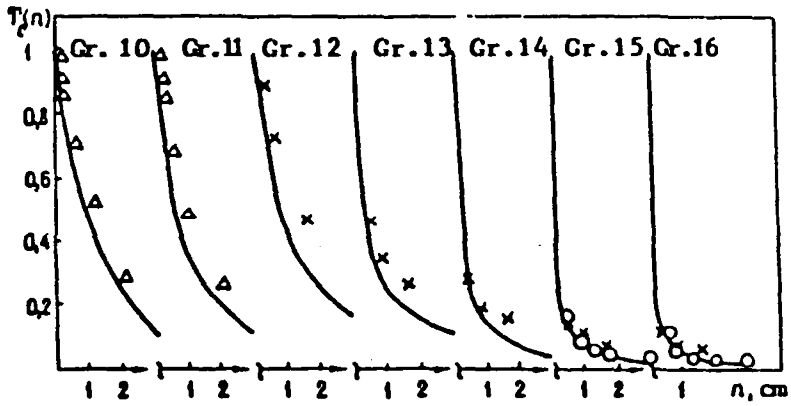

Fig. 2. Experimental and calculated values of capture transmissions $T_c(n)$ in groups 10–16 for ^{238}U. x – data from Ref. [18]; Δ – data from Ref. [17]; o – data from Ref. [19]. —— – values calculated by formula (12).

self-shielding factors f_t, f_c and f_e, which are given in comparison with the data of Ref. [3] in Table 4. It is seen from this table that the error in calculation of the Doppler increments does not generally exceed 15-20%.

The parameters of the method proposed for analysing experimental data on direct and capture transmission are very sensitive to transmissions for large thicknesses. This applies most to the parameters σ_m – the value of the group-averaged minimum cross-section. By analysing the experimental data, we can find the values of this and other parameters with fairly high accuracy and calculate the transmissions for any thickness [5–7].

Here it is significant that the direct transmissions can be used to obtain the capture transmissions only on the basis of data on average cross-

Table 4. Doppler increments in resonance self-shielding factors for groups 15 and 16.

Group No.	Δ	Δf_c for σ_R, equal to			Δf_t for σ_R, equal to			Δf_e for σ_R, equal to		
		I0000	I00	0	I0000	I00	0	I0000	I00	0
I5	Δ_1	0,0I42 (0,0082)	0,090I (0,0945)	0,03II (0,0407)	0,0I48 (0,0II4)	0,035 (0,0394)	0,0343 (0,0555)	0,0065 (0,0055)	0,0393 (0,0425)	0,0293 (0,0299)
	Δ_2	0,0072 (0,0045)	0,0786 (0,082)	0,034 (0,050)	0,0078 (0,0066)	0,0382 (0,0432)	0,0332 (0,0385)	0,0034 (0,0032)	0,037 (0,0402)	0,0324 (0,0274)
I6	Δ_1	0,0I52 (0,0I46)	0,0905 (0,084I)	0,039 (0,028I)	0,0I54 (0,0I55)	0,032 (0,025)	0,0I3I (0,0203)	0,0063 (0,0066)	0,0343 (0,0262)	0,0I40 (0,0I46)
	Δ_2	0,0078 (0,0078)	0,0797 (0,0773)	0,037 (0,0348)	0,0082 (0,0087)	0,0363 (0,0284)	0,024 (0,0I72)	0,0033 (0,0037)	0,0336 (0,0259)	0,0248 (0,0I43)

N.B. The data from Ref. [3] are given in brackets ($\Delta_1 = f(900K) - f(300K)$; $\Delta_2 = f(2100K) - f(900K)$).

sections $<\sigma_c>$ [5]. Unfortunately, there are no reliable experimental data on the direct and capture transmissions for ^{238}U over a wide range of sample thicknesses, including the biggest, which could show all the advantages and disadvantages of the proposed method. However, it can be concluded that the method is fully applicable to the region of unresolved resonances and can be employed to describe the experimental data available here. The situation is somewhat less favourable in the region of resolved resonances, where analysis of transmission data can be further complicated by the strong correlation of parameters S and σ_o. A more correct method which can give values of S, ϕ, σ_o agreeing with the respective theoretical evaluations would obviously be to use, for the cross-section, formula (21), which takes proper account of the effect of the Doppler broadening of resonances; however, it would give rise in the analysis of transmission data to computational difficulties.

The method we proposed for analysing experimental data brings out the importance of new experimentation on total and capture transmissions for large thicknesses in the case of ^{238}U if we are to refine the data in the unresolved resonance region.

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