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ESTIMATION OF TOTAL CROSS-SECTIONS FOR NEUTRON AND PROTON FORMATION IN INTERACTIONS BETWEEN DEUTERONS AND ^{7}Li NUCLEI

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(Translated from Nuclear Constants No. 3, Moscow 1985, Original in Russian distributed as INDC(CCP)-252/G)

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ESTIMATION OF TOTAL CROSS-SECTIONS FOR NEUTRON AND PROTON FORMATION IN INTERACTIONS BETWEEN DEUTERONS AND ^{7}Li NUCLEI

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ABSTRACT

Cubic spline approximation curves were obtained from experimental data. A brief description is given of an evaluation method using spline-functions with due regard for systematic and accidental errors. A method is suggested for representing the curve thus obtained in the form of a table of cubic spline coefficients which are suitable for interpolation calculations.

The continuing interest in controlled thermonuclear fusion reactors has given impetus to work aimed at refining the total cross-sections of various reactions with light nuclei. In addition to defining the constants more accurately for the basic reactions $^2H(d,n)^3He$, $^2H(t,n)^4He$, a lot of effort has been put into obtaining and evaluating total and differential cross-sections for interactions of hydrogen isotope nuclei with lithium nuclei [1, 2].

This paper presents results for the total cross-sections of the reactions $^7Li(d,n)TOT$ and $^7Li(d,p)^8Li$. The source data were presented in the literature mainly in the form of graphs and accordingly needed to be put in numerical form [3]. Random errors in the original cross-section values were made up quadratically from the errors cited by the authors of the original papers, and round-off errors from processing of the numerical data. In the course of an expert evaluation of each particular experimental paper, systematic errors were determined which in the main reflected the degree of confidence of the evaluating physicist in the experimental data considered.

In addition to the evaluated total cross-sections for the reactions mentioned above, we must draw attention to the presentation of the evaluated curves in the form of tables of spline coefficients, and we shall also briefly consider the evaluation method using the spline-approximation.

Evaluation method. The method described in Ref. [4] was essentially the one used to plot the evaluation curve. As a curve approximating the experimental data we took a polynomial spline of some step p on the fixed grid, $E_H = x_o < x_1 \ldots x_{n-1} < x_n = Ek$, where $[E_H, E_k]$ is the energy variation interval of interest to the paper's authors.

A spline defect of 1 [5] is usually used when, at the mesh nodes, all derivatives up to the order of p-1 are continuous. However, it is quite often necessary to make do with less smooth functions (for example because of sharp jumps or abrupt bends in the evaluation curve). In this case, at several nodes a spline defect k > 1 can be used - in other words we require the derivatives to be continuous only to the order p-k.

It follows from spline theory [6] that the linear space formed by a large number of splines specified in a fixed mesh with fixed defect values will be fully determined if any base is defined for it. The elements of the base used in this paper, which have a defect k in the node x_i, are the following functions:

$$Q_{i,k}(x) = \sum_{j=1}^{p+2-k} \frac{(x_{j+i} - x)_+^p}{(x_{j+i} - x_i)^{k-1} \prod\limits_{\substack{m=0 \\ j=0}}^{p+2-k} (x_{j+i} - x_{m+i})}, \tag{1}$$

where $i = -p, -p+1, \ldots, n-1, \quad 1 \leq k \leq p+1,$

$$(x_{j+i} - x)_+^p = \begin{cases} (x_{j+i} - x)^p, & (x_{j+i} - x) > 0; \\ 0 & , (x_{j+i} - x) < 0 \end{cases}$$

It follows from the forms of the neutron angular distributions in various channels of the (^{7}Li+d) interaction [9, 10] - for incident deuteron energies of E_d < 2 MeV - that the total cross-section of the ^{7}Li(d,n)TOT reaction can be expressed to within 10-15% through the differential cross-section at 90° if we multiply by 4π. On this basis, in addition to our own data we have used data from Ref. [8] multiplied by 4π to plot an approximation curve for the energy interval E_d = 0.2-11 MeV.

Figure 1 shows the excitation function of the total cross-section for the integral neutron yield from the reaction ^{7}Li(d,n)TOT.

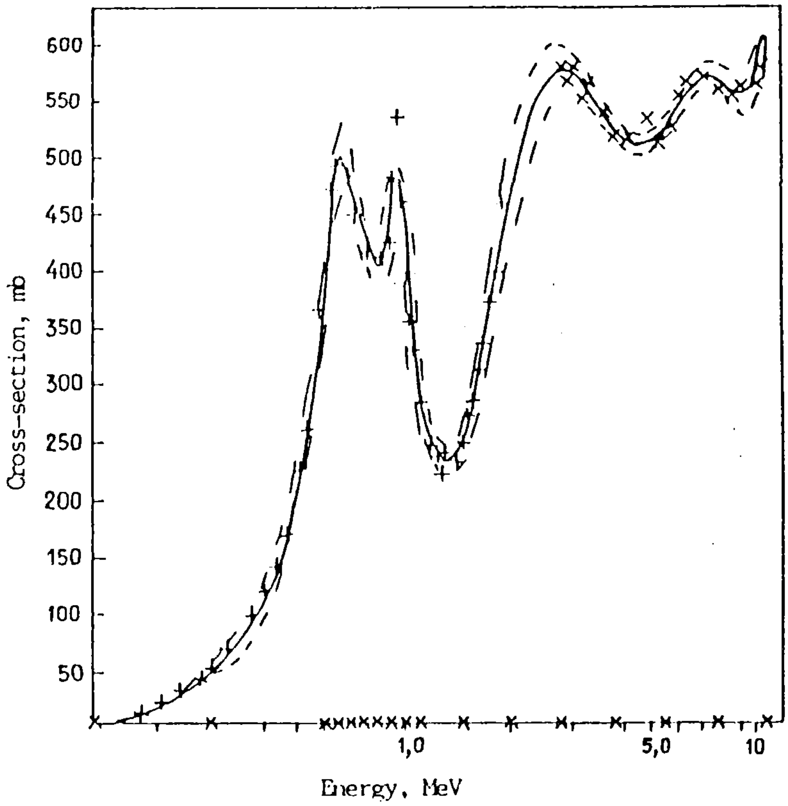

Figure 1: Excitation function of the total cross-section for the $^{7}Li(d,n)TOT.$ reaction:

+ Data from Ref. [8]

* Data obtained by the authors of this paper

— Approximation spline curve with nodes indicated by x on the abscissa

--- 67% confidence interval

Table 1 shows the coefficients of the spline curve and the scatter of the evaluated cross-sections in relation to particular spline nodes.

The evaluated cross-section can be obtained from the formula:

$$\sigma_t(E) = S_0^{i+1} + \sum_{l=1}^{3} S_l^{i+1}\left[\ln(E/E_i)\right]^l, \qquad (4)$$

where $\ln(E/E_i)$ is the natural log of the ratio between the actual energy value and the value for the nearest node of the spline curve meeting the condition $E_i < E$.

Among the non-neutron channels for the (^{7}Li+d) interaction, the cross-section of the $^{7}Li(d,p)^{8}Li$ reaction is of interest because its absolute

Table 1

Coefficients of a spline curve obtained to describe evaluated total cross-sections for neutron formation in channels of the (^7Li+d) interaction

Node number	Node energy, MeV	S_0, mb	S_1, mb	S_2, mb	S_3, mb	$\Delta\sigma$, mb
1	0,134	—	—	—	—	0,669
2	0,283	6,722	−14,60	140,26	−54,82	2,266
3	0,599	51,65	103,3	16,75	541,9	0,238
4	0,664	368,2	1045,4	12111	−94308	0,318
5	0,734	500,2	604,9	−16471	68790	0,292
6	0,812	464,1	−616,9	4376,9	−21921	0,277
7	0,898	423,9	−403,7	−226,7	51996	0,264
8	0,994	413,6	730,3	13492	−142090	0,281
9	1,1	478,6	−89,40	−29570	204890	0,170
10	1,483	297,7	−595,6	1079	1370,3	0,118
11	2,0	252,7	417,3	2308,6	−4488,2	0,478
12	2,809	463,9	−594,3	−592,7	−444,0	0,204
13	3,951	579,9	36,83	−1045,9	1454,9	8,979
14	5,551	528,7	−169,6	−439,1	85,45	9,507
15	7,799	525,2	158,8	526,3	−1638,6	10,71
16	10,957	575,7	−52,06	−1146,1	4754,4	22,99

cross-section is required for fusion reactors [11] and for monitoring the cross-sections of certain other reactions [12] which are important for astrophysical calculations.

There are quite a few papers on $^7Li(d,p)^8Li$ reaction in which the total proton yield was measured. Most of these measurements relied on β-decay of ^8Li. From these papers it can be seen that in the data now available there is a wide scatter around the absolute value. The most reliable data in our opinion are those in Refs [13, 14], but even here there are differences of 24%, which is greater than the sum of the mean square errors.

Discrepancies of this magnitude in the data made it necessary to perform additional measurements of the total cross-section for the $^7Li(d,p)^8Li$ reaction [15, 16] using two different methods: measurement of the proton yield, and ^8Li decay. These measurements [15, 16] gave results in good agreement with each other which were, moreover, close to the values given in Ref. [13].

Table 2

Coefficients of a spline curve obtained to describe evaluated total cross-section of the $^7Li(d,p)^8Li$ reaction

Node number	Node energy, MeV	S_0, mb	S_1, mb	S_2, mb	S_3, mb	$\Delta\sigma$, mb
1	0,405	—	—	—	—	14,84
2	0,695	1,839	110,61	−609,97	1464,3	6,0
3	0,768	113,39	728,56	−4328,9	4373,2	5,5
4	0,849	147,33	−6,289	−3016,3	11732	5,9
5	0,938	128,26	−257,54	504,91	8453,3	6,0
6	1,037	116,02	97,314	3042,0	−25032	5,9
7	1,146	131,13	−45,657	−4471,1	29263	6,0
8	1,266	111,12	−61,583	4311,9	−18053	7,5
9	1,400	130,04	259,11	−1106,4	2580,8	6,7
10	1,962	147,47	115,22	−331,83	496,67	6,5
11	2,750	167,66	60,977	171,14	−484,68	6,4
12	3,317	189,10	10,833	2829,5	−10845	6,8
13	4,000	219,13	−70,895	−3265,7	12010	6,9
14	5,314	170,19	−29,953	−756,69	1241,2	6,9
15	7,059	129,09	−159,43	300,81	−390,66	10,7

The data in the literature on the total cross-section of the $^7Li(d,p)^8Li$ reaction cover the range from the reaction threshold to E_d = 4.0 MeV. At high energies the authors used the data from Ref. [17], which were normalized to E_d = 2.0 MeV on the basis of the data from Ref. [13].

The total cross-section of the $^7Li(d,p)^8Li$ reaction can be found from Eq. (4) by using the spline coefficients given in Table 2.

It can be shown that the functions of the base indicated above are non-zero only over a few grid intervals (p+2-k). This makes it easier to solve the linear systems arising in evaluation problems.

Equation (1) is convenient for programming. It has one deficiency, however: at large p($p \geq 5$), calculations using this formula can lead to large rounding-off errors. Normally p does not exceed 3, and so the limitation we have mentioned is usually not serious. Proceeding from the base in Eq. (1), we can represent any spline in the form

$$S = \sum_{l} \alpha_l Q_l, \qquad (2)$$

where l is the number of the base spline which can be expressed in terms of i,k in formula (1).

Thus, any problem involving the construction of an approximation in abstract linear finite-dimensional functional space will have a solution in spline form. It follows, therefore, that the use of spines in evaluation problems based on the method of maximum likelihood makes possible a solution in a linear approximation - and this substantially simplifies the calculation procedure.

Reference [4] describes, in considerable detail, a statistical model designed for processing the data of a nuclear experiment. The model makes it possible to obtain an evaluation curve from the results of experiments reported by different authors, with allowance for random and systematic errors. It was assumed that the random errors of each author and the systematic errors of a group of authors were distributed according to a normal law with zero mathematical expectation. Since the systematic errors are as a general rule unknown, they had to be determined in the course of critical analysis of specific experimental work. The resulting systematic error was then taken as a first approximation. The final value of the systematic error for the data of a specific author was determined from analysis of all data provided by the different authors.

The error corridor for the evaluation curve was attained in the following manner. The method of maximum likelihood for the approximation S-spline was used to obtain a covariant matrix $C(\alpha_i,\alpha_j)$ relative to the coefficients α_i [see Eq. (2)]. If the covariant matrix is known, then the dispersion of the spline curve will be

$$\sigma^2(S) = \sum_{i,j} C(\alpha_i, \alpha_j) Q_i(x) Q_j(x). \qquad (3)$$

In this particular case the standard deviation was calculated only at the spline nodes with the consequent assumption that the values at intermediate points could be obtained by means of linear interpolation.

If one wishes to obtain a spline curve for practical use, the spline representation in Eq. (2) is not particularly convenient owing to the cumbersome nature of the subsequent calculations using the coefficients α_1. It is much simpler to perform a few simple recalculations in order to obtain coefficients with which the evaluated curve can be represented in the form of a polynomial of power p at each interval of the grid $[x_i, x_{i+1}]$ in powers of $(x-x_i)$.

For example, for the cubic spline used in the reaction cross-section evaluations shown below, we have a set of coefficients $\left\{S_0^{(i+1)}, S_1^{(i+1)}, S_2^{(i+1)}, S_3^{(i+1)}\right\}$, and the value of the spline at point $x_i < x < x_{i+1}$ is calculated from the expression $S(x) = \sum_{l=0}^{3} S_l^{(i+1)} (x-x_i)^l$.

Experimental data. Neutron formation in the ^7Li+d reaction takes place by many channels, and this leads to a complex neutron spectrum [7]. The difficulties involved in studying multiparticle neutron channels for the (^7Li+d) interaction limit the possibilities of a detailed description of each channel. At the same time, for practical work it is often desirable to know, in the first instance, the total neutron yield. As regards specific measurement results for the total neutron yield from the ^7Li(d,n)TOT reaction, the only data that can be called sufficiently reliable here are those of Ref. [8] where the differential cross-section of the reaction was measured in the energy range E_α = 0.2-2 MeV at an angle of 90^o. At higher energies, the cross-section of the total neutron yield for the interaction between deuterons and ^7Li nuclei was measured in the range E_d = 2.76-10.96 MeV.

The cross-section was absolutized in the last case by the ratio method using the familiar $^7Li(p,n)^7Be$ reaction, the accuracy being no worse than 15%.

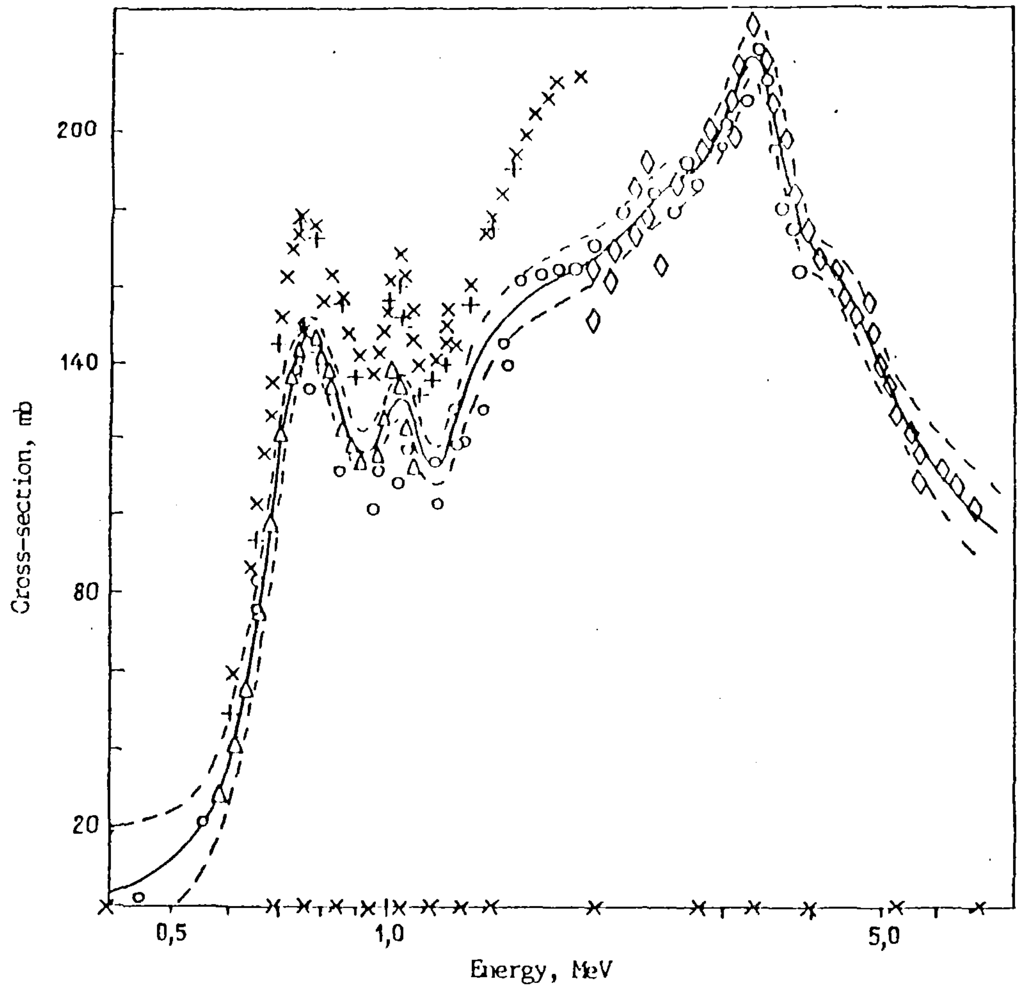

Figure 2: Excitation function of the total cross-section for the $^{7}Li(d,p)^{8}Li$ reaction:

o Data from Ref. [13]

+ Data from Ref. [18]

Δ Data from Ref. [16]

* Data from Ref. [15]

x Data from Ref. [14]

◊ Data from Ref. [17]

—— Approximation spline curve with nodes indicated by x on the abscissa

--- 67% confidence interval

As can be seen from Fig. 2, the evaluated curve follows principally the data of Refs [13, 15-18] and reflects well the resonance nature of the excitation function of the total cross-section for the $^{7}Li(d,p)^{8}Li$ reaction.

Thus, in proposing this spline function as an approximation curve for the evaluation of experimental data, we can sum up by mentioning the following advantages of the method:

- The spline function makes it possible to use the method of maximum likelihood in a linear approximation;
- The evaluated curve obtained in this way can be represented as a set of relatively few coefficients at the nodes of the spline function; and
- The spline method allows us to find an approximation curve satisfying certain boundary conditions, and this in turn is convenient if we wish to match the approximation curve with corresponding analytic continuations.