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THE OPERATIONS RESEARCH AS AN INSTRUMENT FOR ANALYSIS AND PLANNING OF NUCLEAR SPECTROSCOPIC EXPERIMENT

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Abstract

The procedure of nuclear-spectroscopic data analysis by the operations research methods is presented. The fragment of ^{239}Pu decay is considered as an example of application. The conclusion is drawn on nonconformity between experimental data and the adopted system of ^{235}U levels.

In the majority of present-day experiments in the field of nuclear spectroscopy definite initial state of nuclear system (or series of such states) is created and its subsequent decay is observed. Specifically complicated for interpretation, as a rule, are characteristics of electromagnetic transitions accompanied by gamma-quanta, conversion electrons, x-radiation, etc. This is related to the fact that if in a reaction heavy particles are generated, then a study of specific features in their spectrum makes it possible to identify excited states of the nuclear system, its energy levels. An in studying an electromagnetic radiation no nucleus levels themselves are observed but only transitions between them.

The system of nucleus levels and transitions between them forms, from the view-point of the graph theory, a certain acyclic directed network to which it would be reasonable to apply methods developed in operation research where properties of various functionals are determined from its elements according to some characteristics of the network, whereas when applied to the nuclear spectroscopy the problem is reverse - to estimate the network structure and characteristics of relations between its elements according to functionals observed. These methods make it possible, from the common stand to analyse various experiments, to find more probable characteristics for nucleus levels and transitions.

In presenting the suggested methods we shall follow mainly methods and ways described the basic work [1] concerning operation research of an economic and military character.

I. CONSTRUCTION OF MODEL

Let us consider the L + 1 system of nuclear levels, in which the level with number 0 represents a ground state and the number L - initial. Let the system of transitions to and out an energy state be assigned to every energy level by some method. This can be determined, for instance, by means of experiments with gamma-coincidences and methods similar to [2].

Above all for each from these levels the intensities balance equation can be formulated

$$\sum_{j=\ell+1}^{L} T_{j\ell} - \sum_{j=0}^{\ell-1} T_{\ell j} = 0 \tag{1.1}$$

$$\sum_{j=0}^{L-1} T_{Lj} = I \tag{1.2}$$

The physical sense of these equations is rather obvious: the first one describes the fact that the number of transitions from the "l" number level equals to the number of transitions to this level; the second one - all transitions proceed from the "L" state and their complete quantity comes up to I transitions per unit time. The second equation can also be interpreted as an intensity of "L" state formation. Each value T_{lj} - a probability of transition from "l" state into "j" state - represents a sum of all open channels of such a transition. In the case of α-decay for T_{lj} transition the only channel is open and, for example, for electromagnetic transitions lots of them can be available. All further presentation will be based on the assumption that ℓ_j is the electromagnetic transition. The extension of the proposed method to simpler cases involves no difficulties whatever.

If the (l_j) - transition is electromagnetic and in the list of observed quanta there is a quantum λ having according energy E_λ, then

$$T_{\ell j} = \sum_{m=1}^{M} x_{\lambda m}\left(1+\alpha_{\lambda m}\right) \tag{1.3}$$

where $x_{\lambda m}$ is the intensity of E_λ quanta with multipolarity m and $\alpha_{\lambda m}$ is the total conversion coefficient of these quant.

If the quantum E_λ is observed in an experiment with intensity $D_\lambda \pm \Delta_\lambda$, one can write:

$$D_\lambda \pm \Delta_\lambda = \sum_{\ell,j} \sum_{m} x_{\lambda m \ell j} \tag{1.4}$$

In expression (1.4) it is assumed that the quantum can, generally speaking, be placed in several places of the adopted levels system.

The condition (1.4) can be written as a system of linear restrictions

$$\sum_{\ell,j}\sum_{m} x_{\lambda m \ell_j} - y_\lambda + z_\lambda = D_\lambda \tag{1.5}$$

$$0 \leq y_\lambda \leq \Delta_\lambda \tag{1.6}$$

$$0 \leq z_\lambda \leq \Delta_\lambda \tag{1.7}$$

Similar restrictions can be formulated also for other experimental facts. Let us consider, for example, the case of electron conversions. Let the peak be observed, in the conversion electrons spectrum, the energy of which corresponds to a conversion on μ -shell of (l,j) transition with energy E_λ and a conversion on ν -shell of (ik) transition with energy E_ω. Let this peak intensity be $C \mp \xi$. Then we shall obtain linear restrictions as follows

$$\sum_{m}\left(\alpha_{\lambda\mu m}\, x_{\lambda m \ell_j} + \alpha_{\omega\nu m}\, x_{\omega m i k}\right) - y_{\lambda\omega} + z_{\lambda\omega} = C \tag{1.8}$$

$$0 \leq y_{\lambda\omega} \leq \xi \tag{1.9}$$

$$0 \leq z_{\lambda\omega} \leq \xi \tag{1.10}$$

If the ratio K_1/K_2 of intensities for two conversion lines $R \pm \rho$ is known, then:

$$K_1 - RK_2 - y_{12} + z_{12} = 0 \tag{1.11}$$

$$-\rho K_2 + y_{12} \leq 0 \tag{1.12}$$

$$-\rho K_2 + z_{12} \leq 0 \tag{1.13}$$

$$y_{12} \geq 0, \quad z_{12} \geq 0 \tag{1.14}$$

The values K_1 and K_2 represent sums similar to those which are present in (1.8). In like manner one can describe practically any set of nuclear-spectroscopic experiments, presenting experimental data as a system of restrictions analogous to (1.3)-(1.4) and adding to them L+1 intensities balance equation. Let us formulate the optimization problem: to find the minimum $\sum(z_j + y_j)$ when the conditions of all linear restrictions are satisfied. The solution of such a problem gives most probable values $x_{\lambda m \ell_j}$. If the number of restrictions (experiments) is insufficient to determine all $x_{\lambda m l_j}$, then several solutions possessing equal probability will be obtained. By adding in this case to the system of experimental limitations new ones which can logically be named planned ones, reflecting possible results of additionally planned experiments, one can assess usefulness of one or another type of experiments, to find an accuracy required from them.

What has been presented till now represents a linear problem which solution methods are well developed. However, in the majority of real cases the linear restrictions occur which are imposed by the law of conservation of parity. Let us discuss some examples of such limitations. Let x_λ be an intensity of quanta with energy E_λ , responsible for the transition (lj), and

$$x_\lambda = \sum_{m=1}^{M} x_{\lambda m} \tag{1.15}$$

If states l and j have the same parity, then in the sum (1.15) only M1 and E2 etc. components are different from zero. Otherwise one should take into account E1 and M2 etc. solely.

Let us introduce the Boulean variable $P_{lj}=0$, if the parity of l and j states is the same. It is different, $P_{lj} = 1$. Let us correspond even m to components M1, E2... and odd m to E1, M2... Then restrictions (1.2)-(1.14) should be supplemented by limitations of the type

$$\sum_{odd\ m} x_{\lambda m} + P_{l_j} P \leq 0 \tag{1.16}$$

$$\sum_{even\ m} x_{\lambda m} + P_{l_j} P \leq P \tag{1.17}$$

$$P_{l_j} = 0 \text{ or } 1 \tag{1.18}$$

which at a fixed $P > x_\lambda$ ensure automatically an observation of the law of parity conservation and selection of values P_{lj}. But the condition of discreteness of P_{lj} values complicates essentially the solution of the problem, asking for more complicated methods than those needed to solve the problem with restrictions (1.2)-(1.14).

The allowance of the law of parity conservation requires additional limitations which cannot be described by restrictions of the type (1.16)-(1.18).

Let us consider these additional limitations. Let three levels l, k, j be available and transitions (l,k), (k,j), (l,j) be observed. Then 4 restrictions should be fulfilled:

$$
\begin{aligned}
P_{ik} + P_{kj} + P_{ij} &\le 2 \\
-P_{ik} + P_{kj} - P_{ij} &\le 0 \\
-P_{ik} - P_{kj} + P_{ij} &\le 0 \\
P_{ik} - P_{kj} - P_{ij} &\le 0
\end{aligned}
\qquad (1.19)
$$

where every P = 0 or 1.

Thus, having predetermined the parity for one out of states under conditions of sufficiently abundant experimental data one can obtain parities for the rest states of the nuclear system.

If, for some reasons, additional limitations of x_λ value components are known, no doubt, it is reasonable to introduce them into the problem. Thus, if a difference in spins of l and j states is higher, for example, than 2, then $x_{\lambda E1} = x_{\lambda M1} = 0$.

Let us return now to $\sum (Z_j + y)$ and consider the physical sense of this sum.

Since we find a minimum of this sum at non-negative Z_j and y_i it is obvious that either Z_j of y_j equals to zero. That is why a search for the minimum of linear form $(Z_j + y_j)$ is equivalent to a search for a minimum of modulus sum of deviations of solutions of the entire system from experimental data, and the condition that the obtained solution falls within experimental error limits should be fulfilled for each experiment. The requirement of minimization of deviations moduli sum differs, at first glance, from a traditional requirement of minimization of the sum of the square of deviation moduli. But there are two excuses for that:

1. If experimental results are not correlated, the requirement of minimization of the sum of deviation moduli is equivalent to the traditional requirement. If the experimental results are correlated, this correlation can be presented by certain linear restrictions and included into the problem limitations composition.

2. Existing algorithms of a search for the minimum $\sum (Z_j^2 + y_j^2)$ are much more complicated.

2. EXAMPLE OF APPLICATION

Let us consider the ^{239}Pu α-decay. Experiments on measuring the α-spectrum [3], conversion electrons [4], γ-rays [5] are known for it. The problem cannot be solved in a full volume by the method under consideration because of incompleteness of experimental data. Therefore we confine ourselves only to fragment

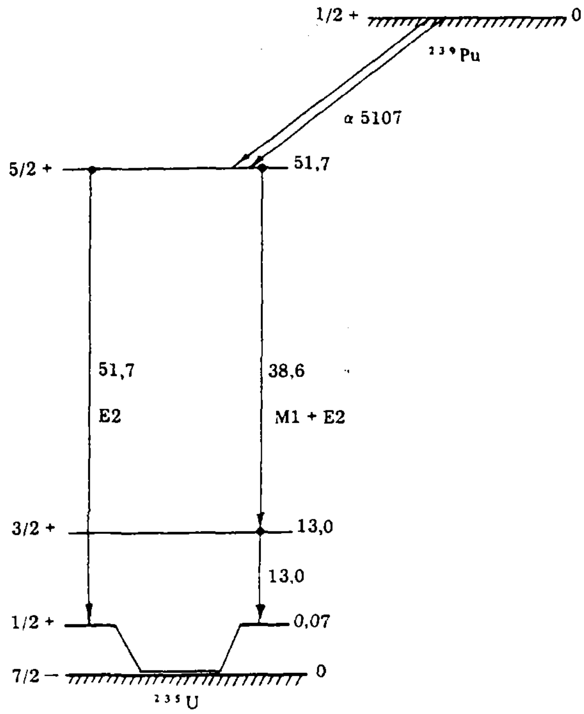

Fig.1. Fragment of ^{239}Pu decay scheme according to data [6] . Energies of levels and transitions are averaged. The energy values are given in kiloelectron volts.

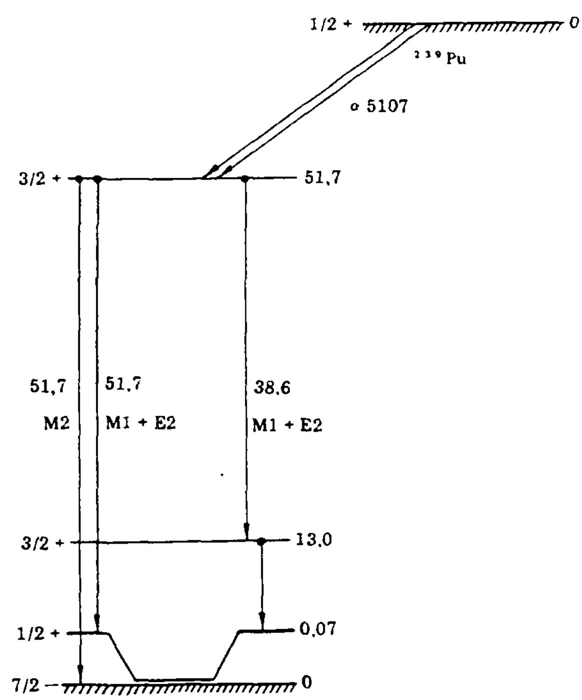

Fig.2. Fragment of the same scheme of ^{239}Pu decay, as it is in Fig.1 but taking into account our results.

the scheme of which is given in Fig.1 and borrowed from [6] . In paper [3] it is reported that a fraction of α-decays to the level 51.7 keV comes up to 11.5%. We shall assume that this portion amounts to $(11.5 \pm 0.5)\%$. The intensity of gamma-quanta with energies 51.7 keV and 38.6 keV is $(2.71 \pm 0.05)\ 10^4$ quanta/10^8 decays and $(1.05 \pm 0.02)\ 10^4$ quanta/10^8 decays, respectively. Intensities of various lines of conversion electrons will be cited from [4] with pointing to a peak number (for instance, T28 means the peak 28 in Table 1). Conversion ratios are taken by us from Tables used while prepared "NUCLEAR DATA SHEETS". Let us turn to setting up equations.

<u>First group</u> - alpha decay

$$X_9 + X_{13} - X_{14} = 1.15.10^7 \tag{2.1}$$

$$X_{13} \leq 5.10^5 \tag{2.2}$$

$$X_{14} \leq 5.10^5 \tag{2.3}$$

In these equations X_9 is the number of α-particles with energy of 5107 keV per 10^8 decays of ^{239}Pu; X_{13} and X_{14} are possible deviations of X_9 intensity from a average experimental value.

<u>Second group</u> - intensity of γ-radiation with energy of 51.7 keV

$$X_1+X_2+X_3+X_4+X_{15}-X_{16} = 2.71.10^4 \tag{2.4}$$

$$X_{15} \leq 500 \tag{2.5}$$

$$X_{16} \leq 500 \tag{2.6}$$

Here, X_1-X_4 is "an amount" of quanta with multipolarity E1, M1, E2 and M2, respectively, i.e. $X_1+X_2+X_3+X_4$ - the intensity of -γ quanta with energy of 51.7 keV X_{15} and X_{16} are possible deviations.

<u>Third group</u> - intensity of γ-radiation with energy of 38.5 keV

$$X_5+X_6+X_7+X_8+X_{17}-X_{18} = 10500 \tag{2.7}$$

$$X_{17} \leq 200 \tag{2.8}$$

$$X_{18} \leq 200 \tag{2.9}$$

Here X_6-X_8 have the same sense as X_1-X_4 in the second group, and X_{17} and X_{18} are possible deviations.

<u>Fourth group</u> - balance of intensities for 51.7 keV level

$$1.724X_1+35.1X_2+318X_3+1431X_4+2.55X_5+80.3X_6+$$

$$+\ 1281X_7+5311X_8-X_9 = 0 \tag{2.10}$$

Coefficients on X_1-X_8 are conversion ratios increased by 1, for corresponding multipoles.

Fifth group - intensity of subshell L_{11} conversion electrons due to 51.7 keV transition

$$0.177X_1+2.65X_2+123X_3+50,5X_4-X_{12}=0 \qquad (2.11)$$

This equation represents, in fact, a designation that X_{12} is the T10 peak intensity. We do not use the value $X_{12} = 2.9 \cdot 10^6$ presented in [4] but employ X_{12} only to normalize the rest data on conversion.

Sixth group - T1 peak

$$0.311X_5+53.5X_6+16.1X_7+2490X_8-0.276X_{12}+X_{19}-X_{20}=0 \qquad (2.12)$$

$$-\ 0.138X_{12} + X_{19} \leqslant 0 \qquad (2.13)$$

$$-\ 0.138X_{12} + X_{20} \leqslant 0 \qquad (2.14)$$

This group of restrictions is representation of the fact that the ratio of intensity of radiation conversion 38.6 keV on subshell L_I amounts to (800 $\pm$ 400)/2900 from a conversion of 51.7 keV radiation of the L_{II} - subshell.

Seventh group - T2 peak ($L_{II}(38.6)/L_{II}(5.7)$)

$$0.39X_5+6.15X_6+486X_7+148X_8-0.31X_{12}+X_{21}-X_{22} = 0 \qquad (2.15)$$

$$-\ 0.138X_{12} + X_{21} \leqslant 0 \qquad (2.16)$$

$$-\ 0.138X_{12} + X_{22} \leqslant 0 \qquad (2.17)$$

Eighth group - T4 peak ($L_{III}(38.6)/(L_{II}(51.7)$)

$$0.456X_5+0.322X_6+439X_7+1200X_8-0.344X_{12}+X_{23}-X_{24} \qquad (2.18)$$

$$-\ 0.138X_{12}+X_{23} \leqslant 0 \qquad (2.19)$$

$$-\ 0.138X_{12}+X_{24} \leqslant 0 \qquad (2.20)$$

Nineth group - T11 peak ($M_{I,II}(38.6)/L_{II}(51.7)$

$$0.155X_5+14.42X_6+133.65X_7+751.1X_8-0.241X_{12}+X_{25}-X_{26} = 0 \qquad (2.21)$$

$$-0.0689X_{12}+X_{25} \leqslant 0 \qquad (2.22)$$

$$-0.0689X_{12}+X_{26} \leqslant 0 \qquad (2.23)$$

Tenth group - T12 peak ($L_{III}(51.7)+M_{III}(38.6)/L_{II}(51.7)$)

$$0.187X_1+0.133X_2+103X_3+286X_4+0.0953X_5+0.0844X_6+$$

$$+122X_7+352X_8-1.0689X_{12}+X_{27}-X_{28}=0 \qquad (2.24)$$

$$-0.103X_{12}+X_{27} \leqslant 0 \qquad (2.25)$$

$$-0.103X_{12}+X_{28} \leqslant 0 \qquad (2.26)$$

Eleventh group - T24 peak ($M_{II}(51.7)/L_{II}(51.7)$)

$$0.0367X_1+0.699X_2+32.7X_3+16.4X_4-0.310X_{12}+X_{31}-X_{32} = 0 \qquad (2.27)$$

$$-0.0344X_{12}+X_{31} \leqslant 0 \qquad (2.28)$$

$$-0.344X_{12}+X_{32} \leqslant 0 \qquad (2.29)$$

Twelth group - T25 peak ($M_{III}(51.7)/L_{II}(51.7)$)

$$0.0414X_1+0.052X_2+28.8X_3+83.6X_4- \quad$$
$$-0.276X_{12}+X_{33}-X_{34}=0 \qquad (2.30)$$
$$-0.0344X_{12}+X_{33}\leq 0 \qquad (2.31)$$
$$-0.0344X_{12}+X_{34}\leq 0 \qquad (2.32)$$

Thirteenth group - T26+T27 peak ($N+O(51.7)/L_{II}(51.7)$)

$$0.0465X_1+2.26X_2+23.7X_3+113X_4-0.2069X_{12}+X_{29}-$$
$$-X_{30}=0 \qquad (2.33)$$
$$-0.0217X_{12}+X_{29}\leq 0 \qquad (2.34)$$
$$-0.0217X_{12}+X_{30}\leq 0 \qquad (2.35)$$

Fourteenth group - the law of conservation of parity

$$X_{10}=0 \text{ or } 1 \qquad (2.36)$$
$$X_{11}=0 \text{ or } 1 \qquad (2.37)$$
$$X_1+X_4-10^5X_{10}\leq 0 \qquad (2.38)$$
$$X_2+X_3+10^5X_{10}\leq 10^5 \qquad (2.39)$$
$$X_5+X_8-10^5X_{11}\leq 0 \qquad (2.40)$$
$$X_6+X_7+10^5X_{11}\leq 10^5 \qquad (2.41)$$

Since no experimental data on the parity of transitions from 13 keV level are available the values X_{10} and X_{11} act as independing.

The following task was set: to minimize $\sum_{j=13}^{34} x_j$ under the condition that restrictions (2.1)-(2.41) are satisfied. It happened that this problem does not have a solution. To make appearance of the solution it was necessary:

1.To drop restrictions (2.33)-(2.35).

2. To double the coefficient on X_{12} in restrictions (2.28) and (2.29), i.e. to increase twoce as high the uncertainty of T24 peak intensity given by authors 4 .

3. To give up the assumption that the 51.7 keV radiation has the certain parity.

As a result, the following solutions were obtained (only 4 figures of an answer are retained):

$X_1 = 0$	$X_5 = 0$	$X_9 = 1.15.10^7$
$X_2 = 8333$	$X_6 = 8718$	$X_{12} = 2.269.10^6$
$X_3 = 17570$	$X_7 = 1981$	
$X_4 = 1689$	$X_8 = 0$	

This solution indicates that radiation with the energy of 51.7 keV "represents" the mixture E2+M1+M2. In [4] the multipolarity E2 is given for this radiation. We tried to solve the same problem by additn the condition $X_1=X_2=X_4=0$. This system turned out to be contradictory. In such a way, a radiation of the M2 type is necessary. The attempt to substitute the multipolarity with M1+E2 by E1+M2 in the radiation 36.6 keV proved a failure.

3. ANALYSIS OF THE SOLUTION FOUND

A connection was been analyzed between the appearance of M2-type multipolarity in the 51.7 keV radiation and the volume of experimental data published in [4] . The 51.7 keV radiation can have the type E2 only in the case if 8-13 group restrictions are dropled, i.e. if the most of experimental data [4] are not used, which are related to transitions with energies of 51.7 keV and 38.6 keV.

The result determined in the previous s ction can be explained by dropping the interpretation [6] of experimental data. By assuming that the scheme of ^{235}U lower states is the same as it is in Fig.2 we succeeded in detecting this system's discrepancies with other type experiments. But if the scheme in Fig.2 is accepted, one should give up the interpretation of uranium-235 levels system as a system of rotational hands, presented in [6] , since the mentioned interpretation relies largely on the premise that the level 51.7 keV has the characteristics 5/2+ and is the third term of rotational band constructed at the level 70 eV.

From the above-presented is seen that the ^{239}Pu decay requires an additional experimental research, especially in reginement of conversion electron spectra.

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