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PREPARATION OF EVALUATED DATA FOR A FISSION BARRIER
PARAMETER LIBRARY FOR ISOTOPES WITH $Z = 82 - 98$, WITH
CONSIDERATION OF THE LEVEL DENSITY MODELS USED

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Translated by the IAEA
(from unpublished manuscript)

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ABSTRACT

The present report summarizes the results of research on the preparation of evaluated data for a fission barrier parameter library for isotopes with $Z = 82 - 98$, with consideration of the level density models used. This activity is a part of the IAEA/NDS project on the development of a Reference Nuclear Parameter Library for Nuclear Data Computation which was initiated by the Agency with the purpose to assist the evaluators in their computational work and to encourage standardization of code input parameters.

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I. INTRODUCTION

The potential fission barrier restrains the atomic nucleus from an energetically favourable breakdown into pieces (fragments) and thus plays a fundamental role in nature. The potential deformation energy determines the most important characteristics of fissile nuclei and of the fission process itself: stability against spontaneous fission and the resulting boundaries of the periodic system, the probability of induced fission and the formation of reaction product properties, especially their mass energy distributions etc. [1-7]. Fission barrier data have become an essential part of the initial experimental data on which contemporary theoretical description of such important characteristics as nuclear mass and energy is based [8-10].

Fission barrier data are important both from the theoretical and the applied point of view. The properties of deformation energy are to a great extent, if not entirely, determined by the magnitude and energy dependence of fission cross-sections, which it is essential to know in order to use the neutron-induced fission process as a source of nuclear energy. A wide range of basic characteristics of fissile nuclei are of practical interest: excitation energy $E < 25\text{-}30$ MeV (or incident neutron energy $E_n < 20$ MeV) and nucleon composition (transactinide nuclei up to californium) while scientific interest extends far beyond this range [11, 12].

In this energy range, together with (n,f) fission of nucleus A formed during neutron absorption, there occur reactions such as (n,xnf) with preliminary emission of $x = 1\text{-}3$ neutrons. The emission of these neutrons indicates the transition from "pure" to "emission" fission in which the fission process takes place for a mixture of isotopes with A to A-3. It is because of this factor that the description of fission cross-sections becomes much more complicated and it is necessary to expand the fission barrier data used in the description [13-15]. Lastly, we should emphasize that, in directly influencing the fission channel of compound nucleus decay, fission barrier parameters indirectly affect the cross-

sections associated with all competing channels: the channels of radiative capture (n,γ), inelastic scattering (n,n'), multiple neutron emission (n,xn), and also charged particle emission.

Extensive experimental data have been accumulated for the most important fissile nuclides widely used in nuclear reactors, although such data far from satisfy the requirements of nuclear engineering in all respects. The general status of experimental data with regard to the nuclei accumulating during nuclear fuel burnup is much worse. They include nuclides, such as ^{232}U , ^{236}Pu , isotopes of Am, Cm, etc. - the so-called minor isotopes, which constitute a radiation hazard for the external fuel cycle and power plant waste reprocessing (transmutation). The experimental information for these isotopes contains many more gaps than actual data.

For this group of hazardous nuclides it is difficult substantially to improve the nuclear data situation by experimental means alone. The latter can and should be supplemented by theoretical calculation and evaluation. Of current interest in this connection, among the various aspects of work carried out by laboratories in different countries, is the organizational effort which is being made to formulate a self-consistent approach. Such an approach is undoubtedly essential since the discrepancies even among the evaluation results in national nuclear data libraries (see, for example, Refs [16, 17]) are unacceptably large.

In the present paper we have prepared evaluated data with a view to setting up a library of fission barrier parameters, which, as we have seen, are among the most important characteristics for calculating cross-sections of reactions induced by fast neutrons.

2. BASIC PROPERTIES OF THE DEFORMATION ENERGY OF NUCLEI

We write the potential energy of deformation of a nucleus in the traditional form for applications of the shell correction method [2, 6]:

$$V(\alpha) = \tilde{V}(\alpha) + \delta W(\alpha) \quad (1)$$

Here $\tilde{V}(\alpha)$ is the smooth macroscopic component corresponding to the liquid-drop model, i.e. to homogeneous distribution of nucleons; $\delta W(\alpha)$ is the shell correction, oscillating with changes in deformation α and also in the number of protons Z and neutrons N due to irregularities in nucleon distribution and with corresponding behaviour of the density of single-particle states near the Fermi energy (with alternating increases and decreases in density); α is the set of deformation parameters determining the shape of the nucleus.

2.1. Properties of $\tilde{V}(\alpha)$

The liquid-drop model forms the basis of fission physics and the "energetics" of nuclei in general. Unfortunately, no single approach yet exists to calculation of $\tilde{V}(\alpha)$, and there are a number of versions of the model in which different macroscopic properties of actual nuclei are considered. The simplest of these is the liquid drop model with a sharp boundary on which the pioneering work in the area of the theory of fission was based [1, 2]. These studies described the shape of the axisymmetric by a series of Legendre polynomials with coefficients α_i : ($i = 2, 3 \dots n$) - independent deformations whereas the deformation energy $\tilde{V}(\alpha)$ is a hypersurface in deformation space $\alpha_2, \dots, \alpha_n$ having the shape of a multidimensional saddle. A noteworthy feature is the presence of a saddle point $\alpha = \tilde{\alpha}_{sp}$ or point of absolute extremum at which $\tilde{V}(\alpha)$ has a minimum in all co-ordinates α_i ($i > 2$) and a maximum along the quadrupole co-ordinate α_2 , responsible for the general elongation of the nucleus. This means that surface $\tilde{V}(\alpha)$, has a valley, at the bottom of which the saddle point, representing its highest point, is created and corresponds to $\alpha_{2m+1} = 0$, i.e. in the liquid-drop model symmetric fission is energetically favourable.

Taking as the origin of energy its value in the ground state of the liquid-drop model $\tilde{\alpha}_g = 0$, we determine the height of the barrier as:

$$\tilde{B}_f = \tilde{V}(\tilde{\alpha}_{sp}) - \tilde{V}(0) = E_{sc} \cdot \xi(X) \sim (1-X)^3, \quad (2)$$

where $X = E_{co}/2E_{s0} \sim Z^2/A$ is the fissionability parameter, $E_{co} \sim Z^2/A^2$ and $E_{s0} \sim A^{2/3}$ are the Coulomb and surface energies of the initial sphere in the liquid-drop model (LDM), $\xi(X)$

is a dimensionless function [18] indicating, on the basis of rough evaluation of Eq. (2), a strongly decreasing X-dependence of \bar{B}_f .

2.2. Phenomenological consideration of shells

Discrepancies were found between Eq. (2) and the experimental data as early as the 50s and 60s, and attempts - not without success - were made to eliminate them by phenomenological consideration of the influence of shells on the energy of the ground state $V(\alpha_g)$ [19], as in the formula for masses of nuclei, with the correction

$$\delta W_g = M - M_{\text{LDM}} \quad (3)$$

where M and M_{LDM} are the experimental and calculated masses of nuclei respectively [8,20].

Instead of Eq. (2) this gives

$$B_f = \bar{B}_f - \delta W_g + \delta W_f - \frac{\hbar \omega_g}{2} \quad (4)$$

where δW_f is a correction which was made, by analogy with (3), for the saddle point and ultimately neglected, $\frac{\hbar \omega_g}{2}$ is the zero-point vibration energy by which the ground state is raised above the bottom of the well for $\alpha = \alpha_g$. Here we should note that the corrections in Eq. (4)

$$\delta W_g = V(\alpha_g) - \bar{V}(0) \quad \delta W_f = V(\alpha_{\text{sp}}) - \bar{V}(\bar{\alpha}_{\text{sp}}) \quad (5)$$

do not coincide with the respective shell corrections $\delta W(\alpha_g)$ and $\delta W(\alpha_{\text{sp}})$ from Eq. (1).

Relationships (3) and (4), which were the starting point in the interpretation and analysis of dependence $B_f(Z,A)$ [11, 12, 19], became the basis for determining the parameters of the phenomenological description of nuclear masses [8-10]. In this connection, additional explanations are required. The assumption that $\delta W_f = 0$, which had become traditional for the whole range of studies cited in this area, subsequently turned out to be unsatisfactory in the actinides region, although perfectly acceptable for pre-actinides in the region of Pb and lighter nuclei. According to the analysis of fragment mass distributions in Ref. [21], $|\delta W_f| \lesssim 1 \text{ MeV}$, i.e. less than 5% of B_f , whereas the second term of Eq. (1) accounts for most of the barrier in the region of Fm. We would therefore emphasize that

experimental values of $B_f(Z,A)$ from the favourable region of pre-actinide nuclei are used in the description of nuclear mass.

2.3. Double-humped fission barrier

In parallel the shell correction method was developed. Its applications clearly demonstrate to what extent theoretical solution can be physically richer than the most successful phenomenology. Calculations of $\delta W(\alpha)$ for a sufficiently simple parametrization of shape and energy and yet realistic single-particle scheme established that the total deformation energy $V(\alpha)$ in the region of the Th - Cm nuclei has two humps - which are most important for practical applications, an inner hump A and an outer hump B - with a minimum (second well) between them. The discovery that the fission barrier of heavy nuclei had a double-humped shape gave fresh impetus to the development of nuclear physics in a number of directions¹. The "second breath" that it gave to fission physics enabled the latter to overcome the difficulties in explaining a number of experimental facts which had previously led it to a deadend. They include the phenomenon of spontaneously fissioning isomers (shape isomerism), vibration resonances and the gross-structure of neutron resonances in fission sub-barrier cross-sections of the anomalous X-dependence of observed fission thresholds and angular anisotropy of fragments [2, 6, 22-24]. No less a role was played by the prediction of an island of stability in the transition region [4], of a triple-humped fission barrier in the thorium region [25], and so on.

Comparison of the calculated and experimental barriers, or more precisely the heights of their humps, B_A and B_B , made parametrization of the shape of the nucleus more

¹ We emphasize that it was not our aim to belittle the phenomenological method. On the contrary, we show that it is more effective as a technical tool and that it is far superior to the shell correction method as regards the accuracy of determination of those quantities for the description of which it was developed (for example, masses, and barriers in the pre-actinide region). Our concern here is to compare theoretical and phenomenological approaches in relation to the significance of the anticipated consequences.

complicated than in the pioneering work [2]. In particular, it was found to be energetically advantageous for the nucleus to violate axial symmetry on hump A [26, 27] and mirror symmetry on hump B [5, 28], and in both cases agreement with experimental data on barriers improved [4, 24, 25, 27, 29, 30]. As has already been pointed out, for nuclei in the vicinity of thorium this led to the appearance of a third well at the peak of hump B. The existence of this new structural feature of the fission barrier would have opened the way to eliminating the so-called thorium anomaly [24, 25].

As parameter X decreases the liquid-drop saddle deformation $\tilde{\alpha}_{sp}$ shifts to critical deformation, and the fissioning nucleus splits. This results in a monotonic decrease in the difference between the heights of barriers ($B_A - B_B$), which is positive in the transuranium region and negative for lighter nuclei. With changes in the nucleon composition of the nucleus the picture changes so rapidly that for the fission of Ra and Ac nuclei, the nearest neighbours of Th in the periodic system, the barrier can be considered virtually single-humped [31, 32]. This is even more true in the case of lighter pre-actinides discussed earlier because of the increase in \tilde{B}_f with the decrease in X in Eq. (2).

Above we talked about changes in deformation energy due to deformation of the nucleus in the direction of fission (mainly α_2). We now consider the role of mass(mirror)-asymmetric deformation (mainly α_3), which is important in the region of light actinides and even of lighter nuclei, where the outer hump becomes predominant in height. In the wide range of deformations α_2 from the second minimum to the point of splitting these nuclei have three minima of $V(\alpha)$ in the direction of deformation α_3 for $\alpha_3 \approx 0$ and $\alpha_3 \neq 0$ of both signs [5]. Two basic fission modes - symmetric and asymmetric ($i = s$ and a) - are associated with their corresponding valleys of $V(\alpha)$, which differ in many characteristics, including saddle point parameters α_{sp}^i , $V(\alpha_{sp}^i)$, B_f^i [5, 7, 30, 32]. Like the difference between the heights of humps A and B, the difference between the heights of the symmetric and asymmetric fission barriers ($B_f^s - B_f^a$) changes rapidly with nucleon composition: it is positive

in the Ra region and negative in the Pb region [7]. In the first case we expect the mass-symmetric saddle point to be axially asymmetric [30]. The valley structure, whether double- or triple-humped, of the fission barrier is caused by the oscillations of the second term in Eq. (1), not in the direction of the main fission co-ordinate but in that of the mass-symmetric co-ordinate [5, 7].

3. DETERMINATION OF FISSION BARRIER HEIGHTS

The main source of experimental data on fission barrier heights B_f is analysis of the energy dependence of fission cross-sections $\sigma_f(E)$ or fissionability $P_f(E)$, which are connected by the following relationships:

$$\sigma_f^{J\pi}(E) = \sum_{J\pi} \sigma_c^{J\pi}(E) \frac{T_f^{J\pi}(E)}{T_f^{J\pi}(E) + \sum_i T_i^{J\pi}(E)} = \sum_{J\pi} \sigma_c^{J\pi}(E) P_f^{J\pi}(E) = \sigma_c(E) \cdot P_f(E) \quad (6)$$

where $\sigma_c^{J\pi}(E)$ and $P_f^{J\pi}(E)$ are the formation cross-section and fissionability of the compound nucleus in states with an excitation energy $E = B_n + E_n$ (B_n is the neutron binding energy), angular momentum J and parity π ; $\sigma_c(E) = \sum_{J\pi} \sigma_c^{J\pi}(E)$; $T_i^{J\pi}(E)$ are the penetrations for the different decay channels, including $T_f^{J\pi}(E)$ for fission decay. Subscript i denoting the penetrations for competing processes, among which neutron penetration $T_n^{J\pi}(E)$ is predominant in the greater part of the energy range of practical interest $E < 30$ MeV.

Fissionability $P_f(E)$ is measured experimentally in fission reactions following a direct reaction of the type (d, pf) , $(p, p'f)$, $(^3\text{He}, tf)$ and so on [29, 30] but can be determined according to Eq. (6) as the ratio of the observed cross-section $\sigma_f(E)$ to the compound nucleus formation cross-section $\sigma_c(E)$, which is usually obtained by calculation, either using the optical model or by other means [12, 23, 33, 34]. Since this quantity is less sensitive to the entrance channel properties, it is more convenient than the fission cross-section for the purposes of both analysis and comparison of fission probability data for different modes of excitation of nuclei.

3.1. Penetration of the single-humped (parabolic) barrier and its properties

The barrier data in Eq. (6) are directly associated with the behaviour of fission penetration, which is generally described with the help of the Hill-Wheeler formula for the parabolic barrier [35]:

$$T_f(E) = \left[1 + \exp\left(-2\pi \frac{E - B_f}{\hbar\omega}\right) \right]^{-1} \quad (7)$$

where the two parameters, B_f and $\hbar\omega$, are the height of the barrier and the energy characterizing its curvature, respectively. The energy dependence $T_f(E)$ has a "threshold"

character - exponential with a rate $\frac{d \ln T_f}{dE} = \frac{2\pi}{\hbar\omega}$ for $E < B_f$ and much slower with an

asymptotic behaviour $T_f \rightarrow 1$ for $E > B_f$. Quantity $\frac{\hbar\omega}{2\pi} \approx 0.1$ MeV is small [23, 24] and corresponds to a drop in T_f in the 0.25 MeV interval by more than an order. We can

therefore speak about a "break" in $T_f(E)$ for $E \approx B_f$ - a very characteristic feature. This is

an extremely favourable factor that ensures a sufficiently high accuracy in determining the

unknown quantity B_f . All the other factors in Eq. (6), to varying degrees, play an

unfavourable role in this sense.

According to the model of the transition state developed by N. Bohr and Wheeler [1] and A. Bohr [36], each fission channel² with given quantum characteristics makes a contribution to barrier penetration $T_f^{j^*}(E)$. If we consider all the varieties of transition state, the quantity $T_f^{j^*}(E)$ can be regarded as the effective number of fission channels so that it is sometimes denoted by $N_f^{j^*}(E)$. It is assumed that the spectra of the lowest fission channels and the lowest levels of a nucleus of equilibrium shape are similar [36]. With increase in energy the fission channel spectrum rapidly becomes denser and can be described statistically, as in the case of the equilibrium state, in terms of level density.

² The term "fission channel" is used variously to mean: (a) the quantum state at the vertex (at the saddle point), synonymous with transition state; (b) the "chance" for the fission process to take place in reactions of the type (n, xnf) , $x = 0, 1, \dots, x_{\max}$; (c) one of the decay channels of the excited compound nucleus.

When describing $T_{f,r}^{J\pi}(E)$ the energy excitation scale can be divided into two regions:

(a) discrete $E < E_d$, where the contributions of each channel are summed; (b) continuous $E > E_d$, where the channel contributions are determined by a statistically-integral relationship, namely

$$T_{f,r}^{J\pi}(E) = \sum_{\lambda} T_{f,r}^{J\pi}(E_{\lambda}) + \int_{E_d}^{\infty} \frac{\rho_f(U, J^{\pi}) dU}{1 + \exp\left(-2\pi \frac{E - B_x - U}{\hbar\omega}\right)}, \quad (8)$$

where λ numbers the discrete transition states with given J and π in the "resolved" region of penetration $T_{f,r}^{J\pi}(E, \rho_{f,r}^{\lambda}, \hbar\omega_{\lambda})$ and $\rho_f(U, J^{\pi})$ is the transition state density. The discrete region is a specific feature of even-even fissioning nuclei owing to the presence of an energy gap $2\Delta_f$ in the excitation spectrum and the existence in it of levels of low density, mainly of a collective nature, i.e. $E_d \approx 2\Delta_f$. For nuclei with a different parity the simplified assumption $E_d = 0$ is generally used. It is sometimes associated with the final theoretical energy resolution for fission channels, which is determined by the curvature parameter $\hbar\omega/2$ in Eq. (7). We should, however, mention the paper by Nemilov and co-workers [37], which considers the effects due to the inhomogeneous spectrum of the lowest fission channels for nuclei with different parity of the number of nucleons.

3.2. Penetration of the double-humped fission barrier

In fission cross-sections the effects due to the double-humped shape of the fission barrier of transactinide nuclei are very varied [22-24, 29, 30], as we have to some extent pointed out. Such effects are determined by the interaction between the fission (collective) degree of freedom and the compound (internal) degrees of freedom in both wells, this interaction being strongly dependent on the excitation energy. At near-threshold energies and above the condition of strong damping of the vibration mode into the internal modes is fulfilled in the case of most nuclei, and this leads to the relationship for fission penetration

$$T_{f,r}^{J\pi}(E) = \frac{T_A(E) \cdot T_B(E)}{T_A(E) + T_B(E)}, \quad (9)$$

where, as in Eq. (7), J , π have been omitted. Incidentally, the Hill-Wheeler correlation for a parabolic barrier (7) with parameters $B_{A(B)}$ and $\hbar\omega_{A(B)}$ is widely used to describe the penetration of the inner (outer) humps $T_{A(B)}(E)$ in Eq. (9).

In the case of strong damping, fission of nuclei is similar to a two-step process, i.e. according to Eq. (9), the probability of penetration of a two-humped barrier can be represented as the product of T_A , penetration of hump A and the probability of entering the second well, and $T_B/(T_A + T_B)$, the probability of not returning to the first well and penetration of hump B. Because of the exponential dependence (7) for a sufficient difference between the heights of humps B_A and B_B compared with $\frac{\hbar\omega_{A(B)}}{2\pi} \approx 0.1-0.2$ MeV, total barrier penetration is approximately equal to the lower penetration of the higher barrier, which thus determines the "threshold" observed in the fission cross-section - the feature ("break") noted above during transition to the sub-barrier energy region.

The transition states of both barrier humps are taken into consideration with the help of Eqs (9) and (8), for which purpose the following transformation is necessary in the parameters of the latter relationship:

$$\lambda \rightarrow \lambda_{A(B)}, E_d \rightarrow E_d^{A(B)}, \rho_s \rightarrow \rho_{A(B)}, B_s \rightarrow B_{A(B)}, \hbar\omega \rightarrow \hbar\omega_{A(B)}$$

The difference in the energy dependences of transition state densities $\rho_A(U)$ and $\rho_B(U)$, which is due to the difference in shape of nuclei for $\alpha = \alpha_A$ and α_B , can lead to the violation of the rule formulated above: $T_f(E) = \min\{T_A(E), T_B(E)\}$, and this minimum corresponds to $\max(B_A, B_B)$. While its first part always holds true, the second part may not, as shown in Refs [29, 30]. These papers, in particular, discuss in detail the description of fissionability of the ^{237}Np nucleus in the $^{236}\text{U}(^3\text{He}, \text{df})$ reaction - an extremely attractive case because of $B_A > B_B$ and the small difference $(B_A - B_B) \approx 0.3-0.4$ MeV. Consequently, near the threshold $T_A/T_B \ll 1$ and $T_f \approx T_A$, as in the trivial case but as energy increases, the T_A/T_B ratio becomes greater than unity and for $E > 10-12$ MeV, $T_A/T_B \gg 1$ and $T \approx T_B$. In other words, we have a situation where one hump (here the greatest B_A), determines the near-

threshold sector of the cross-section energy dependence while the other, although it is lower, determines the sector which is at a sufficient distance from the barrier.

3.3. Description of level density and probability of fission of nuclei

Contemporary, theoretical descriptions of level density generally include nucleon pairing effects and shell and collective effects. The need to take these into account has been demonstrated both theoretically [2, 38-41] and experimentally [12, 30, 38], and we shall not tax the reader's patience by discussing this aspect, nor models that disregard these properties of actual nuclei. There are consistent theoretical descriptions of the nucleon pairing effects and the shell effects [38, 39] but no such description exists for level density which takes into account collective excitations.

Using the adiabatic approximation, we represent level density as thus

$$\rho(U, J) = \rho_{\text{int}}(U, J) \cdot K_{\text{rot}}(U) \cdot K_{\text{vib}}(U), \quad (10)$$

where $\rho_{\text{int}}(U, J)$ is the density of internal (quasi-particle) excitations, K_{rot} and K_{vib} are the coefficients of rotational and vibrational increase in level density. Use of the superfluid model to describe $\rho_{\text{int}}(U, J)$ ensures that account is taken of nucleon pairing correlations and shell inhomogeneity of the single-particle spectrum [38]. The rotational coefficient K_{rot} is the factor which determines the difference between $\rho_A(U, J)$ and $\rho_B(U, J)$ and, consequently, the specific dependence of the fission cross-section on energy and hump heights considered above. It depends strongly on the symmetry of shape of nuclei:

$$K_{\text{rot}} = \begin{cases} 1 & \text{for spherical nuclei} & (11a) \\ \sigma_{\perp}^2 & \text{for axisymmetric and mirror-symmetric nuclei} & (11b) \\ 2\sigma_1^2 & \text{for axisymmetric but mirror-asymmetric nuclei} & (11c) \\ \sqrt{\frac{8}{2}} \sigma_x \sigma_y \sigma_z & \text{for nuclei, symmetric with respect to } 180^\circ & (11d) \\ \sqrt{8\pi} \sigma_x \sigma_y \sigma_z & \text{for nuclei with no rotational symmetry,} & (11e) \end{cases}$$

where $\sigma_i^2 = J_i t$, are the spin dependence parameters, J_i is the moment of inertia of the nucleus

relative to the i -th axis, t the temperature of the nucleus for axisymmetric shapes $J_x = J_y = J_z$, $J_z = J_{\parallel}$, i.e. $\sigma_x \sigma_y \sigma_z = \sigma_{\perp}^2 \sigma_{\parallel}$. Cases (11a) and (11b) are typical for describing the level density of equilibrium-shape nuclei [12, 38, 42] and (11b)-(11e) for describing the density of the transition states [29, 30, 32]. More often than not ρ_A and ρ_B , ρ_n and ρ_f (or $\rho_{A(B)}$ for the two-humped barrier) differ considerably in actual cases. By way of an example we note that $\sigma_{\perp}^2 \approx 50-100$ for $U = B_n$ and $\sigma_{\perp}^2 \gg K_{vib}$ [38, 43].

Expression (10) for level density was obtained within the framework of the so-called generalized superfluid model. The level density systematics of this model are based on relationships (10), (11a) and (11b). Furthermore, in calculating K_{vib} the model uses the liquid drop evaluation [38], while in $\rho_{mt}(U, J)$ shell effects are taken into account phenomenologically with the help of

$$a(U) = \begin{cases} \bar{a} \left[1 + \delta w \cdot \frac{f(U - E_0)}{E - E_0} \right] & \text{for } U \geq U_{cr} \\ a(U_{cr}) = a_{cr} & \text{for } U < U_{cr} \end{cases} \quad (12)$$

where $a(U)$ is the level density parameter, $\bar{a} = \text{const}$ its asymptotic value for large excitations (liquid drop model), δw the shell correction, $f(U) = 1 - \exp(-\gamma U)$ a dimensionless function describing shell restructuring with energy, and E_0 and U_{cr} are the condensation energy and critical energy of phase transition.

$$E_0 = 0,152 a_{cr} \Delta^2 - n \Delta \quad U_{cr} = 0,472 a_{cr} \Delta^2 - n \Delta, \quad (13)$$

where Δ is the correlation function, $n = 0, 1$ and 2 respectively for even-even, A-odd and odd-odd nuclei. Fitting of Eq. (10) to the observed neutron resonance densities yielded $\bar{a} = 0.093 \cdot A \text{ MeV}^{-1}$ and $\gamma = 0.064 \text{ MeV}^{-1}$ [42].

The level density systematics of the generalized superfluid model were applied successfully to the analysis and description of fissionabilities $P_f(E)$ and the determination in this process of the heights of B_f for "light" nuclei from Ac to Pb and further right up to rare earths [12, 32]. It was subsequently found that in the actinide region the above systematics

underestimated level density at low energies and did not ensure agreement between the terms in Eq. (8). This shortcoming was detected when describing the observed fission cross-sections and then directly during the comparison of the calculated total level density curves $\rho_{\text{tot}}(U) = \int_0^{\infty} \rho(U, J) dJ$ with experimental data from low-lying level spectra for equilibrium-shape nuclei [32]. For this reason, the generalized superfluid model systematics [42] were rejected in [13, 14] in favour of a hybrid model [44] linking the generalized superfluid model to the constant-temperature model in the spirit of the Gilbert-Cameron phenomenological description [45], and in Ref. [15] in favour of theoretical calculations of $\rho_{\text{int}}(U, J)$. This difficulty did not arise in the pre-actinide region since here the difference ($B_f - B_n$) is sufficiently large to suppress the influence of the first term in Eq. (8) and, in the end, the more approximate description of the observed values of $\sigma_f(E)$ or $P_f(E)$, which vary much more strongly with energy than in the actinide region, was satisfied. We note that this methodological problem was taken into consideration in all the studies used below directly for the compilation of B_f values.

We now return to the factor $K_{\text{rot}}(U)$, which determines the influence of the symmetry of shape of the nucleus on its fissionability. Unlike the above methodological influence, which is associated with inadequacies in describing $\rho(U, J)$ and is significant mainly in the neighbourhood of thresholds for fission and neutron emission, this is a physical influence occurring in the entire energy region of practical interest. Of the compound nucleus decay channels competing with the fission channel the one that predominates in this region is the neutron channel, for which we can write

$$T_n^J(E) = \frac{2A^{2/3}}{\pi^2} \int_0^{E-B_n} (E-B_n-U) \rho(U, J) dU \quad (14)$$

$$T_i^J(E) = \int_0^{E-B_i} \rho_i(U, J) dU \quad (15)$$

where $\rho_n(U, J)$ is the level density of residual nucleus $(A - 1)$ (after neutron emission), $\kappa = \frac{\hbar^2}{2\mu Z_0^2} \approx 10 \text{ MeV}$ and Eq. (15) simplifies Eq. (8) by taking the barrier penetration to be unity for the transition states for $0 \leq U \leq E - B_i$ and zero for $U > E - B_i$ (as before, $i = f$ in the case of the single-humped barrier and $i = A$ or B in the case of the double-humped barrier). We further simplify Eqs (14) and (15), following Ref. [11], and write the fission to neutron penetration ratio in the form

$$\frac{T_f}{T_n} \approx 5A^{-2/3} t^{-1} \frac{K_{\text{pot}}^b(E - B_f)}{K_{\text{pot}}^n(E - B_n)} \exp\left(-\frac{B_n - B_f}{T}\right) \quad (16)$$

for a single-humped barrier and

$$\frac{T_f}{T_n} \approx 5A^{-2/3} t^{-1} \frac{\exp(B_n/T)}{K_{\text{pot}}^n(E - B_n)} \left[\frac{\exp(B_A/T)}{K_{\text{pot}}^A(E - B_A)} + \frac{\exp(B_B/T)}{K_{\text{pot}}^B(E - B_B)} \right]^{-1} \quad (17)$$

for a double-humped barrier.

On the basis of Eqs (16) and (17) we consider specific situations which have been partly alluded to and which are associated with the occurrence, in the case of fission, of large differences $K_{\text{rot}}^i(E - B_i)$ at various extreme points of deformation energy of nuclei.

1. Fission of nuclei sufficiently distant from Pb.

This region is characterized by transition from a small islet of spherical nuclei in the immediate neighbourhood of doubly magic ^{208}Pb with $K_{\text{rot}}^n = 1$ to deformed nuclei to its left and to its right where $K_{\text{rot}}^n = \sigma_{\perp}^2$. The difference in K_{rot}^n should result in a different rate of growth of fissionability $P_f \sim T_f/T_n$ in the above-threshold sector; this rate is considerably higher for spherical nuclei, as is observed now and was shown for the first time in Ref. [46] (see also more recent work [12, 47]).

2. Fission of nuclei in the Ra region

The group of nuclei accessible for experimental study and analysis is narrow, yet interesting on account of the difference in saddle points and their deformations for mass-

symmetric and mass-asymmetric fission modes. In the first case, the nuclei violate axial symmetry during fission [30], and this, according to Eq. (11), results in a noticeable increase in K_{rot}^s over K_{rot}^a which in its turn leads to an increase in the ratio of symmetric and asymmetric fission probabilities.

$$\frac{P_f^s}{P_f^a} \approx \frac{\kappa_{\text{par}}^s (E - B_{\text{ps}}^s)}{\kappa_{\text{par}}^a (E - B_{\text{ps}}^a)} \cdot \exp\left(\frac{B_{\text{f}}^a - B_{\text{f}}^s}{t}\right) \quad (18)$$

Expressions (11d) and (11c) were used in the analysis of fissionability for symmetric and asymmetric fission of ^{227}Ra and $^{226-228}\text{Ac}$ nuclei [32] for K_{rot}^s and K_{rot}^a , respectively, and it was established that on average $B_{\text{f}}^a - B_{\text{f}}^s \approx -1.5$ MeV. Owing to the difference in the magnitudes of K_{rot}^i and B_{f}^i , the contribution δW_{f}^i ($i = a$ and s) of the shells to the latter is ensured by an increase in the $P_{\text{f}}^s(E)/P_{\text{f}}^a(E)$ ratio by an order when energy increases by 4-5 MeV directly above the threshold [32, 48]. Here $P_{\text{f}}^s/P_{\text{f}}^a$ attains the value of approximately one, whence it follows that description of the fission probability of nuclei in the Ra region requires that both fission modes should be taken into account. On both sides of this specific region, however, fission in the energy range of interest can be considered to be of one mode: asymmetric to the right and symmetric to the left.

3. Actinide fission

Calculations in Refs [27, 49] predict that the inner saddle point of the barrier will be unstable with respect to axial deformation in Th-U nuclei for $N \geq 142-144$ and, consequently, the height of hump A will decrease. On the other hand, an increase in K_{rot}^A when describing the cross-sections (fissionability) results in an increase in the value of B_A obtained from analysis of experimental data [14, 29], and this is easily understandable in the light of Eq. (17). The influence of axial asymmetry on B_A increases from U to Cm [27]. This behaviour is correlated with a specific tendency in the analysis of fission probability: the heavier the nucleus the higher is the sensitivity of description to the magnitude of K_{rot}^A .

(U), i.e. if any of the variants (11b)-(11e) is suitable for U nuclei, and this is ensured by changing B_A , then only the last variant of Eq. (11) will be suitable for Cm [29].

In Eq. (7) the terms in square brackets describe the contribution to fission probability of the transition states of each hump. Using this, we can easily reproduce the specific situations resulting from the differences between B_A and B_B , and K_{rot}^A and K_{rot}^B , which were considered at the end of the preceding section. From Eq. (17) it is also evident that the case of humps of similar heights is unfavourable for determining their parameters, while that of substantially differing B_i is unfavourable for determining the parameters of the lower hump. Independent methods of determining B_i , particularly for the smaller of the humps, are therefore of importance. In the region of transuranics with $B_A > B_B$ such possibilities exist and are associated with experimental study and analysis of the excitation functions of spontaneously fissioning isomers and the group of neutron resonances in the sub-barrier fission cross-section [24, 30].

3.4. The status of data on fission barrier heights

Here we consider "experimental"³ fission barrier data obtained from observed characteristics of fission probability, $\sigma_f(E)$ or $P_f(E)$, taking into account the actual properties of nuclei in the statistical description. This description permits a wide spectrum of internal excitation density models ranging from the superfluid model with its analogues and modifications, including hybrid versions [44, 45], to the constant temperature model described in Ref. [24]. In spite of their different theoretical levels, the points they have in common are similar behaviour at low energies $U < U_{cr}$, and correspondence between the continuous and discrete parts of the different functionals that determine Eq. (6). As we have seen, in different regions of nuclei different requirements are applied to the description of

³ We use inverted commas here since in the description often noticeably differing values of B_i are obtained from the same experimental data, owing to differences in parameters. They constitute the majority of the fission barrier data under consideration and it is the aim of this review to select the best.

$\rho(U, J)$, depending on the relationship between the fission and neutron emission thresholds, the associated magnitude of fissionability and, lastly, the fission barrier shape. For this reason, in considering the status of fission barrier data, we divide the region of nuclei under consideration into two: $Z < 90$ and $Z \geq 90$.

1. Pre-actinide nuclei ($Z < 90$)

Table 1 gives the set of fission barrier (B_f) data for ^{200}Tl - ^{216}Rn nuclei, based principally on the original results contained in Ref. [12], which is also a review of the problem discussed here for pre-actinides. It covers data for 20 spherical nuclei in the ground state obtained from fission reactions induced by light charged particles: electrons, protons, deuterons, ^3He ions and α -particles indicated in the second column. Fission cross-section measurements were carried out in the range from several tens of MeV to the observed threshold in the case of the seven marked in Table 1 and only in the above-barrier energy region for the rest.

In the analysis contained in Ref. [12] covering up to 70-80 MeV, an unsuccessful attempt was made to take into account the reduction in the contribution of rotational modes to $\rho_f(U, J)$ (more detail in Ref. [47]). A new analysis was therefore carried out in Ref. [50] only for the near-threshold and above-barrier energy sector extending 10 to 15 MeV, which is unaffected by the decrease of $K_{\text{rot}}(U)$. In Ref. [11] the collective effects were not taken into account at all in description of the fission cross-sections. It follows from comparison of the results of these papers, given in Table 1, that the original postulates have little effect on them. They agree within error $\delta B_f = \pm(0.4-0.6)$ MeV, evaluated in Ref. [12] for nuclei whose fission has been experimentally investigated up to the threshold.

The failure in Refs [12, 46] to take correct account of the decrease in $K_{\text{rot}}(U)$ was eliminated in Ref. [47], and it was shown that satisfactory description of the fission cross-sections could be obtained using the barriers of the Myers-Swiiatecki phenomenological model [8], which are given in the last column of Table 1. Meanwhile, the data in Ref. [12]

Table 1

Fission barrier parameters for pre-actinide nuclei

Fissioning nucleus	Incident particle	B_f , MeV			
		[12]	[50]	[11]	[8]
^{200}Tl	^3He	22,8			21,6
$^{201}\text{Tl}^*$	α	23,1	23,2	22,3	22,5
^{204}Pb	e, p	23,5			23,1
^{205}Pb	d	24,6			24,2
^{206}Pb	e, p	25,3			25,3
^{207}Pb	e, d	27,0			26,7
^{208}Pb	e	27,4			27,3
^{206}Bi	d	22,4			21,8
$^{207}\text{Bi}^*$	p, α	22,8	22,1	21,9	22,6
^{208}Bi	p, d	23,8			23,8
$^{209}\text{Bi}^*$	e, p, d, α	24,3	23,8	23,3	24,3
^{210}Bi	d	23,6			23,2
^{207}Po	^3He	19,3			19,0
$^{208}\text{Po}^*$	α	19,9	20,7		19,9
^{209}Po	^3He	21,1			21,1
$^{210}\text{Po}^*$	p, ^3He , α	21,2	21,1	20,5; 21,4	21,6
^{211}Po	d, ^3He , α	20,6		19,7	20,5
$^{212}\text{Po}^*$	α	19,6	19,3	19,5	19,6
^{212}At	^3He	18,6			18,1
$^{213}\text{At}^*$	α	17,3	17,8	17,0	17,1
^{216}Rn	^7Li	13,5			13,8

- Remarks:
1. Nuclei marked with * have fission cross-sections measured up to threshold.
 2. Ref. [11] gives two values for ^{210}Po - a lower value for the (α , f) reaction and a higher value for the (p, f) reaction. Analysis in Ref. [12] does not corroborate this difference.
 3. The error for ^{216}Rn is $\delta B_f = \pm 1.0$ MeV.

also show excellent agreement with them, further corroborating the low sensitivity of the sought for quantities to the model used in the analysis. In order not to create the false impression that there is absolutely no sensitivity, we cite Ref. [51], which used a Fermi-gas description of level density, where, the barriers differ from those given in Table 1 on average by 2 MeV, i.e. by approximately four times the error δB_f (more detail in Refs [12, 47]).

With regard to the set of data on $B_f(Z,A)$ given in Table 1 we recommend that, since the phenomenological description [8] based on Eqs (3) and (4), is the best, it should be used. It not only agrees with experimental data but also enables us to evaluate B for any Z and A pair.

Table 2 presents data on fission barriers in the Ra region⁴. They are mainly the results of work carried out at the Radium Institute (St. Petersburg), analysis of which has undergone repeated improvements [37, 52, 53], together with data from Refs [32, 54, 55]. Comparison of the values obtained by the various authors in Table 2 presents some difficulties since the fission process was considered to be of two modes in Refs [32, 54], and so values are given for B_f^a and B_f^s , while in Refs [37, 52-55] it was considered to be of one mode. Since in the latter papers a fundamental role was played by description of the low-energy sector (E) adjoining the observed asymmetric threshold ($B_f^a < B_f^s$), the B_f data in Table 2 should be compared with the B_f^a values.

Table 2 also presents the results of calculations using the shell correction method [56] (see also Ref. [12]) and the values obtained using the phenomenological model [8]. The mean-root-square deviation of the experimental values of B from the calculated values is comparable with the error of analysis and amounts to 0.3-0.4 MeV. In view of the advantages of Ref. [8] for evaluating $B_f(Z,A)$ (see above), we consider that recommendation about the phenomenological description [8] as giving the best values of B_f should be extended to the $Z = 87-89$ region. We note, however, that there would be better agreement between the values in Ref. [8] and experimental values if we increased the former by 0.2 MeV, interpreting this as consideration in general of contribution δW_f in Eq. (4), which was ignored in Ref. [8].

⁴ Here and henceforth the barrier heights are rounded off to one significant digit after the decimal point.

Table 2

Fission barrier parameters for nuclei in the Ra region.

Fissioning nucleus	$B_{\frac{1}{2}}^{\alpha}$, MeV	$B_{\frac{1}{2}}^S$, MeV	$B_{\frac{1}{2}}$, MeV	
			Experiment	Calculation
^{225}Ra			7,6 /52/	8,2 /8/
^{226}Ra			8,5 /52/; 8,4 /53/	8,1 /56/; 8,2/8/
^{227}Ra	8,0 /32/	9,3 /32/	8,4/37; 8,3/5/; 8,3/53/; 8,2/55/	7,9/8/
^{228}Ra	7,8-8,2/54/	9,1-9,3/54/	7,8/53/	7,9/56; 7,9/8/
^{226}Ac	7,8 /32/	8,8 /32/		7,3/8/
^{227}Ac	7,4 /32/	8,5 /32/	7,5/37/; 7,0-7,6/52/; 7,7/53/	7,1/8/
^{228}Ac	7,0 /32/	8,8 /32/	7,1/37/; 7,2/52/	7,1/8/

2. Transactinide nuclei with $Z = 90-98$

In the actinide region there is no such ready-made universal recipe for evaluation as the phenomenological description of masses and fission barrier heights [8-10]. The reason for this is that the general theoretical approach, based on the shell correction method, does not have the degree of accuracy required for this purpose, which is estimated at 1-2 MeV [57]. The structure of the fission barrier for actinides was studied on the basis of the phenomenological approach, together with the possibility of obtaining a corresponding description of hump heights $B_f(Z,A)$, in Refs [24, 58, 59]. In Ref. [59] an attempt was made to construct a simple systematic taking into account fission cross-section $\sigma_f(Z,A)$ with the degree of accuracy required for practical purposes. However, with this approach it has been possible so far to take into account only axisymmetric and mirror-symmetric deformations; it can be used for $Z \geq 92$ and requires a certain amount of caution. Therefore, the experimental data must be analysed in order to evaluate $B_f(Z,A)$ in the region of nuclei under discussion.

For $B_i(Z,A)$ of actinides there are two compilations [24, 30] encompassing more or less the same range of isotopes. The experimental data on barrier heights come mainly from analysis of the fission cross-section energy dependence and fissionability of nuclei, supplemented by data obtained from analysis of the excitation functions of spontaneously fissioning isomers and the group of strong resonances in the sub-barrier fission cross-section. Both compilations for $B_i(Z,A)$, covering approximately 70 nuclei from Th to Fm, were published more than ten years ago. Since then, however, no further data on new nuclei have been added to them and this simplifies our task considerably. We note, nonetheless, the results of analysis of neutron fission cross-sections which appeared later and are given in Refs [13-15].

Comparison of these compilations is of great methodological interest since Ref. [30] used the results of theoretical calculations for $\rho(U,J)$, and the relations of the adiabatic model (11) for $K_{r\alpha}^i(U)$, while Ref. [24] employed a model which was extremely simplified - practically to the limit. This is the constant temperature model in which the J-dependence of level density has been included, while by varying constant C, which is a pre-exponential function in

$$\rho(U,J) = C (2J + 1) \exp \left[- \frac{(J + 1/2)^2}{2\sigma^2} + \frac{U}{t} \right], \quad (19)$$

and also other parameters, account is taken of the difference due to parity of the number of nucleons in the nucleus and the symmetry of its shape at saddle points α_{sp}^i . This comparison will enable us, in the best way possible, to evaluate the description of level density in the problem of determination of B_i .

The entire set of data is shown on Figs 1 and 3. Table 3 gives the results of Ref. [30], obtained by analysis of fission cross-sections and fissionability. The results of Ref. [24] are given as data from the "best parameter" table and as such they already

Table 3

Fission barrier parameters B_i for transactinide nuclei

Fissioning nucleus	B_A , MeV			B_B , MeV		
	/ 24 /	/ 30 /	Recommended value	/ 24 /	/ 30 /	Recommended value
I	2	3	4	5	6	7
^{90}Th 227	5,9		5,9	6,6		6,6
228	6,2	6,2	6,2	6,5	6,2	6,5
229		5,9	5,9	6,5	6,0	6,3
230	6,1	5,5	6,1	6,5	5,5	6,1
231	6,0	6,1	6,0	6,1	6,1	6,1
232	5,8	5,3	5,8	6,2	5,3	6,2
233	6,3	6,0	6,1	6,3	6,0	6,3
234	6,1	6,0	6,1	6,5	6,0	6,3
^{91}Pa 230		5,4	5,4		5,4	5,4
231	5,9	5,6	5,7	5,9	5,6	5,7
232	6,1	6,0	6,0	6,2	6,0	6,1
233	6,1	5,9	6,0	6,1	5,9	6,0
^{92}U 231		5,2	5,2		5,2	5,2
232	5,2	5,5	5,4	5,1	5,4	5,3
233			5,7			5,7
234	5,6	6,0	5,9	5,5	5,8	5,7
235	5,9	6,0	6,0	5,6	5,9	5,8
236	5,6	5,6	5,6	5,5	5,7	5,6
237	6,1	6,3	6,2	5,9	5,9	5,9
238	5,7	6,1	6,0	5,7	5,9	5,8
239	6,3	6,3	6,3	6,1	5,7	6,0
240	5,7	6,4	6,1	5,5	5,8	5,8
^{93}Np 233		5,0	5,0		5,1	5,1
234	5,5	5,5	5,5	5,1	5,4	5,4
235	5,5	5,5	5,5	5,2	5,5	5,5
236	5,8	5,7	5,8	5,6	5,4	5,6
237	5,7	5,7	5,7	5,4	5,5	5,5
238	6,1	6,0	6,0	6,0	5,6	5,9
239	5,9	5,8	5,8	5,4	5,0	5,4
^{94}Pu 235			5,7	5,1		5,1
236			5,7	4,5		4,5
237		5,6	5,6		5,4	5,4
238	5,5	6,0	5,9	5,0	5,3	5,2
239	6,2	6,3	6,2	5,5	5,5	5,5
240	5,6	6,0	5,8	5,1	5,5	5,3

Table 3 (continued)

I	2	3	4	5	6	7	
${}_{94}\text{Pu}$	241	6,1	6,2	6,2	5,4	5,7	5,6
	242	5,6	5,7	5,7	5,1	5,5	5,3
	243	5,9	5,9	5,9	5,2	5,6	5,5
	244	5,4	5,6	5,5	5,0	5,3	5,2
	245	5,6	5,5	5,5	5,0	5,4	5,4
	246		5,4	5,4		5,3	5,3
${}_{95}\text{Am}$	239	6,2	6,3	6,3			4,9
	240	6,5	6,3	6,4	5,2		5,2
	241	6,0	6,4	6,2	5,1		5,1
	242	6,5	6,3	6,4	5,4		5,4
	243	5,9	6,3	6,1	5,4		5,4
	244	6,3	6,2	6,2	5,4		5,4
	245	5,9	6,1	6,1	5,2		5,2
	246			5,8			5,0
	247	5,5	5,9	5,7			4,8
${}_{96}\text{Cm}$	241	6,3	6,5	6,4	4,3		4,3
	242	5,8	6,2	6,0	4,0		4,0
	243	6,4	6,6	6,5			4,6
	244	5,8	6,3	6,1	4,3		4,3
	245	6,2	6,3	6,3			4,9
	246	5,7	6,2	6,0	4,2	4,7	4,7
	247	6,0	6,2	6,1			4,9
	248	5,7	6,0	5,9		5,0	5,0
	249	5,6	5,7	5,7			4,7
	250	5,3	5,5	5,4		4,4	4,4
	${}_{97}\text{Bk}$	244		6,6	6,6		
245			6,4	6,4			4,2
246			6,5	6,5			4,7
247			6,5	6,5			4,6
248			6,3	6,3			4,8
249		6,1		6,1			4,5
250		6,1		6,1	4,1		4,1
${}_{98}\text{Cf}$		250	5,6		5,6		
	251		6,2	6,2			3,9
	252		5,3	5,3			3,5
	253	5,4	5,3	5,4			3,5

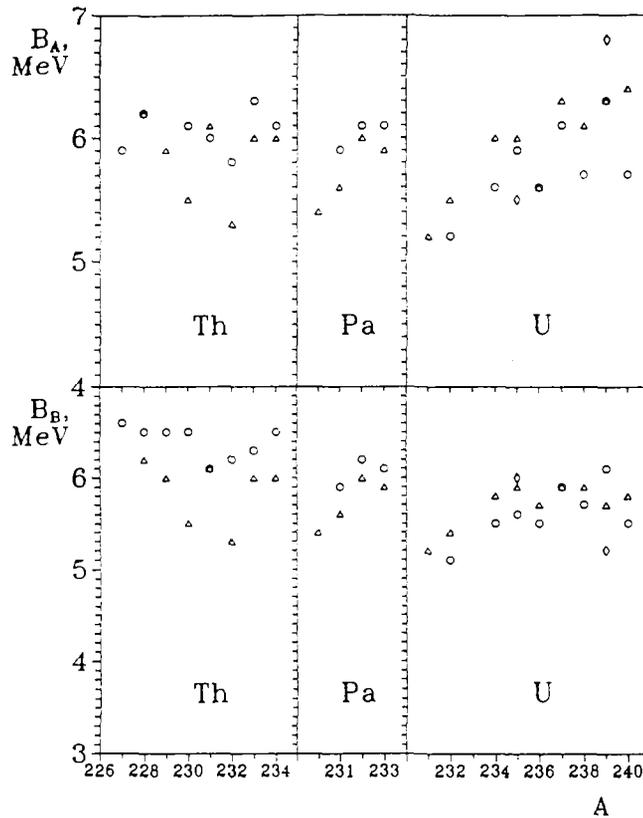


Fig. 1. Fission barrier parameters for Th, Pa and U.
 o: Ref. [24], Δ : analysis of fission probability, \diamond : analysis of the resonance group in sub-barrier fission [30].

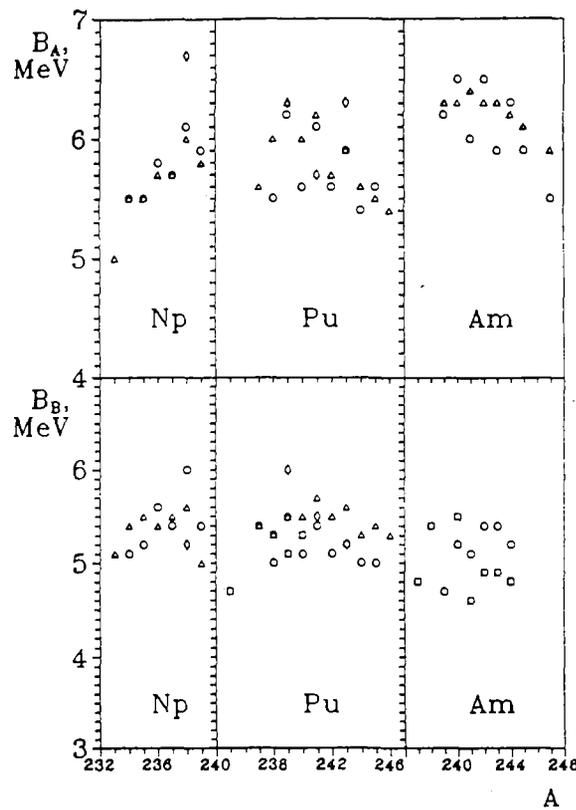


Fig. 2. Fission barrier parameters for Np, Pu and Am.
 o: Ref. [24], Δ : analysis of fission probability, \diamond : analysis of the resonance group in sub-barrier fission, \square : analysis of excitation functions of spontaneously fissioning isomers [30].

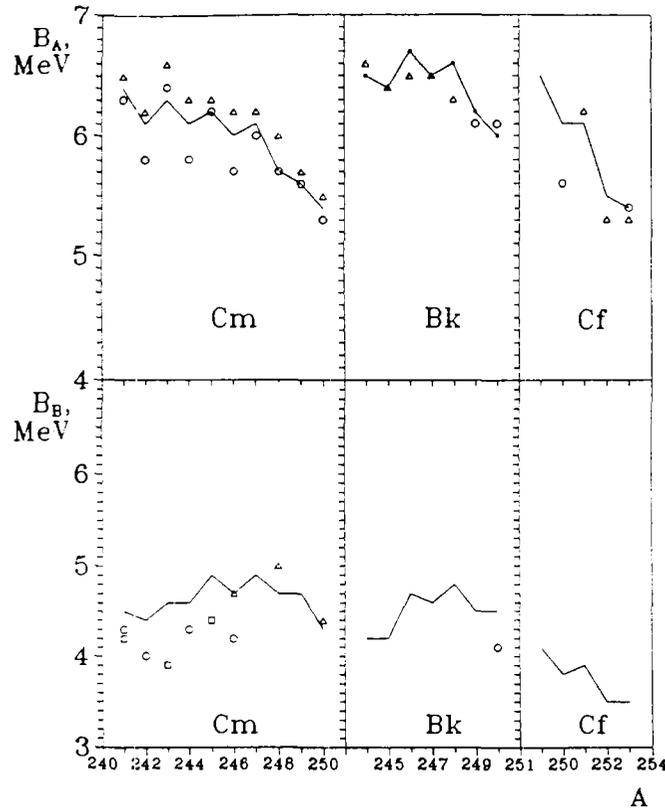


Fig. 3. Fission barrier parameters for Cm, Bk and Cf.
 Broken line: systematics of Ref. [59], remaining notation as for Fig. 2.

represent the product of an evaluation revising, within error $\delta B_i = 0.2-0.3$ MeV, the effects of even-odd differences in $B_i(Z,A)$ in the original analysis results [60]. The data taken from Ref. [30] are original. The root-mean-square deviation between the results of the two compilations in Table 3 is $\Delta B_A = 0.27$ MeV, $\Delta B_B = 0.36$ MeV for the entire data set and 0.29 MeV without Th data. In other words, the values of ΔB_i and the evaluation of error δB_i , attributed to the determination of B_i , are close. Accordingly, we again reach the conclusion that the difference in the models for $\rho(U,J)$, which is large in the case under consideration, does not, within the error of analysis, influence the results for $B_i(Z,A)$.

Table 3 also gives the recommended values of $B_i(Z,A)$, in obtaining them data from Refs [24] and [30] were given equal preference, but in "weighting" them use was made of the systematics of Ref. [59], whose results are shown in Fig. 3. These systematics served as the basis for the values of B_B given in Table 3 for Bk and Cf nuclei, for which there are virtually no experimental data available. The systematics in Ref. [59] can also be used to

extrapolate $B_i(Z,A)$, within small limits, beyond the experimentally investigated region of nuclei.

4. CONCLUSION

Since barrier heights are obtained from experimental data on fission cross-sections or fissionability (necessitated by the characteristic properties of barrier penetration) the sought quantities have a low sensitivity to level density description. This factor ensures the reliability of the obtained fission barrier height data. The uncertainty of ± 0.3 MeV in the barriers, however, is not such a small quantity and its value increases with energy. For this reason, the recommendations made here should be viewed basically as a guide for selection of the initial parameters, which will inevitably need to be adjusted, depending on the specific description (models and parameters).

From the foregoing it is also clear that improvement of barrier data is still a pressing task with emphasis on the applied sphere. Efforts should continue to be made to carry out measurements and analysis of experimental data, and to develop theoretical calculations and fission barrier systematics. It is at least as important, however, to study the sensitivity of description to the various factors discussed in this paper for the complex of interrelated quantities ("barrier + cross-section"). For this purpose, attention should be focused on the second characteristic and its dependence on energy and the nucleonic composition of the nucleus.

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