International Atomic Energy Agency

INDC(CCP)-380
Distr.: L

INDC INTERNATIONAL NUCLEAR DATA COMMITTEE

METHOD FOR ANALYTICAL CALCULATION OF GROUP-AVERAGED FUNCTIONALS OF NEUTRON CROSS-SECTIONS IN THE UNRESOLVED RESONANCE REGION

A.A. Luk'yanov, A.G. Vysotskij, V.F. Ukraintsev
Institute of Atomic Energy, Obninsk, Russia

(Translated from a Russian original published in
Yadernye Konstanty 3-4/1992, p. 71-73)

Translated by the IAEA

August 1994

IAEA NUCLEAR DATA SECTION, WAGRAMERSTRASSE 5, A-1400 VIENNA

Reproduced by the IAEA in Austria
August 1994

INDC(CCP)-380
Distr.: L

94-10590 (12)
Translated from Russian

UDC 539.170

METHOD FOR ANALYTICAL CALCULATION OF GROUP-AVERAGED FUNCTIONALS OF NEUTRON CROSS-SECTIONS IN THE UNRESOLVED RESONANCE REGION

A.A. Luk'yanov, A.G. Vysotskij, V.F. Ukraintsev

Institute of Atomic Energy, Obninsk

ABSTRACT

A new method is suggested for analytical calculations of group-averaged experimentally measurable functionals of neutron cross-sections (transmission functions) in the unresolved resonance region for one neutron channel and many radiative channels. It can be used for correct calculation of these functionals for even-even heavy nuclei such as ^{238}U, ^{240}Pu, etc.

The analytical approach suggested in Refs [1, 2] to construction of average cross-sections in the unresolved region can be generalized formally to the group characteristics used to describe the resonance self-shielding effects if it is taken into account that the total cross-section and the radiative capture cross-section are expressed only in terms of two possible combinations of statistically reproducible quantities $(1 - iR_{nn})^{-1}$ and $(1 + iR^*_{nn})^{-1}$

$$\sigma = 2\pi k^{-2}(1-\mathrm{Re}\, S_{nn}) = \sigma_o\left[\cos^2\varphi - \frac{1}{2}\left(\frac{e^{-2i\varphi}}{1-iR_{nn}} + \frac{e^{2i\varphi}}{1+iR^*_{nn}}\right)\right], \tag{1}$$

where $\sigma_o = 4\pi k^{-2}$, and the radiative capture cross-section

$$\sigma_\gamma = \sigma_o\left[\mathrm{Re}\,\frac{1}{1-iR_{nn}} - \frac{1}{1-iR_{nn}}\cdot\frac{1}{1+iR^*_{nn}}\right]. \tag{2}$$

Then any physical cross-section functional ψ (σ_γ,σ) can be formally represented as a function of variables g = (1 - iR_{nn}) and g^* = (1 - iR_{nn}). Let us assume that there exists an integral relation:

$$\psi\left(\frac{1}{g};\frac{1}{g^*}\right) = \int_0^\infty\int_0^\infty dt\,dt'\,f(t,t')\,e^{-gt}\,e^{-g^*t'} = \int_0^\infty\int_0^\infty dt\,dt'\,f(t,t')\,e^{-(t+t')}\,e^{iR_{nn}t}\,e^{-iR^*_{nn}t'}\,, \tag{3}$$

then the average value of the functional will be

$$<\psi> = \int_0^\infty\int_0^\infty dt\,dt'\,f(t,t')\,e^{-(t+t')}\,F(t,t')\,, \tag{4}$$

where F is the function defined and studied in Ref. [2] with $t = \dfrac{u+v}{2}$ and $t' = \dfrac{u-v}{2}$.

Thus, the problem of finding $<\psi>$ is reduced here to constructing the corresponding function f(t,t') ("original"ψ).

Let us consider the functional

$$<\frac{1}{\sigma_R+\sigma}> = <\frac{1}{\sigma_R + \sigma_o\cos^2\varphi - \frac{1}{2}\sigma_o\,(g^{-1}\,e^{-2i\varphi} + g*^{-1}\,e^{2i\varphi})}> = \tag{5}$$

$$\frac{1}{\sigma_R + \sigma_o\cos^2\varphi}<\frac{gg^*}{gg^* - hg^* - h^*g}>\,,$$

where σ_R is the dilution cross-section, $g = 1 - iR_{nn}$, $g^* = 1 + iR^*_{nn}$

$$\eta = \frac{1}{2}\,\frac{\sigma_o}{\sigma_R + \sigma_o\cos^2\varphi}\,e^{-2i\varphi} = \eta\,e^{-2i\varphi}\,. \tag{6}$$

We denote p = g-h and q = g* -h*, then

$$\frac{gg^*}{gg^*-hg^*-h^*g} = \frac{(p+\eta)\,(q+h^*)}{pq-\eta^2} = 1 + \frac{h^*}{q} + \frac{h}{p} + \frac{\eta^2}{pq-\eta^2}\left(2 + \frac{h^*}{q} + \frac{h}{p}\right). \tag{7}$$

Now averaging the terms in (7) one-by-one, we obtain:

$$<\frac{1}{q}> = <\frac{1}{1+iR^*_{nn}-h^*}> = \int_0^\infty e^{-t(1-h^*)} <e^{-iR^*_{nn}t}>dt = \frac{1}{1-h^*+S_n}, \tag{8}$$

where we used the result of the work, and similarly

$$<\frac{1}{p}> = \frac{1}{1-h+S_n}. \tag{9}$$

Moreover, the representations of the functions in the form of a Laplace transform with respect to two variables are known [3]:

$$\begin{aligned}
\frac{1}{pq-\eta^2} &= \int_0^\infty\int_0^\infty I_o(2\eta\sqrt{tt'})\, e^{-tp}\, e^{-t'q}\, dtdt' \\
\frac{1}{p(pq-\eta^2} &= \frac{1}{\eta}\int_0^\infty\int_0^\infty \sqrt{\frac{t,}{t}}\, I_1(2\eta\sqrt{tt'})\, e^{-tp}\, e^{-t'q}\, dtdq \\
\frac{1}{q\,(pq-\eta^2)} &= \frac{1}{\eta}\int_0^\infty\int_0^\infty \sqrt{\frac{t'}{t}}\, I_1\,(2\eta\sqrt{tt'})\, e^{-tp}\, e^{-t'q}\, dtdt',
\end{aligned} \tag{10}$$

so that

$$<\frac{gg^*}{gg^*-hg^*-h^*g}> = 1 + \frac{h^*}{1-h^*+S_n} + \frac{h}{1-h+S_n} + \eta^2\int_0^\infty\int_0^\infty e^{-t(1-h)}\, e^{-t'(1-h^*)} \times \tag{11}$$

$$dF(t,t')\left[2I_o(2\eta\sqrt{tt'}) + \frac{h^*t'+ht}{\eta\sqrt{tt'}} I_1(2\eta\sqrt{tt'})\right] dtdt' ,$$

i.e. we arrive at a representation of form (4).

This result can be used to construct the average values for a group of transmissions, regarding it as a Laplace image of $e^{-n\sigma}$:

$$\int_0^\infty <e^{-n\sigma}> e^{-np}\, dn = <\frac{1}{p+\sigma}> . \tag{12}$$

The resonance integral in the group can be represented as

$$<\frac{\sigma_\gamma}{\sigma_R+\sigma}> = i\eta <\frac{R^*_{nn}-R_{nn}}{gg^*-\eta g^*-h^*g}> = i\eta <\frac{R^*_{nn}-R_{nn}}{pq-\eta^2} = i\eta <\frac{R^*_{nn}-R_{nn}}{(pq-\eta^2)}> - \eta\int_0^\infty\int_0^\infty dtdt' I_o(2\eta\sqrt{tt'})\times$$

$$e^{-t(1-h)}\, e^{-t'(1-h^*)}\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial t'}\right) F(t,t') = S_n)\eta\int_0^\infty du\, e^{-u(1-\eta\cos 2\varphi)} \times \tag{13}$$

$$\int_{-v}^{v} dv\, I_o(2\eta\sqrt{u^2-v^2}) \cos(\eta\, v \sin 2\varphi)\, sh\, 2f\ \Phi(u,v) .$$

where u = t + t′, ν = t - t′ and function Φ was introduced and studied in Ref. [2]. In the same way we can also determine other group-averaged characteristics in the unresolved region: average cross-sections in experiments with filtered beams (self-indication) and the quantity $<\frac{1}{(\sigma_R+\sigma)^2}>$.

REFERENCES

[1] LUK'YANOV, A.A., Structure of Neutron Cross-sections, Atomizdat, Moscow (1978) [in Russian].

[2] LUK'YANOV, A.A., VYSOTSKIJ, A.N., YANEVA, N.B., in: Problems of Atomic Science and Technology. Ser. Nuclear Constants, No. 1 (1989) 48 [in Russian].

[3] ABRAMOVITS, M., STIGAN, I.M. (Eds), A Handbook of Special Functions, Nauka, Moscow (1979) [in Russian].