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ABSTRACT

The authors describe a method based on the reduction method for the evaluation of photonuclear reaction cross-sections obtained under conditions where there are large systematic uncertainties (different instrumental functions, calibration and normalization errors). The evaluation method involves using the actual instrumental function (photon spectrum) of each individual experiment to reduce the data to a representation generated by an instrumental function of better quality. The objective is to find the most reasonably achievable monoenergetic representation of the information on the reaction cross-section derived from the results of various experiments and to take into account the calibration and normalization errors in these experiments. The method was used to obtain the evaluated total photoneutron reaction cross-section (γ,xn) for a large number of nuclei. Data obtained for ^{16}O and ^{208}Pb are presented.

INTRODUCTION

One of the main difficulties encountered since the beginning of the 1950s until the present time in experimentally studying the interactions of photons with atomic nuclei has been the absence of intensive monoenergetic photon beams. This compels experimenters to resort to various devices in order to obtain conditions under which the effective photon energy spectrum can, with certain assumptions, be considered to be close to a monoenergetic

spectrum. As a direct consequence of this, there arises a situation where the same type of results are obtained in different experiments under widely different experimental conditions.

These differences quite naturally lead to certain, and sometimes very large, systematic discrepancies in the results [1].

The present work is devoted to an analysis of the reasons for such discrepancies and to the development of a method for the combined analysis and evaluation of photonuclear reaction cross-sections in cases where there are large systematic discrepancies.

The source of numerical data on reaction cross-sections was the Photonuclear Data Fund of the Photonuclear Experimental Data Centre of the Moscow State University's Scientific Research Institute for Nuclear Physics. The Fund includes data obtained using bremsstrahlung, as well as quasi-monoenergetic and labelled photon beams (data prepared at the Photonuclear Experimental Data Centre), a data library for reaction cross-sections obtained with quasi-monoenergetic photons (library prepared at the Lawrence Livermore National Laboratory, USA, by Professor B. Berman [2]) and also the set of numerical data on reaction cross-sections prepared by the photonuclear group of Professor E. Fuller (National Institute of Standards and Technology, USA) on the basis of Ref. [3] and transmitted to the Photonuclear Experimental Data Centre.

Since mainly photoneutron cross-sections were obtained owing to the low intensity of the quasi-monoenergetic photon beams used in the experiments, it is these cross-sections that are considered below.

SYSTEMATIC DISCREPANCIES IN THE RESULTS OF PHOTONUCLEAR EXPERIMENTS

Discrepancies caused by the difference in instrumental functions of the experiments

In order to obtain experimentally an effective photon spectrum which can, with certain assumptions, be considered to be close to a monoenergetic spectrum, experimenters have until now had to resort to special instrumental and mathematical devices.

Nowadays there are many methods for creating such conditions, such as:

- The use of difference methods for gamma bremsstrahlung beams: the difference between two or a linear combination of several bremsstrahlung spectra with sufficiently close upper boundaries;
- Experiments with bremsstrahlung and other photons with a continuous spectrum shape and solution of the corresponding transformation problem, for which a large number of methods are used:
  - The inverse matrix method;
  - The Penfold-Leiss method;
  - The regularization method, etc.;
- Use of beams of quasi-monoenergetic radiation, whose spectrum is close to Gaussian or Lorentzian in shape with a width of 250-450 keV:
  - In-flight annihilation of relativistic positrons;
  - Compton scattering of bremsstrahlung;
  - Radiative capture reactions;
  - Use of labelled photons of gamma bremsstrahlung.

The instrumental functions of several methods for generating an effective photon spectrum close to a monoenergetic spectrum are shown in Fig. 1.

Comparison of the instrumental functions shows the complexities that arise when a combined analysis is made of data from different experiments since they turn out to have been obtained under the widely differing conditions of these experiments.

The most widely known [1] systematic differences are those in photonuclear reaction cross-sections obtained for beams of bremsstrahlung and quasi-monoenergetic radiation which occurs during annihilation of relativistic positrons. The absolute majority of photonuclear data have been obtained by these two methods, each accounting for approximately half the data. The main difference is that bremsstrahlung cross-sections have a much more pronounced structure and generally a larger absolute value than the quasi-monoenergetic radiation cross-sections.

The most serious difference - since it is directly linked to such a fundamental physical problem as the existence of an intermediate and fine giant resonance structure - is the difference in the shape of the cross-sections obtained not only in experiments of different types, but often in those of the same type.

Figure 2 illustrates one of the clearest and most obvious examples of such discrepancies discussed in the detailed comparison of the results of bremsstrahlung [4] and quasi-monoenergetic radiation [5] experiments.

In their joint work [4] on the measurement of cross-sections for the $^{18}O((\gamma,n) + 2(\gamma,2n))$ reaction, the Melbourne and Livermore groups essentially investigated whether it was possible in bremsstrahlung experiments to obtain a result that would agree with the results of quasi-monoenergetic radiation measurements. In order to treat the bremsstrahlung yield curve, they employed a method of analysis in which a free parameter could be used to give the relationship between the analysis step and the statistical uncertainty, this parameter

being so chosen that one could speak about "very good agreement" between the results of different methods.

Although in Ref. [4] it is stated that there is very good agreement between the results of quasi-monoenergetic radiation and bremsstrahlung experiments, analysis of the data in that paper leads to the obvious main conclusion that the quasi-monoenergetic radiation cross-section is substantially smoother than the bremsstrahlung cross-section [1]:

- With all the assumptions of the procedure for determining resonance widths in a cross-section with a complex structure, the resonances in the quasi-monoenergetic radiation (QMR) cross-section have larger widths than the resonances in the bremsstrahlung (BS) cross-section: $<\Gamma_{QMR}>/<\Gamma_{BS}> = 1.25$, the number of resonances that appear reliably in the quasi-monoenergetic radiation cross-section being less than in the bremsstrahlung cross-section;
- Practically all the resonances in the quasi-monoenergetic radiation cross-section have, on average, smaller amplitudes than the resonances in the bremsstrahlung cross-section: $<A_{BS}>/<A_{QMR}> = 1.17$, the integral cross-sections in the 8-28 MeV range being also somewhat different: BS - 187 MeV · mb and QMR - 177 MeV · mb.

On the basis of a large number of data for nuclei from ^{6}Li to ^{238}U obtained in various experiments, an analysis was made of the systematics of the different characteristics of (γ,xn) reaction cross-sections. These clearly illustrate the above-mentioned discrepancies in the results, and refine and supplement the data published earlier in Ref. [1].

From the systematics of the values of the special structure parameter S, which describes the degree of deviation of a particular cross-section from a strongly smoothed one,

it was established that all the data can be divided fairly distinctly into two groups on the basis of this parameter: quasi-monoenergetic cross-sections (average value for an arbitrarily chosen normalization - 1.22) and bremsstrahlung cross-sections (4.35). It is found that, with respect to this parameter S, the data obtained in Illinois using labelled photons are (4.22) substantially closer to the bremsstrahlung than to the quasi-monoenergetic radiation data.

We have shown [1] that by making appropriate and consistent allowance for the instrumental functions of the specific experiments, we can bring their results substantially closer in shape: the corresponding resonances of cross-sections obtained in the different experiments became closer both in width and in amplitude.

Discrepancies caused by normalization and calibration errors

Although the differences between the data were reduced after allowance had been made for the instrumental functions of the experiments, i.e. the effective photon spectra, the discrepancies in the absolute value of the results were found to be much greater than those caused by the differences in these functions.

The systematic differences, not only in shape, but also in absolute value, between the photonuclear reaction cross-sections studied in the different experiments are also well known [1]: a large number of quasi-monoenergetic radiation cross-sections, obtained primarily at Livermore, have a smaller absolute value in comparison not only with the bremsstrahlung cross-sections, but also with the quasi-monoenergetic radiation cross-sections obtained in other laboratories [1].

Thus, for the same large number of nuclei, an analysis was made of the ratio of the integral cross-sections for the (γ,xn) reactions obtained in various laboratories to the values obtained in one of them (specifically at Livermore). From the systematics of these ratios $< R >$ it was found that with respect to the absolute value of the cross-sections all the data

can clearly be divided into two groups, one comprising the Livermore data (average value of $< R >$ close to 1) and the other all the rest ($< R > = 1.122 \pm 0.243$), although a considerable scatter is observed within the groups themselves. This scatter indicates that there are appreciable errors in the normalization of the cross-sections in the different experiments.

The possible sources of such errors can be named quite easily. Thus, the reasons for the differences in the absolute value of the cross-sections are:

- The use of neutron detectors with different efficiencies (generally known with an uncertainty of not more than 10%);
- Different methods of determining the photon dose (the measurement of which using, for example, a thin-walled ionization chamber has an uncertainty of about 10%);
- Allowance for photon self-absorption in the target material (uncertainty of only a few per cent);
- Determination of the number of nuclei in the target (uncertainty not above 1%);
- Use of different normalizations (cross-sections of reactions used as a monitor are known with an uncertainty of 5-15%);
- Evaluation of the contributions of various backgrounds;
- Introduction of various corrections and so on.

It should be emphasized that the relevant results of the different experiments differ [1] (sometimes quite substantially) from each other not only in shape and absolute value, but also in the energy position of the structural characteristics observed (and also of the cross-sections as a whole). Moreover, the differences in the energy positions of the resonances and

structural characteristics in the cross-sections depend on the energy of the incident photons - sometimes some cross-section is shifted fully to one or the other side with respect to the other cross-sections, often differently in the region of lower and higher energies; sometimes it is compressed or extended (again differently in different sectors). This can be seen clearly from the data in Fig. 1.

From the systematics of differences between the most probable values of the specific cross-sections for the (γ,xn) reactions of nuclei and the corresponding average values $<E^{mp}>$ it has been found that the most probable energy values for the cross-sections obtained in different experiments (including experiments of the same type) generally do not agree with each other and are shifted with respect to each other by as much as $\Delta E^{mp} \sim 200$ keV (standard deviation $\pm$ 83.6 keV).

Such discrepancies are observed between results of experiments not only of different types, but also of the same type. Thus, E. Fuller's analysis [6] of the data on photoneutron quasi-monoenergetic radiation cross-sections for the ^{12}C, ^{14}N and ^{16}O nuclei, which takes the fullest account of the experimental details and possible systematic uncertainties, has also shown, in particular, that the ratios of the integral cross-sections, as well as the values of total integrals of such cross-sections, differ strongly between the different studies. Table 1 gives the data for the high-energy (HE, E_g = 21-28 MeV) and low-energy (LE, E_g = 18-21 MeV) sectors of the photoneutron cross-sections for the ^{16}O nucleus obtained in Ref. [6] after normalization of all the cross-sections to the same value of the integral cross-section up to a photon energy of 30 MeV (57.6 MeV · mb).

Since the threshold of the $^{16}O(\gamma,2n)$ reaction is 28.9 MeV, the reaction we are talking about in virtually the whole of the incident photon energy region discussed here and below is $(\gamma,xn) = (\gamma,sn) = (\gamma,1n) = (\gamma,n) + (\gamma,np)$.

These data illustrate the presence not only of systematic uncertainties of the constant multiplier type in the normalization of cross-sections, but also of energy-dependent errors of normalization. Thus, the latter errors were interpreted [6], in particular, as a consequence of not taking full account of the loss of a part of low-energy neutrons in water targets with a large effective thickness.

Moreover, in Ref. [6] it is concluded that for agreement to be attained in the position of resonances between all the quasi-monoenergetic radiation data on the total photoneutron cross-section the data from the various studies should be shifted in energy to different sides by different magnitudes. Thus, for example, in the case of the ^{16}O nucleus, the cross-sections obtained in various laboratories [7, 8, 10, 11] should as a whole be shifted towards high energies by 120 keV in relation to the cross-section [9] which was chosen as the best, i.e. the one with the least systematic uncertainties.

The data presented here show that there are appreciable errors in the calibration of the energy scale of the cross-sections in the different experiments, for which the following possible causes can be cited:

- Different methods of calibration with respect to the bremsstrahlung beam energy (use of a procedure of calibration with respect to known thresholds or breaks in the reaction yield curves leads to uncertainties of the order of 100-200 keV);
- Systematic differences in determining the energy scale arising from the use of different methods and a different processing step to solve the inverse problem of determining the reaction cross-section from the experimental value in bremsstrahlung experiments (the accuracy here is half a processing step);

- Systematic differences in determining the energy scale in different quasi-monoenergetic radiation experiments (for example, with respect to positron energy [12] or the most probable energy value [13] of the effective photon spectrum). According to the data in Refs [14, 15] in the region of heavy nuclei (Sn-Au) similar discrepancies for cross-sections obtained at Livermore and Saclay amount to 190-410 keV, etc.

The combined evaluation of several cross-sections obtained in different experiments should obviously take into account that, for the reasons listed above and other similar reasons, the characteristic features of the cross-sections may be shifted in energy with respect to each other by as much as 200-400 keV, and that such shifts may depend on the incident photon energy. If this is not taken into account, the combined evaluation will lead to considerable smoothing of the structural features of cross-sections with a characteristic width smaller than the shift.

This can only mean that information on the structural characteristics of the initial experimental cross-sections will be lost in the evaluated cross-section.

It is also obvious that a combined analysis and evaluation of the photonuclear reaction cross-sections under the conditions described above is meaningful only when the data correspond to a single representation, i.e. if they are converted into the form which they would have if some single instrumental function were used for the experiment and in which the absolute values of both the cross-section and energy scales would coincide (would be close).

In order to make such an analysis and combined evaluation of photonuclear reaction cross-sections in cases where there are large systematic discrepancies between the results of

different experiments, we suggest that the reduction method [1, 16-19] and its modifications be used.

In order to optimize the conditions of the evaluation, which was carried out by a procedure based on the reduction method, we used the data of three systematics of photoneutron cross-section characteristics from the full database of the Photonuclear Experimental Data Centre (Scientific Research Institute for Nuclear Physics, Moscow State University) for integral cross-section, structure parameter and the most probable energy value - which are briefly described above.

EVALUATION OF PHOTONUCLEAR REACTION CROSS-SECTIONS USING THE REDUCTION METHOD

Brief description of the reduction method

From what has been said above, it is clear that in any experiment with incident photons the cross-section of the reaction studied cannot be obtained as a directly measurable quantity, since in no experiment does the instrumental function coincide with the δ-function. The result of each experiment is nothing but the reaction result $Y(e_j)$, i.e. a convolution (1) of the unknown cross-section $\sigma(k)$ with effective photon spectrum $W(E_j,k)$:

$$Y(E_j) = \alpha \int_{E_{threshold}}^{E_j} W(E_j,k)\,\sigma(k)\,dk + \nu_j, \qquad (1)$$

where k is the photon energy and ν_j is the statistical uncertainty (additive noise) described by means of covariance matrix Σ.

Expression (1) can be applied to the results of any experiment if as the kernel of equation $W(E_j,k)$ we choose, not only the Schiff spectrum integrated over angles [20], which is widely used to describe the bremsstrahlung spectrum, but also the corresponding instrumental function of the particular experiment.

It is obvious that in order to unfold the cross-section $\sigma(k)$ from $Y(E_j)$ we need to solve the inverse problem (1) with certain assumptions about the shape of the bremsstrahlung photon spectrum $W(E_j,k)$ or to convert $Y(E_j)$ to the form which it would have if another, more monoenergetic photon spectrum were used in the experiment.

The reduction method [1, 16-19] was used to solve the problem of cross-section interpretation. In essence, this method is as follows.

Expression (1) can also be written in the operator form

$$\xi = Af + \nu, \tag{2}$$

where f is the unknown reaction cross-section, ξ the experimental reaction result, ν the statistical spread (noise) of the result and A the linear integral operator.

In the reduction method the experimental reaction result ξ is transformed into the vector

$$\hat{f} = Uf + \hat{\nu}, \tag{3}$$

which is interpreted as the result of the experiment in which the instrumental function (photon spectrum) W is described by operator U and its properties are given in a special way (for example, the most monochromatic instrumental function possible or a narrower line than that obtained in the specific experimental facility and so on).

Thus, the reduction problem is as follows: from the initial data

ξ - the experimental result,

Σ - the covariance matrix characterizing the uncertainty ν of ξ,

A - the integral operator describing the instrumental function W,

we have to obtain the unknown result:

$\hat{f}$ - the vector describing the reaction cross-section,

G - the covariance matrix describing the uncertainty $\hat{v}$ of vector $\hat{f}$.

In order to solve this problem, we need to find the linear operator R such that vector $f = R\xi$ can be interpreted as the best, in the root-mean-square sense, evaluation for Uf.

It is necessary to introduce the condition of solvability of the problem posed

$$RA = U, \tag{4}$$

since only when this condition is fulfilled the reduction vector

$$\hat{f} = R\xi = Uf + (RA - U)f + Rv \tag{5}$$

does not depend on the unknown solution of f, and the uncertainty of this solution can be evaluated by the covariance matrix Σ of the noise of ξ and by operators A and U.

With this assumption, the magnitude of mathematical expectation $M \| R\nu \|^2$ (where $\| \cdot \|$ is the norm of the operator) of the uncertainty of noise in expression (5) is minimal [16, 17], if

$$R = U(\Sigma^{-1/2} A)^{-} (\Sigma^{-1/2}), \tag{6}$$

where the symbol "-" denotes pseudo-inversion of the operator.

It is operator R which determines the unknown cross-section - vector $f = R\xi$.

Here the covariance matrix G characterizing the uncertainty $\nu = R\nu$ of vector $\hat{f} = R\xi$ is [16, 17] equal to

$$G = R S R^{*}. \tag{7}$$

It is obvious that, while by no means every operator U will satisfy the condition of solvability of the reduction problem (4), not every operator U that can be used to solve the

problem will accord with the experimenter's ideas about the instrumental function of the monochromatic instrument.

In this connection, it is necessary to construct such an operator U which would meet both conditions.

If with the help of operator U_0, we describe the desired instrumental function, for example with a Gaussian shape (with a resolution of, for example, 50 keV as in Fig. 1), the operator U, closest to U_0, can be sought [18] in the form of the product of an arbitrary operator K and operator A (to fulfil the condition of solvability of problem (4)) through minimization with respect to K of the difference between U and U_0 in the sense that

$$\| U - U_0 \|_2^2 = tr((U-U_0)\ (U-U_0)^*). \tag{8}$$

When this minimum is reached, operator U will be determined by the expression

$$U = U_0 A^- A. \tag{9}$$

Consequently, with this approach operator U is found to be dependent only on the experimental operator A and the desired instrumental function U_0.

The operator discrepancy $\| U\text{-}U_0 \|_2$ characterizes the difference between U and U_0.

Here it is very important to note that the reduction method is not a method for transforming, extracting or unfolding the unknown cross-section $\sigma(k)$ from the experimental result $Y(E_j)$. It is a method for converting data obtained with some instrumental function A into the form which they would have if the instrumental function had the form U.

Thus, the reduction method does not solve the unstable inverse problem (1). It is used to obtain the most reasonably achievable monoenergetic representation of the reaction cross-section information. With such an approach, the most reasonably achievable monoenergetic representation means that:

- It is impossible to extract data on a reaction cross-section with an arbitrarily high energy resolution from the specific experimental value of the reaction measured with a fully definite uncertainty in an equally definite energy grid;
- If the uncertainties in the result are too large, the best resolution obtained may be rendered unreasonable.

The potential of applying the reduction method to the problems under discussion can be demonstrated by treating a model cross-section formed by two clearly separated resonances with a width of 10 keV at energies of 11.0 and 11.2 MeV (Fig. 3a).

The result of the quasi-monoenergetic radiation experiment was modelled (Fig. 3a (dots)) with a 100 keV step for the typical energy resolution of 330 keV using the instrumental function shown in Fig. 3b (dots). The result of the reduction (Fig. 3a (broken line)) for an instrumental function with a resolution of 150 keV (Fig. 3b (broken line)) in Fig. 3a is also compared with the accurate model cross-section $U_0\sigma$.

Thus, the reduction method provides a mechanism for eliminating the difference in the data caused by different instrumental functions - a difference that is the most difficult to eliminate. The reduction of the cross-sections obtained in the various experiments to a single representation (single instrumental function) makes it possible in principle to perform a combined analysis and unified evaluation of the cross-sections.

Allowance for calibration and normalization errors

It has already been pointed out that there are two more types of discrepancy which are caused by the systematic errors of calibration and normalization. Firstly, it is known that these are smooth functions of energy and, secondly, the boundaries within which they lie are known.

It was also noted that in view of the observed shift in the characteristic features of comparable cross-sections with respect to each other (the shift being different for different sectors of the cross-sections) by up to 200-400 keV, an additional special correction had to be made to the energy scale to supplement the necessary normalization correction in order to make a combined evaluation that does not lead to appreciable smoothing of the structural characteristics of the cross-section and consequently to a loss of information in the evaluated cross-section on the structural characteristics of the initial cross-sections. It is evident that because of the above-mentioned energy dependences of both the positions of the cross-section characteristics and the normalization error, the required correction should provide for the possibility of introducing minor corrections to various small sectors of the cross-sections. These corrections should bring closer the unambiguously identified cross-section characteristics, but in such a way that no noticeable (on the basis of data on the systematics of the most probable energy values for the analysed cross-sections, not greater than ~ 200 keV) shift occurs in the most probable energy values for the analysed cross-sections.

In the first stage it is desirable to reduce as much as possible the influence of statistical uncertainties on the procedure for determining the values and directions of the corresponding corrections in different sectors of the cross-sections. For this purpose, it is perfectly natural to reduce the data to a very low energy resolution, i.e. in our approach, to the instrumental functions close to each other such that the statistical uncertainties become sufficiently small.

After reducing the data to the same resolution which is so low that the statistical uncertainties become negligibly small and cannot by any means be used to explain the discrepancies between the vectors, it becomes possible to compare the different plots. This is a problem of pattern recognition (identification). Specialized programs using the reduction

method and its implementation [16-18] and information on possible systematic uncertainties were developed in order to resolve this problem.

Here it is necessary to proceed from the fact that there are two types of vector describing the energy dependence of the cross-section.

1. The first occurs in the case where it is easy for the experimenter to identify the shape of the curve and to "guess" what sectors of the different curves correspond to each other. A typical example of this situation is the cross-sections for oxygen [9, 21] containing several resonances which are easy to "recognize".

In this case, it is possible to set ("manual" procedure) the boundaries of the corresponding sectors and to fit the parameters a_i (constant), b_i (correction to the cross-section) and c_i (correction to the energy) of the transform:

$$a_i = (1 + b_i)\sigma(E + c_i), \tag{10}$$

where i varies from 2 to M (M is the number of cross-sections), and $a_i = b_i = c_i = 0$, so that the sum of squares of the deviations of each curve from one selected arbitrarily and denoted by 1 is minimized. Then, the weighted mean values of each of the parameters selected are determined and these means are subtracted from the corresponding parameters found ($\hat{a}_i = a_i - < a >$, $b_i = b_i - < b >$ and $c_i = c_i - < c >$).

Thus are determined the corrections (corresponding to energies c_i) which should be applied to the curves to make them coincide (approach each other).

2. In the second case the plots have only one or two wide maxima, as for example in Refs [22, 23] for praseodymium cross-sections. Here, as in the first case, it becomes difficult to take decisions about identification. The following "automated" procedure was developed in order to combine the plots in the second case.

It was assumed that the transforms (corrections) could take the form

$$(1 + b_i)\sigma(E + c_i). \qquad (11)$$

For energy E_i - 200 keV $< E < E_i$ + 200 keV each graph was approximated by a straight line

$$\sigma(E) = (1 + b)\sigma(E_i) + (E_i)c. \qquad (12)$$

From the condition that the sum of squares of the deviations between each two cross-sections should be minimal, we constructed a linear scheme of $M \cdot n$ equations for $2M \cdot n$ unknowns, where M is the number of cross-sections and n the number of points in each cross-section.

It should be noted in particular that the procedure in both approaches can equally easily cope with one of the most complicated problems encountered in the evaluation of nuclear data - different ranges for different curves (in our example, one of the curves [21] finishes earlier than the other [9]). Usually, in such a situation different kinds of curve "matching" are employed [6], and this generally introduces into the evaluation process a factor that is subjective and rather more speculative than one based on physical considerations. In our approach this problem is resolved automatically.

Another point to note is that this procedure enables the fullest use to be made not only of quasi-monoenergetic radiation data but also of bremsstrahlung data in the evaluation.

Figure 4 shows five quasi-monoenergetic radiation and two bremsstrahlung cross-sections reduced to an energy resolution of 600 keV for the (γ,xn) reaction for the ^{16}O nucleus before (a) and after (b) treatment by the above evaluation method (the statistical uncertainties, the largest of which does not exceed 0.1 mb, fully fit into the symbols of the figure).

It is easy to see that the procedure for taking into account calibration and normalization errors helped us to obtain a substantially better agreement between the quasi-monoenergetic radiation and bremsstrahlung data. The positions of the well-known peak at 17.42 MeV coincided fully for all quasi-monoenergetic radiation and bremsstrahlung data, although for the 22.2 MeV peak there were still some discrepancies - in the bremsstrahlung data it continued to show a slight shift towards higher energies.

RESULTS OF THE TREATMENT

The approach described above was used for the evaluation of the photoneutron reaction cross-sections for a number of nuclei. We illustrated it in detail using, as an example, the evaluation of data for the $^{16}O(\gamma,xn)$ reaction cross-section.

The corrections obtained in the procedure for taking account of calibration and normalization errors in all the data for this cross-section were applied to quasi-monoenergetic radiation data and then all the five corrected quasi-monoenergetic radiation results were included simultaneously in the reduction procedure without trial information. The result of the reduction to the 240 keV resolution - the final evaluation of the photoneutron reaction cross-section for oxygen for this resolution - is shown in Fig. 5b, while Fig. 5a gives, for the purposes of comparison, all the published experimental cross-sections.

It should be noted that the results of our evaluation of the photoneutron reaction cross-section for the ^{16}O nucleus (Table 2) using specific data on three fully determined types of systematic error (errors in the instrumental function, normalization and calibration), together with the mathematical apparatus of pattern recognition, agree on the whole with the results of E. Fuller's analysis [6] based on the most general ideas about the causes of discrepancies in the data obtained from different quasi-monoenergetic radiation experiments and enable a more correct account to be taken of the characteristics of each experimental cross-section.

Table 2 (our results) should be compared with Table 1(E. Fuller's data [6]). It will be seen that E. Fuller's use of only a single normalization coefficient for each initial cross-section and making allowance for calibration differences merely by shifting the data of the four studies discussed towards higher energies by 120 keV in relation to the data in Ref. [9] did not make it possible, from the standpoint of the best agreement of the data, to bring the individual cross-sections sufficiently close, since the cross-section parameter ranges remained fairly large: LE = 2.44-4.19 MeV · mb; HE = 44.8-46.7 MeV · mb, HE/LE = 10.9-19.1. This scatter is perfectly obvious since, as noted earlier, the normalization and calibration differences are energy dependent and cannot be eliminated by means of a procedure in which such rough allowance is made for this non-linearity.

It was for this reason that Fuller [6] could not make an evaluation of all the cross-section data and was compelled to confine himself to selecting, on the basis of some physical considerations, one of the initial cross-sections, namely the one from Ref. [9], as the best cross-section.

The above method, which takes into account a smooth energy dependence of the corrections to the cross-section and the energy itself, gives a much better agreement of the parameters: LE = 3.24-3.67 MeV · mb; HE = 39.64-41.77 MeV · mb; HE/LE = 11.35-12.81, and thereby an evaluated cross-section with a sufficiently high accuracy.

At the same time, it should be noted that, with respect to these parameters, the result of our evaluation is the closest to the Livermore cross-section [9], which was chosen as the best by E. Fuller on the basis of his analysis [6].

It should be stressed in particular that the evaluated cross-section was obtained not by a simple averaging or weighting of the initial cross-sections, but by applying the above method, which takes into account the dependences on incident photon energy not only of

calibration and normalization errors, but also of the instrumental function of each specific experiment. The parameters of the evaluated cross-section correspond to the shape of the evaluated cross-section obtained in the form of the most reasonably achievable monoenergetic representation of the whole set of initial data.

Because of the relatively small uncertainties achieved for the energy resolution of 240 keV, the structural characteristics which appear quite clearly in the evaluated cross-section can be identified reliably. Their parameters are given in the left-hand part of Table 3.

The initial data for the total photoneutron cross-section for the $^{208}Pb(\gamma,xn)$ reaction are presented in Fig. 6a and the evaluation of this cross-section carried out by the above method for an energy resolution of 120 keV is shown in Fig. 6b. For the purposes of comparison, this figure also shows the result of the experiment with a labelled photon beam [24], which was not used in the evaluation procedure. We must stress the good agreement, both in shape and in absolute value, of the obtained evaluated cross-section with the result of the labelled photon experiment performed with the same resolution, which demonstrates the efficiency of the proposed method of analysis and evaluation of the results of bremsstrahlung and quasi-monoenergetic radiation experiments.

Because of the levels of uncertainty achieved for the energy resolution of 120 keV, the structural characteristics which appear quite clearly can be identified reliably also in this evaluated cross-section. Their parameters are shown in the right-hand part of Table 3.

Since the threshold of the $^{208}Pb(\gamma,2n)$ reaction is 14.1 MeV, this means that, as in the case of oxygen, we are talking about the reaction $(\gamma,xn) = (\gamma,sn) = (\gamma,1n) = (\gamma,n) + (\gamma,np)$ over practically the whole incident photon energy region under discussion.

Table 4 shows the integral characteristics of the total photoneutron cross-sections for oxygen and lead nuclei evaluated in the present paper.

The evaluated total photoneutron cross-sections were also obtained for the ^{12}C, ^{27}Al, ^{28}Si, ^{40}Ca, $^{63,65}Ci$, ^{90}Zr and ^{141}Pr nuclei.

CONCLUSION

We have proposed a method for the evaluation of photonuclear reaction cross-sections in cases where there are considerable systematic discrepancies, and it is based on the mathematical reduction method, which is a particular case of the pattern recognition theory.

The method allows consistent and appropriate allowance to be made for three types of uncertainty - in the instrumental functions, the energy calibrations of the experiments and the absolute values of the cross-sections - which are energy dependent, making it difficult to use conventional methods for the evaluation of cross-sections obtained under such conditions.

The proposed method gives an evaluated cross-section with an energy resolution limited only by the energy step of the experimental measurements, and enables us to calculate the total covariance matrix of the evaluated cross-section.

The method has been tested on model problems and used to evaluate the total photoneutron cross-sections for a large number of nuclei.

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Table 1

Comparison [6] of the parameters of photoneutron cross-sections for the ^{16}O nucleus obtained in different quasi-monoenergetic radiation experiments

Ref.	Normalization factor	Integral cross-section LE (MeV · mb)	Integral cross-section HE(MeV · mb)	HE/LE ratio
[7]	0,98±00,04	2,70	46,1	17,1±2,8
[8]	1,06±00,02	3,58	45,8	12,8±3,8
[9]	1,21±00,03	3,88	44,8	11,5±0,8
[10]	1,27±00,05	4,19	45,5	10,9±1,2
[11]	1,22±00,06	2,44	46,7	19,1±5,1

Table 2

Data on the integral photoneutron reaction cross-sections for the ^{16}O nucleus for each of the five quasi-monoenergetic cross-sections after treatment by the method described and for the cross-section evaluated taking into account all five initial cross-sections

Ref.	Integral cross-section LE(MeV · mb)	Integral cross-section HE(MeV · mb)	HE/LE ratio
[7]	3,24±0,12	41,56±0,13	12,81±0,46
[8]	3,67±0,23	40,53±0,58	12,05±0,85
[9]	3,38±0,06	41,77±0,04	12,38±0,22
[10]	3,49±0,28	39,64±0,44	11,35±0,92
[11]	3,30±0,17	40,32±0,19	12,21±0,63
Evaluation	3,83±0,93	41,73±0,08	10,88±0,27

Table 3

Energies and cross-sections at the maximum and their uncertainties for resonances observed in the evaluated cross-sections for the $^{16}O(\gamma,xn)$ $^{208}Pb(\gamma,xn)$ reactions

$^{16}O(\gamma,xn)$			$^{208}Pb(\gamma,xn)$		
Energy (MeV)	Cross-section (mb)	Uncertainty (mb)	Energy (MeV)	Cross-section (mb)	Uncertainty (mb)
16,14	0,347	0,138	7,70	41,0	10,0
16,50	0,163	0,132	8,03	41,5	11,6
17,22	2,925	0,161	8,42	51,1	12,7
19,14	2,003	0,217	8,79	43,0	13,1
19,44	2,078	0,178	9,06	76,3	15,4
20,04	0,863	0,285	9,39	90,8	7,5
20,52	1,632	0,225	9,99	158,2	13,4
20,86	3,240	0,130	10,53	211,5	15,6
22,20	10,81	0,084	10,83	226,9	15,0
22,98	7,512	0,069	11,31	354,8	8,3
23,52	6,719	0,089	11,64	323,9	7,4
23,94	8,781	0,110	12,21	454,9	19,9
24,12	9,658	0,108	12,48	480,3	17,1
24,72	6,639	0,115	13,32	641,0	19,0
25,02	7,487	0,120	13,71	666,6	12,8
25,50	6,882	0,127			
25,86	5,614	0,132			
26,40	5,460	0,138			
27,24	4,540	0,159			
27,66	4,420	0,165			

Table 4

Integral cross-section (MeV, mb)

Energy region (MeV)	$^{16}O(\gamma,xn)$	$^{208}Pb(\gamma,xn)$	Energy region (MeV)
15,91-18,00	1,45±0,30	45,98±15,44	7,50- 9,00
18,00-19,86	2,10±0,38	239,40±23,60	9,00-11,00
19,86-22,68	11,68±0,38	783,53±23,22	11,00-13,00
22,68-27,82	30,34±0,61	1027,62±22,76	13,00-14,82
15,91-27,82	45,56±1,67	2096,53±85,02	7,50-14,82

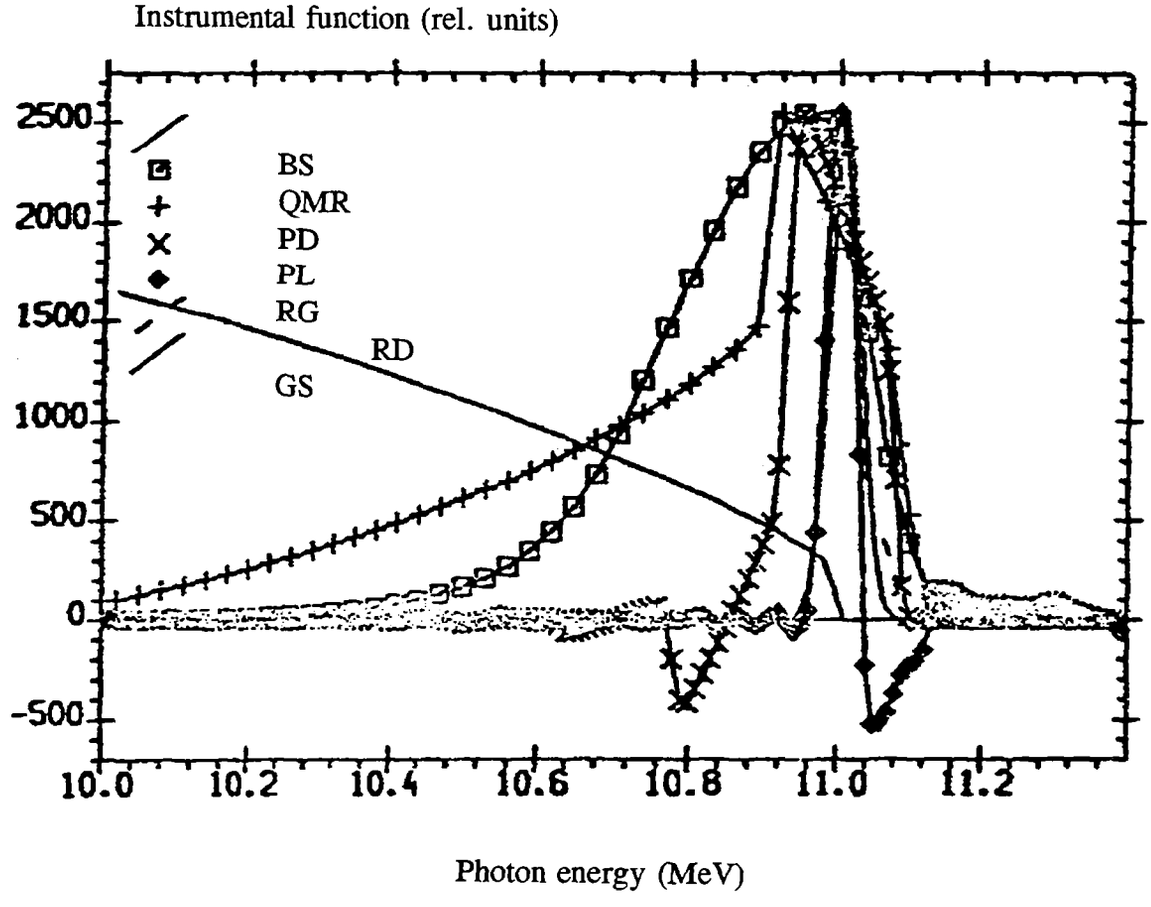


Fig. 1. Instrumental functions of different experiments (BS-bremsstrahlung; QMR - quasi-monoenergetic radiation) and different methods of interpreting the results of experiments with bremsstrahlung beams (PD - photon difference method; PL - Penfold-Leiss method with a 150 keV analysis step; RG - regularization method; RD - reduction method (50 keV energy resolution)).

For comparison the Gaussian (GS) with a width of 50 keV is also given.

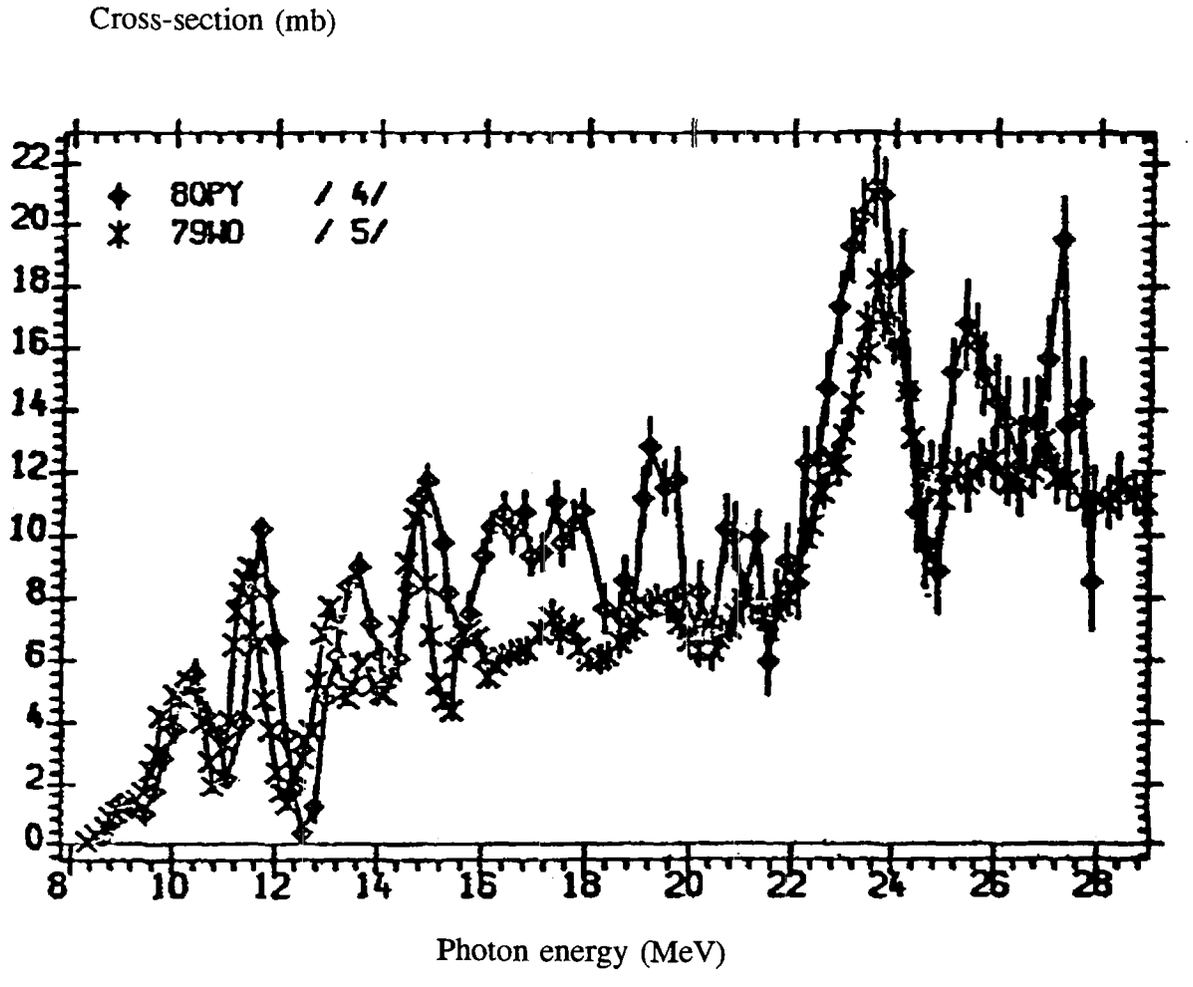
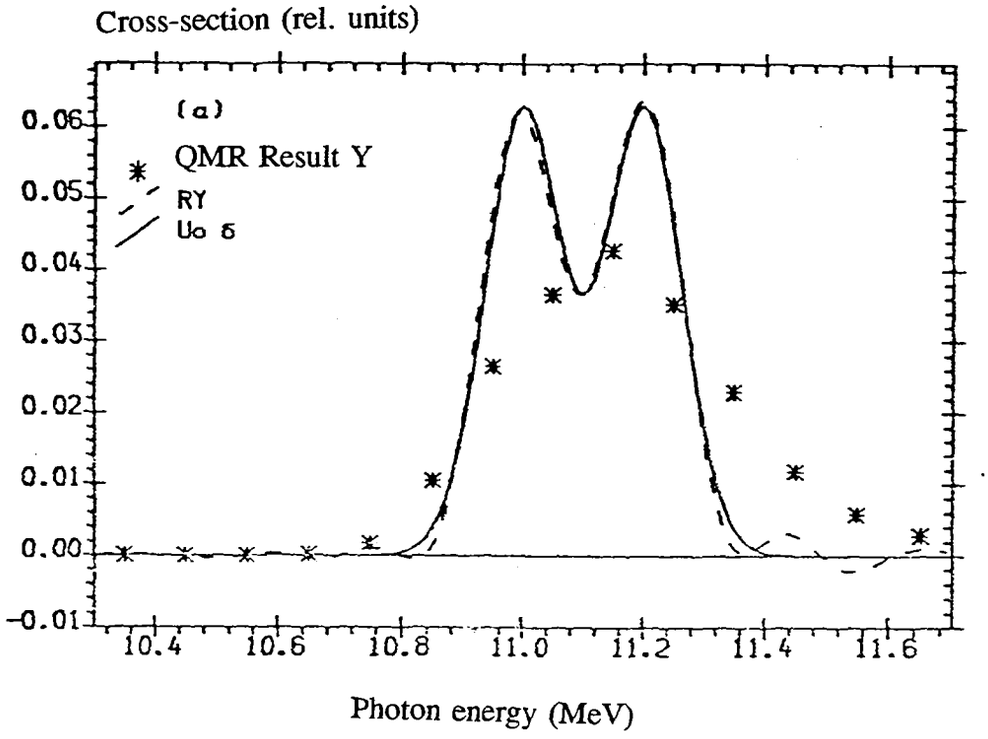


Fig. 2. Comparison of the results of experiments for the $^{18}O((\gamma,n)+2(\gamma,2n))$ reaction using bremsstrahlung [4] and quasi-monoenergetic photons [5].



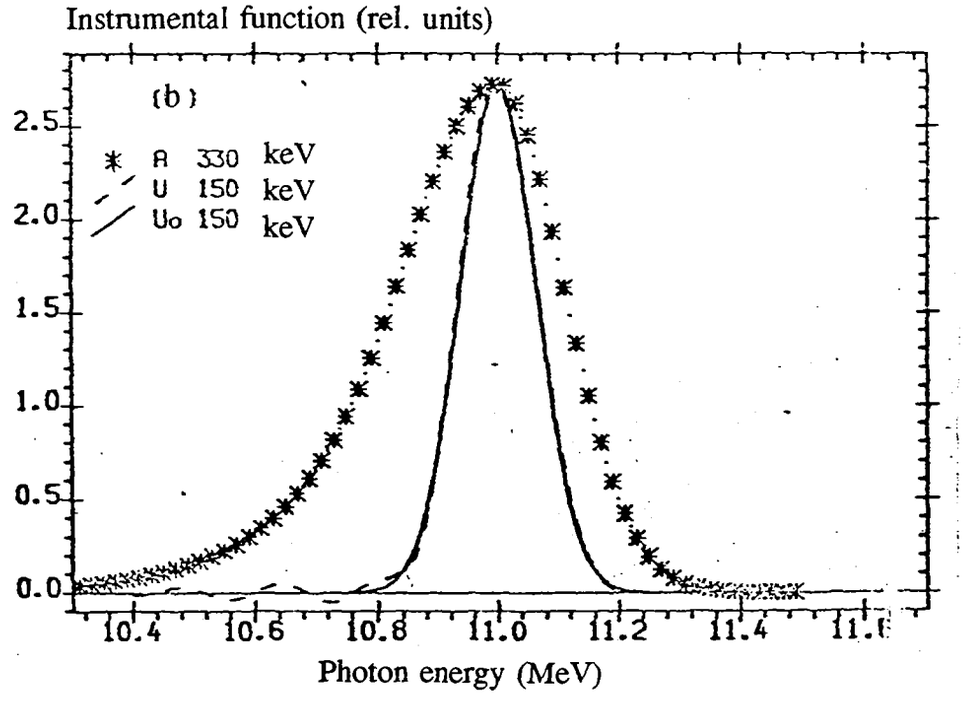


Fig. 3. Possibilities of the reduction method:

(a) Cross-sections ($U_o\sigma$ - accurate cross-section with a resolution of 150 keV (solid line); model of the result of quasi-monoenergetic radiation (QMR) experiment with a resolution of 150 keV and of the corresponding instrumental function from Fig. 3b (dots); result of reduction with a resolution of 150 keV and the corresponding instrumental function (broken line);

(b) Instrumental functions (QMR experiments (dots); desired instrumental function - Gaussian U_o (solid line); constructed instrumental function U (broken line)).

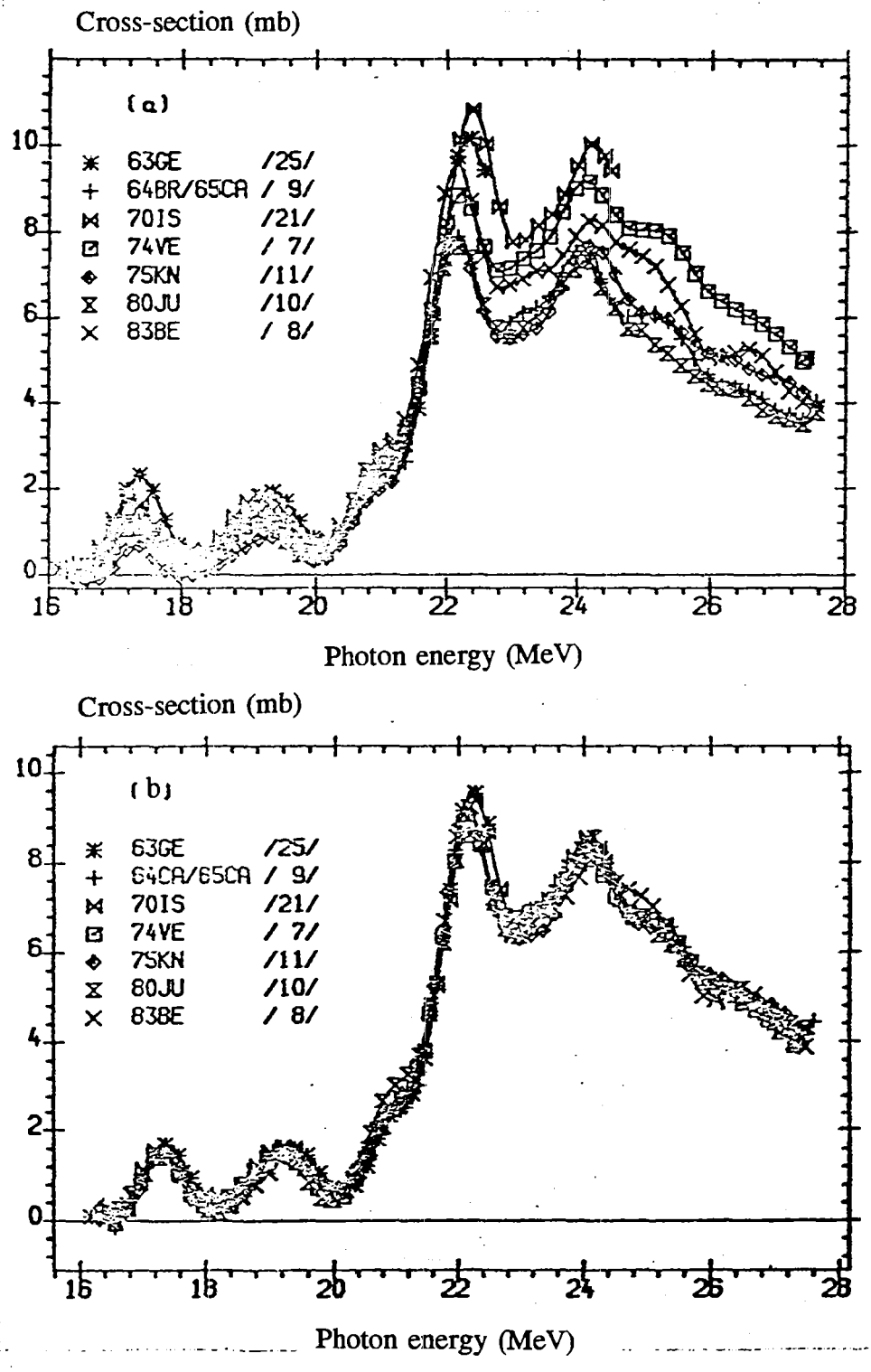


Fig. 4. Results of treatment of the data on photoneutron reaction cross-sections for the ^{16}O nucleus: (a) Five quasi-monoenergetic and two bremsstrahlung cross-sections after reduction to a single resolution; (b) the results of the same seven studies after implementing the procedure for taking account of calibration and normalization errors.

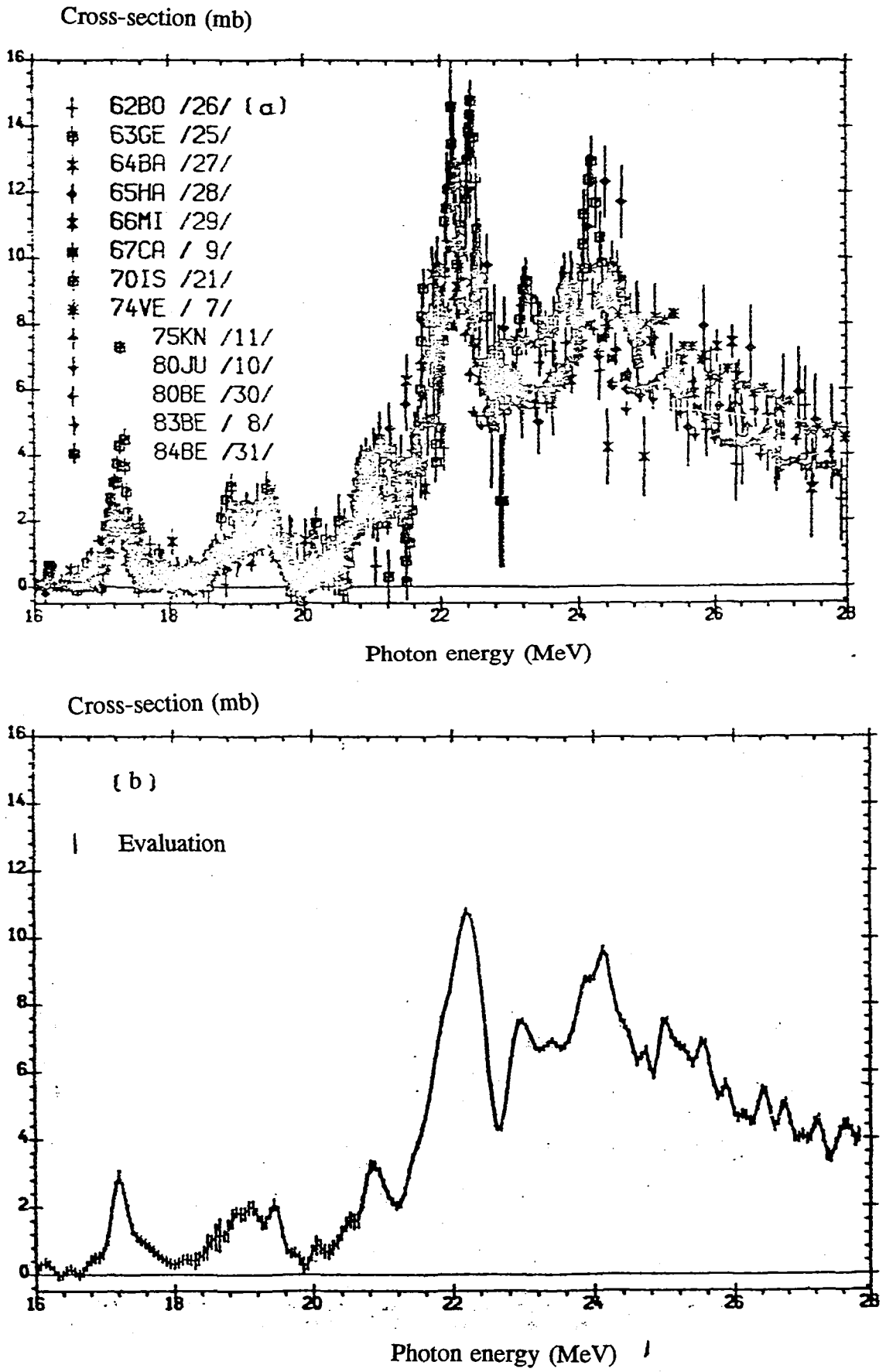


Fig. 5. Comparison of the whole set of initial experimental photonuclear reaction cross-sections for the ^{16}O nucleus (a) with the result of evaluation (b) by the method described for an energy resolution of 240 keV.

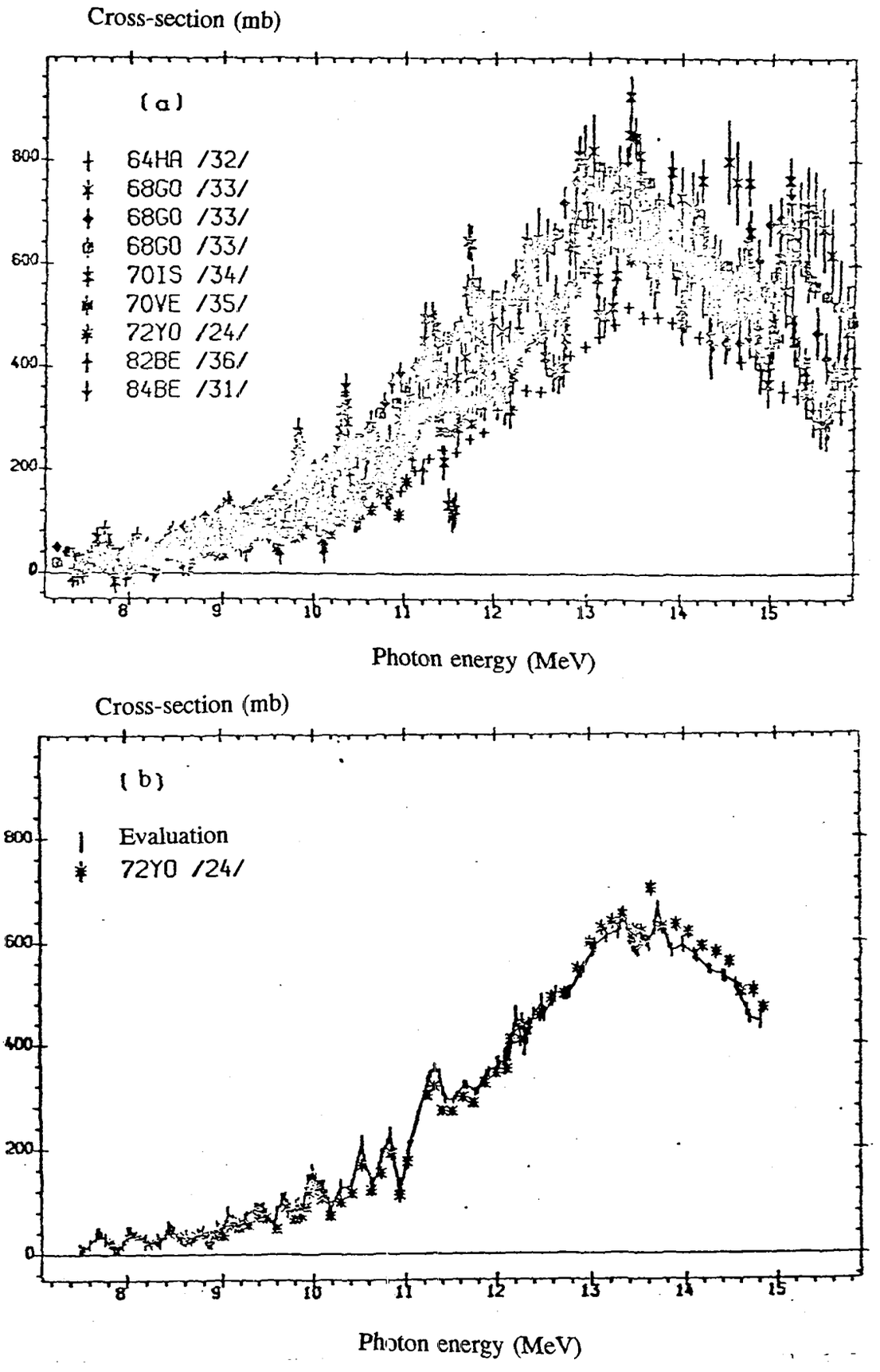


Fig. 6. Comparison of the whole set of initial experimental photoneutron reaction cross-sections for the ^{208}Pb nucleus (a) with the results of evaluation (b) using the method described for an energy resolution of 120 keV. Data obtained using labelled photons [24] are also given for comparison.

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