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Measurements of Critical Parameters of ^{239}Pu and ^{235}U Spherical Assemblies, which contain Nickel as a Reflector and a Filler of the Central Cavity, for the Purpose of Nuclear Data Testing

Final Report of Research Contract 10079

M.I. Kuvshinov, V.P. Gorelov, V.P. Egorov, V.I. Il’yin

Russian Federal Nuclear Center - VNIIEF, Division 04
607190, Mira 37, Sarov (Arzamas-16)
Nizhni Novgorod Region, Russia

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Nizhni Novgorod region, Russian Federation

Annotation

In the report proposed there is presented a description of three test assemblies with their critical spherical models. The assemblies contain nickel layers in reflector and in a central cavity. The cores consist of metal ^{239}Pu (δ,98%), ^{235}U (90%) and ^{235}U(36%). There are mentioned the conditions of measurement performance, the ways of reactivity measurements and calculation techniques used.

There are carefully appreciated the total errors of determination of $\mathbf{k_{eff}}$ values for spherical models in the critical condition, which are less than 0.0033. The experimental components of these errors are less than 0.0026. For the spherical models of the assemblies under consideration before bringing into a critical condition the total errors of $\mathbf{k_{eff}}$ values are less than 0.0025, and their experimental components are less than 0.0014.

The results presented can be used for adjusting neutron constants in the area of fast neutron energies including nickel constants.

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INTRODUCTION

The results of benchmark measurements of critical parameters for various assemblies produced from fission materials are of the utmost significance for increasing the reliability of calculations under the solution of nuclear power applied problems. This is due to the fact, that such results allow to judge the quality of neutron constants used at the calculations and if necessary can be used for adjustment of these constants.

The results of reference measurements of critical parameters of various fission materials assemblies are given in the International Handbook /1/. It is necessary to supplement similar data by new results.

Data presented in the work will serve this purpose. There will be given measurement results of the assemblies which shared the existence of nickel layers in a reflector and in a central cavity. The assembly cores consisted of metallic ^{239}Pu (98%) in δ-phase, ^{235}U (90%) and ^{235}U (36%), where between the brackets the approximate content of the nuclei mentioned in percentage of mass is given.

The work has been carried out on the contract No. 10079 with the IAEA.

1. GENERAL PROPOSITIONS

1.1. BRIEF DESCRIPTION OF THE VNIIEF TEST BENCH CRITICAL ASSEMBLIES

The measurements of the characteristics of fission material assemblies were carried out on the VNIIEF test bench critical assemblies (TBCA). The detailed description of the TBCA is given in the works /2, 3/. Here we shall restrict ourselves to brief information. When mounted, the assembly was divided into two separate units, each was subcritical. The lower (movable) unit laid freely on a truncated-cone-shaped thin-walled steel support (upper support) which based on a pedestal (see the figure for each assembly). The upper (fixed) unit was supported by a thin annular diaphragm of steel. This diaphragm was laid freely on the lower support which also based on a pedestal. Most of the mass of the assembly core was concentrated in the lower unit. Each of the units was assembled manually.

Using the TBCA gear and remote control, the lower unit was raised toward the upper unit to make a near-critical system. Basically, the TBCA gear was built around the electromechanical hoist that made the vertical motion smooth. The TBCA was provided with a safety system that would disconnect the hoisting magnet in case an emergency signal occurred, so that the movable unit would fall by its own weight to its lowest possible position.

The hoisting gear for the lower assembly unit had remotely controlled mechanical stops to limit the height of the unit and to specify its height very accurately. The displacement velocities of the lower assembly unit were 0.001 cm/s, 0.01 cm/s, and 0.1 cm/s, depending on the reactivity level of the assembly.

1.2. DESCRIPTION OF THE ROOM FOR MEASUREMENTS

The measurements for all assemblies were carried out in the same experimental room. The inner dimensions of the experimental room were: 12 m length, 10 m width and 8 m height. The material for the walls, floor, and ceiling was reinforced concrete.

The room contained a bridge crane ≈ 6 m above the floor. The bridge crane resembled a steel beam with effective dimensions of $2 \times 200 \times 1000$ cm. During the measurement it was located close to the wall at the distance of 7 m from the assembly centre. There were no other massive steel structures in the room.

The distance between the assembly center and the room walls was 5 and 6 m, and the height above the floor was 1.9 m. A welded detector pillar composed of steel tubes ~2.5 cm in diameter was installed about 1.5 m from the assembly center. The pillar carried 3 standard neutron detectors (McKibben-type counters). The other dimensions of the polyethylene counter units were 39-cm diameter and 59-cm length. To produce emergency signals, the TBCA was additionally equipped with two measurement channels based on ^{239}Pu fission chambers operating in the current mode. No other equipment units or reflectors that could substantially influence the assembly reactivity were present.

1.3. TECHNIQUES OF REACTIVITY MEASUREMENT USED

The reactivity **R** is connected to effective multiplication factor of neutrons $\mathbf{k_{eff}}$ and effective part of delayed neutrons $\beta_{\mathbf{eff}}$ by a known ratio:

$$\boldsymbol{R} = \frac{\boldsymbol{k}_{eff} - \boldsymbol{1}}{\boldsymbol{k}_{eff}\boldsymbol{\beta}_{eff}}.$$

To increase the reliability of the results a few techniques of reactivity measurement were used:

- technique based on the inverted solution of the kinetics equations (ISKE);
- Sjöstrand method (SM);
- method of a "shooting" source (MSS).

More detailed description of these techniques and the appropriate references are given in works /2, 3/.

Choosing the method was done in terms of the range of the reactivity measured. The ISKE technique was used at $|R| < 0.5$. At $|R| > 0.5$ there were used the SM and MSS methods. The errors of reactivity measurement results and its perturbations are given in Sections 2, 3 and 4 of the work presented for each assembly discussed above.

1.4. TEST ASSEMBLY AND ITS SPHERICAL MODEL

The term “test assembly” used in these sections is concerned with real assembly mounted on the TBCA. Test assembly is divided by a gap into two unequal units. In its core the absence of one or several pieces of small mass, so-called pole or side plugs is possible. The upper unit is located on a steel diaphragm which lays on the upper support. The lower unit is movable in vertical directions and lays on the lower support. The upper and lower supports have their own pedestals. With detectable amounts of plutonium in a core the assembly temperature exceeds the room one.

The results of the test assembly reactivity measurements and their perturbations allowed to turn to the test assembly spherical models. These spherical models are isolated from the influence of the upper and lower support, their pedestals, diaphragm, and the walls of the room. Measurement results also allow to take off conventionally the spherical models to the critical state in terms of delayed neutrons (lower critical condition). For the assemblies with the appreciable plutonium content the temperature of the model can be resulted to the room one. A change-over to such models allows to eliminate the problem of adequate model used at calculations and experimental configuration. Using the most precision models one may be sure that the difference between the experimental and $\mathbf{k_{eff}}$ calculated values are wholly caused by measurement errors and the errors of initial neutron constants.

1.5. OBTAINING OF THE CALCULATED CHARACTERISTICS

There were obtained the characteristics of spherical models for all assemblies under consideration. The models were slightly simplified. It was important to determine the shifts $\mathbf{k_{eff}}$ owing to these simplifications in order to take into account these values when calculating the errors. During the shift calculations there were used 26- group neutron constants /4/ corrected according to the data for integral experiments and the S_n - method (S_{16} - approximation) to solve neutron transport equation. The scheme of solution is presented in work /5/. There was used the Kellog procedure of sequential approximations. The solution was performed with a radial grid (0.1 cm step). The methodical error was $\delta\mathbf{k_{eff}} = 10^{-5}$. The same constants and the method was applied when calculating the shifts $\mathbf{k_{eff}}$ because of the changes of various characteristics of the core and reflector (or other assembly parts consisting of nickel) and when calculating the effective part of delayed neutrons. By characteristics are meant the size, mass, composition of basic fissionable nucleus and composition of ballast impurities.

In calculating nuclear concentrations we used the Avogadro’s number and the atomic masses recommended in the Introduction of the International Handbook /1/.

To calculate the perturbation of assembly reactivity caused by moving the walls to infinity there was used a method described in work /6/. This way is based on the invariant imbedding method and the Monte-Carlo method.

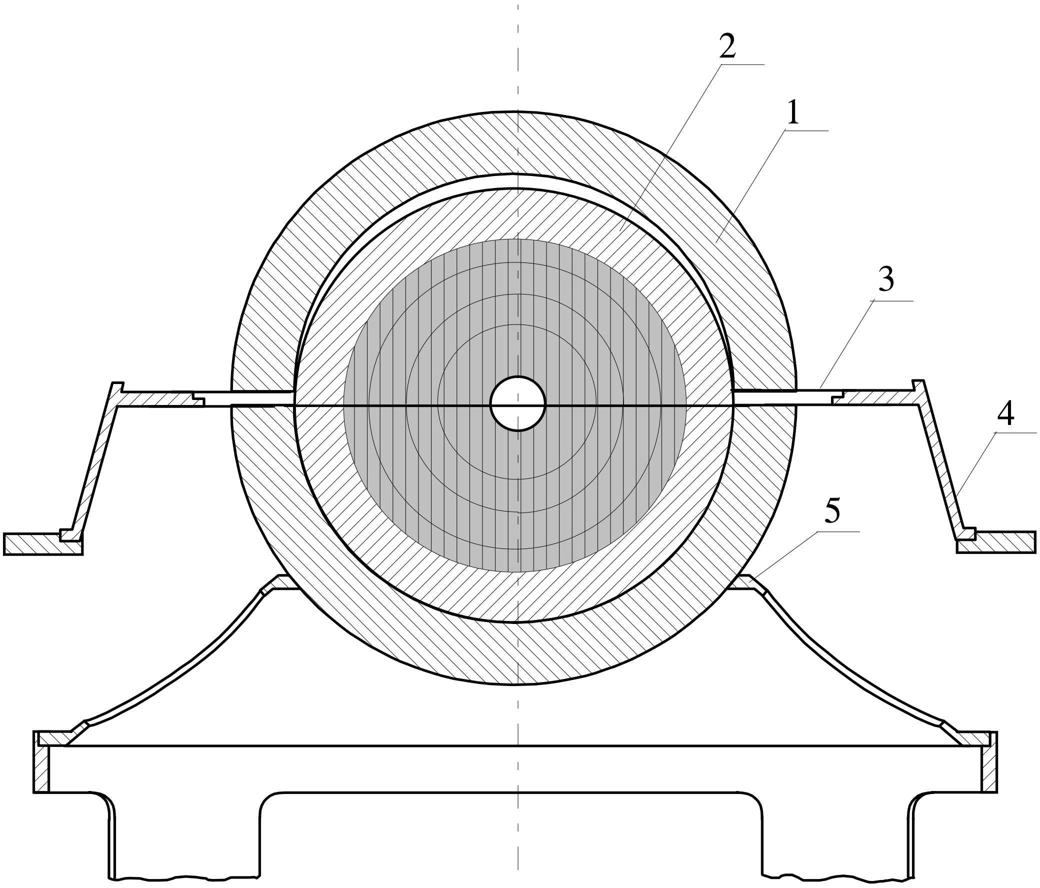
2. ASSEMBLY WITH NICKEL REFLECTOR AND COMPOUND CORE OF ^{239}PU (δ,98%) AND ^{235}U (90%)

2.1. CHARACTERISTICS OF THE TEST ASSEMBLY

2.1.1. Description of the test assembly

Figure 1 is a schematic sectional view of the assembly positioned on the TBCA. The assembly had a central cavity of 1.4-cm radius. The assembly core included 4 spherical layers of metallic ^{239}Pu(δ,98%) and one layer of metallic ^{235}U(90%). The nickel reflector included one spherical layer.

The each layer of ^{239}Pu(δ,98%) was made up of two hemispherical pieces with cylindrical pole holes 2.2-cm in diameter that were plugged during measurements with specialized plugs of the appropriate layer material.



1 - upper assembly unit
2 - lower assembly unit
3 - steel diaphragm
4 - upper support with pedestal
5 - lower support with pedestal

FIG. 1. Sectional view of the assembly on the TBCA.

The layer of ^{235}U(90%) was also made up of two hemispherical pieces with the cylindrical pole holes 2.2-cm in diameter that were plugged during measurements with specialized plugs of this layer material.

The reflector layer was made up of two hemispherical pieces. The upper piece had a cylindrical pole hole 2.2-cm in diameter that was plugged during measurements with specialized plug of this reflector material.

The assembly included two separate units. The upper unit, comprised by one hemispherical piece of nickel, was laid on a steel diaphragm 0.1-cm thick. The other pieces were all included into the lower (movable in vertical direction) unit lifted against the stop in the diaphragm.

2.1.2. Characteristics of a core and reflector of the test assembly

In Table I the sizes and masses of the test assembly layers are given.

TABLE I. SIZES AND MASSES OF THE TEST ASSEMBLY LAYERS

Layer No.	Radius, cm		Mass, g
	inner	outer	
Core, ^{239}Pu(δ, 98%) layers			
1-4	1.400	5.350	9613.4
Core, ^{235}U(90%) layer			
5	5.350	6.000	4747.9
Reflector of Nickel			
6	6.000	9.150	16799

The fission material used in the layers 1-4 of the test assembly was δ-phase plutonium metal. The relation of mass ^{239}Pu to the sum of masses ^{239}Pu and ^{240}Pu average over layers was 98.2%. The percentage of ^{239}Pu isotope in the plutonium parts was known with the relative error of **$(\delta\alpha/\alpha)_{Pu}$ = 0.2%**. The percentage of the ballast impurities was known within the relative error of **$(\delta\varepsilon/\varepsilon)_{Pu}$ = 25%**. The plutonium pieces were all weighed with the relative accuracy of **$(\delta m/m)_{Pu}$ = 0.01%** or better. Uncertainties in the sizes of assembly pieces resulting from temperature measurement errors were added to the relative size error. The total value of the relative size error**,** averaged over 4 plutonium layers, was **$(\delta r/r)_{Pu}$ = 0.06%**. Table II gives a description of the average on 1-4 layers composition of ^{239}Pu(δ,98%).

TABLE II. CHARACTERISTICS OF THE AVERAGE ON 1-4 LAYERS COMPOSITION OF 239Pu (δ,98%)

Composition, Wt. %			
^{239}Pu	^{240}Pu	Heavy (A > 18) impurities	Light (A ≤ 18) impurities
95.23	1.75	2.99	0.03

The fission material used in the layer 5 of the test assembly was uranium metal. The relation of mass ^{235}U to a sum of masses ^{235}U and ^{238}U was 90.6%. The percentage of ^{235}U isotope was known with the relative error of **$(\delta\alpha/\alpha)_U$ = 0.12%**. Impurities were known with the relative error of **$(\delta\varepsilon/\varepsilon)_U$ = 21.5%**. The uranium pieces were all weighed within the relative accuracy of **$(\delta m/m)_U$ = 0.01%** or better. The total value of the relative size error for the uranium layer was **$(\delta r/r)_U$ = 0.04%.** Table III gives a description of the composition of the uranium-layer material.

TABLE III. CHARACTERISTICS OF THE COMPOSITION OF ^{235}U (90%) LAYER

Composition, Wt. %				
^{234}U	^{235}U	^{238}U$^{(a)}$	Heavy (A > 18) impurities	Light (A ≤ 18) impurities
1.05	88.89	9.25	0.76	0.05
(a) The fraction of ^{236}U was added to the ^{238}U fraction, see the evaluation in Section 2.2.3.				

The reflecting material used in the assembly was nickel. In addition to Ni (99.505% by atoms) the reflecting material contained O (0.331% by atoms), Fe (0.115% by atoms) and C (0.049% by atoms). Reflector parts were weighed within the relative accuracy of **$(\delta m/m)_{ref}$ = 0.3%.** The total value of the relative size error for nickel pieces was **$(\delta r/r)_{ref}$ = 0.15%.**

2.1.3. Measurement results of the test assembly reactivity and reactivity perturbations

In Table IV there are presented measurement results of the test assembly reactivity **R_0** and reactivity perturbations **ΔR_n** obtained on the TBCA. There are listed the absolute measurement errors. In Table IV and further the values of reactivity and reactivity perturbations are given in terms of β_{eff} - effective part of delayed neutrons in the assembly under consideration.

TABLE IV. MEASUREMENT RESULTS OF THE TEST ASSEMBLY REACTIVITY AND THE REACTIVITY PERTURBATIONS

Characteristics	Measurement results in terms of β_{eff}
Test assembly reactivity, **R_0**	**- (1.01 ± 0.010)**
Reactivity perturbation caused by removing the upper and lower supports, their pedestals, and diaphragm, **ΔR_{att}**	**- (0.72 ± 0.043)**
Reactivity perturbation conditioned by existing a gap between the upper and lower units without the supports, pedestals and diaphragm, **ΔR_{app}**	**+ (0.49 ± 0.022)**
Correction of reactivity conditioned by reducing the assembly temperature to 20°C, **ΔR_T**	**+ (0.03 ± 0.019)**
Reactivity perturbation caused by moving the walls to infinity, **ΔR_{wall}**	**- (0.02 ± 0.01)**
Reactivity perturbation caused by adding 1 g of plutonium into a central cavity, **ΔR_{cc}**	**+ (5.16 ± 0.155)$\cdot 10^{-2}$**

The value of $\Delta\mathbf{R}_{\mathbf{att}}$ was measured by simultaneous positioning the equivalents of the upper and lower support and their pedestals on the upper unit of the assembly and the second diaphragm under the diaphragm used. To provide nuclear safety in these conditions the lower unit of the assembly was removed downwards at the necessary distance.

The value $\Delta\mathbf{R}_{\mathbf{app}}$ was determined during a series of measurements. At first the lower part was removed downwards at some distance and then it was lifted up to a number of fixed points. At each point the perturbation of lower condition reactivity was measured. Thereafter there was performed the extrapolation of obtained dependence of reactivity perturbation on the distance between the upper and lower assembly units to the contact point. Then there was defined the perturbation value for the assembly with not a gap between the upper and lower units without the supports, pedestals and diaphragm.

The $\Delta\mathbf{R}_{\mathbf{wall}}$ value was defined by calculations.

The $\Delta\mathbf{R}_{\mathbf{cc}}$ value was determined by coating half of a layer with inner radius 1 cm, outer radius 1.4 cm and with composition of a layer 1 in a central cavity. To provide nuclear safety the lower unit of the assembly was removed downwards at the necessary distance.

Assemblies with a core comprised of ^{239}Pu heat up owing to the decay of plutonium nuclei. It leads to the assembly expansion and to reduction of its reactivity. Heating of the assembly comes to some equilibrium temperature $\overline{T}$. The value of $\mathbf{R_0}$ given in Table IV corresponds to this temperature. Temperature correction $\mathbf{R_T}$ was determined in additional measurements. The assembly was broken down into separate parts which were cooled down to the room temperature of 20°C on a special stand. Then the assembly was quickly reassembled at the TBCA, and immediately afterwards the lower and upper units were brought together up to the stop at the diaphragm. Thereafter there were carried out the measurements of reactivity **R** and the temperature **T** depending on time. As a consequence the dependence **R(T)** was obtained. The measurement of temperature was carried out with the help of thermocouples established on the surface of uranium layer. There was no pole plug in the upper unit of the assembly. As soon as the temperature $\overline{T}$ became equilibrium the measurements were finished. Afterwards the meaning of temperature correction was defined as the difference $\Delta\mathbf{R_T} = \mathbf{R}(20^{\circ}\mathbf{C}) - \mathbf{R}(\overline{T})$.

2.2. CHARACTERISTICS OF THE TEST ASSEMBLY SPHERICAL MODEL AT THE LOWER CRITICAL CONDITION

2.2.1. Reactivity of spherical model

Measurement results presented in Table IV allow to define the $\mathbf{R}_{\mathbf{sphere}}$ reactivity of spherical assembly isolated from the influence of the TBCA units (the supports, pedestals and diaphragm) and the walls of the room. The temperature of this assembly was reduced to 20°C. The value of $\mathbf{R}_{\mathbf{sphere}}$ is equal to

$$\mathbf{R}_{\mathbf{sphere}} = \mathbf{R_0} + \Delta\mathbf{R}_{\mathbf{att}} + \Delta\mathbf{R}_{\mathbf{app}} + \Delta\mathbf{R_T} + \Delta\mathbf{R}_{\mathbf{wall}} = -\,\mathbf{1.23}.$$

As one can see, the test assembly spherical model in this case is subcritical.

2.2.2. Extrapolation to the lower critical condition

To remove the spherical model conventionally to the lower critical condition it is necessary to bring into the core an additional quantity of fission material. Additional plutonium mass needed for introduction into the central cavity is equal roughly to

$$\Delta \mathbf{m}_{\mathbf{Pu}} = |\mathbf{R}_{\mathbf{sphere}}|/\Delta \mathbf{R}_{\mathbf{cc}} \approx \mathbf{23.837}\ \boldsymbol{g}.$$

The new radius of a cavity we can calculate by the formula:

$$\bar{r}_{in,1} = [r^{3}_{in,1} - 3\frac{\Delta m_{Pu}}{4\pi\rho_{1}}]^{1/3}.$$

In the this formula $\mathbf{r}_{\mathbf{in,1}}$ = 1.4 cm is the inner radius of a layer 1 of the test assembly, $\rho_1 \approx$ 15.314 g/cm^3 is the density of a this layer. New radius of the cavity is $\bar{r}_{in,1}$ = **1.3337 cm**.

2.2.3. Simplifications of model

There are two groups of impurities in plutonium pieces: one with the relatively low atomic mass numbers (A ≤ 18) and another with the upper-range atomic mass numbers (A > 18). The first group is represented by graphite. The second group represent by nickel. The magnitude of the change in the calculated value of $\boldsymbol{k}_{eff}$ resulting from changing the impurity composition included 17 components by two components is $(\delta \mathbf{k}_{\mathbf{eff}})_{\mathbf{imp,Pu}} \approx$ **0.00023**.

The impurities in the uranium layer are also divided into two groups: one with relatively low atomic mass numbers (A ≤ 18) and another with the upper-range atomic mass numbers (A > 18). The first group is represented by carbon. The second group is represented by nickel. The magnitude of the change in the calculated value of $\boldsymbol{k}_{eff}$ resulting from changing the impurity composition included 15 components by simplified composition is $(\delta \mathbf{k}_{\mathbf{eff}})_{\mathbf{imp,U}} \approx$ **0.00014**.

Isotopes of ^{234}U, ^{235}U, ^{238}U are of prime importance from the standpoint of the weight content and contribution to the critical parameters. The fraction of ^{236}U isotope is small, so it was added to ^{238}U. Calculations show that the magnitude of the change in the calculated value k_{eff} resulted by this action is $(\delta \mathbf{k}_{\mathbf{eff}})_{\mathbf{236}} \approx$ **0.00020**.

A part of the core consisting of plutonium layers was replaced by a homogeneous layer of the same volume and mass. This homogeneous layer contained quantities of particular nuclei equal to the sum of quantities of the same nuclei in initial plutonium layers. It means, that the weight percentages in the homogeneous plutonium core were calculated by the formula:

$$\mathbf{w}_{\mathbf{k}} = \sum_{n} w_{kn} m_{n} / \sum_{n} m_{n}.$$

Here $\mathbf{m}_{\mathbf{n}}$ are the masses of plutonium layers of the spherical model a test assembly in the critical condition, $\mathbf{w}_{\mathbf{kn}}$ - weight percentages in these layers. Calculations show that the magnitude of the change in the value of $\mathbf{k}_{\mathbf{eff}}$ resulted by this action is $(\delta \mathbf{k}_{\mathbf{eff}})_{\mathbf{gom,Pu}} \approx$ **0.00037**.

2.2.4. Characteristics of a core and reflector of spherical model at the lower critical condition

In Table V there are given the sizes and masses of the spherical model layers at the lower critical condition.

TABLE V. SIZES AND MASSES OF THE SPHERICAL MODEL LAYERS AT THE LOWER CRITICAL CONDITION

Layer No.	Radius, cm		Mass m_i, g
	inner	outer	
Core, ^{239}Pu(δ,98%) layer			
1	1.3337	5.350	9637.237
Core, ^{235}U(90%) layer			
2	5.350	6.000	4747.9
Reflector of Nickel			
3	6.000	9.150	16799

The critical mass is equal $\mathbf{m_{cr} = m_1 + m_2 = 14385.137}$ **g**.

The nuclear concentrations in the spherical model layers at the lower critical condition are presented in Table VI.

TABLE VI. NUCLEAR CONCENTRATIONS ((nucl./(barn·cm)) IN THE SPHERICAL MODEL LAYERS AT THE LOWER CRITICAL CONDITION

Core, ^{239}Pu(δ,98%) layer					
Layer No.	^{239}Pu	^{240}Pu	C	Ni	
1	3.6610×10^{-2}	6.6906×10^{-4}	2.8768×10^{-4}	4.6802×10^{-3}	
Core, ^{235}U(90%) layer					
Layer No.	^{234}U	^{235}U	^{238}U	C	Ni
2	4.8710×10^{-4}	4.1060×10^{-2}	4.2188×10^{-3}	4.5197×10^{-4}	1.4059×10^{-3}
Reflector of Nickel					
Layer No.	Ni	O	Fe	C	
3	7.4654×10^{-2}	2.4833×10^{-4}	8.6279×10^{-5}	3.6763×10^{-5}	

2.2.5. Determination of $\mathbf{k_{eff}}$ value error

For the characteristics of the test assembly spherical model presented in Tables V and VI the $\mathbf{k_{eff}}$ value is equal to **1**. It is necessary to calculate the error for these characteristics. It is important not to underestimate these errors. For this reason we shall calculate the errors of quantities the meanings of which depend only on measurement results on the TBCA as the limiting absolute errors (we shall also calculate $\mathbf{\Delta R_{wall}}$ value because of the small contribution). In this case using of the square root of the sum of error squares can cause the reduction of desired error because of neglect of possible correlation of measurement errors.

Let us consider that the operations of weighing, measuring the sizes, determining the percentage of the basic fissile nucleus and determining the percentage of impurity in pieces of various materials are independent of one another, and all of them are independent of a procedure of measuring on the TBCA. Then the total error **k_{eff}** can be calculated by the formula:

$$\delta k_{eff} = [\delta^2 k_{\exp} + \sum_p \delta^2 k_p + \sum_m \delta^2 k_m]^{1/2}. \qquad (1)$$

Here **δk_{exp}** is the component of the total error **k_{eff}** caused by the measurement errors of the test assembly reactivity, reactivity perturbations and by the extrapolation error to the lower critical condition. Sum over **p** is extended for all characteristics of the core and reflector. Their measurement errors are given in Section 2.1.2. Sum over **m** is extended for all model simplifications mentioned in the Section 2.2.3 where the **δk_m** magnitude of the appropriate shifts **k_{eff}** are given. Using these values we shall obtain the meaning of $[\sum_m \delta^2 k_m]^{1/2}$ ≈ **0.00050**. Let us present the separate components of this error: $[\sum_m \delta^2 k_m]^{1/2}_{Pu} \approx 0.00044$, $[\sum_m \delta^2 k_m]^{1/2}_{U} \approx 0.00024$.

The values of **δk_p** are related with **p** characteristics of the core and reflector of the test assembly and their relative errors **$\delta p/p$** by the equality:

$$\delta k_p = p\left|\frac{\partial k_{eff}}{\partial p}\right|(\delta p/p).$$

The $|\partial k_{eff}/\partial p|$ values were defined numerically. In Table VII calculation results of **δk_p** are presented. All designations were interpreted in Section 2.1.2 where the appropriate values of **$\delta p/p$** are given.

TABLE VII. VALUES OF **δk_p**

p→	**m**	**r**	α	ε
Core, ^{239}Pu(δ,98%) layers				
δk_p	0.00008	0.00042	0.00040	0.00015
Core, ^{235}U(90%) layer				
δk_p	0.00005	0.00004	0.00009	0.00015
Reflector of Nickel				
δk_p	0.00043	0.00001	-	-

The value $[\sum_p \delta^2 k_p]^{1/2}$ is about **0.00077**. We shall present the separate components of this error: $[\sum_p \delta^2 k_p]^{1/2}_{Pu} \approx 0.00060$, $[\sum_p \delta^2 k_p]^{1/2}_{U} \approx 0.00018$, $[\sum_p \delta^2 k_p]^{1/2}_{Ni} \approx 0.00043$.

The values of **k_{eff}** and **R** are related by the ratio:

$$k_{eff} = \frac{1}{1 - R\beta_{eff}}.$$

In critical condition at **R** = 0 the limiting absolute error δ$\mathbf{k}_{\mathbf{exp}}$ depends only on error of definition of **R** and β_{eff} value. For calculation δ$\mathbf{k}_{\mathbf{exp}}$ it is possible to receive the following formula

$$\delta k_{exp} = \Delta R_{cc} \beta_{eff} \delta \Delta m_{Pu} \quad .$$

Here δΔ$\mathbf{m}_{\mathbf{Pu}}$ an error of determination Δ$\mathbf{m}_{\mathbf{Pu}}$ because of errors of measurements on TBCA, which it is possible to calculate under the formula

$$\delta \Delta m_{Pu} = \frac{\delta R_0 + \delta \Delta R_{att} + \delta \Delta R_{app} + \delta \Delta R_T + \delta \Delta R_{wall} + \Delta m_{Pu} \cdot \delta \Delta R_{cc}}{\Delta R_{cc}} \; .$$

Using meaning of errors $\delta\mathbf{R}_0$, δΔ$\mathbf{R}_{\mathbf{n}}$ and value Δ$\mathbf{R}_{\mathbf{cc}}$ from Table IV, we shall receive δΔ$\mathbf{m}_{\mathbf{Pu}}$ ≈ **2.73154 g**. The value δΔ$\mathbf{m}_{\mathbf{Pu}}$ is an error of critical mass $\mathbf{m}_{\mathbf{cr}}$ because of errors of measurements on TBCA.

The calculated value of β_{eff} for critical condition of the spherical model is equal to **0.0024**. Using β_{eff} value, the value of δΔ$\mathbf{m}_{\mathbf{Pu}}$ cited above we shall obtain the value of δ$\mathbf{k}_{\mathbf{exp}}$ ≈ **0.00034.** For comparison we shall specify, that $[\sum_m \delta^2 k_m + \sum_p \delta^2 k_p]^{1/2}$ ≈ 0.00092. Turning back to Formula (1), we obtain the value of δ$\mathbf{k}_{\mathbf{eff}}$ ≈ **0.00098**.

Thus, for the spherical assembly with the characteristics given in Tables V and VI and with the temperature 20^{o}C we have the evaluation:

$$\mathbf{k_{eff} = 1 \pm 0.0010}.$$

3. ASSEMBLY WITH NICKEL CENTRAL AREA AND COMPOUND CORE OF ^{235}U (90%) AND ^{235}U (36%)

3.1. CHARACTERISTICS OF THE TEST ASSEMBLY

3.1.1. Description of the test assembly

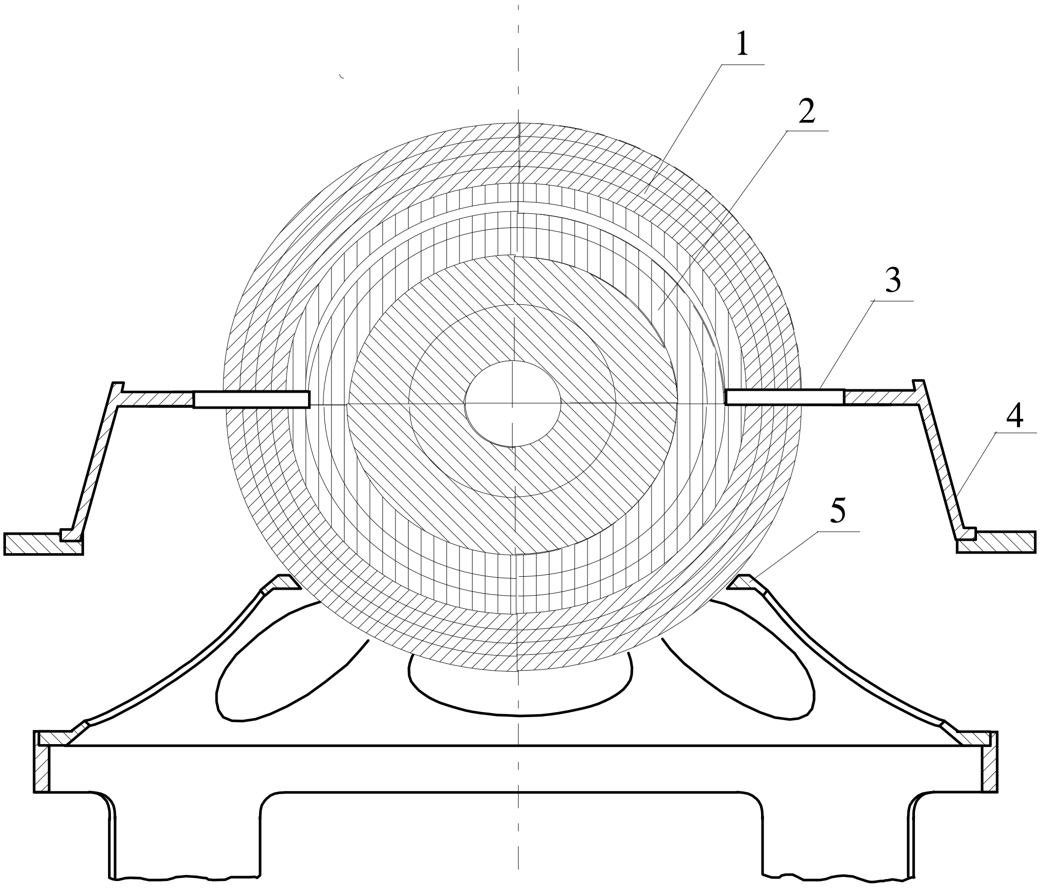
Figure 2 is a schematic sectional view of the assembly positioned on the TBCA. The assembly had a central cavity of 1.4-cm radius, followed by a central area from two spherical layers made of nickel. The assembly core included 3 spherical layers of metallic ^{235}U(90%) and 4 layers of metallic ^{235}U(36%).

The nickel layers were made up of two hemispherical pieces. The upper pieces had the cylindrical pole holes 2.2-cm in diameter that were plugged during measurements with specialized plugs of appropriate layer material.

The layers of ^{235}U(90%) were made up of two hemispherical pieces. The hemispherical pieces had the cylindrical pole holes 2.2 cm in diameter. Specialized plugs of the appropriate layers material were plugged into these pole holes.

Each layer of ^{235}U(36%) was also made up of two hemispherical pieces with cylindrical pole holes 2.2-cm in diameter. The specialized plugs of the appropriate layer material were plugged into all other pole holes.

The assembly included two separate units. The upper unit, comprised by hemispherical piece of ^{235}U(90%), limited on radiuses of 11.465 - 12.25 cm, and hemispherical pieces of ^{235}U(36%), limited on radiuses of 12.25 - 15.3 cm, were laid on a steel diaphragm of 0.2-cm thick. The other pieces were all incorporated into the lower (movable in vertical direction) unit lifted against the stop in the diaphragm.



1 - upper assembly unit
2 - lower assembly unit
3 - steel diaphragm
4 - upper support with pedestal
5 - lower support with pedestal

FIG. 2. Sectional view of the assembly on the TBCA.

3.1.2. Characteristics of a core and central area of the test assembly

In Table VIII the sizes and masses of the test assembly layers are presented.

TABLE VIII. SIZES AND MASSES OF THE TEST ASSEMBLY LAYERS

Layer No.	Radius, cm		Mass, g
	inner	outer	
Central Area of Nickel			
1	1.400	6.000	7146.5
2	6.000	9.150	16799
Core, ^{235}U(90%) layers			
3-5	9.150	12.250	80891.5
Core, ^{235}U(36%) layers			
6-9	12.250	15.300	132295

The material used in the layers 1-2 of the test assembly was nickel. Nickel pieces were weighed with the relative accuracy of **$(\delta m/m)_{Ni}$ = 0.3%.** The value of the relative size error**,** averaged over 2 nickel layers, was (δ**r/r)$_{Ni}$ = 0.15%**. Table IX gives a description of the nickel-layers composition.

TABLE IX. CHARACTERISTICS OF THE NICKEL-LAYERS COMPOSITION

Layer No.	Composition, % by atom			
	Ni	O	Fe	C
1	99.515	0.331	0.105	0.049
2	99.505	0.331	0.115	0.049

The fission material used in the layers 3-5 of the test assembly was uranium metal. The average ratio of ^{235}U mass to a sum of ^{235}U and ^{238}U masses was 90.8%. The percentage of ^{235}U isotope was known with the relative error of **$(\delta\alpha/\alpha)_{U(90)}$ = 0.12%**. Impurities were known within the relative error of **$(\delta\varepsilon/\varepsilon)_{U(90)}$ = 21.5%**. The pieces of ^{235}U(90%) were all weighed within the relative accuracy of **$(\delta m/m)_{U(90)}$ = 0.01%** or better. The total value of the relative size error, averaged over 3 layers of ^{235}U(90%), was **$(\delta r/r)_{U(90)}$ = 0.1%.** Table X gives a description of the average on 3-5 layers composition of ^{235}U(90%).

TABLE X. CHARACTERISTICS OF THE AVERAGE ON 3-5 LAYERS COMPOSITION OF ^{235}U(90%)

Composition, Wt. %				
^{234}U	^{235}U	^{238}U$^{(a)}$	Heavy (A > 8) impurities	Light (A ≤ 18) impurities
0.99	89.34	9.08	0.56	0.03
(a) The fraction of ^{236}U was added to the ^{238}U fraction, see the evaluation in Section 3.2.3.				

The fission material used in the layers 6-9 of the test assembly was uranium metal. The average relation of ^{235}U mass ^{235}U to a sum of ^{235}U and ^{238}U masses was 36.7%. The percentage of ^{235}U isotope was known with the relative error of $(\delta\alpha/\alpha)_{U(36)} = 0.28\%$. Impurities were known within the relative error of $(\delta\varepsilon/\varepsilon)_{U(36)} = 22\%$. The pieces of ^{235}U(36%) were all weighed within the relative accuracy no more than $(\delta m/m)_{U(36)} = 0.01\%$. The total value of the relative size error averaged over 4 layers of ^{235}U(36%) was $(\delta r/r)_{U(36)} = 0.05\%$. Table XI gives a description of the average on 6-9 layers composition of ^{235}U(36%).

TABLE XI. CHARACTERISTICS OF THE AVERAGE ON 6-9 LAYERS COMPOSITION OF ^{235}U (36%)

Composition, Wt. %					
^{234}U	^{235}U	^{238}U$^{(a)}$	Heavy (A ≥ 90) impurities	Mid-range (90<A<18) impurities	Light (A ≤ 18) impurities
0.33	36.31	62.59	0.02	0.64	0.11
(a) The fraction of ^{236}U was added to the ^{238}U fraction, see the evaluation in Section 3.2.3.					

3.1.3. Measurement results of the test assembly reactivity and reactivity perturbations

In Table XII there are presented measurement results of the test assembly reactivity $\mathbf{R_0}$ and reactivity perturbations $\Delta\mathbf{R_n}$ obtained on the TBCA.

The ways of $\Delta\mathbf{R_{att}}$ and $\Delta\mathbf{R_{app}}$ measurement were submitted in Section 2.1.3. The value of $\Delta\mathbf{R_{wall}}$ was defined by calculations. During the measuring the assembly temperature was equal to 20°C.

The $\Delta\mathbf{R_9}$ value was determined by stripping half of a layer 9 from the test assembly.

TABLE XII. MEASUREMENT RESULTS OF THE TEST ASSEMBLY REACTIVITY AND REACTIVITY PERTURBATIONS

Characteristics and it's designation	Result of measurement in terms of $\beta_{\mathbf{eff}}$
Test assembly reactivity, $\mathbf{R_0}$	**- (0.60 ± 0.004)**
Reactivity perturbation caused by removing the upper and lower supports, their pedestals, and diaphragm, $\Delta\mathbf{R_{att}}$	**- (0.38 ± 0.026)**
Reactivity perturbation conditioned by moving apart the assembly units against the stop without diaphragm, supports and their pedestals, $\Delta\mathbf{R_{app}}$	**+ (0.82 ± 0.020)**
Reactivity perturbation caused by removing the walls of the room to infinity, $\Delta\mathbf{R_{wall}}$	**- (0.04 ± 0.02)**
Reactivity perturbation conditioned by removing 1 g of uranium from layer 9, $\Delta\mathbf{R_9}$	$\mathbf{-\ (1.44 \pm 0.025)\cdot 10^{-4}}$

3.2. CHARACTERISTICS OF THE TEST ASSEMBLY SPHERICAL MODEL AT THE LOWER CRITICAL CONDITION

3.2.1. Reactivity of spherical model

Measurement results presented in Table XII allow to define the reactivity $\mathbf{R_{sphere}}$ of the spherical model of the test assembly, isolated from the influence of the TBCA units (the supports, pedestals and diaphragm) and the walls of the room. The value of $\mathbf{R_{sphere}}$ is equal to

$$\mathbf{R_{p.\text{-}sphere} = R_0 + \Delta R_{att} + \Delta R_{app} + \Delta R_{wall} = -0.20}.$$

As one can see, the test assembly spherical model in this case is subcritical.

3.2.2. Extrapolation to the lower critical condition

To remove the spherical model conventionally to the lower critical condition it is necessary to bring into the core an additional quantity of fission material. The mass of ^{235}U(36%) which is necessary to bring in the layer 9 is equal to

$$\mathbf{\Delta m_{U(36)}} = \frac{\left|R_{sphere}\right|}{\left|\Delta R_9\right|} \approx 1388.9\ \mathbf{g}.$$

Addition of this mass is equivalent to changing the outer radius of $\mathbf{r_{out,\,9}}$ = 15.3 cm of layer 9 to the new value of $\bar{r}_{out,9} = \mathbf{15.32661}$ cm on retention of layer 9 composition and its average density.

3.2.3. Simplifications of model

The impurities in the layers of ^{235}U(90%) are divided into two groups: one with relatively low atomic mass numbers ($A \leq 18$) and another having the upper-range atomic mass numbers ($A > 18$) of the periodic Table. The first group is represented by C. The second group is represented by Ni. The magnitude of the change in the calculated value of $\mathbf{k_{eff}}$ when a detailed model (with 15 impurity components) was replaced by a simplified model was no more than $\mathbf{(\delta k_{eff})_{imp,U(90)} \approx 0.00014}$.

The impurity composition in the units of ^{236}U(36%) are divided into three groups. Elements with low atomic mass ($A \leq 18$) were combined and represented by C, those with the mass numbers from the mid-range ($18 < A < 90$) were represented by Ni, and those with high atomic mass numbers ($A > 90$) were represented by W. The magnitude of the change in the calculated value of k_{eff} when a detailed model (with 14 impurity components) was replaced by a model with a simplified composition was $\mathbf{(\delta k_{eff})_{imp,U(36)} \approx 0.00006}$.

In terms of the weight percentage and contribution to the critical parameters the most important uranium isotopes are ^{234}U, ^{235}U, and ^{238}U. The fraction of ^{236}U isotope was very small and therefore it was added to ^{238}U. Calculations showed that the magnitude of the change in the value of k_{eff} in this case was $\mathbf{(\delta k_{eff})_{236} \approx 0.00020}$.

Each layered area of ^{235}U(90%) and ^{235}U(36%) was replaced by homogeneous layer under the condition of retaining their complete volumes, masses and quantities of particular nuclei from their compositions. To account the weight percentage of particular nuclei in each homogeneous area we used the formula similar to the formula from Section 2.2.3. Calculations show that the magnitude of the change in the value of $\mathbf{k_{eff}}$ resulted by this action is $\mathbf{(\delta k_{eff})_{gom,Pu} \approx 0.00002}$.

3.2.4. Characteristics of a core and central area of spherical model in the lower critical condition

In Table XIII there are given the sizes and masses of the spherical model layers at the lower critical condition.

The nuclear concentrations in the spherical model layers in the lower critical condition are presented in Table XIV.

TABLE XIII. SIZES AND MASSES OF THE SPHERICAL MODEL LAYERS AT THE LOWER CRITICAL CONDITION

Layer No.	Radius, cm		Mass m_i, g
	inner	outer	
Central Area of Nickel			
1	1.400	6.000	7146.5
2	6.000	9.150	16799
Core, ^{235}U(90%) layer			
3	9.150	12.250	80891.5
Core, ^{235}U(36%) layer			
4	12.250	15.32661	133683.9

Critical mass is equal $\mathbf{m_{cr} = m_3 + m_4 = 214575.4\ g}$.

TABLE XIV. NUCLEAR CONCENTRATIONS - nucl./(barn·cm) - IN THE SPHERICAL MODEL LAYERS AT THE LOWER CRITICAL CONDITION

Central Area of Nickel						
Layer No.	Ni	O	Fe	C		
1	8.1925×10^{-2}	2.7249×10^{-4}	8.6440×10^{-5}	4.0388×10^{-5}		
2	7.4654×10^{-2}	2.4833×10^{-4}	8.6279×10^{-5}	3.6763×10^{-5}		
Core, ^{235}U(90%) layer						
Layer No.	^{234}U	^{235}U	^{238}U	C	Ni	
3	4.5880×10^{-4}	4.1227×10^{-2}	4.1371×10^{-3}	2.7091×10^{-4}	1.0349×10^{-3}	
Core, ^{235}U(36%) layer						
Layer No.	^{234}U	^{235}U	^{238}U	C	Ni	W
4	1.5386×10^{-4}	1.6848×10^{-2}	2.8678×10^{-2}	9.7424×10^{-4}	1.2172×10^{-3}	8.0791×10^{-6}

3.2.5. Determination of k_{eff} value error

For the characteristics of the test assembly spherical model presented in Tables XIII and XIV $\mathbf{k_{eff}}$ is equal to **1**. It is necessary to calculate the error for these characteristics.

The total error $\mathbf{k_{eff}}$ can be calculated by the Formula (1). Using data from Section 3.2.3 we shall obtain $[\sum_m \delta^2 k_m]^{1/2} \approx$ **0.00025**.

In Table XV calculation results of $\delta\mathbf{k_p}$ are presented. All designations were interpreted in Section 3.1.2 where the appropriate values of $\delta\mathbf{p/p}$ are given.

TABLE XV. VALUES OF $\delta\mathbf{k_p}$

p→	**m**	**r**	α	ε
Central Area of Nickel				
$\delta\mathbf{k_p}$	0.00039	0.00022	-	-
Core, ^{235}U(90%) layers				
$\delta\mathbf{k_p}$	0.00004	0.00001	0.00078	0.00077
Core, ^{235}U(36%) layers				
$\delta\mathbf{k_p}$	0.00003	0.00007	0.00020	0.00049

The value $[\sum_p \delta^2 k_p]^{1/2}$ is approximately equal to **0.00130**. We shall present the separate components of this error: $[\sum_p \delta^2 k_p]^{1/2}_{U(90)} \approx 0.00110$, $[\sum_p \delta^2 k_p]^{1/2}_{U(36)} \approx 0.00054$, $[\sum_p \delta^2 k_p]^{1/2}_{Ni} \approx 0.00045$.

For calculation $\delta\mathbf{k_{exp}}$ it is possible to receive the following formula

$$\delta k_{exp} = |\Delta R_9| \beta_{eff} \delta \Delta m_{U(36)} .$$

Here $\delta\Delta\mathbf{m_{U(36)}}$ an error of determination $\Delta\mathbf{m_{U(36)}}$ because of errors of measurements on TBCA, which it is possible to calculate under the formula

$$\delta \Delta m_{U(36)} = \frac{\delta R_0 + \delta \Delta R_{att} + \delta \Delta R_{app} + \delta \Delta R_{wall} + \Delta m_{U(36)} \cdot \delta \Delta R_9}{|\Delta R_9|} .$$

Using meaning of errors δR_0, $\delta\Delta\mathbf{R_n}$ and value $\Delta\mathbf{R_9}$ from Table XII, we shall receive $\delta\Delta\mathbf{m_{U(36)}} \approx$ **510.7224 g**. The value $\delta\Delta\mathbf{m}_{U(36)}$ is an error of critical mass $\mathbf{m_{cr}}$ because of errors of measurements on TBCA.

The calculated value of β_{eff} for critical condition of the spherical model is equal to **0.0067**. Using β_{eff} value, the value of $\delta\Delta\mathbf{m_{U(36)}}$ cited above we shall obtain the value of $\delta\mathbf{k_{exp}} \approx$ **0.00049**. For comparison we shall specify that $[\sum_m \delta^2 k_m + \sum_p \delta^2 k_p]^{1/2} \approx 0.00132$. Turning back to the Formula (1), we shall obtain the value of $\delta\mathbf{k_{eff}} \approx$ **0.00141**.

Thus, for the spherical assembly with the characteristics given in Tables XIII and XIV and with the temperature 20°C we have the evaluation:

$$\mathbf{k_{eff} = 1 \pm 0.0014.}$$

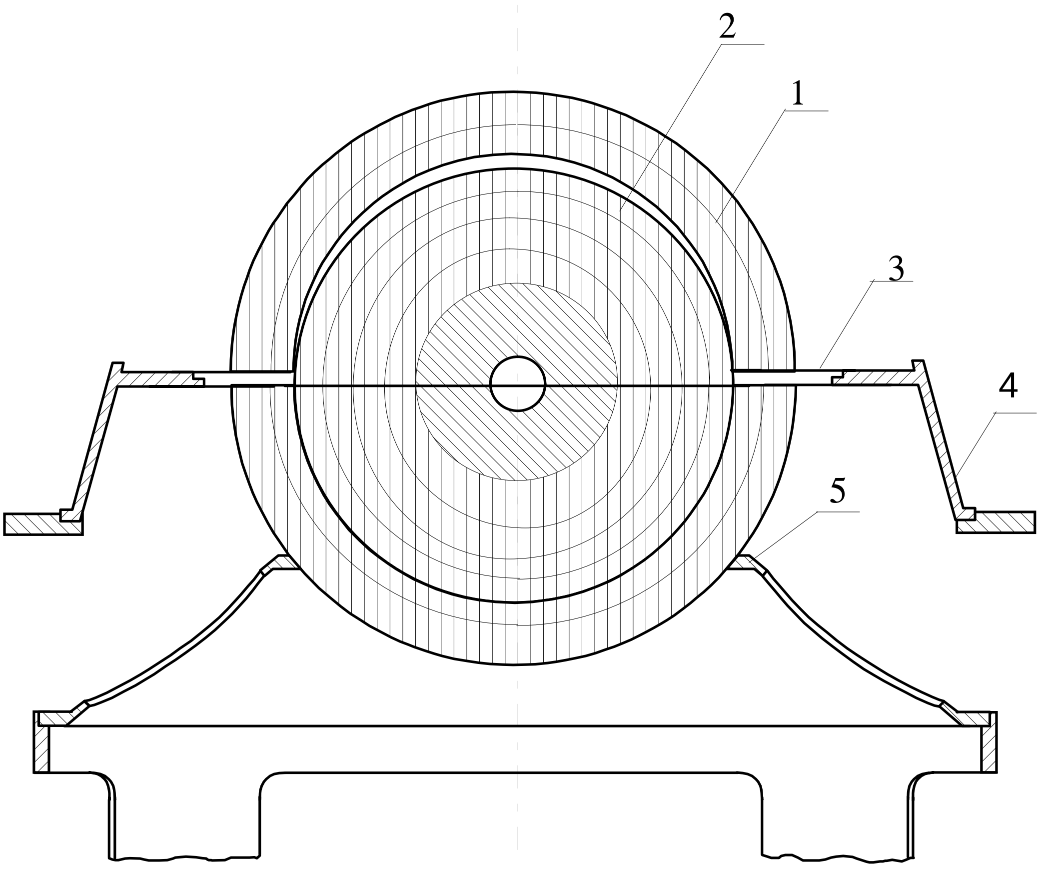
4. ASSEMBLY WITH NICKEL CENTRAL AREA AND THE CORE OF ^{235}U (90%)

4.1. CHARACTERISTICS OF THE TEST ASSEMBLY

4.1.1. Description of the test assembly

Figure 3 is a schematic sectional view of the assembly positioned on the TBCA. The assembly had a central cavity of 1.4-cm radius, followed by a central area from spherical layer made of nickel. The assembly core included 6 spherical layers of metallic ^{235}U(90%).

The nickel layer was made up of two hemispherical pieces. The upper piece had a cylindrical pole hole 2.2-cm in diameter that was plugged during measurements with specialized plug of appropriate layer material.



1 - upper assembly unit
2 - lower assembly unit
3 - steel diaphragm
4 - upper support with pedestal
5 - lower support with pedestal

FIG. 3. Sectional view of the assembly on the TBCA.

The layers of ^{235}U(90%) were made up of two hemispherical pieces. The hemispherical pieces had the cylindrical pole holes with a diameter of 2.2 cm. Specialized plugs of the appropriate layers material were plugged into these pole holes.

The assembly included two separate units. The upper unit consisted of hemispherical pieces of ^{235}U(90%) and limited on radiuses of 9.15 - 11.465 cm was laid on a steel diaphragm of 0.2-cm thick. The other pieces were all incorporated into the lower (movable in vertical direction) unit lifted against the stop in the diaphragm.

4.1.2. Characteristics of a core and central area of the test assembly

In Table XVI the sizes and masses of the test assembly layers are given.

TABLE XVI. SIZES AND MASSES OF THE TEST ASSEMBLY LAYERS

Layer No.	Radius, cm		Mass, g
	inner	outer	
Central Area of Nickel			
1	1.400	6.000	7146.5
Core of ^{235}U(90%)			
2-7	6.000	11.465	98220.3

The material used in the central area (layer 1 of the test assembly) was nickel. In addition to Ni (99.515% by atoms) the central area material contained O (0.331% by atoms), Fe (0.105% by atoms) and C (0.049% by atoms). The nickel pieces were all weighed within the relative accuracy of **$(\delta m/m)_{Ni}$ = 0.3%.** The value of the relative size error averaged over 2 nickel layers was **$(\delta r/r)_{Ni}$ = 0.15%**.

The fission material used in the test assembly core was uranium metal. The average ratio of ^{235}U mass to a sum of masses of ^{235}U and ^{238}U was 90.7%. The percentage of ^{235}U isotope was known with the relative error of **$(\delta\alpha/\alpha)_U$ = 0.12%**. Impurities were known within the relative error of **$(\delta\varepsilon/\varepsilon)_U$ = 21.5%**. The pieces of ^{235}U(90%) were all weighed with the relative accuracy of **$(\delta m/m)_U$ = 0.01%** or better. The total value of the relative size error averaged over 6 layers of ^{235}U(90%) was **$(\delta r/r)_U$ = 0.1%.** Table XVII gives a description of the average on 2-7 layers composition of ^{235}U(90%).

TABLE XVII. CHARACTERISTICS OF THE AVERAGE ON 2-7 LAYERS COMPOSITION OF ^{235}U(90%)

Composition, Wt. %				
^{234}U	^{235}U	^{238}U$^{(a)}$	Heavy (A > 18) impurities	Light (A ≤ 18) impurities
1.02	89.26	9.13	0.55	0.04

(a) The fraction of ^{236}U was added to the ^{238}U fraction, see the evaluation in Section 4.2.3.

4.1.3. Measurements results of the test assembly reactivity and reactivity perturbations

In Table XVIII there are presented measurement results of the test assembly reactivity $\mathbf{R_0}$ and reactivity perturbations $\Delta\mathbf{R_n}$ obtained on the TBCA.

The ways of $\Delta\mathbf{R_{att}}$ and $\Delta\mathbf{R_{app}}$ measurement were submitted in Section 2.1.3. The value of $\Delta\mathbf{R_{wall}}$ was defined by calculations. During measuring the assembly temperature was equal to 20°C.

The value of $\Delta\mathbf{R_7}$ was determined by removing the upper pole plug from layer 7 of the test assembly.

The reactivity perturbation $\Delta\mathbf{R_{out}}$ caused by coating 1 g of a core material on the external surface of the assembly was estimated as follows. There were measured the values of $\Delta\mathbf{R_{pl,I}}$ of the reactivity perturbations caused by removing the upper pole plugs in layers i = 5-7 of the test assembly. When removing one plug from a particular layer the pole plugs in the other two layers were left in place. Then there were calculated the values of $\Delta\mathbf{R_i} = \Delta\mathbf{R_{pl,i}}/\mathbf{m_{pl,i}}$, where $\mathbf{m_{pl,i}}$ is the mass of the upper pole plug in a layer i. In result there was known the dependence:

$$\Delta R(r) = a + b(r_5 - r) + c(r_5 - r)(r_6 - r).$$

Here $\mathbf{r_i}$ is a distance along the plug axis of the geometrical centre of the upper pole plug in a layer i to external surface of the upper part of the test assembly; $\mathbf{r}$ is a distance of any point on this axis to the same surface; $\mathbf{a} = \Delta\mathbf{R_5}$; $\mathbf{b} = (\Delta\mathbf{R_6} - \Delta\mathbf{R_5})/(\mathbf{r_5} - \mathbf{r_6})$; $\mathbf{c} = (\Delta\mathbf{R_7} - \Delta\mathbf{R_5})/[(\mathbf{r_5} - \mathbf{r_7})(\mathbf{r_6} - \mathbf{r_7})] - (\Delta\mathbf{R_6} - \Delta\mathbf{R_5})/[(\mathbf{r_5} - \mathbf{r_6})(\mathbf{r_6} - \mathbf{r_7})]$. Value $\Delta\mathbf{R_{out}} = -(\mathbf{a} + \mathbf{b}\mathbf{r_5} + \mathbf{c}\mathbf{r_5}\mathbf{r_6})$.

TABLE XVIII. MEASUREMENT RESULTS OF THE TEST ASSEMBLY REACTIVITY AND REACTIVITY PERTURBATIONS

Characteristics and it’s designation	Result of measurement in terms of $\beta_{\mathbf{eff}}$
Test assembly reactivity, $\mathbf{R_0}$	**- (3.06 ± 0.054)**
Reactivity perturbation conditioned by removing the upper and lower supports, their pedestals, and diaphragm, $\Delta\mathbf{R_{att}}$	**- (0.57 ± 0.076)**
Reactivity perturbation caused by moving apart the assembly units against the stop without diaphragm, supports and their pedestals, $\Delta\mathbf{R_{app}}$	**+ (0.82 ± 0.046)**
Reactivity perturbation caused by removing the walls of the room to infinity, $\Delta\mathbf{R_{wall}}$	**- (0.04 ± 0.02)**
Reactivity perturbation conditioned by removing 1 g of uranium from layer 7, $\Delta\mathbf{R_7}$	**$-(0.84 \pm 0.045)\cdot 10^{-3}$**
Reactivity perturbation caused by removing 1 g core material on the external surface of the assembly, $\Delta\mathbf{R_{out}}$	**$+(0.44 \pm 0.035)\cdot 10^{-3}$**

4.2. CHARACTERISTICS OF THE TEST ASSEMBLY SPHERICAL MODEL AT THE LOWER CRITICAL CONDITION

4.2.1. Reactivity of spherical model

Measurement results presented in Table XVIII allow to define the reactivity $\mathbf{R_{sphere}}$ of the spherical assembly isolated from the influence of the TBCA units (the supports, pedestals and diaphragm) and the walls of the room. The value of $\mathbf{R_{sphere}}$ is equal to

$$\mathbf{R_{sphere} = R_0 + \Delta R_{att} + \Delta R_{app} + \Delta R_{wall} = -\ 2.85}.$$

As one can see, the test assembly spherical model is subcritical.

4.2.2. Extrapolation to the lower critical condition

To take off conditionally the spherical model to the lower critical condition it is necessary to bring some additional quantity of a fission material to a core. We shall increase the mass of layer 7 by $\mathbf{\Delta m_7 = 1253.6}$ g without changing its sizes. The resulting reactivity $\mathbf{(R_{sphere} - \Delta R_7 \Delta m_7)} \approx -\ 1.79698$ may be compensated by addition of a spherical layer of ^{235}U (90%) on the external surface side. The mass of this layer is equal to

$$\mathbf{\Delta m_{out}} = \frac{\left| \boldsymbol{R}_{sphere} - \Delta \boldsymbol{R}_7 \Delta \boldsymbol{m}_7 \right|}{\Delta \boldsymbol{R}_{out}} \approx \boldsymbol{4084.0}\ \mathbf{g}.$$

Then new mass of layer 7 is equal to $\overline{\boldsymbol{m}}_7 = \mathbf{m_7 + \Delta m_7 + \Delta m_{out} = 32356.9}$ g, where $\mathbf{m_7}$ is the mass of layer 7 of the test assembly. New outer radius of layer 7 is equal to **11.59572** cm.

4.2.3. Simplifications of model

The impurities in the layers of ^{235}U(90%) are divided into two groups: one with relatively low atomic mass numbers ($A \le 18$) and another having the upper-range atomic mass numbers ($A > 18$) of the periodic Table. The first group is represented by C. The second group is represented by Ni. The magnitude of the change in the calculated value of $\mathbf{k_{eff}}$ when a detailed model (with 15 impurity components) was replaced by a simplified model was no more than $\mathbf{(\delta k_{eff})_{imp}} \approx \mathbf{0.00014}$.

In terms of the weight percentage and contribution to the critical parameters the most important uranium isotopes are ^{234}U, ^{235}U, and ^{238}U. The fraction of ^{236}U isotope was very small and therefore it was added to ^{238}U. Calculations showed that the magnitude of the change in the value of k_{eff} in this case was $\mathbf{(\delta k_{eff})_{236}} \approx \mathbf{0.00020}$.

Layered area of ^{235}U(90%) was replaced by homogeneous core under the condition of retaining their complete volume, masses and quantities of particular nuclei from their composition. To account the weight percentage of particular nuclei in the homogeneous core we used the formula similar to the from Section 2.2.3. Calculations show that the magnitude of the change in the value of $\mathbf{k_{eff}}$ resulted by this action is $\mathbf{(\delta k_{eff})_{gom,Pu}} \approx \mathbf{0.00139}$.

4.2.4. Characteristics of a core and central area of spherical model at the lower critical condition

In Table XIX there are given the sizes and masses of the spherical model layers at the lower critical condition.

The nuclear concentrations in the spherical model layers in the lower critical condition are presented in Table XX.

TABLE XIX. SIZES AND MASSES OF THE SPHERICAL MODEL LAYERS AT THE LOWER CRITICAL CONDITION

Layer No.	Radius, cm		Mass m_i, g
	inner	outer	
Central Area of Nickel			
1	1.400	6.000	7146.5
Core of ^{235}U(90%)			
2	6.000	11.59572	103557.9

Critical mass is equal $\mathbf{m_{cr} = m_2 = 103557.9}$ **g**.

TABLE XX. NUCLEAR CONCENTRATIONS - nucl./(barn·cm) - IN THE SPHERICAL MODEL LAYERS AT THE LOWER CRITICAL CONDITION

Central Area of Nickel				
Layer No.	Ni	O	Fe	C
1	8.1925×10^{-2}	2.7249×10^{-4}	8.6440×10^{-5}	4.0388×10^{-5}

Core of ^{235}U(90%)					
Layer No.	^{234}U	^{235}U	^{238}U	C	Ni
2	4.8308×10^{-4}	4.2045×10^{-2}	4.2971×10^{-3}	3.6914×10^{-4}	1.0387×10^{-3}

4.2.5. Determination of $\mathbf{k_{eff}}$ value error

For the characteristics of the test assembly spherical model presented in Tables XIX and XX $\mathbf{k_{eff}}$ is equal to **1**. It is necessary to calculate the error for these characteristics.

The total error of $\mathbf{k_{eff}}$ can be calculated by the Formula (1). Using data from Section 4.2.3 we shall obtain $[\sum_m \delta^2 k_m]^{1/2} \approx \mathbf{0.00141}$.

In Table XXI calculation results of $\delta\mathbf{k_p}$ are presented. All designations were interpreted in Section 4.1.2 where the appropriate values of $\delta\mathbf{p/p}$ are given.

TABLE XXI. VALUES OF $\delta \mathbf{k_p}$

p→	**m**	**r**	α	ε
Central Area of Nickel				
$\delta \mathbf{k_p}$	0.00010	0.00005	**-**	**-**
Core of ^{235}U(90%)				
$\delta \mathbf{k_p}$	0.00011	0.00075	0.00080	0.00080

The value of $[\sum_p \delta^2 k_p]^{1/2}$ is about **0.00136**. We shall present the separate components of this error: $[\sum_p \delta^2 k_p]^{1/2}_{U} \approx 0.00135$, $[\sum_p \delta^2 k_p]^{1/2}_{Ni} \approx 0.00011$.

For calculation $\delta \mathbf{k_{exp}}$ it is possible to receive the following formula

$$\delta k_{exp} = \Delta R_{out} \beta_{eff} \delta \Delta m_{out} \ .$$

Here $\delta\Delta \mathbf{m_{out}}$ an error of determination $\Delta \mathbf{m_{out}}$ because of errors of measurements on TBCA, which it is possible to calculate under the formula

$$\delta\ \Delta m_{out} = \frac{\delta\ R_0 + \delta\ \Delta R_{att} + \delta\ \Delta R_{app} + \delta\ \Delta R_{wall} + \Delta m_7 \delta\ \Delta R_7 + \Delta m_{out} \delta\ \Delta R_{out}}{\Delta R_{out}} \ .$$

Using values of errors $\delta \mathbf{R}_0$, $\delta\Delta \mathbf{R_n}$ and value $\Delta \mathbf{R_{out}}$ from Table XVIII, we shall receive $\delta\Delta \mathbf{m_{out}} \approx$ **898.527 g**. The value $\delta\Delta \mathbf{m_{out}}$ is an error of critical mass $\mathbf{m_{cr}}$ because of errors of measurements on TBCA.

The calculated value of β_{eff} for critical condition of the spherical model is equal to **0.0065**. Using β_{eff} value cited above, the value of $\delta\Delta \mathbf{m_{out}}$ we shall obtain the value of $\delta \mathbf{k_{exp}} \approx$ **0.00257**. For comparison we shall specify that $[\sum_m \delta^2 k_m + \sum_p \delta^2 k_p]^{1/2} \approx 0.00196$. Turning back to the Formula (1), we shall obtain the value of $\delta \mathbf{k_{eff}} \approx$ **0.00323**.

Thus, for the spherical assembly with the characteristics given in Tables XIX and XX and with the temperature 20°C we have the evaluation:

$$\mathbf{k_{eff} = 1 \pm 0.0032.}$$

SUMMARY

In the present work there are given the results of reactivity measurements for three test assemblies containing nickel layers as a reflector and in a central cavity with the core of ^{239}Pu(δ,98%), ^{235}U(90%) and ^{235}U(36%). There are also presented the measurement results of reactivity perturbations.

These results have allowed to come to the test assemblies spherical models at the lower critical condition isolated from the influence of the TBCA units (supports, pedestals, diaphragm) and the walls of the room. With the ^{239}Pu(δ,98%) layers in the composition of the core of the test assembly model temperature was resulted to 20°C.

There have been carried out the estimations of total errors $\mathbf{k_{eff}}$ value at the lower critical condition. For all assemblies being under consideration the values of the latter errors is less than 0.0033. The experimental components of these errors there are less than 0.0026.

Data presented in this work can be used for adjusting neutron constants in the area of fast neutron energies including nickel constants.

It should be noticed, that for such application it will be possible to refuse from conditional transferring the test assembly spherical models discussed above to the lower critical condition and to limit by the values of $\mathbf{k_{eff}} < 1$ and $\mathbf{k_{eff}} > 1$. In this case the value of $\delta\mathbf{k_{exp}}$ should be calculated by the formula:

$$\delta k_{exp} = k_{eff}\sqrt{\beta_{eff}^{2}\delta^{2}R_{sphere} + R_{sphere}^{2}\delta^{2}\beta_{eff}}\ ,$$

where $\delta\beta_{\mathbf{eff}}$ is an error of $\beta_{\mathbf{eff}}$; $\delta\ \mathbf{R_{sphere}}$ is a limiting absolute error of $\mathbf{R_{sphere}}$ determination,

$$k_{eff} = \frac{1}{1-\beta_{eff}R_{sphere}}\ .$$

For example, for spherical model of the assembly from Section 4 until the beginning the taking off to critical condition the value of $\mathbf{k_{eff}} = 0.9818$. In this case we have neglected by the distinction between the values of $\beta_{\mathbf{eff}}$ at subcritical and lower critical condition. To determine $\delta\mathbf{k_{exp}}$ in subcritical condition the error evaluation of $\delta\beta_{\mathbf{eff}}$ is required. Let us consider that $\delta\beta_{\mathbf{eff}} = 3\%$ /7/. Then for subcritical spherical model of the test assembly from Section 4 the value of $\delta\mathbf{k_{exp}} = 0.00139$ (for critical spherical model $\delta\mathbf{k_{exp}} = 0.00257$). Thus, for this model we have the evaluation:

$$\mathbf{k_{eff} = 0.9818 \pm 0.0024.}$$

Comparing this evaluation with the evaluation of $\mathbf{k_{eff}}$ value for spherical model at the lower critical condition from Section 4 we can see, that in a case of subcritical model there was the reduction of the total error $\mathbf{k_{eff}}$ *(*for critical spherical model the total error is $\delta\mathbf{k_{eff}} = 0.0032$).

According to the contents of Sections 2 and 3 it is possible to conclude, as for considered in these sections assemblies the total errors of $\mathbf{k_{eff}}$ values of their spherical models up to a conclusion in a critical condition it is less 0.0025, and experimental components these errors less than 0.0014.

To supplement data presented in this work it is necessary

- to carry out the measurements of the assembly characteristics with nickel layers arrangement inside the core, that is between the layers of fission material;

- to carry out the measurements of neutron spectrum characteristics in the central area filled of nickel and surrounded by a core.

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Nuclear Data Section	e-mail: services@iaeand.iaea.or.at
International Atomic Energy Agency	fax: (43-1) 26007
P.O. Box 100	cable: INATOM VIENNA
A-1400 Vienna	telex: 1-12645
Austria	telephone: (43-1) 2600-21710

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