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OF THE
INFORMATION CENTRE FOR NUCLEAR DATA

(First Issue)

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FOREWORD

This Bulletin is the first of an intended series of regular issues of collected information from the Nuclear Data Centre set up by the State Committe on the Utilization of Atomic Energy to ensure the collation, systematization an exchange of information on nuclear physical constants.

The contents and layout of the Information Centre's bulletins are determi by the need to present in convenient form, in as short a time as possible, the existing information on nuclear data.

The Bulletin will therefore include only information which has been collected and systematized by the groups of the Information Centre at the moment of publication. For this reason the issues of the Bulletin, including this volume, lay no claim to providing a complete and ordered compilation of al the existing information on nuclear data. It is however assumed that the gap in individual issues will be made good in subsequent numbers.

The present Bulletin includes data on the parameters of the elementary interactions of neutrons with a substance (in the energy range of interest for reactor construction, i.e. E not exceeding approximately 15 MeV), and the macroscopic constants, empirical formulae and integral parameters (and characteristics of such parameters) used in calculations for reactors and reactor shieldings, to the extent that such information is not contained in previous publications by the Publishing House for Atomic Literature (Atomizdat Most of the data given were obtained in 1963–64, or became available during this period. The material given varies widely in its degree of completeness. For example, capture and elastic and inelastic scattering cross-sections, and the resonance parameters of neutrons are very fully covered. On the other ha the data on the characteristics of fissionable nuclei, for example, are very incomplete, while the energy dependences of total cross-sections are not given at all. The extent to which the available information is analysed also varie For example, the resonance parameters given were obtained by averaging all the available data, but on inelastic scattering spectra and cross-sections only th data as measured by different authors are given, without a critical discussion of them. Apart from the results of measurements of nuclear data and their

analysis, the Bulletin contains information on the methods used for calculating cross-sections and for analysing experimental data, and also on certain other aspects of the determination or practical use of nuclear physical constants.

The present issue uses more or less generally accepted terminology and notation, and therefore no li. t of symbols or definitions is given. In view of the differences in notation and terminology used in different scientific organizations, it is proposed subsequently to undertake a rational unification of symbols and terms.

The Bulletin is produced by the Scientific and Technical Information Bureau of the Physics and Power Institute of the USSR State Committee on the Utilizatio: of Atomic Energy.

(The Editors)

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PART I

PARAMETERS OF ELEMENTARY INTERACTIONS OF NEUTRONS WITH NUCLEI

I. New data on the interaction of slow neutrons with a substance

1. Generalized spectrum of normal oscillations for H_2O and D_2O

Yu. V. Sokolov and V. F. Turchin

The double-differential cross-section for scattering of slow neutrons may be written in the form [1]

(1)

where $S(\vec{\varkappa},\omega)$ is the scattering law and $\vec{\varkappa} = \vec{R} - \vec{R}_0$: $\omega = (E_0 - E)/h$. Using the concept of the symmetrized scattering law $S(\vec{\varkappa},\omega)$, related to $S(\vec{\varkappa},\omega)$ by the equation

(2)

formula (1) may be presented in the form

(3)

If the scattering system has no separate states in the macroscopic sense (liquid, gas, semi-crystal), the scattering law will not depend on the direction of $\vec{\varkappa}$. Complete information on the scattering cross-sections is then contained in one function of two positive arguments $\tilde{S}(\varkappa^2,\omega)$. This is a basic factor in processing and presenting experimental data, and was first used for this purpose by Egelstaff [2]. Since the double-differential scattering cross-section, measured experimentally, depends on three variables, E_0, E,θ, by passing on to $\tilde{S}(\varkappa^2,\omega)$ (a function of two variables), we increase the density of the experimental points on the graphs of the relationships required, and are able to countercheck the accuracy of the experiment in a more succinct form.

In most cases a rough approximation is adequate for reactor design [1]. In this approximation, the scattering law for a system of N identical atoms takes the form

(4)

where $\chi(\varkappa^2,t)$ is the Fourier transform of the autocorrelation function $G_s(\vec{r},t)$.

For high and low values of time t, the function $\chi(x^2,t)$ takes the Gaussian form

(5)

For an ideal (mono-atomic) gas, a cubic crystal, or a liquid on the continuous diffusion model, equation (5) is valid for any value of t (the function $\Gamma(t)$ differs of course from gas to crystal and from crystal to liquid). For other scattering systems, equation (5) (for arbitrary t values) should be regarded as a (Gaussian) approximation.

Following Egelstaff [8], we take as a basis the symmetrized scattering law $\tilde{S}(x^2,\omega)$ with dimensionless arguments

(6)

(7)

(T is the Kelvin temperature in energy units)

(8)

We designate

(9)

where $S_q(\alpha,\beta)$ is the Gaussian part of the scattering law, corresponding to the function $\chi(x^2,t)$ in the form shown in (5), and $S_n(\alpha,\beta)$ is a non-Gaussian correction.

Using the experimental data on the scattering law, it is possible to determine Γt, and to calculate the Gaussian part of the scattering law $S_q(\alpha,\beta)$ with arbitrary α and β values.

Introducing the dimensionless real function

(10)

the scattering law $S(\alpha,\beta)$ may be written in the form

(11)

Using double integration by parts, we obtain

(12)

(If, when $t' \to \infty$, $W(t')$ does not tend to infinity, it is necessary before integrating by parts to separate the multiplier .....).

Schofield [9] has shown that at low α values the non-Gaussian number is proportional to α^2. Consequently

(13)

We determine the function $\varrho(\beta)$

(14)

Then reverse Fourier transformation gives

(15)

since $W(t')$ is a real function of t'.

It can be shown [1] that for a cubic crystal

(16)

where ..... is the spectrum obtained with normal oscillations of the crystal in temperature units.

In the case of an arbitrary scattering system we may introduce, by analogy with (16), the generalized spectrum of normal oscillations

(17)

which is, broadly speaking, temperature-dependent.

It is not difficult [1] to obtain an expression for $W(t')$. This takes the form

(18)

Since to calculate group constants it is sufficient to know the function $\varrho(\beta)$, the latter is determined from the experimental data on the double-differential cross-sections by extrapolation in accordance with (14). This issue of the Bulletin gives the $\varrho(\beta)$ functions for H_2O and D_2O taken from references [10] [11].

. . . . . (Bibliography)

2. Conditions determining the course of the radiative capture and fission cross-sections of certain elements in the thermal energy range

V. N. Artamkin

In calculating thermalization effects, it is convenient to present the energy dependence of the capture and fission cross-sections in the form

. . .

where n is the number of resonances making a pronounced contribution to the cross-section in the thermal range. Inter-resonance interference is disregarded.

For elements whose cross-section obeys the law $\frac{1}{v}$, only one component - a - remains.

The table gives values of the coefficients a, c_i and e_i for a number of elements [1]. The cross-sections obtained by using them differ from the data in reference [2] by less than 0.25% for energies lower than 0.2 eV, less than 1% for 0.2-0.4 eV, and perhaps a little more for energies above 0.4 eV.

. . . . . (Bibliography and Tables)

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of original)

II. New data on thermal-neutron capture and scattering cross-sections for elements with $Z \leq 91$

D.A. Kardashev

Table I

Isotope element $T_{\frac{1}{2}}$	Gamma cross-section (barns)	Activation cross-section (barn)	Scattering cross-section (barn)
1	2	3	4

. . . . . (Table and Bibliography)

(page 18
of original)

III. Parameters of isolated resonance levels

S.M. Zakharova and A.V. Malyshev

The tables given here have been compiled using the same assumptions as those in the second edition of the Directory [46]. Partial widths obtained in recent work have been averaged with the data in the Directory, if it could be definitely established that they belonged to the resonance in question, and if the discrepancies did not exceed the margins of error indicated. In other cases only the data of the latest work are given.

The parameters given in the tables for resonances for fissionable materials were obtained by one-level analysis. If the data on the total moments of the resonances were contradictory, the results preferred in most cases were those obtained by experiment with polarized neutrons on oriented nuclei. It should be noted that because of the separate averaging of the partial widths, they may sometimes not agree with each other within the limits of the indicated margins of error. The data of some new studies are not given, since the information contained in them is not complete.

. . . . . (Tables and Bibliography)

IV. New experimental data on radiative capture cross-sections and resonance integrals of neutron absorption

Yu.Ya. Stavissky et al.

The radiative capture cross-sections and resonance integrals of neutron absorption are given only for non-fissionable elements. Fissionable elements will be dealt with in the next issue of this Bulletin.

The data on the radiative capture cross-sections ($\sigma_{n\gamma}$) are presented in the form of graphs and tables, which include the results of measurements published up to May 1964.

Most of the experimental data were obtained by recording the capture γ-rays. Among them are the data of F.L. Shapiro et al.....

...

... Yu.Ya. Stavissky et al.....

and R.L. Macklin and J.H. Gibbons ...

The methods and results of the measurements are not discussed in each individual case, as this is done in the literature cited. For most elements, the data agree with each other in overlapping energy ranges, in spite of the differences in measuring methods and energy resolutions. It should, however, be noted that for a number of elements there is a discrepancy between the results obtained from different work using the same methods, and also a systematic discrepancy between the activation measurements and measurements of $\sigma_{n\gamma}$ by recording the capture γ-rays. This discrepancy should obviously not be regarded as a matter of chance, although its nature is not yet clear.

The diagrams show the capture cross-sections $\sigma_{n\gamma}$ as a function of the energy of the bombarding neutron. In a number of cases (in the work of Shapiro et al.) the measurements were made on specimens of different thicknesses. This makes it possible to assess the degree of self-shielding of the capture cross-section in the resonance range, and to determine the energy range where self-shielding becomes unimportant.

For the elements for which new measurements over a wide range of energies are lacking, the values of the capture cross-sections for monokinetic neutrons are given in Table I, which also includes the $\sigma_{n\gamma}$ of Na and K, as measured on the fission spectrum.

Table II gives the results of activation measurements of $\sigma_{n\gamma}$ over wide spectra which are established in the shielding of BR-1 and BR-2 fast reactors (Sh - 1) with a mean effective energy of ~ 150 keV and ~ 30 keV respectively. Wide variations from the average E_{eff} values are marked with an asterisk. Some of the results appearing in Table 3 have already been published [L-3].

To obtain the absolute values of $\sigma_{n\gamma}$, a $\sigma_{n\gamma}$ value for Au^{197} of 940 ± 140 mb measured by transmission in spherical geometry (for E_{eff} = 30 keV) - and a value of 1.74 b as the fission cross-section of Pu^{239} (for E_{eff} = 150 keV) were used as supporting cross-sections.

Table III collates the data on absorption resonance integrals I_γ not included in "Nuclear Physical Constants" by I.V. Gordeev et al. Column 2 shows the values of I_γ in barns, as obtained by direct measurement and also as estimated by the authors of the original papers on the basis of their measurements of capture cros sections (.....).

Column 3 indicates the half-life in cases when measurement was made by the activation method. Column 4 - Comments - indicates which energy range contribute to the resonance integral given. The absence of comment indicates that the total resonance integral is given.

. . . . . (Diagrams, Tables and Bibliography)

V. Cross-sections of neutron reactions accompanied by the escape of charged particles

L.P. Abagyan, S.M. Zakharova and M.N. Nikolaev

This section is a compilation of the experimental data hitherto published on the cross-sections of neutron reactions accompanied by the escape of charged particles, for the elements He, Li, Be, B, C, N, O, F, Ne, Na and Mg.

The results are given in the form of graphs and tables. The graphs generally show the cross-sections of the (n,p) and (n,α) reactions. If the reaction has an energy threshold, its value is also shown on the graph. In some cases, the values of the cross-sections for the fission spectrum are given for threshold reactions. These figures are of direct practical interest in calculating fast reactors.

For C, O and Ne values given are those of the cross-sections of the (n,α_0), (n,α_1), etc. reactions which lead to the formation of a residual nucleus in the ground, first, etc. excited states.

It should be noted that the results of measurement of the cross-sections of the reactions studied agree, within the limits of experimental error. An exception is the cross-section of the B^{10} (n,t) 2α reaction; in paper [F-56] this was measured by studying the tracks in emulsions, and in paper [W-58] by recording the tritium yield by absolute calculation of ß-activity. In spite of the difference in method, the absolute values of the cross-sections agree well. The results of paper [D-61], in which the cross-section was measured by recording α-particles in an ionization chamber, are approximately one third as large in the 4.5-MeV energy range. It is interesting to note that the cross-section of the F^{19} (n,α) N^{16} reaction as measured in the last mentioned work is also considerably (as much as 50%) lower than the results of other works, although in this case the discrepancy does not exceed the limits of experimental error.

In some cases the graphs of cross-sections obtained from inverse reactions are given.

Table I gives some supplementary data on measurements of the cross-sections of various reactions accompanied by the escape of charged particles at an energy of 14 MeV, and also includes cross-sections for the fission spectrum and the reactor spectra; the shape of the latter is similar to that of the former in the high energy range.

It should be noted that data are available on the angular distributions of the charged particles formed in the reactions discussed, but these are not included in the present review.

The authors are grateful to Dr. H. Liskien and Dr. A. Paulsen of EURATOM for the information they kindly gave to S.I. Sukhoruchkin about the (n,p) and (n,α) reactions on Mg and Na. The data they provided on some of the heavier isotopes will together with relevant additional information, be included in the next issue of this Bulletin.

Table I

Reaction	Energy, MeV	Cross-section, mb	Paper	Notes
				A neutron escapes on decay of $_2He^5$
				A neutron escapes on decay of $_2He^5$
	Fission spectrum			A neutron escapes on decay of $_7N^{17}$
	Reactor spectrum			A neutron escapes on decay of $_7N^{17}$

. . . . . (Figures and Bibliography)

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of original)

VI. Elastic scattering of neutrons

N.O. Bazazyants, M.N. Nikolaev and V.I. Popov

This section gives the results of experimental work on the elastic scattering of neutrons.

Tables I - V contain the angular distributions of scattered neutrons given in the form of coefficients of expansion in Legendre polynomials. Using these, it is possible to calculate, by the formula given in the tables, the differential cross-sections for any angle of scattering.

Tables VI - VIII give the differential cross-sections as a function of the angle of scattering. All the data in the above tables refer to work published since 1962.

Figs. 1-11 give experimental data on elastic scattering published in 1963-64 and also before, but not included in the standard work on angular distributions [1,2].

Table IX gives the total elastic scattering cross-sections given by the authors of the corresponding papers, and also (marked with an asterisk) calculated by integrating the differential cross-sections over the total solid angle. When there is a discrepancy between the calculated cross-sections and those listed from [92], as $\sigma_{tot} - \sigma_{non}$, the latter are given in brackets.

The fourth column of this table gives the average cosines of angular distribution, calculated by the formula

These are also given, as a function of the initial energy of the neutrons, in Figs. 12-16 for nuclei of lithium-6, lithium-7, beryllium, boron, carbon, nitrogen and oxygen. When there are large discrepancies in the values obtained by analysing the data of different authors, only the most reliable figures are shown in the graphs, preference being given to work using more modern measuring techniques and a more correct method of analysing the experimental data.

Brief information on the methods used in the experimental programmes whose results are given in this paper is to be found in Table X.

Table XI makes it possible to convert the Legendre polynomial coefficient for the expansion of the angular distribution of scattering in the centre-of-m system to a laboratory system of co-ordinates.

Graphs reprinted from the standard work on angular distributions [1,2] follow; these give, in the most convenient form, all the data published up to October 1962. The present paper thus comprises the results of all the experimental work published on the angular distributions of elastically scattered neutrons up to May 1964.

A complete list of the literature used appears at the end of the article.

. . . . . (Tables I - IX and Figs. 1-16)

METHODS OF EXPERIMENTAL WORK

Table X

Paper	Method	Threshold energy of detector			Correction for multiple scattering	Correction for angular resolution	Notes
1	2	3	4	5	6	7	8
(3,4)	Time of flight, threshold scintillator				Not introduced		Absolute cross-sections given with an accuracy of 15-20%
(5)					Introduced	Assessed as low	
(13)					Not introduced	Introduced	Absolute cross-sections given with an accuracy of 15%.
(37)					Not introduced	Not introduced	Relative cross-sections obtained and normalized to theoretical curve
(38)	Time of flight, annular geometry, threshold scintillator				Introduced	Introduced	
(44)	Time of flight				Not introduced	Not introduced	
(46)	Nuclear emulsions						
(52-53)	Wilson Chamber						
(54)	Annular liquid threshold scintillator						
(55)	Threshold proportional counter and liquid scintillator				Introduced	Introduced	

(pages 140-143 of original)

1	2	3	4	5	6	7	8
(56)	Threshold scintillator				Introduced		Absolute cross-sections given with an accuracy ≈ 20%
(67)	Threshold proportional counter				Introduced		
(77)	Annular geometry, threshold scintillator				Assessed as low		
(87)	Boron counter in a moderator				Not introduced		
(89)	Annular geometry, ionization chamber				Introduced		

. . . . . (Table, Graphs and Bibliography)

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VII. New data on inelastic scattering of neutrons

V. M. Sluchevskaya

This section includes experimental figures for the following: cross-sections for inelastic interactions, cross-sections for inelastic scattering, nuclear "temperatures", angular distributions of inelastically scattered neutrons, cross-sections for excitation of levels in the inelastic scattering of neutrons, cross-sections for emission of gamma rays in the inelastic scattering of neutrons, and spectra of the scattered neutrons.

This material has for the most part been obtained during the last two years, and is therefore not contained in the Directory [58].

. . . . . (Data for lithium, beryllium, boron, etc.)

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Methods of measurement

Neutron spectrometry was in most cases carried out by the time-of-flight method [1, 2, 4, 6-9, 17, 18, 22-24, 26, 28, 29, 31-33, 37, 41, 46, 47, 49, 52-55, 60, 61, 68] and by investigating the spectra of the recoil neutrons using photographic plates [57, 64, 70, 72], ionization chambers [15, 16, 39] and scintillators [36, 40, 45, 69].

Papers [13] and [27] used detectors with an electronic threshold.

Inelastic scattering has also been studied by investigating the gamma rays emitted [4, 5, 12, 14, 19, 25, 34, 35, 50, 62, 63].

In paper [35] the cross-section for inelastic scattering on Mg^{24} is obtained relative to that for the Mg^{24} (n,p) Na^{24} reaction.

In paper [56] the cross-section for excitation of the metastable state of zirconium is measured at 2.3 MeV by gamma-ray emission.

In papers [3], [42], [43] and [67] to [71] the cross-sections for inelastic interactions are determined as the differences between the total cross-sections and the experimentally determined cross-sections for elastic scattering.

Papers [10], [46] and [51] investigated the n'-γ correlation.

. . . . . (Bibliography)

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VIII. Some new data on the characteristics of fissionable nuclei

1. Average number of prompt neutrons emitted on fission of U^{235} by neutrons with an energy of 0.1 - 2.8 MeV

G.N. Smirenkin et al.

. . . . . (Table)

1) Measurements made by the method described in the article by Yu.A. Blumkina et al., Nuclear Physics, 52, 648 (1964).

2) Measurements made by recording the coincidences between a block of boron counters in paraffin and a fission chamber placed inside it containing U^{235}.

2. Average kinetic energy of fission fragments and average number of prompt neutrons in spontaneous fission of Cm^{244}

V.I. Bolshov et al.

For the spontaneous fission of Cm^{244} values were determined for the average kinetic energy of the fission fragments ($\overline{E}_k$ = 182.3 ± 2.3 MeV), the width of distribution at half-height ($\Delta \overline{E}_k$ = 24.8 ± 2.5 MeV) and the average number of prompt neutrons per fission ($\overline{\nu}$ (Cm^{244}) = 2.71 ± 0.4).

The kinetic energy of the fragments was measured using a dual ionization chamber with a grid, by comparison with the well-established figure $\overline{E}_k$ = 166.9 ± 0.8 Me for the fission of U^{235} with thermal neutrons.

$\overline{\nu}$ Cm^{244} was measured relative to the value $\overline{\nu}$ (Pu^{240}) = 2.17 ± 0.015. The method used to measure $\overline{\nu}$ was that of recording coincidences between a neutron detector (a group of counters containing BF_3 in a paraffin block) and a fission ionization chamber containing layers of Cm^{244} and Pu^{240}. The work will be described in greater detail in the journal "Atomnaya Energiya" 1964.

3. Fission cross-sections of U^{233}, U^{235} and Pu^{239} in the neutron energy range 0.3 - 2.5 MeV and of Pu^{240} in the range 0.04 - 4.0 MeV

G.N. Smirenkin et al.

. . . . . (Tables)

4. Fission cross-sections of Pa^{231} and Pu^{239} in the neutron energy range 1.5 - 1500 keV

S.M. Dubrovina and V.A. Shigin

Fig. 1 gives the fission cross-sections of Pa^{231} in the range 150 - 1700 keV.

The measurements were made on an electrostatic accelerator. The method of experiment is described in [1] and [2]. The absolute value of the fission cross-section was determined with an accuracy of 15%.

The authors have discovered three maxima in the curve for the fission cross-section of Pa^{231}, at neutron energies of 330, 550 and 880 keV. Fig. 2 gives the fission cross-section of Pu^{239} in the neutron energy range 1.5 - 1250 keV. The measurements were again made on an electrostatic accelerator. The method of experiment is described in paper [1]. The absolute value of the fission cross-section was determined with an accuracy of 10%.

The dispersion of the neutron energies in measuring the fission cross-section of Pa^{231} is $\pm$ 20 keV, for Pu^{239} $\pm$ 20 - 30 keV at neutron energies of $\geqslant$200 keV, and at neutron energies of 100, 30, 6, and 1.6 keV 10, 8, 3, and 0.8 keV respectively.

. . . . . (Figures and Bibliography)

IX. Investigations of delayed neutrons

B. P. Maksyutenko

A characteristic of this field of investigation is the fact that the first attempts to calculate the probability of the emission of delayed neutrons were not made until 14 years after the experimental discovery of their existence.

This is explained by the insufficiently rapid accumulation of material, as a result of considerable difficulties in experimenting, including low intensity of yield, short lifetimes, and laborious mathematical treatment.

(1) Groups and identification

Theoretical calculations [1, 2] predict that the precursors of delayed neutrons are the fission fragments lying close to the areas with closed neutron shells of 50 and 82 neutrons, occupying the left and right slopes of the peaks of the double-humped curve of mass distribution (light and heavy fragments respectively). A possible set contains about 50. The main contributors must be isotopes of bromine, iodine, rubidium and caesium. The absolute values of the yield as calculated in [1] and [2] sometimes differ by one order of magnitude and cannot be considered accurate enough.

Mathematical analysis of the decay curve [3] has established that all elements and isotopes subjected to fission by thermal neutrons and fission-spectrum neutrons give rise to six groups of delayed neutrons with half-lives of 55, 22, 6, 2, 0.5 and 0.2 seconds. Chemical separation [4] showed a slightly larger number of groups, with half-lives differing from the above. Using fixed criteria of selection, Keepin [1] established for each of the mathematically determined groups a set of precursors, either from among those identified chemically or postulated on the basis of theoretical calculations (Table I). A study of delayed neutrons in the spontaneous fission of californium-252 [5] showed that the groups with a half-life of 55 and 6 seconds had disappeared. Since the groups corresponding to the light-weight peak must disappear - because of its displacement for elements of higher atomic weight - we must conclude that bromine-88 cannot be a contributor to the second group, i.e. the precursor with T≈15.5 seconds must be a heavy fragment, iodine-138 cannot be a main contributor to the third group, etc. The chemical identification [4] must therefore be considered as for the most part erroneous, nor can too

much reliance be placed on what the same paper says regarding the quantitative proportions between the groups. Only bromine-87 and iodine-137 can be regarded as reliably identified. However, it should not be thought that the chemists' work has been in vain. It has shown that there may be more contributors than mathematical breakdown of the decay curve suggests. Unfortunately, even with high statistical accuracy, standard mathematical methods cannot separate more than seven to eight groups. This is because when determining both the half-lives and the initial intensities of the groups, 14 to 16 parameters must be determined. The separation of groups close to each other in half-life and relative contribution is an especially difficult task, requiring, according to our calculations, a statistical accuracy unattainable with existing neutron fluxes. The combined efforts of chemists and mathematicians might improve the situation. If chemists can reliably establish the half-lives (and this task is a simpler one, since either one group is separated in the pure form, or only two need be separated), the number of parameters to be determined for mathematical analysis of the curve can be halved, and the contribution of more groups can consequently be determined. The statistical accuracy at present at our disposal permits the separation of groups with half-lives of 15.5 and 24 seconds. We have made these calculations for plutonium-239 and isotopes of uranium [6, 7 and this paper].

(2) Empirical laws

The empirical laws obtained up to the present time may be summarized as follows:

1. For isotopes of uranium and plutonium [1, 8], and for one and the same energy of fission-inducing neutrons, the total absolute yield of delayed neutrons increases with the increase in the mass number (Fig. 1).

2. On passing from uranium-238 to plutonium-239, there is a sharp drop in total yield. The dependence of the yield on the Z of the nucleus is thus greater than on A.

3. Yields of individual groups show the same dependence on Z and A throughout the group. An exception is the group with T=55 seconds, in which there is an opposite tendency (Fig. 2).

4. We have studied the way in which the ratio between the yields of individual groups (yield of first group taken as unity) changes with a change in the energy of the neutrons causing fission. The way in which the ratio changes is shown for uranium-238 in Fig. 3 and Table II. For most groups the relative yields (i.e. relative to first group) decrease with an increase in energy (this occurs also for uranium-235 and plutonium-239 (Tables III and IV)). In the energy range where the (n, nf) reaction begins there is for uranium-238 a sharp increase in relative yields. We are at present taking measurements for other fissionable nuclei in the same range of energies.

(pages 268-270 of original)

Table I

No.	T sec	Precursors of groups (1)
		Bromine-87 Iodine-137 Bromine-88
		Iodine-138 (bromine-89, bromine-90) Iodine-139, caesium-144, bromine-91, bromine-92 Iodine-140 (bromine-92) Bromine-93, rubidium-97

Table II

Uranium-238
Relative Yields

. . . . . (Table)

Table III

Relative yields of delayed neutrons (uranium-235)

	Thermal neutrons			

T = half-life (sec)

Table IV

Plutonium-239

T sec	Thermal neutrons (1) Relative yields	Fission spectrum (1) Relative yields

Note: In these data the second group is a mixture of the groups with half-lives of 24 seconds and 15.5 seconds.

. . . . . (Remainder of Table IV Figures and Bibliography)

X. Resonance structure of total cross-sections of a number of elements for intermediate and fast neutrons

M.N. Nikolaev et al.

Below are given the characteristics of the resonance structure of the total cross-sections of 20 elements (Na, Mg, Al, P, S, K, Cr, Fe, Ni, Cu, Zn, Zr, Nb, Mo, Cd, W, Pd, Bi, U and Th), these characteristics being obtained by analysis of the transmission curves measured in a good geometry to an attenuation of $T_{min} \simeq 0.01 - 0.001$. The structure of the total cross-section is described in terms of the function $P(\sigma)$ - the probability of finding in the investigated neutron energy range a value of the total cross-section equal to σ. This function is represented approximately in the form of a weighted super-position of the standardized δ functions:

The coefficients a_n and the cross-sections σ_n are unambiguously determined by analysing the transmission curves $T(t)$ by the least squares method according to the form ..... if m and T_{min} are fixed.

Knowing the function $P(\sigma)$, it is possible to find the average values of any functions from the total cross-section ..... . Apart from the magnitudes of a_n and σ_n, Table I gives the average characteristics $\langle\sigma\rangle$ and $\left\langle\frac{1}{\sigma}\right\rangle^{-1}$ (the angular brackets denote averaging over the intervals indicated). The value of the product of $\langle\sigma\rangle$ and $\left\langle\frac{1}{\sigma}\right\rangle$, which is a convenient characteristic of cross-section structure, is also given.

Table I gives the preliminary results of such a breakdown, obtained approximately in graphical form. Accurate analysis will be carried out on a computer. It should also be noted that accurate analysis carried out for the case of uranium (Table II) showed that, at least in the case of a not too considerable cross-section structure ($\langle\sigma\rangle\left\langle\frac{1}{\sigma}\right\rangle \sim 1$), the approximate results give a reasonable estimate of the size of the real structure of the total cross-section. The last column of Table II gives the value of the correlation coefficient for the values of the two cross-sections which describe the resonance structure of the total cross-section of uranium. This figure must be taken into account in calculating errors in the calculated average characteristics. The errors given in Table II do not take into account the effect of indeterminacy in the thickness of the specimens, nor the effect of scattering onto the detector. A detailed description of the experimental method for measuring the

structure of total cross-sections, and of the methods of analysing the data given here, appears in papers [1] to [4].

. . . . . (Tables and Bibliography)

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PART II

REACTOR CONSTANTS

I. Cross-sections averaged over the Maxwell spectrum

A.N. Galanin et al.

Tables I-XII give the cross-sections of uranium and plutonium isotopes averaged over the Maxwell spectrum, which are used in the 2-group method of calculating nuclear reactors. They are given as a function of the neutron gas temperature and of the energy corresponding to the boundary where the Maxwell and Fermi spectra join. The energy dependence of σ_a, $\sigma_f\eta$ and α, and the resonance parameters were taken from papers [1-4]. The cross-sections are normalized to the agreed international values of cross-sections at 2200 m/sec given in papers [4-5].

. . . . . (Tables and Bibliography)

II. Addition to "Group Constants for Designing Nuclear Reactors" [1, 4]

L.P. Abagyan et al.

Some additions to the 26-group system of constants [1] are given below. The first is a system of constants for cerium (Ce), which is used in fast reactors with liquid metal plutonium fuel. Group constants are also given for the steels most widely used in reactor construction (EYa-1T - 73% Fe, 18% Cr, 9% Ni and EI-874 - 67% Fe, 15% Cr, 15% Ni, 3% Mo).

Some corrections to the values of the group parameters of Fe, Ni, U^{235} and Pu^{239} are then given: these are based on multi-group calculations of macroscopic tests on reactors [2] using the system of constants [1], and on new experimental and theoretical data.

These corrections are incorporated in the American edition [4] of the 26-group system of constants.

The system of notation in the group constant tables given here is that used in [1].

Finally, data are given which facilitate the introduction into the slowing-down cross-sections of corrections for the shape of the intra-group spectrum.

In the group constant tables, the cross-section of elastic slowing-down

(1)

contains the correction

(2)

which differs from unity when the intra-group spectrum is not a Fermi spectrum. $b_i(\xi)$ can be calculated by quadratic interpolation of the group spectrum:

(3)

Here φ_i is the integral flux of neutrons in the i^{th} group obtained as a first approximation (with uncorrected slowing-down cross-sections).

The coefficients A_i, B_i and C_i given below were determined in such a way that, by extrapolation of the shape of the neutron spectrum in the region of the

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[4] L.P. Abagyan et al., "Group Constants for Designing Nuclear Reactors", Consultants Bureau Enterprises Inc., New York (in press).

i^{th} group of quadratic form (3), the integral neutron fluxes in the $(i-1)^{th}$, i^{th} and $(i+1)^{th}$ groups are retained.

The coefficients for the first two groups are not given, since quadratic extrapolation of the flux is not well applicable to them.

Furthermore, the reactor spectra in this energy range are always close to the fission spectrum, which was used in paper [1] in claculating the slowing-down cross-sections of groups 1 - 3.

Paper [1], in calculating the slowing-down cross-sections in the third group, accordingly introduced correcting multipliers taking into account the difference between the spectrum used and the Fermi spectrum (taken in the lower energy groups).

The coefficients A, B and C given here for the third group take into account the fact that in order to obtain an accurate slowing-down cross-section it is sufficient to multiply the slowing-down cross-sections given in the table for the third group by b_3, calculated using the coefficients given.

. . . . . (Tables and Bibliography)

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of original)

III. Tables of coefficients for determining group parameters of anisotropy of elastic scattering of neutrons

M. N. Nikolaev et al.

To carry out accurate calculations of reactor systems and radiation shielding, it is often necessary to take into account the anisotropy of neutron scattering with greater accuracy than is the case in the normally used transport approximation with isotropic transfers from group to group. To carry out such calculations it is necessary to determine supplementary group parameters, namely the moments of the cross-section for scattering that results in the passage of neutrons from one energy group to another or in leaving them in the same group. The values for these parameters depend on the width of the energy groups used in the calculation, on the intra-group spectrum of the neutrons, and also on the form of the angular distribution of the scattered neutrons.

For elastic scattering, the expressions for these values may be written in the form

(1)

where

(2)

is the cosine of the angle of scattering in the laboratory system of co-ordinates,

μ' is the cosine of the angle of scattering in the centre-of-inertia system,

$\omega_m^{c.m.}(E')$ is the Legendre polynomial coefficient for the analysis of the elastic scattering indicatrix in the centre-of-inertia system,

σ_e is the cross-section for elastic scattering,

E' is the initial energy of the neutron,

E is the neutron energy after scattering, ...

$\tilde{E}_\ell$ and $\tilde{E}_{\ell-1}$ determine the boundaries of the energy range in group ℓ from which elastic slowing-down to group j is possible,

$\widetilde{E}_j$ and $\widetilde{E}_{j-1}$ are the boundaries of the energy range in group j into which the elastic slowing-down of neutrons from group ℓ take place,

$P_m(\mu)$ are Legendre polynomials of the m^{th} order.

$\varphi_n(E')$ is the spectrum of the n^{th} harmonic of the neutron flux.

The form of this spectrum is correlated with the energy dependence of the cross-section $\sigma_e(E')$ and the coefficients $\omega_m^{c.m.}(E')$ and is determined by expressions given in [1]. This correlation leads to an effect of resonance self-shielding of the slowing-down cross-section and of the parameters of anisotropy of scattering [1]. To separate this effect, which is linked with rapid changes of the neutron spectrum and the cross-sections, we replace the rapidly changing functions . . . . . in expression (1) with their average values in the range $\widetilde{E}_\ell - \widetilde{E}_{\ell-1}$. We then substitute the results of this averaging for the average values in the whole energy range of group ℓ, which makes it easier to allow for the dependence of the group parameters determining the slowing-down of the neutrons on the form of the intra-group spectrum, smoothed out as to its resonance properties [2]. The formulae for this averaging are given in papers [1] and [2]. As a result of this operation, expression (1) takes the form

(3)

In the second integral of formula (3), we now replace integration with respect to E by integration with respect to μ, using relationship (2). Transferring to a laboratory system of co-ordinates in the analysis of the scattering indicatrix, we obtain

(4)

where

(5)

Here ..... is the m^{th} harmonic of the elastic scattering indicatrix in a laboratory system of co-ordinates, averaged according to the spectrum of the n^{th} harmonic of the neutron flux in group ℓ.

$\varphi_n(E')$ is the neutron spectrum smoothed out as to resonance characteristics. The form of this spectrum must be fixed beforehand when compiling group constants.

The tables reproduced below give the coefficients $A_{nm}^{\ell \to j}$ for a wide range of elements used in reactor construction, calculated by formula (5) on an M-20 computer [3]. These coefficients can in principle be used to calculate the parameters of anisotropy of scattering, which must be known in order to make reactor calculations in the P--5 approximation. In cases where this calculation can be made, the degree of accuracy with which the anisotropy of scattering can be taken into account (i.e. the number of terms of the analysis of the scattering indicatrix) corresponds to the P-7 approximation.

The group intervals used in the calculations coincide with those used in paper [2]. The neutron spectrum $\varphi(E')$ is also taken, in accordance with the same paper, as equal to the fission spectrum in groups 1 and 3 and to the Fermi spectrum in the lower energy range. The programme of calculations also envisaged the possibility of finding the coefficients ω_m^{ℓ} for set angular distributions, and also of determining $\bar{W}_n^{\ell \to j}$. No provision is made in this programme, however, for taking into account the resonance self-shielding of the cross-sections. In view of the incomplete nature of the analysis of the experimental data on the angular distributions of elastically scattered neutrons and on the resonance structure of the differential scattering cross-sections, the values of $\bar{W}_n^{\ell \to j}$ are not given here.

For nuclei with comparatively low mass numbers A, the coefficients $A_{nm}^{\ell \to j}$ are given separately for each element. For heavy nuclei, they are given for groups of elements with A values close to one another.

The values of ℓ and j are given above each table. In cases where the coefficients $A_{nm}^{\ell \to j}$ coincide for different values of ℓ and j, a single table is given above which are indicated all the values of ℓ and j.

In the case of heavy nuclei it is possible when calculating equation (4) to use only two terms, corresponding to the values m = n and m = n + 1. To save space, a slightly different layout for presenting the data is used in this case.

. . . . . (Bibliography and Tables)

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of original)

IV. Group cross-sections of some nuclear reactions used for detecting neutrons

V.I. Golubev et al.

The most widely used method of checking the accuracy of multigroup reactor calculations is to compare the experimental distributions of the rates of various reactions having different cross-section vs. energy variations with the calculated distributions, and also to compare the calculated and experimental ratios of the cross-sections of these reactions on reactor spectra.

To be able to perform the calculations for a given type of experiment, we must have a set of the average group cross-sections used for testing the reactions. The number of groups in this set, and also their energy boundaries, must correspond to the breakdown used in calculating nuclear flux.

The most complete system of constants at present in use for reactor calculations is the 26-group system [1]. The group cross-sections given here, for the reactions normally used for macroscopic tests on reactors, presume the use of this system of constants for calculating neutron fluxes. Where the cross-sections of reactions used for detecting neutrons are given in paper [1], no attempt is therefore made to review them here.

The initial data used to compile the tables of the group cross-sections were the published results of individual small-scale experiments, and also information on the parameters of the resonance levels investigated. The papers used to compile the tables given below are listed in reference [2].

In the energy ranges where sufficient experimental data were available, the average cross-sections were obtained from smooth curves passing through the experimental points.

In ranges where there was sufficient information on the parameters of the resonance levels, the resonance capture cross-sections were calculated by adding the integrals of the separate resonances composing the group.

When there were in a particular energy range neither experimental data nor information on the parameters of individual resonances, the radiative capture cross-sections were calculated on the basis of the average resonance parameters. In this case the energy dependence of the contributions made to the capture

cross-section by neutrons with different ortbital movements was taken into account.

The cross-sections of the $Np^{237}(n,f)$ reaction at energies above 10 keV were obtained from the average of the known experimental data. At lower energies, the cross-section was determined from the theory of sub-barrier fission and from the available data on resonance neutrons. In the energy range 0.2 - 1 eV the figure given is the higher estimate for the fission cross-section of Np^{237} according to the parameters of a resonance at 0.49 eV.

Apart from the average group cross-sections, the tables also contain the calculated values for resonance integrals and cross-sections for the fission spectrum. Both these sets of figures are compared with experimental results. It should be noted that all the experimental data used in this work have an accuracy not exceeding 5%. The accuracy of our knowledge of the parameters of resonance levels is, with a few exceptions, even lower. Therefore, when experimental and calculated distributions of nuclear reaction rates are compared, agreement to within 10-20% should be considered satisfactory, the more so since in calculating neutron fluxes the constants used are generally of the same degree of accuracy (especially when the effect of resonance self-shielding of the cross-sections is great).

The proposed average cross-sections of nuclear reactions have been checked in macroscopic tests on a BR-1 reactor [3].

. . . . . (Bibliography and Table)

(page 374 of original - at bottom of table)

Cross-sections for the fission spectrum

Measured

Calculated

Resonance integrals of capture

Measured

Calculated

V. Empirical constants used in calculating radiation shielding for nuclear reactors

1. Coefficients of secondary gamma radiation

S.P. Belov and Yu.A. Kazansky

The intensity of the gamma-radiation caused by the radiative capture of neutrons in the shielding (secondary or capture gamma radiation) is conventionally normalized to the all-wave integral neutron flux behind the shielding. The coefficient of secondary gamma-radiation is determined as a ratio of the total number of gamma quanta to the total number of neutrons leaving the shielding (QB) [1].

Using the coefficient of secondary radiation, it is possible to determine the gamma flux $J(R,\vartheta)$ at a point behind the shielding with co-ordinates R and ϑ: $J(R,\vartheta) = BQbG(R,\vartheta)$ where $G(R,\vartheta)$ is the geometrical factor of attenuation, which can be calculated comparatively easily in each particular case [1, 2, 3].

As has been shown in references [1] and [4], the values of B for a shielding thickness of 150 - 200 g/cm^2 are in practice constant, if the length of the mean free path is less than the analogous figure for the neutrons of the leading group, and depend on the physical properties of the medium, i.e. the linear coefficient of gamma attenuation, the cross-sections of interaction of the neutrons with the nuclei, etc. The coefficients of secondary gamma-radiation can be experimentally determined without resorting to absolute measurement of gamma and neutron fluxes. The accuracy of the measurements of B for gamma-rays with an energy of more than 4 - 6 MeV is ~10%. At lower energies the error increases considerably as a result of the indeterminacy of the number of gamma quanta emitted on the capture of a neutron.

The table gives, in percentages, the experimental asymptotic values of the coefficients of secondary gamma-radiation for a number of materials and two neutron sources [1, 4, 5] (Po-α-Be), and the spectrum of the RIZ reactor [5].

The coefficients of secondary radiation are determined for gamma quanta with energies higher than E_γ, the values of which are given in the table.

. . . . . (Table and Bibliography)

(page 378 of original)

2. Angular distribution of intensity of scattered gamma-rays on the boundary of a semi-infinite medium

Yu.A. Kazansky and E.S. Matusevich

The angular distribution of intensity from the gamma-rays emitted by an isotropic point source on the boundary of a semi-infinite medium is determined by the expression

(1)

where E_0 is the energy of the source, R is the distance between the source and the boundary of the medium, ϑ is the angle between the direction from the source to the observation point and the direction of observation.

This distribution can be represented by an expression with one experimentally determined parameter ϑ_c^T

(2)

where I_0 is the intensity of the source, μ_0 is the linear coefficient of attenuation of the gamma-rays from the source in the material of the medium, B_∞ is the energy factor of accumulation for a point isotropic source in a medium of infinite extent. Representation of J_ϑ^T in the form (2) is valid for the range of angles $5^\circ < \vartheta < 90^\circ$ with an accuracy of the order of 10%. The empirical magnitude ϑ_c^T is independent of R to an accuracy of up to 10% when $\mu R > 1$.

The experimental values of ϑ_c^T from papers [1], [2] and [3] are given in the following table.

Source energy	ϑ_c^T degrees		
	Medium		
	Water	Iron	Lead
0.411 MeV	-	49 ± 3	
1.25 MeV	38 ± 2	33 ± 2	19 ± 1
2.76 MeV	30 ± 3	-	-

The errors take into account a slight change in ϑ_c^T depending on μ_0^R. The values of ϑ_c^T for intermediate Z values of the medium can be obtained by linear interpolation.

For a plane monodirectional source, with normal incidence on a barrier of thickness d, the value of the intensity of radiation per unit solid angle may also be represented by the following expression

(3)

in which J^n_ϑ depends on one parameter ϑ^n_c

(4)

where q_E is the energy emitted by a unit surface of the source and B_d is the energy factor of accumulation for a barrier of thickness d ($B_d \approx B_\infty$). The formula is valid for $15 < \vartheta < 90^\circ$, and values of ϑ^n_c are independent of $\mu_o d$ to an accuracy of up to 10%.

The experimental values of ϑ^n_c obtained in papers [4] to [8] are given in the following table.

Values of ϑ^n_c in degrees

Source energy	Medium		
	Aluminium	Iron	Lead
0.28 MeV	40		32
0.411 MeV	32		14
0.66 MeV	24	21	16
1.25 MeV	21	19	15
2.76 MeV	16.5		13
4.0 MeV			12.5

The error in the ϑ^n_c values is $\pm$ 10%. Linear interpolation according to Z is permissible. In formulae (2) and (4) ϑ^T_c and ϑ^n_c are given in radians.

. . . . . (Bibliography)

3. Some constants determining flux and dose rate of fast neutrons in air at different distances from the source

S.F. Degtyaryov and V.I. Kukhtevich

A. Infinite air medium

If R is the distance from the source to the detector, Σ_s is the macroscopic scattering cross-section of air, Q is the source intensity and E_o is the initial energy of the neutrons, then the expression

(1),

where ..... is the flux of scattered neutrons, and ..... is the flux of direct radiation (.....), will be valid for a point isotropic source and an isotropic detector.

Expression (1) is valid for $0 \leqslant R \leqslant 50$ m and for initial neutron energies from 0.33 to 14 MeV.

For the same geometric conditions, if Σ_t is the total macroscopic cross-section of interaction, then

(2)

where $B(E_o) = 0.195 \pm 0.02$ for $0.33 \leqslant E_o \leqslant 6$ MeV and 0.145 ± 0.015 for $7 \leqslant E_o \leqslant 14$ MeV, $D_{рас}$ is the dose rate due to the scattered neutrons, and $D_{пр}$ is the dose rate of the direct radiation ($D_{пр}$ = ... where E_o is the coefficient converting a single neutron flux into dose rate).

The angular distribution from a point isotropic source, and the spatial distribution from a point monodirectional source, are described by the expressions

(3)

where ϑ is the angle of orientation of the monodirectional source or detector relative to the axis connecting them.

For dose rate the expressions take the form

(4)

The accuracy of expressions (3) and (4) is $\pm$ 20% for distances of $0 \leqslant R \leqslant 50$ m. In (1) - (4) R is expressed in m; Σ_t in m^{-1}, and in (3) and (4) ϑ is expressed in degrees.

B. Air-earth separation boundary

If H is the equidistant height of the isotropic source and the isotropic detector above the earth's surface, and R is the distance from the source to the detector, then

(5)

where

R and H are expressed in m, ... in m^{-1}. The accuracy of expression (5) is $\pm$ 10% in an initial-neutron-energy range from 1 MeV to 5 MeV. The limits of applicability of (5) are $5 \leqslant R \leqslant 30$ m and $1 \text{ m} \leqslant H \leqslant 30$ m. Expressions (1) - (5) are obtained from the results of papers [1] - [6].

. . . . . (Bibliography)

4. Passage of fast neutrons from a reactor through shielding not containing hydrogen

B.I. Sinitsyn and S.G. Tsypin

In media not containing hydrogen, the functions of attenuation of a fast-neutron flux ($E_n > 3$ MeV) at medium distance from the source of a fission spectrum*/ can normally be approximated with sufficient accuracy by functions of the type**/

(1)

where R is the thickness of the shielding and q is an empirical parameter equivalent to the reciprocal neutron relaxation length $\frac{1}{\lambda}$, which will be determined below.

The range of applicability of expression (1) is normally limited to from 2 to 15--20 mean free paths of the neutrons. With high shielding thickness, the effect of filtration of the neutrons by the shielding material may have a considerable effect on the relaxation length. In this range the relaxation length will be determined to an increasing extent by the minimum - for the energy range investigated - cross-section of interaction of the neutrons with the shielding material.

The following values are normally used as the parameter q:

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*/ The spectra of neutrons in reactors in the energy range above 3 MeV normally differ little from that of fission neutrons.

**/ For plane-parallel geometry.

(1) The removal cross-sections Σ_{rem}, determined for the group of neutrons with energy E> 3 MeV [1];

(2) The cross-sections obtained from the reciprocal relaxation lengths $\Sigma_{\frac{1}{\lambda}} = \frac{1}{\lambda}$ for the group of neutrons with an energy E>3 MeV [1];

(3) The asymptotic cross-sections Σ_{as} obtained by solving the one-group kinetic equation, using the expression

(2)

obtained in the transport approximation, using group constants systems [3], [4] for the energy groups 1.4 MeV to ∞ and 2.5 MeV to ∞.

In expression (2) the following symbols are used: Σ_{tr} - the total transport cross-section in the group, and Σ^{s}_{tr} - the transport scattering cross-section in the group. $\Sigma_{tr} = \Sigma^{s}_{tr} + \Sigma_{yb}$, Σ_{yb} being the cross-section for removal from the group.

The table contains the empirical parameters ..... (ϱ is the density of the nuclei), taken from the numerous sources in the literature. It can be seen from the table that these parameters agree satisfactorily (the maximum deviation from the average does not exceed 10%). This makes it possible to use these data for estimates of nuclear reactor shielding.

. . . . . (Table and Bibliography)

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of original)

VI. Critical parameters and nuclear safety

B. G. Dubovsky et al.

The authors have carried out a series of critical experiments aimed at nuclear safety and the study of reactor physics. The results of some of these experiments are given below.

The experiments were conducted with aqueous solutions of $UO_2(NO_3)_2$. The temperature of the solutions varied from 15 to 21°C, and the number of nitrogen nuclei ϱ_N was between 3.0 and 3.1 times as large as the number of uranium nuclei ϱ_U. The uranium concentration of the solution was varied from 40 to 500 g/l.

The experiments were carried out on reactors in the form of spheres, cylinders and rectangular parallelipipeds made of stainless steel 1-1.5 mm thick (3 mm for the parallelipipeds).

All the experiments may be classified in one of the following groups:

1. Critical parameters of reactors of different shapes.
2. Efficiency of neutron absorbers.
3. Interaction of sub-critical systems.

1. Critical parameters of reactors of different shapes

The dependence of the critical volumes of the solution and the critical mass of the uranium on uranium concentration was studied for reactors of different shapes with a water reflector and with no reflector. The results of the experiments with uranium of 90% enrichment are given in Figs. 1 and 2, and those with uranium of 10% enrichment in Table I.

. . . . . (Table I)

2. Efficiency of neutron absorbers

Boron and cadmium absorbers in the form of rods and inserts were studied.

(a) Rods

The absorbing rod was a cylindrical sheath of stainless steel containing either boron carbide powder with a density of 1.25 g/cm^3, or a cadmium tube 0.5 mm thick filled with water.

The dependence of the efficiency of the rod on its diameter, the thickness of the steel casing, the diameter of the core and the position of the rod in the core was studied for various concentrations of uranium in the solution, for 90%-enriched uranium.

The interference of two or more rods as a function of the distance between them was also studied.

The effectiveness of lattices of absorbent rods as a function of the lattice pitch and the number of rods in the core was studied.

Figs. 3-8 give some of the results of experiments with rods and rod clusters.

(b) Inserts

The absorbing inserts were two coaxial stainless steel cylinders with the absorbent placed between them.

Two types were investigated - "thin" ones in which the absorbent was either boron carbide powder with a density of 1.25 g/cm^3, 6 mm thick, or cadmium 0.5 mm thick; and "thick" ones in which the absorbent consisted of two 0.5 mm layers of cadmium on the cylinder walls, with either 25 or 50 mm of water in between.

The efficiency of the absorbing inserts as a function of the average radius of the insert and the thickness of the layer of water was studied for various concentrations of uranium in the solution.

Figs. 9-10 give some of the results of these experiments.

3. Interaction of sub-critical systems

Interaction was studied on reactors in the shape of rectangular parallelipipeds with a 30 cm base, and of cylinders 30 mm in diameter, for various concentrations of 90%-enriched uranium.

Figs. 11-13 give the results of some of the experiments. The diagram shows the dependence of the volume of the solution in one reactor on the distance between reactors, on condition that all the reactors are critical and the amount of solution is the same in each.

The interaction of these reactors through water, graphite and boron carbide was also studied.

A semi-empirical method of calculating the interactions of sub-critical reactors, known as the method of "equivalent dimensions", was developed in the laboratory. This method consists essentially of replacing the whole system of reactors by a single reactor, determining the equivalent dimensions of this reactor and comparing the geometrical specification of the equivalent reactor with that of a reactor of the particular design in question. The formula used to determine the equivalent dimensions of the reactor is $C_{equiv.} = C\,[1 + (n-1)\Omega]$, where C is the dimension of one sub-critical reactor in the direction of interaction, n is the number of interacting reactors in the direction of C, and Ω is the solid angle between two neighbouring reactors in the direction of interaction.

Figs. 11-13 compare the results of calculations by the "equivalent dimensions" method with experiment.

The interaction of a large number of sub-critical reactors of 6-litre capacity was also studied. The reactors were constituted by cylindrical glass vessels 18 cm in diameter and 24 cm high. The wall thickness was 0.5 cm and the concentration of uranium in the solution was 96 g/l.

The vessels were arranged in several planes at different distances from each other, forming a three-dimensional network. The results of these experiments are given in Table II.

As can be seen from the diagrams, with a small number of reactors arranged in one plane the calculation ensures a 10-15% safety margin. In the case of three-dimensional networks (Table II) this margin increases considerably, and may exceed 100%. It emerges from the review paper [1] that the accuracy of the proposed method is comparable with that of other methods of calculating systems of interacting reactors; it is distinguished from them, however, by its considerably greater simplicity and convenience.

. . . . . (Bibliography)

Table II

Arrangement of containers*/	No. of containers		
	Used	Critical	Calculation
In two planes	23	23	12.
d_x = 1 cm	In first plane 4 x 3 = 12		
d_y = 4.5 cm	In second plane /4 x 3/ - 1 = 11		
d_z = 12 cm			
In three planes	52	54 ± 0.5	18
$d_x = d_y$ = 6.5 cm	In first plane 4 x 4 = 16		
d_x = 12 cm	In second plane 4 x 5 = 20	/Extrapolation/	
	In third plane 4 x 4 = 16		
In three planes	39	39.5 ± 0.3	12.
$d_x = d_y$ = 4.5 cm	In first plane 3 x 4 = 12		
d_z = 12 cm	In second plane /3 x 4/ + 3 = 15	/Extrapolation/	
	In third plane 3 x 4 = 12		
In four planes	67	80 ± 5	36
$d_x = d_y$ = 9 cm	In first plane 4 x 4 = 16		
d_z = 11 cm	In second plane 4 x 4 = 16	/Extrapolation/	
	In third plane /4 x 5/ - 1 = 19		
	In fourth plane 4 x 4 = 16		
In four planes	64	72 ± 5	27
$d_x = d_y$ = 7.5 cm	In each plane 4 x 4 = 16		
d_z = 12 cm		/Extrapolation/	

*/ d_x and d_y – distances in plane; d_z – distance between planes. All distances take into account wall thickness, i.e. are distances between solutions.

Captions to diagrams

Fig. 1. Critical masses and volumes of aqueous solutions of $UO_2(NO_3)_2$
(in top right-hand corner)

Cylinders.

With full H_2O reflector:

Without upper reflector:

Rectangular tanks without reflector.

Fig. 2. Critical masses and volumes of aqueous solutions of $UO_2(NO_3)_2$.
(in top right-hand corner)

Spheres without reflector:

- experiment, curve 1.
- calculation of geometric parameter, curve 2.

Spheres with water reflector /curve 8/:

- experiment - calculation of geometrical parameter with extrapolation length ......

Cylinders without reflector:

Fig. 3. Efficiency of central boron carbide rods and water-filled cadmium tubes as a function of rod radius through absorbent. Reactor without reflector. Diameter of core 400 mm.

1. Cadmium tube 0.5 mm thick, water-filled. Thickness of steel casing 2-3 mm.
2. Boron carbide rods. Thickness of steel casing 2-3 mm.

Uranium concentration in the solution 286 g/l.

Fig. 4. Efficiency of central boron carbide rods and water-filled cadmium tubes as a function of rod radius through absorbent. Reactor without reflector. Diameter of core 400 mm. Uranium concentration in the solution 136 g/l. Critical volume of reactor without rod 25.6 l.

1. Water-filled cadmium tube 0.5 mm thick.
2. Boron carbide rod. Thickness of steel casing 1 mm.

Fig. 5. Critical volumes of reactors with and without an absorbing rod as a function of core diameter. Reactor with reflector. Diameter of absorbing rod through boron carbide 50 mm. Thickness of steel casing 4 mm.

1. Reactor with rod.
2. Reactor without rod.

Curves – uranium concentration in the solution 286 g/l
Curves – uranium concentration in the solution 72 g/l.

Fig. 6. Efficiency of one and two absorbing rods as a function of their position along the radius of the core. Reactor without reflector. Diameter of core – 400 mm. Uranium concentration in the solution 286 g/l. Diameter of absorbing rod through boron carbide 24 mm. Thickness of steel casing 4 mm.

1. Efficiency of one rod.
2. Efficiency of two rods.
3. Double efficiency of one rod.

Fig. 7. Efficiency of seven rods. Diameter of core 400 mm. Reactor with side and bottom water reflectors

1. Boron carbide rods 50 mm in diameter. Thickness of steel casing 4 mm. Uranium concentration 281 g/l. Critical volume without rods 18.07 l.
2. Cadmium rods 46 mm in diameter, thickness of cadmium 0.5 mm. Thickness of steel casing 5.5 mm. Uranium concentration 457 g/l. Critical volume without rods 17.56 l.

Fig. 8. Efficiency of a cluster of boron rods. Diameter of core 40 cm. Reactor with side and bottom water reflectors. Uranium concentration 281 g/l.

(at top)

Lattice pitch 40 mm. Diameter of rod 16 mm.
" " " " " " " " "

with central rod
without central rod

Fig. 9. Efficiency of boron and cadmium annular inserts. Cylindrical core 40 mm in diameter. Uranium concentration 288 g/l

O – boron insert
Δ – cadmium insert

Curves 1 and 2: with side and bottom water reflectors
Curves 3 and 4: without reflectors

Fig. 10. Efficiency of cadmium annular inserts as a function of the average radius of the annular layer. Cylindrical core 40 cm in diameter with side and bottom water reflectors. Uranium concentration 281 g/l

(at top)

O – annular insert 25 mm thick
Δ – " " 50 mm "
□ – " " 50 mm " with polyethylene instead of water

With an annular insert of average radius 100 mm and thickness of layer 50 mm, 48 l of solution were poured in. Extrapolation of inverse multiplication curves gives a critical volume of 75 l.

(at bottom)

Critical volume without insert – 18.1 l.

Fig. 11. Interaction of parallelipiped reactors with square base = = 300 mm

1 and 2: two and three reactors respectively.

3: calculation for two reactors.

4: calculation for three reactors.

Uranium concentration in the solution 71 g/l

Arrangement of reactors -

Fig. 12. Interaction of parallelipiped reactors with square base a = b = 300 mm

1, 2, 3: three, four and five reactors respectively.

4: calculation (first approximation).

Uranium concentration in the solution 71 g/l

Arrangement of reactors -

Fig. 13. Interaction of two cylindrical reactors in water.

Diameter of core 30 cm

1. Experiment

2. Calculation

Uranium concentration in the solution 113 g/l.

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PART III

METHODOLOGICAL QUESTIONS

I. Calculating neutron cross-sections from the optical model using a computer

V.E. Kolesov

The development of methods of calculating various cross-sections, and also the carrying out of such calculations, are of great practical as well as theoretical interest. The data obtained may be used, for example, to establish systems of multi-group constants for calculating reactors and their biological shielding.

The optical model of the nucleus [1] is widely used to calculate cross-sections. It satisfactorily describes in a limited energy range not only the total cross-section σ_t, but also its separate components. Apart from σ_t, it also permits calculation of what is known as the cross-section of optical elastic scattering σ_n^s, the cross-section of formation of the compound nucleus (capture cross-section) σ_c, the transport cross-section σ_{tr}, and the differential cross-section of optical elastic scattering $\frac{d\sigma_n^s}{d\Omega}$. "Optical elastic scattering" here means the elastic scattering which proceeds without the formation of a compound state. This cross-section is sometimes called the "cross-section of potential scattering". We follow the text-book [2] for the designation of the various cross-sections.

The optical model is also widely used for calculating the cross-sections of inelastic scattering $\sigma_{n'}$ and radiative capture σ_γ, and in calculating the polarization of neutrons. The data obtained by calculations on this model may also be used to solve other important problems.

The various cross-sections and the degree of polarization of the neutrons are expressed by the coefficients $\eta_{\ell j}$, which determine the ratio between the amplitudes of the outgoing and ingoing spherical waves in the expression for the neutron wave function in the entry channel. For nuclear potentials with a spin-orbital component $\eta_{\ell j}$ depends not only on the orbital moment ℓ of the amount of movement of the neutron, but also on the total moment $j = l\ell \pm \frac{1}{2}l$.

To find the amplitude of $\eta_{\ell j}$, it is necessary to solve Schrödinger's equation for the radial part of the wave function. With the forms of optical potential at

present in general use, this equation is solved approximately, or various numerical methods are used which give a final result of sufficient accuracy. Computers are normally used for carrying out the numerical algorithms.

The present review describes briefly the numerical method of finding the coefficients $\eta_{\ell j}$, and a number of programmes based on this method, which make it possible to calculate the various neutron cross-sections within the framework of the optical model of the nucleus. Information is also given regarding the tables for certain cross-sections, coefficients η, and penetration factors T, calculated according to these programmes with different types of potential.

. . . . . (Acknowledgements)

Method of calculating cross-sections

In the region of space where the nuclear potential $V_{\ell j}(r)$ may be regarded as equal to zero, there exists for Schrödinger's radial equation [3]

(1)

an analytical solution in the form of a linear combination of cylindrical functions. The problem is to find a solution of this equation in the range of action of nuclear forces. Here the physical boundary condition at zero obtains:

(2)

As is known from [3], the coefficients $\eta_{\ell j}$ are expressed by the logarithmic derivative of equation (1)

(3)

at a certain point r = H where the potential is already close to zero with a sufficient degree of accuracy. If equation (3) is regarded as a second boundary condition, equation (1), together with (2) and (3), forms a boundary value problem in which the logarithmic derivative is an unknown factor.

The solution of this problem by the method of finite differences using the simplest 3-point difference scheme leads to a system of high-order linear algebraic levels. The system being necessarily solvable, expressions can be obtained for the logarithmic derivative and the recurrent relationship for calculating the determinant deduced.

For a complex potential, the real and imaginary parts of the logarithmic derivative are equal

where h is the interval of the finite difference system. The values of P_{n-1}, P_{n-2}, Q_{n-1} and Q_{n-2}, which have to be known in order to calculate $f_{\ell j}$, are worked out by the recurrent formulas

(4)

Here

The form of the initial conditions for the recurrent relationships depends on where the boundary condition (2) of the problem is placed. When it is set in the form of a logarithmic derivative of $\pi_{\ell j}$ of the type of (3), at a certain point in the neighbourhood of zero the initial conditions for (4) take the form

where

The function F(r) is completely determined by the form of the potential $V_{\ell j}(r)$.

No particular restrictions are placed on the nuclear potential; its form may vary over a wide range. The method permits simple calculation on a computer, and is satisfactorily accurate, with a comparatively low consumption of machine time. The method described here of finding the logarithmic derivative is set out in more detail in reference [4]. The method has considerable general practicability, and may be used to calculate not only the cross-sections, but also the bound energy states in the nuclear potential.

The amplitudes of the outgoing waves $\eta_{\ell j}$ are expressed as follows via $f_{\ell j}$

The values of Δ_ℓ, S_ℓ and C_ℓ are completely determined by the conditions in the region of space where the potential is equal to zero, and are set using the relationships

....

....

The spherical Bessel functions $j_\nu(x)$ entering into these formulae depend on the variable $x = kH$ and are expressed in their final form by means of trigonometric functions.

Thus, knowing the logarithmic derivative $F_{\ell j}$, it is possible to calculate, by known formulae, the cross-sections for the interaction of neutrons with the nuclei. For example the expressions for the integral cross-sections σ_n^s, σ_c and σ_t take the form

where

(5)

the so-called penetration factors, which represent the probability of a neutron penetrating the surface of the nucleus. These factors are widely used in calculating the cross-sections of inelastic scattering and radiative capture of neutrons.

Characteristics of programmes

The method described above proved very effective, and was used as the basis of a number of programmes for calculating various neutron cross-sections, employing the optical model of the nucleus. The possibilities of some programmes with potentials of different forms for different computers are described below.

1. Programmes for the "Strela" computer

Two types of programme are possible for this machine: (a) for potential without spin-orbit interaction [5] and (b) for potential taking the spin-orbit interaction into account [6].

(a) The original variant of the programme [5] was set up for the potential

(6)

with spin-orbit interaction not taken into account. The programme permits calculation of σ_n^s, σ_c, σ_t and the differential cross-section $\frac{d\sigma_n^s}{d\Omega}$, in this case expressed as follows via η_ℓ and the Legendre polynomials $P_\ell(\mu)$

Certain alterations were subsequently made to the programme [7] so that the formula could also be used to calculate the transport cross-section σ_{tr}:

. . . . . .

The programme was so arranged that after one insertion of the initial information it was possible to calculate the cross-sections for five nuclei with any mass numbers A and any parameters of potential (6) for each A. The number of variants in energy and the maximum orbital moment ℓ_{max} were set. The number of nuclei, and also the number of variants in energy and the number of harmonics for which the calculation is carried out, can be reduced to any figure. Reduction of ℓ_{max} gives in many cases a notable saving of machine time, without detriment to the accuracy of the calculations. To save time, the programme provides for the possibility of not calculating the angular distributions for some, or even for all, energies.

The parameters of potential and the other data and constants necessary for the calculations and for controlling the operation of the computer are inserted into the operative memory of the machine together with the programme, as initial data for the calculation. The final results, printed or on punched cards, are A, E(MeV), the cross-sections σ_n^s, σ_c, σ_t and σ_{tr} in barns, the values of $\mathrm{Re}\eta_\ell$ and the penetration factor T_ℓ (with signs of the corresponding values of $\mathrm{Im}\eta_\ell$) for each ℓ, and the angular distributions $\sigma_n^s(\mu)$ (in b/sr) for μ in the range from -1 to +1 at intervals of 0.1. The value of $\frac{\pi}{k^2}$ is also obtained (in units of 10^{-24} cm^2), which makes it possible to quickly calculate the partial cross-sections $\sigma_c(\ell)$. Since the penetration factors T_ℓ are positive in their physical sense, it is possible, knowing the sign of $\mathrm{Im}\eta_\ell$, to re-establish the imaginary part of the amplitude $\mathrm{Im}\eta_\ell$ from the known $\mathrm{Re}\eta_\ell$ and T_ℓ using (5).

The required accuracy of calculation can be attained by correct choice of the interval h. Experience shows that it takes an average of about a minute to calculate one variant (one energy with a fixed set of parameters of potential) with an interval of h = 0.1, which gives the final results an accuracy of the order of 0.5%. This time varies slightly for light and heavy nuclei, and depends on the energy, i.e. on the value of ℓ_{max}.

The programme makes provision for control of the accuracy of the calculations by machine; this is achieved by repeated checks using magnetic tape. If the control totals of two checks do not agree, the programme provides for a third check. The machine stops if all three totals do not agree.

(b) The programme [6] for the case when the nuclear potential contains a spin-orbital component of the type (for $\ell \neq 0$)

(7)

where V(r) is set by formula (6), is a modification of the previous programme [5]. They therefore both have the same possibilities except that the programme taking into account the spin-orbit interaction does not provide for calculation of the angular distributions and transport cross-sections. The time taken to calculate the cross-sections is roughly doubled in this case. It is also possible to use the programme for calculations with a potential having no spin-orbital component, if χ is taken as equal to 0.

2. Programmes for the BESM-2 computer

Two programmes will be described. One of them [8] affords the possibility of calculating, as well as the other cross-sections, those for inelastic scattering and radiative capture of neutrons. The second [9] provides for calculations according to the optical model in the case of both volume and surface absorption of neutrons.

(a) The programme [8] takes the potential (6) corresponding to the case of volume absorption. For a nucleus with mass number A, particular parameters of potential and the other nuclear characteristics necessary for the calculations, the programme permits calculation of the following cross-sections

where $\sigma_{n'}^{(m)}$ is the cross-section for inelastic scattering with excitation of the m^{th} level and $\sigma_{\gamma}^{(\ell)}$ is the partial cross-section for radiative capture for the ℓ^{th} harmonic. Apart from calculating the above cross-section it is also possible to calculate their various combinations. This must be reflected by setting the appropriate constant in the special sign cell.

The functions of excitation of the separate levels $\sigma_{n'}^{(m)}$ are calculated by the Hauser-Feshbach formula [10], and the cross-section σ_{γ}, taking into account the concurrence of elastic and inelastic scattering, by the formula of Margolis [11]. The theory of these cross-sections assumes that the processes reviewed pass through the stage of a compound nucleus, whose decay does not depend on the method of formation. It is accepted that statistical inspection is valid for a compound nucleus. In the calculations, it is also assumed that the variation of the energy dependence of the neutron penetration factors does not depend on whether the nucleus is in the ground state or in one of its excited states. The energy dependence of the radiation widths is taken into account. For the density of the levels, the simplest relationship which follows from the Fermi-gas model is used.

The initial data required for calculating the cross-sections are placed in the special setting zone on punched tape. The operator sets the parameters of optical potential, energy, spin and symmetry of the levels excited in inelastic scattering, the parameter characterizing density of the levels in the Fermi-gas model, the ratio of the average distance between the levels of the compound nucleus to the average radiation width, multiplied by 2π, at an excitation energy equal to the effective energy (taking into account the energy of pairing) of the neutron bond in the compound nucleus, and the effective energy of the bond, together with certain other data and constants for controlling the operation of the machine. The symmetry and spin characterizing the state of the nucleus are set in combination, the symmetry being set in the form of a sign on the appropriate spin value. The number of levels of the nucleus may be as high as 15. The number of energy variants may be set at any value desired by using a special constant in the setting zone. The number of harmonics contributing to each different type of cross-section is determined automatically using the conditions

where Σ is a positive number, set in the setting zone, determined by the accuracy required in the results.

The final results printed are A, E(MeV), the cross-sections (in barns), and also the values of $\mathrm{Re}\eta_\ell$ and T_ℓ for different values of ℓ and the constant $\frac{\pi}{k^2}$ (in barns). The cross-sections $\sigma_n^s(\mu)$ (in b/sr) are printed for different μ values ($-1 \leqslant \mu \leqslant 1$) at intervals of 0.1. The signs of the coefficients $\mathrm{Im}\eta_\ell$ are used for the penetration factors T_ℓ. This makes it possible to re-establish from known T_ℓ and $\mathrm{Re}\eta_\ell$ figures the values of $\mathrm{Im}\eta_\ell$.

The time taken by the machine to calculate one energy variant for a given A depends on the nature of the cross-sections calculated. Calculating inelastic scattering cross-sections takes longest. Double counting is used as a check that the machine is operating properly. Renewal of the programme in the case of faulty operation is done from a magnetic drum.

(b) The second programme [9] permits calculation for two types of optical potential when spin-orbit interaction is absent: potential of the type (6), and a potential whose real part coincides with (6) and whose imaginary part has the form

(8)

The first of these potentials corresponds to the case of volume absorption and the second to that of surface absorption. The machine is set for one case or the other by setting a special constant in the setting zone.

In its characteristics and possibilities the programme is generally similar to that described above for the "Strela" computer [5], but requires three to four times less machine time for calculating a variant.

3. At present programme 12 has been set up, to calculate various cross-sections including those for inelastic scattering and radiative capture of neutrons. The experience gained in using the programmes described above was taken into account in drawing up this programme. This made it possible to introduce a number of modifications which considerably improved it. In particular, it became possible to considerably shorten the time taken to calculate one variant, especially when calculating the very labour-consuming cross-sections for inelastic scattering (up to several minutes).

As regards type of potential, the programme permits calculation for the following four cases:

(1) Volume absorption without spin-orbit interaction

(2) Volume absorption taking the spin-orbit interaction into account

(3) Surface absorption without spin-orbit interaction

(4) Surface absorption taking the spin-orbit interaction into account.

The form of the potentials is given by expressions (6), (7) and (8). From the point of view of possibilities, this programme combines the advantages of both the programmes for the BESM-2.

Results of the calculations

The programmes described above are at present being used for calculating different types of cross-section, and the data obtained from these calculations are widely employed in the analysis and systematization of experimental data, as well as for other purposes. The programmes have also been used to compile extensive tables of certain cross-sections, the coefficients η and the penetration factors T for a wide range of energies and atomic weights, and for potentials of different forms with certain optimum parameters. These tables can be of use for all kinds of rough calculations.

As has already been observed, to calculate the cross-sections and the polarization of neutrons according to the optical model, it is sufficient to have only complex amplitudes. However, to calculate $\sigma_{n'}$ and σ_{γ} only the penetration factors T, which are certain combinations of η, are necessary. For convenience in calculating these cross-sections, the literature often gives data only for T. Detailed calculations of T for the case of surface absorption, taking into account the spin-orbit interaction, have been carried out by the authors of paper [13], published in 1963 in the form of a report. The calculations cover the energy range from 0.1 MeV to 5 MeV at A = 100 and up to 3 MeV at A = 100 (sic). Some data on calculating the coefficients T are also given in the monograph of P.E. Nemirovsky [1]. However, knowledge of T alone is in many cases insufficient for calculations according to the optical model. It is therefore certainly desirable to have data on the amplitudes η.

We give here some information about the tables containing data on the cross-sections and the coefficients η and T. At present such tables exist for three types of nuclear potential.

1. Volume absorption not taking the spin-orbit interaction into account [14]

The tables include the cross-sections σ^{s}_{n}, σ_{c}, σ_{t}, σ_{tr} and the values of $\frac{\pi}{k^2}$, $\mathrm{Re}\eta_{\ell}$ and T_{ℓ} (with the sign $\mathrm{Im}\eta_{\ell}$) for different values of ℓ. As has already been noted, the absolute value of $\mathrm{Im}\eta_{\ell}$ may be found by formula (5). Some of these results are included as an appendix to the reference book [2] (p.484).

The calculations were carried out by programme [5] for 59 nuclei with mass numbers from A = 7 to A = 248. The selection of A values for which the calculations were made corresponds approximately to an even rate of change in nucleus size. 15 strictly fixed energy values E in the range 100 keV to 15 MeV were taken for each nucleus, the range up to 1 MeV being studied in greater detail. Identical parameters of potential (6) were used for all nuclei and energies. The calculations were carried out for the following sets of these parameters:

The values of a and r_o are given here and subsequently in units of 10^{-13}cm. The parameter r_o is determined by the radius of the nucleus R, assuming that R changes according to the law $R = r_o A^{1/3}$.

The parameters used here agree with the data published by many authors who have carried out calculations according to the optical model.

2. Surface absorption not taking the spin-orbit interaction into account [15]

The calculations were carried out by programme [9] for the same nuclei and energies as in the above case, with parameters of potentials (6) and (8)

The parameter b, like a and r_o is given in units of 10^{-13} cm. The tables contain all the data referred to in the previous case, and also the differential cross-sections $\sigma_n^s(\mu)$ for all the nuclei and energies.

3. Volume absorption taking spin-orbit interaction into account [16]

The calculations were carried out using programme [12]. The same mass numbers A and energies E were used as in the two preceding cases, with the following set of parameters for potentials (6) and (7)

The tables include results for ..... The values of the last three are given for different values of ℓ and j.

. . . . . (Bibliography)

II. Analysis of data obtained using a fast-neutron single-crystal scintillation spectrometer

V. G. Zolotukhin et al.

The fast-neutron single-crystal scintillation spectrometer has in recent years attained wide use, as a result of its ability to separate proton pulses from those caused by electrons. A number of papers [1-5] have discussed the problem of the characteristics of the scintillation detector as a spectrometer - line form and detecting efficiency. Conditions have been discovered under which the simplest approximation of single scattering on protons is applicable, as also are quantitative corrections which the authors consider prevent most of the error caused by distortions of the right-angle distribution of the energies of the recoil protons owing to multiple scattering and boundary effect. The basic equation

(1)

where $P_э(E_p)$ is the observed distribution of the total energy of the recoil protons due to neutrons with the spectrum $N(E_n)$, and $K(E_p, E_n)$ is the analogous distribution from monochromatic neutrons, may be presented, after differentiation in respect of E_p, in the form

(2)

where

H is the thickness of the crystal and $\Sigma_H(E_n)$ is the macroscopic cross-section for n-p scattering.

The extent to which the factor at $N(E_p)$ in the right-hand part of (2) diverges from unity characterizes the effect of distortions of the right-angle form of the line on account of the effects mentioned. Disregarding the integral in curly brackets, the correction is reduced to calculating the quantity $\frac{K(E_p, E_p)}{K_H(E_p, E_p)}$, equal to the proportion in which the height of the plateau decreases or increases when $E_p = E_n$. This correction was calculated in reference [3], assuming the linearity of the luminescence yield and taking into account only twofold scattering of the neutron on the protons.

Similar calculations of the form of the line $K(E_p, E_n)$ for a Stilbene crystal which we have carried out by the Monte Carlo method make it possible to assess the error in the approximation

and to propose matrix methods of solving equation (1) for crystals of considerable size. A detailed description of the method of calculation and of some results is given in references [6] to [8].

Fig. 1 shows the form of the line $\frac{K(E_p, E_n)}{K_H(E_p, E_n)}$ for a cylindrical Stilbene crystal 30 mm in height and 30 mm in diameter (neutron energy E_n = 1.0; 4.15 MeV). The presence of a contribution from multiple scattering is shown by a rise in the probability density. It is of basic importance that this rise, as a result of the considerable non-linearity of the luminescence yield of Stilbene (varying approximately as $E_p^{3/2}$), occurs not in the range $E_p = E_n$, as it would with a linear relationship between the energy of the recoil proton and the amplitude of the light pulse, but in the range $E_p \simeq 0.75\ E_n$. Furthermore, when $E_p = E_n$ there is a reduction in the height of the plateau, so that the value of

is less than unity. This reduction in height can easily be calculated if we take into account that a neutron of energy E_n can give recoil protons with a total energy E_n, either as a result of first, head-on collision (the probability of which is . . . . .), or as a result of multiple scattering on protons, causing the appearance of a light pulse corresponding to the energy E_n of the proton. As a result of the non-linearity of the luminescence yield the latter possibility is not realized, and therefore

(3)

where . . . . . is the macroscopic cross-section for interaction of the neutrons with carbon nuclei. Formula (3) is confirmed by the results of accurate calculation for crystal sizes at which the boundary effect is of no great importance.

The values of . . . . . found by numerical differentiation of the histograms are given in Figs. 2 and 3 for different Stilbene crystal sizes and for energies of E_n = 1.05 and 2.0 MeV. Using these curves and formulae (2) and (3) it is possible to calculate for a particular form of the spectrum the errors introduced by multiple scattering.

It should be noted that these errors are essentially dependent on the form of the spectrum. For a "white" spectrum $N(E_n)/N(E_p)$ = const. = 1 there is a notable compensation of errors. Table I gives the results of a calculation by formulae (2) and (3), together with the data of Brock and Anderson, for a crystal 30 x 30 mm.

Table I

Errors of the differentiation method caused by multiple scattering, %

Data of [3]	Data of present work
. . . . . (Table)	

Thus for smooth, slowly changing spectra the errors of the differentiation method caused by multiple scattering clearly lie within the limits of error due to statistics, accuracy of calibration, etc.

It can be shown that the formula

(4)

where $R(E_n)$ is the path length of a proton of energy E_n, is accurate for the boundary effect. Here, too, there is a certain compensation of the error for $E_p = E_n$. For a crystal with $H \geqslant 30$ mm and $E_p \leqslant 14$ MeV the boundary effect may be disregarded.

To sum up the discussion of the accuracy of the differentiation method, for slowly changing spectra the systematic errors connected with distortion of the line form are within the limits of a few per cent. The spectrometry method itself can hardly guarantee such accuracy, and there are therefore no obstacles to using crystals 20 to 40 mm high.

Fast-changing neutron spectra constitute an exception; in this case the error of the differentiation method may be considerable. Here the development of matrix methods of solving equation (1), taking into account the real line form of the spectrometer and also its energy resolution, is desirable.

The work in question makes an accurate allowance for the energy resolution of the detector, and matrices are calculated for analysing spectrograms. The kernel of equation (1), $K(E_p, E_n)$, taking energy resolution into account, takes the form

where $V(E_p)$ is the average amplitude of the pulse from a proton of energy E_p, $\sigma^2(E_p)$ is the standard deviation in the distribution of the pulses from monokinetic protons, and $K_o(E_p, E_n)$ is the line form found by the Monte Carlo method.

The energy dependence of the standard deviation, according to reference [4], takes the form . . . . . Using the energy dependence of the luminescence yield in the form $V = kE^{3/2}$, we obtain a relationship between σ_o' and σ_o

$$\sigma_o = 3/2\sigma_o'$$

The calculations were carried out for three values of the parameter σ_o - 0.07, 0.13 and 0.19. From the line forms found for 55 values of E_n [8], the counter detection efficiencies for the above three values of σ_o were calculated. Fig. 4 gives one example (calculated) of the extent to which counter detection efficiency is energy-dependent, taking energy resolution into account.

The matrix method used for analysing the spectrogram was the counting efficiencies method [9]. In this, the system of linear algebraic equations takes the form

(7)

where B_i is the energy threshold of the counter. All the units of the quadrature trapezium formula, except the first, coincided with the spacing of energy thresholds. To improve the conditionality of the system of equations [10], the first unit was situated at a point where the effectiveness was in practice zero.

The number of columns in the matrix then became equal to n-1. To give a quadratic matrix, the last row and the corresponding right-hand part were discarded. As a result of these operations the small diagonal elements of the

quadratic matrix of the n^{th} order become semi-diagonal elements in the quadratic matrix of the $n-1^{th}$ order, which gives considerably better conditionality of the system of equations.

Table II gives the direct matrix of the 32nd order for $\Delta E = 0.5$ MeV and $\sigma_o = 0$. The matrix for any σ value different from zero is obtained by displacing the diagonal elements and adding semi-diagonal elements. The diagonal and semi-diagonal elements for the above values of the constant σ_o are given in the same table, and can be obtained for intermediate σ_o values by interpolation. In the same table are given the first diagonal elements, and also the semi-diagonal elements, for a matrix with an interval of $\Delta E = 1.0$ MeV and the same values of the constant σ_o.

Table III gives the inverse transposed matrix of second differences [9] of the 32nd order for $\Delta E = 0.5$ MeV and $\sigma_o = 0$, and Tables IV and V give the inverse transposed matrices of second differences of the 19th order for $\sigma_o = 0.07$ and $\sigma_o = 0.13$. The system of equations for $\sigma_o = 0.19$ is badly conditioned even in the case of a matrix of the 19th order. In this case, the method of regularization [11, 12] must be used; this requires analysis of each spectrogram using a computer.

Fig. 5 gives the results of analysis of the spectrum of fast neutrons from a Po–Be source using matrices corresponding to the above values of the constant σ_o, the regularization method being used in the case of $\sigma_o = 0.19$. It can be seen from Fig. 6 that by taking into account the resolution of the detector it is possible to show the fine structure of the spectrum of neutrons from a Po-Be source. The resolution of the detector in this case was characterized by the constant $\sigma_o = 0.13$. Comparison of the spectrum we obtained and that obtained by Medvetsky [13] shows their good agreement (Fig. 6) and similarity of fine structure. On the same figure are given the results of a calculation [3] of part of the energy spectrum of the neutrons corresponding to the formation of a C^{12} nucleus in the ground state. As can be seen, the three peaks at neutron energies of 6.6 MeV, 7.6 MeV and 9.5 MeV agree well both with the experimental data obtained using photographic plates and with the theoretical spectrum, obtained if account is taken of the anisotropy of the angular distribution of neutrons formed in the Be^9 (α,n) C^{12} reaction.

The results obtained show the great possibilities of matrix analysis of spectra with fairly poor detector line forms and also the possibility of taking the energy resolution accurately into account by direct means, in the elements of an orthogonal matrix. The method is also applicable to the problem of analysing gamma spectra, measured using detectors with either inorganic or organic scintillators.

Figure Captions

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Fig. 1 Line form of a Stilbene crystal (P = $\emptyset$ = 30) for neutron energies of 1.0 MeV (dotted histogram) and 4.15 MeV (continuous histogram).

Fig. 2 Values of ..... as a function of E_n for E_r = 1.05 MeV.

Fig. 3 Values of ..... as a function of E_n for E_p = 2.0 MeV.

Fig. 4 Counter detection efficiency of a Stilbene crystal (30 x 30) (taking resolution into account) for the threshold B = 7.5 MeV. Resolution parameter σ_o = 0 (curve 1), σ_o = 0.07 (curve 2), σ_o = 0.13 (curve 3), σ_o = 0.19 (curve 4).

Fig. 5 Spectrum of neutrons from a Po-Be source, analysed by the matrix method taking resolution into account. The values of the resolution parameter and the numbering of the curves are the same as for Fig. 4.

Fig. 6 Spectra of a Po-Be source, obtained (1) in the present paper, (2) by the photographic plates method (σ_o = 0.13), (3) theoretically [3].

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