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Description of the $^{252}Cf(sf)$ neutron spectrum in the framework of a generalized Madland-Nix model

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Abstract

The Madland-Nix model for the calculation of fission neutron spectra is modified considering the dependence on fragment mass number A. Further, an approximation of this generalized Madland-Nix model (GMNM) which takes into account the different center-of-mass system (cms) spectra for the light (L) and heavy (H) fragment group is discussed. We compare these new calculations with two versions of the original Madland-Nix model (MNM). Especially, the level density parameter becomes more reasonable in the framework of the GMNM. The results of the different model calculations are compared with experimental data on the $^{252}Cf(sf)$ neutron spectrum in the energy range from 0.1 to 20 MeV.

1. Introduction

The MNM for fission-neutron spectrum calculations[1-3] is based on rather rough approximations concerning the residual-temperature distribution of the fission fragments and the level-density description. However, it was successfully applied to describe experimental data on fission neutron spectra by adjusting the level density formulae. The original MNM does not take into account different cms spectra for the light and heavy fragment group.

We propose a generalization of the Madland-Nix model which includes the A dependence of the model parameters. Here, we refer to studies in the framework of a complex cascade evaporation model[4].

In all calculations, the inverse cross section σ_c for compound-nucleus formation is taken into account on the base of optical model calculations. The different versions considered are applied to calculate the ^{252}Cf(sf) neutron spectrum. The aim of this work is to compare the model versions with recent experimental data[5-10] which cover the energy range from 0.01 to 30 MeV. We try to obtain a conformable representation of the ^{252}Cf(sf) standard spectrum for a wide energy range on the base of only one model.

2. Brief review on the original MNM (cp. ref. 1-3)

The MNM is based on the assumption that the distribution P(T) in nuclear temperature T of the residual fission fragments is triangular in shape, i.e.

$$P(T) = \begin{cases} 2T/T_m^2, & T \leq T_m \\ 0, & T > T_m \end{cases} \tag{1}$$

Madland et al. deduce the maximum value T_m from the Fermi-gas model equation

$$T_m = (\langle E^x \rangle / a)^{1/2}, \tag{2}$$

where the level-density parameter is

$$a = A/C. \tag{3}$$

$\langle E^x \rangle$ is the average fission-fragment excitation energy. It is emphasized that this value is approximately equal to the upper edge of the total distribution in residual fragment excitation energy, because the average E^x loss due to first-step neutron emission is comparable with the half width of full maximum of the initial distribution in fragment excitation energy (cp. ref. 4).

Refering on the standard evaporation theory and considering the constant-T description of nuclear level density the cms neutron energy spectrum $\varphi(\varepsilon)$ (ε - cms neutron energy) for a fixed residual nuclear temperature T is given by the approximative formulae

$$\varphi(\varepsilon, \sigma_c) = k(T) \cdot \sigma_c(\varepsilon) \cdot \varepsilon \cdot e^{-\varepsilon/T} \tag{4}$$

with a T-dependent normalization constant k(T). The integration of this spectrum over T considering P(T) and its transformation into the laboratory system (ls) for a fission fragment, which is moving with the average kinetic energy per nucleon E_f, yields

$$N(E, E_f, \sigma_c) = \frac{1}{2\sqrt{E_f} \cdot T_m^2} \int_{(\sqrt{E} - \sqrt{E_f})^2}^{(\sqrt{E} + \sqrt{E_f})^2} \sigma_c(\varepsilon) \cdot \sqrt{\varepsilon}\, d\varepsilon \int_0^{T_m} k(T) \cdot T \cdot e^{-\varepsilon/T}\, dT \tag{5}$$

(E - ls neutron energy).

If the inverse compound-nucleus cross section is assumed to be constant, Eq. 5 is solvable analytically[1,2].

However, we don't consider this approximation. According to the treatment of Madland and Nix, the ls neutron energy spectrum N(E) is given by the average of the spectra calculated for both the light and the heavy fragment group, i.e.

$$N(E) = \frac{1}{2}\left[N(E, E_f^L, \sigma_c^L) + N(E, E_f^H, \sigma_c^H)\right] \tag{6}$$

(version II in paragraph 4). Here, we point out that T_m is taken to be equal for the two representative fragments.

Refering to ref. 1, N(E) can be calculated approximatively using Eq. 5 and inserting the average E_f value concerning the whole fission process ($E_f = \langle TKE \rangle / 252$, $\langle TKE \rangle$ - average total kinetic energy of the fission fragments). The inverse compound-nucleus formation cross section should be deduced for a representative fragment with A = 252/2 = 126. This approximation is considered in paragraph 4 as version I (MNM(appr.)).

3. Generalized Madland-Nix model (GMNM)

As studied in the framework of the complex cascade evaporation model[4,11], the neglection of the model dependence on both A and TKE yields systematic deviations in calculated N(E). Therefore, we generalize the MNM considering the A-dependence of

- the average excitation energy $\langle E^x \rangle(A)$,
- the average kinetic energy of the fission fragment per nucleon E_f (A),
- the inverse cross-section of compound-nucleus formation $\sigma_c(A)$.

Consequently, Eq. 5 is to be modified. One obtains

$$N(E, E_f(A), \sigma_c(A), T_m(A)) = f(A) \tag{7}$$

where

$$T_m(A) = (\langle E^x \rangle (A) \cdot C / A)^{1/2} \qquad (8)$$

(cp. Eq. 2,3). The whole energy spectrum N(E) is given by the sum over all partial spectra taking into account both the fragment yield Y(A) and the average number of neutrons per fission fragment $\bar{\nu}(A)$. This yields

$$N(E) = \sum_A \frac{\bar{\nu}(A)}{\bar{\nu}_{tot}} \cdot Y(A) \cdot N(E, E_f(A), \sigma_c(A), T_m(A)) \qquad (9)$$

(version IV in paragraph 4) where the average number of emitted neutrons per fission $\bar{\nu}_{tot}$ is

$$\bar{\nu}_{tot} = \sum_A \bar{\nu}(A) \cdot Y(A). \qquad (10)$$

Taking into consideration the average γ-ray energy per fission fragment[12] from $^{252}Cf(sf)$

$$\langle E_\gamma \rangle / \mathrm{MeV} = 2.0 + 0.75 \cdot \bar{\nu}(A) \qquad (11)$$

and the total average excitation energy carried away by neutrons

$$\langle E_{ne} \rangle = \bar{\nu}(A) \cdot (B_n(A) + \langle \varepsilon \rangle (A)) \qquad (12)$$

(B_n - neutron separation energy, $\langle \varepsilon \rangle$ - average cms neutron energy), the average initial excitation energy $\langle E^x \rangle$ is given by

$$\langle E^x \rangle (A) = 2.0 \text{ MeV} + (B_n(A) + \langle \varepsilon \rangle (A) + 0.75 \text{ MeV}) \cdot \bar{\nu}(A). \qquad (13)$$

We deduced $\langle E^x \rangle (A)$ accepting the $\bar{\nu}(A)$ curve of ref. 13 and using the $B_n(A)$ as well as $\langle \varepsilon \rangle (A)$ data of ref. 4.

Fig. 1 represents the corresponding $T_m(A)$ data calculated for C = 8.0 MeV (Eq. 8) together with the $E_f(A)$ data[15]. The average 1s neutron energy $\langle E \rangle$ according to

$$\langle E \rangle = \frac{4}{3} T_m + E_f \text{ (cp. ref. 2)} \tag{14}$$

is also shown. Obviously, the maximum residual temperature T_m depends on A rather strongly according to the saw-tooth behaviour of the $\bar{\nu}(A)$ curve. This is neglected in the framework of the original MNM. E_f decreases as A increases. Consequently, $\langle E \rangle (A)$ is nearly constant in the mass number regions corresponding to the light and heavy fragment group.

Further, we propose an approximated GMNM version (number III in paragraph 4) which is based on averaged values of T_m, E_f, A, and $\bar{\nu}$ for the two complementary fragment groups according to

$$\langle X \rangle^M = \sum_{A^M} X(A)\cdot Y(A), \quad M = L,H, \tag{15}$$

i.e. averaging of parameter X multiplied by the fragment yield over the light and heavy fragment group respectively. σ_c is to be calculated for the two average fragment masses $\langle A \rangle^L$ and $\langle A \rangle^H$.

4. Results and comparison with experimental data

As summarized in Table 1, we consider four model versions:

- the approximated MNM - version I -
- the MNM - version II -
- the approximated GMNM - version III -
- the GMNM - version IV -.

$\sigma_c(\varepsilon)$ was calculated on the base of the optical model using the Becchetti-Greenlees-potential[14]. Fragment yields and kinetic energies were taken from ref. 15.

First of all, the different model versions are compared for a fixed C = 8.0 MeV. Fig. 2 shows how the spectrum is changed if using a more complex model for the calculation of fission neutron spectra. The spectrum at high and low energy increases and at intermediate energy decreases as the complexity of the model increases. The deviations are strong at high energy especially. They can be compensated by adjusting C. However, the spectrum shape is changed if requiring an equal average 1s emission energy.

Experimental data on the $^{252}Cf(sf)$ neutron spectrum can be represented approximatively by a Maxwellian distribution

$$N_M(E) = \frac{2}{\sqrt{\pi}\cdot T_M^{3/2}} \cdot E^{1/2}\cdot e^{-E/T_M} \tag{16}$$

with T_M = 1.420 MeV at least for the energy range from 0.2 to 6 MeV (cp. Fig. 3). Deviations from this ideal spectrum shape are less than about 5 % in the stated energy region. Therefore, we fit the calculated N(E) to Eq. 5 and adjust C so that T_M = 1.42 MeV is obtained for the given energy range. The results of this procedure are shown in Table 1 as well as in Fig. 3 for the considered model versions. The parameter C becomes more "physical" if applying a more complex model. According to level density systematics (see for instance ref. 2), C is equal to about 8.0 MeV. Fig. 3 shows the calculated spectra in comparison with recent experimental data[5-10] represented as percentage departures from a Maxwellian with T_M = 1.42 MeV. Inside of the Maxwellian fit range, the calculated spectra are in a quite good agreement. Departures are less than 3 %. It is indicated that the spectrum corresponding to version II (MNM) lies below experimental data at very low and very high energy. In the framework of this model version, a better spectrum description can be obtained at the low-energy or the high-energy end of the distribution by adjusting C, if one tolerates larger deviations from experimental data at the complementary spectrum end.

The spectra calculated in the framework of the model versions III and IV (GMNM and its approximation) are close to each other for energies below 6 MeV. Above 6 MeV, spectrum IV is higher due to the contributions from fragments with high T_m (A around 120), whereas the spectrum III reproduces the NBS evaluation[16] approximatively at the high-energy end. Compared to the spectrum of version II, the spectra III and IV represented in Fig. 3 as percentage departures from a Maxwellian with T_M = 1.42 MeV are flattened, i.e. they are closer to the Maxwellian.

The spectrum calculated in the framework of version IV with a physically more reasonable value of C = 8.0 MeV corresponds to T_M = 1.39 MeV. It is also shown in Fig. 3. In this case, the experimental data on the high-energy end of the spectrum are reproduced. This calculation agrees rather good with experimental data of ref. 5 especially in the whole energy range up to 10 MeV.

5. Summary and conclusions

The study presented in this paper indicates that the consideration of special characteristics of fission neutron emission[4] like the dependence of $\langle E^x \rangle$ and E_f upon A yields somewhat changed fission-neutron spectrum descriptions if using an unchanged level density parameter. This influence is compensated by adjusting the level density parameter, but deviations at high energy remain especially. A similar behaviour was already discussed concerning the dependence on TKE[4]. Compared to the MNM, the GMNM calculations seem to be in a better agreement with experimental data at least at very low and very high emission energies.

For practical purposes, the approximative GMNM (version III) can be applied up to about 10 MeV without large departures from the GMNM calculation. In this case, the computing effort is comparable to version II (MNM).

Finally, it is pointed out that the proposed generalization of the MNM is an attempt to describe experimental data on fission neutron spectra satisfactorily in a wide energy range and basing on a comparatively simple theoretical model. However, calculations of fission neutron emission probabilities for the comparison with multiple-differential experimental data should be based on more complex models to avoid false conclusions regarding the emission mechanism of fission neutrons. The present study considers the predominant mechanism of fission neutron emission, i.e. evaporation from fully accelerated fragments. Other eventual partial spectra of minor occurance probability (cp. ref. 11 concerning scission neutrons) have not yet been taken into account, but they might influence the energy spectrum N(E) also.

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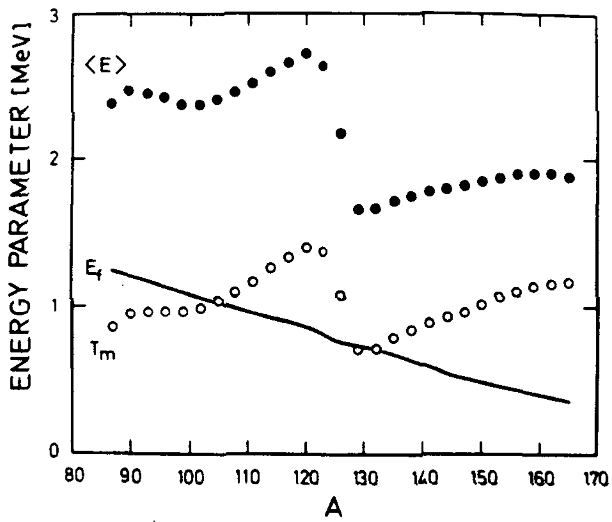
Figure captions

1) The maximum residual temperature T_m of the distribution P(T) (Eq. 1) and E_f as well as the average ls neutron energy $\langle E \rangle$ as a function of A for $^{252}Cf(sf)$. $T_m(A)$ was deduced according to Eq. 8.

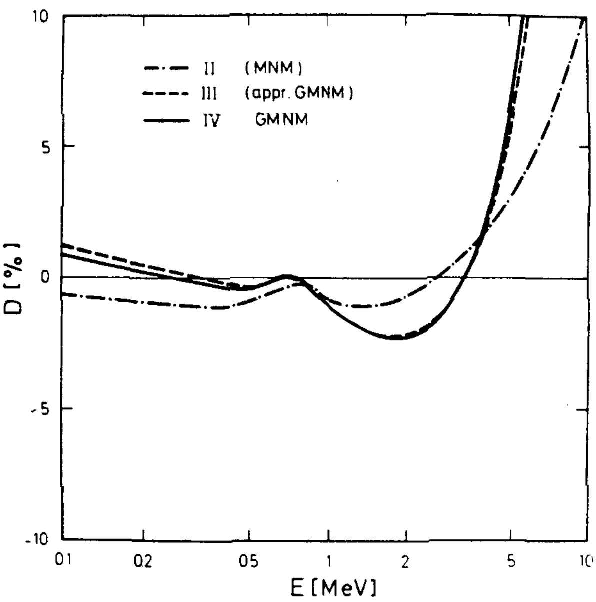
2) The percentage departures of the $^{252}Cf(sf)$ neutron spectra calculated in the framework of the versions II, III, and IV from the version-I spectrum. In all these calculations, C was taken to be 8.0 MeV (cp. Table 1).

3) Percentage departures of recent experimental data (RIL[5], CRIPB/IPPEO[6], ANL[7], PTBB/IRKV[8], TUD[10]) on the $^{252}Cf(sf)$ neutron spectrum and different calculated spectra from a Maxwellian distribution with T_M = 1.42 MeV. Except the calculation IV (8.0000), the C values (included in parenthesis) have been adjusted to obtain T_M = 1.42 MeV if fitting the calculated spectra to a Maxwellian in the shown energy range. The experimental errors are not represented for clearness. We refer to the original papers. Typical values are 5 %, 3 %, 5 %, and (8-30) % (concerning ref. 10 only) for the energy regions below 0.5 MeV, (0.5-5) MeV, (5-10) MeV, and (10-18) MeV respectively.



ENERGY PARAMETER [MeV]
A
⟨E⟩
E_f
T_m

Fig. 1



II (MNM)
III (appr. GMNM)
IV GMNM
D [%]
E [MeV]

Fig. 2

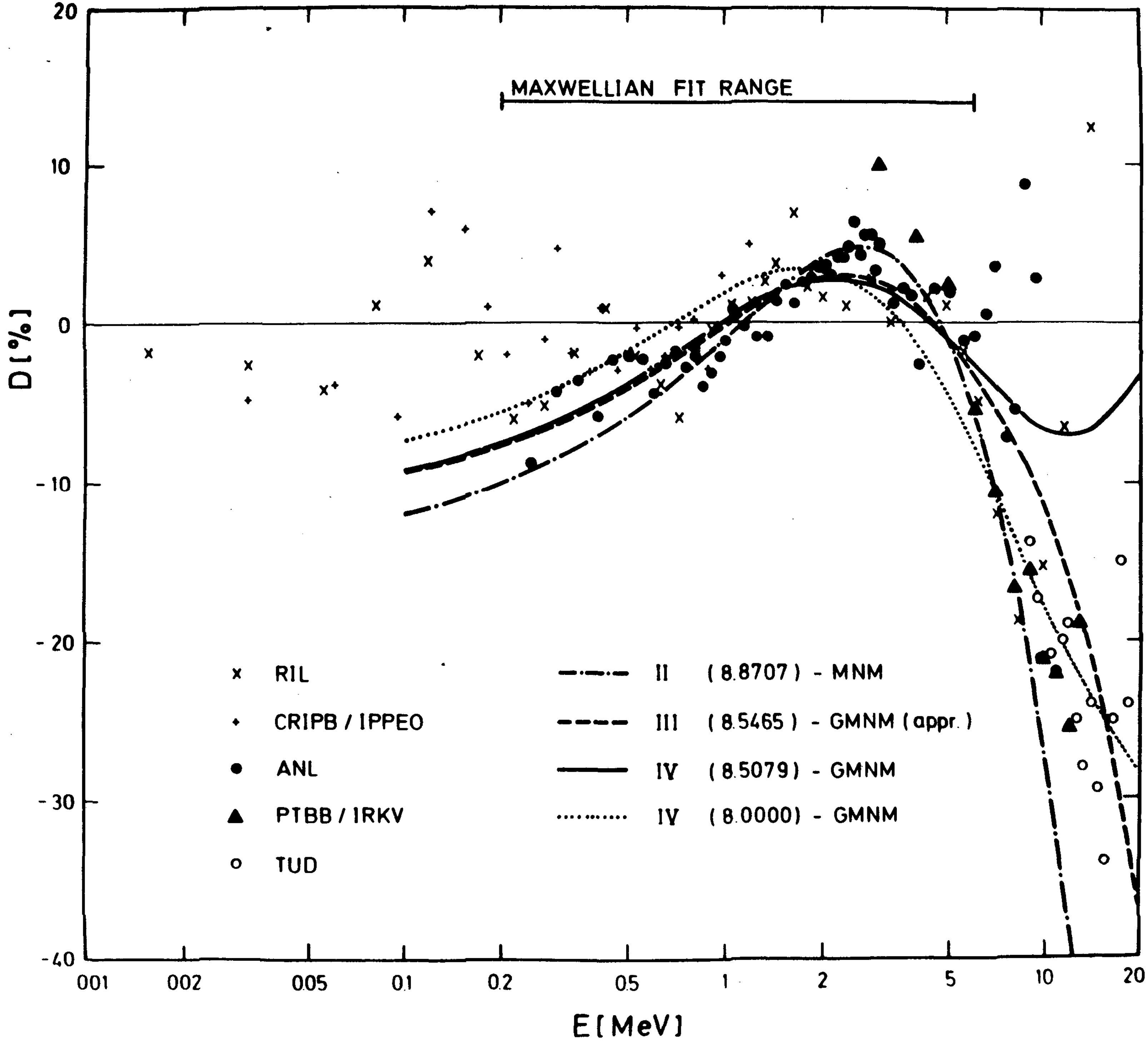

MAXWELLIAN FIT RANGE
D [%]
E [MeV]
20
10
0
-10
-20
-30
-40
001
002
005
0.1
0.2
0.5
1
2
5
10
20
RIL
CRIPB / IPPEO
ANL
PTBB / IRKV
TUD
II (8.8707) - MNM
III (8.5465) - GMNM (appr.)
IV (8.5079) - GMNM
IV (8.0000) - GMNM

Fig. 3

TABLE 1

Characterization of the considered model versions (cp. paragraph 2 and 3) concerning the input data. The C values listed correspond to N(E) whose fit to a Maxwellian distribution between 0.2 and 6 MeV yield a "temperature" parameter T_M = 1.420 MeV

Version	Typ	T_m/MeV	E_f/MeV	Weight W of partial spectra	C/MeV
I	MNM (appr.)	T_m = 1.1065	E_f = 0.7419	-	9.2368
II	MNM	T_m^L = 1.0844 T_m^H = 1.0844	E_f^L = 0.9811 E_f^H = 0.5611	W^L = 0.5 W^H = 0.5	8.8707
III	GMNM (appr.)	T_m^L = 1.1908 T_m^H = 0.9555	E_f^L = 0.9811 E_f^H = 0.5611	W^L = 0.5334 W^H = 0.4666	8.5465
IV	GMNM	T_m = f(A) (Fig. 1)	E_f = f(A) (Fig. 1)	$W(A) = \frac{\bar{\nu}(A) \cdot Y(A)}{\bar{\nu}_{tot}}$	8.5079

Figure captions

1. The maximum residual temperature T_m of the distribution P(T) (Eq. 1) and E_f as well as the average ls neutron energy $\langle E \rangle$ as a function of A for ^{252}Cf(sf). T_m(A) was deduced according to Eq. 8.

2. The percentage departures of the ^{252}Cf(sf) neutron spectra calculated in the framework of the versions II, III, and IV from the version-I spectrum. In all these calculations, C was taken to be 8.0 MeV (cp. TABLE I).

3. Percentage departures of recent experimental data (RIL[5], CRIPB/IPPEO[6], ANL[7], PTBB/IRKV[8], TUD[10]) on the ^{252}Cf(sf) neutron spectrum and different calculated spectra from a Maxwellian distribution with T_M = 1.42 MeV. Except the calculation IV (8.0000), the C values (included in parenthesis) have been adjusted to obtain T_M = 1.42 MeV if fitting the calculated spectra to a Maxwellian in the shown energy range. The experimental errors are not represented for clearness. We refer to the original papers. Typical values are 5 %, 3 %, 5 %, and (8-30) % (concerning Ref. 10 only) for the energy regions below 0.5 MeV, (0.5-5) MeV, (5-10) MeV, and (10-18) MeV respectively.