

Institut für Neutronenphysik und Reaktortechnik Projekt Schneller Brüter

Evaluation of Fast Critical Experiments Using Recent Methods and Data

Compiled by
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Using Recent Methods and Data
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## Abstract

In this work we study systematically the influence of changes in nuciear data on the calculated values for the criticality and other important parameters of critical assemblies. The analysis is done for a variety of assemblies which differ in geometry, material composition, and energy distribution of the normal and adjoint flux. The primary objective is:
(a) to detect deficiencies in nuclear data and calculational methods,
(b) to get first preliminary improvements and
(c) to get indications in which respect and in which way further long-range improvements should be carried on.

The long-range aim of this kind of investigations is to provide or establish satisfactory nuclear data and calculational methods which can reliably be applied to the calculation of fast critical assemblies and of large fast power reactors. In the present investigation the differences between calculated and measured results for the critical assemblies studied are not yet reduced to a satisfactory level. Therefore this study muat be considered as one step in the desired direction but further investigations along the same line will be necessary.

## Zusammenfassung

In dieser Arbeit wird systematisch der Einflub von Anderungen in den nuklearen Daten auf berechnete Werte für die Kritikaiitãt und andere wichtige Parameter kritischer Anordnungen untersucht. Eine Reihe von Anordnungen, die sich in ibrem geometrischen Aufbau und in ihrer Materialzusammensetzung und daher auch in der Energieverteilung des normalen und adjungierten Neutronenflusaes erbeblich unterscheideng bildet die Basis der Untersuchungen. Das Hauptriel ist:
(a) Mängel in den nuklearen Daten und Berachnungsverfahren herauszufinden,
(b) erste vorläufige Verbesserungen zu erreichen und
(c) Hinweise zu erhalten, in welcher Richtung langfristige und langwierige Verbesserungen durchgeführt werden sollen.

Das Endziel dieser Art von Untersuchungen besteht darin, für die Berechnung von schnellen kritischen Anordnungen und groBen schnellen Leistungsreaktoren genügend gate und verläBliche nukleare Daten und Rechenmethoden zur Verfügung zu stellen. Beim gegenwärtigen Stand der Untersuchungen konnten die Differenzen zwischen gemessenen und berechneten Werten für wichtige Parameter von kritischen Anordnungen noch nicht auf ein genügend kleines Maß verringert werden. Diese frbeit muß daher alsein erster Schritt in die gewünschte Richtung angesehen werden. Weitere Schritte werden jedoch notwendig sein, um das Langfristige Ziel zu erreichen.
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I. IIMTRODUCTION

The aim of the present report is twofold:
(i) the results of this study are presented in more detail than it was possible in the paper $\left[^{-1} 7\right.$ presented at the BNES Fast Reactor Enysics Conference in 1969.
(ii) the group constants which have been prepared during this wiok, the data used in the calculations and the comparisons between theory and experiment are documented as a reference for further studies.

In this paper we study systemically the influence of changes in nuclear data on the calculated values for the criticality and other important parameters of critical assemblies. As in the precedine work 127 the analysis is done for a variety of assemblies which differ in Eeometry, material composition, and energy distribution of the normal and adjoint neutron flix (see table 1 ).

The primary objective and short range aim of our work is
(a) to detect deficiencies in nuclear data and/or calculational methods,
(b) to get first improvements in the data and/or methods as f'ar as possible.
(c) to get indications in which respect and in which way further improvements which need longer time should be carried on. .

In the present investigation the differences between the calculated and measured results for the critical assemblies studied are still not reduced to a satisfactory level. This study is only intended as a step in this direction, and further investigations along the same line will be necessary.

The lone range aim of our investigation is to establish satisfactory nuclear data and calculational methods which can be reliably used for the calculation of fast critical assemblies and of large fast power reactors, and to try
to obtein a nore and more precise judgenent of the reliability and conficence Ievel of preaicitions of the nuclear characteristics of fast reactors. The necessity and importance of such a study is illustrated e.g. in [367 which shows to which extent the econorics of large fast breeders are influenced by variations in the besic nuclear data.

In comparins the neasured results for critical assemblies with the correspondine calculated results one should be aware of the possible sources of errors. These may lie in
(i) The nicroscopic nuclear data and the theoretical methods used for their determination.
(ii) The method and model used to analyse the integral measurcment and the accuracy of the experinental results.

To check the reliability of the predictions oniy those integral experimentai cuantities should be used for the comparison
(a) which are(or at least seem to be) free of systematic errors,
(b) for which the experimental uncertainty limit is rather small,
(c) for which the calculational model is (or at least seems to be) most ac̉enuate.

In this respect the criticality of the system is supposed to be the most suitable and most reliable quartity. This is the reason why primarily we are comparine, our calculated value for the criticality factor $k_{\text {eff }}$ with the measured one. A disagrement in this value is a strong indication that there are remarkable deficiencies in the nuclear data or the calculational methods or even in both. Because of possibly compensating effects present in the determination of criticality we are further comparing some calculated quantities whick may be norc instructive with respect to special aspects with the corresponding experimental results. Such quantities are e.g. reaction rate ratios, spatial reaction rate distributions, ratios of material worths and the neutron enerey distribution.

We use this additional information as supplementary indications to necessary inprovements in data and/or methods.

The changes in microscoyic nuclear data and consequent changes in the group constants which heve been performed for the present investigations are outlined in section II.

Section III contains a descrintion of the critical assemblies stuaied.

In section IV the calculational rethods usea are discussed in sone detail.

The detailed results of the rresent study erc eiven in section $V$.
rine general discussion and the conclusions drawn are presented in section VI.

Apnendix I contains the corplete documentation of the new groun constants and Amencix II tire docmentation of the additional date ueed in the calculations.

## II.1. Eri-CTME Set

The wroun constent set used as refercnce set is the sra-Silenf set often nbbreviates as SiEAT-set. Almost all of its parsical basis is outlined extensiven- in the report lif 12c/nart i, the pertinent ricrosconic enera dependent data sre tabulated in reference 1 27_7. The feneration of the FGT-SIEAK set is described in reference $[3]$; tables with the corresponding group constants are siven in reference $/ 4$. 7. For convenience the most important characteristics of this set for the heavy fertile and fissionaiole auclei (see als [25 7) are repeated here. $\underbrace{235}$

The fission cross section values below 20 vev down wo the eV range are based on the data of lifichaudion et al. 1557 from Saclay. Between $20 k e V$ and HeV the date of :inite [E67 and of Pertin et al. $\left[_{7} 77\right.$ were used. Above 3 vev the old Los Alamos data [8_7 were used still without the recent corrections for the long counter efficiency. Setween $i$ and 3 MeV an eye-quide curve through rather scattering data was chosen.

The a values used below lokeV correspond to an unweighted arithmetic averace of direat measurements by de Saussure et al. I9_7. Vang-Shi-di et al. [10_7 end estimates uy Uttley [11] from measured ( $\sigma_{\mathrm{T}}$ : Uttley [11_7; $\sigma_{f}$ : Michaudor et al. [5.7) and calculated ( $\sigma_{n}$ : measured $\sigma_{p o t}=11.7 \mathrm{~b}$ ${ }^{[11} 7$ plus calculated constant average resonance scattering cross section contribution of 0.6 barn) cross sections. Between 10 kev and 1 MeV the highest weight was riven to the rather well agreeing liquid scintillator results of Diven et al. [12_7 and Weston et al. $\underline{I}^{-13} 7$. Below MeV the capture cross sections ver obtained as the product of $a$ and the fission cross sections. Above lijev, where no measurements ere available, $\alpha$ was rather arbitrarily smoothly extrapolated to 10 MeV such as to correspond rather closely to a $1 / E$ dependence of $\sigma_{\gamma}$. Concerning $\stackrel{v}{ }$ the thermal value was taken from the careful evaluation of

Westcott et al. [14_7. At higher energies all measurements available up to mid 1956 were renormalized io the following besic standords

$$
\begin{aligned}
& \bar{v}(2200 \mathrm{~m} / \mathrm{sec})\left(u^{235}\right) \quad=2.430 \pm 0.008 \\
& \text { (Vestcott value [14_7) } \\
& \bar{u}_{p}(2200=/ \mathrm{sec})\left(d^{235}\right) \quad=2.1: 14 \pm 0.00 \mathrm{D} \\
& \left(\bar{v}_{\mathrm{a}}(2200 \mathrm{~m} / \mathrm{sec})\left(\mathrm{U}^{235}\right) \quad=0.016\right) \\
& \bar{v}_{\text {spont. }}\left(C f^{252}\right) \\
& \bar{v}_{\text {spont. }}^{\text {p }}\left(C f^{252}\right) \quad=3.764 \pm 0.012 \\
& \left(\bar{v}_{\text {spont. }}^{\text {a }}\left(\mathrm{Cf}^{252}\right) \quad=0.009\right)
\end{aligned}
$$

and fitted to straight line segments (see extensive description and documentation in reference [15_7). Eelow 2.5MeV down to thermil, i.e. for the most important energies, $\bar{v}(E)$ is represented by

$$
\bar{v}_{25}(E)=2.430+0.106 \mathrm{E} \quad(\mathrm{E} \text { in } \mathrm{MeV})
$$

$\mathrm{v}^{238}$
Below 40 keV the capture cross section is composed by contributions only from $s$ and $p$ wave neutrons without inelastic scattering competition. In this range $\sigma_{Y}$ was calculated from average $s$ and $p$ wave resonance parameters and statistical distributions, viz.

$$
\begin{array}{rl}
\mathrm{F}_{\gamma}= & 2.48 \pm 5.6(\mathrm{meV}) \\
& \text { independent of } 1 \text { and } \mathrm{J} \\
= & (0.90 \pm 0.10) \cdot 10^{-4} \\
\mathrm{~S}_{0}= & (2.5 \pm 0.5) \cdot 10^{-4} \\
\mathrm{~S}_{1} & \text { indenendent of } \mathrm{J} \\
\overline{\mathrm{D}}_{\mathrm{J}=1 / 2}= & 20.6 \pm 2.0(\mathrm{eV}) \\
& \text { independent of } 1 \\
\overline{\mathrm{D}}_{\mathrm{J}=3 / 2}= & 11.4 \pm 1.1(\mathrm{eV}) \\
\nu_{\mathrm{n}} & 1 \text { for } 0.111 \text { and } \mathrm{J} \\
\nu_{\gamma}= & \infty \text { for all } 1 \text { and } \mathrm{J} \\
R & 9.18 \pm 0.13(\mathrm{f})
\end{array}
$$

Corresnonding to a potential scattering cross section

$$
\sigma_{\text {pot }} \quad=10.6 \pm 0.2(\mathrm{~b})
$$

where
1 = neutron orbital angular nomentum,
J $\quad=$ totel compound nucleus angular momentur,
$v_{x} \quad=$ number of dearees of freeciom in a $x^{2}$ tyne distribution for process $x$ ( $x=n ; \gamma ; n=s c a t t e r i n E, \gamma=c a p t u r e)$

The statistical formula used is on energ and half vidth distribution averace of single level Breitwigner resonance temns. The derivation of the statistical data listed above is to be found in reference [16_7. In the SHEAK set there is still some inconsistency in $\sigma_{\gamma}$ (u230) in the eroups 13 and 14 ( $1.0-4.65 \mathrm{keV}$ ). In the whole group 14 and part of the group 13 $\sigma_{\gamma}$ ves calculated fron resolved resonance parameters ( $1.0-3.9 \mathrm{scV}$ ) as contained in the IIDNK file $[16$. 7 . The nurber of $n$ weve resonances analysed and considered in this range and therefore the calculated $\sigma_{\gamma}$ values are too small compared to $\sigma_{\gamma}$ values calculated from the above statistical parameters which actuolly take all. $n$ wave resonances into account.
Above $130 k e V$ the measurements by Barry et al. [17_7 were used up to 10 MeV . As Barry uses the some detector for the neutron flux measurements as White I6_7 in his fission cross section measurements, this choice is consistent with the choice of White's $\sigma_{f}\left(U^{235}\right)$ data.
Eetween 40 and 130 keV the $\sigma_{\gamma}$ values were obtained by smooth interpolation beteween the statistical theory estimates and Barry's data. The inelastic scatterins cross sections for the important enerries velow alov were obtained by a corerul analysis of all available excitation and other inelastic scattorine data; this is extensively described in reference [15.7. Among nore recent exyeriments the highest weight was given to the compehensive excitation cross scetion measurcments by Parnerd et al. [19_7. These five about $20 \%$ larger inelastic scattering cross sections between 1.0 and $1.6 \% \mathrm{lV}$ than obtained in carlier evaluations. The inelastic scatticring nrobabilities were still teaken throughout fron the Russian ADi: set [20_7. The fission cross section values between threshold and 3MeV are based on the results of Lamphere [21_7 and between 3 and 10 HeV on the old Los Alamos data [ 57 also still without the recent corrections for the long counter efficiency.

A weigited least sciuares fit to the available renormalizeã experimental $\bar{v}$ velues led to the following lineer relationsinip for $\bar{v}(E)$ valic between threshold and 15 ify (see reference [18_7)

$$
\bar{v}(E)=2.3576+0.1557 \mathrm{E} \quad(\mathrm{E} \text { in } \mathrm{IfeV})
$$

I239
The fission cross section values below $20 k e V$ are based on the experimentel data of Eollinger et al. [22_7. Consistently with $U^{235}$ the measurements of Mite et al. [-6, 7_7 were used between 20 keV and 1 MeV . Between 1 and 3 jeV we relied on two rather dense and compatible fussian measurement series $\underline{I}^{-28}, 29$ 7. Between 3 and $10: \% \mathrm{fl}$ as for $\mathrm{U}^{235}$ the (still urcorrectea) Los Alamos fission data [8_7 were used.
Below 30 keV the $\alpha$ values are based on the old KAPL integral measurem ents 123_7. Detween 30 keV and 1 iteV the rather well agreeing Oak Ridge 1247 and Los Alamos [25_7 Iiquid scintillator values were used. As for $U^{23 \overline{5}}$ below lleV the capture cross sections were obtained as the product of $\alpha$ and the fission cross sections. Above MeV no measurements are available and, as for $\mathrm{U}^{235}$, $\alpha$ was rather arbitrarily extrapolated to $10 \% \mathrm{ieV}$ in rather close correspondence to a $1 / \Sigma$ behaviour of $\sigma_{\gamma}$.
Concerning $\bar{v}$ the thermal value was obtained as a weighted least squares average of all measurements available up to mid 1966 after renormalization to the $U^{235}$ and $C f^{252} \bar{v}$ standard values listed in the $U^{235}$ section above; the result was

$$
\begin{aligned}
\bar{v}(2200 \mathrm{~m} / \mathrm{sec})\left(\mathrm{Fu}^{239}\right) & =2.392 \\
\bar{v}_{p}(2200 \mathrm{~m} / \mathrm{sec})\left(\mathrm{Pu}^{239}\right) & =2.886 \\
\left(\bar{v}_{d}(2200 \mathrm{~m} / \mathrm{sec})\left(\mathrm{Pu}^{239}\right)\right. & =0.006)
\end{aligned}
$$

Using the same procedures as for $U^{235}$ the energy dependence of $\overline{\mathrm{v}}$ for $\mathrm{Pu}^{239}$ was obtained to

$$
\bar{y}_{49}(E)=289200+0.12791 E+0.00189 E^{2}-0.00010 E^{3}(E \text { in } M e V)
$$

Entensive documentation of these evaluations is to be found in reference [26.7.

Starting fron the KFK-GIEAK set improvements are now introduced by the successive replacement of older data by nore recent, more reliable data (see also reference [-2_7). The changes in basic daia leading to partly important modifications in the KFK-SHEAK Eroup cross sections are discussed in the following. In generating modified group cross sections the same collision density weighting spectrun as for the KFK-SNEAK set was used (see reference [4_7, rigure 1).
II.2. SNEAPM, SIIEPMB Sets

The SIEAPM set differs from the lerk-SIEAK set only in that in the range 10 to 500 keV the $\sigma_{\gamma}\left(\mathrm{U}^{238}\right)$ values are renlaced by the experimental results obtained by Pönitz et al. 1 $30 \_7$ and by Menlove and Pönitz [ 31 _7. This lowers the group $\sigma_{\gamma}^{28}$ values in the croups 6 to 11 ( $10-300 \mathrm{keV}$ ) by up to 13.5方.

In the SIIEPMB set, in addition to these changes for $U^{238} \sigma_{f}$ and $\sigma_{\gamma}$ of $\mathrm{U}^{235}$ ( $\alpha$ is kept konstant) are lowered in group 7 to 10 ( $21.5-400 \mathrm{HeV}$ ) according to a $\sigma_{f}^{25}(E)$ curve mronosed by Beckurts [32_7. This curve was obtained by multiplyine measured $\sigma_{f}^{25} / \sigma_{\gamma}(A u)$ ratios with the $\sigma_{\gamma}(A u)$ shane measurements normalized to an absolute detemination of $\sigma_{\gamma}$ (Au) at 30 seV by Fönitz et al [ 30,33 _7 in the rance 25 to 5001 eV and subsequent averacing. The correspondine modified $\sigma_{\gamma}^{28}, \sigma_{f}^{25}$ and $\sigma_{\gamma}^{25}$ Group cross section values are eiven in tables $A I-1$ and $A I-2$.

## II.3. PU9SGP Set

In this set the SIDEAK set $\alpha\left(1 \times u^{239}\right.$ ) values in the group $11-15$ ( 465 eV to 21.5 keV ) based on the old KAPJ measurements are replaced by values based on the first results of the linear accelerator $\alpha$ ( $\mathrm{Pu}^{239}$ ) measurements by fivin et al. [34.7. The $\sigma_{\gamma}$ values are correspondingly changed for $\sigma_{f}$ kent constant. Among the more recent experimental $\alpha_{49}$ data available at the
time where the RUgSCP set was estaiolished Gwin's date were considered to be the rost reliable ones by the following reasons. From the available measurements Gwin's results were assigned the smallest errors (on the average about $\pm 15 \%$ in $\alpha$ ). The independent $\alpha$ estimates by Pitterle et al.
 cross sections and calculateđ scatterine cross sections agree best with Gwin's data amone the available experimental $\alpha_{49}$ results. The latest available results obtained by Schomberg et al. $I^{-17} 7$ in measurements with considerably improved pulse shape discrimination, background determination and electronics are partly substantially lower than the first preliminary results reported in 1967 [38_7. Below 4zeV Schomberg's more recent data [7] 7 are in good agreement with Gwin's values, above 4keV they are closer to Gwin than Schomberg's first preliminary data, but there are still discrepancies between Schomberg's and Gwin's results particularly in the regions $4-$ TkeV and $10-30 \mathrm{keV}$, which might partly be due to different normalizations. First $\alpha_{49}$ measurements of Ryabov et al. [ 39 _7 with the fast pulsea reactor IBR as neutron source are compatible with Gwin's and Schombere's measurements below 2 keV , but are even below the KAPL measurements above $2 k e V$. Extensive discussions at the Anglo-Russian Seminar at Dubna in June 1968 clarified this discrenancy. Compared to the other more recent measurements Ryabov et al. overestimated the fission rates and underestimated the capture rates by applying too large corrections for neutron scattering before canture. First corrections for both errors led to a considerable enlargement of Ryabov's $\alpha$ values with results coming close to those of Gwin. Nore details can be found in reference [40_7. The nev $\alpha$ and $\sigma_{\gamma}$ values for Pu ${ }^{239}$ in the groups 11-15 are given in table AI-3.

## II.4. SCTAJ $\varnothing$ Set

The inelastic scattering probabilities contained in the ABN set $\mathbf{1}^{-20} 7$ used in the SMEAK set are replaced by Karlsruhe data. In the range of resolved residual nucleus levels excitation cross sections evaluated and documented in various sections of the report KFKK 120/part I are taken from the KEDAK file $\left.\right|^{2} 27$. . In the "continulum" range of residual nucleus levels the Weißkopf evaporation model $/ 41$ - 7 is used with nuclear temperatures as recommended by Swarcbaum et al. / 42, 7. The materials concerned are $\mathrm{C}, \mathrm{O}, \mathrm{Na}, \mathrm{Al}, \mathrm{Cr}$,
$\mathrm{Fe}, \mathrm{Hi}, \mathrm{u}^{235}, \mathrm{U}^{238}$ and $\mathrm{Pu}^{239}, \mathrm{Pu}^{240}, \mathrm{Pu}^{241}, \mathrm{Pu}^{242}$. For the most important nuclide $U^{238}$ the new inelastic scatterine spectra turn out to be sonewhat softer than the ABir set spectra.

The new inelastic scatterine matrices are given in table AI-T. The calculation of the inelestic scattering matrices and their conparison with the ARE set matrices are more cotensively described in anmendix AI.

## II.5. UFUCDR Set

In this set tine sumis set fission data in the group $1-3$ (2.5-10.5iev) for $U^{235}$ and $\mathrm{Pu}{ }^{23 n}$ and in the groups 1 and $2\left(4.0-10.5 \% \mathrm{teV}\right.$ ) for $\mathrm{U}^{238}$ uased on the old Los Nlemos reesurcments by Smith et al. 18 F were downeraded by up to shout 10 if in accord with recent efficiency corrections for the long counter used in the ahove measurenents [ ${ }^{43}$ _7.
Fecently cormrehensive $\bar{v}\left(\right.$ ru $^{239)}$ ) results heceme available by measurerents of Frehaut et al. [ $4 \mu_{2} 7$ in the ronce 1.5 to 15 MeV and Conde et al. [ ${ }^{4}+7$ in the range 1 to 15 :eV. Soth measurcments show very cood areement and close a gan in the higher :NV range. Delow 4 HeV they are in agrement with the scattered earlier measurements. Trom 4 feV unwards they show disferences un to $+4 \%$ at $15: 1 \mathrm{eV}$ from the former evaluated curve [26_7 which forms the uasis of the SIEAK set $\bar{v}$ data.

The nev $\sigma_{f}$ and $\bar{v}$ data arc given in table AI-4.

## II. 6. PUO2PTES

For the higher stable Iu isotones $\mathrm{Pu}^{2 / 10}, \mathrm{Pu}^{241}$ and $\mathrm{Pu}^{242}$ the siveAK set so far contained the groun cross cections and shielding factors as eiven in the ABII set [-20_7. They werd completely replaced by group cross sections and shielding factors calculated from the recently evaluated microscopic cross sections of Yiftah et al. [46 7. After the publication of the evaluation of Yiftah et al. the very comprehensive resonance total and partial cross section measurements for $\mathrm{Pu}^{240}$ by the linear accelerator group of the BCIN Geel [ 47.7 became available. They showed in particular that the average s wave capture width ( 32 meV ) and the s wave strength function $\left(1.37 \cdot 10^{-4}\right)$ as assumed by Yiftah et al. in their: calculations of $\left\langle\sigma_{\gamma}\right\rangle(E)$
for Pu ${ }^{24 C}$ on the basis of previous much less comprehensive measurements had to be replaced by the lower values $\bar{\Gamma}_{\gamma}=23.2 \mathrm{meV}$ and $\mathrm{S}_{0}=1.05 \cdot 10^{-4}$. With these values and a (probably somewhat too low) $\underline{D}$ wave strength function $s_{1}=1.5 \cdot 10^{-4}$ the capture cross sections of $\mathrm{Pu}^{240}$ were recalculated in the range of predominant $s$ and $p$ wave capture above 1 keV and extrapolated to higher energies so as to join smoothly the curve of Vifteh et al. at about 800 keV . These lower $\sigma_{\gamma}^{40}$ values were taken into account in the PUO2RE set. Integral substitution measurements performed by Oosterkamp [487 in SMEAK assemblies, viz. measurements of reactivity differences due to substituion of a $\mathrm{PuO}_{2} / \mathrm{UO}_{2}$ mixture containing $8 \% \mathrm{Pu}^{240}$ by another one containing $22 \%$ Pu ${ }^{240}$ were well reproduced by theoretical calculations using the improved capture data for $\mathrm{P}_{2}{ }^{240}$ mentioned above (see more extensive discussion in reference (50_7).

The much more comprenensive and accurate experimental information used in the evaluation of Yiftah, Schmidt et al. leads to striking differences of the present to the ABII group cross sections particularly for $\mathrm{Pu}^{240}$. The capture data for $P u^{240}$ are about a factor 2 lower than the ABN set values in the energy range 1 keV to 1 MeV . The fission cross sections for $\mathrm{Pu}{ }^{240}$ in the ABII set drop to zero in the keV range with decreasing energy, whereas according to the Gee $]^{-47} \mathbf{7}$ and the Los Alamos bomb shot measurements [497, as a consequence of the phenomenon of intermediate subthreshold fission, $\sigma_{f}^{40}$ on the average over many resonances resp. fission resonance clusters is of the order of 100 mb and higher all the way down to the resolved resonance range.

The new group cross sections, shielding factors and inelastic scatterins matrices for $\mathrm{Pu}^{240}, \mathrm{Pu}^{241}$ and $\mathrm{Pu}^{242}$ are given in table AI-5.

## II.7. M $\$ \times 911$, i $1 \phi X \mathrm{XID}$ Sets

Finally, as trial date the recent measurements of $\sigma_{\gamma}\left(u^{238}\right)$ by Moxon et al - ${ }^{50} 50$ in the range 500 eV to 100 keV were used to replace the SIFAK and STEAPM set $U^{238}$ capture data in the groups 9 to 11 ( $10-100 \mathrm{keV}$, MøX911 set) ard in the whole range, Eroups $9-15$ ( $465 \mathrm{eV}-100 \mathrm{keV}, \mathrm{M} \mathrm{\phi XI} \mathrm{\phi} \phi \mathrm{set}$ ). The
results of these neaure-ents are nearly equal to those of the nrevious measurements of Foxon et aI. [51_7, in particular the former $I^{10}(n, \alpha)$ normalization was once afain carefully checked. The loxon data are on the averace about 20 , maller than the Silinis set data and of the order of $10 \%$ smaller than Pönitz's values in the cormon energy range $25-100 \mathrm{in}$. Tae difference between yoxon and Pünitz is still unexplained. Furthermore time apmerent discrepancies between lloxon's results and the average of a number of absolute and relative determinations at selected energies ( $24 ; 30 ; 65 \mathrm{keV}$ ) have still to be resolved. The modified capture data for $\mathrm{U}^{238}$ are given in table $\Omega I-6$.

In this chapter we will briefly describe the critical assemblies considered for the present study. A more detailed documentation of the assembly characteristics used in the calcuiations and of the integral riata used for comparison between theory and experiment will be given in Appendix II.

Besically we studied 12 fast critical assemblies: SUAK U1B, SUAK UIIP, ZPR III-10, $2 P R$ III-25, ZFR III-4B, ZIR III-48B, ZEBRA 6A, SNEAK 3A1, SHEN: 3AC, SIEAK 3B2, CIMAK 5C, ZPR III-55. Six of these assemblics are fuclled with U235, five with Iu239, and one (Simif 3BR) with a twozored core, the inner zone containing Pu239 and the driver zone U235. ©ome characteristics of the essemblies are given in table 1.

All criticals are of medium or large size, the snallestone, SUAK U1I, havine. a core volume of about 40 liters. Ve ineve not incluced very small asscmblies line GODIVA or JEMTHFL in our sturies becruse the hich enerey cross sections of thosc nateriels wilicla could be checked by these assemblics arc not very much uncertain und are not so imnortanit for the plysics preaiction of large fest power renctors.

The hardness of the neutron enerar spectrum veries considerably as can be seen e.g. from the median fission energ, the neutron lifetime, and the ratio $\sigma_{f}(U 23 B) / \sigma_{c}(U 23 \delta)$ given in table 1 . The micration area $H^{2}$ of the core varies considerably between the lowest value for SUAK UF1B and the higher values for the more dilute assemblies ZPR III48, ZERRA 6A, and SIIENK 3A1. Apart from the $k_{\infty}$-experiments GHEAK 5C and ZPR III55 there is also a large span in the geometric configuration characterized by the geometric buckling $E^{2}$ and the core volume between the small unreflected SUAK-assemblies and the larger well rerlected assemblies ZPR III-25, ZPR III-48, ZEBRA $6 A$ and the three SNLAK criticals, the small but reflected assembly $Z P R$ III- 10 ranging in between. The leakage probability and the probability for the most important reaction ratios in the core are also given in table 1.

SUAF U1B is a metal fuelled uranium assembly with $20 \%$ enrichment of about 30 cm length in each direction of the cube.

SUAK UE1B is similar to SUAK U1B. It is also $\varepsilon$ metal fuelled uranium assembly with $20 \%$ enrichment but containing a relatively large amount of byarocen in foils of polyethyleme. The byarogen to urenium atan ratio is about 0.5 .

ZDP IIIm 10 is a metal fuelled uranium assembly with a rather small core of about $17 \%$ enrichment surrounded by a relatively large reflector of depleted uranium.

ZPR ITI-25 has a larger core. The uraniun metal fuel has an enrichment of about $9 \%$. The core is surrounded also by a relatively large reflector of denzeted uranium.

ZnR III-48 is a well known assembly with plutonium es fissionable meterial. The fuel enrichment is about $18 \%$. In order to simulate the neutron energy spectrum of a sodium cooled fast reactor with ceramic fuel, Na and. $C$ have been added to the core composition to soften the spectrum. The reflector $0_{i}$ about 30 cm thickness consists of depleted uranium metal.

ZPR III-48B is very similar to ZPR III-48. The essential difference is the inner core zone which has a higher content of Pu240 compared to ZPR III-48.

ZEBRA 6 A is a somewhat smaller plutonium assembly with a fuel enrichment of about $24 \%$. iVa and $C$ are also added to influence the neutron energy spectrum in the desired manner. The reflector of about 30 cm thickness consists of natural uranium and graphite.

SNEAK 3 A1 was built to simulate the core of a fast steam cooled power reactor. The enrichment of the uranaummetal fuel 18 about $20 \%$. The polyethylene foils used to simulate the coolant are contained in stainless steel canned platelets. The hyarogen concentration of SNENK $3 A 1$ is about half that considered for a typical stean cooled power reactor characterized by a coolant pressure of about 170 atm and a relatively small coolant
volure fraction of the core. Fithin the unit cell one Al- and one $\mathrm{Al}_{2} \mathrm{O}_{3}$-platelet are used torether with the fuel- and stainless steel canned polyethylene-platelet to influence the neutron enere spectrur. in the desired ranner.

SHEAN 3 A2 is slighty smaller in core volume than EIEAK $3 A 1$ but has still a relativejr lorge core zone. fy aajuusting tine tinckness of the nolyethylene foil one outained in SiFAi 3.2 a kydrogen concentration winch corresponds to thet of a future stem-cooled nower reactor with the ciarcsteristics nentioned before. Jesides the thicliness of the polyetinlene foil the comosition is almost identical to SHENK 3A1.

GIENE 3 , 2 has en inner core zone mich contains plutonim instead of uraniure as fissile raterial. Othervise the comosition of this twozoned core asser:bly is equivalent to cimar $3 A 2$.

SITAIE 5 C is a so-called $\mathrm{F}_{\infty}$-owerirent. It consists of an inner Plutonium zone of about 300 liters anc an outer uroniur driver zone. Fy adiustine the comositions of both zones in the annropriate manner one aims at a flat distribution of the normal and adjoint fluxes across the inner test zonc and to bring the $k_{\infty}$ of the test zone close to 1 . The atom ratio C/U?3? in SIIN: 5C is rather large, about 12, which leads to a "soft" neutron enercy spectrum.

ZPR III-55 is also a $k_{\infty}$-experinent for a plutoniwn composition fron which one tries to get information on the $\alpha$-value of Pu239 in the enerey range from 0.5 to $20 \mathrm{keV}\left(\alpha=\sigma_{c} / \sigma_{f}\right)$. Because of the smaller C/U238 atom ratio of about 2. ${ }^{4}$ the neutron spectrum is not as "soft" as that of SIEAK 5 C .

In addition to these asserblies we consider also the measurements concerning the steam density- and steam void-coefficient which have been performed during the so-called GNAK-3A-series. They are described in [-57 7 and include besides the assemblies SNFAK $3 A 1$ and SIEAK 3 A2. mentioned before, two other assemblies SNEAK $3 A O$ and SNEAK $3 A 3$ which contained no hydrogen and twice the hydrogen concentration of SNEAK 3A2, respectively.

Apart fron the leakage the small and "clean" assemblies SUAK-U1B and SUAK-UH1B provide checks for the higiner energy date of U235 and U236. Because of the increased moderation SUAK UFif. is more sensitive than SUAK U1B to the capture data of U238 and U235 since there is a rapid increase of the canture cross sections to lower energies. For ZPR III-10 the importance of the leakage is reduced compared to SUAK U1B and the importance of the $U 235$ and $U 238$ data in the hundred keV region is increastd. The same tendencies but still more pronounced are valid for ZPF III-25. The larger and well refiected assemblies SNEAK 3A1 and SNEAK $3 A 2$ will give additional information on the uranium data in the lower keV region. The somewhat similar plutonium assemblies ZPR III-48 and $\quad$ IEBRA 6A are both included in the analysis since the information on glutonium assemblies is not vexy extensive. ZPR III-48BB is of interest because of its hieher Pu240 concentration in the inne:: core zone compared to the usually available plutonium isotopic cornosition. The $k_{\infty}$-experiments SNEAK 5C and ZPR III-55 provide a check of the low energy capture data o? U238 and, hopefully, of the capture and fispion data of Pu239 in the keV region. SNEAK 3R2 has been included in the analysis as the first fast critical at Karlsruhe containing plutonium.

With this selection of critical assemblies we are confident to obtain essential information on the reliability of data and methods for the energy range of interest in fast reactor analysis.

TEI CAICUTARIOTAL METHODS USE

IY.1. Ceneral Approsci
In this study we want to deterrine the effect of each change in the nuclear data for the whole variety of criticals. The reason is two fold:

1. we want to fet an insisint into the uncertainty of the calculation of characteristic quantities for these assemblies,
2. we want to obtain information on the sensitivity of the nuentities studiec on various changes of the nuclear deta Of several materials in different energ recions.
Doing this in the most correct way voulc consume a large enount of conuter tire since one hes to nerform e.E. two-aimensional calculations and has to amply corrections as mentioned later e.fe for heterogeneity etc. which in a strict sense would have to be recalculated for each chanse in the nuclear data. In order to avoid this we treated the problem in the following approxinate ranner: For each essembly wich has not essentially a twomoned core, we determined with our reference groun set, the somcalled SMEAK set [3_7, [4_7, the bucklings in each one-dinensional direction (i.e. $B_{x}^{2}, B_{y}^{2}, B_{z}^{2}$ respectively $B_{r}^{2}, B_{z}^{2}$ ) by comparine one-dimensional results for the criticality with the corresponding ones of fundamental mode calculations. These adjusted buckings have then been used throughout the study in the fundamental mode calculations for the rodified Eroun sets. The adequacy of this procedure has been proved by some two-dimensional diffusion calculations with 11 energy rroups esnecially for tinose changes in the nuclear data which caused major cf anges in the calculated criticality. The 19 group constants are condensed from the oricinal 26 croun constents using as weichting syectrum thet of a one-dimensional diffusion calculation in sphericel Eeometry for each modified group set. These two-dinensional results of course have to be considered as the basic ones. The fundariental mode results are used only to obtain the criticality difference of those changes in the nuclear data which lead to relativels small changes in criticality.

## TY.2. Corrections

To ret the finel results (best avaifable values of table Val) some corrections which are pertially calculated by ourselves and partially taken from the literature have to bc applied if necessary: (a) the heterorencity correction using the FrRAmcode $/ 50$ _7. (b) the transnort correction usine a one-̇imensional sr-code, (c) the so-celled penio-correction which arises from a nore elaborate treatrent of the elastic slowing cown [ ${ }^{3}$ _7, $]_{2}^{2} 7,(\mathrm{a})$ a correction for the transport cross section which tekes into account the anisotronic down-scattering of hydroren especially for SUAL UIIB [3_7. [59_7. In the cases where these corrections are calculated by ourselves they werc detemined once with the sfwni-reference set and are then assumed to be only weati- denencent on chanses in the nuclear data and therefore are taken the same for the modified groun sets.

## IV.3. Corment on the Fission Rnectrum

With resnect to the calculational methois we should mention that the standard fission spectrum $x$ in our wroup sets is that beloneine to $v=2.8$ of the Russian ABr-set $\underline{I}^{20} 7$. In reality $x$ depends on the enerey of the fissionine neutron and the type of the fissionable isotope; these dependences cannot routinely be talen into account in our calculations. We studied the magnitude of the effects which can be attributed to changes in the fission spectru in an approximate manner. The main results are (see trble 2): for U235-fuclleत assemblies a reduction of the criticality between $0.001-0.003$ and a decrease of the neutron spectrum by about $6 \%$ only in the energy range from $6.5-10.5 \mathrm{MeV}$ if $\mathrm{f}(v=2.6)$ is used instead of $x(v=2,8)$; for Pu23? fuelled assemblies a slicht increase in criticality (about 0.001 ) and an increase of the neutron spectrum by $11 \%$ in the enerry range mentioned, if $x(v=3.0)$ is used instear of $x(v=2.8)$. The averafe $v$ of U235 fuelled asseriblies is in the ranfe 2.5-2.6, that of Pue39 fuelled assemblies in the rance 2.9-3.0. One sinould mention that the fission ratio $\sigma_{r}(i, 33 \hat{i}) / \sigma_{f}(\mathrm{~J} 23 \mathrm{~F})$ is only changed be 1-2: by the chnares of the fission sinectrur. rentionct above.

## 

Fince the tro-dimensional fincusion results obtaired with the BINXgrocur form the basis on cur hest available criticality gresictions, it res necossary to ciec tie vaicita or tive awrovinctions we in rerivin- these renuts, The First aywoxivation is the emensation

 same sontiel mes' ( 16 nn meshoints) we observell a difference in $k$ eif of n. non wicil was fer less then the accuracy of 0.001 recuired for ench calculation. tine seconi amroxination concerns the aistance between the reshoints for the calculetion of the spece ienerience of the neutron filu, This probler hes been studied for en asseribly siriler to cmari 3A1 by tro-icimensional calculations using $2 G$ grougs. The results are presented in tia followine table.

| Depenience of lieff on the spatial mesh |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | case 1 | case 2 | case 3 |
| Redial तirection | nuriner of mesh noints | 20 | 40 | 68 |
|  | number of mesh intervals in the core per transport mean free neth | :, 04 | 2.16 | 3.64 |
| Axial <br> direction | number of mesh points | 20 | 40 | 60 |
|  | number of mesh intervals in the core per transport mean free path | 1.1 | 2.2 | 3.74 |
| Total rumber of mesh points |  | 400 | 1600 | 4080 |
| $\mathrm{k}_{\mathrm{eff}}$ |  | 0.98600 | 0.98746 | 60.98744 |

From this table one concludes that 2 mesh intervals per transport mean free path is surficient for the two-dinensional calculations if on accuracy or 0.001 in $k$ eff is desired.

א) All results quoted in this paragraph have been provided by W. Höbel (Karlsruhe).

## IV.5. Approximate Treatment of the Anisotropic Scatterine of Iydrogen in SUNT UR1B

Usuelly ve use the diffusion equation for the determination of the criticnlity of the assemblies studied. In deriving the cross sections respectively group constants which ere needed in this equation the usuel transport approximation is anplied. It is very probable that this annroximation is not sufficient for the anisotropic scatterine of hydrogen and leads to errors in the transport cross section and the diffusion constant and therefore also to errors in the leakare probability. Among the hydrogen containing assemblies the leakage probebility is most important for SUAK UIIB. Therefore an improved but still approximate treatment of the problem has been applied for this assembly [3_7 which is indicated in the follorinc.

From the P1-equations the following relation can be derived:

$$
(I V .1) \sigma_{t r}^{i}=\sigma_{t}^{i}-\underset{j \leq i}{s} \sigma_{1}^{j+i} \frac{J_{j}}{J_{i}}
$$

where $\sigma_{1}^{j+i}=\sigma_{0}^{j+i} \mu^{j+i}$ is the second moment of the scattering matrix and $J_{j}$ is the net current in the energy group $j$. For the isotopes with higher atomic weight than 10 , the second term on the right hand side of the above equation (IV.1) can be approximated by $\sigma_{e}^{i} \mu^{i}$. $\sigma_{e}^{i}$ is the elastic scattering cross section in Eroun $i$ and $\mu^{i}$ the average cosine of the scattering ancle. Tihis approximation is of course not adeg̣uate for hyrrofen. Inprovements compared to this averace cosine concept have been made for hydrogen in two steps: in the first sten it was assumed that the currents are weakly enerwrolependent so the ratio $J_{j} / J_{i}$ in equation (IV.1) is set equal to unity, in the second sten the cromp dependent currents of the GUAK UIF1B asseribly have been used to detemine the transrort cross section of lydrogen. Since the droup depentance varics with space coordinates the group currents have been taken at two space noints, located at distances of 6.5 cm and 13.3 cm from the core houndary. The resultinct tronsport cross sections of hydrogen
have been used in the appropriate core regions in the criticality celculations. The influence of these two stens of improvernents on the criticality has been studied for SUN UN1B in diffusion approxiration using a sphericel model for the assembly. The following chenges $\Delta E$ have been obtained for the RBSIT: the first step gives $k=0.011$, the second sten $k=0.0022$ compared to the usual average cosine concept. The reason for these different results is that in the second step the tremsnort cross section of lydrogen is increased. in the high energy rersion even slirhtly more then in the first step since $J_{i} / J_{i}$ is sraller than one. But below 0.5 MeV this tendency in reversed and the transport cross section is calculated even smeller than with the average cosine concept. Since the transport cross section is also used in the boundary condition of the diffusion calculations the calculated change in criticality may also depend on the seometrical model used for the assembly because in sphericel seonetry only one external boundery has to be considered whereas in the real cubic arrangenent three boundaries have to be taken into account.

It seems to us that the maenitude of this correction is still uncertain. We have applied in the present evaluation a value of 0.007 for this correction which is taken from the literature [50_7 and is in between of the two extreme values mentioned before in this paragraph.

Ye will study the effect of the anisotropic scattering of hydrogen once again usine the anpropriate recently established improved version of the DTK-S
IV.6. Weighting Spectrum and $\sigma$-Concept

The effect of the veighting spectrum on criticaility has been stuaied using the REN groun constonts. The results are discussed in [2_7. In all ceses studied there the differences have been smaller then 0.005 in $k$. For the neutron spectrum the swoprocedure generally leads to better arreement of colculations with experiments.
It should be mentioned, hovever, that the ROM-concept is used only for those isotones wich scatter only into the next eroup of the 26 -croup set witin the ABM-rroup structure. The Binib-concept is not well suited for
 of the scatterine netrix, Comperine the scetterine cross sections of Sixtoren in the $A B M-$ end rriv-sct one notices considerable differences which rost mrobably are due to tine weixnting soectrwe used because the besic nuclear तata are well bom. In the Gimaimset the weichtinc. syectrum corresmonds to that of tie nmar 3hi asserbly. Therefore one rav obtain changes in criticality and neutron ssectrus for the SUAF UF1B assembly if the anmomiate veichting snectrum is used for the determination of the scatterine cross sections of hydrogen.

Onc should mention also that the ply-correction is ajplied only for ncutron enercies above 1 keV (croup 1-14). The extension to lower energies will probably brine about son:e influence on the results for the criticality of assemblies with rather soft neutron spectra like ZPR III-55 and especielly SMLA: 5C.

In reactor calculations ve norrally use an averare backaround cross section $\sigma_{0}$ for the determination of resonance self-shielding ( $\sigma_{0}$-concept, see [-20_7, [4_7). The determination of $\sigma_{0}$ in each group is normaily done with the infinite dilute total cross section with the exception of U23 (see [3_7, [4_7). For the test calculations $0_{0}$ has been detemined in a different way usine the effective total cross section with the strongest resonence sclf shielding (sec [-7). Beccuse these two rethods are extreme approvimations to the true situation 0 possible error of 0.002 may arise when usinf the $\sigma_{0}$-concent at least for asserblies with a not too "soft" neutron spectrum because the difference increases as the importance of the low energe and of the spectrum increases.

## IV.7. Accuracy of the Calculated Material Worths and Reaction Rates

a) Comarison of one and two-dimensional results

Jn order to save computer time the material worths and the reaction rates in the core center heve been calculated in spherical geometry using 26 encrev groups. This proccdure has been adopted because it seems more important to us to take into account all energy croups and to approximate
the geometry than to perform calculations for the real geometry but with a reduced number of enerey Erouns. Taking into account the real geometry and the whole number of eroups would necessitate rather large computer times. We expected that the desired quentities can be calculated well enouch with the procedure mentioned and checked this $b_{j}$ two-dimensional calculations for CrFAK-3A1 and SiFAh-3A2 with 4 and 11 Eroups. The results siven in table 3 indicate that for predominantly scattering
 is ver: poor as has been expected. This is essentielly cue to the influence of croup collapsing as is already known in the literature
 For the other nateriels the afreenent is rather cood: for the cese with 11 crouns the differences anount to about 5\%. This indicates that there is no surstematic deviation botween the results for soherical geometry and the tromimensional ones as could e.e. arise from differences in the nomalization interral. this has also been founc byy Pitterle et al. [35.7. For the same assemblies SIPAK 3 A1 and SNEAK $3 A 2$ in the central region of the core the microscopic reaction rate per unit flux and per atom of ij235, this meens the effective group averaged cross section , arrees within $0.15 \%$ and the importont reaction rate ratics $\mathrm{U} 23 \mathrm{e}(\mathrm{n}, \gamma) / \mathrm{U} 235(n, n)$
 one-dimensional 26 group results with two-dimensional 4- and 11-group results.

The results mentioned so far have been obtained using the sman set but it is expected that they are valid for other croup sets and other assemblies as well. Galculations for the two asserblies SIEAK $3 M 1$ and STEAK $3 A 2$ using the : HOXOT -set indeed showed practically the same results as obtained for the EIT:AK set. Therefore, it seens very probable that the one-dimensional results for the central material worth and the central reaction rate are not subject to systematic calculational errors.

## b) Cormarison of results with and without Rwhocorrection

As stated two paragraphs before the REMD-correction generally leads to a better agreement between calculations and experiment for the energy-dependent neutron flux. The differences between the spectrum calculated in the
usual way and that calculated with the RPlo-correction are quite appreciable: For the assembly SiFAK 3A1 e. $\mathrm{E}_{\mathrm{o}}$, they amount to about $\pm 15 \%$ in some eneriw crouns. The question arises wether such changes in the energy dependence of the neutron spectrum affect the calculated material worths to a remarkable extent or not.

The RE:O-correction is based essentially on a correction of the macroscopic elastic and total removal cross section elenents of the scattering matrix and is appropriate for the calculation of the neutron flux. The question if the RFMO-correction is also an adequate procedure for the calculation of the adjoint flux will not be considered here although we adopted this assumption in the present study.

In the following lists we compare the results for the calculated material worths using two different methods:
method a): The normal and adjoint fluxes used in the perturbation calcuLations have been obtained using RE: 0 -corrected group constants. rochod b): The normal and adjoint fluxes used are calculated in the usual manner without the REMO-correction for the group constants.

In both methods we used perturbation cross sections which have been calculated in the usual way without any REMO-correction. For a ricorous comparison of the influence of the REMO-correction one should apply in method e) the REMO-correction to the perturbation cross sections too. This has not been done here because it would have caused inconveniences in handling the programme for the REMO-correction and because we think that the effect is almost negligible for the perturbation calculations of the material worths.

This assumption seems to be reasonable because the REMO-correction has to be applied both to the normal and pertubed core composition and these two compositions are almost identical apart from the relatively small addition of the special material or isotope considered. The amount of material added to the normal mixture is in our cases $10^{20}$ atoms which means for most materials about $1-10 \%$ of the amount which is normally present in the

Results for Assembly RPR III. $1 / 8$ using the SIIFN - set


Results for Assembly GIEAK 3A1 usine SNENK-set

| Material | AL | B10 | C | CR | FE | II | 110 | III | 0 | Pu239 | U235 | U238 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Materiall worth method a$)$ | 0.367 | 1.019 | 1.174 | 0.988 | 0.976 | 0.992 | 1.021 | 1.029 | 1.263 | 0.996 | 0.993 | 0.997 |
| Materiol worth method b$)$ |  |  |  |  |  |  |  |  |  |  |  |  |

Results for Assembly SNEAK 3A? using SHEAK-set

| Material | AL | B10 | C | CR | TE | II | MO | III | 0 | Pu239 | U235 | U238 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\|\frac{\text { Material worth method } a)}{\text { Material worth method } b)}\right\|$ | 0.851 | 1.006 | 1.094 | 0.991 | 0.976 | 0.900 | 1.006 | 1.015 | 1.182 | 1.000 | 0.994 | 0.985 |

mixture. Therefore we expect that the influence of the PEMN-correction on the cross sections of both mixtures will be almost identical and the net effect on the perturbation cross sections will be practically zero because of a compensation effect since the perturbation cross sections are determined as differences between the cross sections oin both compositions which show ałmost icientical PE: 10 -corrections.

It can be seen from this list that for the fertile and fissile isotopes the influence of the REI:O-correction on the calculated material worth is fenerally smaller than $3 F$ and for the predominantly absorbing materials cenerally smaller than $5 \%$ which is not too micin comnared to the existing discrepancies between calculation and experinent. However, one should have this in mind when in the fluture these discrepancies come down to comparable mapnitude. For predoninently scatterinf materials, as expected, the influence of the NEFiO-correction is larger - up to $30 \%$. But for these materials all 26-group results either with or without RE:Ocorrection are somewhat doubtful as indicated in the preceding paragraph.
V. RESUITS AHD DISCUSSION
V.1. Results of Fundamental Mode Calculations forkeff

The results of the fundamental mode diffision calculations for $k$ eff are given in table V-1. The values given refer to the homogenized core composition. For an easy comparison with the measmred values the calcuiated criticality values for the SNEAK-series have been normalized in such a way that the normalized value for $k_{\text {eff }}$ of SNEAK $3 A 2$ is equal to unity for all sets of group constants.
In the figures AII-1 to AII-36 the neutron flux, the adjoint neutron flux and the collision density are shown for 12 assemblies. For an easy intercomparison of corresponding curves for different assemblies we have taken the same scale in the corresponding figures for the different assemblies. The curves have been obtained from the group values using a program which produces a smooth curve through the group values I-61_7. Except for the two-zoned cores SNEAK $3 B 2$ and $2 P R$ III- $48 B$ the values for the normal and adjoint fluxes have been taken from fundamental mode calculation. The normalization is

$$
\int_{0}^{\infty} v \Sigma_{f}(E) \phi(E) d E=1
$$

for the flux density and

$$
\int_{0}^{\infty} x(E) \phi^{+}(E) d E=1
$$

for the adjoint flux. For the two exceptions SNEAK $3 B 2$ and ZPR III $48 B$ the normal and adjoint fluxes at the core center have been taken from one-dimensional calculations. The corresponding curves therefore have been drawn with an arbitrary normalization. For the adjoint flux the arbitrary normalization has been done in the following way: for SNEAK $3 B 2$ the curve has been normalized to agree in the relatively flat region between 50 100 keV with the corresponding curve of SNEAK $3 A 2$; for ZPR III-48B the normalization has been done in the energy range from $0.4-0.8 \mathrm{keV}$ to give agreement with the similar curve of ZPR IIIm4 which is also flat in this energy range.
All results for the collision densities have been taken from one-dimensional calculationss This has been done because the programs involved in these calculations can handle only the results of one-dimensional calculations in the appropriate way. Therefore it would have been very inconvenient
to use the results of the fundamental mode calculation in order to get the corresponding, collision density. The curves given always belong to the inner core zone, sometimes to an even smaller artificial zone around the core center. The normalization for all collision kansity curres is arbitrary.

All results for the figures AII-1 to AII-36 have been obtained using the SNEAK-set.

In the following we will indicate the sensitivity of the criticality of the various assemblies studied to changes in the nuclear data. This will be done for each assembly by giving the criticality chenges for the most important changes of the group constant sets.

SUAK U1B: SNEAK + SIEPMB: $\Delta k=-0.015$
SNEPMB $\rightarrow$ SNEAFM: $\Delta k=+0.021$
This small assembly with a relatively hard neutron spectrum is very sensitive to the reduction of the U235 fission cross section which is implied in the first step and is omitted in the second step. The reduction of the $U^{238}$ capture data causes a criticality increase of $+0.006=0.021-0.015$.

SUAK UH1B: SNEPMB $\rightarrow$ SNEAPM: $\triangle \mathrm{k}=+0.008$
PU9SCP $\rightarrow$ SCTALO: $\Delta k=+0.003$
MOX911 $~$ MOXTOT: $\Delta \mathrm{k}=+0.004$
Due to the much softer neutron spectrum of this assembly compared to SUAK U1B changes in the high energy data are less important. The omission of the reduction of the $U^{235}$ fission cross section gives rise to a criticality increase which is only $1 / 3$ of that obtained for SUAK U1B. Taking into account the lower $U^{238}$ capture cross section of MOXON in the keV-region causes an increase of criticality which is not negligible for this assembly. The increase of criticality obtained with the new scattering probabilites for the inelastic scattering probabilities for the inelastic scattering is due to the special energy dependence of the neutron importance for this assembly which increases considerably below 1 MeV with decreasing energy.

ZPR III-10:

$$
\begin{array}{ll}
\text { SNEAK } \rightarrow \text { SNEPMB: } & -0.015 \\
\text { SNEPAB } \rightarrow \text { SNEAPY: } & \div 0.026
\end{array}
$$

The results for this assembly with a relatively hard neutron spectrum are very similar to that obtained for SUAK U1B.

ZPR III-25:

Apart from the results similar to ZPR III-10 the new inelastic scattering data cause a remarkable reduction of criticality which is due to the special form of the energy dependence of the neutron adjoint which decreases below 1 MeV with decreasing energy contrary to the behaviour for most of the other assembiies. Tine reducticn of the $U^{238}$ capture data in the $10-100 \mathrm{keV}$ region to the Low MOXON-values shows a remarkable effect on the criticality of this assembly because the enrichment is lower and the neutron spectrum somewhat sof'ter than that of the assemblies considered in the preceding paragraphs. The higher sensitivity to the $U^{238}$ capture data in this energy region is also indicated by the difference between the first two steps which amounts to $+0.019=0.029-0.010$ and is considerably larger than the corresponding data for the SUAK assemblies and for ZPRIII-10.

ZPR III-48: SNEAK $\rightarrow$ SNEPMB: +0.009

$$
\text { SNEAPM } \rightarrow \text { PU9SCP: }-0.009
$$

$$
\text { PUO2RE } \rightarrow \text { MOX911: }+0.008
$$

This assembly is rather sensitive to the $U^{238}$ capture data below about 100 keV which have been changed for the sets SNEPNB and MOX911. The new a-values for $\mathrm{Pu}^{239}$ of GWIN used in the PU9SCP-set cause a criticality reduction which is not as large as that of -0.016 reported previously $\leq 2 \_7$ where lower limits of the first preliminary $\alpha$-values of SCHOMBERG had been used.

ZEBRA 6A:

$$
\begin{array}{ll}
\text { SAM } \sim \text { SN: } \\
\text { SNEPMB } \rightarrow \text { PU9SCP: } & -0.010 \\
\text { PUO2RE } \rightarrow \text { MOX911: } & +0.006
\end{array}
$$

As expected this assembly shows a behaviour similar to that observed for ZPR III-48.

$$
\begin{aligned}
& \text { SNEAK } \rightarrow \text { STIEPMB: }-0.010 \\
& \text { SNEHMB } \rightarrow \text { SIEEAPM: }+0.029 \\
& \text { PUSSCP } \rightarrow \text { SCTALX: }-0.011 \\
& \text { PUO2RE } \rightarrow \text { MOXO11: +0.013 }
\end{aligned}
$$

SMEAK-Series: The criticality of the assemblies consiaered within the SKEAK-3A-jeries reacts most sensitive to the first two steps of changes in the group cross section sets. The fact that the absolute values of the criticality differences for the first two steps are of the same magnitude. indicates that the steam density coefficient is much more sensiti.fe to changes in the $y^{235}$ fission cross section tham to those in the $U^{238}$ capture cross section in the energy region considered (i.e. ..etween about $20-500 \mathrm{keV}$ ) . One should of course always have in mind that the criticality values for the SINEAK-series are normalized in such a way as to give a $k_{\text {eff }}$ equal to unity for SNEAK $3 A 2$.

SNEAK 3A1:

$$
\begin{array}{ll}
\text { SNEAK } \rightarrow \text { SINEPMB: } & -0.009 \\
\text { SNEPMB } \rightarrow \text { SIIEAPM: } & +0.018 \\
\text { PUO2RE } \rightarrow \text { MOX911: } & +0.009 \\
\text { MOX911 } \rightarrow \text { MOXTOT: } & +0.006
\end{array}
$$

The criticality change corresponding only to the reduction of the $\mathrm{U}^{238}$ capture cross section in the first step amounts to +0.009 . It is overcompensated by the effect of the reduction of the $U^{235}$ fission cross section which is twice as large as an be seen from the second step leading from a criticality change of $\mathbf{- 0 . 0 0 9}$ for the first step to a total criticality change of +0.009 for the first two steps. From the last two steps it can be seen that for SNEAK 3A1 the changes caused by the MOXON data are more important in the $10-100 \mathrm{keV}$ region than in the energy region below 10 keV .

| SNEAK 3A2: | SNEAK | SNEPMB: | -0.006 |
| :--- | :--- | :--- | :--- |
|  | SNEPMB | SNEAPM: | +0.013 |
|  | PUO2RE | MOX911: | +0.007 |
|  | MOX911 | MOXTOT: | +0.007 |

For SNEAK $3 A 2$ the results are qualitatively similar to those for SNEAK $3 A 1$. The last two steps cause nearly equal criticality changes. This means that for this assembly with respect to criticality the influence of the change to the MOXON data is of equal importance for the energy region below 10 keV as for the $10-100 \mathrm{keV}$ region. This fact gives direct evidence of the softer neutron spectrum of SNEAK $3 A 2$ compared to that of SNEAK $3 A 1$.

```
SNEAK 5C: SNEAK \(\rightarrow\) SMEPMB: +0.008
    SNEAPM \(\rightarrow\) PU9SCP: -0.015
    PUCஜRE \(\rightarrow\) MOX911: +0.008
    MOX911 \(\rightarrow\) MOXTTOT: +0.014
```

The largest criticality change for this assembly is caused by the change to the $P_{u}{ }^{239} \alpha$-שalues of GNIN (SNEAPN $\rightarrow P U 9 S C P$ ). The reduction of the $U^{238}$ capture cross section to the PÖNITZ-data in the first step produces nearly the same criticality increase as that of the subsequent inclusion of the MOXON-data in the $10-100 \mathrm{keV}$ region (PUO2RE $\rightarrow$ MOX911). The criticality effect of the change to the MOXON $U^{238}$ capture data in the whole energy region concerned overcompensates that of the change to the GWIN $\alpha$-data. A comparison of the two last steps illustrates the importance of the energy region below 10 keV for this assembly compared to e.g. SNEAK 3A2 which already has a relatively soft neutron spectrum.

```
ZPR III-55: SNEAK -> SNFPMB: +0.018
    SNEAPM }->\mathrm{ PU9SCP: -0.011
    FU9SCP -> SCTALO: -0.013
    PUO2RE -> MOK911: +0.017
    MOX911 -> MOXIOT: +0.017
```

For this assembly relatively large criticality changes have been obtained. Qualitatively the results are similar to that for SNEAK 5C or even to SNEAK 3A2 with the exception of the importance of the inelastic scattering data for this assembly which is due to the special neutron adjoint as has been discussed for ZPR III-25. For a quantitiative understanding of the effects one should have in mind that the atomic number density of $U^{238}$ is considerably larger than that e.g. for SIJEAK 5C and SNEAK $3 A 2$ and that the neutron spectrum is considerably harder than that of SNEAK 5C. This fact is for both assemblies illustrated for example by the different criticality changes cased by the two last changes in the group cross section sets.

At the end of this chapter we will discuss briefly which assemblies are most sensitive to the changes in the nuclear data which have been performed during this study.

SHEAK $\rightarrow$ STEEPAB:

| ZPR III-55 | $:+0.018$ |
| :--- | :--- |
| SNEAK 5C | $=+0.008$ |
| SUAK UIB | $:-0.015$ |
| ZPR III-10 | $:-0.015$ |
| ZPR III-25 | $:-0.010$ |

Inis change actually consists of two iifferent changes:
a) the reduction of the $U^{238}$ capture cross section
b) the reduction of the $U^{235}$ fission and capture cross section.

The second part has no influence on the Pu-fuelled assemblies. These assemblies show the effect of the first part only and therefore a criticality increase is observed. For all U-fuelled assemblies the effect of the first part is more than compensated by the affect of the second part and therefore we obtained a decrease of criticality for these assemblies. Since in the next step (SNEPMB $\rightarrow$ SNEAPM) the second part has been cancelled the effect of the first part can be determined separately (assuring additivity for the criticality changes of the two parts). This leads to the result that the criticality effect of the first part for the assemblies ZPR III-25 and ZPR III-10 is of comparable magnitude as the corresponding values for ZPR III-55 and SNEAK 5C.
$\begin{array}{ll}\text { SNEPMB } \rightarrow \text { SNEAPM: } & \text { ZPR III-25 } \\ & :+0.029 \\ & \text { ZPR III-10 } \\ & \text { SUAK U1B } \\ & \text { SNEAK 3A1 }:+0.026 \\ & :+0.021 \\ & \text { +0.018 }\end{array}$
This step corresponds to the cancelling of the second part mentioned for the step before i.e. going back from the reduced $U^{235}$ fission cross sections to the previously used values in the $25-500 \mathrm{keV}$ regicn. It produces rather large criticality changes, of course only for the U-fuelled systems.

```
SNEAPM + PU9SCP: SNEAK 5C : -0.015
    ZPR III-55 : -0.011
    ZPR III-48 : -0.010
```

The inclusion of the $\mathrm{Pu}^{239}$ a-values of GWIN results in a criticality reduction of about $\mathbf{- 0 . 0 1}$ for the usual Pu-fuelled assemblies. The reduction becomes even larger if the neutron energy spectrum becomes softer. SNEAK 5C seems
to be as the locrer end of the "soft" assemblies because its median fission energy of 1.2 keV is rather low. This has been demonstrated in our study because a reduction of 0.016 has been obtained in the present study for the somerhat "murder" assembly SNEAR 5B which is described in 5627 and has a median fission energy of about 3 keV .

## PU9SCP $\rightarrow$ SCTALO: $\quad$ ZPR III-55: -0.013 <br> ZPR III-25: -0.011

The new inelastic scattering probabilities generally have only a small effect on the criticality. The two exceptions given above are caused by the special form of the energy dependence of the adjoint flux as mentioned before.

$$
\begin{array}{ll}
\text { SCPALO } \rightarrow \text { UPUCOR: } & \text { SUAK U1B: }-0.006 \\
& \text { ZPR III-10:-0.005 } \\
& \text { SUAK UH1B: }-0.004
\end{array}
$$

The reduction of the fission cross sections in the energy region above 2 MeV causes only relatively small criticality changes for the assemblies considered in the present study. For smaller assemblies with even harder neutron spectra more marked effects are to be expected.

| UPUCOR. $\rightarrow$ PUOZRE: |  |
| :--- | :--- |
| SNEAK 5C | $:+0.004$ |
| ZEBRA 6A $:+0.004$ |  |

Because of the relatively small concentration of higher plutonium isotopes in the presently available assemblies the effect of rather drastic changes in the nuclear data for the higher plutonium isotopes is not very prom nounced.

| PUO2RE $\rightarrow$ MOX911: | ZPRIII-55: |
| :--- | :--- |
|  | +0.017 |
| ZPRIII-25: | +0.013 |

Because of the large $\mathrm{U}^{238}$ atomic number densities present in both assemblies and because of their enersy distributions of the neutron flux which give considerable weight to the energy region between $10-100 \mathrm{keV}$ these tws assem-
blies show the largest effects of the reduction of the $U^{238}$ capture data from the PÖNITZ-values to the MOXON-values in this energy range.

MOX911 $\rightarrow$ MOXTOT: ZPR III-55: +0.017
SNEAK 5C : +0.014
SNEAK 3A2 : +0.007
This step gives only a marked criticality effect for assemblies with soft neutron spectra. The effect is most pronounced for ZPR III-55 with its relatively large $U^{238}$ concentration.
V.2. Results for $k_{\text {fff }}$ from One- and Two-Dimensional Calculations

The results for the criticality of the various assemblies obtained in onesnd two-dixsensional diffusion calculations using different sets of group constants are given in table V-2. These results essentially confirm the results obtained by the fundamental mode calculations and thus provide a check that our general approach is correct which assumes that if no two-dimensional results are available for the criticality differences when going from one group set to the other the corresponding fundamental mode criticality differences can be used. This assumption is valid if the criticality differences are not too lerge.
Its validity is reduced to some extent if the properties of the blankets or reflectors are changed by changes in the group constants. This is expecially true of the reductions in the capture cross section of $U^{238}$ and to a smaller degree of the reduction in the high energy fission cross sections of all materials (SCTALO UPUCOR).
V.3. Resulta for the Best Avai Iable Values or the Criticallty

In this section we have to compare our calculated criticality values for the various essemblies with the corresponding measured values. The calculated values are based on the results of two-dimensional diffusion calculations. If necessary the criticality differences obtained by fundamental mode calculations for successive changes in the group constantisets have been used to determine in an approximate manner criticality values for the
modified data sets which are equivalent to twom, in wis. nal results. In order to establish the best theoretical values "...i eff some corrections have to be applied as outlined in chapter IT-2. 'the numerical values of these corrections for the various assemblies are given in table V-3. The origin of the data is described in Appendix. If for each assembly separately. The corrections have been assumed to be the same for the different sets of group constants used in this study. They have been applied to the exactly or approximately determined criticality values which correspond to two-dimensional diffusion calculations. In this way the best available criticality values of the various assemblies have been determined for the different sets of group constants. They are given in table V-4 together with the experimental results. These results will be discussed in a subsequent chapter.

## V.4. Results for Material Worth Ratios and Central Reaction Rate Ratios

The information on the material worth and reaction rate of various materials is considered in our study as complementary to the information provided by the criticality. The normalization of the material worths relative to that of $\mathrm{U}^{235}$ which is used generally in this study brings the advantage to avoid the trouble with the well known discrepancies in the absolute magnitude of the measured and calculated material worths or reactivity coefficients 1-637. Furthermore the calculated values are independent of the somcalled normalization integral which may perhaps be in error at Least for one-dimensional models of the real two or three-dimensional problems because the geometry is not taken into account properly although our results mentioned in chapter IV-7 seem to indicate that this is not the case. The reaction rates are usually normalized in the experiments to the number of fissions of $\mathrm{U}^{235}$. For an easy comparison with the experimental results this normalization has also been done for the theoretical results.
The theoretical results presented for the central material worth ratios have been obtained by first order perturbation theory using the normal and adjoint fluxes of homogeneous diffusion calculations. In the interpretation of discrepancies between theory and experiment one should be
very cautious because in the calculations the effects of samale size (see e.g. [-64_7) and sample environment have not been taken into account. For those materials which show a large contribution to the material worth by the somealled scattering- or degradation-term the theoretical results can not be considered as very reliable as has been explained in chapter IV-T. Further but periaps less severe aoubts ou the reliability of the calculated material worths arise from the fact that the group constants usually used in the adjoint flux- and perturbation calculations are the same as that used for the normal flux calculetions, i.e. group constents which have generally been obtained by a flux averaging procedure within the groups (see e.g. [607).
The theoretical results for the central reaction rate ratios have been obtained also by homogeneous diffusion calculations. In comparing them with experimental results one should exarine quite carefully if the experimental situation corresponds to this assumption of homogeneity. It is known that the so-called chamber-measurements for the determination of reaction rates do not correspond to the assumption of homogeneity made in the calculations. On the other hand the reaction rates measured with foils (see e.5. [65_7) often can be adequately treated theoretically only by heterogeneous calculations.
Some theoretical results from spherical calculations for SNEAK 3 A1 and SNEAK 3AP are given in table V-5. For the same assemblies the reaction rate ratios $\sigma_{f} U^{238} / \sigma_{f} U^{238}$ and $\sigma_{e} U^{238} / \sigma_{f} U^{235}$ obtained from fundemental mode homogeneous calculations with the various group sets are given in table V-6. Corresponding experimental results are presented in [-65_7, 1-66_7 and 1-67.7. In comparing them with our theoretical results one should be aware of the effect of sample size for the material worth [-64_7 and the fact that the experimental results for reaction rates measured with chambers and foils are sometimes quite different [65_7. Therefore from the tables $V-5$ and $V-6$ only the tendencies with the different group sets can be deduced but no final conclusions can be drawn by the direct comparison with the experimental results.

The theoretical results for the central material worth ratios and central reaction rate ratios for ZPR III-48 are given in table V-7. Generally the agreement between theory and experiment is not too bad. Perhaps a certain amount of the disagreement for the material worth of $\mathrm{B}^{10}, \mathrm{Fe}, \mathrm{Cr}$, Ni, Mo may be due to the sample size effect. This seems at least to be
possible consicering the results shown in 5647 for most of these materials. The calculated vorth for sodium seems to be doubtful since it is apredominantly scattering material.
The theoretical result for the worth of Pu ${ }^{240}$ shows a marked improvement compared to the experimental result when the PUOZRE-set is used i.e. then the upaated data for the higher pilutonium isotopes are included. The theoretical result for the reaction rate ratio $\sigma_{f} P u^{239} / \sigma_{f} v^{235}$ is consistently lower then the measured one. This gives - togetherwith the material worth ratio - an indication that $\mathrm{Pu}^{239}$ is underreactive in the group sets used in this study.
In table $V-8$ the central reaction rate ratios obtained from fundamental mode homogeneous diffusion calculations with various group sets are given.
Table V-9 shows the theoretical results for the central material worth ratios and central reaction rate ratios for assembly ZPR III-48B. The results are essentially the same as those for the assembly ZPR III-48. Therefore no additional or more precise conclusions can be drawn from a comparison of the theoretical and experimental results than those already obtained for the assembly ZPR III-48. Especially no specific results with respect to $\mathrm{Pu}{ }^{240}$ can be deduced by comparing theory and measurement for ZPR III-48B.
It has been checked for assembly SNEAK 3 A2 that the energy dependence of the neutron flux in the core center is practically the same for the diffusion and $S_{N}$-calculations. Therefore the central reaction rates are nearly the same in both calculations. The very small deviations are unimportant compared to the discrepancies which still exist between theory and experiment.

## V.5. Results for Reaction Rate Traverses

The only experimental results for reaction rate traverses considered for a comparison between theory and measurement are those for SNEAK-3A2 [65_7 and [67.7. Earlier measurements have most times been performed using chambers. These measurements are not considered very reliable. Furthermore it is argued that streaming eifects sometimes may lead to erroneous results in the experiments (see [ 67 _ $7 \mathrm{p}, 42$ ) . The presently available foil experiments are considered more reliable (see [67_7p. 42): Foil ex-
perinents which can with some confidence be compared with calculated results are only the measurements of BÖHME and SEUFERT in SNEAK 3A2 described in ! 65 _7 and also in $\mathbf{1}^{67}$.7. These experiments show discrepancies to the chamber measurements, which are attributed to the streaming effect, and discrepancies to the theory especially in the blanket region near the core which are considered as "not yet understoad" in 1-67.7. In the core region the calculated traverses for $\sigma_{f}\left(U^{235}\right), \sigma_{f}\left(U^{238}\right)$, $\sigma_{c}\left(U^{238}\right)$ show a steeper descent to the coremblanket interface than the experimental ones. This effect is less strong for $\sigma_{f}\left(v^{238}\right)$. In the blanket region near the core all three traverses show an increase of the ratio of the theoretical to the experimental result which is most pronounced for $\sigma_{f}\left(U^{238}\right)$.
The three reaction rate traverses mentioned have been calculated for SNEAK-3A2 with the SNEAK-set as the basis of our study and the MOXTOT-set as the final group set of our present study. The ratio of the results obtained with the MOXIOT-set to the corresponding results with the SNEAK-set are shown in fig. 1. All results have been ottained for the axial direction using one-dimensional diffusion theory for the homogeneous case. The radial leakage has been taken into account by a global buckling. For the capture rate in $U^{238}$ and the fission rate in $U^{235}$ the traverses calculated using the MOXTOT-set are larger in the outer core region and especially in the blanket then the traverses calculated using the SNEAK-set. Both traverses are normalized at the core center. The fission rate traverse for $U^{238}$ remains essentially unchanged within the core region. In the blanket region, however, the MOXTOT-set-results are considerably smaller than the SNEAK-setresults. From the results of fig. 1 it can be concluded that all three traverses now show nearly the same tendency within the core region when the MOXTOT-set-results are compared with the experimental results. The agreement between theory and experiment is improved for the core region when the MOXTOT-set is used compared to the SNEAK-set results which have been used for fig. 19 of [ 65 _ 7 respectively fig. 29 of [ 67 _ 7 which is reproduced as fig. 3 in this report for sake of an easy comparison. It seems probable that the calculations will give a slightly steeper descent of the traverses to the core boundary than the experiments, but the differences will become rather small in the core region using the MOXTOT-set.

In the blanket region the peak within the first 5 cm of the blanket near the core which is already observed when comparing the calculated to neasured ratio for the traverses for the SHEAK-set in fig. 3 is increased by about $5 \%$ for $\sigma_{c}\left(U^{238}\right)$ and $\sigma_{f}\left(U^{235}\right)$ when the MOXPO set is used. This leads to an overall deviation between theory and experiment of about $10 \%$ at a distance of about 5 cm from the core-blanket interface for both reaction rate traverses when the MOXTOT-set is used. For the fission rate in $U^{238}$ the situation is reversed. Using the MOXTOT-set reduces the deviation observed between theory and experiment when the SNEAK-set is used. However, one should have in mind that this deviation for $\sigma_{f}\left(U^{238}\right)$ is the most pronounced one of all the three reaction rate traverses studied. Even when the MOXTOT-set is used a deviation of about $15 \%$ will remain between theory and experiment for the $\sigma_{f}\left(U^{238}\right)$ traverse at a distance of about 5 cm from the core-blanket interface. This leads to the result that with the MOXTOT-set all the three reaction rate traverses $\sigma_{c}\left(u^{238}\right), \sigma_{f}\left(U^{238}\right)$, $\sigma_{f}\left(U^{235}\right)$, can be rather well predicted within the core region but are overestimated by 10 to $15 \%$ in the blanket at a distance of about 5 cm from the:oorembanket interface using the usual diffusion theory results. Since this overestimation appears for all three reaction rates it is not very probable that it is due to spatial resonance self-shielding which cannot be taken into account by the presently used $\sigma_{0}$-concept for the homogeneous mixtures. If the neglect of the spatial resonance shielding should be responsible for the deviation between theory and experiment in the core-blanket transition region one would expect that the reaction rate traverses $\sigma_{c}\left(U^{238}\right)$ and $\sigma_{f}\left(U^{235}\right)$ would show opposite tendencies since the atomic number density of $\mathrm{U}^{238}$ is increased that of $\mathrm{U}^{235}$ is decreased when going from the core to the blanket region. Therefore we tried to check whether a transport effect could be responsible for the observed deviations between experiment and diffusion theory reault. Unfortunately at the time of this study no code was available to evaluate reaction rate traverses using the fluxes calculated by an $S_{N}$-code. Therefore we used the fission source traverse instead of the reaction rate traverses. Fig. 2 shows a comparison of the one-dimensional $S_{6}$ and diffusion theory fission source traverses. Both are normalized to give one source neutron in the whole reactor. For the calculations 35 mesh-intervals in the core and 20 mesh-intervals in the blanket have been used. The desired accuracy in $k_{\text {eff }}$ was $10^{-4}$ in both calculations and a source accuracy of $10^{-3}$ was
required in the diffusion calculation. The results shown in fig. 2 have been confirned by doubling the number of mesh-intervels and requiring a ten times higher accuracy. Fig. 2 clearly demonstrates that by using transport theory the fission source distribution in the blanket is changed by about 1 后 compared to the diffusion theory result. This means that at least the fission rate of $U^{238}$ will be changed by about the same amount.
The dip of the curve in fige. 2 at a distance of about 5 cm from the coreblanket interface has just about the same magnitude (about 10\%) as the peak discussed just before which will oe obtained practically at the same position when the MOXIOT-set-results are compared with the experimental results for the reaction rate traverses. The peak shown in figo 2 in the outer blanket region cannot clearly be verified experimentally because the measurements do not have the necessary accuracy in this region because of bad statistics.
For the inner blanket region, nowever, fig. 2 strongly indicates that the reaction rate traverses should be evaluated using transport theory results if an accuracy of better than $10 \%$ is required. It can be expected from the preceding results that doing this and using the MOXTOT-set the deviations between theory an experiment for the reaction rate traverses will become rather small in such regions of the core where the experiments can be considered as reliable.

## V.6. Results for the Neutron Importance

The results for the neutron importance for the assembly SNEAK 3 A2 reported in [ ${ }^{67} 7$ table 10 and 1-67_7 fig. 26 and in 1568_7 show no drastic disagreement between theory and experiment. The most pronounced deviation of about $5 \%$ has been observed for the $\mathrm{Sb} / \mathrm{Be}$ source with an energy of 24 keV . We studied whether the new groups sets would bring about certain improvements. We found that in the energy region of interest the neutron importance bhows a smailer energy slope with the MOXTOT-set compared to the results with the SNEAK-set. The changes are small, of the order of $2 \%$ or less. They tend to decrease the differences between theory and experiment reported in table 10 of $\mathbb{I}^{67} 7$ but for the lowenergy $\mathrm{Sb} / \mathrm{Be}$ source the difference between the measured and calculated result is still larger than the experimental mincertainty.

He wented to stuay if the Findo-correction for the elastic removal group constants causes changes in the calculated neutron importance. We excluded the problem whether the Remo-correction is an appropriate method when applied to the adjoint flux or importance calculation. A calculation vith REMC-corrected Eroup constants resulted in an additional small decrease of the difference between theory and experiment of about $1 \%$ for the $\mathrm{Sb} / \mathrm{Be}$ source but even then the remaining discrepancy is larger than the experimental uncertainty.
The influence of using heterogeneity-corrected cross sections for the adjoint flux calculations has not been studied in this work. A more principal uncertainty in the adjoint flux calculation mentioned e.g in $1^{-60} 7$ is caused by the use of flux-weighted group constants for the calculation of the adioint group fluxes.

## V.T. Heterogeneity Calculations

V.7.1. Introduction

As has been shown e.g. in [58_7, 15 65, 166_7, 167_7, 169_7 heterogeneity calculations are helpful and sometimes necessary to control and improve the accuracy of nuclear data. Beside the calculations which are performed to obtain criticality corrections for most of the assemblies studied in this report using always the SNEAK-set, we started a few investigations to determine the influence of different group sets on the heterogeneity correction of $k_{e f f}$ and to check the method applied. Furthernore we calculated reaction rate distributions within the unit cells and within the fuel platelets itself again for different group sets. On the one hand we wanted to see of what use these data are for the improvement of special cross sections, on the other hand we wanted to get more theoretical data concerning some experiments referred to in 1558_7, 165_7. [66_7, [67_7. Up to now a part of the reported experimental data could not be verified by calculations. The authors supposed that these differencies may be caused by inexact cross sections as well as by insufficient calculational methods.
Our own ivestigations did have the aim to clear some of these discrepancies.
V.T.2.1. Codes, Cross Section Sets, Assemblies Investigated and Their

Geometric and Material Data
All heterogeneity calculations were performed with the code ZERA, described in $=.7$ mad 158_7. ZERA is part of the MUSYS-system and contains evaluation routines to calculate reaction rates and to proauce heterogeneity corrected cross sections for the homogenized unit cells.

For our comparison-calculations we used the old ABH-set and the more recently established sets, called SNEAK, SNEPMB and MOXTOT, presented in $:^{-3} 7$ and in this report.
We investigated the two critical SNEAF assemblies $3 A 1$ and 3A2. Both facilities have two uranium fuelled core zones with about the same homogenized composition in each zone, but with differing structure of the mit cells. The detailed description of both facilities, of their macroscopic geometry and dimensions of their homogenized compositions as well as the microscopic structure of their unit cells and the atom densities of the single platelets are given in ref. $\mathrm{T}^{6} 66$ (3A1) and in ref. 1565 ( 3 A 2 ). We have taken the necessary data exactly from these reports.

The atom densities of the $35 \%$ enriched uranium platelet used in the unit cell of core zone II are taren from ref. $]_{67}^{6}$ 7, while the atom densities of the natural uranium platelet has been determined in such a manner that the homogenized uranium densities in the unit cell of core zone II are identical with that used in core zone I. To explain it in more detail: The unit cell of core $z$ one I has four platelets, one of them is a $20 \%$ enriched uranium platelet, the unit cell of core zone II has 36 platelets, five of them are $35 \%$ enriched uranium platelets and four of them are natural uranium platelets.

For all ZERA-calculations we used the following total bucislings: Assembly $3 A 1: B^{2}=22.01 \times 10^{-4} \mathrm{~cm}^{-2}$, Assembly $3 A 2: B^{2}=25.55 \times 10^{-4} \mathrm{~cm}^{-2}$.
For the one-dimensional radial diffusien calculation for SNEAK $3 A 1$ we used the buckling $B^{2}$ axial $=8.47 \times 10^{-4} \mathrm{~cm}^{2}$.

## V.7.2.2. Cell Structures, Modifications and Motations

All SNEAK 3A platelets have the same thickness of 0.314 cm . Within a single zone the platelets are arranged in periodic sequences. Such sequence is defined as the normal unit cell. In order to enlarge the effect of heterogeneity one can re-arrange the platelets as is done in the bunching experiments. Single bunched cells are built from two normal cells, double bunched cells from four normal cells. The thickness of a single material zone (two identical platelets) amounts to 0.628 cm for the single bunched cell and to 1.256 cm for the double bunched cell (four identical platelets).

## V.7.3. Influence of Heterogeneity on $k$ eff

In order to get the effect of heterogeneity by means of ZERA-calculations it is necessary to compare with results from the homogenized cells. We obtain the heterogeneity correction if we reduce the platelet-thickness by a factor large enough to avoid spatial self-shielding effects. We have chosen in our calculations a factor of 1000. (To avoid numerical difficulties perhaps a factor of 100 is more appropriate.) We denote such a cell as quasihomogeneous.
Now we define the heterogeneity effect of $k_{\text {eff }}$ as difference $\Delta k_{\text {eff }}$ between the ZERA-calculated multipicication factors of the heterogeneous and the quasihomogeneous cells.
Implicitly the ZERA-code relies on the assumption that the source distribution is flat within a single zone. For this reason we divided the uranium zone in five subregions with the following thicknesses: $0.02,0.03$, $0.214,0.03,0.02 \mathrm{~cm}$. In case of bunched cells these thicknesses are duplicated or multiplied by four. This is valid for the unit cells of core zone I.
V.7.3.1. Zera-Calculated $\Delta k$ eff for Different_Cross gection_Spts

These calculations are performed using the normal unit cell of core sone I. The results are given in the following table.

|  |  |  |
| :---: | :---: | :---: |
| Group Set | 3A1 | 3 A 2 |
| ABN | -1.54 | -1.32 |
| SNEAK | +1.14 | +2.53 |
| SIIEPMB | +1.20 | +2.56 |
| M $\overline{\mathrm{XT}}$ ¢ $^{\text {P }}$ | +0.90 | +2.22 |

The table shows at first sight the very strong difference between the results of the ABN-set on the cne hand and the results of the SNEAK-set and the succeeding ones on the other hand.
The variations within the results of the new SNEAK-sets are comparative small.
V.7.3.2. The differences between $A B N$ and SNEAK results increase strongly with increasing heterogeneity. This is apparent from the next table, which contains the results for different degrees of bunching. ${ }^{( }$)
$\Delta k_{e f f}$ (ZERA-calc.) I in units $10^{-3}-7$ for different degrees of bunching (Zone $I$ )

| Assembly | 3A1 |  |  | 3A2 |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Groun Set | normal | single | double | normal | single double |  |
| ABN | -1.54 | -4.24 | -9.21 | -1.32 | -3.98 | -10.51 |
| SNEAK | +1.14 | +0.83 | -0.10 | +2.53 | +3.03 | +2.35 |

K) Meanwhile it has been found that the main reas on for this discrepancy between the results for this two cross section sets comes from the fact that in the SNEAK-set and the succeeding ones the cross section $U^{238}$ used for the determination of the background cross section $\sigma_{0}$ -necessary for the calculation of the resonance-self-shielding- has been set equal to the potential cross section of 10.6 barns in the groups $10-21$ whereas in the $A B N$ met the total cross sections are used for the corresponding values.

## V.7.3.3. One-Dimepsional Diffusion Calculations vith Heterogeneitr Corrested

Cross Sections for two Core Zones
The ZERA-calculated $\Delta t$ eff of the central core-zone I is generally used for the correction of one-dimensional diffusion theory results. This method is justified, if the assembly nay be well described by epolying a fundamental mode calculation. A better aporoximation is the onedimensionel calculation of $\Delta k_{\text {eff }}$ with ZERA-corrected cross sections. In our case the two zones have practically the sane homogeneous compositicn, but a different structure of the unit cells. Therefore it seems to be interesting to check the ZERA-correction $\Delta k_{\text {eff }}$ by calculatiag the same quantity in one-dimensional diffusion theory with ZERA-corrected cross sections for both core zones. Doing this a certain difficulty arises: The unit cell of core zone II is built by 36 platelets. A further splitting in subregions has become a question of calculation-time. Therefore we kept the normal structure using the actual platelet thickness for each region. We have justified this procedure by a test calculation in core zone $I$. Only a negligible change in $\Delta\}_{\text {eff }}$ of about $1 \times 10^{-5}$ which is within the accuracy limit was obtained changing from five subregions to one single region per platelet. $O f$ course, this is valid only for the normal unit cell but not for bunching experiments. In the following table the ZERA-results for each core zone and the onedimensional radial diffusion result for the whole assembly are given.

```
SNEAK 3A1 \(\mathrm{Sk}_{\text {eff }}\) I in units of \(10^{-3} 7\)
\(B^{2}\) totel \(=22.01 \times 10^{-4} \mathrm{~cm}^{-2}, B^{2}\) axial \(=8.47 \times 10^{-4} \mathrm{~cm}^{-2}\). SNEAK-set
```

| ZERA-calc. Zone I | +1.14 |
| :--- | :--- |
| ZERA-calc. Zone II | -0.06 |
| Diff.-calc. whole assembly <br> with ZERA-correctez cross <br> sections in both core zones | +1.80 |

Tae differing ZERA-results for the two core zones demonstrate the necessity to determine $\operatorname{lk}_{\text {eff }}$ at least by a real one-dimensional calculation using the ZERA-corrected cross sections for the two zones of different cell structure.

The somewhat surprising results of the preceaing teble are probably due to the fact that using the heterogeneity-corrected cross section the flux shape is slightly modified which is equivalent to a slight correction of the buckling for the ZERA-calculations.

Generally it will be not sufficient to take only ZERA-results for a central unit cell as heterogeneity correction for a one-dinensional keff"
Y.7.4. Reaction Rate Ratios of Uranium for a Central Unit Cell of SNEAK 3A? and their Depencence an the Degree of Bunching. Comparison with Experiments

The experimental data are reported in 1-65_7 Fig. 15 and 1567. Fig. 15, Table 11. Some theoretical data are given there too, but they did not agree well with the experimental values.

In order to check the theoretical values and to get a deeper insight into the effect of the different nuclear data we re-determined these nuantities by ZERA-calculations. The results of these calculations can be considered as representative for central reaction rate ratios, because in the midde of the core the macroscopic flux spectrum of diffusion calculations is in good agreement with the fundamental mode spectrum using a suitable buckling. Fig. 4 is presented in the same manner as has been done in 1565.7 Fig. 15. We have only replaced the theoretical data.
In case of the $\sigma_{f}^{28} / \sigma_{f}^{25}$ ratio neither the ABN nor the SNEAK set are able to represent the experimental data. Both sets are completely unable to verify the dependence of the degree shown by the experiments. This leads us to the suspicion that the given experimental data are erroneous. In case of the $\sigma_{c}^{29} / \sigma_{f}^{25}$ ratio the $\operatorname{SNEAK}$ set represents the experimental data better, although the theoretical points do not lie completely within the experimental errors.

A lowering of $\sigma_{f}^{25}$ by about $6 \%$ for the normal cell or an increase by about the same amount for the bunched cells would result in a rather good agreement for these ratios between theory and experiment considering only the dependence on the degree of bunching.

## V.7.5. Fine Structure of Reaction Dates

The comparison between ZERA-calculated data and the measured spatial distribution of reaction rates within a single platelet is suggested in the above mentioned references as a suitable method to check end improve the accuracy of cross sections. It was our aim to apply this method to the new group sets. Therefore we investigated the influence of various cross section sets on the rate distributions within the $20 \%$ enriched uranium platelet of $3 A 2$ unit cell (zone I).

The results are given in Fig. 5A and Fig. 5B. All rates are normalized to one in the central subregion of the uranium platelets.

The most essential results and conclusions are the following ones:
a) For the new group sets all reaction rate cell traverses for $U^{238}$ and $\mathrm{U}^{235}$ lie close together. But for $\mathrm{U}^{235}$ a large difference exists bet-ween these curves and that calculated using the ABN set. This is valid for fission rates as well as for capture rates of $u^{235}$.
b) The $U^{235}$ reaction rate distributions calculated with the new group sets show so small differences, that a significant conclusion concerning the accuracy of cross sections seems to be impossible.
c) We feel that the differences in the normalized cell traverses for $U^{238}$ are not so significant between the various group sets that definite conclusions can be drawn by comparing them with the experiment. Especially not because the shape of the curve is influenced by the two quantities, the infinite dilute cross section ( $\sigma_{c}^{\infty}$ ) and the resonance-self-shielding factors (f-factors). However, it seems to us that precise and reliable measurements of the central reaction rate ratio $\sigma_{c}^{28} / \sigma_{f}^{25}$ will give information on the correctness of the $U^{238}$ capture data as can be seen from Fig. 4. This information will be additional and more unique than the information which can be obtained fram a comparison of $k_{\text {eff }}$ between theory and experiment.
V.T.6. General Conclusions

The most striking result of our investigations is the very large discrepancy between ZERA-calculated data using the ABN set on the one hand and using the more recently established sets on the other hand. Compared to this discrepancy the differences within the results of the new group sets are small, although considerable changes of cross sections exist between them. Therefore there seems to exist a fundamental difference between the old ABN- and the new sets and/or the treatment of these data by the ZERAcode.

Furthermore it seems to be impossible to come to a definite conclusion on cross section accuracy by evaluating the fine structure of reaction rates within normal, single or double bunched cell-regions. More information may become available by enlarging the heterogeneity and probably by considering in more detail the energy dependence of the neutron flux in special important energy regions.

## v.8. Results of the $S_{X_{N}}$-calculations

In order to get a reliable value for the calculated multiplication factor which can be compared with the corresponding experimental one, we have to apply certain corrections to the multiplication factor resulting from calculativs using diffusion theory for the homogenized material composition of the different zones of the assembly. An essential correction is the transport- or $S_{N}$-correction which takes into account the difference between an ampropriate treatment of the neutron transport process and the usual diffusion approximation for this process. Although some of the transport corrections are given in the literature and were partially applied in our study we wanted to have an independent check of these data. Therefore we evaluated for most of the assemblies that we have studied $S_{N}$-corrections by means of our own nuclear data and code. All calculations have been done in one-dimensional geometry with the code DTK K 70_7, the Karlsruhe version of the well known DTF-IV-code /T1, 72_7. using the 26 group SNEAK-set as nuclear data basis. ${ }^{*}$ We have taken the original $S_{N}$ constants as published in / T1_7on pages 135 up to 138.

Having had not much experience with the code at the beginning of the present work, first of all some studies have been undertaken to become more familiar with it. Mainly we have been interested in the question how to choose mesh-size and the order $N$ of the $S_{N}$-calculations which are necessary to get a desired accuracy. The essential results of these studies are given in the section entitled "Comments on the $S_{N}$-calculations".
By using the results specified in the section mentioned above, we have done the calculations reported on in the section entitled ${ }^{S} S_{N}$-corrections for various assemblies".

Assuming that the one-dimensional $S_{N}$-corrections for the various space directions could be added up to give the final $S_{N}$-corrections, we also compare in this section the added up value with the $S_{N}$-correction for the equivalent spherical model of the assembly.
*) The calculations have been done on an IBM $360 / 65$ with an Operating System providing multiprogramming with a variable number of tasks (MVT).

Coments on the $S_{n-c a l c u l a t i o n s ~}^{n}$

First we will give some general remarks ebout the dimension and the numeration of zones. Unless otherwise stated the zone dimensions used by DTK-calculetions are identical with those used in one dimensional diffusion calculations. Usually the zone denoted as zone 1 corresponas to the lower or inner part of the assembly. We have assumed in this chapter that the reader is somewhat familiar with the notations used in the DTF-IV-code and therefore we will not explain all notations used here.

Before considering the influence of mesh size on the multiplication factor $k_{e f f}$, we will discuss en effect, which in some circumstances can make the DTK calculated value not a good approximation for the real keff. The code determines $k_{\text {eff }}$ in a sequence of eigenvalues from which the last one is assumed to be identical with $k_{\text {eff }}$, of course within the desired eccuracy. In this sequence the new eigenvalue is obtained from the old one by multiplication of the latter one with a variable factor called $\lambda$. For illustration see table V-10a and table $V-10 b$. The eigenvalue within a row has been determined by multiplicating the factor $\lambda$ in the same row with the eigenvalue in the row above. The iteration process is stopped if $|\lambda-1|$ is less than or equal to $\varepsilon$, with $\varepsilon$ being specified in the input by the user of the code.

After termination of the iteration process there is done afinal step in which one more eigenvalue is calculated using the above described method. In most cases $\lambda$ for this final step is closer to unity than the previous one. This nonoscillating convergence ${ }^{*}$ of $\lambda$ being the usual one is shown in table V-10a; $\varepsilon$ has been specified to $10^{-5}$ so that the $|\lambda-1| \leqq \varepsilon$ condition has been fulfilled after 10 outer iterations with $|\lambda-1|=0.5 \cdot 10^{-5}$. The value of $\lambda$ used in the final step is closer to unity than the previous one. In this case and if some other conditions, which will be discussed later, are also fulfilled, we can have much confidence that the last ei genvalue is a good approximation for keff* But thereare some other cases; one of these is shown in table V-10b. The $|\lambda-1| \leqq \varepsilon$ condition has been satisfied after 3 outer iterations because $|1-1.00005|=0.5 \cdot 10^{-4}$

[^0]is less than $10^{-4}$, but using the lest $\lambda,|\lambda-1|$ becomes greater than $\varepsilon(|\lambda-1|=$ $4.13 \cdot 10^{-4}>10^{-i}$ ). It is evident that in this case the last eigenvalue is not a. good approximation for $k_{\text {eff }}$ and we must check all results on the appearance of this effect.

Keanwhile DTK has been improved. In a first step there was given a detailed printing of the number of inner iterations for each enerev group. If at the iteration break off the distribution of the inner iterations over the eneray groups oscillates not too much and if in addition the number of iterations for one group is low, we may take this also as an indication that the last eigenvalue is a good approximation for $k_{\text {eff }}$. In a second step the possibility has been provided to use
a) Tschebyscheff extrepolation and
b) an improved guess for the source distribution.

Using Tschebyscherf extrapolation causes the effect that the convergence of $\lambda$ $1 s$ oscillating ${ }^{*}$ so that the iteration may be terminated without having reached the desired accuracy for the eigenvalue. To exclude this possibility a new iteration break off condition of the form

$$
\frac{1}{1.4}\left\{\left|1-\lambda_{i}\right|+\left|1-\lambda_{i-1}\right|\right\}<\varepsilon
$$

with $i$ being the outer iteration number has been introduced by the authors of 5707.
Besides $\varepsilon$, which is responsible for the accuracy of $k_{\text {eff }}$, it is possible to fix EPSA in the input of the code to determine the fluxes with a certain accuracy. But being primarily interested in $k_{\text {eff }}$ we have not taken advantage of this possibility.
The following investigations on the influence of mesh size on the accuracy of $k_{\text {eff }}$ were motivated by two reasons: a small number of mesh points saves computing time on the one hand and reduces possible numerical effects, e.g. round off errors, on the other hand. We have made our investigations for SUAK U1B respectively for SUAK UH1B. The results are shown in table $V-11 a$ and table $V-11 b$ and plotted in figure 6A and figure 6B. In figure 6A we have plotted the
w) We have seen that this may also be true if Tschebyscheff extrapolation is not used.
variation of $k_{\text {eff }}$ versus the number of mesh intervals in zone 3 , because this zone is identical with the core and the number of mesh intervals used in the other zones containine no fissionable material is changed most times proportional to tre corresponjing change in the core region. Looking at table V-lla and figure $6 A$ we cen see that $k_{\text {eff }}$ is increasine with rising mesh interval number. But the increment is small if we have more than 30 mesh intervals in zone 3.
Doing the same calculations in spherical geometry usine the assembly suak Unif. (see table $\mathrm{V}-11 \mathrm{~b}$ respectively figure 6 B ) one remarks only two facts different from the prececing results shown in table $V-11 a$ and figure 6A:
a) The part of the curve with small changes has a decreasing tendency ${ }^{*}$ and
b) This part of the curve starts at about 20 mesh intervals.

The first difference is due to changed geometry whereas the second one is less significant as explained below. In the case of ficures $V-11 a$ and $V-11 b$ we have
 calculations because the configuration is not symmetric, whereas in synerical geometry only the radius of the sphere has been used. In order to have a correspondence between the two cases we would have to take in slab geometry only half the core heifht, so that the mesh interval number is helved and the part of the curve with small changes starts in figures 6 A and 6 B at the same number of mesh intervals. To overcome this difficulty we must not relate to the mesh interval number but to the mesh interval size.
To obtain a general rule for fixing the mesh interval size valid for all cases investigated here, we use the transport mean free path $\lambda_{t r}$. We have fiven the mesh interval axis in figures 6 A and 6 B a second notaincm measured in units of $\lambda_{t r}$ divided by the mesh size. If this quantity has a value of about 3 there are only small changes in $k_{\text {eff }}$, if we increase the number of mesh intervals, so that a mesh interval size of about one third of $\lambda_{t r}$ seems to be an appropriate value, at least if the assembly to b investigated is similar to SUAK U1B or SUAK UH1B. This rule has been confirmed by the negative results of all spot checks which we have made additionally.
\%) Looking at table V-11b one may areue that this tendency is not true for hipher mesh interval number. But the changes showing the opposite tendency are only of the order $10^{-5}$ which is less than the accuracy $\varepsilon=10^{-4}$ thereby used.

Hevine spherical or cylindrical Eeometry, one may suspect keff not bein quite correct because of the different size of volume elements at different radial positions ${ }^{\text {Wh }}$ sposing an equal mesh spacing. In order to check this we divided the one zone homogenized SUAK U1B spherical core into 4 zones and varied the number of mesh intervals in this zones. The result of these investigations - listed in table V-12a - indicates that fortunately our suspicion has not proved right. Taking the sane total number of mesh intervals heff rewains macianger within the first four digits alf nough the largest volume elements in the various cases differ by a factor of 2 , with the ratio of largest to smallest volune element being about 270. Cnly by increasinf the total number of intervele (case 4 in table V-12a.) we fot an effect of the order $10^{-4}=$ hut this can be understood if we remember figure V-11b: for spherical feometry an increase of the number of mesh intervals results at a certain point in a small decrease of keff"
we have done the investigations cescribed above for cylincricel geometry too. The results - listed in table $V-12 b-a l s o l e a d$ to the staterent that the size of a volume element has no influence on the accuracy of $k e f f$ if we only choose the mesh size properly, e.E. about one third of the transport mean free path.

Finelly, we have studied the dependence of $k_{\text {eff }}$ on the order if of the $S_{N}-c a l-$ culaiions. We have done this for various assemblies in different feometries. The results are listed in teble $V-13 a$ upt to table $V-131$ and plotted in figure TA up to fifure 7L. Looking at the fipures one may observe some general tendencies. If we compare all calculations done in slab geometry we recoenize that keff computed with a quadrature order $N=2\left(k_{e f f}\left(S_{2}\right)\right.$ ) is alweys considerably smaller than the rest of the values. Furthermore one can see that $k_{e f f}\left(S_{\ell}\right), k_{e f f}\left(S_{12}\right)$ and $k_{e f f}\left(S_{16}\right)$ are not much different from $k_{e f f}\left(S_{6}\right)$, so that a quedrature order $N=6$ would have been sufficient ${ }^{\text {H/K}}$.
«)
In our calculations the volume of inner and outer elements differ by a factor of up to 1000.

36K)
This statement of course is valid only for slab geometry and assemblies resembling those investigated in this report.

Comparing all calculations done in spherical and cylindrical geometry one remarks that $k$ eff $\left(s_{2}\right)$ is always much larger than the other values of $k$ eff. In adaition $k_{e f f}\left(S_{N}\right)$ is not becoming constant repidiy for hither orders of N , but still decreases slightiy. If we exclude numerical effects we cane to the conclusion tinat it is necessary to calculate with the highest possible quadrature order allowed by our nresent code in spherical and cylinarical peometry if $k_{\text {eff }}$ should be determined with hieh accuracy. Because computing time increases very rapidy for high orier $S_{N}$-calculations, an improved version of DTK has been estabiisned by the autinors of $\mathbb{F}_{70} 70$ by which we can calculate the hith quadrature order $k_{\text {eff }}\left(S_{N}\right)$ within a small fraction of the time needed by the old version * This is done by usine.
a) Tschebyscheff extrapolation and
b) the possibility to use the flux of a former case to Eet a reasonable Eness for the sonuce distribution of e successive calculation for exemple with increased order of N .

There are some values in tables $V-13 a$ to $V-131$ which have been determined in this manner. They are characterized by the small number of outer iterations. The effect of the reduction of computing time is remarkable; for example (see table $V-13 i$ ) all given values of $k_{\text {eff }}$ starting with $k_{e f f}\left(S_{2}\right)$ up to $k_{\text {eff }}\left(S_{16}\right)$ have been obtained within 8 minutes and 16 seconds, whereas usine the previously applied method to compute only $k_{\text {eff }}\left(S_{12}\right)$ has taken 10 minutes and 12 seconds. More examples for computing times are given in the tables V-13a to V-13l and if possible, comparisons between computing times obtained with the old and new version of the code.
We have compared our results with those given in [73_7 finding good agreement especially with respect to the dependence of $k_{\text {eff }}$ on the quadrature order $N$ for spherical and cylindrical eeometry. There is eiven in 173_7 no calculation referrinc to $s l a b$ geometry and the mesh interval dependence starts at 13 mesh intervals and is only given for cylindrical geometry so that a detailed corparison with respect to these items is not possible.
*)
This improved version has been mentioned earlier in connection with the iteration break off condition.

On page 30 of $/ 71$ _ 7 there has been made the statement "..., diffusion calculations are often nore accurate than $S_{2}$-calculations". With respect to the assemblies analized in this report this statement is not vaiid for all geometries. If one looks at teble V-14 one can sec that onlr in spinericel ceonetry the $k_{\text {eff }}$ (diff)-value is closer to the $k_{\text {eff }}\left(S_{H}\right)$-value than the $k_{\text {erf }}\left(S_{2}\right)$ value. In cylindricel and especielly in slab eeometry the $k_{\text {eff }}\left(S_{2}\right)$-value is much =ore eecurate then the diffision celeuleted velue ofteff"

## Summary

If we take into account only assemblies similar to those investigated here, we have to choose for the mesh size about one third of $\lambda_{t r}$ to aet a keff with sufficient accuracy. For sleb geometry a quadrature order of $N=6$ is sufficient, whereas in spherical and cylindrical ceometry $\Pi=6$ may be sufficient when only three digits are important, If higher accuracy is desired higher quadrature oraer must be used for these two geometries. The different size of volume elements in non-slab geometry with equal mesh spacing has found to have practically no influence on the accuracy of $k_{\text {eff }}$. Other effects reported here have been taken into account by improvements in the code for exanyle with respect to the convergence condition and the saving of computing time for higher order $S_{N}$-calculations.
$\mathrm{S}_{\mathrm{N}}$-corrections for various assemblies

The $S_{N}$-corrections for the assemblies ZPR-III-10, ZPR-III-25, ZPR-III-48, ZEBRA 6A, SNEAK 3A1, SNEAK 3A2, SUAK U1B and SUAK UH1B are given in table V-15a up to $V-15 \mathrm{~h}$.
Before discussing the results we will make two remarks about mesh size and quadrature order. Not in all cases we have given the mesh size a value of about one third of $\lambda_{t r}$, because some of these calculations have been done before the first section of this report has been finished. But all mesh sizes are situated in such a range that the possible errors in $k_{\text {eff }}$ are only about 0.0001 (see table $\mathrm{V}-11 a$ and $\mathrm{V}-11 \mathrm{~b}$ together with table $\mathrm{V}-15 \mathrm{a}$ to table $\mathrm{V}-15 \mathrm{~h}$ ).

As for the quadrature order we have taken in cylindrical and spherical geometry the maximal values allowed by the code ${ }^{n}$. In some cases we have computed $S_{16}$-values for $k_{\text {eff }}$ also in slab eeometry, but usually we have taken the $S_{6}$-values.
By table V-15a to table V-15h one can see that a $S_{N}$-correction has been necessary for all assemblies if cue aims to come to an accuracy for the criticality of more than $1.0 \cdot 10^{-3}$. The $S_{N}$-ccrrections on $k_{\text {eff }}$ decreases if the size of the assemblies increases. This statement agrees with the results Eiven in [73].
In the part of this report entitled "Comments on the $S_{N}$-calculations", we have made the assumption that the one-dimensional $S_{N}$-corrections on $k$ eff for slab and cylindrical geometry - respectively at SUAK for slab geometry in all three space directions - could be added up to give the final $S_{\mathrm{If}}$-correction. Comparing the added up values with the correspondine spherical values (all given in tables $V-15 a$ to $V-15 h$ ) we see that our assumption is doubtful. It is true that there is agreement in the case of the assembly ZPR-III-25 but otherwise there are differences in the $S_{\mathrm{II}}$-corrections between $8 \%$ and $21 \%$ even up to $60 \%$ for the SUAK-assemblies. The agreement in the ZPR-III-25 case may be fortuitous because due to calculating accuracy all $\mathrm{k}_{\text {eff }}$-values are uncertain by one unit in the last dieit eiven.
We tried to find a reason for the disagreement mentioned above. Our suspicion has been that an inappropriate boundary condition is used in the diffusion calculations especially in spherical geometry. To get an idea of the influence of the boundary condition we calculated $k$ eff for SUAK U1B with one-dimensional diffusion theory in spherical and slab geometry usine two different boundary conditions: the first of these has been the ususally used condition

$$
\frac{\phi^{\prime}(a)}{\phi(a)}=-\frac{1}{0.7104 \cdot \lambda_{t r r}}
$$

and the second one has been

|  | $\phi(a+d)=0$ |
| :--- | :--- |
| with | $d=0.7104 \cdot \lambda_{t r}$ |

*) $N=8$ for cylindrical and $N=16$ for spherical geometry.

The results listed in table V-16 denonstrate the correctness of our suspicion. There is little difference for the slab values ( $0.17 \%$ ) but large difference for the spherical values ( $1.6 \%$ ). If we take into account the uncertainty in the boundery condition the relative error between the added up value and the sphericel value for $S U A K U 1 R$ is reduced considerably. One may expect that the difierences for the other assemilies will be reauced corresponäingly.
The uncertainty in the boundary condition effects very ruch the criticality for the SUAK assemblies because these assemblies are small with respect to the other ones and are unreflected in two space directions so that differences in the extrapolated end point have a larger effect on $k_{\text {eff. }}$.

## Sumary

We have to take into account for all assemblies studied here the $S_{N}$-corrections on $k_{\text {eff, }}$ if we want to obtain an accuracy of better than $10^{-3}$. The calculation of the $S_{N}$-corrections for spherical models may lead to values which differ from those obtained by adding, up the correspondint one-dimensional $S_{N}-$ corrections determined by using the assumption of separability of the flux in different space diractions. But the difference can be attributed to a laree extent to the uncertainty in the boundary condition for the diffusion calculation especially for spherical Eeometry.

- $50-$

| Outer iteration <br> number | Eigenvalue | Lambde |
| :---: | :--- | :--- |
| 0 | 0.8810000 |  |
| 1 | 0.8996826 | 0.8996818 |
| 2 | 0.8939858 | 0.9936674 |
| 3 | 0.8940460 | 1.000067 |
| 4 | 0.8944250 | 1.000423 |
| 5 | 0.8946577 | 1.000259 |
| 6 | 0.8947755 | 1.000131 |
| 7 | 0.8948326 | 1.000064 |
| 8 | 0.8948585 | 1.000029 |
| 9 | 0.8948715 | 1.000014 |
| 10 | 0.8948767 | 1.000005 |
| 11 | 0.8048799 | 1.000004 |


| Assembly | SUAK UH1B |
| :--- | :--- |
| Geometry | sphere |
| Quadrature Order | 6 |
| Number of Zones | 1 |
| Humber of Intervals | 98 |
| $\varepsilon$ | $10^{-5}$ |

Table V-lcz: Variation of Eigenvalues during outer iterations. Non-oscillating convergence of $\lambda$ to 1 a.t the end of the iteration process.

| Outer iteration <br> nurber | Eigenvalue | Lambda |
| :---: | :---: | :---: |
| 0 | 0.8810000 |  |
| 1 | 0.8996648 | 0.8996638 |
| 2 | 0.8938476 | 0.9936436 |
| 3 | 0.8939926 | 1.000050 |
| 4 | 0.8943624 | 1.000413 |


| Assembly | SUAI. UH1B |
| :--- | :--- |
| Ceometry | sphere |
| Quadrature Order | 6 |
| Number of Zones | 1 |
| Number of Intervals | 98 |
| e | $10^{-2}$ |

Table V-10b: Variation of Eigenvalues during outer iterations. Oscillating convergence of $\lambda$ to 1 at the end of the iteration process.

| Number total | $\begin{gathered} \text { of } 10 \\ \text { Zone } \\ 1 \end{gathered}$ | Zone <br> 2 | $\begin{aligned} & \text { Entery } \\ & \left\lvert\, \begin{array}{c} \text { Zone } \\ 3 \end{array}\right. \end{aligned}$ | rals <br> IGone <br> 4 | $k_{\text {eff }}$ | Outer <br> iteretion <br> number | Last $\lambda$ | ${ }^{\text {areff }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | 1 | 1 | 5 | 1 | 0.833436 | 11 | 1.000001 |  |
|  |  |  |  |  |  |  |  | 0.003176 |
| 16 | 2 | 2 | 10 | 2 | 0.836612 | 13 | 1.000003 |  |
|  |  |  |  |  |  |  |  | 0.000575 |
| 24 | 3 | 3 | 15 | 3 | 0.837187 | 14 | 1.000000 |  |
|  |  |  |  |  |  |  |  | 0.000198 |
| 32 | 4 | 4 | 20 | 4 | 0.837385 | 14 | 1.000003 |  |
|  |  |  |  |  |  |  |  | 0.00009: |
| 40 | 5 | 5 | 25 | 5 | 0.837478 | 14 | 1.000003 |  |
|  |  |  |  |  |  |  |  | 0.000048 |
| 48 | 6 | 6 | 30 | 6 | 0.837526 | 14 | 1.000001 |  |
|  |  |  |  |  |  |  |  | 0.000031 |
| 56 | 7 | 7 | 35 | 7 | 0.837557 | 14 | 1.000003 |  |
|  |  |  |  |  |  |  |  | 0.000033 |
| 63 | 6 | 6 | 45 | 6 | 0.837590 | 14 | 1.000003 |  |
|  |  |  |  |  |  | 13 | 1.000003 | 0.000031 |
| 130 | 10 | 10 | 100 | 10 | 0.837621 |  |  |  |
|  |  |  |  |  |  |  |  |  |


| Assembly | SUAK U1B |
| :--- | :--- |
| Geometry |  |
| Quadrature Order | plane in z-direction |
| $\varepsilon$ | 8 |

Table V-11a: Variation of $k_{\text {eff }}$ with the number of mesh intervals. Zone 3 is identical with the core.

| Number of mesh intervals | $k_{\text {eff }}$ | Outer iteration number | Las: $\lambda$ | $\Delta k_{\text {eff }}$ |
| :---: | :---: | :---: | :---: | :---: |
| 3 | 0.889086 | 10 | 0.9999713 | 0.003637 |
|  |  | 10 | $0.9999589$ |  |
| 4 | 0.892723 |  |  | $0.001004$ |
|  |  |  | 0.9099659 |  |
| 5 | 0.893807 | 10 |  | 0.000516 |
|  |  |  | 0.0999690 |  |
| 6 | 0.894323 | 10 |  | 0.000380 |
|  |  |  | 0.9999572 |  |
| 8 | 0.894703 | 9 |  | 0.000130 |
|  |  |  | 0.9999662 |  |
| 10 | 0.894833 | $\bigcirc$ |  | 0.000101 |
|  |  |  | 0.9999708 |  |
| 15 | 0.894934 | 8 |  | 0.000007 |
|  |  |  | 0.9999794 |  |
| 20 | 0.894947 | 7 |  | -0.000057 |
|  |  |  | 0.9909876 |  |
| 25 | 0.894884 | 6 |  | -0.000035 |
|  |  |  | 1.000016 |  |
| 30 | 0.894819 | 6 |  | -0.000023 |
|  |  |  | 1.000044 |  |
| 50 | 0.894796 | 7 |  | -0.000008 |
|  |  |  | 1.000049 |  |
| 60 | 0.894788 | 7 |  | +0.000012 |
| 80 | 0.894805 | 8 | $1.000032$ |  |
|  | 0.894805 | 8 |  |  |


| Assembly | SUAK UH1B |
| :--- | :--- |
| Geometry | sphere |
| Quadrature Order | 6 |
| $\varepsilon$ | $10^{-4}$ |

Table V-11b:
Variation of $k_{\text {eff }}$ with the number of mesh intervals.

| Zone | Right boundary ${ }^{5}{ }^{\mathrm{cm}} 7$ | Number of intervals | Smallest volume element $\left[\mathrm{cm}^{3} 7\right.$ | Largest volume element $\left[\mathrm{cm}^{3}-7\right.$ | Total number of intervals | $k_{\text {eff }}$ | Outer <br> iteration number | Last $\lambda$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8.000 | 5 | 17.157 | 1046.592 |  |  |  |  |
| 2 | 13.000 | 15 | 279.408 | 689.908 |  |  |  |  |
| 3 | 16.000 | 40 | 160.199 | 240.145 |  |  |  |  |
| 4 | 19.364 | 50 | 217.329 | 315.896 | 110 | 0.836799 | 14 | 1.000004 |
| 1 | 8.000 | 5 | 17.157 | 1046.592 |  |  |  |  |
| 2 | 13.000 | 10 | 427.780 | 1021.541 |  |  |  |  |
| 3 | 16.000 | 40 | 160.198 | 240.145 |  |  |  |  |
| 4 | 19.364 | 55 | 197.474 | 287.271 | 110 | 0.836801 | 14 | 1.000004 |
| 1 | 8.000 | 10 | 2.145 | 581.202 |  |  |  |  |
| 2 | 13.000 | 20 | 207.410 | 520.784 |  |  |  |  |
| 3 | 16.000 | 35 | 183.233 | 274.264 |  |  |  |  |
| 4 | 19.364 | 45 | 241.564 | 350.675 | 110 | 0.836772 | 14 | 1.000004 |
| 1 | 8.000 | 20 | 0.268 | 305.881 |  |  |  |  |
| 2 | 13.000 | 30 | 136.853 | $349.43 ?$ |  |  |  |  |
| --3 | 16.000 | 80 | 79.867 | 120.353 |  |  |  |  |
| 4 | 19.364 | 100 | 108.389 | 158.198 | 230 | 0.836704 | 14 | 1.000004 |


| Geometry | sphere |
| :--- | :--- |
| Quadrature Number | 8 |
| $E$ | $10^{-5}$ |

Table V-12a: SUAK U1D. Varying size of volume elements in spherical geometry.

| Mesh intervals in Zone |  |  |  |  | Smallest and largest volume element in zone |  |  |  | $k_{\mathrm{eff}}\left(S_{H}\right)$ | Outer iteration number | Last $\lambda$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 2 | 3 | 4 | 5 |  |  |  |
| 10 | 21 | 9 | 31 | 19 | $\begin{aligned} & 40.322 \\ & 88.998 \end{aligned}$ | $\begin{aligned} & 217.298 \\ & 323.467 \end{aligned}$ | $\begin{aligned} & 127.139 \\ & 185.301 \end{aligned}$ | $\begin{aligned} & 306.498 \\ & 399.387 \end{aligned}$ | 0.970423 | 3 | 1.000007 |
| 10 | 19 | 11 | 29 | 21 | $\begin{aligned} & 44.708 \\ & 98.223 \end{aligned}$ | $\begin{aligned} & 176.803 \\ & 265.643 \end{aligned}$ | $\begin{aligned} & 135.977 \\ & 198.002 \end{aligned}$ | $\begin{aligned} & 277.084 \\ & 361.582 \end{aligned}$ | 0.9704116 | 3 | 1.000004 |
| 10 | 17 | 13 | 27 | 23 | $\begin{array}{r} 50.164 \\ 109.585 \end{array}$ | $\begin{aligned} & 149.024 \\ & 225.354 \end{aligned}$ | $\begin{aligned} & 146.139 \\ & 212.582 \end{aligned}$ | $\begin{aligned} & 252.818 \\ & 330.301 \end{aligned}$ | 0.970485 | 3 | 1.000001 |
| 10 | 15 | 15 | 25 | 25 | $\begin{array}{r} 57.133 \\ 123.915 \end{array}$ | $\begin{aligned} & 128.786 \\ & 195.672 \end{aligned}$ | $\begin{aligned} & 157.941 \\ & 229.486 \end{aligned}$ | $\begin{aligned} & 232.467 \\ & 304.004 \end{aligned}$ | 0.970559 | 5 | 0.9999862 |
| 10 | 13 | 17 | 23 | 27 | 66.346 142.556 | $\begin{aligned} & 113.388 \\ & 172.902 \end{aligned}$ | $\begin{aligned} & 171.815 \\ & 249.297 \end{aligned}$ | $\begin{aligned} & 215.145 \\ & 281.586 \end{aligned}$ | 0.970423 | 3 | 1.000011 |
| 10 | 11 | 19 | 21 | 29 | $\begin{array}{r} 79.09 .1 \\ 167.794 \end{array}$ | $\begin{aligned} & 101.277 \\ & 154.875 \end{aligned}$ | $\begin{aligned} & 188.361 \\ & 272.856 \end{aligned}$ | $\begin{aligned} & 200.217 \\ & 262.244 \end{aligned}$ | 0.970419 | 3 | 1.000009 |
| 10 | 9 | 21 | 19 | 31 | $\begin{array}{r} 97.871 \\ 203.876 \end{array}$ | $\begin{array}{r} 91.502 \\ 140.252 \end{array}$ | $\begin{aligned} & 208.436 \\ & 301.336 \end{aligned}$ | $\begin{aligned} & 187.240 \\ & 245.404 \end{aligned}$ | 0.970418 | 3 | 1.000011 |

> | $\begin{array}{l}\text { Smallest and largest volume element in zone } \\ 3.14159 \quad 59.69019\end{array}$ |
| :--- |

Table V-12b: Varying size of volume elements in cylindrical geometry. All volumes are given in $\mathrm{cm}^{3}$.

| Assembly | 2FiBRA 6A |  |
| :--- | :---: | :---: |
| Ceometry | cylinder |  |
| Quadrature order | 6 |  |
| E | $10^{-4}$ |  |
| Right boundaries of zones | $\left[\mathrm{cm}_{2} 7\right.$ |  |
| $10.00 \quad 23.07$ | 36.15 | 53.37 |
| Total number of mesh intervals | 90 |  |


| Order N of <br> $S_{N}$-calculation | $k_{\text {eff }}$ | Outer <br> iteretion <br> number | Last $\lambda$ |
| :---: | :---: | :---: | :---: |
| 2 | 0.832344 | 13 | 1.000002 |
| 4 | 0.838085 | 14 | 1.000002 |
| 6 | 0.837627 | 14 | 1.000001 |
| 8 | 0.837590 | 14 | 1.000003 |
| 12 | 0.837679 | 14 | 1.000003 |
| 16 |  | 14 | 1.000001 |


| Assembly | SUAK U1B |
| :--- | :---: |
| Geometry | slab in z-dir. |
| $\varepsilon$ | $10^{-5}$ |
| Zone | Mesh intervals |
| 1 | 6 |
| 2 | 6 |
| 3 core | 45 |
| 4 | 6 |

Table V-13a: Variation of $k_{\text {eff }}$ with the quadrature order.

| Order N of <br> $s_{N}$-calculation | keff | Outer <br> iteration <br> number | Last $\lambda$ |
| :---: | :---: | :---: | :---: |
| 2 | 0.961550 | 16 | 1.000009 |
| 4 | 0.960804 | 4 | 1.000048 |
| 6 | 0.960128 | 5 | 0.9999885 |
| 8 | 0.959984 | 4 | 0.9999909 |


| Assembly | ZPR-III-48 |
| :--- | :--- |
| Geometry | cylinder |
| $\varepsilon$ | $10^{-4}$ |
| Zone | Mesh intervals |
| 1 | 40 |
| 2 | 20 |

Total DIK computing time for all $k_{\text {eff }}$ values (with improved code): 5 min 59 sec

Table $V-13 b$ : Variation of $k_{\text {eff }}$ with the quadrature order.

| Creer II of $\mathrm{S}_{\mathrm{H}}$-caleulation | $k_{\text {eff }}$ | Outer iteration number | Lest $\lambda$ |
| :---: | :---: | :---: | :---: |
| 2 | 0.971194 | 12 | 1.000038 |
| 4 | 0.061005 | 12 | 1.000032 |
| 6 | 0.961506 | 12 | 1.000034 |
| 8 | 0.961363 | 12 | 1.000034 |
| 12 | 0.061175 | 12 | 1.000032 |
| 16 | 0.961096 | 12 | 1.000035 |


| Assembly | ZPR-III-48 |
| :--- | :--- |
| Geometry | sphere |
| $\varepsilon$ | $10^{-4}$ |
| Zone | Mesh intervals |
| 1 | 10 |
| 2 | 30 |
| 3 | 20 |

Table V-13c: Variation of $k_{\text {eff }}$ with the quadrature order.

| Order N of $\mathrm{S}_{\mathrm{K}}$-calculation | $k_{\text {eff }}$ | Outer iteration nuriber | Lest $\lambda$ |
| :---: | :---: | :---: | :---: |
| 2 | 0.977436 | 13 | $1.00003 \%$ |
| 4 | 0.979326 | 13 | 1.000047 |
| 6 | 0.970125 | 13 | 1.000027 |
| $\delta$ | 0.970111 | 13 | 1.000046 |
| 12 | 0.979116 | 13 | 1.0000 17 |
| 16 | 0.979127 | 13 | 1.000046 |


| Assembly |
| :--- |
| Geometry |
| $\varepsilon$ |

Table $V-13 d$ : Variation of $k_{\text {eff }}$ with the cuadrature order,

| onder M of $\mathrm{S}_{\text {necalculation }}$ | $k_{\text {eif }}$ | Oater iteretion number | Last $\lambda$ |
| :---: | :---: | :---: | :---: |
| 2 | 0.085757 | 17 | $1.000 c 0$ e |
| 4 | C.085100 | 4 | 1.000037 |
| 6 | 0.984156 | 5 | 0.9909920 |
| $\varepsilon$ | 0.983926 | 4 | $0.0992 ? 95$ |


| Asserbiy | ZPR-III-10 |
| :--- | :--- |
| Geametry | cylinder |
| $\varepsilon$ | $10^{-4}$ |
| Zone | Mesh intervals |
| 1 | 10 |
| 2 | 20 |
| 3 | 30 |


| Comeutine times |  |
| :--- | :--- |
| old method <br> $s_{2}$ and $s_{4}$ | improved netiod <br> $s_{2}$ and $s_{4}$ and $s_{6}$ <br> $5 \mathrm{~min} 12 \mathrm{sec} s_{8}$ $5^{5 \mathrm{~min} 43 \mathrm{sec}}$ |

Table V-13e: Variation of $k$ eff with the quadrature order.

| Oraer N of <br> $S_{\text {ricalculation }}$ | keff | Outer <br> iteration <br> number | Lest $\lambda$ |
| :---: | :---: | :---: | :---: |
| 2 | 1.000214 | 8 | 1.000001 |
| 4 | 0.088320 | 7 | 1.000015 |
| 6 | 0.987458 | 5 | 1.000007 |
| 6 | 0.987207 | 4 | 0.9999957 |
| 12 | 0.986848 | 7 | 1.000021 |
| 16 | 3 | 0.9999930 |  |


| Asserbly | ZPR-III-10 |
| :--- | :--- |
| Geonetry | sphere |
| $\varepsilon$ | $10^{-4}$ |
| Zone | Mesh intervals |
| 1 | 10 |
| 2 | 25 |
| 3 | 25 |

Computinc time
for $S_{2}$ up to $S_{16}$ :
$5 \min 46 \mathrm{sec}$

Table V-13f: Variation of $k$ eff with the quadrature order,

| Order II of <br> $S_{N}$-calculation | $k_{\text {eff }}$ | Cuter <br> iteration <br> number | Lest $\lambda$ |
| :---: | :---: | :---: | :---: |
| 2 | 0.970277 | 17 | 1.000053 |
| 4 | 0.970814 | 17 | 1.000058 |
| 6 | 0.970746 | 17 | 1.000057 |
| 8 | 0.070750 | 17 | 1.000057 |
| 12 | 0.970751 | 17 | 1.000058 |
| 16 |  | 17 | 1.000058 |


| Asserbly | ZPR-III-25 |
| :--- | :---: |
| reometry | slab |
| $\varepsilon$ | $10^{-4}$ |
| Zore | Mesh intervals |
| 1 | $\varepsilon$ |
| 2 | 24 |
| 3 | 18 |

Table V-13r: Variation of $k$ eff with the nuadrature order.

| Order H oit $\mathrm{S}_{\mathrm{in}}$-calculation | $\mathrm{F}_{\text {eff }}$ | Oater <br> iteration nur:cer | Last $\lambda$ |
| :---: | :---: | :---: | :---: |
| 2 | 0.072157 | 19 | 1.000061 |
| 1 | 0.071006 | 19 | 1.000060 |
| 6 | 0.071502 | 19 | 1.000059 |
| $\because$ | 0.071546 | 10 | 1.000058 |


| Assembly | ZPR-III-25 |
| :--- | :---: |
| Geometry | cylinder |
| $\varepsilon$ | $10^{-4}$ |
| Zone | Nesh intervals |
| 1 | 8 |
| 2 | 30 |
| 3 | 22 |

Table $\mathrm{V}-13 \mathrm{~h}:$ Variation of $\mathrm{k}_{\mathrm{eff}}$ with the quadrature order,

| Order N of $\mathrm{S}_{\mathrm{N}}$-celculation | ${ }^{\text {beff }}$ | Outer iteration nurher | Lest $\lambda$ |
| :---: | :---: | :---: | :---: |
| 2 | 0.076101 | 17 | 1.000035 |
| 4 | 0.972702 | 8 | 1.000006 |
| 6 | 0.972421 | 4 | 0.0900071 |
| 3 | 0.972330 | 3 | 0.999098 .2 |
| 12 | 0.972336 | 3 | 0.9900973 |
| 16 | 0.972320 | 3 | 0. 0.900060 |


| Assembly | ZPR-III-25 |
| :--- | :---: |
| Geometry | sphere |
| E | $10^{-4}$ |
| Zone | Mesh intervals |
| 1 | 15 |
| 2 | 45 |
| 3 | 30 |


| Comutine time for <br> $S_{2}$ up to $\mathrm{S}_{16}$ <br> 01d rethod <br> 46 min. |  |
| :--- | :--- |

Table V-13i: Variation of $k_{\text {eff }}$ with the quadrature order.

| Crder IN of <br> $S_{M}$-calculation | $k_{\text {eff }}$ | Outer <br> iteration <br> number | Lest $\lambda$ |
| :---: | :---: | :---: | :---: |
| 2 | 0.965114 | 11 | 1.000023 |
| 4 | 0.967606 | 11 | 1.000026 |
| 6 | 0.967313 | 11 | 1.000026 |
| 8 | 0.967293 | 11 | 1.000027 |
| 12 | 0.967290 | 11 | 1.000024 |
| 16 | 0.967305 | 11 | 1.000026 |


| Assembly | 2EERA 6A |
| :--- | :---: |
| Geometry | slab |
| $\varepsilon$ | $10^{-4}$ |
| Zone | nesh intervals |
| 1 | 25 |
| 2 | 20 |
|  | . |

Table V-13i: Veriation of $k_{\text {eff }}$ with the quadrature order.

| Oraer 1 oi $\mathrm{S}_{\mathrm{F}}$-calculation | Eerf | Outer iteration nuruber | Lpst $\lambda$ |
| :---: | :---: | :---: | :---: |
| ? | 0.072010 | 12 | 1.00003? |
| 4 | 0.971307 | 12 | $1.000 \mathrm{c}^{2}$ |
| 5 | C.070364 | 12 | $1.70002 ?$ |
| 3 | 0.970180 | 12 | 1.neore? |


| Assembly | ZEBRA 6A |
| :--- | :---: |
| ceometry | cylinder |
| $\varepsilon$ | $10^{-4}$ |
| Zone | liesh intervals |
| 1 | 6 |
| 2 | 40 |
| 3 | 30 |

T'able V-13k: Variation of $k_{\text {eff }}$ vith the quadroture order.

| Orier in of <br> $S_{\mathrm{M}}$-calculation | keff | Cuter <br> iteration <br> number | Lest $\lambda$ |
| :---: | :---: | :---: | :---: |
| 2 | 0.085000 | 12 | 1.000032 |
| 4 | 0.973033 | 12 | 1.000028 |
| 6 | 0.973125 | 12 | 1.000025 |
| 6 | 0.972659 | 12 | 1.000025 |
| 12 | 0.972625 | 12 | 1.000026 |
| 16 | 0.972510 | 12 | 1.000026 |


| Assembly | ZERRA 6A |
| :--- | :--- |
| Geometry | sphere |
| $\varepsilon$ | $10^{-4}$ |
| Zone | Mesh intervals |
| 1 | 0 |
| 2 | 32 |
| 3 | 20 |

Table V-131: Variaticn of $k_{\text {eff }}$ with the quadrature order.

| Assembly | Geometry | $k_{e f f}\left(s_{\text {IT }}\right) \underline{k}_{\text {eff }}(\mathrm{diff})$ | II | $\mathrm{keff}^{(S 2)-\mathrm{k}_{\text {eff }}\left(S_{\mathrm{H}}\right)}$ |
| :---: | :---: | :---: | :---: | :---: |
| ZPR-III-48 | cylinder | 0.0050 | 8 | 0.0016 |
|  | scinere | 0.0061 | 16 | 0.0101 |
| ZPR-III-10 | slab | 0.0039 | 16 | -0.0017 |
|  | cylinder | 0.0087 | 8 | 0,0019 |
|  | schere | 0.0116 | 16 | 0.0134 |
| ZPR-III-25 | slab | 0.0008 | 16 | -0.0005 |
|  | cylinder | 0.0015 | 8 | 0.0007 |
|  | sphere | 0.0023 | 16 | 0.0038 |
| ZEBRA 6A | slab | 0.0043 | 16 | -0.0022 |
|  | cylinder | 0.0072 | 8 | 0.0018 |
|  | snhere | 0.0095 | 16 | 0.0135 |

Table V-14: Comparing diffusion calculated $k_{\text {eff }}$ values with $S_{\mathbb{N}}$ calculated $k_{\text {eff }}$ values for various assemblies and different geometries.

| Assembly: ZPR-TII-10 |  |  |  | $\lambda_{t r}=3.663 \mathrm{~cm}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Geometry } \\ & \text { and } \\ & \text { Euckline } \end{aligned}$ | $\begin{aligned} & k_{e f f} \\ & \text { (one-ajm.diff.) } \end{aligned}$ | Zone | Pfesh <br> Interval | Firint <br> Poundary: <br> (cm) | $\begin{aligned} & \text { Yesh } \\ & \text { Cize } \\ & (\mathrm{cm}) \end{aligned}$ | oundroture order and $k_{\text {erf }}\left(G_{H}\right)$ | Tncreose |
|  |  | 1 | 10 | 10.0 | 1.000 | 11016 |  |
| slab | 0.9752 | 2 | 13 | 22.05 | 0.096 |  |  |
| $57.534 \cdot 10^{-4}$ |  | 3 | 27 | 55.05 | 1 .2อ2? | 0.9791 | 0.0039 |
|  |  | 1 | 10 | 10.0 | 1.000 | $N=6$ |  |
| cylinder | 0.9752 | 2 | 20 | 22.11 | 0.606 |  |  |
| $23.839 \cdot 10^{-4}$ |  | 3 | 30 | 63.59 | 1.383 | 0.0839 | 0.0087 |
|  |  | 1 | 10 | 10.0 | 1.000 | $1!816$ |  |
| sphere | 0.9752 | 2 | 25 | 25.011 | 0.600 |  |  |
| - |  | 3 | 25 | 62.011 | 1.480 | 0.9868 | 0.0116 |
| Increase (sphere) |  | $\begin{aligned} & \text { increase (slab) } \\ & + \text { increase (cylinder) } \end{aligned}$ |  | Difference | Relative error |  |  |
| 0.0116 | 0.0126 |  |  | 010 | $7.7 \%$ |  |  |

Table V-15a: $\mathrm{S}_{17}$-Corrections for $\mathrm{ZPF}-$ IIT-10.


Table V-15b: $S_{N}$-(\%orrections for ZPR-JTI-25.


Mable $\mathrm{V}-15 \mathrm{c}: \mathrm{S}_{\mathrm{rI}}$-corrections for 2RETII-4 E .

| Assembly: 2FBrA 6A |  |  |  |  | $\lambda_{t r}=4.810 \mathrm{~cm}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Geometry } \\ & \text { and } \\ & \text { Buckline } \end{aligned}$ | $\begin{aligned} & k_{\text {eff }} \\ & \text { (one-dim.diff.) } \end{aligned}$ |  | Zone | Mesh <br> Intervals | Birtht Boundary (cm) | leah size (cm) | nuadrature order and $k_{e f f}\left(5_{1 H}\right)$ | Incrense |
| slab |  |  | 1 | 2 | 30.08 | 1.203 | $!=16$ |  |
| 22.178 | 0.9630 |  | 2 | 20 | 60.50 | 1.52 .1 | 0.0673 | 0,0013 |
|  |  |  | 1 | 6 | 10.0 | 1.667 | $t=0$ |  |
| cylinder |  |  | ? | 40 | 35.15 | 0.65\% |  |  |
| 12.418 | 0.9630 |  | 3 | 30 | 70.59 | 1.148 | 0.070 ? | 0.0072 |
|  |  |  | 1 | 8 | 10.0 | 1.050 | 17=16 |  |
| sphere | 0.9630 |  | 2 | $3 ?$ | 30.203 | 0.881 |  |  |
| - |  |  | 3 | 20 | 72.613 | 1.75 ? | 0.972 .5 | 0.0095 |
| Increase (sphere) |  | Increase (slab) <br> + increase (cylinder) |  |  | ni fference |  | Relative orror |  |
| 0.0095 |  | 0.0115 |  |  | 0.0010 |  | $8.7 \%$ |  |

Table V-15d: $\mathrm{S}_{\mathrm{N}}$-corrections for ZEBPA 6A.

| Assembly: sHEAK 3A1 |  |  |  | $\lambda_{t r}=4.230 \mathrm{~cm}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Geometry } \\ \text { and } \\ \text { Buckling } \end{gathered}$ | ```keff. (one-dim•diff.)``` | Zoile | $\begin{aligned} & \text { Yesh } \\ & \text { Intervals } \end{aligned}$ | Bicht <br> Roundery (cm) | jesh size (cm) | Func?rature order and. $k_{e f f f}\left(K_{N}\right)$ | Increase |
| slab |  | 1 | 30 | 40.27 | 1.3112 | $\pi=6$ |  |
| $13.540 \cdot 10^{-4}$ | 0.9902 | 2 | 20 | 70.77 | 1.525 | 0.0017 | 0.0015 |
|  |  | 1 | 4 | 5.000 | 1.250 |  |  |
|  |  | 2 | 21 | 32.900 | 1.329 |  |  |
| cylinder |  | 3 | 15 | 51.200 | 1.220 |  |  |
| $8.4606 \cdot 10^{-4}$ | 0.9002 | 4 | 25 | $80.55 ?$ | 1.196 | 0.0925 | 0.0023 |
|  |  | 1 | 30 | 3?.231 | 1.308 | $" 1016$ |  |
| sphere |  | 2 | 10 | 52.604 | 1.345 |  |  |
| - | 0.9211 | 3 | 22 | 22.34? | 1.3he | 0.0045 | 0.0031 |
| Increase (sphere) | Increase (slab) <br> +increase (cylinder) |  |  | Difference |  | Helu, ive error |  |
| 0.0031 | 0.0036 |  |  | 0.9007 |  | 120.4\% |  |

Table V-15e: $\varepsilon_{y}$-corrections for smiak 3 3 . 1 .

| Assembly: SIEEAK 3az |  |  |  | $\lambda_{\text {tr }}=4.173 \mathrm{~cm}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Geometry } \\ & \text { and } \\ & \text { Buckling } \end{aligned}$ | $\begin{aligned} & { }^{k_{\text {eff }}} \\ & \text { (one-dim.diff.) } \end{aligned}$ | Zone | l:esh <br> intervals | Rifht, Boundary (cm) | "tesh size (cm) | Cundratiture order and $k_{\mathrm{erff}}\left(S_{i j}\right)$ | Incrense |
| slab |  | 1 | 35 | 40.27 | 1.151 | $\mathrm{N}=6$ |  |
| $16.949 \cdot 10^{-4}$ | 0.9827 | 2 | 20 | 70.77 | 1.52 .5 | 0.9342 | 0.0015 |
|  |  | 1 | 4 | 5.000 | 1.250 |  |  |
| cylinder |  | 2 | 25 | 33.760 | 1.150 | $\mathrm{H}=3$ |  |
|  |  | 3 | 8 | 44.660 | 1.363 |  |  |
| $8.5998 \cdot 10^{-4}$ | 0.9828 | 4 | 30 | 00.859 | 1.207 | 0.9856 | 0.0020 |
| sphere |  | 1 | 30 | 40.094 | 1.336 | $\mathrm{t}=16$ |  |
|  |  | 2 | 8 | 48.315 | 1.028 |  |  |
| - | 0.9837 | 3 | 30 | 84.515 | 1.207 | 0.9875 | 0.0038 |
| Increase (sphere) | $\begin{aligned} & \text { Tncrease ( } 31 \mathrm{lnb} \text { ) } \\ & + \text { incrense (cylinder) } \end{aligned}$ |  |  | difforenen |  | Inlative orror |  |
| 0.0039 | 0.0043 |  |  | 9.0005 |  | 11.6\% |  |




Trble V-15F: Gn-corrections for sUAK U1B.

| Assembly: SUAK UH1B |  |  |  | $\lambda_{t r}=3.359 \mathrm{~cm}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Geometry and Buckline | $\begin{aligned} & k_{\text {eff }} \\ & \text { (one-din.diff.) } \end{aligned}$ | zone | 1/esh Intervals | Ripht <br> Boundary <br> (cm) | Liesh Bize (cm) | Cuadrature order and $k_{\text {eff }}\left(\sigma_{y}\right)$ | Incrause |
|  |  | 1 | 3 | 3.0 | 1.000 |  |  |
| slab in z-dir. |  | 2 | 4 | 6.5 | 0.1775 |  |  |
| $141.314 \cdot 10^{-4}$ |  | 3 | 34 | 39.55 | 0.972 |  |  |
|  | 0.8807 | 4 | 3 | 42.55 | 1.000 | 0.0942 | 0.0135 |
| slab in $x$-dir. <br> $134.681 \cdot 10^{-4}$ | 0.8808 | 1 | 32 | 32.3 | 1.009 | $\begin{gathered} N=6 \\ 0.8891 \end{gathered}$ | 0.0083 |
| sphere | 0.8808 | 1 | 18 | 19.087 | 1.060 | $0.8937 \quad N=16$ | 0.0129 |
| Increase (sphere) | $\begin{aligned} & \text { Increase (slab z-dir.) } \\ & +2 \cdot i n c r e a s e ~(s l a b ~ x m d i r .) ~ \end{aligned}$ |  |  | Difference |  | Relative error |  |
| 0.0129 | 0.0301 |  |  | 0.0172 |  | 57.1\% |  |

Table V-15h: $S_{N-c o r r e c t i o n s ~ f o r ~ S U A K ~ U H 1 B . ~}^{\text {- }}$

| Geometry | Boundary Condition | $k_{\text {eff }}$ | Difference |
| :---: | :---: | :---: | :---: |
| sphere | (1) $a=19.364 \mathrm{~cm}$ | 0.823752 | -0.015595 |
|  | (2) $\alpha=2.534$ | 0.808157 |  |
| $\begin{aligned} & \text { slab in } \\ & \text { x-direction } \end{aligned}$ | (1) $a=16.15 \mathrm{~cm}$ | 0.824127 | 0.001661 |
|  | (2) $a=2.534$ | 0.822466 |  |

Assembly SUAK U1B

Boundery conditions
(1) $\phi^{\prime}(a) / \phi(a)=-1 /\left(0.7104 \cdot \lambda_{t r}\right)$
(2) $\phi(a+i)=0$
$a=$ boundary of the assembly
$d=0.7104 \cdot \lambda_{t r}$

Table V-16: $k_{\text {eff }}$ determined by diffusion calculations with various boundary conditions.

## VI.

Nost of the criticality changes observed during the present stußy are smaller than 0.01 as can be seen from table V-1 and table V-4. This means that some nodifications of the muclear data can totally cthers at least for a number of assemblies partially be regarded simply as an inolusion of more recent inproved micmosconic enoss section information into our group sets. Of course these less important modifications provide valuable information on the sensitivity of integral parameters on changes in the differential microscoric data.

The detailed information contained in table V-4 has been sumarized in table VI-1. In this table the mean deviation between the measured and best available calculated criticality values for the 12 assemblies studied is given for each of the different group sets used. The corresponding root mean square or stamard deviation is also given in table vi-1. Jhis table gives immediately a general impression of the capability of the different group sets to predict correctly the criticality of the variety of assemblies studied. It shows that at least with respect to criticality the $\mathbb{M D X T O I}-$ set is preferable to all the other group sets which bave been established during this study, a result which of course could have been obtained also by looking at the results of table V-4.

In this general discussion the important effects of the changes in the nuclear data will be summarized. At first the situation at the beginning of the present study that means the results obtained with our referenceset, the somealled SNEAK-set, will be analysed. The most obvious fact is that with this set all assemblies are calculated underreactive. The region of deviation extends from approximately zero to $-3 \%$ with the exception of ZPR III-55 with an underprediction of criticality by more than $4 \%$. The mean criticality deviation amounts to $1.7 \%$ as can be seen from table VI-1. The small assemblies fuelled with U235, i.e. SUAK U1B and ZPR III-10 are well predicted, the subcriticality being less then $0.5 \%$. The hydrogen containing assembly SUAK UH1B represents one exception which is probably caused by insufficient calculational methods, the other exception is the
lareer assembly ZPRIII-25 with its relatively large U230 content. As has been known before $1-27$ the SNEAK-3A assembies, that =eans large U235 fuelled systems with saft meutron spectra, are calculated slightly underreactive, $3 A 1$ by $0.6 \%$ and $3 A 2$ by $1.3 \%$, For the mixed fuelled assembly SNEAK 3E2 an even larger underprediction of $1.6 \%$ has been obtained which is an indication that the worth of Pu239 compared to that of 4235 is underpreaicted dy the SHEAK-set. The same tendency is shown by the normal Pufuelled assemblies ZEBRA 6A, ZPR III-48 and ZPR III-48B which have been calculated 2-3\% underreactive. Out of the two $\mathrm{k}_{\infty}$-experiments ZPR III-55 is predicted more sutcritical, SNEAK 5C less subcritical than the normal Pu-fuelled systems. With respect to the latter experiment it seems very probable that the heterogeneity correction is calculated too large. Therefore the result is rather doubtful and should not be taken too seriously. As a first change of the nuclear data we included the so-called PMB-data that means between $10-500 \mathrm{keV}$ the lower capture data for U 238 and the low fission and capture data for U235 as given by PÖNITZ and others. Our results with the SNEPRB-set confirsed the conclusions of similar earlier studies [2_7 mamely:
Because of the lower U238 capture data the reactivity of the Pu-fuelled assemblies increases. This effect is particularly pronounced for 2PR III-55. For the U235-fleelled assemblies the reactivity decreases. This result indicates that the effect of the reduction of $U 235$ fission overcompensates the effect of the reduction of U238 capture. It is most pronounced for the assemblies with hard neutron spectra.
With the SNEPNQ-set nost of the assemblies are calculated about $1 \%-2 \%$ underreactive. Following the conclusions presented in ! 2_7 we then decided to keep only the low U238 capture data and to return from the low PÖNITZ to the higher WHITE U235 fission and capture data. This led to the SNEAPM-set which predicts the criticality of U235-fuelled systems rather well. A drastic decrease of the mean criticality deviation and a considerable decrease of the root mean square criticality deviation can be seen in table VI-1.

In the next step we improved the $\alpha(P u)$ values by including the Pu- $\alpha$-measurements of GWIN et al. into our group set (PUgSCP-set). As expected, this change caused an increased underprediction of the criticality for the Pu-
systers, especially for the $k_{\infty}$-experiments.
In the SCTALO-set new data are used for the inelastic scattering matrix of most isotopes which have been calculated by ourselves using the data of the KEDAK-file and evaporation spectra (see Appendix I). The spectra of the inelestically scattered neutrons are in general somewhat softer than the ABN-sFectra used before, The criticality changes obtained are generally smell. Oaly for 2PR IIT-25 and ZPR III-55 the effect is more pronounced due to the special energy dependence of the adjoint flux for these assemblies (see chapter III). The next two changes have a rather small effect on criticality. The changes in the LA fission data for U235, U238 and Pup39 above 2 Mev ieading to the UPUCOR-set cause a general reduction of criticality by orly about $0.3 \%-0.4 \%$. Aiso the new cross sections for the higher plutonium isotopes in the PUO2RFmet which replace the old Russian ABM-data used ur to now result in rather small criticality chenees of about $0.3 \%$. As expected, a rather large effect is caused by the inclusion of the MOXOIJ-data for the U238 capture cross section. The replacement has been done in two steps. The effect on criticality of the replacement of the previously used Pönitz valnes in the erouns gall, i.e. from 10 to 100 keV (MめYO11-set) is generally of tife ouder of $+1 \%$. The assemblies with hí U238 content, namely ZPR III-25 and Z.PR III-55 show en even larger reactivity increase. The additional replacement of the u23f cemture data below 10 keV by the Moxon data (MyTDT-set) causes for assemplies with hard neutron spectra a negligible reactivity effect, for asserblies with sonewhat softer spectra the criticality increases by 0.5 tc $i \neq$. For the $k_{\infty}$-experiments studied here with still softer spectra the increase is about $1.5 \%$.

It seems worthwhile to mention that the inclusion of the $10 w$ U238 capture data resulted in an overestimation of the criticality at least for U-fuelled assemblies.

Let us now analyse the situation with the M $\overline{\text { n }}$, section set considered in this study. All assemblies are within a $\pm 2 \%$ region of deviation between calculated and measured criticality. The mean criticality deviation of 0,0022 and the root mean square a criticality deviation of 0.0125 given in table VI-1 are the most favourable results obtained in the present study. Furthermore it is encouraging that there is nearly a clear
separation in the keffresults for U- and Pu-fuelled assemblies: Pu<1.0, U>1.0, the exceptions (SUAK UH1B and SNEAK 5C) being most rrobably due to deficiencies in the calculational methods. This means that according to the fissile material used, it will be possible to predict the probable deviation between celculated and measured eriticality for reactors whose composition and neutron spectrum are similar to the presently studied asseminlies. The remainine uncertainty will be in most cases far less than the $\pm 2 \%$ deviation mentioned above.

With respect to the calculational methods two facts are of major concern and need an improved treatment: (1) For the assembly SUAK UH1B the anisotropic scattering of hyarogen and more appropriate weighting spectrum for the generation of group constants for the elastic slowing down should be taken into account. (2) For SNEAK-5C, the corrections applied are rather large (about 9\%). The largest part is due to heterogeneity, about $7 \%$ and should be further investigated both experimentally and theoretically. In addition, the rather large REMO-correction of about $2 \%$ suggests to study whether a more appropriate weighting spectrum should be used for the generation of group constants for the elastic moderation especially below 1 keV . For this soft spectrum system a considerable part of the neutrons has energies below 1 keV ; here the RElOMcorrection cannot be applied with the presently available code. In the near future the correctness of diffusion calculations using 26 energy groups for the criticals studied here will be checked by P1-calculations using about 200 groups. Major attention will then be Eiven to the determination of the diffusion constant for 26 groups which usually can be calculated only in an approximate manner as explained in 14_7.
With respect to the nuclear data basis the following conclusions can be drawn for the MDXIDT-set:
(1) The fuel-mixture PU239-U238 is predicted underreactive whereas for the U235-U238 fuel mixture the criticality is overestimated. These facts whose consequences are discussed more extensively later on in this chapter almost obviously lead to the very probable conclusion that Pu239 is underreactive. A too low Pu230 fission cross section is also indicated by the underprediction of both the central material worth ratios and the ratios of the central fission rates of Pu239 to U235 in ZPR III-48 and 48B (table V-7 and V-9). The. Pu239 a-data of

GWIN cannot be considered final at the present time but it seems that only minor changes in the $\alpha$-data will have to be expected in the near future. The $\alpha$-values presently used require a change in the resonance parameters compared to the parameters used before. This means that we have still to determine new resonance self-shielding factors (fliactors) for Pu239 based on resonance parameters compatible with the Gwin $\alpha$-values.

For U235 the situation seems not to be so unequivocal. The overestimation of the criticality of U235-U238 fuel mixtures can be caused Either by too large U235 fission cross sections or by too small U238 capture cross sections. Very probably our decision will be not to change the U235 fission data at the present time because the uncertainty in the U238 capture data is still too large so that some changes (slight increase) of these data will be sufficient to diminisil the present discrepancy in the criticality prediction for the U235-j238 fuelled systens without any change in the U235 fission data. Pu-fuelled systems will then become even more underreactive. In the preceding work 127 it was already indicated that the measurements of de Saussure for the $\gamma$-values of U 235 below 300 eV should be included. Since the effect for the assemblies studied here is expected to be of minor inportance (perhaps with the exception of the steam density coefficient fo: the SNEAK-3A-series), this change together with the corresponding changes in the f-factors has been postponed to the future work. For U238 the situation is still more complicated. As explained in chapter II there are some doubts as to the reliability of the absolute magnitude of the MOXOII-data for U238 capture. They are lower than most of the available 30 and 65 keV measurements. Furthermore in the region between 100 and 800 keV the $M \varnothing \mathrm{XI} \varnothing \mathrm{T}$ set still contains the Pönitz data which are lower than the Barry data in this range and do not join smoothly to the Barry data used above 800 keV .

In any case it seems to be quite evident that the capture cross section used for the SNEAK-set is too large and has to be reduced; it may be that this has been slightly overdone by the inclusion of the MOXON-data. Besides the indication from the criticality of U-fuelled assemblies this can be deduced by comparing the experimental results for the ratio $\sigma_{c} \mathrm{U} 238 / \sigma_{\mathrm{f}} \mathrm{U} 235$ for ZPR III-48 and SNEAK $3 \Lambda 2$ and the material worth ratio of U238 to U235 for ZPR III-48 and SHEAK 3A1 with the corresponding theoretical results obtained
with the M $\phi$ XI $\phi$ T-set. Therefore it seems probable that in a further step the U238 capture data will be changed slightly. Hopefully such a change which will be based essentially on differential data will lead to a slight increase of the capture rate of $U 238$ compared to the result with the MøXPør-set but will not be so marked as to bring ebout a result as large as that obtained with the SNEAK-set.
In any case the change in the capture cross section corresponds to a change in the resonance parameters. Therefore it will be necessary in a further step to determine new f-factors using resonance parameters which are in accordance with the capture cross section of 2238 . The influence of cross section uncertainties of the higher Pu-isotopes on the criticality of the systems studied is not very pronounced. Even by comparing $k_{\text {eff }}$ for ZPR III-48 with that for ZPR III-48B which has an inner zone with considerably larger Pualo-concentration one is not able to draw firm conclusions on the correctness of the nuclear data for Pu240 because this inner zone is still too small. This fact has also been observed by PITTERLE [74_7. One should mention, however, that the inclusion of the new data has brought the prediction of the material worth of Pu240 for both assemblies in egreement with measurement within the admittedly large range of experimental uncertainty (tables V-7 and V-9). Using the old ABM-data the calculated value was lower by a factor of about 4. With our new data we attained a considerable improvement but it is obvious that the criticality of presently available plutoniun fuelled assemblies does not provide a check which allows a definite conclusion on the correctness of the unclear data for the higher Pu-isotopes. For this purpose more specific precise experiments have to be performed. The measurements of 00STERKAMP reported in $140 \_7$ (see also $\left[48 \_7\right.$ ) can be considered as an example of this sort of experiments.
In our opinion the correctness of the data for the inelastic scat,tering cannot be judged definitely on the basis of the presently available information. Criticality of the assemblies studied in this work provides no sufficient information. Other experimental information e.g. reaction rates of materials with a fission threshold cannot be considered very reliable because the chamber measurements cannot be compared easily with calculations which are mostly done for homogeneous mixtures. Furthermore
the foil measurements for the U 235 fission rate traverse within a cell cannot be well predicted by the heterogeneity code ZERA; it may be that this disagreement is not caused by the nuclear data used in the group sets, but is due to reasons inherent to the code itself. Since the jeterogeneous U 235 fission rate hes to be used for the determination of the fission rate ratio of $U 238$ to $U 235$ the theoretical result is somewhat doubtful. A thorough examination of the ZERAncode would be helpful in this respect.
Further detailed investigations of the neutron energy spectrum might help to check the correctness of the nuclear data for inelastic scattering. In conclusion it is expected that by the future changes and improvements mentioned before the uncertainty region for the criticality prediction which is now $\pm 2 \%$ can be narrowed hopefully to $\pm 1 \%$. Certain improverents in the calculational methods are indicated e.g. the anisotropy of the elastic slowing down of hydrogen should be taken into account in the $S_{N}$-calculations and the influence of an appropriate weighting spectrum on the removal Eroup constants should be considered for SUAK UH1B and SNEAK 5C. As has been stated before a reinvestigation of the ZERA-code for heterogeneous calculations seems to be necessary. This will be essential for the more correct determination (1) of the criticality for SNEAK 5C, (b) of heterogeneous reaction rates in this assembly and e.g. in SNEAK 3A2, (c) of bunching experiments in SNEAK $3 A 1$ and SNEAK $3 A 2$, (d) rather probably also of the steam density coefficient for the streak-3A-series.
For the quantities which have been studied besides the criticality the following items seem to be important:
( $\alpha$ ) For the calculation of material worth more refined codes than the usually used first order perturbation theory should be applied which take into account the effect of sample size (see e.g. [-64_7) or the effect of the heterogeneous structure of the environment (see e.g. [48_7.
( $\beta$ ) For the calculation of central reaction rates an improved heterogeneity code should be availabie as indicated just before.
( $\gamma$ ) For the calculation of reaction rate traverses especially in the blanket transport theory should be used. At present the agreenent between theory and experiment can be considered fairly good when the MфXTфT-set and transport theory are used. Perhaps slight rodifications of the transport or totel cross section of $U 238$ or of the structural materials in the enerfy region $200 \mathrm{keV}-2 \mathrm{MeV}$ will help ro reduce the existing rather small discrepancies.
( $\delta$ ) With respect to the neutron importance more experimental information and perhaps more appropriate calculational methods seem to be necessary.

The important results and conclusions of the heterogereity- and $S_{H}$-calculations have already been summerized at the end of the corresponding paragraphs in the preceding chapter $V$ and will not be repeated here.

## VII. APPENDIX I

## Documentation of the new group constants

## I. 1 Modified group cross sections

Table AI-1 SNEAPM-set

|  | $\sigma_{r}\left(v^{238}\right)(b)$ |  |  |
| :---: | :---: | :---: | :---: |
| Group | $\Delta E$ | SNEAK | SNEAFM |
| 6 | $400-800 \mathrm{KeV}$ | 0.134 | 0.121 |
| 7 | $200-400$ |  | 0.127 |
| 8 | $100-200$ |  |  |
| 9 | $46.5-100$ | 0.138 | 0.164 |
| 0.190 | 0.282 |  |  |
| 10 | $21.5-46.5$ | 0.286 | 0.457 |
| 11 | $10.0-21.5 \mathrm{KeV}$ | 0.471 | 0.630 |

Table AI-2 SNEPMB-set

| Group | $\triangle \mathrm{E}$ | $\sigma_{f}\left(u^{235}\right)(b)$ |  | $\sigma_{y}\left(u^{235)}(b)\right.$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | SNEAR | STEPMB | SNEAK. | SNEPMB |
| 7 | 200-400 KeV | 1.32 | 1.154 | 0.287 | 0.199 |
| 8 | 100-200 | 1.53 | 1.337 | 0.375 | 0.322 |
| 9 | 46.5-100 | 1.80 | 1.667 | 0.569 | 0.531 |
| 10 | 21.546 .5 KeV | 2.22 | 2.150 | 0.794 | 0.772 |

Table AI-3 PU9SCP-set

| Group | $\triangle E$ | $\alpha\left(\mathrm{Pu}^{239}\right)$ |  | $\sigma_{r}\left(\mathrm{Pu}^{239}\right)(\mathrm{b})$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | SNEAK | PugSce | SNEAK | PU9SCP |
| 11 | 10.0-21.5 KeV | 0.420 | 0.413 | 0.791 | 0.775 |
| 12 | 4.65-10.0 | 0.490 | 0.604 | 1.20 | 1.480 |
| 13 | 2.15-4.65 | 0.537 | 0.905 | 1.76 | 2.957 |
| 14 | 1.0-2.15 | 0.593 | 0.892 | 2.33 | 3.504 |
| 15 | 0.465-1.0 KeV | 0.640 | 0.846 | 4.70 | 6.218 |

Table AI-4 UPUCфR-set

| Group | $\triangle E$ | $\sigma_{f}\left(u^{235}\right)(b)$ |  | $\sigma_{p}\left(u^{238}\right)(b)$ |  | $\sigma_{f}\left(\mathrm{Pu}^{239}\right)(\mathrm{b})$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | SNEAK | UPUCQR | SNEAK | IUPUC6R | SNEAK | UPUC 6 R |
| 1 | $6.5-10.5 \mathrm{MeV}$ | 1.65 | 1.53 | 0.967 | 0.934 | 2.19 | 2.05 |
| 2. | 4.0-6.5 | 1.19 | 1.07 | 0.584 | 0.540v | 1.86 | 1.65 |
| 3 | $2.5-4.0 \mathrm{MeV}$ | 1.28 | 1.17 | 0.572 | 0.572 | 1.97 | 1.82 |


|  | $\bar{U}\left(\mathrm{Pu}^{239}\right)$ |  |  |
| :---: | :--- | :--- | :--- |
| Group | $\triangle E$ | SNEAK | UPUC $\varnothing R$ |
| 1 | $6.5-10.5 \mathrm{MeV}$ | 4.02 | 4.175 |
| 2 | $4.0-6.5 \mathrm{MeV}$ | 3.60 | 3.660 |

Table AI-5 PUOZRE-set (see next pages)


| Group | $\triangle \mathrm{E}$ | $\sigma_{\gamma}\left(U^{238}\right)(b)$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | SNEAK | SNEAPM SNEAPMB | $\begin{aligned} & M \phi \times 911 \\ & M \phi \times T \phi T \end{aligned}$ |
| 9 | 46.5-100 KeV | 0.286 | 0.282 | 0.245 |
| 10 | 21.5-46.5 | 0.471 | 0.457 | 0.405 |
| 11 | 10.0-21.5 | 0.728 | 0.630 | 0.576 |
| 12 | 4.65-10.0 | 1.034 | =SNEAK | 0.790 |
| 13 | 2.15-4.65 | 1.237 |  | 1.18 |
| 14 | 1.0-2.15 | 1.590 |  | 1.72 |
| 15 | 0.465-1.0 KeV | 3.107 | =SNEAK | 2.91 |


|  |  |  |  | 240 |  |  |  | 241 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Group | $\Delta \mathrm{E}$ | $\sigma_{T}(\mathrm{~b})$ | $\\|_{f}(b)$ | ${ }^{\sigma_{r}}$ (b) | ${ }_{0}{ }_{\text {el }}(\mathrm{b})$ | $\sigma_{T}(\mathrm{~b})$ | $\mathcal{L}^{J_{f}(b)}$ | $\sigma_{r}(\mathrm{~b})$ | ${ }^{0}{ }_{\mathrm{e}}{ }^{\text {(b) }}$. | $\sigma_{T}(\mathrm{~b})$ | $\sigma_{f}(b)$ | $\sigma_{y}(b)$ | $\sigma_{\text {al }}{ }^{(b)}$ |
| 26 | 0.0252 | 177.2 | 0.0327 | 175.4 | 1.72 | 1082.3 | 779.5 | 291.0 | 11.77 | 25.04 | 0 | 16.21 | 8.83 |
| 25 | 0.215-0.465 eV | 164.6 | 0.0306 | 164.0 | 0.532 | 925.0 | 561.0 | 348.3 | 15.17 | 18.52 | 0 | 10.02 | 8.49 |
| 24 | $0.465-1.0 \mathrm{eV}$ | 1637.9 | 0.294 | 1570.2 | 67.42 | 66.59 | 44.70 | 12.03 | 9.87 | 17.40 | 0 | 9.50 | 7.90 |
| 23 | 1.0-2.15 eV | 7253.9 | 1.25 | 6734.0 | 518.7 | 36.08 | 24.39 | 2.58 | 9.11 | 25.40 | 0 | 19.97 | 5.42 |
| 22 | $2.15-4.65 \mathrm{eV}$ | 22.82 | 0.0025 | 7.84 | 14.97 | 204.0 | 126.3 | 69.20 | 8.50 | 1312.4 | 0 | 1208.6 | 103.8 |
| 21 | $4.65-10.0 \mathrm{eV}$ | 11.14 | 0.00107 | 0.676 | 10.47 | 262.5 | 214.5 | 37.25 | 10.78 | 11.80 | 0 | 0.606 | 11.20 |
| 20 | 10.0-21.5 eV | 49.01 | 0.934 | 35.77 | 12.31 | 183.9 | 125.6 | 42.21 | 16.10 | 11.44 | 0 | 1.57 | 9.87 |
| 19 | $21.5-46.5 \mathrm{eV}$ | 149.9 | 0.506 | 77.47 | 71.94 | 81.60 | 60.24 | 10.53 | 10.84 | 12.45 | 0 | 3.98 | 8.47 |
| 18 | $46.5-100 \mathrm{eV}$ | 102.50 | 0.726 | 37.96 | 63.81 | 73.20 | 39.27 | 19.89 | 14.04 | 90.13 | 0 | 22.95 | 67.18 |
| 17 | 100-215 eV | 46.94 | 0.448 | 18.37 | 28.12 | 54.51 | 29.10 | 13.66 | 11.74 | 26.76 | 0 | 10.33 | 16.43 |
| 16 | 215 -465 eV | 29.88 | 0.0639 | 6.62 | 23.20 | 41.38 | 20.03 | 8.88 | 12.48 | 33.60 | 0.00821 | 7.93 | 25.67 |
| 15 | $465-1000 \mathrm{eV}$ | 24.30 | 0.0177 | 3.79 | 20.50 | 31.62 | 14.23 | 5.59 | 11.79 | 26.63 | 0.0100 | 4.55 | 22.07 |
| 14 | 1. $0-2.15 \mathrm{keV}$ | 25.65 | 0.155 | 2.48 | 23.01 | 24.55 | 9.38 | 3.10 | 12.07 | 21.47 | 0.0100 | 2.81 | 18.6\% |
| 13 | $2.15-4.65 \mathrm{keV}$ | 21.26 | 0.142 | 1.51 | 19.61 | 20.53 | 6.06 | 1.97 | 12.50 | 18.22 | 0.0100 | 1.94 | 16.27 |
| 12 | $4.65-10.0 \mathrm{keV}$ | 18.89 | 0.104 | 1.02 | 17.77 | 16.97 | 4.47 | 1.50 | 11.01 | 16.09 | 0.0100 | 1.48 | 14.60 |
| 11 | $10.0-21.5 \mathrm{keV}$ | 17.05 | 0.105 | 0.730 | 16.22 | 15.44 | 3.55 | 1.13 | 10.76 | 14.70 | 0.0100 | 1.11 | 13.58 |
| 10 | $21.5-46.5 \mathrm{keV}$ | 15.51 | 0.108 | 0.494 | 14.91 | 14.15 | 2.91 | 0.805 | 10.38 | 13.55 | 0.0100 | 0.758 | \| 12.78 |
| 9 | $46.5-100.0 \mathrm{keV}$ | 13.90 | 0.0845 | 0.287 | 13.39 | 12.75 | 2.42 | 0.538 | 9.44 | 12.90 | 0.0100 | 0.487 | 712.27 |
| 8 | 0.1-0.2 MeV | 11.98 | 0.0986 | 0.209 | 11.08 | 11.73 | 2.04 | 0.314 | 8.77 | 12.06 | 0.0203 | 0.297 | 71.03 |
| 7 | $0.2-0.4 \mathrm{MeV}$ | 9.84 | 0.133 | 0.148 | 8.37 | 10.10 | 1.72 | 0.188 | 7.39 | 10.25 | 0.0636 | 0.222 | 8.89 |
| 6 | 0.4-0.8 MeV | 8.02 | 0.548 | 0.142 | 5.79 | 8.35 | 1.51 | 0.104 | 5.82 | 7.91 | 0.344 | 0.199 | 6.02 |
| 5 | 0.8-1.4 MeV | 7.00 | 1.43 | 0.105 | 4.08 | 7.24 | 1.59 | $0.048 \pi$ | 4.65 | 6.98 | 1.32 | 0.153 | 34.43 |

Table AI-5 PUORREset (continued)

| Group $\triangle E$ |  | $\mathrm{Pu}^{240}$ |  |  |  | $\mathrm{Pu}^{241}$ |  |  |  | $p u^{242}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\sigma_{T}(\mathrm{~b})$ | $\sigma_{p}(b)$ | $\sigma_{\gamma}(b)$ | $\sigma_{\text {el }}(\mathrm{b})$ | $\sigma_{T}(b)$ | $\sigma_{f}(\mathrm{~b})$ | $\sigma_{\sim}(\mathrm{b})$ | ${ }_{\text {cel }}(b)$ | $\sigma_{T}(b)$ | $\sigma_{f}(b)$ | ${ }_{0}{ }_{y}(\mathrm{~b})$ | ${ }_{0}{ }_{e 1}(b)$ |
| 4 | 1.4-2.5MeV | 7.35 | 1.48 | 0.0586 | 4.17 | 7.23 | 1.74 | 0.0245 | 4.63 | 7.34 | 1.46 | 0.0666 | 3.90 |
| 3 | 2.5-4.0MeV | 7.93 | 1.51 | 0.0268 | 4.74 | 7.94 | 1.54 | 0.0109 | 5.31 | 7.92 | 1.51 | 0.0306 | 4.72 |
| 2 | $4.0-6.5 \mathrm{MeV}$ | 7.76 | 1.51 | 0.0118 | 4.63 | 7.96 | 1.54 | 0.00614 | 4.94 | 7.81 | 1.51 | 0.0130 | 4.64 |
| 1 | $6.5-10.5 \mathrm{MeV}$ | 6.54 | 1.89 | 0.00666 | 3.56 | 6.87 | 2.09 | 0.00349 | 3.79 | 6.53 | 1.89 | 0.00753 | 3.55 |

## I. 2 The calculation of the inelastic scatterins matrices

The general expression for the probability of inelastic scattering out of any energy group i to any group $h$ is given by Yiftah, Okrent and Moldauer [91_7 in the following form


Here $E$ is the incident neutron-energy and $E$ the energy of ine scattered neutron. The inaices $L$ and $H$ indicate the lower and higher energy-limits respectively of any group. $\phi(E)$ represents the energy-dependent neutron fiux. $P\left(E+E^{\prime}\right)$ represents the transition probability of the inelastically scattered neutrons. For its determination one has to distinguish between the range of resolved levels of the residual nucleus and the somalled continuum range in which the levels of the residual nucleus are undistinguishsble.

## I. $2 a$ Fnergy groups with discrete excitation levels for inelastic scattering

For given incident neutron energies $E$ in group $i$ and outgoing neutron energies $E^{\prime}$ in group $K$ inelastic scattering can occur only to those excitation levels for which

$$
E^{\prime}=E-E_{j}
$$

where $E_{j}$ is the energy of the $j$ th level.
The transition probability therefore is a $\delta$-function which has the value one, if the above condition is fulfilled and zero, if not. Thus we have

$$
\sigma_{n^{\prime}}(E) P\left(E+E^{\prime}\right)=\sum_{j=1}^{N} \sigma_{n^{\prime}}^{E j}(E) P_{j}\left(E+E^{\prime}\right)=\sum_{j=1}^{N} \sigma_{n^{\prime}}^{E j}(E) \delta\left(E-E^{\prime}-E_{j}\right) \quad(A I-2)
$$

where $N$ is the number of excitation levels of the isotope considered. $P_{j}$ the probability of the transition to the jin level of the residual nucleus and $\sigma_{n}^{E j}$ the inelastic excitation cross section of the jth level. With this special form of the transition-probability the double integration in (AI-1) cen be reduced to a single one over the energy group $K$ to which the scattering occurs.

$$
\begin{equation*}
\sigma_{n^{\prime} i, K}=\frac{\sum \int_{E_{K I}}^{E_{K E}} d E^{\prime} \sigma_{n^{\prime}}^{E j}\left(E^{\prime}+E_{j}\right) \phi\left(E^{\prime}+E_{j}\right)}{\int_{E_{i L}}^{E_{r I}} \phi(E) d E} \tag{AI-3}
\end{equation*}
$$

with the transformation $E^{\prime}+E_{j} \rightarrow E$ it follows:

$$
\begin{equation*}
\sigma_{n^{\prime} i_{0} K}=\frac{\sum \int_{E_{K I L}+E_{j}}^{E_{K H}+E j} d E \sigma_{\sigma^{\prime}}(E) \phi(E) \times\left(E_{i L}<E \leq E_{i H}\right)}{\int_{E_{i L}}^{E_{i H}} \phi(E) d E} \tag{AI-4}
\end{equation*}
$$

By the distribution-fir $t i=n x$ it ras taken into account that the incident neutron-energy $E$ should be contained in a 26 -group $i$. on the other hand it must be of course $E_{K I}{ }^{+E_{j}} \leq E \leq E_{K H}+E_{j}$ because of the integration over this range, The integration therefore has to be extended only over the interval $I_{E_{u}}, E_{0-} 7$ with the following definition

$$
E_{u}=\left\{\begin{array}{l}
E_{i L}, \text { if } E_{K L}+E_{j}<E_{i L}  \tag{AI-5}\\
E_{K L}+E_{j} \text { if } E_{K L}+E_{j}>E_{i I}
\end{array} \quad \text { and } \quad E_{0}= \begin{cases}E_{i H} & \text { if } E_{K H}+E_{j}>E_{i H} \\
E_{K H}+E_{j} & \text { if } E_{K H}+E_{j} \leq E_{i H}\end{cases}\right.
$$

Instead of the neutron flux the collision density in the special form of the SNEAK-3A spectrum was taken as weighting spectrum. Reasons for preferring the collision density to the neutron flux for weighting purposes are outlined in 14.7. The integral was solved numerically by use of the trapezoidal rule. The inelastic excitation cross sections were taken from the NEDAK-tape [27.7.

## I. $2 b$ Energy grouns with unresolved excitation levels for inelastic scattering

In the energy region, where the excitation levels of the residual nucleus are not experimentally resolved, the energy distribution of the inelastically scattered neutrons is described by an evaporation model. The transition probability for neutrons from energy $E$ to energy $E^{\prime}$ by inelastic scattering is then given by

$$
\begin{equation*}
P\left(E+E^{\prime}\right)=N E^{\prime} \exp \left(-E^{\prime} / \theta(E)\right) ;\left(E^{\prime} \leq E\right) \tag{AI-6}
\end{equation*}
$$

Here $\mathbb{V}$ is a normalization constant and $\theta$ is the so-called nuclear temperature of the residual nucleus, which WeiBkopf 17117 derived as

$$
\begin{equation*}
\theta(E)=\sqrt{E} \quad\left[\mathrm{MeV}_{-} \quad \quad\binom{E \text { in } \mathrm{MeV}}{v \text { in } \mathrm{MeV}^{-1}}\right. \tag{AI-T}
\end{equation*}
$$

Here again $E$ is the energy of the incident neutron, $A$ is the atomic yass number of the residual nucleus and $v$ is an adjustable parameter. For a given value of $E P\left(E+E^{\prime}\right)$ has its maximum at $E^{\prime}=\theta(E)$. Yiftah, Okrent and Moldauer [9i_7 chose $v=0.096 \mathrm{MeV}^{-1}$ for all nuclei. We tooir the $v$-value recomenced by Swarcbaum et al. 142.7, which is $v=0.16 \mathrm{MeV}^{-1}$ for all nuclei. Swarcbaum et al. adjusted more recently nuclear temperatures for materials from Al to $\mathrm{U}^{238}$ in the energy region between 2.5 and 7.0 MeV and found that over this whole region $v=0.16$ gives an acceptable overall fit for the energy distribution of inelastically scattered neutrons.

Inserting (AI-6) into(AI-1) the cross section for inelatic scattering from energy group $i$ to energy group $K$ is calculated from:


Following Yiftah et al. [91_7 and Swarcoaum et a1. [42_7 we replaced $\theta$ (E) by an average $\theta_{i}$ value for each group $i$. This was necessary because only a limited amount of computer ccre storage was available for our program, which is part of the whole MIGR $\varnothing$ S-system and was developed originally for the IBM 707 \& computer. For the same reason we had to find a very simple expression for $\theta_{i}$. We chese

$$
\theta_{i}=\sqrt{\frac{E_{i H}+E_{i L}}{2 V A}}
$$

instead of the flux-averaged value taken by Yiftah et al. and Swarcbaum et al. This procedure may be justified if one considers that formula (AI-7) already is an approximation for $\theta(E)$.
With the introduction of an averaged $\theta$ the energy integrations over the groupe i and $K$ can be separated. The E'-integration can be carried out analytically.

The result is

$$
\begin{align*}
\sigma_{n^{\prime} i, K} & =\sigma_{n^{\prime} i} N /-\theta_{i}^{2} \exp \left(-\frac{E_{K L}}{\theta_{i}}\right)\left(\frac{E_{K L}}{\theta_{i}}+1\right)-\theta_{i}^{2} \exp \left(-\frac{E_{K H}}{\theta_{i}}\right)\left(\frac{E_{K H}}{\theta_{i}}+1\right) \_7  \tag{AI-9}\\
& =\sigma_{n^{\prime} i} N \stackrel{\sim}{P}\left(\Delta E_{i}+\Delta E_{K}\right)
\end{align*}
$$

The normalization constant $N$ is determined by the requirement

$$
\begin{align*}
& \sum_{K \geq i} \stackrel{?}{P}\left(\Delta E_{i} \rightarrow \Delta E_{K}\right) N=1 \quad \text { and }  \tag{AI-10}\\
& P\left(\Delta E_{i}+\Delta E_{K}\right)=\frac{\tilde{P}\left(\Delta E_{i} \rightarrow \Delta E_{K}\right)}{\sum_{K \geq i}^{P}\left(\Delta E_{i} \rightarrow \Delta E_{K}\right)}
\end{align*}
$$

## I. 2 c Energy groups, where the excitation levels are partly resolved and and partly unresolved

For nearly all materials the energy of the last discrete excitation level lies within a $26-g r o u p$, so that the lower part of the group has to be treated by the discrete level method and the upper part by the evaporation model. The results for each part are linked together as follows. Let $\Delta \mathbb{E}^{i}{ }_{d}$ be the energy range of group $i$, where discrete excitation levels are known, and $\Delta E_{c}^{i}$ the energy range, where the excitation levels are unresolved. Then for a group with both discrete and "continuous" levels the transition probability of inelastic scattering is calculated according to the formula

$$
\begin{equation*}
P\left(\Delta E_{i} \rightarrow \Delta E_{K}\right)_{d, c}=d_{i} P\left(\Delta E_{d}^{i} \rightarrow \Delta E_{K}\right)+c_{i} P\left(\Delta E_{c}^{i}+\Delta E_{K}\right) \tag{AI-11}
\end{equation*}
$$

In (AI-11) $P\left(\Delta E_{d}^{i} \rightarrow \Delta E_{K}\right)$ is the transition probability to group $K$ of neutrons whose incident energy lies in the range of resolved excitation levels of the residual nucleus within group i, $P\left(\Delta E_{d}^{i} \rightarrow \Delta E_{K}\right)$ is given by $\sigma_{n ' i}, K_{n} / \sigma_{n} i^{\text {w }}$ where $\sigma_{n} i_{i} K$ follows from equation (AI-4). $P\left(\Delta E_{c}^{i} \rightarrow \Delta E_{K}\right)$ is the corresponding quantity for neutrons with energies in the range of unresolved levels of the residual levels of the residual nucleus within group i. $P\left(\Delta E_{d}^{i} \rightarrow \Delta E_{K}\right)$ and $P\left(\Delta E_{c}^{i}+\Delta E_{K}\right)$ are
both normalized to $\sum_{K \geq 1} P\left(\Delta E_{1}+\Delta E_{K}\right)=1$.
The relative contributions of resolved and unresolved levels $d_{i}$ and $c_{i}$ are given by

$$
\begin{align*}
& d_{i}=\frac{\int_{\Delta E_{d}^{i}} \sigma_{n}(E) \phi(E) d E}{\int_{E_{i L}}^{E_{i H}} \phi(E) d E} ; c_{i}=\frac{\int_{c} \sigma_{n}^{i}(E) \phi(E) d E}{\int_{E_{i L}}^{i H} \phi(E) d E}  \tag{AI-12}\\
& \left(\Delta E_{d}^{i} \Delta E_{c}^{i}=\Delta E_{i}\right)
\end{align*}
$$

Of course (AI-6) and (AI-7) are simplified formulas. Taking the same value of $\cup$ for all nuclei and distinguishing between them by their atomic mass numbers only means neglecting individual nuclear properties. The use of an averaged nuclear temperature instead of performing the double integration is another source of error.

## I.2d Transition probabilities for ( $n, 2 n$ ) reactions

So far we did not calculate transition probabilities for ( $n, 2 n$ ) reactions of our own. In energy groups, where ( $n, 2 n$ ) reactions occur, the inelastic transition probabilities of the ABN $26-\mathrm{group}$ set $\left[20 \_7, W\left(\Delta E_{i} \rightarrow \Delta E_{K}\right)\right.$ were used. The inelastic matrix was then calculated according to

$$
\begin{equation*}
\sigma_{n^{\prime}+n, 2 n ; i, K}=\left(\sigma_{n} i^{+2 \sigma_{n, 2 n i}}\right) W\left(\Delta E_{i} \rightarrow \Delta E_{K}\right) \tag{AI-13}
\end{equation*}
$$

## I.2e Discussion

The scattering mairices were obtained by multiplication of the transition probabilities with the total 26 group-cross sections for inelastic scattering. Contrary to the ABN-set $[20-7$ the scattering matrices have been calculated for scattering into all energy groups $K \underline{26}$; no cut because of the smallness of the following matrix-elements was made at any $K$. Our new transition probabilities in general show a softer spectrum than the matrices of the ABN-set as well in the "continuum" as in the range of discrete levels. For the most
important case of $U^{238}$ the main reasons for this fact are the following: The inelastic scattering to higher levels especially to the levels between 0.7 and 1 MeV has been underestimated in the ABN-set. Moreover we have taken into account a wider resolved energy range than in the ABN-set. In the continuum region one reason for the softer spectrum quite clearly is the use of the value of 0.16 for $v_{3}$ which results in a rather small nuclear temperature and therefore shifts the maximu of the energy distribution of the inelastically scattered neutrons $\ddagger 0$ smaller energies compared to the results obtained with $v=0.096$ for example.

ABN also applied an evaporation model in the energy range of unresolved excitation levels. For $\mathrm{U}^{238}$ they used experimental values of the nuclear temperature, from which a value of $v=0.11$ is obtained. Fhe resultinis inelastic scattering matrix was then still modified in the following way. Calculations carried out with this matris should give the correct value for the total number of fissions in an infinite slab of $u^{238}$.

For nearly all other materials individual corrections were applied in the ABN-set. This should be kept in mind by someone whowants to compare our matrices to these of Abagjan et al.

The new scattering matrices are given in table AI-7 on the following pages.

Table AI-T

Inelastic scettering matrices $\sigma_{n}{ }^{\prime} \mathbf{i n g}_{\mathbf{K}}$

Explanation of material names used in the following tables:

$$
\begin{aligned}
& \text { AL2TO }={ }^{27} \mathrm{AL} \\
& \mathrm{C}^{\mathrm{B}} 120={ }^{12} \mathrm{C} \\
& \mathrm{C}
\end{aligned}
$$

AL270


## C.R520

$1=1$
k: $1-10$
k: 11 -
K: 21-2
$1=2$
$K: \quad 2$
K: 2-11
K: 12-21

- 22
$I=3$
$k=3$
K: 13-22
$K: 13-22$
$K: 23-26$
4.018 .02
$=4$
$=4-13$
$: 14-23$
$: 24-26$
9.03E-03
2.39 E $5.25 \mathrm{E}-$
$7.93 E-02$
$5.21 E-05$
$1.14 F-11$
4.10E-02
4.10E-0 $8.57 \mathrm{~F}-05$
$1.88 \mathrm{E}-11$
1.6RE-01 $1.87 E-05$
$4.00 E-12$

| 2.16E-01 | 3.62E-01 | 2.71E-01 | 1.67F-01 |
| :---: | :---: | :---: | :---: |
| 1.13E-05 | 2.42E-06 | $5.25 \mathrm{E}-07$ | 1.14F-n7 |
| $2.43 \mathrm{E}-12$ | 5.25F-13 | 1.14E-13 | 2.02F-14 |
| 3. $78 \mathrm{~F}-01$ | 3.43F-01 | 2.38E-01 | R. $855-02$ |
| 3.99F-06 | 8.65E-07 | 1.88E-07 | 4.00E-OA |
| 8.66E-13 | 1.8BE-13 | $3.76 F-14$ |  |
| 3.91E-01 | 1.30E-01 | 5.95E-02 | 2.345-02 |
| 4.03E-07 | 8.73F-08 | 1.86E-08 | $4.02 \mathrm{E}-00$ |
| 8. 76F-14 | 2.27F-14 |  |  |
| 2.40F-01 | 9.42F-02 | 3.705-02 | 1.54r-n? |
| 8.62E-07 | 1.84E-07 | $3.97 \mathrm{E}-\mathrm{DR}$ | B.62F-09 |
| 2.03E-13 |  |  |  |
| 2.985-02 | 1.06F-02 | 4.24F-03 | A.635-04 |


$l=5$
$\mathrm{K}: 5-11$
7.19E-04
2.56F-02
$1.535-03$
6.51 F-34
2.32r-n4

## C 120

| $I=1$ |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $K:$ | $1-6$ | 0.0 | $5.20 E-02$ | $1.55 F-01$ | $8.65 F-02$ | $2.48 F-04$ | $1.24 F-05$ |
| $I=2$ |  | 0.0 | $3.86 E-02$ | $2.00 F-02$ | $5.50 F-03$ | $R .26 E-05$ |  |

## M0960

| $\mathrm{I}=$ | 1 |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K： | 1－10 | 1．60E－03 | 3．73F－02 | 1．92F－01 | 5．10E－01 | 5．1RE－01 | 3．82F－n1 | 1．48F－01 | 4． $53 \times-n 7$ | 1．254－n2 | 1．119－n？ |
| K ： | 11－20 | 6．82E－04 | 1．49E－04 | 3．26F－05 | 6．97E－06 | 1．51F－06 | 3．24E－ 77 | 6．0rE－ñ | 1．51r－ns | 3.70 ran | h．gne－17 |
| K： | 21－26 | 1．51E－10 | 3．28E－11 | 6．99E－12 | 1．51F－12． | 3．27E－13 | 7．59E－14 |  |  |  |  |
| $\mathrm{I}=$ | 2 |  |  |  |  |  |  |  |  |  |  |
| K： | 2－11 | 1．27E－02 | 1．11E－01 | 4．35F－01 | 5．66F－01 | $4.91 \mathrm{~F}-01$ | 2．10r－ก1 | 6． APF FO | ？${ }^{\text {n7r－n7 }}$ | 4．0．75－73 | 1．n¢E－n3 |
| K： | 12－21 | 2．33E－04 | $5.10 \mathrm{E}-05$ | 1．09E－05 | 2．36F－06 | 5．13E－07 | 1．00F－07 | 2．3GERAR | 5．13F－no | 1．ant．an | 3．7hr－17 |
| K： | 22－26 | 5．13E－11 | 1．09E－11 | 2．37E－12 | $5.13 \mathrm{E}-13$ | 1．27E－13 |  |  |  |  |  |
| $1=$ | 3 |  |  |  |  |  |  |  |  |  |  |
| K： | 3－12 | 4．53E－02 | 3．05E－01 | 5．63E－01 | 6．1nF－01 | 2．99E－01 | 1．055－ก1 | 3．29F－ng | 7．70r－n3 | 1.776 nc | 2．9ク「－＾4 |
| $K$ ： | 13－22 | 8．37E－05 | 1．79E－05 | 3．88F－06 | 8．43E－07 | $1.87 \mathrm{~F}-07$ | 3．RAF－OR | P．44F－ 0 | 1． ANE － OB | 2．aor－10 | r． $44=-11$ |
| K： | 23－26 | 1．80E－11 | 3．89E－12 | A． $44 \mathrm{E}-13$ | 2．17E－13 |  |  |  |  |  |  |
| $\underline{I}=$ | 4 |  |  |  |  |  |  |  |  |  |  |
| K： | 4－13 | $6.63 \mathrm{E}-02$ | 5．63F－01 | 7．13E－01 | 2．22F－01 | $6.21 \mathrm{E}-\mathrm{n} 2$ | 1．76E－0？ | $4.075-03$ | a．n4E－r4 | 1．000－r 4 | 4．745－ra |
| $k$ ： | 14－23 | 9．34E－06 | 2．02E－06 | 4．39F－07 | 9．37E－08 | $2.03 \mathrm{~F}-08$ | $4.40 \mathrm{~F}-79$ | 0．375－10 | 7．n3－1n | 4． 7 ff－11 |  |
| K： | 24－26 | $2.06 \mathrm{E}-12$ | 4．10E－13 | 1．LIE－13 |  |  |  |  |  |  |  |
| $\mathrm{I}=$ | 5 |  |  |  |  |  |  |  |  |  |  |
| K： | 5－14 | 5．53E－02 | 3．37E－01 | 2． $73 \mathrm{~F}-01$ | $6.915-02$ | 1．31F－52 | 2．105－03 | 1．39F－n4 | 9.33 Can | 1．09r－76 | 2．nのローベ |
| K： | 15－24 | 8．46E－08 | 1．84F－08 | 3．92E－09 | 8．48F－10 | 1．84E－10 | 3．92F－11 | D．40F－17 | 1．93F－1？ | 3．RET－13 | 8．705－1／ |
| $k$ ： | 25－26 | 1．65F－14 | 4．63E－15 |  |  |  |  |  |  |  |  |
| $1=$ | 6 |  |  |  |  |  |  |  |  |  |  |
| K： | 6－15 | 6．44F－02 | 7．10F－C2 | 1．17F－02 | 7．23E－03 | $4.665-04$ | 1．5nc－04 | 2．14r－n5 | ？．19rant | A．trront | 1．65r－rs |
| K： | 16－25 | 3．16E－08 | 6．73E－09 | 1．46E－09 | 3．16F－10 | $6.74 \mathrm{~F}-11$ | 1．46E－11 | 3．16F－1？ | h． 74 C－13 | 1．475－13 | 3．11r－14 |
|  | 26－26 | 8．20E－15 |  |  |  |  |  |  |  |  |  |
| I＝ | 7 |  |  |  |  |  |  |  |  |  |  |
| K： | 7－14 | 0.0 | 2．66\％－02 | 9．94F－03 | 2．64F－03 | 6．50E－04 | 1．255．034 | 1．57F－A5 | 3．0n5－0． 7 |  |  |

## FE560



## NA230


$-108-$

## NI 590





## PU410



| PU420 |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & I=1 \\ & K: \quad 1-10 \end{aligned}$ | 0.0 | $8.93 E-03$ | $6.25 F-02$ | 2.59F-01 | $4 \cdot 02 F-01$ | 4.23F-01 | 2.33t-01 | $0 \cdot n \rightarrow r-n ?$ | 7.895-97 | Q.narani |
| $1=2$ |  |  |  |  |  |  |  |  |  |  |
| K: 2-11 | 2.66E-04 | 1.17E-02 | 1.50F-01 | 4.16F-01 | 5. $\mathrm{ABF}=01$ | 3.40F-91 | 1.3nF- 31 | 4.345-n7 | 1.915-9 | 3. 3 Prerin |
| K: 12-21 | 5.14E-04 | 1.13E-04 | 2.42F-05 | 5.25E-06 | 1.14F-06 | 2.43E-n7 | 5.26F-nR | 1.14F-nR | 2.410-70 |  |
| K: 22-26 | $1.14 \mathrm{E}-10$ | 2.43E-11 | 5.26E-12 | 1.14F-12 | 3.06F-13 |  |  |  |  |  |
| $1=3$ |  |  |  |  |  |  |  |  |  |  |
| K: 3-12 | 2. 20E-03 | 6.54F-02 | 3.01E-01 | 5.96E-01 | 4.25E-01 | 1.82F-01 | 6. $2 \mathrm{nF}-\mathrm{n}$ ? | 1.575-ns | 3.5.ir-93 | 9. Mirant |
| K: 13-22 | 1.76E-04 | 3.7BF-05 | 8. $21 \mathrm{E}-06$ | 1.7AE-06 | $3.80 \mathrm{~F}-07$ | R. $23 \mathrm{~F}-\mathrm{OR}$ | 1.7AE-AR | 3.pnr-na | Q.2tF-10 | 1.7er-1n |
| K: 23-26 | 3.80E-11 | 8.23E-12 | 1.79E-12 | $4.70 \mathrm{E}-13$ |  |  |  |  |  |  |
| $1=4$ |  |  |  |  |  |  |  |  |  |  |
| K: 4-13 | 2.33E-02 | 2.065:01 | 6.38E-01 | $5.96 \mathrm{~F}-\mathrm{Cl}$ | 2.96F-n1 | 1.10F-01 | 2. ARE-n2 | G. AREAS | 1.51F-7x | 7.7aran/ |
| K: 14-23 | 7.15E-05 | 1.55E-05 | 3.38E-06 | 7.20E-07 | 1.56F-07 | 3.38E-08 | 7.2nE-79 | 1.56r-nn | 7.77F-10 | 7. 29r-11 |
| K: 24-26 | $1.56 \mathrm{E}-11$ | 3.29E-12 | 9.17E-13 |  |  |  |  |  |  |  |
| $\mathrm{I}=5$ |  |  |  |  |  |  |  |  |  |  |
| K: 5-14 | 3.06F-01 | 3.32F-01 | 2.29F-01 | 1.46F-01 | 4.87F-02 | 1.11F-0? | 2.58F-03 | 5.RAP-04 | 1.205-74 | ?.79r-n5 |
| K: 15-24 | 6.05E-06 | 1.31E-06 | 2. BOE-07 | 6.07E-08 | 1.32E-D8 | P. ALE-00 | 6.07E-10 | 1.7.7r-19 | 2.F1E-11 | 6. $10=-17$ |
| K: 25-26 | İ3IE-12 | 3.30E-13 |  |  |  |  |  |  |  |  |
| $I=6$ |  |  |  |  |  |  |  |  |  |  |
| K: 6-15 | $9.92 E-01$ | 3.19F-01 | 1.80F-02 | 8.89F-03 | 2.98F-03 | R. $145 \mathrm{~F}-74$ | 2.03F-04 | 4.4.1F- 5 5 | 7.5Ar-na | 3.ntranh |
| $K: 16-25$ | 4.47E-07 | 9.53F-08 | 2.06E-08 | 4.47E-09 | $9.53 \mathrm{E}-10$ | 2.06F-12 | 4.47E-11 | O. 53F-17 | 2.ntr-12 | 4.41F-17 |
| K: 26-26 | 1.16E-13 |  |  |  |  |  |  |  |  |  |
| I= 7 |  |  |  |  |  |  |  |  |  |  |
| K: 7-16 | 7.07E-01 | $3.405-01$ | 2.65E-02 | 1.05F-03 | 2.56E-04 | 5.76F-05 | $1.27 E-05$ | 2.735-n6 | 5.030-97 | 1.por-n7 |
| K: 17-26 | 2.75E-08 | 5.95E-09 | 1.29E-09 | 2.75E-10 | 5.95F-11 | 1.29F-11 | 7.75\%-17 | 5. $9910-13$ | 1.975-13 | 3. $35 \times-14$ |
| $1=8$ |  |  |  |  |  |  |  |  |  |  |
| K: 8-17 | 4.07E-01 | $2.96 \mathrm{~F}-01$ | 7.63F-03 | 2.05F-03 | 5. 22E-04 | 1.42r-04 | 3. $80 \mathrm{OF}-05$ | 8. $3 n \mathrm{Fa}-16$ | 1.9A5-nk | 3.paramer |
| K: 18-26 | 8.33E-08 | 1.81E-08 | 3.85E-09 | 8.34F-10 | 1.81F-10 | 3.95F-11 | タ.30F-1? | 1.7RF-12 | 4.hirs-13 |  |
| $1=9$ |  |  |  |  |  |  |  |  |  |  |
| K: 9-14 | 3.77F-02 | 7.13F-02 | $1.42 \mathrm{E}-02$ | 3.37F-03 | A.46F-04 | $1.15 \mathrm{~F} \cdot 74$ |  |  |  |  |
| $1=10$ |  |  |  |  |  |  |  |  |  |  |
| K: 10-17 | 0.0 | 0.0 | 0.0 | 0.0 | 1.69F-03 | i). $2 \mathrm{hF}-74$ | 1.14F-04 | 2.830-nts |  |  |

$\mathbf{U 2 3 5 0}$

| $\begin{aligned} & I= \\ & K: \end{aligned}$ | $\begin{aligned} & 1 \\ & 1-10 \end{aligned}$ | 0.0 | 9．92F－03 | 4．96E－02 | 2．4RE－01 | $4.26 \mathrm{E}-01$ | $5.55 \mathrm{~F}-01$ | 3．47F－n1 | 1．3or－cl | $3.075-73$ | Q．97－－n7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I $=$ | 2 |  |  |  |  |  |  |  |  |  |  |
| K： | 2－11 | 3．99E－04 | 1．64F－02 | 2．02E－01 | 5．43E－01 | 7．53F－01 | 4．30F－01 | 1．64F－01 | 5．32r－n？ | 1．2nE－n？ | 2．9nr－n1 |
| K： | 12－21 | 6．43E－04 | 1．41E－04 | 3．02E－05 | 6．54E－06 | 1．42F－06 | 3．04F－0i | 6．57F－nR | $1.43 \mathrm{~F}-78$ | $3.04 \mathrm{F-nO}$ | 6．575－1n |
| K： | 22－26 | 1．43E－10 | $3.04 \mathrm{E}-11$ | 6．57E－12 | 1．42E－12 | 3．6AF－13 |  |  |  |  |  |
| I＝ | 3 |  |  |  |  |  |  |  |  |  |  |
| $K:$ | 3－12 | 3．16E－03 | 8．84E－02 | 3．93E－01 | 7．61E－01 | 5．34t－01 | 2．27F－01 | 7． $80 \mathrm{~F}-117$ | 1．95F－r？ | 4．45F－17 | 9．02F－nk |
| K： | 13－22 | 2．18E－04 | $4.68 \mathrm{E}-05$ | 1．02E－05 | 2．21E－06 | $4.71 \mathrm{~F}-07$ | $1.02 \mathrm{~F}-97$ | 2．2．1F－na | $4.71 \%-ก 7$ | 1．0？F－70 | 2．215－1才 |
| K： | 23－26 | 4．71E－11 | 1．02E－11 | 2．21E－12 | 5．95E－13 |  |  |  |  |  |  |
| $\mathrm{I}=$ | 4 |  |  |  |  |  |  |  |  |  |  |
| K： | 4－13 | 8．82E－02 | 6．32E－01 | 6．55E－01 | 1．77E－01 | 5．77E－02 | 1．76F－0？ | $4.30 r-03$ | 9．47F－n4 | ？．ARF－n4 | 4．55F－nk |
| $K$ ： | 14－23 | 9．72E－06 | 2．11E－06 | 4．57E－07 | 9．75F－08 | $2.11 \mathrm{~F}-08$ | 4．58E－${ }^{\text {a }}$ | ？．75F－17 | 2．12F－10 | 4．538－11 | 9．35r－17 |
|  | 24－26 | 2．21E－12 | 3．67E－13 | 1．13E－13 |  |  |  |  |  |  |  |
| $1=$ | 5 |  |  |  |  |  |  |  |  |  |  |
| K： | 5－14 | 3．75E－01 | 7．69E－01 | 2．23E－01 | 2．72F－02 | 7．89E－03 | 1．8AR－ 03 | 4．17F－n4 | 9．19F－05 | 2．01く－75 | A．3nE－nt |
|  | 15－24 | $9.33 \mathrm{E}-07$ | 2．02E－07 | 4．32f．08 | 9．34E－09 | 2．03E－09 | $4.32 \mathrm{~F}-10$ | a．30F－11 | 2．กnc－11 | 4．${ }^{\text {BbE－12 }}$ | 1．9nF－12 |
| K： | 25－26 | $1.43 \mathrm{E}-13$ | 4．77E－14 |  |  |  |  |  |  |  |  |
| $\underline{I}=$ | 6 |  |  |  |  |  |  |  |  |  |  |
| K： | 6－15 | 7．33E－01 | 3．45E－01 | 3．45E－02 | 6．58E－03 | 1．45F－03 | 3．16t－04 | 7．01F－n5 | 1．545－95 | 3．3nEーフh | 7．14r－nt |
| $\mathrm{K}:$ | 16－25 | 1．55E－07 | 3．31E－08 | 7．17F－09 | 1．56E－09 | 3．31F－10 | 7．17E－11 | $1.56 \mathrm{E}-11$ | $3.315-12$ | 7．71E－13 | 1．53F－13 |
|  | 26－26 | $4.04 E-14$ |  |  |  |  |  |  |  |  |  |
| $1=$ | 7 |  |  |  |  |  |  |  |  |  |  |
|  | 7－16 | 5．26E－01 | 2．09E－01 | و．05E－03 | T．36F－04 | $4.25 \mathrm{E}-05$ | 9．1nf－06 | ？．01F－ns | 4．71r－n7 | 9．36F－CA | 9．03F－nn |
|  | 17－26 | 4．34E－09 | 9．38E－10 | 2．04E－10 | 4．34E－11 | 9．39F－ 22 | 2．n4E－12 | 4．34F－13 | 0．44F－14 | 7．＾1E－14 | 5．3AF－15 |
| $I=$ | 8 |  |  |  |  |  |  |  |  |  |  |
|  | 8－11 | 3．21E－01 | 1．20F－01 | 9．84E－03 | 6．01E－04 |  |  |  |  |  |  |
| I $=$ | 9 |  |  |  |  |  |  |  |  |  |  |
|  | 9－11 | 1．29E－01 | 2．32F－02 | 6．6．7E－05 |  |  |  |  |  |  |  |
|  | 10 |  |  |  |  |  |  |  |  |  |  |
|  | 10－11 | 1．47E－02 | $4.075-03$ |  |  |  |  |  |  |  |  |
|  | 11 |  |  |  |  |  |  |  |  |  |  |
| K： | 11－11 | 9．00E－06 |  |  |  |  |  |  |  |  |  |

U2380


## VIII. APPETIDIX II

Documentation of the assembly characteristics used in the calculations and of the integral data used for the comparison between theory and experiment

It seems necessary or at least very useful to have a fairly complete documentation of the data used as input data in this work. Such a documentation provides the necessary information on the basis of our work, helps in comparing our results with those of similar studies, eases a continuation of this work by ourselves or other people and serves as a sufficiently detailed basis to which to refer in further studies. Most of the information is provided as tables of a fixed forme. The abbreviations and symbols used in these tables are explained in table AII-1,

SUAK U1B
The data for the material and geometric composition have been taken from 159_7 Other data from the same reference are:
$k_{e f f}=0.86 \pm 0.01$
$\Delta \mathrm{k}_{\text {het }} \approx 0$ (heterogeneity correction negligible)
The fundamental mode buckling has been determined such as to give agreement for $k_{\text {eff }}$ with the one-dimensional calculations. The bucklings used in the one-dimensional calculations have been obtained in an analogous manner by an iteration procedure.

The assembly is symmetric in $X$ and Yadirection. The first data for the onedimensional SLAB-geometry refer to this X- respectively Y-direction. The second data for the one-dimensional SLAB-geometry refer to the z-direction (vertical direction). The radius of the equivalent sphere has been determined such as to give the same $k_{\text {eff }}$ as that obtained by the other one-dimensional calculations.
The REMO-correction and the $\mathrm{S}_{\mathrm{N}}$-correction have been determined during this study to be -0.0001 and +0.031 respectively. The $S_{N_{-}}$-correction has been obtained by adding up the individual $S_{N}$-corrections for the three directions of the cube, All correction calculations mentioned here have been done using the SNEAK-set.
No reflector has been assumed in the spherical calculations because in reality

4 of the 6 sides of the cube are unreflected. The calculations mentioned have been done in diffusion approximation with the exception of the $\mathrm{S}_{\mathrm{N}}$-calculations.

## SUAK UH1B

The data for the material and geometric composition have been taken from [59_7 Ohter data from the same reference are:

```
\(k_{e f f}=0.945 \pm 0.01\)
\(\Delta k_{\text {het }} \approx 0\) (heterogeneity correction negligible)
\(\Delta k_{\text {aniso }}=+0.007\) (correction for the anisotropic downscattering of hydrogen)
```

The fundanental mode bucking has been determined such as to give agreement for $t_{\text {eff }}$ with the one-dimensional calculations. The bucklings used in the one-dimensional calculations have been obtained in the analogous manner by an iteration procedure.
The assembly is symmetric in X - and $Y$-direction. The first data for the one-dimensional SLAB-geometry refer to this $X$ - respectively Y-direction. The second data for the one-dimensional SLAB-geometry refer to the Z -direction (vertical direction). The radius of the equivalent sphere has been determined such as to give the same $k_{\text {eff }}$ as that obtained by the other one-dimensional calculations.
The REMO-correction and the $S_{\mathbb{N}}$-correction have been determined during this study to be +0.0003 and +0.030 respectively. The $S_{N}$-correction has been obtained by adding up the individual $S_{N}$-corrections of the three directions of the cube. All calculations mentioned here have been done using the SNEAK-set. No reflector has been assumed in the spherical calculations because in reality 4 of the 6 sides of the cube are unreflected. The calculations mentioned have been done in diffusion approximation with the exception of the $S_{N}$-calculations.

## ZPR III-10

The data for the material composition have been taken from 575 .7. The data for the geometric configuration have been taken from [767. They are in rather good agreement with those given in [77.7. The $S_{N}$-correction of +0.013 has also been taken from [75.7. It is in very good agreement with that calculated by ourselves of +0.0127 . The heterogeneity correction of +0.0105 has been de-
termined from the corresponding data given in table II of $\mathrm{ITF}_{7} 7$. The REMO-correction has been calculated by ourselves to be -0.0008 . The fundamental mode buckling has been determined such as to give agreement for $k_{\text {eff }}$ with the one-dimensional calculations. The bucklings used in the one-dimensional calculations have been obteined in an analogous manner by an iteration procedure. The radius of the equivalent sphere has been determined such as to give the same $k_{e f f}$ as that of the other onemimensional calculations leading to a skape factor of 0.9297 . The thickness of the blanket of 37 cm for the spherical calculations has been chosen as a guessed average of the real axial and radial blanket thicknesses. Except for the $S_{N}$-calculations all calculations mentioned here have been done in diffusina approximation using the SNEAK-set.

## ZPR=III-25

The data for the material composition have been taken from 575 _7. Whe data for the geometric configuration have been taken from 1 T8_7. They are in rather good agreement with those given in [77_7. The $S_{N}$-correction of +0.002 has been taken also from 575 .7. It is in very good agreement with that calculated by ourselves of +0.0023 .
The heterogeneity correction of +0.0085 has been determined from the corresponding data given in table II of $[77$. 7 . The REMO-correction has been calculated by ourselves to be -0.0013 .
The fundamental mode buckling has been determined such as to give agreement for $k_{\text {eff }}$ with the one-dimensional calculations. The bucklings used in the onedimensional calculations have been obtained in an analogous manner by an iteration procedure. The radius of the equivalent sphere has been determined such as to give the same $k_{\text {eff }}$ as that of the other one-dimensional calculations leading to a shape factor of 0.92166 . The thickness of the blanket of 37 cm for the spherical calculations has been chosen as a guessed weighted average of the real axial and radial blanket thicknesses. Except for the $S_{Y}$-calculations all calculations mentioned here have been done in diffusion approximation using the SNEAK-set.

## ZPR III-48

The data for the material and geometric composition of the assembly have essentially been taken from ${ }^{74} 7$. Vol. I, pp. 95-96. Other information is available from [797 and [807. The mixture number 4 of table AII-6 is used for the blanket in the spherical calculation. Its composition has also been taken from 1-74_7 and, as explained there, represents a weighted average of the axial and radial blanket compositions.
The fundamental mode buckling has been determined such as to give agreement for $k_{\text {eff }}$ with the one-dimensional calculations. The bucklings used in the onedimensional calculations have been obtained in an analogous manner by an iteration procedure. The radius of the oquivalent sphere has been determined such as to give the same $k_{\text {eff }}$ as that of the other one-dimensional calculations leading to a shape factor of 0.933456 . Our value of 45.213 am compares favourably well with
 thickness of the spherical blanket to be equal to that of the radial blanket. In 10747 a $S_{N}$-correction of +0.0053 is reported. This represents the difference in $k_{\text {eff }}$ for spherical diffusion theory and $S_{4}$ transport theory calculations. In our own calculations we found for the same case a value of +0.0065
and a value of +0.0076 by adding up the $S_{N}$-corrections for the axial and radial directions, which shows a $15 \%$ increase compared to the correction for the spherical case. The finally applied value of +0.006 is based on the value 0.0053 but takes into account the $15 \%$ increase just mentioned before. The heterogeneity correction of +0.014 has been taken from [74.7. It is in between other published values for the same assembly of $\div 0.013$ by Eaison [ $81 \_$and +0.016 by Broomfield. The REMO-correction of +0.0046 has been calculated by ourselves. All our cai,culations referred to here have been done using the SNEAK-set. Except for the $S_{N}$-calculations the diffusion approximation has been applied. The experimental results for the central reaction rate ratios and for the material worths relative to that of U235 have been taken from [-74_7. Almost all of these values are also given in [79.7. It is known that there exists a general difficulty in calculating the absolute worth of materials correctly even for such important reactor materials as U235 or Pu239. Furthermore there is a discrepancy in the conversion factor from inhours to $\Delta k / k$ for ZPR III-48: 1002 inhours $=1 \% \Delta k / k$ is published in [79_7 and 942 inhours $=1 \% \Delta k / k$ is used in [74_7. If: one assumes that the reson for the present difficulties in calculating the correct absolute
moterial worth, as is presently supposed, is due to a normalization effect, for example in $\beta_{\text {eff: }}$ one may avoid these difficulaties by comparing the theoretical and experimental deta for the material worths relative to the material worth of U235. This has been done in the present work.

## ZPR III-48B

The assembly ZPR III-48B is very similar to ZPR III-48. The only difference, apart from a small change in core radius, is the central plutonium zone. The plutonium isotopic mixture of this zone contains a larger amount of higher plutonium isotopes, especially of Pu240. The composition of this inner zone has been taken from [83_7. The data for the real core geometry are those reported in [74.7. The core radii for the equivalent spherical madel have been derived by ourselves applying a shape fastor of 0.933456 which we derived for the similar assembly ZPR III-48. The equivalent spherical core radius of 45.7045 is in good agreement with the value of 45.67 published in 174 . The thickness of the spherical blanket has been chosen to be 30.0 cm , the same as that for ZPR III-48, in close agreement with the actual reactor axial and radial blanket thicknesses.

The $S_{N}$ - and heterogeneity-corrections have been taken the same as the corresponding values for ZPR III-48. The REMO-correction of +0.0047 has been calculated for the spherical model of the assembly by ourselves, As usually the SNEAK-set and diffusion approximation have been used for this calculation.
The central fission ratios are taken from / ${ }^{-83} 7 \mathrm{p} .65$, the data for the central material worths from [74_7p. 57. The same arguments as for ZPR III-48 are adopted in using the material worth relative to that of U235 for the comparison between theory and experiment.

A median fission energy of 223.1 keV has been obtained for ZPR III-48B in an analogous manner to that used for the determination of the values given in table 1 .

## ZEBRA=6A

The data for the material and geometric composition have been taken fram 1-84.7. The $S_{N}$-correction of +0.010 has been taken from [75_7. It is almost the same as that of +0.011 reported in $\mathbf{I V}^{-84}$. Our own value of +0.0117 is in reasonable agreement with the above values. The heterogeneity correction has to be taken as zero since the real heterogeneous assembly has been transformed into a homogeneous model for the calculations using an experimentally determined correction for the
heterogeneity. The REMO-correction of $+0,0040$ has been determined by ourselves.
The fundamental mode buckling has been detemined such as to give agreement for $k_{e f f}$ with the one-dimensional calculations. The bucklings used in the onediemensional calculations have been obtained in an analogous manner by an iteration process. The radius of the equivalent sphere has been determined such as to give the same $k_{\text {eff }}$ as that of the other onemaimensional calculations, leading to a shape factor of 0.9496 . The shape factor given in $1 / 84$ is 0.935. We have chosen for the thickness of the spherical blanket the same value as that for the real radial blanket thickness, since leakage in radial direction is more important than in axial direction.
The experimental values for the central reaction rate ratios have also been taken from 584_7.
All our calculations mentioned here hare been done using the SMEAK-set. Except for the $S_{N}$-calculations the diffusion approximation has been applied.

## SNEAK $3 A 1$

The assembly SNEAK $3 A 1$ has first been described in 1-66 7. A comprehensive description of the experiments in the SMEAK-3A assemblies with the exception of SNEAK $3 A 1$ has been compiled by SEHRÖDER $\mathbf{I}^{-67} 7$. The material compositions chosen for our work have been taken from this study $[67,7$, since it is based on a more refined (compared to $[667$ ) evaluation of the experiments. The $C-$ and H-coneentrations of the mixtures 1 and 2 have been taken from / 66_7. The blanket composition, mixture 3, has been the same for SNEAK $3 A 1$ and SNEAK 3A2. Mixture 4 has also essentially been taken from SNEAK $3 A 2$ except for the $C$ and H- concentration. The H-concentration has been determined as volume-average of the H-concentration of the two core zones containing mixture 1 and mixture 2 respectively. The C-concentration then has been adjusted assuming that for polyethylene the hydrogen content is twice that of cearbon ( $\mathrm{CH}_{2}$ ). This way of determining the C concentration seems more appropriate for the calculation of the so-called steam density coefficient and its comparison with the experiment. Mixture 4 has been used in the fundamental mode and one-dimensional axial calculations. With respect to the data on geometry the height of the axial zones has been taken from /666. The outer dimension of the assembly has also been taken from $1-67 / 7$ since these data seem to be more reliable and the outer dimensions of the assembly have not been changed during the SNEAK-3A-experiments [85]. The radii of the inner and
outer core zone have been taken from [66_7.
The $S_{N}$-correction of $+0,003$ has been taken from [66_7. It is in reasonable agreement with the value calculated by ourselves of +0.0038 . The heterogeneity correction of +0.003 has also been taken from [66_7. The REMO-correction of -0.0006 has been calculated by ourselves.
The fundamental mode buckling has been determined such as to give agreement for $k_{\text {eff }}$ with the one-dimensional calculations. The bucklings used in the one-dimensional calculations have been obtained in an analogous manner by an iteration procedure. The outer core radius of the equivalent sphere has been determined such as to give the same $k_{\text {eff }}$ as that of the other one-dimensional calculations leading to a shape factor of 0.9234 . Using this shape factor and the given volume of the inner core zone the radius of the inner core zone has been derived. This value is only of minor importance since both core zones have very similar compositions. We have chosen for the thick.ness of the spherical blanket the same value as that for the real radial blanket thickness since leakage in radial direction is more important than that in axial direction. Furthermore the thicknesses of the radial and axial blanket are nearly equal 29.66 cm and 30.5 cm respectively. The SNEAK-set and diffusion approximation have been used in all cases except those where other information is mentioned explicitly.

SNEAK 3A2

The data for the material and geometric composition have been taken from [67_7. One should mention that the composition of mixture 4 has been obtained as a volume-average of the compositions of mixture 1 and mixture 2 which are present in the inner and outer core zone respectively. Mixture 4 has been used in the fundamental mode and one-dimensional axial calculations. The $S_{N}$-correction of +0.004 has been published in $I^{-2} 7$, p. 29. It is in good agreement with the value of +0.0041 calculated by ourselves. The heterogeneity correction of +0.004 is given in ! ${ }^{2} 7$ p. 29 for a calculation using the so-called H2OPMB-set ${ }^{-2}$-7, which is a modified SNEAK-set very similar to the SNEPMB-set used in our present study. The same value is reported in [57_7, p. 14. Using the SNEAK-set a slightly different value of +0.003 has been obtained in $5^{2} \mathbf{2}$. The small difference of 0.001 in $k$, however, is not imporiant with respect to the final conclusion to be drawn from the comparison between theory and experiment because the differences observed are most times in the order of $1 \%$. The REMOcorrection of $\mathbf{0 . 0 0 0 6}$ has been determined by ourselves. It is rather small but
has a different sign compared to the values obtained in 127 for the SNEAKand E2OPMB-sets. A possible explanation for this difference is the fact that in the present study the REMO correction is applied to all regions of the assembly incluaing the blanket whereas in the previous study $\Gamma_{2} 7$ the REMOcorrection has been applied to the core region only.

The fundamental mode buckiing has been determined such as to give agreement for $k_{\text {eff }}$ with the one-dimensional calculations. The bucklings used in the one-dimensional calculations have been obtained in an analogous manner by an iteration procedure. The outer core radius of the equivalent sphere has been determined such as to give the same keff as that of the other onemdimensional calculations leading to a shape factor of 0.93613 . Using this shape factor and the given volume of the inner core zone the radius of the inner core zone has been derived. This value is only of minor importance since both core zones have very similar compositions. We have chosen for the thickness of the spherical blanket the same value as that for the real radial blanket thickness since leakage in radial direction is more important than that in axial direction. The SNEAK-set and diffusion approximation have been used in all cases except those where other information is mentioned explicitly.

## SMEAK 3B2

Experiments and various calculations for the Uranium-Plutonium fuelled assembly SNEAK 3B2 have been described in [68_7. Our calculations are based on the same data for the material and geometric composition as reported in that paper. For the calculation of the axial buckling we have used an axial saving of 13.81 cm which has been obtained at an early stage of the present calculations for SNEAK 3 AZ. The final correct value of 13.29 cm is not too different from the preliminary value used here. The adoption of the SNEAK $3 A 2$ saving for the aseembly SNEAK 3B2 seems to be reasonable because the core-geometry and composition are very similar. The $S_{N}$-correction of +0.004 is assumed to be the same as that for the similar SNEAK 3 A2 essembly. The heterogeneity-correction of +0.003 is taken from F68 7 . The REMO-correction, calculated in the usual menners is found to be +0.0008 . In all calculations mentioned here the SNEAK-set and diffusion approximation have been used.

A median fission energy of 81.81 keV has been obtained for SNEAK $3 B 2$ in an analogous manner to that used for the determination of the values given in table 1.

## SKEAK-5C

In SNEAK assembly 5 a series of experiments was carried out to determine $k_{\infty}$ and reaction rate ratios. The most important of these experiments is that labelled SNEAK-5C. The composition of its inner most zone was chosen to render a $k_{\infty}$ close to unity. The composition given in table AII-12 has been provided by MEISTER $/{ }^{-86}$ 7. It should be considered as preliminary but it is to be expected that the final composition which will be published in a comprehensive report on these experiments $/ \mathbf{6 2} 7$ will not differ very much from that given in table In this report adaitional detailed information especially on the driver zone will be published.
The heterogeneity-correction of +0.071 has been determined by FISCHER 187.7 as preliminary result. Meanwhile a samewhat smaller value of +0.068 has been calculated by ourselves. But it seems to us that even this value is probably too large. The REMO-correction of +0.016 has been calculated by ourselves using diffusion approximation. All calculaitions mentioneă here have been done using the group constants of the SNEAK-set.

## ZPR III-55

The main incentive for the ZPRIII-55 experiment was to measure the $\alpha$-value of Pu239 by mears of a $k_{\infty}$ experiment ( $\alpha=\sigma_{c} / \sigma_{f}$ ). The composition of the $k_{\infty}$-zone has been taken from [ 88 7. More recent information is published in 159_7. The beterogeneity-correction of +0.010 has been reported by JOURDAN [907. The REMO Correction of +0.011 has been calculated by ourselves using diffusion approximation. All calculations mentioned here have been done using the SNEAKset.

## SNEAK-series

In our work we have included the somcalled SNEAK-series which refers to a series of SNEAK assemblies namely, $3 A O, 3 A 1,3 A 2,3 A 3$ carried out to get information on the steammensity and steam-void coefficient. The assemblies SNEAK 3A1 and SNEAK 3AR have been described above. SNEAK $3 A O$ refers to a core composition
where the polyethylene foils have been removed completely so that the hydrogen concentration is zero; the carbon concentration is reduced by the corresponding amount. In SNEAK 3 A3 the hydrogen concentration of the core has been doubled by doubling the thickness of the polyethylene foils; the carbon concentration is increased by the corresponding amount. The experiments have been evaluated in ! 57.7 by assuming that the core compositions of the different hydrogen contents (and the correspondingly different carbon contents) are placed into the core geometry of the assembly SNEAK 3A2. All experimental and theoretical results therefore are normalized relative to SNEAK 3A2. Especially the calculated criticaility is adjusted so that by ading or subtracting a small amount to the $k_{\text {eff }}$ of all 4 assemblies the adjusted value for $k_{\text {eff }}$ of SNEAK $3 A 2$ is equal to unity.
The core geametry has been simplified for our calculations since it would take too mach computer time to take into account the geometry assumed for the evaluation of the experiments and since we expected that fundamental mode calculations would be sufficiently accurate for a comparison of the theoretical and experimental results for the steam-density and steam voir. coefficient. The bucklings used in the fundamental mode calculations are given in table AII-14. They have been obtained in the following manner. For SNEAK 3A2 the reflector savings of the axial and radial blankets have been letermined from the bucklings given in tables AII-9 and AII-10 and the real core geometry also given in these tables.

For the determination of the savings for SNEAK-3AO by one-dimensional calculations we placed the composition used in the fundamental mode calculations for this assembly into the core geometry of SNEAK 3 A 2 , becalnse $3 A O$ can be considered as the voided version of 3 A 2 (see [57.7). The savings for SNEAK $3 A 3$ have been obtained by placing the composition used in the fundamental mode calculations into the whole core of the SNEAK $3 A 3$ experiment. In the real $3 A 3$ configuration this composition of high hydrogen concentration was contained only in an inner zone of 18.7 cm whereas the core radius was 42.2 cm as given in [67.7. The outer core zones in the real SNEAK 3A3 experiment were the same as in SNEAK $3 A 2$. The core geometry chosen for the determination of the savings is not a crucial point because according to our experience the savings depend more strongly on the core composition (and eventually on the blarket composition if it is changed) than on the core geometry. Furthermore the core geometry does not change drastically for the experiments $3 A O, 3 A 1,3 A 2,3 A 3$. The core height is the same for all 4 experiments and the radius varies between a smallest value of 42.2 cm
and a largest value of 51.2 cm .
The agreement between our values of $k_{\text {eff }}$ for the 4 assemblies determined in the indicated manner and the corresponding values given in $\leq 57$ _ 7 can be considered as a check of the sdequacy of our approximate procedure.
The compositions used in the fundamental mode calculations are derived from mixture 4 given in table AII-10 for SNEAK 3A2. The concentrations of all isotopes or elements are taken the same in all 4 compositions of the furdamental mode calculations with the exception of carbon and hydrogen. The carbon- and hydrogen concentrations used are given in table AII-14.
The $S_{N}$-correction for SNEAK $3 A O$ of $+0,003$ has been taken the same as that for SNEAK 3A1. The $S_{N}$-correction of +0.004 for SNEAK $3 A 3$ has been taken the same as that for SNEAK 3A2. The heterogeneity corrections of $+0.001,+0.003,+0.004$, +0.007 for the assemblies $3 A O, 3 A 1,3 A 2,3 A 3$ respectively have been taken from [57.7. p. 14. The REMO-corrections of $-0.0015,-0.0006,-0.0006,-0.0009$ for the 4 assemblies have been calculated by ourselves. For the assemblies SNEAK $3 A O$, SNEAK $3 A 1$, and SNEAK $3 A 2$ the spherical appropriate models have been applied in the one-dimensional calculations. For SNEAK $3 A O$ this reans a radius of the inner core zone of 41 cm , a radius of the outer core zone of 48.56 cm and a blanket thiakness $8 \pm 36.2 \mathrm{~cm}$.
For SNEAK $3 A 3$ the REMO-correction has been determined for the one-dimensional radial geometry. The composition used in the whole core is that taken in the correspanding fundamental mode calculation.

Table AII-1

Exolanation of the abbreviations and symbols used in tables AII-2 throush AII-13

IIXIURE: The mixtures are numbered consecutively in order to refer on the numbers furtheron

INATERIAE: The material means either elemeni (in natural composition) or isotope. One should mention that only for U235, U238 and Pu239 the cross sections are taten at $300^{\circ} \mathrm{K}$. For all other materials the cross sections are taken at $900^{\circ} \mathrm{K}$ or assumed to be independent of tie temperature.

Fundemental mode calculations

NG: Number of enerey groups
MIXP: Number of mixtures
BUCKo: Buckling used for the fundamental mode calculations

## One-dimensional calculations

GEO: Kind of geometry: slab, cylinder or sphere

NG:
BUCKIING

BC:

NZ:
NP :

Number of energy groups
Buckling used to take into account the leakage into the separated directions
Boundary condition; the number refers to the left hand side, the second number to the right hand side. 2 a net current $=0$, symmetric or reflective boundary condition $3 \pm$ usual diffusion boundary condition
$\phi_{B}^{ \pm} 0.71 \cdot \lambda_{t r} \cdot \phi_{B}^{\prime}=0$
where $\phi_{B}$ and $\phi_{B}^{\prime}$ are the flux and its gradient at the boundary and the plus sign refers to the right hand, the minus sign to the left hand boundary, if the axis is directed to the right hand side.
Number of zones
Number of mesh points

ZONE IZ: The zones are numbered consecutively from 1-NZ to refer on zone number IZ in the twodimensional case

MIXI: Numher of the MIXIURE mentioned above which is present in zone IZ
XL: Left hand abscissa (radius) of zone Iz
IMT Mmber of mesi intervels used in this zone
XR: Right hand asscissa (radius) of zone IZ

Two-Cimensional calculations

| GEO: | Geometry | 2 = RZ-Geometry |
| :---: | :---: | :---: |

NG: Nimber of energy Eroups
NP: Total mumber of mesh points
ROWS: Jlumber of rows
COI: Number of colums
NZ: Number of zones
BCL:
BCR :
BCUP:
BCLOW:
SPEKIRTM FOR COND :

Boundary condition at left hand boundary $2 \hat{=}$ net current $=0$, Boundary condition at right hand boundary symmetric or reflective condition Boundary condition at upper boundary $\quad 3 \hat{=}$ usual diffusion Boundary condition at lower boundary boundary condition Origin of the spectrum used for the condensation to few group constants (taken from one of the one-dimensional calculations)
NEN GROUP: The new groups are numbered consecutively
IGUP OLD GROUPS:Upper group index of those groups which belong to the new group considered
MIXT: Number of the new mixture used in the two-dimensional calculations. it may be different from the original number of tise mixtures in the line MIXIURE because two different condensation spectra are used for the same mixture of the line MIXIUPE. Since different few group cross sections arise from this procedure, the new mixtures characterized by MIXT must be distinguished from each other.

MIXIURE: Number of the original mixture from the line MIXTURE

PEI-1 dim.(IZ): The spectrum of the zone number IZ of the one-dimensional calculation indicated in SPEKIRUM FOR COND is used Ior the condensation of the few group constants of the special mixture (MIXI) considered

ZONE:
YIXT:
RL:
INTH:

RR:
HUP:
INTV:

HLOM:

The two-dimensional zones are mombered consecutively HIXHURE used in special zone consikered Left radius (or abscissa) of the zone considered Number of mesh intervals in the horizontal direction of the zone considered

Right radius (or abscissa) of the zone considered Upper height (or radius) of the zone considered Number of mesh intervals in the vertical direction of the zone considered

Lower height (or radius) of the zone considered

Table AII-2

CRITICAL FACILITY: SUAK U1B

Atom-Densities (in $10^{20}$ atoms $/ \mathrm{cm}^{3}$ )

| Mixture <br> Material | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: |
| Al | 42.77 | 438.0 | - | 600.0 |
| C | - | - | - | - |
| Te | - | - | 844.0 | - |
| H | - | - | - | - |
| Hi | 6.60 | - | - | - |
| U235 | 81.50 | - | - | - |
| U238 | 328.97 | - | - | - |

Fundamental Mode Calculation:

| MG | MIXT | Bucko |
| :--- | :--- | :--- |
| 26 | 1 | $197.898 \cdot 10^{-4} \mathrm{~cm}^{-2}$ |

One-Dimensional Calculations:

| GEO | NG | Buckling | BC | NZ | NP |
| :--- | :--- | :--- | :--- | :--- | :--- |
| SLAB | 26 | $128,097 \cdot 10^{-4} \mathrm{~cm}^{-2}$ | 23 | 1 | 101 |


| Zone IZ | MIXI | $\mathrm{XL} 1 \mathrm{Cm}_{-} 7$ | INT | $\mathrm{XR} 1 \mathrm{~cm}_{-} 7$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 100 | 16.15 |


| $G E O$ | $N G$ | JCKLING | BC | NZ | NP |
| :--- | :---: | :---: | :---: | :---: | :---: |
| SLAB | 26 | $139.601 \cdot 10^{-4} \mathrm{~cm}^{-2}$ | 33 | 4 | 116 |


| Zone IZ | MIXT | XI / $\mathrm{cm}_{2} 7$ | INT | XR / cm 7 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 4 | 0.0 | 15 | 3.0 |
| 2 | 3 | 3.0 | 10 | 6.5 |
| 3 | 1 | 6.5 | 80 | 41.6 |
| 4 | 2 | 41.6 | 10 | 44.6 |


| GEO | NG | BUCKLING | BC | NZ | IP |
| :--- | :--- | :---: | :---: | :---: | :---: |
| SPHERE | 26 | - | 23 | 1 | 101 |


| zone IZ | MIXI | $\mathrm{xL} 1-\mathrm{cm} 7$ | INI | XR - $\mathrm{cm}_{-} 7$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0.0 | 100 | 19.363968 |

Table AII-3

GRIRICAT FACIITMY: SUAF UH1B

Ator-Densities (in $10^{20}$ atons $/ \mathrm{cm}^{3}$ )

| rixture <br> ilaterial | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: |
| Al | 42.77 | 438.0 | - | 600.0 |
| C | 74.22 | - | - | - |
| Fe | - | 54.0 | 844.0 | - |
| H | 148.44 | - | - | - |
| 1 II | 5.20 | - | - | - |
| U 235 | 65.02 | - | - | - |
| U 233 | 262.37 | - | - | - |

Fundamental Yode Calculation:

| NG | NIXI | Bucko |
| :--- | :--- | :--- |
| 26 | 1 | $205 \cdot 333 \cdot 10^{-4} \mathrm{~cm}^{-2}$ |

One-Diemensional Calculations:

| $G E O$ | $N G$ | BUCKLING | BC | NZ | NP |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $G L A B$ | 26 | $134.681 \cdot 10^{-4} \mathrm{~cm}^{-2}$ | 23 | 1 | 101 |


| ZONE IZ | MIXT | $\mathrm{XL}\left[\mathrm{cm}_{-} 7\right.$ | INT | $\mathrm{XR}\left[\mathrm{cm}_{-} 7\right.$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 100 | 16.15 |


| GEO | NG | BUCKLITG | BG | HZ | ITP |
| :--- | :--- | :--- | :--- | :--- | :--- |
| SLAB | 26 | $141.314 \cdot 10^{-4} \mathrm{~cm}^{-2}$ | 33 | 4 | 16 |


| zoure IZ | SIXXI | $\mathrm{xL} 1^{-} \mathrm{Cm} 7$ | INT | $\mathrm{xR} \mathrm{I}^{-\mathrm{Cm}} 7$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 4 | 0.0 | 15 | 3.0 |
| 2 | 3 | 3.0 | 10 | 6.5 |
| 3 | 1 | 6.5 | 80 | 39.55 |
| 4 | 2 | 39.55 | 10 | 42.55 |


| GEO | NG | BUCKLIHG | BC | NZ | NP |
| :--- | :--- | :---: | :---: | :---: | :---: |
| SPHERE | 26 | - | 23 | 1 | 101 |


| ZONE IZ | MIXT | $\mathrm{xL} 1 \mathrm{Cm}_{-7} 7$ | INT | $\mathrm{XR} \quad 1 \mathrm{lcm}_{-7}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0.0 | 100 | 19.087051 |

Tcbie AII-4

CPTMICAI FACILTTV: ZPETII-10

At.on-Densities (in $10^{20}$ atoms $/ \mathrm{cm}^{3}$ )

| Iixture <br> Saterial | 1 | 2 |
| :---: | :---: | :---: |
|  |  |  |
| Cr | 29.6 | 11.2 |
| Fe | 171.1 | 44.0 |
| ITi | 17.8 | 6.7 |
| U 235 | 56.83 | 0.01 |
| $\mathrm{~J} 23^{?}$ | 277.9 | 390.3 |

Fundamental Yode Calculation:

| IG | YTXY | BUCKO |
| :--- | :---: | :--- |
| 26 | 1 | $81.373=10^{-4} \mathrm{~cm}^{2}$ |

One-Dimensional Celculetions:

| GEO | NG | SUCKLING | BC | NZ | NP |
| :--- | :--- | :--- | :---: | :---: | :---: |
| SLAB | 26 | 57.534 | 23 | 3 | 113 |


| ZONE IZ | MIXI | XI ICM_7 | INT | XR ICM 7 |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 0. | 20 | 10.0 |
| 2 | 1 | 10.0 | 26 | 22.95 |
| 3 | 2 | 22.95 | 66 | 55.95 |


| GEO | NG | BUCKLING | EC | NZ | NF |
| :--- | :--- | :--- | :--- | :--- | :--- |
| CYL | 26 | $23.839 \cdot 10^{-4} \mathrm{~cm}^{-2}$ | 23 | 3 | 129 |


| ZOUE IZ | : IXT | XL $1 \mathrm{Cm}_{2} 7$ | ITrI | $\mathrm{yR}\left[\mathrm{cm}_{-} 7\right.$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 20 | 10.0 |
| 2 | 1 | 10.0 | 24 | 22.11 |
| 3 | 2 | 22.11 | 84 | 63.59 |


| GEO | IG | BUCKLING | BC | NZ | NP |
| :--- | :--- | :---: | :---: | :---: | :---: |
| SPHERE | 26 | - | 23 | 3 | 125 |


| ZONE IZ | ITIXT | XL 1 | INT | XR I $\mathrm{cm}_{-}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 20 | 10.0 |
| 2 | 1 | 10.0 | 30 | 25.011 |
| 3 | 2 | 25.011 | 74 | 02.011 |

Tro-Dimensional Calculations:

| GMO | NG | IJP | ROWG | COL | NZ | BCL | BCR | BCUP | BCLOW | SFECTRUM FOR COND |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 11 | 1152 | 32 | 36 | 5 | 2 | 3 | 2 | 3 | 1 - $\lim$. SPIIERE |


| NEW GIOUP | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IGUP OLD GROUPA | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 15 | 20 | 26 |


| MIXI | $I$ | $I I$ | $I I I$ |
| :--- | :---: | :---: | :---: |
| MIXTURF | 1 | 1 | 2 |
| PYI-1 dim $(I Z)$ | 1 | 2 | 3 |


| zowe | :10xe | RL [-Cm 7 | ITT | Rr [ Cor_7 | Ifle [ $\mathrm{cm}_{-7}$ | Tive | Store 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | I | 0. | 6 | 10.0 | 10.0 | 6 | D. |
| 2 | II | 10.6 | 8 | 22.11 | 22.95 | 14 | 0. |
| 3 | II | 0. | 6 | 10.0 | 22.95 | 9 | 10.0 |
| 4 | III | 22.11 | 21 | 63.59 | 55.95 | 31 | 0. |
| 5 | ITI | 0. | 14 | 22.11 | 55.95 | 17 | 22.95 |

Table AII-5

CRITIMCAI FACITITY: ZPRIII-25

$$
\text { Atom-Densities (in } 10^{20} \text { atoms } / \mathrm{cm}^{3} \text { ) }
$$

| Mixture <br> Inaterial | 1 | 2 |
| :--- | :---: | :---: |
| Cr | 14.0 | 11.2 |
| Fe | 55.5 | 44.0 |
| JFi | 8.4 | 6.7 |
| U 235 | 34.42 | 0.91 |
| U 238 | 356.0 | 309.8 |

## Fundamental Iode Calculation:

| ISG | MIXT | BUCKO |
| :--- | :---: | :---: |
| 26 | 1 | $31 \cdot 725 \cdot 10^{-4} \mathrm{~cm}^{-2}$ |

One-Dimensional Calculations:

| GFO | NG | BUCKIING | BC | NZ | NP |
| :--- | :--- | :--- | :---: | :---: | :---: |
| SIAB | 26 | $20.981 \cdot 10^{-4} \mathrm{~cm}^{-2}$ | 23 | 3 | 137 |


| ZONE IZ | MIXT | $\mathrm{XL} / \mathrm{cm}$ | INT | XR [ $\mathrm{Cm} /$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 20 | 10.0 |
| 2 | 1 | 10.0 | 56 | 33.175 |
| 3 | 2 | 38.175 | 60 | 68.675 |


| GEO | NG | BUCKIING | BC | NZ | NP |
| :--- | :---: | :--- | :---: | :---: | :---: |
| CYI | 26 | $10.744 \times 10^{-4} \mathrm{~cm}^{-2}$ | 23 | 3 | 145 |


| ZONE IZ | MIXI | XI $/ \mathrm{cm}$ | INT | XR $\mathrm{Icm}^{7}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 20 | 10.0 |
| 2 | 1 | 10.0 | 64 | 42.60 |
| 3 | 2 | 42.60 | 60 | 83.40 |


| GEO | HG | BUCKLING | BC | IVZ | TP |
| :--- | :--- | :---: | :---: | :---: | :---: |
| SPHERE | 26 | - | 23 | 3 | 149 |


| 70IEE IZ | !!IXI | xI 1-m 7 | InTr | xR 1-cm 7 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 20 | 10.0 |
| 2 | 1 | 10.0 | 70 | 45.753 |
| 3 | 2 | 45.753 | 58 | 82.753 |

Two-Dimensional Calculations:

| CEO | NTE | in | ROw: | COI, | WZ | $\mathrm{BCL}_{2}$ | BCR | BCUS | 3CLO: | SPELTEUS FOR CORD |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 11 | 1584 | 36 | 44 | 5 | 2 | 3 | 3 | 2 | 1 dim. SPHERE |


| NE GBOUN | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| TMJN OLD GROUDC | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 11 | 16 | 26 |


| IIXI | $I$ | $I T$ | $I T I$ |
| :--- | :---: | :---: | :---: |
| IIXTURE | 1 | 1 | 2 |
| PUI-1 dim (IZ) | 1 | 2 | 3 |


| 20NE | MILXT | RL [ $\mathrm{cm}_{2} 7$ | INT | RR - $\mathrm{cm}_{-} 7$ | HUP [ $\mathrm{Cm}_{7} 7$ | INT | $\mathrm{HLO}_{1} \mathrm{~T}^{-} \mathrm{Cm}_{-} 7$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | I | 0. | 5 | 10.0 | 10.0 | 5 | 0. |
| 2 | IT | 10.0 | 17 | 42.6 | 38.175 | 20 | 0. |
| 3 | II | 0. | 5 | 10.0 | 38.175 | 15 | 10.0 |
| 4 | III | 42.6 | 21 | 83.4 | 68.675 | 35 | 0. |
| 5 | III | 0. | 22 | 42.6 | 68.675 | 15 | 38.175 |

Table AII-́́

CRITICAI TACILITY: ZPRIII-48

Atom-Densities (in $10^{20}$ atores/ $\mathrm{cm}^{3}$ )

| Mixture Slaterial | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| AI | 1.00 | - | - | - |
| C | 207.67 | - | - | - |
| Cr | 26.81 | 14.81 | 11.93 | 12.8 |
| Fe | 99.85 | 55.15 | 44.44 | 47.7 |
| 110 | 2.06 | - | - | - |
| Na | 62.31 | - | - | - |
| IVi | 13.30 | 7.29 | 5.87 | 6.3 |
| Pu239 | 16.45 | - | - | - |
| Pu240 | 1.06 | - | - | - |
| Pu241 | 0.11 | - | - | - |
| Pu242 | 0.004 | - | - | - |
| U235 | 0.16 | 0.82 | 0.83 | 0.83 |
| U238 | 74.27 | 383.77 | 397.98 | 393.0 |

Fundamental Mode Calculation:

| IVG | MIXT | BUCKo |
| :--- | :--- | :--- |
| 26 | 1 | $27.015 \quad 10^{-4} \mathrm{~cm}^{-2}$ |

One-Dimensional Ca?culations:

| GEO | NG | BUCKLING | BC | NZ | NP |
| :--- | :--- | :--- | :--- | :---: | :---: |
| SLAB | 26 | $18.132 \quad 10^{-4} \mathrm{~cm}^{-2}$ | 23 | 2 | 91 |


| ZOitE IZ | MIXT | $X L \leq \mathrm{cm}_{-} 7$ | INT | $\mathrm{XR}]^{-\mathrm{cm}} 7$ |
| :---: | :---: | :---: | :---: | :---: |
| . 1 | 1 | 0. | 60 | 38.18 |
| 2 | 2 | 38.18 | 30 | 68.66 |


| GFO | NG | BUCKLTNG | BC | NZ | NP |
| :--- | :--- | :--- | :--- | :--- | :--- |
| CYT | 26 | $8.88310^{-4} \operatorname{cm}^{-2}$ | 23 | 2 | 91 |


| ZONE IZ | SIIXI | xL 1-cm 7 | InT | XR I $\mathrm{Cm}_{-7}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 60 | 41.58 |
| 2 | 3 | 41.58 | 30 | 71.53 |


| GEO | NG | BUCKLIIG | BC | NZ | ITP |
| :--- | :---: | :---: | :---: | :---: | :---: |
| SFHERIS | 26 | - | 23 | 3 | 91 |


| zorre IZ | UIXT | XL 1-cm 7 | INT | xS [-cm 7 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 14 | 10.0 |
| 2 | 1 | 10.0 | 46 | 45.213 |
| 3 | 4 | 45.213 | 30 | 75.213 |

TWODTMETSIONAL CALCULATTONS:

| CRO | ING | NP | ROIS | COL | NZ | BCL | BCR | BCUP | ECLO: | SPEKRRU: FOR COID |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 11 | 1296 | 36 | 36 | 5 | 2 | 3 | 3 | 2 | 1 -dim. SFHERE |


| IEN GROUP | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ICUP OLD GROUSS | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 16 | 26 |


| MIXI | I | II | III | IV |
| :--- | :--- | :---: | :---: | :---: |
| MIXTURE | 1 | 1 | 2 | 3 |
| PHI-1dim (IZ) | 1 | 2 | 3 | 3 |


| ZOIE | "ITM | $\mathrm{RL}[\mathrm{Com} 7$ | ITT | RR ! Cm_7 | HUP $\mathrm{l}_{\text {cmi }} 7$ | In | ELOT [ $\mathrm{cra}^{7}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | I | 0. | 4 | 10.0 | 10.0 | 5 | 0. |
| 2 | II | 10.0 | 16 | 41.58 | 38.18 | 20 | O. |
| 3 | II | 0. | 4 | 10.0 | 38.18 | 15 | 10.0 |
| 4 | III | 0. | 20 | 41.58 | 68.66 | 15 | 38.18 |
| 5 | IV | 41.58 | 15 | 71.53 | 68.65 | 35 | 0. |

Table AII-T

CRITITCAL FACIITRY: ZPRIII-43B
Atcan Densities (in $10^{20}$ atoms $/ \mathrm{cm}^{3}$ )

| IVixture | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Yaterial |  |  |  |  |  |
| Al | 1.13 | 1.09 | - | - | - |
| C | 207.67 | 207.57 | - | - | - |
| Cr | 51.92 | 26.81 | 14.81 | 11.93 | 12.3 |
| Fe | 102.23 | 99.85 | 55.15 | 44.44 | 47.7 |
| Mo | 2.06 | 2.06 | - | - | - |
| He | 62.37 | 62.31 | - | - | - |
| Ii | 11.13 | 13.30 | 7.29 | 5.87 | 6.3 |
| Pu239 | 14.36 | 16.45 | - | - | - |
| Pu240 | 3.11 | 1.06 | - | - | - |
| Pu241 | 0.59 | 0.11 | - | - | - |
| Pu242 | 0.07 | 0.004 | - | - | - |
| Si | 1.25 | - | - | - | - |
| U235 | 0.16 | 0.16 | 0.82 | 0.83 | 0.83 |
| U238 | 74.05 | 74.27 | 383.77 | 397.98 | 393.0 |


| GEO | NG | BUCKLING | BC | INZ | NP |
| :--- | :---: | :---: | :---: | :---: | :---: |
| SPYERE | 26 | - | 23 | 3 | 86 |


| ZONE IZ | MIXT | $\mathrm{XL}_{1} \mathrm{~T}^{-} \mathrm{Cm}_{2} 7$ | IMT | XP. $\mathrm{I}^{-\mathrm{cm}} 7$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 30 | 19.758 |
| 2 | 2 | 19.75\% | 25 | 45.7045 |
| 3 | 5 | 45.7045 | 30 | 75.7045 |

Tro-Dirensional Calculations:

| ern | me | \% | nous | COL | NZ | Rer | P.CR | 3CUP | BCLSV | STEXTPIT: FD: COMD |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 11 | 1296 | 36 | 35 | 5 | 2 | 3 | 3 | 2 | 1-dim. Spmpri: |


| IRP CROUP | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 5 | 9 | 10 | 11 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| ICUR NLD CROUS | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 16 | 26 |


| IIXP | I | II | III | IV |
| :--- | :---: | :---: | :---: | :---: |
| IIXIURE | 1 | 2 | 3 | 4 |
| IUI-1 din (IE | 1 | 2 | 3 | 3 |


| ZONE | MIXT | RL $1 \mathrm{Cm}_{-} 7$ | INTH | RR I ICm 7 | Hup [ $\mathrm{cm}_{-7} 7$ | Iniv | HLON $\mathrm{ICm}^{-} 7$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | I | 0. | 3 | 19.0 | 15.26 | 7 | 0. |
| 2 | II | 19.0 | 12 | 42.26 | 38.18 | 20 | 0. |
| 3 | II | 0. | 8 | 19.0 | 33.9 | 13 | 15.26 |
| 4 | III | 0. | 20 | 42.26 | 63.66 | 15 | 38.18 |
| 5 | IV | 42.26 | 15 | 71.58 | 68.56 | 35 | 0. |

Teble AII-8

CRITICAL FACILITY: ZEBRA-6A

Atom Densities (in $10^{20}$ atoms $/ \mathrm{cm}^{3}$ )

| Mixture Material | 1 | 2 |
| :---: | :---: | :---: |
| A1 | 25.04 | - |
| C | 295.90 | 234.10 |
| Cr | 12.70 | 0.13 |
| Cl 2 | 8.20 | - |
| Fe | 42.55 | 32.53 |
| Na | 44.74 | - |
| 㴤 | 4.43 | 3.18 |
| Pu239 | 19.79 | - |
| Pu240 | 1.44 | - |
| Pu241 | 0.16 | - |
| U235 | 0.46 | 1.94 |
| U235 | 63.53 | 268.10 |

Fundamental Mode Calculation:

| NG | MIXT | BUCK |
| :--- | :---: | :--- |
| 26 | 1 | $34 \cdot 596 \cdot 10^{-4} \mathrm{~cm}{ }^{-2}$ |

One-Dimensional Calculations:

| GEO | IIG | BUCKLING | BC | NZ | NP |
| :--- | :--- | :--- | :---: | :---: | :---: |
| SLAB | 26 | $22 \cdot 178 \cdot 10^{-4} \mathrm{~cm}^{-2}$ | 23 | 2 | 101 |


| ZONE IZ | M1XT | $\mathrm{XI} \underline{1}^{-1} 7$ | INT | XR [ $\mathrm{Cam}_{-7}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 60 | 30.08 |
| 2 | 2 | 30.08 | 40 | 60.50 |


| GEO | NG | BUCKLTMG | BC | IVZ | IIP |
| :--- | :---: | :--- | :---: | :---: | :---: |
| CYL | 26 | $12.418 \cdot 10^{-4} \mathrm{~cm}^{-2}$ | 23 | 3 | 147 |


| zone Iz | STXT | xL [-cm 7 | InT | XR $1-\mathrm{cr} 7$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | O. | 20 | 10.0 |
| 2 | 1 | 10.0 | 52 | 36.15 |
| 3 | 2 | 36.15 | 63 | 70.59 |


| GEO NG | BUCKLTNG | BC | NZ | INP |
| :--- | :--- | :---: | :---: | :--- |
| GPHERE 26 | - | 23 | 3 | 131 |


| ZONE IZ | MIXT | $\mathrm{xL} 1 \mathrm{Cm}_{2} 7$ | InT | 观 [-cm 7 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 26 | 10.0 |
| 2 | 1 | 10.0 | 74 | 38.203 |
| 3 | 2 | 33.203 | 30 | 72.643 |

Two-Mimensional Calculations:

| GEO | ATG | IP | ROTI | COL | NZ | BCL | BCR | BCIT | BCLOV | SPEKTRUT FOR COND |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 11 | 1152 | 32 | 36 | 5 | 2 | 3 | 3 | 2 | 1 -djm. SPHERE |


| NEM GROUP | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| IGUP OLD GROUPS | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 16 | 26 |


| IIXI | I | II | III |
| :--- | :---: | :---: | :---: |
| IIXXURE | 1 | 1 | 2 |
| XITI-1 Cim (IZ) | 1 | 2 | 3 |


| ZOME | SITXI | RL [-Cm 7 | ITTH | $\mathrm{RR} \mathrm{[-C7} 7$ | EUP $\underline{1}^{\text {cm_ }}$ | ITrTU | HLOTE $\mathrm{F}^{-\mathrm{Cm}} 7$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | I | 0. | 5 | 10.0 | 10.0 | 5 | 0. |
| 2 | II | 10.0 | 13 | 36.15 | 30.08 | 16 | 0. |
| 3 | II | 0. | 5 | 10.0 | 30.08 | 11 | 10.0 |
| 4 | III | 36.15 | 17 | 70.59 | 60.50 | 31 | 0. |
| 5 | III | 0. | 18 | 36.15 | 60.50 | 15 | 30.08 |

## Table AII-9

## CRIMICAI FACILITY: SHEAK 3A1

Atam-Densities (in $10^{20} / \mathrm{cm}^{3}$ )

| Mixture Material | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| AI | 129.10 | 129.36 | - | 129.211 |
| C | 4.12 | 4.13 | 0.14 | 4.063 |
| Cr | $36.47^{17}$ | $36.32{ }^{\text {a }}$ | 11.95 ${ }^{3}$ ) | 36.380 |
| Fe | $122.04^{4)}$ | 1.91.51 ${ }^{5)}$ | 39.55 | 121.730 |
| 1 | 7.40 | 7.42 | - | 7.412 |
| Ms | 0.64 | 0.64 | - | 0.640 |
| Mo | 0.39 | 0.39 | 0.19 | 0.390 |
| Ni | 18.54 | 18.27 | 9.84 | 18.424 |
| 0 | 145.29 | 145.67 | - | 145.510 |
| Si | 1,88 | 1.86 | 0.46 | 1.870 |
| Ti | 0.40 | 0.39 | - | 0.394 |
| U235 | 20.31 | 20.25 | 1.625 | 20.270 |
| U238 | 81.04 | 81.21 | 399.414 | 81.140 |

## Fundamental Mode Calculation:

| NG | MIXT | BUCK |
| :--- | :--- | :--- |
| 26 | 4 | $22.010 \cdot 10^{-4} \mathrm{~cm}^{-2}$ |

One-Dimensional Celculations:

| GEO | NG | BUCKLTNG | BC | NZ | NP |
| :--- | :--- | :--- | :---: | :---: | :---: |
| SIAB | 26 | $13.540 \cdot 10^{-4} \mathrm{~cm}^{-2}$ | 23 | 2 | 91 |

1) includes 1.94 of Mn
2) includes 0.19 of Co
3) includes 1.06 of Mn
4) includes 0.18 of Co
5) includes 0.87 of Mn

| zowe Iz | MIXT | XI $L^{-\mathrm{cm}_{-} 7}$ | InT | xR - $\mathrm{CrO}_{2} 7$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 4 | 0. | 60 | 40.27 |
| 2 | 3 | 40.27 | 30 | 70.77 |


| CEO | W | 3UCYLTiT | 3 C | IN2 | m |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CYL | 26 | $8.470 \cdot 10^{-4} \mathrm{~cm}^{-2}$ | 23 | 4 | 121 |


| FONE IZ | MIXT | $\mathrm{xL} \underline{L}^{-\mathrm{cm}} 7$ | INV | $\mathrm{XR} \mathrm{[ } \mathrm{~cm}_{2} 7$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 30 | 5.0 |
| 2 | 1 | 5.0 | 30 | 32.9 |
| 3 | 2 | 32.9 | 30 | 51.2 |
| 4 | 3 | 51.2 | 30 | 80.86 |


| GEO | NG | BUCELUNG | BC | HZ | IPP |
| :--- | :--- | :---: | :---: | :---: | :---: |
| SPHERF. | 26 | - | 23 | 3 | 147 |


| ZONE IZ | MIXTI | $\mathrm{xL} \underline{1-c m}^{-1}$ | INT | $\mathrm{XR} \mathrm{[-m7}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 48 | 39.231 |
| 2 | 2 | 39.231 | 48 | 52.684 |
| 3 | 3 | 52.684 | 50 | 82.344 |

Two-Dimensional Calculations:

| GEO | IIG | IT | ROWS | COL | NZ | BCL | BCR | BCUP | BCLOW | SPEKTRUP FOR COND |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 11 | 1600 | 40 | 40 | 4 | 2 | 3 | 3 | 2 | 1 dim - SPHERE |


| NEY GROUP | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IGUP OLD GROUPS | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 11 | 16 | 26 |


| IIXI | $I$ | II | III |
| :--- | :---: | :---: | :---: |
| IIXPURE | 1 | 2 | 3 |
| PII-T-dim (IZ) | 1 | 2 | 3 |


| 20IE | MIXT | RL $\underline{1}^{\text {cra }} 7$ | INPH | $\mathrm{RR} \underline{1}^{-\mathrm{cm}} 7$ | HUS 1-Cm_7 | INTU | HLOT [-Cm7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | I | 0. | 16 | 32.90 | 40.27 | 20 | 0. |
| 2 | II | 32.90 | 9 | 51.20 | 40.27 | 20 | 0. |
| 3 | III | 0. | 25 | 51.20 | 70.77 | 19 | 40.27 |
| 4 | III | 51.20 | 14 | 80.86 | 70.77 | 39 | 0. |

Table AII-10

## CRITICAL FACILITY: SIEEAI 3A2

At.am-Densities (in $10^{20}$ atoms/ $\mathrm{cm}^{3}$ )

| Yixture 4aterial | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| AI | 129.10 | 129.36 | - | 129.211 |
| c | 9.32 | 9.0 i | 0.14 | 9.217 |
| Cr | $36.47^{1)}$ | $35.32^{2)}$ | 11.953) | 35.330 |
| Fe | 122.04 | 121.515) | 39.55 | 121.730 |
| H | 17.92 | 17.45 | - | 17.719 |
| Ie | 0.64 | 0.54 | - | 0.640 |
| Wi | 18.54 | 18.27 | 9.34*) | 19.424 |
| Mo+isb | 0.30 | 0.39 | 0.19 | 0.390 |
| 0 | 145.29 | 145.67 | - | 145.510 |
| Si | 1.88 | 1.86 | 0.46 | 1.870 |
| Ti | 0.40 | 0.39 | - | 0.394 |
| U235 | 20.31 | 20.25 | 1.625 | 20.270 |
| U238 | 81.04 | 81.21 | 399.414 | 81.140 |

Fundamentel Mode Calculations:

| WG | MTCI | BUCKo |
| :--- | :--- | :--- |
| 26 | 4 | $25.549 \cdot 10^{-4} \mathrm{cr}^{-2}$ |

One-Dimensional Calculations:

| GEO | IIG | BUCKLING | BC | NZ | INP |
| :--- | :--- | :--- | :--- | :--- | :--- |
| SLAB | 26 | $16.942 \cdot 10^{-4} \mathrm{~cm}^{-2}$ | 23 | 2 | 91 |

1) includes 1.94 of Mn
2) includes 0.19 of Co
3) includes 1.96 of Mn
4) includes 0.18 of Co
5) includes 0.87 of Mn
*) The number given by R. Böhric and H. Scufert in KFK-811 and in Nuclear Applications \& Technolocy Vol. 7. p. 494, 1969 is in error.

| ZONE IL | ITIXI | xL $1^{-}=7$ | IITT |  |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 4 | 0. | 60 | 40.27 |
| 2 | 3 | 40.27 | 30 | 70.77 |


| SEO | HC | BUCKITNG | 3C | IVZ | HD |
| :--- | :--- | :--- | :--- | :--- | :--- |
| CYI | 26 | $8.600 \cdot 10^{-4} \mathrm{~cm}^{-2}$ | 23 | 4 | 121 |


| ZORE IZ | SIXP | xL $\mathrm{I}^{-\mathrm{cm}} 7$ | INT | $\mathrm{XR} \underline{1}^{-c \mathrm{~cm}} 7$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 30 | 5.0 |
| 2 | 1 | 5.0 | 30 | 33.76 |
| 3 | 2 | 33.76 | 30 | 44.66 |
| 4 | 3 | 44.66 | 30 | 80.86 |


| GEO | IC | BUCKIING | BC | HZ | NP |
| :--- | :--- | :--- | :--- | :--- | :--- |
| SEHERE | 26 | - | 23 | 3 | 131 |


| ZONE IZ | MIXT | $\mathrm{XL} 1_{[ } \mathrm{Cm}_{-} 7$ | INT | $\mathrm{XR}_{1} \mathrm{Cm}_{-7}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 83 | 40.0914 |
| 2 | 2 | 40.094 | 17 | 48.315 |
| 3 | 3 | 48.315 | 30 | 8.4 .515 |

Two-Dimensional Calculations:

| GEO | NG | NP | ROTS | COL | NZ | BCL | BCR | BCUP | BCLOT | SPEKTRUM FOP COND |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 11 | 1600 | 40 | 40 | 4 | 2 | 3 | 3 | 2 | 1 dim- GPHERE |


| IIEW GPOUP | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IGUP OLD GPOUP | 3 | 4 | 6 | 7 | 8 | 0 | 10 | 11 | 13 | 15 | 26 |

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| VXI | $I$ | $I I$ | III |
| :--- | :---: | :---: | :---: |
| IIXIURE | 1 | 2 | 3 |
| PHI-1 dim (IZ) | 1 | 2 | 3 |


| 20ice | SIXT | PI 1-cm 7 | ITHY | $\mathrm{RR}\left[\mathrm{cm}_{-7} 7\right.$ | HUP $1-\mathrm{cm} 7$ | InTU | HLOM [-cm 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | I | 0. | 16 | 33.76 | 40.27 | 20 | 0. |
| 2 | II | 33.76 | 6 | 44.66 | 40.27 | 20 | 0. |
| 3 | III | 0. | 22 | 44.66 | 70.77 | 19 | 40.27 |
| 4 | III | 44.56 | 17 | 80.36 | 70.77 | 39 | 0. |

Table AII-11

CRIPICAI EACIIITY: SNEAK-3B2
Ator-Densities (in $10^{20}$ atoms $/ \mathrm{cm}^{3}$ )

| Mixture Material | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| Al | 125.6 | 129.66 | 129.47 | - |
| C | 9.73 | 8.76 | 9.00 | 0.14 |
| Cr | $35.95{ }^{1)}$ | $36.08^{2)}$ | 36.28 ${ }^{3}$ ) | $11.95^{4}$ |
| Fe | 119.845 | $120.65^{6)}$ | 121.35 ${ }^{\text {7 }}$ | 39.55 |
| 11 | 18.49 | 16.81 | 17.29 | - |
| ME | 1.31 | 0.65 | 0.65 | - |
| Mo | $0.38^{8)}$ | 0.40 | 0.40 | - |
| Ni | 17.55 | 18.43 | 18.23 | 9.84 |
| 0 | 122.2 | 146.28 | 145.80 | - |
| Si | 2.54 | 1.84 | 1.86 | 0.46 |
| Ti | 0.30 | 0.38 | 0.39 | 0.19 |
| U235 | 0.56 | 20.404 | 20.25 | 1.625 |
| U238 | 81.86 | 81.39 | 81.26 | 399.414 |
| Pu239 | 14.76 | - | - | - |
| Pu240 | 1.33 | - | - | - |
| Pu241 | 0.11 | - | - | - |
| Pu242 | 0.06 | - | - | - |

1) includes 2.23 Mn
2) includes 0.14 Co
3) includes 1.99 Mn
4) includes 0.18 Co
5) includes 1.97 Mn
6) includes 0.19 Co
7) includes 0.87 Mn
8) includes 0.09 Nb

| $G E O$ | YG | EUCKIIING | BC | NZ | IP |
| :--- | :--- | :--- | :--- | :--- | :--- |
| CYI | 26 | $8.46 \cdot 10^{-4} \mathrm{~cm}^{-2}$ | 23 | 4 | 129 |


| ZONE IZ | ITTXT | XIS $\mathrm{Cr}_{2} 7$ | ITH | $x \mathrm{x}$ 1-m] |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 60 | 29.910 |
| 2 | 2 | 29.91 | 12 | 36.180 |
| 3 | 3 | 36.18 | 16 | 44.530 |
| 4 | 4 | 44.53 | 40 | 82.120 |

Two-Dinensional Calculations:

| GEO | ING | IJP | RONE | COL | NZ | BCL | BCR | BCUP | BCLON | SFERTRUN FOR COMD |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 11 | 1600 | 40 | 40 | 5 | 2 | 3 | 3 | 2 | 1 dim- CYL |


| IES CROTP | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| IGUP OID GROUFS | 3 | 4 | 6 | 7 | 8 | 9 | 10 | 11 | 13 | 15 | 26 |


| YIXI | I | II | III | IV |
| :--- | :---: | :---: | :---: | :---: |
| YIXMURE | 1 | 2 | 3 | 4 |
| PHI- 1 dim (IZ) | 1 | 2 | 3 | 4 |


| ZONE | SIIXT | $\mathrm{RL} \mathrm{Im}_{-7} 7$ | INTH | RR 1-cm_7 | HUP [ $\mathrm{cm}^{\text {c }} 7$ | INTU | HLOW 1-cm_7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | I | 0. | 14 | 29.91 | 40.2 |  | 0. |
| 2 | II | 29.91 | 6 | 36.18 | 40.2 |  | 0. |
| 3 | III | 36.18 | 6 | 44.53 | 40.02 |  | 0. |
| 4 | IV | 44.53 | 13 | 82.12 | 40.2 |  | 0. |
| 5 | IV | 0. | 40 | 82. 12 | 70.7 |  | 40.2 |

Table AII-12

CRITICAL FACILITY: SHEAK-5C

$$
\text { Atom Densities (in } 10^{20} \text { atoms } / \mathrm{cm}^{3} \text { ) }
$$

| Mixture |  |
| :--- | ---: |
| Material | 1 |
| Al | 0.0145 |
| C | 612.1704 |
| Cr | 13.1268 |
| Fe | 46.7934 |
| ME | 0.0065 |
| Mo | 0.1116 |
| Nb | 0.0860 |
| Ni | 7.5235 |
| 0 | 39.4747 |
| Pu239 | 4.7656 |
| Pu240 | 0.4281 |
| Pu241 | 0.0389 |
| Pu242 | 0.0020 |
| Si | 0.5399 |
| U235 | 0.3307 |
| U238 | 50.5434 |
| V 1) | 0.9165 |

1) Vanadium has been taken instead of manganese

## Fundamental Mode Calculation:

| NG | IIXT | BUCK 0 |
| :--- | :---: | :--- |
| 26 | 1 | 0.0 |

## Table AII-13

CRITICAI FACIIITY: ZPRIII-55

Atom-Deasities (in $10^{20}$ atoms/ $\mathrm{cm}^{3}$ )

| Glixture | 1 |
| :--- | ---: |
| Glaterial |  |
| Al | 1.11 |
| C | 372.69 |
| Cr | 16.95 |
| Fe | 61.77 |
| Hi | 8.39 |
| Pu239 | 10.68 |
| Pu240 | 0.51 |
| Pu241 | 0.05 |
| U235 | 0.33 |
| U238 | 152.82 |

Fundamental Mode Calculation:

| NG | IIIXT | BUCKO |
| :--- | :---: | :--- |
| 26 | 1 | 0.0 |

Table AII-14

Savings and Bucklings, Carbon- and Fydrogen-Concentrations used in the Fundamental Mode Calculations for SHEAK-series, $3 A O, 3 A 1,3 A 2,3 \Lambda 3$

| Assembly | $\begin{aligned} & S_{a x} \\ & L_{\mathrm{cm}} 7 \end{aligned}$ | $\begin{aligned} & S_{\mathrm{rad}} \\ & \underline{I}_{\mathrm{cm}} 7 \end{aligned}$ | $\begin{aligned} & \mathrm{H}_{\mathrm{c}}^{\mathrm{E}} \\ & \underline{\mathrm{Cm}} 7 \end{aligned}$ | $\left\lvert\, \begin{aligned} & R_{c} \\ & \underline{c_{m}} 7 \end{aligned}\right.$ | $\left[\begin{array}{l} \mathrm{B}_{\mathrm{a} \cdot}^{2} \cdot 10^{4} \\ I \mathrm{~cm}^{-2} 7 \end{array}\right.$ | $\begin{aligned} & \mathrm{rad}^{2} \cdot 10^{4} \\ & \underline{\mathrm{~cm}}^{-2} 7 \end{aligned}$ | $\begin{aligned} & \mathrm{B}_{\text {tot } \cdot 10^{4}}^{1} \\ & 1^{-2} \mathrm{~cm}^{7} \end{aligned}$ | $\begin{aligned} & \text { C-concentration } \\ & \text { in } 1020 \text { atoms } / \mathrm{cm}^{3} \end{aligned}$ | 11-concentrations 3 in 1020 atoms $/ \mathrm{cm}^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3n02 | 14.1566 | 14.6162 | 80.54 | 114.66 | 8.329456 | 16.161194 | 24.790950 | 0.36 | 0. |
| 3 A 1 | 13.7045 | 14.1538 | 80.54 | 44.66 | 8.469593 | 16.718.531 | 25.103124 | 4.063 | 7.412 |
| ЗА2 | 13.2914 .4 | 13.7573 | 30.54 | 44.65 | 8.509800 | 16.949119 | 25.548919 | 9.217 | 17.719 |
| 3 A 3 | 12.7101 | 12.9645 | 80.54 | 44.66 | 8.790509 | 17.413687 | 26.209196 | 13.28 | 35.84 |

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Table 1: Characteristics of Fast Critical Assemblios otudied *)

|  |  | EUN: U1B | $\begin{aligned} & \text { CUAK } \\ & \text { Im:1D } \end{aligned}$ | $\begin{aligned} & \text { 2P? } \\ & \text { III-10 } \end{aligned}$ | $\begin{aligned} & \operatorname{ZPR} \\ & \operatorname{III-25} \end{aligned}$ | $\begin{aligned} & 2 \operatorname{PR}- \\ & \operatorname{III}-48 \end{aligned}$ | $6 \mathrm{CmA}$ | $3 \mathrm{~mA}$ | $\begin{aligned} & 112 N R- \\ & 3 A 2 \end{aligned}$ | $\left[\begin{array}{l} \mathrm{EmR} \mathrm{~K}- \\ \mathrm{C}, \end{array}\right.$ | $\begin{aligned} & 2 \mathrm{PR} \\ & \operatorname{III-55} \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Med |  | 611.1 | 532.1 | 375.8 | 3 S 5.6 | 225.0 | 186.0 | 100.0 | 55.77 | 1.20 | 14.3 .3 |
|  | utron lifetime $3.107!-\mathrm{sec} 7$ | 0.286 | 1.738 | 0.515 | 0.723 | 2.776 | 2.681 | 2.942 | 3.725 | 16,483 | $16.18 \%$ |
|  | U238/ $\mathrm{c}_{\mathrm{c}}^{\mathrm{U} 238}$ | 0.59 | 0.36 | 0.40 | 0.25 | 0.21 | 0.25 | 0.21 | 0.21 | 0.07 | 0.12 |
|  | cration area $1 ?^{2}!\mathrm{cm}^{2}$ | 47.7 | 28.1 | 64.0 | 62.7 | 179.7 | 170.0 | 148.2 | 126.7 | 141.1 | 97.5 |
|  | oretric buckling $\mathrm{B}^{2} \cdot 10^{+} 4 / \mathrm{cm}^{-2} 7$ | 197.9 | 205.3 | 31.4 | 31.7 | 27.0 | 34.6 | 22.0 | 25.5 | 0 | 0 |
|  | ${ }^{2} \cdot 3^{2}$ | 0.94 | 0.58 | 0.53 | 0.20 | 0.49 | 0.59 | 0.33 | 0.32 | 0 | 0 |
|  | [-liters_7 | 36.6 | 34.5 | 70.5 | 435.3 | 4, 14.7 | $23+0$ | 663.3 | 501.7 | - | - |
|  | re fissile mass $\underline{1}^{k_{E_{-}} 7}$ | 116.4 | 87.5 | 156.4 | 584.6 | 273.3 | 183.6 | 524.7 | 399.4 | - | - |
|  | Total leakage | 0.645 | 0.1468 | 0.396 | 0.205 | 0.415 | 0.166 | 0.295 | 0.293 | - |  |
|  | Total fission | 0.386 | 0.394 | 0.389 | 0.398 | 0.3410 | 0.339 | 0.398 | 0.399 | 0.348 | 0.344 |
|  | Total capture | 0.187 | 0.276 | 0.244 | 0.422 | 0.294 | 0.238 | 0.321 | 0.329 | 0.720 | 0.727 |
| $\sim$ | Capture in U238 | 0.136 | 0.156 | 0.181 | 0.378 | 0.205 | 0.154 | 0.2011 | 0.193 | 0.500 | 0.623 |
| 第 | Fission in U238 | 0.081 | 0.057 | 0.073 | 0.093 | 0.01 .4 | 0.030 | 0.043 | 0.012 | 0.036 | 9. 0.0 |
| $\sim$ | Centure in U235 | 0.051 | 0.120 | 0.058 | 0.060 | 10.001 | 0.002 | ก. 099 | 0.114 | 0.012 | 0.003 |
| E | Fission in U235 | 0.305 | 0.337 | 0.316 | 0.295 | 0.003 | 0.008 | 0.355 | 0.357 | 0.025 | 0.010 |
| U | Capture in Pu239 | - | - | - | - | 0.059 | 0.760 | - | - | 0.151 | 0.076 |
|  | Fission in Pu239 | - | - | - | - | 0.285 | 0. 28.4 | - | - | 0.281 | 0.256 |

*) Taken fron fundamental mode horogeneous calculations with the crienk-sot

1) Determined from one-dimensional calculations
2) Normalized to one fission source ne:itron in the reactor $\int_{0}^{\infty} \nu \Sigma_{f}(\delta) \delta(E) d E=1$

Gable 2: Infiuence of tie Fission Spectrum

| Assembly | SUAL UIZ | Suat ilie | ZPS III-10 | ZP? III-25 | STEAK 3A1 | ZEBRA 6A |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Actual $v$ of the assembly | 2.59 | 2.54 | 2.57 | 2.55 | 2.51 | 2.94 |
| Yodified fission spectrum corresponds to $v=$ of the ABi-set | 2.6 | 2.6 | 2.6 | 2.6 | 2.6 | 3.0 |
| ```\Deltak=l:(modified fission spectrum) -l(standard fission spectrum)``` | -0.002 | -0.001 | -0.002 | -0.003 | -0.001 | +0.001 |
| $\begin{aligned} & \text { Change in } \\ & \left.\sigma_{f} \text { (U230 }\right) / \sigma_{f}(\mathrm{U} 235) \end{aligned}$ | -1.2\% | -1.2\% | -1.2\% | -1.2\% | -1.2\% | +2\% |
| $\begin{aligned} & \text { Change in } \\ & \phi(1)=\int_{6.5 \mathrm{MeV}}^{10.5(5) d \Sigma} \end{aligned}$ | -6\% | -5.5\% | -5.5. | -5.5\% | -6\% | +11\% |

Table 3: Comparison of central naterial wortis for SNEAK 3A1 calculated in different rass (sricsy-set has been useà)

| Alaterial | $\frac{2 \text {-din. } 4 \text { frouns }}{1-\text { dim.sphere } 26 \text { Eroups }}$ | $\frac{2-d i n .11 \text { erouns }}{1-d i m, \text { snhere } 20 \text { Eroups }}$ |
| :---: | :---: | :---: |
| AI | 1.005 | 0.974 |
| $\mathrm{P}^{10}$ | 0.915 | 0.959 |
| C | 0.025 | 0.653 |
| Cr | 1.071 | 1.035 |
| Fe | 0.968 | 0.972 |
| F | 0.098 | 0.684 |
| Mg | 3.264 | 1.389 |
| Ii | 1.082 | 1.067 |
| 0 | 0.129 | 0.784 |
| So | 0.934 | 0.967 |
| Pu239 | 1.043 | 1.019 |
| บ235 | 1.069 | 1.030 |
| U23? | 0.953 | 0.970 |

Table V-1: $k_{\text {eff }}$ of fundamental mode homoreneous diffusion calculations
for the various assemblies with different sets of groun contants

| Assembly | SHEAK | SJIEP:TB | CIIEAPM | FUOSCP | SCIAS | UPUC $\quad$ R | PU02RT | $1: 7 \% y 011$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SUAK U1B | 0.8238 | 0.8087 | $0.8295 ?$ | 0.82958 | 0.02759 | 0.802167 | 0.62167 | 0.824531 | 0.82466 |
| SUAK UIL1D | 0.8808 | 0.2782 | 0.8 .8572 | 0.88572. | 0.3888 .3 | 0.8845? | 0.38452 | 0.0888290 | 0.89277 |
| ZPR III~10 | 0.9752 | 0.959864 | 0.90570 | 0.93579 | 0.98 .3153 | 0.97811 | 0.973110 | 0.984030 | 0.984469 |
| ZPP. III-25 | 0.9700 | 0.35992 | 0.98 .932 | 0.98932 | 0.978624 | 0.97389 | 0.273080 | 0.986780 | 0.987716 |
| Z PR IIIm 48 | 0.95497 | 0.063581 | 0.063737 | 0.055207 | 0.952540 | 0.943864 | 0.951737 | 0.959456 | 0.964963 |
| ZEERA $6 \Lambda$ | 0.9530 | 0.269439 | 0.060760 | 0.050724 | 0.05800 | 0.05190 | 0.950542 | 0.9614453 | 0.969320 |
| (3A0 | 0.913 | 0.034 | 0.014 | 0.914 | 0.01111 | 0.94!4 | 0.944 | 0.946 | 0.942 |
| SNEAK- 3 A1 | 0.95? | 0.966 | 0.970 | 0.970 | 0.070 | 0.970 | 0.970 | 0.971 | 0.970 |
| Series $\quad 3 A^{2}$ ) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $3 \mathrm{~A}, 3$ | 1.036 | 1.03 ? | 1.031 | 1.034 | $1.03{ }^{1}$ | 1.035 | 1.035 | 1.033 | 1.033 |
| SITENK 3 A 1 | 0.990164 | 0.9 ? 0.1076 | 1.000429 | 1.0004 ? 5 | 0.990654 | 0.906736 | 0.996736 | 1.006054 | 1.012547 |
| SNEAK 3N2 | 0.982758 | 0.076798 | 0.391394 | $0.9013 \mathrm{I}_{4}$ | 0.921260 | 0.983418 | 0.988448 | $0.99565 \%$ | 1003036 |
| SHENK 5C | 0.937010 | 0.945313 | 0.045666 | 0.930798 | 0.020670 | 0.926926 | 0.930385 | 0.938632 | 0.952949 |
| ZPR III-55 | 0.935316 | 0.953645 | 0.954084 | 0.943182 | 0.23001 | 0.926116 | 0.022143 | 0.944763 | 0.961705 |

1) All values of the s:TAN-series nomalized so as to rive keff (SHLN: $3 N 2)=1$

Table V-R: $\mathrm{F}_{\text {eff }}$ of one- and twodimensional diffusion calculations for the various assemblies with different bets of croun constants


Table V-3: Criticality Corrections for the Various Asserbilies \%)

| Assembires | DEA <br> Corr. | Corr. | Lieteraraneit:Corrections | Other Corr. |
| :---: | :---: | :---: | :---: | :---: |
| SUAK U1S | -0.0002 | 0.031 | - |  |
| STAET UTIT | -0.0004 | 0.030 | - | +0.007 |
| ZPR III-10 | -0.000: | 0.013 | 0.0105 |  |
| 2 FR III-25 | -0.0009 | 0.002 | 0.0085 |  |
| Zrr III-48 | +0.0047 | 0.006 | 0.014 |  |
| 2PR III-483 | +0.0048 | 0.006 | 0.014 |  |
| ZEBRA 6A | +0.0040 | 0.010 | - |  |
| SHENK-series 340 | -0.0015 | 0.003 | 0.001 |  |
| 341 | -0.0006 | 0.003 | 0.003 |  |
| 3 A 2 | -0.0006 | $+0.004$ | 0.004 |  |
| 3 A 3 | -0.0009 | 0.004 | 0.007 |  |
| SIIEAK-3A1 | -0.0006 | 0.003 | 0.003 |  |
| SILEAK-3A? | -0.0006 | +0.004 | 0.004 |  |
| SUEAK-3B2 | $+0.0003$ | +0.001 | 0.003 |  |
| SNFAK-5C | +0.0185 | - | $0.071^{1)}$ |  |
| ZPR III-55 | +0.0123 | - | 0.010 |  |

*) Corrections calculated using EnLAN-set or taken from the literature

1) Heterogeneity correction is probably too large

Table V~4: Dest available criticality values for the various assemblies with different sets of nuclear zroun constants


[^1]Table V-5: Central naterial morth- and central reaction rate-ratios for srieak 3A1 ana sreaf 3A2 (obtained from homoseneous spinerical diffusion calculations)
( $\Delta \mathrm{k} / \mathrm{k}$ ner atom of the raterial considered $/ \Delta t / k$ per atom of U235)

| Taterial | Assembly |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | SIEAT 3 AT |  | SIITAF 3A2 |  |
|  | S\एAK-Set | MOXTOT-Set | SITMY-Set | MOXTOT-Set |
| 15235 | 1.0 | 1.0 | 1.0 | 1.0 |
| U23? | -0.080 | $-0.069$ | -0.062 | -0.070 |
| ㄲu239 | +1.419 | +1.377 | $+1.420$ | $+1.374$ |
| T 510 | -1.210 | -1.289 | $-1.759$ | -1.814 |
| Fe | -0.0087 | -0.007? | -0.0097 | -0.0089 |
| Cr | -0.0067 | -0.0062 | -0.0072 | -0.0063 |
| Ti | -0.0165 | -0.0159 | -0.0188 | -0.0182 |
| Yo | -0.0691 | -0.0709 | -0.0931 | -0.0944 |
| C | $+0.0030$ | +0.0030 | +0.0036 | +0.0039 |
| $\pm 5$ | -0.0007 | +0.0003 | -0.0001 | +0.0005 |
| AI | -0.0027 | -0.0021 | -0.0022 | -0.0017 |
| 0 | +0.0025 | +0.0032 | +0.0032 | +0.0034 |
| IT | +0.1519 | +0.1400 | +0.1304 | $+0.1167$ |
| $\sigma_{\underline{1}} \mathrm{U} 238$ | 0.0306 | $0.029,4$ | 0.0202 |  |
| $\mathrm{o}_{\mathrm{f}} \mathrm{J} 235$ | 0.0306 | $0.028,4$ | 0.0292 | 0.0272 |
| $\sigma_{c}$ U233 |  |  |  |  |
| $0_{\text {d }}$ U235 | 0.1435 | 0.1271 | 0.1383 | 0.1237 |

Table V-C: Reaction rate ratios for EIIEM $3 A 1$ amn Simair 3i? (results of fundamental moie honoreneous ciffusion calculetions)

| Sroum-Set | $0_{5} 1133^{2} / \sigma_{0} 0335$ |  | $\sigma_{c} 0230 / c_{4}$ U235 |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Assemblir |  | Assembly |  |
|  | SIENT 3A1 | SITMA 312 | SITAAF 31. | SIIEA 3 A2 |
| SIFAE | 0.0305646 | 0.0291813 | 0.1435001 | 0.1393348 |
| Gmulu | 0.0305555 | 0.0293001 | 0.14291:07 | 0.13712420 |
| clitare | 0.03020257 | 0.0288892 | 0.13634168 | 0.13232973 |
| fuoser | 0.0302023 | 0.0208802 | S.1353394 | 0.13232976 |
| $\operatorname{scmat}$ | 0.02921786 | 0.02797753 | 0.13687611 | 0.13272677 |
| UPUC\%. | 0.02393525 | $0.027700 \cap 7$ | 0.13725300 | 0.13307827 |
| TyOnje | 0.02893525 | 0.02770097 | 0.13725308 | 0.13307827 |
| \%n>911 | 0.02362756 | $0.027)<573$ | 0.13105388 | 0.12516419 |
| Shxan | 0.02C14725 | 0.0272414 | 0.1270707 | 0.1237131 |

iable V-T: Central materiel worth- and central reaction ratemratios for ZPR III-4 (obtained from homozeneous spherical diffusion calculations)

| Yaterial | Saterial worth ner atom norralized to ! 235 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Evoeriment | STEAE-Set | UPUCDR-Set | FUO2SE-Set | IngXT $\quad$ T-Set |
| U235 | 1. | 1. | 1. | 1. |  |
| I238 | $-0.074096 \pm 0.00237$ | -0.07799903 | -0.07400296 | -0.0743288 | -0.0677507 |
| P4239 | $+1.3347 \pm 0.02761$ | +1.3360672 | +1.2851539 | $+1.28430262$ | $+1.276766$ |
| Puc! 0 | $+0.2439 \pm 0.06305$ | +0.06455886 | +2.04634762 | +0.2088790 | +0.200401 |
| TJA | $-0.00162 \pm 0.000107$ | -0.00391285 | -0.00411021 | -0.00398746 | $-0.00374$ |
| 510 | $-1.12219 \pm 0.02330$ | -1.038983 | -1.0163986 | -1.0260348 | -1.060586 |
| FE | $-0.00876 \pm 0.000364$ | -0.0102505 | -0.00988528 | $-0.00984360$ | -0.0094751 |
| CR | $-0.00613 \pm 0.000334$ | -0.0085428 | -0.00831905 | -0.00873844 | -0.0084667 |
| III | $-0.013396 \pm 0.000305$ | -0.01655626 | -0.01627882 | -0.01620403 | -0.0161596 |
| 210 | $-0.05226 \pm 0.001102$ | -0.0656256 | -0.06350217 | -0.06383736 | $-0.064600$ |
| $\frac{\sigma_{f} U 2.32}{\sigma_{f} U 235}$ | $0.0307 \pm 0.0003$ | 0.030994 | 0.029793 | 0.029635 | 0.028852 |
| $\frac{\sigma_{f} \mathrm{Pu} 240}{\sigma_{f} U 235}$ | $0.976 \pm 0.070$ | 0.913747 | 0.903357 | 0.901930 | 0.894870 |
| $\frac{\sigma_{f} \text { Fu240 }}{\sigma_{f} U 235}$ | $0.243 \pm 0.002$ | 0.215419 | 0.203743 | 0.212505 | 0.207663 |
| $\frac{a^{2} 238}{\sqrt{6}+5}$ | $0.130 \quad \pm 0.007$ | 0.146049 | 0.140746 | 0.140771 | 0.130090 |

Table V-8: Reaction rate-ratios for ZPR III-40
(results of fundamental mode homogeneous diffusion calculations)

|  | Exneriment | SIJTAK-Set | SIETYTS | SHEAPli | [USGCי | $\operatorname{SCT} \Lambda \square \varnothing$ | UPUC\&D | 1U02n: | $\therefore 8 \times 011$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{\sigma_{f} U 238}{\sigma_{f} I J 235}$ | $0.0307 \pm 0.0003$ | 0.031022311 | 0.03186160 | 0.03065153 | 0.031170523 | 0.030143929 | 0.02983008 | 0.02967420 | 0.029281124 | 0.028992113 |
| $\sigma_{r} \mathrm{Pu} 239$ | $0.976 \pm 0.010$ | 0.91553033 | 0.9418422475 | 0.91227008 | 0.018097626 | 0.90623265? | 0.90277856 | 0.90332037 | 0.89085206 | $0.896862 \% 4$ |
| $\sigma_{f} P u 240$ | $0.243 \pm 0.002$ | 0.21614874 | 0.22208147 | 0.21364155 | 0.21725915 | 0.201108605 | 0.204437765 | $0.2131 / 218$ | $0.210 \% 31188$ | 0.20882071 |
| ${ }^{0}{ }_{c}$ | $0.138 \pm 0.007$ | 0.14504459 | 0.14418948 | $0.13 \cap 68333$ | 0.13916081 | 0.140.3?37 | 0.14066813 | 0.11600100 | 0.131433686 | $0.12909 \sim$ |

Table V-9: Central material worth- and central reaction rete-ratios for ZPR III-43B
(obtained from homoeneous spherical diffusion calculations)


Table VI-1: The mean and root mean square or standerd deviation for criticalitio between experirent and theory with verious croun scts

| ?uantity | Crounmet |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | STENT. | ETEPM3 | EMEAP:i | ETOCOP |  | UPUCDR | PUJO2nJ | 1¢x, 11 | :1 6 NT |
| $\frac{1}{I T} \sum_{i=1}^{I T}\left(k_{\text {eff }} \text { exp.,i-k off calc., } i\right)$ | 0.0173 | 0.0175 | 0.0069 | 0.0123 | 0.0150 | 0.0194 | 0.0179 | 0.0090 | 0.0022 |
|  | 0.0270 | 0.0193 | 0.0132 | 0.0189 | 0.0224 | 0.0249 | 0.02 .32 | 0.0158 | 0.0125 |





Fig. All-3 Neutron flux for 2PR-III-10


Fig. All-4 Neutron flux for ZPR-1ll-25


Fig. All-5 Neutron flux for 2PR-1II-48



Fig. All-7 Neutron flux for ZEBRA 6A


Fig. AII-8 Neutron flux for SNEAK 3 AI



Fig. All-10 Neutron flux for SEAK $3 B 2$


Fig. All-11 Neutron flux for SNEAK 5C



























Fig. 1 Comparison of axial reaction rate traverses for SNEAK-3A2



Fig. 3


FIG. 4 CENTRAL REACTION RATE RATIOS FOR DIFFERENT DEGREES OF BUNCHING SNEAK 3A-2


fig. 5 B RATE DISTRIBUTIONS WITHIN THE URANIUM ZONE OF THE DOUbLE BUNCHED UNIT CELL



Fig. 6B Variation of $k_{\text {eff }}$ with the number of mesh intervais MESH SIZE


Fig. 7A Variation of $\mathrm{k}_{\text {eff }}$ with the quadrature order. SUAK UIB, slab geometry. Complete listing of data see table V-13a


Fig. 7B Variation of $k_{\text {eff }}$ with the quadrature order. Complete listing of data see table $\mathrm{V}-13 \mathrm{~b}$. ZPR-III-48, cylindrical geometry.


Fig. 7C Variation of $k_{\text {eff }}$ with the quadrature order.
Complete listing of data see table V-13c. ZPR-III-48, spherical geometry.


Fig. 7 D Variation of $k_{\text {eff }}$ with the quadrature order. Complete listing of data see table V -13d.
ZPR-III-10, slab geometry.


Fig. 7E Variation of $\mathrm{k}_{\mathrm{eff}}$ with the quatrature arter. Complete listing of data see table V-13e. ZPR-III-10, cylindrical geometry.


Fig. 7F Variation of $k_{e f f}$ with the quadrature order.
Complete listing of data see table V-13f.
ZPR-III-10, spherical geometry.


Fig. 7 G Variation of $\boldsymbol{k}_{\text {eff }}$ with the quadrature order. Complete listing of data see table V-13g. ZPR-III-25, slab geometry.


Fig. 7 H Variation of $k_{\text {eff }}$ with the quadrature order. Complete listing of data see table V - 13 h . ZPR-III-25, cylindrical geometry.


Fig. 71 Variation of $k_{\text {eff }}$ with the quadrature order.
Complete listing of data see table $\mathrm{V}-13 \mathrm{i}$.
ZPR-III-25, spherical geometry.


Fig. 7J Variation of $k_{\text {eff }}$ with the quadrature order. Complete listing of data see table V - 13 j .
Zebra 6A, slab geometry.


Fig. 7K Variation of $k_{\text {eff }}$ with the quadrature order. Complete listing of data see table V -13k. Zebra 6A, cylindrical geometry.


Fig. 7 L Variation of $k_{\text {eff }}$ with the quadrature order.
Complete listing of data see table V - 13 l .
Zebra 6A, spherical geometry.


[^0]:    Nonmoscillating means nonmoscillating at the end and some steps before the end of the iteration process. For example, if you take the values of table $\mathrm{V}-10 \mathrm{a}$, $|1-\lambda|$ converges monotonously to zero only after the third outer iteration.

[^1]:    1) Breliminary experimental result
    2) Homelization point for the sNPMr-set
    3) This correction is probably too laree
