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# A SURVEY OF VALUES OF THE 2200 m/s CONSTANTS FOR FOUR FISSILE NUCLIDES

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# ОБЗОР ВЕЛИЧИН ЯДЕРНЫХ КОНСТАНТ ПРИ СКОРОСТИ НЕЙТРОНОВ 2200 м/сек ДЛЯ ЧЕТЫРЕХ ДЕЛЯЩИХСЯ ИЗОТОПОВ, К.Г.Весткотт, К.Экберг, Дж.Ч. Ханна<sup>1</sup>, Н.Дж.Паттенден<sup>2</sup>, С.Санатани и П.М. Эттри (Международное агентство по атомной энергии, Вена)

Аппроксимируя способом наименьших квадратов, проделан анализ наиболее вероятных величин ядерных констант при скорости нейтронов 2200 м/сек для делящихся изотопов U<sup>233</sup>, U<sup>235</sup>, Pu<sup>239</sup> и Pu<sup>241</sup>. Путем тщательной обработки многочисленных экспериментальных данных получены следующие величины, которые можно рекомендовать как наиболее точные в настоящее время.

	<sup>233</sup> U	<sup>235</sup> U	<sup>239</sup> Pu	<sup>241</sup> Pu			
σ <sub>a</sub> σf	576,3 ± 2,3 527,7 ± 2,1	$679,9 \pm 2,3$ $579,5 \pm 2,0$	1008,1 ± 4,9 742,4 ± 3,5	1391 ± 22 1009 ± 9			
σγ	48,6 ± 1,5	100,5 ± 1,4	265,7 ± 3,7	382 ± 21			
α η	0,0921 ± 0,0029 2,284 ± 0,008	0,1734 ± 0,0025 2,071 ± 0,007	0,3580 ± 0,0054 2,114 ± 0,010	0,379 ± 0,021 2,154 ± 0,036			
ν	2,494 ± 0,009	2,430 ± 0,008	2,871 ± 0,014	2,969 ± 0,023			
$\nu$ ( <sup>252</sup> Cf) = 3,772 ± 0,015							

### РЕКОМЕНДУЕМЫЕ ВЕЛИЧИНЫ ЯДЕРНЫХ КОНСТАНТ ПРИ СКОРОСТИ НЕЙТРОНОВ 2200 м/сек<sup>а</sup>

<sup>а</sup> Значения сечений даны в барнах.

Подробно рассмотрен круг вопросов, связанных с обсуждаемой проблемой, в частности оценка точности отдельных измерений и выходных величин. Указанные выше ошибки, являющиеся среднеквадратичными отклонениями, учитывают возможность наличия систематических и регулярных погрешностей в результатах первоначальных измерений, которые могут вкрасться во входные данные, где их бывает невозможно установить.

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# A SURVEY OF VALUES OF THE 2200 m/s CONSTANTS FOR FOUR FISSILE NUCLIDES

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ABSTRACT. A study of the most probable values of the 2200 m/s constants for the fissile nuclides  $U^{233}$ ,  $U^{235}$ ,  $Pu^{239}$  and  $Pu^{241}$  has been undertaken, using a least-squares fitting method. The various experimental data have been carefully reviewed, and the following values resulted.

	U <sup>233</sup>	U235	Pu <sup>239</sup>	Pu241			
σ a σc	576.3±2.3	679,9±2,3	$1008.1\pm4.9$ 742.4+3.5	$1391 \pm 22$ 1009 ± 9			
σγ	48.6±1.5	100, 5 ± 1, 4	265.7±3.7	382±21			
α η	0.0921±0.0029 2.284±0.008	0. $1734 \pm 0.0025$ 2. $071 \pm 0.007$	0.3580 $\pm$ 0.0054 2.114 $\pm$ 0.010	0.379±0.021 2.154±0.036			
$\nu$ 2.494±0.009 2.430±0.008 2.871±0.014 2.969±0.023							
$\nu$ (Cf <sup>252</sup> ) = 3. 772 ± 0. 015							

### RECOMMENDED VALUES FOR 2200 m/s CONSTANTS<sup>a</sup>

<sup>1</sup> Cross-section values in barns (b).

A detailed discussion of the problems involved, especially in assessing the accuracy of the individual measurements and of the final output values, is given. The errors shown above are quoted as standard deviations, and include some allowance for possible systematic or non-random contributions to the errors of the original measurements that, although not identifiable in the input data, may nevertheless exist.

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### 1. INTRODUCTION

The cross-sections for 2200 m/s neutrons for the fissile nuclides  $U^{233}$ .  $U^{235}$ .  $Pu^{239}$  and  $Pu^{241}$ , as well as the related quantities  $\nu$  (the neutron yield per fission) and  $\eta$  (the neutron yield per absorption), are very important quantities which are basic to many reactor calculations. The cross-sections needed are those for fission and radiative capture, the sum of these crosssections (the absorption cross-section) and their ratio (frequently called  $\alpha$ ); of less importance is the scattering cross-section  $\sigma_s$ , but since the total cross-sections (including scattering) have been carefully measured,  $\sigma_{\rm c}$  must also be included in any study of the best values. It is true that  $Pu^{241}$  is generally less important than the other nuclides mentioned, and the data for this nuclide are less well determined, but there is now a significant mass of data for this isotope so that we have decided to include it in our study. Since these particular constants are basic not only to thermal reactor calculations, but are also often used as the basis for normalizing other measurements (e.g. fast-neutron fission cross-sections) needed for the design of intermediate and fast reactors, it is important that the best available values of these 2200 m/s constants be made known and used.

There have been a number of surveys of the 2200 m/s constants since the first United Nations Conference<sup>3</sup> on the Peaceful Uses of Atomic Energy (Geneva, 1955), at which values from various national sources were first collected and compared [1]. Up to that time these values had generally been kept secret in the countries where most of the work had been done, but when they were released, satisfaction was expressed with the agreement found. Now that more sophisticated criteria are being applied, the overall position on these constants is, however, seen to exhibit some continuing unsatisfactory features.

In addition, therefore, to the series of revisions of the table of "world's best values" contained in successive issues of the well-known Brookhaven National Laboratory compilation BNL-325 [2], special surveys have been undertaken from time to time. Those of EVANS and FLUHARTY [3], SAFFORD and HAVENS [4] and LEONARD [5] were separate studies of these constants for the nuclides  $U^{233}$ ,  $U^{235}$  and  $Pu^{239}$ , respectively, but since some of the quite accurate measurements were made of one nuclide relative to another, several other authors [6, 7, 9 and 10] have made a simultaneous study of all these three nuclides. Generally the method used was to make a least-squares fit for three independent quantities for each of the three nuclides considered, based on all the available reliable measurements, each weighted according to its accuracy. This is also the procedure used in the present study, as was explained in the 3rd Geneva Conference paper [11] in which preliminary results of the present work were presented.

In the present work this process of simultaneously fitting for nine independent variables has been supplemented by a subsequent fit for the three independent variables for  $Pu^{241}$ . It was unnecessary to fit the data for this nuclide simultaneously with the others since the  $Pu^{241}$  measurements were

<sup>&</sup>lt;sup>3</sup> In this text the United Nations International Conferences on the Peaceful Uses of Atomic Energy (1st in 1955 with Proceedings in 1956, 2nd in 1958, 3rd in 1964) will be referred to as 1st, 2nd or 3rd Geneva Conference, respectively.

much fewer and their accuracy much lower. The results for Pu<sup>241</sup> thus would not have reacted significantly on the values for the other three nuclides in a simultaneous twelve-parameter fit. Indeed, the Pu<sup>241</sup> constants were not much over-determined, while the measurements for the other three nuclides form a highly-interlocking over-determined set.

### 2. GENERAL CRITERIA USED IN OUR STUDY

The underlying basis of the least-squares procedure used in the present study is that the errors involved in the various measurements are of a random nature and are not correlated or systematic. It is also of fundamental importance to try to assess all errors on a uniform basis, so that the relative weights to be attached to, say, a direct measurement of a fission crosssection or its indirect determination via  $\alpha (= \sigma_{\gamma} / \sigma_{f})$  and  $\sigma_{a} (= \sigma_{f} + \sigma_{\gamma})$  are correctly represented. Therefore the accuracy claimed for all the measurements has been reviewed. The "weight" of any value in the least-squares fit is proportional to the inverse square of its error, so that careful review is considered essential.

A number of general principles which we adopted in our review of errors must be mentioned. Without making any <u>a priori</u> assumptions concerning the superiority of measurements with monokinetic (2200 m/s) neutrons over those in a "thermal" spectrum which is believed to be accurately Maxwellian, it is realized that adequate justification of this 'Maxwellian spectrum' assumption is needed for correcting the latter values to 2200 m/s, and uncertainties due to this, as well as in the g- and s-factors [12] used to convert thermal to 2200 m/s values, have been included in our estimates of the error. Thus all "thermal" measurements are to some extent automatically down-weighted in our survey, and for those in reactor spectra containing an appreciable (definite) fraction of epithermal neutrons the downweighting will be correspondingly greater. It is of course obvious that measurements made in ill-defined spectra generally have to be ignored.

Unfortunately difficulties arise, especially in connection with the older measurements, in ascertaining all the factors needed for an adequate review of the accuracy of measured values. We have therefore adopted rather strict criteria, e. g. expecting that all but the most recent measurements should by now have been written up in adequate detail and either published<sup>4</sup> or issued as a report that is generally available. When adequate documentation is lacking, the errors of the results concerned have been arbitrarily increased (often, to give half-weight, by a factor of  $\sqrt{2}$ ) to provide some downweighting in the least-squares procedure. Where the lack of documentation is such that one is really unable to assess the reliability of the work or to make needed corrections to 2200 m/s values, there is no possible alternative to rejecting the value completely. Most of the values rejected for this reason are from measurements made before 1955; it appears that some of the measurements made before this date were never adequately described in the open literature. Although a few of these results may be of real value,

<sup>&</sup>lt;sup>4</sup> In this text 'unpublished' refers to articles that exist as internal documents in a particular organization, and that were not made generally available.

there is no general way of judging which these are, and some were found to be of little value. For most of the older measurements examined it was clear that a lack of an adequate description of the method used or of knowledge of the neutron spectrum or other quantities (such as what values were used for normalization) rendered the results obtained almost valueless. In view of the general advance in techniques since the period 1944-1954, we believe we have not unduly discriminated against the older measurements by totally rejecting some 80% of the results obtained at that time.

Some of the known measurements of more recent date have never been adequately documented. In such cases we have applied a down-weighting factor unless by inquiries we could obtain all details necessary to interpret the result or find out why a full report had not been written. For example, the authors may not have been satisfied with the results, but it may not have been possible to carry the work further. Where the difficulties in interpreting a measurement appeared serious, the values obtained have been ignored. Pile-oscillator or criticality measurements may be treated as determinations of  $\eta$ ,  $\eta \sigma_a$  or  $\sigma_a(\eta - 1)$  (the latter being strictly  $\sigma_a(W\eta - 1)$  where usually  $W \approx 1$ ), but the resulting values are only taken into account if the analysis of the measurements leads fairly directly to the quantity deduced. In fact, since the use of nuclear data for predicting criticality is an important application, we have tried not to exclude all measurements with imperfectly-documented spectra, though they are naturally treated as having errors of the appropriate magnitude, but on the other hand we cannot regard every zero-power critical experiment as a measurement of  $\eta$ . Those giving  $\eta$  or some related quantity reasonably directly were therefore selected, generally in this case from published or freely-available papers or documents.

One or two general points of difficulty, which will be discussed later in connection with particular input values, deserve mention at the outset. Those connected with the form of a neutron spectrum are considered in section 2.1 below. The other main problems concern possible correlations between different measurements in the input data for the least-squares procedure. One case, the scattering cross-section correction to measured total cross-sections used as input to our analysis as values of  $\sigma_a$ , is dealt with in sections 4.2 and 4.3. Other cases are more difficult to express satisfactorily, in that the factors concerned cause correlations to arise between inputs for different measured quantities. While it is possible to devise special least-squares routines when correlated inputs are known to exist, this would considerably complicate the analysis, as well as the chosen statistical criteria of goodness of fit, and we believe the degree of correlation which exists is insufficient to justify the additional effort involved.

Examples of a situation where the errors attributed to the input data need to be carefully considered, is when three non-independent ratios (e.g. a/b, b/c, c/a) are all measured, or when three quantities a, b, and c are each measured but with an appreciable normalization error such that the ratios are in fact known more accurately than would be deduced from the errors quoted for the corresponding absolute values separately. In the former example we have to allow for the accuracy given for any one ratio in virtue of the existence of an indirect value derived from the other two measurements; one has to know whether in fact the three measured ratios were independent (in which case they may be inconsistent) or not. Only in the latter case need the errors be revised to exclude the additional weight due to the accuracy of the indirect value. In the second example cited above we can represent the excess accuracy of the knowledge of the relative values by inserting experimental values for the ratios as well as for the absolute values. In all such cases we have been careful not to include the "weight" of any measurement more than once in the input to the least-squares fitting routine.

There are several other possible difficulties due to correlations in the set of measurements under study. One arises in connection with the measurements of  $\nu$ , most of which are made relative to the spontaneous fission  $\nu$ of Cf<sup>252</sup>. Since the preliminary publication of the present study[11], results for  $\nu$  (Cf<sup>252</sup>) have been obtained whose scatter is considerably larger than would be expected from the claimed errors. The resulting uncertainty led to a decision to treat  $\nu$  (Cf<sup>252</sup>) as an independent variable in the least-squares treatment, using a ten-parameter fit in place of the nine-parameter fit used in Ref. [11] and mentioned in section 1. In this way the main correlation between the  $\nu$  values for the three nuclides being studied was automatically allowed for. There are other correlations which remain in the tenparameter fit, but, except for those involving Maxwellian spectra, which are considered below, the effects of these appear relatively minor, and error adjustment appears to suffice to express their effects. Increasing the number of independent parameters further to deal with these remaining factors does not appear justified.

We have also avoided the perhaps dubious process of rejecting input data whose deviations from acceptable values are unacceptably large. This process may sometimes be justified if the basic assumption of the leastsquares method, that all errors are of a random nature, is felt to be inapplicable for a particular value which is a "flier", but we have preferred to re-examine any measurement of this type for possible unexpected sources of error, and in each case a reasonable degree of downweighting has been found to be justified, as discussed in section 4 below.

### 2.1. Treatment of Maxwellian spectra measurements

It has already been mentioned that the correction of values obtained with Maxwellian or reactor neutron spectra to 2200 m/s gave rise to additional errors. It has also been necessary to estimate how reliably the neutron spectrum in question is known to be Maxwellian; the spectrum in a thermal column or a large tank of D<sub>2</sub>O, for example, is much better known than the spectrum near or in a reactor lattice, apart from the need, in the latter case, to correct for the epithermal (1/E) component, or to allow for the departure of this spectrum from a 1/E form.

A more difficult problem is the possible error in the g-factors [12] used for correcting Maxwellian results to thermal. Studies of the variability of g-factors as tabulated at different dates, based on different sets of  $\sigma(E)$ data from time-of-flight or crystal spectrometer measurements, could only yield a lower limit to the uncertainty of g-factors, which in the best cases was about  $\pm 0.1\%$ . VOGT [13] is presently studying the question of what are the probable uncertainties of g-factors in the light of resonance theory and the existing experimental knowledge of the variation of the various crosssections (and of  $\eta$ ) with energy, but the results of this work are not yet available. Since there must be some correlation between  $\sigma_f$  and  $\sigma_a$  variations, because  $\sigma_a$  includes  $\sigma_f$ , the errors to be attached to  $g_{\eta}$  should be less than the quadrature sums of the (fractional) errors of  $g_f$  and  $g_a$ , even when experimental data for  $\eta$  as a function of E are not of appreciable importance. It is clear that the experimental results from which g factors are obtained vary in quality, being, for example, relatively good for  $\sigma_a$  (U<sup>235</sup>), but quite poor for  $\sigma_f$  (Pu<sup>241</sup>), where JAMES [14] and the US results [15, 16] differ by about 6% in the ratio between a 2200 m/s and a 0.26 eV resonance crosssection.

The values chosen for the g-factors used in correcting thermal spectrum measurements to 2200 m/s are taken from WESTCOTT [12] as revised by CRITOPH [17], and are given with their estimated errors (taken to be standard deviations) in Table I, for 20°C (for other temperatures the correction to 20°C is assumed to involve no additional error).

### TABLE I

Parameter	U 233	U 235	Pu 239	Pu 241
ga	0, 9983 ± 0, 20%	0.9771±0.13%	1.0723±0.13%	1.030±0.3%
g f	1,0003±0,28%	0.5781±0.17%	1.0487±0.17%	1.039 <sub>5</sub> ±0.6%
£η	1.0020±0.30%	1.0010±0.20%	0.9780±0.20%	1.0095 ± 0.6%

### VALUES OF g-FACTORS FOR 20°C

The errors are taken to be about  $\sqrt{2}$  times the above-mentioned variability of g-factors as tabulated at different dates. The value of  $g_f$  for  $Pu^{241}$  is the result of a separate compilation, weighting the James and the US  $\sigma_f$  results equally, pending clarification of the discrepancy, and its error is estimated taking this discrepancy into account.

### 3. NUMERICAL TREATMENT

### 3. 1. U<sup>233</sup>, U<sup>235</sup> and Pu<sup>239</sup> data; 10-parameter fitting system

For the present study, as we shall see in section 4 below, there exist the measured parameters  $\eta$ ,  $\nu$ ,  $\sigma_f$ ,  $\sigma_a$ ,  $\alpha$  and  $\sigma_\gamma$  for each of the three nuclides  $U^{233}$ ,  $U^{235}$  and  $Pu^{239}$ , as well as the parameters  $\eta$ ,  $\nu$ ,  $\sigma_f$ ,  $(\eta - 1)\sigma_a$ ,  $\eta\sigma_a$ , measured as ratios for the nuclide pairs  $U^{233}/U^{235}$ ,  $Pu^{239}/U^{233}$ ,  $Pu^{239}/U^{235}$ . Of the parameters,  $\eta$ ,  $\nu$ ,  $\sigma_f$  were taken to be independent, and all the other parameters can be expressed in terms of these, giving 9 independent and 24

### TABLE II

Parameter <sup>a</sup>	U <sup>233</sup>	U <sup>235</sup>	Pu <sup>239</sup>	U233 U <sup>235</sup>	Pu 239 U <sup>233</sup>	Pu <sup>239</sup> U <sup>235</sup>
η ν σ <sub>1</sub> α α σ <sub>γ</sub> (η - 1)σ <sub>a</sub>	1 -1 -7 -11 -14 	2 5(35) 8 19 15 18 -	3 6 9 13 16 19 -	20 23 26 - - 29	21 24 27 - - 30	22 25 - - - 31
no <sub>a</sub>	-	-	-	32	33	34

### PARAMETER NUMBERING USED

a Parameter 10 is  $\nu(Cf^{252})$ .

dependent parameters. This was the system used in the 3rd Geneva Conference paper [11] but, as explained above, it was found preferable to add  $\nu$  (Cf<sup>252</sup>) to the parameters to be fitted, since many measurements of  $\nu$  were made relative to  $\nu$  for this nuclide. There was only one input datum (for U<sup>235</sup>) for a  $\nu$  measured absolutely, apart from the measurements of  $\nu$  (Cf<sup>252</sup>).

The parameters were therefore numbered as shown in Table II, where the 10th independent parameter is  $\nu$  (Cf<sup>252</sup>). The estimated values for the parameters to be fitted are denoted by  $X_i + x_i$  (X is a "base" value and x the small increment to be determined), where  $1 \le i \le 10$ , while the values obtained by measurement are  $Y_i + y_i$ , where  $1 \le i \le 35$ . For  $4 \le i \le 6$ , while the X's are actually  $\nu$ 's, the correspondingly-numbered Y's are ratios to the  $\nu$  of Cf<sup>252</sup>, and  $Y_{35} + y_{35}$  is the absolute  $\nu$  (U<sup>235</sup>) datum. For the i-th measured parameter there may be  $N_i$  experimental values with their associated errors, i.e.  $\hat{Y}_n \pm \sigma_n$ , where  $\sigma_n$  includes the weighting factor. In a few cases no experimental data exist for one of the nuclides (or one of the ratios) so that a guess is used, in which case a very large error is assumed for the quantity concerned, to give it a quite negligible weight in the result. In addition there is, associated with some of the parameters, another quantity,  $\delta_{err}^2$ , which represents an increment to the variance required for this parameter (generally due to a correlated effect), see section 4 below.

The following quantities are calculated for each parameter (i = 1, 2...35).

(i) Weighted mean value,

$$Y_{\text{mean } i} = \frac{\sum_{1}^{N_{i}} (\widehat{Y}_{n} / \sigma_{n}^{2})}{\sum_{1}^{N_{i}} (1 / \sigma_{n}^{2})}$$
(1)

and its error,  $\sigma_{\text{mean i}} = \sqrt{v_i}$ , where

$$\mathbf{v}_{i} \text{ (variance)} = \left\{ \sum_{1}^{N_{i}} (1/\sigma_{n}^{2}) \right\}^{-1} + \delta_{err}^{2}$$
(2)

These quantities (1) and (2) are used in the least-squares fitting process rather than the individual experimental values.

(ii) Internal consistency ratio (ICR),

$$ICR_{i} = \sum_{i}^{N_{i}} \frac{(\hat{Y}_{n} - Y_{mean i})^{2}}{\sigma_{n}^{2}} / (N_{i} - 1)$$
(3)

This is the normal  $\chi^2$  for the distribution of the input measured values for any one parameter divided by (N<sub>i</sub> - 1), and expresses their spread in comparison with the originally stated errors. This quantity can only be determined if there is more than one experimental value for any given parameter.

A standard least-squares correlation process [18] is used for computing the final values of the parameters on the basis of a first order approximation to a predetermined set of base values. The quantity to be minimized (apart from the  $\delta_{err}^2$  terms) is

$$\sum_{i=1}^{35} \sum_{n=1}^{N_i} \frac{[\hat{Y}_n - (Y_i + y_i)]^2}{\sigma_n^2},$$

but in order to include the  $\delta_{err}^2$  terms we use the otherwise equivalent expression

$$\sum_{1}^{35} [Y_{\text{mean } i} - (Y_i + y_i)]^2 / \sigma_{\text{mean } i}^2,$$

where (see Eq. (2) above)  $\sigma_{mean\,i} = \sqrt{v_i}$ . In the first iteration of the fitting procedure the base values of the ten independent variables are taken as equal to the  $Y_{mean}$ 's as already defined, except that, for  $4 \leq i \leq 6$ ,  $X_i = Y_{base\,i} \times Y_{base\,10}$ , and from these a consistent set of  $Y_{base}$  values for the other 25 parameters is determined.

A set of simultaneous equations is set up in matrix form, y = Ax, where: (i) y is a vector of 35 elements,  $y_i$  representing increments to the base values of each parameter indicated by the direct measurements of that parameter.

$$y_i = Y_{\text{mean } i} - Y_{\text{base } i}$$
(4)

10

(ii) x is a 10 element vector of the unknown quantities  $x_j$ , representing the increments by which the base values of the independent parameters must be increased to give improved values of the parameters.

(iii) A is a  $35 \times 10$  matrix of the coefficients of  $x_j$  in the first order expansion of the dependent parameters; i.e. for any given parameter f  $(x_1, x_2, \dots, x_{10})$ ,

$$\mathbf{y}_{i} = \sum_{j=1}^{10} \frac{\partial \mathbf{f}}{\partial \mathbf{x}_{j}} \mathbf{x}_{j}, \qquad (5)$$

where the differentials are evaluated with all  $x_j$ 's = 0. For the 10 independent parameters,  $y_i = x_i$ , except that for  $4 \le i \le 6$ ,

$$\mathbf{y}_{i} = \frac{1}{X_{10}} \mathbf{x}_{i} - \frac{X_{i}}{X_{10}^{2}} \mathbf{x}_{10}.$$

For the 17th parameter,  $\sigma_v$  for U<sup>233</sup>, for example,

$$y_{17} = -\frac{X_4 X_7}{X_1^2} x_1 + \frac{X_7}{X_1} x_4 + \left(\frac{X_4}{X_1} - 1\right) x_7.$$

In order to perform a weighted least-squares fit, one more matrix is required. This is a  $35 \times 35$  diagonal matrix, V, whose diagonal elements are  $v_i$ , defined in Eq. (2).

The solution vector,  $\overline{x}$ , is then given by

$$\overline{x} = (A'V^{-1}A)^{-1}A'V^{-1}y.$$

The improved parameter values  $X_i$  are then calculated from the elements  $\overline{x_i}$ of the solution vector just obtained for the 10 independent variables using  $\overline{X_i} = X_i + \overline{x_i}$ , the dependent  $\overline{Y_i}$  are calculated, and thus a consistent set of improved parameters is obtained. To complete the set of  $\overline{Y}$ 's, we note that for  $i \leq 10$ ,  $\overline{Y_i} = \overline{X_i}$ , except that for i = 4, 5, 6,  $\overline{Y_i} = \overline{X_i}/\overline{X_{10}}$ . In order to calculate the errors on these improved values, the elements of the variancecovariance matrix are used:

$$C = (A^{\dagger} V^{-1} A)^{-1}$$

The general expression for the variance of any one of the parameters,  $f(x_1, x_2, \dots, x_{10})$  is

$$\operatorname{var} \mathbf{f} = 2 \sum_{i=1}^{9} \sum_{j=i+1}^{10} \left( \frac{\partial \mathbf{f}}{\partial \mathbf{x}_{i}} \right) \left( \frac{\partial \mathbf{f}}{\partial \mathbf{x}_{j}} \right) \mathbf{c}_{ij} + \sum_{i=1}^{10} \left( \frac{\partial \mathbf{f}}{\partial \mathbf{x}_{i}} \right)^{2} \mathbf{c}_{ii},$$

where  $c_{ij}$  are the elements of variance-covariance matrix, C. Thus the variance of the 10 independent parameters is equal to  $c_{ii}$  and, for example, for  $\alpha$  for U<sup>233</sup> (the 14th parameter) for which  $f_{14} = X_4/X_1 - 1$ ,

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var 
$$f_{14} = \frac{1}{\overline{X}_1^2} \left[ c_{44} + \frac{\overline{X}_1^2}{\overline{X}_1^2} c_{11} - \frac{2\overline{X}_4}{\overline{X}_1} c_{14} \right].$$

The variances of the ratios of  $\nu$  to  $\nu(Cf^{252})$  are given by

$$\left(\frac{\overline{X}_{i}}{\overline{X}_{10}}\right)^{2} \left[\frac{\underline{c}_{i,i}}{\overline{X}_{i}^{2}} + \frac{\underline{c}_{10,10}}{\overline{X}_{10}^{2}} - \frac{2\underline{c}_{i,10}}{\overline{X}_{i}\overline{X}_{10}}\right],$$

for i = 4, 5, 6. The errors can then be calculated from the variances

$$\overline{\sigma}_i = \sqrt{\operatorname{var} \overline{Y}_i}$$
.

We now have an improved set of values,  $\overline{Y}_i \pm \overline{\sigma}_i$ .

Further improvements can be made by replacing  $Y_{base i}$  by  $\overline{Y_i}$  in Eq. (4) and repeating the whole process. Any number of iterations can be made, but it was found that, in practice, only 2 were needed. Further iterations made insignificant improvements.

The programme was written in FORTRAN IV for an IBM 7040 computer. In the matrix inversion, the method of elimination, pivoting on the largest element of each column, and back substitution was used. For programme testing purposes, data from Table 4.22, page 48 of reference [7] were used, and their results reproduced  $[(\eta-1)\sigma_a \text{ replaced } \sigma_y \text{ for testing purposes}]$ .

### 3.2. Extension to 16-parameter fit

A further extension of the programme was tried, in which the g-factors for  $\eta$  and fission for each of the three nuclides were treated as parameters to be fitted, the values of Table I being used as input values of the fit. This was done to ascertain whether any changes in the g-factors from the nominal values chosen would allow the fit to be improved. Any such changes would of course introduce correlated changes in those quantities which were measured in a Maxwellian spectrum. The effects of these possible correlations were shown to be negligible by this test (see section 6 below), so that this extension to the programme is not described in detail.

### 3.3. Pu<sup>241</sup> data

As there were no data available for  $\eta$  for  $Pu^{241}$ , the independent parameters for this fitting process were chosen to be  $\sigma_a$ ,  $\nu$  and  $\sigma_f$ , and the dependent parameters were  $\eta$ ,  $\sigma_{\gamma}$ ,  $\eta\sigma_a$  and  $\alpha$ , making 7 parameters in all. In some cases a value of a parameter had been measured as a ratio to the corresponding quantity for one of the three nuclides  $U^{233}$ ,  $U^{235}$  and  $Pu^{239}$ , e.g.  $\sigma_f (Pu^{241})/\sigma_f (Pu^{239})$ . Under these circumstances the relevant best values from the computation involving the three nuclides were used in conjunction with the measured ratio, to provide a value with error, for the Pu<sup>241</sup> input, as explained in detail in section 6 below.

In all other ways the method of fitting for this nuclide is parallel to that used for the other three nuclides.

### 2200 m/s CONSTANTS

### 4. MEASURED VALUES USED IN THE LEAST-SQUARES FIT

In this section are given the values used as inputs for the least-squares procedure, derived from various investigations, with notes on any important corrections or reassessments of the accuracy of any result. The "additional error on the mean", denoted by  $\delta_{err}$  in Eq. (2) above, is also given where it exists, and the weighted mean input for each quantity is also shown, this being the actual input quantity used in the least-squares method for each parameter. The values are given in Tables IV-X, accompanied by the relevant references, for ready identification, but any more extensive comments are made in the text. In these tables an asterisk(\*) is placed after the error to label those values (measured in Maxwellian spectra) for which an allowance for the g-factor error is made when a with-g-errors fit is performed.

Those measurements or classes of measurements whose results were discarded are briefly considered in section 5 below, for the sake of completeness.

### 4.1. Standard absorption cross-sections

In the present study it was necessary to review the standard crosssections which the authors used to interpret their measurements. These standard values fall into two classes. The first consists of those standard values relative to which the measurement has been made or to which the results have been renormalized and include the absorption cross-sections of cobalt, sodium and gold. We have consistently renormalized any values which depend on these quantities to correspond to our standards. The other class consists of various constants, usually cross-sections, which have been used in the course of calculations, often to evaluate a correction factor, and which have only an indirect and rather small effect on the results. In these cases we have only satisfied ourselves that the values used are reasonable, that is, that a recalculation would not give any significant changes.

4.1.1. Absorption cross-section for gold. The following four precise measurements have been considered:

98.7±0.6 b	[19]	Carter <u>et al</u> .
97.7±0.9 b	[20]	Egelstaff
98.8±0.3 b	[21]	Gould <u>et al</u> .
98.6±0.2 b	[22]	Als-Nielsen, Dietrich

Our preferred standard value, obtained from these, is  $98.7 \pm 0.2$  b, which also agrees satisfactorily with the value  $98.8 \pm 0.3$  b recommended in the latest available version (2nd ed., 1958) of BNL-325.

4.1.2. Absorption cross-sections for hydrogen, sodium and cobalt. The value recommended in BNL-325 [2nd ed., suppl. 2 (1964)], Z = 1 to 20, for sodium,  $0.534 \pm 0.005$  b, has been adopted, but for hydrogen their value (332 mb) has been modified slightly, in view of the JARVIS [23] study on this

isotope, and a value of  $331 \pm 2$  mb has been assumed. For cobalt, we have adopted the value  $37.8 \pm 0.7$  b, recommended by SJÖSTRAND [8] for the absorption cross-section, which implies  $37.7 \pm 0.7$  b for the activation cross-section.

4.2. The estimation of scattering cross-sections

With one exception, all of the absorption cross-section values used in the present evaluation are derived from measurements of total crosssections from which an estimated value of the scattering cross-section has been subtracted. The experimental information available on the low energy scattering cross-sections is very sparse, and the only course of action open to us is to make intelligent guesses of values, on which we must place arbitrary, and therefore relatively large, uncertainties. There are two problems: firstly to estimate the free-atom scattering cross-section at 2200 m/s, and secondly to correct this for a possible coherent scattering effect in the sample being measured.

4.2.1. Free atom scattering cross-sections. There are three types of information available: (1) direct measurements of scattering cross-sections; (2) values of potential cross-section derived usually from total cross-section measurements at higher energies; and (3) values of the scattering cross-section calculated by multi-level analysis methods.

The scattering cross-section of  $U^{233}$  has been measured by OLEKSA [24] at spot energy points between 0.27 and 3.31 eV, and by MOORE and SIMPSON [25] between about 1.7 and 19 eV; in both sets of measurements the samples were metal foils. In the overlapping region, the agreement between them is reasonably good. The measurements of Oleksa showed that, below 1 eV, the cross-section was not energy dependent and had a value of  $12.5 \pm 0.5$  b.

The  $U^{235}$  scattering cross-section has been measured by FOOTE [26] using the same apparatus as Oleksa, at spot energies between 0.27 and 7.7 eV, with a metal foil sample. The cross-section showed an increase with decreasing neutron energy, with a value 14.7±0.4 b at 0.27 eV, the error referring to the counting statistics only.

The scattering cross-section at thermal energies consists of the potential scattering together with contributions from neighbouring levels; in particular the interference between the scattering components of bound levels and potential scattering has an effect over a relatively wide range. However, the potential scattering is important in our estimation because, for the uranium isotopes, values have been assumed in the multilevel analyses mentioned below, and, for the plutonium isotopes, no other data are available. Some values of potential scattering cross-sections for nuclides in this mass region are shown in Table III.

Multilevel fits to experimental data on  $U^{233}$ ,  $U^{235}$  and  $Pu^{239}$  have been described by VOGT [31]. For  $U^{233}$ , using a value for the potential scattering cross-section of 12.7 b, he obtained a scattering cross-section value at 2200 m/s of 12.1 b. For  $U^{235}$  the respective values were 12.0 and 16.8 b. No scattering cross-sections for  $Pu^{239}$  are given.

SHORE and SAILOR [32] fitted their own measurements of  $U^{235}$ , assuming a potential scattering cross-section of 10.3 b, using a single-fission-

### 2200 m/s CONSTANTS

### TABLE III

Th <sup>232</sup>	U <sup>235</sup>	U <sup>238</sup>	Pu 239	Reference
12.0±0.3		10.7±0.3		[27] Seth <u>et al</u> .
<b>11</b> .0±0.3		10,6±0,3		[28] Lynn
12.0±0.3	11.7±0.1	10.6±0.3	10.3±0.2	[29] Uttley
			10.5±0.5	[J0] Bollinger <u>et al</u> .

### POTENTIAL SCATTERING CROSS-SECTIONS

channel analysis method of Reich and Moore. Shore and Sailor obtained a 2200 m/s value of 17.0 b. A comparison with the experimental scattering data of FOOTE [26] showed that a potential scattering of 9.5 b would give a better fit.

MOORE and REICH [33], quoted in [25], performed a multilevel analysis of their own data on  $U^{233}$ , assuming a potential scattering cross-section of 12.6 b, and obtained a scattering cross-section at 2200 m/s of 11.5 b.

The lack of uniqueness and the sensitivity of the scattering cross-section to the chosen parameters make the multilevel analyses an unreliable guide to the absolute value of the cross-section. It'is more reasonable to use them only as an indication of the shape of the energy dependence of the scattering cross-section.

We consider that the best procedure for  $U^{233}$  and  $U^{235}$  is to use the multilevel analysis curves to extrapolate to lower energies from the observed values. We must accept that there is a possibility of small coherence effects at 0.27 eV (the lowest energy experimental point). This, together with the uncertainty involved in the extrapolation, gives us for the free atom scattering cross-sections at 2200 m/s, the values 13.0±2.0 b for  $U^{233}$  and 16.0±2.0 b for  $U^{235}$ .

For  $Pu^{239}$ , we increase the potential scattering value slightly to allow for a possible contribution from negative energy resonances, and increase the uncertainty. For  $Pu^{241}$ , we can only assume, from optical model predictions [27], that the value is the same as for  $Pu^{239}$ , and we assign a still larger uncertainty. Thus, for the free atom scattering cross-sections at 2200 m/s, we use the values 11.0±2.0 b for  $Pu^{239}$ , and 11.0±3.0 b for  $Pu^{241}$ .

4.2.2. Possible coherent scattering corrections. The scattering from crystalline materials can be coherent, which results in sharp discontinuities in the energy dependence of the scattering cross-section in the thermal neutron region. The discontinuities are due to the increased coherent scattering which occurs when the neutron wavelength becomes shorter than twice the lattice spacing of a particular set of lattice planes. For a particular wavelength and plane, the angle of coherent scattering is well defined.

In metal samples, depending on the method of fabrication and heat treatment of the sample, the crystallites may not be randomly oriented, they may vary in size and have a different mosaic spread. A variety of effects can occur in these situations (e.g. the preferred orientation can reduce the number of crystallites which are available, for coherent scattering at a particular wavelength, or extinction effects can occur in a sufficiently large crystallite or one with a sufficiently small mosaic spread). The net effect is to reduce the amount of coherent scattering.

There are a number of possible sources of incoherence, of which spin and isotopic incoherence are the only two to concern us. S-wave neutron interactions can proceed via compound nuclear states of spin  $I + \frac{1}{2}$  or  $I - \frac{1}{2}$ , where I is the target nucleus spin. The scattering amplitudes of the two states are different, and they are randomly distributed, so spin-incoherent scattering will occur. Spin incoherence is not possible in the case of nuclei with I = 0, for example Th<sup>232</sup> and U<sup>238</sup>, but it can occur in the fissile nuclides considered here.

Where a mixture of isotopes is present in an elemental sample, in general their scattering amplitudes are different, and they are randomly distributed throughout the sample. This produces isotopic incoherence. This source of incoherence cannot occur in mono-isotopic materials.

The coherent scattering cross-sections for thorium, uranium and plutonium have been studied by ROOF et al. [34]. They obtained values of 12.0±0.1, 8.9±0.2 and 6.0±0.3 b, respectively. The low value for plutonium is due, presumably, to the influence of spin incoherence. For  $Pu^{239}$  at least, coherent effects are likely to be much smaller than for Th<sup>232</sup> or  $U^{238}$ .

There are few data available from which to estimate the size of the effect in the total cross-section measurements considered in this evaluation. CARTER et al. [19] observed that rolled metal samples of gold gave thermal total cross-section values about 2 b lower than those obtained with powdered samples. This corresponds to a reduction in scattering cross-section of about 20%. Gold is a material in which spin incoherence can occur, but not isotopic incoherence, and the coherent scattering is about three quarters of the total scattering (Ref. [2], 1958 ed.). Consequently, we estimate that, in metal foil samples of U<sup>233</sup> and U<sup>235</sup>, there is a reduction in the scattering cross-section of 1.5 $\pm$ 2.2 b, due to coherent effects. In other uranium and in all plutonium samples, there is no significant reduction.

The error of  $\pm\sqrt{5}$  b quoted for the scattering correction is such that the overall error in the quantity to be subtracted becomes  $\pm 3$  b for metal foil samples of both uranium isotopes. However, this cannot be treated in the same way as the original uncertainty of  $\sigma_s$ , since the latter is necessarily the same for all samples. The additional  $\pm\sqrt{5}$  b only applies to those samples which are rolled metal foils, and, on account of differences in fabrication methods, the actual value of  $\sigma_s$  may not be the same for different metal foils. Therefore the  $\pm\sqrt{5}$  b is added to the uncertainty of an individual value (in quadrature) as an uncorrelated error, whereas the  $\pm 2b$  (or  $\pm 3b$  for Pu<sup>241</sup>) error is an error correlated for all measurements of  $\sigma_T$  for any one isotope, and is added as a  $\delta_{err}^2$  term (see Eq. (2) above).

### 4.3. Measured values for $\sigma_{T}$ and $\sigma_{a}$ ; input for absorption cross-sections

The experimental results used for the uranium isotopes and  $Pu^{239}$  are considered in this and the following five sections; in section 4.9 the input for the  $Pu^{241}$  fit is considered.

The input data for  $\sigma_a$  are shown in Table IVa-IVc. The only direct determination of  $\sigma_a$  was measurement No. 3 of Table IVa which used a pile oscillator technique, and since this already had a quite large error it was felt that its weight would not be seriously affected by the  $\pm 2$  b error which is being added as a  $\delta_{err}$  term to the weighted mean, as explained at the end of section 4.2, to allow for the uncertainty in  $\sigma_s$ . For reasons already explained, we have distinguished rolled metal foil samples from solutions or oxide powder samples, and in the former case an extra error of  $\pm \sqrt{5}$  b is added in quadrature. It was noted that the foils used for measurements numbered 5 and 7 of IVa were all fabricated at Oak Ridge National Laboratory, but No.4 came from a different source, and as far as is known all the samples of Table IVb were fabricated independently.

Adjustments were made to the Safford <u>et al</u>. value No. 5 of Table IVb since the choice of a 12th degree polynomial to fit the liquid sample data did not seem justified. The Safford results in both Tables IVa and IVb used statistical criteria for rejecting data which seem of doubtful validity and the errors are therefore somewhat increased.

### 4.4. Fission cross-sections and ratios

Table V shows the data used as input for  $\sigma_f$ , and describes some of the adjustments made to the authors' values. In the case of Raffle (No. 7 in the table) only the values measured with monokinetic neutrons were used; those made in a reactor spectrum or in a beam from a thermal column, quoted in the same reference, were rejected.

The work of Fraysse and Prosdocimi (No. 12) was only available to us in a preliminary form and, due to some possible uncertainty in the normalization, the results were interpreted only as a ratio between  $\sigma_f(Pu^{239})$ and  $\sigma_f(U^{235})$ . A correction to 2200 m/s and a reassessment of the halflives [60] involved have been made and the error slightly increased for halflife uncertainties. Because the documentation is still preliminary it was decided to downweight this value by multiplying the error by  $\frac{4}{3}$ .

The interpretation of the work of Bigham <u>et al.</u> (No.4 in Table V) also calls for an explanation. Since one of us is also an author of the paper concerned, a fundamental revaluation was possible, but to counteract any claim that, because of a common authorship, this measurement had been considered particularly favourably, all calculated errors were increased by 10%. The main problem for re-evaluation was concerned with the question whether the three independently measured ratio values  $(U^{233}/U^{235}, Pu^{239}/U^{235})$  and  $Pu^{239}/U^{233}$ ) should be used, or the "best values" obtained by averaging direct and indirect results. The former would have been used, as independent inputs, were it not that uncertainties in other factors (half-life and g-factors) introduced complications. For  $Pu^{239}$ , for example, the half-life and g-factor errors were common to both the Pu/U ratios and the accuracy of the ratio for a Maxwellian spectrum was relatively high so that the common

### TABLE IV a

Authors	Year, Ref.	Authors' o <sub>T</sub> (b)	Reassessed $\sigma_{\mathrm{T}}$ (b)	σ <sub>a</sub> (b)	Comments
1. Nikitin, Galanina, Ignatiev, Okorokov, Sukhoruchkin	1956 [35]	580±20	580 ± 24	568 ± 24	Description very brief; error × 1, 2
2. Pattenden	1956 [36]	590 ± 15	590 ± 20	578 ± 20	Error increased from 15 to 20 b for discontinuity in the cross-section curve in the region around 0, 025 eV
3. Green, Small, Glanville	1957 [37]	578±17(o <sub>a</sub> )	582±20(o <sub>a</sub> )	582 ± 20	Renormalized to our preferred gold $\sigma_a$ and error increased from 17 to 20 b to allow for neutron spectrum uncertainties
4. Simpson, Moore, Simpson	1960 [38]	587 ± 6	587 ± 5. 1	575. 5 ± 5. 6	The authors' errors included an uncertainty due to $\sigma_s$ , which we have subtracted before applying our own corrections
5. Block, Slaughter, Harvey	1960 [39]	587±3	587 ± 3	575.5±3.8	
6. Safford, Havens, Rustad	1960 [40]	587±5	587 ± 6	574 ± 6	Liquid sample; error × 1.2 (see text)
7. Safford, Havens, Rustad	1960 [40]	586±2	586±2.4	574.5±3.3	Metal sample, error $\times$ 1.2 (see text)
Additional error on mean (δ <sub>err</sub> )		-	-	±2	
Weighted mean		-	-	574.95±2.90	
ICR	<u></u>	-	-	0.0509	

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# TOTAL AND ABSORPTION CROSS-SECTIONS FOR U<sup>233</sup>

### TABLE IVb

	Authors	Year, Ref.	Authors' o <sub>T</sub> (b)	Reassessed $\sigma_{\mathrm{T}}$ (b)	σ <sub>a</sub> (b)	Comments
1.	Melkonian, Havens, Levin	1953 [41]	691 ± 5	694 ± 14	679, 5 ± 14	Because of large discrepancies between samples and because the work is unpublished the error is increased from 5 to 14 b
2.	Palevsky, Carter, Eisberg, Hughes	1954 [42]	700±5	700±10	685, 5 ± 10, 3	Very short and rather inadequate description does not permit a reasonable assessment; error $\times 2$
3,	Egelstaff	1954 [43]	$729 \pm 15$	729 ± 26	714 ± 26	Description brief and rather inadequate; error increased from 15 to 26 b
4.	Nikitin, Galanina, Ignatiev, Okorokov, Sukhoruchkin	1956 [35]	710±20	710±24	695 ± 24	Description very brief, error x 1.2
5.	Safford, Havens, Rustad	1959 [44]	695.0±1.8	696, 5±2, 8	680, 5 ± 2, 8	Liquid sample (see text)
6.	Safford, Havens, Rustad	1959 [44]	698.7±4.7	$698.7 \pm 5.5$	684.2±5.9	Metal sample (see text)
7.	Simpson, Moore, Simpson	1960 [38]	690 ± 10	690±9.7	675.5±10	The authors' errors included an uncertainty due to $\sigma_{\rm S}$ , which we have subtracted before applying our own corrections
8.	Block, Slaughter, Harvey	1960 [39]	693 ± 5	693 ± 5	678.5±5.5	
9.	Şaplakoğlu	1961 [45]	694 ± 1, 5	694 ± 2, 5	679.5±3.4	Measurements performed over a narrow energy range, so a coherent scattering effect may cause a fluctuation of the cross-section, and claimed accuracy for sample thickness not justified; error increased from 1, 5 to 2, 5 b
A	lditional error on mean (Serr)		-	-	±2	
Weighted mean		-	-	680, 57 ± 2, 70		
IC	R		-	-	0.3908	

# TOTAL AND ABSORPTION CROSS-SECTIONS FOR $\mathrm{U}^{235}$

# T'ABLE IVc

# TOTAL AND ABSORPTION CROSS-SECTIONS FOR $\mathrm{Pu}^{239}$

Authors	Years, Ref.	Authors' o <sub>T</sub> (b)	Reassessed o <sub>T</sub> (b)	σ <sub>a</sub> (b)	Comments
<ol> <li>Nikitin, Galanina, Ignatiev, Okorokov, Sukhoruchkin</li> </ol>	1956 [35]	1040±30	1040±42	1029±42	Description is brief, especially as regards samples; error x 1.4
2.' Pattenden	1956 [46]	$1015\pm30$	$1015 \pm 30$	$1004 \pm 30$	
3. Bollinger, Coté, Thomas	1958 [30]	1015±10	1015±12	1004 ± 12	Inadequate description, especially concerning details of samples; error $\times 1.2$
4. Safford, Havens	1961 [47]	1018±8	1018±7.4	1007 ± 7.4	We preferred to take their $\sigma_{\rm T}$ and apply our $\sigma_{\rm s}$ correction
Additional error on mean (ô <sub>err</sub> )		-	-	±\ 2	
Weighted mean		-	-	1006.56±6.42	
ICR		-	-	0. 1139	

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### TABLE V

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# FISSION CROSS-SECTIONS

Authors		New Def		Absolute values (b)		
		lear, kei,	U 233 5	U 235	Pu <sup>239</sup>	Comments
1.	Popovic, Grimeland	1953 [48]	-	586±19*	-	The authors give $1072 \pm 2\%$ for the ratio $\sigma_f (U^{239})/\sigma_a$ (Na), sub-cadmium value in a thermal column spectrum. This has been converted using our standard Na cross-section and corrected to 2200 m/s; error $\times \sqrt{2}$ , following our general criteria
2.	Popovic, Saeland	1955 [49]	525 ± 24*	-	-	Authors' sub-cadmium value $\sigma_f$ (U <sup>233</sup> )/ $\sigma_a$ (Na) is 985±3%. Treated similarly to No. 1 above
3.	Friesen, Leonard, Seppi	1956 [50]	-	$555 \pm 14$	-	See text
4.	Bigham, Hanna, Tunnicliffe, Campion, Lounsbury, MacKenzie	1958 [51]	517.5±13*	-	742.7 ± 5.6*	See text
5.	Şaplakoğlu	1958 [52]	-	602.6 ± 10.3	-	See text
6.	Cocking	1958 [53]	-	-	760 ± 30	The author measured $1 + \alpha$ at 0,001 eV, and this datum has been used with $\sigma_a$ at 0,001 eV, and the ratio $\sigma_f(0.0253 \text{ eV})/(0.001 \text{ eV})$ to obtain $\sigma_f$ . The error is almost entirely due to error in the measured $1 + \alpha$
7.	Raffle	1959 [54]	508 ± 17	586 ± 18	702 ± 20	We have adopted the values and errors given by SJÖSTRAND and STORY [7], only renormalized to our preferred gold cross-section, see text.

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TABLE V (cont.)

Authors Mass Def		Absolute values (b)		
Authors Year, Rer.	U 233	U 235	Pu 239	Comments
8. Safford, Melkonian 1959 [55]	-	586. 2± 8. 0	-	The authors' value 590, $8 \pm 5$ , 3 has been re- evaluated using different data to correct from 0,00291 eV. The uncertainty in this correction appears to be greater than the authors' estimate, and an allowance has been made for uncertainty in the fission fragment counting efficiency
9. Deruytter 1960 [56]	-	590±8 -	_	A correction of $+$ 0.5% for neutron scattering in the gold foil has been introduced, and the partial errors have been reassessed, which increases the final error from 6 to 8 b
10. Maslin, Moore, Reichelt 1964 [57] Crowde	-	574±7 <sup>a</sup>	-	As there seems to be no uniquely correct way of combining the data from the different foils at different orientations the error has been increased from 6 to 7 b
Additional error on mean (δ <sub>err</sub> )	-	± 2	-	
Weighted mean	515.71±9.49	583.49±4.23	740.38±5.31	
ICR	0, 1872	See text	2. 1410	

<sup>a</sup> Note added in proof: in a manuscript accepted in the Physical Review the value has been modified to 572b.

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Australia			Ratios	· 2	
Authors	iear, kei.	U <sup>233</sup> /U <sup>235</sup>	Pu <sup>239</sup> /U <sup>233</sup>	Pu <sup>239</sup> /U <sup>235</sup>	Comments
4. Bigham, Hanna, Tunnicliffe, Campion, Lounsbury, MacKenzie	1958 [51]	0.9108±0.0012*	-	1.3072±0.0098*	See text
11. Auclair, Galula, Hubert, Jacrot, Joly, Netter, Vendryes	1956 [58]	- *	1.417±0.0284*	-	This datum has been re-evaluated. The main change was in the estimate of epithermal absorption (Westcott $s_4$ assumption). The authors quote an experimental error of $\pm$ 1%, but the uncertainties in spectrum corrections are larger than this, and the result has been somewhat downweighted because of the brief account
12. Fraysse, Prosdocimi	1965 [59]	-	-	1.264±0.032*	We used only a ratio based on the $\sigma_f$ values measured at 0, 0322 eV (see text)
Weighted mean		0.9108±0.0012	1. 417 ± 0. 0284	1. 3035 ± 0. 0094	
ICR		-	-	~1. 6662	

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errors were dominant and caused a serious correlation. Several alternatives were considered, but finally it was decided to present the results as two ratios only,  $Pu^{239}/U^{235}$  and  $U^{235}/U^{233}$ , so as to avoid correlated inputs as far as possible. The  $Pu/U^{235}$  ratio was chosen rather than the  $Pu/U^{233}$  one since the g-factor errors are larger for  $U^{233}$  than  $U^{235}$ . The correlation between the two ratios used, which would apparently arise due to the  $U^{235}$  half-life, is actually not present, since the  $U^{235}/U^{233}$  ratio was measured by an isotopic dilution technique and was therefore independent of half-lives (the half-lives of  $U^{233}$  and  $U^{235}$  were also tied together experimentally by this measurement). The correlation due to g-factor errors in  $U^{235}$  will be present in the fit with g-errors included (see section 6), but the 16-parameter fit trials mentioned in section 3.2 showed that this was unimportant.

The other problem concerns the fact that the Bigham et al. work also included a measurement of  $\sigma_f(U^{233})$  in terms of  $\sigma_a$  for gold. This measurement therefore determines, with the ratios, both the other  $\sigma_i$ 's as absolute values. However, due to the U<sup>233</sup> half-life uncertainty being greater than that of  $Pu^{239}$ , we can consider  $\sigma_f(Pu^{239})$  as being a more accurate result than the  $U^{233} \sigma_{f}$ ; the  $U^{235}$  value is tied to the  $U^{233}$  one (independently of the half-lives), so that it does not represent any extra accuracy over that given by the ratio and the U<sup>233</sup> value. Of course, when g-factor errors are introduced, further correlations arise which give us cause for even more caution in treating these results. The final decision was to use the value for  $\sigma_f(Pu^{239})$  as the principal absolute fission cross-section input datum, its accuracy (without g-errors) being 0.753%. Then the inputs including the ratios would imply a knowledge of  $\sigma_f(U^{233})$ , which, adding the errors in quadrature, appears accurate to  $\pm 1.07\%$ , whereas the actual error for this quantity is  $\pm 0.907\%$ . This procedure thus represents a 28% loss of weight for the  $\sigma_f$  (U<sup>233</sup>) input. The weight so lost can only be inserted (by adding a second absolute  $\sigma_f$  input datum) at the expense of using a set of correlated inputs, and for this reason we had considered omitting such an input for simplicity. However, the half-life error of Pu<sup>239</sup> introduced some correlation in any case, and the best compromise seemed to be to re-insert almost half the missing "weight" by using as an input datum a  $\sigma_{\rm f}$  (U<sup>233</sup>) value with an increased error  $(\pm 13 \text{ b})$ ; this of course means that we can have an indirect value as well as a direct value for some of the ratios and for  $\sigma_f$  (Pu<sup>239</sup>), but the errors with our compromise are such that in the worst cases the existence of the extra input only adds about 7 to 8% to the weight of any input used. Since we have previously multiplied all errors by 1.1, as explained above, and thereby discarded 20% of the weight for each value, this seems acceptable. It has the advantage that the 517.5 b input for  $U^{233}$ is now given about 85% of its true "weight" (after applying the 1.1 factor). A more exact treatment may have been possible, but the one adopted appears adequate.

Two other values deserve some comment; these are No.3, Table V (Friesen <u>et al.</u>) and No.5 (Saplakoglu). Both values lie far from the weighted mean and were therefore studied carefully. For the Friesen value, an erroneous foil assay may be responsible, since Raffle [54] quotes a 3% lower mass for this sample, but another US assay gave a somewhat higher value. A somewhat larger correction ( $\approx 1\%$ ) should perhaps also be made for fission counts lost below the bias level. We have therefore renormalized to our

standard  $\sigma_a$  for gold, increased the error by 1.5%, to allow for the uncertainties just mentioned, and further downweighted because of non-publication by multiplying the resulting error by  $\sqrt{2}$ . The Saplakoglu value was obtained by a method similar to one since used by Maslin (No. 10), who demonstrated that an angular correlation between fission product and neutron direction could introduce a considerable error in the result if it was not allowed for. Unfortunately the Maslin and the Saplakoglu geometries differ sufficiently to make the use of the former investigation invalid for correcting the results from the latter. However, enquiries have shown that Saplakoglu appears to have considered the possibility of such an effect and checks with bearing on this point were made. As a result of our re-examination of this work, reasons were found for increasing the author's correction for the dependence of the fission chamber efficiency on fission-fragment pulse height from  $(-1.1\pm0.5)\%$  to  $(-1.5\pm0.7)\%$ . Since the  $\pm0.7\%$  is based on assuming a linear 'pulse height-efficiency' relationship, the error has been further increased to  $\pm 1\%$ . Further, the spread of the values obtained in the seven different runs is considerably greater than one would expect from the errors quoted for the individual runs; this, together with the fact that the description is very brief for an experiment claiming such high accuracy, has led us to downweight the measurement by multiplying the reassessed error by  $\sqrt{1.5}$ .

One further comment concerning the set of input values for  $\sigma_f(U^{235})$  is needed. A question arises as to whether the two discordant measurements, those of Friesen and Saplakoglu, actually add to our knowledge of this quantity; their inclusion in fact would decrease the error of the weighted mean from  $\pm 4.17$  to  $\pm 3.73$  b. We do not feel that this is a reasonable representation of the situation, but rather that their existence indicates the difficulties involved in  $\sigma_f$  measurements, and we have therefore added a  $\delta_{err}$  of  $\pm 2$  b to bring the error of the mean to  $\pm 4.23$  b, or essentially the same error as would have existed had these discordant values been rejected. This modest (about 22%) down-weighting factor was adopted rather than using an error of the mean based on the observed spread of the values, since it seems undesirable to allow the weights in the least-squares fit of the various input mean values to be subject to statistical fluctuations. This downweighting of the  $\sigma_f(U^{235})$  inputs causes the ICR [section 3.1, Eq.(3)] to decrease from 1.706 to 1.504.

### 4.5. $\eta$ data and ratios

Table VI shows the data for  $\eta$ . The measurements of Macklin et al. (No. 1) and Smith <u>et al.</u> (No. 3) were both made with a manganese bath method using thick samples. Although the incident neutrons in the Macklin work had a thermal spectrum, the correction factor used with a thick sample is not the g-factor (so this is <u>not</u> shown in the table with an asterisk); errors in the correction process are included in the errors shown. The Smith results are from a private communication and are preliminary, the errors being increased to about 1% to allow for this fact, from the 0.8% which the authors quote.

Both the Macklin (No. 1) and Gwin (No. 2) results are given as absolute values and ratios. Following the principles of section 2 above, we are

# TABLE VI

# NEUTRON YIELD PER ABSORPTION $(\eta)$

Authors Ver Ref			Absolute values	Commente	
	1ea nei.	U <sup>233</sup>	U 235	Pu <sup>239</sup>	Comments
1. Macklin, DeSaussure, Kington, Lyon	1960 [61] 1962	2.289±0.0104	2.071±0.0117	2. 139±0. 0158	Manganese resonance absorption correction has been recalculated, a small spectrum-shape correction has been applied; errors have been somewhat increased for spectrum shape uncertainties
2. Gwin, Magnuson	1962 [62]	2, 284 ± 0, 0140*	2, 071 ± 0, 0140*	-	The values have been corrected to 2200 m/s, and to the preferred hydrogen cross-section. The errors are decreased by this new normalization (from $\pm 0.015$ )
3. Smith, Reeder, Fluharty	1965 [63]	2.305±0.023	2.090±0.021	2,118±0.022	See text
Weighted mean		2, 2893 ± 0, 0079	2.0739±0.0083	2,1319±0,0128	
ICR		0, 3051 **	0. 3461	0.6011	

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TABLE	VI	(CONT.)	
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	•		Ratios	_	
Authors	Year, Ret.	U233/U235	Pu239/U233	Pu239/U235	Comments
1. Macklin, DeSaussure, Kington, Lyon	1960 [61] 1962	1.105±0.010	-	1.033±0.0211	Manganese resonance absorption correction has been recalculated, a small spectrum-shape correction has been applied; errors have been somewhat increased for spectrum shape uncertainties
2. Gwin, Magnuson	1961 [62]	1, 103±0, 0202*	-	-	The values have been corrected to 2200 m/s, and to the preferred hydrogen cross-section.
4. DeBoisblanc, Fast	1961 [64]	1, 114±0, 0120*	-	-	The value has been corrected to 2200 m/s and the error increased by a factor of $\sqrt{2}$ because of inadequate documentation
Weighted mean		1. 1080±0. 0072	-	1.033±0.0211	
ICR		0. 2006		-	

here representing the fact that certain errors in the absolute values are common to measurements of two or three nuclides, so that the ratios are better known than the absolute values. The errors quoted for the ratios thus represent only the difference between the accuracy of the ratios and that deduced from the accuracy of the absolute values, although in fact this procedure slightly over-weights the set of data to which it is applied.

### 4.6. $\nu$ data and ratios

Table VII shows the data for  $\nu$ ; the rather difficult situation concerning the absolute value for  $Cf^{252}$  is considered in section 4.6.1. The only other absolute value is that of Kenward (No.1), which is based on the calibration [76] of the Harwell standard  $Pu^{240}$  source, which gave  $1.989 \times 10^4 \pm 0.8\%$  neutrons/s. The value for  $Cf^{252}$  by Moat <u>et al.</u> (No. 2) is also based on the same calibration, and since the source strength error is dominant, we have had to treat these two values by dividing between them the weight available from the known accuracy of this source strength calibration, so that each appears as if based on a less accurate calibration than in fact exists.

Also, in connection with measurement No. 4 (Hopkins <u>et al.</u>) there is a situation similar to that described in section 4.5 above for Macklin and for Gwin; the absolute values of Hopkins are less accurately known than the ratios between values for different nuclides. In this case this fact arises from certain errors being systematic and common to all the measurements made, so that we have inserted the ratio values with errors which represent only the excess accuracy needed to supplement the absolute values.

4.6.1. Basic normalizing value,  $\nu$  (Cf<sup>252</sup>). In section 2 it was already mentioned that, since the 3rd Geneva Conference preliminary publication of this study [11], new measurements had produced a rather unsatisfactory situation concerning the absolute values of  $\nu$ . The results of Nos. 3 and 4 of Table VII, which were both obtained using the liquid scintillator technique agree quite well, but the Harwell boron-pile value (No. 5) with a claimed accuracy of  $\pm 0.4\%$  differs<sup>5</sup> from the other two by about 2%.

It seems unlikely, though not impossible, that the difference between the results is due to statistical fluctuations. Indeed Colvin and Sowerby stated (in Ref. [69], No.5 of Table VII) that their result indicated a possible systematic error in one, or both, of the techniques used. After discussions with a number of the physicists involved, we decided, as the preferred option, to give the boron-pile value the same weight as the two liquid scintillator results combined, and to do this by increasing its error, leaving the other errors unchanged.

However, to examine what effect would be produced, we have made the fit also for two other options: (a) downweighting the  $\nu$  (Cf<sup>252</sup>) mean value further because of this difficulty, and (b) accepting the Harwell accuracy as claimed. The results are considered in section 6 below. The downweighting used in option (a) corresponds to multiplying the error of the mean by a factor of 1.6; this factor is equal to the square root of the internal

<sup>&</sup>lt;sup>5</sup> At the time the publication [11] of this work was in preparation only a preliminary boron-pile result was available, which agreed much better with the liquid scintillator values.

### TABLE VII

# NEUTRON YIELD PER FISSION ( $\nu$ ) Delayed neutrons are included everywhere

	-	Absolute values					
Author	Year, Ref.	U <sup>233</sup>	U <sup>235</sup>	Pu <sup>239</sup> Cf <sup>252</sup>		Comments	
1. Kenward, Richmond, Sanders	1958 [65]	-	2. 373 ± 0. 029	-		The value has been corrected $0.32 \pm 0.3\%$ upwards for difference in fission spectrum between U <sup>235</sup> and Pu <sup>240</sup> , see text	
2. Moat, Mather, McTaggart	1961 [66]	-	-	-	3,680±0,067	See also MATHER 1964 [71] below, see text.	
3. Asplund-Nilsson, Condé, Starfelt	1963 [67]	-	-	-	3.808±0.034	Authors' value and error accepted	
4. Hopkins, Diven	1963 [68]	-	-	-	3.780±0.031	Authors' value and error accepted	
5. Colvin, Sowerby	1965 [69]	-	-	-	3.713±0.0229	The error of the $Cf^{252}\nu$ -value has been in- creased to give this input equal weight to those of Asplund-Nilsson and Hopkins put together	
Weighted mean		-	2.373±0.029	-	3.7488±0.0157		
ICR			-	-	2. 5144		

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NOTE ON OPTIONS: the table above shows the "middle" option.

For option (a) add a  $\delta_{err}$  term equal to 0.0194 (multiplies error of mean by 1.6); resulting input value is then 3.7488±0.0250. For option (b) reduce error of Colvin (No. 5) value to ±0.0150; resulting input value is 3.7350±0.0123.

TABLE	VII	(cont.)
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Authors - Mary Daf			Ratios to $\nu$ of Cf <sup>252</sup>		
Autors	Ital, Rei.	U <sup>233</sup> /Cf <sup>252</sup>	U <sup>235</sup> /Cf <sup>252</sup>	Pu <sup>239</sup> /Cf <sup>252</sup>	Comments
4. Hopkins, Diven	1963 [68]	0.6521±0.0053	0. 6423 ± 0. 0046	0.7566±0.0081	Authors' zero energy fitted values for $U^{233}$ and $U^{235}$ were transformed to ratios to $\nu$ of Cf <sup>252</sup> . The Pu <sup>239</sup> value was obtained from an extrapolation to zero energy of the experimental points below 1 MeV
5. Colvin, Sowerby	1965 [69]	-	0. 6423±0.0029	-	Authors' value and error accepted
6. Meadows, Whalen	1962 [70]	-	0.6458±0.0080	-	Authors' value and error accepted
7. Mather, Fieldhouse, Moat	1964 [71]	0.6700±0.0082	0.6421±0.0034	0. 7747±0. 0091	Authors' values and errors have been accepted
8. Condé, Holmberg	1965 [72]	-	0. 6427 ± 0. 0053	-	Authors' value and error accepted
Weighted mean		0.6574±0.0045	0.6425±0,0018	0.7646±0.0061	
ICR		3. 3611	0. 0480	2. 2073	

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TABLE VII (cont.)

Authors	Year, Ref.	Ratios c	of one fissile nuclide t	Comments	
		U233/U235	Pu239/U233	Pu239/U235	
4. Hopkins, Diven	1963 [68]	1.015±0.0222	1.160±0.0372	1.178±0.0341	Ratios between fissile nuclides were calculated and used as inputs, with their errors accordingly adjusted
5. Colvin, Sowerby	1965 [69]	1.020±0.0060	-	1.182±0.0080	Authors' values and errors accepted
9. Sanders	1956 [73]	1.006±0.0200	-	1.179±0.0400	Original values later amended by author. We have used values as quoted by SJÖSTRAND and STORY [7] but increased errors because the amendments are not documented
10. Jacob	1958 [74]	-	1.160±0.0240	-	Author's value accepted and error slightly increased
11. DeSaussure, Silver	1959 [75]	1.020±0.0120	-	-	Authors' value accepted and error slightly increased
Weighted mean		1.0189±0.0051	1.1600±0.0202	1.1817±0.0076	
ICR		0. 1629	0	0. 0089	~

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consistency ratio (Eq. (3), in section 3.1) for these input values, so that the effect of this change is to keep the relative weights of the different measurements unchanged but to base the error used in the least-squares fit on the actual spread between the measured values.

We therefore present as our 'middle' or preferred option a value of 3.749 for our  $\nu$  (Cf<sup>252</sup>) input; the 'downweighted from spread' value is the same, with a larger error, while the acceptance of the claimed accuracy for the Colvin value (No. 5) gives 3.735 for the input for  $\nu$  (Cf<sup>252</sup>). Further discussion is in section 6 below.

### 4.7. Capture cross-sections and ratio of capture to fission ( $\alpha$ )

Table VIIIa shows the data used for the least-squares input for  $\alpha$ , and Table VIIIb that for  $\sigma_{\gamma}$ . Most of the important measurements in this group were made in reactor spectra and for this reason careful assessment of their accuracy has been necessary.

### 4.8. Other combinations of constants for U isotopes and Pu<sup>239</sup>

There remain measurements of combinations of  $\eta$  with cross-sections, made in various arrangements, the values used being shown in Table IX. It was mentioned earlier that "pile oscillator" and similar measurements are difficult to interpret in terms of fundamental constants, and especially for the older values (Nos. 1 and 2 of Table IX) we have taken a more cautious approach in stating errors, since our philosophy (section 2) is to be more critical and expect fuller documentation than was earlier demanded by SJÖSTRAND and STORY [7]. In the case of these first two measurements, although the results were presented as  $\eta$  measurements, we have taken advantage of the work of Ref. [7] in recomputing these data into the  $(\eta - 1)\sigma_a$ form which was, at least very nearly, what was actually measured. For value No. 2 (Table IX) in particular the publication was made so long after the measurement that this method seems preferred. In this case a separate evaluation, of which we have no particulars, has also been made by SHER and FELBERBAUM [10], who obtain slightly different values; this is another reason why we felt unable to accept the values with smaller errors than those shown in the table. Also, while the Cabell value (No. 3) is not so old as the others the report concerned is unpublished; the additional downweighting of the GLEEP results is due to the spectrum in this reactor departing further from a Maxwellian form than in DIMPLE.

It should also be remarked that, on account of doubts concerning calibration, and the relative statistical weight of fast and slow neutrons in the neutron balance for the arrangements used, we have felt it desirable to present these results only as ratios of one nuclide to another, and have discarded all the absolute  $(\eta - 1)\sigma_a$  values. The  $\eta\sigma_a$  measurements (No. 4 of Table IX) were of course only presented as relative values by Gwin and Magnuson.

For both results 3 and 4 of Table IX, three ratios are presented, of which one is strictly redundant. However, the measurements were symmetrical in the three quantities concerned, so following the principles given in section 2 above, all ratios have been retained but with errors increased by a factor  $\sqrt{3/2}$ .

### TABLE VIIIa

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CAPTURE-TO-FISSION CROSS-SECTION RATIOS ( $\alpha$ )

Authors	Year, Ref.	U233	U235	Pu <sup>239</sup>	Comments
1. Inghram, Hess, Hayden, Stevens	1956 [77]	0.0980±0.00283*	-	-	This result has been recalculated using modern data, and the measured error $\times \sqrt{2}$ . Further details of this measurement are available from Chalk River reactor record
2. Cocking	1958 [53]	0. 1130±0.0212	0.172±0.025	-	Author's measured values at 0, 0011 eV have been used as values at 0, 0253 eV, assuming that $\alpha$ is constant within this energy range and allowing a $\pm 1\%$ error on $(1 + \alpha)$ for the uncertainty of this assumption
3. Safford, Melkonian	1959 [55]	-	0.171±0.014	-	Authors' measured value at 0.0029 eV has been used at 0.0253 eV (same assumption as for Cocking). Error has been further increased since fission cross-section value based on this work is also used as an input
4. Cornish	1960 [78]	-	0.190±0.014*	-	Details of measurement in private communication. The value has been recalculated for recent g- and s-values and the error X 1.4
5. Cabell, Slee	1962 [79] 1963	0.0909±0.0023*	0.1715±0.0015* -	0.356±0.0092*	Authors' values have been recalculated and the errors re-assessed. It has been possible to reduce their error on the U <sup>235</sup> value con- siderably, where they had overlooked that the errors of some of the constants used in the correction from $\hat{\alpha}$ to $\alpha_0$ were correlated and tended to cancel

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Authors Year, Ref.	U 233	U 235	Pu 239 ·	Comments
6. Okazaki, Lounsbury, 1964 [80] Durham, Crocker; also Okazaki, Lounsbury, Durham	0.0936±0.00125*	0.1718±0.0015*	-	The error of the U <sup>233</sup> value has been somewhat increased because it was felt that the allowance for spectrum uncertainties was insufficient. The U <sup>235</sup> value has been given the same weight as the datum by CABELL[79, ii]. The U <sup>233</sup> result differs slightly from published value because of a recalculation with a revised value for the resonance-capture integral
<ol> <li>Durham, Crocker, Hart, 1964 [81] Jones, Lounsbury, Bigham, Hanna; also Durham, Crocker, Harz, Lounsbury, Hanna</li> </ol>	-	0. 1765±0. 0015*	0.362±0.0092*	The claimed accuracy for $U^{235}$ has been accepted but that for $Pu^{239}$ (± 0, 006) has been increased so as to give it equal weight with the Cabell, Slee [79] determination which involved more measurements, but in an inferior spectrum
Weighted mean	0.09369±0.00102	0.17332±0.00086	0.3590±0.0065	
ICR	1.5419	1. 6886	0. 2127	

TABLE VIIIa (cont.)

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### TABLE VIII b

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	Authors	Year, Ref.	U <sup>238</sup>	U235	Pu <sup>239</sup>	Comments .
8.	Cornish, Lounsbury	1956 [82]	-	-	283±10*	Recalculated with recent cross-section values and g- and s-values.
9.	Halperin, Johnston, Stoughton, Oliver, Blevins, Druschel, Harkness, Swarz	1963 [83]	52.7±3.0*	-		Renormalized to our preferred cobalt activation cross-section value $37.7 \pm 0.7$ b; error slightly increased since the account is very brief
10.	Hanna	1962 [84]	-	-	265 ± 12*	A value derived from mass spectrometric studies of irradiated fuel. The claimed error $(\pm 7 \text{ b}) \times \sqrt{3}$ , because the interpretation of the work is complex and unpublished
We	ighted mean		52.7±3.0	-	275.6±7.7	
ICF	2			_	1. 3279	

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# CAPTURE CROSS-SECTIONS IN BARNS

Authors	Year, Ref.	U <sup>233</sup> /U <sup>235</sup>	Pu <sup>239</sup> /U <sup>233</sup>	Pu <sup>239</sup> /U <sup>235</sup>	Comments
1. Alkhanov, Vladimirsky, Nikitin	1956 [85]	1.014±0.036*	-	1.456±0.098 <sup>*</sup>	The values adopted are those quoted by SJÖSTRAND [7]; errors (from the same source) × 2, see text
2. Muelhouse	1959 [86]	0.977±0.038*	-	1,408±0,096*	The values adopted are those quoted by SJÖSTRAND [7]; errors (from the same source) x 2, see text
3. Cabell, Rose, Tattersall	1960 [87]	1,027±0,025*	1,474±0.039*	1,513±0.037*	These results have been recalculated, using modern cross-sections; resulting errors, which were based on author's claim, $\times \sqrt{2}$ for DIMPLE results and $\times 2$ for GLEEP results. These sets of results have then been averaged, and the errors further $\times \sqrt{3/2}$ because they form a non- independent triangle
Weighted mean		1,0124±0.0181	1.474±0.039	<b>1.4946</b> ±0.0326	
ICR		0. 6054	-	0. 6081	

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# TABLE IXb

# $\eta \sigma_a$ RATIOS

Authors	Year, Ref.	U 233/U 235	Pu <sup>239</sup> /U <sup>233</sup>	Pu <sup>239</sup> /U <sup>235</sup>	Comments
4. Gwin, Magnuson	1962 [62]	0.9434±0.0168 <sup>**</sup>	$1.5714 \pm 0.0279$	1, 4857±0, 0264*	A symmetric input has been obtained using the data in Table II of their paper. Values have been corrected to 2200 m/s, errors $\times \sqrt{3/2}$ because the ratios form a non-independent triangle
Weighted mean		0.9434±0.0168	1,5714±0.0279	1.4857±0.0264	
ICR		-	-	-	

### TABLE X

# INPUT DATA FOR Pu<sup>241</sup>

Authors	Year, Ref.	Datum	Value	Normalization value	Normalized value	Weighted mean	Comments
1. Simpson, Schuman; Simpson, Marshall	1961 [88]	o <sub>a</sub>	1379 ± 50	-	-		Since the authors quote no 2200 m/s value, a value has been obtained by averaging $\sigma_T \times \sqrt{E}$ for 40 points between 0.02 eV and 0.03 eV. The error is mainly due to sample thickness uncertainty
2. Craig, Westcott	1964 [89]	σ <sub>a</sub>	1372±40	_	-		Since the error allowed for uncertainty of sample thickness was rather small and no precautions were taken against possible water vapour in the sample, error increased from 30 to 40 b
3. Kalashnikova, Lebedev, Mikaelyan, Spivak, Zakharova	1955 [90]	ν (Ρυ <sup>241</sup> )/ν (Pu <sup>239</sup> )	1.048±0.014	2.8701±0.0116	3.008±0.042		This is an early work and the account is brief, so that only the $Pu^{241}/Pu^{239} \nu$ ratio is used; quoted error $x \sqrt{2}$
4. Sanders	1956 [73]	ν (Pu <sup>241</sup> )/ν (U <sup>235</sup> )	1.240±0.080	2.4302±0.0061	3,013±0,195	2. 9727 ± 0. 0213	Author's value accepted and error increased from 0.052 to 0.080, mainly because for this ratio the revision mentioned in Table VII (No. 9) has not been made
5. DeSaussure, Silver	1959 [75]	ν (Pu <sup>241</sup> )/ν (Pu <sup>239</sup> )	1,059±0,019	2.8701±0.0116	3,039±0.056		Although these authors give a ratio $Pu^{241}/U^{235}$ they did not actually measure it but calculated it from their data. We have instead calculated a $Pu^{241}/Pu^{239}$ ratio, where a possible error due to differences between fission neutron spectra would be small
6. Colvin, Sowerby	1965 [69]	$v (Pu^{241}) / v (U^{235})$	1.210±0.011	2,4302±0,0061	2, 941 ± 0, 028	-	Authors' value and error accepted

	Authors	Year, Ref.	Datum	Value	Normalization value	Normalized value	Weighted mean	Comments
7.	Jaffey, Studier, Fields, Bentley	1955 [91]	σ <sub>f</sub> (Pu <sup>241</sup> )/σ <sub>f</sub> (Pu <sup>239</sup> )	1.372±0.032*	742.52±3.10	1018.7±24.1	ł	This measurement has been re-interpreted; resulting error $\times \sqrt{2}$ , following our general criteria
8.	Raffle	1956 [92]	σf (Pu <sup>241</sup> )/σf (Pu <sup>239</sup> )	1.332±0.057	742, 52±3, 10	989.0±42.5	1012. 20 ± 8. 55	A ratio is obtained from the quoted figures 935 b and 702 b, and the error is taken to 40 b in 935, which implies some down- weighting, because no details are given
9.	Bigham, Hanna, Tunnicliffe, Campion, Lounsbury, MacKenzie	1958 [51]	σf (Pu <sup>241</sup> )/σf (Pu <sup>239</sup> )	1,367±0.0115*	742.52±3.10	i315.0±9.5		See text
10.	Leonard	1959 [93]	σ <sub>f</sub> (Pu <sup>241</sup> )/σ <sub>f</sub> (U <sup>235</sup> )	1,618±0.087	579.18±1.60	937, 1±50, 5	± .	Insufficient documentation; error $\times \sqrt{3/2}$
11.	Fields, Pyle, Inghram, Diamond, Studier, Manning	1956 [94]	σγ	374 ± 86*	-	-	374 ± 86	Corrected to 2200 m/s using current g-factors
12,	Jaffey, Hibdon, Sjoblom	1959 [95]	$\eta \sigma_{a} (\mathrm{Pu}^{241}) / \eta \sigma_{a} (\mathrm{Pu}^{235})$	1.387±0.032*	2131.4±12.1	2956, 3 ± 70, 2	2956.3±70.2	Only the results for Pu <sup>241</sup> and Pu <sup>239</sup> have been used for this ratio
13.	Cabell	1965 [96]	α	0.388±0.023*	-	-	0.3SS±0.023	

# TABLE X (cont.)

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### 4.9. Input values for Pu<sup>241</sup> fit

Table X shows the input measured values which we use for the Pu<sup>241</sup> fit. Column 4 of this table contains the values from the fit for the other nuclides which we use to interpret the ratio measurements; details of how these are obtained are discussed in section 6. It has already been remarked that the data for this nuclide are of much lower quality than for Pu<sup>239</sup> or the fissile U isotopes, but we have endeavoured to maintain the same criteria of acceptability. The g-factors for this nuclide have been re-calculated from up-to-date data; the older absorption g-factor values [12] were based on  $\sigma_{\rm f}$  measurements as representing the best data then available. In this table, as elsewhere, the measurements made in a Maxwellian spectrum are marked by an asterisk. Here, however, the errors of the values in column 3 contain an allowance for the error of the relevant g-factor.

### 5. RESULTS NOT USED FOR THE LEAST-SQUARES FITTING PROCESS

The list of those references which were examined, but for which the values obtained were discarded is given as an Annex, although this is rather a collection of the more important examples, and the list may not be somplete. Section A is for those references where it was difficult or impossible to obtain the document, or for which the document was such an incomplete description of the work that it was felt unwise to use the values given in any way. In this connection, we had personal discussions with Dr. J.S. Story (co-author of Ref. [7]) and were thus able to ascertain some facts about some documents which we could not obtain. Section B of the table i cludes short papers or similar documents where the work was described in some, but we felt insufficient, detail, or where the techniques have advanced so greatly since that the work was felt to be of little use at the present time. Section C is for measurements where uncertainties of the neutron spectrum, or other factors indicated, were such that it seemed unwise to attempt to interpret the result as a 2200 m/s value, and section D covers measurements superseded by later work. This table is given more as an illustration of how difficult cases were dealt with and may be incomplete, but we can supplement it by saying that every reference in Sjöstrand and Story's [7] valuable and exhaustive earlier compilation was considered again in the course of this work.

### 6. RESULTS AND DISCUSSION

The weighted mean values derived in section 4, which constitute the input data to the least-squares fitting routine for the uranium isotopes and  $Pu^{239}$ , are summarized in Table XI. In this table are shown also the errors (E\*) as they are modified by allowing for, where appropriate, the errors of the g-factors used to correct Maxwellian values to 2200 m/s. The least-squares fitting procedure was carried out with and without the inclusion of these g-factor errors, to study what effects resulted from each assumption (see Tables XII and XIII). A third alternative type of fit is possible:

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# TABLE XI IN PUT DATA FOR U<sup>233</sup>, U<sup>235</sup> AND Pu<sup>239</sup><sup>a</sup>

#### [] 233 [] 235 Pu 239 Е\* E \*\* E\* Value Quantity Value Error Value Error Error 1006,56 574,95 2,90 2,90 680, 57 2,70 2,70 6,42 6,42 σa 740.38b 515,71 9,49 9.49 583,49 4,23 4,23 5.31 5,43 Øf 52,70 3,00 3,00 275.62 7.68 7.68 no value σγ 0,3990 0,0937 10 34 0,1733 9 25 65 71 α 0.6425 0,7646 61 0,6574 18 18 61 <sup>ν/ν</sup>Cf 45 45 2, 0739 <sup>b</sup> 2. 2893<sup>b</sup> 79 81 83 84 2,1319 128 128 η

		Ratios									
	U <sup>233</sup> /U <sup>235</sup>			Pu <sup>239</sup> /U <sup>233</sup>			Pu <sup>239</sup> /U <sup>235</sup>				
Quantity	Value	Error	E*	Value	Error	E %	Value	Error	E*		
σ <sub>f</sub>	0, <b>91</b> 08	12	32	1.4170	284	286	1, 3035 <sup>b</sup>	94	98		
ν	1,0189	51	51	1, 1600	202	202	1, 1817	76	76		
η	1, 1080 <sup>b</sup>	72	75	n n	o value		1.0330	211	211		
ησ <sub>a</sub>	0, 9434	168	172	1, 5714	279	286 •	1.4857	264	268		
$(\eta - 1)\sigma_a$	1.0124	181	181	1.4740	390	390	1.4946	326	326		
· ·	1			1							

Options	Option (a) "Downweight for spread"	Middle option (preferred)	Option (b) Full weight as claimed		
Quantity	Value Error	Value Error	Value Error		
ν (Cf <sup>252</sup> ) ν (U <sup>235</sup> )	3. 7488 250 2. 3730 460	3.7488     157       2.3730     290	3. 73501232. 3730290		

 $E^*$  = error used when g-errors are included. When g-errors are much smaller than other errors, we neglect this allowance.

<sup>a</sup> In this table all cross-sections and their errors are in barns, while ratios and other quantities are dimensionless and their errors are in units of  $10^{-4}$ .

<sup>b</sup> These values change slightly when g-errors are included, due to the relative weight of 2200 m/s and Maxwellian values changing. The 'with g-error' values are respectively:

a mixed set of input data, some measured with monokinetic neu 1 some with Maxwellian spectra, it is, in principle, possible to dedu ...er a 2200 m/s set of values or a set applicable to a 20.4°C Maxwellian neutron spectrum. To perform the latter process, the errors applicable to the gfactors used for conversion have to be added (in quadrature) to the errors of all measurements not made in a Maxwellian spectrum (except for  $\nu$  measurements, which are spectrum-insensitive). This process was carried out on one set of input data, quite close to that finally adopted, and it was found that the output values from this fit did not differ significantly from the results obtained ignoring all g-factor errors, and their errors, in comparison to the case with no g-error, were never greater by more than one unit in the last place shown in Table XII and then only due to rounding-off, since the actual difference was usually less than half a unit in this place.

The input for the  $Pu^{241}$  fitting programme, Table X, includes some results from the fit for the other three nuclides, which have to be used to interpret the ratio measurements. For those  $Pu^{241}$  input values ( $\sigma_f$  and  $\eta\sigma_a$ ), which were measured in Maxwellian spectra, the value for the comparison isotope was taken from the 'g-errors omitted' fit. With the error figure increased by one unit in the last place shown, to be on the safe side, such a value represents the value for the comparison isotope in a Maxwellian spectrum. Then we can obtain a fit with g-errors included for  $Pu^{241}$  (for which no 'no g-error' fit has been made) by adding the error for the  $Pu^{241}$  g-factor when interpreting a ratio measurement input datum, but no allowance for any g-factor error for the comparison isotope is required.

For the three nuclides U<sup>233</sup>, U<sup>235</sup> and Pu<sup>239</sup> three separate sets of fits were made, corresponding to the three options for the value and accuracy of the absolute  $\nu$  (Cf<sup>252</sup>) measurement, which arose as described in section .4.6.1 above; these alternative inputs for  $\nu$  (Cf<sup>252</sup>) are shown at the bottom of Table XI. The results of the least-squares fit for the middle option, and for g-errors included, is given in Table XII, while Table XIII gives the results obtained on a 'no g-error' basis, and Table XIV the Pu<sup>241</sup> results. It will be seen that the inclusion or omission of the g-factor error does not significantly affect the values or errors for  $\nu$  or  $\eta$ , but that for some of the cross-sections and  $\alpha$  a difference comparable with the standard deviation is found between the two sets of output values, and the accuracy of  $\alpha$  and  $\sigma_{\gamma}$  for the uranium isotopes is particularly affected by the inclusion of gfactor errors. The worst cases are for  $\alpha$  and  $\sigma_{y}$  for U<sup>233</sup> where the standard deviation of the 'no g-error' fit is small and almost equal to the difference between the two output values; in all other cases the differences are much smaller than the errors. On the whole, therefore the differences between the values obtained with and without g-errors are small enough to enable us to feel confident that the procedures chosen are satisfactory in this respect.

As was already mentioned in section 3.2, a 16-parameter fit was tried to ascertain whether the effects of correlations due to possible g-factor errors could be serious. In such a fit a change of the g-factors from the nominal values used as input would indicate such an effect. The changes found were negligible, the largest being for the g-factor for  $\eta$  of U<sup>233</sup>, which decreased 0.15% from its nominal value, the other changes being 0.1% or less. The other quantities fitted were not appreciably affected;  $\eta$  (U<sup>233</sup>) itself increased

### TABLE XII

### OUTPUT OF LEAST-SQUARES FIT FOR 2200 m/s PARAMETERS<sup>a</sup>, g-ERRORS INCLUDED

Quantity	Value	Error	Value	Error	Value	Error	
ν (Cf <sup>252</sup> )	<b></b>		3. 7724	107			
	U <sup>23</sup>	3	U <sup>23</sup>	15	թս <sup>2:</sup>	39	
σ <sub>a</sub>	576.3	2. 0	679,9	2. 0	1008.1	4, 3	
$a_{\mathrm{f}}$	527.7	1. 9	579.5	1. 8	742.4	3. 1	
σγ	48,6	1, 3	100,5	1.2	265.7	3, 3	
α	0.0921	25	0.1734	22	0,3580	48	
ν	2, 4943	78	2. 4296	66	2.8705	118	
η	2, 2839	63	2, 0707	55	2, 1138	86	
$\nu/\nu$ (Cf <sup>252</sup> )	0.6612	21	0.6441	15	0.7609	31	
	U <sup>233</sup> /1	J <sup>235</sup>	Рц <sup>239</sup> /	Pu <sup>239</sup> /U <sup>233</sup>		Pu <sup>239</sup> /U <sup>235</sup>	
٥f	0,9107	27	1, 4067	68	1. 2811	56	
ν	1.0266	30	1.1509	51	1, 1815	46	
η	1, 1030	33	0.9255	41	1,0208	43	
$\eta \sigma_{a}$	0, 9349	35	1, 6189	89	1, 5135	75	
(η-1)σ <sub>a</sub>	1,0165	56	1. 5174	122	1.5424	116	

### Middle $\nu$ option

<sup>a</sup> In this table all cross-sections and their errors are in barns, while all ratios are dimensionless and their errors are in units of  $10^{-4}$ 

by 0.0014 from the value obtained with the 10-parameter fit, its error being about  $\pm 0.0075$ , while  $\sigma_a(U^{233})$  decreased by about 0.25 b and  $\sigma_f$  for the same isotope increased by 0.2 b. The changes for  $U^{235}$  were smaller, and for  $Pu^{239}$  were smaller or at most comparable to those for  $U^{233}$ , although the accuracy of the values for this nuclide was lower than for the uranium isotopes, so the changes were less significant. It was therefore concluded that, unless the estimated errors of the g-factors, given in Table I, were

### TABLE XIII

### OUTPUT OF LEAST-SQUARES FIT FOR 2200 m/s PARAMETERS<sup>a</sup>, g-ERRORS EXCLUDED

Quantity	Value	Error	Value	Error	Value	Error	
ν(Cf <sup>252</sup> )			3. 7733	103 ——			
	U23	3	U <sup>23</sup>	5	Pu <sup>z</sup>	Pu <sup>239</sup>	
σ <sub>a</sub>	576.8	1.7	679.6	1, 9	1008.3	4.3	
σf	527.5	1.5	579. 2	1. 6	742.5	3.0	
cγ	49.3	0.5	100.4	0, 5	265. 8	3. 2	
α	0,0934	10	0, 1734	8	0,3580	46	
ν	2, 4967	67	2, 4302	60	2,8706	115	
η	2, 2833	60	2. 0711	52	2. 1139	85	
ν/ν (Cf <sup>252</sup> )	0, 6617	20	0. 6441	15	0.7608	31	
	IJ <sup>233</sup> /(	J235	Pu <sup>239</sup> /U <sup>233</sup>		Pu <sup>239</sup> /U <sup>235</sup>		
σ <sub>f</sub>	0.9107	12	1.4077	60	1. 2820	54	
ν	1,0274	27	1, 1498	48	1, 1813	45	
η	1, 1025	29	0. 9258	40	1.0207	41	
ησ <sub>a</sub>	0. 9357	27	1. 6185	84	1.5144	74	
$(\eta - 1)\sigma_{a}$	1.0169	50	1.5174	119	1.5430	114	

### Middle $\nu$ option

<sup>a</sup> In this table all cross-sections and their errors are in barns, while all ratios are dimensionless and their errors are in units of  $10^{-4}$ 

to be considerably increased, there was no need to change from the 10parameter to the 16-parameter fitting procedure.

The effects of the choice between the three options for  $\nu$  (Cf<sup>252</sup>), which are mentioned earlier in this section and described in section 4.6.1, are shown in Table XV, which shows the results obtained for  $\eta$  and  $\nu$ . In option (a) the Kenward absolute  $\eta$  measurement is downweighted similarly to the input mean value of  $\nu$  (Cf) since we feel that the unreliability (indicated by the

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### TABLE XIV

### OUTPUT OF LEAST-SQUARES FIT FOR Pu<sup>241</sup> 2200 m/s PARAMETERS, g-ERRORS INCLUDED<sup>2</sup>

Quantity	Value	Error		
σ <sub>a</sub>	1390.8	19.5		
σf	1008.8	7.8		
σγ	382.0	18. 2		
α	0. 3787	184		
υ	2. 9693	204		
η	2. 1538	321		
ησa	2995.4	29.4		

<sup>a</sup> In this table all cross-sections and their errors are in barns, while the errors of dimensionless quantities are in units of  $10^{-4}$ 

spread of values) may affect all absolute measurements of  $\nu$  similarly. Option (b) resembles the 'middle' (or preferred) option except that the Colvin and Sowerby result is given its full claimed 0.4% accuracy. This table shows the 'g-errors included' fits which are more sensitive to the choice of option than the 'no g-error' results.

A significant difference related to this choice of option is that some of the errors shown in Table XV are noticeably smaller for option (b). We do not feel that the (b) option errors are physically meaningful, since the situation for  $Cf^{252}$  is not satisfactory, and in fact, although the values from our 'middle' option appear a good compromise, the errors should perhaps be taken from option (a) of Table XV, to take into account this spread of the  $Cf^{252}$  values. However, the differences are not very large and this may be only a matter of principle. The main effects of the choice of option is to vary all the  $\nu$ 's, and also the  $\eta$ 's, substantially in the same proportion; this is evidently due, at least in part, to the high accuracy of the  $\alpha$  measurements.

The quantities not shown in Table XV are rather little affected by the choice of option. The most sensitive is  $\alpha$ , for which the range within which the three values lie is about 0.0012 wide for the uranium isotopes but only 0.0015 wide for Pu<sup>239</sup>, while for  $\sigma_{\gamma}$  it is about 0.5 b wide for uranium isotopes and 0.8 b for Pu<sup>239</sup>. The ratios between different nuclides for  $\nu$  and  $\eta$  lie within a range which is largest for U<sup>233</sup>, but never more than 0.0013 wide, and the range for  $\sigma_{a}$ 's and  $\sigma_{f}$ 's is only of the order of 0.2 and 0.5 b

respectively, or at most 0.0002 in the ratios. What is significant is that these quantities are so little affected, and especially that  $\sigma_f$ 's and  $\sigma_a$ 's are sensibly independent of the  $\nu$ -option chosen.

### 6.1. Statistical criteria

There is a possible criticism of the present study, based on the feeling that the output errors are unreasonably small; for example, some physicists with considerable experience in measuring cross-sections have expressed doubts as to whether  $\sigma_a$  or  $\sigma_f$  for the uranium isotopes can now be considered as known to the accuracy (about  $\pm 2$  b) claimed. This criticism is basic to the way we use the least-squares method, and can be directed to the question of whether, as we have assumed, the errors of measurement are truly random and independent. Certainly systematic errors may exist, and when these are considerably smaller than the random errors of a particular investigation, the authors may tend to ignore them in presenting their results. When many such results are averaged these systematic factors could become important. For our work, however, the experimental methods used were so various in character, that any serious systematic error is unlikely to be common to more than a rather small fraction of the set of input data used. The methods of measurement have been reviewed for such possible systematic errors, but none was discovered which it was felt could have been significant.

Statistical tests have also been made on our values, and for this purpose we have had the advantage of consultations with Professor L. Schmetterer of Vienna University, who has contributed to the Appendix which deals with these points. The tests used are essentially standard  $X^2$  and F-type tests. but some explanation of their application in this work seems appropriate, and is given in the Appendix cited. Unfortunately, due to the small sample size which is available, these tests are limited in what they can say. The first test is a  $X^2$  test which measures the spread between values obtained for measurements of the same parameter, and is a sum of terms of the type (N-1) times the internal consistency ratio (Eq. (3), section 3.1) taken over all parameters for which at least two measurements exist. The second is also a  $X^2$  test, but one which measures the spread of the  $Y_{mean}$  input values from the  $\overline{Y}$  set finally obtained, and the third is an overall X<sup>2</sup> for the sum of both these spreads and can be used to answer the question of whether all the assumed standard errors attributed to the measurements need to be scaled upward or downward to be statistically acceptable as an independent set of random errors in view of the actual spread of the values. Finally, the F-test shows whether this type of assumption is reasonable for this case.

The application of these tests is complicated by any correlations which may exist and these would only be fully removed by adding additional parameters, as was done in a test (section 3.2) for the g-factors. However, for practical purposes the tests seem useful, with some allowance, in the form of a reduction of the nominal number of degrees of freedom, for those cases where both absolute values and derived ratios were used as inputs. It is then found that: 2200 m/s CONSTANTS

(i) the spread of the values within the sets of values for each input parameter is somewhat smaller than would be expected from the errors attributed to the results;

(ii) the spread between input mean values and output values is larger than would be expected on the same basis;

(iii) the spread as a whole is rather close to its expectation on the same assumptions.

The values for the g-errors-in fit, middle  $\nu$  (Cf) option, are:

(i)  $X^2 = 53.3$  versus expectation 59, lower limit 39.6

(ii)  $X^2 = 33.3$  versus expectation 22, upper limit 36.8

(iii) X<sup>2</sup> = 86.7 versus expectation 81, upper limit 107.7

(the limits are for 95% probability, or 2.5% of being outside the limits at either end of the range).

The values of F are subject to a slight uncertainty (perhaps  $\pm 0.02$ ) due to the allowance made, as already mentioned, in the effective number of degrees of freedom because of the use of "extra" ratio inputs which are known to be consistent with the other inputs for the same quantity. Keeping this in mind, we note that we obtain F = 0.57 for the middle option (g-in), whereas the 95% probability limits within which F should lie are 0.52 and 2.17, so that the value obtained is uncomfortably near the lower limit. If we take the option (b) (no  $\nu$  (Cf) downweighting) F becomes 0.51 and for this case X<sup>2</sup> for test (ii) becomes 38.3 which also lies outside the 95% probability limit.

Therefore we must conclude that, even allowing for the fact that the downweighting which was applied when full information was lacking was admittedly somewhat arbitrary, comparison of different results of measurements of the same quantity indicates that we have only downweighted our overall results slightly more than the spread would suggest; on the other hand, the consistency of all the different quantities with one another is on the borderline of being statistically unacceptable, and this is a tendency in the other direction (i.e. the differences are too large to be very probable on the basis that the errors of the weighted means represent truly statistical fluctuations with the standard deviations given). This can be due to various forms of systematic error which we have not been able to identify.

### 6.2. Further comments on results

In addition to performing these statistical tests, the data have been examined to see where the differences between the input mean values and the fitted output are large compared with the errors concerned. Such studies being effectively studies of single values, are of much less cogency than tests on a sample of statistically significant size, and the most important case,  $\nu$  (Cf<sup>252</sup>), has, of course, already been mentioned at length. However, it seems worth while to point out that some rather large input-output differences seem to have occurred in connection with  $\sigma_f$  values and ratios, especially for the Bigham et al. Pu<sup>239</sup>/U<sup>235</sup> ratio (No. 4 in Table V, Ref. [51]). The Bigham absolute value of  $\sigma_f$  (U<sup>233</sup>) which, as mentioned in section 4.4, has been put into the present study with a somewhat reduced weight, gives a measured value 517.5 (±4.7 b, on the basis of present correction factors, g-factor error excluded) and lies about 10 b below our output value. However, these may both be due to statistical fluctuations. It is true that for plutonium there is a lack of reliable measurements of  $\sigma_f$  with monokinetic 2200 m/s neutrons, which is unfortunate since those monokinetic values which do exist all seem appreciably below the values indicated by the Maxwellian spectrum measurements. It must in any case be admitted that measurements on Pu<sup>239</sup> generally are somewhat more difficult than for the uranium isotopes, and the errors given in our output tables are noticeably higher.

For  $Pu^{241}$  the accuracy attainable is considerably worse, if only because reasonably pure samples of this isotope have only rather recently become available, and its short half-life is also a disadvantage. The new (unpublished)  $\alpha$  measurement of Cabell and Slee (Table VIII, No.5) for this nuclide has changed the output values noticeably since the preliminary 3rd Geneva Conference [11] publication of the present work, as have some reassessments of other data, but for this nuclide almost every parameter could be measured with higher accuracy, and only the future can show whether the present values are reliable.

For the other nuclides there are also some changes since the Geneva presentation of this work, but these are to a great extent connected with the  $\nu$  (Cf<sup>252</sup>) problem already discussed. Perhaps a final comment may be made on this problem. As Colvin and Sowerby stated in their Salzburg Conference paper (Ref. [69], No.5 in Table VII), the value which is obtained indirectly from, for example, measurements of  $\eta$  and  $\alpha$  for U<sup>235</sup>, lies close to the liquid scintillator value. This fact is fully borne out in the present study; thus when the  $\nu$  (Cf<sup>252</sup>) input value [option (a)] is 3.749 the output value is 3.784; for option (b), with no downweighting, the input value goes down to 3.735 and carries a higher weight while the output value only falls to 3.759. From our results it is clear that if the input for  $\nu$  (Cf<sup>252</sup>) were given only a very low weight, our fit would give an output value of about 3.79.

6.3. Conclusions: numerical values recommended (Table XVI)

It has to be admitted that the problem we have been studying has not reached finality - not only is there the difficulty concerning the values of  $\nu$  and  $\eta$  arising from  $\nu$  (Cf<sup>252</sup>) measurements, but also that final values may be produced for those measurements [Smith,  $\eta$  (U<sup>233</sup>), or Cabell  $\alpha$  (Pu<sup>241</sup>), for example] for which we now have only provisional figures. The work of Vogt [13] on g-factor accuracy is also awaited with interest.

Nevertheless we feel that one can always find something incomplete in this field, and a "cut-off date" must be set to enable some results to be published. The values we now recommend are essentially those from Tables XII and XIV, with some upward adjustments of errors, so as to allow for the spread shown in Table XV. However, a further small increase of all the errors was made to allow for the possible presence of some errors of a non-random type, indicated by the X<sup>2</sup> and F tests. For the X<sup>2</sup> test (b) of section 6.1 a factor of 1.23 (= $\sqrt{33.3/22}$ ) applied to all errors would have caused the X<sup>2</sup> obtained to become equal to its expectation; we have chosen

### TABLE XV

### CHANGES FOR DIFFERENT $\nu$ (Cf<sup>252</sup>) INPUT DATA

OPTIO	N (a)		(Downw	veight for spread)		
ν			Cf <sup>252</sup>	3.7842±0.0127		
ν	U <sup>233</sup>	2.4984±0.0081	U <sup>235</sup>	2, 4354±0, 0072	Pu <sup>239</sup>	2.8759±0.0121
η	U <sup>233</sup>	2. 2865 ± 0. 0065	U <sup>235</sup>	2.0744 ± 0.0058	Pu <sup>239</sup>	2.1166±0.0088

PREFERRED OPTION								
ν			Cf <sup>252</sup>	3,7724±0.0107				
ν	U <sup>233</sup>	2. 4943 ± 0. 0078	U <sup>235</sup>	2.4296±0.0066	Pu <sup>239</sup>	2.8705±0.0118		
η	U 233	2, 2839 ± 0, 0063	U <sup>235</sup>	2.0707±0.0055	Pu <sup>239</sup>	2,1138±0,0086		

OI	TION (b)		(Full we	rights as claimed)		
ν			Cf 252	3.7592±0.0094		
υ	U 233	2,4902±0,0077	U <sup>235</sup>	2,4244±0,0062	Pu 239	2,8652±0,0116
η	U 233	2, 2813 ± 0. 0063	U <sup>235</sup>	2.0672±0.0054	Pu 239	<b>2.1111</b> ±0.0086

to apply to the output errors an intermediate factor of 1.125, as being a reasonable compromise in this situation. The resulting errors are quoted as standard deviations, and any readers who desire to deduce "limits of error" from them may apply whatever conventional factor they may choose.

We thus obtain the recommended values and their errors as given in Table XVI (also shown in the abstract).

### 7. COMPARISON WITH EARLIER VALUES

It is instructive to consider whether the values now obtained represent any important changes from previously published figures. We therefore present Table XVII, which gives a summary of values from a number of previous surveys and compilations, for comparison with our own results. We would like to remark that the high values of  $\nu$  occurring in the Sher and Felberbaum 1965 revision might be at least partly explained by the fact that the recent value of  $\nu$  (Cf<sup>252</sup>) by Colvin and Sowerby was not known to them.

### TABLE XVI

	U <sup>293</sup>	U235	Pu239	Pu241					
σ <sub>a</sub>	576.3±2.3	679.9±2.3	1008.1±4.9	1391 ± 22					
ο <sub>f</sub> σγ	$527.7 \pm 2.1$ 48.6±1.5	$579.5\pm 2.0$ 100.5±1.4	$742, 4 \pm 3, 5$ 265. 7 ± 3. 7	1009±9 382±21					
α η	0.0921±0.0029 2.284±0.008	0. 1734 ± 0. 0025 2. 071 ± 0. 007	0. 3580 ± 0. 0054 2. 114 ± 0. 010	$0.379 \pm 0.021$ $2.154 \pm 0.036$					
ν	2.494±0.009	2.430±0.008	2.871±0.014	2.969±0.023					
	$\nu$ (Cf <sup>252</sup> ) = 3.772±0.015								

### RECOMMENDED VALUES FOR 2200 m/s CONSTANTS<sup>a</sup>

<sup>a</sup> Cross-section values in barns (b).

While the changes in many of these quantities now appear to be fluctuations rather random in time, there are some clear trends; for example  $\sigma_a(Pu^{239})$  has tended to decrease, as have  $\alpha$  and  $\sigma_\gamma$  for the same nuclide, while  $\eta$  for this isotope has definitely risen. For  $U^{235}$ ,  $\nu$  decreased about 1% between 1958 and 1960, but has been fairly steady since, and  $1 + \alpha$  decreased roughly proportionately; other changes are generally less than 1%. Thus the main changes in recent years have concerned Pu<sup>239</sup>, and as we have seen  $\sigma_a$  and  $\sigma_f$  for this isotope may still need to be revised although  $\sigma_\gamma$  seems now to be fairly closely determined, mainly by the  $\alpha$  measurements, and any changes should only be proportional to those for  $\sigma_a$ .

For the reactor designer, two of the more important outstanding problems concern the absolute values of  $\nu$  and  $\eta$ , and these depend on the  $\nu(Cf^{252})$  situation as already discussed. In connection with the  $\eta(Pu^{239})$ , it may be noted that in the discussion in Session 3.1 of the 3rd Geneva Conference, Dr. Chernick especially expressed the view that a lower value would fit better with some US reactor physics measurements, although Dr. Sanders supported a value close to the one we now recommend (our value [11] was then a little higher,  $2.123 \pm 0.009$ , mainly due to the higher  $\nu$  (Cf) at that time). It is unfortunate that some reactor physics measurements have too involved an interpretation for us to regard them as measurements of  $\eta$  in the present study. But one might remark on this point that some of the older natural uranium burn-up studies [97] which were made with a lower  $\eta$  value for Pu<sup>239</sup>, were also made with lower fission-product absorption cross-sections [98], and in a natural uranium reactor these two factors tend to cancel one another out. In the recent burn-up studies [99] the inclusion of some epithermal neutron capture effects in the fission products and a higher  $\eta(Pu^{239})$  value seem to be giving reasonably satisfactory explanation of the observations. The presently advocated value for this quan-

### 2200 m/s CONSTANTS

tity is only about 0.006 higher than the value adopted in 1963 by Critoph [17] and it is possible that the present value might help to remove some of the remaining discrepancies such as that shown in Figure 4 of Ref. [99] in the region of (1 to 2) $\times 10^{20}$  n/cm<sup>2</sup> integrated irradiation.

The remaining constants do not appear to call for detailed comment. There have been some changes since the preliminary version of this work [11] was presented to the 3rd Geneva Conference, mainly due to the new data becoming available, but these are all within the errors and are greatest for  $Pu^{241}$ , where even a single new measurement can produce significant changes.

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### APPENDIX

### STATISTICAL TESTS

Let  $\hat{Y}_{ij}$ ,  $1 \leq j \leq N_i$ ,  $1 \leq i \leq k$  be a set of independent "random variables" (measurements), split up into k groups having  $N_1$ ,  $N_2$ , ...  $N_k$  members respectively. Suppose  $\hat{Y}_{ij}$  has a normal distribution with mean value  $f_i$ , and variance  $\sigma_{ij}^2$ , where  $f_i$  is a function of m independent parameters  $f_i(X_1, X_2, \ldots, X_m)$ ; the first order variation equations for the f's are given in Eq. (5), section 3.1 above, the coefficients being the elements  $A_{ri}$  of a matrix A (where  $1 \leq r \leq m$ ). Then the weighted means  $Y_{mean i}$  are given by

$$\mathbf{Y}_{\text{mean } i} = \left(\frac{\widehat{\mathbf{Y}}_{i1}}{\sigma_{i1}^2} + \ldots + \frac{\mathbf{Y}_{iN_i}}{\sigma_{iN_i}^2}\right) / \left(\sum_{j=1}^{N_i} \frac{1}{\sigma_{ij}^2}\right)$$

which is readily shown to have a normal probability distribution with expectation  $f_i$ , and variance  $v_i = \sigma_{meani}^2$  given by Eq. (2) above (the  $\delta_{err}^2$  term is omitted in the present simplified theory; its physical reason for existence indicates how it should be treated in the application of these tests). Then

### TABLE XVII

# COMPARISON OF OUR RECOMMENDED RESULTS WITH EARLIER VALUES<sup>a</sup>

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Quantity	BNL-325 1958 [2]	BNL-325 1960 [2]	Sjöstrand and Story 1961 [7]	Leonard <sup>b</sup> 1962 [9]	Sher and Felberbaum 1962 [10]	Sher and Felberbaum 1965 [10]	Present work 1965
				1			
$\sigma_{a}^{(U^{233})}$	581 ± 7	578±4	573.7±2.5	575±4	576.1±2.7	573, 1 ± 2, 1	576,3±2,3
$\sigma_a(U^{235})$	694 ± 8	683±3	680.5±2.9	678±5	682.0±2.6	678, 2±2, 2	679.9±2.3
σ <sub>a</sub> (Pu <sup>239</sup> )	1026 ± 13	$1028 \pm 8$	1026.7±7.5	$1008 \pm 6$	1030,1±7,4	1014,5±4,2	1008.1±4.9
σ <sub>f</sub> (U <sup>233</sup> )	527 ± 4	525 ± 4	524.5±2.7	526 ± 4	527.5±2.4	524, 5±1, 9	527.7±2,1
σ <sub>f</sub> (U <sup>235</sup> )	$582 \pm 6$	577 ± 4 <sup>c</sup>	579.9±2.7	579±6	582. 2 ± 2. 2	577.1±1.9	579.5±2.0
σ <sub>f</sub> (Pu <sup>239</sup> )	746 ± 8	742±4	740.6±5.5	754 ± 9	748.2±4.9	740.6±3.5	742.4±3.5
$\nu (U^{233})$	$2,51 \pm 0.03$	2.51±0.02	2,505±0,012	$2.502 \pm 0.014$	2, 503 ± 0, 010	2,504±0.008	2.494±0.009
$\nu  ({ m U}^{235})$	2.47±0.03	2.45±0.02 <sup>C</sup>	2,438±0,011	$2.434 \pm 0.019$	2,430±0,009	2.442±0.006	2.430±0.008
ν (Pu <sup>239</sup> )	2,90±0.04	2.89±0.03	2.901±0.018	2.89±0.05	2.882±0.016	2.898±0.011	2.871±0.014
η(U <sup>233</sup> )	2,28±0,02	2,28±0,02	2, 290 ± 0, 008	$2.288 \pm 0.010$	2.292±0.008	2,292±0.006	2.284±0.008
$\eta(\mathrm{U}^{235})$	2.07±0.02	2.07±0.01	2.078±0.007	2.077±0.010	2.074±0.006	2.078±0.005	2.071±0.007
η(Pu <sup>239</sup> )	2.10±0.02	2.08±0.02	2. 093 ± 0. 019	2.16±0.05	2.093±0.014	2.116±0.009	2.114±0.010

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Quantity	BNL-325 1958 [2]	BNL-325 1960 [2]	Sjöstrand and Story 1961 [7]	Leonard <sup>b</sup> 1962 [9]	Sher and Felberbaum 1962 [10]	Sher and Felberbaum 1965 [10]	Present work 1965
α(U <sup>233</sup> )	0.102±0.005	0,101±0.004	0.0938±0.0047	0.0935±0.0038	0.092±0.003	0,0926±0,0027	0.0921±0.0029
α(U <sup>235</sup> )	0.19±0.01	0.184±0.010 <sup>C</sup>	0, <b>1734 ± 0, 0</b> 050	0.172±0.007	0.171±0.003	0.175±0.002	0.1734±0.0025
α(Pu <sup>239</sup> )	0.38±0.02	0.39±0.03	0, 386 ± 0. 013	0.337±0.017	0.377±0.011	$0.370 \pm 0.006$	0.3580±0.0054
σ <sub>γ</sub> (U <sup>233</sup> )	54 ± 3	53 ± 2	49.2±2.5	49±2	48.6±1.6	48.6±1.5	48.6±1.5
$\sigma_{\gamma}^{(U^{235})}$	112±6	101±6	100, 6±2, 9	99 ± 4	99, 8±1, 8	101, 1 ± 1, 0	100,5±1,4
σ <sub>γ</sub> (Pu <sup>239</sup> )	280 ± 15	286 ± 9	286.1±9.3	254 ± 11	281.9±8.9	273.9±4.7	265.7±3.7

### TABLE XVII (cont.)

<sup>a</sup> In this table, all cross-sections and their errors are in barns.

<sup>b</sup> It should be noted that the values given for Leonard in this table are the first of three alternative sets listed in Table VI of his paper, which - in the light of later developments - appears the most probable of the three sets presented by him.

<sup>c</sup> The values for  $\sigma_f(U^{235})$ ,  $\nu(U^{235})$  and  $\alpha(U^{235})$  on page 4 of Ref. [2] (1960 supplement) were subsequently corrected by the authors; the corrected values are given here.

using the regression theory, outlined in section 3.1, with the notation  $x_i = X_i - X_{base i}$  for the first order increment to the base values, we obtain the elements  $\overline{X_i}$  of the solution vector  $\overline{X}$ . The full set of k improved values,  $\overline{Y_i}$ , is then obtained from solution vector  $\overline{X}$ , and to the first order this process is the same as is given in terms of the elements  $A_{ri}$  of the matrix A by

$$\overline{\mathbf{Y}}_{i} = \mathbf{Y}_{base i} + \sum_{r=1}^{m} \mathbf{A}_{ri} \mathbf{x}_{r}.$$

It is not difficult to show that the following break-down is correct:

$$\sum_{i=1}^{k} \sum_{j=1}^{N_{i}} \left( \frac{\widehat{Y}_{ij} - f_{i}}{\sigma_{ij}} \right)^{2} = P + Q + \left( \frac{\widetilde{x}_{1} - \overline{x}_{1}}{\ldots \ldots} \right)^{r} A \left( \frac{\widetilde{x}_{1} - \overline{x}_{1}}{\ldots \ldots} \right)^{r} A \left( \frac{\widetilde{x}_{1} - \overline{x}_{1}}{\widetilde{x}_{m} - \overline{x}_{m}} \right)^{r} A \left( \frac{\widetilde{x}_{1} - \overline{x}_{1}}{\widetilde{x}_{m} - \overline{$$

where  $\tilde{x}_r$ 's are the (unknown) differences between the true values and the base values  $X_{base r}$ . Also P and Q are:

$$\cdot \mathbf{P} = \sum_{i=1}^{k} \sum_{j=1}^{N_{i}} \left( \frac{\widehat{\mathbf{Y}}_{ij} - \mathbf{Y}_{meani}}{\sigma_{ij}} \right)^{2}$$

and

$$Q = \sum_{i=1}^{k} \frac{\left(Y_{\text{mean } i} - Y_{\text{base } i} - \sum_{r=1}^{k} A_{ri} \overline{x}_{i}\right)^{2}}{\sigma_{\text{mean } i}^{2}}$$

Using the well-known lemma of Cochran it follows that P has a  $X^2$  dis-

tribution with  $\sum_{i=1}^{k} N_i$  - k degrees of freedom and that Q, often called the re-

sidual term, has the same distribution with (k-m) degrees of freedom. Furthermore, these two terms are independently distributed. It follows as a byproduct that the mathematical expectation

$$E\left(\frac{Q}{k-m}\right)=1.$$

It also follows that P + Q is distributed according to  $X^2$  with  $\sum_{i=1}^{K} N_i$  - m degrees of freedom. Furthermore, the quotient

$$\frac{P}{k} \left(\frac{Q}{k-m}\right)^{-1}$$

$$\sum_{i=1}^{m} N_i - k$$

is distributed according to F with  $\sum_{i=1}^{k} N_i - k$  and k - m degrees of freedom.

These last two statements are the basis of the statistical tests.

The significance of these last two tests is that if all errors are underor over-estimated by a constant numerical factor,  $\gamma$ , say, the X<sup>2</sup> test on P + Q can provide indications of what values of  $\gamma$  would be statistically acceptable. However, if F lies outside the range of values acceptable for the number of degrees of freedom concerned, not only does such a hypothesis of a constant- $\gamma$  not provide a satisfactory basis for the observed distribution of input values, but we also have to conclude that some systematic errors or other factors may exist which throw doubt on the statistical assumptions made.

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### ÉTUDE DES VALEURS DES CONSTANTES POUR UN FLUX DE 2200 m/s POUR QUATRE NUCLEIDES FISSILES, C.H. Westcott, K. Ekberg, G.C. Hanna<sup>1</sup>, N.J. Pattenden<sup>2</sup>, S. Sanatani et P.M. Attree (Organisation internationale de l'énergie atomique, Vienne)

Les auteurs ont entrepris une étude des valeurs les plus probables des constantes pour un flux de 2200 m/s, pour les nucléides fissiles <sup>233</sup>U, <sup>235</sup>U, <sup>239</sup>Pu et <sup>241</sup>Pu, en recourant à un ajustement par la méthode des moindres carrés. Ils ont soigneusement étudié les diverses données expérimentales et ont obtenu les valeurs suivantes:

	<sup>233</sup> U	235 U	<sup>239</sup> Pu	<sup>241</sup> Pu			
σ <sub>a</sub>	576,3 ± 2,3	679,9 ± 2,3	$1008, 1 \pm 4, 9$ $742, 4 \pm 3, 5$	$1391 \pm 22$			
σ <sub>f</sub>	48,6 ± 1,5	100,5 ± 1,4	265,7 ± 3,7	382 ± 21			
α η	0,0921 ± 0,0029 2,284 ± 0,008	0,1734 ± 0,0025 2,071 ± 0,007	0,3580 ± 0,0054 2,114 ± 0,010	0,379 ± 0,021 2,154 ± 0,036			
$\nu$ 2,494 ± 0,009 2,430 ± 0,008 2,871 ± 0,014 2,969 ± 0,023							
$\nu$ ( <sup>252</sup> Cf) = 3,772 ± 0,015							

### VALEURS RECOMMANDÉES POUR DES CONSTANTES DE 2200 m/sª

a Valeurs des sections efficaces en barns (b).

Les auteurs discutent en détail les problèmes qui se posent surtout étant donné qu'il faut déterminer l'exactitude des mesures individuelles et des valeurs définitives à la sortie. Les erreurs indiquées sont les valeurs de l'écart type et comprennent une certaine marge pour le cas où il y aurait une erreur systématique ou non accidentelle dans les erreurs sur les mesures originales, qui, bien qu'il soit impossible de l'identifier dans les données à l'entrée, peut cependant exister.

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<sup>&</sup>lt;sup>2</sup> Atomic Energy Research Establishment, Harwell, Berks., Royaume-Uni

### ESTUDIO DE VALORES CORRESPONDIENTES A 2200 m/s DE LAS CONS-TANTES DE CUATRO NUCLIDOS FISIONABLES, C.H. Westcott, K. Ekberg, G.C. Hanna<sup>1</sup>, N.J. Pattenden<sup>2</sup>, S. Sanatani y P.M. Attree (Organismo Internacional de Energía Atómica, Viena)

Los autores han estudiado por el método de los cuadrados mínimos los valores más probables de las constantes de los núclidos <sup>233</sup>U, <sup>235</sup>U, <sup>239</sup>P<sub>u</sub> y <sup>241</sup>P<sub>u</sub> correspondientes a 2200 m/s. Revisaron cuidadosamente los diversos datos experimentales, obteniendo los valores siguientes:

	<sup>233</sup> U	235 U	<sup>239</sup> Pu	<sup>241</sup> Pu		
σ <sub>a</sub>	576,3 ± 2,3	679,9 ± 2,3	1008,1 ± 4,9	1391 ± 22		
σ <sub>f</sub> σγ	527,7 ± 2,1 48,6 ± 1,5	579,5 ± 2,0 100,5 ± 1,4	742,4 ± 3,5 265,7 ± 3,7	1009 ± 9 382 ± 21		
α	0,0921 ± 0,0029	0,1734 ± 0,0025	0,3580 ± 0,0054	0,379 ± 0,021		
η	2,284 ± 0,008	2,071 ± 0,007	2,114 ± 0,010	2,154 ± 0,036		
ν	2,494 ± 0,009	2,430 ± 0,008	2,871 ± 0,014	2,969 ± 0,023		
$\nu (^{252}Cf) = 3,772 \pm 0,015$						

VALORES RECOMENDADOS PARA LAS CONSTANTES A 2200 m/sa

a Valores de las secciones eficaces en barns (b)

La memoria discute detalladamente los problemas que se plantean, especialmente al evaluar la exactitud de los resultados de cada medición y de los valores finales obtenidos. Los errores que figuran en el cuadro anterior se indican como desviaciones tipo e incluyen un margen para posibles contribuciones sistemáticas o no aleatorías a los errores de las mediciones originales, que pueden cometerse incluso si no son identificables en los datos de entrada.

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