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FISSION AND ABSORPTION g-FACTORS OF ^{241}Pu

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Abstract—The effective cross section of a Maxwellian neutron spectrum can be given as the product of the cross section at 0·0253 eV and the 'g-factor'. Fission and absorption g-factors of ^{241}Pu are calculated from various cross-section measurements and tabulated in the temperature range 0°–2000°C. Some discrepancies between different measurements are discussed.

1. INTRODUCTION

THE effective cross section $\hat{\sigma}(T)$ for neutrons having a pure Maxwellian distribution of energies $n(E; T)$ for temperature T is given as the product of the cross section at 0·0253 eV and the so-called g-factor (WESTCOTT, 1955):

$$\hat{\sigma}(T) = \sigma(0{\cdot}0253 \text{ eV}) \,.\, g(T). \tag{1}$$

The g-factor is a function of temperature and can be calculated from the cross-section curve $\sigma(E)$ in the region of thermal energies. Especially for the fissile nuclides, the g-factors g_f for the fission cross section and g_a for the absorption cross section are of considerable importance for the physics of thermal reactors.

During the preparation of the survey on the 2200 m/sec neutron data for fissile nuclides (WESTCOTT *et al.*, 1964; WESTCOTT *et al.*, 1965), a need for better values of the g-factors for ^{241}Pu, for both absorption and fission, was encountered. The earlier values calculated in 1960 and reprinted with corrections in 1962 (WESTCOTT, 1960), were based on the 1958 and 1960 editions of BNL-325, and at that time no reliable measurements of the total cross section $\sigma_T(E)$ were available; it was therefore assumed that the g-factors for absorption and fission were equal.

Since new measurements have been made for the absorption cross sections (see Section 2.1 below), more exact calculations based on these 1964 measurements are now possible; preliminary results of this work were reported to the International Nuclear Data Scientific Working Group (INDSWG) meeting in September 1965. With the encouragement of this group these calculations have now been extended to higher temperatures and, in the case of the Hanford measurements, modified after receiving more detailed information by private communication.

2. THE MEASUREMENTS

For the calculation of ^{241}Pu g-factors, the following measurements were used, whilst some others were found to be superseded with respect to their accuracy.

2.1 *Total cross section*

(i) SIMPSON and SCHUMAN (1961), and other references cited in this paper, describe a time-of-flight measurement on an 81 per cent enriched sample using the MTR fast chopper in the energy ranges 0·016–0·48 and 1·5–2000 eV. The detailed tabulated data for these measurements were obtained from Report IDO-16679 (unpublished).

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(ii) CRAIG and WESTCOTT (1964) [for tabulated data see Report AECL-1948 (1964)], a time-of-flight transmission measurement on an 80 per cent enriched sample using the BNL–AECL fast chopper at Chalk River in the energy ranges 0·025–0·8 and 13·8–1000 eV.

2.2 *Fission cross section*

(iii) ADAMCHUK *et al.* (1955), tabulated data given by EGELSTAFF (1957), a time-of-flight measurement on an 88·5 per cent enriched sample using a mechanical selector at the U.S.S.R. RTP reactor and an ionization fission chamber in the energy range 0·01–800 eV.

(iv) HANFORD, 1957–1959. Three measurements were made using the Hanford crystal spectrometer and a gas ionization fission chamber. The first (SEPPI *et al.*, 1957) was performed in the energy range 0·025–1·0 eV on a sample containing 19·24 per cent ^{241}Pu. The two other measurements using a 96·6 per cent enriched sample, covered the energy ranges 0·0025–0·005 eV (SEPPI *et al.*, 1958) and 0·1–23 eV (LEONARD and FRIESENHAHN, 1959). The tabulated data of the first and third measurements were received by private communication.

(v) JAMES (1964), tabulated data by private communication, a time-of-flight measurement on samples enriched to 95 and 97 per cent ^{241}Pu, using the Harwell electron linac and a surface barrier semiconductor as a fission fragment detector in the energy range 0·0084–3000 eV.

(vi) WATANABE and SIMPSON (1964) [for tabulated data see IDO-16995 (1964)], a time-of-flight measurement on an 80 per cent enriched sample using the MTR fast chopper and a gas scintillation detector in the energy range 0·024–100 eV.

3. THE PROCEDURE

The g-factors $g(T)$ were calculated from the cross section $\sigma(E)$ according to

$$g(T) = \frac{2}{\sqrt{\pi}\sqrt{E_0}\,\sigma(E_0)} \int_0^\infty \sqrt{E}\,\sigma(E) \,.\, \sqrt{\frac{E}{E_T}} \exp\left(-\frac{E}{E_T}\right) \frac{\mathrm{d}E}{E_T} \tag{2}$$

where $E_0 = 0{\cdot}0253$ eV, $E_T = E_0\, T/T_0$, $T_0 = 293{\cdot}6$°K.

The experimental cross-section values of each measurement were fitted to

$$\sqrt{E}\,\sigma(E) = a_{n0} + a_{n1}E + a_{n2}E^2 + a_{n3}E^3 + a_{n4}E^4 + a_{n5}E^5 + R(E). \tag{3}$$

$R(E)$ is a single-level Breit–Wigner formula

$$R(E) = \frac{h}{1 + \left(\dfrac{E - E_r}{\Gamma/2}\right)^2} \tag{4}$$

with resonance energy $E_r = 0{\cdot}256$ eV, width $\Gamma = 0{\cdot}112$ eV, and amplitude h as a free parameter; for the polynomial fit, the energy scale was split up into a number of energy ranges indicated by the subscript n in equation (3), and the six polynomial coefficients, a, were fitted to six representative cross-section values chosen by eye, separately in each energy range.

As the lowest energy of most of the measurements was near 0·02 eV, special care was needed in the extrapolation of the measured values to zero energy (see Fig. 1). To estimate the influence of the extrapolation, g-factors were calculated for low, middle and high extrapolation, but only mean values are listed in the results (Tables 1 and 2).

A constant scattering cross section of $\sigma_s = 11$ barns was assumed and subtracted from the total cross section for the calculation of the absorption g-factor g_a.

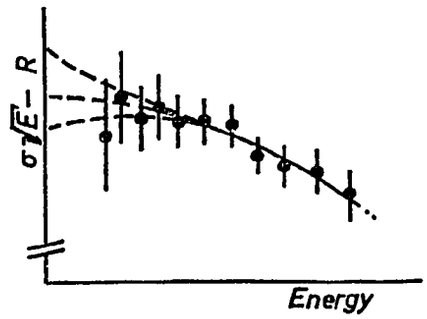

FIG. 1.—Typical low-energy experimental points after subtraction of the Breit–Wigner fit R, showing three possible extrapolations to zero energy.

4. RESULTS AND DISCUSSION

For the following discussion, it is useful to distinguish three main sources of error which, however, can be interdependent.

(i) *Due to normalization.* If the normalization of the entire $\sigma(E)$-curve is incorrect, this does not affect the calculation of g-factors, as $\sigma(E)$ in the numerator and $\sigma(E_0)$ in the denominator of equation (2) are equally affected. However, statistical fluctuations around the normalization point cause an uncertainty in the normalization value $\sigma(E_0)$ which may increase or decrease the whole $g(T)$-curve by a constant factor.

(ii) *Due to extrapolation to zero energy (Fig. 1).* This error could be estimated by using different extrapolations for the calculation of g-factors; its magnitude decreases considerably at higher temperatures. The errors due to extrapolation and normalization are not independent but can be distinguished by their different temperature dependence.

(iii) *Due to the shape of the resonance.* Statistical errors in the $\sigma(E)$-curve are eliminated by the integration of equation (2). However, any systematic error resulting in different peak heights or peak shapes of the 0·256 eV resonance, may have considerable influence on the g-factors, which can only be estimated by comparison of the g-factors of different measurements.

4.1 *Absorption*

Table 1 shows the g-factors resulting from the absorption cross-section measurements of SIMPSON and SCHUMAN and CRAIG and WESTCOTT at various temperatures. Both measurements agree quite well. The estimated standard error due to normalization is in both cases ±0·3%. The estimated standard error due to extrapolation is in both cases ±0·08% at 20°C, 0·007% at 600°C, 0·002% at 2000°C. The error due to the resonance shape is—by comparison of both measurements— ±0·02% at 20°C, 1·3% at 600°C, 1·5% at 2000°C.

TABLE 1.—THE g-FACTOR $g_a(T)$ FOR THE ^{241}Pu ABSORPTION CROSS SECTION AS FUNCTION OF TEMPERATURE T

T (°C)	Simpson	Craig	60% Craig +40% Simpson	Estimated standard error
0	1·024	1·022	1·023	
20	1·032	1·029	1·030	±0·2%
40	1·040	1·038	1·039	
60	1·050	1·047	1·048	
80	1·062	1·059	1·060	
100	1·075	1·071	1·073	±0·2%
120	1·089	1·085	1·086	
140	1·105	1·101	1·103	
160	1·122	1·117	1·119	
180	1·140	1·135	1·137	
200	1·160	1·153	1·156	
220	1·180	1·173	1·176	
240	1·201	1·193	1·196	
260	1·223	1·214	1·218	
280	1·245	1·236	1·240	
300	1·268	1·258	1·262	±0·4%
320	1·290	1·280	1·284	
340	1·314	1·302	1·307	
360	1·337	1·325	1·330	
380	1·360	1·347	1·352	
400	1·383	1·369	1·375	
420	1·406	1·391	1·397	
440	1·428	1·413	1·419	
460	1·450	1·435	1·441	
480	1·472	1·456	1·462	
500	1·493	1·477	1·483	
520	1·514	1·497	1·504	
540	1·534	1·516	1·523	
560	1·554	1·536	1·543	
580	1·573	1·554	1·562	
600	1·591	1·572	1·580	±0·7%
650	1·635	1·614	1·622	
700	1·674	1·652	1·661	
750	1·709	1·686	1·695	
800	1·740	1·716	1·725	
850	1·766	1·742	1·752	
900	1·790	1·765	1·775	
950	1·809	1·784	1·794	
1000	1·825	1·800	1·810	
1100	1·849	1·822	1·833	
1200	1·861	1·835	1·845	
1300	1·865	1·839	1·849	
1400	1·861	1·835	1·846	
1500	1·852	1·826	1·836	±0·8%
2000	1·749	1·727	1·735	

TABLE 2.—THE g-FACTOR $g_f(T)$ FOR THE ^{241}Pu FISSION CROSS SECTION AS FUNCTION OF TEMPERATURE T

T (°C)	(Adamchuk)	Hanford	James	(Watanabe)	75% Hanford +25% James	Estimated standard error
0	1·038	1·037	1·027	1·038	1·035	
20	1·048	1·046	1·036	1·046	1·044	±0·4%
40	1·059	1·057	1·045	1·056	1·054	
60	1·071	1·069	1·056	1·067	1·066	
80	1·084	1·082	1·068	1·080	1·079	
100	1·098	1·097	1·081	1·093	1·093	±0·6%
120	1·112	1·112	1·095	1·108	1·108	
140	1·128	1·129	1·110	1·124	1·124	
160	1·144	1·147	1·126	1·141	1·142	
180	1·161	1·166	1·143	1·159	1·160	
200	1·179	1·185	1·160	1·178	1·179	
220	1·197	1·206	1·179	1·197	1·199	
240	1·216	1·227	1·197	1·217	1·220	
260	1·235	1·248	1·216	1·238	1·240	
280	1·254	1·270	1·236	1·259	1·262	
300	1·274	1·292	1·255	1·280	1·283	±1·2%
320	1·293	1·314	1·275	1·302	1·304	
340	1·313	1·336	1·295	1·323	1·326	
360	1·333	1·358	1·314	1·345	1·347	
380	1·352	1·380	1·334	1·366	1·369	
400	1·371	1·402	1·353	1·387	1·390	
420	1·390	1·424	1·372	1·408	1·411	
440	1·409	1·445	1·391	1·429	1·432	
460	1·427	1·466	1·409	1·449	1·452	
480	1·444	1·487	1·427	1·469	1·472	
500	1·462	1·507	1·444	1·488	1·491	
520	1·478	1·526	1·461	1·507	1·510	
540	1·495	1·545	1·477	1·525	1·528	
560	1·510	1·563	1·492	1·542	1·545	
580	1·526	1·580	1·507	1·559	1·562	
600	1·440	1·597	1·522	1·576	1·578	(±1·5%)
650	1·574	1·637	1·556	1·614	1·617	
700	1·604	1·673	1·586	1·648	1·651	
750	1·631	1·704	1·612	1·678	1·681	
800	1·654	1·731	1·634	1·705	1·707	
850	1·674	1·755	1·654	1·728	1·730	
900	1·691	1·775	1·669	1·747	1·749	
950	1·704	1·791	1·682	1·763	1·764	
1000	1·715	1·805	1·693	1·775	1·777	
1100	1·729	1·823	1·705	1·792	1·794	
1200	1·735	1·831	1·709	1·800	1·801	
1300	1·733	1·831	1·706	1·799	1·800	
1400	1·725	1·825	1·697	1·792	1·793	
1500	1·713	1·813	1·684	1·779	1·781	(±3%)
2000	1·609	1·703	1·575	1·669	1·671	

As the measurement of Craig appears a little better with respect to resolution and statistics, we would recommend a weighted averaging of 60% Craig + 40% Simpson.

4.2 *Fission*

Although there are more measurements on the fission cross section than on the total cross section, there are, in the case of fission, still some discrepancies in the peak shape of the resonance near 0·256 eV and also in the shape of the cross-section curve in the thermal energy range. Table 2 shows the g-factors for fission calculated from various measurements.

The measurement of Adamchuk *et al.* showed, after subtraction of the Breit–Wigner fit, another peak at 0·303 eV, probably due to another isotope. This peak was

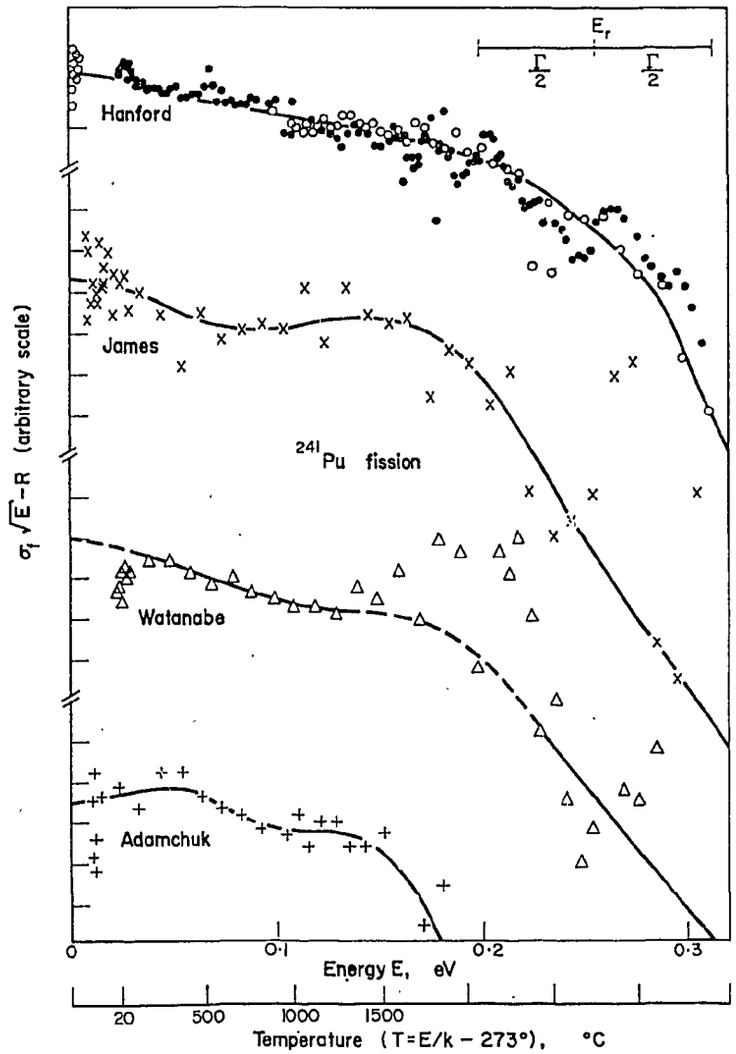

FIG. 2.—Illustration of the accuracy of four ^{241}Pu fission cross-section measurements in the energy range up to 0·3 eV, after subtraction of a single-level Breit–Wigner fit R at the resonance energy $E = 0{\cdot}256$ eV. The curves represent the fits finally used. Note that the curves cannot be easily compared, since the subtracted Breit–Wigner fits R are different.

subtracted and neglected, and the g-factors calculated from this measurement were used for comparison only.

The Hanford measurements were performed with two samples of different qualities with respect to the contamination of other isotopes. Both measurements agree quite well, except for some discrepancies in the peak shape of the 0·256 eV resonance which, however, have little influence on the g-factors. Table 2 shows the averaged g-factors of both Hanford measurements, which agree almost completely in the temperature ranges 0–100°C and 700–800°C and differ from their average by only $\pm 0{\cdot}001$ in the range 100–700°C and $\pm 0{\cdot}005$ at 1500°C. The Hanford measurements have good statistics, but the accuracy is limited by higher order contaminations in the neutron beam generated by the crystal spectrometer. The resulting higher order peaks are most evident when the data are plotted after subtraction of the Breit–Wigner fit (see Fig. 2). In the calculation of g-factors, a smoothed curve was used, where the higher order peaks were cut off. However, the higher order contamination may still cause some error because of a possible influence on the monitoring and normalization.

The measurements of JAMES (1964) extends to 0·008 eV, but bad statistics at this low energy still cause some uncertainty in the extrapolation to zero energy. The peak shape of James does not agree with that of the Hanford measurement, so that there are considerable discrepancies in the g-factors at higher temperatures.

The measurement of WATANABE and SIMPSON is apparently wrong in the energy region below 0·04 eV (see Fig. 2). Thus, an extrapolation to zero energy and a determination of the normalization value at 0·0253 eV can only be done by comparison with one of the other measurements.

Figure 2 illustrates the accuracy of the ^{241}Pu fission cross-section measurements in the energy range up to 0·3 eV. Plotted is $\sigma\sqrt{E}$ after subtraction of the Breit–Wigner fit $R(E)$ to the 0·256 eV resonance. The uncertainties of the cross sections give rise to the following estimated standard errors of the g-factors.

(i) *Due to normalization*
Adamchuk: $\pm 1\%$, Hanford: $\pm 0{\cdot}1\%$, James: $\pm 0{\cdot}4\%$, Watanabe: uncertain.

(ii) *Due to extrapolation to zero-energy*
Adamchuk: $\pm 0{\cdot}15\%$ at 20°C, $\pm 0{\cdot}02\%$ at 600°C,
Hanford: $\pm 0{\cdot}02\%$ at 20°C, negligible at higher temperatures,
James: $\pm 0{\cdot}07\%$ at 20°C, $\pm 0{\cdot}01\%$ at 600°C,
Watanabe: uncertain.

(iii) *Due to the shape of the resonance.* Comparison of the Hanford and James $\sqrt{E}\,\sigma(E)$-curve shows a difference of 10 per cent in the ratio of the peak cross section to that at 0·0253 eV. This gives rise to a difference of g-factors of 1 per cent at 20°C, 5 per cent at 600°C, and 7·5 per cent at 1500°C. This is a systematic error, and it is difficult to decide which measurement is better. The peak height of the Watanabe measurement seems to support that of Hanford, whilst the Adamchuk measurement tends to confirm that of James, at least after neglecting Adamchuk's spurious 0·303 eV peak. However, the following arguments suggest that the Hanford measurement is the better one (compare Fig. 2): (1) it extends to the lowest energy; (2) it has good statistics even at low energies; (3) it allows the best fit to a single-level Breit–Wigner

formula; (4) measurements made on two different samples are in agreement. Thus, we have listed in Table 2 a weighted average of 75 per cent Hanford and 25 per cent James. This average agrees excellently with the g-factors calculated from Watanabe's measurement, which, of course, are somewhat arbitrary because of the doubtful low-energy cross section values. It should be pointed out, however, that at temperatures above about 300°C, where the systematic error is dominant, the use of an averaged value cannot be recommended since the g-factors from various measurements differ too much. The discrepancy of the fission cross section in the range of the 0·256 eV resonance deserves further study; preferably further measurements should be made.

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