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Radiative Losses and Electron Cooling Rates of Hydrogen, Helium, Carbon and Oxygen

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Abstract

Atomic rates relevant to fusion plasma modelling are calculated from a simplified collisional radiative model, using recommended atomic data. The rates calculated are the radiative loss and the electron cooling rates for individual ionisation stages of hydrogen, helium, carbon, and oxygen. The results are presented graphically as a function of the temperature, for various electron densities. They are also approximated by analytic fits convenient to use in computer simulation models.

Contents

1	Introduction	5
2	Atomic model description	5
3	Atomic data base	7
4	Analytic Fits	7
5	Summary	9

List of Tables

1	Definiton of the variables and rates used in the calculations.	7
2	Parameters for the analytic fits: Hydrogen; Electron Cooling Rates . .	11
3	Parameters for the analytic fits: Hydrogen; Radiative Loss Rates . . .	12
4	Parameters for the analytic fits: Helium; Electron Cooling Rates . . .	13
5	Parameters for the analytic fits: Helium; Radiative Loss Rates	14
6	Parameters for the analytic fits: Helium; Electron Cooling Rates . . .	15
7	Parameters for the analytic fits: Carbon; Electron Cooling Rates . . .	16
8	Parameters for the analytic fits: Carbon; Radiative Loss Rates	17
9	Parameters for the analytic fits: Oxygen; Electron Cooling Rates . . .	18
10	Parameters for the analytic fits: Oxygen; Radiative Loss Rates	19

List of Figures

1	Electron cooling rate: H; Density = 10^{-19} m^{-3}	20
2	Electron cooling rate: H; Density = 10^{-20} m^{-3}	21
3	Electron cooling rate: H; Density = 10^{-21} m^{-3}	22
4	Radiative Loss rate: He; Density = 10^{-19} m^{-3}	23
5	Radiative Loss rate: He; Density = 10^{-20} m^{-3}	24
6	Radiative Loss rate: He; Density = 10^{-21} m^{-3}	25
7	Electron cooling rate: He; Density = 10^{-19} m^{-3}	26
8	Electron cooling rate: He; Density = 10^{-20} m^{-3}	27
9	Electron cooling rate: He; Density = 10^{-21} m^{-3}	28
10	Radiative Loss rate: He; Density = 10^{-19} m^{-3}	29
11	Radiative Loss rate: He; Density = 10^{-20} m^{-3}	30
12	Radiative Loss rate: He; Density = 10^{-21} m^{-3}	31
13	Electron cooling rate: C; Density = 10^{-19} m^{-3}	32
14	Electron cooling rate: C; Density = 10^{-20} m^{-3}	33
15	Electron cooling rate: C; Density = 10^{-21} m^{-3}	34
16	Radiative Loss rate: C; Density = 10^{-19} m^{-3}	35
17	Radiative Loss rate: C; Density = 10^{-20} m^{-3}	36
18	Radiative Loss rate: C; Density = 10^{-21} m^{-3}	37
19	Electron cooling rate: O; Density = 10^{-19} m^{-3}	38
20	Electron cooling rate: O; Density = 10^{-20} m^{-3}	39
21	Electron cooling rate: O; Density = 10^{-21} m^{-3}	40
22	Radiative Loss rate: O; Density = 10^{-19} m^{-3}	41
23	Radiative Loss rate: O; Density = 10^{-20} m^{-3}	42
24	Radiative Loss rate: O; Density = 10^{-21} m^{-3}	43

1 Introduction

The physics associated with atomic processes is important to fusion research, particularly with regard to the edge and divertor region of tokamaks which are characterized by low temperatures and potentially high density plasmas. A good understanding, a capacity to model edge plasmas, recycling, and plasma-material interaction necessitates a comprehensive and accurate knowledge of atomic, molecular, and plasma-material processes. These processes can have important physical consequences in the plasma, such as electron cooling associated with inelastic collisions and energy dissipation by radiation. The rate at which the various ion species ionize and recombine is also important as it determines the number of free electrons per nucleus and, hence, fuel dilution. Considering the key role expected from atomic processes in certain scenarios of power exhaust in an eventual fusion reactor it is crucial that computer models of reactor plasmas be equipped with accurate and complete atomic rates. Over the years, several calculations have been made of these rates; the earlier ones being motivated primarily by astrophysical applications [1-3]. More recently, several calculations have been made specifically for fusion applications [4-13]. In this report, we present results of atomic rate calculations for hydrogen and light impurity ions for fusion plasmas. These calculations are based on a simplified collisional radiative model very similar to one already described in Refs. [12,13]. In addition to hydrogen, the ion species considered are those of helium, carbon, and oxygen. The rates calculated are the radiative loss and electron cooling rates. In each case, the rates are represented graphically as a function of temperature, for various electron densities. Analytic fits are also given, which permit the use of these rates in actual plasma simulation models. The rates presented here are for the most part very close to the ones given in Ref. [13]. The new analytic fits, however, should be used instead of the ones of Ref. [13], because the latter have been found to be systematically in error.

The remainder of this report is organized as follows. In Sec. 2, a brief description of the atomic model is given. In Sec. 3, a brief description of the atomic data base is presented. In Sec. 4 the various rates are presented both graphically, and with analytic fits. Finally, a summary and some concluding remarks are given in Sec. 5.

2 Atomic model description

The simplified collisional-radiative model considered here is very close to one already described in Refs. [12,13]. The main points are repeated here for completeness. For more details the reader is referred to aforementioned references.

The radiative loss rate is calculated as

$$P_r = \sum_{i,\alpha} n_i^i P_{r,\alpha}^i, \quad (1)$$

where n_i is the density of a given element in ionisation stage i , and $P_{r,\alpha}^i$ is called the radiative loss coefficient. For a given ionisation stage, using the definitions in Table

1, this coefficient is calculated as

$$n_{\alpha}^i P_{r,\alpha}^i = \sum_{\beta} n_{\alpha}^i [A_{\beta\alpha}^i \epsilon_{\beta\alpha}^i + n_e \int d^3v \left(\frac{1}{2} M v^2 + \epsilon_{\beta}^{i-1} + \epsilon_{0\alpha}^i \right) v \sigma_{rr,0\alpha}^{i-1} f(v) + R_{\beta\alpha} \epsilon_{0\beta}^{i-1} + n_e B_{\alpha}^i] \quad (2)$$

In Eq. (2), the first term represents line radiation associated with radiative decay. The second term represents continuum radiation associated with radiative recombination. The third term represents the radiative cascade which follows radiative recombination to an excited state, and the last term represents bremsstrahlung. We note that in this approximation, recombined ions are assumed to decay radiatively to the ground state without any possibility of excitation or ionisation from an excited state. That is, recombination is treated in the coronal approximation. The contribution to radiative losses associated with a radiative cascade which would follow charge exchange to an excited state is not included here for simplicity. This contribution should be calculated separately and added to the radiative loss coefficient calculated here. This contribution would add a dependence of the rates on particle neutral densities.

The electron cooling rate P_e is defined similarly in terms of the densities of ionisation stage i n_i and the electron cooling coefficients $P_{e,\alpha}^i$ as

$$P_e = \sum_{i,\alpha} n_{\alpha}^i P_{e,\alpha}^i, \quad (3)$$

where

$$n_{\alpha}^i P_{e,\alpha}^i = \sum_{\beta} n_{\alpha}^i n_e [X_{\alpha\beta}^i \epsilon_{\alpha\beta}^i + \int d^3v \frac{1}{2} M v^2 v \sigma_{rr,0\alpha}^{i-1} f(v) + \sum_{\gamma} \int d^3v \frac{1}{2} M v^2 v \sigma_{dr,\alpha\beta\gamma}^{i-1} f(v)] + n_{\alpha}^i n_e [S_{\alpha}^i \epsilon_{\alpha}^i + B_{\alpha}^i]. \quad (4)$$

The first term in this equation represents cooling associated with collisional excitation. The second term represents the loss of energy associated with radiative recombination. The third and fourth terms represent the loss of electron energy associated with collisional ionisation and bremsstrahlung respectively.

The calculation of the expressions given in the above equations requires a knowledge of the densities n_{α}^i for individual excited states, as well as accurate values for cross sections and rates which appear in the right hand sides. As in Refs. [12,13], the densities are obtained by solving the system of equations

$$\frac{\partial n_{\alpha}^i}{\partial t} = \sum_{\beta} [-n_{\alpha}^i (A_{\alpha\beta}^i + n_e X_{\alpha\beta}^i) n_{\alpha}^i (A_{\alpha\beta}^i + n_e X_{\alpha\beta}^i)] - n_{\alpha}^i S_{\alpha}^i + \delta_{0\alpha} \sum_{\beta} n_{\beta}^i n_e S_{\beta}^i. \quad (5)$$

This constitutes a simplified collisional radiative model in which the populations of the various excited states are determined by radiative decay, collisional excitation and deexcitation, and by ionisation. The main difference between the model

Table 1. Definition of the variables and rates used in the calculations.

$A_{\alpha\beta}^i$	Transition probability for transition $\alpha \rightarrow \beta$ in ionisation stage i .
B_{α}^i	Power radiated by bremsstrahlung per unit ion ionisation stage i .
S_{α}^i	Rate of ionisation for ionisation stage i in state α .
$X_{\alpha\beta}^i$	Rate of excitation (including resonance excitation) for transition $\alpha \rightarrow \beta$ in ionisation stage i .
n_{α}^i	Density of ions in ionisation stage i and state α .
n_e	Electron density.
ϵ_{α}^i	Ionisation energy for ionisation stage i in state α .
$\epsilon_{\alpha\beta}^i$	Excitation energy for transition $\alpha \rightarrow \beta$ in ionisation stage i .
σ_{rr}^i	Cross section for radiative recombination in state β from an ion in ionisation stage i and in state α .
$\sigma_{dr,\alpha\beta\gamma}^i$	Cross section for dielectronic recombination in Rydberg state γ of ionisation stage i associated with a core transition $\alpha \rightarrow \beta$.

considered here and the one presented in Refs. [12,13] is that in these references, multistep processes were considered only for metastable states. Specifically, collisional deexcitation and ionisation from an excited states were considered only from excited metastable states. In practice, this can be imposed by setting $A_{\alpha\beta}^i$, S_{α}^i , and $X_{\alpha\beta}^i$ to zero whenever the initial state is not the ground state or a metastable state. In the calculations considered here, multistep processes are considered for both metastable and non metastable states.

3 Atomic data base

For carbon and oxygen, the atomic data base used for the detailed rates and cross sections in this calculation is essentially the same as described in Refs. [12,13]. For hydrogen, we use rates and cross sections given by Janev, et al. [14]. For neutral helium, the ionisation rate from the ground state is calculated from fits produced by Lennon, et al. [15]. Ionisation from excited states is calculated from recommended and semiempirical formulas taken from Ref. [14]. For excitation rates, we use rates taken from Refs. [16-20] when available, and rates obtained from Ref. [14] otherwise.

4 Analytic Fits

In many applications, plasma modeling for example, cross sections or rate coefficients of different processes are required over a wide energy region. Analytic fits to the data serve this purpose, because with few parameters and a given function, the rate coefficients can be generated easily. Compared to polynomial fits (which are linear

in the fitting parameters), non-linear analytic fits do not have spurious oscillations and can represent the low and high energy limits properly.

In this work, we have made analytic fits to all data presented. The functions used are (T refers to temperature in keV):

Expression 1:

$$P_{r,e}(T) = a_1 \frac{e^{-a_2/T^{a_3}}}{T^{a_4} + a_5 T^{a_6}} \quad (6)$$

Expression 2:

$$P_{r,e}(T) = a_1 \frac{e^{-a_2/T^{a_3}}}{1 + a_4 T^{a_5} + a_6 T^{a_7}} \quad (7)$$

Expression 3:

$$P_{r,e}(T) = a_1 \frac{e^{-a_2/T^{a_3}}}{T^{a_4} + a_5 T^{a_6} + a_7 T^{a_8}} + a_9 \frac{e^{-a_{10}T}}{T^{a_{11}}} \quad (8)$$

Expression 4:

$$P_{r,e}(T) = a_1 \frac{e^{-a_2/T^{a_3}}}{T^{a_4} + a_5 T^{a_6}} + a_7 T^{a_8} \quad (9)$$

Expression 5:

$$P_{r,e}(T) = a_1 T^{a_2} + a_3 T^{a_4} \quad (10)$$

Expression 6:

$$P_{r,e}(T) = a_1 T^{a_2} + a_3 T^{a_4} + a_5 T^{a_6} \quad (11)$$

Expression 7:

$$P_{r,e}(T) = a_1 \frac{e^{-a_2/T^{a_3}} \ln(1 + T^{a_4})}{T^{a_5} + a_6 T^{a_7}} \quad (12)$$

Expression 8:

$$P_{r,e}(T) = a_1 e^{-a_2/T^{a_3}} (1 + a_4 T^{a_5} + a_6 T^{a_7}) \quad (13)$$

Tables 2-11 give the parameters for the fits shown in Figs. (2)-(37).

5 Summary

A simplified collisional radiative model has been used to calculate atomic rate coefficients for radiative loss, and electron cooling associated with atoms and ions in fusion plasmas. The species considered are hydrogen, helium, carbon, and oxygen. The results are presented graphically as a function of temperature, for three densities (10^{19} m^{-3} , 10^{20} m^{-3} , 10^{21} m^{-3}). They are also expressed in terms of analytic fits, convenient for use in numerical plasma simulation models. The agreement between the actual rates and the analytic fits is generally excellent (within 10%), except at low temperatures ($\approx 1 \text{ eV}$) where some of the fits can be in error by more than a factor two. The density dependence of the rates is found to be generally weak. For densities in the range 10^{19} m^{-3} , 10^{21} m^{-3} , this dependence can be well approximated from a logarithmic interpolation of the rates presented. The strongest density dependence is observed for the radiative cooling rate with neutral or weakly ionized species. In comparison, the electron cooling rate is relatively insensitive to the electron density.

Acknowledgements

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Table 2: Parameters for the analytic fits: Hydrogen.

Hydrogen: $P_e, n = 10^{19} m^{-3}$

	I	II
Fit*	1	6
a ₁	.1247E+03	3180E-03
a ₂	2111	2332
a ₃	5982	- 9388E-02
a ₄	.4063	8617
a ₅	9640E-03	9663E-02
a ₆	1 4523	8573

Hydrogen: $P_e, n = 10^{20} m^{-3}$

	I	II
Fit*	1	6
a ₁	.1246E+03	.3180E-03
a ₂	2078	2332
a ₃	6007	- 9388E-02
a ₄	4071	8617
a ₅	8290E-03	9663E-02
a ₆	1 4738	8573

Hydrogen: $P_e, n = 10^{21} m^{-3}$

	I	II
Fit*	1	6
a ₁	3184E+03	.3180E-03
a ₂	2087	2332
a ₃	5986	- 9388E-02
a ₄	4155	8617
a ₅	1 5270	9663E-02
a ₆	4155	8573

* Refers to expressions in Eqs (6)-(13)

Table 3: Parameters for the analytic fits.**Hydrogen: $P_{r, n} = 10^{19} \text{m}^{-3}$**

	I	II
Fit*	1	6
a ₁	60.3763	1300E-04
a ₂	1795	- 1775E-02
a ₃	5985	.9000E-05
a ₄	3382	- 6615
a ₅	8330E-03	5760E-03
a ₆	1 5886	5019

Hydrogen: $P_{r, n} = 10^{20} \text{m}^{-3}$

	I	II
Fit*	1	6
a ₁	50.5610	1300E-04
a ₂	.1667	- 1775E-02
a ₃	6020	9000E-05
a ₄	.3109	- 6615
a ₅	.1555E-02	5760E-03
a ₆	1 4469	.5019

Hydrogen: $P_{r, n} = 10^{21} \text{m}^{-3}$

	I	II
Fit*	1	6
a ₁	26 7831	1300E-04
a ₂	.1017	-.1775E-02
a ₃	6343	9000E-05
a ₄	1250	- 6615
a ₅	5904E-01	5760E-03
a ₆	7304	5019

* Refers to expressions in Eqs (6)-(13)

Table 4: Parameters for the analytic fits.

Helium: $P_e, n = 10^{19} m^{-3}$

	I	II	III
Fit*	1	4	6
a ₁	6623E+03	3476E+03	.2730E-01
a ₂	9476E-01	1214	.6004
a ₃	7456	7974	- 2772E-01
a ₄	- 2592	4819	5925
a ₅	3 8098	1 4066	3060E-02
a ₆	4026	- 3639E-02	3590
a ₇		9720E-03	
a ₈		4078	

Helium: $P_e, n = 10^{20} m^{-3}$

	I	II	III
Fit*	1	4	6
a ₁	.6547E+03	2487E+03	2730E-01
a ₂	9272E-01	1194	6004
a ₃	7505	8003	- 2772E-01
a ₄	- 2589	- 8375E-02	.5925
a ₅	3 7644	7251	.3060E-02
a ₆	4028	4797	3590
a ₇		9720E-03	
a ₈		.4078	

Helium: $P_e, n = 10^{21} m^{-3}$

	I	II	III
Fit*	1	4	6
a ₁	6540E+03	3459E+03	2730E-01
a ₂	9454E-01	1194	.6004
a ₃	7480	8007	- 2772E-01
a ₄	-.2498	.4812	5925
a ₅	3 7483	1 4009	3060E-02
a ₆	4036	- 5582E-02	3590
a ₇		9720E-03	
a ₈		.4078	

* Refers to expressions in Eqs (6)-(13)

Table 5: Parameters for the analytic fits.

Helium: $P_{r, n} = 10^{19}m^{-3}$

	I	II	III
Fit*	1	4	6
a ₁	1248E+03	1263E+03	1601E-03
a ₂	5112E-01	1213	-.4784
a ₃	8413	.8023	8600E-04
a ₄	-.1881	8712E-01	- 6062
a ₅	3 1032	2276	.2284E-02
a ₆	3749	.5996	5051
a ₇		1800E-04	
a ₈		- 6236	

Helium: $P_{r, n} = 10^{20}m^{-3}$

	I	II	III
Fit*	1	4	6
a ₁	88.9673	.5847E+03	.1610E-03
a ₂	7567E-01	1261	- 4784
a ₃	7895	7963	.8600E-04
a ₄	-.9620E-01	6092	- 6062
a ₅	2 2503	4 7760	.2284E-02
a ₆	3787	9214E-01	5051
a ₇		1800E-04	
a ₈		- 6236	

Helium: $P_{r, n} = 10^{21}m^{-3}$

	I	II	III
Fit*	1	4	6
a ₁	47 7193	1253E+03	1610E-03
a ₂	7062E-01	1283	- 4784
a ₃	7941	7894	8600E-04
a ₄	- 1261	7467E-01	- 6062
a ₅	1 5861	2628	.2284E-02
a ₆	3484	.5770	5051
a ₇		1800E-04	
a ₈		- 6236	

* Refers to expressions in Eqs (6)-(13)

Table 6: Parameters for the analytic fits.

Carbon: $P_e, n = 10^{19} m^{-3}$

	I	II	III	IV	V	VI	VII
Fit*	1	1	2	1	4	4	6
a ₁	6448E+03	.4519E+03	4368E+03	1738E+03	1173E+03	52 5191	1662
a ₂	3824E-02	.6083E-01	9100E-04	1640E-03	2612	3274	8353
a ₃	1 1008	7313	1 6421	1.5417	9629	.9951	- 1868
a ₄	3730	3943	2334E-01	3624	-.4263	- 3099	8126
a ₅	2766	4318	-1 1336	4363	9247	7766	6259E-01
a ₆	- 1836	- 4402E-01	1 1203	- 2909	3279	3331	
a ₇			4493		6987E-02	.2229E-01	
a ₈					2506	3049	

Carbon: $P_e, n = 10^{20} m^{-3}$

	I	II	III	IV	V	VI	VII
Fit*	1	1	2	1	4	4	5
a ₁	6341E+03	4478E+03	.4327E+03	1705E+03	1179E+03	52.3327	.1662
a ₂	3301E-02	.5713E-01	1070E-03	1540E-03	2662	3331	.8353
a ₃	1 1241	.7411	1.6144	1 5495	.9619	9924	- 1868
a ₄	3714	3931	.2300E-01	3576	- 4202	- 2995	8126
a ₅	2852	4323	-1 1340	4116	9264	7610	6258E-01
a ₆	- 1795	- 4881E-01	1 0992	-.3073	.3286	3362	3902
a ₇			4513		6893E-02	2211E-01	
a ₈					.2483	3034	

Carbon: $P_e, n = 10^{21} m^{-3}$

	I	II	III	IV	V	VI	VII
Fit*	1	1	2	1	4	4	6
a ₁	.5742E+03	4248E+03	.4112E+03	1532E+03	1231E+03	52 2594	1662
a ₂	2407E-02	4082E-01	3440E-03	1450E-03	.3113	3285	.8353
a ₃	1 1731	.7937	1 4249	1.5344	.9340	9982	- 1868
a ₄	3583	3856	2365E-01	3296	-.3729	- 3069	8126
a ₅	2523	4098	-1 1104	2883	9319	7675	6259E-01
a ₆	- 1921	- 8080E-01	1.0141	- 4230	3357	3345	3902
a ₇			4552		6788E-02	2195E-01	
a ₈					.2455	3020	

* Refers to expressions in Eqs (6)-(13)

Table 7: Parameters for the analytic fits.

Carbon: $P_{r, n} = 10^{19} m^{-3}$

	I	II	III	IV	V	VI	VII
Fit*	1	1	2	1	3	3	6
a ₁	3355E+03	3149E+03	.3409E+03	1061E+03	59 5004	33 1965	2866E-01
a ₂	4649E-02	3120E-01	2100E-04	4890E-03	1643E-01	4860E-01	4204
a ₃	1 0747	8327	1 8872	1.3696	1 6944	1 4735	3 5997
a ₄	3842	4229	1408E-01	.3394	3499	3219	- 4611
a ₅	2871	6912	-1 1895	2132	1 2847	1 2237	-3 5712
a ₆	- 5545E-01	3375E-01	1 4539	-.2981	- 3530	- 4093	- 4598
a ₇			4354		2396	1113	
a ₈					-1 4526	-1 7343	
a ₉					5520E-03	7283E-02	
a ₁₀					- 8682E-01	- 2828E-01	
a ₁₁					6945	7062	

Carbon: $P_{r, n} = 10^{20} m^{-3}$

	I	II	III	IV	V	VI	VII
Fit*	1	1	2	1	3	3	6
a ₁	2833E+03	2657E+03	3311E+03	1046E+03	55 6466	32 8181	2866E-01
a ₂	3145E-02	2266E-01	2300E-04	4830E-03	1813E-01	5149E-01	4204
a ₃	1 1257	.8745	1 8755	1 3696	1 6565	1 4601	3 5997
a ₄	3741	3982	1381E-01	3360	3377	.3201	- 4611
a ₅	3987	5639	-1 1871	2061	1 1879	1 2001	-3 5712
a ₆	- 6201E-01	- 2280E-01	1 4124	- 3100	- 4060	- 4145	- 4598
a ₇			4355		1811	1063	
a ₈					-1.5368	-1 7338	
a ₉					4470E-03	.7279E-02	
a ₁₀					4600E-04	-.2418E-01	
a ₁₁					7276	7063	

Carbon: $P_{r, n} = 10^{21} m^{-3}$

	I	II	III	IV	V	VI	VII
Fit*	1	1	2	1	3	3	6
a ₁	2246E+03	2006E+03	2780E+03	95.2822	59 3701	32 9139	2866E-01
a ₂	1683E-02	9595E-02	2400E-04	5410E-03	1441E-01	5305E-01	4204
a ₃	1 2048	9961	1.8652	1 3340	1 7419	1 4521	3 5997
a ₄	3635	3647	1133E-01	3155	3514	3211	- 4611
a ₅	1 6126	5052	-1.1977	1690	1.3364	1 2109	-3 5712
a ₆	- 4745E-01	- 9863E-01	1 1938	- 3957	- 3553	- 4123	- 4598
a ₇			4354		.2434	1055	
a ₈					-1 5597	-1.7384	
a ₉					5620E-03	7269E-02	
a ₁₀					- 9486E-01	- 2339E-01	
a ₁₁					6916	7065	

* Refers to expressions in Eqs (6)-(13)

Table 8: Parameters for the analytic fits.

Oxygen: $P_e, n = 10^{19} m^{-3}$

	I	II	III	IV	V
Fit*	1	2	2	4	4
a ₁	4675E+03	8565E+03	.6388E+04	.3009E+03	161 8436
a ₂	3565E-01	1319E-01	3855	4673E-01	1130E-01
a ₃	7942	1 0022	4629	.7865	1.7609
a ₄	.3707	1209	8117E-02	.3472	.3169
a ₅	4492	- 6863	-1 3193	.3230	1448
a ₆	- 1616	1.4778	10 1092	- 3553	- 7023
a ₇		4233	4055	.3500E-01	.200E-02
a ₈				.2500E-02	200E-01

	VI	VII	VIII	IX
Fit*	7	4	4	6
a ₁	95 0706	62.5956	27 8644	6100
a ₂	7180E-03	3643	5154	6191E-01
a ₃	1 3730	1 0742	9911	.6844E-02
a ₄	9117	- 5975	- 4207	8010
a ₅	6831	6774	5982	- 5485
a ₆	- 3251	2791	3010	.4822E-01
a ₇	6173	2049E-01	4235E-01	
a ₈		2504	2687	

Oxygen: $P_e, n = 10^{20} m^{-3}$

	I	II	III	IV	V
Fit*	1	2	2	4	4
a ₁	4614E+03	.8452E+03	6383E+04	3045E+03	161 5876
a ₂	3625E-01	1123E-01	.3848	5247E-01	1100E-03
a ₃	.7919	1 0302	.4631	7688	1 7649
a ₄	.3696	1221	.8037E-02	3518	3202
a ₅	4486	- .6900	-1 3222	3326	1462
a ₆	- 1633	1.4651	10 1209	- 3384	- 7034
a ₇		4228	4051	.2000	2000E-02
a ₈				3300	2200E-01

	VI	VII	VIII	IX
Fit*	7	4	4	6
a ₁	1039E+03	62.6109	28 2469	6100
a ₂	.3320E-03	.3628	5280	.6191E-01
a ₃	1 5185	1 0770	9869	6844E-02
a ₄	9160	- 6024	- 4111	8100
a ₅	7166	6815	6015	- .5485
a ₆	- .2689	2779	3024	4822E-01
a ₇	7166	2044E-01	4233E-01	
a ₈		2500	.2686	

* Refers to expressions in Eqs (6)-(13)

Table 9: Parameters for the analytic fits.

Oxygen: $P_e, n = 10^{21} \text{m}^{-3}$

	I	II	III	IV	V
Fit*	1	2	2	4	4
a ₁	4505E+03	8401E+03	6356E+04	3089E+03	161 3996
a ₂	.3395E-01	8584E-02	3847	6222E-01	1180E-03
a ₃	8009	1 0813	4622	7457	1 7511
a ₄	3673	1303	7127E-02	3553	3200
a ₅	4348	- 6774	-1 3532	3397	1467
a ₆	- 1745	1 4928	10 1544	- 3134	- 7060
a ₇		4202	4039	1800	2000E-02
a ₈				3900	2000E-02

	VI	VII	VIII	IX
Fit*	7	4	4	6
a ₁	1858E+05	63 9077	28.5565	6100
a ₂	4 8333	3787	5445	6191E-01
a ₃	1532	1 0640	9824	6844E-02
a ₄	-.1044E-03	- 5898	- 3948	8100
a ₅	8467	6901	5955	- 5485
a ₆	1718E-10	2796	3062	4822E-01
a ₇	-3 6156	2038E-01	4230E-01	
a ₈		2494	2685	

Oxygen: $P_r, n = 10^{19} \text{m}^{-3}$

	I	II	III	IV	V
Fit*	1	1	1	4	4
a ₁	1606E+05	7378E+04	3452E+03	.2647E+04	104 4110
a ₂	7736E-01	5324E-01	3006	1235	3230E-03
a ₃	6957	8260	4783	6843	1 5838
a ₄	1 2057	9278	3761	7279	3390
a ₅	2657E+03	48 3739	8210E-03	16 8926	9594E-01
a ₆	2327	2833	-1 3370	2554	- 6673
a ₇				9000E-01	3600
a ₈				- 1000E-01	2000

	VI	VII	VIII	IX
Fit*	8	3	3	5
a ₁	6964	40 1658	20 8457	1815E-01
a ₂	2392	5266E-01	4104	7137
a ₃	.6039	1.7029	8141	1084
a ₄	5147	.2631	- 4310	- 5546
a ₅	-1 8233	1 1736	1 0111	
a ₆	1218E+03	- 6624	.2823	
a ₇	- 3048	1530	4229E-01	
a ₈		-1 9360	-2 5273	
a ₉		2412E-02	.2708E-01	
a ₁₀		- 5328E-01	- 1033E-01	
a ₁₁		7565	7351	

* Refers to expressions in Eqs (6)-(13)

Table 10: Parameters for the analytic fits.

Oxygen: $P_r; n = 10^{20} m^{-3}$

	I	II	III	IV	V
Fit*	7	1	1	4	4
a ₁	39 4024	1345E+03	3416E+03	2654E+04	104 0786
a ₂	.3864E-02	4326E-01	3009	1228	2990E-03
a ₃	.9753	8545	4775	.6850	1 5985
a ₄	4329	.2597	3741	7285	3424
a ₅	4615	2252E-01	.7840E-03	17.0219	9792E-01
a ₆	3977E-02	9026	-1 3462	2544	-.6664
a ₇	- 9034			.7800E-01	3500
a ₈				-.9500E-02	.2000

	VI	VII	VIII	IX
Fit*	8	3	3	5
a ₁	7469	39 1740	21 2873	1815E-01
a ₂	.2243	5429E-01	4270	7137
a ₃	6134	1 6643	8102	1084
a ₄	4095	2579	- 4204	- 4456
a ₅	-1 8347	1.1645	1 0269	
a ₆	1126E+03	- 7047	2833	
a ₇	- 3030	1067	4284E-01	
a ₈		-2 1320	-2 5143	
a ₉		2386E-02	2708E-01	
a ₁₀		1016E-02	- 2266E-01	
a ₁₁		7582	7351	

Oxygen: $P_r; n = 10^{21} m^{-3}$

	I	II	III	IV	V
Fit*	2	1	1	4	4
a ₁	18 0865	2751E+04	3199E+03	2092E+04	103 0567
a ₂	9879E-02	2563E-01	2895	1062	3120E-03
a ₃	8327	9304	4798	7093	1 5909
a ₄	7514	7847	3640	7032	3408
a ₅	2978	29 0309	6440E-03	13 8388	9837E-01
a ₆	7021E-02	.2139	-1 3915	2367	- 6663
a ₇	-1 2861			6600E-01	3400
a ₈				- 8000E-02	2000

	VI	VII	VIII	IX
Fit*	8	3	3	5
a ₁	8581	39 6638	23 8166	1814E-01
a ₂	1325	5398E-01	4862	.7137
a ₃	7040	1 6911	7748	1084
a ₄	3782E-01	.2612	-.4105	-.5546
a ₅	-2.1886	1 1596	1 1507	
a ₆	95 0512	- 6693	2831	
a ₇	- 2953	1486	.4359E-01	
a ₈		-1 9502	-2 5220	
a ₉		2402E-02	2709E-01	
a ₁₀		-.2655E-01	-.2156E-01	
a ₁₁		7572	7350	

Electron Cooling Rate: H

Density = 10^{19} m^{-3}

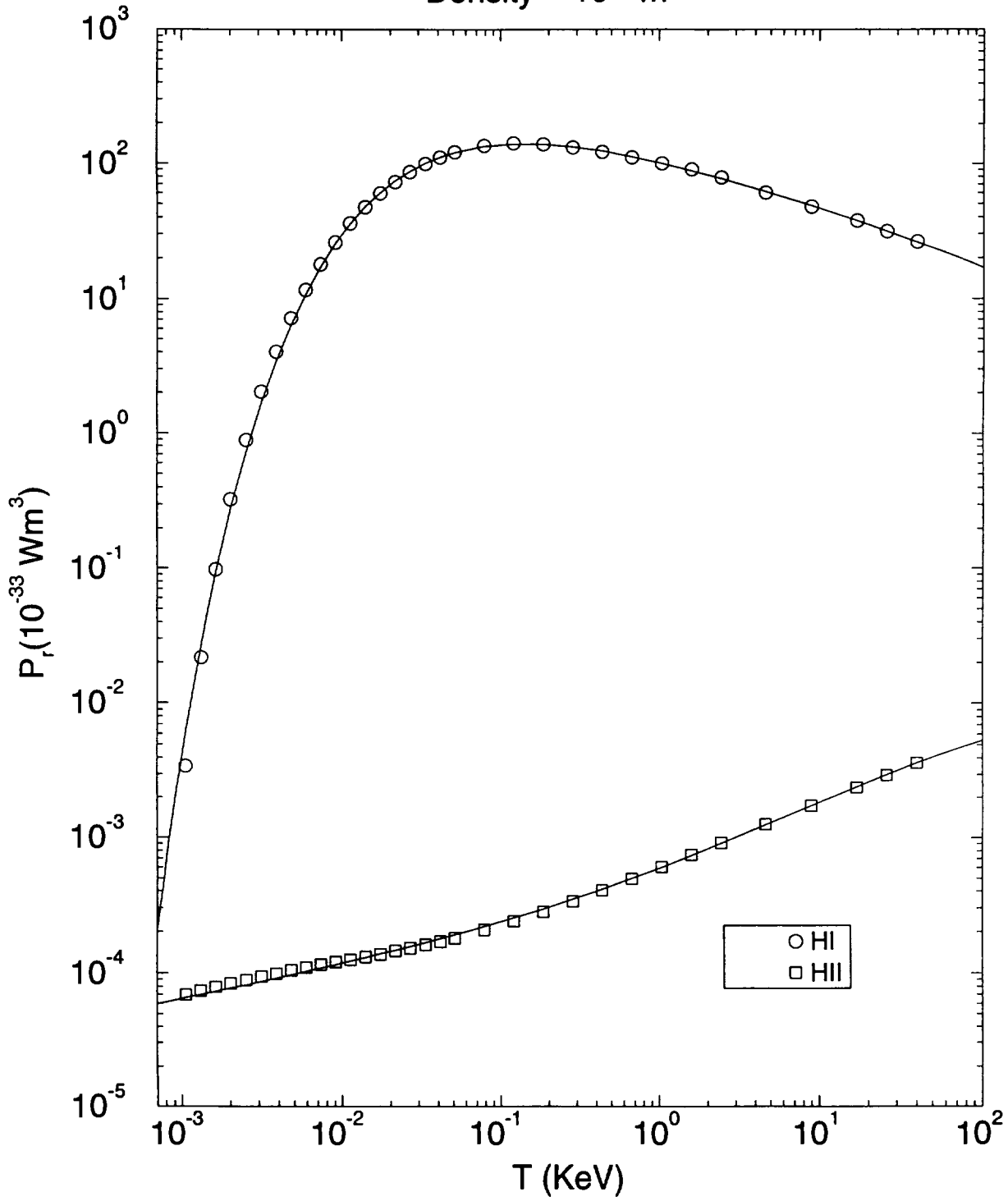


Figure 1

Electron Cooling Rate: H

Density = 10^{20} m^{-3}

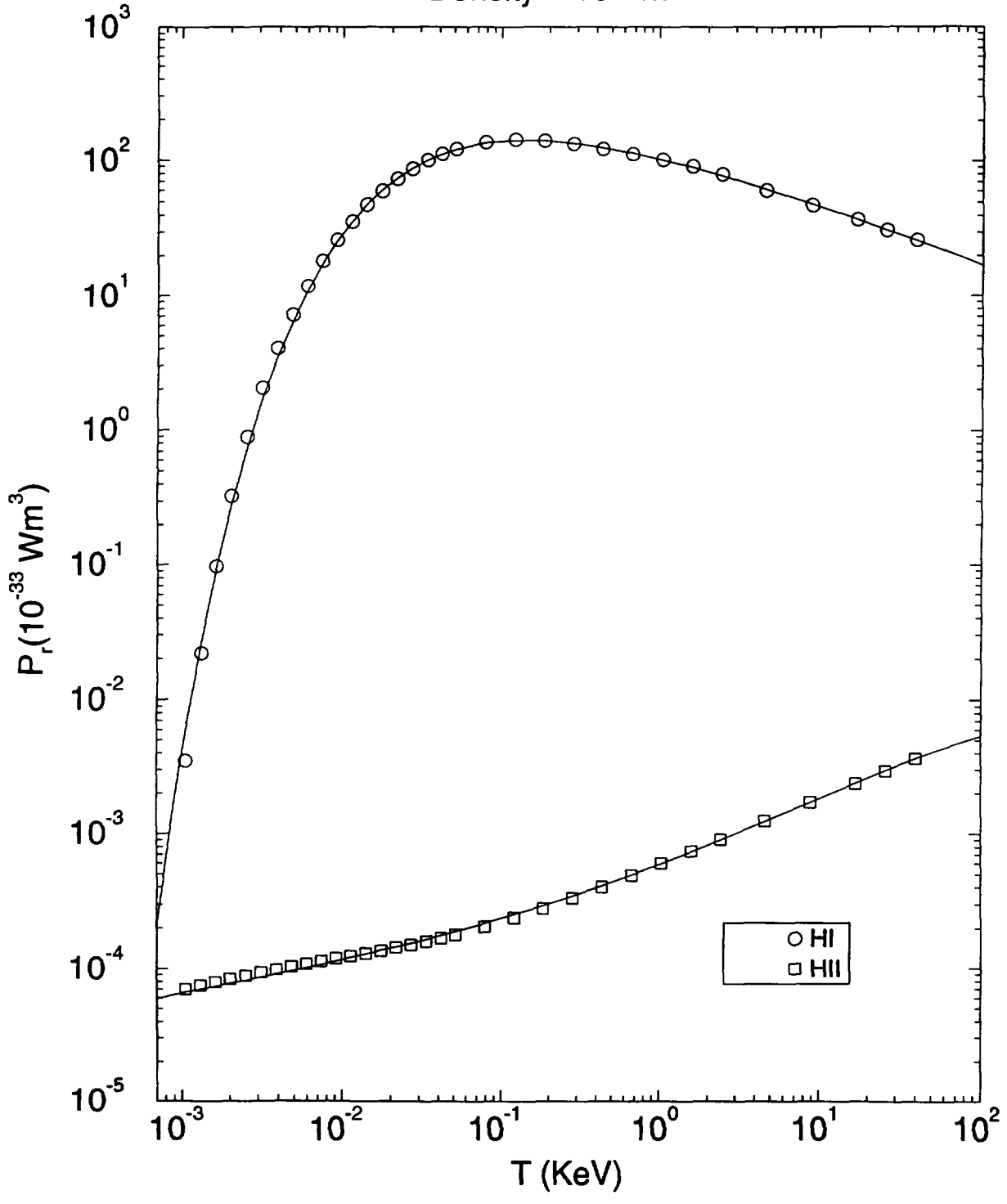


Figure 2

Electron Cooling Rate: H

Density = 10^{21} m^{-3}

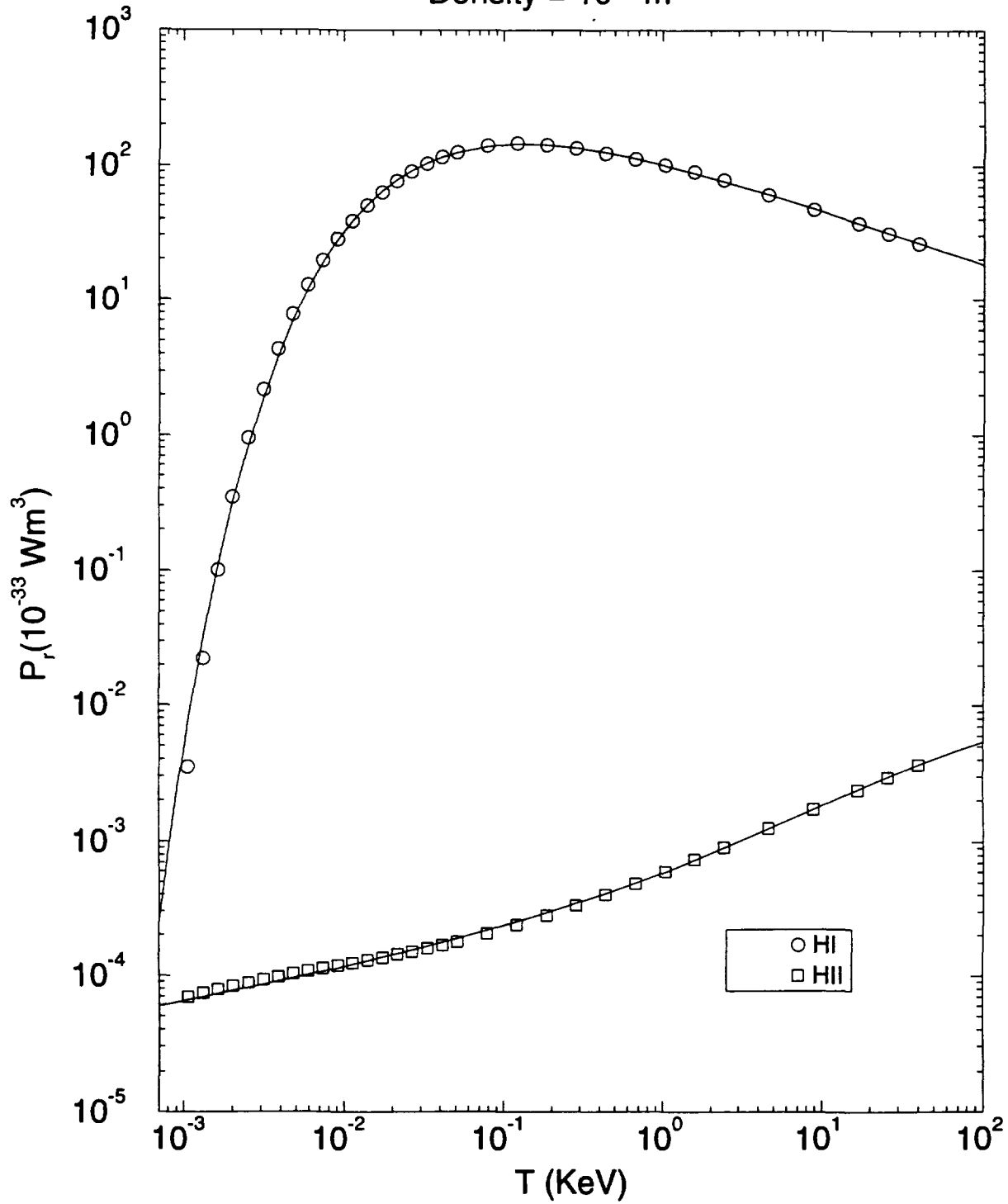


Figure 3

Radiative Loss Rate: H

Density = 10^{19} m^{-3}

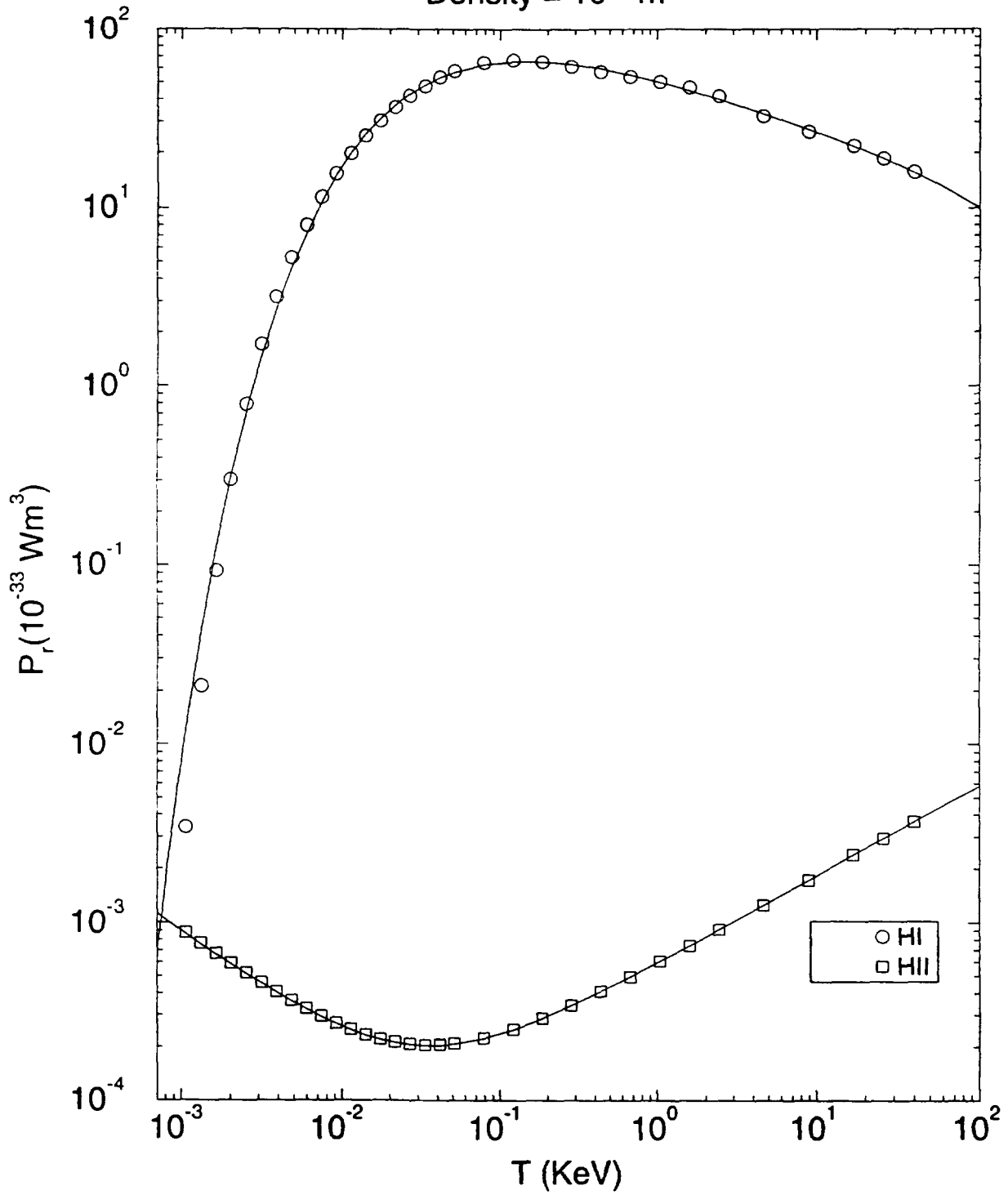


Figure 4

Radiative Loss Rate: H

Density = 10^{20} m^{-3}

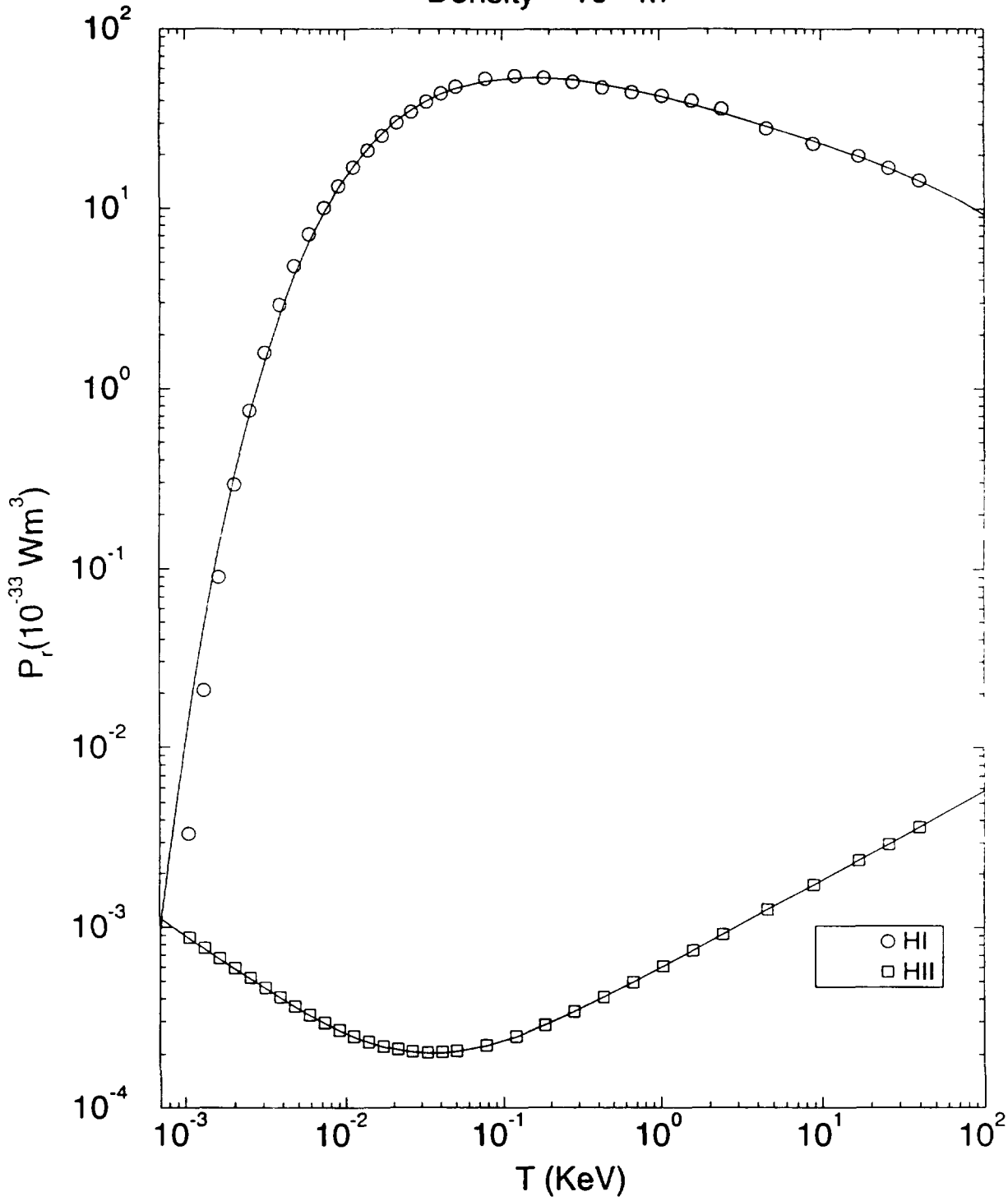


Figure 5

Radiative Loss Rate: H

Density = 10^{21} m^{-3}

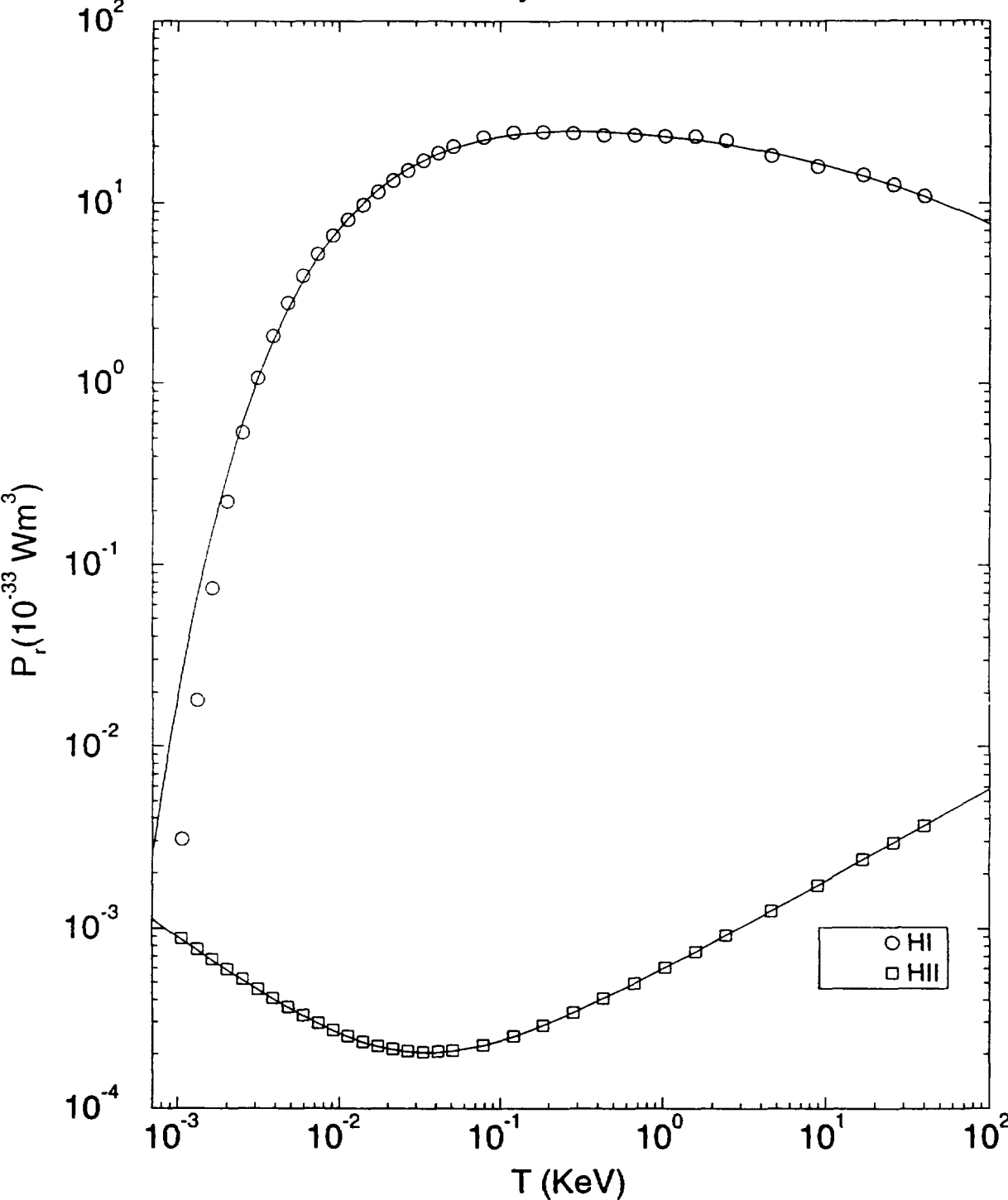


Figure 6

Electron Cooling Rate: He

Density = 10^{19} m^{-3}

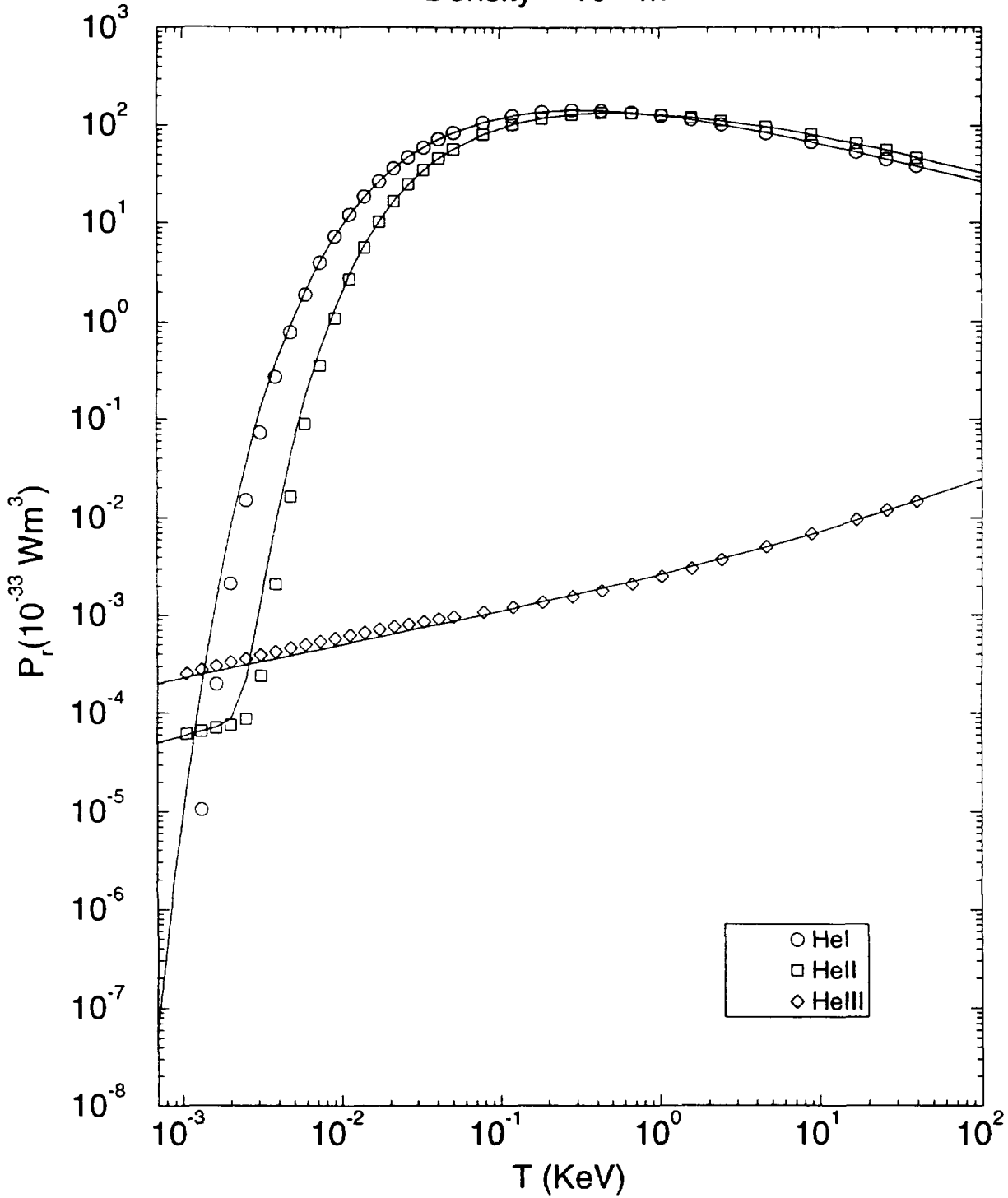


Figure 7

Electron Cooling Rate: He

Density = 10^{20} m^{-3}

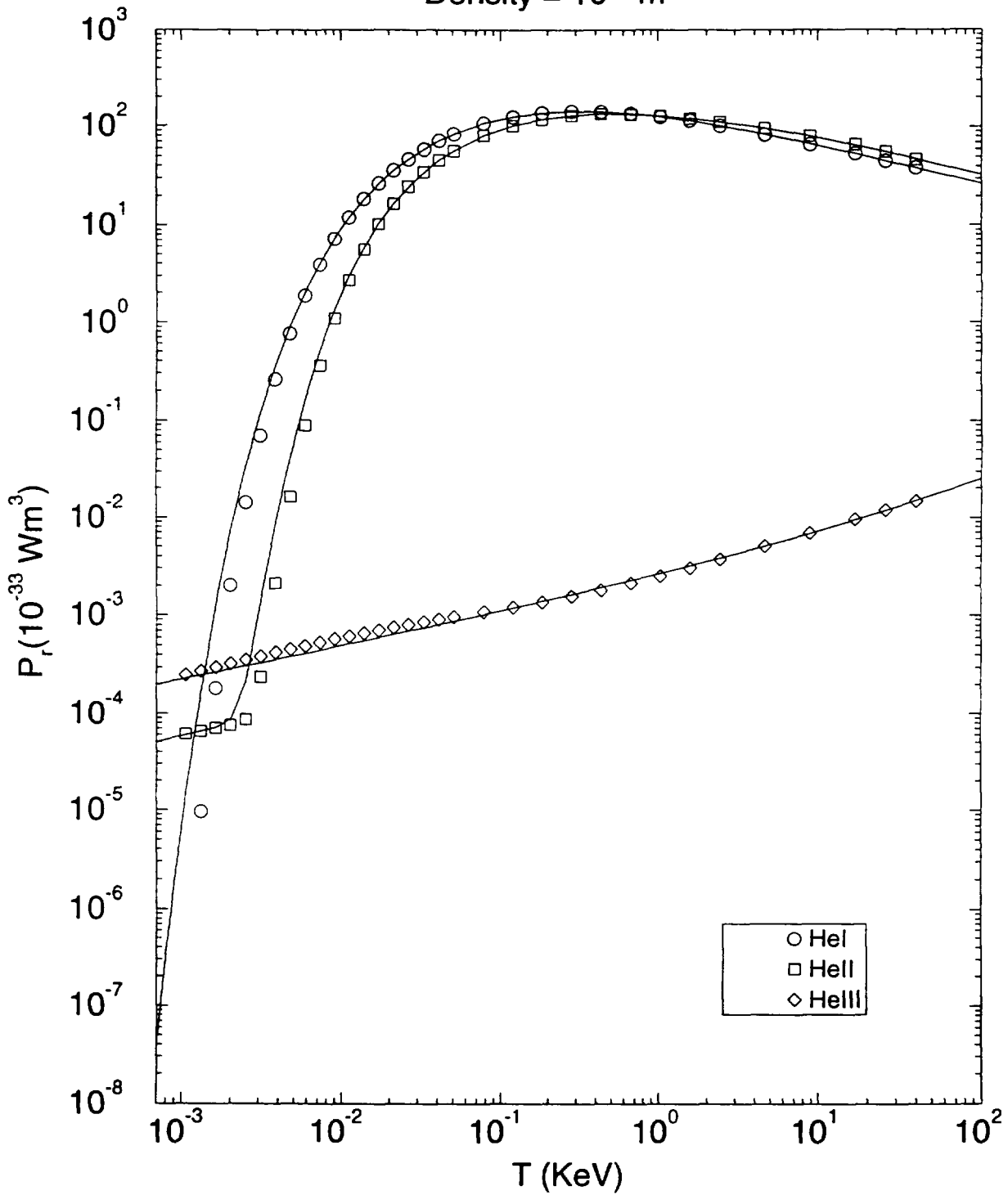


Figure 8

Electron Cooling Rate: He

Density = 10^{21} m^{-3}

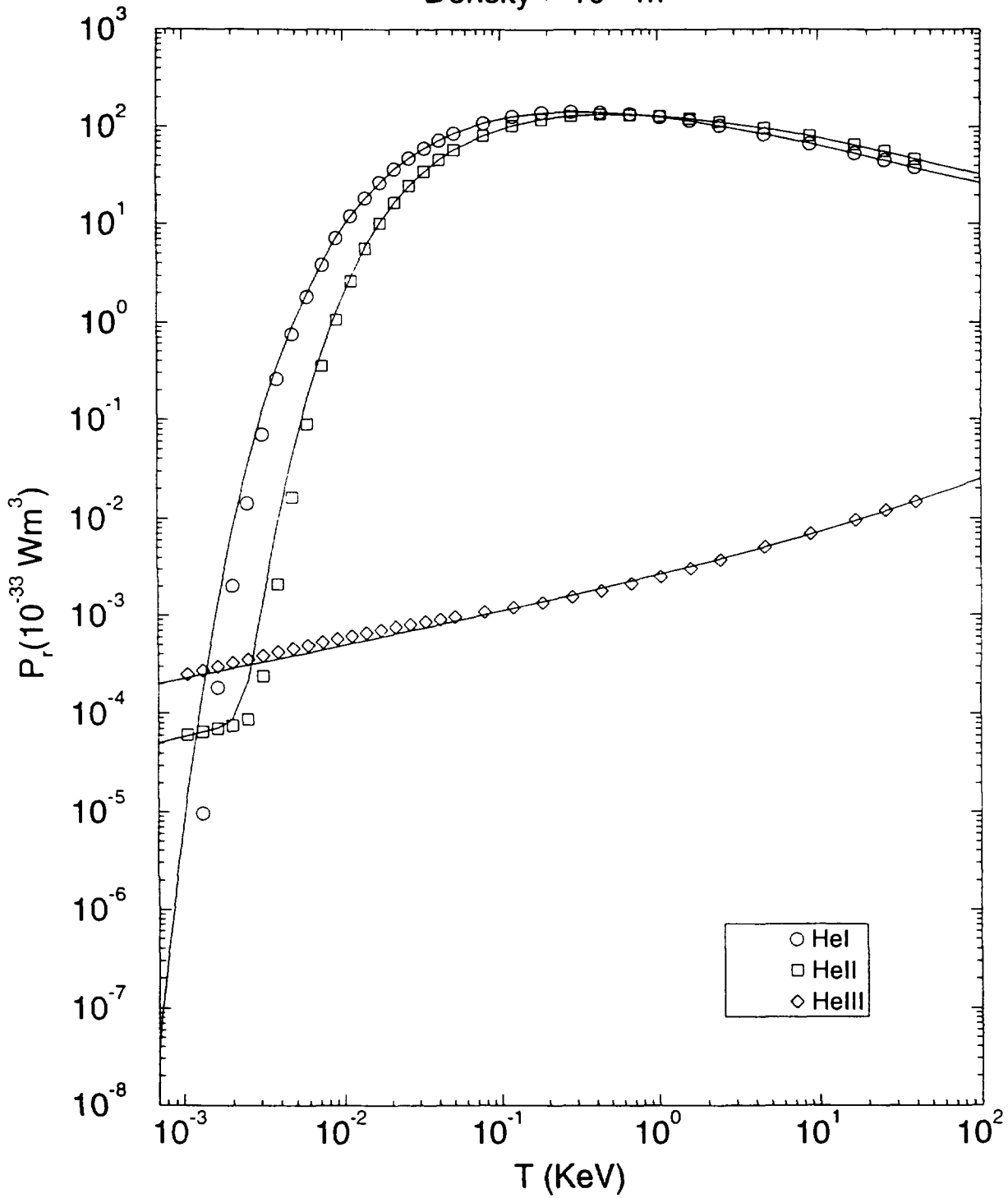


Figure 9

Radiative Loss Rate: He

Density = 10^{19} m^{-3}

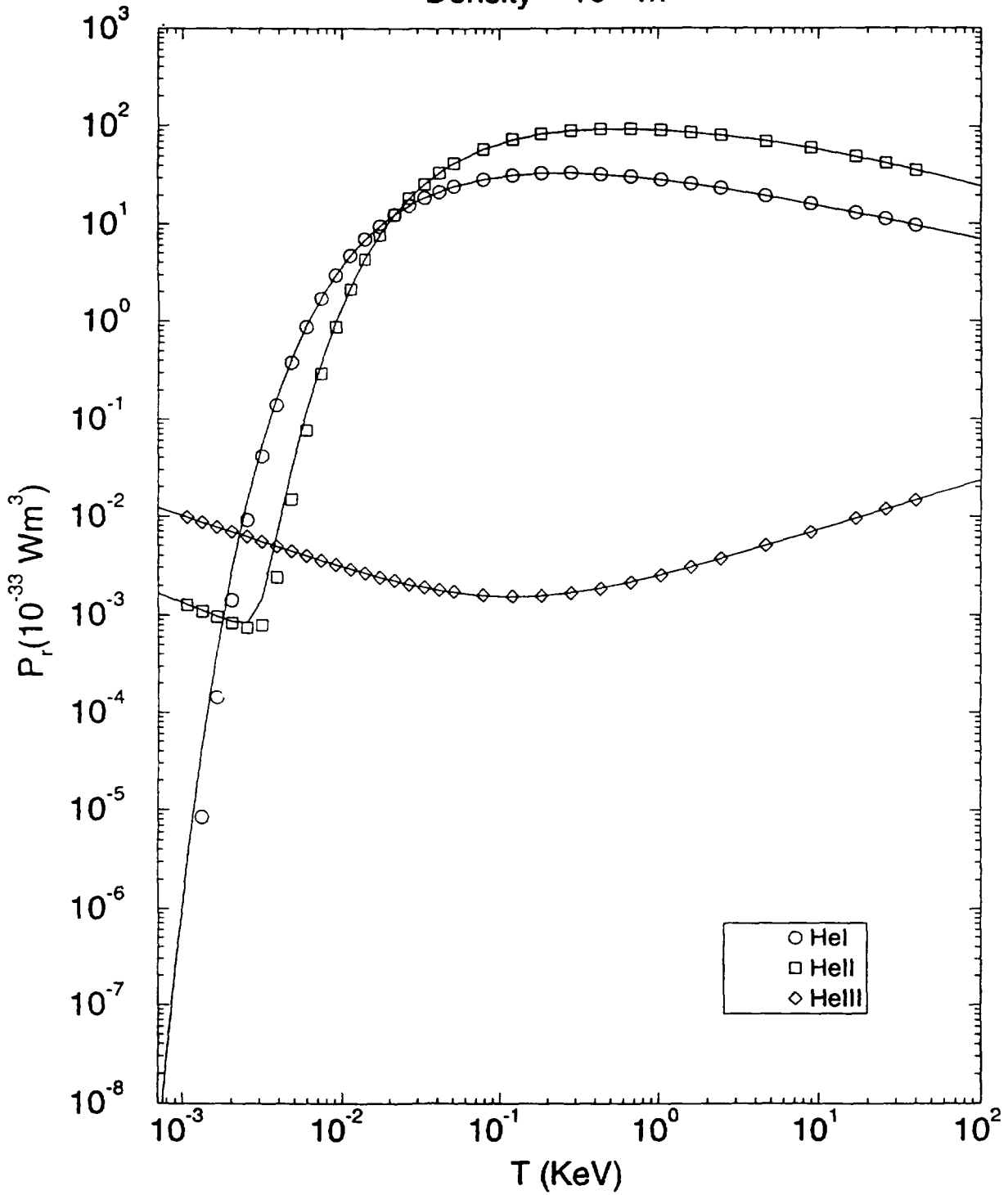


Figure 10

Radiative Loss Rate: He

Density = 10^{20} m^{-3}

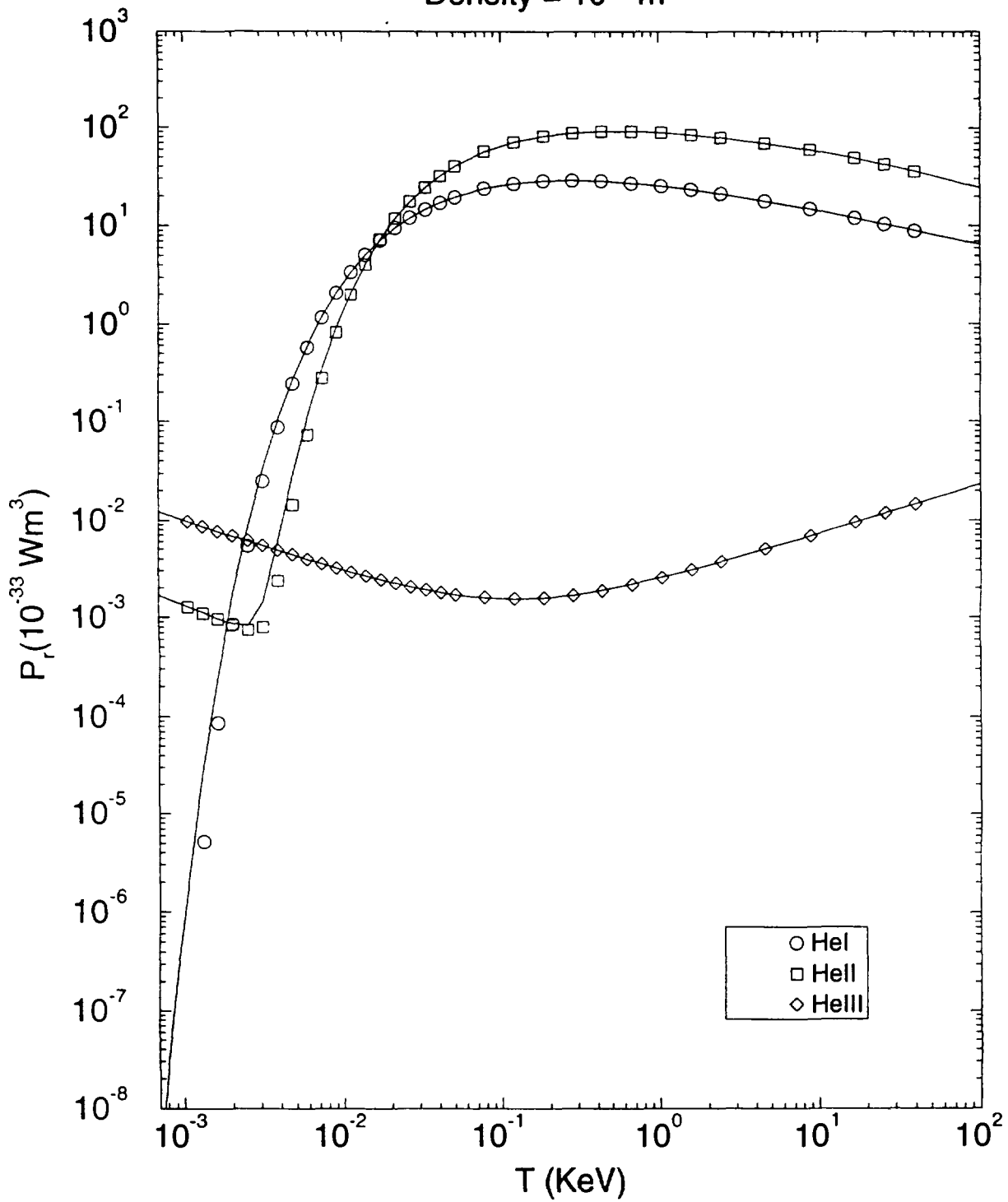


Figure 11

Radiative Loss Rate: He

Density = 10^{21} m^{-3}

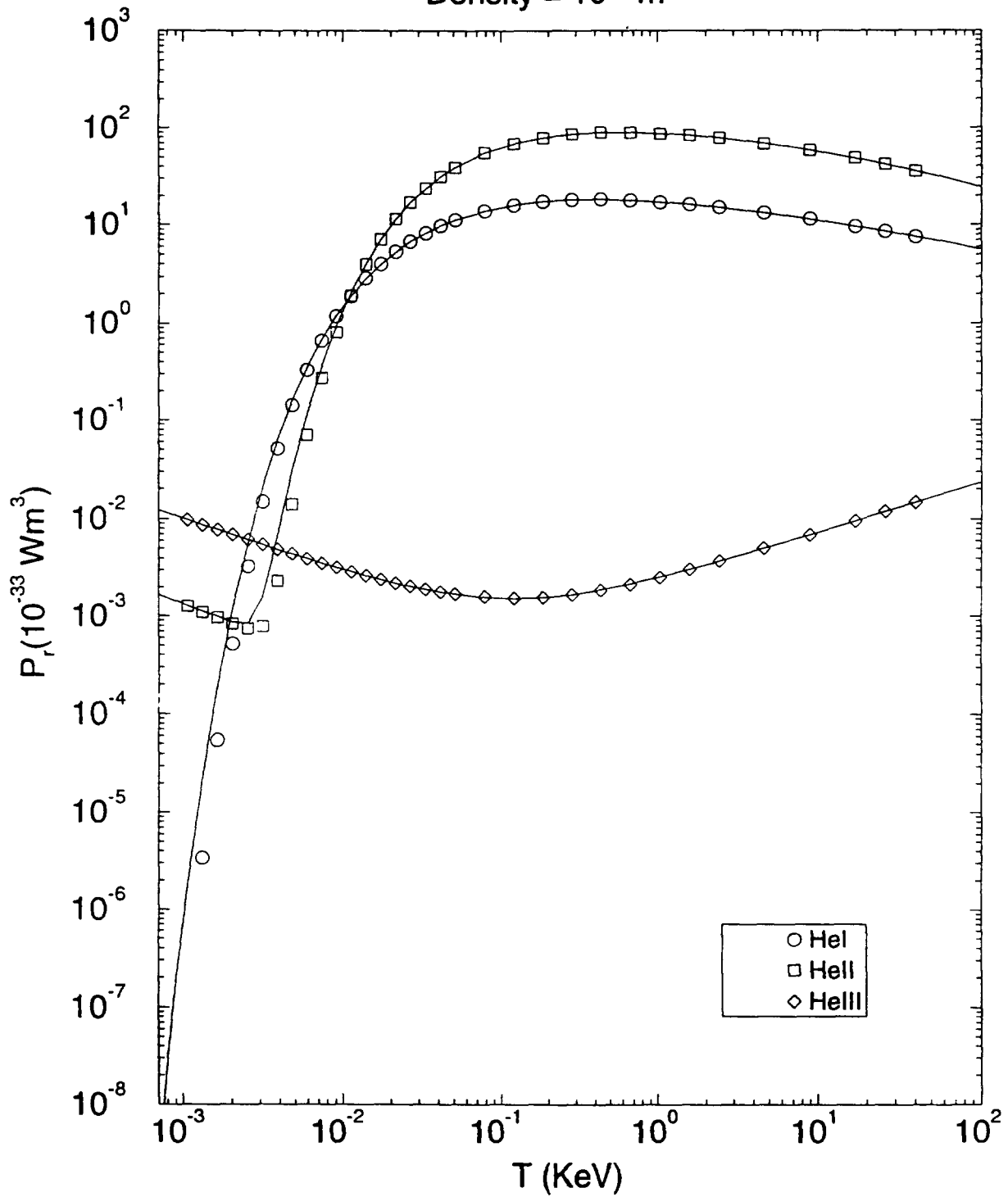


Figure 12

Electron Cooling Rate: C

Density = 10^{19} m^{-3}

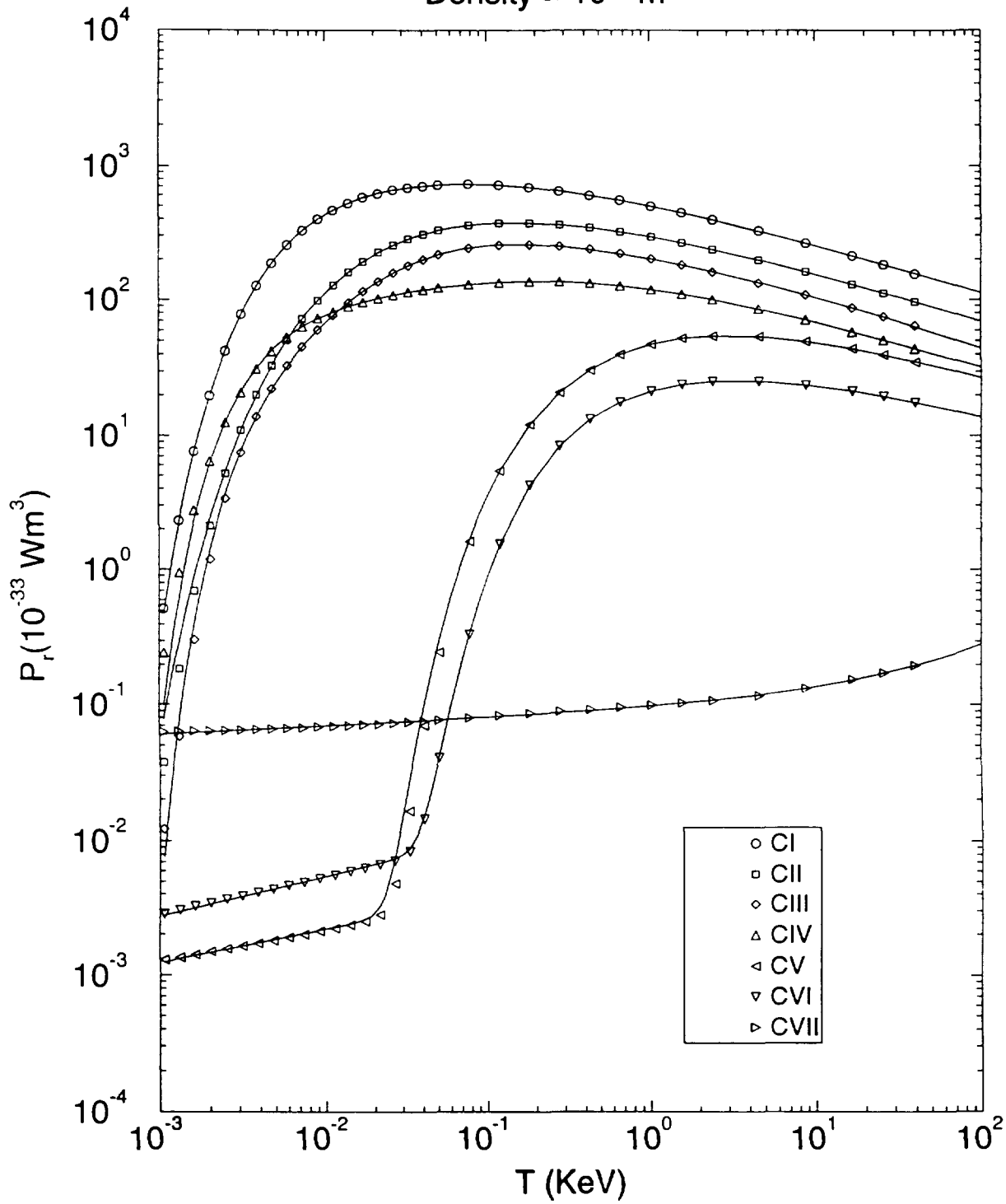


Figure 13

Electron Cooling Rate: C

Density = 10^{20} m^{-3}

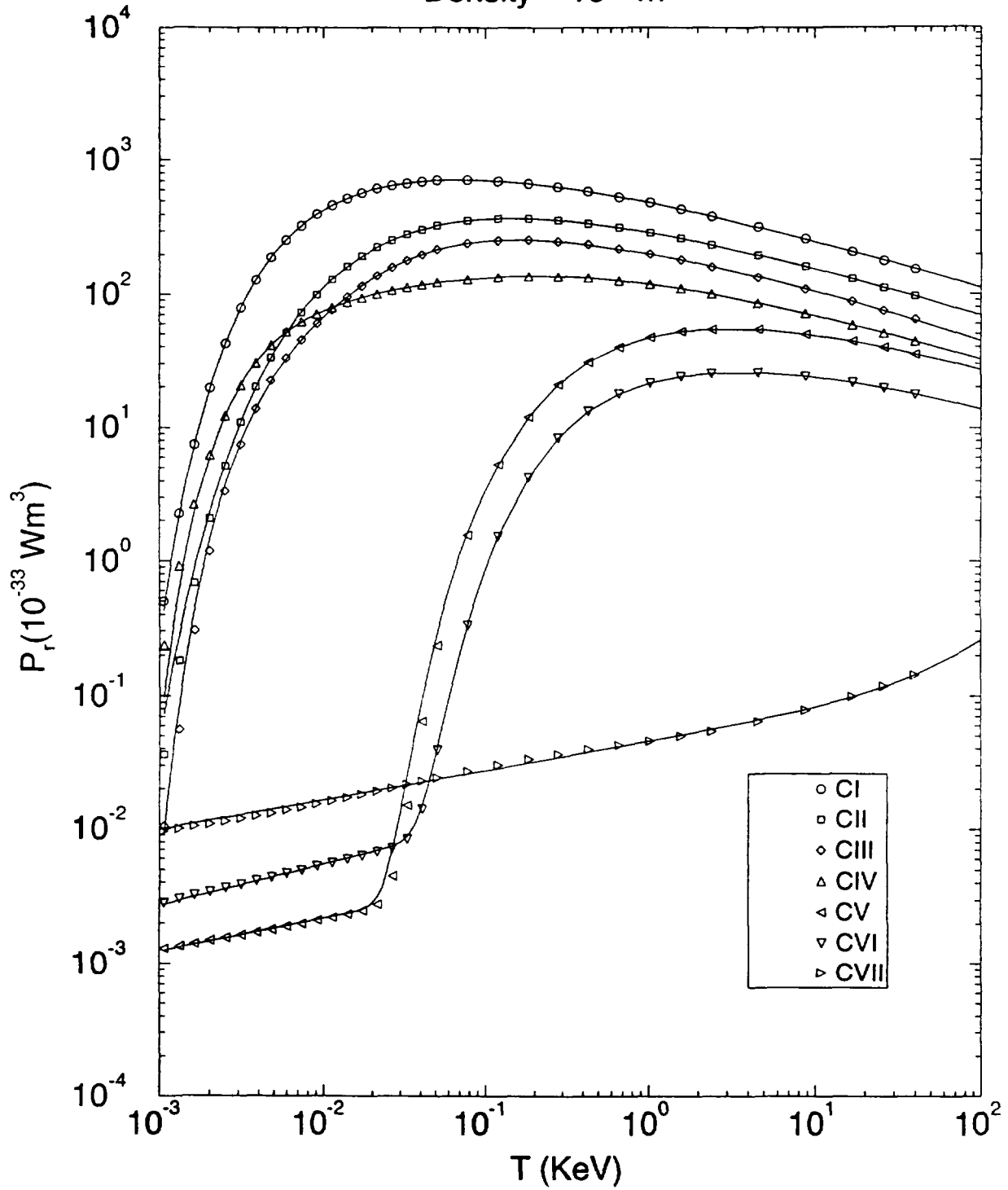


Figure 14

Electron Cooling Rate: C

Density = 10^{21} m^{-3}

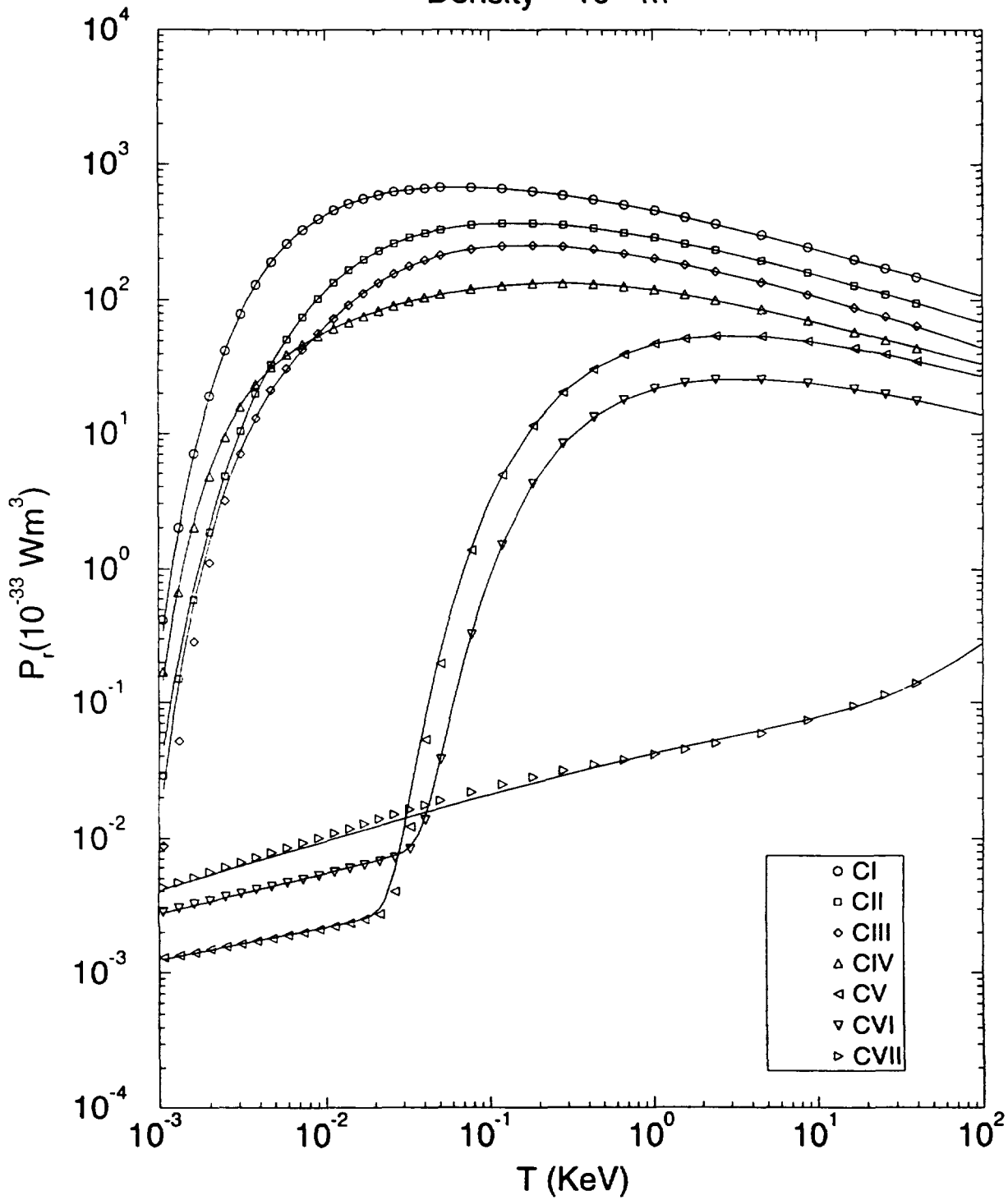


Figure 15

Radiative Loss Rate: C

Density = 10^{19} m^{-3}

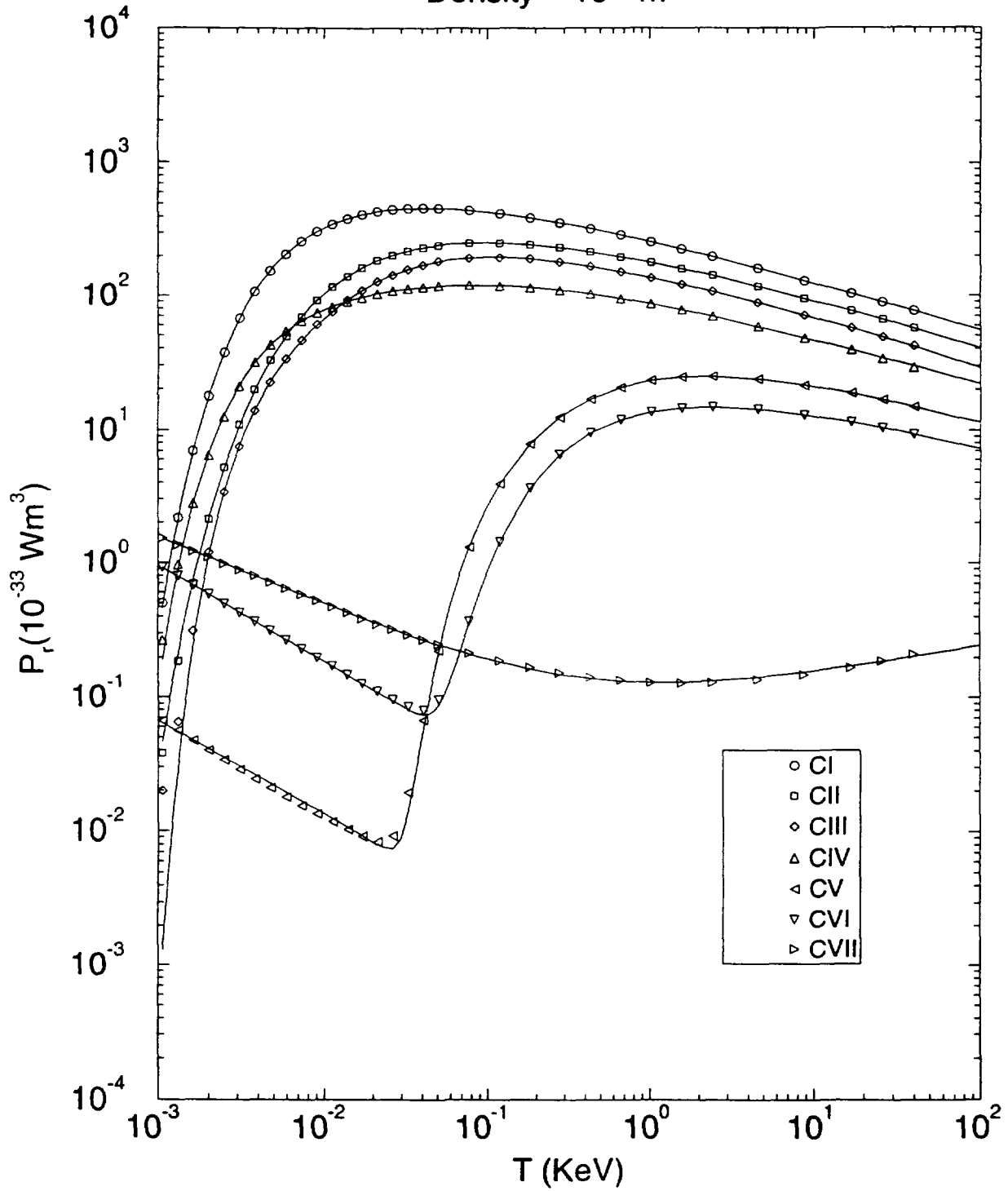


Figure 16

Radiative Loss Rate: C

Density = 10^{20} m^{-3}

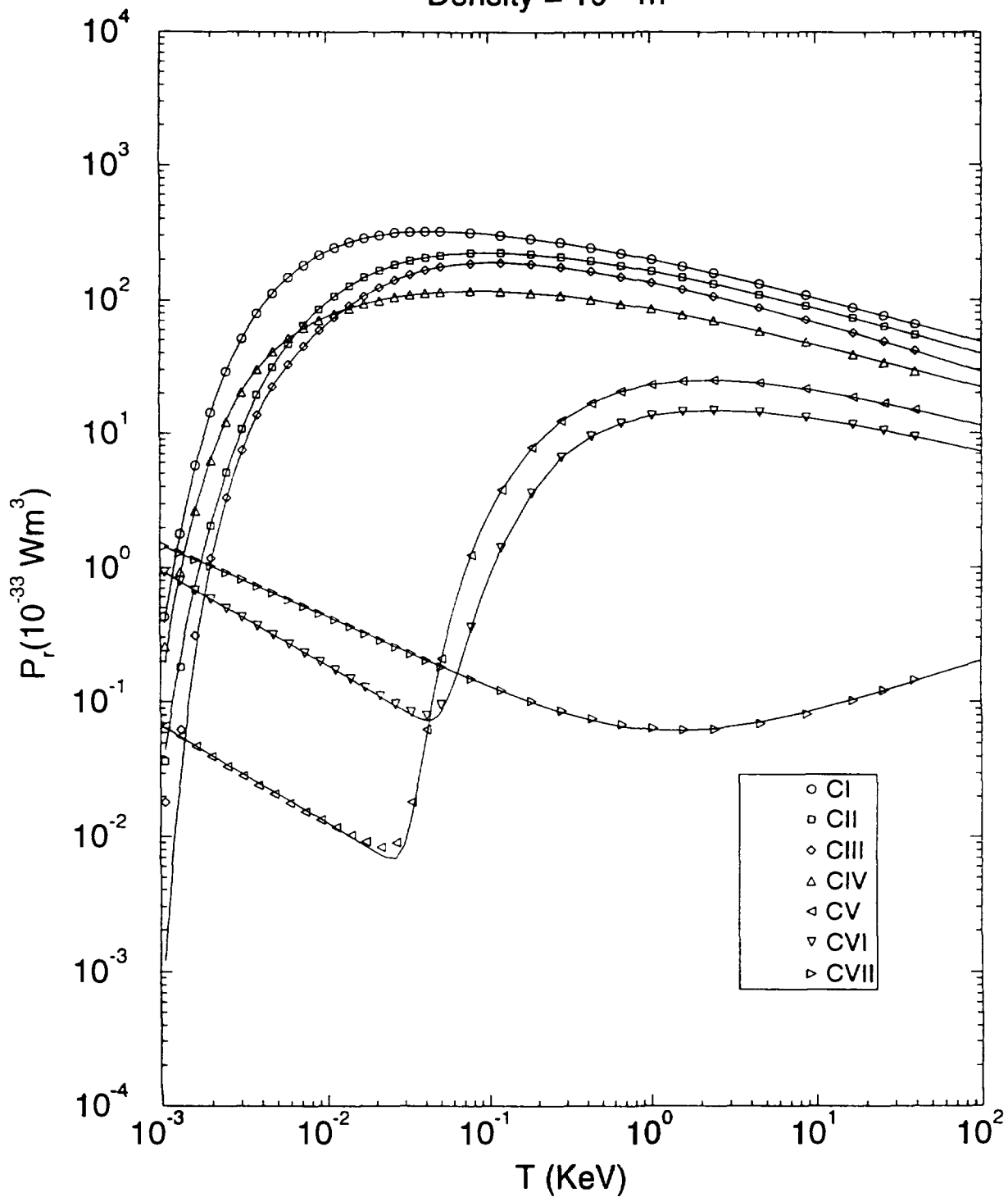


Figure 17

Radiative Loss Rate: C

Density = 10^{21} m^{-3}

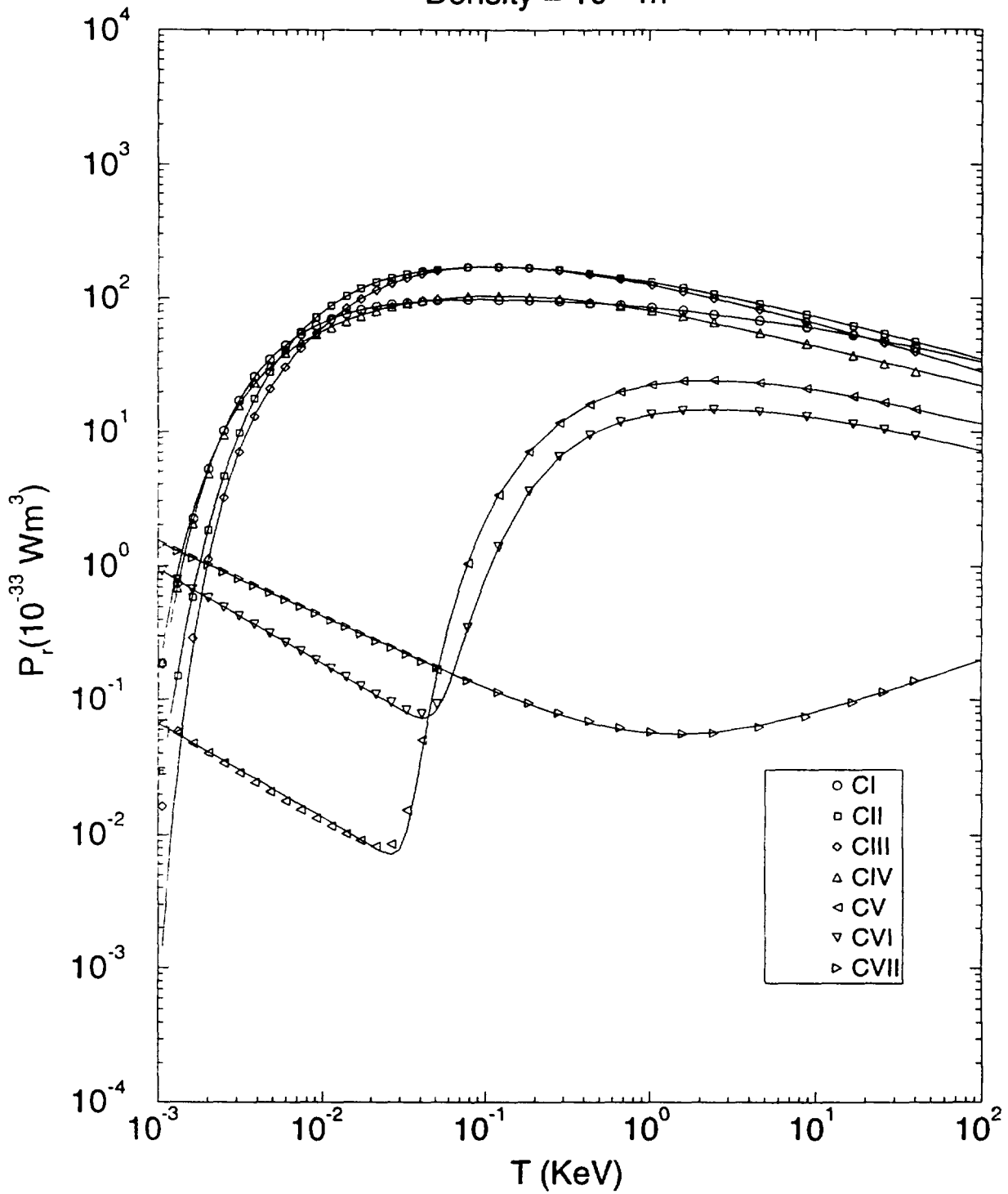


Figure 18

Electron Cooling Rate: O

Density = 10^{19} m^{-3}

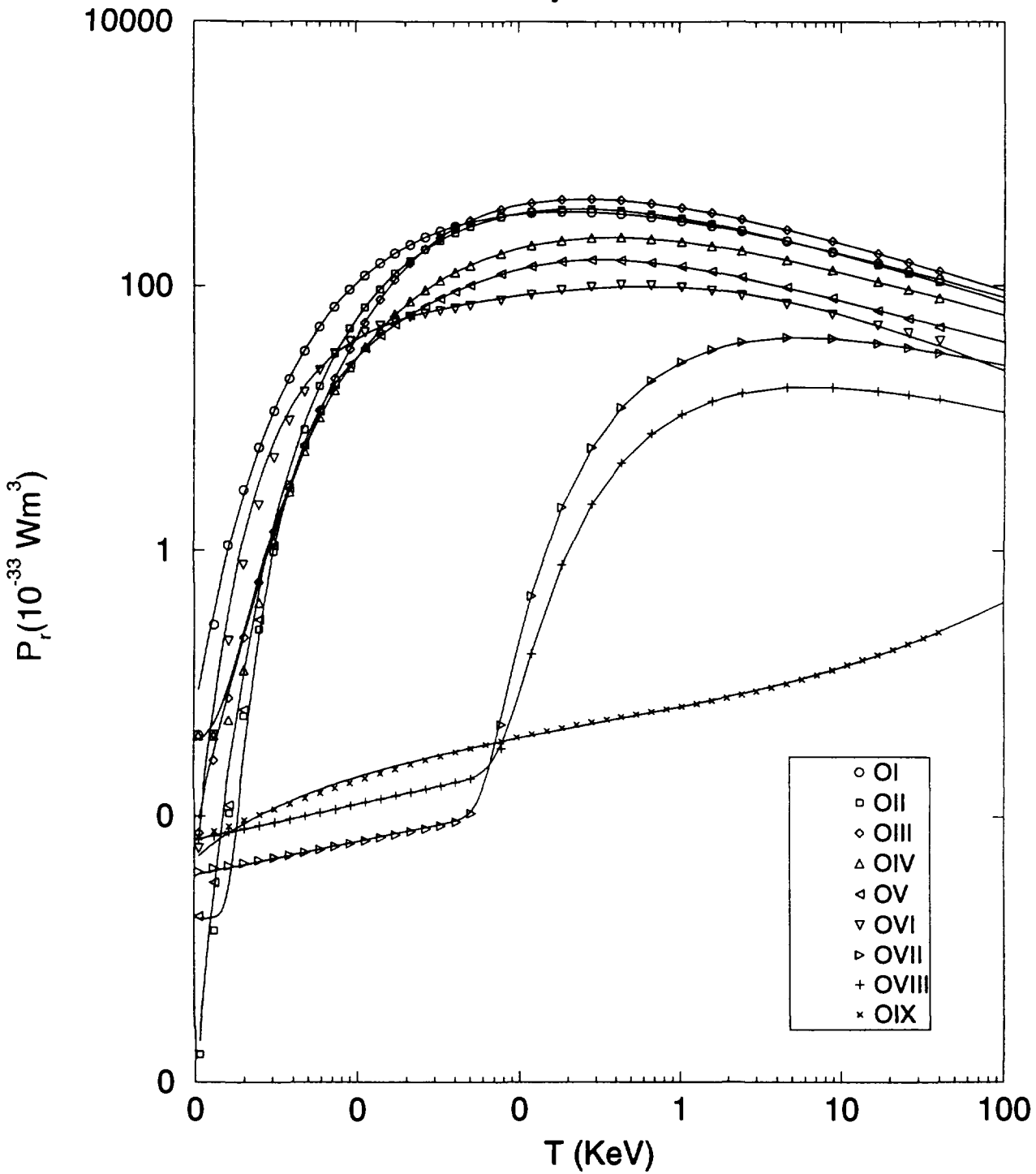


Figure 19

Electron Cooling Rate: O

Density = 10^{20} m^{-3}

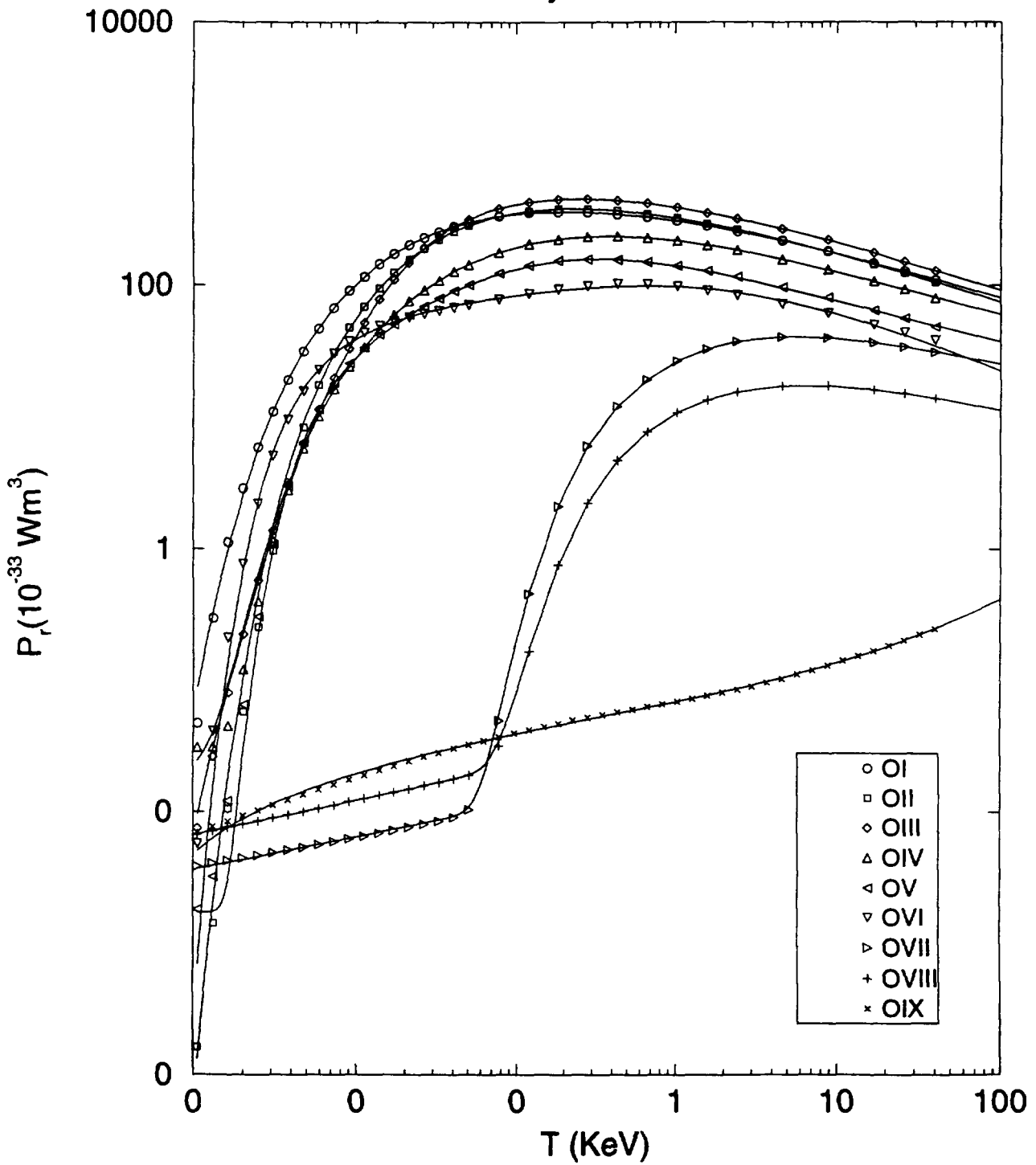


Figure 20

Electron Cooling Rate: O

Density = 10^{21} m^{-3}

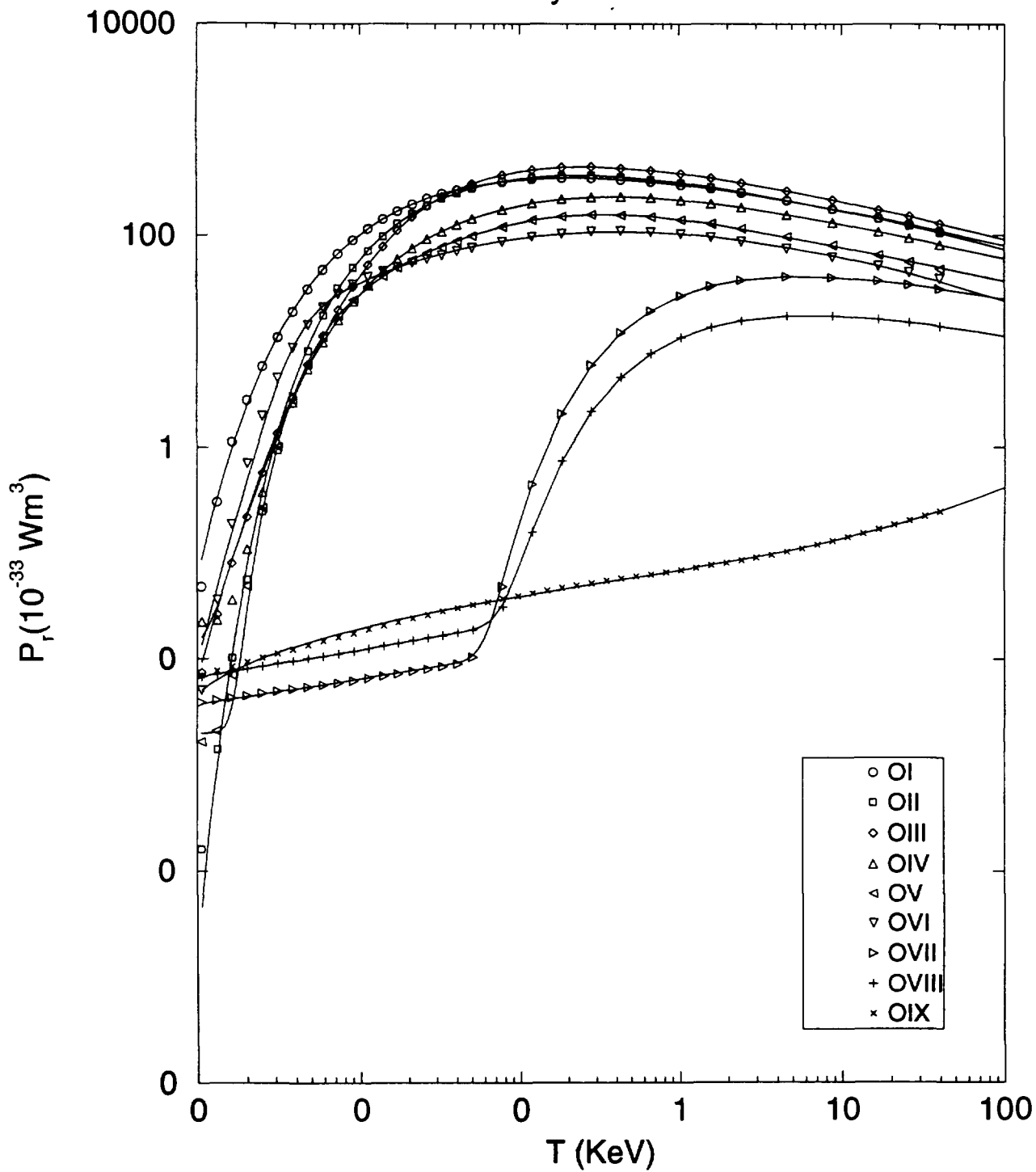


Figure 21

Radiative Loss Rate: O

Density = 10^{19} m^{-3}

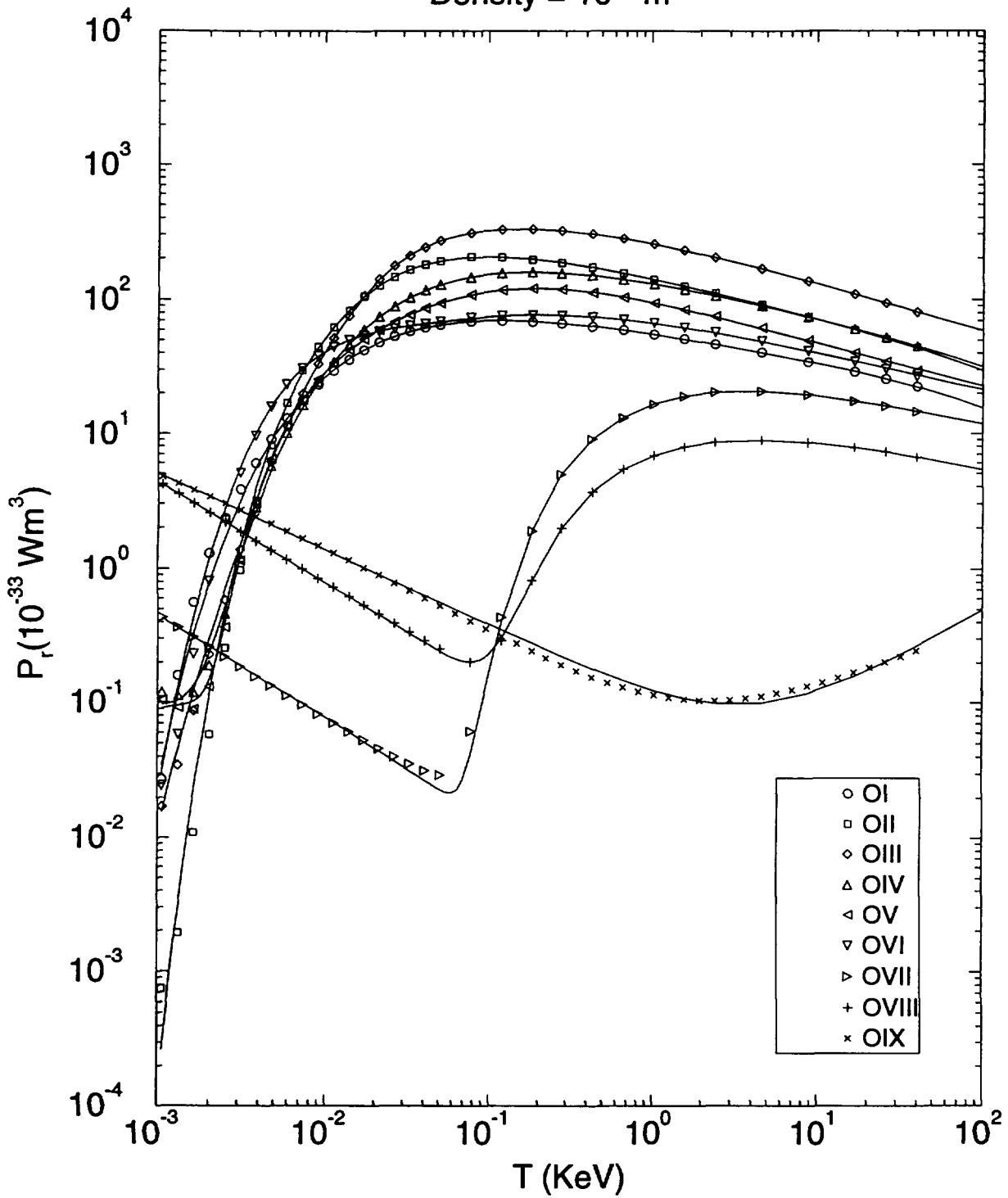


Figure 22

Radiative Loss Rate: O

Density = 10^{20} m^{-3}

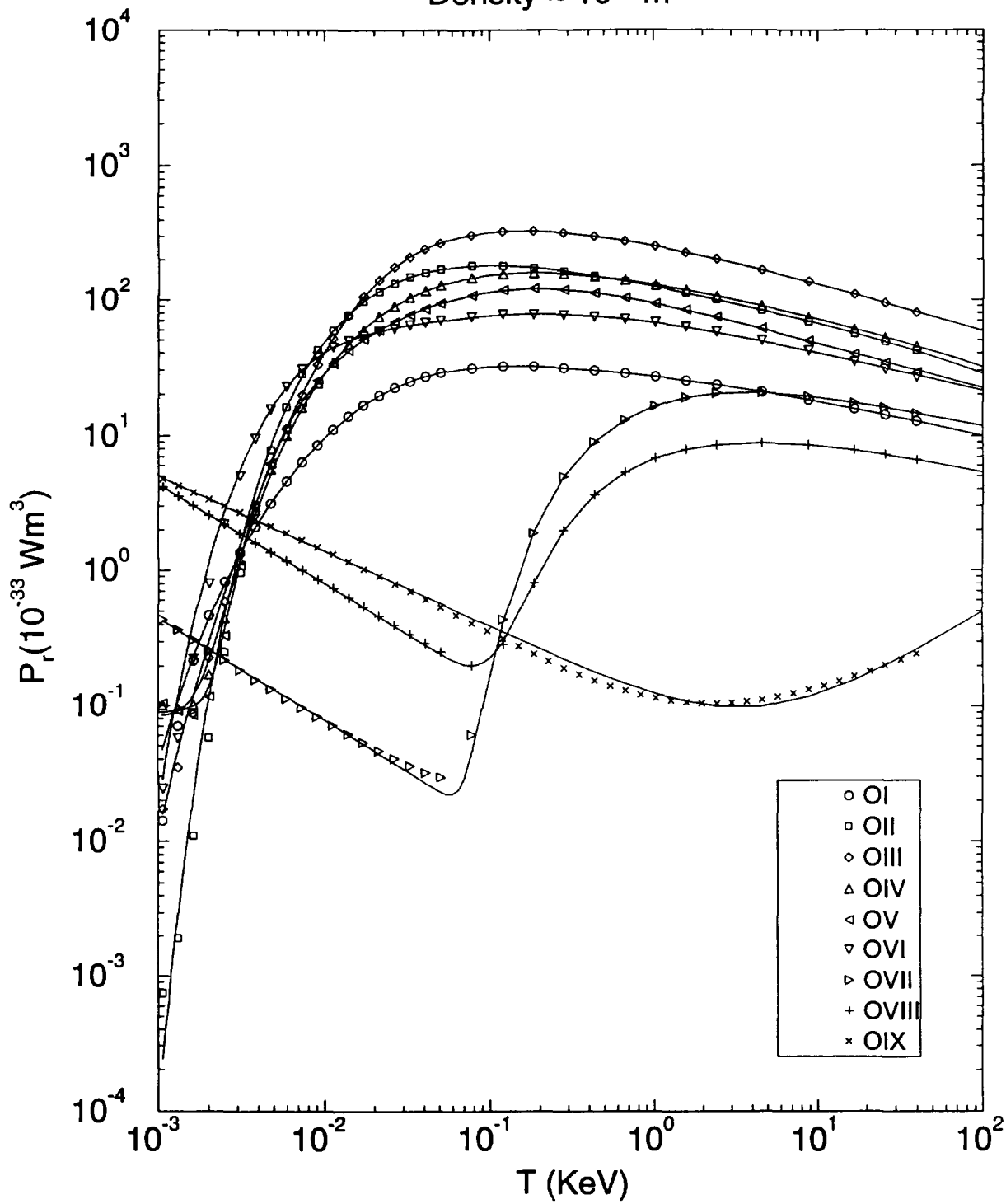


Figure 23

Radiative Loss Rate: O

Density = 10^{21} m^{-3}

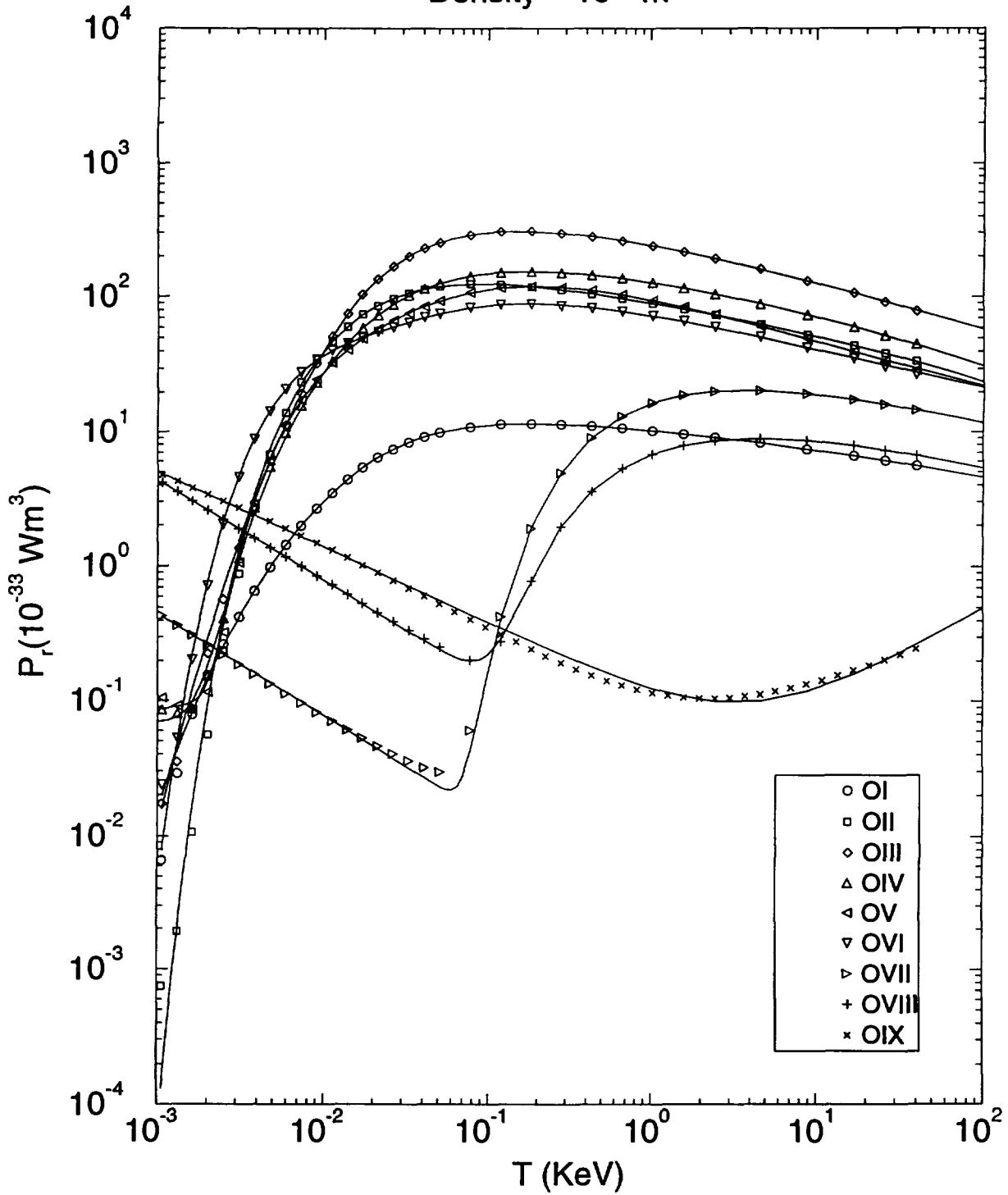


Figure 24