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I N D C INTERNATIONAL NUCLEAR DATA COMMITTEE

WORKSHOP
ON NUCLEAR STRUCTURE AND DECAY DATA:
THEORY AND EVALUATION
ADDENDUM - 2006

Editors: A.L.Nichols and P.K.McLaughlin
IAEA Nuclear Data Section
Vienna, Austria

June 2006

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WORKSHOP ON NUCLEAR STRUCTURE AND DECAY DATA: THEORY AND EVALUATION ADDENDUM - 2006

ICTP Trieste, Italy

20 February – 3 March 2006

Editors

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Vienna, Austria

Abstract

A two-week Workshop on Nuclear Structure and Decay Data under the auspices of the IAEA Nuclear Data Section was organised and administrated at the Abdus Salam International Centre for Theoretical Physics (ICTP) in Trieste, Italy from 20 February to 3 March 2006. This workshop constituted a further development of previous Nuclear Structure and Decay Data Workshops held in 2002, 2003 and 2005. The aims and contents of this workshop are summarized, along with the agenda, list of participants, comments and recommendations. Most of the workshop material can be found in the INDC report of the equivalent workshop of 17 to 28 November 2003 (INDC(NDS)-452). Some new material was prepared for 4 to 15 April 2005, as a first addendum (INDC(NDS)-473), furthermore, new and modified lectures from the 20 February to 3 March 2006 workshop have been brought together in this second addendum report. All of this material is freely available on CD-ROM (all relevant PowerPoint presentations and manuals along with appropriate computer codes):

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June 2006

TABLE OF CONTENTS

WORKSHOP
ON NUCLEAR STRUCTURE AND DECAY DATA: THEORY AND EVALUATION – ADDENDUM, 2006

Summary

ICTP Trieste, Italy

20 February – 3 March 2006

Prepared by

A.L. Nichols

IAEA Nuclear Data Section

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Abstract

Basic aspects of a two-week Workshop on Nuclear Structure and Decay Data: Theory and Evaluation are outlined in this short summary note for the record. The aims and contents of this workshop are summarized, along with the agenda, list of participants, comments and recommendations. Further consideration will be given to holding this specific workshop at various time intervals for training purposes (with agreed changes and regular modifications) on the advice of the International Nuclear Data Committee (INDC) and the International Network of Nuclear Structure and Decay Data Evaluators.

June 2006

1.1 OBJECTIVES

The International Atomic Energy Agency sponsored a two-week Workshop on "Nuclear Structure and Decay Data: Theory and Evaluation" at the Abdus Salam International Centre for Theoretical Physics (ICTP) in Trieste from 20 February to 3 March 2006. This workshop was organised and directed by A.L. Nichols (IAEA Nuclear Data Section), J. Tuli (NNDC, Brookhaven National Laboratory, USA) and A. Ventura (ENEA, Bologna, Italy).

As with earlier workshops [1,3], the primary objective was to familiarize nuclear physicists and engineers from both developed and developing countries with

(i) modern nuclear models;
(ii) relevant experimental techniques;
(iii) statistical analyses procedures to derive recommended data sets;
(iv) evaluation methodologies for nuclear structure and decay data;
(v) international efforts to produce the Evaluated Nuclear Structure Data File (ENSDF).

Reliable nuclear structure and decay data are important in a wide range of nuclear applications and basic research. Participants were introduced to both the theory and measurement of nuclear structure data, and the use of computer codes to evaluate decay data.

Detailed presentations were given by invited lecturers, along with computer exercises and workshop tasks. Participants were also invited to contribute their own thoughts and papers of direct relevance to the workshop.

1.2 PROGRAMME

The workshop programme is briefly summarised below.

1.2.1 Agenda

<u>Monday, 20 February 2006</u>

08:30 - 9:30	Registration
10:30 - 12:30	Opening Session Welcome (Alan Nichols (IAEA) and Jag Tuli (BNL)) Aims (Jag Tuli) NSDD – general features (Jag Tuli) IAEA-NDS – NSDD network and recent relevant CRPs (Alan Nichols)
12:30 - 14:00	Lunch break
14:00 – 15:30	Introduction to ICTP computer facilities (Johannes Grassberger/ Kevin McLaughlin)
15:30 – 16:00	Coffee break
16:00 – 17:30	Introduction (cont.) Web capabilities + NUDAT (Tom Burrows and Alan Nichols)

Tuesday, 21 February 2006

09:00 – 10:30	Nuclear theory (Piet Van Isacker)
10:30 – 11:00	Coffee break
11:00 – 12:30	ENSDF format + model exercises (Jag Tuli)
12:30 - 14:00	Lunch break
14:00 – 15:30	Bibliographic databases and ENSDF programs (Tom Burrows)
15:30 – 16:00	Coffee break
16:00 – 17:30	Students' presentations

Wednesday, 22 February 2006

09:00 – 10:30	Nuclear theory (Piet Van Isacker)
10:30 – 11:00	Coffee break
11:00 – 12:30	ENSDF – evaluation techniques (Jagdish Tuli)
12:30 - 14:00	Lunch break
14:00 – 15:30	ENSDF programs+model exercise (Tom Burrows)
15:30 – 16:00	Coffee break
16:00 – 17:30	Students' presentations

Thursday, 23 February 2006

09:00 – 10:30	Experimental techniques (Filip Kondev)
10:30 – 11:00	Coffee break
11:00 – 12:30	ENSDF – decay (Eddie Browne)
12:30 - 14:00	Lunch break
14:00 – 15:30	ENSDF- reaction (Coral Baglin)
15:30 – 16:00	Coffee break
16:00 – 17:30	Workshop activities (Jagdish Tuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)

Friday, 24 February 2006

09:00 – 10:30	Experimental techniques (Filip Kondev)
10:30 – 11:00	Coffee break
11:00 – 12:30	Model exercise – decay (lead by Eddie Browne)
12:30 - 14:00	Lunch break
14:00 onwards	Free time

Monday, 27 February 2006

09:00 – 10:30	ENSDF – Theory (Yogendra Gambhir)
10:30 – 11:00	Coffee break
11:00 – 12:30	Model exercise – reaction (lead by Coral Baglin)
12:30 - 14:00	Lunch break
14:00 – 15:30	Workshop activities (Jagdish Tuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)
15:30 – 16:00	Coffee break
16:00 – 17:30	Workshop activities (Jagdish Tuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)

Tuesday, 28 February 2006

09:00 – 10:30	ENSDF – Theory (Yogendra Gambhir)
10:30 – 11:00	Coffee break
11:00 – 12:30	ENSDF- adopted (Coral Baglin)
12:30 - 14:00	Lunch break
14:00 – 15:30	Model exercises- adopted (Coral Baglin)
15:30 – 16:00	Coffee break
16:00 – 17:30	Workshop activities (Jagdish Tuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)

Wednesday, 1 March 2006

09:00 – 10:30	ENSDF – Experimental techniques (Tibor Kibedi)
10:30 – 11:00	Coffee break
11:00 – 12:30	Data analyses (Desmond MacMahon)
12:30 - 14:00	Lunch break
14:00 – 15:30	Workshop activities (Jagdish Tuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)
15:30 – 16:00	Coffee break
16:00 – 17:30	Workshop activities (JagdishTuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)

Thursday, 2 March 2006

09:00 – 10:30	ENSDF – Other data considerations (Tibor Kibedi)
10:30 – 11:00	Coffee break
11:00 – 12:30	Data analyses (Desmond MacMahon)
12:30 - 14:00	Lunch break
14:00 – 15:30	Workshop activities (Jagdish Tuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)
15:30 – 16:00	Coffee break
16:00 – 17:30	Workshop activities (Jagdish Tuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)

Friday, 3 March 2006

09:00 – 10:30	Workshop activities (Jagdish Tuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)
10:30 – 11:00	Coffee break
11:00 – 12:30	Review of workshop (Jagdish Tuli; Thomas Burrows; Eddie Browne; Alan Nichols)
12:30 - 14:00	Lunch break
14:00 – 15:30	Workshop activities (Jagdish Tuli; Thomas Burrows; Coral Baglin; Eddie Browne; Kevin McLaughlin)
15:30	Close of workshop

1.2.2 Participants

Twenty-three participants (predominantly from developing countries) with full or partial support from the IAEA were selected to attend the workshop in February 2006. Selection was undertaken by Nuclear Data Section staff in association with the workshop directors and ICTP staff.

First row, seated from left to right:
Zelia Maria DA COSTA LUDWIG (Brazil), Hanane SAIFI (Algeria), Yusuf Aminu AHMED (Algeria), Alan NICHOLS (IAEA), Lamia AISSAOU (Algeria), GAMBHIR Yogendra (India), Monica Galan (Spain), Zhaleh GHAEMI BAFGHI (Iran), Coral M. BAGLIN (USA)

Second row, standing from left to right:
Edgardo BROWNE-MORENO (USA), Thomas W. BURROWS (USA), MERIC Niyazi (Turkey), Enkhold SANSARBAYAR (Mongolia), Ali Asghar MOWLAVI (Iran), Srijit BHATTACHARYA (India), Tibor Kibedi (Australia), Muhammad Obaidur RAHMAN (Bangladesh), Jagdish K. TULI (USA), Jameel-Un NABI (Pakistan), Filip Kondev (USA)

Third row, standing from left to right:
Huibin SUN (China), Wang Jimin (China), Kevin MCLAUGHLIN (IAEA), Desmond MACMAHON (UK), Pavlo GRYGOROV (Ukraine), Bayarbadrakh BARAMSAI (Mongolia), Fouad Attia MAJEED (Iraq), Faustin Laurentiu ROMAN (Romania), Kumar SURESH (India), Lilya ATANASOVA (Bulgaria)

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1.3 PRESENTATIONS AVAILABLE IN ELECTRONIC FORM ON CD-ROM

Presentations by Lecturers

Aims of the Workshop - General features of NSDD, J. Tuli

Nuclear Theory:
Nuclear Shell Model, P. Van Isacker (November 2003)
Interacting Boson Model, P. Van Isacker
Nuclear Structure: Single-particle models, P. Van Isacker (February 2006)
Nuclear Structure: Collective models, P. Van Isacker (February 2006)
Structure of the odd-even nuclei in the interacting boson model, S. Brant (April 2005)
High spin states in the interacting boson and boson-fermion model, S. Brant (April 2005)
Structure of odd-odd nuclei in the interacting boson-fermion-fermion model, S. Brant (April 2005)
β decay in the interacting boson-fermion model, S. Brant (April 2005)
Geometrical Symmetries in Nuclei – An Introduction, A. Jain ((November 2003)
Geometrical Symmetries in Nuclei, A. Jain (November 2003)
Lectures on Geometrical Symmetries in Nuclei, A. Jain (November 2003)
Hartree-Fock-Bogoliubov Method, D. Vretenar (November 2003)
Self-consistent Mean-field Models – Structure of Heavy Nuclei, D. Vretenar (November 2003)
Quasiparticle OR BCS method, Y. Gambhir (February 2006)
Hartree-Fock (HF) Mean Field Theory. Y. Gambhir (February 2006)

Experimental Nuclear Spectroscopy:
Introduction, P. Von Brentano
Lecture I – Nuclear Shapes, P. Von Brentano
Lecture II – Measurement of Lifetimes, P. Von Brentano
Lecture I – Experimental Nuclear Structure Physics, F. Kondev (April 2005)
Lecture II – Experimental Nuclear Structure Physics at the extreme, F. Kondev (April 2005)
Lecture I – Experimental techniques to deduce J^{π}, T. Kibedi (February 2006)
Lecture II – New developments in characterizing nuclei using separators, T.Kibedi (February 2006)

Statistical Analyses:
Evaluation of Discrepant Data I, D. MacMahon

Evaluation of Discrepant Data II, D. MacMahon
Convergence of Techniques for the Evaluation of Discrepant Data: D. MacMahon, A. Pearce, P. Harris
Techniques for Evaluating Discrepant Data, M.U. Rajput, D. MacMahon
Possible Advantages of a Robust Evaluation of Comparisons, J.W. Muller (presented by D. MacMahon)

ENSDF:
Evaluated Nuclear Structure Data Base, J.K. Tuli
Evaluations – A Very Informal History, J.K. Tuli
Evaluated Nuclear Structure Data File – A Manual for Preparation of Data Sets, J.K. Tuli
Guidelines for Evaluators, M.J. Martin, J.K. Tuli
Bibliographic Databases, T.W. Burrows

ENSDF Analysis and Utility Codes, T.W. Burrows:
- Their Descriptions and Uses, T.W. Burrows
- FMTCHK (Format and Syntax Checking), T.W. Burrows
- PowerPoint presentations, T.W. Burrows
- LOGFT (Calculates log *ft* for beta decay), T.W. Burrows
- GTOL (Gamma to Level), T.W. Burrows
- HSICC (Hager-Seltzer Internal Conversion Coefficients), T.W. Burrows

ENSDF – Decay Data, E. Browne
Model Exercises – Decay, E. Browne
ENSDF – Reaction Data, C. Baglin
ENSDF – Adopted Levels and Gammas, C. Baglin
ENSDF – Examples 1, 2, 3, 4 and 5, C. Baglin

Additional Material:
IAEA: NSDD Network, Recent Relevant CRPs and Other Activities (PowerPoint presentation), A.L. Nichols
IAEA: NSDD Network, Recent Relevant CRPs and Other Activities (draft paper), A.L. Nichols
Nuclear Structure and Decay Data: Introduction to Relevant Web Pages (draft paper), T.W. Burrows, P.K. McLaughlin, A.L. Nichols

Presentations by Participants

2003 Workshop

ETFFS calculations of the low-lying strength in Ca isotopes, E.Litvinova
A=193 Mass Chain evaluation: A summary, Guillermo V. Marti
Fission of ^{210}Po and ^{198}Hg Nuclei at Intermediate Excitation Energies, Houshyar Noshad
Neutron Cross Sections of Er Isotopes, A.K.M. Harun-Ar-Rashid
Comparison of Rotating Finite Range Model and Thomas-Fermi Fission barriers, K. Mahata
Target/Projectile Structure Dependence In Transfer Reactions, P.K.Sahu
^{152}Gd collective states, V.Pronskikh

2005 Workshop

Compton Add-Back Protocols for use with the EXOGAM Array, A. Garnswothy

Experimental determination of photon emission probabilities, A. Luca
Nuclear data activities for Astrophysics at Oak Ridge National Laboratory, C. Nesaraja
Tandar Laboratory, CNEA. Argentina, D. Abriola
Experimental approach to the dynamics of fission, G. Ishak Boushaki
Laboratoire National Henri Becquerel, M.M. Be
Nuclear structure by gamma-ray spectroscopy, a completeness perspective, N. Nica
Radioactive beam spectroscopy of ^{212}Po and ^{213}At with the EXOGAM array, N. Thompson
Developing ^{152}Eu into a standard for detector efficiency calibration, R.M. Castro

2006 Workshop
Photo-Nuclear Reaction Cross Sections for Some Isotopes of Ti and Mo, E.Sansarbayar
Evolution of Massive Stars, Jameel Un Nabi
Pulsed beam method for half-life time measurements M.R. band head in Pb^{197}, S. Kumar
g-factor measurement at RISING: The case of ^{127}Sn, Liliya Atanasova
BANDRRI, National Database at CIEMAT (SPAIN), M. Galan
An appropriate treatment of the Centre-Of-Mass motion in finite nuclei, P.Grygorov
Giant Dipole Resonances: Present & future perspectives at VECC, India, S. Bhattacharya

1.4 OTHER WORKSHOP MATERIALS ON CD-ROM

Atomic Masses
Access to NSDD Resources

NNDC Online Data Service Manual and Data Citation Guidelines

Introduction to International Nuclear Structure and Decay Data Network
Contact names and addresses

Access to ENSDF Format Summary and Examples

Nuclear Structure Manuals

1.5 ADDENDUM MANUAL

Significant quantities of written material have been prepared for the Nuclear Structure and Decay Data workshop. Their accumulation in various forms acted as an aid to the participants in their understanding of nuclear theory, measurement techniques, data analysis and ENSDF mass-chain evaluations, representing an important combination of technical information for future reference and other NSDD workshops. Therefore, a relatively large fraction of these presentations, background papers and manuals have been assembled for further use in the form of earlier documents [2,3] and this Addendum report.

Our intention is to use and develop this material in the years to come, particularly for other workshops of this type. Another aim is to ensure that such presentations are not lost, and can be readily at hand for new mass-chain and decay-data evaluators to assist them in their preparation of recommended data for the ENSDF files.

1.6 RECOMMENDATIONS AND CONCLUSIONS

A number of important points can be made concerning the workshop:

1. Twenty-three participants were selected and attended a two-week workshop that covered nuclear theory and modeling, relevant experimental techniques, statistical analyses, and the philosophy and methodology for comprehensive mass chain evaluations. Support materials and information were also provided on the International Network of Nuclear Structure and Decay Data Evaluators and the most relevant CRPs organized by the IAEA Nuclear Data Section.

2. Workshop participants were introduced to mass chain evaluations through group and individual PC/computing activities (over 50% of the agenda of the second week) CD-ROM and hardcopy materials were provided by IAEA staff for all students/lecturers.

3. Administrative functions leading up to and during the course of the workshop worked smoothly, including visa arrangements, travel and subsistence payments to students and lecturers, additional banking transactions, and hotel/guest-house accommodation.

4. Specific participants were identified for future involvement in NSDD and mass chain evaluations.

5. Various important lessons were learnt by the IAEA staff and lecturers involved in this ICTP workshop. Students were given the opportunity to review the workshop through a written questionnaire and direct discussions (on 3 March). Their major recommendations are as follows:

(a) provision of all lecture materials prior to the workshops – all available lecture materials can be found on the ICTP website withing one to two weeks of the workshop (ICTP and lecturers to note);

(b) forewarn participants that they will be asked to give a short presentation on their own nuclear physics studies - this warning was made in the advertising material for the workshop, but not all students were aware (ICTP to note);

(c) begin PC activities earlier in the course (although this would pose difficulties with respect to students' awareness of the nature of the work through the series of eight necessary ENSDF lectures);

(d) questioned the need for the Friday afternoon break at the end of the first week (although requested by participants at previous workshops);

(e) requested outside activities during the middle weekend (ICTP to note);

(g) introduce ENSDF format to participants prior to the workshop (through IAEA-NDS web pages?);

As before, this combination of Wednesday/Thursday written questionnaire and Friday face-to-face review session produced significant feedback. The overall opinion of the majority of the students was that they had thoroughly enjoyed the 2-week workshop, made useful new contacts, and learnt much about nuclear structure and decay data:

ACKNOWLEDGEMENTS

The authors wish to thank our fellow co-directors of the NSDD Workshop for their support leading up to February 2006, and particularly the lecturers (all experts in their fields) for their enthusiasm during the workshop and provision of the various technical input to this document. Administrative aspects of the workshop were considerable leading up to and during the course – as an ICTP-supported activity, all such features and problems were handled by Ms Elizabeth Brancaccio (ICTP), and her efforts were much appreciated.

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2.

Nuclear Structure

(I) Single-particle models

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Nuclear Structure
(I) Single-particle models

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Overview of nuclear models

- *Ab initio* methods: Description of nuclei starting from the bare nn & nnn interactions.
- Nuclear shell model: Nuclear average potential + (residual) interaction between nucleons.
- Mean-field methods: Nuclear average potential with global parametrisation (+ correlations).
- Phenomenological models: Specific nuclei or properties with local parametrisation.

Nuclear shell model

- Many-body quantum mechanical problem:

$$\hat{H} = \sum_{k=1}^{A} \frac{p_k^2}{2m_k} + \sum_{k<l}^{A} \hat{V}_2(\boldsymbol{r}_k, \boldsymbol{r}_l)$$

$$= \underbrace{\sum_{k=1}^{A} \left[\frac{p_k^2}{2m_k} + \hat{V}(\boldsymbol{r}_k) \right]}_{\text{mean field}} + \underbrace{\left[\sum_{k<l}^{A} \hat{V}_2(\boldsymbol{r}_k, \boldsymbol{r}_l) - \sum_{k=1}^{A} V(\boldsymbol{r}_k) \right]}_{\text{residual interaction}}$$

- Independent-particle assumption. Choose V and neglect residual interaction:

$$\hat{H} \approx \hat{H}_{\text{IP}} = \sum_{k=1}^{A} \left[\frac{p_k^2}{2m_k} + \hat{V}(\boldsymbol{r}_k) \right]$$

Independent-particle shell model

- Solution for one particle:

$$\left[\frac{p^2}{2m} + \hat{V}(\boldsymbol{r}) \right] \phi_i(\boldsymbol{r}) = E_i \phi_i(\boldsymbol{r})$$

- Solution for many particles:

$$\Phi_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) = \prod_{k=1}^{A} \phi_{i_k}(\boldsymbol{r}_k)$$

$$\hat{H}_{\text{IP}} \Phi_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) = \left(\sum_{k=1}^{A} E_{i_k} \right) \Phi_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A)$$

Independent-particle shell model

- Anti-symmetric solution for many particles (Slater determinant):

$$\Psi_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_A)=\frac{1}{\sqrt{A!}}\begin{vmatrix}\phi_{i_1}(\boldsymbol{r}_1) & \phi_{i_1}(\boldsymbol{r}_2) & \cdots & \phi_{i_1}(\boldsymbol{r}_A)\\ \phi_{i_2}(\boldsymbol{r}_1) & \phi_{i_2}(\boldsymbol{r}_2) & \cdots & \phi_{i_2}(\boldsymbol{r}_A)\\ \vdots & \vdots & \ddots & \vdots\\ \phi_{i_A}(\boldsymbol{r}_1) & \phi_{i_A}(\boldsymbol{r}_2) & \cdots & \phi_{i_A}(\boldsymbol{r}_A)\end{vmatrix}$$

- Example for $A=2$ particles:

$$\Psi_{i_1 i_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\frac{1}{\sqrt{2}}\left[\phi_{i_1}(\boldsymbol{r}_1)\phi_{i_2}(\boldsymbol{r}_2)-\phi_{i_1}(\boldsymbol{r}_2)\phi_{i_2}(\boldsymbol{r}_1)\right]$$

Hartree-Fock approximation

- Vary ϕ_i (*ie* V) to minize the expectation value of H in a Slater determinant:

$$\delta\frac{\int \Psi^*_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_A)\hat{H}\Psi_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_A)d\boldsymbol{r}_1 d\boldsymbol{r}_2\ldots d\boldsymbol{r}_A}{\int \Psi^*_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_A)\Psi_{i_1 i_2 \ldots i_A}(\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_A)d\boldsymbol{r}_1 d\boldsymbol{r}_2\ldots d\boldsymbol{r}_A}=0$$

- Application requires choice of H. Many global parametrizations (Skyrme, Gogny,…) have been developed.

Poor man's Hartree-Fock

- Choose a simple, analytically solvable V that approximates the microscopic HF potential:

$$\hat{H}_{\mathrm{IP}} = \sum_{k=1}^{A} \left[\frac{p_k^2}{2m} + \frac{m\omega^2}{2} r_k^2 - \zeta \boldsymbol{l}_k \cdot \boldsymbol{s}_k - \kappa l_k^2 \right]$$

- Contains
  - Harmonic oscillator potential with constant ω.
  - Spin-orbit term with strength ζ.
  - Orbit-orbit term with strength κ.
- Adjust ω, ζ and κ to best reproduce HF.

Harmonic oscillator solution

- Energy eigenvalues of the harmonic oscillator:

$$E_{nlj} = \left(N + \tfrac{3}{2}\right)\hbar\omega - \kappa \hbar^2 l(l+1) + \zeta \hbar^2 \begin{cases} -\tfrac{1}{2} l & j = l + \tfrac{1}{2} \\ \tfrac{1}{2}(l+1) & j = l - \tfrac{1}{2} \end{cases}$$

$N = 2n + l = 0,1,2,\ldots$: oscillator quantum number

$n = 0,1,2,\ldots$: radial quantum number

$l = N, N-2, \ldots, 1$ or 0 : orbital angular momentum

$j = l \pm \tfrac{1}{2}$: total angular momentum

$m_j = -j, -j+1, \ldots, +j$: z projection of j

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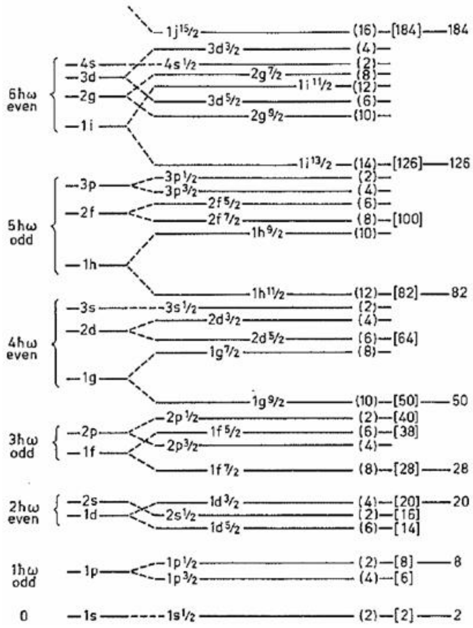
Energy levels of harmonic oscillator

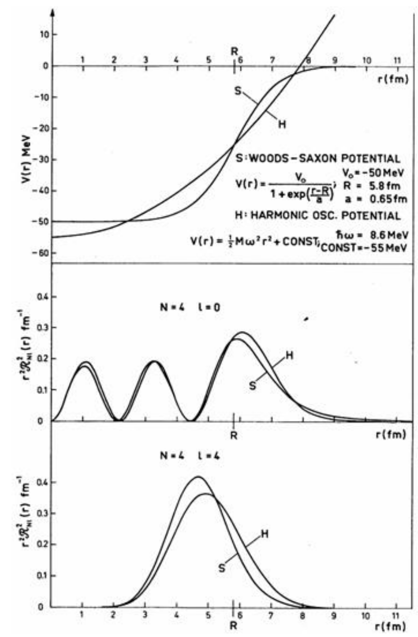


- Typical parameter values:

$$\hbar\omega \approx 41\,A^{-1/3}\ \mathrm{MeV}$$
$$\zeta\hbar^2 \approx 20\,A^{-2/3}\ \mathrm{MeV}$$
$$\kappa\hbar^2 \approx 0.1\ \mathrm{MeV}$$
$$\therefore b \approx 1.0\,A^{1/6}\ \mathrm{fm}$$

- ‘Magic’ numbers at 2, 8, 20, 28, 50, 82, 126, 184,…

Why an orbit-orbit term?



- Nuclear mean field is close to Woods-Saxon:

$$\hat{V}_{\mathrm{WS}}(r) = \frac{V_0}{1+\exp\dfrac{r-R_0}{a}}$$

- $2n+l=N$ degeneracy is lifted $\Rightarrow E_l < E_{l-2} < \ldots$

Why a spin-orbit term?

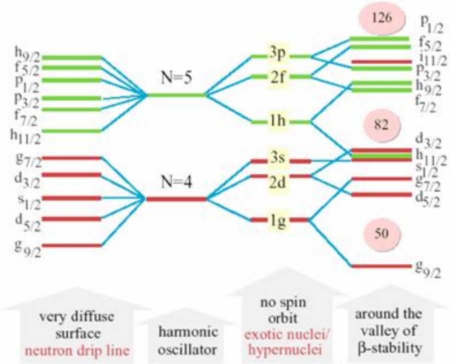
- Relativistic origin (*ie* Dirac equation).
- From general invariance principles:

$$\hat{V}_{\rm so} = \zeta(r)\boldsymbol{l}\cdot\boldsymbol{s}, \quad \zeta(r) = \frac{r_0^2}{r}\frac{\partial V}{\partial r}\,[= \zeta \text{ in HO}]$$

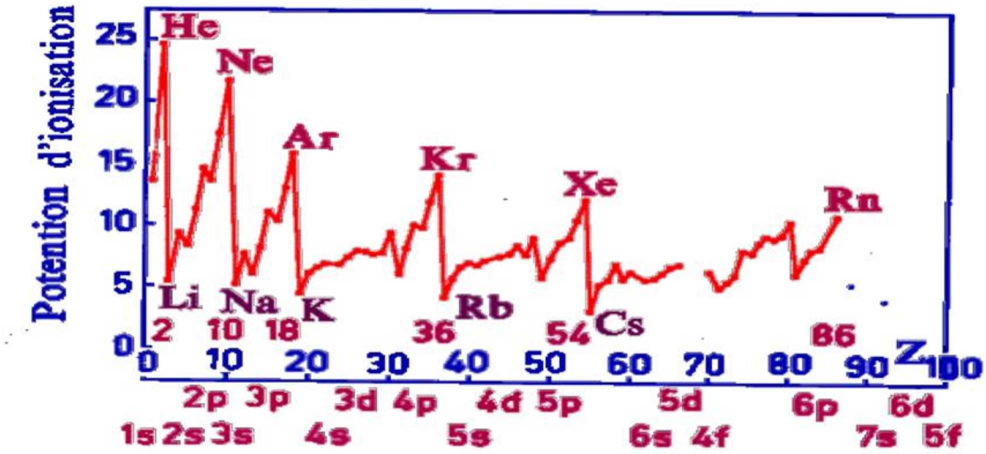
- Spin-orbit term is surface peaked $\Rightarrow$ diminishes for diffuse potentials.

Evidence for shell structure

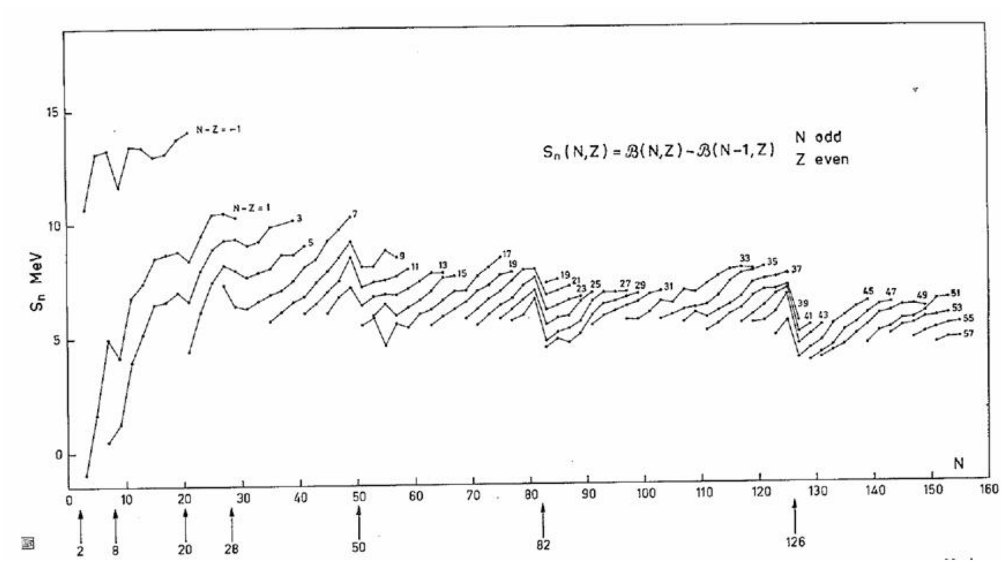
- Evidence for nuclear shell structure from
  - 2^+ in even-even nuclei [$E_{\rm x}$, B(E2)].
  - Nucleon-separation energies & nuclear masses.
  - Nuclear level densities.
  - Reaction cross sections.
- *Is nuclear shell structure modified away from the line of stability?*



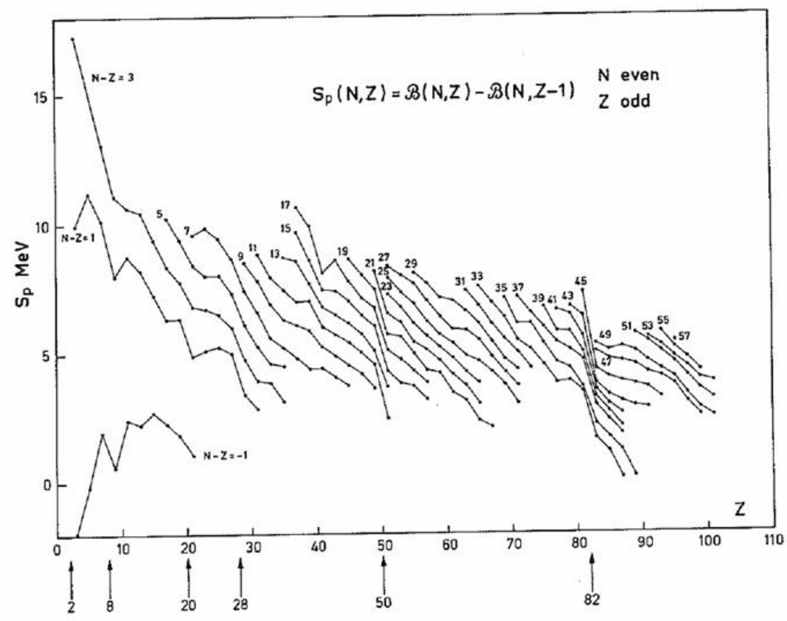
Ionisation potential in atoms



Neutron separation energies



Proton separation energies



Liquid-drop mass formula

- Binding energy of an atomic nucleus:

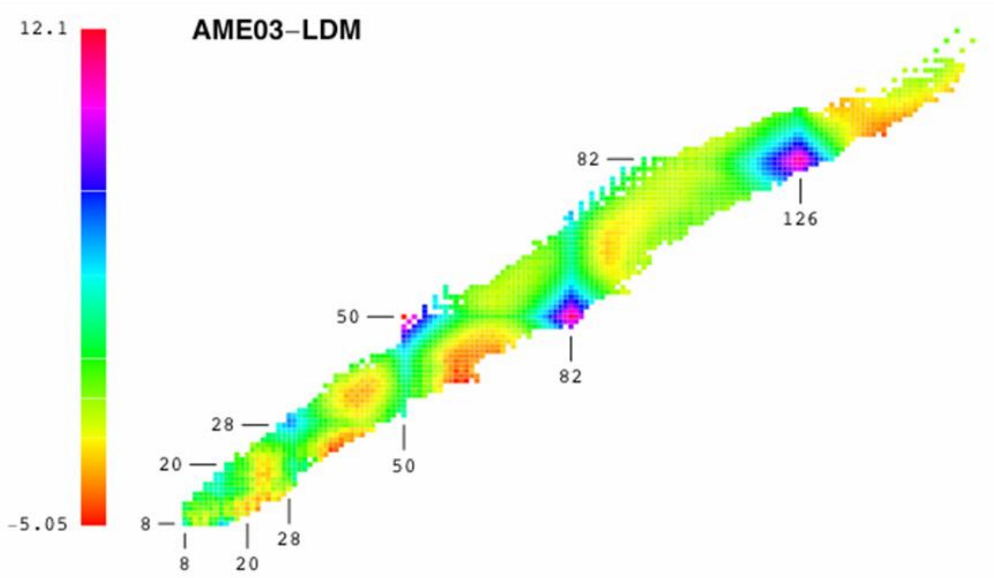
$$B(N,Z) = a_{\text{vol}}A - a_{\text{sur}}A^{2/3} - a_{\text{cou}}\frac{Z(Z-1)}{A^{1/3}} - a_{\text{sym}}\frac{(N-Z)^2}{A} + a_{\text{pai}}\frac{\delta(N,Z)}{A^{1/2}}$$

- For 2149 nuclei ($N,Z \geq 8$) in AME03:

$a_{\text{vol}} \approx 16$, $a_{\text{sur}} \approx 18$, $a_{\text{cou}} \approx 0.71$, $a_{\text{sym}} \approx 23$, $a_{\text{pai}} \approx 13$
$\Rightarrow \sigma_{\text{rms}} \approx 2.93$ MeV.

C.F. von Weizsäcker, Z. Phys. **96** (1935) 431
H.A. Bethe & R.F. Bacher, Rev. Mod. Phys. **8** (1936) 82

Deviations from LDM



Modified liquid-drop formula

- Add surface, Wigner and 'shell' corrections:

$$B(N,Z) = a_{\text{vol}}A - a_{\text{sur}}A^{2/3} - a_{\text{cou}}\frac{Z(Z-1)}{A^{1/3}} - a_{\text{vsym}}\frac{4T(T+r)}{A}$$

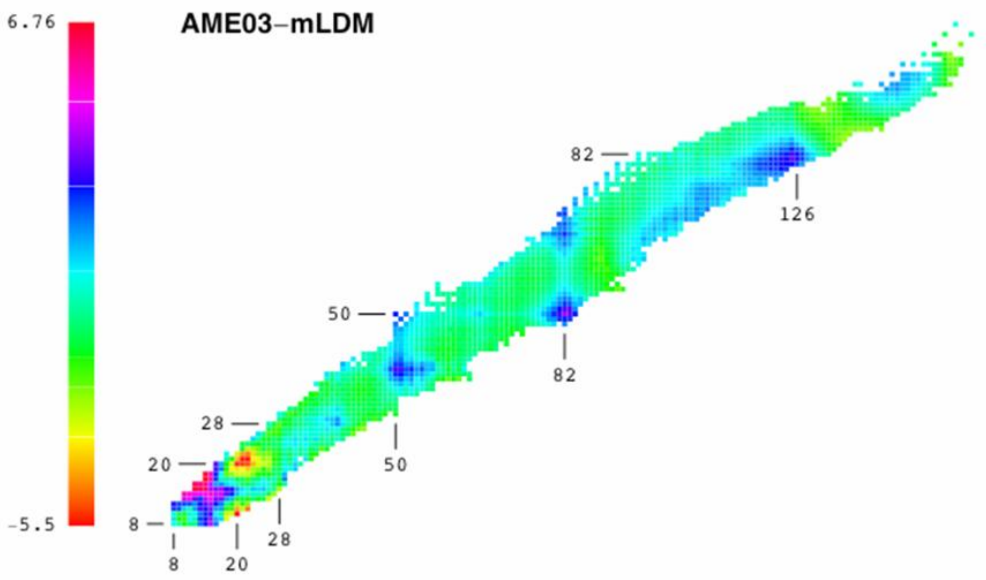
$$+ a_{\text{ssym}}\frac{4T(T+r)}{A^{4/3}} + a_{\text{pai}}\frac{\delta(N,Z)}{A^{1/2}} - a_{\text{f}}F_{\text{max}} + a_{\text{ff}}F_{\text{max}}^2$$

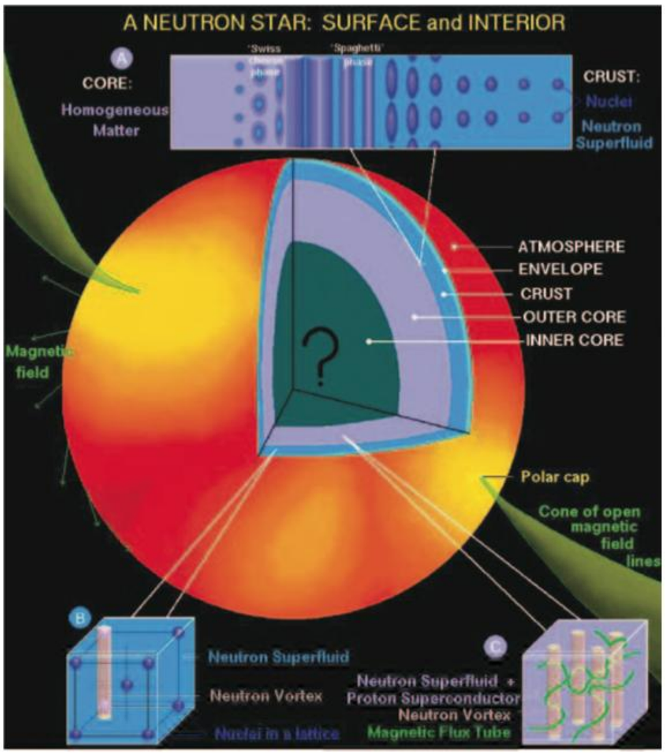
- For 2149 nuclei ($N,Z \geq 8$) in AME03:

$a_{\text{vol}}\approx 16$, $a_{\text{sur}}\approx 18$, $a_{\text{cou}}\approx 0.72$, $a_{\text{vsym}}\approx 32$, $a_{\text{ssym}}\approx 79$, $a_{\text{pai}}\approx 12$, $a_{\text{f}}\approx 0.14$, $a_{\text{ff}}\approx 0.0049$, $r\approx 2.5$

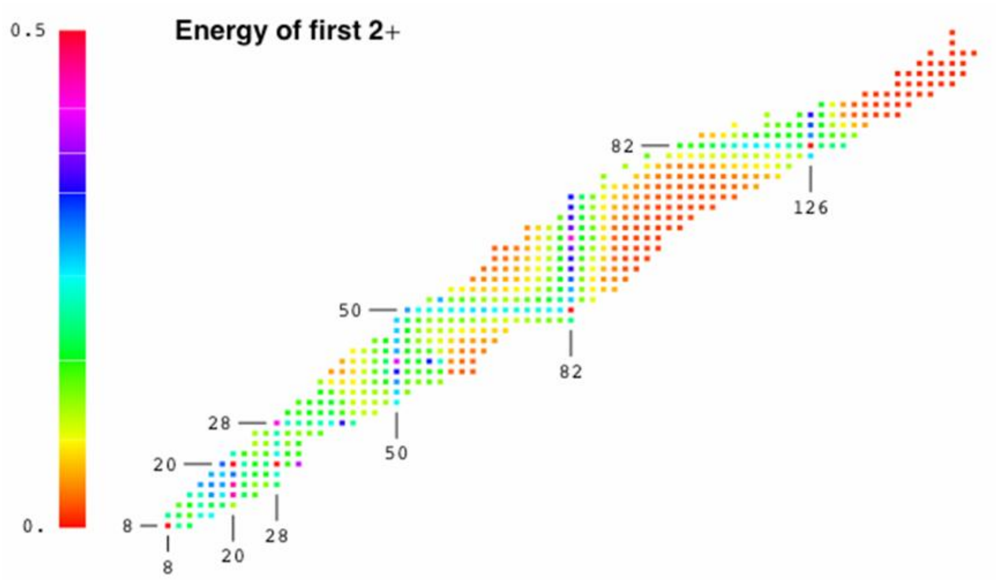
$\Rightarrow \sigma_{\text{rms}}\approx 1.28$ MeV.

Deviations from modified LDM





Shell structure from $E_x(2_1)$



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Evidence for IP shell model

Z	Isotope	Observed J^π	Shell model nlj
3	^{9}Li	$(3/2^-)$	$1p_{3/2}$
5	^{13}B	$3/2^-$	$1p_{3/2}$
7	^{17}N	$1/2^-$	$1p_{1/2}$
9	^{21}F	$5/2^+$	$1d_{5/2}$
11	^{25}Na	$5/2^+$	$1d_{5/2}$
13	^{29}Al	$5/2^+$	$1d_{5/2}$
15	^{33}P	$1/2^+$	$2s_{1/2}$
17	^{37}Cl	$3/2^+$	$1d_{3/2}$
19	^{41}K	$3/2^+$	$1d_{3/2}$
21	^{45}Sc	$7/2^-$	$1f_{7/2}$
23	^{49}Va	$7/2^-$	$1f_{7/2}$
25	^{53}Mn	$7/2^-$	$1f_{7/2}$
27	^{57}Co	$7/2^-$	$1f_{7/2}$
29	^{61}Cu	$3/2^-$	$2p_{3/2}$
31	^{65}Ga	$3/2^-$	$2p_{3/2}$
33	^{69}As	$(5/2^-)$	$1f_{5/2}$
35	^{73}Br	$(3/2^-)$	$1f_{5/2}$

- Ground-state spins and parities of nuclei:

$$\left.\begin{array}{l} j \text{ in } \phi_{nljm_j} \Rightarrow J \\ l \text{ in } \phi_{nljm_j} \Rightarrow (-)^l = \pi \end{array}\right\} \Rightarrow J^\pi$$

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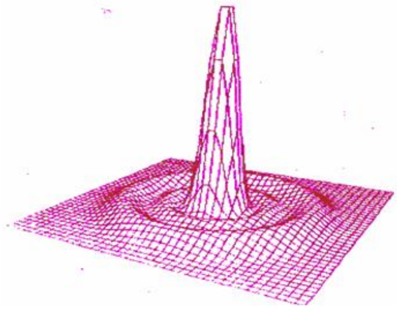
Validity of SM wave functions

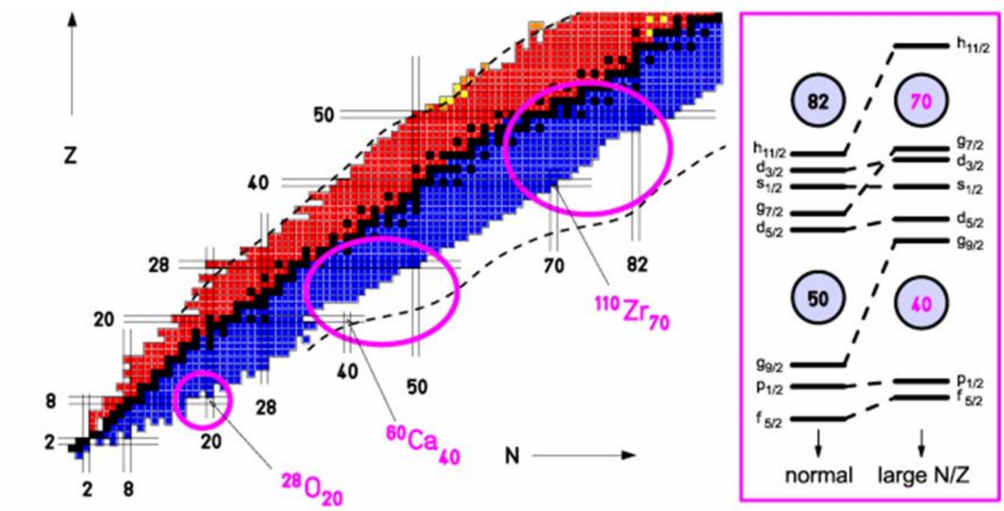
- Example: Elastic electron scattering on ^{206}Pb and ^{205}Tl, differing by a 3*s* proton.
- Measured ratio agrees with shell-model prediction for 3*s* orbit.

QuickTime™ et un décompresseur TIFF (non compressé) sont requis pour visionner cette image.



J.M. Cavedon *et al.*, Phys. Rev. Lett. **49** (1982) 978

Variable shell structure



Beyond Hartree-Fock

- Hartree-Fock-Bogoliubov (HFB): Includes pairing correlations in mean-field treatment.
- Tamm-Dancoff approximation (TDA):
  - Ground state: closed-shell HF configuration
  - Excited states: mixed 1p-1h configurations
- Random-phase approximation (RPA): Correlations in the ground state by treating it on the same footing as 1p-1h excitations.

Nuclear shell model

- The full shell-model hamiltonian:

$$\hat{H} = \sum_{k=1}^{A}\left[\frac{p_k^2}{2m} + \hat{V}(\boldsymbol{r}_k)\right] + \sum_{k<l}^{A}\hat{V}_{\mathrm{RI}}(\boldsymbol{r}_k,\boldsymbol{r}_l)$$

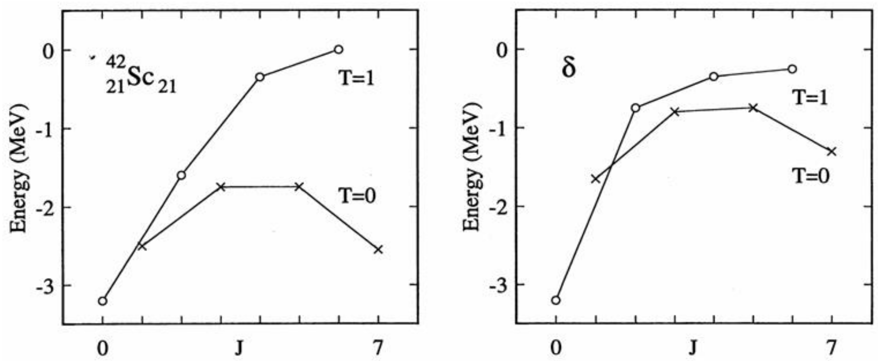
- Valence nucleons: Neutrons or protons that are in excess of the last, completely filled shell.
- Usual approximation: Consider the residual interaction V_{RI} among valence nucleons only.
- Sometimes: Include selected core excitations (‘intruder’ states).

Residual shell-model interaction

- Four approaches:
  - Effective: Derive from free nn interaction taking account of the nuclear medium.
  - Empirical: Adjust matrix elements of residual interaction to data. Examples: *p*, *sd* and *pf* shells.
  - Effective-empirical: Effective interaction with some adjusted (monopole) matrix elements.
  - Schematic: Assume a simple spatial form and calculate its matrix elements in a harmonic-oscillator basis. Example: δ interaction.

Schematic short-range interaction

- Delta interaction in harmonic-oscillator basis:
- Example of $^{42}Sc_{21}$ (1 neutron + 1 proton):

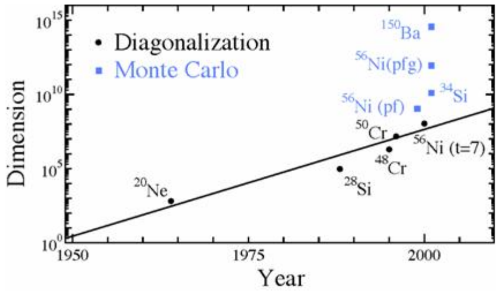


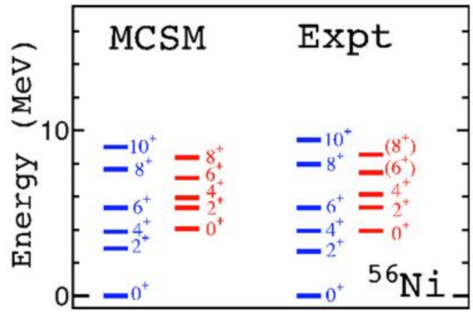
Large-scale shell model

- Large Hilbert spaces:

$$\left\langle \Psi_{i'_1 i'_2 \ldots i'_A} \left| \sum_{k<l}^{n} \hat{V}_{\mathrm{RI}}(\boldsymbol{r}_k,\boldsymbol{r}_l) \right| \Psi_{i_1 i_2 \ldots i_A} \right\rangle$$

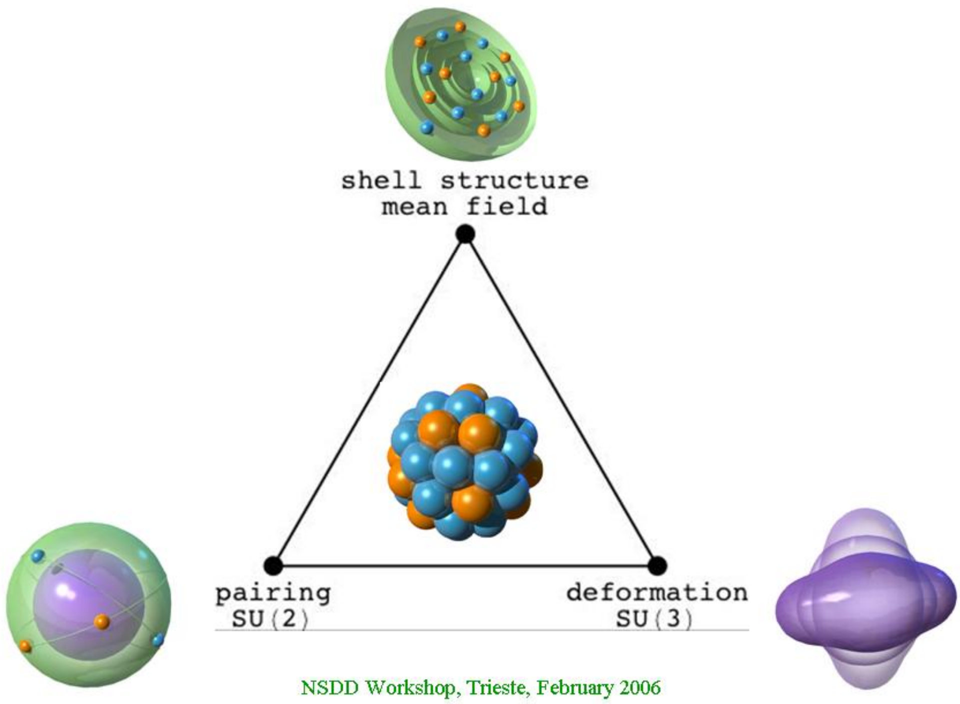
  - Diagonalisation : $\sim 10^9$.
  - Monte Carlo : $\sim 10^{15}$.
  - DMRG : $\sim 10^{120}$ (?).
- Example : 8n + 8p in $pfg_{9/2}$ (^{56}Ni).





M. Honma *et al.*, Phys. Rev. C **69** (2004) 034335

The three faces of the shell model

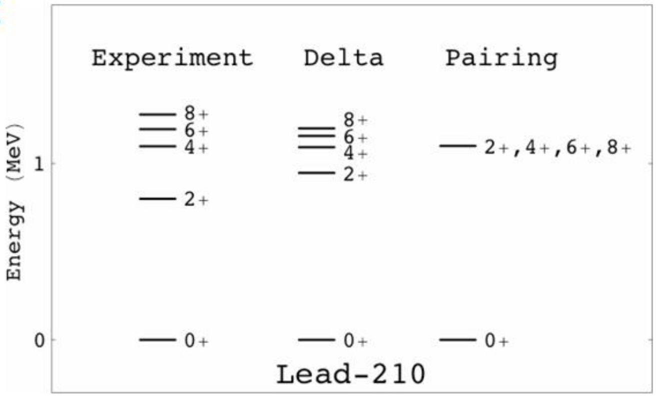


Racah's SU(2) pairing model

- Assume pairing interaction in a single-j shell:

$$\left\langle j^2 J M_J \left| \hat{V}_{\text{pairing}}(\boldsymbol{r}_1, \boldsymbol{r}_2) \right| j^2 J M_J \right\rangle = \begin{cases} -\frac{1}{2}(2j+1) g_0, & J = 0 \\ 0, & J \neq 0 \end{cases}$$

- Spectrum ^{210}Pb:



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Solution of the pairing hamiltonian

- Analytic solution of pairing hamiltonian for identical nucleons in a single-j shell:

$$\left\langle j^n \upsilon J \left| \sum_{1 \le k < l}^{n} \hat{V}_{\text{pairing}}(\boldsymbol{r}_k, \boldsymbol{r}_l) \right| j^n \upsilon J \right\rangle = -g_0 \tfrac{1}{4} (n - \upsilon)(2j - n - \upsilon + 3)$$

- Seniority υ (number of nucleons not in pairs coupled to J=0) is a good quantum number.
- Correlated ground-state solution (*cf.* BCS).

G. Racah, Phys. Rev. **63** (1943) 367

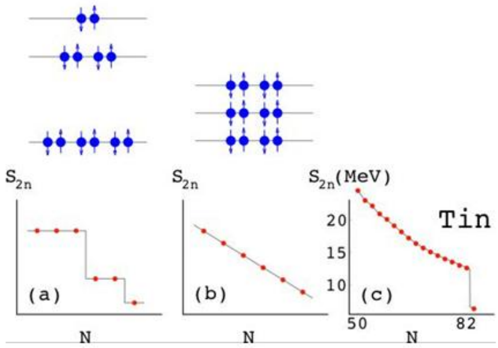
NSDD Workshop, Trieste, February 2006

Nuclear superfluidity

- Ground states of pairing hamiltonian have the following *correlated* character:
  - Even-even nucleus (υ=0): $\left(\hat{S}_+\right)^{n/2}|o\rangle$, $\hat{S}_+ = \sum_m \hat{a}^+_{m\downarrow}\hat{a}^+_{\bar{m}\uparrow}$
  - Odd-mass nucleus (υ=1): $\hat{a}^+_{m\updownarrow}\left(\hat{S}_+\right)^{n/2}|o\rangle$
- Nuclear superfluidity leads to
  - Constant energy of first 2^+ in even-even nuclei.
  - Odd-even staggering in masses.
  - Smooth variation of two-nucleon separation energies with nucleon number.
  - Two-particle (2n or 2p) transfer enhancement.

Two-nucleon separation energies

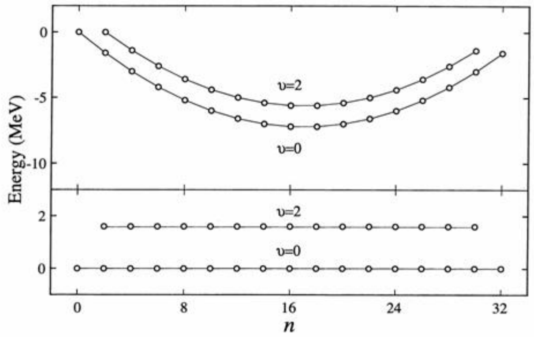
- Two-nucleon separation energies S_{2n}:
  - (a) Shell splitting dominates over interaction.
  - (b) Interaction dominates over shell splitting.
  - (c) S_{2n} in tin isotopes.



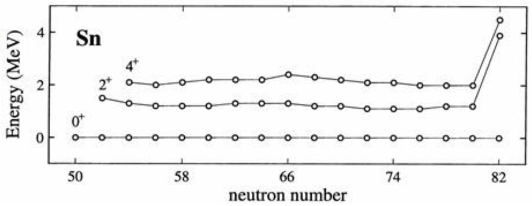
Pairing gap in semi-magic nuclei

- Even-even nuclei:
  - Ground state: $\upsilon=0$.
  - First-excited state: $\upsilon=2$.
  - Pairing produces constant energy gap:

$$E_x\left(2_1^+\right)=\tfrac{1}{2}(2j+1)G$$



- Example of Sn isotopes:



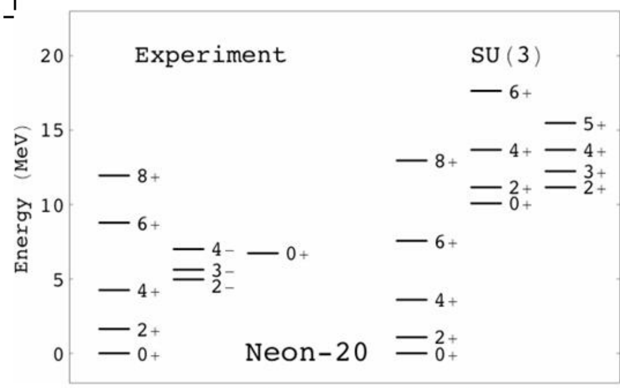
Elliott's SU(3) model of rotation

- Harmonic oscillator mean field (*no* spin-orbit) with residual interaction of quadrupole type:

$$\hat{H}=\sum_{k=1}^{A}\left[\frac{p_k^2}{2m}+\frac{1}{2}m\omega^2 r_k^2\right]-g_2\hat{Q}\cdot\hat{Q},$$

$$\hat{Q}_\mu \propto \sum_{k=1}^{A} r_k^2 Y_{2\mu}(\hat{\boldsymbol{r}}_k)$$

$$+\sum_{k=1}^{A} p_k^2 Y_{2\mu}(\hat{\boldsymbol{p}}_k)$$



J.P. Elliott, Proc. Roy. Soc. A **245** (1958) 128; 562

Nuclear Structure

(II) Collective models

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Nuclear Structure
(II) Collective models

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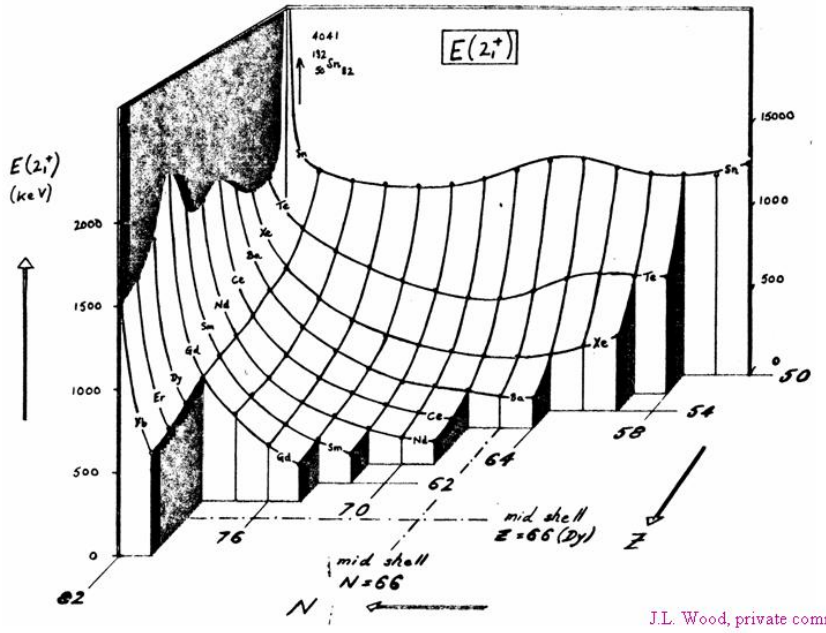
Overview of collective models

- (Rigid) rotor model
- (Harmonic quadrupole) vibrator model
- Liquid-drop model of vibrations and rotations
- Interacting boson model
- Particle-core coupling model
- Nilsson model

Evolution of $E_\mathrm{x}(2^+)$



J.L. Wood, private communication

Quantum-mechanical symmetric top

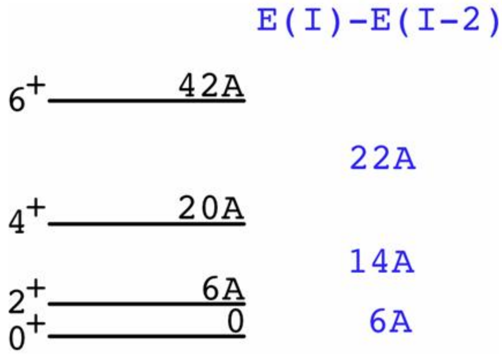
- Energy spectrum:

$$E_{\mathrm{rot}}(I) = \frac{\hbar^2}{2\mathfrak{I}} I(I+1)$$

$$\equiv A\, I(I+1), \quad I = 0, 2, 4, \ldots$$

- Large deformation $\Rightarrow$ large $\mathfrak{I} \Rightarrow$ low $E_\mathrm{x}(2^+)$.
- R_{42} energy ratio:

$$E_{\mathrm{rot}}(4^+)/E_{\mathrm{rot}}(2^+) = 3.333\ldots$$



Rigid rotor model

- Hamiltonian of quantum-mechanical rotor in terms of 'rotational' angular momentum ***R***:

$$\hat{H}_{\text{rot}} = \frac{\hbar^2}{2}\left[\frac{R_1^2}{\mathfrak{I}_1} + \frac{R_2^2}{\mathfrak{I}_2} + \frac{R_3^2}{\mathfrak{I}_3}\right] = \frac{\hbar^2}{2}\sum_{i=1}^{3}\frac{R_i^2}{\mathfrak{I}_i}$$

- Nuclei have an additional intrinsic part H_{intr} with 'intrinsic' angular momentum ***J***.
- The total angular momentum is $\boldsymbol{I}=\boldsymbol{R}+\boldsymbol{J}$.

Rigid axially symmetric rotor

- For $\mathfrak{I}_1=\mathfrak{I}_2=\mathfrak{I}\neq\mathfrak{I}_3$ the rotor hamiltonian is

$$\hat{H}_{\text{rot}} = \sum_{i=1}^{3}\frac{\hbar^2}{2\mathfrak{I}_i}(I_i - J_i)^2 = \underbrace{\sum_{i=1}^{3}\frac{\hbar^2}{2\mathfrak{I}_i}I_i^2}_{\hat{H}'_{\text{rot}}} - \underbrace{\sum_{i=1}^{3}\frac{\hbar^2}{\mathfrak{I}_i}I_iJ_i}_{\text{Coriolis}} + \underbrace{\sum_{i=1}^{3}\frac{\hbar^2}{2\mathfrak{I}_i}J_i^2}_{\text{intrinsic}}$$

- Eigenvalues of H'_{rot}:

$$E'_{KI} = \frac{\hbar^2}{2\mathfrak{I}}I(I+1) + \frac{\hbar^2}{2}\left(\frac{1}{\mathfrak{I}_3} - \frac{1}{\mathfrak{I}}\right)K^2$$

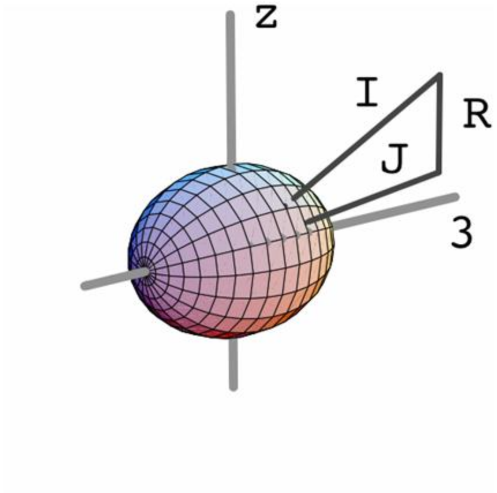
- Eigenvectors $|KIM\rangle$ of H'_{rot} satisfy:

$$I^2|KIM\rangle = I(I+1)|KIM\rangle,$$
$$I_z|KIM\rangle = M|KIM\rangle,\quad I_3|KIM\rangle = K|KIM\rangle$$

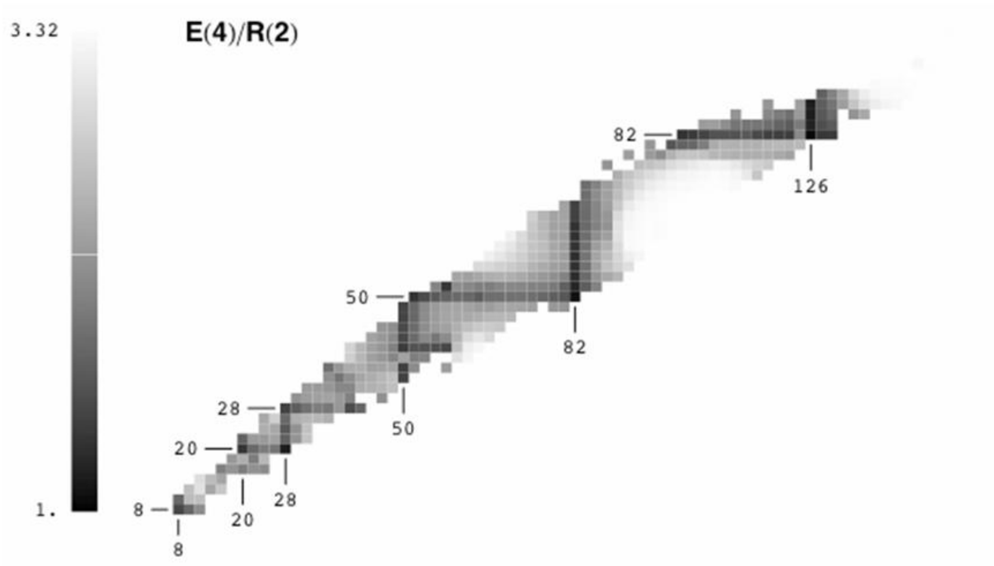
Ground-state band of an axial rotor

- The ground-state spin of even-even nuclei is I=0. Hence K=0 for ground-state band:

$$E_I = \frac{\hbar^2}{2\Im} I(I+1)$$



The ratio R_{42}



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Electric (quadrupole) properties

- Partial γ-ray half-life:

$$T^{\gamma}_{1/2}(\mathrm{E}\lambda)=\ln 2\left\{\frac{8\pi}{\hbar}\frac{\lambda+1}{\lambda[(2\lambda+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1}B(\mathrm{E}\lambda)\right\}^{-1}$$

- Electric quadrupole transitions:

$$B(\mathrm{E2};I_\mathrm{i}\rightarrow I_\mathrm{f})=\frac{1}{2I_\mathrm{i}+1}\sum_{M_\mathrm{i}}\sum_{M_\mathrm{f}\mu}\left|\left\langle I_\mathrm{f}M_\mathrm{f}\left|\sum_{k=1}^{A}e_k r_k^2 Y_{2\mu}(\theta_k,\varphi_k)\right|I_\mathrm{i}M_\mathrm{i}\right\rangle\right|^2$$

- Electric quadrupole moments:

$$eQ(I)=\left\langle IM=I\left|\sqrt{\frac{16\pi}{5}}\sum_{k=1}^{A}e_k r_k^2 Y_{20}(\theta_k,\varphi_k)\right|IM=I\right\rangle$$

Magnetic (dipole) properties

- Partial γ-ray half-life:

$$T^{\gamma}_{1/2}(\mathrm{M}\lambda)=\ln 2\left\{\frac{8\pi}{\hbar}\frac{\lambda+1}{\lambda[(2\lambda+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1}B(\mathrm{M}\lambda)\right\}^{-1}$$

- Magnetic dipole transitions:

$$B(\mathrm{M1};I_\mathrm{i}\rightarrow I_\mathrm{f})=\frac{1}{2I_\mathrm{i}+1}\sum_{M_\mathrm{i}}\sum_{M_\mathrm{f}\mu}\left|\left\langle I_\mathrm{f}M_\mathrm{f}\left|\sum_{k=1}^{A}\left(g_k^l l_{k,\mu}+g_k^s s_{k,\mu}\right)\right|I_\mathrm{i}M_\mathrm{i}\right\rangle\right|^2$$

- Magnetic dipole moments:

$$\mu(I)=\left\langle IM=I\left|\sum_{k=1}^{A}\left(g_k^l l_{k,z}+g_k^s s_{k,z}\right)\right|IM=I\right\rangle$$

E2 properties of rotational nuclei

- *Intra*-band E2 transitions:

$$B(\mathrm{E2}; KI_\mathrm{i} \to KI_\mathrm{f}) = \frac{5}{16\pi} \langle I_\mathrm{i}K\ 20 | I_\mathrm{f}K \rangle^2 e^2 Q_0(K)^2$$

- E2 moments:

$$Q(KI) = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} Q_0(K)$$

- $Q_0(K)$ is the 'intrinsic' quadrupole moment:

$$e\hat{Q}_0 \equiv \int \rho(\boldsymbol{r}') r'^2 (3\cos^2\theta' - 1) d\boldsymbol{r}', \quad Q_0(K) = \langle K | \hat{Q}_0 | K \rangle$$

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E2 properties of ground-state bands

- For the ground state (usually $K=I$):

$$Q(K=I) = \frac{I(2I-1)}{(I+1)(2I+3)} Q_0(K)$$

- For the gsb in even-even nuclei ($K=0$):

$$B(\mathrm{E2}; I \to I-2) = \frac{15}{32\pi} \frac{I(I-1)}{(2I-1)(2I+1)} e^2 Q_0^2$$

$$Q(I) = -\frac{I}{2I+3} Q_0$$

$$\Rightarrow \left| eQ(2_1^+) \right| = \frac{2}{7} \sqrt{16\pi \cdot B(\mathrm{E2}; 2_1^+ \to 0_1^+)}$$

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Generalized intensity relations

- Mixing of K arises from
  - Dependence of Q_0 on $\boldsymbol{I}$ (stretching)
  - Coriolis interaction
  - Triaxiality
- Generalized *intra*- and *inter*-band matrix elements (*eg* E2):

$$\frac{\sqrt{B(\mathrm{E2};K_\mathrm{i}I_\mathrm{i}\rightarrow K_\mathrm{f}I_\mathrm{f})}}{\langle I_\mathrm{i}K_\mathrm{i}\;2K_\mathrm{f}-K_\mathrm{i}|I_\mathrm{f}K_\mathrm{f}\rangle}=M_0+M_1\Delta+M_2\Delta^2+\cdots$$

with $\Delta=I_\mathrm{f}(I_\mathrm{f}+1)-I_\mathrm{i}(I_\mathrm{i}+1)$

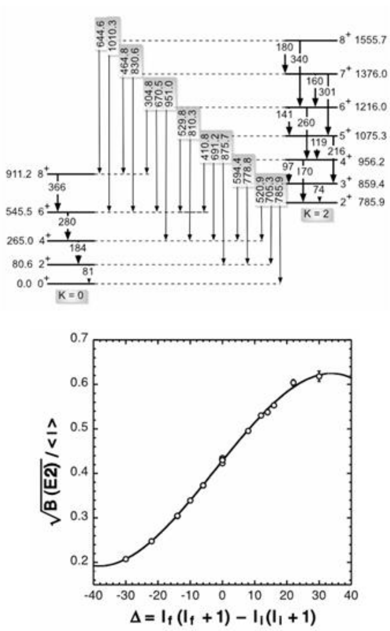
Inter-band E2 transitions

- Example of $\gamma\rightarrow$g transitions in ^{166}Er:

$$\frac{\sqrt{B(\mathrm{E2};I_\gamma\rightarrow I_\mathrm{g})}}{\langle I_\gamma 2\,2-2|I_\mathrm{g}0\rangle}$$

$$=M_0+M_1\Delta+M_2\Delta^2+\cdots$$

$$\Delta=I_\mathrm{g}(I_\mathrm{g}+1)-I_\gamma(I_\gamma+1)$$



W.D. Kulp *et al.*, Phys. Rev. C **73** (2006) 014308

Modes of nuclear vibration

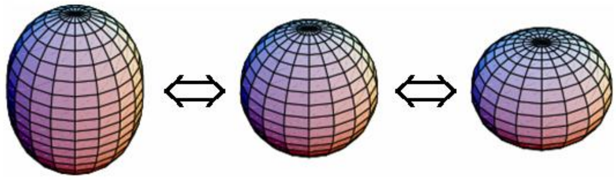
- Nucleus is considered as a droplet of nuclear matter with an equilibrium shape. Vibrations are modes of excitation around that shape.
- Character of vibrations depends on symmetry of equilibrium shape. Two important cases in nuclei:
  - Spherical equilibrium shape
  - Spheroidal equilibrium shape

Vibrations about a spherical shape

- Vibrations are characterized by a multipole quantum number λ in surface parametrization:

$$R(\theta,\varphi)=R_0\left(1+\sum_{\lambda}\sum_{\mu=-\lambda}^{+\lambda}\alpha_{\lambda\mu}Y^*_{\lambda\mu}(\theta,\varphi)\right)$$

  - λ=0: compression (high energy)
  - λ=1: translation (not an intrinsic excitation)
  - λ=2: quadrupole vibration



Properties of spherical vibrations

- Energy spectrum:

$$E_{\text{vib}}(n) = \left(n + \tfrac{5}{2}\right)\hbar\omega,\ n = 0,1\ldots$$

- R_{42} energy ratio:

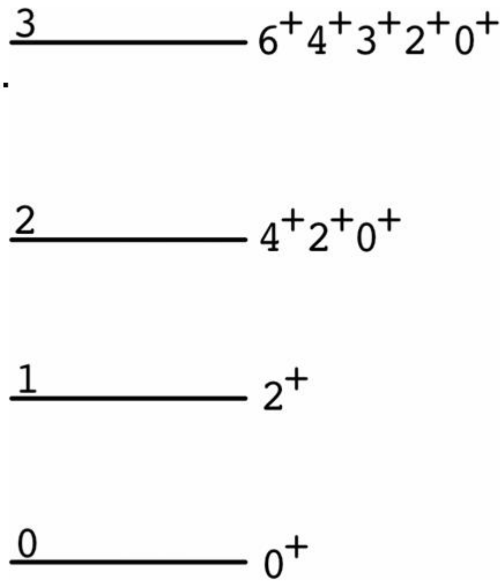
$$E_{\text{vib}}(4^+)/E_{\text{vib}}(2^+) = 2$$

- E2 transitions:

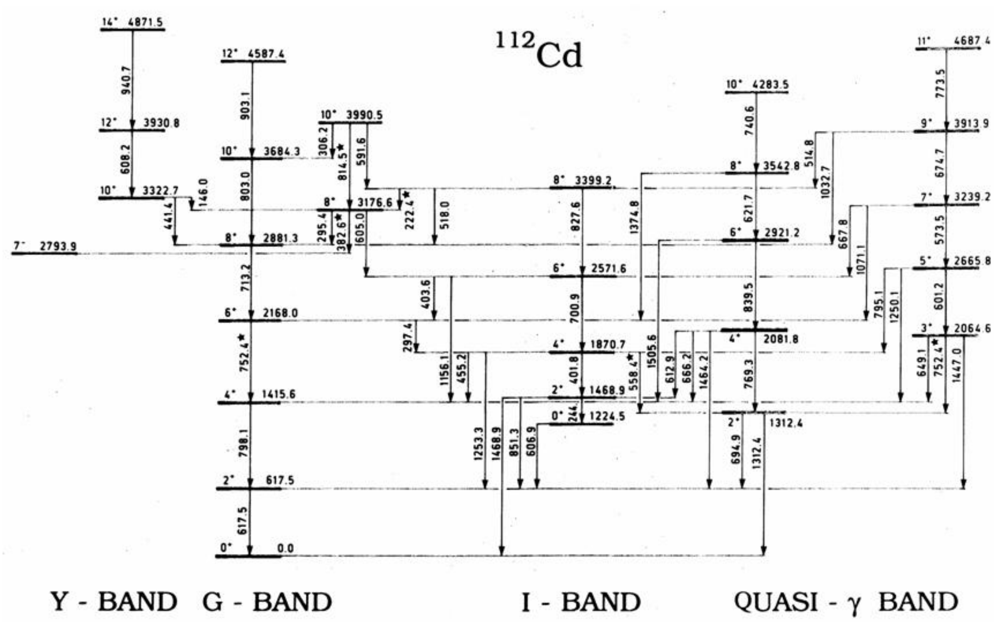
$$B(\text{E2}; 2_1^+ \to 0_1^+) = \alpha^2$$

$$B(\text{E2}; 2_2^+ \to 0_1^+) = 0$$

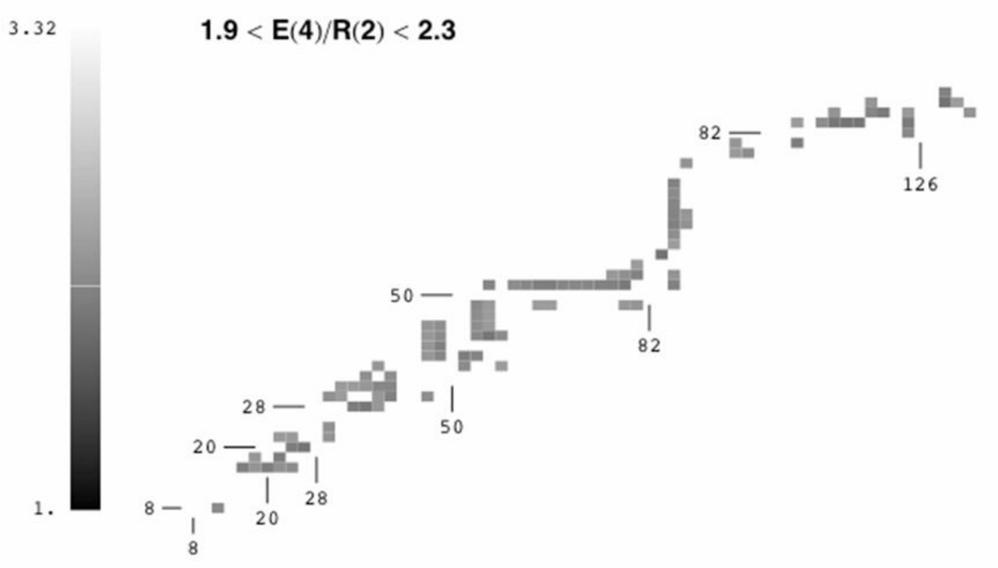
$$B(\text{E2}; n = 2 \to n = 1) = 2\alpha^2$$



Example of ^{112}Cd



Possible vibrational nuclei from R_{42}



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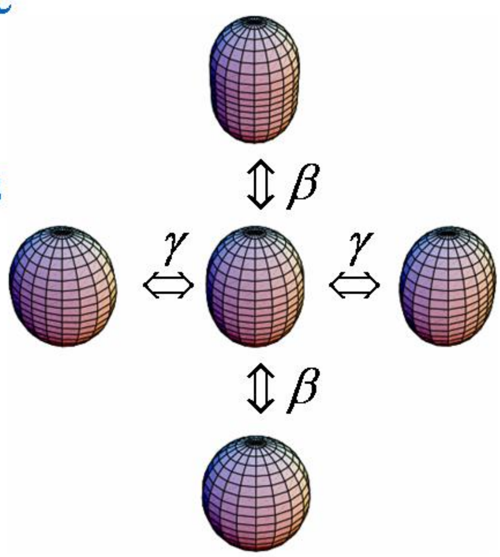
Vibrations about a spheroidal shape

- The vibration of a shape with axial symmetry is characterized by $a_{\lambda\nu}$.
- Quadrupole oscillations
  - ν=0: along the axis of symmetry (β)
  - ν=±1: spurious rotation
  - ν=±2: perpendicular to axis of symmetry (γ)



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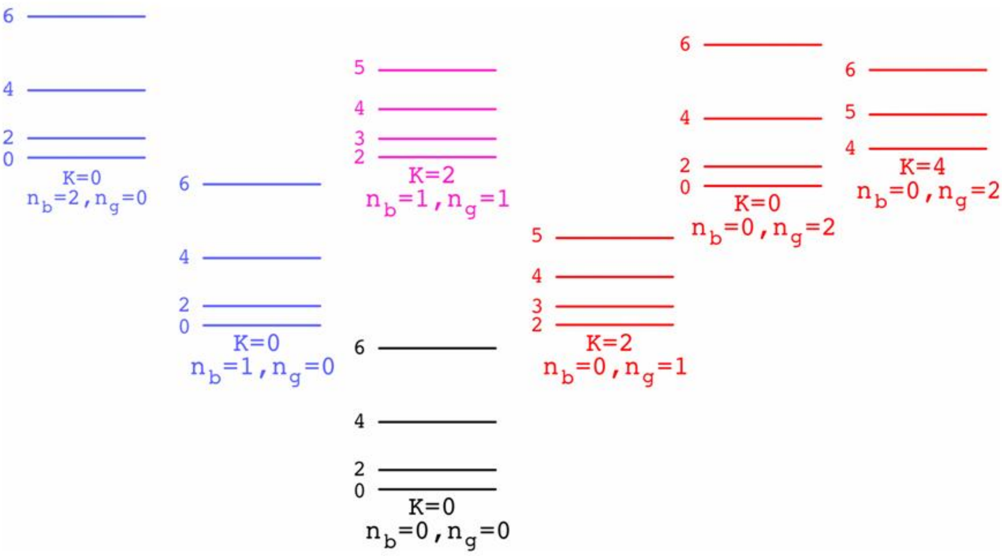
Spectrum of spheroidal vibrations



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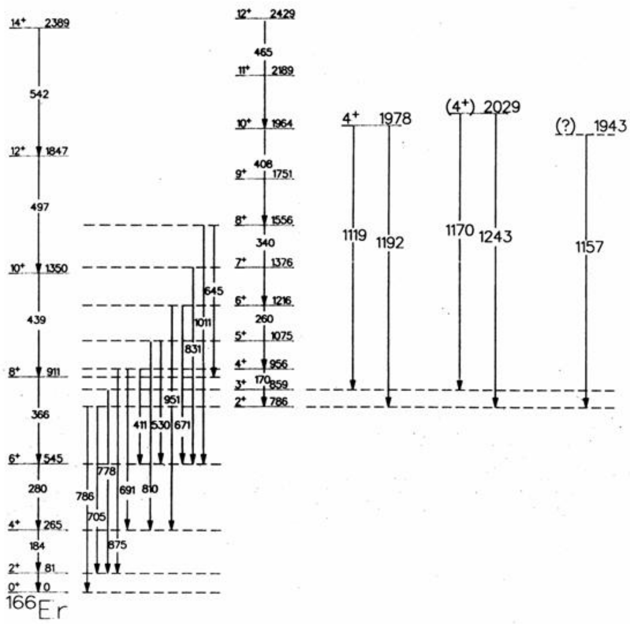
Example of ^{166}Er



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Rigid triaxial rotor

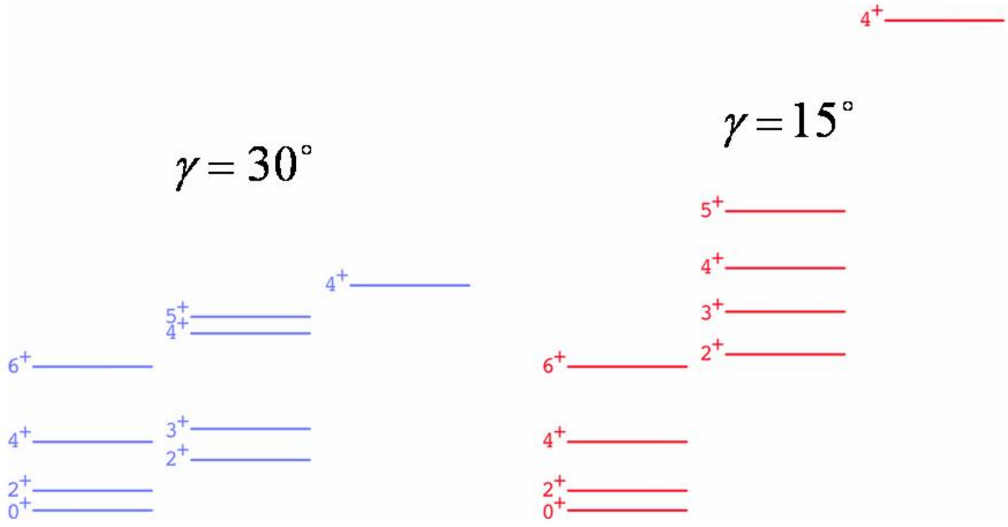
- Triaxial rotor hamiltonian $\mathfrak{I}_1 \neq \mathfrak{I}_2 \neq \mathfrak{I}_3$:

$$\hat{H}'_{\text{rot}} = \sum_{i=1}^{3} \frac{\hbar^2}{2\mathfrak{I}_i} I_i^2 = \underbrace{\frac{\hbar^2}{2\mathfrak{I}} I^2 + \frac{\hbar^2}{2\mathfrak{I}_f} I_3^2}_{\hat{H}'_{\text{axial}}} + \underbrace{\frac{\hbar^2}{2\mathfrak{I}_g}\left(I_+^2 + I_-^2\right)}_{\hat{H}'_{\text{mix}}}$$

$$\frac{1}{\mathfrak{I}} = \frac{1}{2}\left(\frac{1}{\mathfrak{I}_1} + \frac{1}{\mathfrak{I}_2}\right), \quad \frac{1}{\mathfrak{I}_f} = \frac{1}{\mathfrak{I}_3} - \frac{1}{\mathfrak{I}}, \quad \frac{1}{\mathfrak{I}_g} = \frac{1}{4}\left(\frac{1}{\mathfrak{I}_1} - \frac{1}{\mathfrak{I}_2}\right)$$

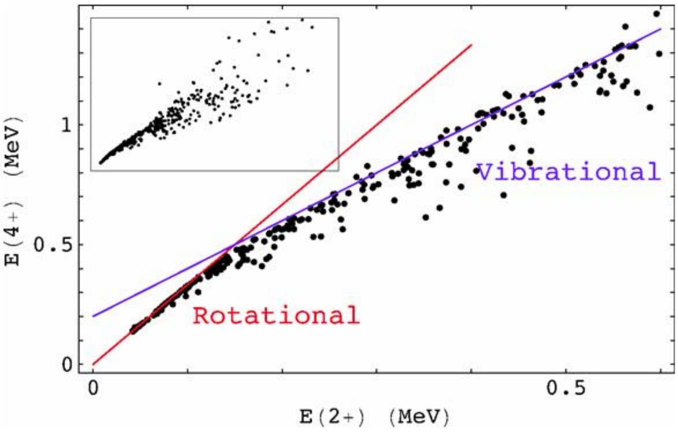
- H'_{mix} non-diagonal in axial basis $|KIM\rangle \Rightarrow K$ is *not* a conserved quantum number

Rigid triaxial rotor spectra



Tri-partite classification of nuclei

- Empirical evidence for seniority-type, vibrational- and rotational-like nuclei:



- Need for model of *vibrational* nuclei.

N.V. Zamfir *et al.*, Phys. Rev. Lett. **72** (1994) 3480

Interacting boson model

- Describe the nucleus as a system of N interacting s and d bosons. Hamiltonian:

$$\hat{H}_{\mathrm{IBM}} = \sum_{i=1}^{6} \varepsilon_i \hat{b}_i^+ \hat{b}_i + \sum_{i_1 i_2 i_3 i_4 = 1}^{6} \upsilon_{i_1 i_2 i_3 i_4} \hat{b}_{i_1}^+ \hat{b}_{i_2}^+ \hat{b}_{i_3} \hat{b}_{i_4}$$

- Justification from
  - Shell model: s and d bosons are associated with S and D fermion (*Cooper*) pairs.
  - Geometric model: for large boson number the IBM reduces to a liquid-drop hamiltonian.

Dimensions

- Assume Ω available 1-fermion states. Number of n-fermion states is $\binom{\Omega}{n} = \frac{\Omega!}{n!(\Omega-n)!}$
- Assume Ω available 1-boson states. Number of n-boson states is $\binom{\Omega+n-1}{n} = \frac{(\Omega+n-1)!}{n!(\Omega-1)!}$
- Example: $^{162}Dy_{96}$ with 14 neutrons (Ω=44) and 16 protons (Ω=32) ($^{132}Sn_{82}$ inert core).
  - SM dimension: ~$7 \cdot 10^{19}$
  - IBM dimension: 15504

Dynamical symmetries

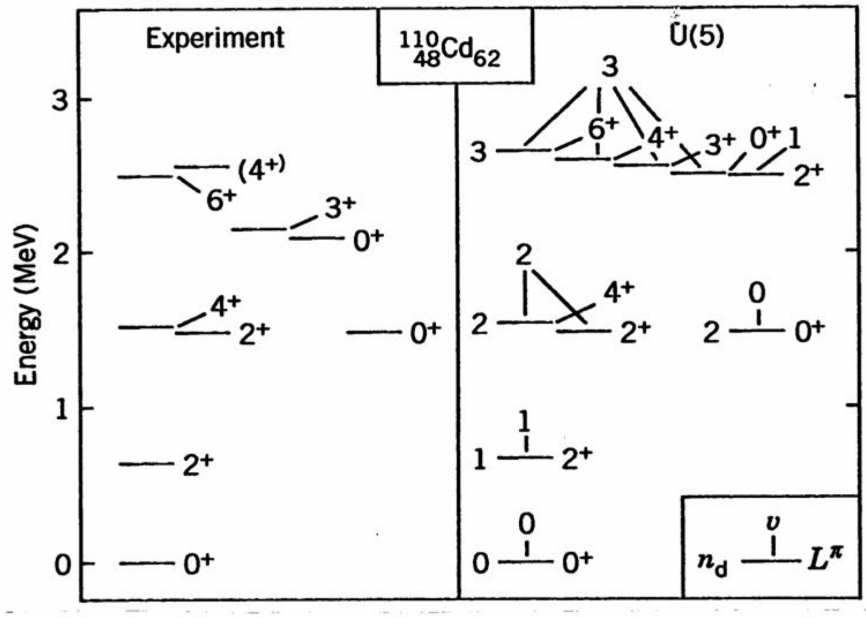
- Boson hamiltonian is of the form

$$\hat{H}_{\mathrm{IBM}} = \sum_{i=1}^{6} \varepsilon_i \hat{b}_i^{+} \hat{b}_i + \sum_{i_1 i_2 i_3 i_4 = 1}^{6} \upsilon_{i_1 i_2 i_3 i_4} \hat{b}_{i_1}^{+} \hat{b}_{i_2}^{+} \hat{b}_{i_3} \hat{b}_{i_4}$$

- In general not solvable analytically.
- Three solvable cases with SO(3) symmetry:

$$\mathrm{U}(6) \supset \mathrm{U}(5) \supset \mathrm{SO}(5) \supset \mathrm{SO}(3)$$
$$\mathrm{U}(6) \supset \mathrm{SU}(3) \supset \mathrm{SO}(3)$$
$$\mathrm{U}(6) \supset \mathrm{SO}(6) \supset \mathrm{SO}(5) \supset \mathrm{SO}(3)$$

U(5) vibrational limit: $^{110}Cd_{62}$



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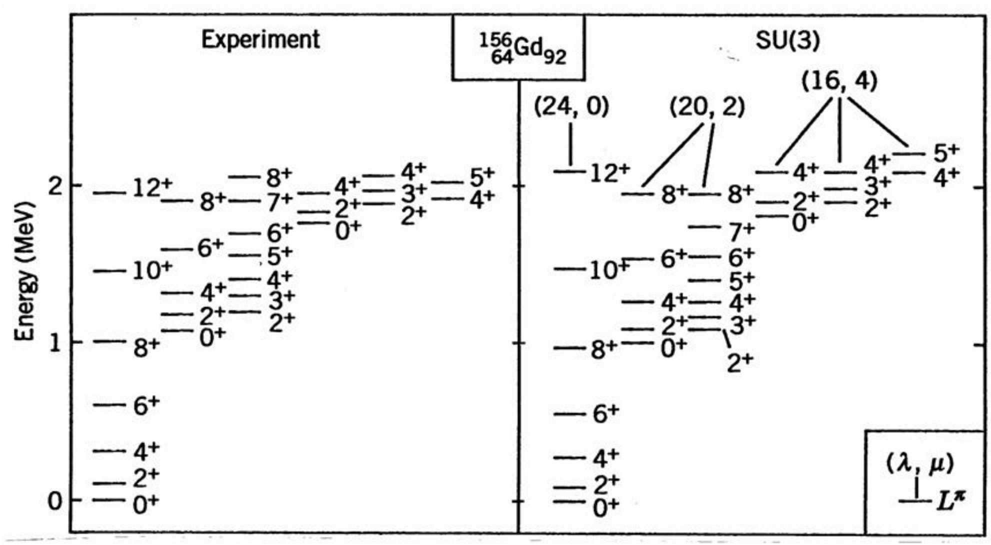
SU(3) rotational limit: $^{156}Gd_{92}$



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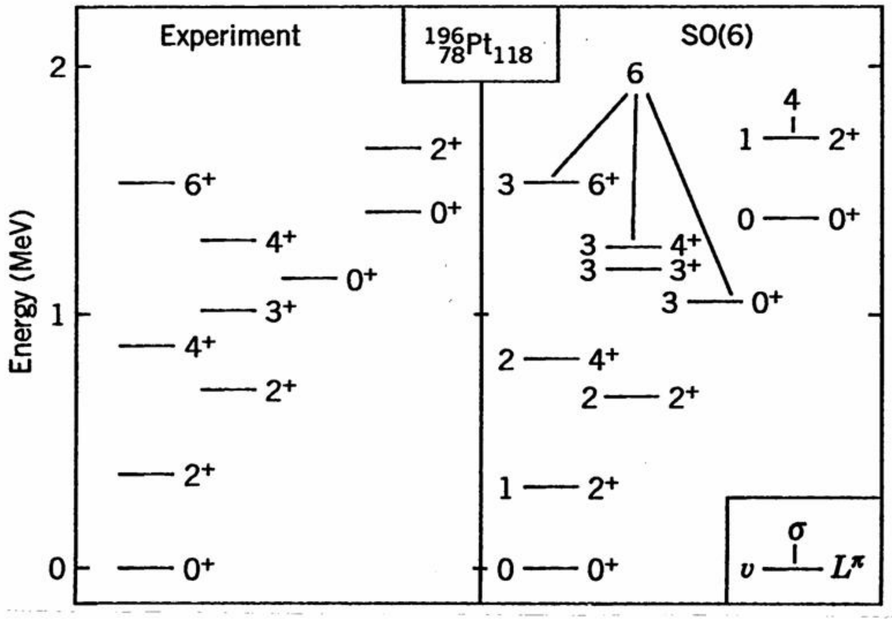
SO(6) γ-unstable limit: $^{196}Pt_{118}$



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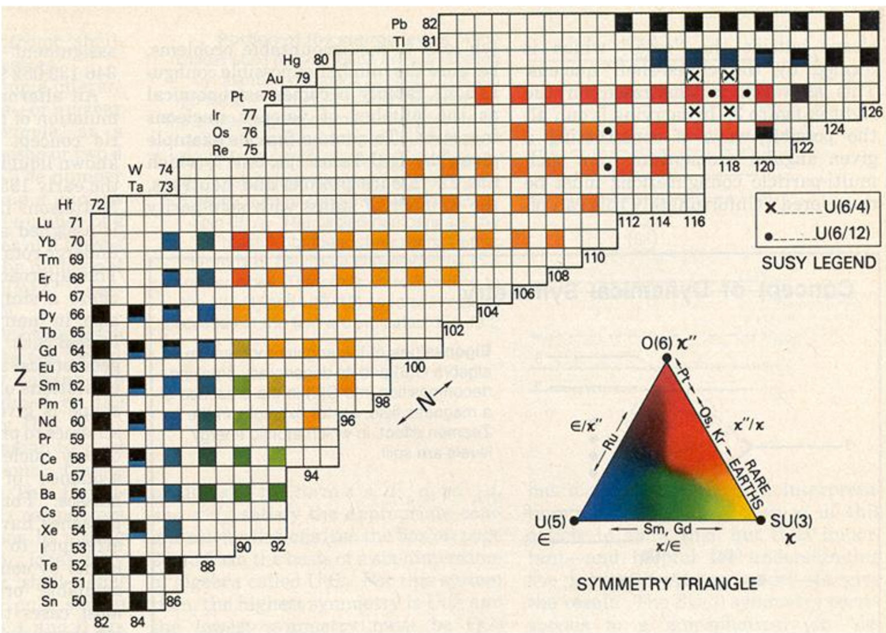
Applications of IBM



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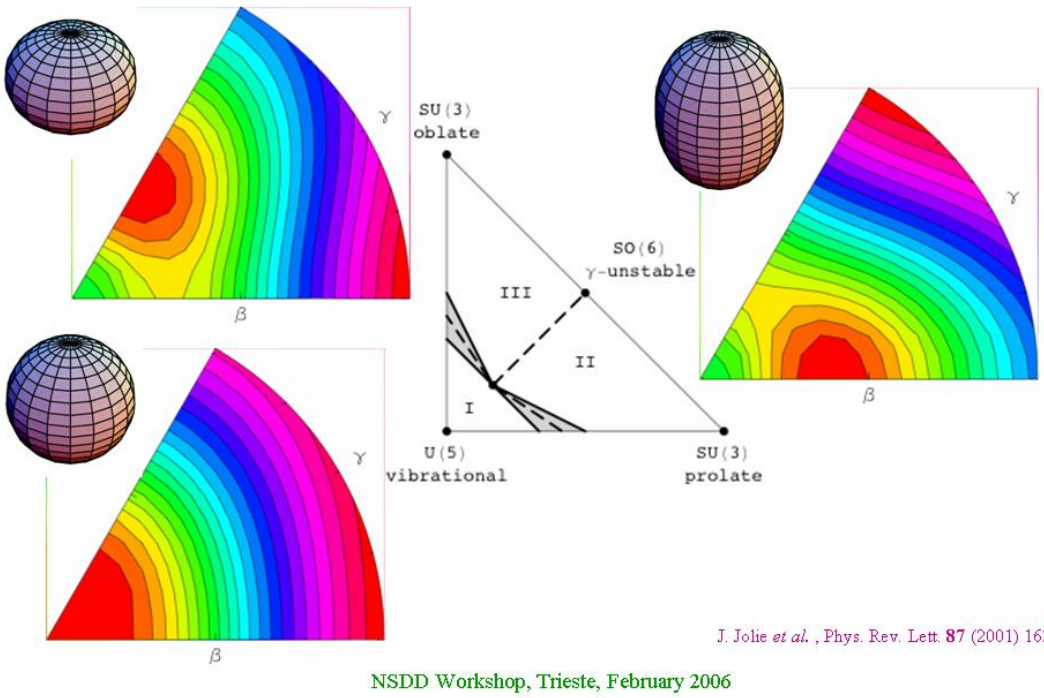
Classical limit of IBM

- For large boson number N the minimum of $V(\beta,\gamma)=\langle N;\beta\gamma | H | N;\beta\gamma\rangle$ approaches the exact ground-state energy:

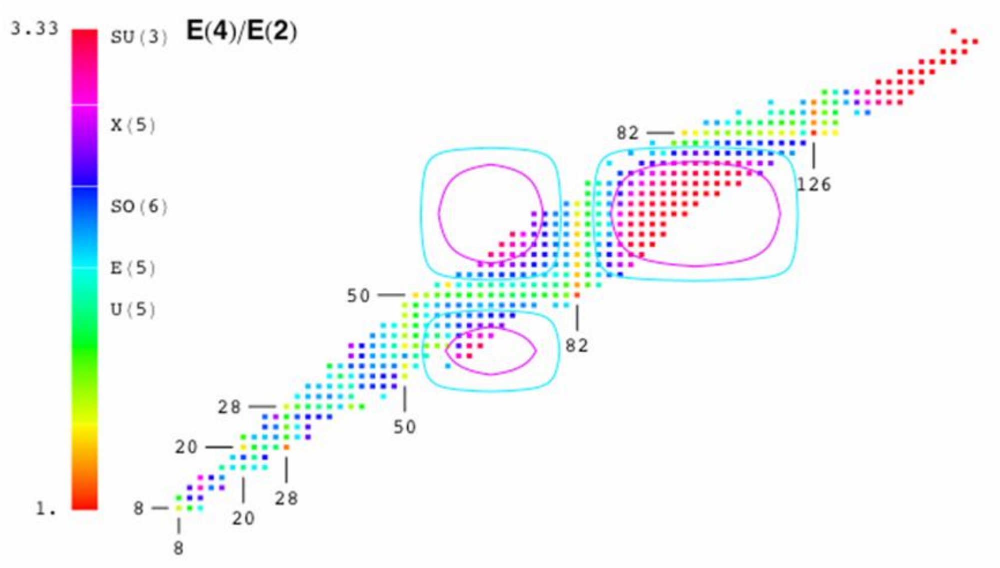
$$V(\beta,\gamma)\propto\begin{cases}\mathrm{U}(5): & \dfrac{\beta^2}{1+\beta^2}\\[2ex] \mathrm{SU}(3): & \dfrac{\beta^4-4\sqrt{2}\beta^3\cos 3\gamma+8\beta^2}{8\left(1+\beta^2\right)^2}\\[2ex] \mathrm{SO}(6): & \left(\dfrac{1-\beta^2}{1+\beta^2}\right)^2\end{cases}$$

Phase diagram of IBM



J. Jolie *et al.*, Phys. Rev. Lett. **87** (2001) 162501.

The ratio R_{42}

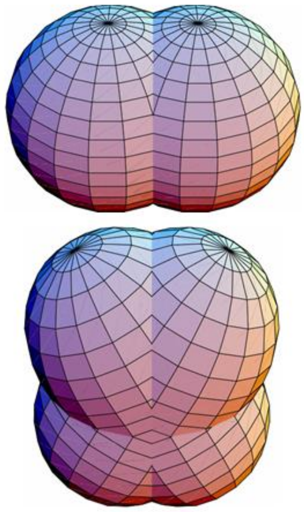


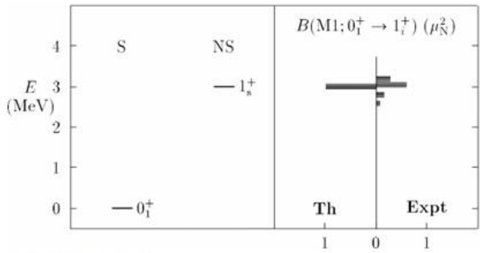
Extensions of IBM

- Neutron and proton degrees freedom (IBM-2):
  - F-spin multiplets ($N_\nu+N_\pi$=constant)
  - Scissors excitations
- Fermion degrees of freedom (IBFM):
  - Odd-mass nuclei
  - Supersymmetry (doublets & quartets)
- Other boson degrees of freedom:
  - Isospin T=0 & T=1 pairs (IBM-3 & IBM-4)
  - Higher multipole (g,…) pairs

Scissors mode

- Collective displacement modes between neutrons and protons:
  - Linear displacement (giant dipole resonance): R_ν-$R_\pi \Rightarrow$ E1 excitation.
  - Angular displacement (scissors resonance): L_ν-$L_\pi \Rightarrow$ M1 excitation.





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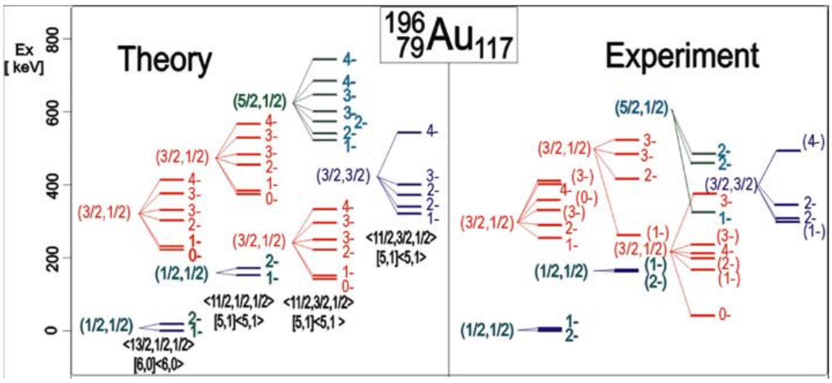
Supersymmetry

- A simultaneous description of even- and odd-mass nuclei (doublets) or of even-even, even-odd, odd-even and odd-odd nuclei (quartets).
- Example of ^{194}Pt, ^{195}Pt, ^{195}Au & ^{196}Au:



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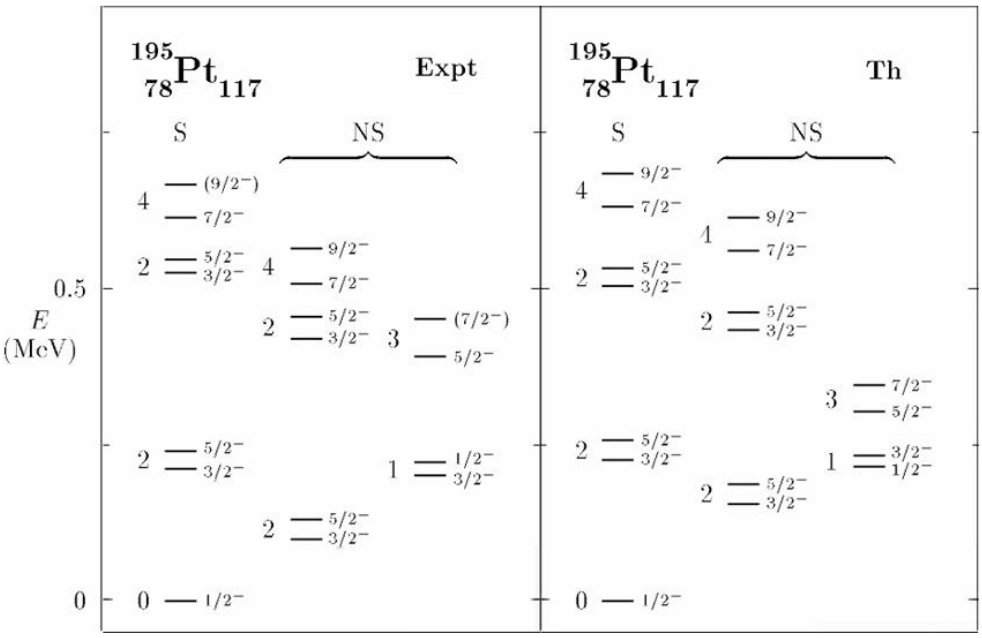
Bosons + fermions

- Odd-mass nuclei are fermions.
- Describe an odd-mass nucleus as N bosons + 1 fermion mutually interacting. Hamiltonian:

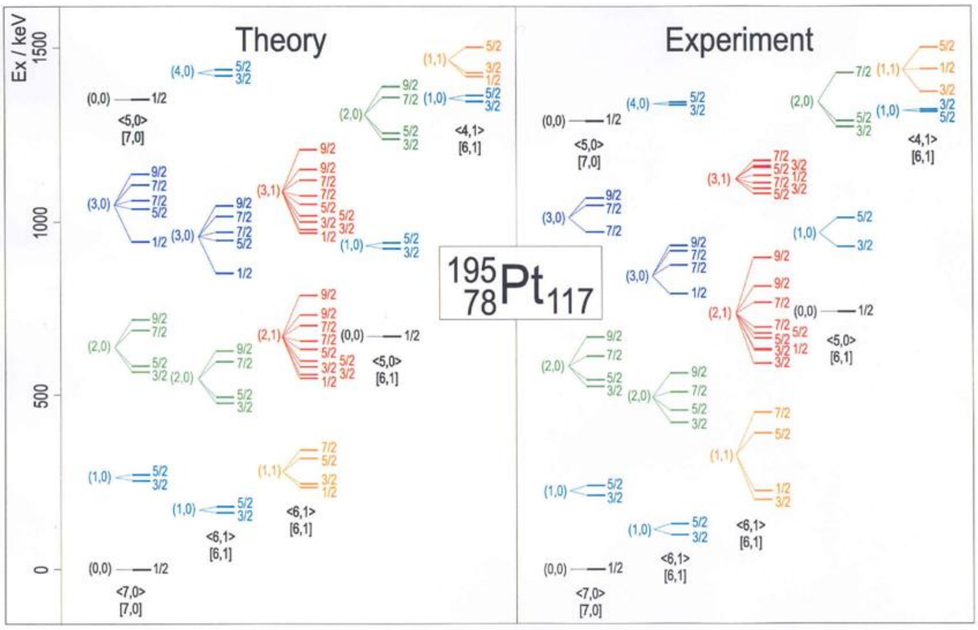
$$\hat{H}_{\mathrm{IBFM}} = \hat{H}_{\mathrm{IBM}} + \sum_{j=1}^{\Omega} \bar{\varepsilon}_j \hat{a}_j^{+} \hat{a}_j + \sum_{i_1 i_2=1}^{6} \sum_{j_1 j_2=1}^{\Omega} \bar{\upsilon}_{i_1 j_1 i_2 j_2} \hat{b}_{i_1}^{+} \hat{a}_{j_1}^{+} \hat{b}_{i_2} \hat{a}_{j_2}$$

- Algebra: $$\mathrm{U}(6) \oplus \mathrm{U}(\Omega) = \left\{ \begin{matrix} \hat{b}_{i_1}^{+} \hat{b}_{i_2} & \\ & \hat{a}_{j_1}^{+} \hat{a}_{j_2} \end{matrix} \right\}$$
- Many-body problem is solved analytically for certain energies ε and interactions υ.

Example: $^{195}\mathrm{Pt}_{117}$



Example: $^{195}\mathrm{Pt}_{117}$ (new data)



Nuclear supersymmetry

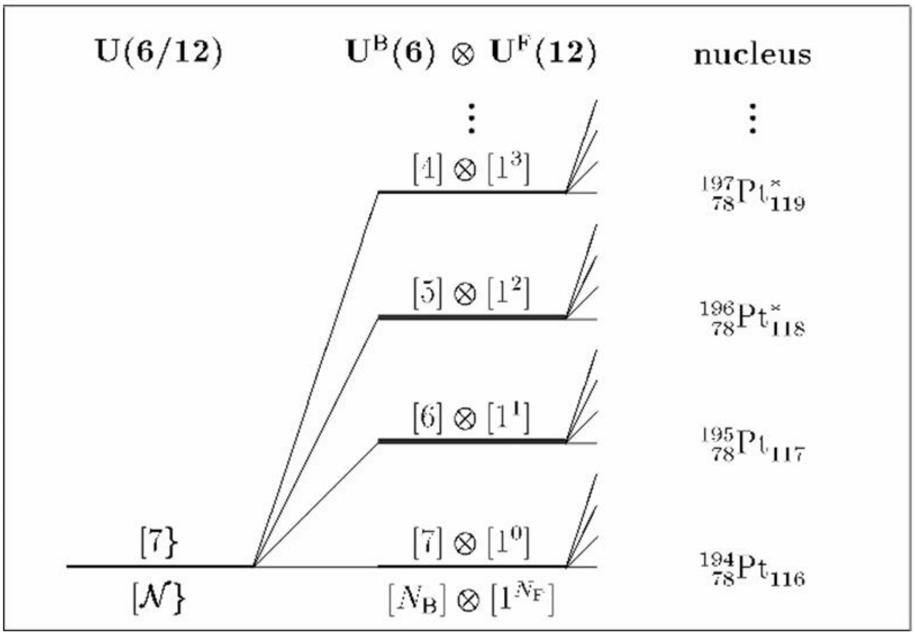
- Up to now: separate description of even-even and odd-mass nuclei with the algebra

$$\mathrm{U}(6)\oplus\mathrm{U}(\Omega)=\left\{\begin{matrix}\hat{b}^{+}_{i_1}\hat{b}_{i_2} & \\ & \hat{a}^{+}_{j_1}\hat{a}_{j_2}\end{matrix}\right\}$$

- Simultaneous description of even-even and odd-mass nuclei with the superalgebra

$$\mathrm{U}(6/\Omega)=\left\{\begin{matrix}\hat{b}^{+}_{i_1}\hat{b}_{i_2} & \hat{b}^{+}_{i_1}\hat{a}_{j_2} \\ \hat{a}^{+}_{j_1}\hat{b}_{i_2} & \hat{a}^{+}_{j_1}\hat{a}_{j_2}\end{matrix}\right\}$$

U(6/12) supermultiplet



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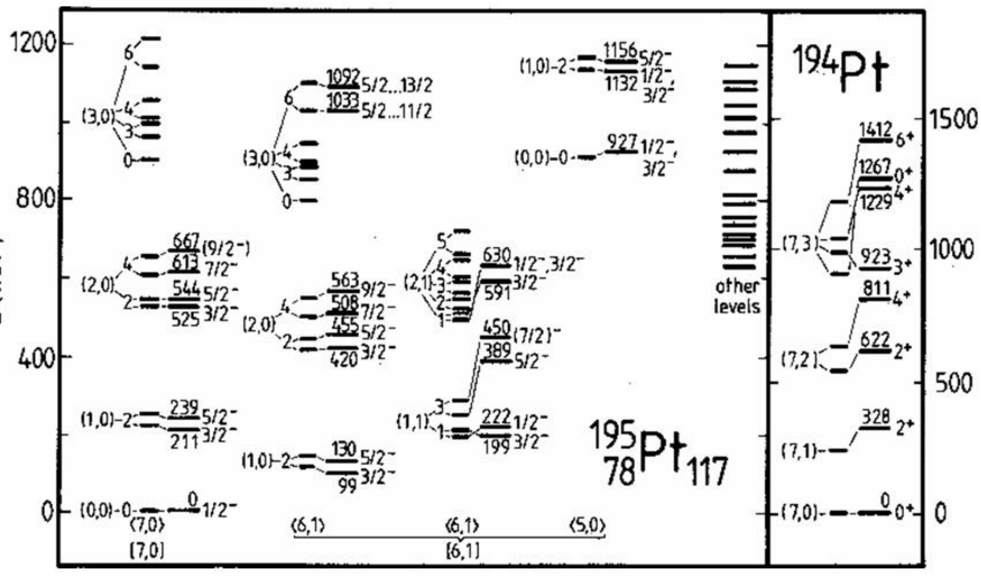
Example: $^{194}Pt_{116}$ &$^{195}Pt_{117}$



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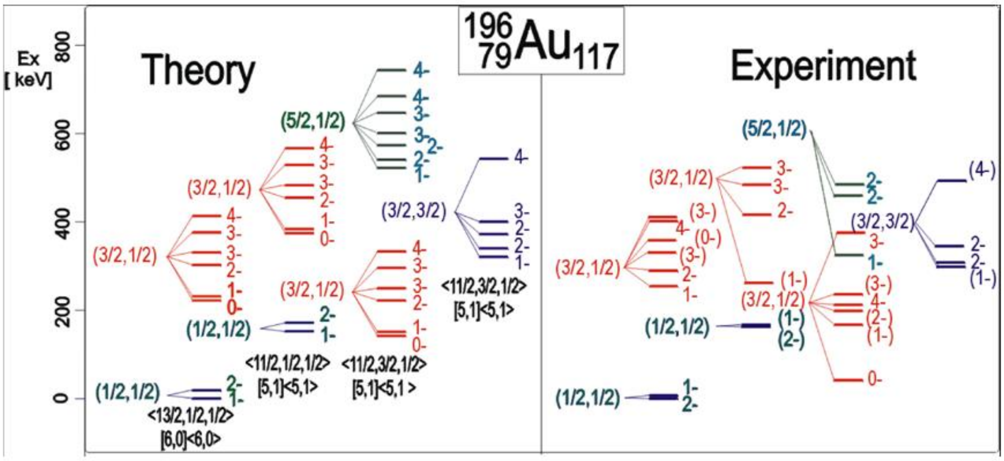
Example: $^{196}Au_{117}$



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3.

Nuclear Theory: Introduction

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Nuclear Structure

THEORY

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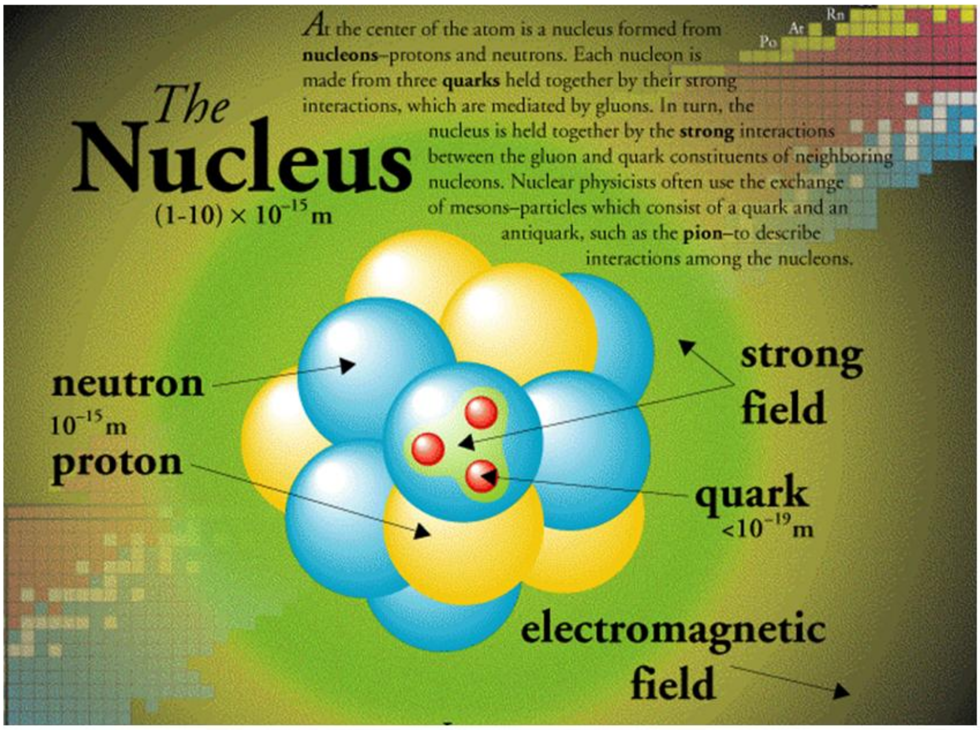
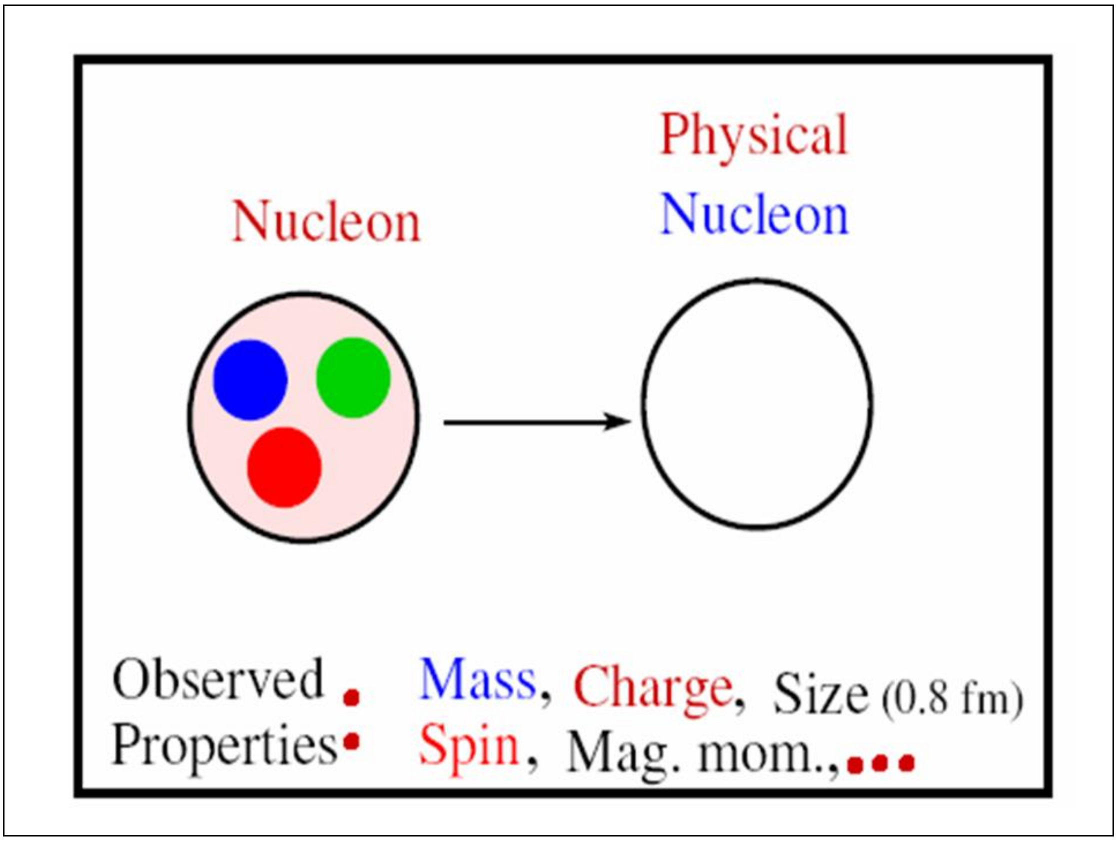
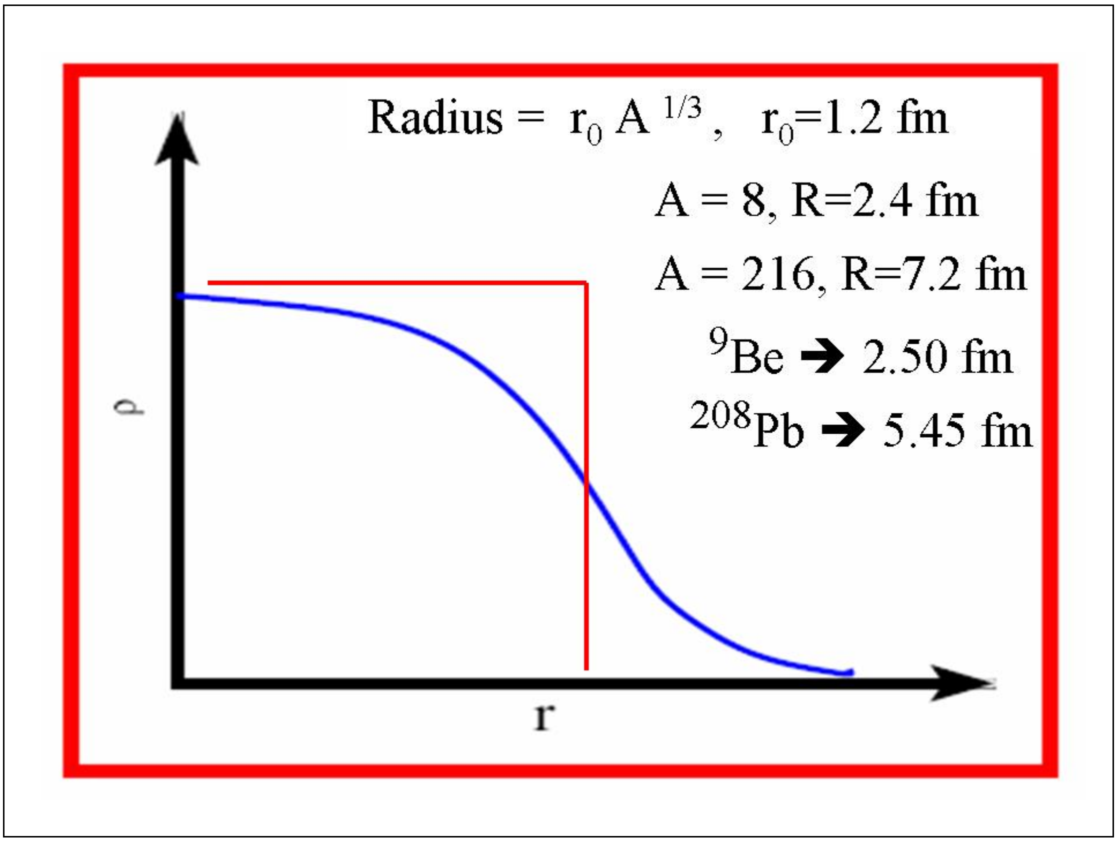
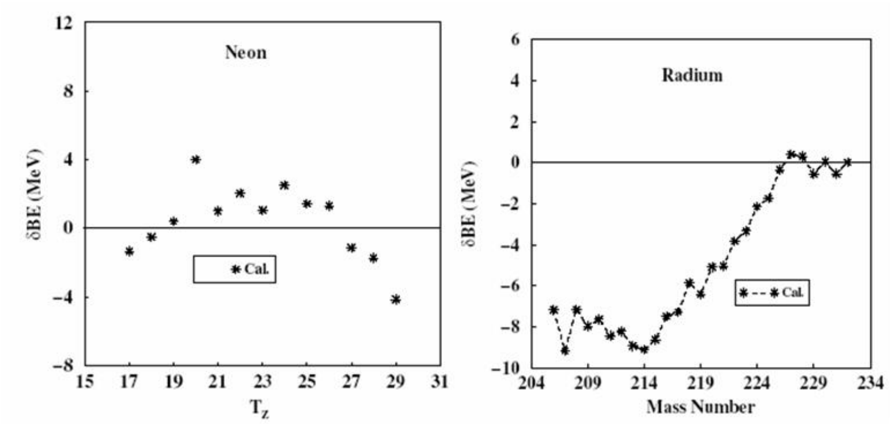
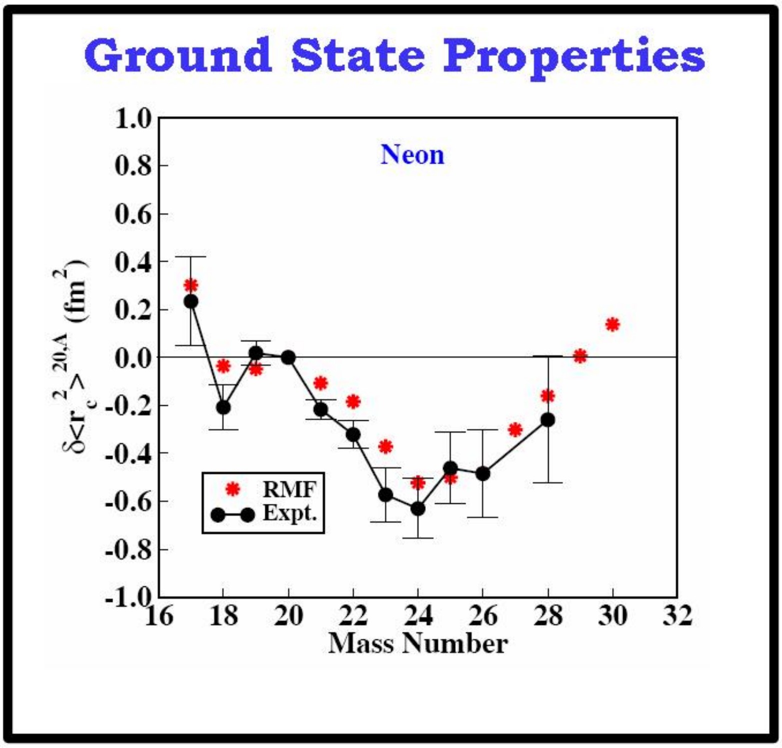
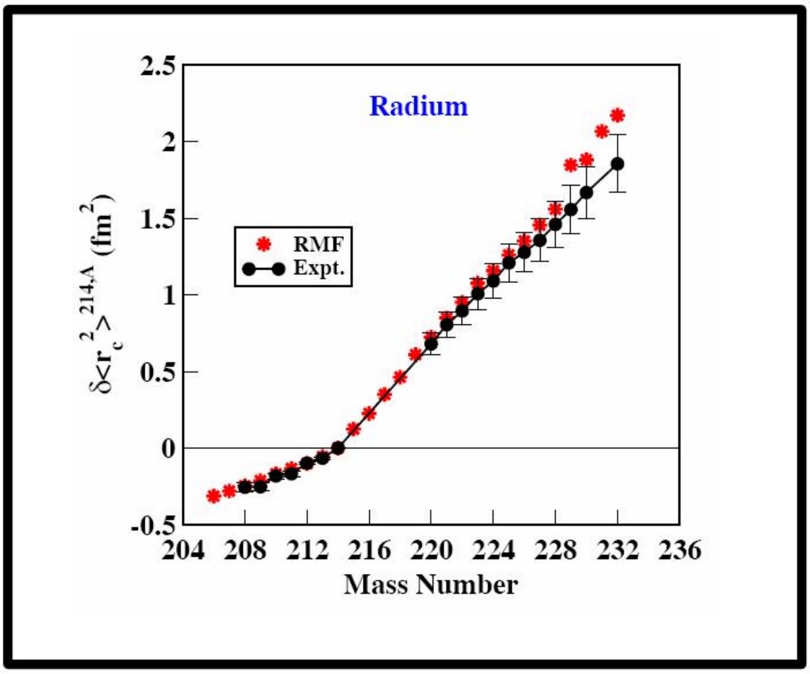



Physical
Nucleon
Nucleon
Observed
Properties
Mass, Charge, Size (0.8 fm)
Spin, Mag. mom.,

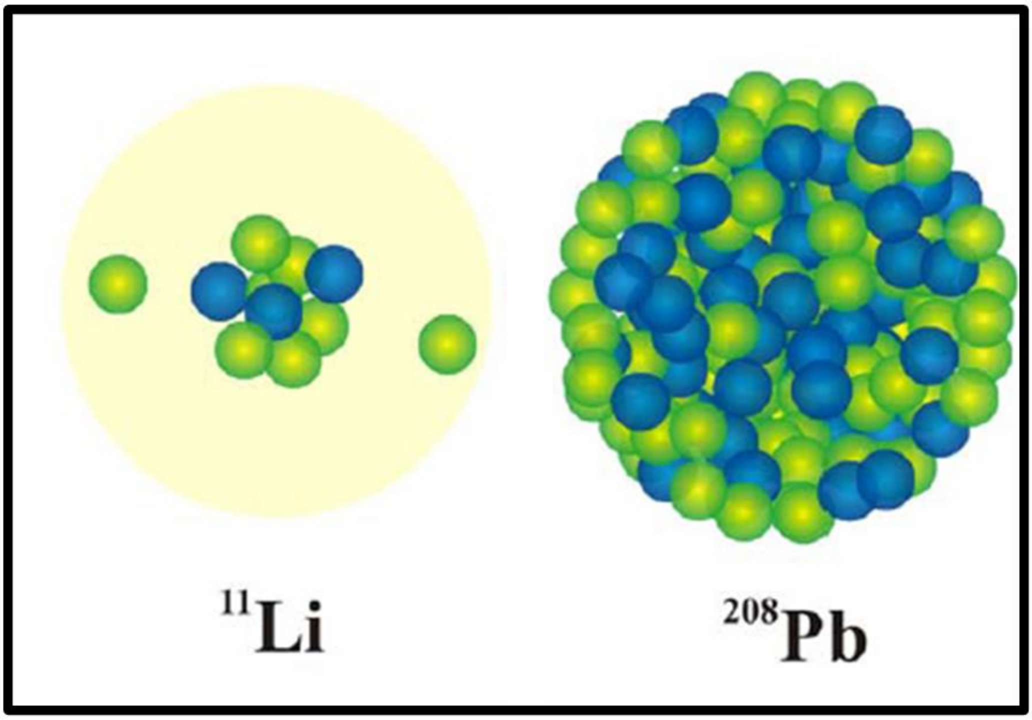

Radius = $r_0 A^{1/3}$, r_0=1.2 fm
A = 8, R=2.4 fm
A = 216, R=7.2 fm
^{9}Be → 2.50 fm
^{208}Pb → 5.45 fm
ρ
r


Neon
δBE (MeV)
Cal.
T_z
Radium
Mass Number


Ground State Properties
Neon
$\delta\langle r_c^2\rangle^{20,A}$ (fm^2)
RMF
Expt.
Mass Number



Radium
RMF
Expt.
$\delta\langle r_c^2\rangle^{214,A}$ (fm^2)
Mass Number



^{11}Li
^{208}Pb

$$\frac{\text{Nucl. Den}}{\text{Atom. Den}} = \frac{\text{Atom. Vol}}{\text{Nucl. Vol}} = \frac{(10^{-8})^3}{(10^{-13})^3} \approx 10^{14}$$

VERY VERY DENSE MATTER

$$\frac{\text{A- nu. Vol.}}{\text{Nucl. Vol.}} = \frac{A.\,(4\pi/3)\,(0.8)^3}{(4\pi/3)\,(1.2\,A^{1/3})^3} = \frac{8}{27} \approx 30\%$$

Most of Nucl. Vol. is Empty

N-N int.:

V. Strong, Net Attractive

Short range, State Dep.

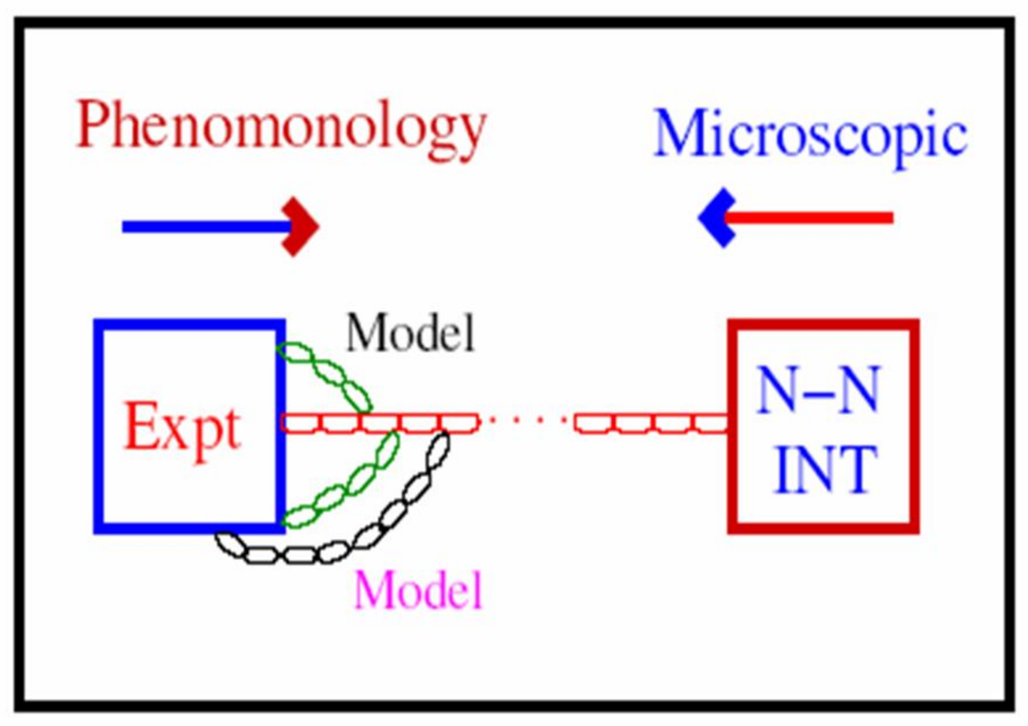
Non - Central

- **Nucleus: A (N+Z) – Body Problem**

$$\mathcal{H}\Psi_\lambda = \left[\sum_i \frac{-\hbar^2}{2m_i}\nabla_i^2 + \sum_{i>k} V_{ik}\right]\Psi_\lambda = E\Psi_\lambda$$

Can Not be Solved:
Difficulties:
● Mathematical
● Two-Body Interaction
(in the Nucleus)

Approximate Methods:
Models Developed:
Many Models Exists



Mean Field Concept:

$$
\begin{aligned}
\mathcal{H} &= \sum_i \frac{-\hbar^2}{2m_i}\nabla_i^2 + \sum_{i>k} V_{ik} \\
&= \underbrace{\sum_i \left(\frac{-\hbar^2}{2m_i}\nabla_i^2 + \mathcal{O}_i\right)}_{\mathcal{H}_o^i} + \underbrace{\left(\sum_{i>k} V_{ik} - \sum_i \mathcal{O}_i\right)}_{h_I} \\
&= \sum_i \mathcal{H}_o^i + h_I = \mathcal{H}_o + h_I
\end{aligned}
$$

Advantage
is
Freedom to Choose $\mathcal{O}_i$

Choose $\mathcal{O}_i \rightarrow h_I$ Zero (Minimum)

Mean Field Helps to reduce
A-Body Problem $\longrightarrow$ One Body Problem

Phenomenological (Shell Model,…)
Microscopic (BBHF)

Phenomenological

- **H.O. + l.s**

$$\mathcal{O}_i = \frac{1}{2} m\omega^2 r^2 + \alpha_{ls}\, \hat{l} \cdot \hat{s}$$

$$\psi_{nljm_j} = R_{nl}(r) \left[Y_l \bigotimes \chi_{1/2} \right]_{jm}$$

Plan

- Mean Field Concept
- Shell Model
- Magic Nuclei : TDA - RPA
- Open Shell Nuclei

a. Configuration Mixing
b. Truncations: Seniority, BPA ….
c. BCS – Quasiparticle Method
d. HF, HFB, PHF, PHFB

Plan

NO CORE

- ab intio Shell Model
- DDHF – Skyrme Type Interaction
- RMF – Rel. Mean Field

Schrodinger Equation

$$\mathcal{H}\Psi_\lambda = \left[\sum_i \frac{-\hbar^2}{2m_i}\nabla_i^2 + \sum_{i>k} V_{ik}\right]\Psi_\lambda = E\Psi_\lambda$$

Bound State Problem

Basis Expansion Method

Basis Expansion Method

$$\mathcal{H}\Psi_\alpha = E_\alpha\Psi_\alpha \; : \; \mathcal{H} = \mathcal{H}_o + \mathcal{V}$$

$$\mathcal{H}_o\Phi_I^\alpha = e_I\Phi_I^\alpha \qquad \Psi_\alpha = \sum_I x_\alpha^I \Phi_I^\alpha$$

$$\sum_I \left[e_I\delta_{IK} + \langle\Phi_K^\alpha|\mathcal{V}|\Phi_i^\alpha\rangle - E_\alpha\delta_{IK}\right] x_\alpha^I = 0$$

$$\mathcal{H}_{IK} = e_I\delta_{IK} + \langle\Phi_K^\alpha|\mathcal{V}|\Phi_I^\alpha\rangle$$

$$\left|\Psi^{v}_{\alpha J^{\pi} M}\right\rangle = \sum_{I} \chi^{v}_{\alpha J^{\pi} M}(I) \left|\Phi^{I}_{\alpha J^{\pi} M}\right\rangle$$

$$\mathcal{H} \left|\Psi^{v}_{\alpha J^{\pi} M}\right\rangle = E_{\alpha J^{\pi} M} \left|\Psi^{v}_{\alpha J^{\pi} M}\right\rangle$$

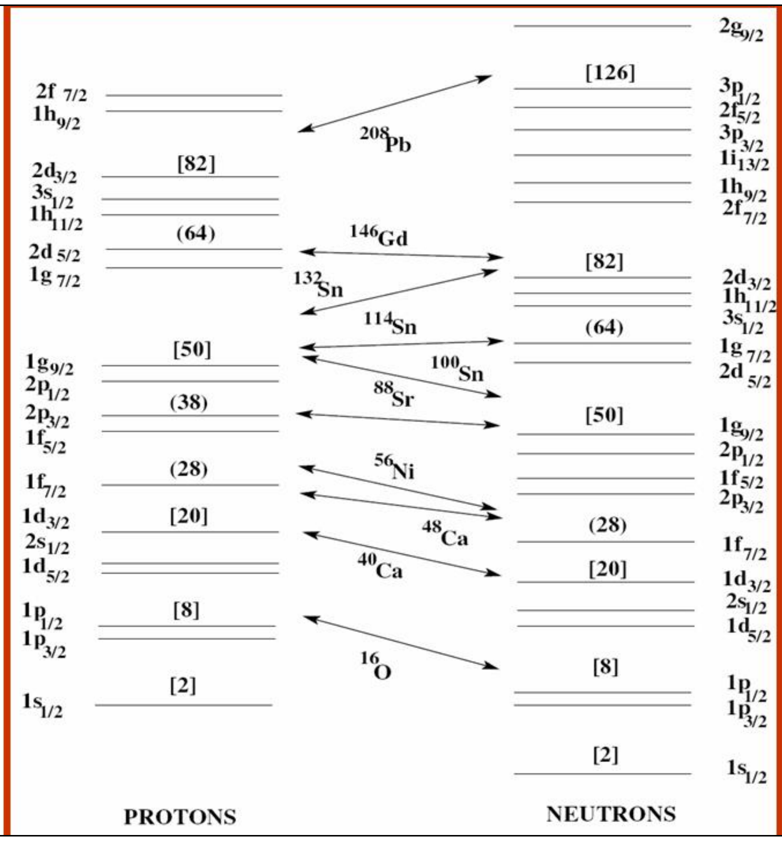
Step I: Choice of Basis (Mean Field)

Step II: Construction of Φ_I - A Nucleons

Unperturbed Energies ε_I

Step III: Setting of Hamiltonian Matrix $\mathcal{H}$

Step IV: Diagonalization of $\mathcal{H}$



Step I: Choose Core, Valence Level, s.p. Energy ε_I

ε_I: Expt., Calculated or Parameters

$\hbar\omega = 41A^{-1/3}$

Step II: Orthonormal Basis Set Φ_I

Group Theoretical Method

Step III: Setting up Hamiltonian Matrix

Requires Two –Body Matrix Elements

Realistic, Phenomenological, Empirical

Step IV: Diagonalization of H-Matrix

Repeat for each J^{π}

Hamiltonian:

$$\sum_{\alpha} \varepsilon_{\alpha} C_{\alpha}^{\dagger} C_{\alpha} + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | V | \gamma\delta \rangle \, C_{\alpha}^{\dagger} C_{\beta}^{\dagger} C_{\delta} C_{\gamma}$$

$$\begin{aligned} \langle \alpha\beta | V | \gamma\delta \rangle &= -\langle \beta\alpha | V | \gamma\delta \rangle \\ &= -\langle \alpha\beta | V | \delta\gamma \rangle \\ &= \langle \beta\alpha | V | \delta\gamma \rangle \end{aligned}$$

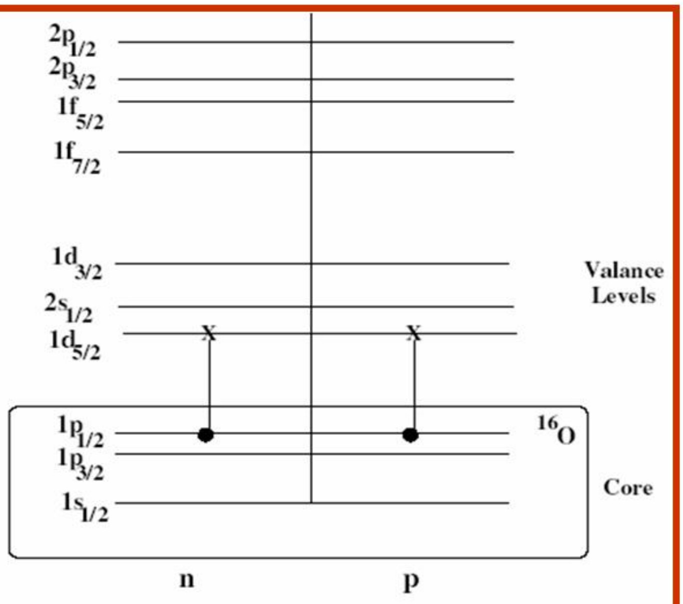
$C^{\dagger}(C)$: Particle Creation (destruction) Operator

Vacuum Obeys $\quad C_{\alpha} |0\rangle = 0$

$C^{\dagger}(C)$ obey anti commutation relations

$$\begin{aligned} \left\{ C_{\alpha}, C_{\beta}^{\dagger} \right\} &\equiv C_{\alpha} C_{\beta}^{\dagger} + C_{\beta}^{\dagger} C_{\alpha} = \delta_{\alpha\beta} \, , \\ \{ C_{\alpha}, C_{\beta} \} &\equiv \left\{ C_{\alpha}^{\dagger}, C_{\beta}^{\dagger} \right\} = 0 \quad . \end{aligned}$$

Application to Closed Shell Nuclei



Hole Levels : $h_1, h_2, h_3, \ldots.$
Particle Levels : $p_1, p_2, p_3, \ldots$

- **1p - 1h : Lowest Energy – Excitation**

$$(C_p^{\dagger} C_h)$$

- **Higher Order (Energy) Excitations**
  2p - 2h, 3p – 3h, …..

Equation of Motion Method

Operator $Q_\alpha^\dagger$ **Obeys:**

$$\iota \frac{\partial Q_\alpha^\dagger}{\partial t} = \left[H, Q_\alpha^\dagger\right] = E_\alpha Q_\alpha^\dagger,$$

$$H Q_\alpha^\dagger |\psi\rangle = (E_\alpha + E) Q_\alpha^\dagger |\psi\rangle$$

$Q_\alpha^\dagger$ (Q_α) acts as step up down Operator

Vacuum (g.s.) is **defined by** $Q_\alpha |\psi_o\rangle = 0$.

if Set of Operators $\boxed{a_i^\dagger \ (i = 1,\ 2,\ 3,\ \ldots,\ \mathrm{N})}$ **Obey**

$$\left[H, a_i^\dagger\right] = \sum_{j=1}^{N} M_{ij} a_j^\dagger$$

Step up operator: $\boxed{Q_\alpha^\dagger = \sum_j x_j^\alpha a_j^\dagger}$

$$\sum_j \tilde{M}_{ij} x_j^\alpha = E_\alpha x_i^\alpha$$

We require $[H, C_p^\dagger C_h]$ and its HC

It Contains Two Terms:

$$\sum_\alpha \epsilon_\alpha \left[C_\alpha^\dagger C_\alpha, C_p^\dagger C_h\right]$$

$$\sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|\mathcal{V}|\gamma\delta\rangle \left[C_\alpha^\dagger C_\beta^\dagger C_\delta C_\gamma, C_p^\dagger C_h\right]$$

Use

$$\begin{aligned}[A, BC] &= -[BC, A] = [A, B]C + B[A, C\\ &= \{A, B\}C - B\{A, C\}\end{aligned}$$

and $\quad C_p|0\rangle = C_h^\dagger|0\rangle = 0$

Where $\quad \{A, B\} = AB + BA$

Notice

$$\left[C_\alpha^\dagger C_\alpha, C_p^\dagger C_h\right] \;=\; \delta_{\alpha p} C_\alpha^\dagger C_h \;-\; \delta_{\alpha h} C_p^\dagger C_\alpha$$

$$\left[C_\alpha^\dagger C_\beta^\dagger, C_p^\dagger C_h\right] \;=\; \left(1 \;-\; P(\alpha \leftrightarrow \beta)\right) C_p^\dagger C_\alpha^\dagger \delta_{\beta h}$$

Define $\bar{P} \;=\; 1 \;-\; P$

$$\boxed{\begin{aligned}\left[H, C_p^\dagger C_h\right] \;&=\; (\epsilon_p \;-\; \epsilon_h)\, C_p^\dagger C_h \\ &-\; \frac{1}{2}\sum_{\alpha\gamma\delta} \langle \alpha h|\mathcal{V}|\gamma\delta\rangle C_\alpha^\dagger C_p^\dagger C_\delta C_\gamma \\ &+\; \frac{1}{2}\sum_{\alpha\beta\delta} \langle \alpha\beta|\mathcal{V}|p\delta\rangle C_\alpha^\dagger C_\beta^\dagger C_\delta C_h\end{aligned}}$$

$$
\begin{aligned}
C_\alpha^\dagger C_p^\dagger C_\delta C_\gamma \;&=\; :C_\alpha^\dagger C_p^\dagger C_\delta C_\gamma:
\end{aligned}
$$

$$
\begin{aligned}
&+\langle C_\alpha^\dagger C_p^\dagger\rangle :C_\delta C_\gamma: +\langle C_\delta C_\gamma\rangle :C_\alpha^\dagger C_p^\dagger: +\langle C_\alpha^\dagger C_\gamma\rangle :C_p^\dagger C_\delta: \\
&+\langle C_p^\dagger C_\delta\rangle :C_\alpha^\dagger C_\gamma: -\langle C_p^\dagger C_\gamma\rangle :C_\alpha^\dagger C_\delta: -\langle C_\alpha^\dagger C_\delta\rangle :C_p^\dagger C_\gamma: \\
&+\langle C_\alpha^\dagger C_p^\dagger\rangle\langle C_\delta C_\gamma\rangle + \langle C_p^\dagger C_\delta\rangle\langle C_\alpha^\dagger C_\gamma\rangle - \langle C_\alpha^\dagger C_\delta\rangle\langle C_p^\dagger C_\gamma\rangle
\end{aligned}
$$

$$
\begin{aligned}
\Big[H, C_p^\dagger C_h\Big] \;&=\; \sum_{p'}\left(\epsilon_p \delta pp' + \sum_{h_1}\langle p'h_1|\mathcal{V}|ph_1\rangle\right) C_{p'}^\dagger C_h \\
&- \;\sum_{h'}\left(\epsilon_h \delta_{hh'} + \sum_{h_1}\langle hh_1|\mathcal{V}|h'h_1\rangle\right) C_p^\dagger C_{h'} \\
&+ \;\sum_{p'h'}\Big(\langle hp'|\mathcal{V}|ph'\rangle C_{p'}^\dagger C_{h'} + \langle hh'|\mathcal{V}|pp'\rangle C_{h'}^\dagger C_{p'}\Big)
\end{aligned}
$$

$$\left[H, C_p^\dagger C_h\right] = \sum_{p'h'} ((\tilde{\epsilon}_p - \tilde{\epsilon}_h)\delta_{pp'}\delta_{hh'}$$
$$+ \langle hp'|\mathcal{V}|ph'\rangle)\, C_{p'}^\dagger C_{h'} + \sum_{p'h'} \langle hh'|\mathcal{V}|pp'\rangle C_{h'}^\dagger C_{p'}$$
$$= \sum_{p'h'} \left(A(p'h,' ph) C_{p'}^\dagger C_{h'} \; + \; B(p'h', ph) C_{h'}^\dagger C_{p'} \right)$$

The Matrices

$$\begin{aligned} A(p'h', ph) &= (\tilde{\epsilon}_p - \tilde{\epsilon}_h)\delta_{pp'}\delta_{hh'} \; + \; \langle hp'|\mathcal{V}|ph'\rangle \\ B(p'h', ph) &= \langle hh'|\mathcal{V}|pp'\rangle \end{aligned}$$

In Coupled Representation: J^Π T

$$\begin{pmatrix} \left[H, \Omega^\dagger\right] \\ \left[H, \hat{\Omega}\right] \end{pmatrix} = \begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} \Omega^\dagger \\ \hat{\Omega} \end{pmatrix}$$

where

$$\hat{\Omega}_{J^\pi M T M_T}(p_i, h_i) =$$
$$(-1)^{J-M+T-M_T} \Omega_{J^\pi -M T -M_T}(p_i, h_i)$$

$$\begin{aligned} A_{ij}^{J^\pi T} &= (\tilde{\epsilon}_{p_i} - \tilde{\epsilon}_{h_i})\delta_{p_i p_j}\delta_{p_i h_j} \; + \; F(p_i h_i p_j h_j J^\pi T) \\ B_{ij}^{J^\pi T} &= (-1)^{j_{p_i} + j_{h_j} + J + T} F(p_i h_i h_j p_j J^\pi T) \; , \end{aligned}$$

Hole Particle Matrix Element

$$F(acdbJ^{\pi}T) =$$
$$\sum_{J'T'}(2J'+1)(2T'+1)W(j_aj_bj_cj_d;J'J)W\left(\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2},T'T\right)$$
$$\times \quad (-1)^{j_a+j_b+j_c+j_d}\langle abJ'T'|\mathcal{V}|dcJ'T'\rangle$$

W is Racah Coefficient

Step Up Operator

$$Q^{\dagger} = X\Omega^{\dagger} - Y\hat{\Omega}$$

X , Y are Eigen Vectors of the Matrix

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix}$$

With Norm

$$X^2 - Y^2 = 1$$

Both X and Y can be large

Illustration ^{16}O

- **Step I:**

Hole levels (h):	$1p_{3/2}$, $1p_{1/2}$
$(\tilde{\epsilon})$ (MeV):	21.8, 15.65 (for neutrons) 18.44, 12.11 (for protons)
Particle levels (p):	$1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$
$(\tilde{\epsilon})$ (MeV):	-4.5, -3.27, 0.93 (for neutrons) -0.59, -0.08, 4.65 (for protons)

- **Step II: Construction of h-p Basis**

For $J^\pi = 0^-$ $\quad T = 1$ and $T = 0$

$$(1d_{3/2} 1p^{-1}_{3/2})_{0^-},\ (2s_{1/2} 1p^{-1}_{1/2})_{0^-}$$

For $J^\pi = 1^-$ $\quad T = 1$ and $T = 0$

$$(1d_{5/2} 1p^{-1}_{3/2})_{1^-},\ (1d_{3/2} 1p^{-1}_{3/2})_{1^-},\ (2s_{1/2} 1p^{-1}_{3/2})_{1^-}$$

$$(2s_{1/2} 1p^{-1}_{1/2})_{1^-},\ (1d_{3/2} 1p^{-1}_{1/2})_{1^-}$$

For 0^- State

$1.\Rightarrow\ 1d_{3/2}\ 1p^{-1}_{3/2}$ $3.\Rightarrow$ hc of 1;

$2.\Rightarrow\ 2s_{1/2}\ 1p^{-1}_{1/2}$, $4.\Rightarrow$ hc of 2;

T		E(MeV)	1	2	3	4
	TDA	11.2	0.001	1.000		
	RPA	11.2	0.001	1.000	-0.002	-0.002
	TDA	23.1	1.000	-0.001		
	RPA	23.0	1.000	-0.001	-0.034	-0.002
	TDA	13.7	-0.045	0.999		
	RPA	13.7	-0.048	0.999	-0.012	-0.012
	TDA	25.7	0.999	0.055		
	RPA	25.6	1.000	0.053	-0.040	-0.015

Step III: Two – Body Interaction

M=W=0.15, H=0.4 and B=0.3
Gaussian Shape, Strength = - 40 MeV

Step IV: Diagonalization

Results for $J^\pi = 0^-$
Both for $T = 1$ and $T = 0$

Similar Results for Other States

Open Shell Nuclei

Illustration ^{58}Ni

Step I: Mean Field

Core: $^{56}_{28}\mathrm{Ni}\ (Z=N=28)$

Valence Levels: $2p_{3/2},\ 1f_{5/2},\ 2p_{1/2}$

s. p. Energies : **0.0, 0.78 and 1.08 MeV**

Step II: Orthonormal Basis Set

2 Valence Neutrons

$$\left(1p_{3/2}\right)^2_{J^\pi=0+\ 2+}\ ;\quad \left(2p_{3/2}\,1f_{5/2}\right)_{J^\pi=1+,2+,3+,4+}\ ;$$

$$\left(2p_{3/2}2p_{1/2}\right)_{J^\pi=1+,2+}\ ;\quad \left(1f_{5/2}\right)^2_{J^\pi=0+,2+,4+}\ ;$$

$$\left(1f_{5/2}\,2p_{1/2}\right)_{J^\pi=2+,3+}\ ;\quad \left(2p_{1/2}\right)^2_{J^\pi=0+}$$

No of Basis States are:

$$0^+(3),\ \ 1^+(2),\ 2^+(5),\ 3^+(2),\ 4^+(2),$$

Step III: Kuo – Brown Inte. M.E.

Step IV: Diagonalization.

Results for ^{58}Ni, ^{60}Ni, ^{62}Ni and ^{64}Ni.

	J^π	0_1^+	0_2^+	2_1^+	2_2^+	4_1^+	4_2^+
^{58}Ni	EMS	0.0	2.56	1.41	2.86	2.30	
	EXPT.	0.0		1.45	2.78	2.46	
^{60}Ni	EMS	0.0	2.30	1.50	2.20	2.18	
	EXPT.	0.0	2.29	1.33	2.16	2.50	
^{62}Ni	EMS	0.0	2.11	1.56	2.29	2.15	
	EXPT.	0.0	2.05	1.17	2.30	2.34	
	J^π	$1/2_1^-$	$1/2_2^-$	$3/2_1^-$	$3/2_2^-$	$5/2_1^-$	$5/2_2^-$
^{59}Ni	EMS	0.24	1.10	0.0	0.82	0.21	1.47
	EXPT.	0.47	1.32	0.0	0.89	0.34	

Is Nuclear problem solved? NO
Reason : Huge number of Basis Φ:
For ^{112}Sn: 12 neutrons in five s.p. states

$$(2d_{5/2},\ 1g_{7/2},\ 3s_{1/2},\ 2d_{3/2},\ 1h_{11/2})$$

The Number of States Φ are for

$J^\pi = 0^+$ is 55,907,
$J^\pi = 2^+$ is 267,720
$J^\pi = 4^+$ is 426,558.

Solution: Truncation Schemes:
Seniority Truncation Scheme
Seniority (ν): No. of Nucleons Left After all Pairs Coupled to J = 0 are Removed.
Even – Even: ν = 0, 2, 4 are OK
Odd - Even : ν = 1, 3, 5 are OK

Seniority Decomposition (in %) ^{61}Ni

State J^π	Energy Theo.	Energy Expt.	$\nu=1$	$\nu=3$	$\nu=5$
$1/2_1^-$	0.02	0.28	96.9	2.9	0.2
$1/2_2^-$	1.02	—	24.1	74.5	1.4
$3/2_1^-$	0.0	0.0	92.4	7.0	0.6
$3/2_2^-$	1.03	0.66	31.2	65.7	3.1
$5/2_1^-$	0.12	0.07	97.1	2.7	0.2
$5/2_2^-$	0.93	0.91	24.3	71.3	0.4
$7/2_1^-$	0.92	1.02	—	94.9	5.1
$9/2_1^-$	1.00	—	—	99.3	0.7

Seniority Decomposition (in %) ^{62}Ni

State J^π	Energy Theo.	Energy Expt.	$\nu=0$	$\nu=2$	$\nu=4$	$\nu=6$
0_1^+	0.0	0.0	99.7	—	0.3	—
0_2^+	2.11	2.05	87.3	—	12.7	—
1_1^+	3.57	—	24.7	70.0	5.3	
2_1^+	1.56	1.17		99.4	0.5	0.1
2_2^+	2.29	2.30		89.1	10.7	0.2
3_1^+	2.84	—		40.6	59.3	0.1
4_1^+	2.15	2.34		92.9	7.0	0.1
4_2^+	2.76	—		41.6	58.3	0.1

Still Problem is Not Solved:
For ^{112}Sn the ν=0 States are 110
While ν=2 States Approach Thousand

Solution:
Quasiparticle (BCS) Theory
Broken Pair Approximation (BPA)

<u>Quasiparticle OR BCS Method</u>

This Takes Into Account the Strong Pairing Part of the Effective Two-Body Interaction.

The Idea is to Go From Particle Picture to Quasiparticle Picture (New Mean Field) Through Bogoliubov or Quasiparticle (qp) Transformation. This Leads in the Lowest Approximation to Independent Quasiparticle Picture - Incorporates the Pairing Interaction.

Nuclear Theory:

Quasiparticle OR BCS Method

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Quasiparticle OR BCS Method

This Takes Into Account the Strong Pairing Part of the Effective Two-Body Interaction.

The Idea is to Go From Particle Picture to Quasiparticle Picture (New Mean Field) Through Bogoliubov or Quasiparticle (qp) Transformation. This Leads in the Lowest Approximation to Independent Quasiparticle Picture - Incorporates the Pairing Interaction.

The Quasiparticle/BCS Transformation :

$$\boxed{a_\alpha^\dagger = U_\alpha C_\alpha^\dagger - V_\alpha \tilde{C}_\alpha} \; ; \; \boxed{\tilde{a}_\alpha = U_\alpha \tilde{C}_\alpha + V_\alpha C_\alpha^\dagger}$$

The Inverse Transformation Is:

$$\boxed{C_\alpha^\dagger = U_\alpha a_\alpha^\dagger + V_\alpha \tilde{a}_\alpha} \; ; \; \boxed{\tilde{C}_\alpha = U_\alpha \tilde{a}_\alpha - V_\alpha a_\alpha^\dagger}$$

Here:

$$\boxed{\tilde{C}_\alpha = S_\alpha C_{-\alpha}} \; ; \; \boxed{\tilde{a}_\alpha = S_\alpha a_{-\alpha}}$$

With:

$$\boxed{S_\alpha = (-1)^{j_\alpha - m_\alpha}}$$

The qp (New) Operators $a^\dagger_\alpha$ (a_α) **also Obey Fermion Commutation Rules. This Requires**

$$\boxed{U_\alpha^2 \; + \; V_\alpha^2 \; = \; 1} \qquad \boxed{V_\alpha = V_{-\alpha}\,,\, U_\alpha = U_{-\alpha}}$$

The New or qp (Particle) Vacuum $|qp>$ $(|0>)$ **is Defined Through**

$$\boxed{\begin{aligned} a_\alpha|qp> \; &= \; 0\;, \text{ and} \\ C_\alpha|0> \; &= \; 0. \end{aligned}}$$

The qp or BCS State can be Expressed as

$$\boxed{|\mathrm{BCS}\rangle \; = \; \prod_{\alpha>0}\left(\mathrm{U}_\alpha \; + \; \mathrm{V}_\alpha \mathrm{S}_\alpha \mathrm{C}^\dagger_\alpha \mathrm{C}^\dagger_{-\alpha}\right)|0\rangle}$$

The qp (BCS) Transformation Does Not Conserve the Nucleon Number. λ **Therefore Introduce Lagrange Multiplier and Use the Hamiltonian** H'

$$\boxed{H' \rightarrow H - \lambda\hat{N}}\;, \text{where, } \; \hat{N} \; = \; \sum_\alpha C^\dagger_\alpha C_\alpha$$

H' **Can be Written as:**

$$\boxed{H' \; = \; H - \lambda\hat{N}}$$

$$\boxed{= \; \sum_\alpha(\epsilon_\alpha - \lambda)C^\dagger_\alpha C_\alpha \; + \; \frac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|\mathcal{V}|\gamma\delta\rangle C^\dagger_\alpha C^\dagger_\beta C_\delta C_\gamma}$$

Various Ways to Derive qp Equations:
We Follow Here the Conventional Procedure.
Step I: Use Wick's Theorem to Write the One Body and Two Body Particle Operators of the Hamiltonian in Terms of Normal Products and Expectation Values / Contractions.
Step II: Express All These in Terms of qp Operators using qp Transformation. Evaluate the Expectation Values wrt qp Vacuum.
The Transformed Hamiltonian Contains Three Terms:

• H_0 a Constant without any qp Operators

• Terms With Two qp Operators. This Contains Two Parts. The First H_{11} Contains Only $a^+ a$ Terms (Required For New Mean Field) While the Second $H_{20}(H_{02})$ Involves the Terms $a^+ a^+$ (a a). This Dangerous Term has to be Equated to Zero

• Terms Involving Four qp Operators (Hint) Arising from $:C^+C^+CC:$, The Residual qp Interaction Needed While Going Beyond Mean Field

The Resulting qp Hamiltonian is:

$$\begin{aligned} H' &= H - \lambda\hat{N} \\ &= H_0 + H_{11} + H_{20} + H_{02} + H_{\text{int}} \end{aligned}$$

Where

$$\begin{aligned} H_0 &= \sum_\alpha \left(\varepsilon_a - \lambda + \frac{1}{2}\sum_\gamma V_c^2\langle\alpha\gamma|\mathcal{V}|\alpha\gamma\rangle\right) V_a^2 \\ &+\frac{1}{2}\sum_\alpha U_a V_a \left(\frac{1}{2}\sum_\gamma \langle\alpha-\alpha|\mathcal{V}|\gamma-\gamma\rangle S_\alpha S_\gamma V_c U_c\right) \\ &= \sum_\alpha \left((\tilde{\varepsilon}_a - \lambda)V_a^2 - \frac{1}{2}\Delta_\alpha U_a V_a\right) \end{aligned}$$

$$\boxed{\mathbf{H_{11}} = \sum_\alpha \left((\tilde{\varepsilon}-\lambda)_a \left(U_a^2 - V_a^2\right) + 2\Delta_\alpha U_a V_a\right) a_\alpha^\dagger a_\alpha}$$

$$\boxed{\mathbf{H_{20}} = \sum_\alpha \left((\tilde{\varepsilon}-\lambda)_a U_a V_a - \frac{1}{2}\Delta_\alpha\left(U_a^2 - V_a^2\right)\right) S_a a_a^\dagger a_{-a}^\dagger}$$

$$\mathbf{H_{20} = H_{02}}^{+}$$

$$\boxed{\begin{aligned} \bar{\varepsilon}_\alpha &= \varepsilon_\alpha - \lambda + \Gamma_\alpha \\ \Gamma_\alpha &= \frac{1}{2}\sum_{\alpha\beta} <\alpha\beta\,|V\,|\,\alpha\beta> V_\beta^2 \end{aligned}}$$

Γ is Self Energy Contribution to New Mean Field. It is Usually Small and is Ignored

$$\Delta_a = -\frac{1}{2}\sum_{\beta} < \alpha - \alpha | V | \beta - \beta > S_\alpha S_\beta V_d U_d$$

Step III: We Need to Retain $H_0 + H_{11}$. Equate $H_{20} = H_{02}^{+}$ to Zero. This gives

$$(\overline{\varepsilon_a} - \lambda) U_a V_a = \frac{\Delta_a}{2}(U_a^2 - V_a^2)$$

Put $V_a = Sin\vartheta_a$, $U_a = Cos\vartheta_a$

Use $U_a^2 + V_a^2 = 1$ **To Get**

$$Tan(2\vartheta_a) = \frac{\Delta_a}{(\overline{\varepsilon_a} - \lambda)}$$

$$U_a^2 - V_a^2 = Cos(2\vartheta_a) = \frac{\overline{\varepsilon_a} - \lambda}{E_a}$$

$$E_a = ((\overline{\varepsilon_a} - \lambda)^2 + \Delta_a^2)^{1/2}$$

$$V_a^2 = \frac{1}{2}(1 - \frac{(\overline{\varepsilon}_a - \lambda)}{E_a})$$

We Get (Gap Eq.)

$$\Delta_a = -\frac{1}{2}\sum_{\beta} < \alpha - \alpha | V | \beta - \beta > S_\alpha S_\beta V_d U_d$$

$$= -\frac{1}{4}\sum_{c} < j_a^2 0 | V | j_c^2 0 > \left[\frac{2j_c + 1}{2j_a + 1} \right]^{1/2} \frac{\Delta_c}{E_c}$$

The Lagrange Multiplier λ is Obtained Through the Requirement That

$$\sum_{\alpha} \langle C_\alpha^\dagger C_\alpha \rangle = \sum_{\alpha} V_\alpha^2 = N$$

N is the Nucleon Number (Number Eq.)

These qp or BCS (Gap and Number) are Coupled Highly Non-linear Set of Eqs. → Are to be Solved Self-Consistently

Interpretation of λ:

The Expression $\langle C_\alpha^\dagger C_\beta \rangle = \delta_{\alpha\beta} V_\alpha^2$

→ V_a^2 $(U_a^2 = 1 - V_a^2)$

Occupation (Non-Occupation) Probability

$$V_a^2 = \frac{1}{2}\left[1 - \frac{(\tilde{\varepsilon}_a - \lambda)}{\sqrt{(\tilde{\varepsilon}_a - \lambda)^2 + \Delta_a^2}}\right]$$

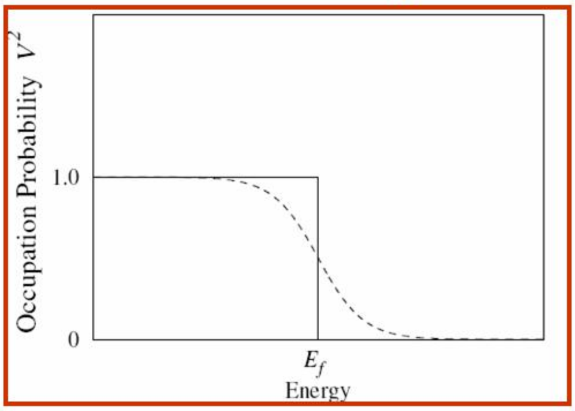
For $\tilde{\varepsilon}_a \gg \lambda$ $\quad V_a^2 \approx 0$

$\tilde{\varepsilon}_a \ll \lambda$ $\quad V_a^2 \approx 1$

AS $\tilde{\varepsilon}$ Approaches λ, V_a^2 Deviates From Unity (zero)

$\tilde{\varepsilon}_a \leq \lambda,\ V_a^2 \geq 0.5;$
$\tilde{\varepsilon}_a \geq \lambda,\ V_a^2 \leq 0.5$ and
$\tilde{\varepsilon}_a = \lambda,\ V_a^2 = 0.5$.

This Gives



Interpretation of Δ

Inserting the Values of Vs (Us), H_{11} becomes

$$H_{11} = \sum_{\alpha} \left[\frac{(\tilde{\varepsilon}_a - \lambda)(\tilde{\varepsilon}_a - \lambda)}{E_a} + 2\Delta_a \frac{\Delta_a}{2} \frac{(\tilde{\varepsilon}_a - \lambda)}{((\tilde{\varepsilon}_a - \lambda) E_a)} \right] a_{\alpha}^{\dagger} a_{\alpha}$$

$$\equiv \sum_{\alpha} E_{\alpha} a_{\alpha}^{\dagger} a_{\alpha}$$

Neglect H_{int} → $H = H_0 + H_{11}$

Zero qp or |BCS> State Satisfies

$$a_{\alpha}|\mathrm{BCS}\rangle = 0$$

Even – Even Nuclei → 0, 2, 4 … qp
Odd-A Nuclei → 1, 3, 5 … qp

qp Energy $E_a = \sqrt{\left((\tilde{\varepsilon}_a - \lambda)^2 + \Delta_a^2\right)} \geq \Delta_a$

Take Δ to be independent of α

For $\tilde{\varepsilon}_a \approx \lambda$ → $E_a \approx \Delta$

The 2qp State Will be at least 2Δ above g.s.

For e – e Nuclei → gap 2Δ Between g.s & first Exc. State → Agree With Expt.

For Odd-A Nuclei: The g.s. → 1qp State Nearest to λ
Energy $E_a \approx \Delta$, As $\tilde{\varepsilon}_a \approx \lambda$
There Exist Other 1 qp Levels With Energy

$$E_{\beta \neq \alpha} = \sqrt{(\tilde{\varepsilon}_b - \lambda)^2 + \Delta^2}$$

$(\tilde{\varepsilon}_b - \lambda)$ Being Small, So several 1 qp States Will Lie Close to Each Other → No Gap Between the g.s. and First Excited State.

Rough estimate of Δ

For N Valence Nucleons, The Energy of |BCS> or g.s. is:

$$E_N = \langle H \rangle = \langle H' \rangle + \lambda \langle \hat{N} \rangle = H_0(N) + \lambda N$$

The g.s Energy for Nuclei With N+(-) one Nucleons:

$$E_{N+1} = \langle H' \rangle + \lambda \langle \hat{N} \rangle \simeq H_0(N) + \lambda(N+1) + \Delta$$
$$E_{N-1} = \langle H' \rangle + \lambda \langle \hat{N} \rangle \simeq H_0(N) + \lambda(N-1) + \Delta$$

Thus $E_{N+1} + E_{N-1} - 2E_N = 2\Delta$

So , For a Given N the Gap Δ Can be Obtained From Odd-Even Mass Difference Its Approximate Value is:

1.5 MeV for Ni isotopes and $N=50$ isotones
1.2 MeV for Sn isotopes and $N=82$ isotones
0.9 MeV for Pb isotopes.

Illustration: Ni - Isotopes

Core: ^{56}Ni → Z=28, N=28

Valence Levels: $1p_{3/2}$, $0f_{5/2}$ and $1p_{1/2}$

Energies: $\tilde{\varepsilon}_{3/2} = \varepsilon_{3/2} = 0.0$, $\tilde{\varepsilon}_{5/2} = \varepsilon_{5/2} = 0.78$
$\tilde{\varepsilon}_{1/2} = \varepsilon_{1/2} = 1.08$ MeV

Interaction: Empirical and Pairing

Table → Results For ^{60}Ni
( ) → Results With Pairing Int.

	λ	Δ	E	V
$1p_{3/2}$	0.064 (0.008)	1.352 (1.444)	1.353 (1.444)	0.724 (0.708)
$0f_{5/2}$		1.249 (1.444)	1.440 (1.637)	0.501 (0.597)
$1p_{1/2}$		1.352 (1.444)	1.691 (1.798)	0.447 (0.578)

Excited States: qp Configuration Mixing → H_{int}

Even – Even Nuclei → 0, 2 ,4 qp

Odd – A Nuclei → 1, 3 may be 5 qp

Advantages: Up to $\nu = 4$ (5) Space

Drawback: Non-conservation of N
→ Spurious States

Remedy → Number Projection
Broken Pair approximation (BPA)

BROKEN PAIR APPROXIMATION (BPA)

The SM gs State for 2 Identical Nucleons

$$s^{+}|0\rangle = \sum_{a} \frac{\hat{a}}{2} x_a A_{00}^{+}(aa)|0\rangle$$

$$\hat{a} = (2j_a + 1)^{1/2}$$

$$A_{JM}^{+}(ab) = \left[C_a^{+} \otimes C_b^{+}\right]_{JM}$$

gs - Φ_0 : P Pairs of Identical Nucleons

$$\Phi_0 \Rightarrow (s^{+})^{P}|0\rangle = \left(\sum_{a} \frac{\hat{a}}{2} x_a A_{00}^{+}(aa)\right)^{P}|0\rangle$$

$$\Rightarrow \tau_{+}^{P}|0\rangle = \frac{1}{P!}\left(\prod_{a} u_a \frac{\hat{a}^2}{2}\right)(s^{+})^{P}|0\rangle$$

$$x_a = v_a / u_a \; ; \; u_a^2 + v_a^2 = 1$$

The gs Parameters x (v or u) are obtained by:

$$\delta\left(\langle\Phi_0 \mid H \mid \Phi_0\rangle / \langle\Phi_0 \mid \Phi_0\rangle\right) = 0$$

Φ_0 :

Special Seniority 0 State

> 98% of ESM gs

2P – Particle Component of BCS State

If v/u → v/u of BCS

<u>Excited States: BPA Basis States</u>

$$\tau^{+} \rightarrow A^{+}_{JM}(ab)$$

1 BPA Basis:

$$|\Phi_{JM}(ab)\rangle \Rightarrow A^{+}_{JM}(ab)\tau_{+}^{P-1}|0\rangle$$

Special Seniority 2 State

Diagonalise → Eigenvalues, Eigenvectors

^{60}Ni					
J^π	Expt.	ESM	$\nu \leq 2$	1bp	BCS2qp
0^+	0	0 (99.8)	0	0	0
	2.29	2.323 (95.8)	2.414	2.455	1.933
		3.268 (86.7)	3.415	3.645	2.977
2^+	1.33	1.421 (99.8)	1.418	1.421	0.946
	2.16	2.171 (76.6)	2.425	2.533	2.068
		2.578	2.866	3.481	2.994
3^+		2.758 (55.5)	3.439	3.506	2.991
		3.370 (30.0)	3.872	3.976	3.509
4^+	2.50	2.205 (91.9)	2.296	2.299	1.863
		2.798 (23.9)	3.497	3.565	3.205

^{64}Ni					
J^π	Expt.	ESM	$\nu \leq 2$	1bp	BCS2qp
0^+	0	0 (99.8)	0	0	0
	2.27	2.156 (98.8)	2.180	2.188	1.720
		3.559 (81.2)	3.659	3.768	3.417
2^+	1.34	1.560 (99.7)	1.556	1.559	1.110
	2.89	2.371 (78.7)	2.479	2.492	2.084
		2.597 (64.9)	3.277	3.308	2.753
3^+		3.069 (36.6)	3.445	3.454	2.946
		3.477 (72.7)	3.766	3.804	3.340
4^+	2.61	2.257 (96.3)	2.292	2.307	1.835
		2.725 (34.1)	3.352	3.396	2.861

BE(2) Transition and Quadrupole Moments of Ni Isotopes

	$B(E2, 0_1^+ \rightarrow 2_1^+);\ e^2\,fm^4$			$Q(2_1^+); e\,fm^2$		
	ESM	**1bp**	**BCS2qp**	**ESM**	**1bp**	**BCS2qp**
^{58}Ni	**233**	**233**	**183**	**-14**	**-14**	**-8**
^{60}Ni	**386**	**390**	**303**	**-2**	**-5**	**-3**
^{62}Ni	**458**	**474**	**383**	**+2**	**+1**	**+1**
^{64}Ni	**410**	**431**	**343**	**+6**	**+8**	**+5**

1 (2) BPA: Good Approximation to Seniority 2 (4) Shell Model

Nuclear Theory:

Hartree Fock (HF) Mean Field Theory

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Hartree Fock (HF) Mean Field Theory

We have

$$H \mid \Psi_J\rangle = E_J \mid \Psi_J\rangle \, ,$$
$$J^2 \mid \Psi_J\rangle = J(J+1) \mid \Psi_J\rangle \, .$$

HF Does not deal directly with $\mid \Psi_J >$ instead determines by minimizing E_J. In independent particle picture, Many Body HF Wave Function Φ is

$$\Phi = \mathcal{A}\prod_i^A \phi_i \; ; \quad H \rightarrow H_{eff} = \sum_i^A h(i) \, , \quad h \mid \phi_i\rangle = \varepsilon_i \mid \phi_i\rangle$$

Φ Is obtained through $\delta\langle\Phi \mid H \mid \Phi\rangle = 0 \, .$

The HF s.p. states ϕ_i is expanded in terms of s.p. basis states $\mid \alpha >\equiv\mid nljm\tau_3 >$

$$\mid \phi_i >\equiv\mid i >= \sum_\alpha x_\alpha^i \mid \alpha >,$$

The Real Expansion Coeff. $x_\alpha^i =< \alpha \mid i >$ are Variational Parameters

These Orthonormal Sets of s.p. States Satisfy:

$$< i \mid i' >= \delta_{ii'} = \sum_{\alpha} x_{\alpha}^{i*} x_{\alpha}^{i'}$$

$$< \alpha \mid \beta >= \delta_{\alpha\beta} = \sum_{i} x_{\alpha}^{i*} x_{\beta}^{i}$$

One Uses

$$\delta[\langle \Phi \mid H \mid \Phi \rangle - \sum_{i} \varepsilon_i \sum_{\alpha} x_{\alpha}^{i*} x_{\alpha}^{i}] = 0$$

Where ε_i are Lagrange Multipliers

The Expectation Value

$$\begin{aligned} \langle \Phi \mid H \mid \Phi \rangle &= \sum_{i}^{occ} < i \mid t \mid i > + \frac{1}{2} \sum_{ii'}^{occ} < ii' \mid v \mid ii' > \\ &= \sum_{i}^{occ} \sum_{\alpha} e_{\alpha} x_{\alpha}^{i*} x_{\alpha}^{i} + \frac{1}{2} \sum_{ii'}^{occ} \sum_{\alpha\beta\gamma\delta} x_{\alpha}^{i*} x_{\beta}^{i'*} < \alpha\beta \mid v \mid \gamma\delta > x_{\gamma}^{i} x_{\delta}^{i'} \end{aligned}$$

e_{α} is Eigen Energy of $\mid \alpha >$

'occ' Stands for Sum Over Lowest A Occupied States

With

$$\rho_{\delta\beta} = \sum_{i'}^{occ} x_{\beta}^{i'*} x_{\delta}^{i'}$$

And Manipulation of Summation Indices . One Gets

$$e_{\mu} x_{\mu}^{k} + \sum_{\beta\gamma\delta} < \mu\beta \mid v \mid \gamma\delta > \rho_{\delta\beta} x_{\gamma}^{k} = \varepsilon_{k} x_{\mu}^{k}$$

Define One Body HF Potential

$$\Gamma_{\mu\gamma} = \sum_{\beta\delta} < \mu\beta \mid v \mid \gamma\delta > \rho_{\delta\beta}$$

The HF Equation Now Becomes

$$\sum_{\gamma} [(e_{\gamma} - \varepsilon_{k})\delta_{\mu\gamma} + \Gamma_{\mu\gamma}] x_{\gamma}^{k} = 0$$

This is an Eigen Value Equation With One Body HF Hamiltonian h_{HF}

$$< \mu \mid h_{HF} \mid \gamma >= e_{\gamma} \delta\mu\gamma + < \mu \mid \Gamma \mid \gamma >$$

The Diagonalization of this HF Matrix Yields HF s.p. Energies ε **and Wave Functions (Through Vectors X) Defining** ϕ

The HF Matrix Requires Γ Which in Turn Requires ρ .
It has to be Solved Iteratively. One starts With Initial Guess $\rho^{(i)}$ (of Nilsson Hamiltonian) and calculates New $\rho^{(f)}$, Which Forms the New Input.
This Procedure is Continued Until the Desired Convergence is Achieved.

HF Total Energy

$$
\begin{aligned}
E_{HF} &= \sum_{i}^{occ} < i \mid t \mid i > + \frac{1}{2} \sum_{ii'}^{occ} < ii' \mid v \mid ii' > \\
&= \sum_{i} [\varepsilon_i - \frac{1}{2} < i \mid \Gamma \mid i >] \\
&= \sum_{i} \varepsilon_i + |\delta E|
\end{aligned}
$$

δE is Positive for Attractive Potential. Thus HF Overestimates the Total Energy.

For Spherical Nuclei the Summation in the Expansion is Over Nodal Quantum Number n

HF Wave function $|\Phi>$ is not an Eigen State of Total Angular Momentum J^2 . The State with Good J and Projection M (on the Lab. Fixed z-axis is written as

$$\boxed{|\Psi_{JM}>= n_J P^J_{MK} \mid \Phi\rangle}$$

n_J is normalization and Projector P is

$$\boxed{P^J_{MK} = \frac{2J+1}{8\pi^2}\int d\Omega D^{J*}_{MK}(\Omega)R(\Omega)}$$

D is the Well Known Rotation Matrix

The Energy becomes (Choose M=K)

$$\begin{aligned}
E_{JK} &= < JK|H|JK > / < JK|JK > \\
&= \frac{< \Phi|P^{J\dagger}_{KK} H P^J_{KK}|\Phi >}{< \Phi|P^{J\dagger}_{KK} P^J_{KK}|\Phi >} \\
&= \frac{< \Phi|H P^J_{KK}|\Phi >}{< \Phi|P^J_{KK}|\Phi >}
\end{aligned}$$

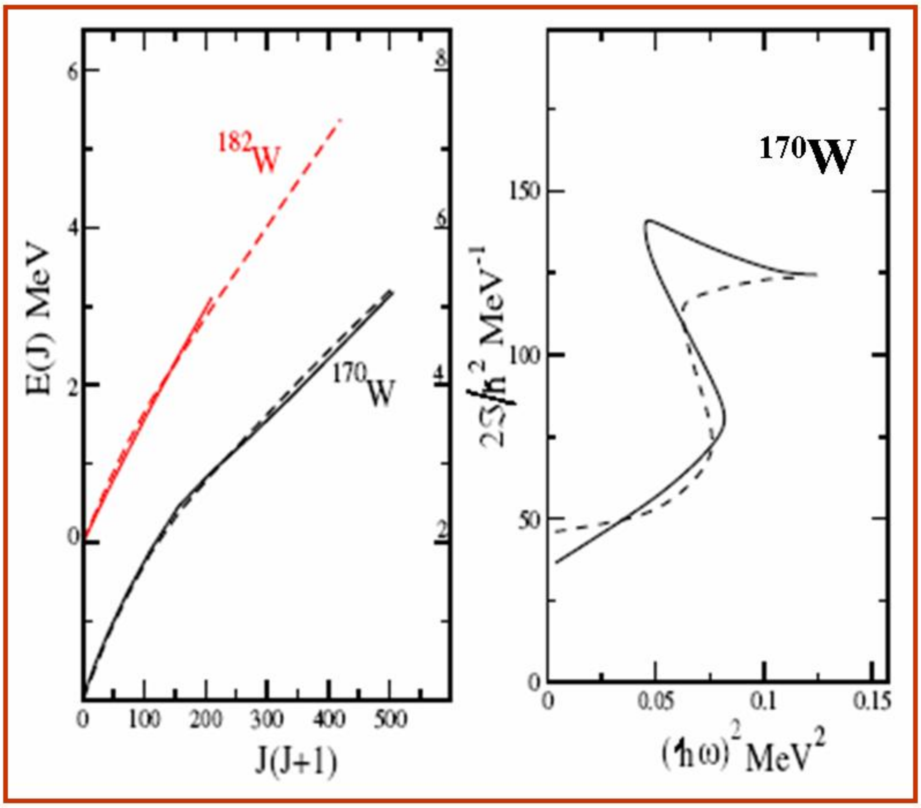
Illustration: **W Isotopes (Z = 74)**

Step 1: Core: Z = 40, N = 70
Valance Levels: $2\hbar\omega$, Both for p & n

Step 2: HF Basis States

Step 3: Interaction – Pairing + Q.Q

Step 4: Diagonalisation of HF Matrices



Backbending

$$\boxed{E_I = \frac{\hbar^2}{2\Im_I} I(I+1)}$$

$$\frac{\hbar^2}{2\Im_I} = \frac{\partial E_I}{\partial (I(I+1))} = \boxed{\frac{E_I - E_{I-2}}{2(2I-1)}}$$

$$\Im_I \omega_I = \hbar\sqrt{(I(I+1))} = \hbar \bar{I} \ ;$$

$$\hbar\omega_I = \frac{\partial E_I}{\partial \bar{I}} \approx \boxed{\frac{E_I - E_{I-2}}{\left(2 - \frac{1}{2I}\right)}}$$

Hartree-Fock-Bogoliubov (HFB) Theory

The Quasi-particle Operators b are Defined in terms of the Basis Space Operators c

$$\boxed{\begin{aligned} b_i^\dagger &= \sum_\alpha (A_{\alpha i} A_\alpha^\dagger + B_{\alpha i} c_\alpha) \\ b_i &= \sum_\alpha (B_{\alpha i}^* c_\alpha^\dagger + A_{\alpha i}^* c_\alpha) \end{aligned}}$$

The Inverse Transformation Reads

$$\boxed{\begin{aligned} c_\alpha^\dagger &= \sum_i (A_{\alpha i}^* b_i^\dagger + B_{\alpha i} b_i) \\ c_\alpha &= \sum_i (B_{\alpha i}^* b_i^\dagger + A_{\alpha i} b_i) \end{aligned}}$$

The HFB g.s. is Defined Through

$$b_i \mid HFB\rangle = 0 \qquad |HFB >= \prod_i b_i|0>$$

$|0>$ **Being the Real Vacuum**

|HFB > is not an Eigen Function of the Particle Number Operator

$$\hat{N} = \sum_\alpha c^\dagger_\alpha c_\alpha$$

This can be Rectified by Introducing Lagrange Multiplier and Working with the New Hamiltonian

$$\boxed{H' \;=\; H - \lambda\hat{N}}$$

$$= \sum_\alpha (e_\alpha - \lambda) c^\dagger_\alpha c_\alpha + \frac{1}{4}\sum_{\alpha\beta\gamma\delta} < \alpha\beta|v|\gamma\delta > c^\dagger_\alpha c^\dagger_\beta c_\delta c_\gamma$$

In the Independent Quasiparticle Picture, we have

$$[H', b^\dagger_i] = E_i b^\dagger_i$$
$$[H', b_i] = -E_i b_i$$

Evaluating These Commutators

$$[H', b_i^\dagger] \;=$$

$$\sum_{\alpha\gamma}\{[((e_\alpha-\lambda)\delta_{\alpha\gamma}+\Gamma_{\alpha\gamma})A_{\gamma i}+\Delta_{\alpha\gamma}B_{\gamma i}]c_\alpha^\dagger$$
$$-[((e_\alpha-\lambda)\delta_{\alpha\gamma}+\Gamma^*_{\alpha\Gamma})B_{\gamma i}+\Delta^*_{\alpha\gamma}A_{\gamma i}]c_\alpha\}$$

HF Potential Γ and Pairing Potential Δ are:

$$\Gamma_{\alpha\gamma}=\sum_{\beta\delta}<\alpha\beta|v|\gamma\delta>\rho_{\delta\beta}$$

$$\Delta_{\alpha\beta}=\frac{1}{2}\sum_{\gamma\delta}<\alpha\beta|v|\gamma\delta>\kappa_{\delta\gamma}$$

One Body HF Density ρ and Pairing Matrix κ are;

$$\begin{aligned}\rho_{\delta\beta} &= <HFB|c_\beta^\dagger c_\delta|HFB> \\ &= (B^*\tilde{B})_{\delta\beta} \\ \kappa_{\delta\gamma} &= <HFB|c_\delta c_\gamma|HFB> \\ &= (AB^\dagger)_{\delta\gamma}\ ,\end{aligned}$$

Here $\tilde{B}$ Stands for Transpose of B

The Commutator Should be Equated To

$$E_i\sum_\alpha(A_{\alpha i}c_\alpha^\dagger+B_{\alpha i}c_\alpha)$$

For Each Value of α

This Equality Leads To HFB Equations:

$$\boxed{\begin{pmatrix} \bar{\Gamma} & \Delta \\ -\Delta^* & -\bar{\Gamma}^* \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = E \begin{pmatrix} A \\ B \end{pmatrix}}$$

With : $\bar{\Gamma}_{\alpha\gamma} = (e_\alpha - \lambda)\delta_{\alpha\gamma} + \Gamma_{\alpha\gamma}$

The Total HFB Energy can be Calculated as:

$$E_{HFB} =$$

$$\boxed{\sum_{\alpha\gamma}(e_\alpha \delta_{\alpha\gamma} + \frac{1}{2}\Gamma_{\alpha\gamma})\rho_{\gamma\alpha} + \frac{1}{2}\sum_{\alpha\beta}\Delta_{\alpha\beta}\kappa^*_{\beta\alpha}}$$

- **HFB Equations Have to be Solved Iteratively.**
- **Initial Guess for Bogoliubov Transformation Coefficients A and B →**
- **Through Hartree – Nilsson OR Nilsson – BCS Calculations Such That**

$$A_{\alpha i} = x^i_\alpha U_i \quad \textbf{and} \quad B_{\alpha i} = x^i_\alpha V_i$$

Where x are the HF Expansion Coeff. and V, U are the Standard BCS Occupation Parameters.

Nuclear Theory:

Density Dependent Hartree Fock (DDHF)

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Density Dependent Hartree-Fock (DDHF)

The 3-body Zero-range Density Dependent Skyrme Int.

$$\begin{aligned}
V_{Sk} = \; & t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}_i - \mathbf{r}_j) \\
& + \tfrac{1}{2} t_1(1 + x_1 P_\sigma)\{\mathbf{p}_{12}^2 \delta(\mathbf{r}_i - \mathbf{r}_j) + \delta(\mathbf{r}_i - \mathbf{r}_j)\mathbf{p}_{12}^2\} \\
& + t_2(1 + x_2 P_\sigma)\mathbf{p}_{12} \cdot \delta(\mathbf{r}_i - \mathbf{r}_j)\mathbf{p}_{12} \\
& + \tfrac{1}{6} t_3(1 + x_3 P_\sigma)\rho^\alpha(\bar{\mathbf{r}})\delta(\mathbf{r}_i - \mathbf{r}_j) \\
& + iW_0(\sigma_\mathbf{i} + \sigma_\mathbf{j}) \cdot \mathbf{p}_{12} \times \delta(\mathbf{r}_\mathbf{i} - \mathbf{r}_\mathbf{j})\mathbf{p}_{12}
\end{aligned}$$

$\mathrm{p}_{12} = \mathrm{p}_\mathrm{i} - \mathrm{p}_\mathrm{j}$ is the relative momentum

P_σ → **Spin Exchange Operator** $\sigma_\mathbf{i} \leftrightarrow \sigma_\mathbf{j}$

σ → **Spin Pauli Matrices**

$\bar{\mathrm{r}} = \frac{1}{2}(\mathrm{r}_\mathbf{i} + \mathrm{r}_\mathbf{j})$

t_3 → **3 – Body Contact Interaction Term**

W_o → **Spin – Orbit Term**

Advantage: Expectation value of E wrt Slater Determinant (Mean Field) WF
→ In Analytical Form

The Total Energy Functional E → Skyrme, Coulomb, Pairing Parts + Spurious CM Motion

$$E = E_{Sk} + E_{Coul} + E_{Pair} - E_{c.m.}$$

Spherical Nuclei → s.p. WF

$$\varphi_\beta(\mathbf{r}) = \frac{R_\beta(r)}{r}\mathcal{Y}_{j_\beta l_\beta m_\beta}(\theta,\phi)$$

and

$$\mathcal{Y}_{j_\beta l_\beta m_\beta}(= [Y_{l_\beta} \times \chi_{1/2}]_{j_\beta m_\beta})$$

Are Spinor Spherical Harmonics

$$E_{Sk} =$$

$$\begin{aligned}
&4\pi \int dr r^2 \{ \frac{\hbar^2}{2m}\tau + \tfrac{1}{2}t_0(1+\tfrac{1}{2}x_0)\rho^2 - \tfrac{1}{2}t_0(\tfrac{1}{2}+x_0)\sum_q \rho_q^2 \\
&+\tfrac{1}{4}[t_1(1+\tfrac{1}{2}x_1) + t_2(1+\tfrac{1}{2}x_2)]\rho\tau \\
&-\tfrac{1}{4}[t_1(\tfrac{1}{2}+x_1) - t_2(\tfrac{1}{2}+x_2)]\sum_q \rho_q\tau_q \\
&+\tfrac{1}{16}[3t_1(1+\tfrac{1}{2}x_1) + t_2(1+\tfrac{1}{2}x_2)]\sum_q \rho_q\nabla^2\rho_q \\
&-\tfrac{1}{16}[3t_1(1+\tfrac{1}{2}x_1) - t_2(1+\tfrac{1}{2}x_2)]\rho\nabla^2\rho \\
&-\tfrac{1}{2}W_0[\rho\nabla\cdot\mathbf{J} + \sum_q \rho_q\nabla\cdot\mathbf{J}_q]\}
\end{aligned}$$

$$\nabla^2 = \partial_r^2 + \tfrac{2}{r}\partial_r \quad , \quad \partial_r \rightarrow \frac{\partial}{\partial r}$$

The Spherical Densities and Currents are

$$\rho_q(r) = \sum_{n_\beta j_\beta l_\beta} n_\beta^q \frac{2j_\beta + 1}{4\pi} \left(\frac{R_\beta}{r}\right)^2$$

$$\tau_q(r) = \sum_{n_\beta j_\beta l_\beta} n_\beta^q \frac{2j_\beta + 1}{4\pi} [\left(\partial_r \frac{R_\beta}{r}\right)^2 + \frac{l(l+1)}{r^2} \left(\frac{R_\beta}{r}\right)^2]$$

$$\nabla \mathbf{J}_q(r) = (\partial_r + \frac{2}{r}) J_q(r)$$

$$J_q(\mathbf{r}) = \sum_{n_\beta j_\beta l_\beta} n_\beta^q \frac{2j_\beta + 1}{4\pi} (j_\beta(j_\beta + 1) - l_\beta(l_\beta + 1) - \tfrac{3}{4}) \frac{2}{r} \left(\frac{R_\beta}{r}\right)^2$$

The Occupation Probabilities n_β^q → **Independent of** m_β . q **Runs over n and p**

$$\rho = \rho_p + \rho_n \qquad \tau = \tau_p + \tau_n$$

$$\nabla \cdot \mathbf{J} = \nabla \cdot \mathbf{J}_p + \nabla \cdot \mathbf{J}_n$$

The Variation of E wrt R_β → **HF Equations**

$$h_q R_\beta = \epsilon_\beta R_\beta$$

The Mean Field Hamiltonian

$$h_q = \partial_r \mathcal{B}_q \partial_r + U_q + U_{ls,q} \mathbf{l}\sigma$$

$$\mathcal{B}_q = \frac{\hbar^2}{2m_q} + \tfrac{1}{8}[t_1(1+\tfrac{1}{2}x_1) + t_2(1+\tfrac{1}{2}x_2)]\rho - \tfrac{1}{8}[t_1(\tfrac{1}{2}+x_1) - t_2(\tfrac{1}{2}+x_2)]\rho_q$$

$$U_{ls,q} = \tfrac{1}{4}W_o(\rho+\rho_q) + \tfrac{1}{8}(t_1 - t_2)J_q - \tfrac{1}{8}(x_1 t_1 + x_2 t_2)J$$

$$\begin{aligned}
U_q \;=\;& t_0(1+\tfrac{1}{2}x_0)\rho - t_0(\tfrac{1}{2}+x_0)\rho_q \\
&+ \tfrac{1}{12}t_3\rho^\alpha[(2+\alpha)(1+\tfrac{1}{2}x_3)\rho \\
&- 2(\tfrac{1}{2}+x_3)\rho_q - \alpha(\tfrac{1}{2}+x_3)\frac{\rho_p^2+\rho_n^2}{\rho}] \\
&+ \tfrac{1}{4}[t_1(1+\tfrac{1}{2}x_1) + t_2(1+\tfrac{1}{2}x_2)]\tau \\
&- \tfrac{1}{4}[t_1(\tfrac{1}{2}+x_1) - t_2(\tfrac{1}{2}+x_2)]\tau_q \\
&- \tfrac{1}{8}[3t_1(1+\tfrac{1}{2}x_1) - t_2(1+\tfrac{1}{2}x_2)]\nabla^2\rho \\
&+ \tfrac{1}{8}[3t_1(\tfrac{1}{2}+x_1) + t_2(\tfrac{1}{2}+x_2)]\nabla^2\rho_q \\
&- \tfrac{1}{2}W_o(\nabla\cdot\mathbf{J} + \nabla\cdot\mathbf{J}_q) + \delta_{q1/2}\, U_{Coul}
\end{aligned}$$

U_{Coul} → Direct + Exchange Terms.

Direct Term is Trivial, Exchange Term is Taken As

$$U_{coul}(exchange) = \left(-\frac{3}{\pi}\right)^{1/3} 4\pi \int drr^2 \rho_p^{4/3}$$

$$E_{c.m.} = \frac{\langle P^2_{c.m.} \rangle}{2AM}$$

For H.O. s.p. Basis $E_{c.m} = \frac{3}{4}\hbar\omega$

BCS Type Occupation Probabilities are obtained through Gap and Number Equations. Then

$$E_{Pair} = -\sum_q \mathcal{G}_q \left[\sum_{\beta\in q} \sqrt{n_\beta^q (1-n_\beta^q)}\right]^2$$

Illustration:

Skyrme Int. Parameters

t_o = -1057, t_1 = 235.9, t_2 = -100, t_3 = 14463.5,

W_o = 120, x_o = 0.56 , $x_1 = x_2$=0.0, x_3 = 1.0

$\alpha = 1$

Gap Parameter:

$$\Delta_q = 11.2 MeV/\sqrt{A} \qquad A = A_p + A_n$$

$$r_c = \sqrt{r_p^2 + 0.64} \quad , \quad r_m = \left[\left(Z r_p^2 + N r_n^2\right)/\left(Z + N\right)\right]^{1/2}$$

	${}^{16}_{8}O$	${}^{40}_{20}Ca$	${}^{48}_{20}Ca$	${}^{90}_{40}Zr$	${}^{208}_{82}Pb$
BE/A	-8.22 (-7.98)	-8.64 (-8.55)	-8.93 (-8.67)	-8.81 (-8.71)	-7.89 (-7.87)
r_c	2.68 (2.73)	3.41 (3.49)	3.46 (3.48)	4.22 (4.27)	5.44 (5.50)
r_m	2.55	3.29	3.43	4.17	5.45

Calculations Reproduce Expt. Well.

Relativistic Mean Field (RMF) Approach

Non-Relativistic Analysis Indicates That

$$\mathrm{U} \sim 50 \text{ MeV} \ll mc^2 \ (\sim 1000 \text{ MeV}).$$

Question: Why Relativistic Formulation?

Reasons::
**** Nuclear l.s. splitting is 30 times larger than that of the Atomic Case and is of Opposite Sign.**

** Convential Optical Model (OPM) Fails to Describe Spin Observables in the Intermediate Energy Polarized Proton – Nucleus (p-A) Scattering.

Dirac Phenomenology: Use of Dirac Eq. With Lorentz Scalar and Vector Potentials in Place of Schrodinger Eq. Remarkably Successful. Scalar Pot. U and Vector Pot. V are of the Order of -400 and 350 MeV – Their Diff. Yields Required 50 MeV. This Success Triggered the Application of RMF to Nuclear Structure.

RMF-Formulation:

Nucleon Interacts With the
Meson (σ,ω and ρ) and e.m. (γ) Fields.
The Lagrangian : Free Baryon (L_B) , Mesons (L_M) and the INT. (L_{BM}) Terms.

$$L_B = \bar{\psi}_i \left(\iota \gamma^\mu \partial_\mu - M \right) \psi_i$$

$$\begin{aligned} L_M = \; & \frac{1}{2}\, \partial^\mu \sigma \, \partial_\mu \sigma - U(\sigma) \\ & -\frac{1}{4}\, \mathbf{\Omega}^{\mu\nu}\, \mathbf{\Omega}_{\mu\nu} + \frac{1}{2}\, m_\omega^2\, \omega^\mu \omega_\mu \\ & -\frac{1}{4}\, \vec{\mathbf{R}}^{\mu\nu}\, \vec{\mathbf{R}}_{\mu\nu} + \frac{1}{2}\, m_\rho^2\, \vec{\rho}^{\,\mu}\, \vec{\rho}_\mu - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \end{aligned}$$

$$L_{MB} = \begin{aligned} & -g_\sigma\ \bar{\psi}_i \psi_i\ \sigma \\ & -g_\omega\ \bar{\psi}_i \gamma^\mu \psi_i\ \omega_\mu \\ & -g_\rho\ \bar{\psi}_i \gamma^\mu \vec{\tau} \psi_i\ \vec{\rho}_\mu \\ & -e\ \bar{\psi}_i \gamma^\mu \frac{(1+\tau_3)}{2} \psi_i\ A_\mu \end{aligned}$$

With $$U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4$$

The Field Tensors

$$\begin{aligned} \Omega^{\mu\nu} &= \partial^\mu \omega^\nu - \partial^\nu \omega^\mu \\ \vec{\mathrm{R}}^{\mu\nu} &= \partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu - g_\rho (\vec{\rho}^\mu \times \vec{\rho}^\nu) \\ \mathrm{F}^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu \end{aligned}$$

The Classical Variation Principle Gives the Eqs. Of Motion. Replacing the Fields By Their Expectation Values →
Dirac Eq. With Pot. Terms for Nucleons and KG Type Eqs. With Sources For Meson and the Photon

Non Linear Walecka Model

Relativistic Mean Field: RMF

Ann. Phys. 198 (1990) 132
Gambhir, Ring and Thimet

Nucleons Interact With Meson Fields

σ, ω, ρ and γ

Lagrangian { Masses: M, m_σ, m_ω, m_ρ
Couplings: g_σ, g_ω, g_ρ
g_2, g_3

$\Downarrow$ Classical Variation Principle

Equations of Motion

Approx: Field Operators $\Rightarrow$ C – Numbers

$\Downarrow$

Dirac Equation: Nucleons
K. G. Equation: Mesons & Photon

For: Static Case + Charge Conserv + Time Reversal

$\Downarrow$

Dirac Eqn: $\Rightarrow$ Scalar + Time like vector
$M^* = M + \sigma g_\sigma$; ω^o, ρ^{oo}, A^o

K. G. Eqn: $\Rightarrow$ Sources: Baryon Densities and Currents

Closed Set of Equations (RMF Equations)

$\Longrightarrow$ **To be solved Self – Consistently**

The Dirac Eq.:

$$\left(-\iota\alpha\cdot\nabla+\beta(M\ +\ g_\sigma\sigma)+g_\omega\omega^o\right.$$
$$\left.+\,g_\rho\tau_3\rho_3^o+e\frac{1\ +\ \tau_3}{2}A^o\right)\psi_i\ =\ \epsilon_i\ \psi_i.$$

The KG Eqs.:

$$\begin{aligned}
\left\{-\nabla^2\ +\ m_\sigma^2\right\}\sigma &= -g_\sigma\rho_s\ -\ g_2\sigma^2\ -\ g_3\sigma^3\\
\left\{-\nabla^2\ +\ m_\omega^2\right\}\omega^o &= g_\omega\rho_v\\
\left\{-\nabla^2\ +\ m_\rho^2\right\}\rho_3^o &= g_\rho\rho_3\\
-\nabla^2 A^o &= e\rho_c
\end{aligned}$$

$m_\sigma\ (g_\sigma)$, $m_\omega\ (g_\omega)$, $m_\rho\ (g_\rho)$ **are Meson Masses (Coupling Constants)**
g2 (g3) : Coupling Constants of Cubic (Quartic) Non-Linear Terms

Currents and Densities are:

$$\begin{aligned}
\rho_s &= \sum_i n_i\bar{\psi}_i\psi_i\\
\rho_v &= \sum_i n_i\psi_i^\dagger\psi_i\\
\rho_3 &= \sum_i n_i\psi_i^\dagger\tau_3\psi_i\\
\rho_c &= \sum_i n_i\psi_i^\dagger\left(\frac{1+\tau_3}{2}\right)\psi_i
\end{aligned}$$

Pairing: **Important**

- **Constant Gap Approx.**

Δ_n (Δ_p) **Fixed** $\Rightarrow$

Simple BCS Type

- **Self Consistent:**

Bogoliubov Transform $\Rightarrow$

Relativistic Hartree Bogoliubov (RHB) Equations

RHB Equations:

$$\begin{pmatrix} h_D - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -h_D^* + \lambda \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix}_k = E_k \begin{pmatrix} U \\ V \end{pmatrix}_k$$

h_D: **Dirac Hamiltonian**

$\hat{\Delta}$: **Pairing Field**

U_k,V_k : **Dirac Super-Spinors**

Dirac Hamiltonian:

$$\mathbf{h_D} = -\iota\alpha\cdot\nabla + \beta\left(\mathbf{M} + \mathbf{g}_\sigma\sigma\right)$$
$$+ g_\omega\omega^o + g_\rho\tau_3\rho_3^o + e\frac{1 - \tau_3}{2}A^o$$

$$\int\left(U_k^\dagger U_{k'} + V_k^\dagger V_{k'}\right) = \delta_{kk'}$$

Kernel of Pairing Field $\hat{\Delta}$ is:

$$\triangle_{ab}\left(\mathbf{r},\mathbf{r}'\right) = \frac{1}{2}\sum_{c,d} V^{pp}_{abcd}\left(\mathbf{r},\mathbf{r}'\right)\kappa_{cd}\left(\mathbf{r},\mathbf{r}'\right)$$

κ **: Pairing Tensor** $\rightarrow$

$$\kappa_{cd}\left(\mathbf{r},\mathbf{r}'\right) = \sum_{E_k>0} U^*_{ck}\left(\mathbf{r}\right) V_{dk}\left(\mathbf{r}'\right)$$

V^{pp}**: Interaction in** pp **channel**

Constant Gap Approximation:

$\hat{\Delta}$ Diagonal

RHB $\rightarrow$ RMF with FROZEN GAP:

Occupancies: given by BCS equation:

$$\mathrm{n_k} = v_k^2 = \frac{1}{2}\left[1 - \frac{\epsilon_k - \lambda}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}\right]$$

$\lambda \rightarrow$ Lagrange Multiplier
$\Delta \rightarrow$ Pairing Gap
$\epsilon_k \rightarrow$ Single Particle Energies

RMF / RHB Calculations

Input:

Lagrangian Parameter Set (NL3)

Δ / Pairing Interaction

Δ: Odd – Even Mass Difference

OR

Determine so as to Reproduce

RHB Proton / Neutron Pairing Energies
With Gogny D1S Interaction

RMF / RHB Calculations

Output:

Dirac Spinors / Mesonic Fields

Single Particle Energies

Binding Energies

Densities

Radii

NL3 Parameter Set

Masses	m	939	m_σ	508.194
(MeV)	m_ω	782.501	m_ρ	763.0
Coupling	g_σ	10.217	g_ω	12.868
Constants	g_ρ	4.474	g_2	-10.431
	g_3	-28.885		(fm^{-1})

•**Zero Range Density Dependent:**

$$V(\mathbf{r_1},\mathbf{r_2}) = \frac{1}{4}V_o\delta(\mathbf{r_1}-\mathbf{r_2})(1-\sigma_1\sigma_2)\left(1-\frac{\rho(r)}{\rho_o}\right)$$

ρ_o =**0.152** fm^{-3}

V_o $\approx$**-700 MeV-**fm^{-3}

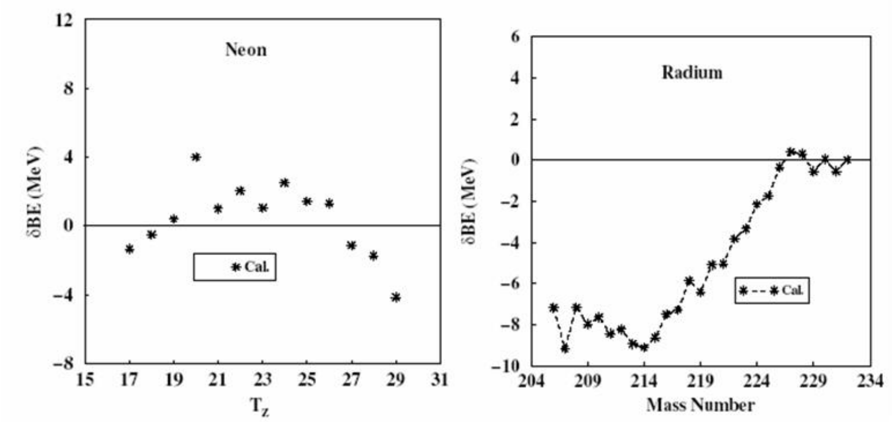
$\mathbf{V^{pp}}(\mathbf{r},\mathbf{r}')$:**Non Relativistic:**

•**Gogny D1S:**

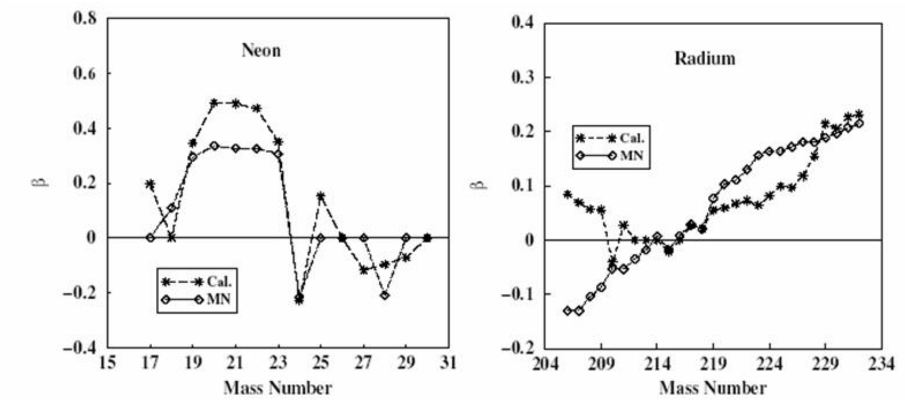
$$V(\mathbf{r_1},\mathbf{r_2}) = \sum_{i=1,2} e^{-\{(\mathbf{r_1}-\mathbf{r_2})/\mu_i\}^2}(W_i+B_iP^\sigma - H_iP^\tau - M_iP^\sigma P^\tau)$$

D1S Parameters:

Parameter	$i = 1$	$i = 2$
μ_i	0.7	1.2
W_i	-1720.30	103.64
B_i	1300.00	-163.48
H_i	-1813.53	162.81
M_i	1397.60	-223.93

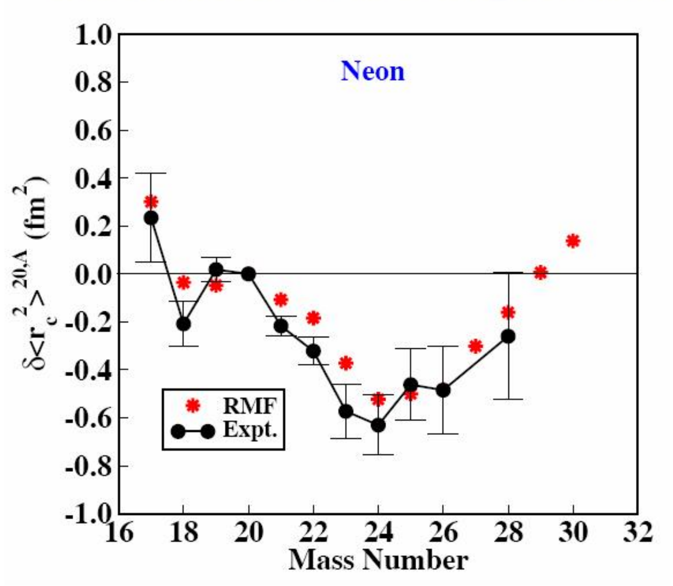


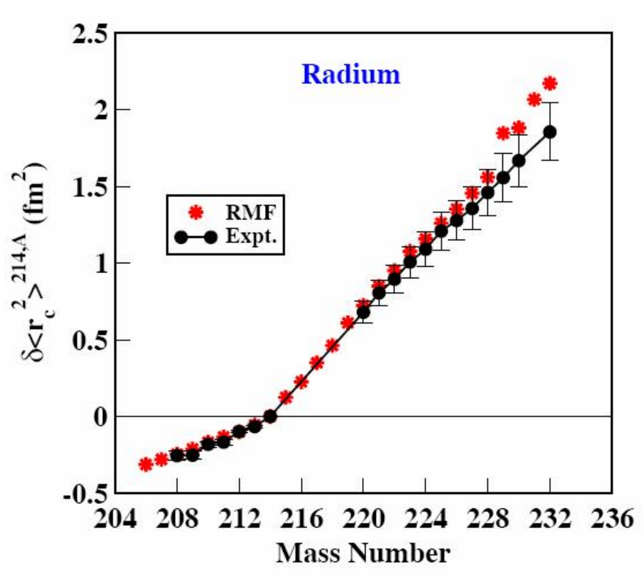
Neon
δBE (MeV)
Cal.
T_Z
Radium
Cal.
Mass Number



Neon
β
Cal.
MN
Mass Number
Radium
Cal.
MN
Mass Number

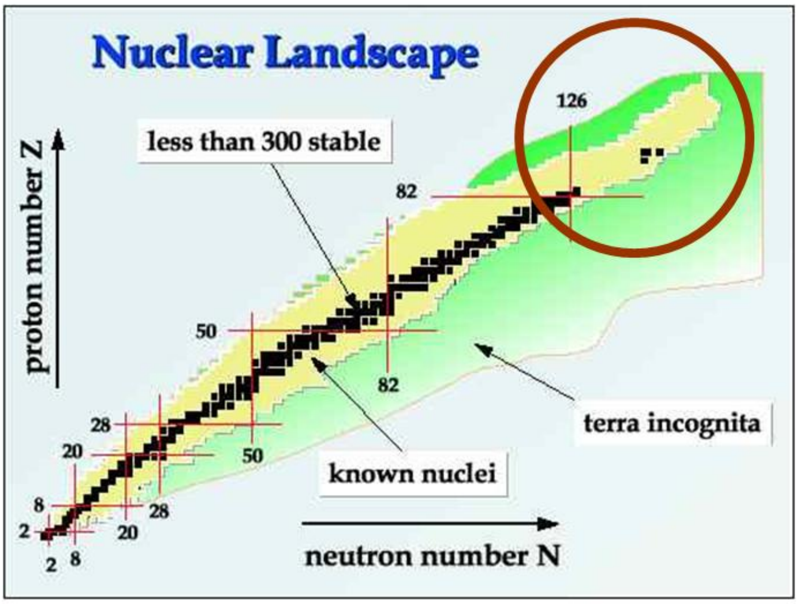
Ground State Properties

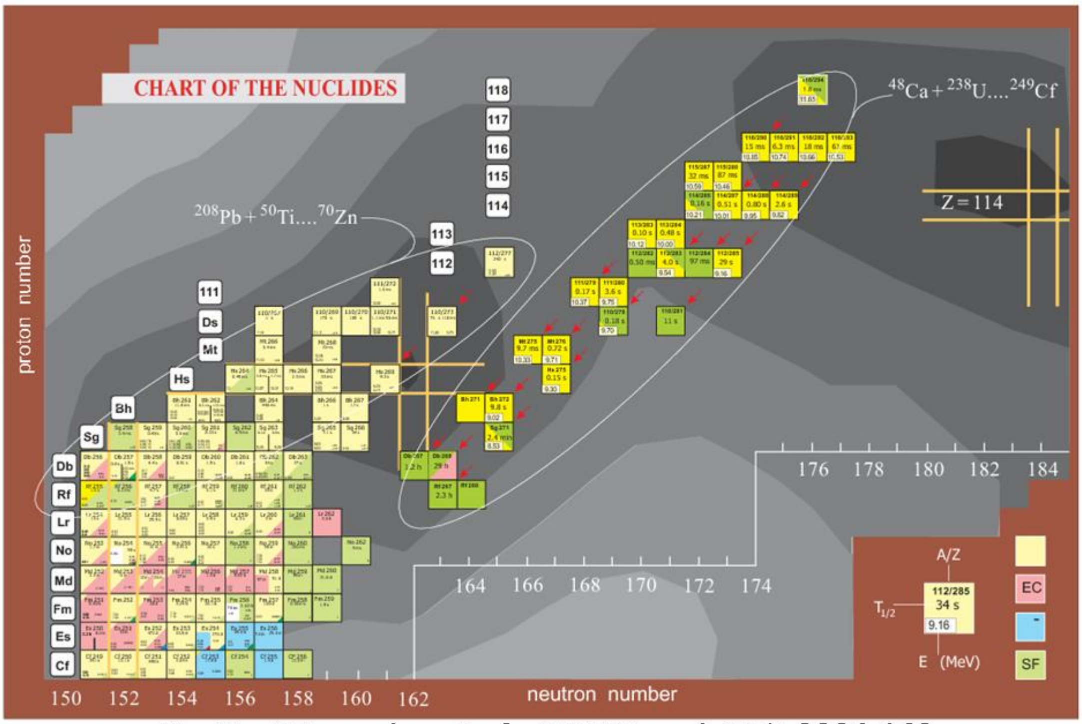




Ground State Properties

- **Binding Energies:** Well Reproduced ~ 0.25% of Expt. (On Average)
- **β:** Reasonable: Consistent with Moller – Nix Systematics
- **Charge Radii:** Upto 2nd Decimal
- **Isotopic Shifts:** Reproduced





Yu. Ts. Oganessian *et al.*, JINR Preprint E7-2004-160

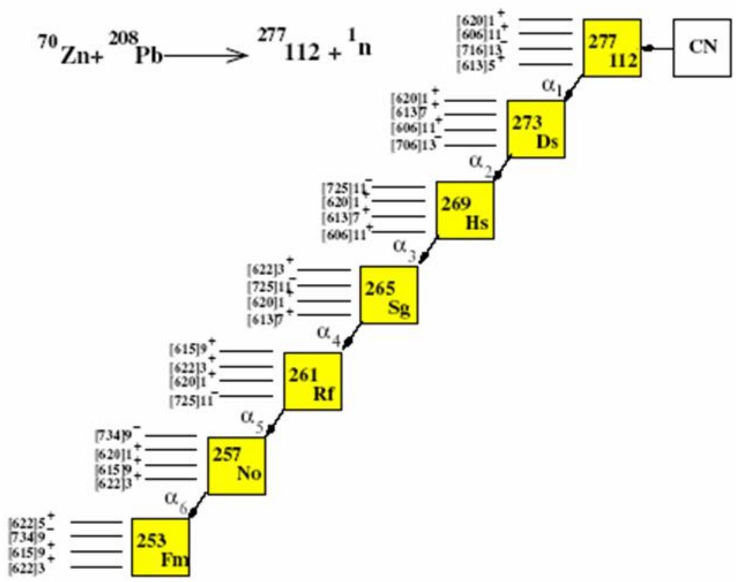
Production:

(Cold Fusion)

Decay:

(α Emission)

http://ie.lbl.gov/education/glossary/glossaryf.htm



S. Hofmann et al, Z. Phys. A 354 (1996) 229.

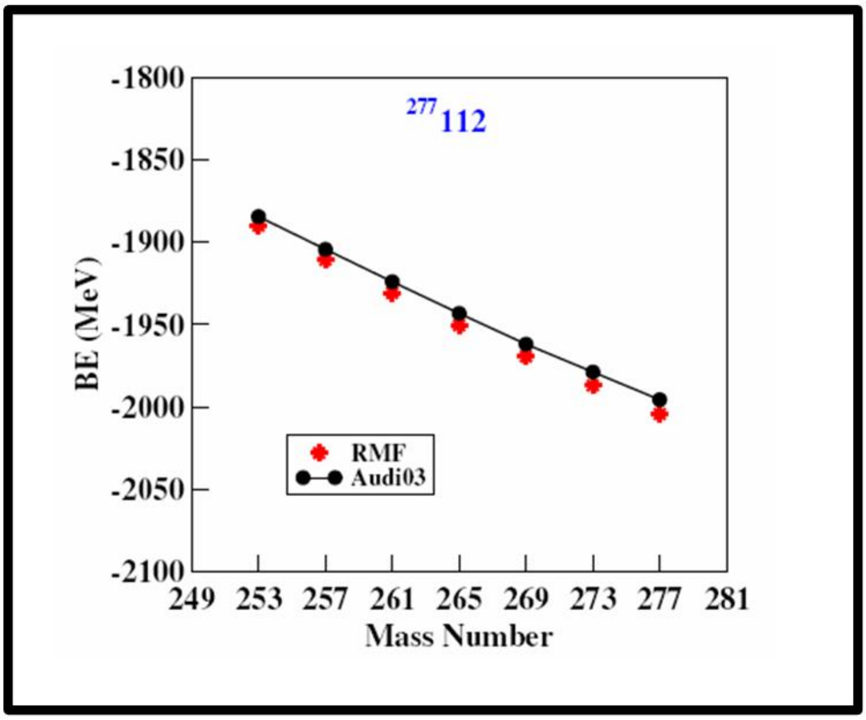
Superheavy Nuclei
(Half Lives)

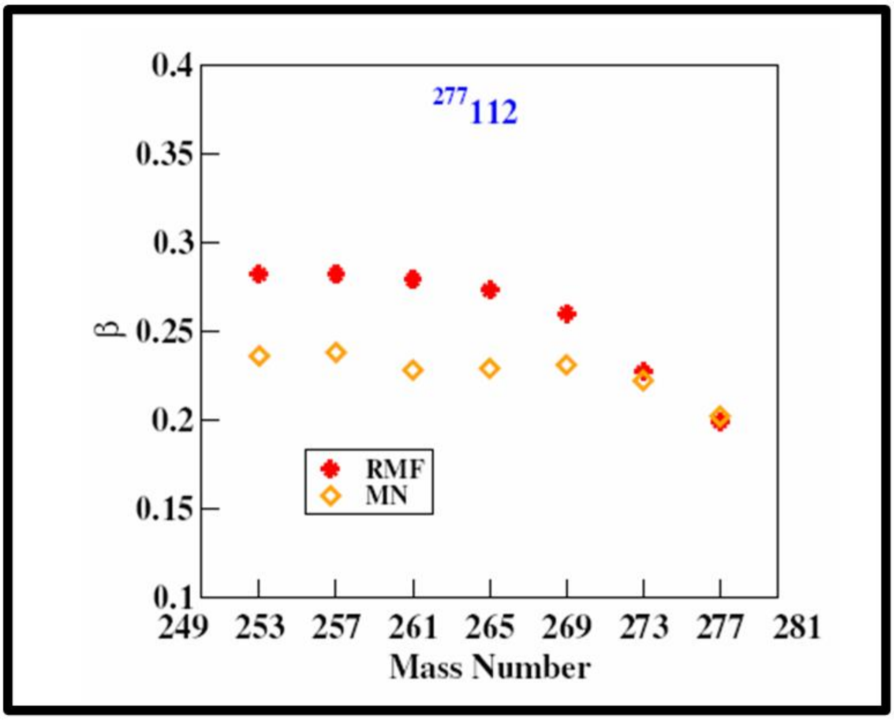
Calculation:

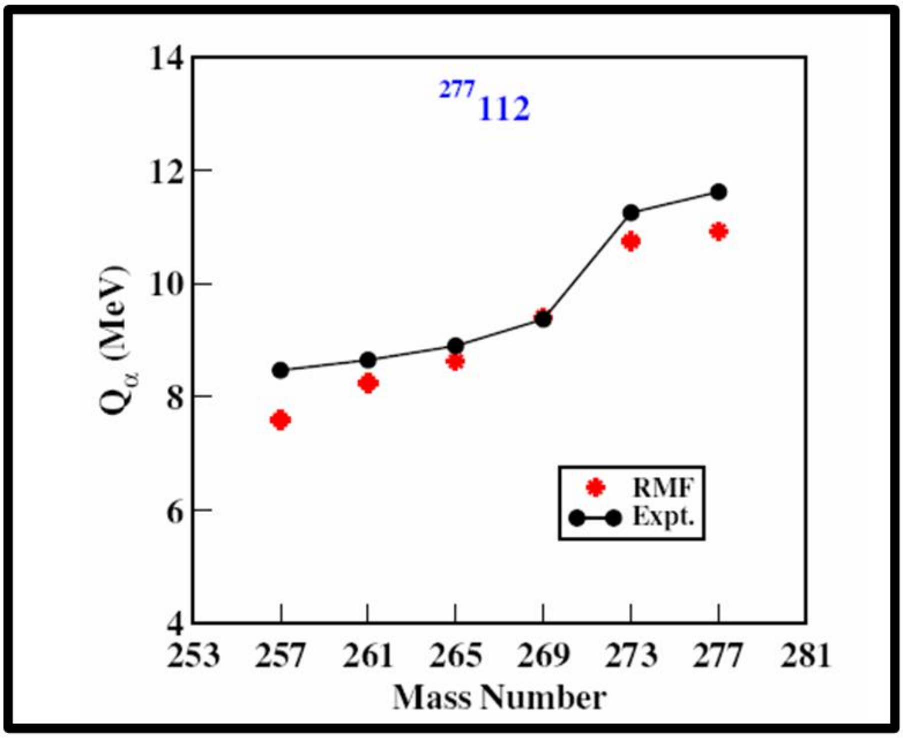
Ground State Properties: RMF
-- Well Reproduced (BE, β, etc.)

Half Lives: WKB Approximation
-- Requires: Q Values + Potentials

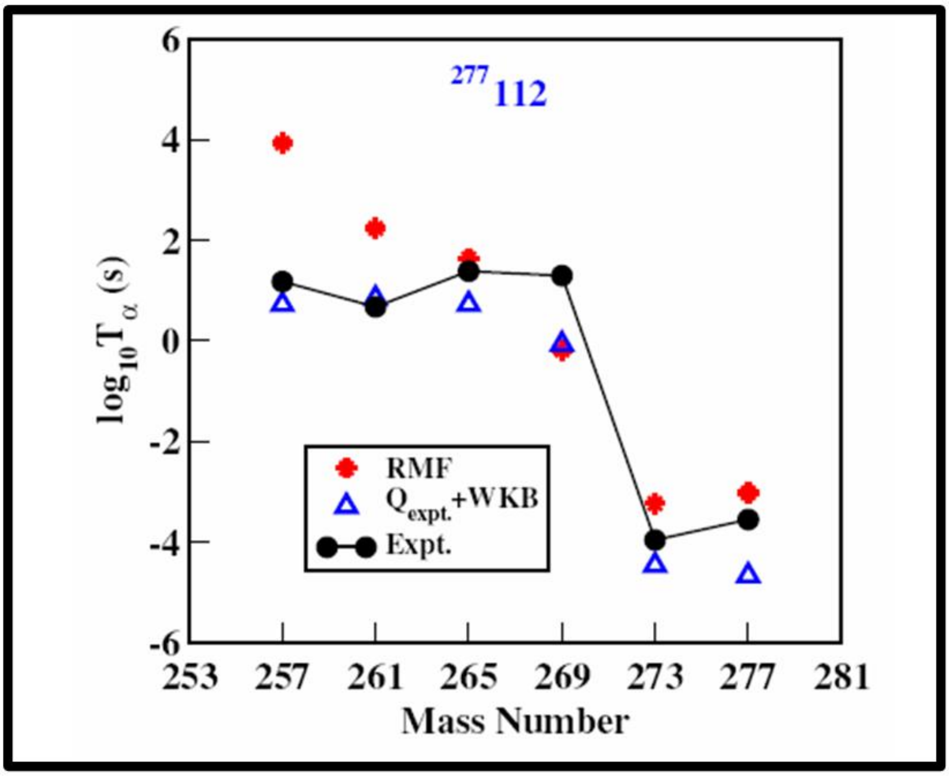
→ α - Daughter Interaction Potentials
(Double Folding Model)


277112
BE (MeV)
-1800
-1850
-1900
-1950
-2000
-2050
-2100
249
253
257
261
265
269
273
277
281
Mass Number
RMF
Audi03


277112
β
0.4
0.35
0.3
0.25
0.2
0.15
0.1
249
253
257
261
265
269
273
277
281
Mass Number
RMF
MN



^{277}112
RMF
Expt.
Q_α (MeV)
Mass Number



^{277}112
RMF
$Q_{expt.}$+WKB
Expt.
$\log_{10}T_\alpha$ (s)
Mass Number

Summary & Conclusions

- **Q Values: Well Reproduced**
- **Experimental Q values + WKB →**
  Reproduces half lives well →
  Double Folding Potential Reliable
- **Half Lives Depend Sensitively on**
  Q values

RMF IS SUCCESSFUL

Recent Developments:

Large Shell-Model Calculations:

Large Dimensions
Effective Interaction

ANTOINE (E.Courier, Strasburg, France)

OXBASH (B.A.Brown; Michigan, USA)

DUSM (Vallieres + Novoselsky, Phil., USA)

sd – shell (^{16}O Core) → Good

pf – shell (^{40}Ca Core) →
Yet to be Achieved Fully

Limits: 10^7 – 10^8 Basis States

How Much Dependence on Effective Interaction?

What can we learn from Eigenvectors with Billions of Components?

Exotic Nuclei: Asymptotics is Important
Continuum Shell Model

Ab – Initio : No Core Shell Model (NCSM)
With nn, nnn Interaction

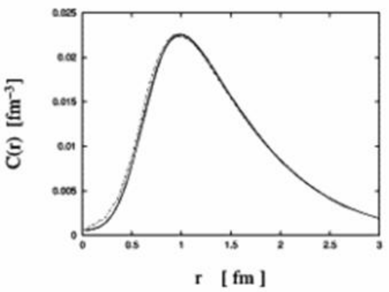
- Hyperspherical harmonic variational:
- Green's function Monte Carlo: $A \le 7$
- No-core shell model: $A \le 12$

Benchmark calculation for $A=4$

- Test calculation with realistic interaction: all methods agree.

$$\langle \Psi | \sum_{k<l}^{4} \delta(r - r_{kl}) | \Psi \rangle$$

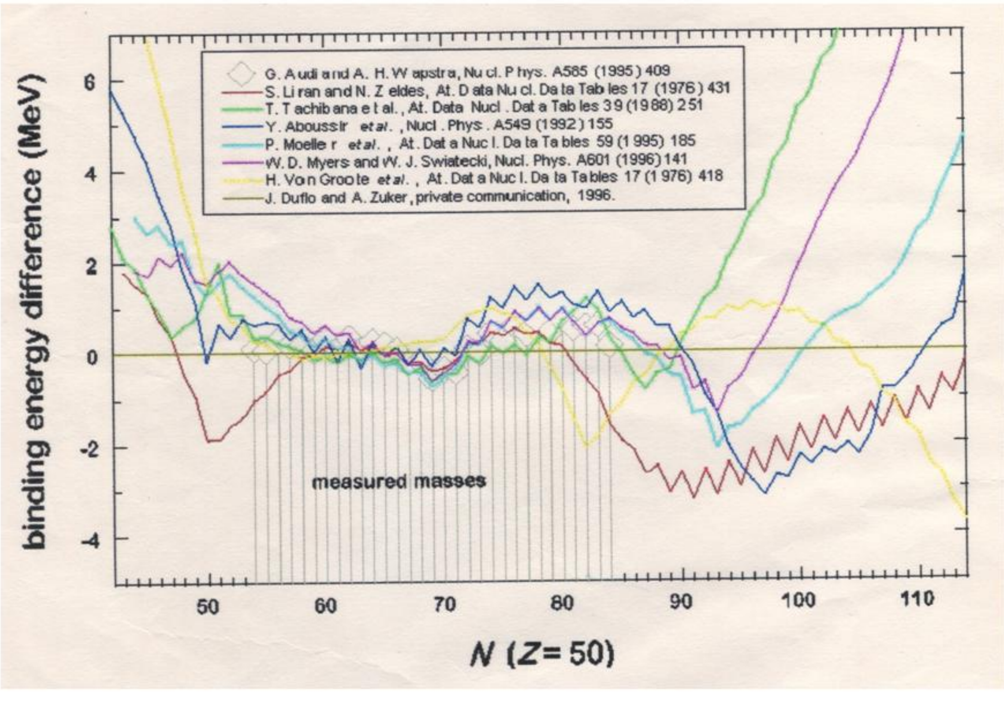
Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486



- But E_{expt}=-28.296 MeV ⇒ need for three-nucleon interaction.

H. Kamada *et al.*, Phys. Rev. C **63** (2001) 034006

^{16}O Calculations in 2010!



4.

Experimental Nuclear Structure Physics:

Experimental Techniques

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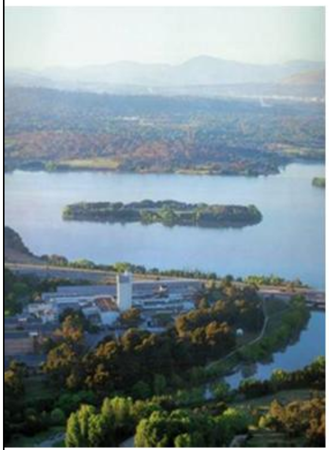
Experimental techniques to deduce J^{π}



T. Kibédi

Dept. of Nuclear Physics, Australian National University, Canberra, Australia

Workshop on "Nuclear Structure and Decay Data: Theory and Evaluation" Trieste, Italy, 2006

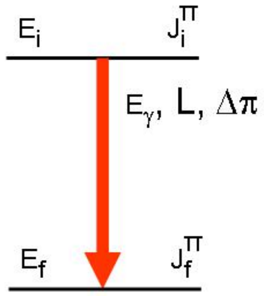
Outline:

Lecture I: *Experimental techniques to deduce J^{π} from*

- *Angular distributions and correlations*
- *Directional Correlations from Oriented nuclei (DCO)*
- *Gamma-ray linear polarizations*
- *Internal conversion coefficients*

Lecture II: *New developments in characterizing nuclei using separators*

Electromagnetic Decay and Nuclear Structure



Energetics of γ-decay:

$E_i = E_f + E_\gamma + T_r$

$0 = p_R + p_\gamma$

where $T_R = (p_R)^2/2M$; usually $T_R/E_\gamma \sim 10^{-5}$

Angular momentum and parity selection rules & multipolarities

$\lvert J_i - J_f \rvert \leq L \leq \lvert J_i + J_f \rvert$	$L \neq 0$
$\Delta\pi$ = no	M1, E2, M3, E4, …
$\Delta\pi$ = yes	E1, M2, E3, M4, …
$\lvert J_i - J_f \rvert \leq L \leq \lvert J_i + J_f \rvert$	$L \neq 0$
$\Delta\pi$ = unknown	D, Q, O, H, ...
$J_i = J_f$	$L = 0$
$\Delta\pi$ = no;	E0

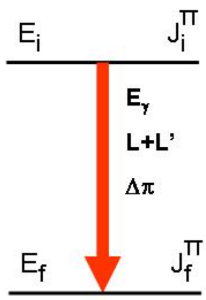
Multipolarity known

ΔJ may not be unique

$\Delta\pi$ unique

T. Kibèdi, NSDD Workshop, Trieste 2006

More on EM transitions



Mixed multipolarity & Mixing ratio

$\delta(\pi`L`/\pi L) = I_\gamma(\pi`L`) \;/\; I_\gamma(\pi L)$

$I\gamma = I\gamma(\pi L) + I\gamma(\pi' L')$

Or in terms of transition probability

$\lambda_\gamma = \lambda_\gamma(\pi L) + \lambda_\gamma(\pi' L')$

Total transition probability

$\lambda_T = \lambda_\gamma + \lambda_{CE} + \lambda_\pi + \lambda_{\gamma\gamma} + \dots$

λ_{CE} – conversion electrons, K,L1,L2… shells

λ_π – electron-positron pair production; $E_\gamma > 2\, m_o c^2$

$\lambda_{\gamma\gamma}$ – 2 photon emission; very rare (10^{-5})

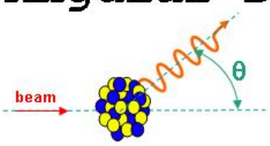
T. Kibèdi, NSDD Workshop, Trieste 2006

Determining transition multipolarity

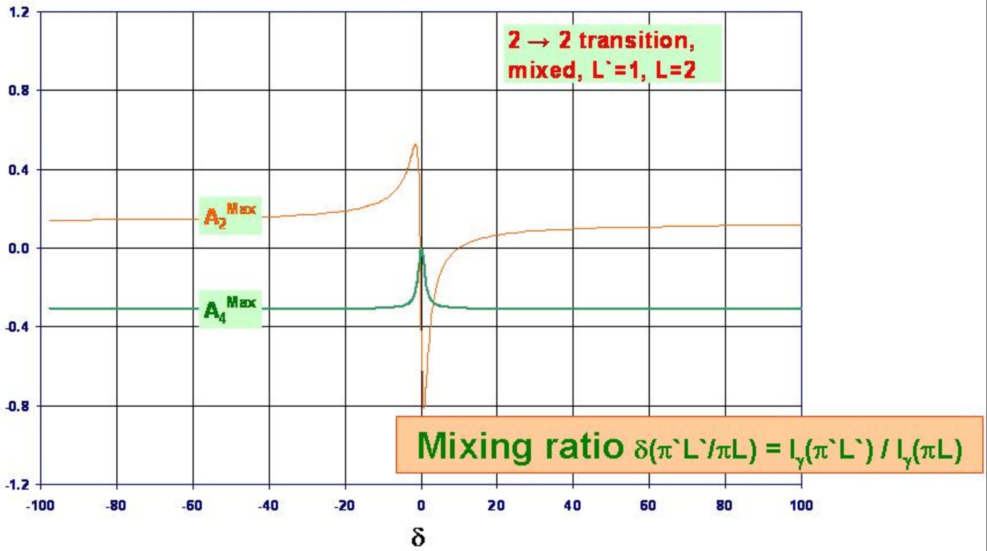
- **Gamma rays** — Angular distribution with spins oriented; Angular correlations; Polarization effects
- **Conversion electrons** — Electron conversion coefficients; E0 (L=0) transitions
- **Electron positron pairs** — Pair conversion coefficients

T. Kibèdi, NSDD Workshop, Trieste 2006

Angular Distributions of Gamma Rays



$$W(\theta) = 1 + A_{22}P_2(\cos\theta) + A_{44}P_4(\cos\theta)$$



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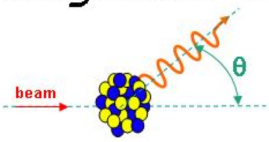
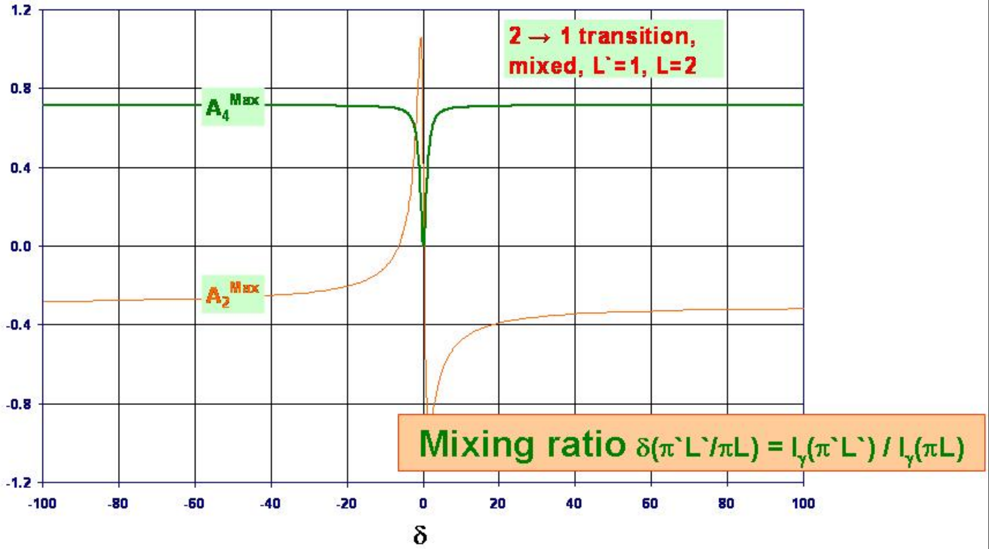
Angular Distributions of Gamma Rays



$$W(\theta) = 1 + A_{22}P_2(\cos\theta) + A_{44}P_4(\cos\theta)$$



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Angular Distributions of Gamma Rays

$$W(\theta) = 1 + A_{22}P_2(\cos\theta) + A_{44}P_4(\cos\theta)$$

Attenuation due to relaxation of nuclear orientation

$$0 \le A_{kk} \le A_k^{\max}(J_i, J_f, L); k = 2,4\ldots$$

$$A_k^{\max}(J_i, J_f, L) = \frac{F_k(LLJ_fJ_i) + 2\delta \times F_k(LL{+}1J_fJ_i) + \delta^2 \times F_k(L{+}1L{+}1J_fJ_i)}{1+\delta^2}$$

For $F_k(LL`J_fJ)$ see E. Der Mateosian and A.W. Sunyar, ADNDT 13 (1974) 407

$$A_{kk} = B_k(J_i) \times A_k^{\max}(J_i, J_f, L)$$

Nuclear orientation can be achieved

- by interaction of external fields (E,B) with the static moments of the nuclei at low temperatures
- by nuclear reaction

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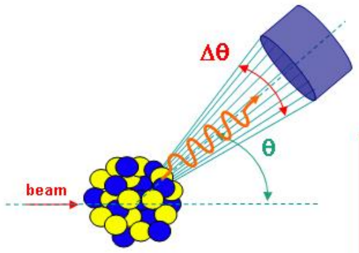
Attenuation of Angular Distributions

Detector finite size (Δθ): solid angle attenuation



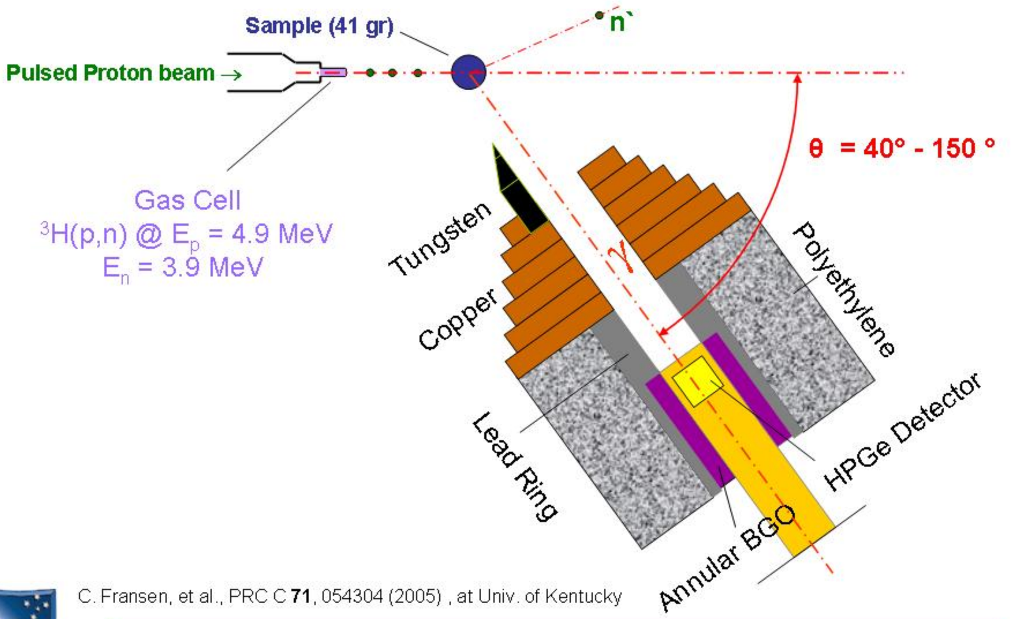
Beam defines a symmetry axis

$$W(\theta) = 1 + \Sigma_{k=2,4} Q_k \times B_k(J_i) \times A_k^{\max}(J_i, J_f, L, \delta)$$

where $B_k(J)$ is the statistical tensor

$$B_k(J_i) = \sum_{m=-J_i}^{+1} (-1)^{J_i+m} \sqrt{(2k+1)(2J_i+1)} \times \begin{pmatrix} J_i & J_i & k \\ -m & m & 0 \end{pmatrix} \times \underbrace{\frac{Exp(-m^2/2\sigma^2)}{\sum_{m=-1}^{J_i} Exp(-m^2/2\sigma^2)}}_{\text{Approximation with Gaussian distribution}}$$

Angular Distributions of Gamma Rays (n,n`) reaction on ^{92}Zr



C. Fransen, et al., PRC C **71**, 054304 (2005) , at Univ. of Kentucky

Figure courtesy of S.W. Yates

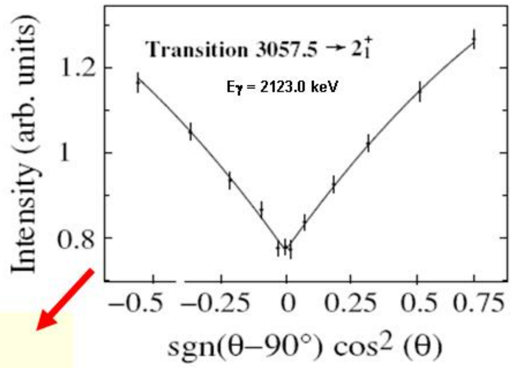
Angular Distributions of Gamma Rays

$^{92}Zr(n,n`)$ *reaction*

12 angles and

12 hours / angle

γ-spectrometer at 1.4 m



C. Fransen, et al., PRC C **71**, 054304 (2005), at Univ. of Kentucky

Fit to data

$$W(\theta) = A_0 + A_{22}P_2(\cos\theta) + A_{44}P_4(\cos\theta)$$

Deduced $A_2 = A_{22}/A_0$

$A_4 = A_{44}/A_0$

Typical values	A_2	A_4
ΔJ=2 (stretched quadrupole)	+0.3	-0.1
ΔJ=1 (stretched dipole)	-0.2	0
ΔJ=1, D+Q	+0.5 to -0.8	>0

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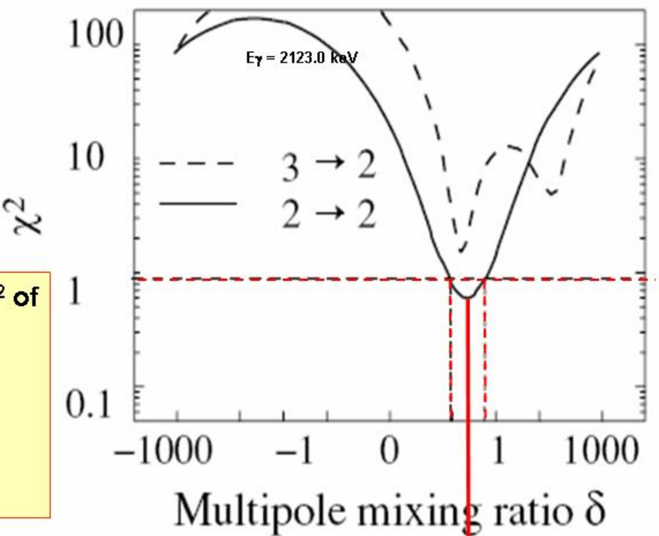
Angular Distributions - mixing ratio

C. Fransen, et al., PRC C **71**, 054304 (2005) , at Univ. of Kentucky



Mixing ratio, δ deduced from χ^2 of $\frac{W_{exp}(\theta)}{W_{calc}(\theta)}$ as a function of δ

But no information on Electric or Magnetic character E1+M2 or M1+E2

δ= 0.69(16) (D+Q)

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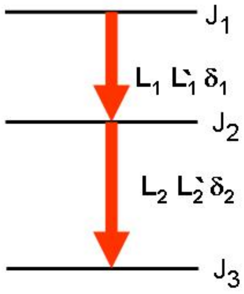
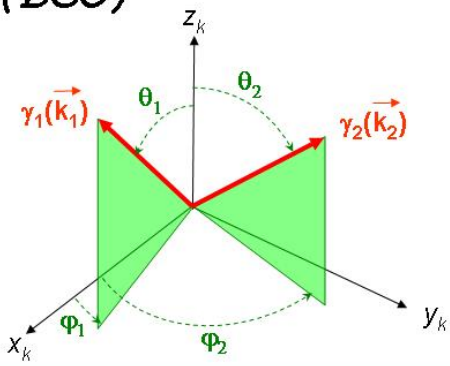
Directional Correlations from Oriented nuclei (DCO)





For a $J_1 \rightarrow J_2 \rightarrow J_3$ cascade *(see A.E. Stuchbery, Nucl. Phys.* ***A723*** *(2003) 69)*

$$W(\theta_1,\varphi_1,\theta_2,\varphi_2) = \sum_{k,q,k_1,q_1,k_2,q_2} \rho_{k_1 q_1}(J_1)(-1)^{k_1+q_1}\sqrt{(2k+1)(2k_1+1)}\begin{pmatrix} k_1 & k & k_2 \\ -q_1 & q & q_2 \end{pmatrix}$$

$$\times A_k^{k_2 k_1}(\delta_{\gamma 12} L L' J_2 J_1) Q_k(E_{\gamma 12}) D^{k*}_{q0}(\varphi_1,\theta_1,0)$$

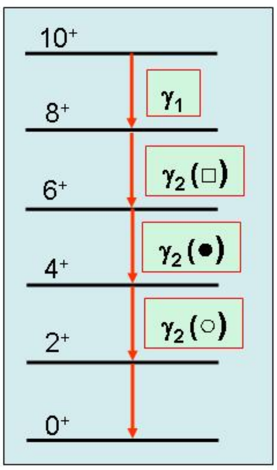
$$\times A_{k2}(\delta_{\gamma 23} L L' J_3 J_2) Q_{k_2}(E_{\gamma 23}) D^{k_2 *}_{q_2 0}(\varphi_2,\theta_2,0)$$

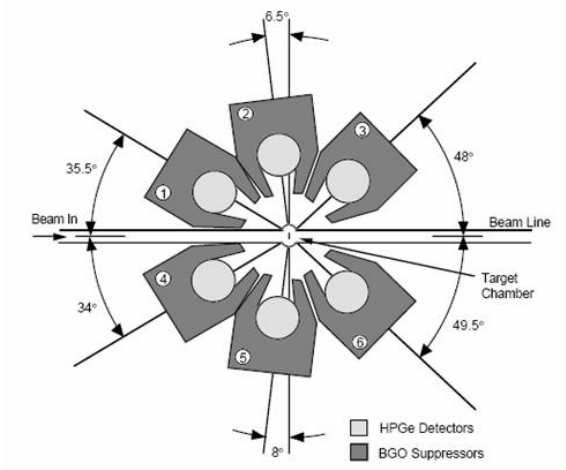
T. Kibèdi, NSDD Workshop, Trieste 2006

Directional Correlations from Oriented nuclei (DCO) - example

^{184}Pt from ^{nat}Gd + ^{29}Si @ 145 MeV CAESAR array (ANU)

M.P. Robinson et al., Phys. Lett B530 (2002) 74





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Directional Correlations from Oriented nuclei (DCO) - example

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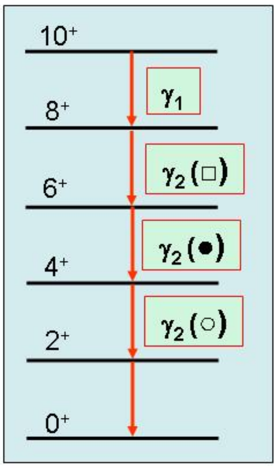
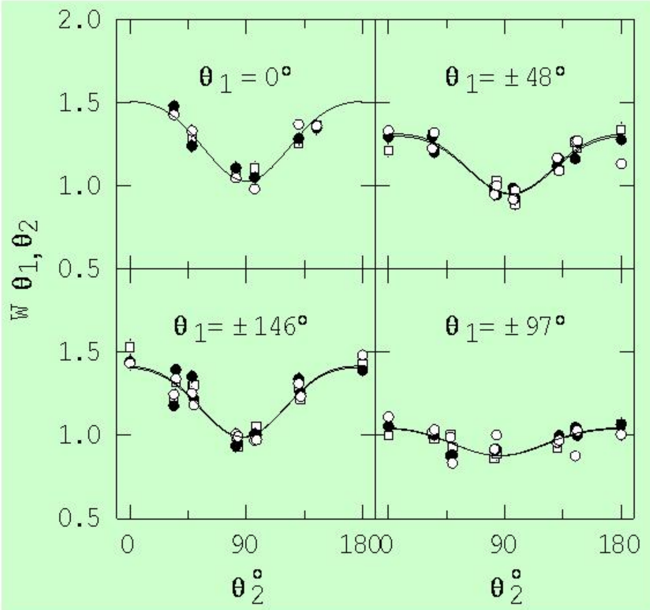
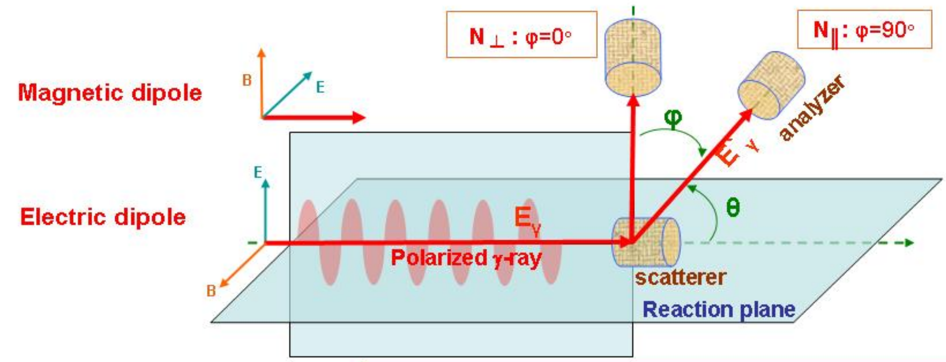
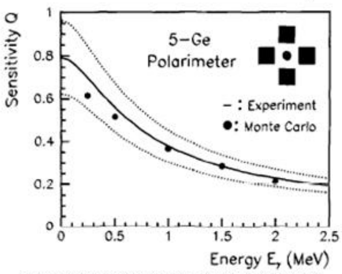




Gamma-ray linear polarization

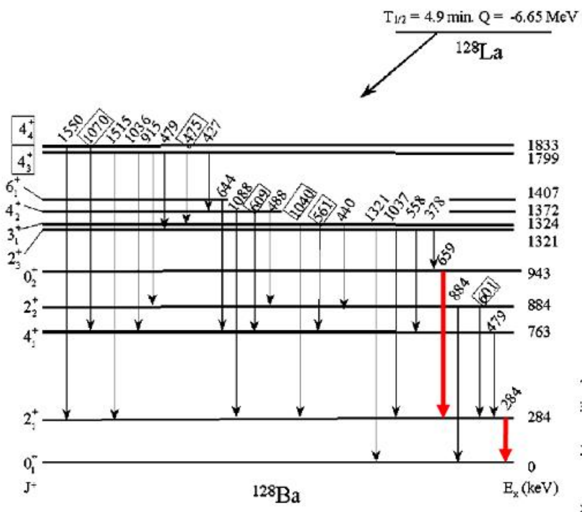




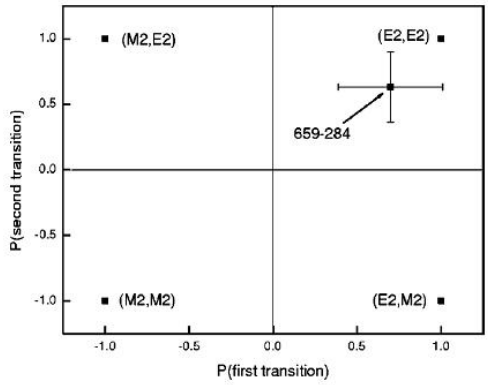
L.M. Garcia-Raffi, et al. NIM ***A391*** *(1997) 461*

Compton-scattering	$E_{\gamma'} = \dfrac{E_\gamma}{1+\alpha(1-\cos\theta)}$ with $\alpha = \dfrac{E_\gamma}{m_\circ c^2}$
Klein-Nishina formula	$\dfrac{d\sigma}{d\Omega} = \dfrac{1}{2} r_\circ^2 \dfrac{E_{\gamma'}^2}{E_\gamma^2}\left(\dfrac{E_{\gamma'}}{E_\gamma} - \dfrac{E_\gamma}{E_{\gamma'}} - 2\sin^2\theta\cos^2\varphi\right)$
Polarization	$P = A / Q$
Asymmetry	$A = (N_\perp - N_\parallel) / (N_\perp + N_\parallel)$
Polarization sensitivity	Q

Gamma-ray linear polarization – example

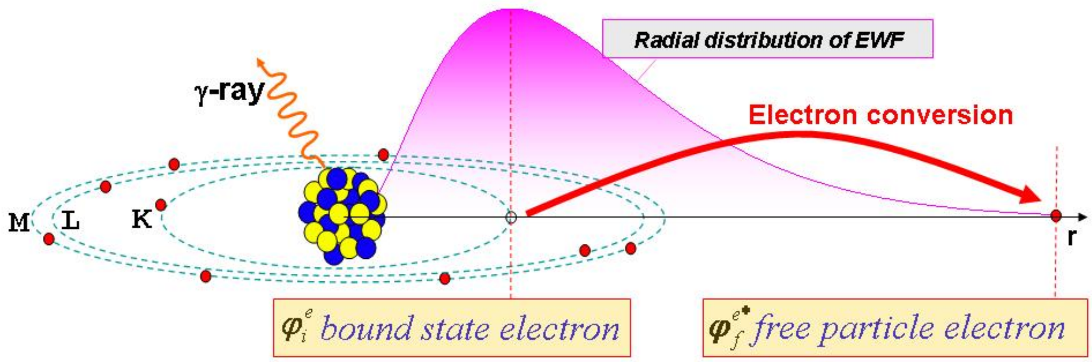


659–284 keV, $0_2 \rightarrow 2_1 \rightarrow 0_1$



A. Wolf, *et al. Phys. Rev.* C **66**, 024323 (2002)

Conversion electrons (CE)



Energetics of CE-decay (i=K, L, M,….)

$E_i = E_f + E_{ce,i} + E_{BE,i} + \cancel{T_r}$

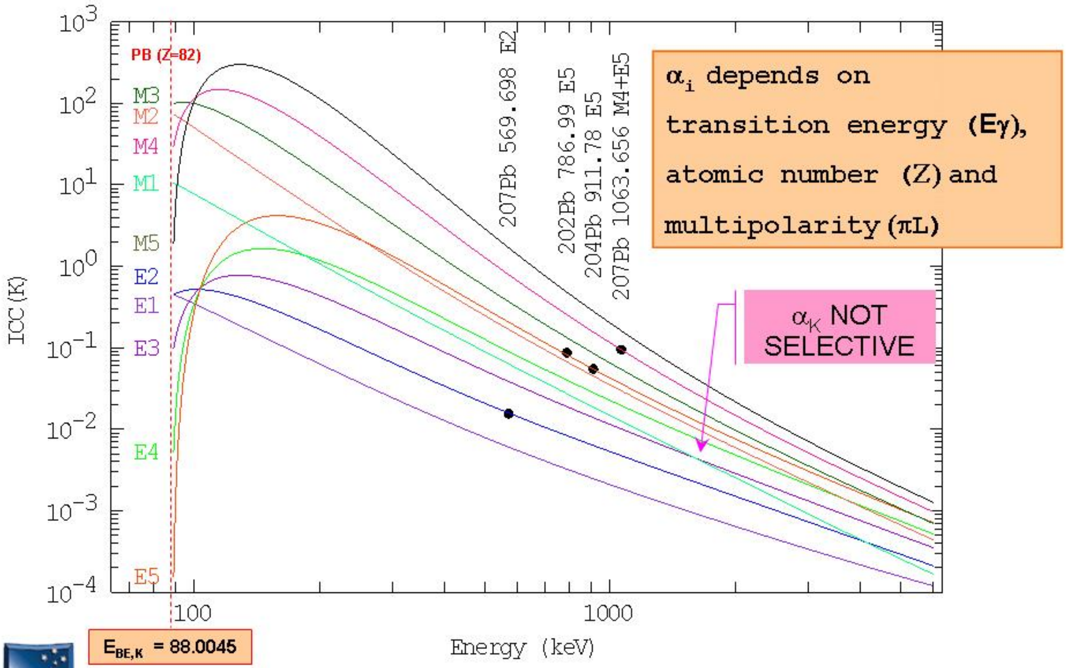
γ- and CE-decays are independent; transition probability (λ ~ Intensity)

$\lambda_T = \lambda_\gamma + \lambda_{CE} = \lambda_\gamma + \lambda_K + \lambda_L + \lambda_M \ldots\ldots$

Conversion coefficient

$\alpha_i = \lambda_{CE,i} / \lambda_\gamma$

Sensitivity to multipolarity



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The physics of conversion coefficients

$$\alpha_K \equiv \frac{\lambda_{e,K}}{\lambda_\gamma}$$

$$\lambda_e = \frac{2\pi}{\hbar}\left|m_{fi}\right|^2 \frac{d\rho}{dE} \qquad \textit{Fermi's golden rule}$$

Density of final electron state (continuum)

$$m_{fi} = \psi_f^{N*}\ \varphi_f^{e*}\ F_{\ell,m}\ \psi_i^N\ \varphi_i^e$$

Nuclear

Electron

Multipolar source

Same for γ and CE

φ_i^e *bound state electron*

φ_f^{e*} *free particle electron*

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Theoretical Conversion Coefficients

Current tabulations:

- **Hager and Seltzer** (1968)
  Relativistic Hartree-Fock-Slater, WITH Hole, NO dynamic effect
  Z=30-103; K, L, M only; limited energy range
- **Rösel-Fries-Paul** (1978)
  Relativistic Hartree-Fock-Slater, NO Hole, NO dynamic effect
  Z=30-104; All shells; wider energy range
- **Band-Trzhaskovskaya** (1978)
  Relativistic Hartree-Fock-Slater, WITH Hole, WITH dynamic effect
  Z=10-104; K, L, M; wider energy range
- **Band-Trzhaskovskaya-Nestor-Tikkanen-Raman** (2002)
  Relativistic Dirac-Fock, NO Hole, WITH dynamic effect
  Z=10-126; ALL shells; wider energy range
- **BrIcc** (2005)
  Relativistic Dirac-Fock, With Hole, WITH dynamic effect
  Z=10-95; ALL shells; improved accuracy

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Higher order and atomic effects

- **Atomic many body correlations: factor ~2 for E_{kin}(ce) < 1 keV (BrIcc single particle approximation)**
- **Partially filled valence shell: non-spherical atomic field**
- **Shake effect: increases ICC**
- **Resonance internal conversion: E_{kin}(ce) ≈ BE**
- **Binding energy unc.: <0.5% for E_{kin}(ce) > 10 keV**
- **Chemical effects: <<1%**
- **Penetration:**
  ***n s1/2* shells (K, L1, M1,..); M1, M2, M3.. multipolarities**
  For M1 transition:
  0.01% (Z=10)
  ~15% (Z=112)

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Mixed multipolarity and E0 transitions

$$\delta^2 = \frac{I_\gamma(E2)}{I_\gamma(M1)} \qquad \alpha^{M1/E2} = \frac{\alpha_{M1} + \delta^2 \alpha_{E2}}{1+\delta^2}$$

In some cases the mixing ratio can be deduced

$$\delta^2 = \frac{\alpha_{M1} - \alpha^{\exp}}{\alpha^{\exp} - \alpha_{E2}}$$

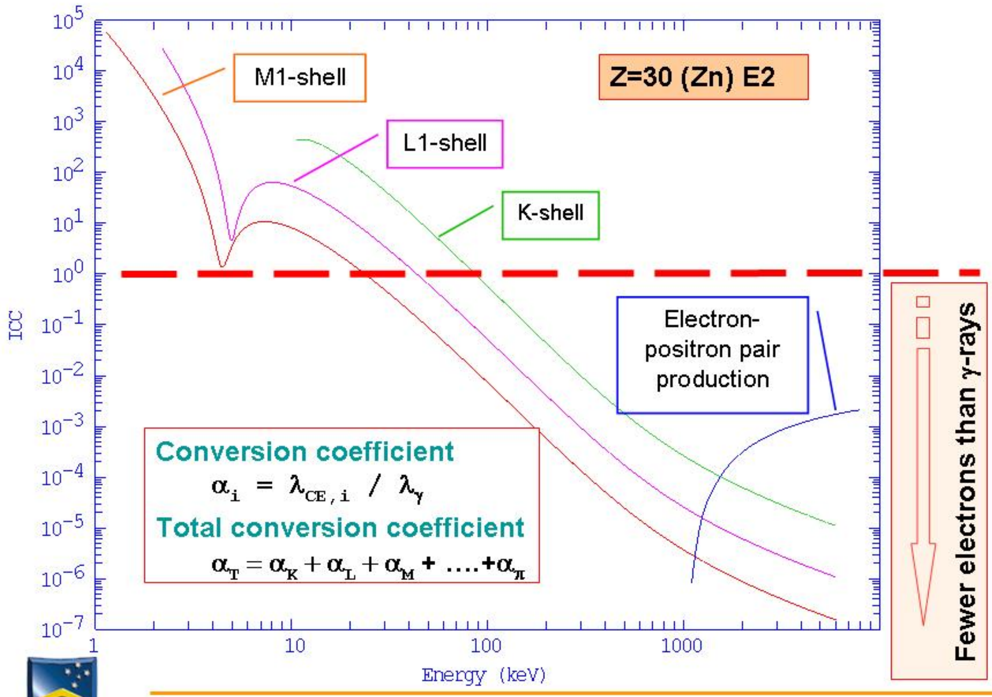
E0 transitions – pure penetration effect; no γ rays (I_γ=0)

$$\alpha = \frac{I_{CE}}{I_\gamma} = \infty$$

- Pure E0 transition: $0^+ \rightarrow 0^+$ or $0^- \rightarrow 0^-$
- J → J (J≠0) transitions can be mixed E0+E2+M1

$$\alpha = \frac{I_{CE}(E0) + I_{CE}(E2) + I_{CE}(M1)}{I_\gamma(E2) \quad + I_\gamma(M1)}$$

More on conversion coefficients



Conversion coefficient

$$\alpha_i = \lambda_{CE,i} / \lambda_\gamma$$

Total conversion coefficient

$$\alpha_T = \alpha_K + \alpha_L + \alpha_M + \dots + \alpha_\pi$$

Measuring conversion coefficients – methods

- NPG: normalization of relative CE ($I_{CE,i}$) and γ (I_γ) intensities via intensities of one (or more) transition with known α

$$\alpha_i = \frac{I_{CE,i}}{I_\gamma} \times \left[\frac{I^*_\gamma}{I^*_{CE}} \times \alpha^* \right]_{KNOWN}$$

- CEL: Coulomb excitation and lifetime measurement

$$\alpha_T = \frac{2.829 \times 10^{11} \times E_\gamma^{-5}(keV)}{B(E2)\uparrow (e^2b^2) \times T_{1/2}(ns)} - 1$$

- XPG: intensity ratio of K X-rays to γ rays with K-fluorescent yield, ω_K

$$\alpha_K = \frac{I_{KX}}{I_\gamma} \times \frac{1}{\omega_K}$$

And many more, see Hamilton`s book

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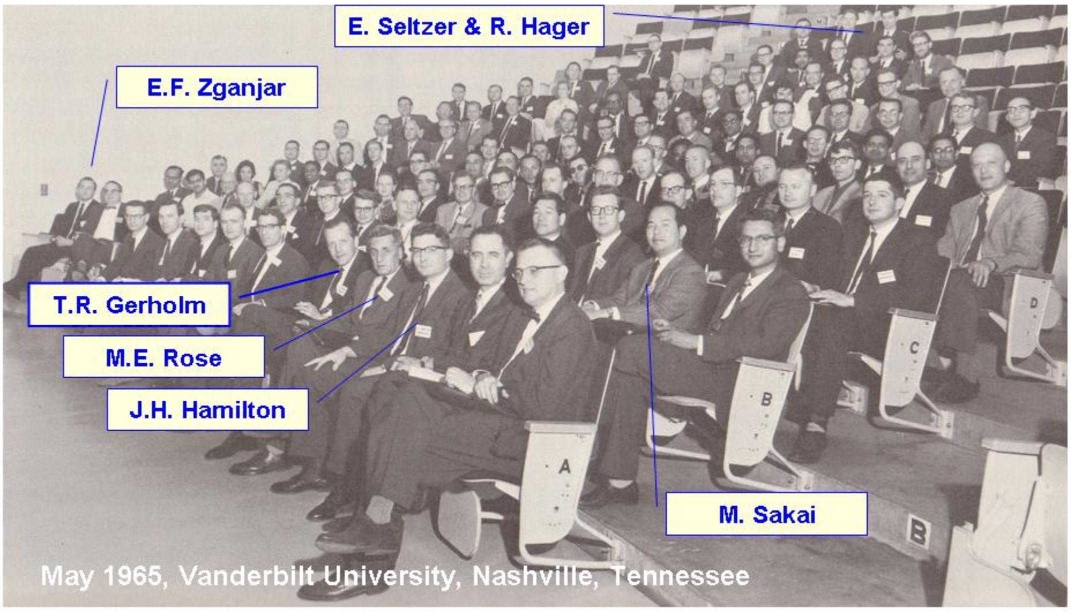
Internal Conversion Process – The Pioneers



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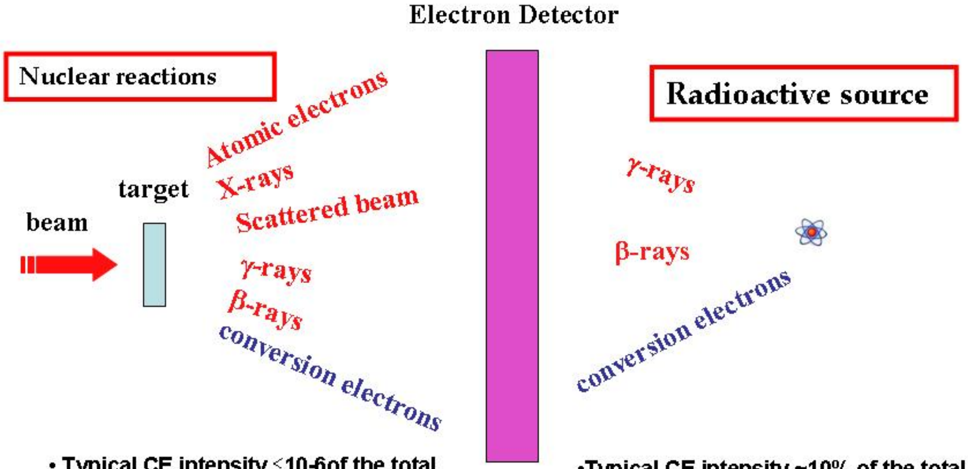
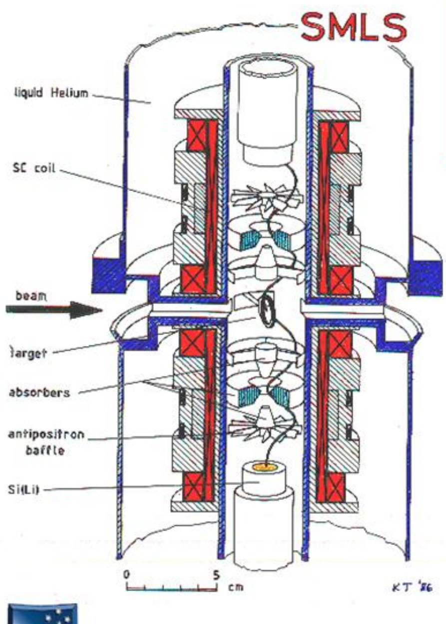
Direct ICC measurements



- Typical CE intensity $\leq 10^{-6}$ of the total *(scattered beam excluded)*
- *Energy resolution affected by absorption in target; FWHM for a Si(Li): 3-5 keV*
- *Angular distribution of conversion electrons – usually neglected(?)*
- *Typical* $\Delta\alpha/\alpha \sim 10-20\%$

- Typical CE intensity ~10% of the total
- Thin source, better energy resolution; *FWHM for a Si(Li): 1.6-2.5 keV*
- *No/attenuated angular distribution*
- *Typical* $\Delta\alpha/\alpha \sim 5-10\%$

Basic electron transporters

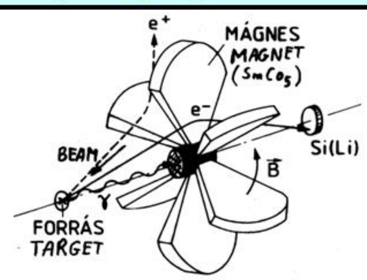


Superconducting solenoid

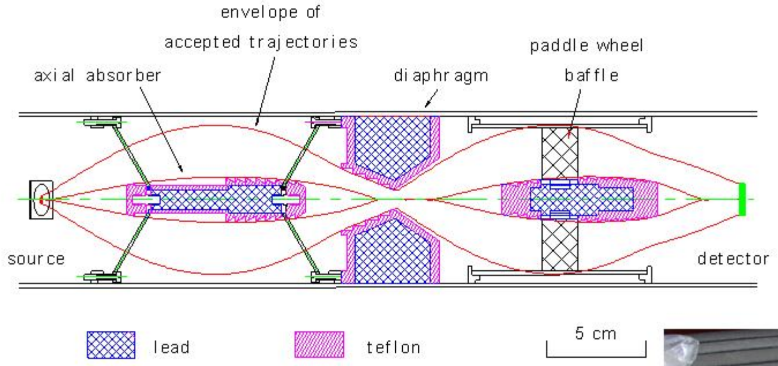
- Broad-range mode – 100 keV up to a few MeV
- Lens mode – finite transmitted momentum bandwidth ($\Delta p/p \sim 15\text{-}25\%$) – high peak-to-total ratio

Mini-orange (looks like a peeled orange)

- transmission > 20%
- small size and portability, but poorer quality



ANU Superconducting Solenoid Spectrometer



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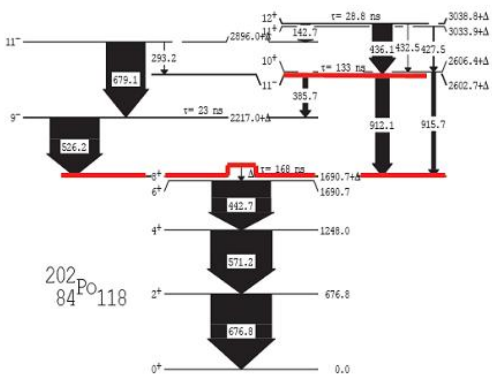
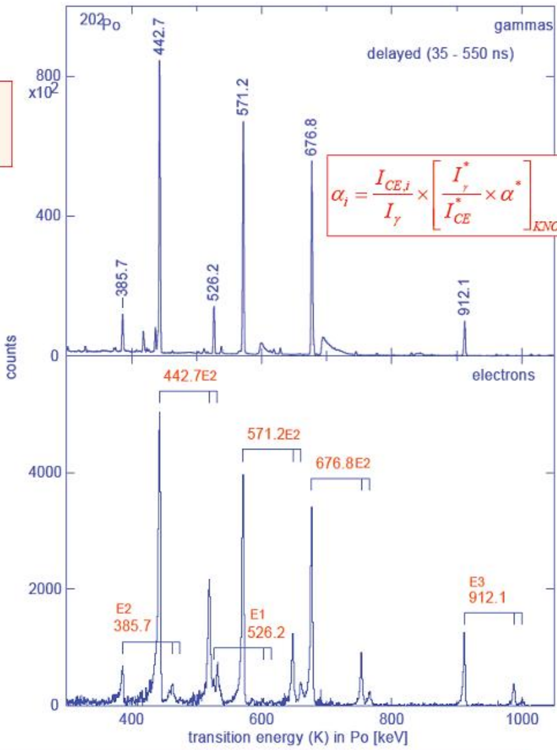
Super-e Lens (ANU)

^{194}Pt(^{12}C,4n)^{202}Po @ 76 MeV

Pulsed beams (~1 ns) with 1.7 μs separation





$$\alpha_i = \frac{I_{CE,i}}{I_\gamma} \times \left[\frac{I^*_\gamma}{I^*_{CE}} \times \alpha^* \right]_{KNOWN}$$

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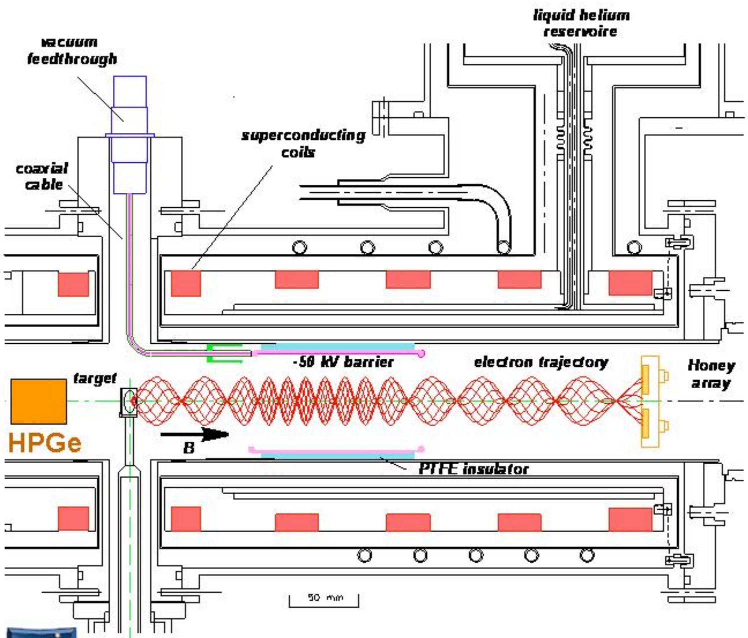
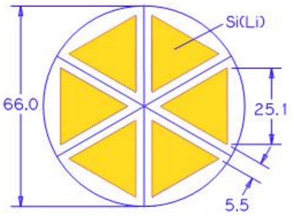
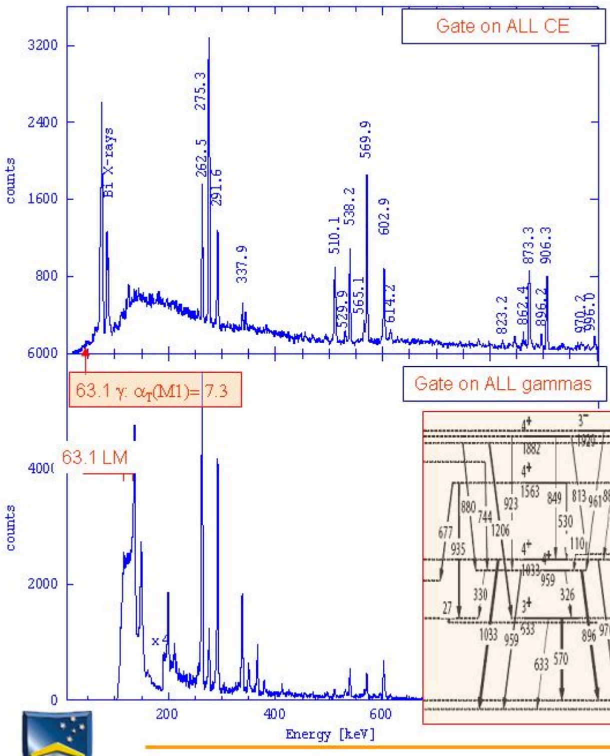
Super-e Honey (ANU)



Electrons from atomic collisions are the major difficulty in low energy CE spectroscopy using ion induced reactions



eeγ coincidences



Super-e Honey (ANU)

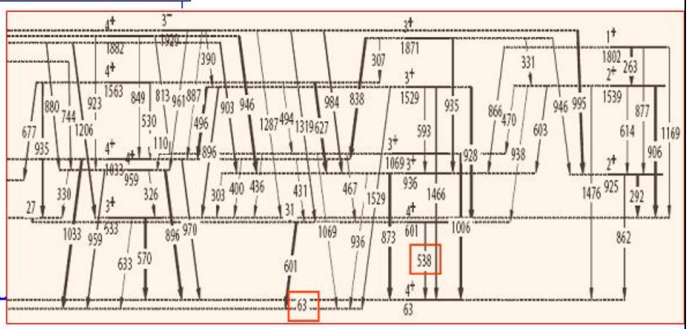
eeγ coincidences

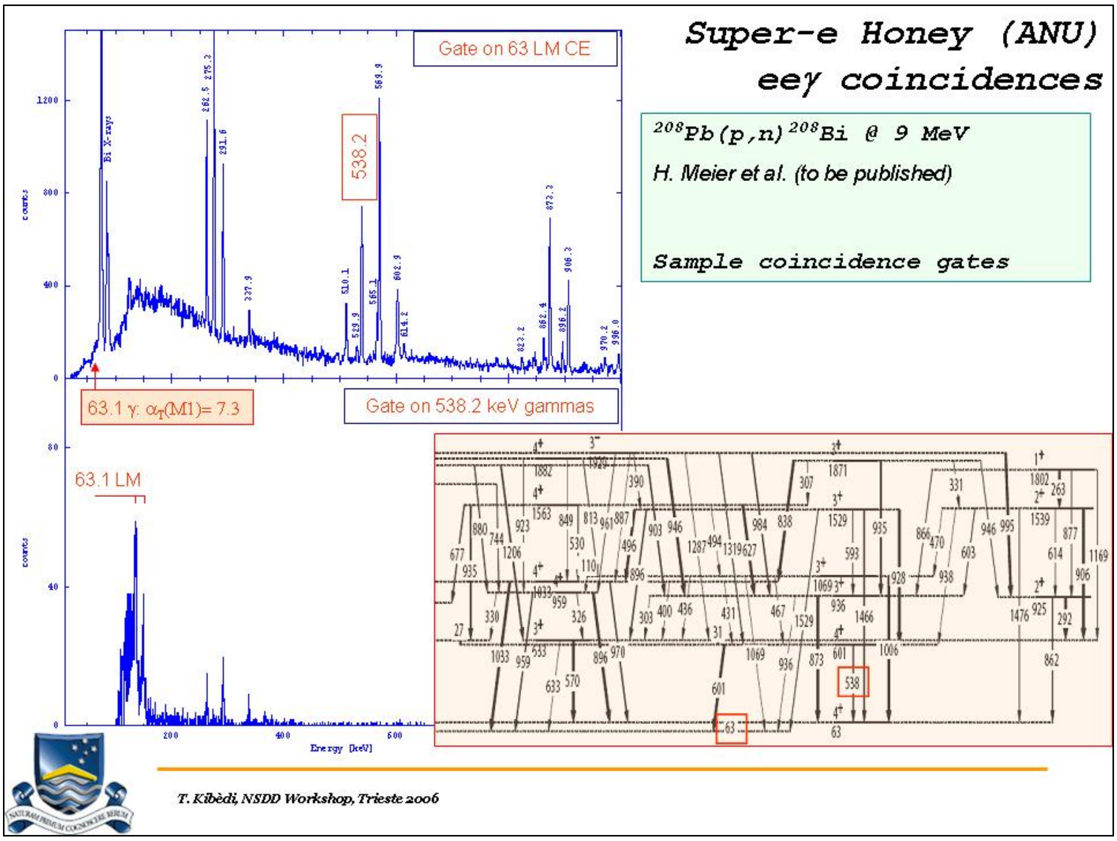
$^{208}Pb(p,n)^{208}Bi$ @ 9 MeV

H. Meier et al. (to be published)

E_γ=63.1 keV transition not visible in gamma spectra.

No K conversion; BE_K=90.5 keV





ICC from total intensity balances – example 1

In out-of-beam (or decay) coincidence data

$$I^{tot}_{\gamma_{in}} = I_{\gamma_{in}} \times (1+\alpha^{tot}_{in}) \equiv I^{tot}_{\gamma_{out}} = I_{\gamma_{out}} \times (1+\alpha^{tot}_{out})$$

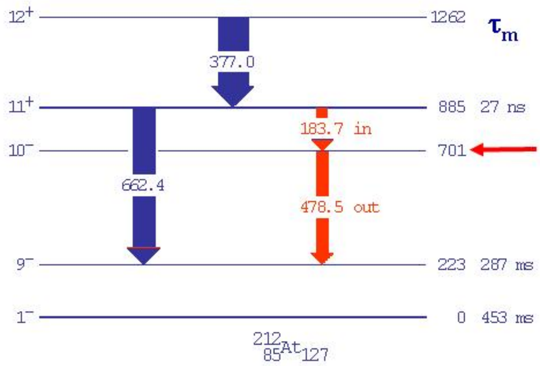
$$\alpha^{tot}_{in} \equiv (I_{\gamma_{out}} / I_{\gamma_{in}}) \times (1+\alpha^{tot}_{out}) - 1$$

$$\alpha^{tot}_{out}(478.5, M1) = 0.166$$

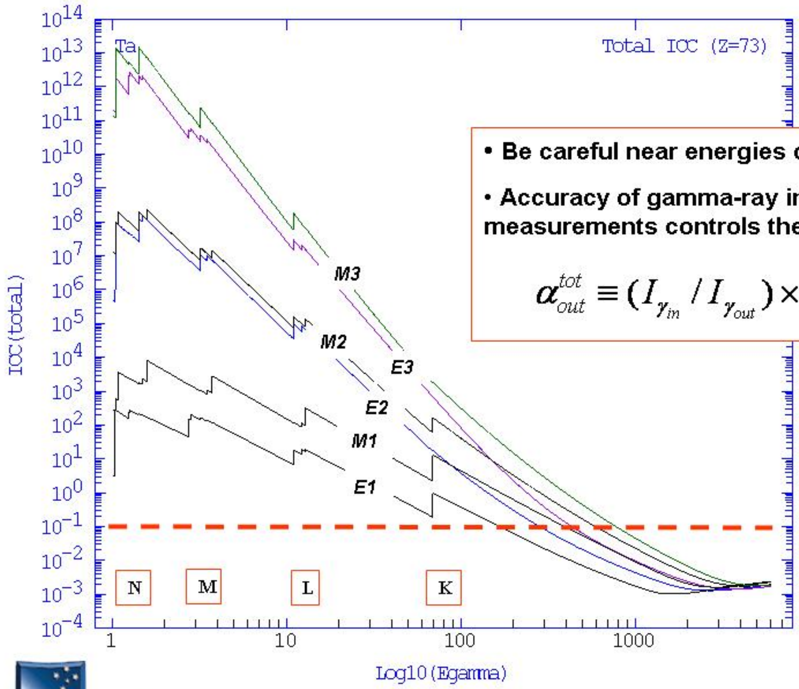
$\alpha^{tot}_{in}(183.7) = 0.23(5)$ $E1: 0.103$ $M1: 2.15$ $E2: 0.674$ $M2: 10.7$

E1(+M2)

S. Bayer, et al., Nucl. Phys. A650, 3 (1999)



ICC from total intensity balances – when to use



- **Be careful near energies close to shell binding**
- **Accuracy of gamma-ray intensity measurements controls the range of its use**

$$\alpha_{out}^{tot} \equiv (I_{\gamma_{in}} / I_{\gamma_{out}}) \times (1 + \alpha_{in}^{tot}) - 1$$

$\delta I_\gamma / I_\gamma$ ~ 10%

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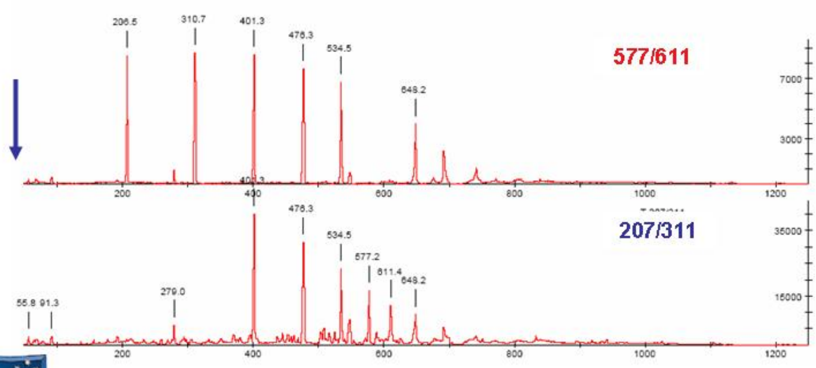
ICC from total intensity balances – example 2

In-beam: only when gating from "above"

$$I_{\gamma_{in}}^{tot} = I_{\gamma_{in}} \times (1 + \alpha_{in}^{tot}) \equiv I_{\gamma_{out}}^{tot} = I_{\gamma_{out}} \times (1 + \alpha_{out}^{tot})$$

$$I_{\gamma_{out}}^{tot} = I_{\gamma_{in}}^{tot} + I_{sf}$$



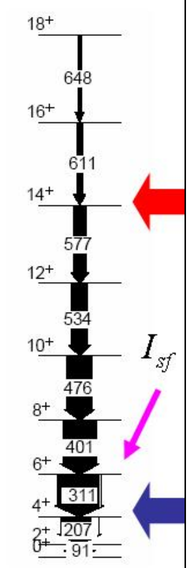


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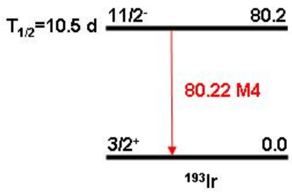
Figure courtesy of F.G. Kondev (ANL)

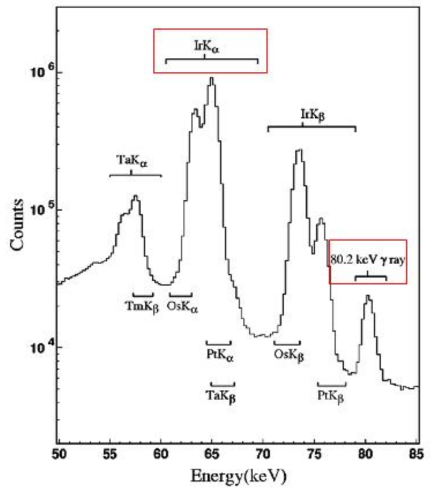
ICC from intensity ratio of K X-rays to γ rays – ^{193m}Ir



$$\alpha_K = \frac{N_{KX}}{N_\gamma} \times \frac{\varepsilon_\gamma}{\varepsilon_{KX}} \times \frac{1}{\omega_K}$$

Looks simple but….

- source preparation (purity)
- efficiency (ε) calibration
- coincidence summing
- etc.



N. Nica, et al., Phys. Rev. ***C 70****, 054305 (2004)*

Determined: α_K = 103.8(8)

Note: α_T = 21333(373)

T. Kibèdi, NSDD Workshop, Trieste 2006

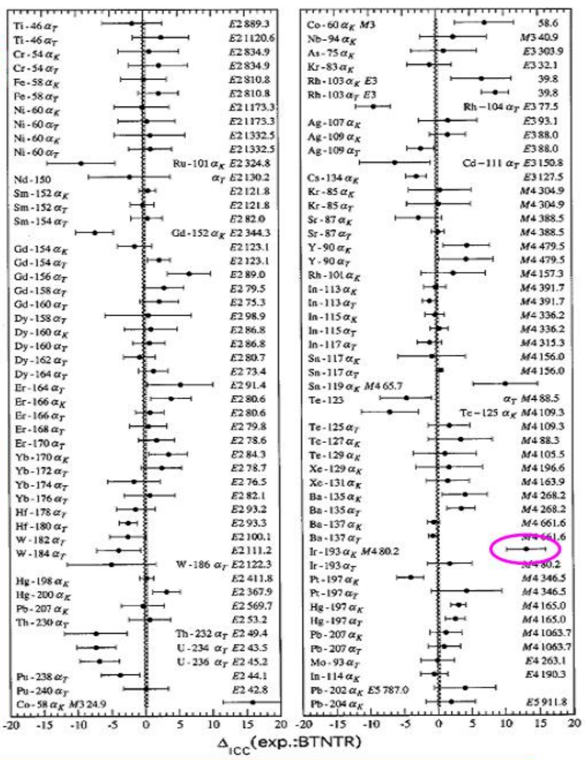
Raman et al. *(2002)*

"How good are the internal conversion coefficients now?"

- 100 experimental ICC
- Deviation of ICC

$$\Delta ICC(Exp:Theory) = \frac{[Icc(Exp) - Icc(Theory)]}{Icc(Theory)} \times 100$$

Hager-Seltzer:	**-3.01(24)%**
Rössel et al:	-2.71(24)%
BTNTR NO Hole	+0.19(26)%
RNIT(1) With Hole	-0.94(24)%
RNIT(2) With Hole	-1.18(24)%



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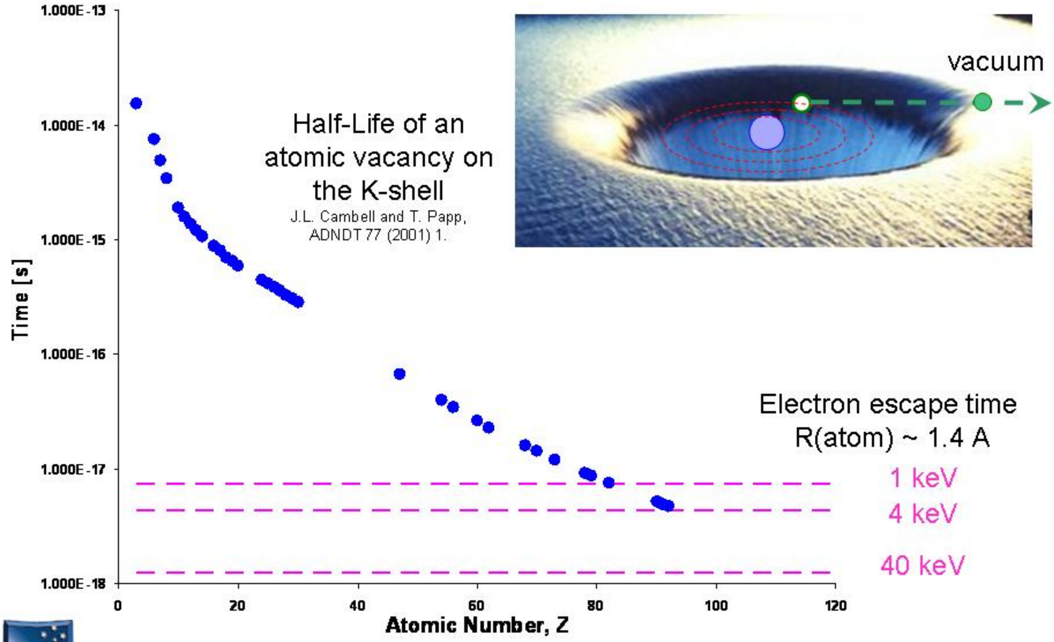
Can we ignore the atomic vacancies?



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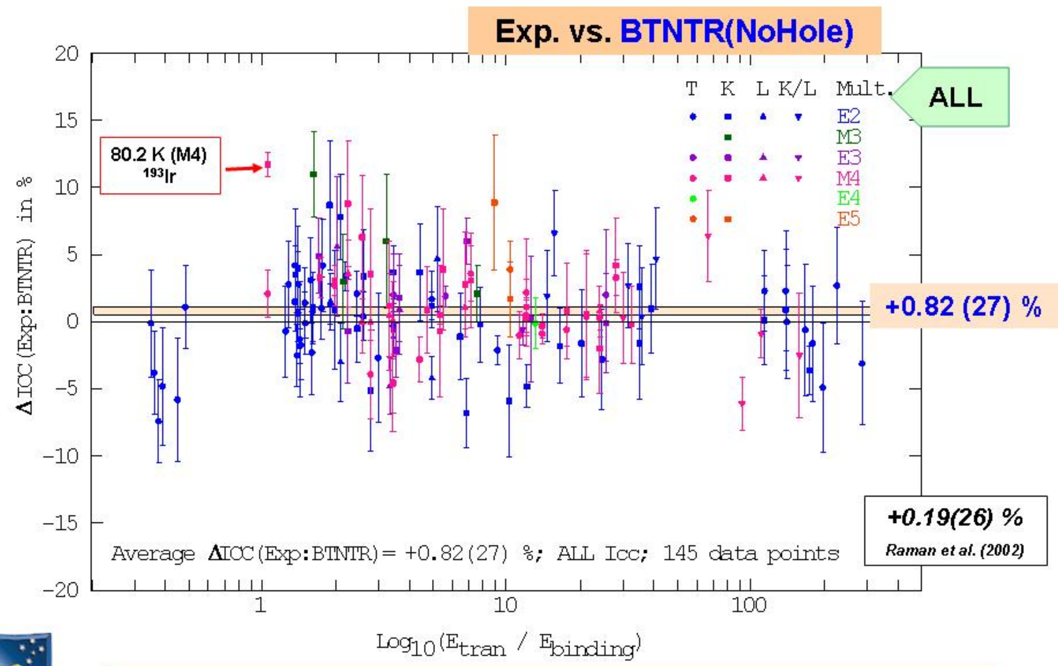
How good are the internal conversion coefficients now?



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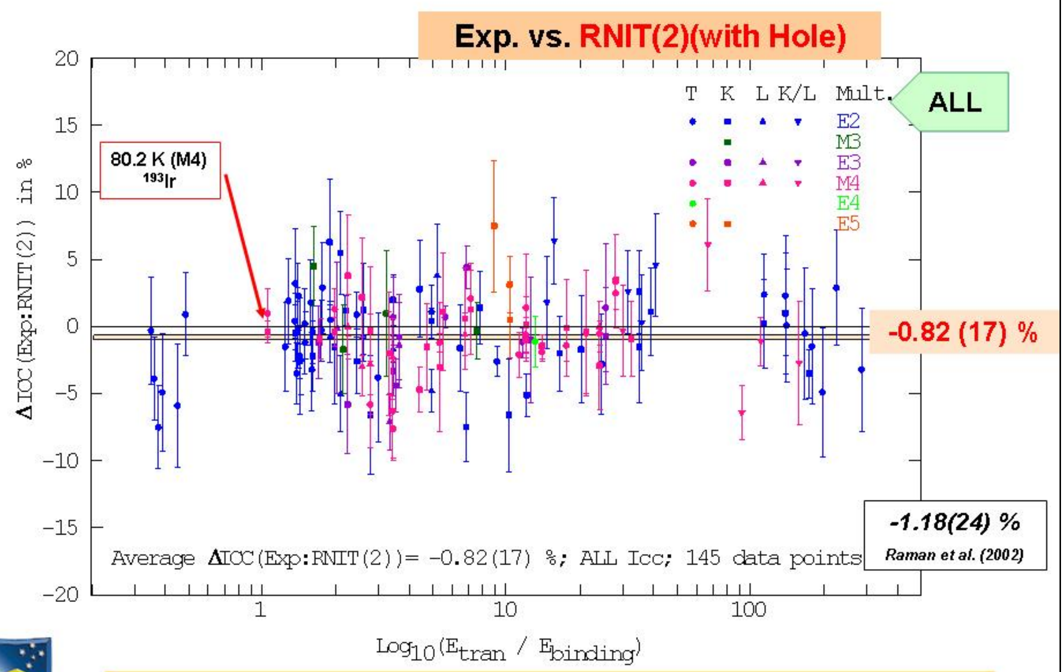
How good are the internal conversion coefficients now?



T. Kibèdi, NSDD Workshop, Trieste 2006

Acknowledgements

G.D. Dracoulis , G.J. Lane, P. Nieminen, H. Maier (ANU)

F.G. Kondev (ANL)

T.W. Burrows (NNDC)

P.E. Garrett (*University of Guelph and TRIUMF)*

S.W. Yates *(University of Kentucky)*

P. Greenlees (*University of Jyväskylä*)

P.M. Walker *(University of Surrey)*

T. Kibèdi, NSDD Workshop, Trieste 2006

Experimental Nuclear Structure Physics:

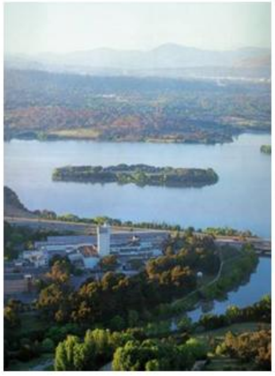
Other Data Considerations

T. Kibédi

Australian National University, Australia

E-mail: Tibor.Kibedi@anu.edu.au

New developments in characterizing nuclei using separators



T. Kibédi

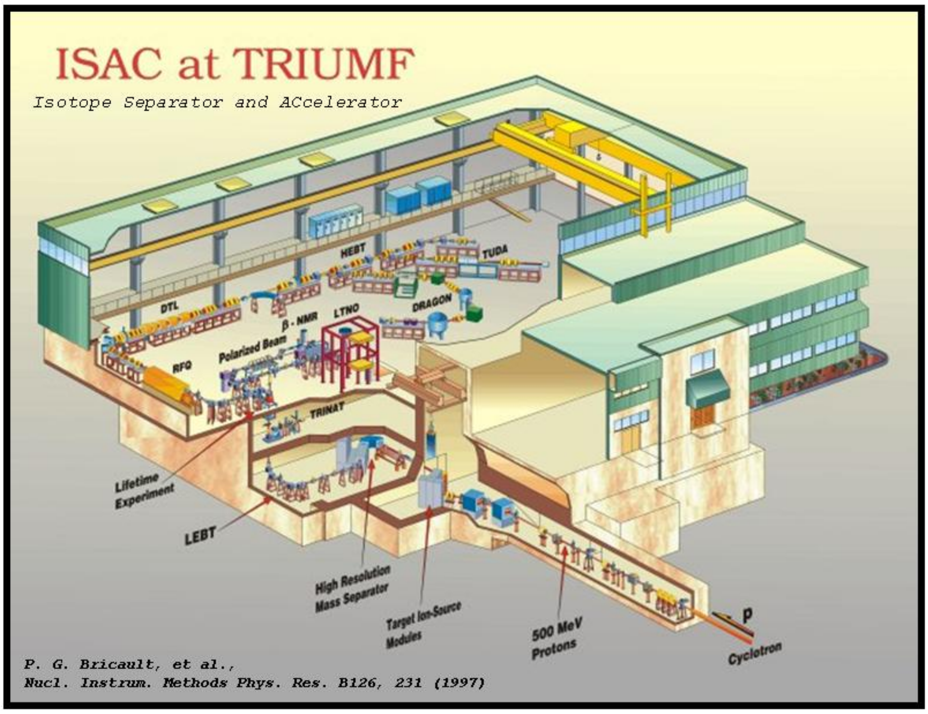
Dept. of Nuclear Physics, Australian National University, Canberra, Australia

Workshop on "Nuclear Structure and Decay Data: Theory and Evaluation" Trieste, Italy, 2006

Outline:

Lecture II: New developments in characterizing nuclei using separators

- *TRIUMF-ISAC*
- Heavy Element Spectroscopy at JYFL
- New compact recoil separator at the ANU
- Future – radioactive beam facilities



T. Kibèdi, NSDD Workshop, Trieste 2006 Courtesy of P.E. Garrett

Gamma-Ray Spectroscopy at TRIUMF-ISAC

The 8π Collaboration

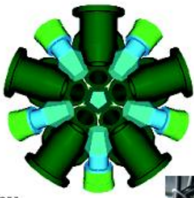
- **A. Andreyev, G.C. Ball, R. Churchman, G. Hackman, R.S. Chakrawarthy, C. Morton, C.J. Pearson, M.B. Smith, *TRIUMF***
- **P.E. Garrett, C.E. Svensson, C. Andreoiu, D. Bandyopadhyay, G.F. Grinyer, B. Hyland, E. Illes, M. Schumaker, A. Phillips, J.J. Valiente-Dobon, J. Wong, *University of Guelph***
- **J.C. Waddington, L.M. Watters, *McMaster University***
- **R.A.E. Austin, *St. Mary's University***
- **S. Ashley, P. Regan, S.C. Williams, P.M. Walker, *University of Surrey***
- **J.A. Becker, C.Y. Wu, *Lawrence Livermore National Laboratory***
- **W.D. Kulp, J.L. Wood, *Georgia Tech.***
- **E. Zganjar, *Louisiana State***
- **J. Schwarzenberg, *University of Vienna***
- **F. Sarazin, C. Matoon, *Colorado School of Mines***
- **J.J. Ressler, *Simon Fraser University***
- **J.R. Leslie, *Queens University,***

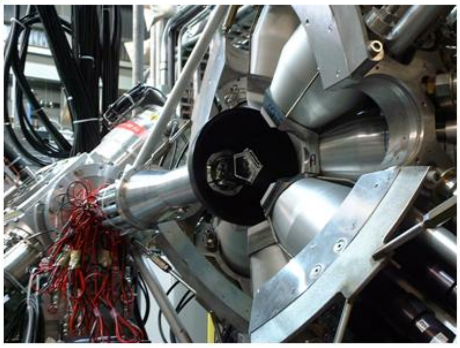
T. Kibèdi, NSDD Workshop, Trieste 2006 Courtesy of P.E. Garrett

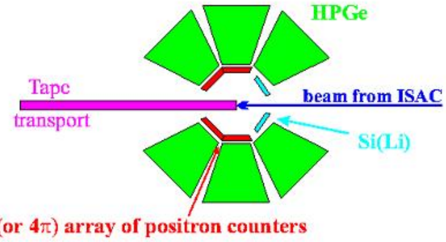
The 8π spectrometer – a versatile tool for nuclear physics

8π Spectrometer at ISAC

20 Compton-Suppressed HPGe detectors
and 10 BaF2 detectors for γ-ray detection
20 plastic scintillators for β detection
5 Si(Li) detectors for conversion electron spectroscopy
Fast, in-vacuum tape transport system







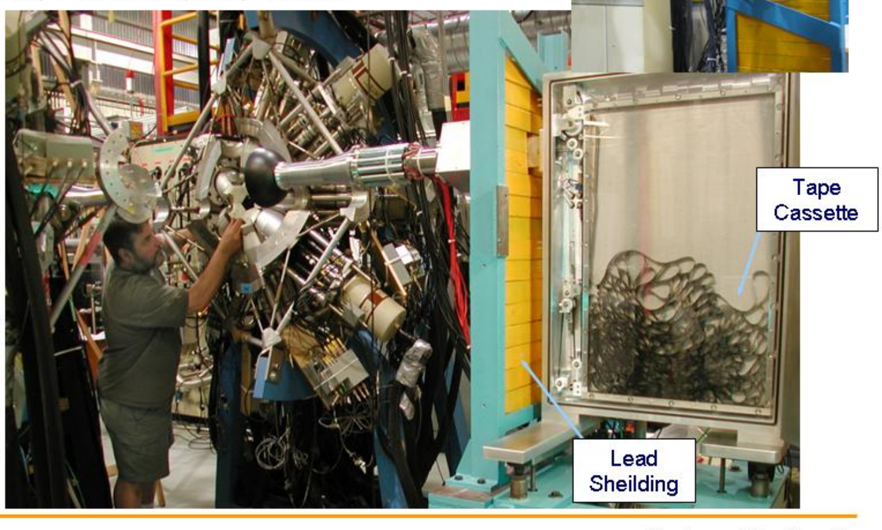
Trigger rate of ~30 kHz; data transfer 5 MB/s

The 8π spectrometer is a unique device for these types of studies. Simultaneous collection of γ-singles, γγ coincidences, β tagging, conversion electrons, and lifetime measurements

Courtesy of P.E. Garrett

Moving tape collector for transport of activity

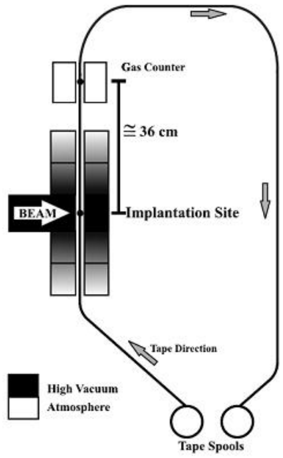
- Beam implanted onto a moving tape
- Allows for movement of long-lived activity out of focus of spectrometer
- Beam collection time variable (down to ~ 10 ms)
- Tape speeds, dwell times, etc., variable

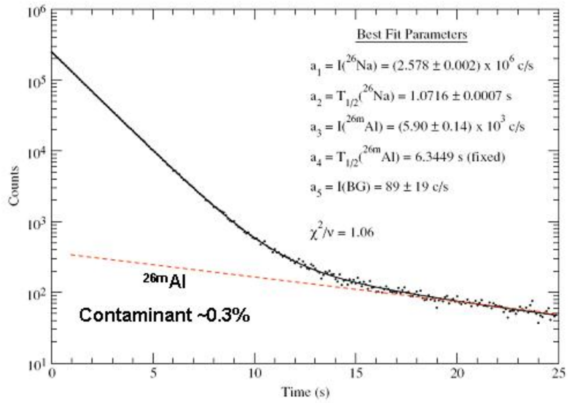


Built by
E. Zganjar, LSU

Courtesy of P.E. Garrett

β decay half-life measurements at a new level





$T_{1/2}$ (^{26}Na) = 1.07128 ± 0.00013 ± 0.00021 s
{Statistical} {Systematic}

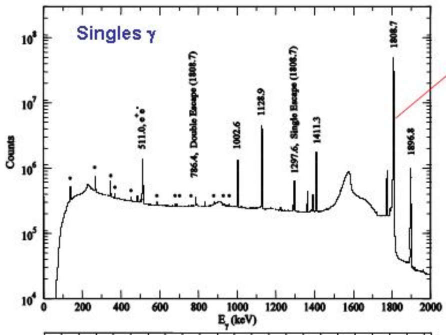
*G.F. Grinyer et al., Phys. Rev. C **71**, 044309 (2005)*

T. Kibèdi, NSDD Workshop, Trieste 2006

Courtesy of P.E. Garrett

Sensitivity to β branches at 10^{-6} level: ^{26}Na → ^{26}Mg

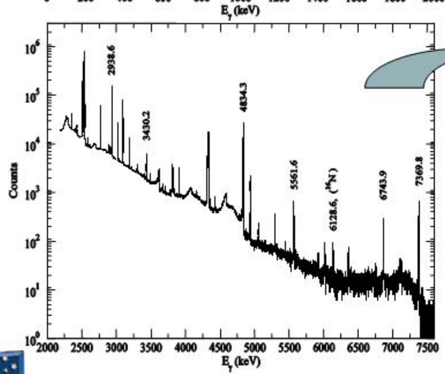
^{26}Na beam: 10^6 s^{-1} for 12 hours, trigger rate 24 kHz

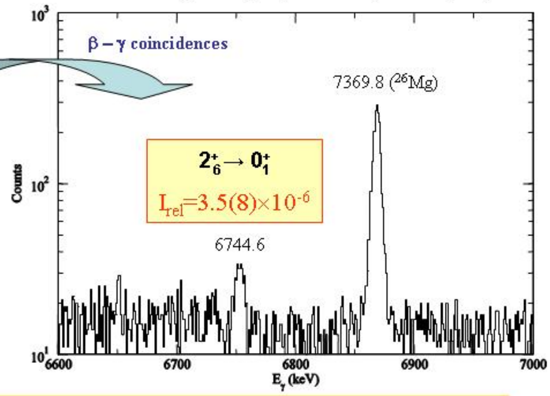


$2_1^+ \rightarrow 0_1^+$
1.54×10^8 counts
I_{rel}=1.0

8π HPGe array
β – γ coincidences

*G.F. Grinyer et al., Phys. Rev. C **71**, 044309 (2005)*





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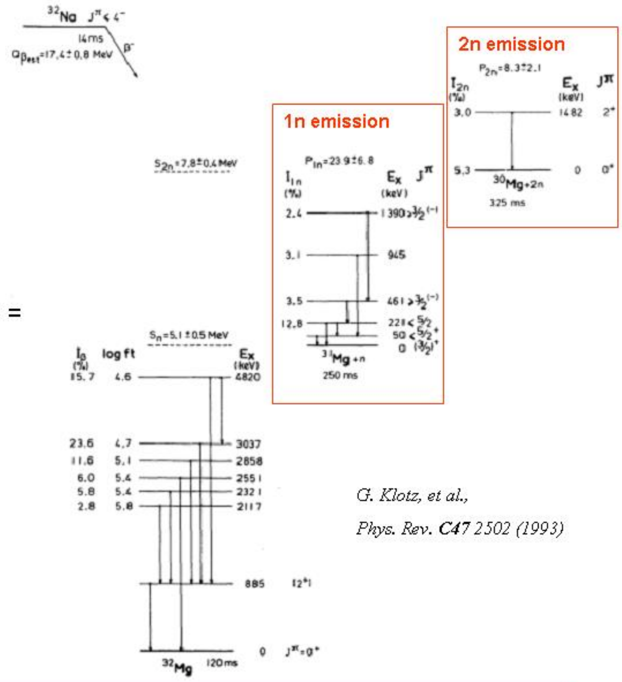
Courtesy of P.E. Garrett

$^{32}Na \rightarrow {}^{32}Mg$ decay – "Island of inversion"

^{32}Na (Z=11, N=21) extra 9 neutrons

N=20 magic number may disappear around Z=11

- ***Large $B(E2; 2^+ \rightarrow 0^+)$***
- Large quadrupole moments
- ***Competition of normal and intruder configurations*** (excitations across the N = 20 shell gap)
- ***Call for detailed spectroscopy ($T_{1/2}$, multipolarity, branching ratio)***

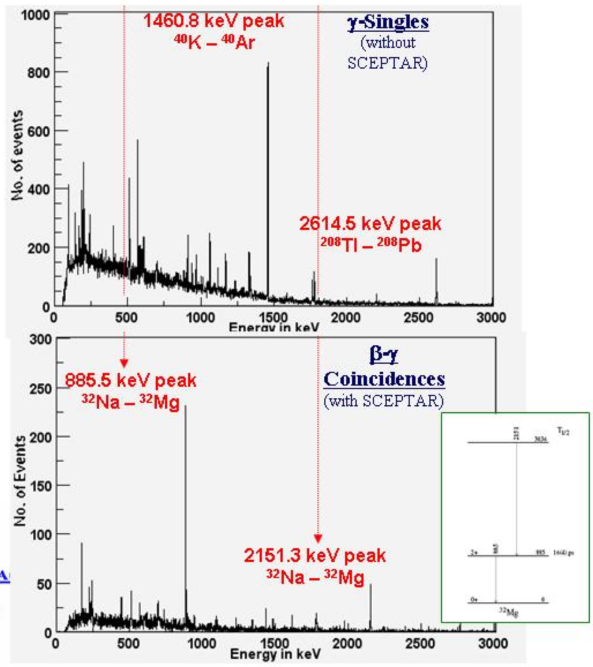


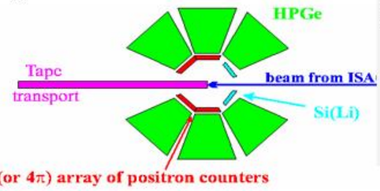
G. Klotz, et al., Phys. Rev. ***C47*** *2502 (1993)*

Courtesy of P.E. Garrett

Preliminary experiment to examine ^{32}Na decay

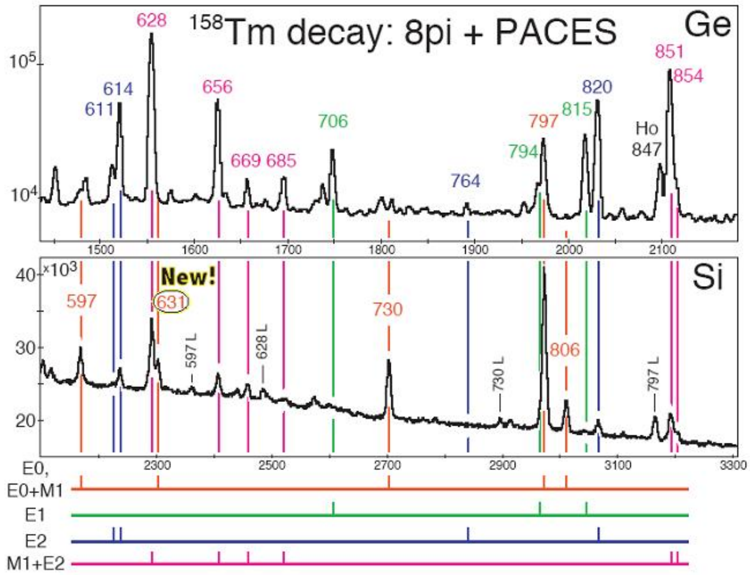
- **^{32}Na decay investigated as a means to study the excited nuclear states of ^{32}Mg (Z=12, N=20).**
- **Investigate the breakdown of shell closures far from stability.**
- **β-γ coincidences measured with 8π and SCEPTAR.**
- **Reduce background and allow weak ^{32}Na decay spectrum to be measured. (^{32}Na beam rate at ~2 ions/s).**
- **Beam production with Ta target insufficient for detailed study, but expect boost of 2-3 orders of magnitude with actinide target**



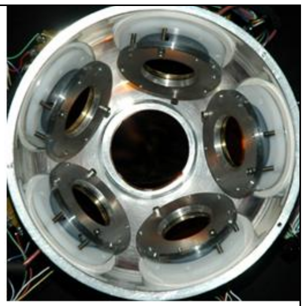


Courtesy of P.E. Garrett

Preliminary experiment on ^{158}Er



W.D. Kulp, et al.



Data are for 1/3 of the sample, 1 detector

- Part of study of N=90 isotopes examining shape transition
- Goal is to search for vital $E0$ transitions
- 7d run collected 1 TB of data with 8π, SCEPTAR, and PACES

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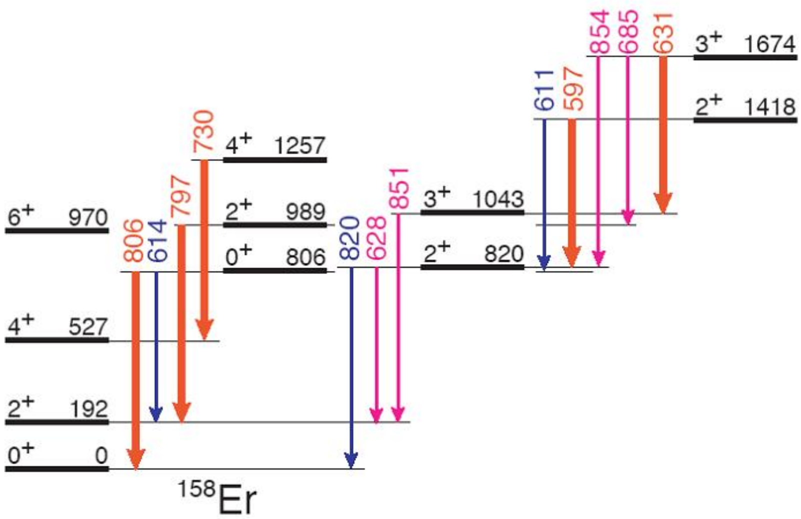
Courtesy of P.E. Garrett

Enhanced E0 transitions observed

E0 transitions: mixing between coexisting shapes of different deformation



Must await lifetime measurements for ρ^2(E0)

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Courtesy of P.E. Garrett

Heavy Element Spectroscopy at JYFL

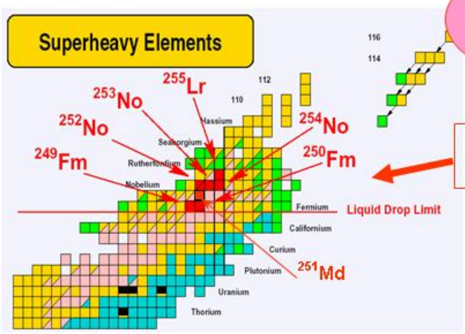
RITU

Next predicted closed shells

Nilsson/Woods Saxon:
Z = … 50, 82, *114*
N =… 50, 82, 126, *184*

Skyrme-Hartree-Fock:
Z = … 50, 82, *126*
N = … 50, 82, 126, *184*

Relativistic mean-field:
Z = … 50, 82, *120*
N = … 50, 82, 126, *172*



Not accessible yet

New spectroscopic data can be used to improve predictions

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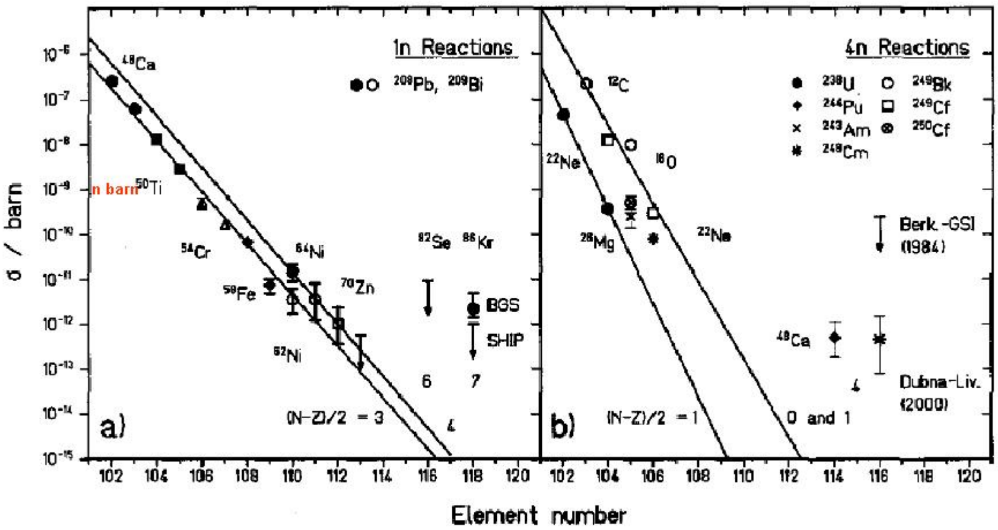
Courtesy of P. Greenlees

Decay Spectroscopy at the limits



S. Hofmann / Prog. Part. Nucl. Phys. 46 (2001) 293.

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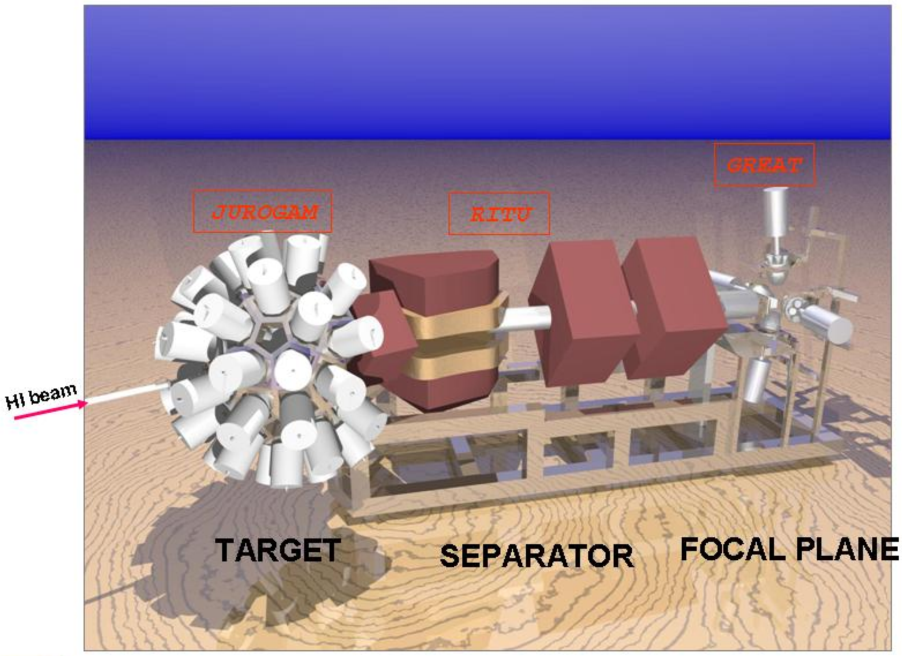
RITU + JUROGAM + GREAT



M. Leino et al., Nucl. Instr. and Meth. B 99 (1995) 653.

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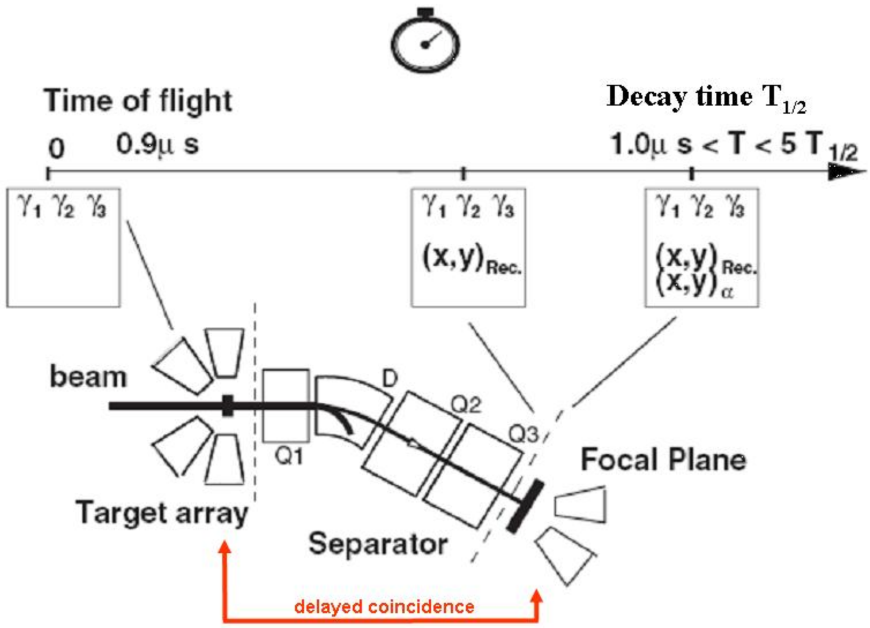
Recoil Decay Tagging (RDT)



R-D Herzberg, J. Phys. G 30 (2004) R123.

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JuroGam

43 Phase I and GASP-type detectors – Ex. EUROBALL and UK-France loan pool

Efficiency ~ 4.2% @ 1.3 MeV

TDR data acquisition system – Data rate ~ 5 MB/s @ 10 kHz

Software BGO suppression

Auto fill system built by University of York, part of GREAT Project

Online/Offline Sorting – Grain developed by P. Rahkila

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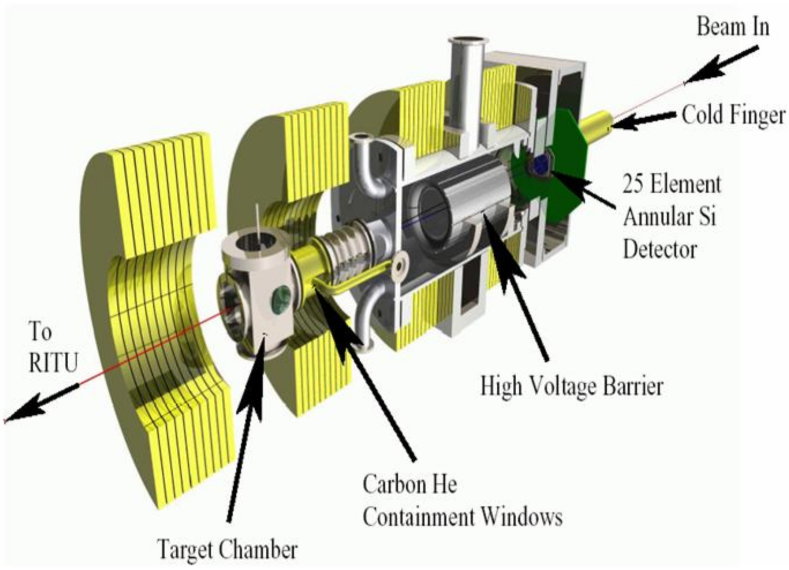
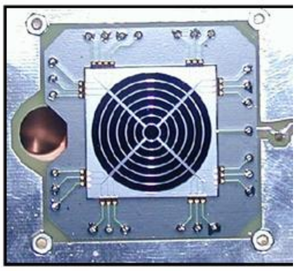
Courtesy of P. Greenlees

The SACRED Electron Spectrometer





THE UNIVERSITY *of* LIVERPOOL

UNIVERSITY OF JYVÄSKYLÄ

H. Kankaanpää, et al., Nucl. Instrum. Meth. Phys. Res. ***A534*** *(2004) 503*

see also P.A. Butler, et al., Nucl. Instrum. Meth. Phys. Res. ***A381*** *(1996) 433*

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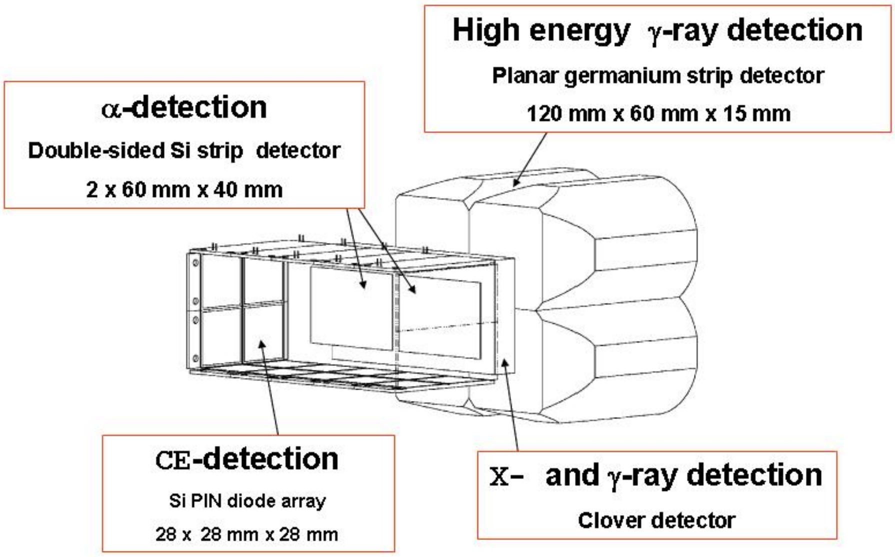
Courtesy of P. Greenlees

The GREAT Spectrometer



Total Data Readout (TDR) Acquisiton System

*R.D. Page, et al., Nucl. Instrum.Meth.Phys.Res. **B 204** (2003) 634.*

T. Kibèdi, NSDD Workshop, Trieste 2006

The SACRED - example (Liverpool-JYFL)

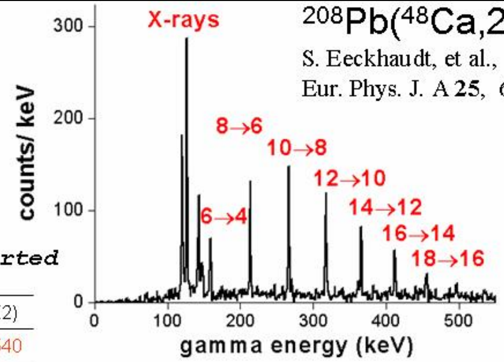
Z=102

BE_K=149.2 keV

Lowest transitions fully converted

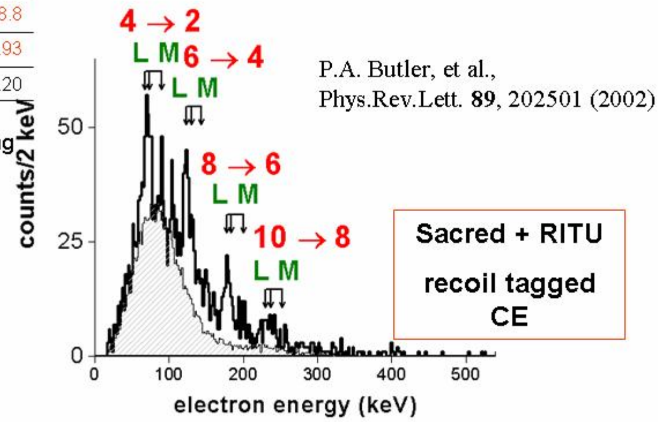
	E_γ (keV)	α_K(E2)	α_L(E2)	α_T(E2)
$2^+ \to 0^+$(*)	44(1)	N/A	1100	1540
$4^+ \to 2^+$(*)	102(1)	N/A	20.6	28.8
$6^+ \to 4^+$	159.5(2)	0.108	2.74	3.93
$8^+ \to 6^+$	214.1(1)	0.122	0.772	1.20

(*) transition not seen; from extrapolation using the Harris formulae



^{208}Pb(^{48}Ca,2n) ^{254}No

S. Eeckhaudt, et al., Eur. Phys. J. A **25**, 605 (2005)



P.A. Butler, et al., Phys.Rev.Lett. **89**, 202501 (2002)

Figure courtesy of P. Greenlees

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Collective nuclear properties

Dynamic Moments of Inertia

$$J^{(2)} = \frac{4\hbar^2}{E_\gamma(I) - E_\gamma(I-2)}$$

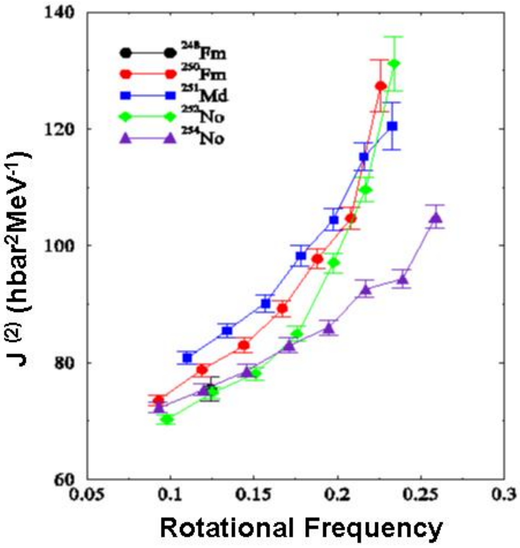
^{250}Fm
Experiment: J.E. Bastin et al.,
Phys. Rev. C73 (2006) 024308

$B\uparrow(E2;2^+ \to 0^+) = 17.9(23)\quad e^2b^2$

$\beta_2 = 0.28(2)$

Raman, et al.,
Atomic Data and Nuclear Data Tables **78**, 1–128 (2001)
GLOBAL FIT to data:

$B\uparrow(E2;2^+ \to 0^+) = 14.7(31)\quad e^2b^2$



Courtesy of P. Greenlees

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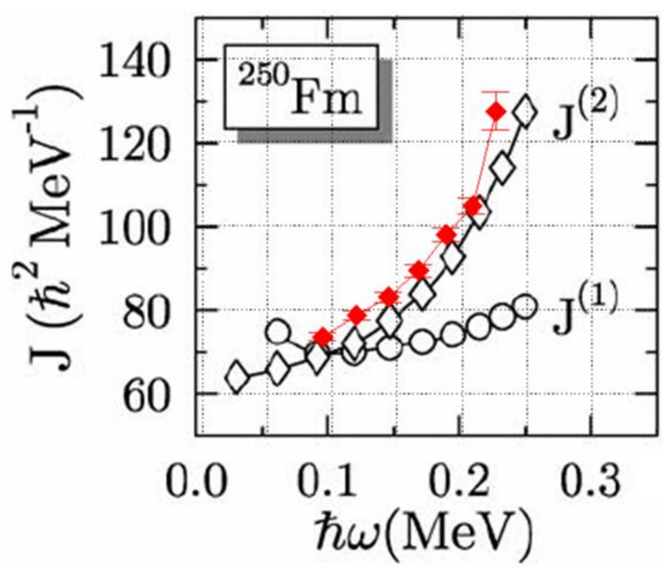
^{250}Fm Dynamic Moment of Inertia $J^{(2)}$



Theory:

M. Bender et al.,
NPA 723 (2003) 354

(mean-field)

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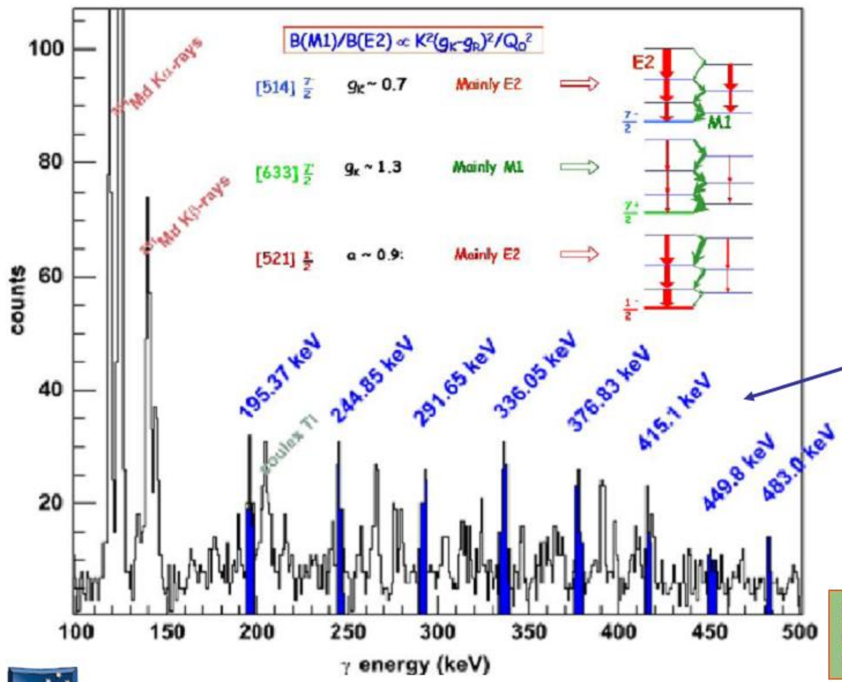
Courtesy of P. Greenlees

ROTATIONAL BAND OF THE ^{251}Md NUCLEUS



Rotational band

Call for prompt e-γ coincidence measurements

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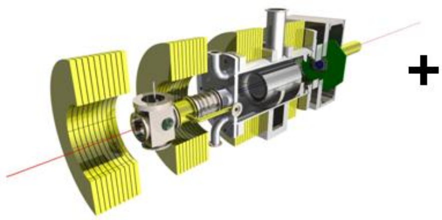
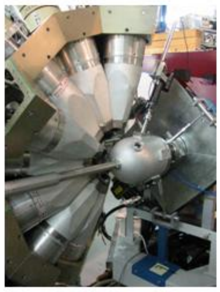
Courtesy of P. Greenlees

Future Developments

 +  = **SAGE**

- Combined In-beam electron and gamma spectroscopy
- Electron Efficiency ~ 10%, Gamma Efficiency ~ 4%
- Liverpool/Daresbury/JYFL Collaboration
- Digital Electronics
- New Vacuum Separator

T. Kibèdi, NSDD Workshop, Trieste 2006

Courtesy of P. Greenlees

RITU + JUROGAM + GREAT Collaborators

University of Liverpool, UK
DAPNIA/SPhN CEA Saclay, France
GSI Darmstadt, Germany
IReS Strasbourg, France
ANL Argonne, USA
University of Helsinki, Finland
University of Oslo, Norway
Ludwig Maximilians Universität, Germany
Niels Bohr Institute, Denmark

New compact recoil separator at the ANU

Australian Research Council, Discovery Grant (Dracoulis, Lane, Kibédi)
ANU Major Equipment Grant (Lane, Kibédi, Dracoulis)

P. Nieminen, G.J. Lane, G.D. Dracoulis, T. Kibédi, D.J. Hinde and N. Dasgupta

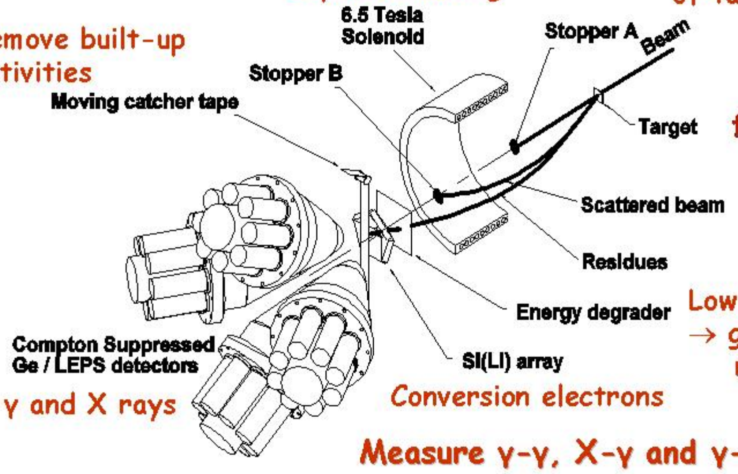
T. Kibèdi, NSDD Workshop, Trieste 2006

System design

Gas-filled, superconducting

Gas window in front of target

Remove built-up activities



$$f = \frac{2\pi m_p v_z \cos(\theta_{lab})}{qB_z}$$

q : average charge state in gas

Low implantation depth → good electron resolution

γ and X rays

Conversion electrons

Measure γ-γ, X-γ and γ-e coincidences

* No recoil detector in use during experiments, possibly a removable array of solar cell detectors for image size measurements during setup

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A compact separator: properties of short-lived isomeric states...

Short flight path to focal plane compared to other separators:

Device	Length	t_{flight}, v/c ~ 2%	t_{flight}, v/c ~ 4%
RITU (Jyväskylä)	4.8 m	800 ns	400 ns
FMA (Argonne)	8.2 m	1370 ns	685 ns
SOLITAIRE	1.7 m	280 ns	140 ns

Sample reactions for ^{189}Pb

^{164}Er(^{29}Si,4n) @ 140 MeV ; v/c = 1.5 %, t_{flight} ~ 380 ns

^{100}Mo(^{91}Zr,2n) @ 350 MeV ; v/c = 4.3 %, t_{flight} ~ 130 ns

* Symmetric reactions and high v/c using heavy beams from the combined tandem + LINAC system; beams being developed

...and longer-lived states

* Very flexible pulsing of high-quality tandem (+LINAC) beams
* Time correlation techniques

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Setup with one unsuppressed Ge detector (May 2005)

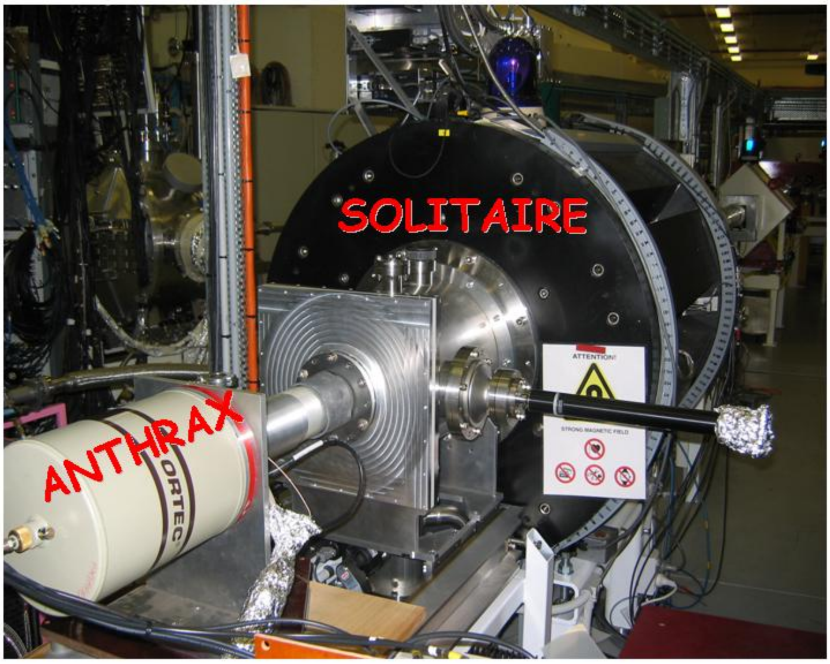
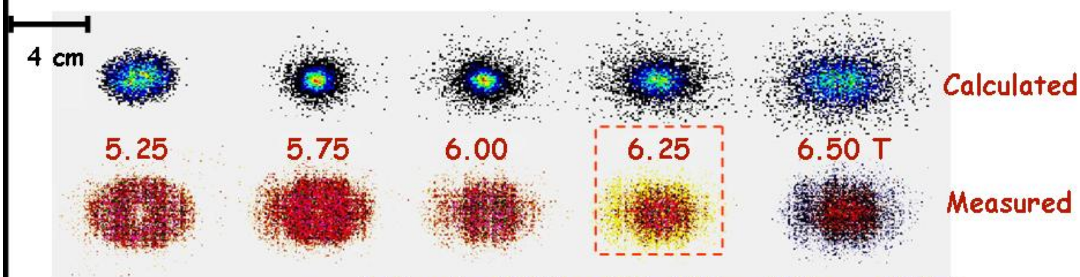


Image size at focal plane < 3 cm (diameter)



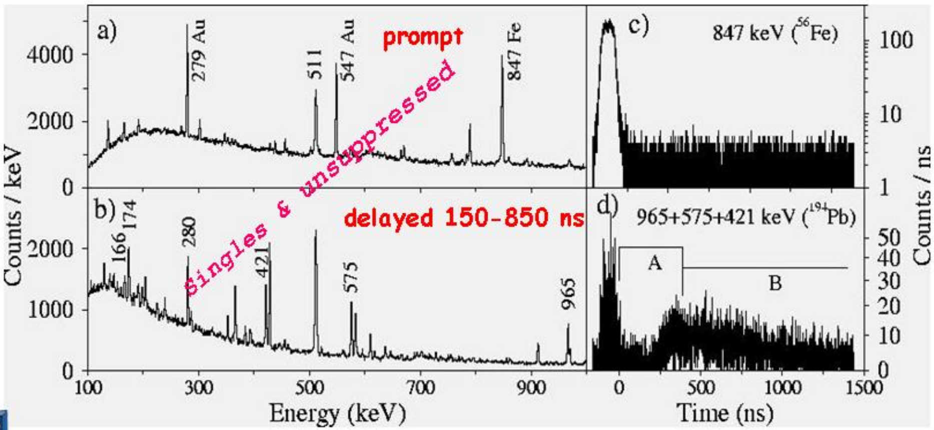
570 μg/cm^2 ^{109}Ag foil: ^{58}Ni + ^{109}Ag → ^{167}Re*

Low energy CE detection – minimize implantation depth by using energy degrader foil

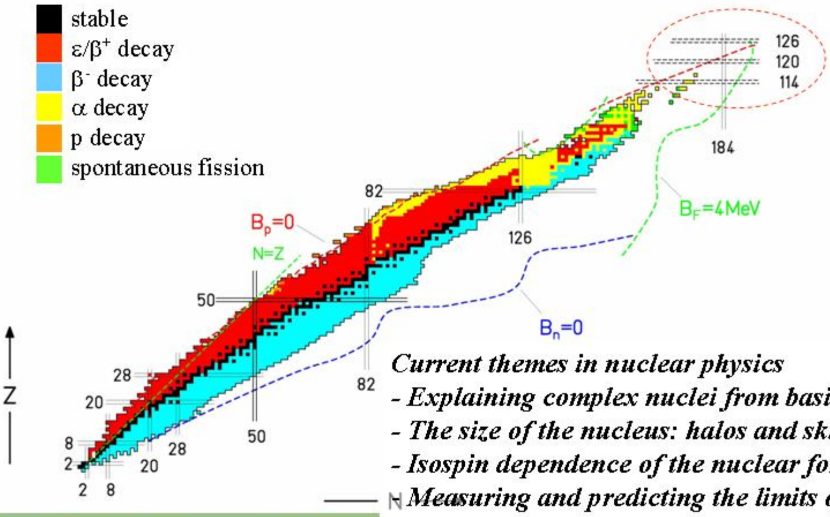
Transport Efficiency
Beam Pulsing and Observation of Gamma Rays at the Focal Plane

$^{170}Er(^{29}Si,5n)^{194}Pb$ @ 145 MeV; Beam ~0.1 μs on / ~1.6 μs off

(a) Delayed (150-850 ns) γ rays: ^{194}Pb transitions
(b) Time spectrum gated by 847-keV transition in ^{56}Fe (beam-related)
(c) Summed time spectrum gated by transitions in ^{194}Pb
Region A: flight time (390 ns) & accumulation during beam-on period (107 ns)
Region B: de-excitation of 565-ns (meanlife) 12^+ isomer in ^{194}Pb



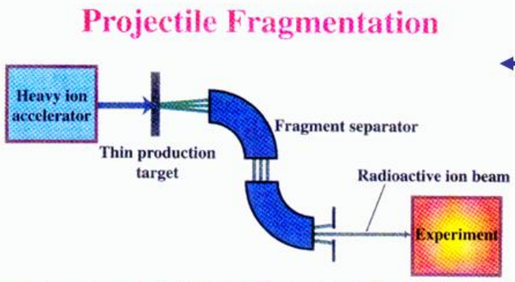
The present chart of nuclei – exploring Super Heavy nuclei



Current themes in nuclear physics
- *Explaining complex nuclei from basic constituents*
- *The size of the nucleus: halos and skins*
- *Isospin dependence of the nuclear force*
- *Measuring and predicting the limits of nuclear existence*
- *Doubly-magic nuclei and shell structure far from stability*
- *The end of Mendeleev's table: superheavies*
- *Understanding the origin of elements*
- *Testing the Standard Model*
- *Applications in materials and life sciences*

Around 3000 of the expected 6000 nuclei have been observed

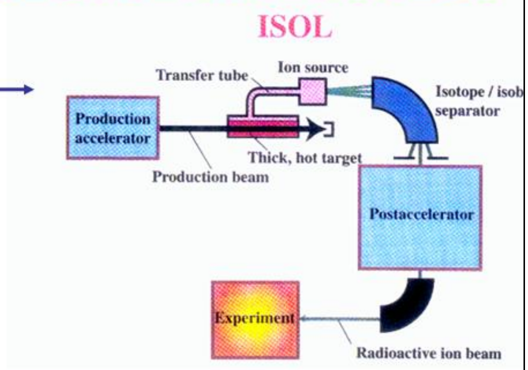
Radioactive beam production: Two complementary methods



GANIL/SISSI, GSI, RIKEN, NSCL/MSU

High energy, large variety of species, Poor optical qualities

GANIL/SPIRAL, REX/ISOLDE, ISAAC/TRIUMF



Low energy, chemistry is difficult, good beam qualities

First generation Radioactive Beam Projects

Location	Start	Driver	Post-accelerator	Upgrade planned
CRC, Louvain-la-Neuve, Belgium	1989	cyclotron p, 30 MeV, 200μA	cyclotrons K = 44 and 110	
SPIRAL, GANIL, Caen, France	2001	2 cyclotrons heavy ions up to 95 MeV/u 6 kW	cyclotron K = 265 2 - 25 MeV/u	new driver
REX-ISOLDE, CERN, Geneva, Switzerland	2001	PS booster p, 1.4 GeV, 2 μA	linac 0.8 - 2.2 MeV/u	energy upgrade 4.3 MeV/u
HRIBF, Oak Ridge, USA	1998	cyclotron p, d, α, 50 -100 MeV 10 - 20 μA	25 MV tandem	
ISAC, TRIUMF, Vancouver, Canada	2000	synchrotron p, 500 MeV, 100 μA	linac 1.5 MeV/u	energy upgrade 6.5 MeV/u

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Selected themes in nuclear physics

- ***How are complex systems built from a few, simple ingredients?***
  - **Our Universe seems quite complex yet it is constructed from a small number of objects**
  - **These objects obey simple physical laws and interact via a handful of forces**
- ***The study of nuclear structure plays a central role here.***
  - **A two-fluid (neutrons and protons), finite N system interacting via strong, short-range forces**
- ***The Goal***
  - **A comprehensive understanding of nuclear structure over all the relevant parameters [Temp., Ang. momentum, N/Z ratio, etc.]**
- ***The Opportunity***
  - **If we can generate high quality beams of radioactive ions we will have the ability to focus on specific nuclei from the whole of the Nuclear Chart in order to isolate specific aspects of the system**

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