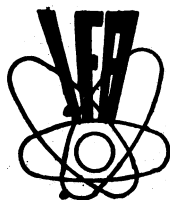


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MULTILEVEL CALCULATION OF ^{235}U NEUTRON

FISSION CROSS-SECTION

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MULTILEVEL CALCULATION OF ^{235}U NEUTRON

FISSION CROSS - SECTION

I.M. Mihăilescu, M. Petraşcu

In the R-matrix theory of Wigner and Eisenbud [1] the relation between the R-matrix and the collision matrix implies the inversion of a channel matrix. Relying themselves on the theory of Bohr [2], according to which the fission proceeds through a limited number of channels, Reich and Moore [3] obtained the multilevel formula of the fission cross-section, applicable in the low energy range.

For thermal neutrons one can assume a single neutron channel a single fission channel and very many capture channels, resulting the following neutron fission cross-section expression [4]:

$$\sigma_{nf} = \frac{4\pi}{k^2} \sum_J g_J \left| \frac{\sum_{\lambda} \frac{\rho_{\lambda n} \rho_{\lambda f}}{E_{\lambda} - E - \frac{i}{2} \Gamma_{\lambda R}}}{\left(1 - i \sum_{\lambda} \frac{\rho_{\lambda n}^2}{E_{\lambda} - E - \frac{i}{2} \Gamma_{\lambda R}}\right) \left(1 - i \sum_{\lambda} \frac{\rho_{\lambda f}^2}{E_{\lambda} - E - \frac{i}{2} \Gamma_{\lambda R}}\right) + \left(\sum_{\lambda} \frac{\rho_{\lambda n} \rho_{\lambda f}}{E_{\lambda} - E - \frac{i}{2} \Gamma_{\lambda R}}\right)^2} \right|^2$$

where the sum on J implies but two terms for $J = I \pm \frac{1}{2}$, I being the spin of the nucleus target and the $\rho_{\lambda i}$ parameters are

defined by the relations :

$$\beta_{\lambda\lambda'}^2 = \frac{1}{2} \Gamma_{\lambda\lambda'} \quad (2)$$

Using the cross-section formula of Reich and Moore, Shore and Sailor [5, 6], analysed the ^{235}U fission cross-section in the energy range of neutrons less than 1.5 eV, admitting the interference between the resonance levels at the neutron positive energy and supposing the existence of two bound levels of the opposite spin state. The interference between the resonances in the same spin state needs, besides the energies and the widths of the levels, an additional parameter to be determined for each resonance, s_{λ} , representing the sign of the product $\beta_{\lambda\lambda'} \beta_{\lambda\lambda''}$ relative to any other resonance.

The present paper is meant to present a code to calculate the ^{235}U thermal fission cross-section, using the experimental data of Shore and Sailor [6] and starting from the formula (1). Taking into account that for s-wave neutrons the statistical weight factor has two possible values, corresponding to the two possible values of the angular momentum, J , of the compound nucleus, and since the spin of the ^{235}U nucleus is 7/2, we can use a statistical weight factor $g = \frac{1}{2}$, introduced in the definition of the neutron reduced width :

$$\Gamma_{\lambda\lambda'}^c = 2g \Gamma_{\lambda\lambda'} E^{-1/2} \quad (3)$$

Replacing $\frac{c}{E}$ for $\frac{\pi}{k_n^2}$, we obtained the following fission cross-section formula :

$$\sigma_{nf} \sqrt{E} = \frac{c}{M^2 + N^2} \sum_{\lambda\lambda'} s_{\lambda} s_{\lambda'} \frac{(\Gamma_{\lambda\lambda}^c \Gamma_{\lambda'\lambda'}^c \Gamma_{\lambda\lambda'}^c \Gamma_{\lambda'\lambda}^c)^{1/2} [(E_{\lambda} - E)(E_{\lambda'} - E) + \frac{1}{2} \Gamma_{\lambda\lambda'} \Gamma_{\lambda'\lambda}]}{D_{\lambda} \cdot D_{\lambda'}} \quad (4)$$

where $c = 3.26 \cdot 10^5$, when the cross-section and neutron energy are given in barns and electron-volts, respectively. The quantities

D_λ are defined by the expressions :

$$D_\lambda = (E_\lambda - E)^2 + \frac{1}{4} \Gamma_\lambda^2 \quad (5)$$

and the quantities M and N represents the real and imaginary part of the denominator of the formula (1), having the forms :

$$M = 1 + \frac{1}{4} \left\{ \sum_\lambda \frac{(\Gamma_{\lambda\pi}^0 E^{\pi/2} + \Gamma_{\lambda f}) \Gamma_{\lambda f}}{D_\lambda} - \sum_{\lambda\lambda'} \frac{[\Gamma_{\lambda\pi}^0 \Gamma_{\lambda f} - g_\lambda g_{\lambda'} (\Gamma_{\lambda\pi}^0 \Gamma_{\lambda'\pi}^0 \Gamma_{\lambda f} \Gamma_{\lambda' f})^{\pi/2}]}{D_\lambda \cdot D_{\lambda'}} \left[(E_\lambda - E)(E_{\lambda'} - E) - \frac{1}{4} \Gamma_{\lambda f} \Gamma_{\lambda' f} \right] E^{\pi/2} \right\} \quad (6)$$

$$N = \frac{1}{2} \left\{ \sum_\lambda \frac{(\Gamma_{\lambda\pi}^0 E^{\pi/2} + \Gamma_{\lambda f}) (E_\lambda - E)}{D_\lambda} + \sum_{\lambda\lambda'} \frac{[\Gamma_{\lambda\pi}^0 \Gamma_{\lambda f} - g_\lambda g_{\lambda'} (\Gamma_{\lambda\pi}^0 \Gamma_{\lambda'\pi}^0 \Gamma_{\lambda f} \Gamma_{\lambda' f})^{\pi/2}][\Gamma_{\lambda f} (E_{\lambda'} - E) + \Gamma_{\lambda' f} (E_\lambda - E)] E^{\pi/2}}{D_\lambda \cdot D_{\lambda'}} \right\} \quad (7)$$

The numerical calculations were carried-out on a IBM-360D-30 electronic computer. The program computes the fission cross-section in two cases, using the multilevel expression (4) and the singlelevel formula :

$$\sigma_{ff} \sqrt{E} = c \sum_\lambda \frac{\Gamma_{\lambda\pi}^0 \Gamma_{\lambda f}}{(E_\lambda - E)^2 + \frac{1}{4} \Gamma_\lambda^2} \quad (8)$$

In figure (1) the curves computed with the multilevel and singlelevel fission cross-section formula are compared with the experimental data. The ^{235}U resonance parameters used in computation are listed in tabel (1). One can notice the existence of a good fit between the curve (a) and the experimental points for the first two resonance levels at 0.29 and 1.14ev. The curve (a) was obtained assuming the interference between the first seven resonance levels, according to the formula (4), to which we added the contribution of two bound levels, of the opposite spin state, using the formula (8). The curve (b) represents the computed fission cross-section assuming

a sum of contributions from each resonance term, according to the formula (8). It is seen the failure of the single level formula in obtaining the fission cross-sections.

Figure (2) illustrates the values computed for the ^{235}U fission cross-section in the proximity of the thermal energy of neutrons, according to the multilevel formula (4) and the experimental points found in the measurements performed in our laboratory to determine absolutely the ^{235}U fission cross-section for 2200m/sec neutrons [7]. The experimental values were normalised to the ^{235}U fission cross-section value for thermal neutrons determined in our absolute measurements :

$$\sigma_{nf}^T = (581.7 \pm 7.8) \text{ barns}$$

It is seen a very good agreement in the energy range from 0.017ev to 0.6ev, but for the low energy less than 0.021 ev the experimental errors become greater than 2%. For energies above 0.6ev, a poor fit resulted because our measurements were performed in conditions which permitted a very good determination of the ^{235}U fission cross-section for 0.023ev neutrons, when the rotation speed of the chopper was chosen as being low enough.

Finally, we have studied the importance of choosing of the resonance levels and parameters in the computation of the fission cross-section value for the 2200 m/sec neutrons. It has been settled that the bound resonance level at 1.45 ev has a great influence in the low energy range. In figure (3) were drawn the multilevel curves computed for three different values of the neutron width of this level.

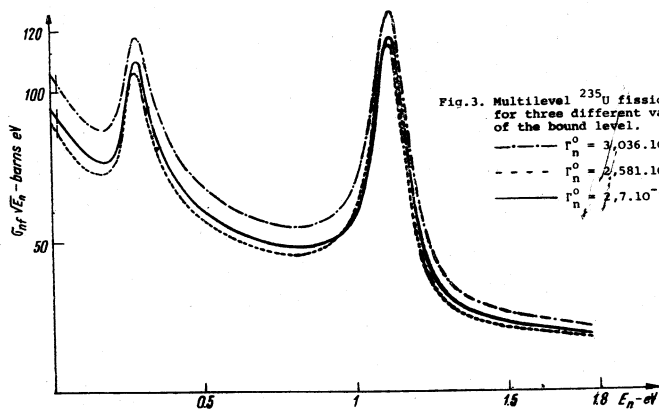
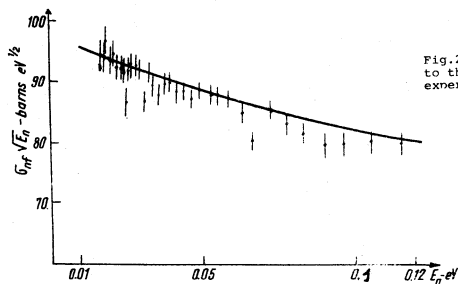
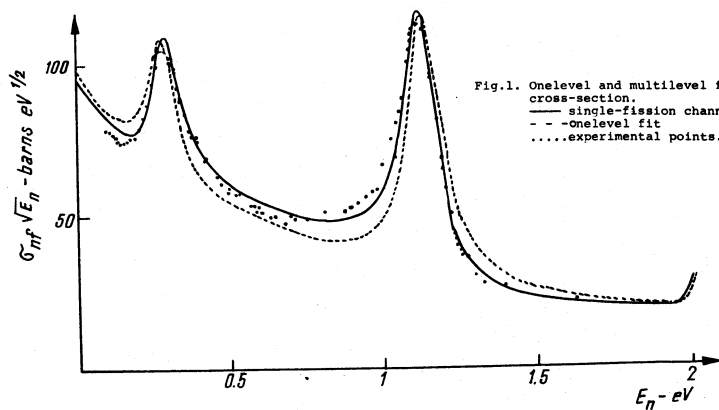
- a). the curve with $\Gamma_n^0 = 3.036 \times 10^{-3} \text{ eV}$, value used by Shore and Sailor [6] ;
- b). the curve with $\Gamma_n^0 = 2.581 \times 10^{-3} \text{ eV}$, value used by de Saussure [2]
- c). the curve with $\Gamma_n^0 = 3.77 \times 10^{-3} \text{ eV}$; value which we have used to

obtain the best fit of the computed thermal fission cross-section with the experimental value.

As can be seen from this figure, a very great difference among this three curves exists in the low energy region. In this respect, it is also obvious table (2), in which ^{235}U fission cross-sections values for 2200 m/sec neutrons are listed for different neutron width values of the bound level at - 1.45 ev. Taking into account the difficulty of the determination of this level from experiences and the great influence that it exerts on the thermal fission cross-section value, we preferred to use in computing the fission cross-section a value of the neutron width fitting the value found experimentally for the fission cross-section.

The second bound level, at - 0.02 ev presents a minor influence on the fission cross-section value for thermal neutrons. Thus, neglecting this level in computing the fission cross-section, a value 562.83 barns was obtained, that means a 3.5% variation in the fission cross-section values.

As concerns the influence of the distant resonance levels on the thermal fission cross-section value, we found it to be a very little one. Thus, the neglect of the level at 6.40 eV gives a variation in the fission cross-section value for 2.200 m/sec neutrons less than 0.02 %.



Abstract: A multilevel calculation of the ^{235}U neutron fission cross-section is presented. The values for neutron energy less than 2 eV are computed with a program written in FORTRAN IV. The curves obtained using the multilevel and single level fission cross-section formula are compared with experimental data. It is obvious the failure of the single-level formula. The values computed for neutron energies less than 0.6 eV are in good agreement with the experimental points found in our measurements to determine absolutely the ^{235}U fission cross-section for 2200 m/sec neutrons. The influence of the bound resonance levels in computing the fission cross-section values for low energy neutrons is studied.

Table 1.

^{235}U resonance parameters. The values were used to generate the curves shown in Figure 1.

E_λ (eV)	Γ_λ (10^{-3} eV)	$\Gamma_{\lambda r}$ (10^{-3} eV)	$\Gamma_{\lambda f}$ (10^{-3} eV)	$2g \Gamma_{\lambda n}$ (10^{-6} eV)	Relative sign of the product $\beta_{\lambda i} \beta_{\lambda j}$
-1.45	256	33	223	2770	
-0.02	97	34	63	0.72	
0.282	99.7	27.9	71.8	4.49	+
1.138	134	41.9	92.1	12.9	-
2.036	41.4	34.6	668	5.3	-
3.599	81.4	37	45	23.4	-
4.847	27.8	25.5	2.3	25	-
6.40	42	33	9	100	-
8.795	93	33	60	257	-

Table 2.

^{235}U thermal neutron fission cross-section values. In computing it was used five different

half-widths of the bound level, $\Gamma_{\lambda n}$ (10^{-3} eV)	σ_{np} (barns)
2.581	543.07
2.650	557.45
2.700	567.37
2.770	582.40
3.036	637.90

A P P E N D I X

By the following program, written in FORTRAN IV, we have computed the sums (4) and (8) for neutron energies less than 2eV.

The identifiers GAMA1, GAMA2, GAMA3 and GAMA4 stand for $\sqrt{\lambda_{\pi}}$, $\sqrt{\lambda_f}$, $\sqrt{\lambda_{\gamma}}$ and $\sqrt{\lambda}$, respectively. The sign s_{λ} , of the product $\beta_{\lambda i} \beta_{\lambda j}$ relative to the first ^(resonance) was identified by ISEMN.

The neutron energy parameter, identified as E1, was varied by steps of 0.001 eV from 0.01 to 0.15 eV, except the interval from 0.025 to 0.026 eV where the step was 0.0001 eV. In the energy range from 0.15 eV to 2 eV we have chosen a step of 0.01 eV.

At the energy value corresponding to 2.200 m/sec neutrons, besides the above mentioned sums, the fission cross-section value was listed.

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DIMENSION R(9,9),E(9),GAMA1(9),GAMA2(9),GAMA3(9),GAMA4(9),D(9),H(9)
1) O(9),P(9),C(9),G(9),ISEMN(9)
REAL E1,SUM,SIGMA1,SIGMA2,A,M,N
WRITE(3,2)
2) FORMAT(' ',40X,'THE THERMAL NEUTRON FISSION CROSS-SECTION ',35X,/,
140X,'*****',32X,/,30X,/,
1'ENERGY',30X,'CROSS-SECTION',39X,/,50X,'ONELEVEL',11X,'MULTILEVEL'
1,7X,'THERMAL',26X,/,32X,'EV',13X,'BARN*SQRT(EV)',6X,
1'BARN*SQRT(EV)',6X,'BARN',27X,/)
50) FORMAT(' ',30X,F6.4,15X,F6.2,14X,F6.2,42X)
READ(1,3)(E(I),GAMA1(I),GAMA2(I),GAMA3(I),GAMA4(I),ISEMN(I), I=1,9)
3) FORMAT(F5.3,4E10.4,12)
E1=0.01
8) SUM=0.
DO 15 I=1,9
15) SUM=SUM+(GAMA1(I)*GAMA2(I)/((E(I)-E1)**2 +GAMA4(I)**2/4.))
SIGMA1=0.326E+06*SUM
A=0.
M=0.
N=0.
DO 30 J=3,9
D(I)=(E(I)-E1)**2+GAMA3(I)**2/4.
H(I)=(ISEMN(I)*SQRT(GAMA1(I)*GAMA2(I)))/D(I)
O(I)=(GAMA1(I)*SQRT(E1)+GAMA2(I))*GAMA3(I)/D(I)
P(I)=(GAMA1(I)*SQRT(E1)+GAMA2(I))*((E(I)-E1)/D(I)
N=M+O(I)/4.
N=N+P(I)
DO 30 J=3,9
5) C(J)=(E(J)-E1)**2+GAMA3(J)**2/4.
G(J)=(ISEMN(J)*SQRT(GAMA1(J)*GAMA2(J)))/C(J)
A=A+H(I)*G(J)*((E(I)-E1)*(E(J)-E1)+GAMA3(I)*GAMA3(J)/4.)
M=M+{0.25*SQRT(E1)*ISEMN(I)*ISEMN(J)*SQRT(GAMA1(I)*GAMA2(I)*
IGAMA1(J)*GAMA2(J))-GAMA1(I)*GAMA2(J))*((E(I)-E1)*(E(J)-E1)-
IGAMA3(I)*GAMA3(J)/4.)/D(I)*C(J)}
N=N+{0.5*SQRT(E1)*(0.5*GAMA1(I)*GAMA2(J)*GAMA3(J)*((E(I)-E1)+
IGAMA3(I)*((E(J)-E1))-ISEMN(I)*ISEMN(J)*SQRT(GAMA1(I)*GAMA2(I)*
IGAMA1(J)*GAMA2(J)*GAMA3(J)*((E(I)-E1))/D(I)*C(J))}
30) CONTINUE
M=1.+M
N=N/2.
SUM=0.
DO 40 I=1,2
40) SUM=SUM+(GAMA1(I)*GAMA2(I)/((E(I)-E1)**2+GAMA4(I)**2/4.))
SIGMA2=0.326E+06*(SUM+A/(M**2+N**2))
WRITE(3,50) E1,SIGMA1,SIGMA2
IF(ABS(E1-0.0253)-0.00001)13,18,18
13) B=SQRT(E1)
SIGMA2=SIGMA2/B
WRITE(3,51) SIGMA2
51) FORMAT(' ',86X,F6.2,27X)
18) IF(E1-2.)10,10,9
10) IF(E1-0.14907)11,12,12
11) IF(E1-0.0259)16,14,14
16) IF(ABS(0.0260-E1)-0.0018)17,14,14
17) E1=E1+0.0001
GO TO 8
14) E1=E1+0.0010
GO TO 8
12) E1=E1+0.01000
GO TO 8
9) STOP
END

```