ÇNAEM 34

December 1965

PARAMETERS FOR THE FIRST NEUTRON RESONANCE IN Cd^{113} *

R. Ö. Akyüz ***, Ç. Cansoy †, F. Domaniç

* Work performed under the auspices of the Turkish Atomic Energy Commission.

*** Turkish Navy Training Command
Heybeliada, Istanbul, Turkey

† Present Address: New York State University
Buffalo, N.Y., U.S.A.

ABSTRACT

The total neutron cross section of Cd^{113} has been measured in the energy range from 0.025 eV to 1 eV with a crystal spectrometer using the Be($10\bar{1}1$) and NaCl(200) crystal planes as monochromators. A new method for second order correction was applied, and the resonance at 0.181 eV was fitted to a Breit-Wigner single-level formula by the method of least squares. The resonance parameters obtained from the analysis are: $E_0 = 0.181 \pm 0.003$ eV, $\sigma_0 = 7847 \pm 187$ barns, $\Gamma = 0.1087 \pm 0.0033$ eV and $\Gamma_n = (0.0791 \pm 0.0032) \times 10^{-3}$ eV assuming that the statistical weighing factor is $g = 3/4$.

I. INTRODUCTION

The slow neutron cross sections of most heavy nuclei have been measured and the Breit-Wigner parameters for many of the observed resonances have been evaluated.[1] However, accurate values of the parameters for many resonances are still lacking. One such example is the first resonance in Cd^{113}, which is a resonance of particular interest because of the frequent use of cadmium as an absorber in neutron experiments. Previous published work on the first cadmium resonance[2-5] appear inconsistent and disagreed significantly with some unpublished data obtained by Shore.[6]

---

[1] Many of the results are summarized in D. J. Hughes and R. B. Schwartz, "Neutron Cross Sections" BNL 325, 2nd Edition, July 1, 1958, Brookhaven National Laboratory; and in D. J. Hughes, B. A. Magurno, and N. K. Brussel, "Neutron Cross Sections", Supplement No. 1 to BNL 325, January 1, 1960.

[2] L. J. Rainwater, W. W. Havens, Jr., C. S. Wu, and J. R. Dunning, Phys. Rev. 71, 65(1947).

[3] E. Bagge and H. Kürston, Atomkern Energie 9 No. 2, 47(1964).

[4] P. Höhne, Ann. Physik 7, 50(1961).

[5] B. N. Brockhouse, Can. J. Phys. 31, 267(1953).

[6] F. J. Shore, Brookhaven National Laboratory (private communication).

When a new spectrometer was constructed at ÇNAEM† and installed at the TR-1 Reactor†, a new measurement of the cadmium cross section was undertaken as one of the initial experiments. Since the goal of these measurements was to obtain more precise values of the resonance parameters, great care was taken in all phases of the work to eliminate or reduce errors and uncertainties.

II. EXPERIMENTAL DETAILS

A detailed description of the ÇNAEM spectrometer appears elsewhere,[7] therefore only a few pertinent characteristics are listed here. The neutron beam is collimated by a pair of Soller collimators** placed symmetrically before and after the monochromator. These each have an angular divergence of 7 minutes of arc giving a net angular resolution width of approximately 5 minutes of arc. The spectrometer resolution was 1.405 μsec/m, using the Be(10$\bar{1}$1) planes as monochromator, and 2.287 μsec/m using NaCl(200)

The energy of the spectrometer is determined from the absolute Bragg angle. This angle is measured on a steel tape placed on an arc at a

---

† ÇNAEM is the Çekmece Nuclear Research and Training Center (Çekmece Nükleer Araştırma ve Eğitim Merkezi) of the Turkish Atomic Energy Commission located near Istanbul, Turkey, and is the site of the TR-1, the first Turkish research reactor.

7 ÇNAEM Report No. 33.

** These collimators were fabricated at Brookhaven National Laboratory under the auspices of the BNL-ÇNAEM Cooperative Program.

distance of 171.974 cm from the axis of rotation. At this distance, an angle of 6017 minutes of arc corresponds to 301 cm on the tape. Errors in absolute Bragg angle are less than ± 1 minutes of arc due to uncertainties in the placement of the radius of the measuring arc. The zero angle was determined by putting the spectrometer arm in approximately zero angle position when reactor operates at low power and plotting count rates vs arm scales, which gave a triangle shaped curve. The arm scale corresponding to the vertex of the triangle determines the zero angle. The precision in the zero angle is ± 0.2 minutes of arc. With Be($10\bar{1}1$) monochromator the Bragg angle of the Cd^{113} resonant energy, E_0 = 0.181 eV, is 11 degs 11 mins. Thus at this energy the uncertainty in the energy scale is ± 0.00047 eV.

Samples of three different thicknesses were prepared from cadmium plate of natural isotopic abundance. The thickness of each sample was determined by precise measurements of the foil area and the weight. Uniformity of the thinnest sample was checked by cutting it into 24 separate parts and determining the thickness of each part. The arithmetic mean of these thicknesses gave a value of 0.0243 gr/cm^2 and the standard deviation from the mean value was found to be of 0.0005 gr/cm^2. Consequently error in cross section at the resonant energy is 16 barns due to disturbances in the uniformity of the sample. This kind of error is far less than 16 barns at energies other than resonant energy and also for the thicker samples. The samples used had thicknesses of 0.01252±0.00005, 0.002809±0.000016, 0.07308±0.00034 cm which correspond respectively to the inverse thickness, 1/N, of

1722±7, 7676±45, and 295±1.4 barns per atom. To minimize error, all cross section values were determined from transmission values lying in the range $0.20 < T < 0.85$, the appropriate sample being selected at each energy to meet this condition. The three samples mentioned above covered the energy region 0.025 to 0.062 eV, 0.062 to 0.3, and 0.3 to 1 eV respectively.

At each energy, a series of counts were taken with the sample in and out of the beam and with the monochromator set "on-Bragg" and 5 degrees "off-Bragg" (background). Each set of counts were repeated many times and in all cases the statistical uncertainties in the observed transmission are less than 1.3%.

III. SECOND ORDER CONTAMINATION

The observed transmission of a sample placed in the spectrometer beam is

$$T_{obs}(E) = \sum_i f_i(E) T_i(E) \tag{1}$$

where $f_i(E)$ is the fraction of the neutrons of order i, and $T_i(E)$ is the true transmission of the sample at energy $E_i = i^2E$. The $f_i(E)$ and $T_i(E)$ satisfy the following identities:

$$\sum_i f_i(E) \equiv 1 \tag{2}$$

$$T_i(E) \equiv T_1(i^2E)^{\dagger\dagger} \tag{3}$$

In most practical cases, one needs consider only $f_1(E)$ and $f_2(E)$, since $f_2(E)$ is of the order of a few percent, and all higher orders are much smaller. Thus the equations (1) and (2) can be rewritten in the following form.

$$T_{obs}(E) = f_1(E)\ T_1(E) + f_2(E)\ T_1(4E) \tag{4}$$

$$f_1(E) + f_2(E) \equiv 1\ . \tag{5}$$

By introducing the new function

$$f(E) \equiv \frac{f_2(E)}{f_1(E)} \tag{6}$$

and using the identity (5), equation (4) becomes

$$T_1(E) = T_{obs}(E) - f(E)\ [T_1(4E) - T_{obs}(E)] \tag{7}$$

which contains only $T_1(E)$ as an unknown.

---

††This is valid only for ideal resolution. If resolution is finite $T_i(E) \neq T(i^2E)$ since the resolution function for i^{th} order neutrons is sharper than for first order at i^2E.

A. Measurement of the second-order contamination

The second-order fractions $f_2(E)$ were measured for the Be(10$\bar{1}$1) planes using TR-1 reactor and our single-crystal spectrometer. The measurement was made at the energies 0.062 eV and 0.07 eV by using a Gd sample. For the energy interval 0.1-0.2 eV, a Cd sample was used. Finally for 0.275 eV and 0.364 eV we had to use Hf and In samples respectively.

Fig. 1 shows the measured values of the second-order contamination as a function of neutron energy for the Be(10$\bar{1}$1) planes. One notes that the curve is constant for energies above 0.25 eV, and that it has minimum at the point 0.07 eV. The values of $f_2(E)$ are somewhat higher than those obtained at the Brookhaven graphite moderated reactor (BGRR).[8] To account for this difference[8] we have plotted the count rate spectrum as a function of neutron energy for Be(10$\bar{1}$1) planes in Fig. 2. Throughout the region between 0.3 and 1.5 eV our count rate falls as $E^{-0.55}$ rather than as E^{1} as observed at BGRR.[8] Below 0.2 eV the influence of the Maxwellian distribution of the pile flux is apparent. The function $f^*(E)$, which is the ratio of the count rate at 4E to that at E, is determined from Fig. 2. The variation of $f^*(E)$ with energy is plotted in Fig. 3. It should be noted that the higher energy asymptotic value for $f^*(E)$ is 0.467 compared with 0.25 at BGRR[8] which corresponds to $4^{-0.55}$ and 4^{-1} respective The pronounced minimum at 0.07 eV is mainly due to the Maxwellian peak. Since $f_2(E)$ is proportional to $f^*(E)$, the larger values of f (E) at TR-1 are due to the larger values of $f^*(E)$.

The measured second-order functions $f_2(E)$ for the NaCl(200) planes are also shown in Fig. 1. As in the case of Be(10$\bar{1}$1) the curve is

---

[8] R. Haas and F. J. Shore, Rev. Sci. Instr. 30, 17(1959

constant for energies above 0.25 eV, and has minimum 0.07 eV.

B. Performance of the second-order correction

In calculating $T_1(E)$ from Eq. (7) one can substitute $T_{OBS}(4E)$ instead of $T_1(4E)$ as a first approximation. Thus, one obtains

$$T_1(E) \cong T_{OBS}(E) - f(E)[T_{OBS}(4E) - T_{OBS}(E)] \ . \qquad (7')$$

In order to obtain a better approximation, Eq. (7) may be written as,

$$\begin{aligned} T_1(E) = {} & T_{OBS}(E) - f(E)[T_{OBS}(4E) - T_{OBS}(E)] \\ & + f(E)f(4E)[T_{OBS}(16E) - T_{OBS}(4E)] \\ & - f(E)f(4E)f(16E)[T_1(64E) - T_{OBS}(16E)] \ . \qquad (8) \end{aligned}$$

The last term in Eq. (8) is always less than 0.001, since all the f values are less than 10% (Fig. 1) and the expressions in the brackets are always less than 1. Therefore, this term can be neglected to our purposes. Now, as there remains on the right-hand side of Eq. (8) only the quantities which were actually measured, the values of $T_1(E)$ can be obtained with an error less than 0.001.

It should be noticed that the additional information fed in through $T_{OBS}(16E)$ improves the accuracy on the calculation of $T_1(E)$.

For Be(10$\bar{1}$1) plane and for the Cd samples with the values of 1/N 7676 barns and 295 barns the third term of Eq. (8) is less than 0.001 and so it has been neglected. On the other hand, for NaCl(200) plane and for the Cd sample with the value of 1/N 1722 barns the third term varies between 0.002 and 0.003 and therefore has been taken into account.

IV. RESULTS AND ANALYSIS

Observed resonance curves differ from the single-level Breit-Wigner formula in that they are broader in width and lower in peak value. Distortions in experimental data result from the Doppler broadening of the resonance and instrument resolution. Analysis to determine the true resonance parameters requires careful consideration of these effects.

At the energy of the first resonance for Cd^{113}, $E_{max} = 0.177$ eV, the spread in energy of the monochromatic beam, $\Delta E = 0.00286$ eV for the Be(10$\bar{1}$1) reflection. Using this resolution width, the resolution correction was performed[9,10] for the second- order corrected transmission curve $T_1(E)$ at the resonant energy. The resolution correction at the center of the resonance turns out to be only 1.00025 which is quite trivial and can therefore be neglected. The observed total Doppler-broadened cross sections were calculated by means of

$$\sigma_\Delta(E) = -\frac{1}{N} \ln T_1(E) \quad ,$$

where $\sigma_\Delta(E)$ is the Doppler-broadened cross section which should be identical with the experimental data. The results are shown in Fig. 4.

The theoretical total Doppler-broadened cross section[10] is

$$\sigma_\Delta(x,t) \cong \sigma_{\gamma_0} \left(\frac{E_0}{E}\right)^{1/2} \psi(x,t) + \sigma_p \qquad (9)$$

---

[9] V.L.Sailor, H.L.Foote, H.H.Landon, and R.E.Wood, Rev. Sci. Instr. 27, 26(1956).

[10] S.A.Cermabei, L.B.Borst, and V.L.Sailor, Nucl. Sci. Eng. 12, 63(1962).

where

$$x \equiv \frac{E-E_o}{\Gamma/2}, \qquad t \equiv \left(\frac{\Delta}{\Gamma}\right)^2, \qquad \Delta = 2\left(\frac{m}{M}\, kTE_o\right)^{1/2}$$

But according to the studies of Bethe[11] on Doppler Broadening, this formula should not be satisfactory in the region where $E < 0.5$ eV. Therefore some calculations are needed for resonances which lie in this region. Calculations show that, the differences from the expression (9), apparently, are not significant[12]. So one can use expression (9) for correction.

The function $\psi(x,t)$ has been evaluated and is available in tables[13]. In equation (9) additional terms contributed by resonant scattering have been omitted since the ratio of these terms to the first term is less than 1% for Cd^{113} over the resonance region[1]. The potential scattering cross section σ_p was calculated from the relations

$$\sigma_p = 4\pi R_{eff}^2, \qquad R_{eff} = 1.35\, A^{1/3} \times 10^{-13} \text{ cm.}$$

which gives the value $\sigma_p \cong 5$ barns.

The observed width which is also the Doppler-broadened width Γ_Δ and the observed maximum cross section $\sqrt{E_o}\sigma_{\Delta o}$ of the $\sqrt{E}\sigma_\Delta(E)$ curve were determined from the data by using the method of least squares[14]. It is not practical to fit the data directly to the

---

[11] Hans A. Bethe, Revs. Modern Phys. 9, 69(1937

[12] Joseph P. Roberge and Vance L. Sailor, Nucl.Sci.Eng. 7, 502(1960)

[13] Westinghouse Atomic Power Division Report SR-506, 1954(unpublished)

[14] [illegible]wood, H.H.Landon, and V.L.Sailor, Phys. Rev. 98, 639(1955)

Doppler-broadened formula, so an arbitrary function must be derived which adequately approximates the shape of the Doppler-broadened resonance. The arbitrary function chosen to serve this purpose was the capture term of the Breit-Wigner formula[14],

$$\sqrt{E}\,(\sigma_\Delta(E) - \sigma_p) \cong \sqrt{E_o}\,\sigma_o\,\psi(x,t) \cong \frac{\sqrt{E_o}\,\sigma_{\Delta o}\,\Gamma_\Delta^2}{4(E-E_o)^2+\Gamma_\Delta^2} \tag{10}$$

The observed parameters, Γ_Δ and $\sigma_{\Delta o}$ derived from this analysis, were converted to Γ and σ_o by means of the curve of Γ_Δ/Δ vs Δ/Γ , i.e.

$$\frac{1}{2}\,\psi[0,(\tfrac{\Delta}{\Gamma})^2] = \psi[(\tfrac{\Gamma_\Delta}{\Delta})(\tfrac{\Delta}{\Gamma}) \;,\; (\tfrac{\Delta}{\Gamma})^2] \quad , \tag{11}$$

and relation

$$\sigma_o = \frac{\sigma_{\Delta o}}{\psi(0,t)} \tag{12}$$

The values of Γ and σ_o obtained in this way agreed with the values obtained from the "trial and error" method.

V. DISCUSSION

The quantities obtained from analysis of the data are listed in Table I. The agreement between the experimental points and the derived Doppler-broadened Breit-Wigner curve is reasonably good over the entire range (Fig. 4).

Comparison between the parameters in Table I and older measurements[1] disclose an agreement within experimental error. However, the error in σ_o is less than that in older measurements. Furthermore, we hope that we have obtained a better fit to the Breit-Wigner single-level formula.

The effect of resolution correction is negligible and the effect of Doppler correction at the resonant energy is 200 barns. On the other hand, the effect of second-order correction at the resonant energy is 600 barns. Therefore, we have paid a special attention to this correction.

VI. ACKNOWLEDGEMENT

The authors wish to thank Dr. V. L. Sailor from Brookhaven National Laboratory for suggesting to them a new total cross-section measurement and determination of Breit-Wigner parameters for the first neutron resonance in Cd^{113}, and for a critical perusal of the manuscript.

TABLE I

Breit-Wigner Parameters for the Cadmium Resonance
Derived from Analysis of Experimental Data

E_o	= 0.181 ± 0.003 eV
Γ	= 0.1087 ± 0.0033 eV
Γ_γ	= 0.1087 ± 0.0033 eV
Γ_n	= 0.0000791 ± 0.0000032 eV ; $g = 3/4$ *
$\Gamma_n^o \equiv \Gamma_n / E_o^{1/2}$	= 0.0001858 ± 0.0000060 $(eV)^{1/2}$
σ_o	= 7847 ± 187 barns
$\sigma_o \Gamma$	= 853 ± 20 barn x eV
$\sigma_o \Gamma^2$	= 93 ± 5 barn x $(eV)^2$

*See Reference 1.

FIGURE CAPTIONS

Fig. 1 : The measured second-order contamination $f_2(E)$ as a function of energy for Be($10\bar{1}1$) and NaCl(200).

Fig. 2 : Net open-beam count rate obtained for Be($10\bar{1}1$) reflection as a function of neutron energy. Above 0.3 eV the rate falls as $E^{-0.55}$.

Fig. 3 : The ratio $f^*(E)$ of the net count rate observed for Be($10\bar{1}1$) at energy 4E, to that observed at E as a function of E. At $E > 0.3$ eV, $f^*(E)$ approaches 0.467 asymptotically.

Fig. 4 : The total cross section of elemental cadmium as a function of neutron energy in the region of the 0.181-eV resonance. The curve is the Doppler-broadeneded single-level formula calculated from the parameters listed in Table I.

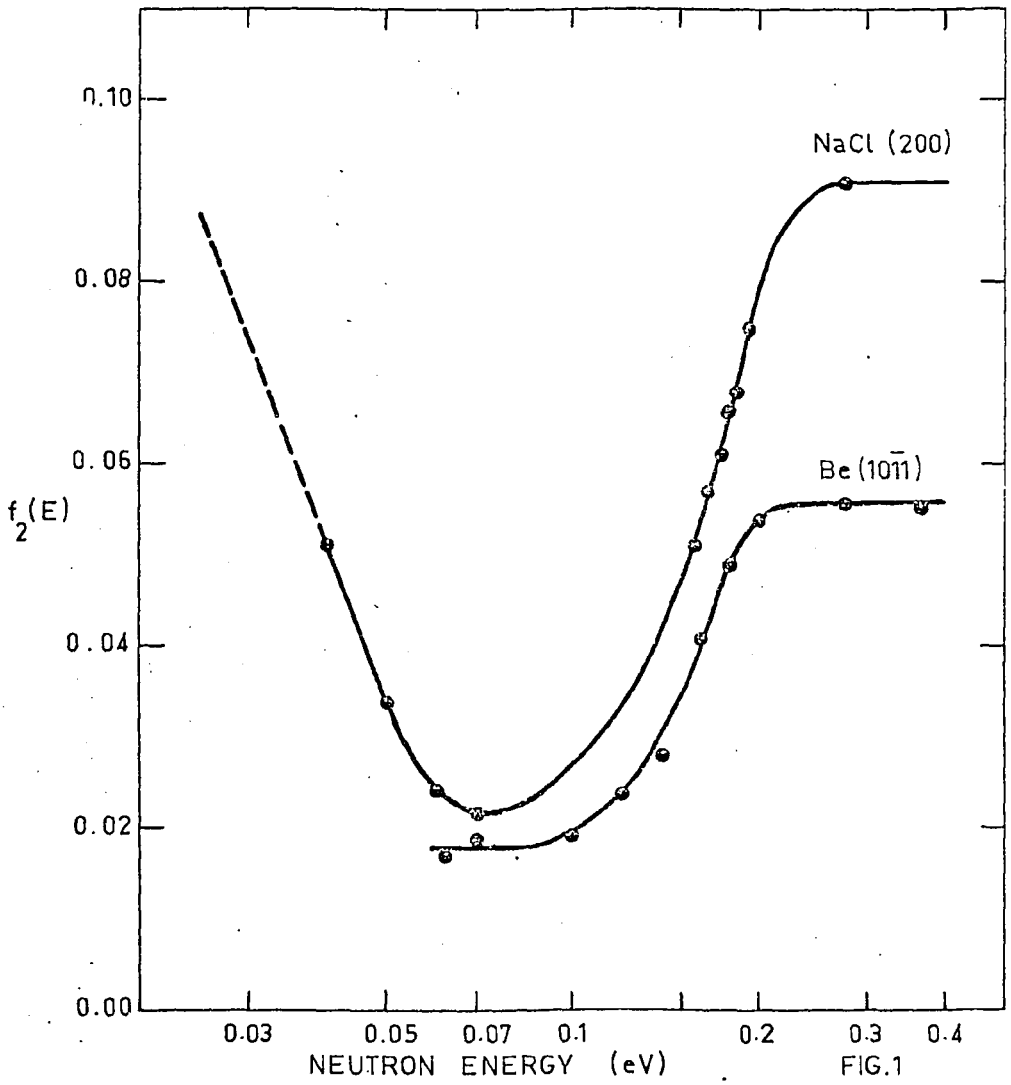
0.10
0.08
0.06
0.04
0.02
0.00
$f_2(E)$
NaCl (200)
Be (10$\bar{1}$1)
0.03
0.05
0.07
0.1
0.2
0.3
0.4
NEUTRON ENERGY (eV)

FIG.1

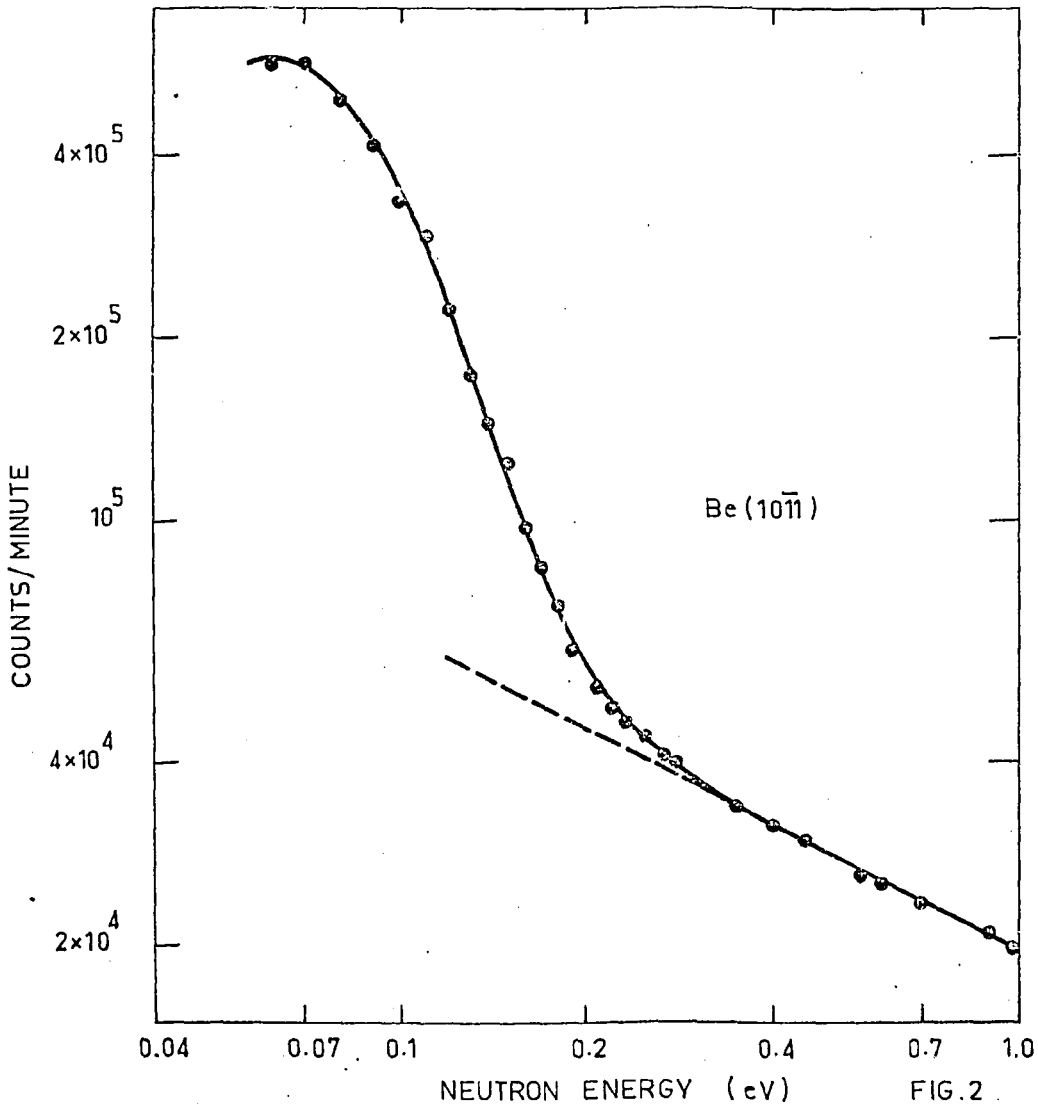
COUNTS/MINUTE
4×10^5
2×10^5
10^5
4×10^4
2×10^4
Be (10$\bar{1}$1)
0.04
0.07
0.1
0.2
0.4
0.7
1.0
NEUTRON ENERGY (eV)

FIG.2

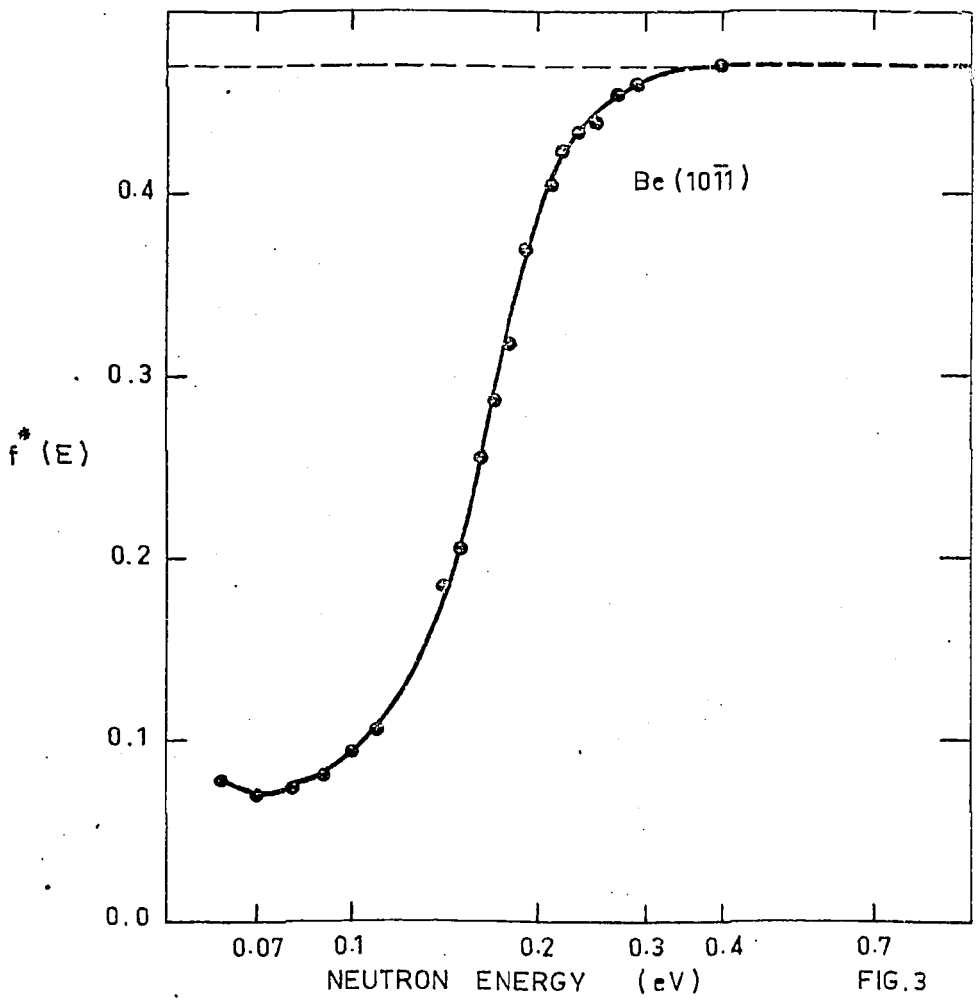
Be (10$\bar{1}\bar{1}$)
0.4
0.3
0.2
0.1
0.0
$f^*(E)$
0.07
0.1
0.2
0.3
0.4
0.7
NEUTRON ENERGY (eV)

FIG.3

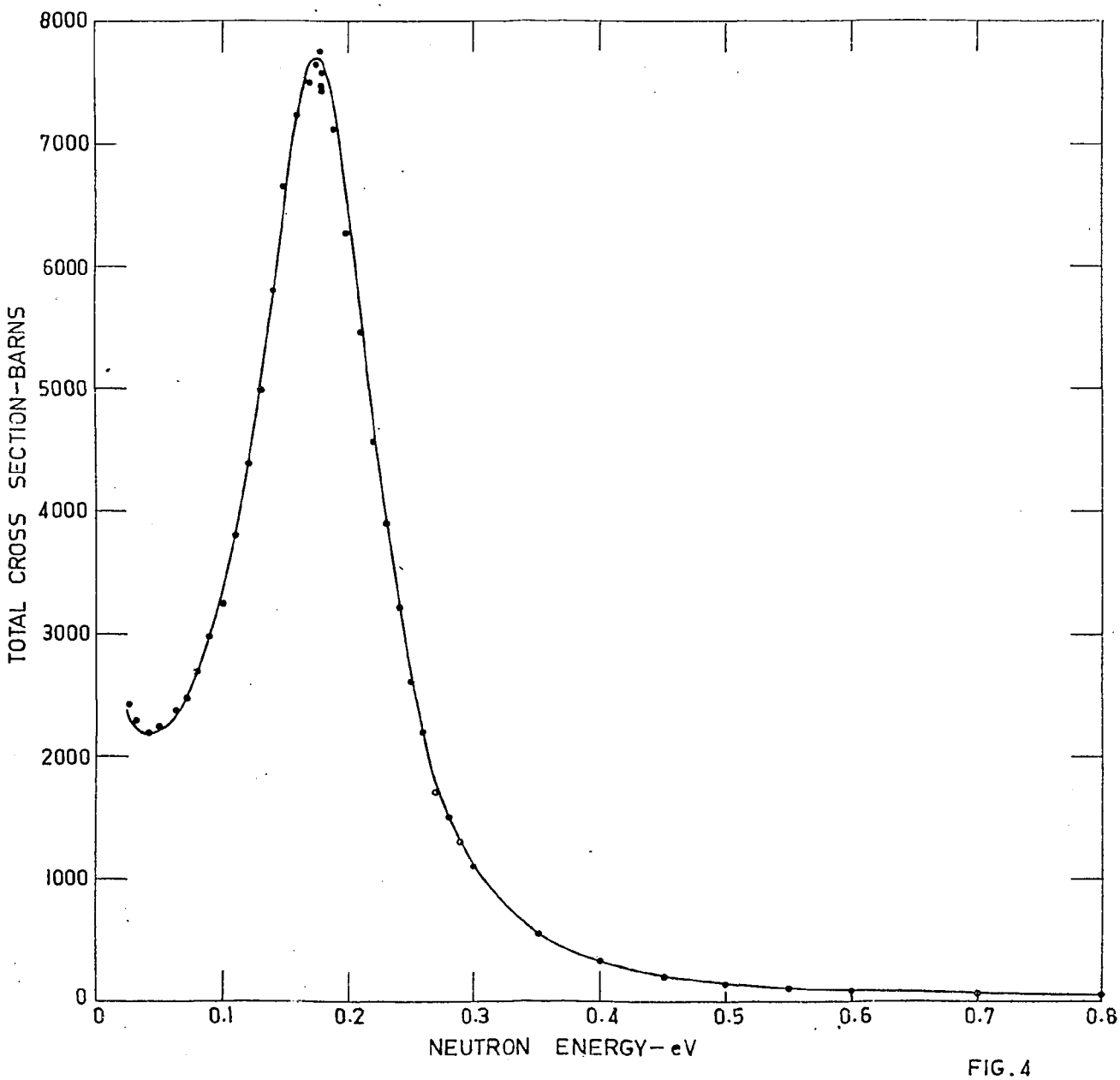
8000
7000
6000
5000
4000
3000
2000
1000
0
TOTAL CROSS SECTION-BARNS
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
NEUTRON ENERGY-eV

FIG. 4