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**R-MATRIX EVALUATION OF NEUTRON CROSS SECTIONS  
AND AVERAGE RESONANCE PARAMETERS OF  $^{236, 238}\text{U}$   
IN UNRESOLVED RESONANCE REGION**

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# R-matrix Evaluation of Neutron Cross Sections and Average Resonance Parameters of $^{236, 238}\text{U}$ in Unresolved Resonance Region

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## Abstract

The extended review of works carried out in Kiev Institute for Nuclear Research devoted to R-matrix analysis of experimental data on averaged over resonances total and partial neutron cross sections and resonance selfshielding effects for  $^{236, 238}\text{U}$  is given. The essential part of the analyzed experimental data was obtained by using of the most intensive and low-background experimental techniques, namely the filtered neutron beams at research reactors and the multiplicity detector at the neutron pulse source based on accelerator.

The  $^{236, 238}\text{U}$  average resonance parameters are determined. The sensitivity of inelastic scattering  $\langle\sigma_{in}\rangle$  and radiative capture  $\langle\sigma_{\gamma}\rangle$  neutron cross sections of  $^{238}\text{U}$  to the form of the R-matrix boundary conditions is investigated. The energy dependence of s- and p- wave neutron strength functions  $S_{n0}$ ,  $S_{n1}$  and potential scattering radius  $R'_0$  are determined for  $^{238}\text{U}$  in the energy range  $E_n \leq 400\text{keV}$ . Based on the obtained parameters, the  $^{236}\text{U}$  neutron inelastic scattering cross section is calculated, for which experimental data are not available.

## 1. Introduction

Using the modern experimental technique, the resonance structure of neutron cross sections  $\sigma$  can be investigated for heavy nuclei in a relatively narrow energy range which does not exceed a few keV. At higher energies only cross sections averaged over resonances  $\langle\sigma\rangle$  or more complicated functionals of  $\sigma$  are measurable experimentally.

As to the averaged cross sections, they have rather smooth energy dependence. Therefore, if a lot of differential experimental data on  $\langle\sigma\rangle$  are available, the evaluated values of  $\langle\sigma\rangle$  may be obtained by model-independent approximation of the data with smooth function, e.g. power polynomial. For example, this approach is used often for evaluated data file preparation. But such a way gives no possibilities for:

- extrapolation of neutron cross sections to the energy range where experimental data are not available,
- finding the possible systematic errors of experimental data,
- checking the total and partial neutron cross section data on selfconsistency,
- prediction of the cross sections of those reactions, which experimental data are lacking for.

The indicated disadvantages are partially or totally missing at consecutive evaluation performed on the R-matrix theory base. Such evaluation can be realized most reliably in the range of isolated resonances (up to the neutron energies  $E_n \sim 0.5$  MeV for heavy nuclei with  $A \geq 100$ ). In this range the formalism of  $\langle \sigma \rangle$  parameterization based on the one-level approach of the R-matrix theory is quite right and no uncertainties exist in the region of overlapping resonances, where various versions of optical model are used. At the same time, the use of R-matrix approach out of the experimentally resolved resonance region needs some features to be taken into account. For example, there are the optimal choice of boundary conditions of R-matrix theory, right account of the local fluctuations of the data to be analysed, etc.

The present work is an extended review of our studies [2-11], which were devoted to R-matrix analysis of experimental data on averaged over resonances total and partial neutron cross sections and resonance selfshielding effects for  $^{236,238}\text{U}$ . These studies were carried out in Kiev Institute for Nuclear Research of the National Academy of Sciences of Ukraine during last ten years. There was a set of unique algorithms and codes developed to calculate and to analyse the averaged neutron cross sections  $\langle \sigma \rangle$  and its various functionals  $F(\sigma)$  for heavy nuclei in the range of unresolved resonances. The essential part of the analyzed experimental data was obtained by using the most intensive and low-background experimental techniques, namely the filtered neutron beams at research reactors [2-4,6,7,13,14,28] and the multiplicity detector at the neutron pulse source based on accelerator [32].

In the present work the  $^{236,238}\text{U}$  average resonance parameters are determined. The sensitivity of the inelastic scattering  $\langle \sigma_{in} \rangle$  and radiative capture  $\langle \sigma_{\gamma} \rangle$  neutron cross sections of  $^{238}\text{U}$  to the form of the R-matrix boundary conditions is investigated. The energy dependence of s- and p- wave neutron strength functions  $S_{n0}$ ,  $S_{n1}$  and potential scattering radius  $R'_0$  are determined for  $^{238}\text{U}$  in the energy range  $E_n \leq 400\text{keV}$ . Based on the obtained parameters, the  $^{236}\text{U}$  neutron inelastic scattering cross section is calculated, experimental data for which are not available.

## 2. Parameterization of experimental data in unresolved resonance region

The analysed data [13,14] on resonance selfshielding effects in the total neutron cross sections of  $^{236,238}\text{U}$  were obtained in the range of unresolved resonances by transmission method at the filtered reactor neutron beams. The transmission  $\langle T \rangle$  measured in these experiments can be parameterized as

$$\langle T \rangle = \frac{\int \varphi(E) \exp[-n\sigma_t(E)] dE}{\int \varphi(E) dE}, \quad (1)$$

where  $\varphi(E)$  - function of neutron spectrum separated by a filter,  $\sigma_t(E)$  - nonaveraged total cross section,  $n$  - sample thickness. The transmission  $\langle T \rangle$ , as well as the observed total neutron cross section  $\tilde{\sigma}_t$ :

$$\tilde{\sigma}_t = -\frac{1}{n} \ln \langle T \rangle \quad (2)$$

are the values averaged over respectively wide energy range  $\Delta E$ , which includes a lot of compound resonances.

The observed effect of a resonance selfshielding (dependence of  $\langle T \rangle$  and  $\tilde{\sigma}_t$  on  $n$ ) is essentially defined by the detailed resonance structure of  $\sigma_t(E)$ . Therefore, to parameterize the experimentally observed values, the simulation of this structure by Monte-Carlo method [5,15] was carried out. In this procedure the initial values of the average distance between resonances  $\bar{D}_0$ , neutron strength functions  $S_{nl}$ , and average radiative widths  $\bar{\Gamma}_\gamma$  were taken from [21]. The distribution of simulated distances between levels was assumed as Wigner's one and distribution of neutron widths as this of Porter-Thomas. The radiative widths of all resonances were assumed as unfluctuated and to be equal to the average values  $\bar{\Gamma}_\gamma$ . Dependence of  $\bar{\Gamma}_\gamma$  on the excitation energy of compound nucleus was taken into account in form [12],

$$\bar{\Gamma}_\gamma = \bar{\Gamma}_\gamma(0) \left(1 + \frac{E}{W}\right)^{2.5}, \quad (3)$$

where  $W$  - neutron binding energy,  $\bar{\Gamma}_\gamma(0)$  - average radiative width in the resolved resonances range ( $E \sim 0$ ).

The average distance between resonances  $\bar{D}_j$  was calculated using the Fermi-gas model with normalisation to the average distance between  $s$  - resonances  $\bar{D}_0$  observed in the resolved range [5]. The energy dependence of the average neutron widths of the elastic  $\bar{\Gamma}_n^J$  and inelastic  $\bar{\Gamma}_n^{J'}$  channels was parameterized via neutron strength functions  $S_{nl}$ :

$$\bar{\Gamma}_{n(n)}^J = \frac{S_{nl}}{d_l(E_{n(n)})} \bar{D}_j n_{n(n)}^J \nu_l(E_{n(n)}) \sqrt{E_{n(n)}}, \quad (4)$$

where  $E_{n(n)}$  - neutron energy in the channel of elastic (inelastic) scattering,  $\nu_l$  - optical penetration factor of the neutron wave with momentum  $l$ ;  $n^J$  - degeneration factor corresponding to a freedom degree number of the total momentum  $J$ ;  $d_l$  - renormalization factor:

$$d_l \equiv [1 - R_l^\infty(s_l - B_l)]^2 - (p_l R_l^\infty)^2. \quad (5)$$

Here  $p_l$  and  $s_l$  are optical factors of unreduced penetration and shift,  $B_l$  - boundary condition parameter;  $R_l^\infty$  - parameter of potential scattering, determined by the contribution of far resonances. The parameter  $R_l^\infty$  defines the phase of scattering  $\varphi_l$ :

$$\varphi_l = \phi_l - \arctan \frac{p_l R_l^\infty}{1 - R_l^\infty(s_l - B_l)}, \quad (6)$$

where  $\phi_l$  is optical phase. The potential scattering radii  $R'_l$  used in practical applications are determined via  $R_l^\infty$  as:

$$R'_l = 1.35A^{1/3} \left( 1 - \frac{(2l+1)R_l^\infty}{1 + (l+B_l)R_l^\infty} \right). \quad (7)$$

To calculate the resonance structure  $\sigma_l(E)$ , the isolated resonance approximation [16] was used taking into account the Doppler effect [1,17]. After simulation of  $\sigma_l(E)$  and calculation of its functionals the average resonance parameters were corrected by fitting to the measured values of  $\langle T \rangle$ . To determine the final optimised values of average parameters, the whole procedure was repeated step by step. Simultaneously with the determination of average resonance parameters, calculation of the average total neutron cross section  $\langle \sigma_t \rangle$  at the each measured energy point was carried out corresponding to the cross section  $\tilde{\sigma}_t$  observed for the sample of one nuclear layer thickness.

If relatively thin samples are used to measure, for which effects of resonance selfshielding are small or negligible, then average cross sections can be determined just from experimental data by the model independent way. In this case, the integral expressions of type (1) transform to simple analytical formulae and the procedure of data analysis does not need the Monte-Carlo simulation of the resonance structure to determine the average resonance parameters. The resonance-averaged neutron cross-sections of inelastic scattering  $\langle \sigma_{in} \rangle$ , radiative capture  $\langle \sigma_\gamma \rangle$  and total  $\langle \sigma_t \rangle$  are parameterized as [1]:

$$\langle \sigma_{in} \rangle = \frac{2\pi^2}{k^2} \sum_{j,n} \frac{g_j}{D_j} \frac{\bar{\Gamma}_n^j}{\bar{\Gamma}_j} \sum_{i'} \bar{\Gamma}_{n'}^{j'} F_{in}, \quad (8)$$

$$\langle \sigma_\gamma \rangle = \frac{2\pi^2}{k^2} \sum_{j,n} \frac{g_j}{D_j} \frac{\bar{\Gamma}_n^j \bar{\Gamma}_\gamma}{\bar{\Gamma}_j} F_\gamma, \quad (9)$$

$$\langle \sigma_t \rangle = \frac{4\pi}{k^2} \sum_l (2l+1) \sin^2 \varphi_l + \frac{2\pi^2}{k^2} \sum_{j,n} \frac{g_j}{D_j} \sum_{i'} \bar{\Gamma}_{n'}^{j'} \cos 2\varphi_l, \quad (10)$$

where  $F_{in}$ ,  $F_\gamma$  - fluctuation factors, taking into account a difference between averaged multiplication of values and multiplication of those averaged. In this approach the average resonance parameters  $\bar{D}_0$ ,  $S_{nl}$ ,  $R_l^\infty$  and  $\bar{\Gamma}_\gamma$  can be obtained by direct fitting of the expressions (8-10) to experimental data. By this way in [7] the joint analysis of the available experimental data on  $\langle \sigma_{in} \rangle$  and  $\langle \sigma_\gamma \rangle$  of  $^{238}\text{U}$  was carried out in the energy range  $E_n = 0.1 \div 300$  keV. The similar analysis of the data on  $\langle \sigma_t \rangle$  and  $\langle \sigma_\gamma \rangle$  in the energy range  $E_n = 0.1 \div 30$  keV for  $^{238}\text{U}$  was performed in [11].

### 3. The choice of R-matrix boundary conditions

The formal R-matrix approach does not impose restriction on a choice of the boundary condition parameters  $B_l$ . If experimental data within the relatively narrow resolved resonance range are parameterized, then an energy dependence of the renormalization factor  $d_l$  is negligible and its value is usually taken as  $d_l = 1$ . It means, that the parameters  $B_l$  have the form:

$$B_l = s_l. \quad (11)$$

However, these conditions are energy dependent. To simplify the calculations of cross sections, the energy independent conditions, such as

$$B_l = \text{const} \quad (12)$$

are more preferable. The particular value of constant  $B_l$  does not influence results of the fitting to experimental data in a wide energy range up to  $\sim 1$  MeV. According to (4)-(6), a choice of various constants in (12) leads only to renormalization of the neutron strength functions  $S_{nl}$  and potential scattering parameters  $R_l^\infty$ :

$$S_{nl} = \frac{S_{nl}^*}{(1 - \text{const} \cdot R_l^{\infty*})^2}, \quad R_l^\infty = \frac{R_l^{\infty*}}{1 - \text{const} \cdot R_l^{\infty*}}, \quad (13)$$

where  $R_l^{\infty*}$  and  $S_{nl}^*$  are the parameters, which corresponds to conditions

$$B_l = 0. \quad (14)$$

A sensitivity of experimental data to a form of the boundary conditions was investigated in a joint analysis of the  $^{238}\text{U}$  inelastic scattering and radiative capture neutron cross sections  $\langle\sigma_{in}\rangle$ ,  $\langle\sigma_\gamma\rangle$  in the energy range  $E_n = 0.1+300$  keV. In this analysis the contribution of direct processes to inelastic scattering cross section evaluated in [18] was taken into account, as well as cross section fluctuational uncertainties of connected with the limited number of averaged resonances [5,18]. A contribution of s-, p-, d- and f- partial neutron waves to interaction with nucleus was taken into consideration in our calculations.

The experimental data on  $\sigma_{in}$  [3,4,7,24-28] and  $\sigma_\gamma$  [32] were approximated by the least square method with the expressions of type (1),(2) under the boundary conditions (14) or (11). In a fitting procedure the neutron strength functions  $S_{n0}$ ,  $S_{n1}$ ,  $S_{n2}$  of s-, p- and d- waves and the radiative widths  $\bar{\Gamma}_{\gamma 0}$ ,  $\bar{\Gamma}_{\gamma 1}$  for s- and p- neutrons were varied.

The fitting results are presented in Fig.1. As it is seen from the figure, in the energy range  $E_n = 0.1+50$  keV there is no difference between the calculated curves of  $\langle\sigma_\gamma\rangle$  obtained with different boundary conditions. The curves differ noticeably at  $E_n > 50$  keV, where relatively high uncertainties of  $\langle\sigma_\gamma\rangle$  give no

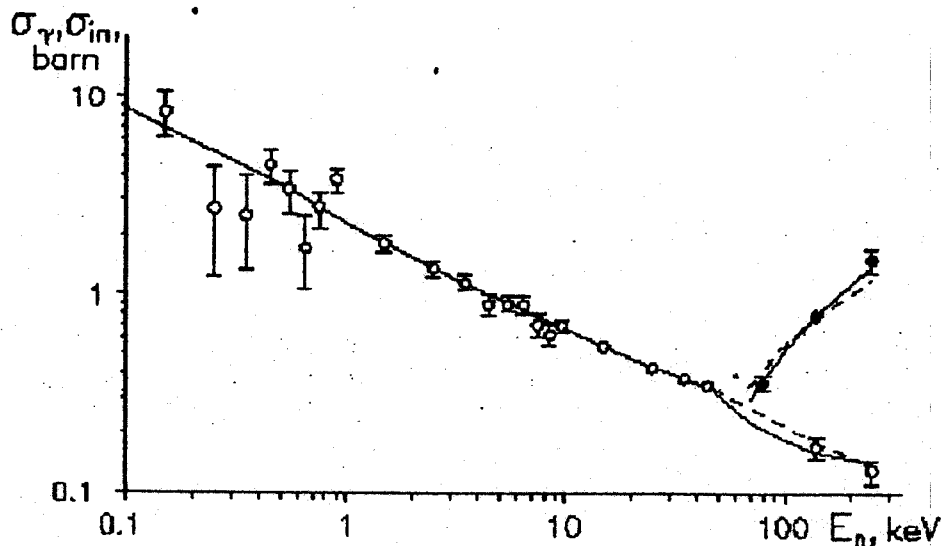


Fig.1. Inelastic scattering (•) and radiative capture (o) neutron cross sections of  $^{238}\text{U}$ . The solid and dashed curves represent approximations of experimental data, using the boundary conditions (14) and (11) respectively.

possibility to optimize a form of boundary conditions. For the whole capture cross section data a root-mean square deviations of calculated values  $\sigma^{th}$  from the experimental points  $\sigma^{ex}$ :

$$Q^2 = \frac{1}{n} \sum_{i=1}^n \frac{(\sigma_i^{ex} - \sigma_i^{th})^2}{(\Delta\sigma_i)^2}, \quad (15)$$

were obtained as  $Q_y^2 = 1.20$  with the boundary conditions of the form (11) and  $Q_y^2 = 1.12$  for the conditions of the form (14).

At the same time, the fitted values of the inelastic scattering cross section  $\langle\sigma_{in}^{th}\rangle$  calculated with the boundary conditions (14) are in much better agreement with experimental data, then those calculated with the conditions (11) (Fig.1). The  $Q_{in}^2$  values (15) for the conditions (11) and (14) were obtained as  $Q_{in}^2 = 1.74$  and 0.71 accordingly. The last result allows to conclude, that the energy independent boundary conditions of the general form (12) are most optimal for cross section parameterization. As to the choice of the particular values of constants, the most preferable ones are:

$$B_l = -l. \quad (16)$$

At low energies the form (16) leads to the renormalization factor

$$d_l(E_n \rightarrow 0) = 1, \quad (17)$$

that allows to compare directly parameters obtained at higher energies with those from the resolved resonance range.

#### 4. The energy dependence of neutron strength functions and potential scattering radius

It is generally agreed that the neutron strength functions of heavy nuclei are energy independent in the range of isolated resonances  $E_n \leq 0.5$  MeV [21]. This relies on the optical-statistical model, which parameterizes quite adequately the resonance-averaged neutron cross sections under the assumption of the full fragmentation of one-particle strength over compound resonances [22]. At the same time, some semi-microscopic and microscopic models predict intermediate structure in distribution of a one-particle neutron strength even at high energies above the neutron binding energy (see, for example, [23]). Appearance of such intermediate structure in the  $S_n$  energy dependence would lead to the necessity of revision of some model-dependent calculations, in particular evaluations of the neutron cross sections for unstable transuranium nuclei, fission products nuclei, etc.

To investigate the possible energy dependence of the s-wave strength function  $S_{n0}$  and the s-wave potential scattering radius  $R'_0$  of  $^{238}\text{U}$ , the data on resonance selfshielding of total neutron cross sections, measured at filtered reactor beams in the energy range  $E_n = (2 \div 300)$  keV [13,14] were analysed in [10]. The analysis was performed by Monte-Carlo simulation of the cross section resonance structure. Since the s-wave gives the main contribution to the total cross section in the analyzed energy range, only the values of  $S_{n0}$  and  $R'_0$  were varied. Other



parameters were taken from [7] as:  $\bar{D}_0 = 20.4$  eV [21],  $\Gamma_\gamma = 0.018$  eV,  $S_{n1} = 1.66 \cdot 10^{-4}$ ,  $R_1^\infty = 0.204$ ,  $S_{n2} = 2.5 \cdot 10^{-4}$ ,  $R_2^\infty = 0.311$  [7]. The average total neutron cross sections  $\langle\sigma_t\rangle$  and local values of  $S_{n0}$  and  $R'_0$  were determined independently at each energy point, measured in [13,14] (Table 1). In addition to statistical

Table 1.  
The averaged total cross sections, s-wave neutron strength functions and potential scattering radius of  $^{238}\text{U}$

$E_n, \text{keV}$	$\Delta E, \text{keV}$	$\langle\sigma_t\rangle, \text{barn}$	$S_{n0}, 10^{-4}$	$R'_0, \text{fm}$
1.8	0.7	24.31(34)	1.30(43)	9.18(14)
1.9	2.2	23.5(12)	1.34(22)	9.64(9)
3.5	1.5	20.89(45)	1.44(30)	9.83(9)
5.2	2.4	18.35(25)	0.91(14)	9.46(10)
12.0	0.7	15.81(35)	1.45(40)	9.21(14)
24.5	2.1	14.01(18)	0.81(13)	9.56(10)
45.0	~3	13.50(20)	1.04(41)	9.68(13)
53.5	0.9	13.33(12)	1.27(32)	9.53(12)
55.0	~3	13.35(70)	1.21(31)	9.64(9)
59.0	2.3	13.00(3)	0.99(14)	9.52(8)
66.8	~3	12.45(15)		9.30(9)
97.0	~3	11.61(12)		9.19(9)
106.0	~3	11.89(9)		9.40(10)
132.0	~3	11.85(18)		9.39(12)
136.0	~3	11.54(18)		9.41(16)
144.0	25.0	11.46(4)	1.46(10)	9.52(2)
152.0	1.2	11.48(12)	1.95(61)	9.44(18)
155.0	~3	11.53(17)		9.65(32)
179.0	~3	11.03(4)	1.35(69)	9.50(21)
194.0	~3	11.46(53)		9.36(8)
231.0	~3	10.97(40)		
250.0	~3	10.48(32)	1.11(30)	9.67(18)
272.0	~3	10.13(23)	1.39(28)	9.47(15)
312.0	1.6	9.81(16)	1.43(24)	9.38(9)

contribution to  $\langle\sigma_{in}\rangle$ . Because inelastic scattering cross section depends weakly on other parameters but  $S_{n1}$ , their values were fixed according to the data [7,12]

The  $S_{n1}$  local values from the  $\langle\sigma_{in}\rangle$  analysis in the energy range  $E_n = 80 \div 275$  keV and  $S_{n1}$  value determined from the resolved resonances range  $E_n \leq 5$  keV [21] are presented in Fig.3 and Table 2.

As it is seen from Figs.2,3, the local  $S_{n0}$ ,  $S_{n1}$  and  $R'_0$  values don't demonstrate the energy dependence out of uncertainties in the analyzed energy range. According to the optical model calculations, the  $^{238}\text{U}$  s-neutron strength

uncertainties, the errors for  $S_{n0}$  and  $R'_0$  include also fluctuational parts. The results of the  $S_{n0}$  and  $R'_0$  energy dependence analysis are shown in Fig. 2.

To investigate the possible energy dependence of the p-neutron strength function  $S_{n1}$ , the analysis of experimental data [3,4,7,24-28] on neutron inelastic scattering with the excitation of the first level of  $^{238}\text{U}$  ( $I_1^\pi = 2^+$ ,  $E_1 = 45$  keV) was carried out. In the analyzed energy range  $E_n \leq 300$  keV the p-neutron partial wave gives the main

Table 2  
The p - wave neutron strength functions of  $^{238}\text{U}$

$E_n, \text{keV}$	$S_{n1}, 10^{-4}$
5	1.7(3)
82	1.51(10)
100	1.57(26)
106	1.34(23)
112	2.12(39)
120	1.7(4)
126	2.16(23)
128	2.17(33)
135	2.34(43)
140	1.99(22)
143	1.7(23)
145	1.65(7)
157	1.86(25)
200	1.53(19)
275	2.01(34)

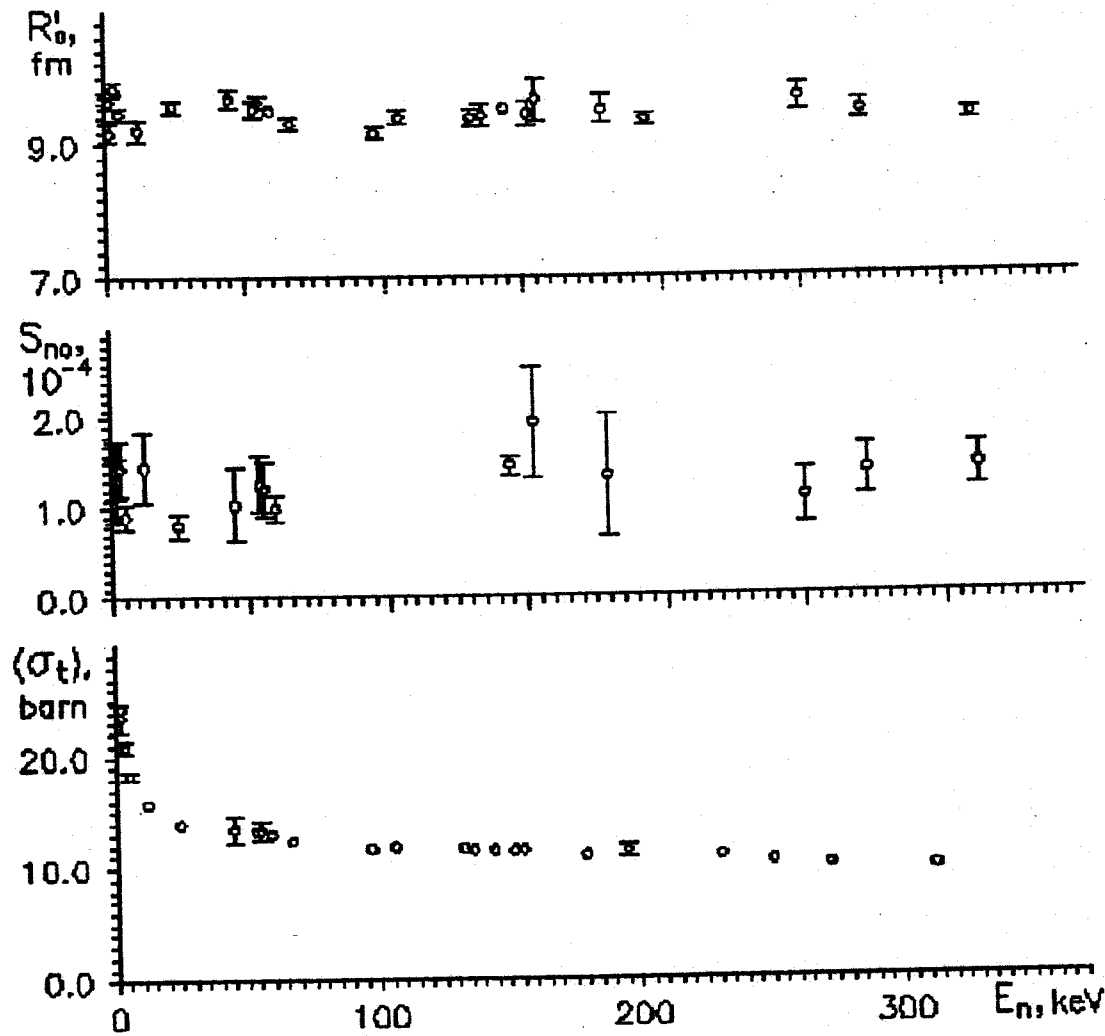


Fig.2. Energy dependence of the averaged total cross section  $\langle \sigma_t \rangle$ , s-neutron strength function  $S_{n0}$  and potential scattering radius  $R'_0$  for  $^{238}\text{U}$ . The errors of  $S_{n0}$  and  $R'_0$  besides experimental include fluctuational ones, which correspond to averaging interval.

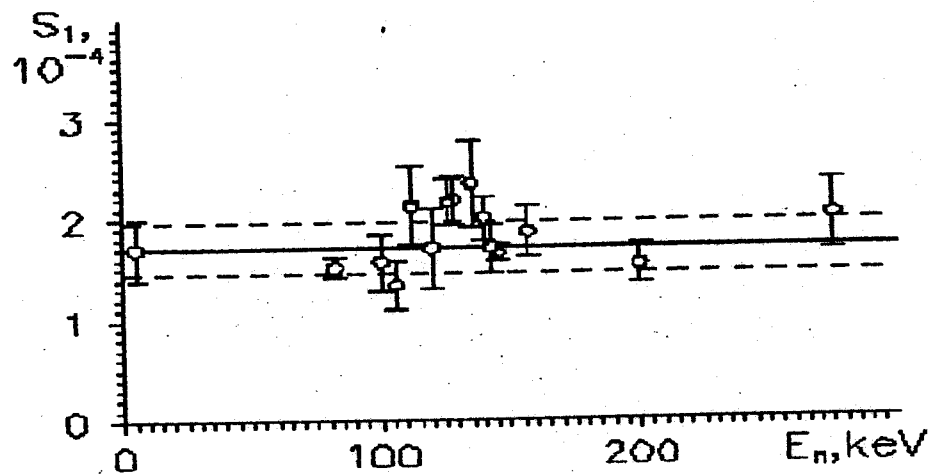


Fig.3. Energy dependence of p-neutron strength function  $^{238}\text{U}$ . The dash lines show the 15%-band of errors.

function is defined by contributions of the two nearest one-particle states  $4s_{1/2}$  and  $5s_{1/2}$  which both distant as about 8MeV from the neutron binding energy  $B_n$ . If an intermediate structure in the strength distributions of these states is absent, the variation of  $S_{n0}$  predicted by the optical model in the energy range  $\Delta E = 300$  keV does not exceed 0.5%. As to the p-neutron strength function of  $^{238}\text{U}$ , its value is defined mainly by contribution of the  $4p$ -state lied near  $B_n$ . The expected variation of the  $S_{n1}$  within the energy range  $E_n = 300$  keV does not exceed  $\sim 10\%$ . Because of the resulting uncertainties of the  $S_{n0}$ ,  $S_{n1}$  local values are comparable with the variations, predicted by optical-statistical model, no conclusions about its presence or absence are possible. However, the results obtained reliably testify that no nonstatistical effects exist, which would lead to the intermediate structure to be visible out of the variations predicted by optical-statistical model.

### 5. $^{238}\text{U}$ resonance parameters averaged over a wide energy range

The data on average resonance parameters of the nuclei  $A \sim 230-240$  have rather limited character. The local values of the s-wave average resonance parameters  $S_{n0}$ ,  $R'_0$ ,  $\bar{D}_0$ ,  $\bar{\Gamma}_{\gamma 0}$  are determined reliably from the resolved resonance range [21]. However, these values have large fluctuational errors due to limited number of measured neutron resonances. As to the parameters of the p-wave resonances, their determination from the resolved range is not so reliable. At the same time, such information for the nuclei  $A \sim 230-240$  is of great interest because of presence of the  $4p$ - one-particle maximum of p-neutron strength function  $S_{n1}$  in this mass range [21].

The present analysis of experimental data on the  $^{238}\text{U}$  cross sections in the energy range 0.1-300 keV was based on the two sets of data:

- inelastic scattering and radiative capture cross sections [7] and
- resonance selfshielding of total neutron cross sections [5,10].

In both cases, the cross section data were analysed jointly in the whole considered energy range to determine the parameter values averaged over as many resonances as possible. The analysis of the data on angular distribution of elastic scattering at the neutron energies  $E_n = 53.5$  and 144 keV was carried out in our earlier work [4].

The average resonance parameters obtained by these different ways are given in Table 3 in comparison with the data of other authors. According to (13),

Table 3.

The average resonance parameters of  $^{238}\text{U}$

Parameter	Reference								Recomm values
	[4]	[7]	[10]	[20]	[21]	[29]	[30]	[31]	
$S_{n0} \cdot 10^{-4}$		1.3(4)	1.09(5)	1.05	1.2(1)	0.93(3)	0.75(6)	0.98	1.09(5)
$S_{n1} \cdot 10^{-4}$		1.66(12)	1.47(6)	1.68(2)	1.7(3)	1.95(6)	1.8(4)	2.01	1.66(12)
$S_{n2} \cdot 10^{-4}$		2.5(4)	2.8(9)	0.76(4)		3.0(3)			2.5(4)
$R'_0, \text{fm}$	9.144		9.63(4)	9.30(5)	9.6(1)	9.35(6)	9.15(24)	9.43	9.63(4)
$R_1^\infty$	0.257								
$R_2^\infty$	0.82								
$D_0, \text{eV}$					20.9(11)				
$\bar{\Gamma}_{\gamma 0}, \text{meV}$		19.6(15)			23.2(3)	23.1(10)	23.0(20)		23.2(3)
$\bar{\Gamma}_{\gamma 1}, \text{meV}$		18.0(15)				16.6(6)	23.2(20)		18.0(15)

all data were renormalized to the unified boundary conditions of the form (16) to compare them directly with the data from the resolved resonances range.

There were two independent sets of the s-wave average resonance parameters obtained by us ([7] and [10]). Because of the resonance selfshielding data are most sensitive to  $S_{n0}$  and  $R'_0$ , the results [10] for these values seem more reliable, than [7]. Some difference between the  $S_{n0}$  value obtained in our work [10] and those from the resolved resonance region [21] can be explained by the fact, that the energy region analyzed in [10] is much more wider than [21]. If only the selfshielding data, obtained at  $E_n \leq 12$  keV are analyzed (Table 1), it leads to the value  $S_{n0} = (1.18 \pm 0.10) \cdot 10^{-4}$ , which is practically equal to the result [21].

For  $S_{n1}$  there were also two results obtained by different ways (Table 3). To our mind the  $S_{n1}$  value from [7] is most reliable due to a high sensibility of  $\langle \sigma_{in}^{(1)} \rangle$  to  $S_{n1}$ , as against of other methods [4,5,10].

The p-wave potential scattering parameter  $R_1^\infty$  was defined from analysis of the angular distribution of elastic neutron scattering. Besides of our work [4], its value was determined by the same method in [30] as well as from the analysis of  $\langle \sigma_t \rangle$  in the widest energy range  $E_n \leq 1$  MeV in [20] (Table 3). The present results and the data [30] seem essentially more reliable due to high sensitivity of analysed data to  $R_1^\infty$  as compared with the approach [20]. In spite of the wider energy range of averaging ( $\Delta E_n \sim 400$  keV) the result [30] is not much more reliable, than those from [4], corresponded to the range  $\Delta E_n \sim 25$  keV because of negligible fluctuation uncertainties of  $R_1^\infty$  in both cases. In the same time such essential advantages of the method [4], as elastic and inelastic scattered neutron separation and high intensity of the filtered neutron beams, exists in comparison with the method [30].

The finally recommended values of the  $^{238}\text{U}$  average resonance parameters are given in Table 3.

## 6. Average resonance parameters of $^{236}\text{U}$

The average resonance parameters of  $^{236}\text{U}$  were determined in our earlier works by two methods. The analysis of the data on resonance selfshielding in the

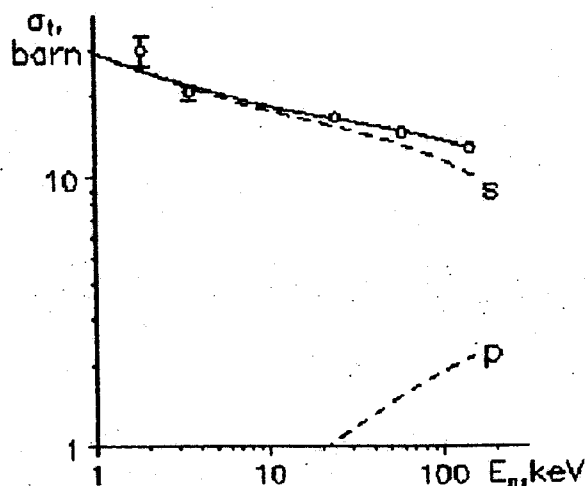


Fig.4. The experimental data on average total neutron cross section of  $^{236}\text{U}$ .

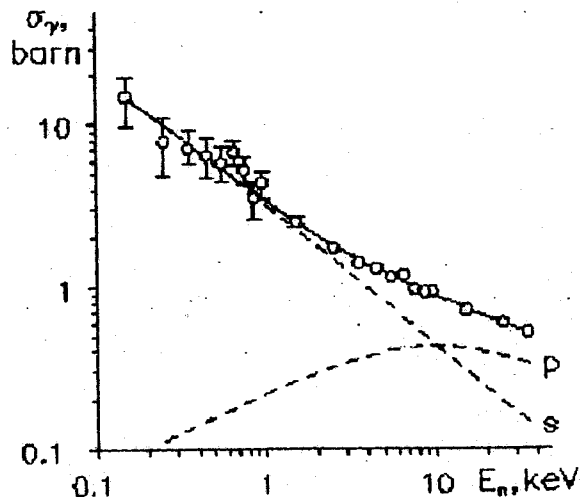


Fig.5. The experimental data on average radiative capture cross section of  $^{236}\text{U}$ .

total neutron cross sections measured at the filtered neutron beams was carried out in [8]. A joint analysis of the experimental data on  $\langle\sigma_t\rangle$  [8] (Fig.4) and  $\langle\sigma_\gamma\rangle$  [32] (Fig.5) was performed in [11].

The results of [8] and [11] are given in Table 4 in comparison with the data of other authors. As seen from the table the obtained values of s-wave average resonance parameters are in agreement with the results of other works.

Table 4.

The average resonance parameters of  $^{236}\text{U}$

Parameter	Reference						Recomm values
	[8]	[11]	[21]	[33]	[34]	[35]	
$S_{n0}, 10^{-4}$	1.23(12)	1.22(15)	1.0(1)	1.0(1)	1.164	0.95(9)	1.23(12)
$S_{n1}, 10^{-4}$	2.3(5)	1.73(19)	2.3(6)	2.4(2)	1.74	1.96(16)	1.73(19)
$S_{n2}, 10^{-4}$	2.8(9)	2.8(9)		2.8(7)	1.164		2.8(9)
$R'_0, \text{fm}$	10.32(12)	10.40(17)		10.2(2)	9.53		10.32(12)
$R_1^\infty$	0.15*	0.15*		0.11			
$R_2^\infty$	0.0*	0.0*					
$D_0, \text{eV}$	14.7*	14.7*	14.7(8)	14.7	14.1		
$\Gamma_{\gamma 0}, \text{meV}$	23.0*	23.2(20)	23.0(15)	22.8(23)	22.8	23.0(20)	23.2(20)
$\Gamma_{\gamma 1}, \text{meV}$	23.0*	19.2(20)		19.8(10)		22.3(20)	19.2(20)

\* - this parameter was fixed in the least-square procedure

Our earlier value of  $S_{n1}$  [8] and those from [21] have relatively high uncertainties. As to the result of [34], it is given without errors, that makes it incomparable at all. Therefore, the most reasonable data to be compared are the results [33,35] and our result [11]. The  $S_{n1}$  values from [11,35] are close one to another, but they essentially differ from the data [33]. It should be noted, that the value, obtained by us ( $\bullet$ ), is in a better agreement with systematics of the  $S_{n1}$  data in the mass range  $A \sim 230-240$  than others ( $\circ$ ) (Fig.6).

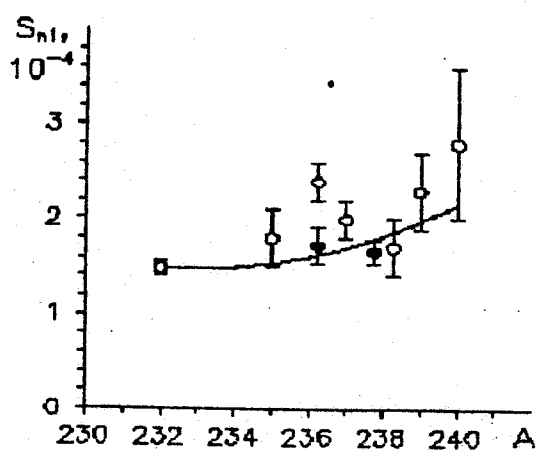


Fig.6. Systematics of the  $S_{n1}$  data

As it is seen from the Table 4, the existing data [11,33,35] on the average radiative widths  $\bar{\Gamma}_{\gamma 1}$  are in agreement within the uncertainties. In addition it should be noted, that the results [11,33] suggest a possible difference between the widths  $\bar{\Gamma}_{\gamma 0}$  and  $\bar{\Gamma}_{\gamma 1}$  of s- and p-  $^{236}\text{U}$  resonances, though it can't be regarded as a reliable confirmation. The similar results were obtained in [7] for  $^{238}\text{U}$  (Table 3). If such effect really exists, it would testify to a noticeable dependence of level density upon the parity.

The finally recommended values of the  $^{236}\text{U}$  average resonance parameters are given in the Table 4.

## 7. Inelastic neutron scattering cross section of $^{236}\text{U}$ in the energy range $E_n \leq 0.3 \text{ MeV}$

No experimental data on  $\langle \sigma_{in} \rangle$  of  $^{236}\text{U}$  in the low energy range  $E_n \leq 0.3 \text{ MeV}$  exist up to now. At the same time this range is very important for breeder reactor calculations, since the open channels of inelastic scattering play an essential role in forming of neutron spectrum. The  $\langle \sigma_{in} \rangle$  calculation for  $^{236}\text{U}$  was carried out by us using the evaluated values of the  $^{236}\text{U}$  average resonance parameters (Table 4). The results of calculation are shown in Fig.7 and Table 5.

Table 5		
Calculated values of $\langle \sigma_{in} \rangle$ $^{236}\text{U}$		
	$\langle \sigma_{in} \rangle$ , mb	
$E_n$ , keV	45keV	150keV
50	65(8)	
60	180(20)	
70	285(30)	
80	375(40)	
90	455(45)	
100	525(55)	
120	650(65)	
140	760(75)	
160	855(85)	0.28(11)
180	945(95)	1.85(55)
200	1030(100)	5.0(15)
220	1100(110)	10(3)
240	1170(120)	17(5)
260	1240(130)	26(8)
280	1300(140)	36(11)
300	1360(140)	48(14)

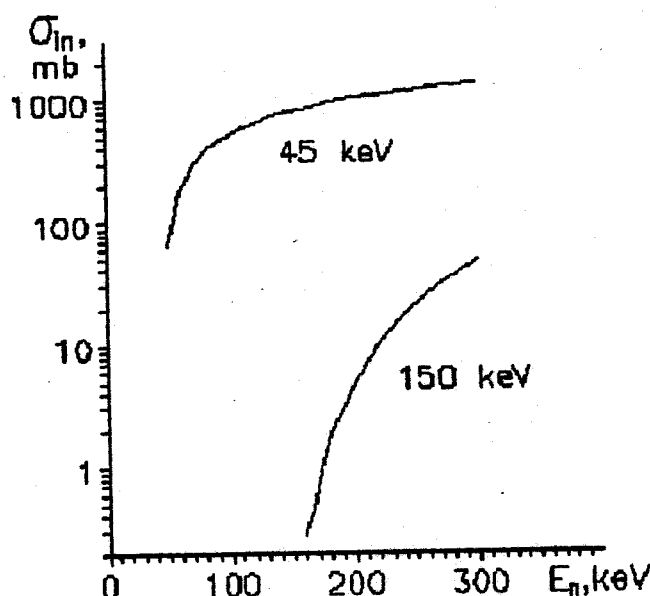


Fig.7. Calculated values of  $\langle \sigma_{in} \rangle$   $^{236}\text{U}$ .

In the considered energy range the inelastic scattering with excitation of the first level  $I_1^* = 2^+$ ,  $E_1 = 40 \text{ keV}$  is the most important process to be taken into account for reactor calculations. Due to the big spin difference  $\Delta I = 4$  between the ground and the second excited level ( $I_2^* = 4^+$ ,  $E_2 = \text{keV}$ ), the inelastic scattering with excitation of the last one is characterised by a small cross section  $\langle \sigma_{in}^{(2)} \rangle$ , that leads to its minor influence on the reactor calculations. Similar to  $^{238}\text{U}$ , the cross section with the first level excitation  $\langle \sigma_{in}^{(1)} \rangle$  of  $^{236}\text{U}$  depends strongly on the  $S_{n1}$  value. In present work the analysis of the existed  $S_{n1}$  data was carried out (see item 5) and the most reliable value was used (Table 4).

## 8. Conclusions

The present review demonstrates, that the R-matrix evaluation of the existing experimental data on averaged neutron cross sections can be carried out by using of a quite limited set of the average resonance parameters, applicable for both the resolved and the unresolved isolated resonances regions.

A few peculiarities have to be taken into account to use the R-matrix approach in the unresolved resonance range. First of all there are a right choice of the boundary conditions in the R-matrix theory and correct consideration of the fluctuation uncertainties of the estimated values. However, as it was demonstrated, even the analysis, taking into account these peculiarities, not always yields an absolute result, if the analyzed data are limited by one type of reaction with neutrons. Therefore to determine the average resonance parameters reliably, the joint analysis of the cross sections of all opened reaction channels in resonance energy range has to be carried out.

### References

1. Lanc A.M. and Thomas R.G., *Rev. Mod. Phys.*, 1958, vol.30, no.2, p.257.
2. Vertebnyi V.P., Libman V.A., Litvinsky L.L. et al., in: *Neutron Physics* [in Russian] (TsNIIAtominform, Moscow, 1988), vol.2, p.175.
3. Litvinsky L.L., Vertebnyi V.P., Libman V.A. et al., in: *Neutron Physics* [in Russian] (TsNIIAtominform, Moscow, 1988), vol.2, p.179.
4. Litvinsky L.L., Vertebnyi V.P., Libman V.A. et al., *Russian Jour. Atomnaya Energiya* (Atomic Energy), 1987, vol.63, p.192.
5. Novosiolov G.M., Litvinsky L.L. and Murzin A.V., *Preprint of Kiev Inst. for Nuclear Research*, Kiev, 1989, no. 89-25.
6. Purtov O.A., Shkarupa A.M., Archipov V.N. et al., *Preprint of Kiev Inst. for Nuclear Research*, Kiev, 1989, no. 89-26.
7. Litvinsky L.L., Murzin A.V., Novosiolov G.M. et al., *Russian Jour. Yadernaya Fizika* (Physics of Atomic Nuclei), 1990, vol.52, no.4(10), p.1025.
8. Purtov O.A., Litvinsky L.L., Murzin A.V. et al., *Russian Jour. Atomnaya Energiya* (Atomic Energy), 1993, vol.75, no.5, p.396.
9. Litvinsky L.L. and Libman V.A., *Russian Jour. Pis'ma ZhETF*, 1994, vol.59, no.7, p.442.
10. Novosiolov G.M., Litvinsky L.L., Muravitsky A.V. et al., *Russian Jour. Atomnaya Energiya* (Atomic Energy), 1995 (to be published).
11. Litvinsky L.L., Vlasov M.F. and Novosiolov G.M., *Russian Jour. Pis'ma ZhETF*, 1994.
12. Belanova T.S., Ignatjuk A.V., Pashenko A.B. et al., *Radiatsionnyi zahvat neitronov* (Neutron radiative absorption), Moscow: Energoatomizdat, 1986.
13. Litvinsky L.L., Libman V.A. and Murzin A.V., *Preprint of Kiev Inst. for Nuclear Research*, Kiev, 1985, no. 85-35.
14. Murzin A.V., Vertebnyi V.P., Gavriljuk V.I. et al., *Russian Jour. Atomnaya Energiya* (Atomic Energy), 1989, vol.67, p.216.
15. Novosiolov G.M. and Vertebnyi V.P., *Preprint of Kiev Inst. for Nuclear Research*, Kiev, 1977, no. 77-9.
16. Novosiolov G.M. and Kolomietz V.M., *Unambiguous Parametrization of Neutron Cross-Sections in the Low-Energy Region*. INDC(CCP) 186, Vienna, 1983.
17. Atta S.E. and Harvey J.A., *Numerical Analysis of Neutron Resonances*. ORNL-3205, 1962.
18. Lagrange Ch., *Results of Coupled Channels Calculations for the Neutron Cross Sections of a Set of Actinide Nuclei*, NEANDC(E) 228 "L"-INDC(FR) 56/L, France: Commissariat AL'Energie Atomique, 1982.

19. Novosiolov G.M., Russian Jour. Yadernaya Fizika (Physics of Atomic Nuclei), 1995, vol.58, no.1, p.23.
20. Tsubone I., Nakajime Y. and Furuta Y., Nucl. Scien. and Eng., 1984, vol.88, p.579.
21. Mughabghab S.F., Neutron Cross Sections, New York, Academic Press, INC, vol.1, 1984.
22. Bohr A. and Mottelson B.R., Nuclear Structure, New York, Amsterdam, vol.1, 1969.
23. Soloviev V.G., Teorya Atomnogo Yadra. Quazichastitzy, Fonony (Nuclear Theory. Quasi-particles and Phonons), Moscow: Energoatomizdat, 1989.
24. Horton C.A., Low Energy Neutron Scattering from Heavy Nuclei (Abstract of a Dissertation), Madison, University of Wisconsin, 1969.
25. Barnard E., Ferguson A. and Murrey W., Proc. 2nd Int. Conf. on Nuclear Data for Reactor, Helsinki 1970, Vienna: IAEA, 1970, p.103.
26. Kegel G.H., Beghien L.B., Couchell G.P. et al., Rep. to DOE NDC, BNL-NCS-24273, Brookhaven: BNL, 1978, p.151.
27. Winters R.R., Hill N.W., Macklin R. et al., Nucl. Sci. Eng., 1981, vol.78, no.2, p.147.
28. Tsang F.I. and Brugger R.M., Nucl. Sci. Eng., 1978, vol.65, no.1, p.70.
29. Manturov G.N., Lunev V.P. and Gorbachev L.V., in: Neutron Physics [in Russian] (TsNIIAtominform, Moscow, 1984), vol.2, p.231.
30. Kuznetzova L.V., Popov A.B. and Samosvat G.S., in: Neutron Physics [in Russian] (TsNIIAtominform, Moscow, 1988), vol.2, p.258.
31. Frohner F.H., Nucl. Scien. and Eng., 1988, vol.103, p.119.
32. Adamchuk Ju.V., Voskanjan M.A., Muradjan G.V. et al., in: Neutron Physics [in Russian] (TsNIIAtominform, Moscow, 1988), vol.2, p.242.
33. Kasakov L.E., Kononov V.N., Manturov G.N. et al., Russian Jour. Atomnaya Energiya (Atomic Energy), 1988, vol.4, p.152.
34. Klepatskyi A.V., Konshin V.A., Maslov V.M. et al., Preprint of Inst. Nuclear Energ. AS BSSR, Minsk, 1987, no.2.
35. Lunev V.P., Manturov G.N., Lipunkov A.O. et al., in: Neutron Physics [in Russian] (TsNIIAtominform, Moscow, 1984), vol.1, p.97.





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