

# ANGULAR DISTRIBUTIONS IN NEUTRON-INDUCED REACTIONS VOLUME $I, Z=1$ to 20 

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## INTRODUCTION

This is the Third Edition of a compilation of significant experimental observations on the angular distributions of scattered neutrons and of the products of the reactions of fast neutrons with nuclei.

The compilation is designed to be of use to theorists and experimentalists interested in neutron interactions and to those concerned with the behavior of neutrons in nuclear reactors and their shields. It gives in considerable detail the results of angular distribution experiments and indicates where they are published. It attempts to portray compactly the experimentally known facts in a manner facilitating interpolation into regions hitherto not covered by experiment.

The First Edition of this compilation appeared in 1956 and the Second Edition, in 1962. As in these earlier editions, data on elastically scattered neutrons predominate; but there are many distributions for inelastically scattered neutrons, for gamma rays resulting from inelastic scattering, and for charged particles produced in neu-tron-indyeed reactions. Angular distributions of fission fragments or fission neutrons are not included nor are distributions for which the neutron source is a fission spectrum. Also excluded are distributions for the scattering of slow ( $\mathrm{E} \leq 1 \mathrm{eV}$ ) neutrons, where molecular binding effects are more important than nuclear effects.

This book differs markedly from the earlier editions in that all of the graphical displays were prepared by a computer and photographed from a CALCOMP cathode-ray tube (CRT) plotter. The use of the computer to produce substantial portions of the material is regarded as a significant advance in publications of this Center and should lead to considerable time and cost savings in future compilations. The source of the individual datum points was the National Neutron Cross Section Center's neutron data storage library, SCISRS.

## ARRANGEMENT

The present compilation is published in two volumes. The first volume encompasses the data from hydrogen to calcium $(1 \leqslant Z \leqslant 20)$, while the second volume covers the remainder of the periodic table $(21 \leqslant Z \leqslant 94)$. A schemetic representation of the compilation is illustrated below.


In each volume, following the introductory remarks, the data are grouped into sections, each corresponding to the element of the nuclear target (increasing Atomic Number Z).

Within a given Element section, data pertaining to the naturally occurring element come first, and the data pertaining to the various isotopes follow in order of increasing Mass Number $\mathbf{A}$.

The reader is to be warned that certain arbitrary, but reasonable, choices of data placement have been made in the interest of reducing ambiguity. For an element like carbon, which contains one dominant isotope, all of the elastic scattering distributions have been collected under natural carbon, regardless of whether the original measurer drew conclusions from the data regarding isotopic structure. By the same rationale, all inelastic scattering distributions have been collected under
the appropriate isotope, in this case $\mathrm{C}^{12}$. However, care should be taken to look in both elemental and isotopic sections for the desired information when such clear distinctions cannot be made.

In a given Natural-Element (or Isotope) section, data are represented in four different ways:
(1) A graph is constructed of the differential cross section vs cosine theta of all points in a particular data set. A data set is the collection of datum points belonging to the same Element/Isotope section and having the "identical" reaction type, incident neutron energy, secondary energy, and kinematic coordinates. A data set may contain data of many authors. The graph produced includes the various datum points plotted, with error bars; a label box which annotates the set; and a fitted curve to the data that is defined over a range of cosine theta where data exist.

These graphs come first in the Element/Isotope section and are placed six on a page, ordered from top to bottom, column by column, until exhausted.
(2) If a data set consists of three or less points, a graph (of type 1) is not constructed, but the set is entered into a data table. The data tables follow the graphical data. As a guide to the reader, a flag "[REFER TO TABLE FOR ADDITIONAL DATA SETS]" appears above the graph where a data table entry would have been plotted.
(3) Where sufficient differential elastic scattering data exist, a three-dimensional orthographic display of the above mentioned fitted curves vs neutron energy is constructed. This is to display compactly the variation of the elastic scattering angular distributions with energy. Where available, evaluated curves of the variation of the elastic scattering cross section (divided by $4 \pi$ ) with energy were projected onto the cross section vs energy plane $(\cos \theta=+1)$ of the three-dimensional plots. This is to show the relationship between measured distributions and the structure in the neutron cross section.
(4) Finally, for a few elements a body of data exists in the form of coefficients of Legendre polynomials. The coefficients are presented in tabular form, and the differential cross sections synthesized from these coefficients are presented in an orthographic representation (similar to 3). Legendre coefficients have been converted to units of millibarns/ steradian. Energy is expressed in MeV .
The graphically represented data (1) and the tabulated data (2) are internally organized in the same way. The data are grouped according to reaction type. The reaction types found in this book are listed in the Glossary in the same order in which the data are presented.

Within a given reaction type, data are ordered by incident neutron energy.
Data sets for measurements with differing outgoing particle energies or for differing level energies in the residual nucleus are then sorted by "Secondary Energy."

The secondary energy can be specified in separate but equivalent ways (see Glossary). It should be noted that no attempt has been made to recognize the identity of two data sets where a reaction with a particular secondary energy has been specified in the library in separate, but equivalent, ways (for example, 4.43L and $4.43 Q$ for the first excited state of $\mathrm{C}^{12}$ ). For explanation of $L$ and $Q$, see discussion of Secondary Energy in Glossary.

Finally, separate, but adjacent, plots (or table entries) are made of center-of-mass or laboratrycoordinate data sets. It should be noted, however, that all differential elastic data have been transformed into center-of-mass coordinates.

## Element//sotope

The first entry of the label box or data table is the nuclear target under consideration: cither the natural element or Isotope-A. Elements which naturally contain only one isotope are marked with the mass number of that isotope ( $\mathrm{Al}^{27}, \mathrm{Bi}^{20 \Omega}$ ). Radiogenic target materials such as "radio-lead" are also noted.

## Reaction Type

The reaction types to be found in this book are listed below:

| $\operatorname{DIF}(N, A L P H A)$ | $D I F S C A T T E R$ | $\operatorname{DIF}(N, N P)$ |
| :--- | :--- | :--- |
| $\operatorname{DIF}(N, D)$ | $\operatorname{DIF}$ ELASIIC $+\mathbb{N}$ | $\operatorname{DIF}(N, N T)$ |
| $\operatorname{DIF}(N, H E 3)$ | $\operatorname{DIF} N O N-E L A S$ | $\operatorname{DIF}(N P+N N P)$ |
| $\operatorname{DIF}(N, P)$ | $\operatorname{DIF}(N, N$ ALF $)$ | $\operatorname{DIF}(N, 2 A L P H)$ |
| $\operatorname{DIF}(N, T)$ | $\operatorname{DIF}(N, N D)$ | $\operatorname{DIF}(N, 2 N)$ |
| $\operatorname{DIF} E L A S T I C$ | $\operatorname{DIF}(N, N G A M)$ | $\operatorname{DIF}(N, 2 P)$ |
| $\operatorname{DIF} \operatorname{INELASIIC}$ | $\operatorname{DIF}(N, N H E 3)$ | $\operatorname{DIF} G A M E M I S$ |

Most of these are self-explanatory and follow the normal process definitions. DIF SCATTER is the sum of the clastic plus inelastic scattering cross sections, while DIF ELAST + IN is the elastic scattering cross section with some significant (though usually unspecified) amount of inclastic scattering contamination. The values for the $(n, n \alpha),(n, n d)$, etc., reaction types are for the angular distribution of the resulting charged particle. DIF( N , NGAM) is for the inelastic scattering cross section determined by detecting gamma rays, that is, the ( $\mathrm{n}, \mathrm{n}^{\prime} \gamma$ ) reaction, while DIF GAM EMIS is the cross section either for gamma rays produced by an unspecified reaction or, more usually, for inelastic scattering gammas with contributions from cascade gammas resulting from deexcitations.
It should be noted that the information on total scattering (elastic + inelastic scattering) angular distributions, which was contained in appendices as excitation functions in the Sccond Edition, is here plotted as angular distributions and labeled DIF SCATTER.
Incident Neutron Energy: $\mathrm{E}=$, or $\mathrm{E}(\mathrm{MEV}$ )
Laboratory energy of incident neutron in MeV.
Secondary Energy: E2 $=$, or SEC.EN.
Reactions which proceed to differing excited states in the final nucleus or emit particics of differing encrgy are differentiated by a secondary energy quantity. The secondary energy is indicated by a number followed by a designating letter or phrase which indicates the meaning of the number. The meaning of the letters is as follows:
N - Inelastically scattered neutron energy, $\mathrm{E}_{\mathrm{n}^{\prime}}$, in MeV .
$X$ - Emitted particle (other than neutron) energy, $\mathrm{E}_{\mathrm{p}}$, $\mathrm{E}_{\mathrm{a}}, \mathrm{E}_{\gamma}, \mathrm{ctc}$., in MeV .
L - Level excited in inelastic scattering, in MeV.
M - Mixture of unresolved excited levels. Average energy, in MeV , sometimes listed.
Q - Q -value of reaction, in MeV .
[REFER TO COMMENT] - Meaning expanded upon in Comment.
A data set may contain datum points from a variety of sources or laboratories. Also, data from a given source may at times be differentiable into sub-sets. Therefore, the following items are keyed to particular datum points of the graphical data by a symbol.





## Reference and Laboratory

(Self-explanatory)

## Energy Uncertainty

Noted in the legend box, when known, is the incident energy uncertainty, indicated by $\Delta \mathrm{E}$. Two forms of $\Delta \mathrm{E}$ appear: $\Delta \mathrm{E}_{\mathrm{R}}$ implies that the following number is an energy resolution, and $\Delta E_{E}$ implies that the following number represents an uncertainty in energy assignment or a range of incident energies. The letters R and E are used in the same sense following the energy uncertainty (DE) in the data tables. Neither quantity is cither precise or consistently uscd, since the meaning is not always clear in a reporting document. The number is included here therefore only as a rough measure of energy uncertainty.

## See Comment No.

$\qquad$
Occasionally, additional text has been appended to the compilation, cither to explain some detail or to add some additional information. When a data set has such an additional Comment, the legend box contains the statement SEE COMMENT NO. $\qquad$ or an identical statement appears under the appropriate data list in the data tables. The comments referred to are enumerated and collected into an Appendix at the end of each Volume under the cle-ment-isotope combination involved, each such combination having a separate enumeration.

## __n $^{\text {nh }}$ Order Legendre Fit

A single curve is fit to a data set over the range of cosines where data exist. The logarithm of the differential cross section is expanded in a set of Legendre polynomials. The final item contained in the legend box is a polynomial order number. This number indicates the highest order ( $l$-valuc) used in the Legendre polynomial fitting sum (see section on Fitting Procedure) that appears on the graph $(0,1,2, \ldots)$.

## Miscellaneous

The Coordinate System in which the data arc plotted is noted on the abcissa of each graph. All elastic scattering angular distributions have been presented in the center-of-mass system. For other reaction types, the coordinate system originally presented by the measurer is generally employed. In the data tables, the system involved is noted immediately following the numerical value of "cos(theta)," an "L" denoting the laboratory system and a blank denoting the center-of-mass system.

The pagination of the book reflects the element/isotope arrangement of the data. A threc-number set is used in which the first number is $Z$, the second is $A$ (with the definition that $A=0$ for the natural element), and the third is merely the page sequence number for the $\mathrm{Z}, \mathrm{A}$ involved. Thus, page 5-0-3 is the third page of natural B, and page 9-19-5 is the fifth page of ${ }_{9} F^{19}$. The element or isotope is also noted in an upper corner of each page, as well as on each angular distribution graph.

If a data set consists of four or more points, a graphical plot is constructed and presented. If there are less than four points involved, they are entered into a data table which follows. Such table entries are flagged in the appropriate order between graphs by the phrase [REFER TO TABLE FOR ADDIIIONAL DATA SETS]. (Refer to section on Arrangement.)

## COMPUTER PREPARATION

The data appearing in this compilation were obtained from the computerized data library SCISRS. A retrieval from this data library consists of a series of datum lines, each line containing a single measured datum point and all the information associated with it.

Computer programs were written to collect datum points having the same Z, A, Reaction Type, E, E2, and Coordinate System into data sets. CRT plots with fitted curves were generated if these sets contained four or more points; otherwise data tables were constructed (also by CRT). All decisions as to scale selection, referencing, fitting criteria, etc., were programmed decisions. The fitting procedure that was used is defined below. It was used unchanged to produce $99.5 \%$ of the graphs. Only $0.5 \%$ of the plots generated in this manner were found to be unsatisfactory (usually too oscillatory). These few plots were re-run with a lower value of $l_{\max }$ (see below). The only other human intervention was in the orthographic data display construction. Often the programmed decisions as to energy ranges were overridden for composition reasons.

## Fitting Procedure

The "eye guides" were produced* by making a least-squares fit of the logarithms of the experimental data to a sum of Legendre polynomials, i.e.,

$$
\ln \sigma=\sum_{l=1}^{l_{\max }} \mathrm{a}_{l} \mathrm{P}_{l}(\cos \theta)
$$

where $\sigma=\mathrm{d} \sigma / \mathrm{d} \Omega$. In this logarithmic mode the experimental error was defined as

$$
\Delta \sigma=\ln (\sigma+\delta \sigma)-\ln \sigma
$$

where $\delta \sigma$ is the assigned experimental error. Each datum point was assigned a weight, W , of the form $W=1 /(\Delta \sigma)^{2}$ in those cases where experimental errors were given by the measurer. When some of the points within a data sub-set of a particular reference had no errors stated, an average of the weights for near-neighbor points of the same sub-set was arbitrarily assigned as the weight for a particular point. If no crrors at all had been given by the measurer, all points in the data set were given equal weight. Absolute cross section errors were treated differently from statistical errors. Corrections were made for normalization errors, etc., in order to extract a "shape" error, the pertinent one for "eyeguide" fitting.

The fitting was performed for all integer values of $l$ up to $l_{\text {max }}$. The value of $l_{\text {max }}$ used was determined as follows:
(1) The linear momentum ( $p$ ) of the incident neutrons was computed. Its classical angular momentum was then calculated assuming an impact parameter ( $r$ ) equal to 1.5 $\mathrm{A}^{1 / 3}$ fermi. A tentative value of $l_{\max }$ proportional to the product ( $\mathrm{r} \times \mathrm{p}$ ) was calculated. If this value of $l_{\max } \leqslant 6, l_{\max }$ was tentatively set equal to 6 .
(2) This tentative value of $l$ was then compared to the total number of datum points N in the fitting set. If $l_{\max } \geqslant \mathrm{N}-1$, the value of $l_{\max }$ was set at $\mathrm{N}-2$ (except for only $\mathrm{N}=4$ datum point fits, where $l_{\max }$ was always set equal to 3 ).
(3) $l_{\text {max }}$ must be less than an upper limit, ordinarily 20. However, a lower upper limit could be input, and this was the mode of intervention mentioned above.

[^0]The selection of the "best" fit was made with a $\chi^{2}$ test, where

$$
\chi^{2}=\frac{1}{\mathrm{~N}} \sum\left(\frac{\sigma_{\mathrm{i}}^{\mathrm{e}}-\sigma_{\mathrm{i}^{\mathrm{e}}}}{\Delta \sigma_{\mathrm{i}}^{\mathrm{e}}}\right)^{2} .
$$

Here N is the number of degrees of freedom (nurnber of datum points); $\sigma_{i}{ }^{\mathrm{c}}$ is the calculated value, $\sum a_{l} \mathrm{P}_{l}\left(\cos \theta_{\mathrm{i}}\right)$, for the $\mathrm{i}^{\text {th }}$ pointat $\cos \theta_{i} ; \sigma_{\mathrm{i}} \mathrm{e}$ is the experimental value at $\cos \theta_{i} ;$ and $\Delta \sigma_{\mathrm{i}}{ }^{\mathrm{e}}$ is the logarithmic experimental error in $\sigma_{\mathbf{i}}{ }^{\mathbf{e}}$, as defined above.

Three separate criteria are applied by the computer in order to choose the best fit:
(1) $\chi^{2} \leqslant \mathrm{C}$, where C is an empirically determined constant defining a "good" fit, the first (lowest $l$ ) polynomial expansion satisfying this was chosen. The value of C used was 1.5 .
(2) If (1) could not be fulfilled, a check was applied to test whether $\chi^{2}$ was constant (within an assigned limit of $\pm 25 \%$ ) over a range of $l$-values. If so, the first (lowest $l$ ) polynomial expansion on this $\chi^{2}$ plateau was chosen.
(3) If neither (1) nor (2) could be fulfilled, the polynomial expansion corresponding to the lowest value of $\chi^{2}$ was chosen.

For angular distributions peaked in the forward direction, a linear extrapolation was made to $\cos \theta=+1\left(\theta=0^{\circ}\right)$, and the resulting fictitious datum point was used in the fitting procedure. Since the fitting was performed only in the angular region where experimental data existed, the Legendre polynomial coefficients obtained should not be used to extrapolate outside this range, nor should they be used to calculate a total integrated cross section.

## ACKNOWLEDGMENTS

A data compilation such as this could not be prepared without the cooperation and assistance of the many people whose work is being presented. We are grateful to the experimentalists, too numerous to name here, who made special efforts to provide us their data in a useful form and answered our questions regarding their techniques.

We hope for the continued cooperation of experimentalists and other users of this book. Since supplements containing new information will be published from time to time, we trust that newly acquired data will be sent to the National Neutron Cross Section Center, here at Brookhaven, as soon as they become available. We shall appreciate criticisms, suggestions, and notes of any errors in graphs or text of this compilation.

Both ours and the reader's tasks have been greatly lightened by the admirable work of the Photography and Grephic Arts Division of this Laboratory. Special thanks are due to R.P. Brown and his group for their work in the publication of the compilation.

The considerable work involved in obtaining error-free and properly formatted data sets from the SCISRS library was ably performed by Miss Helene Bauman.


[^0]:    *Parts of the fitting programs have been extracted from E.M. Pennington, et al., "Programs for Analysis of Scattering Angular Distributions," Argonne National Labera ory Report ANL-7306 (1967).

