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SIGMA CENTER

NEUTRON CROSS SECTION EVALUATION GROUP

Least Squares Analysis of the 2200 m/sec Parameters of U^{233}, U^{235}, and Pu^{239}

FINAL REPORT

March 1965

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Least Squares Analysis of the 2200 m/sec Parameters of U^{233}, U^{235}, and Pu^{239} *

[Final Report]

Rudolph Sher** and Joan Felberbaum
Brookhaven National Laboratory
Upton, New York

I. Introduction

In 1962 "best" values of the 2200 m/sec cross sections of the principal fissionable isotopes U^{233}, U^{235}, and Pu^{239} were obtained[1] by the method of least-squares applied to about 60 measurements, published and unpublished. The present report is a revision of the earlier work in which new data are included, some old data revised or eliminated, and a few errors rectified. The total number of measurements used has increased to 82. All data in the BNL Sigma Center files in December, 1964 have been considered; most have been included, the exceptions being some results which were superseded by later data from the same authors.

For a discussion of the least-squares technique, treatment of the experimental data, and interpretation of the results, the reader is referred to Reference 1. As part of the present study, Ibarra and Sher[2] have shown that taking account of quadratic terms in the expansion of the observational equations leads to negligible changes in the final results; therefore, the present calculations continue to use the linearized equations.

* This work was performed under subcontract with the BNL Sigma Center.
** Present address: Stanford University, Stanford, California

II. Results

The measured values used in the calculation are listed in Appendix A. The first column lists the experimental value, expressed either in barns for cross sections, or as pure numbers for η, ν, $(1+\alpha)$, and ratios. The second column lists the relative (percent) error on the measurement, usually as quoted by the experimenter, but occasionally changed by us to account for estimated spectral uncertainties, etc. The third column lists the assigned weight, usually $10^{-4}/\sigma^2$, but also occasionally changed from this value. References and remarks are in the last column.

For $\sigma_a(Pu^{239})$ we have listed individual measurements based on total cross sections with a scattering cross section of 10 barns subtracted. The average of these results is in close agreement with the value obtained graphically in Reference 1, but the weighting may be more realistic here. For σ_a in U^{233} and U^{235}, the graphically obtained input data of Reference 1 have been retained, except that the scattering cross sections have been changed slightly to agree with the recommended values in the current (1965) edition of BNL-325.

The coefficients of the normal equations and the solutions of the normal equation and the error matrix are tabulated in Appendix B.

The final results are listed in Table I.

Table I

	U^{235}	U^{233}	Pu^{239}
σ_f	577.1 ±1.9	524.5 ±1.9	740.6 ±3.5
σ_a	678.2 ±2.2	573.1 ±2.1	1014.5 ±4.2
ν	2.442±0.006	2.504 ±0.008	2.898±0.011
η	2.078±0.005	2.292 ±0.006	2.116±0.009
α	0.175±0.002	0.0926±0.0027	0.370±0.006

With few exceptions, these results all agree within the listed uncertainties with the values in Reference 1. $\sigma_f(U^{235})$ has changed by an amount barely outside the combined errors; this reflects among other things the elimination in this calculation of 2 or 3 suspect values used in (1). The largest change occurs in $\sigma_a(Pu^{239})$, where the change is about 1.5%.

In Figure 1 the probability integral curve[1] is plotted. Agreement with the calculated distributions seems to be somewhat better than in (1).

There has been little attempt to determine and correct systematic errors in the various experiments, unless these have been fairly obvious. One exception has been in the measurements of ν; these have been corrected, if necessary, to include delayed neutrons; and for those measurements which are normalized to ν of Cf^{252}, the value $\nu(Cf^{252}) = 3.779 \pm 0.010$ has been adopted and the results renormalized to this value, which is the weighted average of several recent measurements.[3]

It is worthwhile comparing the least squares results with the individual measurements. In Table II the results are compared with the weighted averages of the experimentally measured input data. With the exception of $\sigma_f(Pu^{239})$, for which only a single measurement with a quoted uncertainty of ± 3.0% exists, the agreement is quite good.

Table II

	Average of measured values	Least squares value
$\sigma_f(U^{235})$	582.6 ± 4.1	577.1 ± 1.9
$(1+\alpha)(U^{235})$	1.1738 ± 0.0010	1.1756 ± 0.0022
$\eta(U^{235})$	2.076 ± 0.006	2.078 ± 0.005
$\nu(U^{235})$	2.438 ± 0.004	2.442 ± 0.006
$\sigma_f(U^{233})$	518 ± 5	524.5 ± 1.9
$(1+\alpha)(U^{233})$	1.0937 ± 0.0004	1.0926 ± 0.0027

Table II (con't)

	Average of measured values	Least squares value
$\eta(U^{233})/\eta(U^{235})$	1.106 ± 0.002	1.103 ± 0.003
$\eta(U^{233})$	2.295 ± 0.004	2.292 ± 0.006
$\nu(U^{233})/\nu(U^{235})$	1.0215± 0.0029	1.0255± 0.0028
$\sigma_f(U^{233})/\sigma_f(U^{235})$	0.911 ± 0.005	0.909 ± 0.003
$\eta(Pu^{239})$	2.119 ± 0.030	2.116 ± 0.009
$\eta(Pu^{239})/\eta(U^{235})$	1.0175± 0.0064	1.0183± 0.0041
$\sigma_f(Pu^{239})/\sigma_f(U^{235})$	1.2936± 0.0311	1.2837± 0.0055
$\nu(Pu^{239})/\nu(U^{235})$	1.1835± 0.0070	1.1862± 0.0037
$\sigma_f(Pu^{239})/\sigma_f(U^{233})$	1.430 ± 0.021	1.412 ± 0.007
$\nu\sigma_f(Pu^{239})/\nu\sigma_f(U^{235})$	1.5278± 0.0155	1.5229± 0.0059
$\sigma_a(Pu^{239})$	1010.4 ± 3.7	1014.5 ± 4.2
$\eta\sigma_a(U^{233})/\eta\sigma_a(U^{235})$	0.9338± 0.0047	0.9321± 0.0033
$(1+\alpha)(Pu^{239})$	1.370 ± 0.014	1.370 ± 0.006
$\eta\sigma_a(Pu^{239}) = \nu\sigma_f$	2073 ±51	2148 ±10
$\eta\sigma_a(U^{233})$	1312 ± 2	1314 ± 5
$\eta\sigma_a(U^{235})$	1413 ± 5	1409 ± 5
$\nu(Pu^{239})$	2.929 ± 0.038	2.898 ± 0.011
$\nu(U^{233})$	2.538 ± 0.036	2.504 ± 0.008
$\nu(Pu^{239})/\nu(U^{233})$	1.163 ± 0.012	1.157 ± 0.004
$\sigma_f(Pu^{239})$	704 ±21	740.6 ± 3.5
$\sigma_a(U^{233})$	576 ± 4	573.1 ± 2.1
$\sigma_a(U^{235})$	681 ± 4	678.2 ± 2.2

ACKNOWLEDGMENTS

We wish to thank many readers of BNL-722 and experimentalists who have kindly sent us criticisms, corrections, and new data. We hope that these have been properly incorporated in the present report. For more specific help and the use of the Sigma Center facilities we wish to thank J. R. Stehn and B. Magurno, and in particular, Jack Chernick, for suggesting the usefulness of a revision of BNL-722 and arranging the support which made it possible.

REFERENCES

1. R. Sher and J. Felberbaum, "Least Squares Analysis of the 2200 m/sec Parameters of U^{233}, U^{235}, and Pu^{239}," BNL-722 (1962).

2. H. Ibarra and R. Sher, unpublished (1964).

3. Mather, Moat, and Fieldhouse, Phys. Rev. 133, 1403 (1963); Hopkins and Diven, Nuclear Phys. 48, 433 (1963); Colvin and Sowerby, private comm. to BNL Sigma Center (1964); Asplund-Nilsson, Conde, and Starfelt, Nuclear Sci. Eng. 16, 124 (1963).

PROBABILITY INTEGRAL CURVE

$$F(r_m/\sigma_m)=\frac{1}{2\pi}\int_{-r_m/\sigma_m}^{r_m/\sigma_m} e^{-\frac{x^2}{2\alpha^2}}dx$$

— $\alpha^2=\frac{N-n}{N}$ $\qquad N=82 \quad n=9$

---- $\alpha^2=\sum\frac{(r_m/\sigma_m)^2}{N}$ $\qquad \sum(r_m/\sigma_m)^2=92.273$

—⌐ Experimental Distribution

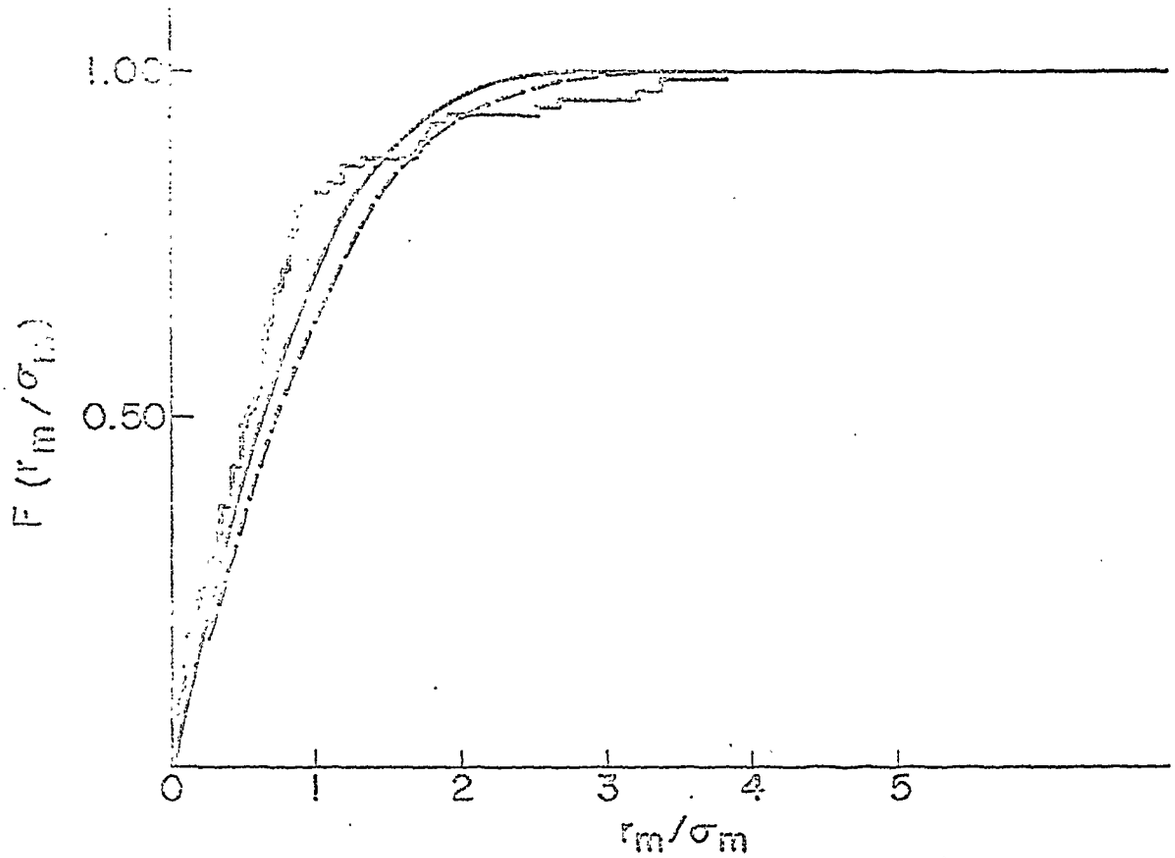

Figure 1

APPENDIX A

		REL[%] ERROR	WT	
				MEASUREMENT OF SIGMA F 235
1	582.00000	2.50	0.16	RAFFLE,JNE 10,8[1959]
1	587.00000	1.00	1.00	DERUYTTER, JNE 15,165 [1961]
1	574.00000	1.00	0.50	MASLIN,QUOTED BY E.R.RAE,GENEVA,[1964]
				MEASUREMENT 1&ALPHA 235
2	1.17160	1.00	1.00	SAFFORD, REFERENCE [3]
2	1.16577	0.50	1.00	WILLIAMS, QUOTED IN REF.[3]
2	1.17400	3.00	0.11	KANNE ET AL, GENEVA CONF.4,315,[1956]
2	1.17200	2.00	0.25	COCKING, QUOTED IN REF.[3]
2	1.17500	2.00	0.25	TINGEY AND VANCE, QUOTED IN REF.[3]
2	1.17300	0.40	6.25	JONES&LOUNSBURY,EANDC[CAN]11[1961]
2	1.17250	0.50	4.00	CABELL&SLEE,J.INORG NUCL CHEM24,1493 [1962] INCLUDES EST.ERROR IN G
2	1.17180	0.50	4.00	OKAZAKI&LOUNSBURY,AECL-1965[APR 1964] INCLUDES EST. ERROR IN G
2	1.17670	0.50	4.00	QUOTED IN OKAZAKI&LOUNSBURY,AECL-1965 [APR 1964] INCLUDES EST ERROR IN G
				MEASUREMENT OF ETA 235
3	2.05000	2.50	0.16	LITTLER&LOCKETT, QUOTED IN REF. [3]
3	2.06500	1.25	0.64	SPIVAK&YEROZITLIMSKY,GENEVA 4,295[1956]
3	2.12000	2.50	0.16	ALICHANOV ET AL, GENEVA 4,301[1956]
3	2.07700	0.48	4.34	MACKLIN ET AL, NSE 8,210[1960]
				MEASUREMENT OF NU 235
4	2.44000	5.00	0.04	SNYDER&WILLIAMS,LA-102[1944], REVISED VALUE QUOTED IN REF.[3]
4	2.42000	1.53	0.43	KENWARD,RICHMOND&SANDERS,QUOTED IN [3]
4	2.44300	0.70	2.04	COLVIN&SOWERBY,P.C. SIGMA CENTER[1964]
4	2.43000	0.80	1.56	MATHER ET AL, PR,133,1403[1963]
4	2.44600	1.00	1.00	HOPKINS&DIVEN,NUCL.PHYS.,48,433[1963] LAST 3 MEASUREMENTS NORMALIZED TO 3.779 FOR NU CALIFORNIUM 252
				MEASUREMENT OF SIGMA F 233
5	518.00000	1.00	1.00	BIGHAM ET AL, GENEVA 16,125[1958]
				MEASUREMENT OF 1&ALPHA 233
6	1.09300	0.50	4.00	INGRAHAM ET AL, GENEVA 4,105[1956]
6	1.09400	0.60	2.78	KUKAVADSE ET AL, GENEVA 4,230[1956]
6	1.09420	0.50	4.00	CABELL&SLEE,JNE 16,195[1962] INCLUDES EST. ERROR IN G
				MEASUREMENT OF RATIO ETA 233/ETA 235
7	1.10500	0.50	4.00	MACKLIN ET AL, NSE 8,210,[1960]
7	1.10800	2.00	0.25	RICHMOND, QUOTED IN REF.[6]
7	1.11400	1.50	0.44	ANDERSON&NAGLE, QUOTED IN REF.[6]
7	1.11500	1.50	0.44	CALLIHAN ET AL, GENEVA P/834 [1955]
7	1.10200	1.30	0.59	CABELL,ROSE,TATTERSALL,TNCC[UK]77[1960
7	1.10100	2.00	0.25	GAERTTNER ET AL, NSE 3,1758[1958]

				MEASUREMENT OF ETA 233
8	2.29600	0.44	5.17	MACKLIN ET AL, NSE 8,210[1960]
8	2.28000	1.50	0.44	SPIVAK ET AL, GENEVA 4,295[1955]
				MEASUREMENT OF RATIO NU 233/NU 235
9	1.02010	1.00	1.00	DESAUSSURE&SILVER,NSE 5,49[1959]
9	1.03000	1.00	1.00	KALASHNIKOVE ET AL, PROC.ACAD.SCI.USSR 156[1955]
9	1.01700	3.00	0.11	MCMILLAN ET AL, KAPL-1464[1955]
9	1.02000	0.60	2.08	COLVIN&SOWERBY,EANDC[UK]30U
9	1.01000	1.70	1.05	SANDERS,JNE 2,247[1956]
9	1.04300	2.00	0.25	DIVEN ET AL, PHYS. REV. 101,1012,[1956
9	1.01600	1.50	0.44	HOPKINS&DIVEN,NUCL. PHYS 48,433[1963] CORRECTED FOR DELAYED NEUTRONS
				MEASUREMENT OF RATIO SIGMA F 233/ SIGMA F 235
10	0.91100	0.50	4.00	BIGHAM ET AL, GENEVA 16,125[1958] INCLUDES ERROR IN G FACTOR
				MEASUREMENT OF ETA 239
11	2.08100	1.25	0.34	SPIVAK ET AL, GENEVA 4,295[1955] INCLUDES G FACTOR CORRECTION
11	2.14300	1.00	1.00	MACKLIN ET AL, NSE 14,101[1962]
				MEASUREMENT OF RATIO ETA 239/ETA 235
12	1.00000	1.10	0.83	ROSE,COOPER,TATTERSALL,TNCC[UK]77[1960
12	1.00700	3.00	0.11	RICHMOND, QUOTED IN REF.[6]
12	1.00000	3.00	0.11	ANDERSON&NAGLE, QUOTED IN REF.[6]
12	1.03200	1.00	1.00	MACKLIN ET AL, NSE 14,101,[1962]
12	1.01000	2.00	0.25	GAERTTNER ET AL, NSE 3,1758[1958]
				MEASUREMENT OF RATIO SIGMA F 239/ SIGMA F 235
13	1.30140	1.50	0.44	BIGHAM ET AL, GENEVA 16,125[1958]
13	1.23500	1.50	0.40	PRATT ET AL, ORNL-2081[1956]
13	1.34100	1.50	0.44	SELLERS ET AL, ANL-5411[1955] CORRECTED FOR G FACTOR
				MEASUREMENT OF RATIO NU 239/NU 235
14	1.23000	3.00	0.11	DIVEN ET AL, PHYS.REV101,1012[1956]
14	1.22500	1.00	1.00	DESAUSSURE&SILVER, NSE 5,49[1959]
14	1.16300	1.00	1.00	SANDERS ET AL, JNE 2,247[1956]
14	1.17700	1.00	1.00	WILSON,LA-104[1944]
14	1.18200	0.70	2.04	COLVIN&SOWERBY, P.C. TO SIGMA CENTER
14	1.18500	1.00	1.00	KALASHNIKOVA ET AL, PROC.ACAD. SCI. USSR [1955]
14	1.17000	2.00	0.25	SNYDER&WILLIAMS QUOTED IN REF.[6]
14	1.16200	1.50	0.44	HOPKINS&DIVEN,NUCL.PHYS,48,433[1963] CORRECTED FOR DELAYED NEUTRONS
14	1.16400	2.00	0.25	JACOB,QUOTED IN TNCC[UK]43[1959]
				MEASUREMENT OF RATIO SIGMA F 239/ SIGMA F 233
15	1.43000	1.50	0.44	BIGHAM ET AL, GENEVA 16,125[1958] INCLUDING G FACTOR CORRECTION

				MEASUREMENT OF RATIO NU SIGMA F 239/ NU SIGMA F 235
16	1.[illegible]000	1.50	0.44	GWIN&MAGNUSON, NSE 12,359[1962] CORRECTED BY G FACTOR
16	1.53400	0.60	2.78	JAFFEY,HIBDON&SJOBLOM QUOTED IN REF.[5
				MEASUREMENT OF SIGMA A 239
19	1060.[illegible]	[illegible]	1.5[illegible]	SAFFORD&HAVENS
19	[illegible]	[illegible]	[illegible]	BOLLINGER
19	1040.[illegible]	3.[illegible]	0.11	PATTENDEN
19	1030.[illegible]	3.[illegible]	0.11	NIKITIN
19	1015.00000	1.00	1.00	PALEVSKY
				MEASUREMENT OF RATIO ETA SIGMA A 233/ ETA SIGMA A 235
20	0.94240	1.50	0.44	GWIN&MAGNUSON, NSE 12,359[1962] INCLUDES G FACTOR CORRECTION
20	0.95120	0.82	1.49	GWIN&MAGNUSON,NSE 12,364[1962]
				MEASUREMENT OF 1&ALPHA 239
21	1.37000	1.30	1.00	[illegible] AND SLEE, J. INORG.NUCL.CHEM 25 607[1963] INC.EST.G FACTOR ERROR
				MEASUREMENT OF ETA SIGMA A 239
22	2073.00[illegible]	2.50	0.16	MUEHLHAUSE, NSE 5,225[1959]
				MEASUREMENT OF ETA SIGMA A 233
23	1307.[illegible]	1.50	0.44	MUEHLHAUSE, NSE 5,225[1959]
23	1313.[illegible]	0.66	2.30	GWIN&MAGNUSON,NSE 12,364[1962]
				MEASUREMENT OF ETA SIGMA A 235
24	1423.00000	1.5[illegible]	0.44	MUEHLHAUSE, NSE 5,225[1959]
24	1410.00000	0.75	1.78	GWIN&MAGNUSON, NSE 12,364[1962]
				MEASUREMENT OF NU 239
25	2.92900	1.30	0.59	MATHER,FIELDHOUSE&MOAT,EANDC[UK]49S
				MEASUREMENT OF NU 233
26	2.53800	1.40	0.51	MATHER,FIELDHOUSE&MOAT,EANDC[UK]49S
				MEASUREMENT OF RATIO NU 239/NU 233
27	1.16300	1.00	1.00	COLVIN&SOWERBY,TNCC[UK]43[1959]
				MEASUREMENT OF SIGMA F 239
28	704.00000	3.00	0.11	RAFFLE,AERE-R-2998 [1959]
				MEASUREMENT OF SIGMA A 233
17	576.00000	0.70	2.00	FROM GRAPH
				MEASUREMENT OF SIGMA A 235
18	681.00000	0.58	3.00	NEW VALUE

REFERENCES

[3] SAFFORD&HAVENS,NUCLEONICS17,134[1959
[5] LEONARD,HW-69342[1961][UNPUBLISHED]
[6] EGELSTAFF,MORTON,SANDERS, AERE NP/R2140 [1957][UNPUBLISHED]

APPENDIX B

COEFFICIENTS OF THE NORMAL EQUATIONS

48.77	20.96	-11.[illegible]	-7.91	-6.81	[illegible]	5.98	-34.44	2.30	-0.23064
20.96	30.05	-7.91	-13.84	-5.52	-11.[illegible]	5.98	-13.58	2.30	-0.01159
-11.[illegible]	-7.91	32.48	16.26	-0.44	0.[illegible]	-22.37	5.98	0.00	[illegible].11763
-7.91	-13.84	16.[illegible]	23.70	0.00	-1.00	-11.5[illegible]	5.98	0.00	0.04745
-6.81	-5.52	-0.44	[illegible].00	10.17	7.32	0.00	2.30	-4.94	0.15001
-5.52	-11.32	0.00	-1.00	7.32	16.01	0.00	2.30	-3.94	0.05797
5.98	5.98	-22.37	-11.[illegible]	0.[illegible]	0.00	24.37	-5.98	0.00	-0.11509
-34.44	-13.58	5.98	5.98	2.30	2.30	-5.98	37.44	-2.30	0.13093
2.30	2.30	0.00	0.00	-4.94	-3.94	0.00	-2.30	8.72	-0.07082

ERROR MATRIX BY ROWS

0.10578386E 00-0.30102927E-01 0.61170736E-01-0.24124192E-01 0.71805535E-01
-0.26286612E-01 0.47508951E-01 0.87219921E-01 0.31845157E-01

-0.30102927E-01 0.65109111E-01-0.23802971E-01 0.43350765E-01-0.23852960E-01
0.56077686E-01-0.12572556E-01-0.11207211E-01-0.36448939E-03

0.61170736E-01-0.23802971E-01 0.13470614E 00-0.49096689E-01 0.51592077E-01
-0.20456001E-01 0.10345286E 00 0.50210671E-01 0.20661341E-01

-0.24124192E-01 0.43350765E-01-0.49096689E-01 0.94964646E-01-0.24743836E-01
0.44676762E-01-0.86970707E-02-0.16608698E-01-0.32831690E-02

0.71805535E-01-0.23852960E-01 0.51592077E-01-0.24743836E-01 0.23045539E 00
-0.85902867E-01 0.37564398E-01 0.55997839E-01 0.93864270E-01

-0.26286612E-01 0.56077686E-01-0.26456001E-01 0.44676762E-01-0.85902867E-01
0.14256015E 00-0.13308619E-01-0.12067451E-01 0.47078235E-02

0.47508951E-01-0.12572556E-01 0.10345286E 00-0.86970707E-02 0.37564398E-01
-0.13308619E-01 0.13434913E 00 0.45078001E-01 0.17942415E-01

0.87219921E-01-0.11207211E-01 0.50210671E-01-0.16608698E-01 0.55997839E-01
-0.12067451E-01 0.45078001E-01 0.10407843E 00 0.33673706E-01

0.31845157E-01-0.36448939E-03 0.20661341E-01-0.32831690E-02 0.93864270E-01
0.47078235E-02 0.17942415E-01 0.33673706E-01 0.17055990E 00

SOLUTION VECTOR

-0.50539367E-02 0.51234443E-02-0.90656684E-03 0.17293692E-02 0.14561774E-01
-0.58309089E-03-0.50225900E-02-0.11134371E-02-0.44758975E-03