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__Abstract:__ The optical reaction cross sections for n, p, ^{2}H, ^{3}H, ^{3}He and ^{4}He for several global optical potentials available in the literature have been parameterized in terms of simple empirical expressions which are smooth functions of the target mass number and the projectile energy. The empirical forms are 5-10% accurate over the whole periodic table and energy range upto 50 MeV. They can be conveniently used in applications where the optical reaction cross-sections are required as input such as in statistical evaporation and preequilibrium emission calculations.

__Key words__ Optical reaction cross sections for n, p, ^{2}H, ^{3}H, ^{3}He, ^{4}He; empirical parameterisation; evaporation and preequilibrium models.

1. Introduction

The non-elastic part of the interaction between the incident particle and the target nucleus manifests itself as the reaction cross-section. As a first approximation this interaction is described by a complex (optical) potential, the imaginary part of which leads to a finite reaction cross-section. In the literature the projectile-target interaction for target nuclei over the periodic chart has been given in terms of global optical parameter-sets. The reaction cross-sections generated using these global parameters are employed in several further calculations. In the statistical evaporation theory and the preequilibrium model the break-up of the total reaction cross section into partial modes involving the emission of one or more particles is described by emission rate expressions which involve the inverse cross-section of the emitted particle as a function of energy. The inverse cross-section is the optical reaction cross-section of the excited residual nucleus with the emitted particle as the projectile. This is approximated as the optical reaction cross-section between the residual nucleus in its ground state and the (emitted) particle. Thus optical reaction cross sections are required for the incident channel at the projectile energy and for the exit channels over the energy range of the emitted particles. The statistical evaporation theory is a widely-used description for reactions at low energies. At energies in the range of several

tens of MeV the preequilibrium models have been successful. Reaction cross-sections are also used in intra-nuclear-cascade calculations (Kikuchi and Kawai 1968). In this description of high-energy nuclear reactions, the projectile is assumed to interact with the nucleons in the target nucleus in a series of successive two body collisions until it escapes or is captured by the nuclear potential. Residual nuclei left after this cascade process undergo particle evaporation, and the evaporation rates are calculated using inverse cross-sections. Another application of reaction cross-sections is in the calculation of the nuclear-reaction efficiency correction for detectors (Makino et al 1968). In the detection of charged particles, an efficiency loss f arises from events where the particles undergo nuclear interactions before they are completely stopped. For a charged particle incident with energy E_i on a target with mass number A, f is given by ,

$$f = \frac{N_0}{A} \int_0^{E_i} \sigma_R(\epsilon) \left(\frac{dE}{dx}\right)^{-1} dE \qquad (1)$$

where σ_R is the reaction cross-section, $\frac{dE}{dx}$ is the stopping power of the particle in the detector material and N_0 is Avogadro's number.

Usually the optical reaction cross sections at all the required energies and particle-target combinations are obtained by a numerical solution of the Schrodinger equation for several partial waves. This involves a large amount of computation time. Also, where more than one global optical potential appears in the literature for the same projectile it is quite laborious to repeat the calculations for each of the sets to see the effect on the final results.

In this work the optical reaction cross-sections for n, p, ^{2}H, ^{3}H, ^{3}He and ^{4}He are directly parameterized in a simple form. The parameterization predicts the optical reaction cross sections for nuclei over the periodic table and energies upto 50 MeV. In the energy space these empirical forms are simple polynomials and consequently the emission rate expressions (using simple forms for the level density) are integrable over the spectrum of final states in a closed form. This work can be considered as a major improvement over an earlier parameterization (Dostrovsky et al 1959) where the continuum model cross sections were the input. A preliminary report of this work has been given earlier (Chatterjee and Gupta 1978b, Murthy et al 1979a and 1979b).

In section 2 the global optical model potentials considered in this work are discussed. The parametric forms which describe the reaction cross section generated by these potentials are described in section 3. The application of these empirical expressions in statistical model calculations is discussed in section 4.

2. Optical Model Potential

The form of the optical potential is (Perey and Perey 1974)

$$U(r,E) = -U_N(r,E) + V_C(r) + U_{SO}(r), \qquad (2)$$

where V_C is the Coulomb potential due to a uniformly charged sphere with radius r_C, U_{SO} is the spin-orbit potential and U_N is the complex central potential taken as

$$U_N(r,E) = V_R(E)\, f(x_R) + i\left[W_V(E)\, f(x_V) + W_S(E)\, g(x_S)\right], \qquad (3)$$

where $f(x_R) = (1 + e^{x_R})^{-1}$ with $x_R = (r - r_R A^{\frac{1}{3}})/a_R$

$g(x_S) = -4 \frac{d}{dx_S} f(x_S)$ (Woods-Saxon derivative)

or $\exp(-x_S^2)$ (Gaussian).

Here x_V and x_S are defined in the same manner with appropriate radius parameters r_S, r_V and diffuseness parameters a_S, a_V. The spin-orbit potential U_{SO} is given by

$$U_{SO}(r) = \lambda_\pi^2 (\vec{l}\cdot\vec{\sigma}) \frac{1}{r}\frac{d}{dr} f(x_{SO})\, V_{SO}. \qquad (4)$$

Using this form of the potential several extensive analyses of differential elastic, polarization and reaction cross section data have been carried out by various workers resulting in global parameter sets. For neutrons the global set of Wilmore and Hodgson (1964) can be considered to be the most useful one, This is the local equivalent version of the non-local potential of Perey and Buck (1962). The potentials of Becchetti and Greenlees (1969) both for protons and neutrons are also well known. We have also included the optical potentials of Mani et al both for protons and neutrons since the tables (Mani et al 1963) using these sets are often used. For protons the parameter-sets of Perey (1963) and Menet (1971) are also considered. For the other projectiles the parameter-sets used are those of Lohr and Haeberli (1974) and Schwandt and Haeberli (1969) for deuterons; and Becchetti and Greenlees (1971) for ^3H and ^3He. For alpha particles there does not exist a clear-cut global phenomenological parameter set. Here the cross sections listed in Huizenga and Igo's (1961) table were used directly in fitting the parametric forms.

3. Parametric forms for the reaction cross section

Using the global optical potential parameters described above, the code SCAT (Smith 1969) was used to generate the reaction cross-sections for the projectiles n, p, ^2H, ^3H and ^3He from 1-50 MeV for 23 nuclei in the mass range 20-214. For ^4He the reaction cross sections over an energy range 2-46 MeV and mass range 20-206 were taken from the existing tabulation (Huizenga and Igo 1961). The parameters of the empirical forms were determined by a fit to these cross sections.

3.1 Neutrons

For energy dependence in the case of neutrons the optical reaction cross section σ_R (in millibarns) is written as

$$\sigma_R = \lambda\varepsilon + \mu + \nu/\varepsilon \quad , \tag{5}$$

where λ, μ, ν are mass-dependent parameters and ε is the neutron laboratory energy in MeV. The addition of the linear term $\lambda\varepsilon$ greatly improves the fit as compared to the parameteric form of Dostrovsky et al (1959). Separating out the energy dependence as in (5), the dependence of λ, μ, ν on target mass number, A were obtained empirically as

$$\begin{aligned} \lambda &= \lambda_0 A^{-1/3} + \lambda_1 \\ \mu &= \mu_0 A^{1/3} + \mu_1 A^{2/3} \\ \nu &= \nu_0 A^{4/3} + \nu_1 A^{2/3} + \nu_2 \quad , \end{aligned} \tag{6}$$

The seven parameters λ_0 through ν_2 were determined by a linear-least-squares fit of σ_R and are given in Table.1. A comparison of the σ_R predicted by the empirical parameterization and that generated from the global optical potentials for neutrons is shown in Fig.1.

3.2 Charged Particles

For charged particles an adequate description of the energy dependence of σ_R and particularly the rapid fall below the Coulomb barrier was obtained by using an energy dependence as in (5) above the Coulomb barrier, E_c and a quadratic energy dependence

$$\sigma_R = p\varepsilon^2 + q\varepsilon + r \quad (mb) , \qquad (7)$$

for $\varepsilon < E_c$. Here E_c is taken as

$$E_c = 1.44 zZ/(1.5A^{1/3} + \delta) \quad (MeV) , \qquad (8)$$

where z and Z are the charge numbers of the projectile and target respectively and $\delta = 0$ for protons and 1.2 for the other charged projectiles. Matching expressions (5) and (7) for their value and derivative at $\varepsilon = E_c$, q and r can be eliminated,

$$q = \lambda - \nu/E_c^2 - 2pE_c$$
$$r = \mu + 2\nu/E_c + pE_c^2 , \qquad (9)$$

and σ_R is expressed over the full energy domain as

$$\sigma_R = p(\varepsilon-\xi)^2 + \lambda\varepsilon + \mu + \nu(2-\varepsilon/\xi)/\xi \qquad (10)$$

where

$$\xi = max(\varepsilon, E_c) .$$

Here the parameters p and ν play a dominant role in the description of σ_R below the Coulomb barrier and were parameterized in terms of E_c . The other parameters λ and μ were parameterized in terms of the target mass number A. The following parameterization was found successful

$$p = p_0 + p_1/E_c + p_2/E_c^2$$
$$\lambda = \lambda_0 A + \lambda_1$$
$$\mu = \mu_0 A^{\mu_1}$$

$$\nu = A^{\mu_1}\left[\nu_0 + \nu_1 E_c + \nu_2 E_c^2\right] , \qquad (11)$$

The parameters p_0 through ν_2 were determined by a least squares fit to the optical cross sections σ_R for various global parameter sets for the projectiles p, ^2H, ^3H, ^3He and ^4He. In the fitting process μ_1 is the only non-linear parameter. Its value was obtained by a search in small steps while the other parameters were determined by a linear-least-squares code. Using these 10 parameters the mass number and energy dependence of σ_R is adequately described. The success of the parameterization for protons is illustrated in Fig.2. For ^2H, ^3H, ^3He and ^4He also these parametric forms are equally successful. The values of the parameters for all these projectiles and various optical potential parameter sets is given in Table 1.

Equation (10) predicts unphysical or negative values of σ_R over a small domain $0<\varepsilon<E_0$. In using (10) σ_R should be set to zero in this domain. The energy E_0 can be determined from the roots ε_1, ε_2 of the quadratic equation, $p\varepsilon^2+q\varepsilon+r=0$ as,

$$\begin{aligned} E_0 &= \max(\varepsilon_1, \varepsilon_2) && (p>0) \\ &= \min(\varepsilon_1, \varepsilon_2) && (p<0) , \end{aligned} \qquad (12)$$

where E_0 is zero when expression (12) is negative.

4. Applications

In both the statistical evaporation (Weisskopf 1937) and preequilibrium (Blann, 1975) models the emission rate for a particle x is written as

$$W_x = \frac{(2s_x+1)m_x}{\pi^2 \hbar^3 \rho_c(E_c)} Q_x \int_{\varepsilon_0}^{E} \varepsilon \sigma_x(\varepsilon) \rho_R(E-\varepsilon) d\varepsilon , \quad (13)$$

where s_x and m_x are the spin and mass of the particle; Q_x is a combinatorial factor (Kalbach 1977), which is unity for the evaporation model; $E = E_c - S_x$ where E_c is the compound system excitation energy and S_x is the binding energy of the particle; ρ_c and ρ_R are the level densities of the compound and residual systems; σ_x is the inverse reaction cross section approximated as the optical reaction cross section of x with the residual nucleus; and $\varepsilon_0 = 0$ is the minimum energy of x. The same expression can be used with $\varepsilon_0 > 0$ to describe the emission of a second particle with the assumption that the second particle is emitted whenever it is energetically possible.

4.1 Statistical Evaporation Theory

For the statistical evaporation theory with a level density such as (Bethe 1937)

$$\rho(E) = \rho_0 \exp 2\sqrt{a(E-\Delta)} , \quad (14)$$

the integration in (13) is analytic when the empirical expressions (5) and (7) are used for σ_x, provided the energy dependence of ρ_0 is neglected. In eq.(14) a is the level density parameter and Δ is the pairing energy. For charged particle emission, the integral is split into contributions below and above E_c, and the lower limit ε_0 is modified to E_0 in case $\varepsilon_0 < E_0$. The integral in (13) is proportional to

$$\int_{\varepsilon_0}^{U} \varepsilon\sigma_x(\varepsilon) \exp 2\sqrt{a(U-x)}\, d\varepsilon =$$

$$\frac{1}{2a}\left[\sum_{i=0}^{3} \frac{(-)^i}{i!\,(4a)^i} J_i \frac{d^i}{dU^i} F(U) + \sum_{j=0}^{2} \frac{(-)^j}{j!\,(4a)^j} K_j \frac{d^j}{dU^j} G(U)\right], \qquad (15)$$

where

$$U = E - \Delta,$$
$$F(U) = pU^3 + qU^2 + rU,$$
$$G(U) = \lambda U^2 + \mu U + \nu,$$
$$J_n = \int_{t(E_c)}^{t(\varepsilon_0)} x^{2n+1} e^x\, dx \quad \text{and} \quad K_n = \int_0^{t(E_c)} x^{2n+1} e^x\, dx, \qquad (16)$$

with

$$t(x) = 2\sqrt{a(U-x)}.$$

Here the integrals J_n and K_n can be evaluated from

$$\int x^{2n+1} e^x\, dx = -(2n+1)!\, e^x \sum_{r=0}^{2n+1} (-)^r x^r / r! \qquad (17)$$

For neutron emission also eq.(15) applies with $E_c = 0$ and the first summation term in the parenthesis set to zero.

4.2 Preequilibrium Model

For the preequilibrium model, the state density of an n-exciton state with p-particles and h-holes (n=p+h) is given by (Williams 1971)

$$\rho(p,h,E) = g^n (E - A_{ph})^{n-1} / p!\,h!\,(n-1)! \quad , \tag{18}$$

where g is the single particle level density, usually taken as $A/13\ \mathrm{MeV}^{-1}$ and A_{ph} is a Pauli correction term. In this case also the use of the empirical σ_x yields a closed expression for the emission rate. For charged particles the integral in (13) is oroportional to

$$\int_{\varepsilon_0}^{U} \varepsilon\sigma_x(\varepsilon)(U-\varepsilon)^{n-1} d\varepsilon = \sum_{i=0}^{3} \frac{(-)^i}{i!} P_i \frac{d^i}{dU^i} F(U) + \sum_{j=0}^{2} \frac{(-)^j}{j!} Q_j \frac{d^j}{dU^j} G(U) \quad , \tag{19}$$

where $U = E - A_{ph}$,

$$P_m = \int_{U-E_c}^{U-\varepsilon_0} x^{m+n-1} dx \quad \text{and} \quad Q_m = \int_0^{U-E_c} x^{m+n-1} dx \,.$$

For neutron emission Eq.(19) applies with $E_c = 0$ and the first summation term set to zero.

These closed form integrals have been employed earlier in preequilibrium model calculations (Chatterjee and Gupta 1978a and 1979).

5. Discussion

The present empirical parameterization of reaction cross-sections for n, p, ^2H, ^3H, ^3He and ^4He reproduces the optical model values satisfactorily over the mass range 16-214 and energies upto 50 MeV. Thus this parameterization should be useful for several applications. For neutrons the parametric form gives a description of the gross behaviour but does not reproduce the oscillatory behaviour of σ_R as a function of A at low energies. These oscillations are due to the size resonance effect. For $\varepsilon > 3$ MeV the agreement is generally better than 5% except for the oscillations. For $\varepsilon <$ 2MeV the reaction cross-sections are not well reproduced particularly for the optical cross-sections of Mani et al (1963) at low mass numbers. For protons the size resonance effect is not prominent because of the Coulomb barrier. The parametric form reproduces the optical cross-sections to better than 5% accuracy over the entire mass-energy domain considered, except at $\varepsilon \sim 4$ MeV. Here the agreement is better than 10% except for $A > 60$, but in this case the optical cross-sections are small and do not contribute significantly in applications where an integral over a large energy domain is computed. For ^2H, ^3H, ^3He and ^4He the cross sections are reproduced to better than 5% accuracy except for heavy targets at energies well below the Coulomb barrier where σ_R is small. The present parametric form cannot be extrapolated far beyond $\varepsilon = 50$ MeV. This is because of the linear term $\lambda\varepsilon$ in eq.(5) which becomes increasingly dominant at higher energies. However this parametrization can be extended to heavier projectiles. This should be useful in view of the great potential for applications.

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Figure captions

Fig.1. Comparison of the neutron optical reaction cross-sections (solid curve) with the predictions of the empirical formula (dotted curve) as a function of the target mass A for the global potentials of (a) Wilmore and Hodgson (1964) (b) Mani et al (1963) and (c) Becchetti and Greenlees (1969). The numbers to the right of the curves indicate the lab. energy in MeV. For the sake of clarity some of the curves have been displaced as indicated.

Fig.2. Comparison of the proton optical reaction cross-sections (solid curve) with the predictions of the empirical formula (dotted curve) as a function of the target mass A for the global potentials of (a) Becchetti and Greenlees (1969) (b) Menet et al (1971) (c) Perey (1963) and (d) Mani et al (1963). The numbers on the curves indicate the lab. energy in MeV.

Table 1. Parameters for empirical reaction cross sections. The numerical values listed here yield σ_R in mb when the energies are in MeV.

	p_0	p_1	p_2	λ_0	λ_1	μ_0	μ_1	ν_0	ν_1	ν_2
Neutrons										
Mani (1963)	-	-	-	15.30	-14.98	297.7	30.58	0.320	-47.08	74.19
Becchetti (1969)	-	-	-	18.57	-22.93	381.7	24.31	0.172	-15.39	804.8
Wilmore (1964)	-	-	-	31.05	-25.91	342.4	21.89	0.223	0.673	617.4
Protons										
Mani (1963)	12.97	15.65	-348.5	0.0247	-7.01	300.4	0.4	330.0	-223.6	-3.251
Perey (1963)	10.53	52.53	-414.0	0.0310	-10.16	327.9	0.4	323.7	-243.4	-3.037
Becchetti (1969)	15.72	9.65	-449.0	0.00437	-16.58	244.7	0.503	273.1	-182.4	-1.872
Menet (1971)	16.99	46.13	-1298	-0.0795	-11.90	127.8	0.66	306.8	-118.5	0.225
Deuterons										
Schwandt (1969)	-43.73	934.1	-2518	-0.0761	-8.83	337.5	0.49	417.5	-353.0	5.515
Lohr (1974)	-38.21	922.6	-2804	-0.0323	-5.48	336.1	0.48	524.3	-371.8	5.924

Tritons										
Becchetti (1971)	-11.04	619.1	-2147	-0.0426	-10.33	601.9	0.37	583.0	-546.2	1.718
He-3										
Becchetti (1971)	-3.06	278.5	-1389	-0.00535	-11.16	555.5	0.4	687.4	-476.3	0.509
He-4										
Huizenga (1961)	10.95	-85.2	1146	0.0643	-13.96	781.2	0.29	-304.7	-470.0	-8.580

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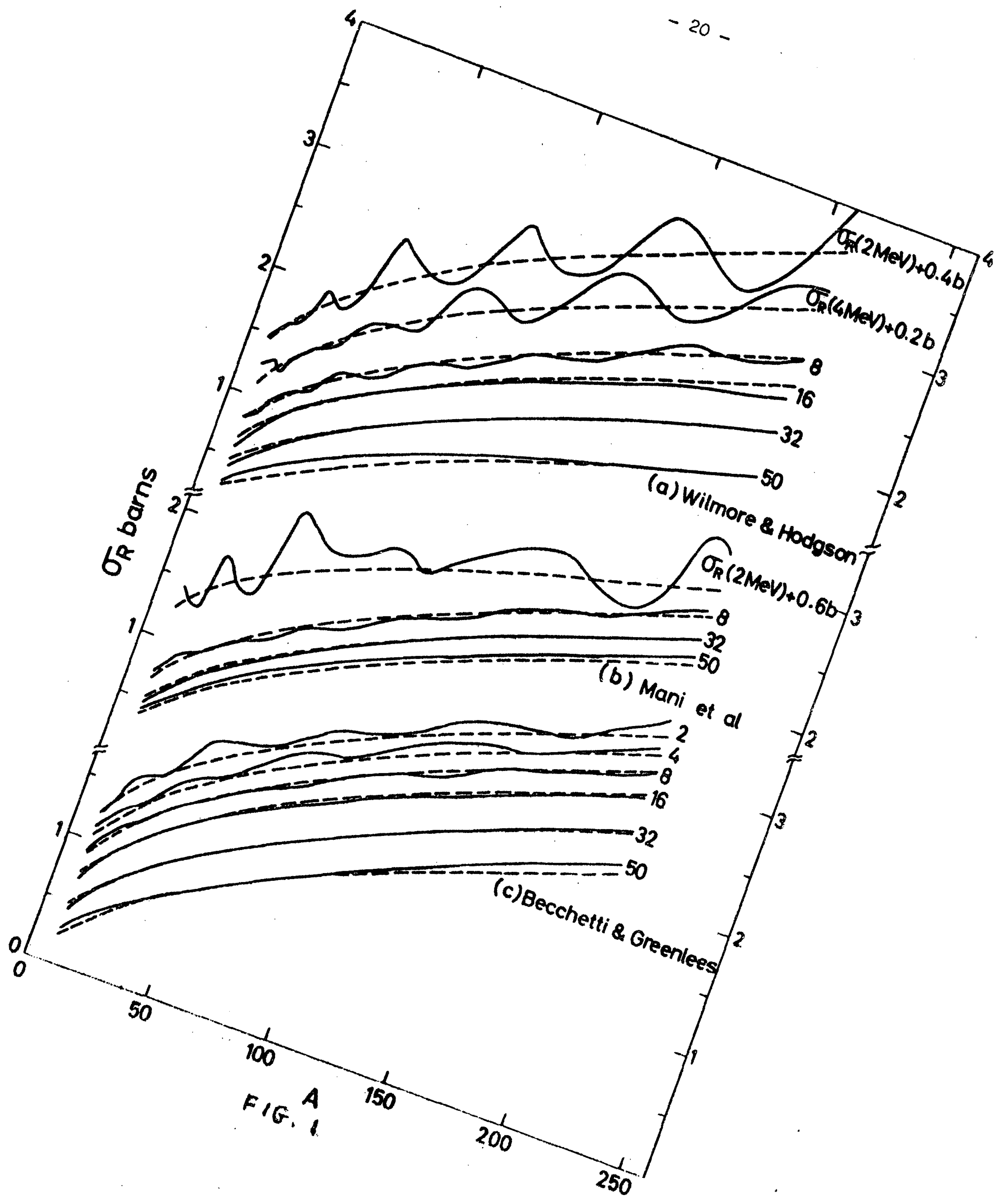

σ_R barns
σ_R(2MeV)+0.4b
σ_R(4MeV)+0.2b
8
16
32
50
(a) Wilmore & Hodgson
σ_R(2MeV)+0.6b
8
32
50
(b) Mani et al
2
4
8
16
32
50
(c) Becchetti & Greenlees
A
0
50
100
150
200
250

FIG. 1

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PROTON REACTION CROSS SECTIONS

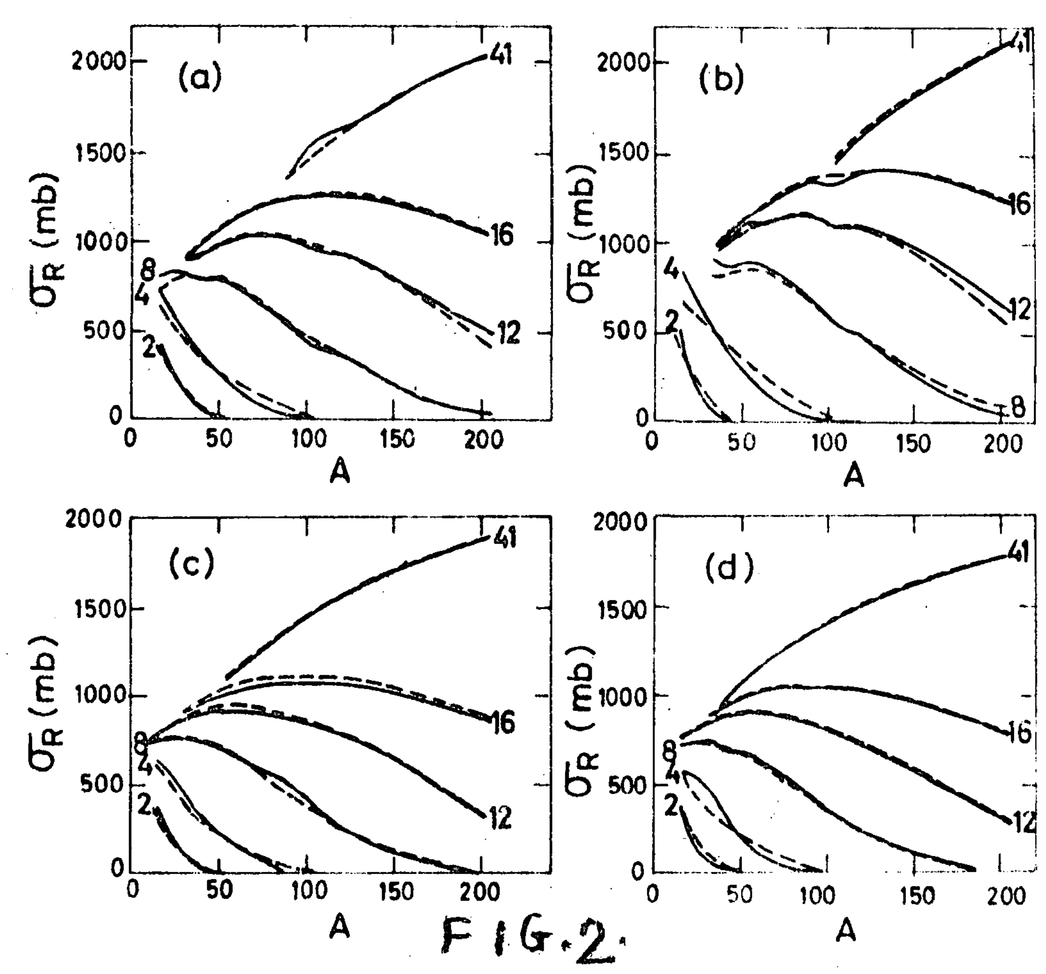

FIG. 2.