## UNFOLDING OF NEUTRON SPECTRA FROM ACTIVATION

DATA USING THE RFO1 AND RFOT CODES
L. Turi and A. Fischer

Hungarian Academy of Sciences
Central Physics Research Institute
Budapest

Translated by the IAEA
May 1972

IAEA NUCLEAR DATA SECTION, KÄRNTNER RING 11, A-1010 VIENNA
unfolding of neutron spectra from activation data using the rfol and rfot codes
L. Turi and A. Fischer

Hungarian Academy of Sciences Central Physics Research Institute Budapest

The authors describe two methods for unfolding neutron spectra from activation data. The RFOl code can be used only with threshold detectors, but the use of the RFO7 code involves no such limitations; if suitable foils are used, neutron spectra can be unfolded over the whole energy range at one time.

## Introduction

Knowledge of the energy distribution of neutrons plays an important part in the study of different reactor types.

The use of activation foils to measure neutron spectra has a potential that is not shared by other methods; the foils are small in size and not normally sensitive to temperature change. They are used to measure spectra as a function of position; moreover, such measurements provide information on the entire energy range at one time. The latter fact is of particular importance in that the results of measurements conducted in different energy ranges can therefore be normalized with respect to one another, and detailed energy distributions determined by other methods in various energy ranges can be fitted into the approximate spectrum obtained from the activation data. It should be noted that in processing the data we do not take into account the fact that the measurement was made with foils; the processing technique can be applied whenever one measures the rate of interaction between neutrons and matter (in fission chambers, $\mathrm{BF}_{3}$ counters, semiconductor detectors and so on). For the sake of simplicity we will henceforth always use the expression "foil activation".

When applying the foil activation method the values one measures are certain integrals of the products of flux and foil cross-sections. The spectrum unfolded from these data is somewhat arbitrary, since we have only a small quantity of data available - namely, the measured foil activities. The spectrum should satisfy the activation equations (1)

$$
\begin{equation*}
A_{i}=\int_{0}^{\infty} p(E) \Sigma_{i}(E) d E \tag{1}
\end{equation*}
$$

where $A_{i}$ is the measured reaction rate in the $i-t h$ foil;
$\phi(E)$ is the spectrum as a function of energy;
$\Sigma_{i}(E)$ is the energy-dependent cross-section for the reaction occurring during activation of the i-th foil.

There is, of course, an infinite number of spectra satisfying these equations. Hence, some more initial information is required to determine the spectrum. It can be said that, on the whole, the more measured data we have, the less initial information is required. However, the microstructure of a spectrum depends greatly on the initial information, which is therefore of great importance and is usually given in the form of an initial spectrum.

Methods of processing activation data can be divided into two groups. The first group comprises methods where it is assumed that one is dealing with a spectrum given in a simple analytical form (Maxwellian distribution, I/E, fission spectrum, some polynomial or certain combinations of these) and that it is only the parameters of this analytical form that are unknown. A method of this kind was first applied by Lanning $[1]$, who expanded a spectrum in an orthonormalized system of functions. The limits of applicability of this method were studied by Gold [2].

In the thermal neutron region, the commonest method is the use of a Laguerre polynomial as the orthonormalized system of functions. If the spectrum is expanded in a series and only the first few terms are considered, the theoretical spectrum closely approximates the actual spectrum. In the fast and epithermal neutron region one cannot determine categorically the form in which the solution should be sought, particularly when the entire neutron energy region is taken into account. In such cases, we therefore consider it better to apply the methods belonging to the second group.

The second group comprises methods with which the spectrum is determined purely numerically by means of iteration. A method of this kind was used in Ref. [4] for the initial measurements of a fast neutron spectrum; it was assumed that the cross-section of threshold detectors was described by means of functions whose values were equal to zero up to a threshold energy, and to certain constants above that threshold. From the foil activities the integral flux was determined for each threshold energy (this quantity is taken to mean a certain flux integral between the threshold energy and infinity), after which the spectrum was determined by differentiation. The values of $\sigma_{i}(E)$ available at that time were only very approximate and more accurate calculations were therefore not possible.

Over the past few years, the availability of more exact cross-section values and the use of computers have made it possible to develop more accurate methods based on these principles. A typical example is the SAND-I code developed by McElroy and co-workers [4]. The SAND-II code, also developed by McElroy and co-workers, is based on different principles [5]. Both these methods have been adapted by us for use with our ICT-1905 computer.

As the SAND-I code is described in detail in Ref. [4], we give only a brief description of the RFOl code in this report. The RFO7 code developed by us is similar to the SAND-II code (of which only a very short description is available) only as far as the main features are concerned.

Further on we describe the RFOI and RFO7 codes developed by us. It should be noted that the sphere of application of the former is limited (it is used only with threshold detectors), whereas there is no limitation of this kind in the case of the RFO7 code; moreover, the RFO7 code gives considerably better results, so that, apart from certain special cases (for example, when no initial information on the spectrum is available), it is better to use the RFO7 code. The entire energy region can be divided into a maximum of 500 points. The initial spectrum can be given in analytical or numerical form; in the latter case, it can be given for arbitrary points. The initial spectrum actually considered will consist of straight lines joining these points. During the calculations, the initial spectrum is varied until the deviation of the measured from the theoretical activities is less than a value specified in the input data.

## RFO1 code

Each iterative step consists of the following operations:

1. Determination of the reaction rate for each foil from the given iteration spectrum and cross-sections:

$$
\begin{equation*}
B_{i}^{(k)}=\int_{0}^{\infty} \phi^{(k)}(E) \Sigma_{i}(E) d E \quad / i=1,2, \ldots, I / \tag{2}
\end{equation*}
$$

where $B_{i}^{(k)}$ is the reaction rate for the $i-t h$ foil as calculated at the k-th iterative step;
$\phi^{(k)}(E)$ is the spectrum at the k-th iterative step;
I is the number of foils.
2. Assignment of a threshold energy to each foil (this value is taken to be the energy above which $95 \%$ of all reactions occur in the given spectrum) in accordance with the following equation:

$$
\begin{equation*}
\int_{E_{i t h r}}^{\infty} \phi^{(k)}(E) \Sigma_{i}(E) d E=\beta \int_{0}^{\infty} \phi^{(k)}(E) \Sigma_{i}(E) d E \tag{3}
\end{equation*}
$$

```
where E (ithr is the threshold energy of the i-th foil as
    determined from Eq. (3);
    B=0.95.
```

3. Determination of the integral flux for each foil:

$$
\begin{equation*}
\Phi_{i}^{(k)}=\int_{E_{i t h r}}^{\infty} \phi^{(k)}(E) d E \tag{4}
\end{equation*}
$$

4. Determination of the new integral flux:

$$
\begin{equation*}
\Phi_{i}^{(k+1)}=\Phi_{I}^{(k)} \frac{A_{i}}{B_{i}^{(k)}} \tag{5}
\end{equation*}
$$

5. Determination of the new spectrum by differentiation of the curve fitted to the points denoting the integral flux values.

The most difficult stage of the method is the fitting of the curve to the integral points. McElroy first carried out this operation by hand. With the method we have developed, use is made of parabolas passing through three integral points. Subsequently, when differentiating, only the segment of the parabola between the first and second points is used. The curve derived in this fashion is broken, but that does not greatly spoil the shape of the spectrum, and after the sixth or seventh iterative step the curve, when corrected manually, gives approximately the same values of $B_{i}^{(k)}$ as an uncorrected broken curve.

The RFOI code has two substantial disadvantages: first, the integral flux is determined only at as many points as there were irradiated foils; second, if the threshold energies in two foils are close together or coincide, some of the information is lost, for the integral flux is determined only at one point by means of two foils.

## RFO7 code

Each iterative step consists of the following operations:

1. Determination of the reaction rates from the given spectrum:

$$
\begin{equation*}
B_{i}^{(k)}=\sum_{n=1}^{N} \phi_{n}^{(k)} \Sigma_{i n} \Delta E_{n} \quad / i=1,2, \ldots, I / \tag{6}
\end{equation*}
$$

> where $\phi_{n}^{(k)}$ is the flux in the $n$-th energy range at the $k$-th step; $\Sigma_{i, n}$ is the cross-section for the $i-t h$ reaction in the $n$-th energy range;
$\Delta E_{n}$ is the width of the $n$-th energy range;
$N$ is the number of energy ranges.
It should be noted that this formula corresponds to formula (2). The quantities appearing in formula (6) are of the group type, but the ranges are narrow and the quantities can therefore be calculated by means of simple formulas, which we will not consider here.
2. Determination of the weight factor for each foil and each range:
3. Determination of the correction factor for each range:

$$
\begin{equation*}
C_{n}^{(k)}=\frac{N}{I} \sum_{i=1}^{I}\left(W_{i, n}^{(k)} \frac{A_{i}-B_{i}^{(k)}}{A_{i}}\right) \tag{8}
\end{equation*}
$$

4. Determination of the new flux for each range:

$$
\begin{equation*}
\phi_{n}^{(k+1)}=\phi_{n}^{(k)}\left(1+d_{n}^{(k)}\right) \tag{9}
\end{equation*}
$$

After each iterative step the flux is normalized in such a way that the sum of the squares of the deviations of the measured and computed activity ratios from unity, i.e. the expression

$$
\sum_{i=1}^{I}\left(\frac{B_{i}^{(k)}}{A_{i}}-1\right)^{2}
$$

is at a minimum. If the flux is multiplied by a number $f$ in each energy range, each activity increases by a factor $f$, so that this number should be determined in such a way as to satisfy the condition

$$
\begin{equation*}
\sum_{i=1}^{I}\left(\frac{f B_{i}^{(k)}}{A_{i}}-1\right)^{2}=\text { minimum } \tag{10}
\end{equation*}
$$

If we differentiate this expression with respect to $f$, we get, after simple transformations

$$
\begin{equation*}
\mathbf{f}=\frac{\sum_{i=1}^{I} \frac{B_{i}^{(k)}}{A_{i}}}{\sum_{i=1}^{I}\left(\frac{B_{i}^{k}}{A_{i}}\right)^{2}} \tag{11}
\end{equation*}
$$

This means that one should calculate this quantity after each step, and then multiply the flux by it.

Detailed examination of the code algorithm shows clearly that at those points where the cross-section of all the detectors used is zero the spectrum varies only as a result of normalization, since the detectors do not carry any information about these ranges.

The applications of the code and some of its properties, and also the interpretation of the results obtained with it, are discussed by us in detail in Ref. [6].

The cross-sections used for the calculations are recorded on RFSPLIBRARY tape. The cross-sections of the cadmium-covered foils were calculated with the aid of a separate code. The data on the magnetic tape can be supplemented or altered as new data are received. Table 1 shows the designations of those reactions for which we have the cross-sections on tape. For the code we use RFSP-LIBRARY tape and one auxiliary magnetic tape. The input data can be inserted on 8-track punched tape or on punched cards.

Using the code, we first seek data on the foil cross-sections; after appropriate processing, these data are then recorded on the auxiliary magnetic tape. During the rest of the operation only the auxiliary tape is used. If there are no data on a particular foil (or foils) on the RFSP-LIBRARY tape, the code indicates this fact and calculates only with the data on the remaining foils. If there are no data on any of the foils, the code reads the available data for the problem being solved and then begins reading the data relating to the next problem.

## Input data for programmes

All the lines of input data are compiled in accordance with the rules for the FORTRAN format. The sequence for these lines is as follows:
lst type: 2II, I2

2nd type: I2 NSIG
3rd type: [2A8, 2E12.5]

4th type: 2 [7A8, A4]

5th type: I4, El4.5

MI = $1 \quad-$ New problem;
3 - Fnd of punched tape.
NANAL $=1$ - The initial spectrum is given in analytical form;

2 - The initial spectrum is given in numerical form.

KIRAJZ - The number of iterations after which the results should be printed out. The results of the first and last iterations are printed out irrespective of this.

- Number of foils used.
- The first two data items indicate the foil designation. This designation must agree with that given in Table 1. The designation is followed by the measured reaction rates in the spectrum being studied and the calibration spectrum. The calibration spectrum need not be read, but if the foils have not been calibrated; we have to give absolute foil activities for one atom. This type is read NSIG times.
- Any text in two lines with 60-60 alphanumerical symbols in each.

LEPMAX - Maximum number of iterations;
DMEG - Permissible error in the activities. Iteration terminates when

$$
\sum_{i=1}^{T} \frac{1}{A_{i}}\left|B_{i}^{(k)}-A_{i}\right| \leqslant \operatorname{DMEG}
$$



After the input data have been issued as first-iteration results, the code prints out the normalized initial spectrum; the spectrum is then
printed out after each KIRAJZ iteration. The measured and calculated reaction rates, their ratios and percentage deviations, are also printed out. The spectrum is displayed schematically on a logarithmic scale.

The Appendix contains a set of input data and a photocopy of results printed out by the RFO7 code.

Table 1
Reactions recorded on RFSP-LIBRARY tape and their energy limits

| U235/NF/FP-CD | $0,7 \cdot 10^{-8}-18 \mathrm{MeV}$ |
| :---: | :---: |
| U233/NF/FP-CD | 1, 0, $10^{-8}-18$ |
| PU239/NF/FP-CD | $0,6.10^{-8}-18$ |
| NA23/NG/NA24-CD | $0,5 \cdot 10^{-8}-18$ |
| MG24/NP/NA24 | 0,05 - 18 |
| IN115/NN/IN115M | 0,05 - 18 |
| S32/NP/P32 | 0,05 - 18 |
| AL27/NP/MG27 | 0,05-18 |
| AL27/NHE/NA24 | 0,05 - 18 |
| RH103/NN/RH103M | 0,05 - 18 |
| FE56/NP/MN56 | 0,05 - 18 |
| NI58/NP/C058 | 0,05 - 18 |
| P31/NP/SI31 | 0,05-18 |
| PU239/NF/FP | $0,5.10^{-10}-10^{-5}$ |
| DY164/NG/DY165 | $0,33.10^{-10}-10^{-5}$ |
| LU176/NG/LUl77 | $0,5,10^{-10}-10^{-5}$ |
| EU151/NG/EU152 | $0,5,10^{-10}-10^{-5}$ |
| IN115/NG/IN116 | $0,5.10^{-10}-10^{-5}$ |
| AU197/NG/AU198 | $0,5.10^{-10}-2.10^{-5}$ |

## REFERENCES

[1] LANNING, W.D., Trans. Am. Nucl. Soc. 6 (1963) 35.
[2] GOLD, R., Nucl. Sci. Eng. 20 (1964) 493.
[3] HUGHES, D.J., Pile Neutron Research, Cambridge (Mass.) (1953).
[4] McELROY, W.N. et al., Nucl. Sci. Eng. 27 (1967) 533.
[5] McELROY, W.N. et al., AFWL-TR-67-41 (September 1967).
[6] FISCHER, A., TURI, L., to be published.

```
222
    5
PU239(NP)FP 8.76059E 02
DY164(NG)DY165 1.80564E+03
LU176(NG)LU177 4.21792E+03
EU151(NG)EU152 2.46769E+03
IN115(NG)IN116 6.08475E+02
    - TEST CASE FOR REPORT -
- }15\mathrm{ JULY 1970. -
    100 0.01000E 00
    1.00000E-09 3.00000E-06 100
    0.0 0.0 0.0
        39
    3. E-06 1.7 E-02 2.75 E-06 1.925 E-02 2.2 E-06 2.32 E-02
    1.9 E-06 2.68 E-02 1.6 E-06 3.19 E-02 1.4. E-06 3.64 E-02
    1.2 E-06 4.25 E-02 1.05 E-06 4.85 E-02 0.9 E-06 5.67 E-02
    0.85109E-06 6.8781 E-02 0.68412E-06 6.8781 E-02 0.68411E-06 9.1563 E-02
    0.51234E-06 9.1563 E-02 0.51233E-06 1.2353 E-01 0.384B2E-06 1.2353 E-01
    0.38481E-06 1.658 E-01 0.27553E-06 1.658 E-01 0.27552E-06 2.4184 E-01
    0.21278E-06 2.4184 E-01 0.21277E-06 3.4906 E-01 0.17104E-06 3.4906 E-01
    0.17103E-06 4.8664 E-01 0.13385E-06 4.8664 E-01 0.13384E-06 8.3368 E-01
    0.10121E-06 8.3368 E-01 0.10120E-06 1.3658 E 00 7.31180E-08 1.3658 E 00
    7.31170E-08 2.139 E-00 4.95890EO08 2.139 E 00 4.95880E-08 2.6911 E-00
    3.0614 E-08 2.6911 E O0 3.06130E-08 2.684 E 00 1.7853, E-08 2.687 E 03
    1.7852 E-08 2.1198 E 00 9.1081 E-09 2.1198 E 00 9.1080 E-09 1.318 E 00
    4.0481 E-09 1.318 E 00 4.0480 E-09 2.8335.E-01 1.0 E-09 2.8335 E-01
3
```

I afed

## －＊RFOT＊＊

$$
\text { Number of iterations: } 1
$$


Page 2
17198／09


|  | A（I） | Be！） | B（I）／a（1） | $B(1) / A(1)-1$ |
| :---: | :---: | :---: | :---: | :---: |
| $0=3.948 F=02$ |  |  |  |  |
| DU239（NE）FD | 8．7nns9f 02 | 8．68173E 02 | 9．90098E－01 | －0．9002 |
| OYT64TNGJDY＊ 67 | 9．80r．66r．03 | －． 88985 F 03 | 1．04642F On | 4.4419 |
| tU976（NG）t1497 | 4．－29702r 03 | $\because 91747 \mathrm{~F} 03$ | －． 37585 ECOH | －7．2417 |
| f⿴囗十介（NG）：Nイ96 | R．08i．79F 02 | A．33400F 02 | 9．A¢098EOA | 4.9989 |
| EU454（NG）f！195？ | ？．69309\％ 03 | P．41761F03 | 9． 77 AOOFOOS | －2．2600 |


| $1.000 E=09$ |
| :---: |
|  |
| 2．245E |
| 3．364\％00 |
| 5. |
| 7.557 F |
| 1．1321m 0 （8 |
| 9．695E08 |
| 2．54nF－08 |
| 3.806 |
| $5.703 \mathrm{~F}=08$ |
| 8.5 |
| 1．28nt－07 |
| 1．9181：07 |
| 2．875\％－07 |
| 4.3075007 |
| 6．457Em？ |
| 9．6701：07 |
| 1．640r． |
| 17 |

[^0]Page 3

## - REOT * *

## w




$\omega$





| 1)/A(1)-1 |
| :---: |
| 0.0978 |
| 2.4415 |
| -0.4565 |
| 0.0020 |
| -2.0789 |



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[^0]:    Measured and counted activities，activity ratios and percentage errors
    II／A（9）

    -0.8002
    6.449
    -7.2497
    4.9989
    -2.2600
    $=5 i c c$
    0
    0
    0
    0

