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THE RFSP PROGRAMME FOR UNFOLDING NEUTRON SPECTRA FROM ACTIVATION DATA
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ABST'RACT
The authors describe the RFSP programme for unfolding neutron spectra from activation data. With spectra satisfying the activation equations they use the programme to unfold that spectrum for which the functional characterizing deviations from the initial spectrum is at a minimum. The RFSP programe is an advanced and more rapid version of the SPECTRA code. The results obtained with the codes are identical, but RFSP requires less computer time. In this report the authors describe the algorithm of the method, the input/output specifications and typical results.

## I. Introduction

When unfolding neutron spectra from activation data, one measures the integrals

$$
\begin{equation*}
A_{i}=\int_{0}^{\infty} \phi(E) \sigma_{i}(E) d E, \quad / 1=1,2, \ldots, n / \tag{1}
\end{equation*}
$$

where $A_{i}$ represents the measured activities of the i-th foil,
$\phi(E)$ is the differential neutron spectrum,
$\sigma_{i}(E)$ is the cross-section for the reaction occurring during activation of the i-th foil, as a function of energy,
and the spectrum $\phi(E)$ has to be determined for known $\sigma_{i}(E)$ and $A_{i}$. Since we are here dealing with the solution of mathematically indeterminate systems of equations (the number of points at which the spectrum is determined is greater than the amount of measurement data available to us), certain initial information is required for the solution in addition to the measurement data.

The published theoretical methods differ mainly in the form that this initial information takes; in practice, these methods can be divided into two groups. The first group comprises methods for which it is assumed that one is dealing with a spectrum given in a simple analytical form and that it is only the parameters of the analytical form that are unknown. In the case of the second group, an initial spectrum given in purely numerical form is varied in the course of an iterative process until the activities agree with the measured values.

The method described in this report belongs to the second group. The problem is formulated as follows: the initial spectrum $\phi_{0}(E)$ is given, and from spectra satisfying the activation equations (i.e. those reproducing the measurement results) we determine the spectrum for which the functional characterizing the difference between the initial spectrum and the derived spectrum is at a minimum.

$$
\begin{equation*}
\int_{E_{\min }}^{E_{\max }}\left[\frac{\phi_{0}(E)-\phi_{0}(E)}{\phi_{0}(E)}\right]^{2} d E=\operatorname{minimum} \tag{2}
\end{equation*}
$$

( $E_{\text {min }}$ and $E_{\text {max }}$ stand for the energy values above and below which the spectrum is taken as zero). At this point the question arises as to what accuracy should be stipulated when solving the activation equations. If the derived spectrum is required to satisfy the activation equations exactly, the problem can be solved by applying Lagrange's method of multipliers. The cross-sections of the foils used for the measurements (first and foremost those of the threshold detectors) are known only to within $\pm 10 \%$, so it is not always advisable to stipulate that the activation equations should be satisfied exactly. Greer and co-workers [1] have described an iterative method of which the result agrees with that obtained by using Lagrange's method of multipliers. During the iteration process the spectrum is varied in such a way that the difference between the measured and theoretical activities gradually decreases. The iterations thus result in a spectrum satisfying the activation equations only within the limits of the measurement accuracy (taking into account the degree of indeterminacy of the cross-sections). As will be demonstrated further on, the spectmom derived as the iteration limit may be obtained directly, without iteration, by Greer's method. The question as to which variant is best used in which particular case and the overall questions involved in applying the programme are discussed in Ref. [2].

The method developed by us is a more advanced and faster version of Greer's method.

## II. Mathematical principles underlying the RFSP code

1. The problem is to find the spectrum in a form such that the product of the spectrum and energy $E \phi(E)$ is a piecewise linear function of the energy; in such a case, the system of activation equations can be transformed into the matrix equation

$$
\begin{equation*}
a=Q \phi \tag{3}
\end{equation*}
$$

where $a$ is a column vector consisting of $n$ elements, the measured activities $A_{i}$ being the elements;
$\phi$ is a column vector consisting of $m$ elements, these being the spectrum values sought at individual energy - and hence lethargy - points;
$Q$ is a matrix of size $n \times m$, the elements of which are definite integrals of the cross-sections, $m$ being greater than $n$ (see Appendix I).

If we divide the rows of the matrix $Q$ into the corresponding elements of the vector $a$, we arrive at the system of equations

$$
\begin{equation*}
c \phi=\left(I_{n}\right) \tag{4}
\end{equation*}
$$

where ( $l_{n}$ ) is a column vector consisting of $n$ elements, each element being equal to 1 .

Assuming an initial spectrum (a vector consisting of $m$ elements) that does not satisfy Eq. (3), let us normalize it so that the sum of the squares of the deviations of the elements of the vector $C \phi_{0}$ from unity is at a minimum (see Appendix II).

Let us now formulate a function that will characterize the deviations of the measured activities from the activities calculated in the unknown spectrum, on one hand, and of the initial from the unknown spectrum, on the other:

$$
\begin{equation*}
\Delta_{1}=\left(a_{1}-\left(1_{n}\right)\right)^{T} F^{2}\left(a_{1}-\left(l_{n}\right)\right)+\left(\phi_{1}-\phi_{0}\right)^{T} G^{2}\left(\phi_{1}-\phi_{0}\right) \tag{5}
\end{equation*}
$$

where $a_{1}=c \phi_{1}$
$G^{2}$ and $F^{2}$ are diagonal matrices used for normalization:

$$
\operatorname{det}(G) \neq 0, \operatorname{det}(F) \neq 0
$$

It is advisable, though not essential, to select these matrices in such a way that the elements of the matrix $G$ represent the reciprocals of the corresponding elements of the initial spectrum, while those of the matrix $F$ represent the reciprocals of the relative errors in measuring the activity of the corresponding foils; the superscript $T$ is used to designate the transposition operation.

Let us now attempt to find the spectrum for which this function is at a minimum. To do so we shall differentiate expression (5) with respect to $\phi_{1}$, and then equate it to zero; from which we obtain

$$
\begin{equation*}
\phi_{1}=\left(C^{T} F^{2} C+G^{2}\right)^{-1}\left(C^{T} F^{2}\left(1_{n}\right)+G^{2} \phi_{0}\right) \tag{6}
\end{equation*}
$$

Where the superscript -1 is used to designate the matrix inversion operation.

We now introduce the notation

$$
\begin{aligned}
\underline{C} & =F C G^{-1} \\
\text { and } \underline{B} & =\left(\underline{C}^{T} \underline{C}+I\right)^{-1}
\end{aligned}
$$

where this time I designates a unit matrix of the order m. On the basis of a brief computation it can easily be seen that expression (6) now assumes the following form:

$$
\begin{equation*}
\phi_{1}=G^{-1} \underline{B}\left(\underline{c}^{T} F\left(1_{n}\right)+G \phi_{0}\right) \tag{7}
\end{equation*}
$$

It can be demonstrated (see Appendix III) that the matrix $\partial^{2} \Delta_{1} / \partial \phi_{1}^{2}=C^{T} F^{2} C+G^{2}$ is positive and therefore transformable; hence, there is a solution of expression (6), and with this solution the function $\Delta_{1}$ does assume a minimum value. It can also be demonstrated [1] that, if $\phi_{0}$ is not a solution of Eq. (3), neither is $\phi_{1}$.

As with expression (5), we can now formulate a new function in which we write $\phi_{1}$ instead of $\phi_{0}$ and $\phi_{2}$ instead of $\phi_{1}$. If this process is repeated several times, we arrive, after $k$ steps, at the following expression for the next spectrum approximation:

$$
\begin{equation*}
\phi_{k+1}=G^{-1} \underline{B}\left(\underline{C}^{T} F\left(l_{n}\right)+G \phi_{k}\right) \tag{8}
\end{equation*}
$$

Here we have derived a formula which is suitable for the iteration process, and by applying formula (8) several times we obtain progressively better spectrum approximations. These approximations tend to a limit. Further calculations are required for a direct determination of the limit. If we express $\phi_{k}$ in Eq. (8) by means of the iteration spectra referred to, we arrive at the expression

$$
\begin{equation*}
\phi_{k+1}=G^{-1}\left(I+\underline{B}^{-1} \underline{B}^{2}+\ldots+\underline{B}^{k}\right) \underline{B} \underline{C}^{T} F\left(I_{n}\right)+G^{1} \underline{B}^{k+1} G \phi_{O} \tag{9}
\end{equation*}
$$

It can be demonstrated (see Appendix III) that, if some vector is an eigenvector of the matrix $\underline{C}^{T} \underline{C}$ and it has an eigenvalue $\lambda$, then this vector is at the same time an eigenvector of the matrix $B$, and in this case the eigenvalue is $1 /(1+\lambda)$. Since the rank of the matrix $\underline{C}^{T} \underline{C}$ is equal to $n$, zero is its $m-n-f o l d$ eigenvalue. Thus, $l$ is the $m-n-f o l d$ eigenvalue of the matrix $B$, while the other eigenvalues lie between 0 and 1 (see Appendix III).

The matrix $\underline{B}$ is symmetric, so that its right-hand and left-hand eigenvectors are identical and it can be written in the form PLP ${ }^{T}$, where $L$ is the diagonal matrix containing eigenvalues and $P$ is the matrix containing the orthonormalized eigenvectors as column vectors. Let us arrange the elements of the matrix $L$ in ascending order and let the order of the column vectors in the matrix $P$ follow the same order. Equation (9) can be rewritten in the following form:

$$
\begin{equation*}
\phi_{k+1}=G^{-1} P\left(I+L+L^{2}+\ldots+L^{k}\right) P^{T} \underline{B C}^{T} F\left(1_{n}\right)+G^{-1} P L^{k+1} P^{T} G \phi_{O} \tag{10}
\end{equation*}
$$

(here we have made use of the identity resulting from the fact that the eigenvectors have been orthonormalized: $P^{T}=P^{-1}$, so $P^{T}=1$ ).

Let us determine the limit of this expression for $k \rightarrow \infty$. It is easy to determine the limit of the second term: $\lim L^{k+1}=H$, where $H$ represents a diagonal matrix, of which the first $n{ }^{k \rightarrow \infty}$ diagonal elements are equal to 0 and the others equal to 1.

To determine the limit of the first term, in expression (10) let us look at the matrix product $P^{T} \underline{B C}^{r}{ }^{\Gamma}$. Since the matrix $\underline{B}$ is symmetric, $\underline{B}=\underline{B}^{T}$, so that
 there are only zeros (see Appendix IV), so that, if we take the diagonal matrix $L_{0}$, the first $n$ elements of which coincide with the corresponding elements of the matrix L, while the other elements are equal to zero, then the following relationship is valid for any $k$ :

$$
L^{k} P^{T} B C^{T}=L_{O}^{k} P^{T} B C^{T}
$$

Consequently, we can everywhere write $L_{0}$ instead of $L$ in the first term of expression (10) without affecting the result. The limit of the expression in brackets, $\left(I+L_{o}+L_{o}^{2}+\ldots+L_{o}^{k}\right)$, for $k \rightarrow \infty$ can easily be determined; the individual diagonal elements are the sums of infinite geometric series. If in this case, as above, the non-zero eigenvalues of the matrix $\underline{C}^{T} \underline{C}$ are designated by $\lambda_{1}, \lambda_{2}, \ldots, \lambda_{n}$, then the non-zero diagonal elements of the matrix $L_{0}$ will be $1 /\left(1+\lambda_{1}\right), 1 /\left(1+\lambda_{2}\right) \ldots, 1 /\left(1+\lambda_{n}\right)$, while the elements of the matrix $K$ corresponding to the limit of the matrix sequence will be $\left.\left(1+\lambda_{1}\right) / \lambda_{1},\left(1+\lambda_{2}\right) / \lambda_{2}, \ldots,\left(1+\lambda_{n}\right) / \lambda_{n}\right)$.

Thus, the spectrum limit for $k \rightarrow \infty$ is

$$
\begin{equation*}
\phi=\lim _{k \rightarrow \infty} \phi_{k}=G^{-1} P_{K P}^{T} \underline{E C}^{T} F\left(l_{n}\right)+G^{-1} P H P^{T} G \phi_{0} \tag{11}
\end{equation*}
$$

It can be demonstrated that this spectrum satisfies either Eq. (3) or
Eq. (4) and agrees with the solution obtained by Lagrange's method of multipliers; i.e. for this spectrum the integral (2) does assume its minimum value. However, the proof is complicated and, as it is fully described in Ref. [1] , we do not include it here.
2. Complicated and laborious matrix operations are required to determine the limit by means of Eq. (1l). Later on we shall show that there is a simpler but rapid method of determining it. By appropriate conversion of the iteration formula one obtairs a formula with which, after only one iteration, one can find the limit with the desired accuracy (if necessary, the convergence rate can be adjusted to the desired value).

Let us write the result of the first iteration and the limit expression of the iteration process, with the restriction that matrix $G$ contains the reciprocals of the initial spectrum $\phi_{0}$; the diagonal elements of matrix $G^{-1}$ then coincide with the elements of the vector $\phi_{0}$. The fact that the problem concerns a diagonal matrix rather than a vector is underscored by designating the matrix by $\phi_{0}^{M}$. In this case, the product $G \phi_{0}$ appearing in Eqs (7) and (11) is a column vector with each element equal to unity. We will denote this vector by ( $1_{m}$ ). Formulas (7) and (11) can then be written in the following form:

$$
\begin{align*}
\phi_{1} & =\phi_{o}^{M}\left(\operatorname{PIP}^{T} \underline{B}^{T} \underline{C}^{T} F\left(1_{n}\right)+P L P^{T}\left(I_{m}\right)\right)  \tag{12}\\
\phi & =\phi_{o}^{M}\left(\operatorname{PKP}^{T} \underline{B}^{T} \underline{C}^{T} F\left(1_{n}\right)+\operatorname{PHP}^{T}\left(1_{m}\right)\right) \tag{13}
\end{align*}
$$

The difference between the two spectra is

$$
\begin{equation*}
\phi-\phi_{1}=\phi_{0}^{M}\left(P\left(L_{0}+L_{o}^{2}+\ldots\right) P^{T} \underline{B}^{T} \underline{C}^{T} F\left(1_{n}\right)+P(H-L) P^{T}\left(1_{m}\right)\right) \tag{14}
\end{equation*}
$$

Further on we will describe a method by means of which, without affecting the flux $\phi$, the difference $\phi-\phi_{1}$ can be reduced in such a way that the flux can be derived with sufficient accuracy after only one iterative step.

Let us go back to Eq. (4). Physically, this equation means that, when multiplied by the corresponding cross-sections, the flux should give the measured activities. If both sides of the equation - i.e. all the elements of the matrix $C$ and of the vector $\left(l_{n}\right)$ - are multiplied by the same number $\omega$, there will be no change in the physical meaning of the equation; i.e. the spectrum obtained as the iteration limit remains unchanged. This operation comes to the same thing as writing $C^{\prime}=\omega \underline{C}$ in Eqs (5)-(14) instead of the matrix $C, \underline{B}^{\prime}=\left(\underline{C}^{\prime} \underline{T}^{\prime}+1\right)^{-1}$ instead of $\underline{B}$, and the column vector ( $\omega_{n}$ ) - all the elements of which are equal to $\omega$ instead of the vector $\left(1_{n}\right)$.

If we multiply all the elements of some matrix by the same number, the eigenvectors of the matrix remain unchanged, while the eigenvalues are multiplied by the same number. Having multiplied the matrix $\underline{C}$ by $\omega$, we multiply each element of the matrix $\underline{C}^{T} \underline{C}$ by $\omega^{2}$; accordingly, the eigenvalues of $\underline{C}^{\prime} \underline{\mathrm{C}}^{\mathrm{C}}$ will be $\omega^{2}$-fold values of the eigenvalues of $\underline{C}^{T} \underline{C}$, and the eigenvalues of $\underline{B}^{\prime}$ will therefore be $1 /\left(1+\omega{ }^{2} \lambda_{i}\right) / i=1,2, \ldots n /$. As was pointed out, the introduction of these changes does not affect $\phi$, but $\phi_{1}$, and therefore $\phi-\phi_{1}$, are changed. In this case the difference will be

$$
\begin{equation*}
\phi-\phi_{1}^{\prime}=\phi_{0}^{M}\left[P\left(L_{o}^{\prime}+L_{o}^{\prime 2}+\ldots\right) P^{T_{B}}{ }^{\prime}{ }^{T} C^{\prime} T_{F}\left(\omega_{n}\right)+P(H-L) P^{\prime}\left(1_{m}\right)\right] \tag{15}
\end{equation*}
$$

where the matrix $L^{\prime}$ contains all the eigenvalues of $\underline{B}^{\prime}$, while the matrix $L_{0}^{\prime}$ contains only the first $n$ of these values.

After multiplication and addition, the square brackets in Eq. (15) contain one column vector of $m$ elements. As this column vector is multiplied from the left by the diagonal matrix $\phi_{o}^{N}$, any element of the column vector can be multiplied by only one corresponding element of the initial spectrum; each element of the column vector $\phi-\phi_{i}$ is therefore obtained as a result of only one multiplication of this kind.

In Appendix $V$ it is demonstrated that for any positive $\varepsilon$ there will be a $\omega$ for which the absolute value of the largest element of the column vector standing in square brackets in expression (15) will be smaller than $\varepsilon$. In other words, if we use a large enough value of $\omega$, then the difference $\phi_{-} \phi_{1}$ can be reduced to the desired value. Appendix $V$ also contains formula ( $V-11$ ), which can be used to determine the required value of $\omega$ for a given value of $\varepsilon$.

Hence, if we multiply the matrix C by $\omega$ according to formula ( $V-11$ ) at the very beginning of the computation, $\phi$ is determined with an accuracy of the order $\boldsymbol{\varepsilon}$ in the first iteration step. It should be pointed out here that the phrase "with an accuracy of the order $\varepsilon$ " does not in this case mean that the following expression is satisfied for any element of the vectors $\phi, \phi_{1}, \phi_{0}$ :

$$
\begin{equation*}
\left|\frac{\phi_{\mathrm{s}}-\phi_{1 \mathrm{~s}}}{\phi_{\mathrm{s}}}\right| \leq \varepsilon \tag{16}
\end{equation*}
$$

but that only

$$
\begin{equation*}
\left|\frac{\phi_{s}-\phi_{1 S}}{\phi_{O S}}\right| \leq \varepsilon \tag{17}
\end{equation*}
$$

is satisfied, since the s-th element of the exact solution $\phi_{s}$ may be very small or equal to zero, in which case the inequality (16) cannot be satisfied, even if $\phi_{s}-\phi_{1 \mathrm{~s}}$ is very small.

It should also be noted that, when calculating (1), we always took into account the most unfavourable conditions as regards the accuracy of the result. In practice, the accuracy of this method is much better than it would seem from Eqs (V-11) and (V-12).
3. With the method described by Greer []$\left._{-}\right]$, the limit is determined with the help of Eq. (11), in which one has to calculate all the eigenvalues and eigenvectors of a matrix whose dimensions coincide with those of matrix $\underline{\mathrm{C}} \underline{\mathrm{C}}$; one must then still orthogonalize the eigenvectors belonging to the eigenvalue 0 .

Examining the sequence of the calculations, we see that, when our method is used, after forming the matrix $\underline{C}^{T} \underline{C}$ we have to calculate its $n$ non-zero eigenvalues and from this determine the multiplier $\omega$; we can then determine the limit of the iteration process by multiplying the matrix $\underline{C}$ by this multiplier, using the simple formula (7). Thus, the use of our method helps to save a considerable number of cells in the computer's internal memory and a great deal of computer time.

## III. RFSP code

The RFSP code, which is written in FORTRAN IV, was developed for the translator of our Institute's ICT-1905 computer. Two regimes (modes of operation) are possible with the code; in the first it determines the neutron spectrum using the method described in Section II, while in the second it performs functions connected with the storing of cross-sections:
(a) It compiles a new magnetic tape;
(b) It records new c̣ross-sections on library tape already available;
(c) It erases the cross-section for a particular reaction from the tape;
(d) It prints out the cross-section for a particular reaction on a line printer.

At present, the cross-sections are stored on "RFSP LIBRARY" tape.
In the first regime, the sequence of operations is as follows:
After compiling the matrix $\underline{\underline{C}}^{T} \underline{C}$, the code determines $\omega$. The value $\omega$ may sometimes be too large, in which case some elements of the matrix $\underline{C}^{\prime} \underline{C}^{\prime}$ exceed $10^{12}$ and in the sum $\underline{C}^{\prime T} \underline{C}^{\prime}+1$ the addition is not performed accurately. To avoid this situation, the programme (after determining $\omega$ ) checks whether or not this value is too high. If it is, the highest value of $\omega$ not leading to inaccuracy is determined. Then, on the basis of formula (V-12), the programe determines the number of iteration steps required to calculate the limit with the desired accuracy using the new value of $\omega$.

It may sometimes prove necessary to study the way in which a spectrum varies as one approaches an exact solution. In such a case, of course, the method we
describe (i.e. determining $\omega$ and multiplying the matrix $\underline{C}^{\mathrm{T}} \underline{\mathrm{C}}$ by $\omega^{2}$ is not applicable, since it gives the limit at once. However, bearing in mind that the rate of convergence is determined by $\omega$, we can take some value $\omega^{\prime}$ such that $1<\omega^{\prime}<\omega$, multiply the matrix $\underline{C}^{T} \underline{C}$ by $\omega^{\prime 2}$, and thereby vary the rate of convergence which would be obtained by using Greer's method - i.e. if we used $\omega^{\prime}=1$.

As part of the input information we have to indicate whether we want to apply this iteration method (see further on). If we do, the rate of convergence is controlled by means of keys. If neither the third nor the fourth key is inserted then $\omega^{\prime}=\sqrt{\omega}$; if only the third key is inserted, then $\omega^{\prime}=1$; if the fourth key is inserted, then $\omega^{\prime}=\frac{1}{\sqrt{\omega}}$ (the latter may be required if the iteration process converges very rapidly even for $\omega^{\prime}=1$; i.e. for $\omega^{\prime}=1$ a solution that is "too good" is obtained after the first step).

In certain cases it may prove necessary not to normalize the initial flux i.e. not to make the calculation described in Appendix II. There is no normalization when key 1 is inserted. When key 2 is inserted, the code - before calculation begins - prints out the response functions of the individual foils in a given spectrum - i.e. the elements of the matrix $C \phi_{0}^{M}$. Questions connected with calculating a specific spectrum, using the different regimes and interpreting the results are described in detail in Ref. [2].

When compiling the RPSP code we drew considerably on the SPECTRA code. The input data for both codes are virtually identical, but our code nevertheless differs greatly from the SPECTRA code. Apart from the mathematical differences described in Section II, the two codes are dissimilar in the following respects:
(a) In the RFSP code $E \phi(E)$, and not $\phi(E)$, is the piecewise linear energy function. In the case of conventional reactors, $\phi(E)$ varies within eight orders of magnitude and we have to describe the spectrum by a limited number of points (a maximum of 50 in this case). Taking $\not \subset(E)$ as the piecewise linear function, we can permit an error of up to $10-12 \%$ in the activities. If $E \not \subset(\mathbb{E})$ is taken as the piecewise linear function, the error is virtually eliminated, but this is only a slight modification from the standpoint of programming and mathematics;
(b) As already mentioned in Section II, with the SPECTRA code we calculate all the eigenvectors of a matrix of size $m \times m$ and then orthogonalize some of them, whereas there is no calculation of eigenvectors with the

RFSP code. As a result of this difference, the number of cells in the computer's internal memory required to accommodate the RFSP programme is considerably smaller than for the SPEC'TRA programme. With the ICT-1905 computer programmed with the RFSP code, a spectrum can be unfolded at 50 points, whereas the SPECTRA code can be accommodated in the computer only wher the number of points drops to 25.

The RFSP programme occupies 25500 cells in the computer's internal memory. The computer time required to solve one problem is a function of the size of the matrix $\underline{C}$ and - when an iteration process is used - of the number of iterations. In the case of 10 foils and 50 energy points it takes $\sim 250$ seconds to solve one problem, i.e. to determine the limit.

## Introduction of the input data

The input data are introduced on punched tape or punched cards in the form described below, as required by the rules for using FORTRAN; each format corresponds to one line.

Format I/10A8/ - This line stipulates the operational regime for the programme. The information recorded in this line consists of only one of the following words and has to be recorded at the beginning of the line.

RUN - Unfolding of neutron spectra by the method described in Section II.

NENTAPE - Compilation of a new magnetic tape library storing the cross-section data.

ADD - Recording of new reactions and cross-sections on existing magnetic tape.

DELETE - Erasure of any reaction or cross-section from the tape.
EDIT - Print-out of any reaction or cross-section on a line printer.

ENDEND - End of punched tape containing input data.
The information given after this line depends on the operational regime set in the first line.
(a) In the case of RUN

Format II/10A8/

- Any text; 80 alphanumerical symbols; the title of the problem may be put in here. The programme prints out this line at the beginning of each page on the line printer.

Format III / 3I3, El2.5,I5,I2/ - IFOIL - Number of foils $\leqslant 30$
IENER - Number of energy points $\leqslant 50$
MODE - May have the following meanings:
(i) If the LIMIT regime is used by itself, only the three variables mentioned (IFOIL, IENER and MODE) need be given in this line, while the other spaces remain empty; in this case, if MODE $=1$, the code produces a linear representation of the spectrum, while if MODE $=2$, the mode represents it logarithmically (this only relates to a schematic representation of the spectrum and does not affect the code's operation);
(ii) If the REPETE regime is used, then the value of the remaining variables must be given and, furthermore, one must indicate whether the REPETE regime is being used by itself or whether the REPETE and LIMIT regimes are being used in sequence. If $\mathrm{MODE}=1$, the spectrum is unfolded only during the REPETE regime; if MODE $=2$, it is unfolded in both regimes. The remaining variables are

$$
\operatorname{ERRE}=\frac{1}{n} \sum_{i=1}^{n}\left|\frac{A_{i}^{k}-A_{i}}{A_{i}}\right|
$$

or the permissible mean error, where $\mathbb{A}_{i}^{k}$ is the activity of the i-th foil at the k-th iteration step:

MITE - Maximum number of iterations
IPLT - If it is equal to 1 , the spectrum is represented linearly; if it is equal to 2 , it is represented logarithmically. The REPETE regime is terminated when the mean deviation of the activities is less than ERRE, or when the number of iterations attains MITE. If a separate
error is given for the foils (see type V), the operation is also terminated when each activation error is less than the corresponding measurement error.
(iii) If MODE = 4 and we do not assign a value to the variables ERRE, MITE and IPLT, the programme determines only the mean cross-section for the reactions under consideration in the spectrum given as the programme input; in this case, the activation values have to be given a value of unity (see type V).

Format IV/6El2.5/

- $E_{1}, \phi\left(E_{1}\right), \ldots E_{m}, \phi\left(E_{m}\right)$, in all $2 x$ IENER variables; energy and input flux values. The energy should be given in ascending order. This format is fed in as many times as the IENBR value requires.

Format V/2A8, E12.5, E13.5/ - The first two variables contain the foil designation, whicn should tally with the name of the reaction given in Table 1. This is followed by the measured foil activity and by the measurement error, which does not necessarily have to be determined; if the measurement error is not stipulated the diagonal elements of the matrix $F$ are equal to unity. The measurement error has to be stipulated for all the foils or for none of them. This type is inserted IFOIL times. Then comes the line in which the next regime is given (Format I).
(b) In the case of NEWTAPE or ADD

| Format VI/2A8,14/ $\quad$ | The first two variables contain the foil |
| ---: | :--- |
|  | desigriation, after which comes the number of |
|  | data (i.e. a number twice the number of energy |
|  | points used to determine the cross-sections); |
|  | it should not exceed 3500. |

This is followed either by the line giving the regime (Format I) or again by a line of Format IV.
(c) In the case of $D E L E T E$ or $A D D$

Format VIII/2A8/ - The name of the reaction that it is intended to erase or to print-out on the line printer. This is followed either by a line of Format VIII or by the line of Format I giving the regime.

The programme enables separate regimes to be run one after the other, following the order of the input data. After the final data the punched tape or card bearing the data should be terminated with the line of format I containing the word ENDEND.

## The input data

In the NEWNAPE, $A D D$ and DELETPE versions of the RFSP programme, the input data include the name of a given reaction, while for EDIT they also contain the reaction cross-section at all points.

In the RUN version, the title of the problem is followed by the designations of the foils, their measured activities and (if stipulated) the activity measurement errors. After this, the initial flux, $\phi(E)$ and $E \not \subset E)$, and then the activity ratios, are printed out.

After the normalization described in Appendix. II, there come the normalized fluxes and the corresponding normalized activity ratios. This is followed by $\omega$ and then (if there are any) by the results of the LIMIT version - i.e. the theoretical fluxes $\phi(E)$ and $E \dot{\phi}(E)$, the activity ratios and the percentage errors.

In the case of the REPETE version, the output is the same as for LIMIT; each of the first ten iterations is printed out, after which every fifth iteration up to 100 and then every hundredth iteration. However, when REPETE is used, not more than 50 iterations are usually required.

Appendices VI and VII describe the set of input data and the result as calculated and printed out by the RFSP programme.

APPENDIX I: DETERMINATION OF THE ELEMENTS OF THE MATRIX C
As pointed out, the flux is given in such a way that $E \phi(E)$ is a piecewise linear energy function. Let $E_{1}, E_{2}, \ldots, E_{m}$ be the corresponding energy values,
and $\phi_{1}, \phi_{2}, \ldots, \phi_{m}$ the corresponding fluxes. The flux values between $E_{j}$ and $\mathrm{E}_{\mathrm{j}+1}$ are then

$$
\begin{equation*}
E \phi(E)=\phi_{j+1} E_{j+1} \frac{E-E_{j}}{E_{j+1}-E_{j}}+\phi_{j} E_{j} \frac{E_{j+1}-E}{E_{j+1}-E_{j}} \tag{I-1}
\end{equation*}
$$

Let us assume that below $\mathrm{E}_{1}$ and above $\mathrm{E}_{\mathrm{m}}$ the flux is zero, i.e. that $\phi_{1}=\phi_{\mathrm{m}}=0$. Substituting the expression given above into Eq. (I) for the activity of the i-th foil, we get the following equation:

$$
\begin{align*}
& A_{1}=\int_{E_{1}}^{E_{2}} E \phi(E) \frac{\sigma_{1}(E)}{E} d E+\int_{E_{2}}^{E_{3}} E \phi(E) \frac{\sigma_{1}(E)}{E} d E+\ldots+\int_{E_{m-1}}^{E_{m}} E \phi(E) \frac{\sigma_{1}(E)}{E} d E= \\
& =\int_{E_{1}}^{E_{2}}\left[\phi_{2} E_{2} \frac{E-E_{1}}{E_{2}-E_{1}}+\phi_{1} E_{1} \frac{E_{2}-E}{E_{2}-E_{1}}\right] \frac{\sigma_{1}(E)}{E} d E+\ldots+\int_{E_{m-1}}^{E_{m}}\left[\phi_{m} E_{m} \frac{E-E_{m-1}}{E_{m}-E_{m-1}}+\right. \\
& \left.+\phi_{m-1} E_{m-1} \frac{E_{m}-E}{E_{m}-E_{m-1}}\right] \frac{\sigma_{1}(E)}{E} d E \tag{I-2}
\end{align*}
$$

If the integral sign is removed from $E_{1}, E_{2}, \ldots, E_{m}$ and $\phi_{1}, \phi_{2}, \ldots, \phi_{m}$, which are independent of the variable of integration, we get the following expression:

$$
\begin{aligned}
& A_{i}=\phi_{1} E_{1}\left[\int_{E_{1}}^{E} \frac{E_{2}-E}{E_{2}-E_{1}} \frac{\sigma_{1}(E)}{E} d E\right]+\phi_{2} E_{2}\left[\int_{E_{1}}^{E_{2}} \frac{E-E_{1}}{E_{2}-E_{1}} \frac{\sigma_{1}(E)}{E} d E+\int_{E_{2}}^{E_{3}} \frac{E_{3}-E}{E_{3}-E_{2}} \frac{\sigma_{1}(E)}{E} d E\right]+\ldots \\
& \ldots+\phi_{j} E_{j}\left[\int_{E_{j-1}}^{E_{j}} \frac{E-E_{j-1}}{E_{j}-E_{j-1}} \frac{\sigma_{i}(E)}{E}+\int_{E_{j}}^{E_{j+1}} \frac{E_{j+1}-E}{E_{j+1}-E_{j}} \frac{\sigma_{1}(E)}{E} d E\right]+\ldots+\phi_{m} E_{m}\left[\int_{E_{m-1}}^{E_{m}} \frac{E-E_{m-1}}{E_{m}-E_{m-1}} \frac{\sigma_{i}(E)}{E} d E\right]
\end{aligned}
$$

The values in square brackets are the corresponding elements of the matrix $Q$. When one actually calculates these matrix elements, the functions $\sigma_{i}(E)$ are given at very closely grouped points, while linear interpolation is applied. between points. Since the points are very close together, linear interpolation. provides satisfactory accuracy.

## APPENDIX II: NORMALIZATION OF THE INXTIAL FLUXES AND SOME PROPERTIES OF NORMALIZED ACTIVITIES

Let us assume we are given an initial flux $\phi_{0}^{\prime}$ and let it produce the activities $d_{1}, d_{2}, \ldots, d_{n}$ when multiplied by matrix $C$. If each element of the flux vector is reduced by the constant $x$, then the activities are multiplied by this constant. It is desirable to normalize the flux in such a way that the following condition is satisfied:

$$
\begin{equation*}
\sum_{i=1}^{n}\left(x d_{i}-1\right)^{2}=\text { minimum } \tag{II-1}
\end{equation*}
$$

Differentiating this expression with respect to x and then equating it to zero, after simple transformations we arrive at the following result:

$$
\begin{equation*}
x=\frac{{ }_{i}^{n}{ }_{\underline{=}}^{1} d_{i}}{i_{i}^{n}{ }_{1}^{n} d_{i}^{2}} \tag{II-2}
\end{equation*}
$$

In this report the normalized flux is designated by $\phi_{0}$, i.e. $x \phi_{0}^{\prime}$.
Let us see what can be said of the activities derived with the aid of the normalized flux. We shall calculate the sum of the normalized activities and the sum of their squares

$$
\begin{align*}
& \sum_{i=1}^{n} x d_{i}=x \sum_{i=1}^{n} d_{i}=\frac{\left(\sum_{i=1}^{n} d_{i}\right)^{2}}{\sum_{i=1}^{n} d_{i}^{2}}  \tag{II-3}\\
& \sum_{i=1}^{n}\left(x d_{i}\right)^{2}=x^{2} \sum_{i=1}^{n} d_{i}^{2}=\frac{\left(\sum_{i=1}^{n} d_{i}\right)^{2}}{\sum_{i=1}^{n} d_{i}^{2}}
\end{align*}
$$

This means that the two values are equal. If we substitute x from Eq. (II-2) into expression (II-1), then after the corresponding transformations we arrive at the following expression:

$$
\begin{equation*}
n-\frac{\left(\sum_{i=1}^{n} d_{i}\right)^{2}}{\sum_{i=1}^{n} d_{1}^{2}} \tag{II-5}
\end{equation*}
$$

If we bear in mind that in Eq. (II-1) this value is derived as a sum of squares, it is clear that

$$
\begin{equation*}
\frac{\left(\sum_{i=1}^{n} d_{i}\right)^{2}}{\sum_{i=1}^{n} d_{i}^{2}}<n \tag{II-6}
\end{equation*}
$$

On the left-hand side we have the sum of the normalized activities. If both sides are divided by the number of foils (i.e. by $n$ ), we find that both the arithmetic mean and the arithmetic mean of the squares of the normalized activities are less than unity.

APPEINDIX III: THEOREMS CONGERNING AND PROOF OF THE EXISTPENCE OF THE MATRIX B AND ITS EIGENVALUES

Let us first demonstrate that each matrix of the form $\underline{C}^{T} \underline{C}$ is a positive semi-definite matrix.

We know that the scalar product of any vector multiplied by itself cannot be negative: $(V, V)=V^{T} V \geqslant 0$. Let $V$ be an eigenvector of the matrix $\frac{C^{T}}{T}-$ and $\lambda$ the eigenvalue pertaining to that vector. Then $\lambda(V, V)=\lambda(V, V)=\left(\underline{C}^{T} C V, V\right)=$ $V^{T} \underline{C}^{T} \underline{C V}=(\underline{C V})^{T}(\underline{C V})=(\underline{C V}, \underline{C V}) \geq 0$.

Thus, $\lambda(V, V) \geqslant 0$, and therefore $\lambda \geqslant 0$. We have thereby proved that the eigenvalue of $\underline{C}^{T} \underline{C}$ cannot be negative, i.e. that this matrix is a positive semidefinite one.

If $V$ is an eigenvector of the matrix $\underline{C}^{T} \underline{C}$, then it must at the same time be an eigenvector of the matrix $\underline{C}^{T} \underline{C}+I$, and the corresponding eigenvalue will be $\lambda+1$, for if

$$
\underline{c}^{\mathrm{T}} \underline{\underline{C}}=\lambda V
$$

then

$$
\left(\underline{c}^{T} \underline{\underline{c}}+I\right) v=\lambda V+V=(\lambda+1)
$$

If $\lambda \geqslant 0$, it follows that $\lambda+1>0$, i.e. the matrix $\underline{C}^{T} \underline{C}+I$ is a positive definite matrix.

On the other hand (see determination of the matrix $\underline{C}$ ),

$$
\begin{equation*}
\frac{\partial^{2} \Delta_{1}}{\partial^{2} \phi_{1}^{2}}=C^{T} F^{2} C+G^{2}=G\left(\underline{C}^{T} \underline{c}+I\right) G \tag{III-1}
\end{equation*}
$$

If we multiply any positive definite matrix by a diagonal matrix whose diagonal contains only positive elements, we again obtain a positive definite matrix, so that $\underline{C}^{\mathrm{T}} \underline{C}+I$ is positive definite.

Let us now go back to the matrix $\underline{C}^{T} \underline{C}+I$. It is positive definite with an inverse matrix that can be designated as $\underline{B}$.

Let $V$ again be one of the eigenvectors of the matrix $\underline{C}^{T} \underline{C}+I$ and let the corresponding eigenvalue be $\lambda+1$, i.e.

$$
\begin{equation*}
\left(\underline{c}^{\mathrm{T}} \underline{c}+\mathrm{I}\right) \mathrm{v}=(\lambda+1) \mathrm{v} \tag{III-2}
\end{equation*}
$$

Let us multiply from the left both sides of this equation by the matrix $\underline{B}$ :

$$
\begin{equation*}
\underline{B}\left(\underline{C}^{T} \underline{C}+I\right) v=(\lambda+1) \underline{B} V \tag{III-3}
\end{equation*}
$$

from which we get

$$
\begin{equation*}
V=(\lambda+1) \underline{B} v, \quad \underline{B} V=1 /(\lambda+1) V \tag{III-4}
\end{equation*}
$$

Consequently, the vector that is an eigenvector of $\underline{C}^{T} \underline{C}$ and $\underline{C}^{T} \underline{C}+I$ is also an eigenvector of $B$, and the corresponding eigenvalue will be $1 /(\lambda+1)$. It follows from the relationship $\lambda \geqslant 0$ that any eigenvalue $1 /(\lambda+1)$ of the matrix $\underline{B}$ is less than or equal to unity.

APPENDIX IV: ONE OF THE PROPERTIES OF EIGENVECTORS OF THE MARRIX $C^{T} \underline{C}$ AND THE FORN OF THE PRODUCT PBC

1. As already pointed out, the rank of $\underline{C}^{T} \underline{C}$ is $n$, so that in the m-dimensional space of the flux vectors there is a subspace containing all the linear combinations of $m-n$ linearly independent vectors which, when multiplied by $\underline{C}^{\mathrm{T}} \underline{C}$, give us zero. The matrix $\underline{C}$ also possesses a subspace with the property indicated above. If we consider any element of the subspace of the matrix $\underline{C}$, we find that this element is also an element of the subspace belonging to the matrix $\underline{C}^{T} \underline{C}$, for if $\underline{C} \phi=0$, then $\underline{C}^{\mathrm{T}} \underline{C} \phi=0$.

Both the above subspaces are m-n-dimensional. Hence, if for any element of the first subspace there is one, and only one, corresponding element in the second subspace, the opposite statement (i.e. if $\underline{C}^{T} \underline{C} \phi=0$, then $\underline{C} \phi=0$ ) will also be true.
2. Let us now consider the product PBC .

The matrix $P$ consists of eigenvectors of $\underline{B}$, so that the columns of the matrix $\underline{B} P$ contain eigenvectors of $\underline{B}$ multiplied by the corresponding eigenvalues. The last m-n columns simply contain eigenvectors, since the corresponding eigenvalues of $B$ are equal to unity.

These eigenvectors are also eigenvectors of matrix $\underline{C}^{T} \underline{C}$, and the corresponding eigenvalues are equal to zero. Hence, it can be stated on the basis of the theorem proved in the first part of Appendix IV that, if these vectors are multiplied by the matrix $\underline{C}$, the result will be zero; consequently, there are zeros in the last m-n columns of the matrix PBC .

APPENDIX V: CALCULATION OF THE MULTIPLIER $\omega$
For any positive $\varepsilon$ there will be a $\omega$ for which even the absolute value of the element with the greatest absolute value of the column vector - in square brackets in expression (15) - will be less than $\varepsilon$; in other words, if a high enough $\omega$ is used, $\phi-\phi_{1}$ can be reduced to the desired value.

To prove this statement let us consider separately the two terms of the sum in square brackets in expression (15). In the first term we shall write out the matrix $\underline{C}^{\prime T}$ in greater detail: $\underline{C}^{\prime T}=\phi_{o}^{M} C^{T} F\left(\omega_{n}\right)$. Since $F$ and $\phi_{o}^{M}$ are diagonal matrices, i.e. $F=\mathrm{F}^{\mathrm{T}}$ and $\phi_{0}^{\mathrm{M}}=\phi_{0}^{\mathrm{MT}}$ we do not designate the transposition operation for these matrices. In that case, the first term becomes

$$
\begin{equation*}
\mathrm{P}\left(\mathrm{~L}_{0}^{\ell}+L_{o}^{r}+\ldots\right) \mathrm{P}^{\mathrm{T}} \underline{B}^{T} \phi_{O}^{M} \underline{C}^{T} F^{2}\left(\omega_{n}^{2}\right) \tag{V-1}
\end{equation*}
$$

The sum of the infinite geometric series contained in brackets is a diagonal matrix of which the elements are in sequence $1 /\left(\lambda_{1} \omega^{2}\right), 1 /\left(\lambda_{2} \omega^{2}\right), \ldots, 1 /\left(\lambda_{n} \omega^{2}\right)$. Each element of the vector $\left(\omega_{n}^{2}\right)$ is equal to $\omega^{2}$, so that multiplication by this vector can also be performed in such a way that each element of the diagonal matrix obtained as the sum of the matrix series is multiplied by $\omega^{2}$, and instead of the column vector ( $\omega_{n}$ ) we write ( $1_{n}$ ). In this case, the first term can be written as

$$
\begin{equation*}
\mathrm{P} \overline{\mathrm{~L}}_{o} \mathrm{P}^{\mathrm{T}} \underline{B}^{\prime T} \phi_{O}^{M} C^{T} \mathrm{~F}^{2}\left(1_{n}\right) \tag{V-2}
\end{equation*}
$$

where the elements of the diagonal matrix $\bar{L}_{0}$ are in sequence $1 / \lambda_{1}, 1 / \lambda_{2}, \ldots, 1 / \lambda_{n}$.

Let us now consider the individual components of this product.

1. As the matrix $P$ consists of eigenvectors of the matrix $\underline{B}^{\prime}$ and as $P^{T} \underline{B}^{\prime}{ }^{T}=L^{\prime} P^{T}$, we can multiply the diagonal matrix $L^{\prime}$ directly by $\bar{L}_{0}$, while the diagonal matrix $\bar{L}_{o} L^{\prime}$ contains the following elements:

$$
\begin{equation*}
\frac{1}{\lambda_{1}\left(1+\omega^{2} \lambda_{1}\right)}, \frac{1}{\lambda_{2}\left(1+\omega^{2} \lambda_{2}\right)}, \cdots, \frac{1}{\lambda_{n}\left(1+\omega^{2} \cdot \lambda_{n}\right)} \tag{V-3}
\end{equation*}
$$

2. The i-th column of the matrix $\phi_{0}^{M} c^{T}$ contains the following elements

$$
c_{i 1} \phi_{01}, c_{12} \phi_{o 2}, \ldots, c_{i m} \phi_{o m}
$$

The sum of these elements is equal to $x_{i}$, i.e. to the normalized activity ratio pertaining to the i-th foil. If we divide each column by the corresponding activity ratio, then in the i-th column of the new matrix (designated $C_{d}^{T}$ ) we will find the elements

$$
\begin{equation*}
\frac{c_{11} \phi_{01}}{x d_{1}}, \frac{c_{12} \phi_{02}}{x d_{1}}, \ldots, \frac{c_{1 m} \phi_{0 m}}{x d_{i}} \tag{v-4}
\end{equation*}
$$

Thus, the sum of the elements in any column is equal to unity, so that the sum of the squares of the elements cannot be greater than unity:

$$
\begin{equation*}
\sum_{j=1}^{m}\left(\frac{c_{i j} \phi_{o j}}{x d_{1}}\right)^{2} \leq 1 \tag{V-5}
\end{equation*}
$$

for all the elements are positive and less than unity, so that their squares are smaller than the numbers themselves. For this reason, the sum of their squares is greater than their sum.

To obtain the matrix $\phi_{0}^{\mathrm{M}} C^{\mathrm{T}}$, the matrix $C_{d}^{T}$ has to be multiplied by a diagonal matrix of size $n \times n$ whose elements are the corresponding normalized activity ratios $x_{i}$. This diagonal matrix is designated by $D$. In this case the expression ( $V-2$ ) takes the following form:

$$
\begin{equation*}
P \bar{L}_{o} L^{\prime} P^{T} C_{d}^{T} D^{2}\left(1_{n}\right) \tag{V-6}
\end{equation*}
$$

3. The matrices $D$ and $F^{2}$ are diagonal matrices of the order $n$. Let us use $f^{2}$ to designate the sum of the squares of the elements in the diagonal of the matrix $\mathrm{DF}^{2}$. The sum of the squares of the elements of the column vector
$D F^{2}\left(l_{n}\right)$ is then also equal to $f^{2}$. It should be noted that, if the errors of the individual activation measurements are taken as identical (i.e. if no weight is attributed to the foils), the elements of the matrix $F$ are equal to unity, and then $f^{2} \leqslant n$, since the sum of the squares of the normalized activity ratios cannot exceed $n$ (see Appendix II).

Let us now multiply together the products considered separately in paragraphs 1-3.
$P^{T}$ is an orthonormalized matrix, so that, when we multiply $\bar{L}_{o} L^{\prime}$ from the right by $P^{T}$, the lengths of the row vectors of the matrix $\bar{L}_{o} L^{\prime} P^{T}$ will be equal to the lengths of the corresponding row vectors of the matrix $\bar{L}_{o} L^{\prime}$, i.e. to the values of its corresponding diagonal elements. Hence, the sum of the squares of the elements of the i-th row of the matrix $\bar{L}_{o} L^{\prime} P^{T}$ is not greater than the square of the i-th diagonal element of matrix $\bar{L}_{0} L^{\prime}$, and if the matrix $\bar{L}_{0} L^{\prime} P^{T}$ is multiplied by $C_{d}^{T}$, no el ement of the i-th row of the matrix $\bar{L}_{0} L^{\prime} P^{T} C_{d}^{T}$ can be greater than $1 /\left(\lambda_{i}\left(1+\lambda_{i} \omega^{2}\right)\right)$, and for $i>n$ all the elements are equal to zero. In view of the fact that in this matrix there are $n$ elements in one row, the sum of the squares of the elements of the i-th row is not greater than

$$
\frac{n}{\lambda_{i}^{2}\left(1+\lambda_{i} \omega^{2}\right)^{2}}
$$

If the matrix $\bar{L}_{o} L^{\prime} P^{T} C_{d}^{T}$ is multiplied by the column vector $D F^{2}\left(l_{n}\right)$, the elements of the resulting column vector of $m$ elements are equal to the scaler products of the rows of the matrix $\bar{L}_{o} L P^{T} C_{d}^{T}$ as vectors and the column vector $D F^{2}\left(l_{n}\right)$. Because of the properties of the scalar product, the i-th element of this vector cannot be greater than

$$
\frac{£ \sqrt{n}}{\lambda_{i}\left(1+\lambda_{1} \omega^{2}\right)}
$$

and for $i>n$ the value of the elements is zero.
Thus, the sum of the squares of the elements of this vector is not greater than

$$
\mathrm{f}^{2} n \sum_{i=1}^{n} \frac{1}{\lambda_{i}^{2}\left(1+\lambda_{1} \omega^{2}\right)^{2}}
$$

while its length is not greater than

$$
\begin{equation*}
f \sqrt{n \sum_{i=1}^{n} \frac{1}{\lambda_{i}^{2}\left(1+\lambda_{1} \omega^{2}\right)^{2}}} \tag{v-7}
\end{equation*}
$$

$P$ is an orthonormalized matrix, so that, when the matrix is multiplied by any vector, the vector length remains unchanged. The maximum value of the element with the greatest absolute value cannot exceed the length of the vector, so that the absolute value of the largest element of the first column vector in square brackets in Eq. (15) cannot be greater than the value of expression (V-7).

Let us now consider the second term in square brackets in Eq. (15):

$$
\begin{equation*}
P\left(H-L^{\prime}\right) P^{T}\left(1_{m}\right) \tag{v-8}
\end{equation*}
$$

The diagonal elements of the matrix $\mathrm{H}-\mathrm{L}$ ' are

$$
-\frac{1}{1+\lambda_{1} \omega^{2}},-\frac{1}{1+\lambda_{2} \omega^{2}}, \cdots,-\frac{1}{1+\lambda_{n} \omega^{2}}
$$

The sum of the squares of the elements of the vector $\left(1_{m}\right)$ is $m$, which fact remains unchanged when the vector is multiplied by the orthonormalized matrix $\mathrm{P}^{\mathrm{T}}$; consequently, the sum of the squares of the elements of a column vector consisting of $m$ elements, $P^{T}\left(I_{m}\right)$, will also be $m$. Thus, by multiplying it by the diagonal matrix -L', the maximum value of the sum of the squares for the resulting column vector consisting of $m$ elements cannot exceed

$$
m \sum_{i=1}^{n} \frac{1}{\left(1+\lambda_{i} w^{2}\right)^{2}}
$$

Multiplication by matrix $P$ does not alter the sum of the squares in this case either so that the length - and at the same time the element with the greatest value - of the vector $P\left(H-L L^{\prime}\right) P^{T}\left(l_{m}\right)$ cannot exceed

$$
\begin{equation*}
\sqrt{m \sum_{i=1}^{n} \frac{1}{\left(1+\lambda_{i} \omega^{2}\right)^{2}}} \tag{V-9}
\end{equation*}
$$

The absolute value of the largest element of the column vector in square brackets in Eq. (15) cannot be greater than the sum of expressions (V-7) and (V-9).

Let us now determine which value of $\omega$ is required for known $n, m, f$ and $\lambda_{1}, \lambda_{2}, \ldots, \lambda_{n}$ for this sum to be less than a certain $\varepsilon$. This condition can be written in the following form:

$$
\begin{equation*}
\varepsilon>f \sqrt{n \sum_{i=1}^{n} \frac{1}{\lambda_{i}^{2}\left(1+\lambda_{i} \omega^{2}\right)^{2}}}+\sqrt{m \sum_{i=1}^{n} \frac{1}{\left(1+\lambda_{i} \omega^{2}\right)^{2}}} \tag{V-10}
\end{equation*}
$$

If we regard the unity in each denominator as negligible compared to $\lambda_{i} \omega^{2}$, the square root sign can be removed from $\omega^{2}$ and we arrive directly at the required value of $\omega$ :

$$
\begin{equation*}
\omega>\sqrt{\frac{1}{\varepsilon}\left[f \sqrt{n \sum_{i=1}^{n} \frac{1}{\lambda_{i}^{4}}}+\sqrt{m \sum_{i=1}^{n} \frac{1}{\lambda_{i}^{2}}}\right]} \tag{V-11}
\end{equation*}
$$

It should be noted that, if we wish to find out what the greatest error can be after the k-th iterative step, by disregarding the unity in each denominator we arrive at an inequality similar to expression ( $V-10$ )
$\varepsilon>\frac{1}{\omega^{2 k}}\left[f \sqrt{n \sum_{i=1}^{n} \frac{1}{\lambda_{i}^{2 k+2}}}+\sqrt{m \sum_{i=1}^{n} \frac{1}{\lambda_{i}^{2 k}}}\right]$

## REFERENCES

1. C.I. Greer, J.A. Halbleib, J.V. Walker, "A Technique for Unfolding Neutron Spectra from Activation Measurements" SC-RR-67-746, Nov. 1967
2. A. Fischer, L. Turi, to be published

Table 1
Reactions recorded on RFSP LIBRARY MAGNETIC TAPE and their energy limits in MeV

| U235/NF/FP-CD | $0,7.10^{-8}-18$ |  |
| :--- | :--- | :--- |
| U235/NF/FP-CD | $1,0.10^{-8}-18$ |  |
| PU239/NF/FP-CD | $0,6.10^{-8}-18$ |  |
| NA23/NG/NA24-CD | $0,5.10^{-8}-18$ |  |
| MG24/NP/NA24 | 0,05 | -18 |
| IN115/NN/IN115M | 0,05 | -18 |
| S32/NP/P32 | 0,05 | -18 |
| AL27/NP/MG27 | 0,05 | -18 |
| AL27/NHE/NA24 | 0,05 | -18 |
| RH103/NN/RH103M | 0,05 | -18 |
| FE56/NP/MN56 | 0,05 | -18 |
| NI58/NP/C058 | 0,05 | -18 |
| P31/NP/SI31 | 0,05 | -18 |
| PU239/NF/FP | $0,5.10^{-10}-10^{-5}$ |  |
| DY164/NG/DY165 | $0,33.10^{-10}-10^{-5}$ |  |
| LU176/NG/LU177 | $0,5.10^{-10}-10^{-5}$ |  |
| EU151/NG/EU152 | $0,5.10^{-10}-10^{-5}$ |  |
| IN115/NG/INI16 | $0,5.10^{-10}-10^{-5}$ |  |
| AU197/NG/AU198 | $0,5.10^{-10}-2.10^{-5}$ |  |

APPENDIX VI: Set of input data

```
RUN
TEST CASE FOR RFSP - REPCRT.
    9 50 2
    5.00OOOE-O4 O.0000OE OO 1.000JOE-03 5.50000E+10 1.40,OOOE-03 5.50000E+10
    2.1000JE-03 5.50000E+10 3.10JJOE-03 5.50000E+10 4.600.JJE-03 3.00000E+10
    5.50JJOE=03 1.5700OEE+10 6.50J00E=03 1.53000E+10 8.50000E=03 1.30000E+10
    1.00000E=02 3.00000E+09 1.20000EE02 2.50000E+09 1.40000E=02 2.50000E+09
    1.80000E三02 2.50000E+09 2.10000EEO2 3.00000E+09 4.60000EE02 9.50000E+09
    1.00000E=01 9.00000E+09 1.20000EE01 8.50000E+09 1.40000EE01 8.20000E+09
    1.80000EE01 7.50000E+09 2.00000E=01 T.30000E+09 2.80000E=01 6.60000E+09
    4.00000E=01 5.70000E+09 6.00000E=01 4.80000E+09 B.00000E:01 4.20000E+09
    9.00000E=01 2.70000E +09 9.50000E=01 1.70000E+09 1.06000EE00 1.30000E+09
    1.20100E-OO 9.92000E+0S 1.35100E=00 9.59300E+08 1.51900E 00 9.096T4E+08
    1.70900E 00 8.48295E+08 1.92300E 00 7.66264E+08 2.16300E 00 6.54626E+08
    2.43300E 00 5.19910E+08 2.73700E OO 3.49759E+08 3.07900E 00 1.95804E+08
    3.46300E 00 1.69401E+08 3.89600E 00 9.77683E+07 4.38300E 00 5.33872E+07
    4.93000E 00 4.34825E+07 5.54600E 00 4.44581E+07 6.23900E 00 3.36522E+07
    7.01900E 00 1.79139E+07 7.89600E 00 8.45628E+06 8.88200E 00 4.52701E+05
    9.99200E 00 1.88431E+06 1.12400E+01 6.61172E+05 1.26400E+01 2.02586E+05
    1.42200E+01 5.39000E+04 1.80000E+01 0.00000E 00
IN115(NN)IN115M 3.69900E 08 5.-2
NI58(NP)C058 1.87500E OB 5.#2
AL27(NHE)MA24 1.38100E 06 5.#2
MG24(NP)NA24 2.91160E 06 5.#2
PE56(NP)M#56 2.21020E 06 5.E2
AL27 (NP)KG27 6.56420E 06 5.E2
RH103(NN)RH1O迆 1.97220E 09 5.#2
U235(NF)FP-CD 9.68650E+09 5.#2
NA23(NG)NA24-CD 1.28500E O7 5.E2
ENDENTD = -
```

- RFSP *
TEST CASE POR
TEST CASE FOR RESD - REPORT
MG2GSMDSHAZ4 $2.01960 E$ n6 $5.00000 E-02$
3.6990nE O8 5.00000E-02
6.56420E 06 5.00000E-02
1.38900E 06 5.000NOE-02
9.9722nE n9 5.000NOE-02
2.21020E 06 S.00000E-02
1.87500E O8 S.00ODOE-02
9.68650E 09 5.000NOE-02
1.28500E 07 5.00000E-02
APPENDIX VII: Results printed out by RFSP code (page 1)
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| INITIAL CALCULA HGZA(NP)NA? | M. ${ }^{\text {nctivity Ratios }}$ |  |
| :---: | :---: | :---: |
|  |  |  |
| IN195(NNSINT15M. | 9.12167E 00 |  |
| AL27(ND)MG27 | 9.80240E-09 |  |
| Al27(NHE)NA? | 9.87434E-09 |  |
| RH103(NN)RHTO3K | 9.22295 00 |  |
| EESG(np)Mnsio | $9.99607 \mathrm{E}-09$ |  |
| H158(ND)COSB | 9.03413 E on |  |
| U235(nf)FP-C.O | 1.19182E 00 |  |
| naz3(ng)nazi-cd | 1.12186E 00 |  |





$6 \div \therefore \infty+\cdots m+n$

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MNN～N天下： $6 O O N O F=$
SOAAE





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| $\underset{x}{c}$ |
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activity ratios
CALCULATED
$.2994 \angle E-0$
$03622 E 00$
NORHALIてED INITIAL

0.50E 09
$0.60 E 01$

0.20 E 01
0.90 E 01
APPENDIX VII: Results printed out by RFSP code (page 4)

## H624(NP)NA24







INT15(NN)!H915M




 AL27(NP)MG27




 AL27(NNE)NAZ4



 8.69282E-02 1.79538E-01 ?.72546E-09 2.336nSE-01 1.48234E-01 7.13066E-02 2.70457E-02 7.30286E-03

APPENDIX VII: Results printed out by RFSP code (page 5)
PABE NO 5




 FE56(NP)MAS6






## N: $58(N P) \operatorname{COSB}$





 U23S(NF)FP-CD





APPENDIX VII: Fesults printed out by RFSP code (page 6)

## NA23(NG)NATG-CD







> the eigenvalues or (c-p)*c arf

number of iteration steds is 6
OHEGA $=2.88646 E 09$
APPENDIX VII: Results printed out by RFSP code (page 7)






MMNNGーECCOCO 4. $4 O O O O E=0$
$5.5 O O O O F=O$














$$
\begin{aligned}
& 106 \\
& 0161
\end{aligned}
$$

phe average activity farak is >.87i,beg-0?

$$
\begin{aligned}
& \text { - DERCENTAGE ERRORS } \\
& 0.0000 \\
& 0.00009
\end{aligned}
$$

APPENDIX VII: Results printed out by RFSP code (page 8)


APPENDIX VII: Results printed out by RFSP code (page 9)

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$\left.\begin{array}{l}\wedge \propto \infty \\ c\end{array}\right)$



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| Calculated activ | ES. ACT | VI | Q.it10 | AND PfrCENTAGE | FRRORS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| MG24(NP)NA24 | 2.919605 | On | 1.10000 | -10.0000 |  |
| IHITS(NN)INITSM | 3.6090 niz | 02 | 1.10000 | 0.0009 |  |
| AL27(nf)mgri | 6.5642 AE | 06 | 1. inon | 0.0000 |  |
| AL27(NME)NASG | 1.33100E | 06 | $1 .: 3000$ | -0.0000 |  |
| RH103(NN)RNIO.3M | 1.972.2nf | 09 | 9.inoo | 0.0000 |  |
| FESG(NP)NN5Ci | 2.2:n2nf | 176 | $1 . \therefore 1000$ | - 0.0100 |  |
| W158(ND)COSA | 1.87s0ne | 08 | 1. | 0.0000 |  |
| U23S(NF)FF-CO | 9.6A6SOF | 09 | 1. inno | 0.0000 |  |
| nat3(NG)NA24-CD | 1.2350nt. | 07 | 1.10000 | 0.08180 |  |
| phe averacie ar.ti | it Eaknk | IS | 7.87 i | 6E-07 |  |


differential flux fef
1.00E O9:
$0.90 E 09$
0.80 E 01
$0.70 E 09$
0.60 E 09
$\because$.
-•


80
70
60


$$
\because
$$

* 

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