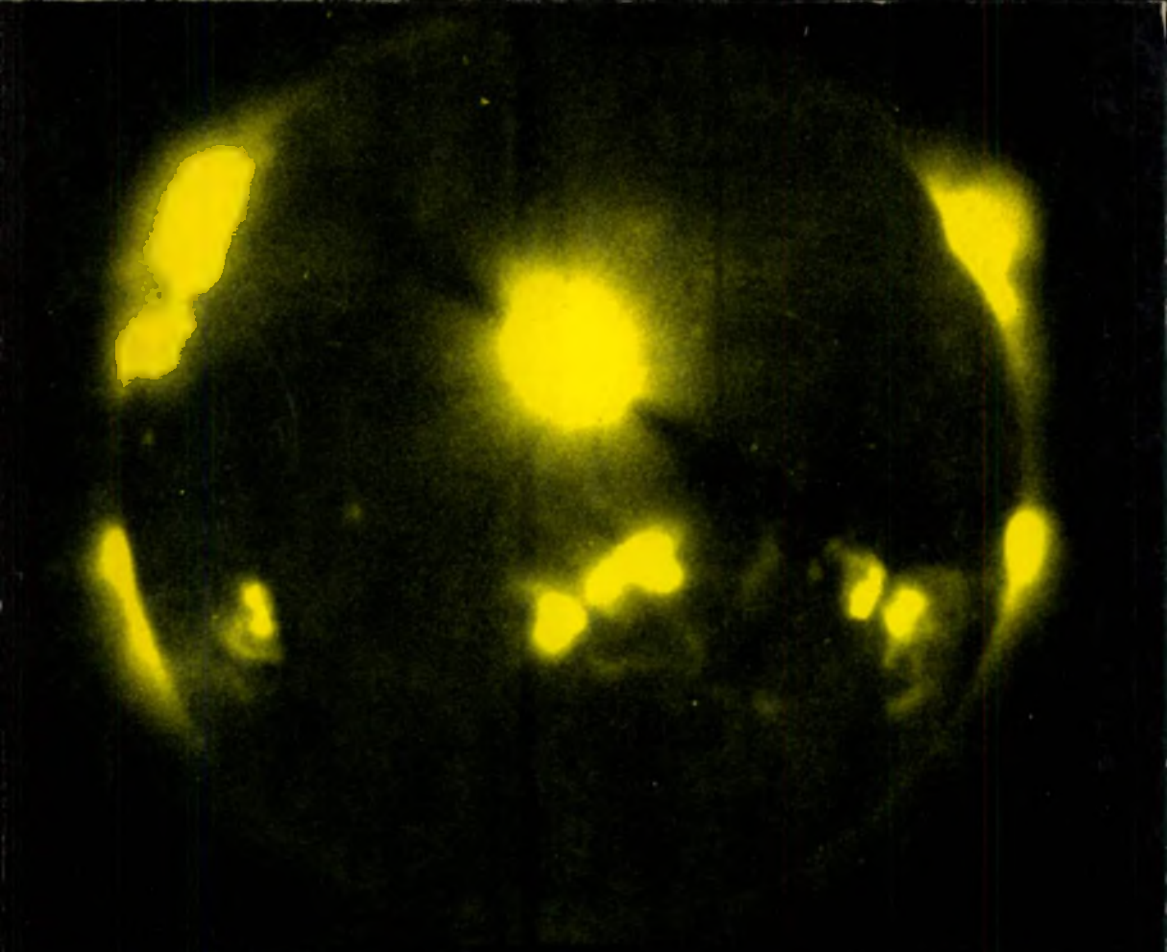

CONTEMPORARY PHYSICS VOL.II

TRIESTE SYMPOSIUM 1968

PROCEEDINGS OF THE INTERNATIONAL SYMPOSIUM
TRIESTE, 7-28 JUNE 1968, ORGANIZED BY THE
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS, TRIESTE

High-energy physics and elementary particles
Nuclear physics
Foundations of quantum mechanics and fundamentals of field theory

Scientific Secretary: L. FONDA Symposium Director: ABDUS SALAM

INTERNATIONAL ATOMIC ENERGY AGENCY, VIENNA, 1969

CONTEMPORARY PHYSICS:
TRIESTE SYMPOSIUM 1968

VOL. II

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ON CONTEMPORARY PHYSICS
ORGANIZED BY AND HELD AT THE
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS, TRIESTE
FROM 7 TO 28 JUNE 1968

SCIENTIFIC SECRETARY: L. FONDA
SYMPOSIUM DIRECTOR: ABDUS SALAM

In two volumes

VOL.II

HIGH-ENERGY PHYSICS AND
ELEMENTARY PARTICLES

NUCLEAR PHYSICS

FOUNDATIONS OF QUANTUM MECHANICS AND
FUNDAMENTALS OF FIELD THEORY

INTERNATIONAL ATOMIC ENERGY AGENCY
VIENNA, 1969

The cover shows a solar X-ray photograph
taken at the time of an important flare

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FOREWORD

In June 1968, the International Centre for Theoretical Physics was privileged to hold an extended symposium on contemporary physics – an event probably unique in that its aim was to range over and review not just one specialized aspect of modern physics but its entire spectrum. The symposium brought together a group of 300 leading specialists in particle physics, theory of condensed matter, astrophysics, relativity, plasma physics, cosmology, nuclear physics, quantum electronics and biophysics; they lived together for three weeks, to deliver, to discuss and to listen to lectures in depth on their various specialities. The intention was to acquire, if possible, a deeper sense of the scope and unified nature of the general subject – physics – by sharing the insights of its fascinatingly diverse disciplines.

The present two volumes are a record of the proceedings of the symposium. The first groups papers presented on condensed and plasma matter, on earth and in the universe; the second deals with theory of matter at very small distances and fundamentals of quantum theory.

How far did we succeed in achieving the objectives of the symposium? Did we succeed in communicating to those in other disciplines the present situation in our own – the problems, the prospects? Did we emerge, three weeks after, as universal in our outlook and competence as the great physicists of even forty years ago, who could, with felicity, pass from one discipline to another enriching each? How much cross-fertilization was achieved? To what extent were the hierarchy of disciplines and the barriers between generations broken? In retrospect – and particularly in retrospect – the answers to these questions appear positive: we succeeded perhaps partially; but far more than anyone anticipated. There is no doubt that communication is a difficult art. There was so much ground to be covered before the treasures of one's own discipline could be unfolded. We could not easily forget those in the audience belonging to our own specialities. We were all guilty of exaggerating the importance of the problems of just this day – this will be found reflected in the proceedings, where survey lectures designed expressly for those in other disciplines are juxtaposed with contributions unabashedly specialized. We debated long on whether to publish the latter. In the end we decided to do so, for these contributions do sample in depth the present day's preoccupations with a vital, living subject in all its kaleidoscopic aspects, and they give a perspective to the general surveys.

Notwithstanding these specialist contributions, however, there did shine through the symposium the recognition that physics is still very much a unity – that even though they may appear under different names and guises in differing specialities, the conceptual constructs are the same; that results and techniques, both experimental and theoretical, developed for one discipline are relevant in others. If anything, we grew fonder of, more enthusiastic about our own discipline, after recognizing the problems which the others face.

But, even more than the scientific, for most of us the symposium was a human experience of a lifetime. I do not believe there ever has assembled, at one time and place and for so long, such a galaxy of first-rate physicists from so diverse fields; most lecturers were the men who had been responsible for creating their subjects. Then there was the civilizing influence of the presence among us, over three weeks, of some of the grand old men of physics – the founders of modern quantum theory – who so warmly responded to the invitation to participate. We were privileged to hear evening lectures from some of them (to be published as a Supplement to the IAEA Bulletin) on their life of physics. These were magical evenings and perhaps the most memorable part of the symposium. Finally, there was the physical beauty of the surroundings – we had just moved into the Centre's new premises, tastefully completed on the warm Adriatic shores just before the symposium began and graciously presented to the Centre by the Community of Trieste.

The symposium was made financially possible by generous grants from: Ford Foundation; Italian Ministry of Education; Region of Friuli-Venezia Giulia; Consorzio per l'Incremento degli studi e delle ricerche degli Istituti di Fisica dell'Università di Trieste; UNESCO; Academy of Sciences of the USSR; Royal Society (London); US Atomic Energy Commission; National Science Foundation (US); and Gulf General Atomic Inc.; as well as a personal donation from Mr. Cecil H. Green of Texas Instruments Inc. We received warm hospitality from His Highness Prince Raimondo Torre e Tasso and from the City of Trieste – this and an infinity of other courtesies we owe to the tireless organization of Professor Paolo Budini.

An international Organizing Committee, chaired till his tragic death by Professor J.R. Oppenheimer, was responsible for drawing up the programme – it was shaped finally through the dedicated efforts of the monitors, Professors G. Burbidge, D. Pines and M. Rosenbluth, of the Scientific Secretary of the symposium, Professor L. Fonda, and, particularly, of Professor R.E. Marshak. The symposium would never have been held if, at all stages, we had not had the assured warm support of Dr. S. Eklund.

The members of the Organizing Committee were as follows: Professors H. Alfvén, V. Ambartsumyan, N.N. Bogolyubov, Aage Bohr, H. Bondi, S. Brenner, A. de-Shalit, J.F. Dyson, M. Gell-Mann, M. Goldberger, I.M. Khalatnikov, R.E. Marshak, A. Matveyev, D. Pines, I. Prigogine, R.Z. Sagdeev, M. Sandoval Vallarte, V. Soloviev, J. Tiomno, L. Van Hove, and V.F. Weisskopf.

Abdus Salam

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HIGH-ENERGY PHYSICS AND ELEMENTARY PARTICLES

FUNDAMENTAL THEORY OF MATTER
A survey of results and methods

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Abstract

FUNDAMENTAL THEORY OF MATTER: A SURVEY OF RESULTS AND METHODS. Introduction; Part 1. Classification of particles and charges; 1.1. Poincaré description; 1.2. Classification charges; 1.3. Distinguished particles and elementarity; Part 2. Relativistic quantum mechanics; 2.1. Kinematical considerations; 2.2. Ingredients of the theory; 2.3. Postulated canons of relativistic quantum mechanics; 2.4. Deductions from canons of relativistic quantum mechanics; 2.5. Hadron spectrum and the S-matrix program; Part 3. The dynamical role of charges and sharing of currents between strong, weak and electromagnetic interactions; 3.1. Dynamical role of charge; 3.2. Dynamical role of currents; 3.3. Universality of currents; 3.4. Low-frequency dynamics; 3.5. Particle spectrum and the symmetry algebra program; Part 4. The structure of hadrons, still higher dynamical symmetries; Summary.

INTRODUCTION

Our systematic knowledge of stable and semi-stable forms[1] (particles and resonances, lifetimes $\geqslant 10^{-24}$ sec) in which matter seems to exist, extends at present to forms created in laboratory collision experiments with proton beams of energies less than 30 GeV. A like situation holds in respect of the fundamental forces which govern the behaviour of these forms of matter; our systematic empirical knowledge extends no further than these same relatively low energies. I wish to give a rapid survey of what we believe are some of the "truths" and "insights" about fundamental laws of physics which, though abstracted from these low-energy data, may, hopefully, survive in a future theory.

The course which theory of matter would take in future centuries - its particle aspect and the hierarchy of forces between particles - was forecast in a remarkable prediction made by Isaac Newton:

"Now the smallest particles of matter may cohere by the strongest attractions, and compose bigger particles of weaker virtue; and many of these may cohere and compose bigger particles whose virtue is still weaker, and so on for divers successions, until the progression ends in the biggest particles on which the operations in chemistry and the colours of natural bodies depend, and which by cohering compose bodies of a sensible magnitude.

"There are, therefore, agents in nature able to make the particles of bodies stick together by very strong attractions. And it is the business of experimental philosophy to find them out."

In Newton's day, the only virtue which particles of matter were known to possess was gravitational. Subsequently, we have learnt that there are at least three other virtues. These are: (1) strong; (2) electromagnetic; (3) weak; and, possibly, (4) super-weak.

[1] Lifetime $\tau \sim \frac{\hbar}{\text{width}\,\Gamma} \sim \frac{6.6 \times 10^{-22}\ \text{MeV sec}}{\text{width in MeV}}$. Thus 10^{-24} sec lifetime corresponds to resonances of width ~ 100 MeV.

There are two important points to be made about these fundamental forces:

1. They have vastly different strengths; typically the ratios[2] are

strong	:	E.M.	:	weak	:	super-weak	:	gravitational
1	:	10^{-2}	:	10^{-5}	:	10^{-8}	:	10^{-34}

2. Even more important, there is a sharp selectivity about the other forces in contrast to the universality of gravitation. Thus,

(a) strong forces divide matter sharply into hadrons (strongly interacting matter) and leptons (with no strong forces);

(b) E.M. forces divide matter into electrically charged and uncharged;

(c) weak forces divide matter into "left-spinning" and "right-spinning". They act selectively between "left-spinning" and "right-spinning" matter in a sense I shall define more precisely later.

As in the familiar case of electrodynamics, it turns out that this selectivity is best expressed by assigning to different varieties of matter a number of fully or approximately conserved charges (strong charges, E.M. charge, weak charges). The assignment of charges serves two roles:

1. Kinematic; since the charges defined are (fully or partially) conserved, they serve to classify single-particle states.

2. Dynamic; as for the case of the Coulomb force, the long-range part of physical forces is determined by the magnitudes of the corresponding charges.

Since the concept of strong, weak and E.M. charges is so crucial to particle physics, nearly half of my lecture is devoted to it. Indeed, if I were asked to list the major recent "truths" in particle physics, the list would be somewhat like this:

1. A clearer comprehension of the charge concept, both for classifying particles and for the dynamical role it plays. In particular, the discovery that there is a unifying principle running through strong, weak and E.M. forces, in that these forces share the same basic charges and their corresponding currents (scaled differently, of course, in strength).

2. Recognition of the essential correctness of basic laws of relativistic quantum mechanics (RQM) up to the energies available at present. Quantum mechanics was invented for systems of typical dimensions $\approx 10^{-8}$ cm, typical energies measured in eV. Non-relativistic quantum mechanics continued to work in the nuclear domain with typical lengths $\approx 10^{-12}$ cm and MeV energies - a vast extrapolation[3] from the situation it was invented for. Relativistic quantum theory - the strongly operative word is relativistic - seems to hold down to 10^{-14} cm and GeV energies.

---

[2] These are ratios (of squares) of the well-known dimensionless coupling constants, normalized (where necessary) with proton mass. There has been the conjecture (due, I believe, to Dirac) that the strengths of these forces and presumably also their relative ratios may have varied with the age of the universe (GAMOW, G., Phys. Rev. Lett. 19 (1967) 759). Evidence to show that this is not likely for some of the constants has been adduced recently (DYSON, F.J., Phys. Rev. Lett. 19 (1967) 1291; PERES, A., ibid. 19 (1967) 1293; BAHCALL, J.N., SCHMIDT, M., ibid. 19 (1967) 1294; and CHITRE, S.M., PAL, Y., ibid. 20 (1968) 278.

[3] There was never a conference held in the nineteenthirties, where some of the founders of quantum theory did not express doubts about this extrapolation. The fundamental length at which quantum mechanics was expected to break down has continually diminished in size as time has gone on.

With quantum theory, we appear to have built ourselves a house with no doors and no windows and with walls so high that (in Jost's phrase) it is hard to know if it is a house or a prison we have inherited. It will be clear as I go on how tight and restrictive the structure of relativistic quantum mechanics is; for example, it does not appear to permit an easy mixing of space-time degrees of freedom with the "internal" degrees of freedom represented by the charges.

3. Renormalized quantum electrodynamics (Dirac-Maxwell theory of electrons and muons) which agrees with the highest experimental accuracy achieved at present. Since Professor Low reviews this in these Proceedings, I shall not discuss this most beautiful of theories which makes everyone of us purr with elation.

4. Developments in weak interaction theory, initiated by the discovery of their extraordinary space-time reflection properties, which were first correlated through the recognition of the crucial role of the two-component (left-handed) nature of the neutrino, and through its later generalization, to the concept of chirality (handedness) for all matter. Professor Lee covers this subject in these Proceedings and I shall not discuss this beautifully connected development.

The plan of my lecture is as follows. First we go over the Rosenfeld table (Appendix A) of stable and semi-stable particles and resonances, introducing at the same time the various classifying charges; next describe the theoretical apparatus used, setting down also the canons of relativistic quantum mechanics (RQM); finally, turn back to the charges in their dynamical role and consider other dynamical symmetries not associated - to our present knowledge - with charges. We end with speculations on whether these symmetries foreshadow discovery of newer forms of matter interacting with super-strong forces.

PART 1. CLASSIFICATION OF PARTICLES AND CHARGES

1.1. POINCARE DESCRIPTION

Throughout we make the assumption that laws of physics are translation- and space-time rotation symmetric. This assumed Poincaré symmetry of space-time implies in turn conservation of energy, momentum (P_μ) and angular momentum[4] ($J_{\mu\nu}$), respectively. [The argument connecting symmetries and conservation laws, though well known, is worth repeating. A symmetry in quantum theory is represented (Table I) by a unitary transformation U relating a given state of the system to the one obtained by the symmetry operation. Infinitesimally, let $U = 1 + i\alpha_e X_e$ (J_e Hermitian). A symmetry is exact if U commutes with the full Hamiltonian of the system; it is a partial symmetry if it commutes with only a part (hopefully the dominant part) of the Hamiltonian. Thus for an exact symmetry,

[4] Conservation law of angular momentum is verified empirically, for example, in $0^+ \to 0^+$ internal conversion in electro-transitions in ^{22}Ge; Sunyar (FEINBERG, G., GOLDHABER, M., Proc. natn. Acad. Sci. 45 (1959) 1301) found a ratio $\gamma/e^- < 1/1000$. Since γ-rate for a $1^\pm \to 0^+$ transition would be $\approx 10^6$ faster than e^- rate for $0^+ \to 0^+$ transition, one concludes that the amplitude for the admixture of spin one state with spin zero state is less than one part in 10^{-4}.

TABLE I. INVARIANCE OR SYMMETRY REPRESENTED BY UNITARY TRANSFORMATION

$U = 1 + i\alpha_\ell X_\ell$
X_ℓ observables
If symmetry is exact
$[U, H] = 0 = [X_\ell, H]$
then
$\frac{d}{dt} X_\ell(t) = 0$
If
$\frac{d}{dt} X_\ell(t) \neq 0$
symmetry is broken

$[U, H] = 0 = [X_e, H)$, i.e. $dX_e/dt = 0$ so that X_e is a time-independent operator; its eigenvalues do not change with time and serve to classify one-particle states. If the symmetry is only partial, i.e. if the Hamiltonian consists of two pieces, $H_{inv.} + gH_{non\text{-}inv.}$, $dX_e/dt \neq 0$; i.e. X_e is time-dependent, its time rate of change being proportional to the symmetry breaking parameter g.]

In the following it is our aim to specify a set of quantum operators whose eigenvalues serve to classify single-particle states. A complete characterization of those involving the Poincaré operators P_μ and $J_{\mu\nu}$ ($\mu = 0, 1, 2, 3$) was given by Wigner as follows:

Consider the rest frame of the particle $P_i = 0$, $i = 1, 2, 3$. Single-particle states are labelled according to:

(a) Rest mass (eigenvalues of the operator P_0).

(b) Spin.

The three spin operators[5] J_i correspond to space rotations close on the algebra of SU(2)

$$[J_i, J_j] = i\epsilon_{ijk} J_k \quad i = 1, 2, 3$$

Thus from standard group theory, quantum mechanical states (in the rest frame) can be labelled with the spin eigenvalues J and J_3 of the two (Casimir) invariants of SU(2), i.e. of the operators

$$\vec{J}^2 = J_1^2 + J_2^2 + J_3^2 = J(J + 1) \text{ and } J_3$$

[5] J_1, J_2, J_3 are the same as J_{23}, J_{31}, J_{12} of the set $J_{\mu\nu}$.

where

$$J = 0, \tfrac{1}{2}, 1, \ldots$$

$$J_3 = 0, \pm\tfrac{1}{2}, \begin{matrix}\pm 1\\ 0\end{matrix}, \ldots$$ [6]

(c) Particle-antiparticle duality

One of the gifts of local relativistic quantum theory and Poincaré symmetry is the assertion that all particles possess antiparticles[7] (with the same mass, same lifetime and opposite charge). If experiment shows that this symmetry does not hold, either the Lorentz-invariance or the locality of the theory[8] must be abandoned.

Summarizing, the Poincaré classification of physical states proceeds with the ingredients[9] given in Table II.

1.2. CLASSIFICATION CHARGES

The spin operators J_i (and their group-theoretic properties) provide prototypes for other classification charges. We now go over these.

1.2.1. Electric charge (Q)

Empirically the electric charge Q

(a) is conserved $\dot{Q} = 0$; $\tau(\text{electron}) > 2 \times 10^{21}$ yr;

(b) possesses (like J_3) the eigenvalues 0, ± 1, ± 2, ... in units of electron charge. (In algebraic language Q is the generator of an internal symmetry group U(1).) All known charges exist in units of electron's charge;

(c) the most remarkable manifestation of this is the charge equality of electron and proton verified to better than one part in 10^{20}.

1.2.2. Strong (hadronic) charges

Unlike the electric case with but one type of charge[10] known, there exist a variety of strong charges which manifest themselves for strongly

[6] If the precise value of J_3 is not specified, the spin label J denotes collectively a $(2J+1)$ dimensional spin multiplet of particles. In subsequent work, spin J will be specified by indicating the multiplicity; thus multiplicities $\underset{\sim}{1}$, $\underset{\sim}{3}$, $\underset{\sim}{5}$, ... will indicate spins 0, $\frac{1}{2}$, 1, ..., respectively.

[7] The reason for this is subtle; real Poincaré invariance in a local theory (defined more precisely in Part 2.2) implies - it so happens - full complex Poincaré invariance ($U = 1 + i\alpha_\ell X_\ell$; α_ℓ, X_ℓ complex). A part of the complex Poincaré group connected to real Poincaré is space-time reflection $\vec{x} \to -\vec{x}$ (parity or P-operation) and $t \to -t$ (time reflection, T). This, together with C (conjugation of particles with antiparticles), defines the CTP symmetry which converts incoming particle states to the corresponding outgoing antiparticle states. CTP symmetry is thus an in-built part of a Poincaré-Lorentz-symmetric local relativistic quantum mechanics.

[8] Present limits on mass and lifetime equalities of π^+ and π^-, for example, set (not too stringent) limits on the experimental validity of CTP$((I_{\pi^+}/I_{\pi^-}) - 1 = 0.064 \pm 0.069)$.

[9] The impatient reader will get a reasonable notion of the concepts of the subject by reading only the tables.

[10] There could, for example, exist particles with magnetic charges (monopoles), but at present there is no evidence for these.

TABLE II. POINCARÉ DESCRIPTION

(a) Rest mass

(b) Spin

Spin algebra SU(2) $\Longrightarrow$

$$[J_i, J_j] = i\epsilon_{ijk} J_k$$

$$J = 0, \tfrac{1}{2}, 1, \ldots\ldots$$

$$J_3 = 0, \pm\tfrac{1}{2}, \pm 1, \ldots$$

Multiplets of dimensionality $\longrightarrow$ $\underset{\sim}{1}, \underset{\sim}{2}, \underset{\sim}{3}, \ldots\ldots$

(c) Particle $\overset{CTP}{\longleftrightarrow}$ antiparticle duality

interacting particles, the so-called hadrons. These are baryon charge (B), hypercharge (Y), isotopic charges ($\vec{I}$) and unitary charges ($\underset{\sim}{F}$).

(a) Baryon charge B

Like electric charge, baryon charge B carried by protons, neutrons, etc. (see Table IV for more baryons)

(i) is absolutely conserved[11]

$$\tau_{\text{free proton}} > 10^{21}\ \text{yr}$$

$$\tau_{\text{bound protons}} > 4 \times 10^{23}\ \text{yr}$$

(ii) possesses (like J_3) eigenvalues 0, ± 1, ± 2, ... in units of proton baryonic charge. A remarkable (but not understood) empirical relation seems to exist connecting spin J and baryon charge B; empirically $(J - \frac{1}{2}B)$ takes integer but not half-integer values[12].

(b) Hypercharge Y

The discovery of hypercharge Y for strongly interacting particles is the outstanding post-war discovery of experimental hadron physics. Like Q and B, Y is associated with the algebra of group structure U(1) (eigen-

[11] These are estimates of Reines, Cowan and Goldhaber (see FEINBERG, G., GOLDHABER, M., Proc. natn. Acad. Sci. 45 (1959) 1301), who looked for possible proton decays in large hydrogenous scintillation counters C_7H_8.

[12] This empirical relation has led to a certain confusion of terminology. To clarify: All strongly interacting particles are called hadrons. Among these are half-integer (Fermi) as well as integer-spin (Bose) particles. Empirically all baryons interact strongly and are therefore hadrons. From the relation $J - \frac{1}{2}B$ = integer, clearly zero baryon charge B=0 implies integer (including zero) spin. Such particles are called mesons. A deuteron (J = 1, B = 2) is a hadron, a boson, a baryon, but not a meson. When a particle is called baryon without qualification, the normal implication is baryon charge B = 1 and (from $J - \frac{1}{2}B$ = integer) half-integer spin.

values $Y = 0, \pm 1, \pm 2, \ldots$). Unlike Q and B, particles of large Y value have not been reported so far; the largest firmly-established value of Y is -2 for Ω^-. Whereas Q and B are (to all intents and purposes) absolutely conserved, with Y we enter the domain of partially-conserved charges $\dot{Y} \neq 0$. A K^0-meson with $Y = +1$ decays in 10^{-10} sec into a pair of π-mesons with $Y = 0$, i.e. a unit of hypercharge disappears (into the vacuum) in 10^{-10} sec. Since the characteristic times involved in strong interactions are of the order of 10^{-23} to 10^{-24} sec ($\Delta E \approx$ several hundred mega electron volts), it is clear that this relatively slow hypercharge violation[13] ($\tau \sim 10^{-10}$ sec) is irrelevant (to one part in a trillion) in strong interaction physics.[14]

(c) Isotopic charges

The concept of the three isotopic charges I_i (which generate, like the spin-operators J_i, the algebra SU(2)) has been familiar since the 1930s:

$$[I_i(t), I_j(t)] = i\epsilon_{ijk} I_k(t)$$

(i) Like spin, multiplets of isotopic charge are labelled with two numbers:

$$|\underset{\sim}{I}| = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$$

$$I_3 = 0, \pm\frac{1}{2}, \left.\begin{matrix}\pm 1\\ 0\end{matrix}\right\}, \begin{matrix}\pm\frac{3}{2}\\ \pm\frac{1}{2}\end{matrix}, \ldots$$

with dimensionalities $\underset{\sim}{1}, \underset{\sim}{2}, \underset{\sim}{3}, \underset{\sim}{4}, \ldots$

(ii) Like hypercharge, isotopic charge is not conserved

$$\dot{\underset{\sim}{I}} \neq 0$$

if forces other than the strong forces are taken into account. As is well known from nuclear physics, the isotopic symmetry is broken by electromagnetism. Thus

$$\frac{d}{dt}|\underset{\sim}{I}| \propto g^2_{E.M.}$$

---

[13] The lack of hypercharge conservation proceeds from that part of the Hamiltonian which corresponds to weak forces, $\dot{Y} \propto g^2_{weak}$. This is qualitatively clear since the ratio of hypercharge-violating decay times ($K^0 \to 2\pi$) to the hypercharge-conserving decays ($\rho^0 \to 2\pi$) is (apart from phase space factors) $|g^2_{weak}/g^2_{strong}|^2 \sim 10^{-11}$.

[14] This is perhaps an appropriate place to make the essentially semantic distinction between semi-stable particles and resonances. When a hadron decays through E.M. or weak forces (like $K^0 \to 2\pi$), we have called it a (semi-stable) particle, the word resonance being reserved for hadrons decaying through strong forces themselves (like $\rho \to 2\pi$).

A quantitative measure of $|\underset{\sim}{I}|$ non-conservation is the mass difference among members of the same I-multiplet; typically for I = 1

	Mass (MeV)	I_3
Σ^+	1189	1
Σ^0	1192	0
Σ^-	1197	- 1

so that $\Delta m < 1$ part in 10^2 $\left(\frac{\Delta m}{m} \sim \frac{g^2_{E.M.}}{g^2_{strong}}\right)$.

(iii) For hadron physics there exists the empirical relation[15] which expresses electric charge Q as a linear sum of hypercharge and one of the isotopic charges

$$Q = I_3 + \tfrac{1}{2} Y$$

This is the first example of charges shared between two different types of interaction, strong and E.M.

(d) Unitary charges

A remarkable synthesis of isotopic charges and hypercharge was achieved when it was recognized, between 1959 and 1961, that these four charges were part of a larger set of eight, the so-called unitary charges F_i (i = 1, 2, 3, ..., 8) which close on the algebra of the SU(3) group

$$[F_i(t), F_j(t)] = i f_{ijk} F_k(t)$$

provided one identifies F_1, F_2, F_3 with I_1, I_2, I_3 and Y with $2/\sqrt{3}\, F_8$. The charge-sharing relation for hadrons, mentioned above, now reads

$$Q = F_3 + \frac{1}{\sqrt{3}} F_8$$

The multiplets of unitary charge correspond to the representations of the SU(3) algebra; these typically possess dimensionalities $\underset{\sim}{1}$, $\underset{\sim}{3}$, $\underset{\sim}{\bar{3}}$, $\underset{\sim}{8}$, $\underset{\sim}{10}$, $\underset{\sim}{27}$, $\underset{\sim}{35}$, ...

Since the algebras of the hypercharge ($U_Y(1)$) and isotopic charge ($SU_I(2)$) are contained in the algebra of SU(3), each SU(3) multiplet contains in a specified manner a number of $U_Y(1)$ and $SU_I(2)$ multiplets. Listed below are some of the known $\underset{\sim}{8}$'s and $\underset{\sim}{10}$'s of SU(3) - the existence[16] of the latter being splendidly confirmed by the discovery of the Ω^-. The fundamental triplet representations $\underset{\sim}{3}$ and $\underset{\sim}{\bar{3}}$ of SU(3) from which, according to standard group theory, all other representations can be made by repeated

---

[15] In all such numerical sharing relations, and throughout this lecture, the charges are specified after scaling in their "natural" units.

[16] A puzzle on a par with the puzzle of non-existence of particles with $|Y| > 2$ is the non-appearance in particle spectrum of higher SU(3) multiplets $\underset{\sim}{27}$, $\underset{\sim}{35}$, ...

TABLE III. ISOTOPIC, HYPERCHARGE AND CHARGE CONTENT OF QUARKS AND ANTIQUARKS

		I_3	Y	$Q = I_3 + \frac{1}{2}Y$
Quark	$\underset{\sim}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$
		$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$
		0	$-\frac{2}{3}$	$-\frac{1}{3}$
Antiquark	$\underset{\sim}{\bar{3}}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$
		$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$
		0	$\frac{2}{3}$	$\frac{1}{3}$

TABLE IV. SUMMARY OF ROSENFELD'S TABLE FOR HADRONS

Eight-folds					Ten-fold	
Mesons B = 0				Baryons B=1	B = 1	
J^P	0^-	1^-	1^+	$\frac{1}{2}^+$	$\frac{3}{2}^+$	Mass (MeV)
I = 1Y = 0	π^+, π^0, π^-	$\rho^+\rho^0\rho^-$	A_1^+, A_1^0, A_1^-	$\Sigma^+\Sigma^0\Sigma^-$	$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$	1236
$I = \frac{1}{2}Y = 1$	K^+K^0	$K^{*+}K^{*0}$	K_A^+, K_A^0	pn	$Y_1^+Y_1^0Y_1^-$	1385
$I = \frac{1}{2}Y = -1$	$\bar{K}^0K^-$	$\bar{K}^{*0}K^{*-}$	$\bar{K}_A^0, K_A^-$	$\Xi^0\Xi^-$	$\Xi^{*0}\Xi^{*-}$	1530
I = 0Y = 0	η^0	ϕ	D	Λ	Ω^-	1672

Electric charges indicated as superscripts; J^P = spin^{parity}.

multiplications[17], have been named quark and antiquark representations. Their isotopic, hypercharge and charge content (based on the empirical formula $Q = I_3 + \frac{1}{2}Y$) would be as shown in Table III.

Note the fractional charges one may expect quarks and antiquarks to possess; no physical particles with these charges are known to exist to date. Table IV summarizes some of the better-known hadrons in Rosenfeld's table.

The well-identified SU(3) multiplets with their spins J and parities P are:

(i) boson octets $J^P = 0^-, 1^-, 1^+, 2^+$
(ii) baryon octet $J^P = \frac{1}{2}^+$
(iii) baryon 10-fold $J^P = \frac{3}{2}^+$

Figure 1 is an important plot of still higher spin particles, some already identified, others conjectured on the assumption that all hadrons form SU(3) multiplets of $\underset{\sim}{8}$'s and $\underset{\sim}{10}$'s (octets and decuplets). The plots give spin versus (mass)^2. Notice the important empirical result: these plots (known

[17] E.g. $\underset{\sim}{3} \times \underset{\sim}{\bar{3}} = \underset{\sim}{8} + \underset{\sim}{1}$

$\underset{\sim}{3} \times \underset{\sim}{3} \times \underset{\sim}{3} = \underset{\sim}{1} + \underset{\sim}{8} + \underset{\sim}{8} + \underset{\sim}{10}$.

as Regge trajectory plots) are essentially straight lines and are continually rising. One of the most important unknowns in hadron physics is how far in the mass scale may one expect the rising of these trajectories, how high values of spin J are obtained in the particle spectrum; is there an "ionization limit" as there indeed is if we make a like Bohr plot of J versus M^2 for the hydrogen atom levels.

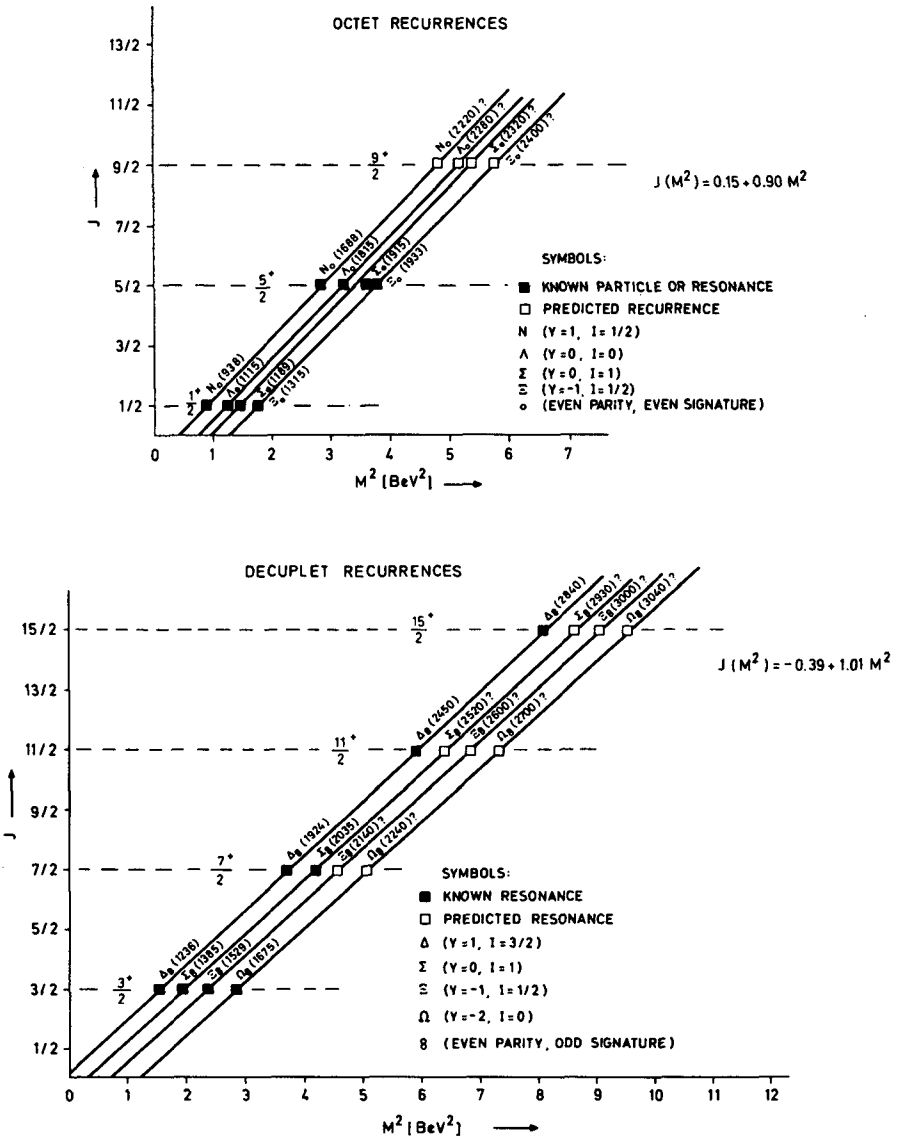

FIG. 1. Regge trajectory assignments of known particles in 8's and 10's of SU(3).

Returning to unitary symmetry, SU(3) (like the isotopic SU(2)) is a broken symmetry; unitary charge is only partially conserved. A measure of dF/dt is provided by the mass difference among members of the same multiplet, e.g. Table IV lists the mass differences of successive members of the 10-fold. There is a (nearly constant) mass increase[18] of ~ 145 MeV (equal-spacing rule) when we go from $Y = 1$ to $Y = -2$. A rough estimate of dF/dt is provided by $\Delta m/m \sim 1/10$, this rather large number giving a measure of the (medium-strong) coupling strength of SU(3)-breaking forces relative to the strongest SU(3)-symmetric forces. The nature of these symmetry breaking forces (unlike the case of isotopic symmetry where we know that the symmetry is broken by electromagnetism) is one of the important unresolved problems.

To summarize, we recognize a succession - a hierarchy - of strong charges and associated symmetries; SU(3) unitary charges, SU(2) isotopic charges, hypercharge and (in a category by itself) baryon charge. If all but the strongest forces in nature are neglected, these charges are fully conserved. Their lack of exact conservation is a reflection of the existence of medium-strong, electromagnetic and weak forces, respectively.

(e) Left and right unitary charges

Even if SU(3) triplets (quarks) do not exist, in so far as they constitute the fundamental representation of SU(3), all other representations, as stated before, can be made as mathematical composites from them.

To construct fermions as well as bosons as composites, quarks must carry spin $\frac{1}{2}$. This circumstance makes it possible to extend the notion of unitary charge in a most fruitful direction. A spin-$\frac{1}{2}$ particle has two polarization directions for a particle in motion, spin along and opposite to its direction of motion, the so-called left spin and right spin. For a particle of rest mass zero (particles travelling at the speed of light) these two polarization states are completely independent states[19]. There is no rest frame for a massless particle in which a rotation may transform right-spinning particles to left-spinning ones. There are thus not just three, but six zero-mass quarks, three spinning right, three spinning left. Corresponding to each spin polarization, there are two distinct types of unitary charges defining two independent algebras, $SU(3)_{left}$ and $SU(3)_{right}$. The symmetry represented by $SU(3)_L \times SU(3)_R$ is of course a badly broken one, because it holds only in the idealized limit of zero mass for the fundamental representation (the quark). We shall see later that this symmetry (also called chiral symmetry) is not a good kinematical symmetry for classifying

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[18] We shall not have time here to consider this SU(3) symmetry breaking medium-strong force in any detail. An important clue towards its group-theoretic specification is provided by the equal-spacing rule for masses mentioned above, which is most simply explained if we assume that the part of the Hamiltonian breaking SU(3) transforms like hypercharge. A certain amount of group theory is involved in deriving this result which is not immediate.

[19] The archetypal example is the neutrino (see under leptons). Left (spinning) neutrinos exist; right (spinning) neutrinos do not. This (two-component) aspect of the neutrino gave rise to the concept of left and right (chiral) charges.

particle states; surprisingly, however, it turns out to be an excellent symmetry in its dynamical aspects, at least for low-frequency phenomena (Part 3).

(f) Nature of the internal symmetries ($U_Q(1)$, $U_B(1)$, $U_Y(1)$, $SU_I(2)$, $SU_F(3)$)

The spin quantum numbers have their origin in Poincaré-Lorentz symmetry of space and time. The other charges Q, B, Y, $\vec{I}$, F, in so far as they are dissociated from space-time structure, presumably represent, in some sense, symmetries associated with internal degrees of freedom. But are these charges really that dissociated from space-time? I shall come back to this problem in Part 2.5.

Table V gives a summary of the classifying charges for hadrons.

TABLE V. SUMMARY OF CLASSIFYING CHARGES FOR HADRONS

Charges	Algebra	Typical multiplets	
Electric Q	U(1)		$\dot{Q} = 0$
Baryonic B	U(1)		$\dot{B} = 0$
Hypercharge Y	U(1)		$\dot{Y} \propto g^2_{weak}$
Isotopic $\underset{\sim}{I}$	SU(2)	$\underset{\sim}{1}, \underset{\sim}{2}, \underset{\sim}{3}, \ldots$	$\dot{I} \propto g^2_{electromagnetic}$
Unitary F^i	SU(3)	$\underset{\sim}{3}, \underset{\sim}{8}, \underset{\sim}{10}, \ldots$	$\dot{F} \propto g^2_{medium\text{-}strong}$
Left and right unitary	$SU_L(3) \times SU_R(3)$	$(3, \bar{3}) + (\bar{3}, 3), \ldots$	
	Sharing of charges $Q = I_3 + \frac{1}{2}Y = F^3 + \frac{1}{\sqrt{3}} F^8$		

1.2.3. Leptons

So far we have dealt with hadrons. Consider now leptons, i.e. particles without strong interactions. The known lepton spectrum[20] is meagre in contrast to the richness of the hadron spectrum. There appear to be but four leptons differentiated from each other by two types of charge. These are:

(a) μ^-, ν_μ,
(b) e^-, ν_e.

Both ν_μ and ν_e are left-spinning objects; μ^-, ν_μ each carry unit muonic charge; e^-, ν_e each carry unit "electronic" charge. Both these leptonic varieties of charge are (individually) conserved. Leptons exhibit E.M.,

[20] Recent systematic search at Stanford has failed to reveal any other leptons with masses less than a GeV with fairly low production cross-sections (BARNA, A. et al., Phys. Rev. Lett. 18 (1967) 360).

weak and gravitational interactions. One of the not understood empirical facts is the remarkable identity of muonic and electronic interactions - roughly speaking, the equality of the two types of leptonic charges - notwithstanding the different masses[21] of these particles. This is somewhat analogous to the surprising numerical equality of the electron's and proton's electric charges mentioned earlier.

1.2.4. Weak charges

Both hadrons and leptons interact weakly. In analogy to strong charges, there are weak charges, defined more precisely in Part 3, where we shall see that (apart from scale) these are related, for leptons, to the (left) leptonic charges introduced above and, for hadrons, to certain combinations of $SU_L(3)$ charges.

1.3. DISTINGUISHED PARTICLES AND ELEMENTARITY

It has been remarked that all hadrons could be considered as (mathematical) composites of three quarks; together with the four leptons above, one could say that this set of seven objects - the last remnants of the elementarity idea - constitutes in some sense a distinguished set (of particles or fields) from which all other particles could be made.

If one wished to substitute for the unobserved quarks a set of particles observed physically, numerous other choices are possible. One economical set would be the Sakata set p, n, Λ - but the SU(3) symmetry would be harder to build in with this as the basic set. To build in SU(3) credibly, the best hadron set is still the familiar old favourite, the octet of baryons N, Λ, Σ, Ξ, of which the nucleon is a member. This, however, would be only one of the many choices possible.

The arbitrariness of such a choice brings us back full cycle to the dilemma of elementarity. The historical tradition of physics - and the view found profitable when dealing with leptons - lies along an identification of a distinguished - an aristocratic - set of particles (as few in number as possible) of which all others are made. The other viewpoint could be that there is no distinguished set at all, at least not for hadrons, that there is full democracy in hadron physics and that it is more profitable to consider all hadrons as composites of each other. We shall consider these two contrasting views of elementarity - aristocratic versus democratic - in more detail in Part 2.

PART 2. RELATIVISTIC QUANTUM MECHANICS

In this part we discuss the structure of relativistic quantum mechanics (RQM). The ideas I shall be wrestling with are some of the prettiest, also some of the profoundest, in the whole range of physics. I have tried very

[21] The large muonic mass (nearly as large as that of the pion) has always led to the suspicion that muons may possess strong charges as well as weak ones.

hard to achieve a clear exposition but the sheer number of concepts makes the task nearly impossible. Perhaps an initial listing here of the topics may help:

Relativistic kinematics;
Ingredients of two approaches to RQM, field theory and S-matrix theory;
Accepted canons of RQM as abstracted from the two approaches;
Some far-reaching consequences of the canons;
Attempts to reproduce the hadron spectrum within the framework of RQM.

2.1. KINEMATICAL CONSIDERATIONS

In Part 1 and Appendix A we have introduced the personae of the cast, the particles and resonances. All members of this cast (except protons, neutrons and electrons of which normal matter is composed) were created and discovered in cosmic rays or accelerator-beam collisions. Appendix B is a note which Dr. G.H. Stafford of the Rutherford Laboratory, Harwell, has kindly prepared for me; this lists typical beams and beam intensities of the present generation of accelerators. Since collision experiments using these beams are our sole systematic means of the study of semi-stable particles, resonances, their masses, their decay widths, as well as the details of the interaction processes, it is important to familiarize oneself with the kinematical and theoretical constructs employed. This is perhaps the dullest part of the lecture; I need, however, the notation we introduce here. Anyone familiar with it may read just the summary at the end of the section and pass on.

2.1.1. The masses of semi-stable particles and resonances

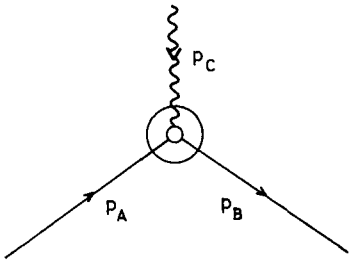
One of the major - and in contrast to non-relativistic quantum mechanics - essentially unsolved problems of particle theory is the prediction of the particle spectrum - the masses, spins, parities and other charges of semi-stable particles and resonances. The problem has been attacked both from the "aristocratic" and "democratic" points of view in Part 1.3[22] and we return to it in 2.2.

2.1.2. Decay widths, coupling constants and form factors

The second task of the theory is to give a description of decay widths. The kinematical tool for two-body decays is the so-called three-point

[22] Internal symmetries discussed in Part 1 simplify the problem to the extent that if one member of a multiplet is discovered, and if we have some idea of the symmetry breaking forces, we can make a reasonable guess at the masses of the other members of the multiplet.

function or the vertex function $F(p_A, p_B, p_C)$



Denote the four momenta of the three objects A, B, C involved as p_A, p_B, p_C. From energy momentum conservation $p_B = p_A + p_C$. The general vertex function is a function of the spins and momenta of the three objects A, B, C which interact. From Lorentz-invariance it can be written in the form of a product of spin factors X^i and the so-called invariant factors F^i. The invariant functions (1) contain no spin, and (2) are functions of the Lorentz-scalars which can be formed from the momenta p_A, p_B, p_C. The spin factors X^i are kinematical objects; the dynamics is expressed by the functions F^i.

$$V = \sum_i X^i \text{ (spins) } F^i (p_A^2, p_B^2, p_C^2)$$

Consider the following special cases:

(i) A, B, C represent three stable physical particles of masses m_A, m_B, m_C

$$(p_A^2 = m_A^2 \quad p_B^2 = m_B^2 \quad p_C^2 = m_C^2)$$

The limiting values $F^i(m_A^2, m_B^2, m_C^2)$ of the invariant function F^i are known as the coupling constants.

(ii) If $m_A^2 > (m_B + m_C)^2$, the particle A is unstable. The vertex function in this case directly gives the decay amplitude[23] $A \to B + C$.

---

[23] If A, B and C all belong to SU(2) (or SU(3)) multiplets, it is clear that the amplitude above is related by simple group theoretic Clebsch-Gordan factors to an amplitude $A' \to B' + C'$, where primed objects are other members of the multiplets. It is the experimental verification of this type of kinematic relation which, in general, gives the empirical SU(2) (or SU(3)) assignments of particles.

(iii) When A and B are the same particle, $p_A^2 = p_B^2 = m^2$ and C is a (virtual) photon (i.e. electromagnetic field frequency), the F^i's are functions of just one variable, the momentum transfer $t = (p_C)^2 = (-p_A + p_B)^2$, and give the so-called electromagnetic form factors[24].

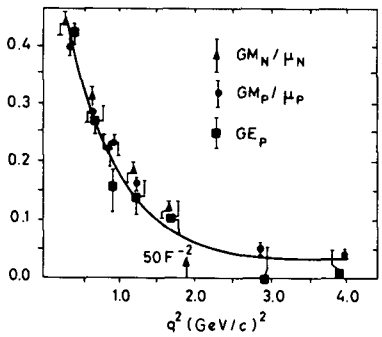


FIG.2. Electric and magnetic form factors for the proton and for the neutron.

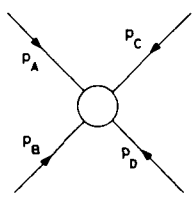


FIG.3. Diagrammatic representation of a four-point function.

2.1.3. The scattering amplitude

The kinematical construct here is the so-called four-point function (Fig. 3). Consider four interacting objects with momenta p_A, p_B, p_C, p_D. From energy-momentum conservation

$$p_A + p_B + p_C + p_D = 0$$

[24] The Fourier transforms of these functions in configuration space represent the spatial extension of charge, magnetic and other E.M. moment densities, for the particle A. One of the most beautiful of recent experimental results (Fig. 2) is that apart from a scale change, the magnetic and electric charge form factors of the proton and the magnetic form factor of the neutron - and thus the spatial charge and magnetic moment densities - are identical when plotted as functions of momentum transfer up to $|(p_A - p_B)^2|$ around $(4\,\mathrm{GeV})^2$. We shall see later (Part 4) how this identity of charge and magnetic densities finds its readiest explanation in terms of dynamical symmetries (like $\tilde{U}(12)$) higher still in hierarchy than the ones considered in Part 1.

Like the vertex function, the *four-point function* $T(p_A, p_B, p_C, p_D)$ can be written in the form

$$T(p_A, \ldots) = \sum_i \chi^i \text{ (spins) } F^i(p_A^2, p_B^2, p_C^2, p_D^2, s, t)$$

where

$$s = (p_A + p_B)^2$$

$$t = (p_A + p_C)^2$$

The F^i's are the invariant functions of the six independent Lorentz scalars which can be formed from the momentum vectors p_A, p_B, p_C, p_D (of which but three are independent, since $p_D = -(p_A + p_B + p_C)$).

We shall mainly - though not exclusively - be interested in the situations where the four objects are physical particles on their mass shells, i.e. when $p_A^2 = m_A^2$, $p_B^2 = m_B^2$, $p_C^2 = m_C^2$, $p_D^2 = m_D^2$. Consider the following special cases, all represented by Fig. 3:

(a) Three-body decays

If $p_A^2 = m_A^2 > (m_B + m_C + m_D)^2$, A is unstable and the amplitude represents its three-body decay $A \to B + C + D$.

(b) Scattering

Channel I. Let momenta p_A and p_B be incoming; p_C and p_D outgoing. The variables s and t on which $F^i(s, t)$ depend are well-known physical variables[25]; thus

$$s = (p_A + p_B)^2 = E_c^2 = 4(k_c^2 + m^2)$$

$$t = (p_A + p_B)^2 = -2k_c^2(1 - \cos\theta_c)$$

i.e. s equals the square of c.m. energy (E_c) and t is the momentum transfer. It is important to note that scattering occurs only when in the (s, t) plane

$$s \geq (m_A + m_B)^2 > 0$$

and

$$t \leq 0$$

Channel II. Figure 3 could represent a second physical situation, where p_A and p_C are incoming momenta and p_B and p_D outgoing. In this case

$$(\text{c.m. energy})^2 = t = (p_A + p_C)^2 \geq (m_A + m_C)^2 > 0$$

$$\text{momentum transfer} = s \leq 0$$

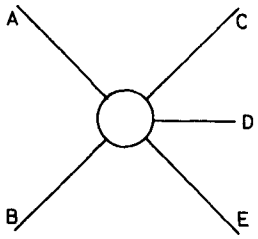
The roles of s and t are reversed for the two channels[26].

---

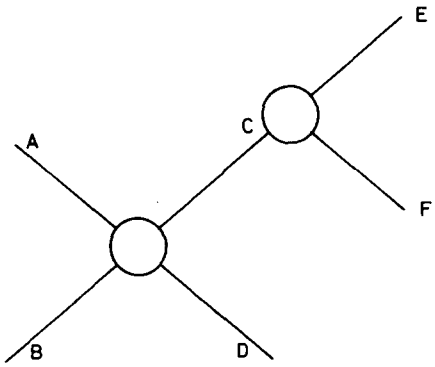
[25] Assume for simplicity all particles have equal masses ($m_A = m_B = m_C = m_D$).

[26] Note that Channel I and Channel II can be distinguished either by specifying which particles are incoming and which outgoing, or by specifying the regions in the (s, t) plane where $s > 0$ and $t < 0$ and vice-versa.

One could go on to the five-point and higher functions



which represent production processes. In the study of these processes one has tended to make a two-stage approximation. For example, most work on the five-point function assumes that it is dominated by the sequence of processes:



$$A + B \rightarrow C + D$$
$$\hookrightarrow E + F$$

Here C is an intermediate resonance which is first formed, decaying subsequently $C \rightarrow E + F$. This approximation has been surprisingly successful in analysis of data.

Summarizing: The theoretical problems investigated at present in particle physics are the following:

(1) The mass spectrum of the particles, their spins, parities and internal charges - a problem towards whose theoretical understanding we have had the least success.

(2) A coherent description of three-body coupling constants, decay constants and momentum transfer dependence of form factors.

(3) The variation of scattering amplitudes $F^i(s, t)$ as functions of energy s and momentum transfer t in Channel I (with the roles of s and t reversed in Channel II). By and large we have been more successful in describing decay constants and the behaviour of scattering amplitudes than that of particle spectra.

2.2. INGREDIENTS OF THE THEORY[27]

Historically, relativistic quantum mechanics (RQM) was more or less contemporaneous with the epic days of Einstein's field theory of gravitation, which itself had before it the great model of another field theory - the electrodynamics of Maxwell and Faraday. Inevitably, the first realization of RQM was carried through in terms of a local field concept, with the following ingredients:

2.2.1. Local fields

Associate with each particle a local[28] field operator $A(x)$ defined at all space-time points x; the electromagnetic fields $\vec{E}(x)$ and $\vec{H}(x)$ are the archetypal examples.

2.2.2. Elementarity versus compositeness

If A is composite of B and C, the field $A(x)$ equals a polynomial product of $B(x)$, $C(x)$ and their derivates.

2.2.3. Scattering amplitudes

The amplitude for scattering of $A + B \to C + D$, and in particular the invariant functions $F^i(s, t)$[29] which describe scattering (see 2.1), can be written in terms of matrix elements of products of field operators associated with A, B, etc. and their derivatives.

It is indeed no exaggeration to say that the local relativistic field concept is one of the most fruitful concepts invented by man. It is not just that the concept is an ideal vehicle for implementing RQM; it would be truer

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[27] This section describes rather complicated ideas, perhaps not too well. The harassed reader may pass directly on to 2.3.

[28] By locality we mean the commutation relation postulate that

$$[A(x),\ A(y)] = 0 \qquad \text{(C.R.1)}$$

whenever x and y are space-like to each other. This relation - called the causality relation - guarantees, in a vague manner, that field influences do not propagate with velocities greater than light. If ever a fundamental length ℓ needs to be introduced, the best place for it may possibly be through a modification of (C.R.1), for example, to a form

$$[A(x),\ A(y)] = 0$$

when $(x - y)^2 \leqslant \ell^2$. The fact that $\ell = 0$ in (C.R.1) is (vaguely) an indication of no fundamental length in RQM.

[29] What are the consequences of locality? First, as we noted in Part 1, locality plus real Lorentz symmetry imply complex Lorentz symmetry and, in particular, CTP symmetry. Thus CTP operation and particle-antiparticle duality is the first gift of locality. Second, one can show that the causality postulate

$$[A(x),\ B(y)] = 0, \quad (x - y)^2 < 0$$

is powerful enough to guarantee that the amplitude for a scattering process - a function like $F^i(s, t)$ - is an analytic function of complex s and t, in a certain domain of s and t. This is an astonishingly powerful result. What this domain in (s, t) space is, we shall discuss later. The important feature to note here is that no detailed dynamics, no precise law of force, no compositeness or structure relation like $(C(x) = A(x)\,B(x))$, etc. has gone into the deduction of analyticity of $F^i(s, t)$.

to say that the concept, once formulated, came to acquire a life of its own; it led inexorably to the creation of a canon of RQM. Years later, attempts were made by a process of abstraction to state these canons independently of the local field concept. The emphasis shifted to the quantities of direct physical interest - the elements of the scattering matrix themselves - the invariant functions $F^i(s,t)$ previously mentioned. Something had then to be substituted for the locality postulate of the field idea. Since the two major deductions from the locality postulate were (1) CTP-symmetry, and (2) analyticity of $F^i(s,t)$ (see footnote 29), it was only natural to propose that these two (in a still stronger version) may be elevated to the status of basic canons of RQM rather than enter through the back door via the field-theoretic version of RQM. This is the second - the so-called scattering-matrix (S-matrix) - approach to relativistic quantum mechanics.

To summarize, then, there are at present two theoretical constructs - two methodologies - embodying RQM: (1) the historic method of local field theory, and (2) the (maximally) analytic S-matrix method[30]. We now describe these approaches in somewhat greater detail.

A. Field theories

There are two distinct classes of field theories corresponding to the two notions of elementarity - aristocratic (Class I) or democratic (Class II) - as discussed in Part 1.

Class I - theories of distinguished fields

Accept from the outset the existence of a set of distinguished fields of which all other fields can be constructed compositively. An example is quantum electrodynamics of leptons and photons:

(a) To each lepton and to the photon, a separate field is assigned, for example the Dirac field $\psi(x)$ for electrons and the Maxwell field $A_\mu(x)$ for photons.

(b) A distinguished form of Hamiltonian[31] is written down; its interaction part equals

$$e j_\mu(x) \quad A_\mu(x)$$

where $j_\mu(x) = i\bar{\psi}(x)\gamma_\mu\psi(x)$ is the electron current. This Hamiltonian is distinguished, not just because Maxwell postulated it, but, more perhaps,

[30] Whereas field theory patiently deduces what the analyticity domain of $F^i(s,t)$ should be in any given configuration of scattering particles using (C.R.1) as its main tool, the S-matrix approach starts with the postulate that $F^i(s,t)$ is as analytic as it possibly can be, with singularities only at those values of s and t which can be associated with the physical particle spectrum. We shall define this maximal analyticity concept more precisely later; note here the amazing circumstance that the postulated domains of the S-matrix approach have found confirmation - if anything - from the detailed (and very involved) calculations within the field-theoretic framework, at least on the so-called physical Riemann sheet of the complex (s,t) surface. When differences - so far minor - do arise, one may take one's choice if to believe in the firm and connected - though sometimes heavy - logical development from field theory or in the attractive simplicity of what one may consider is reasonable.

[31] The Hamiltonian completely determines the field equations and equal-time commutation relations of fields. As is well known, matrix elements of (appropriate) products of field operators define Green's functions of the theory, while operator field equations provide infinite sets of interconnecting relations among them.

since it satisfies a number of additional dynamical constraints (gauge invariance) with important physical significance. The current $j_\mu(x)$ is conserved $((\partial/\partial x_\mu)\, j_\mu(x) = 0)$; e is the coupling strength[32].

This is the prototype of all Class I theories. Though the one "bound state" in this theory, positronium, may be represented by a separate field, there is no need to postulate field equations for it. At any rate, the field equations for the distinguished electron and photon fields form a closed set; they do not contain any piece depending on the positronium field. Clearly this theory was specially favoured for the following reasons:
(i) the number of particles which the theory describes is small; and
(ii) the theory inherited from Maxwell a distinguished nearly unique Hamiltonian with a small coupling parameter e.

In hadron physics with its multitudes of particles, a repetition of any of these fortunate circumstances could not be expected to recur. Field theories of Class II are thus the more appropriate.

Class II theories

(a) Associate with each particle a local field operator and assume that all particles are composites of each other.

(b) To proceed with detailed dynamics we need field equations. By the compositeness assumption, every particle interacts locally with every other. It is thus profitless to try to conjecture unique (infinite) sets of equations of motion; there is just too much arbitrariness.

(c) A part of the dynamics, however, can be specified by postulating specific equal-time commutation relations[33] (C.R.2) of the form:

$$[A(\vec{x}, t), B(\vec{x}', t)] = C(\vec{x}, t)\, \delta(\vec{x} - \vec{x}') \qquad \text{(C.R.2)}$$

For several classes of operators A and B, and in particular when A and B represent current operators associated with internal symmetry groups of Part 1, plausible conjectures are possible from the structure of the symmetry algebras; we shall study examples in Part 3.[34]

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[32] As is well known, the smallness of the coupling parameter e allows for a complete perturbation solution (the Feynman solution). As is also well known, some of the integrals in the solution turn out to be infinite - of the form $\int^{\infty} dx/x$. The famous renormalization theorem of Dyson demonstrated that these infinities affect only self-mass and self-charge (making it impossible, within the theory, to compute m_0/m and e_0/e - the ratios of "bare" to "physical" constants).

[33] Equal-time commutation relations between a field and its time derivative

$$[\phi(x,t), \dot{\phi}(\vec{x}', t)] = i\,\delta(\vec{x} - \vec{x}') \qquad \text{(C.R.3)}$$

(analogous to the relation $[q(t), p(t)] = i$ between the position operator $q(t)$ and its conjugate momentum $p(t)$) are special cases of (C.R.2). In a theory where distinguished sets of fields exist, and one knows the composition law, a single relation like (C.R.3) would suffice to give the entire set of relations (C.R.2).

[34] A commutation relation through its very structure cries out for saturation procedures to be applied. By this we mean the following: If $|n\rangle$ denotes a quantum mechanical state, the completeness relation asserts that $\sum_n |n\rangle\langle n| = 1$. A commutation relation like $[A, B] = C$ on saturation gives rise to the sum rule:

$$(\alpha|C|\beta) = \sum_n [(\alpha|A|n)(n|B|\beta) - (\alpha|B|n)(n|A|\beta)]$$

It is this class of sum rules obtained from Class II field theories which we shall study mainly in Part 3 for cases when A, B and C are charge operators corresponding to the internal symmetries considered in Part 1. The important point to emphasize in such cases is that equal-time commutation relations and presumably the sum rules obtained from them are exact statements even when the symmetry is broken.

B. Summary

Table VI summarizes the theoretical approaches.

TABLE VI. SUMMARY OF THEORETICAL METHODS

A. FIELD THEORETICAL APPROACHES

Class I. Assume the existence of a distinguished set of fields of which other fields are made compositely. Assume a distinguished form of Hamiltonian for these special fields. A complete perturbation solution to the scattering matrix may be constructed if the coupling parameter is small; the only undetermined parameters in the theory are the masses and coupling constants of the distinguished fields. All other parameters, like masses of bound states, if any, are in principle obtainable from a (non-perturbative) expression for the scattering matrix.

Class II. Associate each particle democratically with a local field. If we assume that all particles are composites of each other, no local field equations can be written with profit. The locality postulate, however, determines the analytic structure of the scattering amplitudes. A part of the dynamics is specified by postulating equal-time commutation relations, which in turn give rise to testable sum rules.

B. THE ANALYTIC S-MATRIX METHOD

Give up the field concept. Postulate maximum analyticity of scattering amplitudes in place of locality of fields. Assume that any singularities of the scattering matrix are intimately related to and determined by the spectrum of the physically observed particles. This democratic approach to all particles is more akin in spirit to field theories of Class II.

2.3. POSTULATED CANONS OF RELATIVISTIC QUANTUM MECHANICS

In this section we set down what one has come to accept as the canons of relativistic quantum mechanics (see Table VII). As stated earlier, some of these were inexorable consequences of a local field theoretic formulation of RQM; others were postulated more readily and plausibly from the logic of the S-matrix approach. The most astonishing aspect of these latter postulates has been that in no important case has a deeper study from the field theoretic method uncovered situations where they are contradicted.

2.3.1. Unitarity of the S-matrix; or the law of conservation of probabilities

This basic law - a non-linear statement -

$$\sum_n (a|S|n)(n|S^\dagger|b) = \delta_{ab}$$

constitutes one of the most powerful constraints[35] on physical theory that we know of. In hadron physics, if there is one principle which it is criminal to approximate too drastically, it is this. One of the most useful

[35] In electrodynamics, in a perturbation expansion, it becomes essentially a linear relationship and considerably loses its restraining power.

TABLE VII. CANONS OF RELATIVISTIC QUANTUM MECHANICS

1. Crossing

incoming particle $\xrightarrow{CTP}$ outgoing antiparticle

I. $A+B \rightarrow C+D$

II. $A+\bar{C} \rightarrow \bar{B}+D$

III. $A+\bar{D} \rightarrow \bar{B}+C$ $\Longrightarrow$ Reactions I, II, III described by the same master function $F(s,t)$

2. Unitarity $\Longrightarrow$ optical theorem

$$\mathrm{Im}\, F(s,0) = k\sigma_{total}$$

3. Analyticity

of $F(s,t)$ for complex s and t, except for singularities (poles, branch points) determined by the physical particle spectrum

relations arising from unitarity is the so-called optical theorem connecting the total two-body collision cross-section to the imaginary part of the elastic forward scattering amplitude

$$\mathrm{Im}\, F(s,\ \theta = 0) = k_c \sigma_T$$

2.3.2. Crossing symmetry or substitution law

Recall that in Part 1.1. we stated that locality (of field theory) plus Lorentz invariance implies the existence of a CTP operation which converts an incoming particle to an outgoing antiparticle and vice-versa. Following and generalizing from this, the crossing property of RQM states that, given a reaction

$$A+B \rightarrow C+D$$

and the scattering amplitudes $F^i(s,t)$ for it, the same function $F^i(s,t)$ also describes the amplitude for the process where we interchange one of the incoming particles with an outgoing antiparticle. In detail, the same set of functions $F^i(s,t)$ describes both

$$A+B \rightarrow C+D$$

and

$$A+\bar{C} \rightarrow \bar{B}+D$$

where $\bar{C}$ and $\bar{B}$ are antiparticles of C and B, respectively. Likewise

$$A+\bar{D} \rightarrow \bar{B}+C$$

Note, as remarked earlier, that when we speak of the same functions $F^i(s,t)$ describing reactions in

Channel I $\qquad A + B \rightarrow C + D$

and

Channel II $\qquad A + \bar{C} \rightarrow \bar{B} + D$

we are considering different regions of the (s, t) plane, since for Channel I the physical region is $s > 0$, $t < 0$, while for Channel II, it is $s < 0$, $t > 0$. An analytic continuation must therefore be carried through before we can read off from the knowledge of F(s, t) for Channel I the values of the amplitude for Channel II. We consider this continuation further on; remark here, however, the economy brought about the crossing relations; there exists just one master function for all related channels. These relations give RQM a power that non-relativistic theory never possessed.

2.3.3. Singularities of the S-matrix (Table VIII)

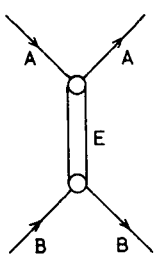
Before any analytic continuation from Channel I to Channel II can be carried out, we need to know the singularity structure of the S-matrix.

A. Poles

One of the basic postulates of S-matrix theory (shared by local field theory) is that the poles of the S-matrix correspond to physical particles exchanged in a reaction and conversely. There are two well-known types of poles:

(a) The Yukawa poles[36]. Poles in (complex) momentum-transfer (t)-plane; their contribution to the scattering matrix equals

$$\frac{g_{AAE}\, g_{BBE}}{t - m_E^2}$$



Here m_E is the mass of the Yukawa particle exchanged and g_{AAE} and g_{BBE} are the coupling constants at the two vertices.

[36] The Yukawa pole contributions (also called the Born terms in potential scattering) determine in configuration space the "potential" produced by the exchanged particle. Thus, every particle E that is exchanged, produces its share of "force" between the interacting particles A and B. The strength of this force depends, of course, on the coupling parameters g_{AAE} and g_{BBE} and its range on the mass m_E of the particle exchanged (the smaller the mass, the longer the range). In hadron physics, where every coupling constant is (nearly) equal to every other, the concept of a "fundamental force" produced by a "fundamental exchange" becomes nebulous. This is another way to restate the dilemma of BI(b').

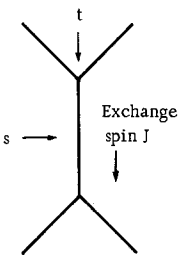
TABLE VIII. SINGULARITIES OF THE S-MATRIX

1. Yukawa poles

$$F(s,t) = g^2 \frac{P_J(\Theta_t)}{t - m_J^2}$$

$$\cos\Theta_t = 1 + \frac{2s}{t - 4m^2}$$

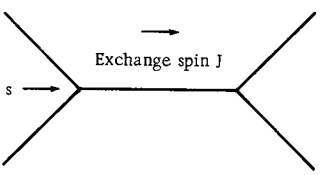
(c.m. energy)^2 = s
momentum transfer = t



2. Resonance poles

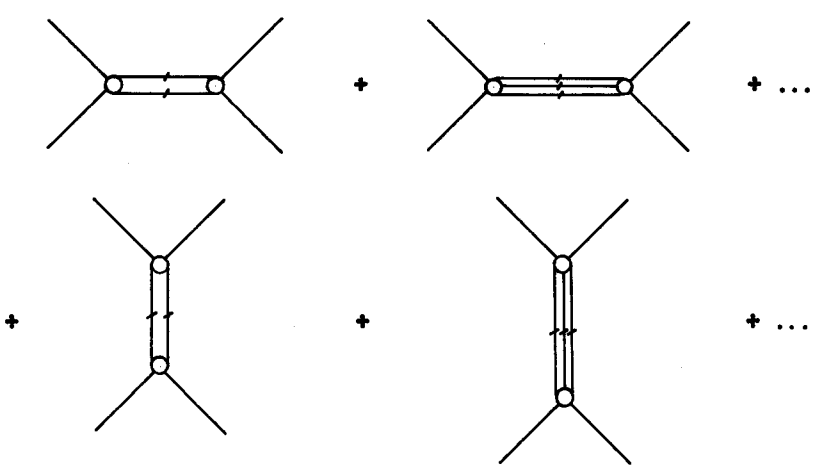
$$F(s,t) = g'^2 \frac{P_J(\Theta_s)}{s - m_J^2}$$

$$\cos\Theta_s = 1 + \frac{2t}{2 - 4m^2}$$



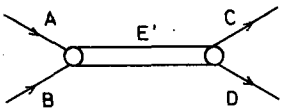
3. Branch points

at two-particle, three-particle, ... thresholds in all the relevant channels



(b) The Breit-Wigner poles. These are poles in complex energy (s)-plane

$$\frac{g_{AE'B}\ g_{E'CD}}{s - m_E^2}$$



(c) Case of spin. It is easy to show that if the exchanged particles E or E' carry spin, the pole contributions modify to the forms:

$$gg' \frac{P_J(\theta_t)}{s - m_E^2} \qquad \cos\theta_t = 1 + \frac{2s}{t - 4m^2}$$

$$gg' \frac{P_J(\theta_s)}{s - m_E^2} \qquad \cos\theta_s = 1 + \frac{2t}{s - 4m^2}$$

Note the crossing symmetry typified by interchange of s and t.

To determine the other singularities of the scattering matrix, unitarity proves to be the crucial tool.

B. Branch points

It is impossible in a general lecture like this to go over details of how the pole structure, together with the quadratic unitarity relation $SS^\dagger = 1$, forces the branch point singularity structure of the scattering amplitudes. This is covered in more detail by Professor Eden in these Proceedings. I shall simply state the postulated result from maximum analyticity for the four-point function (demonstrated to varying degrees of rigour using field theory). The scattering matrix possesses branch-point singularities at two-particle, three-particle, ... thresholds in all channels, the singularities lying along the real and positive s-axis in the s-plane, along the real and positive t-axis (for the channel where t is the energy) and likewise for the third channel. Note the elegance, as well as the simplicity, of this conjectured singularity structure so intimately related to physical particle spectrum.

2.4. DEDUCTIONS FROM CANONS OF RELATIVISTIC QUANTUM MECHANICS

In this section we list some of the important deductions that have been made from the basic principles and which provide the practical working tools of hadron theory.

2.4.1. Dispersion relations (consequences of the assumed analyticity of the S-matrix)

If one knows the singularity structure of the scattering amplitudes (in particular that the singularities lie along the real axes in complex s- and t-planes), one may write integral representations - the so-called dispersion

relations - connecting real and imaginary parts of the scattering amplitudes. For example, from Cauchy's theorem, infer

$$F(s, t) = \frac{1}{2\pi i} \int_C ds' \frac{F(s', t)}{s' - s - i\epsilon}$$

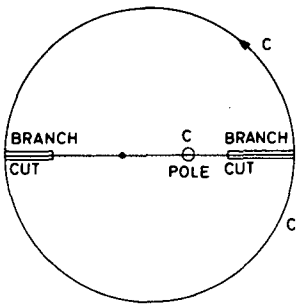


FIG. 4. Singularity structure of the scattering amplitude F(s, t) in the complex s-plane.

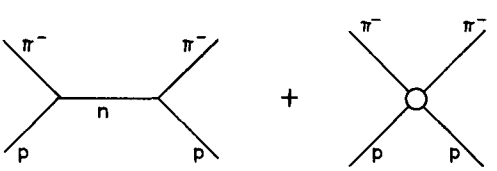
with the contour C as shown in Fig. 4. To evaluate the integral along the large circle, we need to know the behaviour of F(s, t) as $|s| \to \infty$. If F(s, t) falls sufficiently fast (and we consider this in more detail in the next subsection), we may expect the following to hold:

$$F(s, t) = \frac{1}{\pi} \int \left(\frac{F(s', t)}{s' - s + i\epsilon} - \frac{F(s', t)}{s' - s - i\epsilon} \right) ds'$$

plus pole term contribution. Rewrite in the Hilbert form:

$$\text{Re } F(s, t) = \frac{g^2}{s - s_0} + \frac{1}{\pi} P \int \frac{\text{Im } F(s', t)}{s' - s} ds'$$

This is the typical structure of a typical dispersion relation; one may exhibit the pole and the integral contributions diagrammatically (for πp scattering) thus:



Since Re F(s, t) and Im F(s, t) are both experimentally accessible quantities, the dispersion equation provides a determination of the pion-nucleon coupling parameter g^2.

2.4.2. High-energy behaviour of F(s, t) (consequences of combined unitarity and analyticity)

Combining analyticity with unitarity, it has been possible to obtain restrictions governing the growth of physical amplitudes for large s or t. These again have no counterpart in non-relativistic theory. It is impossible to show how these results are derived either in field theory or in the S-matrix dispersion approach. We shall simply exemplify by stating one of the outstanding results.

Limitation growth of forward scattering amplitude

$$|F(s, t = 0)| < C\, s \log^2 (s/s_0)$$

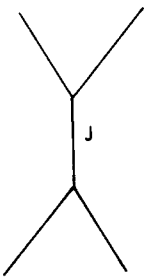
Using the optical theorem[37], this implies that total cross-sections in physics can only grow less fast than

$$\sigma_T < C' \log^2 (s/s_0)$$

Experimentally, up to 30 GeV (and, if cosmic ray data are believed, up to 1000 or more GeV), total cross-sections appear to approach constant limits asymptotically. Contrast this with the theoretical prediction above; clearly we are still far from gleaning from theory the best possible bound. The fact, however, that such bounds exist at all (and are after all not outrageously weak) makes the physicist in this field feel truly arrogant at the power of RQM.

2.4.3. Regge trajectory exchanges

Perhaps the most powerful deduction from crossing and high-energy bounds has been the demonstration that hadrons must lie on Regge trajectories. The argument goes as follows (for more detail, see Professor Van Hove's paper in these Proceedings). Consider in a scattering problem the exchange of a spin J particle; the pole approximation to the amplitude gives:



$$F(s, t) \approx g^2 \frac{P_J(\cos\theta_t)}{t - m_J^2}$$

$$\rightarrow g^2 \frac{(\cos\theta_t)^J}{t - m_J^2} \approx g^2 \frac{s^J}{t - m_J^2}$$

for large s.

If J > 1, this will violate the bound derived above from RQM. Either there are no particles with spins >1, or there is some physical mechanism which smears out the s^J contribution.

[37] The constant C' can actually be evaluated from field theory; for example, one finds the theoretical estimate $C' \leqslant 12/m_\pi^2$ for πN scattering.

Remarkably, with the hadron spectrum known at present, one does not have to seek far for such a mechanism. As pointed out in Part 1 and specifically in Fig. 2, hadrons appear to occur in Regge families with their masses increasing with spin. Each exchanged hadron of mass m_J will contribute a term of the above type; the total pole contribution to the amplitude equals

$$F(s, t) = \sum_J \frac{g_J^2 s^J}{t - m_J^2}$$

This sum can be approximated to by an integral, if the number of particles involved is fairly large. Thus,

$$F(s, t) \approx \int g_J^2 \frac{s^J}{\sin \pi J} \frac{dJ}{t - m_J^2}$$

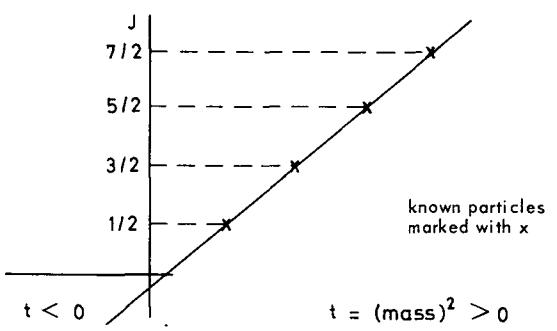
To evaluate the integral, solve $t - m_J^2 = 0$ in the form

$$J = \alpha(t)$$

Thus

$$F(s, t) \approx \int \frac{g_J^2 s^J}{\sin \pi J} \frac{dt}{t - m_J^2} \frac{dJ}{dt} \sim \frac{\alpha'(t)}{\sin \pi\alpha(t)} g_{\alpha(t)}^2 s^{\alpha(t)}$$

The high-energy behaviour of the amplitude is controlled not by the spin of any of the particles exchanged but by an effective spin $J = \alpha(t)$ dependent on momentum transfer t. And since in the scattering region $t \leqslant 0$, one must make a continuation of J from the positive values of t (given by Rosenfeld's tables) to negative t.



Consider now for this continuation one specific trajectory, for example, the nucleon trajectory in Fig. 1. Extrapolate (as a plausible continuation) the known linear plot of the trajectory in the first quadrant, backwards to negative t-values. Clearly empirically $\alpha(t) \leqslant 1$ for the scattering region. No contradiction with high-energy theorems could thus possibly arise

whenever the nucleon trajectory is exchanged in a scattering process (for example, in $\pi + M \to N + \pi$). Likewise, for all known trajectories, the extrapolated values of $J = \alpha(t)$ always empirically satisfy the rule $\alpha(t) \leqslant 1$ $(T \leqslant 0)$.

The moral of this for the hadron spectrum is perhaps that: (1) all hadrons lie on Regge trajectories[38]; and (2) for all trajectories, $\alpha(t)$ must lie lower than unity when continued to negative t. There are few places in particle theory where the power of RQM has evidenced itself to a greater effect than in this beautiful Regge synthesis of known hadron spectra $(t > 0)$ with asymptotics of scattering amplitudes $(t < 0)$.

2.5. HADRON SPECTRUM AND THE S-MATRIX PROGRAM

We have now surveyed the structure of RQM; we can see that in dispersion theory and in the Regge trajectory exchange models of high-energy scattering there is a tremendous amount of predictive power. The chief problem however remains. Where in all this is the analogue of the familiar non-relativistic Schrödinger theory, to yield the particle spectrum?

TABLE IX. DEDUCTIONS FROM GENERAL PRINCIPLES OF RQM

1. Examples of bounds from unitarity and analyticity

$$|F(s, t=0)| < C\, s \log^2(s/s_0)$$

$$\sigma_{total} < C' \log^2(s/s_0)$$

2. A typical dispersion relation

$$\mathrm{Re}\, F(s, 0) = \frac{g^2}{s - s_0} + \frac{1}{\pi} \int_C \frac{\mathrm{Im}\, F(s', 0)}{s' - s}\, ds'$$

3. Regge trajectory's contribution to a scattering amplitude

A trajectory contributes

$$\sum_J g_J^2 \frac{P_J(\cos\theta_t)}{t - m^2(J)} \approx \beta(t) s^{\alpha(t)}$$

for large s, where $\alpha(t) = J \Longleftrightarrow t - m^2(J) = 0$

4. Some achievements of the S-matrix approach

(a) Accurate determination from dispersion relations of $g_{\pi NN}$, $g_{KN\Lambda}$, etc.

(b) Bootstrap generation of certain resonances in pion-nucleon sector

(c) Regge analysis of high-energy scattering data

[38] In Professor Toller's paper in these Proceedings we learn that Regge trajectories (possibly) occur in tribes and families as a further consequence of some unexplored aspect of Lorentz symmetry and analyticity at $t = 0$.

The closest in spirit to the traditional Schrödinger method are field theories of Class I. Except for the masses associated with the distinguished fields and their coupling parameters, the parameters of all bound states could, in principle, be read off from the scattering matrix, computed using the given field equations. In Part 4 we shall see that it may yet be that aristrocratic fields do exist and that the future of particle physics lies along this traditional path. The milieu of our age, however, is somehow against this.

What could substitute for field equations in field theories of Class II or in the S-matrix approach? Since for both approaches the basic assumption is that all particles are equally elementary or equally composite, there is but one way to attack the problem – rely on self-consistency of any assumed spectrum in satisfying the relations provided by the theory. Among those relations with which one has worked are: (1) relations in Class II field theories provided by equal-time commutation rules (one can test if they are self-consistently saturated by an assumed spectrum), or (2) the so-called "superconvergence relations"[39].

One cannot say that such bootstrapping self-consistency ideas have had more than marginal success in the past, though with the greater use of the superconvergence relations the situation may improve.

The bootstrap idea is attractive. Part of its attraction lies in the possibility that the internal symmetries themselves may possibly arise as necessary preconditions for the existence of a particle theory satis-

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[39] These are dispersion relations for amplitudes which fall so fast in s that the relation reduces to $\int \mathrm{Im}\, A(s',t)\, ds' = 0$. This happens, for example, when incoming and outgoing particles carry spin; one can show from RQM that, for a general amplitude,

$$|F_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}(s,t)| < C s^{1-\max(|\lambda|, |\mu|)}$$

where $\lambda = \lambda_1 - \lambda_2$, $\mu = \lambda_3 - \lambda_4$ and the λ's are the spin-polarizations of the incoming and outgoing particles. This important dependence of high-energy behaviour on spin means that large spin-flip amplitudes fall fast for large s. For such amplitudes (and for appropriate values of $|\lambda|$ and $|\mu|$), the normal dispersion relation

$$A(s,t) = \frac{1}{2\pi} \int \frac{\mathrm{Im}\, A(s',t)\, ds'}{s' - s + i\epsilon}$$

reduces to its superconvergent form $\int \mathrm{Im}\, A(s',t) ds' = 0$. To see how such relations work, take the example of π-ρ scattering. Saturating the appropriate relation with ω and ϕ resonances only, one converts the integral $\int \mathrm{Im}\, A\, ds' = 0$ into an algebraic consistency formula which reads:

$$g^2_{\pi\rho\omega}(m^2_\omega - m^2_\rho - m^2_\pi) + g^2_{\pi\rho\phi}(m^2_\phi - m^2_\rho - m^2_\pi) + \ldots = 0$$

Empirically, we know that the masses of ω, ρ and π mesons satisfy

$$m^2_\omega - m^2_\rho - m^2_\pi = 0 \qquad \mathrm{I}$$

while

$$g^2_{\pi\rho\phi} = 0 \qquad \mathrm{II}$$

($\phi \to \rho + \pi$ decay is nearly suppressed). The relation is thus (miraculously) satisfied, showing either: (i) that RQM is a wonderful theory, or (ii) that the existence of ω (and the hypothesis of saturation of the superconvergence relation with a few resonances) implies that ϕ must also exist. One may take one's choice.

fying the very stringent restrictions of crossing, analyticity and unitarity of RQM. We may yet find that we are living (with Voltaire) not only in the best of all possible worlds but indeed in the only possible world.

Table IX gives a summary of the deductions from general principles of relativistic quantum mechanics.

PART 3. THE DYNAMICAL ROLE OF CHARGES AND SHARING OF CURRENTS BETWEEN STRONG, WEAK AND ELECTROMAGNETIC INTERACTIONS

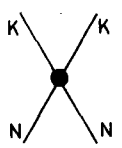
In Part 2 we spoke of abstracting from field theory the general canons of relativistic quantum mechanics. I now wish to speak of the second important idea, again an abstraction from field theory, but this time from one special theory[40] — the quantized Maxwell-Dirac field theory of electrons interacting with photons. This is the notion of field-theoretic currents associated with the charges introduced in Part 1, the dynamical role of charges and currents, particularly for low-energy phenomena, and the sharing of the currents between strong, weak and E.M. interactions.

3.1. DYNAMICAL ROLE OF CHARGE

The dynamical role of electric charge is familiar from the Coulomb force law, where the sign and magnitude of charge determines the long-range Coulomb force ($\cong e_1 e_2/r^2$). The same applies for the gravitational force which is proportional to the product of gravitational charges (mass) ($\cong m_1 m_2/r^2$).

Less familiar, but equally well established, is the role of hyper-, isotopic and other charges in the same context; the long-range parts of the relevant forces are proportional to the charge strengths.

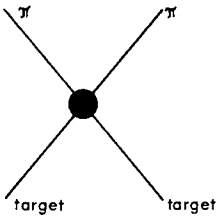
3.1.1. Hypercharge



The long-range part of K^-N force is attractive, while for K^+N this force is repulsive. This corresponds to the sign of the product of hypercharges $Y_K Y_N$ which is -1 for the K^-N and +1 for the K^+N case.

[40] We shall need familiarity with field-theoretic notation in this part of the lecture.

3.1.2. Isotopic charge



The scattering length for pions (isotopic charge $\vec{I}_\pi$) interacting with a target of isotopic charge $\vec{I}_t$ is experimentally found proportional to the scalar product of the two isotopic charges

$$\vec{I}_\pi \cdot \vec{I}_t = \frac{1}{2}\left[(\vec{I}_\pi + \vec{I}_t)^2 - \vec{I}_\pi^{\,2} - \vec{I}_t^{\,2}\right]$$

The conserved charges thus do appear to play a dynamical role similar to the electric charge for low-frequency phenomena. For the E.M. case, one knows one can go further. In Part 2 we have seen that the Maxwell-Dirac Hamiltonian uses currents $j_\mu(x)$ associated with the electric charge. Could this analogy be taken over for the other charges as well?

3.2. DYNAMICAL ROLE OF CURRENTS

Given a charge Q(t), it is well known that one can construct a local four-vector current operator $J_\mu(x)$ associated with it

$$Q(t) \Rightarrow J_\mu(x)$$

where

$$Q(t) = \int_t J_0(\vec{x}, t)\, d^3x$$

If the current is conserved, i.e. $\partial_\mu J_\mu = 0$, then the charge is a constant of motion ($\dot{Q}(t) = 0$) and vice-versa.

For spin-half charged particles (electrons or muons, for example), one can go further; we may define left and right currents corresponding to left and right charges in the zero-mass idealization. The sums and differences of these are the vector and axial-vector currents:

$$V_\mu(x) \equiv J_\mu^V(x) = J_\mu^L(x) + J_\mu^R(x)$$

$$A_\mu(x) \equiv J_\mu^A(x) = -J_\mu^L(x) + J_\mu^R(x)$$

The same applies for right and left hypercharges, isotopic and unitary charges.

One of the important experimental discoveries is that these currents – and also their divergences – if considered as fields, appear to possess particles associated with them (see Table X).

TABLE X. CURRENTS AND ASSOCIATED PARTICLES

	Charge	Conserved status of current	Associated particle
	Gravitational	$\partial_\mu \theta_{\mu\nu} = 0$	$\theta_{\mu\nu} \Rightarrow 2^+$ (graviton?)
	Electromagnetic	$\partial_\mu J_\mu = 0$	$J_\mu \Longrightarrow 1^-(\gamma)$
Strong	Baryon	$\partial_\mu J_\mu = 0$	$J_\mu \Longrightarrow 1^-(\omega)$
Strong	SU(3)	$\partial_\mu J_\mu^V = 0$ if we neglect medium-strong symmetry breaking	$J_\mu \Longrightarrow 1^-$ octet $(\rho, \phi, K^*, \bar{K}^*)$
Chiral Strong	$SU_L(3) \times SU_R(3)$	$\partial_\mu J_\mu^A \neq 0$	$J_\mu^A \Rightarrow 1^+$ octet $(A_1, D, K_A, \bar{K}_A)$ $\partial_\mu J_\mu^A \Longrightarrow 0^-$ octet $(\pi, \eta, K, \bar{K})$
Weak $\Delta Y = 0$	Weak currents J_μ^V (weak) J_μ^A (weak)	 $\partial_\mu J_\mu^V$ (weak)=0 $\partial_\mu J_\mu^A$ (weak) $\neq 0$	$J_\mu^{weak} \Longrightarrow$ Intermediate vector bosons (?)
	Sharing of (hadronic) currents between strong, E.M. and weak forces $J_\mu^{E.M.} = J_\mu^3 + \frac{1}{\sqrt{3}} J_\mu^8$ (J_μ^3, J_μ^8 refer to SU(3) currents) $J_\mu^{weak}(\Delta Y = 0) = J_{\mu L}^{1,2}$		($J_\mu^1, J_\mu^2, J_\mu^3$ form isotopic SU(2) $J_\mu^1 \pm iJ_\mu^2$ are isotopic lowering and raising currents)

Also, one of the recent insights of local field theory is that all operators representing a particle are equivalent so long as they possess the requisite quantum numbers. One may therefore express this association of particles and currents by writing a set of (approximate) field-current identities. Considering isotopic $SU_L(2) \times SU_R(2)$, for example, one may write:

$$\vec{J}_\mu^V = \frac{m_\rho^2}{g_\rho} \vec{\rho}_\mu + \ldots, \quad \vec{J}_\mu^A = \frac{m_A^2}{g_A} \vec{A}_\mu + \ldots$$

$$\frac{\partial \vec{J}_\mu^A}{\partial x_\mu} = c_\pi \vec{\pi} + \ldots$$

The last relation is called the PCAC (short for partially-conserved-axial-current hypothesis) relation.

Where does dynamics come into this?

Ideally, one might hope that, since for every current a corresponding particle exists, by analogy with the two well-known classical Hamiltonians:

(a) Gravitational $g\theta_{\mu\nu} G_{\mu\nu}$

(b) E.M. $e J_\mu^{E.M.} A_\mu$

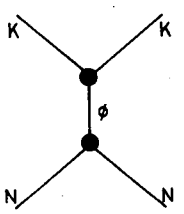
which are simple products of currents and associated particle fields ($\theta_{\mu\nu}$ = stress tensor, $G_{\mu\nu}$ = graviton, A_μ = photon), the strong and weak Hamiltonians[41], may also be written in the form

$$H^{strong} = g_V J_\mu^i V_\mu^i + g_A J_\mu^{Ai} A_\mu^i$$

$$H^{weak} = g^{weak} J_{\mu L} W_\mu^{weak}$$

where V^i and A^i are the strongly interacting 1^- and 1^+ octets of particles; W^{weak} are the weak intermediate vector mesons of Table X. For exact SU(3)×SU(3), $g_A = \pm g_V$. See Table XI for the theoretical prediction when brokenness of the SU(3)×SU(3) symmetry is taken into account (expressed by $\partial J_\mu^A/\partial x_\mu \neq 0$ in contrast to $\partial J_\mu^V/\partial x_\mu = 0$).

This is an attractive hypothesis[42]. From this point of view, it would be easy to understand why, for example, the long-range part of $\left.\begin{matrix} K^- - N \\ K^+ - N \end{matrix}\right\}$ force is attractive or repulsive. One would expect the long-range part of the interaction to be (dominantly) produced by a Yukawa exchange



of the ϕ-meson just like the Coulomb case, where the potential is a consequence of a single photon exchange. Unfortunately this argument is not more than suggestive, since, for a strong interaction theory, the largeness of the coupling parameter g makes it next to impossible, in practice, to decide on one "fundamental" Hamiltonian in contrast to

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[41] We have written H^{weak} in the form: current multiplied by intermediate boson mediating weak interactions. No such intermediate boson has yet been experimentally discovered. Present experiments test only the effective interaction $H_{eff} \simeq J_\mu^{weak} J_\mu^{weak\dagger}$.

[42] One great virtue of such a strong Hamiltonian is that the dilemma presented by whether nature prefers field theories of Class II (democratic) or Class I (aristocratic) no longer arises. The currents $J_\mu(x)$ could be made up either of just the distinguished fields or of all fields democratically. A slightly different version is the one very recently proposed

$$H^{strong} = \frac{g}{m^2}(J_\mu^i J_\mu^i + J_\mu^{Ai} J_\mu^{Ai})$$

where even the aristocracy of the 1^- and 1^+ octet particles V_μ^i and A_μ^i is ended.

another (see footnote 36). To show that the currents defined in Table X do play a role in strong dynamics, we use a different technique, as discussed in Part 3.4. Before going over this, however, we must understand the second aspect of these currents: their property of being shared between disparate forces.

3.3. UNIVERSALITY OF CURRENTS

(a) For the electric charge of hadrons we noted the remarkable relation

$$Q^{\text{hadron}} = I_3 + \frac{1}{2} Y = F^3 + \frac{1}{\sqrt{3}} F^8$$

A similar relation would naturally exist for the corresponding <u>hadron currents</u>

$$J_\mu^{\text{E.M.(hadrons)}} = J_\mu^3(x) + \frac{1}{\sqrt{3}} J_\mu^8(x)$$

where $J_\mu^3(x)$ is the (neutral) isotopic and $J_\mu^8(x)$ is the hypercharge current. This relation is remarkable enough. Even more remarkable is the postulate that the two remaining isotopic <u>left</u> currents $J_{\mu L}^1$ and $J_{\mu L}^2$, or rather their combinations

$$J_{\mu L}^{\pm}(x) = J_{\mu L}^1 \pm i J_{\mu L}^2(x)$$

are precisely the <u>weak</u> hadron currents responsible for $\left.\begin{matrix} n \to p \\ p \to n \end{matrix}\right\}$ transitions in β-decay. In other words, nature believes in an economy of charges and currents; once the isotopic and hyper – or more generally, SU(3)×SU(3) – sets of charges and currents were invented, it was decreed that they would serve not only as strong charges and strong currents but also as E.M. charges and E.M. currents (with a change of scale, of course) as well as for weak charges and weak currents (again with another change of scale), whenever hadron matter was involved.

(b) The statement that the same currents make their appearance in strong, weak and E.M. interactions of hadrons has been confirmed in numerous situations; to take just one representative example:

Consider the decay $\pi^+ \to \pi^0 + (e^+ + \nu)$
compared with $n \to p + (e^- + \bar{\nu})$

If the ideas outlined are correct, the ratio of the two hadronic transitions[43] $\left.\begin{matrix} n \to p \\ \pi^+ \to \pi^0 \end{matrix}\right\}$ should be proportional to the isotopic charges of pions and nucleons. This indeed is the case experimentally. If the concept of isotopic charge for hadrons had not already emerged from strong interaction physics, it would surely have been invented from the weak hadronic phenomena alone.

(c) Electric charge and weak charges are more universal in character than strong charges. Electric charge (as well as weak charges) is shared properties of leptons as well as hadrons. Thus the total electric charge equals

$$Q = Q^{\text{lepton}} + Q^{\text{hadron}}$$

and the equality $Q^{\text{hadron}} = I_3 + Y/2$ naturally holds only for its hadronic part. The same applies for the weak charges and weak currents[44].

One of the remarkable features (noted in Part 1) of electric charge is the exact equality of electron and proton charges (in other words, Q^{leptons} and Q^{hadrons}, and therefore also J_μ^{leptons} (E.M.) and J_μ^{hadrons} (E.M.), possess the same scale). Is this also true of J_μ^{weak}(leptons) and J_μ^{weak} (hadrons)?

Before we can answer this question, recall that weak forces affect left-spinning matter only; more precisely, (primitive) weak currents are left currents $J_{\mu L}$, which are equal mixtures of vector and axial currents ($J_{\mu L} = J_{\mu V} - J_{\mu A}$) both for leptons and hadrons. And of these two types only the vector ones are conserved, with the consequence that only the "vector" weak charges are time-independent.

We can now answer the question posed. The weak vector lepton charge does indeed equal in magnitude the weak vector hadron charge; one of the striking confirming pieces of evidence is the well-verified equality of the vector (Fermi) decay constant $n \to p + (e^- + \bar{\nu}_e)$ with the constant[45] determining μ-decay $\mu^- \to \nu_\mu + (e^- + \bar{\nu}_e)$.

3.4. LOW-FREQUENCY DYNAMICS

Turn back to strong interactions. We said, though the form of the E.M. and weak Hamiltonian $eJ_\mu^{E.M.} \cdot A_\mu$ and $g_{\text{weak}} J_{\mu L} W_\mu$ could be tested — since the coupling constants e and g_{weak} are rather small (and these have indeed

---

[43] More precisely, since $\pi^+ \to \pi^0 + (e + \nu)$ proceeds purely through the vector current, for comparison one must consider only the vector (the so-called Fermi) part of neutron β-decay.

[44] The full weak current is made up of three left-spinning parts

$$J^{\pm}(\text{weak}) = J_{\mu L}^{\pm}(\text{leptonic}) + J_{\mu L}^{\pm}(\text{hadronic } \Delta Y = 0)$$
$$+ J_{\mu L}^{\pm}(\text{hadronic } \Delta Y = \pm 1)$$

The last part $J_{\mu L}^{\pm}(\Delta Y = \pm 1)$ is that part of the weak current which induces transitions of the type $\left.\begin{matrix} \Lambda \to n \\ n \to \Lambda \end{matrix}\right\}$ where hypercharge changes by one unit ($\Delta Y = \pm 1$). The currents involved here are once again made up of $SU_L(3)$ currents. The fact that one is dealing with left currents only in weak interactions implies that weak forces are not left-right symmetric and that they do not preserve space-reflection (parity) symmetry. We must unfortunately omit this fascinating aspect of weak interactions.

[45] The axial constants do not display this equality; this corresponds to the lack of J_μ^A conservation.

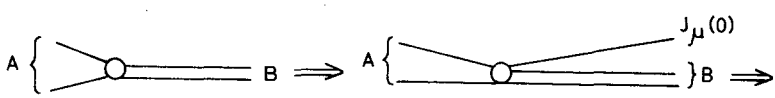
been found to be in accord with experiment) – no immediate tests of the form of the strong Hamiltonian are possible on account of the large coupling constant involved. How could one then be sure that the SU(3) × SU(3) currents do play a role in strong interaction physics? Fortunately a powerful set of tests are available. This is the set of low-energy theorems which exist whenever a symmetry exists. And when, in addition, the currents have physical particles associated with them (field-particle identity of Table X), the experimental verification of these low-energy theorems becomes not too difficult. The theorems are of the following variety:

Theorem I

Whenever a symmetry exists, broken or exact, and a corresponding charge and therefore current $J_\mu(p)$ exists ($J_\mu(p)$ is the Fourier transform of $J_\mu(x)$), one can derive relations connecting the processes

$$\left.\begin{array}{l} A \to B \\ A \to B + J_\mu(0) \\ A \to B + J_\mu(0) + J_\nu(0) \\ \cdots\cdots\cdots\cdots \end{array}\right\} \quad \text{soft } J_\mu\text{-emission}$$

$J_\mu(0)$ are the zero-frequency components of $J_\mu(p)$.



We shall specify the detailed structure of the relation in three examples.

Theorem II

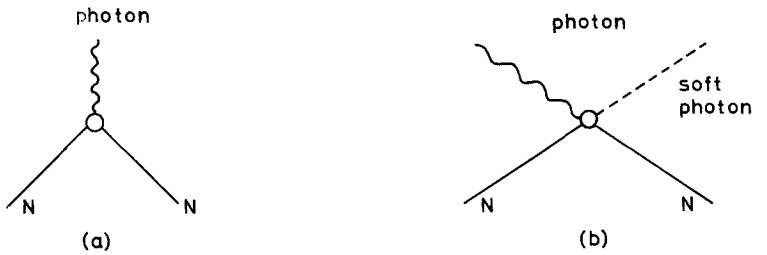
If the current is not conserved but $\partial_\mu J_\mu = \chi$, then similar relations hold for the sequences of processes

$$\left.\begin{array}{l} A \to B \\ A \to B + \chi(0) \\ A \to B + \chi(0) + \chi(0) \\ \cdots\cdots\cdots\cdots \end{array}\right\} \quad \text{soft } \chi\text{-emission}$$

Examples

(a) Electrodynamics

The theorem states that the vertex (a) is related to the process (b)



Write $F(\omega)$ for the "Compton" scattering of photons on nucleons represented in (b). Then, according to the theorem, the kinematic structure of $F(\omega)$ at low frequencies is specified completely by parameters of the vertex (a). In detail,

$$F(\omega) = F_1(0)\,\vec{\epsilon}\,'\cdot\vec{\epsilon} + \omega F_2(0)\, i\,\vec{\sigma}\cdot(\vec{\epsilon}\,'\times\vec{\epsilon})$$

where

$$F_1(0) = \frac{\alpha}{m} Q^2 \qquad \alpha = e^2 = 1/137$$

$$F_2(0) = -\frac{\alpha}{2m} X^2$$

are the two parameters relating to the process (a). Here $\vec{\epsilon}$, $\vec{\epsilon}\,'$ are the polarizations of the photons, $\vec{\sigma}$ is the spin of the nucleon, eQ is its charge and Ke/2m its anomalous magnetic moment[46].

(b) Ratio of nucleon-vector and nucleon-axial-vector coupling constants from low-frequency techniques

One of the most celebrated low-energy relations is the one which connects the nucleon-J_μ^V-vertex to soft pion-nucleon scattering and computes thereby the physically important ratio of the (effective) coupling

[46] This low-energy theorem, a generalization of the well-known Thomson limit theorem in classical electrodynamics can be used as a boundary condition on a dispersion relation one may postulate for $F(\omega)$; thus

$$\frac{\alpha K^2}{2m^2} = \frac{1}{4\pi^2}\int_0^\infty \frac{d\omega}{\omega}\left(\sigma_P(\omega) - \sigma_A(\omega)\right)$$

Here σ_P and σ_A are the photon-nucleon cross-sections with photon polarization parallel and antiparallel to nucleon spin. The relation appears experimentally satisfied. This combination of a (postulated) dispersion relation and a low-energy theorem illustrates the theoretical methodology that has come to be employed more and more in particle physics.

TABLE XI. RATIO OF VECTOR AND AXIAL-VECTOR COUPLING PARAMETERS DERIVED USING LOW-ENERGY THEOREMS

(1) $\partial_\mu \vec{J}_\mu^A = m_\pi^2 c_\pi \vec{\pi}$ PCAC

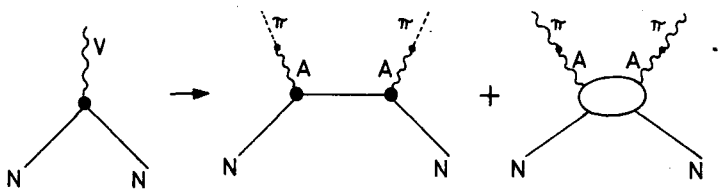
From weak interactions, estimate $(p|J_\mu^A|n)$, to obtain

$$c_\pi = \sqrt{2}\,\frac{g_A}{g_V}\,\frac{m_N}{g_{\pi NN}}$$

(2) Use the equal-time commutation relation

$$(N|V|N) = (N|[A, A]|N)$$

and techniques of field theories of Class II to relate



deriving thereby

$$c_\pi^2 \left[\frac{g_{\pi NN}^2}{2m_N^2} + \int \frac{d\omega}{\omega}(\sigma^+ - \sigma^-)\right] = 1$$

Use c_π above to predict

$$|g_A/g_V| = 1.19$$

Experiment $|g_A/g_V| = 1.18$

constants $g_V = g_{NNV}$ and $g_A = g_{NNA}$ (g_{NNV} and g_{NNA} are the zero-frequency limiting values of the appropriate vertex functions, see 2.1.2). The derivation of the relation is sketched in Table XI; its most important aspect is the purposeful use one must make of both strong and weak data to verify it, once again confirming: (a) that the same currents are operative in both interactions, so far as hadrons are concerned, (b) that these currents – or rather their charges – define the algebra of SU(3)×SU(3), and (c) that the techniques of RQM – as formulated through field theories of Class II – and dispersion theory are marvellously effective in capable hands.

(c) Soft-pion processes

A host of low-frequency relations have been derived in strong, weak and E.M. physics, connecting amplitudes like $A \to B$ with amplitudes for soft-pion emission $A \to B + \pi(0)$, $A \to B + \pi(0) + \pi(0), \ldots$ I shall merely mention that these relations exist and are well verified. Quite recently, a new technique (using non-linear representations of the chiral group $SU_L(3) \times SU_R(3)$) has been perfected, which makes their derivation a relatively painless task. We learn more on this from the papers in these Proceedings by Professors Weinberg and Zumino.

3.5. PARTICLE SPECTRUM AND THE SYMMETRY ALGEBRA PROGRAM

The correlation of low-frequency data achieved by the use of the field-theoretic method (the essential ingredient being the equal-time commutation rules of $SU(3) \times SU(3)$ algebra) was impressive enough, so that one felt tempted – just as the S-matrix theorist had been sorely tempted earlier with dispersion and superconvergence relations – to consider taking the $SU(3) \times SU(3)$ algebra as substitute for a complete dynamical theory. The hope was that one may possibly derive the hadron spectrum by attempting to satisfy the identities provided by the $SU(3) \times SU(3)$ commutation rules (see Part 2.2.3.A(c)). Like the S-matrix theorist's attempt to derive the full particle spectrum by bootstrap procedures, the symmetry algebraist's attempt has also met with scant success. The symmetry algebras play an important role in low-frequency dynamics; apparently they do not constitute the complete theory.

PART 4. THE STRUCTURE OF HADRONS, STILL HIGHER DYNAMICAL SYMMETRIES

In Part 3 we have been concerned with the low-frequency dynamics which the existence of any given symmetry (isotopic, unitary, etc.) implies. Specifically, we saw that one can derive relations between processes involving bosons associated with the (symmetry) currents (0^-, 1^-, 1^+ octets). But these bosons by no means exhaust the full spectrum of hadrons. What about the multitude of other objects the hadron spectrum consists of, and in particular higher spin baryons? Is it conceivable that there are new dynamical symmetries still to be discovered for an elucidation of the rest of the particle spectrum, particularly as $SU(3) \times SU(3)$ appears to be undistinguished for classifying particles?

What exactly do we mean by a dynamical symmetry?

To answer this, consider the familiar case of atomic physics. The relativistic Maxwell-Dirac quantized electrodynamics may indeed be the fundamental theory of charged particle interactions, but to obtain the hydrogen spectrum, one still goes back to the non-relativistic Schrödinger equation with just the static Coulomb potential. And, as is well known, this equation possesses a completely unsuspected O(4) symmetry, first studied by Fock. The emergence of the O(4) is purely "accidental" in

that it arises from the particular form of the (Coulomb) Schrödinger equation. However, it is this dynamical symmetry, rather than the fundamental Dirac-Maxwell form of the Hamiltonian, which dominates the hydrogen spectrum. Table XII shows how the symmetry arises and the classification of the hydrogen levels using O(4).

TABLE XII. THE HYDROGEN ATOM

Hamiltonian	$E = \frac{1}{2}p^2 - \frac{1}{r}$	
Define	$\vec{L} = \vec{r} \times \vec{p}$	
	$\vec{M} = \sqrt{-8E}\,(\vec{L} \times \vec{p} - \vec{p} \times \vec{L}) + \frac{\vec{r}}{r}$	
Operators	$\vec{I} = \frac{1}{2}(\vec{L} + \vec{M})$	generate independent
and	$\vec{K} = \frac{1}{2}(\vec{L} - \vec{M})$	$O_3 \times O_3 \sim O_4$ rotations
Since	$\vec{L} \cdot \vec{M} = 0$ so $\lvert\vec{I}\rvert = \lvert\vec{K}\rvert$	
Verify	$-4E = [\vec{I}^2 + \vec{K}^2 + \frac{1}{2}]^{-1}$	
	$= [i(i+1) + k(k+1) + \frac{1}{2}]^{-1}$	
Since	$\lvert I\rvert = \lvert\vec{K}\rvert = 0, \frac{1}{2}, 1, \ldots$ (I, k generate O_3)	
obtain	$E = -\frac{1}{2n^2}$	
where	$n = (2i+1) = (2k+1)$	

A tower of levels, $(i,k) = (0,0); (\frac{1}{2},\frac{1}{2}); (1,1), \ldots$ corresponds to a single representation of the non-compact group O_{4+1}.

The same thing seems to be happening in particle physics where we appear to find a dynamical symmetry SU(6) more successful than SU(3) × SU(3) for particle classification and for a description of vertex functions.

The development of ideas which I shall now sketch started with Wigner and his postulated supermultiplet symmetry SU(4) of nuclear physics. Starting with the notion of spin and isotopic charge independence of nuclear forces, Wigner came upon the dynamical group SU(4) as a natural completion of the isotopic $SU_I(2)$ and the spin $SU_J(2)$ groups ($SU(4) \subset SU_I(2) \times SU_J(2)$). For particle physics, replace isotopic by the unitary symmetry; the simplest completion group, which includes both the unitary $SU_F(3)$ and the spin $SU_J(2)$, is SU(6). Assume that SU(6) (like the O(4) for the hydrogen atom case) is the dynamical symmetry responsible for the hadron spectrum. Each representation of SU(6) would combine within it particles of different spins and unitary charges.

The fundamental representation of SU(6) is $\underset{\sim}{6}$ dimensional. This would correspond to three quarks of spin up and three quarks of spin down. Other representations are

$\underset{\sim}{35}$, $\underset{\sim}{56}$, $\underset{\sim}{70}$, $\underset{\sim}{405}$, ... with content

$SU(6) \rightarrow SU_F(3) \times SU_J(2)$

$\underset{\sim}{35} = \underset{\sim}{1} \times \underset{\sim}{3} + \underset{\sim}{8} \times \underset{\sim}{3} + \underset{\sim}{8} \times \underset{\sim}{1}$ (i.e. a spin 1 nonet + a spin zero octet)

$\underset{\sim}{56} = \underset{\sim}{10} \times \underset{\sim}{4} + \underset{\sim}{8} \times \underset{\sim}{2}$ (i.e. a 3/2 decuplet + 1/2 octet)

A glance at the Rosenfeld table, where the lowest mass boson entries are just the 1^- nonet + the 0^- octet (constituting together a $\underset{\sim}{35}$ of SU(6)) and the lowest mass baryons are precisely the $3/2^+$ decuplet $+1/2^+$ octet (together constituting the $\underset{\sim}{56}$ of SU(6)), convinces one that SU(6) makes very good sense.

I shall not describe here the extension of SU(6) to U(6)×U(6) symmetry which distinguishes the fundamental quark representation 6 from the antiquark $\bar{6}$, nor its formulation $\tilde{U}(12)$ needed to give correctly the relativistic kinematics of vertex functions. Professor Gürsey deals with the subject in these Proceedings.

Table XIII lists predictions from SU(6) and $\tilde{U}(12)$. In Tables XIV to XVIII are given some of the large variety of predictions – all reasonably well substantiated. The most noteworthy are the predictions of the ratio of magnetic moments of protons and neutrons – obtained essentially as a kinematic prediction of the theory – and the immediate explanation of the scaling law of E.M. form factors mentioned in Part 2.1.

The problem which immediately arises with the undoubted successes of SU(6) and $\tilde{U}(12)$ symmetries for the multiplet and vertex structures of the well-known mesons and baryons is how to reconcile this with, for example, SU(3)×SU(3). What is the nature of charges associated with SU(6)? Are they conserved? What, if any, are the currents? Does spin act as charge, and anyway what is the precise definition of spin used in SU(6)? Or is it that we are perhaps trying to force totally unrelated and distinct ideas into the same mould? A number of answers to these challenging problems have been advanced. Nobody seems to know for sure.

TABLE XIII. PREDICTIONS FROM SU(6) AND $\tilde{U}(12)$

(1) $\frac{\mu_p}{\mu_n} = -3/2$ (see Table XVII)

(2) Host of coupling constant relations (see Table XVIII)

(3) Scaling law for form factors (see Table XVIII)

(4) Mass formulae (see Tables XIV and XV)

$$M^2 = M_0^2 + a\,J(J+1) + b(\vec{I}^2 - \tfrac{1}{4}Y^2) + cY$$

TABLE XIV. MASS FORMULAE

SU(3)

Mass formula for SU(3) multiplets:

$$M = a + bY + c\,[I(I+1) - \tfrac{1}{4}Y^2]$$

Baryon octet

Predict $M_\Sigma + 3M_\Lambda = 2(M_N + M_\Xi)$

Experiment 4539.7 ~4512.8 MeV

Decuplet

Predict equal spacing rule

Theory $M_{Y_1^*} - M_{N^*} = M_{\Xi^*} - M_{Y_1^*} = M_{\Omega^-} = M_{Y_1^*}$

Experiment 149 ~ 145 ~ 142 MeV

Electromagnetic mass differences

Theory $\Sigma^- - \Sigma^+ - n + p = \Xi^- - \Xi^0$

Experiment 6.6 ± 0.1 MeV = 6.5 ± 1.0 MeV

TABLE XV. MASS FORMULAE

SU(6)

Baryon $\underset{\sim}{56}$

Theory $M = M_{00} + M_1\, J(J+1) + M_2 Y + M_3[I(I+1) - \tfrac{1}{4}Y^2]$

An average value of M_{00} ~1065 MeV gives the masses of SU(3) octet and decuplet correctly

Meson $\underset{\sim}{35}$

SU(6) reproduces SU(3) result

$$4K^2 - \pi^2 = 3\eta^2$$

and gives in addition

$$\rho^2 - \pi^2 = K^{*2} - K^2$$

Experiment 0.571 MeV ~0.553 MeV

Electromagnetic mass differences

Theory $n - p = 1/3\,(\Delta^- - \Delta^{++})$

Experiment 1.3 MeV ~2.7 ± 1.3 MeV

TABLE XVI. ELECTROMAGNETIC MASS DIFFERENCE OF HADRONS IN STATIC SU(6) MODEL USING QUARKS

		THEORY	EXPERIMENT
Baryons		MeV	MeV
	$\Xi^- - \Xi^0$	6.3 ± 0.3	6.5 ± 1
	$\Xi^{*-} - \Xi^{*0}$	3.1 ± 0.3	6 ± 3
	$N^{*0} - N^{*++}$	0.7 ± 0.6	0.4 ± 0.8
	$N^{*-} - N^{*++}$	3.6 ± 0.6	0.6 ± 5
	$Y_1^{*-} - Y_1^{*+}$	4.4 ± 0.5	4.3 ± 2
Bosons			
	$\pi^+ - \pi^-$	4.2 ± 0.4	4.6 ± 0.007
	$K^0 - K^+$	-0.9 ± 0.2	4.2 ± 0.5

TABLE XVII. MAGNETIC MOMENTS

		THEORY	EXPERIMENT
SU(3)			
	μ_Λ	-0.96	$-0.73 \pm 0.16 \left(\frac{e\hbar}{2m_p c}\right)$
	μ_{Σ^+}	2.78	2.5 ± 0.7
SU(6)			
	$\frac{\mu_p}{\mu_n}$	$-\frac{3}{2}$	-1.46
	$\frac{\mu}{N^* \to N + \gamma}$	$\frac{2\sqrt{2}}{3}\mu_p$	$(1.28 \pm 0.02)\frac{2\sqrt{2}}{3}\mu_p$

In the meanwhile, however, a different approach – the aristocratic approach to hadron dynamics – has emerged, an approach which frankly negates all one's notions about relativistic quantum mechanics, but one which is amazingly simple and fruitful. Treat known mesons and baryons as composites of basic quarks in an unashamed non-relativistic sense; the composites are listed below:

$$q \to \underset{\sim}{6}$$

$$q\bar{q} \to 6 \times \bar{6} = \underset{\sim}{35} + \underset{\sim}{1}$$

$$qqq \to \underset{\sim}{6} \times \underset{\sim}{6} \times \underset{\sim}{6} = \underset{\sim}{56} + \underset{\sim}{70} + \underset{\sim}{70} + \underset{\sim}{20}$$

Assume a soft (oscillator) potential for the constituent quarks. The baryon multiplet $\underset{\sim}{56}$ represents a three-body symmetric wave function, the meson $\underset{\sim}{35}$ is a two-body quark-antiquark composite. The model goes on to describe scattering and production phenomena in meson-baryon scattering. The fundamental assumption made is that there are just two independent amplitudes for q-q and q-$\bar{q}$ scattering, all amplitudes for composite particles being obtained by simple additivity. The only ingredient of RQM the quark model needs, is the superposition principle.

As I said earlier, the model seems to negate the S-matrix notion that hadrons are made up of each other. Since within its lights it succeeds, it poses one of the many mysteries of our subject. If physical quarks were discovered (and so far the search has not been successful – their massiveness presumably makes their production with present accelerator energies impossible), one would really have to come to grips with the new (dynamical) problem of reconciling the aristocratic with the democratic approach[47].

SUMMARY

This concludes our brief survey of what has been achieved in theoretical understanding in particle physics. To recapitulate, up to the energies for which systematic experimentation has been possible, the highly restrictive and tight structure of RQM seems to hold. We believe that we know what some of the internal symmetries of physical particles are; we know that hadronic matter exhibits the same internal symmetries whether acting strongly, weakly or electromagnetically, and we know that these symmetries not only serve to classify particle but also dominate low-frequency dynamics.

So much for the ideas and principles that have reasonably succeeded. We have been less successful in understanding the architecture of matter itself. The mathematically intractable, though aesthetically attractive, S-matrix bootstrap idea which considers all hadrons as composites of each other, appears at present irreconcilable with the simpler additive aristocratic quark model. The attractive (dynamical) higher symmetries of hadrons, like SU(6), U(6)$\times$U(6), $\widetilde{U}(12)$, . . . , which <u>bypass</u> these difficulties (essentially treating quarks as mathematical entities and giving quark-model-like results without quarks), still need to be reconciled with symmetries like $SU_L(3)\times SU_R(3)$.

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[47] To take an example of the type of problem posed, consider Regge trajectories, treated characteristically differently in S-matrix theory, in the higher symmetry schemes and in the quark model. The quark model explanation is the simplest; the particles on a boson Regge trajectory, for example, are orbital angular momentum ℓ-excitations of a bound $q\bar{q}$ system. The higher symmetry schemes employ a description – as an abstraction from the dynamics – associated with infinite-dimensional unitary representations of non-compact groups like U(6,6) or U(6,6) $\times$ O(3). For analogy, one may draw once again on the hydrogen spectrum, where the hydrogen levels (in O(4) language) are given by the representations

$$(0,0),\ (\tfrac{1}{2},\tfrac{1}{2}),\ (1,1),\ \ldots$$

This set constitutes <u>one single unitary irreducible</u> representation of the non-compact group O(4,1). The exploitation of these notions together with infinite-component field equations, first discussed by Majorana, is the subject of other papers in these Proceedings, particularly that by Professor Fronsdal.

TABLE XVIII. COUPLING CONSTANT RELATIONS FROM $\tilde{U}(12)$

(1) $F^c(q^2)/F^M(q^2) \sim 1 + \frac{m}{2M}$

$F^{c,M}$ are Sachs E.M. form factors

(2) $\mu_p = 1 + 2M/m$, $\mu_n = -\frac{2}{3}(1 + 2M/m)$

m is mean mass of 1^- multiplet and M nucleon mass

(3) Meson baryon vertex

$(V = 1^-$ nonet : $D = \frac{3^+}{2}$ decuplet):

Predict

$$g_{PNN} : g^{ch}_{VNN} : g^{Mag}_{VND} : g_{PND} : g_{VND} : g_{PDD} : g^{ch}_{VDD}$$

$$= \left(1 + \frac{2M}{m}\right)_{3D+2F} : \left(1 - \frac{m^2}{4M^2}\right) : \left(1 + \frac{2M}{m}\right)_{3D+2F} : \left(1 + \frac{2M}{m}\right) : \left(1 + \frac{2M}{m}\right) : \left(1 - \frac{m^2}{4M^2}\right)$$

Also predict

$$g_{N^*N\pi} = \frac{3}{5}\left(1 + \frac{2M}{m}\right) g_{NN\pi}$$

Taking $\langle m \rangle = 700$ MeV $\langle M \rangle = 1300$ MeV

Predict $\Gamma_{N^*N\pi} \sim 110$ MeV

Experiment ~ 100 MeV

Notwithstanding some notable successes in weak interaction theory (the demonstration of the two-component nature of the neutrino, the sharing of $SU_L(3)$ (left, V-A) currents between hadronic strong and weak forces, the well-verified postulate of a suppression of $\Delta Y = \pm 1$ weak effects relative to weak $\Delta Y = 0$, important topics which I have had no time to discuss), there is much that is dark: the recently discovered superweak CP-violation and the possible existence of exotic types of yet undiscovered matter (A-matter of T.D. Lee, introduced specifically to explain CP-violation, magnetic monopoles of Dirac to explain why electric charge must be quantized), the behaviour of weak forces at higher energies; all these are question-marks needing much experiment and deep thought. And then, as I said in the very beginning of my lecture, the energies to which our systematic experimentation extends are painfully low on the cosmic scale. There could be nothing more pretentious than the (unqualified) title of this lecture – "A fundamental theory of matter" – but such, fortunately indeed, is the encompassing conceit of the human mind!

TABLE XIX. SOME UNRESOLVED PROBLEMS

(1)	Origin and reconciling of dynamical symmetries like SU(6) with $SU_L(3) \times SU_R(3)$.
(2)	Nature of internal symmetries; do the postulates of RQM, together with hadron democracy idea, imply the existence of these symmetries?
(3)	Theory of CP-violation; behaviour of weak interactions at high energies. Origin of the suppression of $\Delta Y = \pm 1$ weak forces relative to $\Delta Y = 0$ forces.
(4)	The large numbers like $\frac{m_\mu}{m_e}$, α^{-1}, $(g^2_{weak})^{-1}$, ...
(5)	Exotic forms of matter; quarks, magnetic monopoles.

Then there are the really deep problems: the origin of different types of forces – strong, weak, E.M., gravitational – their outrageously different couplings, the selective division they impose on matter into leptonic and hadronic, into charged and uncharged, into left- and right-spinning varieties. Considering what we have already achieved, one feels proud of the work of our generation. Considering what still remains to be solved, one feels truly humble. Some of the unresolved problems are listed in Table XIX.

I should like to end with a quotation from Oppenheimer, who helped in the planning of this Symposium and whose warmth, whose insight and inspiration I personally miss so deeply to-day: "We are so engulfed by the changes in the current scene in physics: by their ferocity, their brashness, their virtuosity, their diffusion, that we don't understand them very well, and it may not be possible for us to understand them. .... The future will be only more radical and not less, only more strange and not more familiar, and it will have its own new insights for the inquiring mind."

ACKNOWLEDGEMENTS

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APPENDIX A

STABLE PARTICLES. January, 1968.

From Data on Particles and Resonant States, UCRL-8030. Printed at LRL and CERN

A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, Matts Roos, Paul Söding, W. J. Willis, C. G. Wohl

Quantities in italics have changed by more than one standard deviation since January, 1967.

	$(I^G J^P)C$	Mass (MeV)	Mass difference (MeV)	Mean life (sec) $c\tau$(cm)	Mass^2 (GeV)^2	Decays: Partial mode	Fraction	Q(MeV)	p or p_{max} (MeV/c)	Partial Rates (sec^{-1})
γ	$0,1(1^-)^-$	0		stable	0	stable				
ν_e ν_μ	$J=\frac{1}{2}$	0(<0.2 keV) 0(<2.1 MeV)		stable	0	stable				
e	$J=\frac{1}{2}$	0.511006 ±.000002		stable ($>2\times10^{21}$y)	0.000	stable	$\mu_e = 1.001159596 \pm .000000023 \frac{e\hbar}{2m_e c}$			
μ	$J=\frac{1}{2}$	105.659 ±.002		2.1983×10^{-6} ±.0008 $c\tau = 6.592\times10^4$	0.011	$e\nu\bar{\nu}$	100	105	53	
						$e\gamma\gamma$	(<1.6)10^{-5}	105	53	
						3e	(<1.3)10^{-7}	104	53	
						$e\gamma$	(<6)10^{-9}	105	53	
$\mu_\mu = 1.0011666 \pm .0000005 \frac{e\hbar}{2m_\mu c}$			-33.920 ±.014							
$\pi^\pm$	$1(0^-)$	139.579 ±.014		2.604×10^{-8} ±.007, S=2.3* $c\tau = 781$ $(\tau^+-\tau^-)/\bar{\tau}=(.4\pm.2)\%$ (test of CPT)	0.019	$\mu\nu$	100 %	34	30	
						$e\nu$	(1.24±0.03)10^{-4}	139	70	
						$\mu\nu\gamma$	(1.24±0.25)10^{-4}	34	30	
			4.6041 ±.0037			$\pi^0 e\nu$	(1.03±0.07)10^{-8}	4	5	
						$e\nu\gamma$	(3.0 ±0.5)10^{-8}	139	70	
π^0	$1^-(0^-)^+$	134.975 ±.014		[d] 0.89×10^{-16} ±.18, S=1.6* $c\tau = 2.67\times10^{-6}$	0.018	$\gamma\gamma$	(98.83±0.04)%	135	67	
						γe^+e^-	(1.17±0.04)%	134	67	
						$\gamma\gamma\gamma$	(< 5)10^{-6}	135	67	
						$e^+e^-e^+e^-$	[b] (3.47)10^{-5}	133	67	
$K^\pm$	$\frac{1}{2}(0^-)$	493.83 ±0.11		1.235×10^{-8} ±.005 S=2.1* $c\tau = 370$ $(\tau^+-\tau^-)/\bar{\tau}=(.09\pm.12)\%$ (test of CPT) S=1.3*	0.244	$\mu\nu$	[c] (63.58±0.29)%	388	236	[c] (51.49 ±0.27)10^6 S=1.1*
						$\pi\pi^0$	(20.94±0.28)% S=1.1*	219	205	(6.96 ±0.23)10^6
						$\pi\pi^-\pi^+$	(5.57±0.04)%	75	126	(4.51 ±0.03)10^6
						$\pi\pi^0\pi^0$	(1.70±0.05)%	84	133	(1.38 ±0.04)10^6
						$\mu\pi^0\nu$	(3.38±0.17)% S=1.6*	253	215	(2.74 ±0.14)10^6 S=1.6*
						$e\pi^0\nu$	(4.83±0.12)% S=1.4*	358	229	(3.91 ±0.10)10^6 S=1.4*
			-3.93 ±0.17			$\pi\pi^\mp e^\pm\nu$	(3.8 ±0.8)10^{-5}	214	204	
						$\pi\pi^\pm e^\mp\nu$	(<2)10^{-6}	214	204	
						$\pi\pi^\mp\mu^\pm\nu$	(≤1.4)10^{-5}	109	151	
						$\pi\pi^\pm\mu^\mp\nu$	(<3)10^{-6}	109	151	
						$e\nu$	(1.24±0.40)10^{-5}	493	247	
						$\pi\pi^0\gamma$	(2.2 ±0.7)10^{-4}	219	205	
						$\pi\pi^+\pi^-\gamma$	(10 ±4)10^{-5}	75	126	
						$\pi e\nu\gamma$	(6 ±4)10^{-4}	354	227	
						πe^+e^-	(<1.1)10^{-6}	353	227	
						$\pi\mu^+\mu^-$	(<2.4)10^{-6}	143	172	
						$\pi\gamma\gamma$	(<1.1)10^{-4}	354	227	
K^0	$\frac{1}{2}(0^-)$	497.75 ±0.18		50%K_{short}, 50%K_{long}						
K^0_S	$\frac{1}{2}(0^-)$		$-0.48\times\frac{1}{\tau_S}$ ±0.02	0.874×10^{-10} ±.011 S=1.3* $c\tau = 2.61$	0.248	$\pi^+\pi^-$	(68.4 ±1.0)%	219	206	(0.783±0.015)10^{10}
						$\pi^0\pi^0$	(31.6)%	228	209	(0.362±0.012)10^{10}
K^0_L	$\frac{1}{2}(0^-)$			5.30×10^{-8} ±.13 $c\tau = 1593$	0.248	$\pi^0\pi^0\pi^0$	[c] (25.5 ±1.9)%	93	139	[c] (4.81 ±0.41)10^6
						$\pi^+\pi^-\pi^0$	(12.1 ± 0.4)%	84	133	(2.29 ±0.09)10^6
						$\pi\mu\nu$	(27.3 ±1.3)%	253	216	(5.15 ±0.26)10^6
						$\pi e\nu$	(35.1 ±1.4)%	358	229	(6.62 ±0.29)10^6
						$\pi^+\pi^-$	(0.149 ± 0.006)%	219	206	(0.028±0.001)10^6
						d) $\pi^0\pi^0$	still uncertain	228	209	
						$\pi^+\pi^-\gamma$	(< 0.3)%	219	206	
						d) $\gamma\gamma$	still uncertain	498	249	
						$e\mu$	(< 0.8)10^{-5}	392	238	
						$\mu^+\mu^-$	(< 1.5)10^{-6}	287	225	
						e^+e^-	(< 1.7)10^{-5}	497	249	

General Atomic and Nuclear Constants [a]

N	$= 6.02252\times10^{23}$ mole^{-1} (based on A_C 12=12)
c	$= 2.997925\times10^{10}$ cm sec^{-1}
e	$= 4.80298\times10^{-10}$ esu $= 1.60210\times10^{-19}$ coulomb
1 MeV	$= 1.60210\times10^{-6}$ erg
$\hbar$	$= 6.5819\times10^{-22}$ MeV sec
	$= 1.05449\times10^{-27}$ erg sec
$\hbar c$	$= 1.9732\times10^{-11}$ MeV cm = 197.32 MeV fermi
k	$= 8.6171\times10^{-11}$ MeV deg^{-1} (Boltzmann const.)
α	$= e^2/\hbar c = 1/137.0388$
m_e	$= 0.511006$ MeV/$c^2 = 1/1836.10\ m_p$
m_p	$= 938.256$ MeV/$c^2 = 1836.10\ m_e = 6.721\ m_{\pi^\pm}$
	$= 1.00727663\ m_1$ (where $m_1 = 1$ amu $= \frac{1}{12} m_C{}^{12}$ $= 931.478$ MeV/c^2)
r_e	$= e^2/m_e c^2 = 2.81777$ fermi (1 fermi $= 10^{-13}$ cm)
λ_e	$= \hbar/m_e c = r_e \alpha^{-1} = 3.86144\times10^{-11}$ cm
a_∞ Bohr	$= \hbar^2/m_e e^2 = r_e \alpha^{-2} = 0.529167$ A (1 A $=10^{-8}$ cm)
σ Thomson	$= \frac{8}{3}\pi r_e^2 = 0.66516\times10^{-24}$ cm^2 = 0.66516 barn
μ Bohr	$= e\hbar/2m_e c = 0.578817\times10^{-14}$ Mev gauss^{-1}
μ nucl	$= e\hbar/2m_p c = 3.1524\times10^{-18}$ Mev gauss^{-1}

$\frac{1}{2}\omega_{cyclotron}$ (rad sec^{-1} gauss^{-1})

$= e/2m_e c = 8.79404\times10^6$

$= e/2m_p c = 4.7895\times10^3$

[a] Based mainly on E. R. Cohen and J. W. M. DuMond, Rev. Mod. Phys. 37, 537 (1965).

Note that there are some disagreements with the results of recent experiments.

CP violation parameters

$$\eta_{+-\,(00)} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \simeq |\eta_{+-}|_{(00)} e^{i\Phi_{+-\,(00)}}$$

$|\eta_{+-}| = (1.89\pm0.09)\times10^{-3}$

$\Phi_{+-} = (65\pm11)°$ S=1.1*

$|\eta_{00}|$ still uncertain, see data listings

										Magnetic Moment ($e\hbar/2m_pc$)	Decay Parameters† Measured α	Measured Φ(degree)	Derived γ	Derived Δ(degree)
η	$0^+(0^-)^+$	548.8 ±0.6		$\Gamma=(2.3\pm0.5)$keV		Neutral decays 71.0%: $\gamma\gamma$	(42 → 34)% (d)	549	274					
						$\pi^0\gamma\gamma$	(1 → 19)% (d)	414	258					
						$3\pi^0$	(28 → 18)% (d)	144	179					
						Charged decays 29.0%: $\pi^+\pi^-\pi^0$	(23.4 ± 1.1)%	135	174					
						$\pi^+\pi^-\gamma$	(5.5 ±0.5)%	269	236					
						$\pi^0e^+e^-$	(< 0.04)%	413	258					
						$\pi^+\pi^-e^+e^-$	(0.1 ±0.1)%	268	236					
p	$\frac{1}{2}(\frac{1}{2}^+)$	938.256 ±0.005	-1.2933 ±.0001	stable (>6×10^{27}y)	0.880					2.792763 ±.000030				
n	$\frac{1}{2}(\frac{1}{2}^+)$	939.550 ±0.005		$(1.01\pm0.03)10^3$ cτ = 3.03×10^{13}	0.882	$pe^-\nu$	100 %	1	1	-1.913148 ±.000066	$\frac{g_A}{g_V}$ = -1.198 ± .022(e),†			
Λ	$0(\frac{1}{2}^+)$	1115.50 ±0.08		2.52×10^{-10} ±.04 S=1.4* cτ = 7.61	1.245	$p\pi^-$	(65.3 ±1.2)%	38	100	-0.73 ±.16 S=1.2*	0.647±0.016	(-6±7)° S=1.2*	0.75	(7±8)°
						$n\pi^0$	(34.7 ±1.2)%	41	104		0.71 ±0.18			
						$pe\nu$	(0.88±0.15)10^{-3}	177	163		0.06 ±0.19	For $\Lambda\to pe\nu$, $\frac{g_A}{g_V} = -1.14^{+0.23}_{-0.33}$ †		
						$p\mu\nu$	(1.35±0.60)10^{-4}	72	131					
Σ^+	$1(\frac{1}{2}^+)$	1189.47 ±0.08	-7.95 ±.11	0.810×10^{-10} ±.013 cτ = 2.43	1.412	$p\pi^0$	(52.8 ±1.5)%	116	189	2.5 ±.7	-.955±0.070←S=1.1*			
						$n\pi^+$	(47.2 ±1.5)%	110	185		+.017±0.037	(180±30)°	-0.99	(0±85)°
						$p\gamma$	(1.9 ±0.4)10^{-3}	251	225					
						$n\pi^+\gamma$	(≈1.)10^{-3}	110	185					
						$\Lambda e^+\nu$	(2.2 ±0.7)10^{-5}	73	72					
				$\frac{\Gamma(\Sigma^+\to\ell^+n\nu)}{\Gamma(\Sigma^-\to\ell^-n\nu)} = .04^{+.04}_{-.02}$ ←		$n\mu^+\nu$	(< 0.7)10^{-4}	144	202					
						$ne^+\nu$	(< 0.3)10^{-4}	249	224					
Σ^0	$1(\frac{1}{2}^+)$	1192.54 ±0.10	-4.88 ±.06	<1.0×10^{-14} cτ<3×10^{-4}	1.422	$\Lambda\gamma$	100 %	77	75					
						Λe^+e^-	b(5.45)10^{-3}							
Σ^-	$1(\frac{1}{2}^+)$	1197.41 ±0.09		1.66×10^{-10} ±.03 S=1.3* cτ = 4.95	1.434	$n\pi^-$	100 %	118	193		-.06 ±0.05 S=1.6*	(22±30)°	0.90	
						$ne^-\nu$	(1.25±0.17)10^{-3}	257	230					
						$n\mu^-\nu$	(0.62±0.12)10^{-3}	152	210					
						$\Lambda e^-\nu$	(0.66±0.11)10^{-4}	81	79			For $\Sigma^-\to\Lambda e\nu$, $\frac{g_A}{g_V} = 0.3\pm0.3$†		
						$n\pi^-\gamma$	(≈1)10^{-3}	118	193					
Ξ^0	$\frac{1}{2}(\frac{1}{2}^+)$	1314.9 ±0.8	-6.3 ±.7	2.9×10^{-10} ±.4 S=1.2* cτ=8.85	1.728	$\Lambda\pi^0$	100 %	64	135		-.33 ±0.10			
						$p\pi^-$	(< 0.5)%	237	299					
						$pe^-\nu$	(< 0.6)%	376	323					
						$\Sigma^+e^-\nu$	(< 0.7)%	125	119					
						$\Sigma^-e^+\nu$	(< 0.6)%	117	112					
						$\Sigma^+\mu^-\nu$	(< 0.7)%	20	64					
						$\Sigma^-\mu^+\nu$	(< 0.6)%	12	49					
						$p\mu^-\nu$	(< 0.6)%	271	309					
Ξ^-	$\frac{1}{2}(\frac{1}{2}^+)$	1321.3 ±0.2		1.73×10^{-10} ±.05 cτ=5.20	1.746	$\Lambda\pi^-$	100 %	66	139		-.402±.051 S=1.4*	(6±8)° S=1.3*		
						$\Lambda e^-\nu$	($0.90^{+0.71}_{-0.43}$)10^{-3}	205	190					
						$n\pi$	(< 5)10^{-3}	242	303					
						$\Lambda\mu^-\nu$	(< 1.2)%	100	163					
						$\Sigma^0e^-\nu$	(< 0.3)%	128	122					
						$\Sigma^0\mu^-\nu$	(< 0.5)%	23	70					
						$ne^-\nu$	(< 1.0)%	381	327					
Ω^-	$0(\frac{3}{2}^+)$? ?	1672±1		$1.1^{+0.6}_{-0.5}\times10^{-10}$ cτ=3.3	2.795	$\Xi^0\pi^-$		217	293					
						$\Xi^-\pi^0$		216	289					
						ΛK^-		63	210					

†The definition of these quantities is as follows:

$$\alpha = \frac{2\,\mathrm{Re}(S^*P)}{|S|^2+|P|^2}\,;\quad \beta = \frac{2\,\mathrm{Im}(S^*P)}{|S|^2+|P|^2}\,;\quad \gamma = \frac{|S|^2-|P|^2}{|S|^2+|P|^2}$$

$$\tan\Phi = \beta/\gamma \qquad \tan\Delta = -\beta/\alpha$$

g_A/g_V defined by $M \propto \langle B_f|\gamma_\lambda(g_V - g_A\gamma_5)|B_i\rangle$

* S = Scale factor = $\sqrt{\chi^2/(N-1)}$ where N ≈ number of experiments. S should be ≈ 1. If S > 1, we have enlarged the error of the mean, δx, i.e., δx → S δx. This new convention is still inadequate, since if S > 1, the real uncertainty is probably even greater than Sδx. See text of January 1967 edition.

‡ In decays with more than two bodies, P_{max} is the maximum momentum that any particle can have.

b. Theoretical value, see also data card listings.

c. For comparison with predictions of the $\Delta I=\frac{1}{2}$ rule, see Appendix I.

d. See note in data card listings.

e. This value, not listed in the data cards, was obtained by G. Conforto (Acta Phys. Acad. Hungar. 22, 15 (1967).)

Quantities in italics have changed by more than one standard deviation since January, 1967.

	Symbol (J^P)	$I^G(J^P)C_n$ ⊢⊣ estab. ?— guess	Mass M (MeV)	Width Γ (MeV)	$\pm\Gamma M^2_{(a)}$ $(GeV)^2$	Partial decay modes: Mode	Fraction %	Q (MeV)	p or $p_{max}(b)$ (MeV/c)
$\pi^\pm(140)$	$\pi(0^-)$	$1^-(0^-)+$	139.58		0.019	See Table			
$\pi^0(135)$			134.98		0.018				
$\eta(549)$	$\eta(0^-)$	$0^+(0^-)+$	548.8 ±0.6	2.3 kev ±.5 keV	0.301 ±.000	all neutral	71	See Table	
						$\pi^+\pi^-\pi^0+\pi^+\pi^-\gamma$	29		
$\rho^\pm(765)$	$\rho(1^-)$	$1^+(1^-)-$	755-775	110-140	0.585 ±.095	$\pi\pi$	≈ 100	491	359
						$\pi^\pm\pi^0\pi^-\pi^0$	< 0.2	212	247
						$\pi^+\pi^-\pi^+\pi^-$	< 0.15	212	247
						$\pi^0\gamma$	< 0.4	630	372
$\rho^0(765)$			760-780	90-150	0.590 ±.090	$\eta\pi^\pm$	< 0.8	82	146
						e^+e^-	.0055 ± .0007	769	385
						$\mu^+\mu^-$	.0058 ± .0010	559	370
$\omega(783)$	$\phi^1(1^-)$	$0^-(1^-)-$	783.3 ±0.7 S=1.9*	12.2 ±1.3	0.614 ±.009	$\pi^+\pi^-\pi^0$	≈ 90	369	328
						$\pi^+\pi^-$	seen	504	366
						$\pi^0\gamma$	9.5 ± 0.9	648	380
						η+neutral	< 1.5	234	199
						$\pi^+\pi^-\gamma$	< 5	504	366
						$\pi^0\pi^0\gamma$	< 1	513	368
						e^+e^-	$.0053^{+.0015}_{-.0020}$	782	392
						$\mu^+\mu^-$	< 0.10	572	377
$\eta'(958)$ or X^0	$\eta'(0^-)$	$0^+(0^-)+$	958.3 ±0.8	< 4	0.918 <.004	$\eta\pi\pi$	67 ± 10	131	232
						$\pi^+\pi^-\gamma$(incl. $\rho^0\gamma$)	22 ± 3	679	458
						neutrals (excl. $\eta\pi\pi$)	11 ± 9		
$\delta(962)$	?( )	≥1 ( )	962 ±5	<5 <.005	0.927	$\delta^\pm \to$ 1 charged+neutral(s) ≈ 60			
Seen in only one experiment						$\delta^0 \to$ ≥ 3 charged+neutral(s) ≈ 40			
H(990)	$\phi(A)$?	$0^-(1^+)-$ ⊢? ?	≈990	≲80	0.98	3π	only mode seen	575	440
existence in doubt									
$\pi_V(1016)$ $\to K\bar{K}$	$\pi(0^+)$	$1^-(0^+)+$	1016 ±10 (if resonance)	25 ±5	1.032 ±.025	$K^\pm K^0$	only mode seen	24	110
						$\eta\pi$	< 70	328	342
Resonance, virtual bound state, or antibound state, still not distinguished.									
$\phi(1019)$	$\phi(1^-)$	$0^-(1^-)-$	*1019.3* ±0.6 S = 1.5*	3.4 ±0.8	1.039 ±.004	K^+K^-	47.3 ± 3.2	32	126
						$K_L K_S$	38.9 ± 3.1	24	109
						$\pi^+\pi^-\pi^0$ (incl. $\rho\pi$)	13.8 ± 4.3	605	462
						e^+e^- indic. seen,	< 0.2	1018	510
						$\mu^+\mu^-$ seen,	< 0.5	808	499
$\eta_V(1070)$ $\to K_S K_S$	$\eta(0^+)$	$0^+(0^+)+$	1069 ±10 S=2.3*	~80	1.14 ±.09	$\pi\pi$	< 70	793	516
(Some data still favor large scattering length)						$K\bar{K}$	> 30	76	198
A1(1070)	$\pi(1^+)$	$1^-(1^+)+$	1070 ±20	80 ±35	1.12	3π	≈ 100	651	488
Existence in doubt		($J^P = 2^-$ not yet excluded)				$K\bar{K}$	< 0.25, $G=(-1)^{\ell+1}$ forbids this		
B(1220)	$\rho(A)$? ($J^P = 2^-$ not excluded)	$1^+(1^+)-$ ⊢? ?⊣	1220 ±11	129 ±14 S=1.3*	1.46 ±.14	$\omega\pi$	≈ 100	297	339
f(1260)	$\eta'(2^+)$	$0^+(2^+)+$	1263 ±10	*141* ±13 S = 2.1*	1.57 ±.15	$\pi\pi$	large	984	616
						$2\pi^+2\pi^-$	< 4	705	552
						$K\bar{K}$ indic. seen,	< 2.5	267	388

D(1285)	η'(A)	$0^+(1^+)+$	1285 ±4	32 ±8	1.65 ±.04	$K\bar{K}\pi$ (mainly $\pi\gamma(1016)\pi$) only mode seen		154	304
						$K^0\bar{K}+\bar{K}^0K$		-100	
						$\pi\pi\rho$	not seen	256	356
	$(J^P = 0^-, 2^-$ not excluded$)$								
A2(1300)	$\pi(2^+)$	$1^-(2^+)+$	1305 ±10 S=2.2*	90 ±10 S=1.2*	1.70 ±.12	$\rho\pi$	85.1 ± 2.2	395	410
						$K\bar{K}$	4.0 ± 1.1 S=1.1*	313	424
Some evidence for > 1 meson (see data listing).						$\eta\pi$	10.5 ± 2.1	617	526
						$\eta'\pi$	< 10	207	275
						$\pi^+\pi^-\pi^0$(excl. $\rho\pi$)	< 17	891	615
E(1420)	η(A)	$0^+(0^-)+$	1424 ±6	71 ±10 S=1.2*	2.03 ±.10	$K^0\bar{K}+\bar{K}^0K$	50 ±10	38	157
						$\pi\gamma(1016)\pi$	50 ±10	268	328
						$\pi\pi\eta$	< 88	596	569
	$(J^P = 1^+$ not yet excluded$)$					$\pi\pi\rho$	not seen	384	455
f'(1514)	$\eta(2^+)$	$0^+(2^+)+$	1514 ±5	73 ±23 S=1.8*	2.29 ±.11	$K\bar{K}$	72 ± 12	518	570
						$K^0\bar{K}+\bar{K}^0K$	10 ± 10	128	294
						$\pi\pi$	< 14	1235	744
						$\eta\pi\pi$	18 ± 10	686	624
						$\eta\eta$	< 40	417	522
π_A(1640) → 3π	π(A) ?	1^-[A)+ ?	1654 ±12 S=1.6*	109 ±30	2.73 ±.18	3π	appears dominant	1245	797
						[$\rho\pi$†	< 40]	745	637
						[$f\pi$†	appears dominant]	252	320
not yet a completely established resonance						$K\bar{K}$	< 40	654	658
ρ_V(1650) g→ 2π	ρ(V)	1^+(V)−	1660 ±17 S=1.4*	169 ±30	2.76 ±.28	2π	appears dominant	1385	820
						4π } $\rho\pi\pi$ }	probably observed	1106	773
	$(J^P = 1^-, 3^-, \ldots$ with 3^- favored$)$							615	617
						$K\bar{K}$ indication seen, < 10		668	667
						$\omega\pi$ indication seen		741	638

→The remaining data, --e. g., R, S, T, U mesons--in this mass region are too confused to tabulate.

K^+(494) K^0(498)	K(0^-)	1/2(0^-)	493.83 497.75		0.244 0.248	See Table			
K^0(890)	K(1^-)	1/2(1^-)	892.7 ±0.5 S=1.2*	49.2 ±1.0	0.797 ±.044	$K\pi$	≈ 100	259	289
						$K\pi\pi$	< 0.2	120	216
	$m_0 - m_\pm = 6.3 \pm 4.1$								
K_A(1230) or C	K(A) Not yet completely established. $J^P = 2^-$ not yet ruled out.	1/2(1^+)	1230	60	1.51 ±.07				
K_A(1320)	K(A)	1/2(1^+)	1320	60	1.74 ±.08				
	$J^P = 2^-$ not yet completely ruled out.								
K_V(1420)	K(2^+)	1/2(2^+)	1418.6 ±3.2 S=1.3*	89.1 ±5.1 S=1.1*	2.012 ±.126	$K\pi$	51 ± 3	787	615
						$K^0\pi$	33 ± 3	388	413
						$K\rho$	12 ± 3 S=1.5*	156	318
						$K\omega$	2.4 ± 1	143	304
						$K\eta$	2.1 ± 1	378	482
K_A(1800) or L	K(A) ?	1/2(A) ?	1781 ±14 S=1.7*	72 ±24 S=1.3*	3.17 ±.13	$K\pi$	< 2.3	1167	825
						$K^0\pi$	24 ± 8	772	670
						$K\gamma(1420)\pi$	16 ± 8	245	317
						$K\rho$	10 ± 6	532	632
						Remaining $K\pi\pi$	45 ± 15	1032	807
						$K\omega$	4.8 ± 2	519	624
						$K\eta$	< 1	754	732

→ The following bumps, excluded above, are listed among the data cards: σ(410); ϵ(730); $A2_{I=2}$(1320)→$\rho^-\pi^-$; $\rho\rho$(1410); K_SK_S(1440); η(1600)→4π; ρ(1700)→4π, $K^0\bar{K}$; $R_{I=4}$(1630-1830); η(1830)→4π; ϕ(1830)→$\omega\pi\pi$; S(1930); T(2200); $NN_{I=0}$(2380), U(2380); κ(725); K_V(1080); K_A(I=3/2)(1175); K_A(I=3/2)(1265); K_A(I=1/2)(1280); K_V(I=1/2)(1660).

* Quoted error includes scale factor $S = \sqrt{\chi^2/(N-1)}$. See footnote to Table S.

† Square brackets indicate a subreaction of the previous (unbracketed) decay mode.

	Particle or resonance	$I(J^P)$ — = estab.	Beam π, K (BeV) (BeV/c)	Mass (MeV)	Γ (MeV)	$M^2 \pm \Gamma M$ (BeV^2)	Partial decay modes: Mode	Fraction (%)	p or $p^{\dagger}_{max}$ (MeV/c)	$4\pi k^2$ (mb)
N	p n	1/2(1/2+)		938.3 939.6		0.880 0.883		See Table		
	N' (1470)	1/2(1/2+) P_{11}	T=0.53πp p=0.66	1470	210	2.16 ±0.31	Nπ Nππ [Nσ][a]	65 35 [domin]	420	27.8
	N(1518)	1/2(3/2−) D_{13}	T=0.62 p=0.75	1525	115	2.33 ±0.18	Nπ Nππ [Δ(1236)π] Nη	55 45 [domin] ~0.5	460 414 229 161	23.2
	N(1550)	1/2(1/2−) S_{11}	T=0.66 p=0.79	1550	130	2.40 ±0.20	Nπ Nη Nππ	30 70 small	477 210 434	21.5
	N(1680)	1/2(5/2−) D_{15}	T=0.88 p=1.02	1680	170	2.82 ±0.29	Nπ Nππ [Δ(1236)π] ΛK Nη	40 dom. inel. [?] <1.6 <2.5	567 533 365 218 379	15.2
	N(1688)	1/2(5/2+) F_{15}	T=0.90 p=1.03	1690	130	2.86 ±0.22	Nπ Nππ [Δ(1236)π] ΛK Nη	65 dom. inel. [?] <.13 <1.5	574 540 374 234 390	14.9
	N' (1710)	1/2(1/2−) S_{11}	T=0.94 p=1.07	1710	300	2.92 ±0.51	Nπ	80	587	14.2
	N(2190)	1/2(7/2−) G_{17}	T=1.96 p=2.10	2200	250	4.84 ±0.55	Nπ	30	894	6.13
	N(2650)	1/2(? −)	T=3.12 p=3.26	2650	360	7.02 ±0.95	Nπ	(J+1/2)x=0.45[b]	1154	3.67
	N(3030)	1/2(?)	T=4.26 p=4.40	3030	400	9.18 ±1.21	Nπ	(J+1/2)x=0.05[b]	1377	2.62
Δ	Δ(1236)	3/2(3/2+) P_{33}	T=0.195 p=0.304 $m_0 - m_{++} = 0.45 \pm 0.85$	(++) 1236.0 ±0.6 $m_- - m_{++} = 7.9 \pm 6.8$	120 ±2	1.53 ±0.15	Nπ $N\pi^+\pi^-$	100 0	231 89	91.9
	Δ(1640)	3/2(1/2−) S_{31}	T=0.81 p=0.94	1640	180	2.69 ±0.30	Nπ Nππ	30 dom. inel.	540	16.8
	Δ(1920)	3/2(7/2+) F_{37}	T=1.41 p=1.54	1950	220	3.80 ±0.43	Nπ ΣK	40 seen	741 453	8.91
	Δ(2420)	3/2(11/2+)	T=2.50 p=2.64	2420	310	5.86 ±0.75	Nπ	11	1024	4.67
	Δ(2850)	3/2(?+)	T=3.71 p=3.85	2850	400	8.12 ±1.14	Nπ	(J+1/2)x=0.25	1266	3.05
	Δ(3230)	3/2(?)	T=4.94 p=5.08	3230	440	10.4 ±1.4	Nπ	(J+1/2)x=0.05	1475	2.24
Z	Z_0(1865)	0(?)	p=1.15 K^+p	1865	180	3.47 ±0.34	NK	(J+1/2)x=0.35	579	14.6

Resonance interpretation not established.

Λ	$0(1/2^+)$		1115.5		1.24		See Table		
Λ(1405)	$0(1/2^-)\ S_{01}$	p<0 K^-p	1405	50	1.97 ±0.07	Σπ	100	140	
Λ(1520)	$0(3/2^-)\ D_{03}$	p=0.392	1518.8 ±1.5	16 ±2	2.31 ±0.02	$N\bar{K}$ Σπ Λππ	45±4 45±4 10±1 } S=1.8*	235 258 251	83.6
Λ'(1670)	$0(1/2^-)\ S_{01}$	p=0.74	1670	18	2.79 ±0.03	$N\bar{K}$ Λη	$K^-p \to \Lambda\eta$ seen	410 66	28.5
Λ'(1690)	$0(3/2^-)\ D_{03}$	p=0.78	1690	45	2.86 ±0.08	$N\bar{K}$ Σπ	20 58	429 403	26.1
Λ(1815)	$0(5/2^+)\ F_{05}$	p=1.05	1816 ±2 S=1.3*	74 ±5	3.30 ±0.14	$N\bar{K}$ Σπ Σ(1385)π Λη	63 11 11 ~ 1	538 500 359 346	16.7
Λ(1830)	$0(5/2^-)\ D_{05}$	p=1.08	1827	76	3.34 ±0.14	$N\bar{K}$ Σπ	8 42	547 508	16.0
Λ(2100)	$0(7/2^-)\ G_{07}$	p=1.68	2100	140	4.41 ±0.29	$N\bar{K}$ Σπ Λη ΞK Λω	33 4 < 3 ~ 1 <10	748 699 617 483 443	8.68
Λ(2350)	0(?) —	p=2.29 Seen in total c. s.	2350	210	5.52 ±0.49	$N\bar{K}$	(J+1/2)x=0.7	913	5.85
Σ	$1(1/2^+)$		(+)1189.5 (0)1192.5 (-)1197.4		1.41 1.42 1.43		See Table		
Σ(1385)	$1(3/2^+)\ P_{13}$	p<0 K^-p	(+)1382.2±0.8 S=1.6* S=4.8* ! ⟷(-) 1388.0±3.0	(+)37±3 S=2.1* (-)38±8, S=3.7* !	1.92 ±0.05	Λπ Σπ	91±3 9±3 S=1.4*	208 117	
Σ(1660)	$1(3/2^-)\ D_{13}$	p=0.72 Decay modes of these two states not separated yet.	1660	50	2.76 ±0.08	Λ(1405)π $N\bar{K}$	large small for both	197 400	29.9
Σ(1690)	1 (? ?) —	p=0.80	1690	120	2.89 ±0.19	Λπ Σπ	large not disentangled	455 395	25.1
Σ(1770)	$1(5/2^-)\ D_{15}$	p=0.95	1767 ±4 S=1.5*	95 ±12 S=2.3*	3.13 ±0.16	$N\bar{K}$ Λπ Λ(1520)π Σ(1385)π Ση Σπ	46 15 15 14 0.5 ~ 1	497 519 190 317 140 463	19.4
Σ(1910)	$1(5/2^+)\ F_{15}$ —	p=1.25 A small effect. Not established beyond all question.	1910	60	3.65 ±0.11	$N\bar{K}$ Λπ Σπ	8 10 3	612 619 568	12.9
Σ(2030)	$1(7/2^+)\ F_{17}$	p=1.52	2030	120	4.12 ±0.24	$N\bar{K}$ Λπ Σπ ΞK	11 36 9 < 2	700 700 652 412	9.92
Σ(2250)	1(?) —	p=2.04 Seen in total c. s.	2250	200	5.06 ±0.45	$N\bar{K}$	(J+1/2)x=0.3	849	6.76
Σ(2455)	1(?) —	p=2.57 Seen in total c.s.	2455	~ 140	6.03 ±0.34	$N\bar{K}$	(J+1/2)x=0.26	979	5.08
Σ(2595)	1(?) —	p=2.95 Seen in total c.s.	2595	~ 140	6.73 ±0.36	$N\bar{K}$	(J+1/2)x=0.26	1064	4.30

Ξ		1/2(1/2$^+$)	(0)1314.7 (-)1321.2		1.73 1.75	See Table		
Ξ(1530)		1/2(3/2$^+$) p-wave	(0)1528.9±1.1 (-)1533.8±1.9	7.3 ±1.7	2.34 ±0.01	Ξπ	100	145
Ξ(1815)		1/2(?)	1815 ±3	16 ±8 S=2.2*	3.29 ±0.03	ΛK̄ Ξπ Ξππ [Ξ(1530)π] ΣK̄	~65 ~ 10 ~25 [~20] <2	391 409 351 220 299
		Branching ratios poorly known.						
Ξ(1930)		1/2(?)	1930	140	3.72 ±0.27	Ξπ ΛK̄	seen	501 504
		Needs confirmation.						
Ω$^-$		0(3/2$^+$)	1672		2.80	See Table		

Ξ

Ω$^-$

APPENDIX B

by G.H. Stafford, Rutherford Laboratory,
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Take two machines as typical: (a) CERN PS (strong focusing)
(b) Nimrod (weak focusing)

CERN PS

Maximum energy	28.5 GeV
Protons per pulse	$\sim 10^{12}$ internal
Pulse repetition rate	12 per minute

Remarks: There is an agreed improvement program which will double the repetition rate and increase the circulating beam to $\sim 10^{13}$ protons per pulse.

Secondary beams are available both from internal targets and external targets. It is usually possible to have several experiments running on the machine at any time.

Nimrod

Maximum energy	7 GeV
Protons per pulse	1.5×10^{12} internal
Pulse repetition rate	23 per minute

TABLE XX. NIMROD BEAM FIGURES

Particle	Beam	Yield per pulse with 1.5×10^{12} protons circulating with $\Delta P/P = 1\%$ FWHH	Momentum
π^-		1.5×10^6	1.5
π^+	2 tank separated beam	2.5×10^6	1.5
K^-		2.5×10^3	1.5
K^+		1.5×10^4	1.5

Typical beams

The number of usable particles can vary enormously because usually it is necessary to make many compromises to fit in all the experimental beams. Table XX gives some actual Nimrod beam figures. Yields are obtained at the end of the beam line, so decay has been taken into account.

THEORY OF SOURCES

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Abstract

THEORY OF SOURCES. 1. Introduction; 2. Source theory; 3. Application of the source theory to quantum electrodynamics; 4. Strong interactions; 5. Conclusion.

1. INTRODUCTION

I would like to propose a new theory of particles. That may seem like a rather special topic but the particles I have in mind include the photon, which gives one a new approach to quantum electrodynamics. They include the graviton so that we can also deal with gravitational phenomena and the elusive, if uninteresting, quantum corrections to it. Also, though it has not been considered, the methods I am going to outline should be helpful in connection with the non-relativistic problems of many-particle physics, so that in a sense, perhaps, we come close to that feeling of universality which has been the inspiration for this meeting.

However, despite the generality of the theory and its variety of applications, it remains true that it is high-energy physics and the problems of high-energy physics that mark, so to speak, the stimulus for the construction of this theory and so I want first to outline what I see as the difficulties of the conventional approaches and, since the conventional approaches to high-energy physics form a continuum, I will try to polarize this by giving what I consider two extreme viewpoints within which antipodes my own approach will be located.

The two extreme positions, in my opinion, are operator field theory and S-matrix theory. Operator field theory is very familiar; its foundations are the basic ideas of quantum mechanics and relativity. That in fact will be characteristic of all theories we will discuss, but operator field theory is characterized by the assumption that a detailed space-time description of phenomena is possible and, corresponding to that, the fundamental dynamical variables in terms of which this mechanical dynamical system is characterized, are the field-operator functions of space and time. The specific formal operator field theory that I am describing here is one which is based fundamentally upon Hamiltonian or Lagrangian formulations and, as such, a specific Lagrangian is made use of (a function of the fundamental dynamical variables) and so there is a specific dynamics or, as Professor Salam refers to it, a distinguished dynamics. In other words, a specific form of a Hamiltonian or Lagrangian characterizes by hypothesis a particular system of fields as the fundamental dynamical variables. The idea of local action is that there is a Lagrange function and that operator products at the same point are the fundamental structures out of which the

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dynamics is constructed. The particle is then a secondary, derived concept from this point of view. The particle is a stable or quasi-stable excitation of the fields, that is, as in any quantum mechanical system, one looks for the quasi-stationary states and finds them appropriately in terms of the dynamical variables which, by themselves, do not necessarily have any immediate physical interpretation. Since experimentally it is the particle that we deal with and not this fundamental description in terms of fields, to make contact with experiment it is necessary to carry out a transition from the initial fundamental field theory to the phenomenological or the physical particles. That leads to the concept of renormalization because we know that a field operator for an interacting system does not create simply a particle, it is just a way of producing a localized excitation and it will generate every combination of particles with the same set of quantum numbers. If there is a particle with that set of quantum numbers it will generate it, but only for a fraction of the time and so, if one is interested in that particle, then the process of renormalization must be carried out, which is to change the emphasis from the field which generates the particle for only part of the time to the particles supposed always to be in existence.

Renormalization is more fundamental than it is usually considered in connection with perturbation theory which is just a particular mathematical technique, an important but limited one, and it is very familiar that, when one proceeds to convert this general formulation into specific concrete terms, one usually falls back on perturbation theory, and then divergences appear. I leave open the question whether the original theory is mathematically consistent and whether it is divergent or not, but certainly divergences do appear as soon as perturbation theory is used. It is very familiar that in electrodynamics all the divergences are compressed into a very few renormalization constants and therefore the actual physical predictions of the theory are entirely finite and I need not remind you in what excellent accord they are with experiment. But when we come to strongly interacting systems, which is, of course, of principal interest, then it is simply impractical to carry out the transition from the field to the particle point of view. For weakly interacting systems such as one meets in electrodynamics it is usually taken for granted that the transformation from field to particle is not a large one, that is to say, that the fields really are rather closely associated with the particles of interest, and for that reason perhaps perturbation theory is roughly relevant, and at least the logical connection between the particle and field is not very remote. But it seems very plausible that for strongly interacting systems we have no way of even being sure that we have the correct connections between particles and fields; in other words, specific hypotheses of the particle structure are required. We must assume, for example, that it takes the products of, say, three fundamental fields or two fundamental fields perhaps, to make this or that baryon or meson; this is a hypothesis and it is upon these hypotheses that the further development of the theory is based. In other words, I make here the rather serious objection that in order to be able to talk about the physically interesting phenomena at all one must begin with a speculation about how these particles are formed. And so I have the very logical, the very strong logical, objection to this formulation (quite apart from its practical difficulties) that it mixes together phenomenology in the sense of how the observed properties of the particles are correlated with an

unwarranted speculation (which is possibly correct, but the probability is small) about the actual nature of the particles.

Contrasting with this is the second point of view, that of S-matrix theory, which arose not unnaturally as a reaction to field theory and its difficulties. Again, it is based upon the faithful servants of quantum mechanics and relativity, but here we have the complementary point of view; the description is now embedded in momentum space and space time is made use of not at all or perhaps only in a sort of macroscopic correlation limit. The point of view is not an operator one but rather the S-matrix is taken as fundamental and the labels of states are the variables of various particles. In principle, since one is not allowed to use a time description, it is presumably only stable particles with which one is concerned, those that have no time scale attached to them. The principles that have entered into this theory are basically unitarity plus analyticity. These are very general principles, and certainly no specific dynamics in the analogous sense of the Lagrangian or Hamiltonian field theory is used, but one hopes, that some requirement of self-consistency between the various particles will eventually distinguish our world from the infinity of possible other worlds. This is an interesting idea; I have no way of judging how successful it actually has been. But in a way also I would draw an objection to this, that again, in order to be able to discuss phenomenologically the properties of the particles, there is built into this procedure a hypothesis, namely that the particles have no underlying structure. The self-consistency or bootstrap idea is the notion that the observed particles are fundamental, that they create each other and that there is no deeper level in terms of which the particles are to be understood or explained. And that again is a hypothesis. It may be correct but I assume that the probability of that is also small. In any event, such a hypothesis should not, if possible, intervene in the day-by-day task of correlating data and arriving at a phenomenological theory in terms of the useful correlation and description of phenomena.

2. SOURCE THEORY

It is on the basis of these two extreme positions that we come to an intermediate position which is the idea of source theory. It is based again upon quantum mechanics and relativity. It is an intermediate position in the sense that while it is formulated in space-time (it recognizes that measurements are space-time operations), nevertheless it leaves open the question of to what extent an extrapolation down to a point is physically meaningful. In short, it is left open to test experimentally how far down in space and time the possibility of a detailed description can be pushed. Of course, it also makes use of momentum description (that we can hardly avoid), but it leaves open the question of how far throughout the totality of momentum space this is to be applied; that is to say, it is a more experimentally motivated theory, it does not take for granted that infinite momentum space is available from the very beginning, it is an extrapolation upward in momentum, downward in space and time, always testing through experimental comparison whether these extrapolations are in fact valid.

The basic notion is the source and the source is introduced as an attempt to incorporate into the mathematical formulation, as much as possible, what the practising experimental physicist does and to try to find a correlation between actual experimental practice and mathematical theory. We try to abstract what is relevant and to put that into the formulation of the theory. The source is introduced as an abstraction of those realistic collision processes which in fact one either can, or for unstable particles must, make use of in order to generate the particles in which one is interested. Perhaps this theory has arisen at this time as a natural reaction to the totally new state of affairs. We are familiar with the earlier stages of the development of quantum mechanics and relativity in which we were concerned only with a few stable particles. Now the situation is entirely different. The particles we are concerned with are almost all unstable. And instead of regarding this as an aberration I choose to regard it as a definition of particle, that in general, if we are interested in the particle, we must create it to study it. This is certainly true of the high-energy particles we are concerned with; they are not normally found but must be created. The creation of the particles of interest is the first stage of an experiment and in the final stage the particle must be detected and, in a general sense, that is the annihilation of a particle. The detection methods are invariably the transformation of that particle into other forms of matter more amenable to experimentation, the detection process is the destruction of the particular particle of interest. The purpose of the source theory is rather two-fold; firstly the sources, since they are associated with particle creation and annihilation acts, serve as identifying instruments for the particle in question when it enters into the theoretical framework. Secondly, the source of a particular particle unites all of those dynamical properties that various mechanisms for creating that particular particle have in common. However, it does not embody the individual peculiarities of this or that mechanism. But nevertheless, when the whole theory is constructed, it must be verified that the various mechanisms which can create a particle indeed have in common those starting points, those hypotheses of a notion of the source. So in that sense it is a very useful way of talking about dynamics because it incorporates from the very beginning a survey of all dynamical processes and then it begins to put in the details step by step.

What are the constructive principles that we will make use of in the formulation of this theory? They are basically two. Firstly causality, which is the idea that in fact the experimentalist must create the particles and then he must let time elapse as the particles get near each other and interact and then more time must elapse as the particles separate and are eventually detected. This idea of being able to control the space-time arrangement, to be able to make sure that one act precedes another act and so on, is the basic idea of any physics and it occurs here as a creative principle in the sense that we take initial causal arrangements which are so arranged because we can produce them through our experimental knowledge or abilities and use this as a process of extrapolation, always testing, of course, whether the extrapolation and the new physical phenomena that it predicts are indeed in accord with experiment. The second principle is the idea of space-time uniformity, by which I mean that when a particle is created by one source and then travels and is detected by another source, the initial coupling inferred from a certain causal arrangement is supposed

to be more general than that original arrangement. The sources can be moved around and the coupling will still continue to be meaningful and so give logical connections between phenomena in which particles actually do exist as real particles and other phenomena in which they are not actually present, in which perhaps not enough energy occurs to actually create the particles. I also mean that sources occupying different regions of space are not independent physical phenomena, that they are simply different manifestations of the same underlying laws of physics.

The development of the theory starting from these constructive principles begins initially with the idea of particle as fundamental and the creation and detection of the particles, so to speak, as unanalysed acts but upon which is superimposed the self-consistency requirement. Out of this emerges, as a derived concept, a field theory; it is a phenomenological field theory because the fields are attached to the observed particles, and it is most interestingly a non-operator field theory. There are field equations in this theory but there are no commutation relations and no operators. Most of the complicated mathematical difficulties of the initial operator field theory attest to the enormously difficult concept of operators which are functions of points and operators multiplied together at points.

It is a phenomenological theory but it is a physical one which describes how couplings occur through the transport or the exchange of particles. As such it is non-local. The word phenomenological as I use it here, I think, does not have the same associations for me that it did for Professor Heisenberg. I regard this as a phenomenological theory in the sense that we are dealing with the actual phenomena, but it is a creative theory in the sense that different phenomena are connected by fundamental principles. The procedures used, however, are flexible and reflect the complexity of the physical problem and the amount of information available. If one talks about weakly interacting systems, and I think of electrodynamics or gravitational phenomena perhaps, then one begins with a rather specific primitive interaction, as I call it, by which I mean that there is essentially no detail about the inner structure of this primitive interaction; it is represented in fact by a local product of fields. By the repetition of processes in which particles are exchanged among sources one elaborates the dynamics.

The dynamics is not stated once and for all, but it grows organically by considering more and more complicated processes, as a result of which one comes closer and closer to the actual dynamics which is, of course, non-local. For example, beginning with the simplest properties of photons and electrons, one can, out of such a primitive interaction which refers to an experimentally conceivable, if idealized, situation, reconstruct quantum electrodynamics (as has, of course, been obtained first by operator field theories with all the well-known and verified experimental consequences) without any reference to either renormalization or divergences, and in the language of a numerical field theory. If one begins with the graviton (which is a rather more hypothetical beginning, but one can adduce rather strong arguments for the existence of the graviton indirectly) one obtains a reconstruction and a quantum extension of gravitational dynamics – a quantum extension because one is now enabled to discuss, as I think one could not really before, some of the quantum corrections analogous to electrodynamic corrections to gravitational phenomena. It is a purely conceptional problem, of no practical interest, but it has been a vexing question for very many years; I would like to think that it is now solved

in principle. When we turn, however, to more interesting strongly interacting systems then, of course, we have no possibility of thinking that we know a simple dynamical mechanism (as we do in electrodynamics) out of which all the complicated machinery of interacting systems really evolves. We have to recognize the fact that there is an unlimited number of particles with all kinds of complicated interactions and we must begin in a very modest way. And so I suggest that we first turn to low-energy phenomenology, the known properties in the relatively low-energy domain of pions, nucleons, mesons, and so on, and try to codify what is known about these particles in terms of the notion of partial symmetry.

It is well known that in high-energy physics there are many conservation laws, but they are approximate. There are symmetries or conservation laws valid only in certain limited senses and one must, of course, in any formulation use them as a basis for codification or systematization of the phenomena. But I suggest doing this by means of creating a primitive interaction out of which the more elaborate theory will evolve. The important thing is that it be done entirely at the phenomenological level and no hypotheses about inner particle structure be involved. From there one tries to extrapolate to higher energies; one does this by beginning with the local couplings and using them as primitive interactions from which the non-local phenomena actually emerge. When we turn to very high energies, and this is really a totally untouched subject from this point of view, there is a natural analogy that I think I ought to point to. I like to think that what we are doing in this field (as we have done throughout all physics) is observing a certain kind of physical system (molecule, atom, nucleus, whatever it may be), initially beginning from the outside, observing only certain general features. As we move up in energy, we come closer, we explore more and more details. For example, consider the stage of nuclear physics in which at first we recognize only charges and magnetic dipole moments. We come closer and we begin to learn something about electric quadrupole moments. We come closer still and we get higher and higher moments. I would like to suggest that there is an analogy with high-energy physics in a sense that, as we go to higher energies we begin to recognize particles of higher and higher spins. The interactions which these higher and higher spin particles carry are, mathematically speaking, more and more singular just as multipole fields are more and more singular. But we know very well what the story is with multipole fields and multipole expansions; they are useful, if only a few are involved (low energy). If we get very close to the system or if we get inside it, the multipole expansion is useless; in fact, the series of multipole moments diverges. It is simply a way of writing the actually well-defined interaction of the physical systems which have a structure, and when we penetrate seriously into the structure the multipole expansion is useless. And so I suggest that we are meeting the same situation again; the present particles, the few low-energy low-spin particles we presently know about are, so to speak, analogous to low multipole moments of the more fundamental interaction. As we move up in energy we explore closer and closer and the multipole expansion will be useful for a while; eventually it will fail and must be replaced by the more fundamental dynamics in which no longer the particles which we observe, but rather the fundamental structures out of which they are constructed, are the instruments. But I submit to you that we have no right to think that we can guess this more fundamental interaction without the clues which will

be obtained by moving ever inward, ever upward in energy, correlating the data until we have enough information to actually carry out that quantum jump Professor Heisenberg describes. The jump is usually not from an infinitely remote position but from as close as possible to the final goal. It can only be made by finding a formalism which does not have incorporated into it a hypothesis about the more fundamental structure but does enable one to make contact with it in the sense that it is flexible enough in its framework, and that is what I like to think the theory of sources is.

I have said before that, although the phenomenological theory of sources is designed to correlate data and to be as independent of speculations about inner structure as possible, nothing prevents one from speculating independently. What goes on in the innards of the particles we observe may be a manifestation of an entirely new form of matter, not merely fractionally electrically charged but also magnetically charged, and perhaps out of these incredibly strong but, amazingly enough, electromagnetic forces, the actual nature of strongly interacting systems is to be understood. This is a speculation and I have absolutely no idea of how to bring into contact this speculation at the one extreme and the evolving phenomenology of the actually observed particles at the other. But it indicates the eventual goal.

Now we come to the discussion of the formalism of the source theory. Figure 1 represents a realistic collision process whereby particular particles of interest, indicated with heavy lines, are either created or annihilated. The source theory recognizes that if we are concerned only with the particle and if a collision is there simply to produce it or detect it, everything else that enters into the collision serves only the purpose of supplying the properties of that particle. So we abstract from everything and we indicate that, as far as what we are interested in is concerned, a particle emerges or disappears within a more or less limited space-time region. (We have some ability to control this – there is some control over the various momenta that the particle will have.) There is a source in space and time describing the effectiveness of carrying out some kind of space-time localization – that is certainly possible – and, to some extent, the localization in energy and momentum. Certainly we can do that, but we leave open the question of how far it can be pushed. To the extent of our concern with that particle and only that particle we begin with the vacuum. In other words, we disregard everything else and recognize then that the source operates. It is a realistic process, but for our present purposes it simply creates or annihilates a particle. Consider

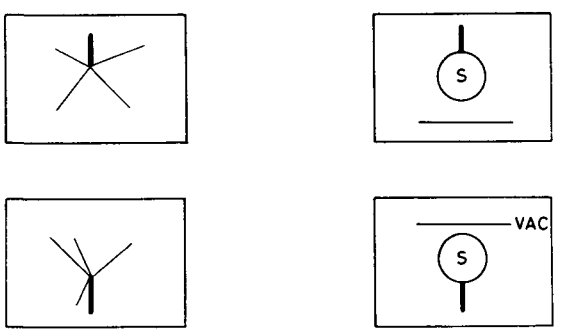

FIG. 1. Collision process.

a particular case, a very simple one, a spin zero stable particle, with no properties other than mechanical, and a weak source which at most can create a single particle which is specified by a momentum p. The subscripts on the zero which represents the vacuum state simply indicate the causal arrangement, the time sequence. We are talking about the creation of the particle and so initially (before the source operates) we have a vacuum. We then create the particle and for this there is a quantum mechanical probability amplitude. The quantum mechanical probability will be proportional to a certain invariant element of momentum space – so this enters as a kinematical factor and the rest is simply the definition of the source. (I use the symbol k to refer to spin zero specifically.)

$$\langle 1_p | 0_- \rangle^k = i\sqrt{d\omega_p}\, K(p) \qquad d\omega_p = \frac{1}{(2\pi)^3}\frac{d\vec{p}}{p^0}$$

But the notion of quantum mechanical complementarity is that the effectiveness of the source in the momentum language is complementary in a four-dimensional sense to the effectiveness of the source in terms of the description of space-time localizability.

$$K(p) = \int (dx)\, e^{-ipx} K(x)$$

In fact very little of the ideas of quantum mechanics is really needed beyond this and some of the general composition properties of probability amplitudes. Finally, there is the reversed process, the annihilation of the particle and the fact that the source is now being used as a sink; instead of emitting the momentum p it absorbs the momentum p.

$$\langle 0_+ | I_p \rangle^k = i\sqrt{d\omega_p}\, K(-p)$$

The appearance of the minus sign involves no hypothesis about time reflection or anything of the sort. One will recognize later from the way in which it is actually used that if we make use of any other assumption we violate translational invariance in space and time. So in fact it is simply space-time uniformity which demands that. This is how the definition of the source is introduced in a simple context where there is only a weak source. We can then go on and consider the description of the first and simplest experimental arrangement which is a beam of particles.

As the experimentalist produces them, a number of particles may be present at a given time and they are non-interacting, not because we mentally switched off any interaction but because we have arranged the experimental circumstances so that the particles are far enough apart and do not interact. The abstraction of this is indicated in Fig. 2. We begin with nothing, we create by a strong source, which is effectively just a weak source acting many times, those particles we are interested in on the condition of non-interaction and then we detect them again. We may graduate this into an act of emission, a time of propagation and an act of

detection and we put together the two basic acts of the emission and absorption as this is generalized to a strong source and the generalization is in terms of an exponential structure.

$$\langle 0_+ | 0_- \rangle = e^{iW}$$

I think it will be recognized that the exponential structure is the familiar way of describing a number of physically independent processes; we are just multiplying them and if there is an unlimited number of them, an exponential sum arises.

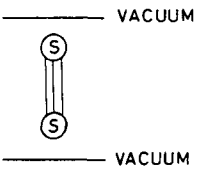

FIG. 2. Non-interacting particles.

In fact, the structure of W just puts together in a space-time language these individual acts. There is a reference to two sources; one will act individually as an emission source, the other as a detection source. But they are all part of one general mechanism and all put together into the general framework of source. W is symmetrical because it is a quadratic form and so the matrix that occurs between states is effectively symmetrical. In the causal arrangement with which we begin we infer what this propagation function is; it has the form

$$W = \frac{1}{i} \int (dx)(dx')\, K(x)\, \Delta_+(x - x')\, K(x')$$

$$\Delta_+(x - x') = \Delta_+(x' - x)$$

and

$$\Delta_+(x - x') = i \int d\omega_p e^{ip(x - x')} \quad x^0 > x^{0\prime} \quad p^2 = -m^2$$

which, you may recognize, puts together the square root of the invariant momentum amplitude and the plane wave that probes the emission source to pick out the effectiveness of emitting a certain particle, and the second square root factor and the plane wave that probes the detection source to measure its effect in detecting the particle. These two factors come together which is the origin of what is known as a propagation function, but the word propagation as such has entered nowhere. All that has been used is the fact that the particle that leaves the emission source is the same particle that hits the detection source; nothing has happened to it. And so one arrives in a very elementary kinematic way at the structure of a very familiar function starting out from these quite simple quantum mechanical relativistic notions. I mentioned an important physical requirement.

This vacuum-to-vacuum probability amplitude describes the whole process. We begin with the vacuum, we end with the vacuum and we describe complete acts of emission and absorption. But this quantity by itself has a physical significance; it is a vacuum-to-vacuum probability amplitude and its absolute square expresses the physical probability that if we begin with the vacuum we should end with the vacuum despite the intervention, the disturbance, produced by the sources. And as such this must be a number less than one.

$$|\langle 0_+|0_-\rangle^k|^2 = \exp\left(-\int d\omega_p |K(p)|^2\right) \leq 1$$

This particular structure meets all those requirements. One instantly checks that it has an exponential form, it is certainly less than or equal to one and it is qualitatively correct in that it is less than one by a measure of the extent to which the source does create a particle and the exponential is just a familiar representation of the multi-particle creations that may go on.

One may simply erect the whole theory of non-interacting particles of any spin on the same basis.

3. APPLICATION OF THE SOURCE THEORY TO QUANTUM ELECTRODYNAMICS

Let me turn to some interesting special features which occur for familiar particles. Consider the photon. To represent a unit spin we must use a vector, and so the coupling will necessarily be of the form

$$W = \frac{1}{i}\int (dx)(dx')\, J^\mu(x)\, D_+(x - x')\, J_\mu(x')$$

$$|\langle 0_+|0_-\rangle|^2 = \exp\left[-\int d\omega_p J^\mu(p)^* J_\mu(p)\right]$$

The exponential quadratic structure is just the statement of the basic act of the emission, propagation and absorption acts endlessly iterated without interaction. The propagation function is here called D_+ just as a reminder that we are talking of massless particles, the vector is the statement that we are interested in spin one. Now suppose we compute the vacuum persistence probability. We shall have exactly the same structure as for spin zero but now it contains a reference to the vectorial nature of the source and the indefinite matrix of space-time. Now we have something which by itself is certainly not less than one. If the vectorial source is unlimited, the exponent could be positive or negative and so we shall have a violation of quantum mechanical positive definiteness which I take to be a very basic requirement in the structure of the theory. It is necessary therefore to restrict the source so that the time component never exceeds the space component. It is also necessary to do that to have a proper account of spin; a necessary and in fact sufficient restriction is that the

scalar, which can be constructed in the only possible way given the ingredients available, be equal to zero

$$p_\mu J^\mu(p) = 0$$

or, more familiarly in space and time, $\partial_\mu J^\mu(x) = 0$. This must hold and then in fact we satisfy all quantum mechanical restrictions and we correctly come back to just those modes of excitation which are the two transverse polarizations of the photon. The important thing, however, is that we arrive at this condition on sources out of general quantum mechanical restrictions. We therefore have a general property of sources for photons which must be characteristic of all possible mechanisms. If the source has a restriction, then every possible realization of the source must have that restriction and so we are uncovering a general law of physics. We know what it is — it is the conservation of charge. We are here reversing the more familiar discussion, the existence of the photon is now taken as the basis of the inference that a property like charge which satisfies conservation laws must exist in nature. To illustrate the fact that this is not merely phenomenology, that we are not just describing the fact that there are photons, I mention the constructive principles of, in particular, the notion of space-time extrapolation. We begin with sources in time-like, or actually, for the photon, in null arrangement — but let us say time-like, it is the more general situation — and we end up with a coupling with the quantity W = JDJ. Nothing in that space-time structure tells us that the component sources have to be in any particular arrangement. Therefore give it a wider validity, let it be meaningful also if the source is made up of two component parts, for example, which are in space-like relation (Fig. 3). These sources may in particular be considered in the limit where they vary very slowly with time, so we are, so to speak, roughly dealing with the situation where two static charge and current distributions are existing simultaneously and are interacting with each other. If we look at the structure of W, we will find that under those conditions of quasi-static sources it reduces to a single time integral of a quantity which is clearly identified from the connection between probability amplitudes or wave functions, as the energy that this system possesses under those particular quasi-static conditions.

$$\langle 0_+ | 0_- \rangle^J = \exp\left(-i \int dx^0 E\right)$$

where energy E is

$$E = \frac{1}{i} \int d\vec{x}\, d\vec{x}' \, \frac{J^0 J^0 - \vec{J} \cdot \vec{J}}{4\pi |\vec{x} - \vec{x}'|}$$

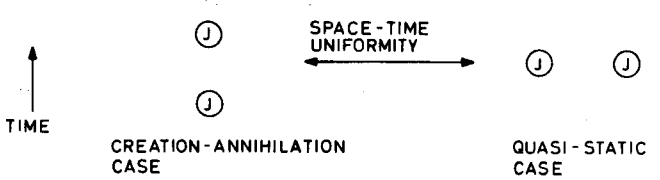

FIG. 3. Source made up of two component parts in space-like relation.

You will recognize the Coulomb and the Ampèrian laws of interaction of static or quasi-static charge distributions, derived here not from Maxwell's equations – we have not seen them yet – but from this notion of space-time extrapolation beginning with the properties of the photon as basic, representing them by couplings between sources, mediated by the photon, giving that coupling a wider space-time significance, extrapolating it here in space-time arrangements and arriving at the interaction laws of Coulomb and Ampère. We may express that a little more generally in the following way. We have begun with action at a distance, we may say the sources are coupled at remote space-time intervals; we could, however, ask whether that could be replaced by a local action – which of course is the introduction of the field concept except that, since we are not dealing with operators but numbers, it is a numerical field – and introduce the field just as one does it in classical physics. We will, so to speak, add a test source into a pre-existing arrangement of sources, and we will add a very weak source δJ^{μ}. That source, of course, must by itself – since it is superimposed on those already existing – satisfy the necessary restrictions that any photon source has to obey; it must be a vector which is divergenceless:

$$\partial_{\mu}\, \delta J^{\mu} = 0$$

If we ask how W alters, it will be linear in this new test source and there will be a quantity which measures the effect on that source of the pre-existing sources:

$$\delta W = \int (dx)\ \delta J^{\mu} A_{\mu}(x)$$

It is, of course, a vector potential, and we recognize immediately that the vector potential so defined is not uniquely determined, because δJ^{μ} is not arbitrary but subject always to the conservation condition. Therefore, A is arbitrary to the extent of an additional gradient which will have no effect on integration by virtue of the divergenceless condition of δJ^{μ}. In short, we recognize that vector potentials are in principle subject to arbitrary gauge transformations.

$$A_{\mu}(x) \rightarrow A_{\mu}(x) + \partial_{\mu}\lambda(x)$$

What is A? From the expression W we can read it directly, and we find

$$A_{\mu}(x) = \int (dx')\, D_{+}(x - x')\, J_{\mu}(x') + \partial_{\mu}\lambda(x)$$

The arbitrariness is restricted somewhat since, if we take the divergence of A_{μ}, then, since the divergence of J_{μ} is zero (no violation of that is thinkable), we find

$$\partial_{\mu} A^{\mu} = \partial^{2}\lambda(x)$$

and since the propagation function satisfies

$$-\partial^2 D_+(x - x') = \delta(x - x')$$

as an expression of the fact that energy and momentum are related by the d'Alembertian equation, we find, upon introducing $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, the Maxwell field equations:

$$\partial_\nu F^{\mu\nu} = J^\mu$$

derived from the starting point of photons and sources.

Once we have a local description, fields being introduced as derived or secondary quantities, we want to recognize the fundamental significance of the quantity W which I have so far refrained from giving a name; action is the name. Let us recognize various reformulations of this structure W. We begin with action at a distance; source, propagation function, source.

$$W = \frac{1}{2}\int (dx)\, J^\mu D_+ J_\mu = \frac{1}{2}\int (dx)\, J^\mu A_\mu = \frac{1}{4}\int (dx) F^{\mu\nu} F_{\mu\nu}$$

All these are equivalent by virtue of the interrelations of the definitions of the fields. Also

$$W = \int (dx)\,[J^\mu A_\mu + \mathscr{L}]$$

where

$$\mathscr{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} = -\frac{1}{2} F^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

The point is the following. Think how W depends upon the variables; think of J, the source and the fields as being independent. Suspend your information about the connection that actually exists. Then the variation of W will consist first of the variation of J times A and then there will be the additional terms that come from varying the fields wherever they are. If only the vector potential is used we get

$$\delta W = \int (dx)\,[\delta JA + \delta_A(\mathscr{L} + JA)]$$

But remember the definition of vector potential; it is the expression of the way in which the quantity W depends upon J and so

$$\delta_A(\mathscr{L} + JA) = 0$$

In other words, we derive a principle of stationary action. So we have an action formulation, at least in the limited physical circumstances of the existence of a local Lagrange function, that is to say of a local function of fields, which, however, we must expect to be eventually superseded by non-local interactions, by non-local Lagrangians as the dynamics develops. How does the dynamics develop? It develops by first extending the source concept. Let us think in terms of the electron. Our starting point is what I now call a simple source (Fig. 4). I use the symbol η for electron source, but I ask you to remember what this circle with the symbol in it stands for. It is an idealization of the realistic collision process in which electrons are generated. And how are they generated? Either by perhaps a beta decay process or more simply by a collision in which an initially slow electron is made an energetic electron by some kind of collision act. In other words, the process of creating the particle is inevitably abrupt and we know that the one dynamical feature of electrons and the electric charge they carry is that an accelerated charge radiates photons.

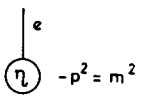

FIG. 4. A simple source.

In short, the same physical act that creates an electron by some kind of collision process unavoidably can and will create an electron and a photon. We can avoid that only by a control of energy and momentum impossible in practice. The point is particularly acute here because the photon has no mass, and photons of arbitrarily low energy exist, but nevertheless I point to this as a general feature. A collision that can create a particle of certain quantum numbers can in principle (given enough energy and momentum) create any other combination of particles with the same over-all quantum numbers if indeed those particles have knowledge of each other dynamically. If they do not, then nothing of this kind happens. But the photon is intimately related to the electric charge that the electron carries, and so, while in principle it would be possible to separate the two acts of the creation of an electron and an electron and a photon, in practice it would be madness because we would then sever the dynamical connection that actually exists between the photon and the electron. And so therefore we are led to extend the concept of source, saying that under those conditions in which more energy is available under the same set of quantum numbers, the electron <u>and</u> the photon can be created. On this qualitative basis one can erect the whole of quantum electrodynamics. If we say this, if we introduce the idea of extended source, then we have a new coupling. We have now a source that creates two particles which can be detected by <u>their</u> respective sources. So we now have cubic couplings among sources. We do not want to regard a simple source and an extended source as different entities; we want to say that the only thing that is different about them is the purely accidental feature of how much energy is available. If enough energy is available to create a photon, it will be created. If it is not available it will not be created. But nothing else distinguishes the sources and so we try to unify them.

Consider Fig. 5 which suggests the unity of the sources. An electron source has a particle travelling to it. This expresses the fact that it is going to detect that particle, therefore the lower extended electron source should also have a particle travelling from it. But that particle, of course,

is not a real particle. We are now extending the particle concept off the mass shell, as the language goes, which simply means, we are taking the same mathematical structure and continuing to give it a meaning in its space-time form. But it no longer has the same meaning in terms of particle structure. The point is that only by this symmetry could we then consider another arrangement in which one source is simple, detecting a real electron, and the other source becomes extended, emitting an electron and photon. I have added symbols like A and ψ to express the fact that the coupling is after all in terms of the idea of more or less local action and is going to be expressed not in terms of the sources, which may be infinitely remote, but rather in terms of the fields, which indicate how the action of the sources is brought into this more or less limited space-time region. This is not, however, an operator field theory although fields <u>are</u> involved because we are talking about particles with certain kinematical properties

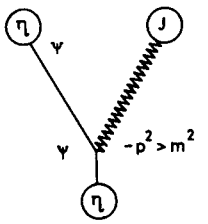

FIG. 5. Diagram showing the unity of sources.

and they had always the same expression. But the mechanism of dealing with these fields and their associated particles is entirely different. A particular example is

$$\mathscr{L} = \ldots + e\bar{\psi}\gamma^{\mu}\psi\, A_{\mu}$$

$$\mathscr{J}^{\mu} = e\bar{\psi}\gamma^{\mu}\psi \qquad \partial_{\mu}\mathscr{J}^{\mu} = 0$$

This is a cubic coupling with appropriate vectorial properties and so on, introduced in order to express the idea of the extended source. But at the same time we are now realizing for the first time, in terms of an explicit dynamical mechanism, an example of a general photon source. $\mathscr{J}^{\mu}$ is an electron current but it is now a specific dynamical mechanism. Here is the simplest example of self-consistency between the general framework and the specific realization. So here, for example, is a primitive interaction; it is not the end of the theory, it is the beginning.

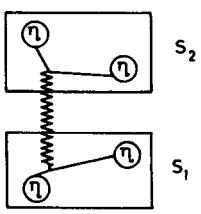

FIG. 6. Extended sources.

Once we have introduced cubic couplings, we infer logically the existence of quartic, etc. In other words, we now have more and more complicated phenomena that necessarily are called into being. Consider the extended source, S_1 (Fig. 6). What does it do? It radiates an electron which is detected and then, out of that limited space-time region, S_1, a photon occurs. In other words, here we have a more complicated realization of a photon source. Now it has inner structure, but out of it comes a photon and it must be represented as a photon source by the same general machinery as any other photon source. Also shown is its inverse, S_2.

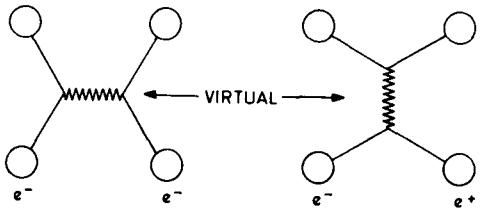

FIG. 7. Scattering skeletons.

These extended sources can now be applied by a process of space-time extrapolation to other arrangements. Figure 7 shows two such arrangements. I ask you to appreciate that the topology of the connections here is the same. We have just moved the sources around. But as we do that we now have an arrangement in which the four electron sources are used to emit or absorb real electrons. We have the process of electron-electron scattering, in which the photon is now virtual, merely by the process of coupling and applying the same space-time arrangement in other forms. The second possibility has instead of a space-like photon a time-like photon. It is the same arrangement and it is the familiar process of electron-positron virtual annihilation as part of the contribution to scattering. What we arrive at in this way, I would have called scattering skeletons. They will be further corrected and improved at later stages. As we have more complicated couplings, we have more complicated possibilities for sources, more complicated interactions and still more complicated possibilities for sources.

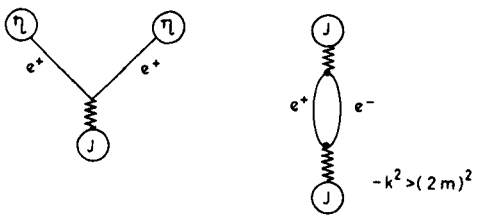

FIG. 8. Creation of electron-positron pairs.

Now there is a next stage, in which we recognize that where single-particle exchange could occur, given enough energy and momentum, two particles could also occur. Consider again the primitive interaction which is a coupling between two electron sources and a photon source. One application of it is to say, let the photon source emit enough mass; it is then a virtual photon, so that it can create an electron-positron pair (see Fig. 8). If we consider the coupling between photon sources, starting out from our initial point of view in which we allowed only enough energy and momentum

(or mass) to emit a photon and then detect it, if these sources are given a wider meaning so that they emit more energy or more mass corresponding to the pair threshold, this pair, so to speak, details the mechanism; the virtual photon field creates the electron-positron pair which, after perhaps a macroscopic interval, recombines to a virtual photon which is detected by the source. This means, therefore, that the two sources are now coupled together by a more elaborate function which represents not only the possibility of a single photon exchange but this and yet more complicated possibilities. Its very familiar momentum form which refers only to the existence of the photon must now be extended to describe a wider range of physical circumstances, not by modifying the initial description of the photon, because that remains intact – it is our phenomenological starting point – but by recognizing the existence of new modes of excitation if the energy and momentum are available to excite them. And since these modes of excitation simply represent the propagation from one space-time region to another of a certain amount of energy-momentum or mass, it looks just like (and the whole formalism automatically supplies this) a new spectrum of particles with a variable mass in which all the dynamics is comprised in a weight function which must be real and positive and describes the particular phenomena involved.

$$D_+(k) = \frac{1}{k^2 - i\epsilon} \rightarrow \bar{D}(k) = \frac{1}{k^2 - i\epsilon} + \int dM^2 \frac{a(M^2)}{k^2 + M^2 - i\epsilon}$$

In other words, there is no such thing as renormalization because our starting point is not a more fundamental field but the particle itself and what we are doing is not changing the initial description of the photon but widening the experimental circumstances. For example, if we actually compute the function $a(M^2)$ for the simplest process, we find (I indicate only the limit where the virtual mass involved is large compared to the electron mass) it is the fine structure constant divided by m^2 as the only available mass that occurs (it could be no other from purely dimensional considerations). This leads to a perfectly convergent integral for $\bar{D}(k)$. Suppose that there actually did exist – they do not – massless charged particles. Then you might recognize that this integral over virtual masses will have a rather singular structure and in fact it would influence the Coulomb law. It would then be altered in the characteristic way

$$\frac{1}{r} \rightarrow \frac{1}{r^{1+\alpha A}} \qquad r > \frac{1}{m}.$$

associated with the possibility of creating pairs of oppositely charged massless particles, had they existed. Coulomb would never have discovered his law if indeed massless charged particles had existed. I mention this to indicate the fact that the formalism we are discussing could equally well have begun with a source appropriate to gravitons, supposed to be massless particles of spin two. Instead of a vector source it would be a tensor source and the whole theory would evolve in the same way. It is perhaps of interest to mention that, after all, for gravitons massless charged particles do exist because charge here means gravitating

charge, that is mass, and all particles carry mass in a physical sense. I do not mean rest mass but mass. And so, for example, a graviton source can also emit pairs of gravitons which are massless gravitating particles and can emit pairs of photons and the formalism can be and has been worked out; what goes on is very easily understood on dimensional grounds – instead of the dimensionless fine structure constant we have the dimensional gravitational constant and the square of the wave number of the virtual particles must occur to keep the dimensions. And in fact here is the corresponding asymptotic form, very interestingly, Newton's potential is changed, but it is changed not by an altered power but by an additional cubic interaction.

$$\frac{1}{r} \to \frac{1}{r} + c\,\frac{G}{r^3} \qquad r > \frac{1}{m}$$

$$\sqrt{G} \sim 10^{-32}\ \text{cm}$$

A student of mine has computed what this constant is; it is known for graviton pairs and for photon pairs. I remind you of the fact that the basic unit of length which the phenomenological gravitation constant introduces is the incredibly small number 10^{-32} cm. This modification could be regarded as saying that we do not really know what distances are to within an uncertainty of plus or minus that amount. It is obviously of no physical practical interest, but I mention this really to indicate that we now have the possibility of answering these conceptually interesting quantum questions concerned with gravitational phenomena. We have a theory which can cover the whole range from the super-microscopic to the galaxies in the sense of recapturing the Einstein field equations, and so on.

Now, continuing with this development, we see, once we have recognized that as we ascend the scale of energy, more and more complicated multiparticle exchange processes can occur, that the photon propagation function D_+ must be replaced by $\bar{D}_+$ to describe the fact that if enough energy is available we will have an electron-positron pair instead of a photon and more complicated things. This must occur everywhere because, after all, the propagation function refers to idealized sources and all sources have at least that property and then more besides, which represents their peculiar individual features. Suppose, for example, that we consider the primitive interaction itself and ask what happens as we move up the scale of energy or mass that the photon source emits. If enough energy is available, the virtual photon will convert to a pair and then reconvert again to finally produce the electron-positron pair that is actually detected. This is in fact exactly that mechanism of pair exchange which is the origin of the modification of the propagation action. Wherever the photon field occurs, related to its source by the propagation function D, it must be replaced by $\bar{D}$. Of course, it must be done also for the electron fields and their sources. Let me not discuss this. But more than this occurs. Because once we have created an electron-positron pair, it not only interacts by re-annihilation but it interacts by normal Coulomb scattering, so that we have the scheme of Fig. 9, where A is localized near the source J and the

ψ's are localized in a macroscopically different space-time interval. So we recognize the existence of a non-local coupling between the ψ's and A which occurs by the propagation of an excitation, represented by a propagation function with a continuous mass spectrum. It is just like a particle, only it is a whole spectrum of particles. And so one infers in a

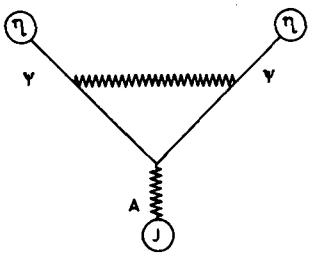

FIG. 9. Interaction of electron-positron pair by normal Coulomb scattering.

purely intuitive way the existence of spectral forms (more familiarly, dispersion relations) which in fact represent what is here the modification of the electromagnetic properties. The vector potential and the electron fields are no longer coupled locally but non-locally. And in addition to an electric form factor, new processes come into existence; the current of the anomalous magnetic moment of familiar value of $\alpha/2\pi$ appears here automatically, simply by keeping in mind the causal and physical arrangement that one has described. I beg you to remember again that we have arrived at all these things by a numerical and not an operator field theory.

Now, continuing this process of evolution, we see that we have here a theory which I think is in close accord with what the physicist actually does. He does not begin with the final theory and draw mathematical consequences, he elaborates the theory, he describes a certain class of phenomena and then he extrapolates and sees how far it works. Then, if it does not work, he discovers new phenomena and so on. The final theory is never in hand, when it is in hand the subject has become uninteresting.

I have mentioned, for example, the phenomena of photon-photon scattering. How do we arrive at all these processes? We do not consider what is actually of interest. In the case of the anomalous magnetic moment of the electron, we consider a causal arrangement in which the sources we are actually interested in are involved, but are involved in a way in which real particles occur. And so also in photon-photon scattering we are interested in the coupling among four photon sources which emit only photons, but consider an arrangement in which four photon sources are involved in a process in which real electrons occur. Figure 10 is an extended photon source. It creates a pair. Then these pairs propagate at macroscopic intervals and they are scattered in such a way that they reconvene and they annihilate and are detected by that extended photon source. This is a particular coupling and we can describe it completely, in fact, very simply, because the kinematic theory is so restricted that this process occurs in one and only one way. There is nothing to integrate over; it is an elementary kinematical procedure, and by studying it we arrive at a coupling among the four photon sources referring to this

arrangement. Now we try to put it in terms of space and time and extrapolate it. It still gives us information about how four photon sources are coupled under different arrangements. What this amounts to is that one derives in an intuitive way the existence of a double spectral form, and in fact it is the Mandelstam representation with the rules of Cutkosky found by considering this physical process.

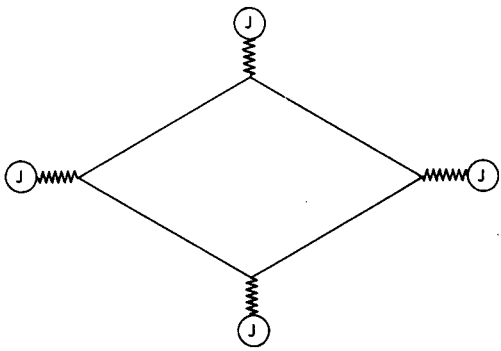

FIG. 10. An extended photon source.

One could go on in this way and still get more complicated processes and so on, in the sense of recognizing new processes produced by new multiparticle exchanges. Among these multiparticle exchange systems are bound systems, say positronium, where one must consider not only the exchanges of single photons once or twice but endlessly. So the next stage is a consideration not of a finite number of particle exchanges but the iteration of this process. One expresses that iteration by writing down integral equations, and one is naturally led in terms of the space-time picture to multiparticle Green's functions and all the rest of the phenomenological description which has been expressed in that language.

4. STRONG INTERACTIONS

We come now to what started all this, i.e. strong interactions. First we must look for unifying principles and I point to the idea of partial symmetry. Partial symmetry has various applications; consider partial symmetry on masses. I remind you of the Lagrange function of the π field as a spin-zero field, of the Lagrange function of the ρ regarded as a spin-one field and I ask what would the world be if one were to omit the mass term of the π (one can do this only in the fairly explicit language of an action or a Lagrangian formulation in which individual terms can be considered).

$$\pi : \qquad \mathscr{L} = -\tfrac{1}{2}(\partial_\mu \pi)^2 - \tfrac{1}{2} m_\pi^2 \pi^2$$

$$\rho : \qquad \mathscr{L} = -\tfrac{1}{4}(\rho_{\mu\nu})^2 - \tfrac{1}{2} m_\rho^2 (\rho_\mu)^2$$

$$\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$$

If the pion mass were zero, then only derivatives of the field would occur and there would be invariance under displacements by constants which I will call $\delta\phi$. If the ρ mass term were absent, then one gets the photon and one has invariance under gauge transformation. But in fact this Lagrange function is not invariant and the actual response to an infinitesimal displacement of the π field and to an infinitesimal gauge transformation on the ρ field is

$$\delta\mathscr{L} = - m_\pi^2 \pi \cdot \delta\phi - \frac{m_\rho^2}{g} \rho^\mu \partial_\mu \delta\omega$$

We want to start with phenomenology. Let us talk about πN and the related particle system. We may analyse it, and the brief discussion of photon-photon scattering and the connection with the Mandelstam representation is an indication that we can, in fact, here also consider multiparticle exchanges and in particular those few processes which can be regarded as produced by single-particle exchange, such as the exchange of one nucleon between pion-nucleon combinations or the exchange of a ρ between nucleon pairs or pion pairs, elementary mechanisms which will give rise to (but certainly not the whole story of) pion-nucleon scattering. My point is, to make a start, we think of these single-particle exchanges as the analogy of primitive interactions, not the final theory, but as the basis from which by iterations the complete dynamics will be evolved. So consider then these single-particle exchanges and think of them as primitive interactions, represented by local products of fields, for example, the $NN\pi$, $NN\rho$, $\rho\pi\pi$, and so on. Here are familiar structures for them now in the language of this phenomenological field theory:

$$\frac{f}{4\pi} \bar{N}\gamma^\mu\gamma_5 \tau N \partial_\mu \pi \qquad g\bar{N}\gamma^\mu \tfrac{1}{2}\tau N \rho_\mu \qquad g' \rho^\mu \partial_\mu \pi \times \pi$$

This is simply an incorporation of the various quantum numbers for local structure, but now beyond this we have to add some hypotheses. One possible hypothesis is that the primitive interactions, not the whole theory, obey the partial symmetries that I have noted; and so, for example, that the primitive interactions are invariant under displacements of the ρ field, which is just a way of saying, as is familiar, that the ρ field must interact with the current of isotopic spin. Also, if we go on and include all carriers of isotopic spin including the ρ itself so that we have self-couplings of the ρ, then we have introduced in this simple way the idea that the ρ dynamics is that of a non-Abelian gauge theory, that is, ρ coupled to itself in a way that corresponds to gauge transformations, but gauge transformations of a non-Abelian kind. We then arrive at the following transformation properties for the ρ, π, and N fields:

$$\delta\rho_\mu = - \delta\omega \times \rho_\mu + \frac{1}{g} \partial_\mu \delta\omega \qquad \delta\pi = \delta\phi$$

$$g\rho^\mu \cdot \partial_\mu(\pi \times \delta\phi) \Rightarrow \delta\rho_\mu = \frac{g}{m_\rho^2} \partial_\mu(\pi \times \delta\phi)$$

Consequently,

$$\delta N = i \frac{g^2}{2m_\rho^2} \tau \cdot \pi \times \delta\phi \, N$$

The form of the ρ transformation follows simply from the fact that ρ is a gauge field and carries isotopic spin. The displacement of the π field implies that the $\rho\pi$ coupling is not invariant and since it changes in a manner exactly like a ρ gauge transformation, it may be encountered by a change in ρ gauge. In view of this transformation the ρN coupling is not invariant, but this may be countered by a redefinition of the nucleon field, which is a set of three transformations because of isotopic spin multiplicity. Then there are isotopic spin transformations. We put all these together and we recognize that we have a group of six parameters which show the structure of the group as that of the four-dimensional Euclidean group which factors in a familiar way into two three-dimensional Euclidean groups or the equivalent SU(2) groups; the parameters which characterize these individual SU(2) groups are fixed, there is no freedom here. These transformations are unique, the group structures uniquely defined and the parameters are given by

$$\delta\omega_\pm = \delta\omega \pm \frac{\sqrt{2}\,g}{m_\rho} \delta\phi$$

What we are recognizing here is, at the phenomenological strong interaction level, the existence of chiral groups, because the $\delta\phi$ as a π displacement has opposite parity from $\delta\omega$ as a purely inert mechanical rotation. We now set up a new hypothesis which seems to be hardly avoidable. We have recognized that instead of isotopic spin we have two isotopic spin groups. We should then extend the fields to be carriers of them, replace ρ by two ρ fields, which are given by

$$\rho_\pm = \rho \pm a \qquad \rho \sim 1^- \qquad a \sim 1^+$$

with

$$\delta(\rho \pm a) = -\delta\omega_+ \times (\rho \pm a) + \frac{1}{g} \partial \delta\omega_\pm$$

$$\delta a_\mu = \frac{\sqrt{2}}{m_\rho} \partial_\mu \delta\phi(x) + \text{homogeneous transformations}$$

Then we have the possibility of insisting even further on the invariance under these groups of the primitive interactions. For example, here is the π Lagrange function:

$$\mathscr{L} = -\left(\partial_\mu \pi - \frac{m_\rho}{\sqrt{2}} a_\mu\right)^2 - \tfrac{1}{4}(\rho_{\mu\nu})^2$$

$$- \tfrac{1}{4}(a_{\mu\nu})^2 - \tfrac{1}{2} m_\rho^2 (\rho_\mu^2 + a_\mu^2)$$

This is not the whole Lagrangian, it is a simplified part which keeps only quadratic terms. Here we have partial invariance, partial symmetry, under the non-Abelian chiral groups. This is a definite structure. We see, however, that π and a are mixed together; we must carry out a diagonalization process. Here it is:

$$a_\mu \rightarrow A_{,\mu} + \frac{1}{\sqrt{2}\, m_\rho} \partial_\mu \pi$$

The Lagrangian function of the few particles that are being considered so far then reduces to the correct and previously written π and ρ Lagrange function,

$$\mathscr{L} = \mathscr{L}_\pi + \mathscr{L}_\rho - \tfrac{1}{4} (A_{,\mu\nu})^2 - \tfrac{1}{2} 2m_\rho^2 A_{,\mu}^2$$

and since everything is completely fixed, we have now predicted the mass of A_1. It is a well-known prediction first found by Weinberg in quite different ways which involve hypotheses about high-energy behaviour rather than the low-energy behaviour exploited here. It is this famous $\sqrt{2}$ ratio which seems very well obeyed in nature. I think there is hardly any doubt any longer about the physical existence of A_1.

$$\frac{m_{A_1}}{m_\rho} = \sqrt{2}$$

Now, continuing with the dynamics evolved in this way, we have a much more complicated scheme. a is mixed with π and so there are more complicated couplings between ρ and π's. Just to describe what the situation is, we find a new coupling constant effective in the actual decay of ρ into two π's. It turns out to be three-quarters of the initial coupling constant g. Let us make a temporary assumption of ρ dominance. I tried to emphasize in the beginning that we are talking about primitive interactions which are single-particle exchanges, and it is not a necessary assumption. If indeed we had a more detailed analysis of the experimental data, we could separate that part of the S-wave pion-nucleon low-energy scattering which is produced by ρ exchange from that part which is not. At the moment this analysis does not yet exist and it is conventionally assumed that it is dominated by ρ exchange. It must be nearly dominated. Let us assume that it is. It is a familiar assumption but it is merely an approximate one for our purposes. It then gives rise to a definite value of

$$\frac{g^2}{4\pi} = 3.4 \pm 0.3$$

coming from the known S-wave πN scattering. If we use this known value of g and use the effective coupling constant of 3/4 we predict a ρ width of 95 MeV. We also predict, of course, the effective coupling constant of $\rho\pi\pi$ decay $g'/g = 3/4$. The colliding beam experiments which have been

done in Novosibirsk have shown us very directly that the ρ meson is an intermediary between electron-positron colliding partners and charged pion pairs, and the preliminary data have given a value of the width $\Gamma = 93 \pm 15$. Their derived value of g'/g is 0.77 ± 0.10. These numbers are in striking agreement, I think, with these anticipations from the simple chiral group. The simplest chiral invariant theory also predicts a width of A_1. It is too large from the most recent clear-cut measurements made on the neutral A_1. There are various possibilities. I mention an interesting one here, which has to do with the particle we have not mentioned yet, the sigma meson, which now seems to be known experimentally as a very broad S-wave resonance somewhere around 700 MeV close to the ρ mass. There is the theoretical possibility that the observed decay of A_1, which is after all into three pions, can be a process of $\rho\pi$ or $\sigma\pi$ decay and one can exhibit models in which that decay shows destructive interference. Through this interference between the rather sharp $\rho\pi$ decay and the very broad $\sigma\pi$ background, one can produce, depending on the model, a fairly substantial reduction of the A_1 width, without in any way, of course, affecting the ρ. I do not think one can use this to account entirely for the observed width, but for the rest we can use further chiral invariant interactions which are analogous to magnetic moment, like interactions, Pauli terms which can always be added independently of the particular invariant structure and can be introduced to reduce the A_1 width. This increases the ρ width and then we must use a small value of g. So the final story on this will only be available when one has analysed all the relevant experimental data including in particular the question of what is the value of g as inferred from only the ρ exchange in pion-nucleon S-wave scattering, without this simplifying assumption.

I mention merely some additional topics to which these notions could be applied; for example, the possibility of introducing electromagnetic coupling of the strongly interacting particles by simply insisting upon gauge invariance, balancing the gauge invariance of the vector potential against the gauge invariance of the ρ field (neutral ρ field as well as ω and so on) which gives one a very immediate derivation of the now familiar coupling of vector potentials to fields in which the field plays the role of the current. The same procedures are, of course, used for weak interactions in particular, since the phenomenology of strong interactions have already given us a chiral group.

The known chiral behaviour of strong interactions can then be used immediately to make predictions about the chiral behaviour anticipated for weak interaction. Here we are completely reversing the logical order made use of in current algebra which begins with hypotheses about weak interactions and turns them about to make predictions about strong interactions. I think we must agree that logically, in the sense of the natural order of things, the strong interactions dominate the physics and not the other way round. I also mentioned the fact that the primitive interactions I have written down, $\rho\pi\pi$ and so on, are not the end of the story, they are the starting point, and out of this one must consider multiparticle exchanges in far more realistic dynamics in which requirements such as unitarity will automatically come out from the causal structure and use them, so to speak, to interpolate between the resonances. This naturally raises the question whether in fact, when we have interpolated and come to the position of a new resonance, the strong interaction dynamics already available could

not be used to predict new, but of course already recognized, resonances; that is, the scheme has some aspect of self-consistency by interpolating from one resonance to others where we could not insist that we get back to the one we started with. These ideas of self-consistency have been exploited before and I anticipate that the general structure will not be basically different, but nothing has been done so far.

I mention the fact that the ideas of partial symmetry, of considering a restricted domain of physics or of omitting temporarily certain parts of Lagrange functions as in higher symmetries, can be used in other ways, in particular if we consider physical processes where no momentum is exchanged. We can recognize already within the framework of the non-interacting particles that a wider group opens up. It is the group which has been suggested long before as a means of uniting spins and internal degrees of freedom. If we transfer the symmetry so recognized, as a testing hypothesis to interactions, we then find some of the very successful predictions previously obtained from SU(6) and one new prediction: the ratio of the axial vector to the vector coupling in weak decay, $G_A/G_V = 5/3\sqrt{2} = 1.18$. This is a very striking number indeed and represents, I think, one of the as yet unchallenged successes of this particular point of view.

5. CONCLUSION

I have tried to indicate that here is a new and a very physical, intuitive and practical approach not only in high-energy physics but in the whole range of physical phenomena.

Source theory is not an engineer's theory with fixed calculational rules, not a mathematician's theory with emphasis on irrelevant subtleties but a physicist's theory which recognizes the evolutionary growth of experimental knowledge and helps pave the way for an eventual more fundamental theory by not confusing phenomenology with speculation.

THE S-MATRIX

An Introduction

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Abstract

THE S-MATRIX: AN INTRODUCTION. The author briefly summarizes the history of the S-matrix by way of introducing the papers by Feshbach, Eden and Omnès.

The Heisenberg S-matrix was invented by John Wheeler in 1937 in a discussion of nuclear reactions involving α-particles. The reason it is called the Heisenberg S-matrix, and properly so, is that Heisenberg drew attention in 1943 to the fact that in atomic and nuclear physics one deals with scattering amplitudes and energy levels and that these aspects of physical systems could be conveniently characterized by a certain scattering matrix he called S. Heisenberg was particularly enthusiastic about trying to construct a theory involving only measurable quantities (like scattering amplitudes) and was hopeful that some of the diseases plaguing quantum field theory could thereby be avoided.

Since there are supposed to be some non-experts here, let me indicate how the S-matrix is introduced into collision theory. We imagine describing the physical system under consideration by time-independent state vectors $|\psi\rangle$; suppose we have a state, call it $|\psi_i^{(+)}\rangle$ which is characterized by the property that in the remote past we have a collection of particles (described by wave packets) moving towards each other. If by "experiment" (i.e. using the laws of quantum mechanics) we examine the state $|\psi_i^{(+)}\rangle$ at some time in the remote future we will find wave packets describing particles moving away from each other (with amplitudes decreasing inversely with distance from the interaction region), the original set having interacted and scattered or perhaps producing new particles. The superscript (+) is meant to imply the normal experimental time sequence of a collision experiment of incoming beams and outgoing reaction products. Now consider another Heisenberg state $|\psi_f^{(-)}\rangle$ characterized by a measurement in the remote future which says that particles are moving away from each other in well-defined beams. This is a kind of space-time reversed situation in which the scattered particles are carefully aimed at each other to produce nice outgoing beams and we use the superscript (-) to designate this rather unusual experiment. The amplitude for the transition from the initial configuration to the final one is

$$S_{fi} = \langle \psi_f^- | \psi_i^+ \rangle$$

This set of numbers, S_{fi}, is the S-matrix. From the S-matrix elements, one can compute cross-sections, transition probabilities, etc.

There is no question that there is an S-matrix and one can show that it is a unitary matrix and has certain well-defined behaviour under Lorentz

transformations. It is an eminently desirable object to obtain. The question is how do you get it. Within the framework of non-relativistic quantum mechanics or in models of relativistic quantum theory (such as quantum electrodynamics) there is a well-defined procedure for constructing the matrix element. It was Heisenberg's hope, however, that one might be able to throw away that conventional underlying machinery and start afresh with a formalism which admittedly forgoes some of the intimate space-time structure implicit in the ordinary theory and thereby construct a more physical theory, hopefully less sick.

The way in which this might be done began to emerge about 15 years ago when still within the framework of quantum field theory people began to study the analytic dependence of S-matrix elements as a function of variables like energy and momentum transfer (in two body scattering processes). This development, which has been a very intense one, goes under the name of dispersion theory or in more modern terminology, analytic S-matrix theory. There has developed what one might call a half-axiomatic S-matrix theory which attempts to present itself as a logical alternative to quantum field theory, substituting analyticity, unitarity, singularity structure, and connectedness postulates for the familiar axioms of quantum field theory.

I am myself strongly partial to the pure S-matrix approach but abhor the extremism with which some proponents and opponents discuss the relative merits of S-matrix theory as compared to local quantum field theory. Axiomatic quantum field theory has the virtue of being precisely and concisely formulatable and the difficulty that it is hard to make contact with experiment. S-matrix theory is more phenomenologically and physically oriented, is more directly applicable to experiment, but as yet vaguely formulated. I know of no conflict between the two theories, if they can be called that, and I feel that they are in fact the same. Whether this is true is somewhat irrelevant at present - the problem of extending the basic laws of physics into the sub-atomic regime is what we are really interested in and I think we should try to get on with the job without polemics or dogma and use any tools that seem appropriate.

In other papers in these Proceedings you will have an opportunity to judge for yourself how much progress (or lack of it) has been made in the last 15 years. Professor Feshbach reviews the role of the S-matrix in nuclear physics; Professor Eden discusses the S-matrix in relativistic quantum theory; and Professor Omnès describes S-matrix methods in dynamical calculations in the elementary particle regime.

RELATIVISTIC S-MATRIX THEORY *

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Abstract

RELATIVISTIC S-MATRIX THEORY. 1. *Introduction;* 2. *Properties of Scattering Amplitudes;* 2.1. Kinematics and physical regions; 2.2. Analytic properties and crossing symmetry; 2.3. Dispersion relations; 2.4. Unitarity and asymptotic behaviour; 2.5. Partial wave amplitudes; 2.6. Symmetries; 2.7. Spin properties; 3. Theoretical Foundations; 3.1. Axiomatic S-matrix theory; 3.2. Axiomatic quantum field theory; 3.3. Perturbation theory; 3.4. Heuristic assumptions, Mandelstam representations, Regge theory; 4. Results and Experimental Tests; 4.1. Effective range formulae; 4.2. High energy bounds; 4.3. Dispersion relations; 4.4. Asymptotic relations between cross-sections; 5. Discussion of Further Objectives.

1. INTRODUCTION

The S-matrix approach to a theory of elementary particles was suggested by Heisenberg in 1943 [1]. In this review, I will begin by describing the properties of S-matrix elements which form the general framework that is used in applications. Secondly I will outline the various approaches to a theoretical basis for S-matrix theory. Thirdly I will state briefly a few applications, some of which will be discussed in more detail in other talks, and will conclude by mentioning some of the more distant objectives of the theory.

Most of the applications of S-matrix theory concern scattering amplitudes, so I will begin by giving the relation between an S matrix, a scattering amplitude and a differential cross-section, in the simple case of equal-mass two-particle scattering. The S-matrix element is the transition amplitude from an initial incoming state α to a final outgoing state β,

$$S_{\beta\alpha} = \langle \beta, \text{out} | \alpha, \text{in} \rangle = \langle \beta, \text{in} | S | \alpha, \text{in} \rangle$$

Each initial and final particle is on its mass shell

$$p_r^2 = m^2 + \vec{p}_r^{\,2}$$

Each S-matrix element contains a δ function factor that conserves total energy and momentum,

$$\delta\left(\sum p'_r - \sum p_r\right) = \prod_1^4 \delta\left(\sum p'_{ri} - \sum p_{ri}\right)$$

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The scattering amplitude $T_{\beta\alpha}$ is defined by

$$S_{\beta\alpha} = \delta_{\beta\alpha} + i(2\pi)^4 \delta\left(\sum p'_r - \sum p_r\right) T_{\beta\alpha}$$

where the first term on the right corresponds to zero interaction of the colliding particles. With covariant normalization, $T_{\beta\alpha}$ is an invariant. In the particular case of equal mass scattering $T_{\beta\alpha}$ is the scattering amplitude F(s, t), which depends on the invariant energy and momentum transfer variables s and t. This is related to the elastic differential cross-section by

$$\frac{d\sigma}{dt} = \frac{|F(s,t)|^2}{16\pi s(s - 4m^2)}$$

In this review we will be concerned with the properties of S-matrix elements, which will be illustrated and discussed mainly in terms of the properties of scattering amplitudes.

There are several recent books on S-matrix theory [2] to which the reader is referred for details, which are clearly beyond the scope of this brief review. These books also give references to the original literature.

2. PROPERTIES OF SCATTERING AMPLITUDES

The scattering amplitude F(s, t) for spinless particles is directly related to cross-sections when the variables s and t correspond to physically allowed kinematics. I will begin by discussing this for general masses in two-body collisions. Then I will describe analytic properties of F, and crossing symmetry. An important aspect of analyticity is represented by dispersion relations, and if use is made of unitarity they lead to bounds on high energy behaviour of differential cross-sections. Symmetries and spin properties are of great importance in both the theory and application of S-matrix theory and I will indicate some of their features. The final point, that I will mention in this review of properties, is the distinction between kinematic and dynamic properties.

2.1. Kinematics and physical regions

Let process (I) denote the collision of initial particles 1 and 2 leading to final particles 3 and 4,

$$\text{(I)} \quad 1 + 2 \rightarrow 3 + 4 \tag{2.1}$$

with 4-momenta as shown in Fig.1(a), denoted by

$$p_1, \; p_2, \; -p_3, \; -p_4, \tag{2.2}$$

with

$$\sum_1^4 p_i = 0 \qquad\qquad p_i^2 = m_i^2 \tag{2.3}$$

The scattering amplitude F(s, t) may be expressed as a function of any two of the three invariants,

$$s = (p_1 + p_2)^2$$
$$t = (p_1 + p_4)^2 \qquad (2.4)$$
$$u = (p_1 + p_3)^2$$

which, by Eqs (2.3), satisfy the relation

$$s + t + u = \sum_1^4 m_i^2 \equiv K \qquad \text{(say)} \qquad (2.5)$$

For process (I) to be physical, the energy in the centre-of-mass system must exceed the rest energy of particles 1 and 2, and the scattering angle must be real. These conditions give

$$(s)^{\frac{1}{2}} > \max\,[(m_1 + m_2),\ (m_3 + m_4)] \qquad (2.6)$$

$$D = \begin{vmatrix} p_1^2 \ , & p_1 p_2 , & p_1 p_3 \\ p_1 p_2 , & p_2^2 \ , & p_2 p_3 \\ p_1 p_3 , & p_2 p_3 , & p_3^2 \end{vmatrix} \geqslant 0 \qquad (2.7)$$

In terms of s, t and u, the latter condition has the form [3]

$$stu \geqslant (s + t + u)^2 (as + bt + cu) \qquad (2.8)$$

where the dimensionless parameters a, b, c, depend on the masses m_i.

It is convenient to illustrate the symmetry that exists between the variables by using homogeneous co-ordinates s, t and u. Then the regions where the inequality (expression (2.8)) is satisfied are those labelled I, II, III, and IV in Fig.1(b). In the region I, F(s, t) is the scattering amplitude for process (I) (Eq.(2.1)), in which $\sqrt{s}$ is the centre-of-mass energy, provided that it is evaluated in the limit

$$F(s, t) = \lim_{\epsilon \to 0+} F(s + i\epsilon,\ t) \qquad (2.9)$$

The regions shown as II and III in Fig.1(b), correspond to the collision processes in which $\sqrt{t}$ and $\sqrt{u}$, respectively, are the centre-of-mass energies, namely

$$\text{(II)} \qquad 1 + \bar{4} \to \bar{2} + 3 \qquad (2.10)$$

$$\text{(III)} \qquad 1 + \bar{3} \to \bar{2} + 4 \qquad (2.11)$$

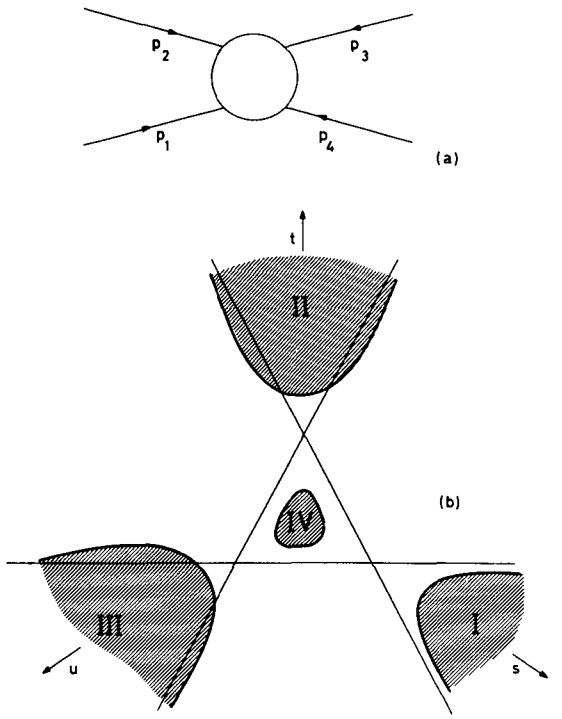


FIG.1. (a) Momentum four vectors in a two-body collision giving $s = (p_1 + p_2)^2$, $t = (p_1 + p_4)^2$, $u = (p_1 + p_3)^2$. (b) The Mandelstam s, t, u diagram showing Kibble curves surrounding the physical regions I, II, III, IV. Note that the region IV does not exist if each particle 1, 2, 3, 4 is stable against decay.

If one of the particles is unstable, say,

$$m_1 > m_2 + m_3 + m_4 \tag{2.12}$$

there is a non-zero region IV in Fig.1(b), which corresponds kinematically to the decay process

$$1 \rightarrow 2 + 3 + 4 \tag{2.13}$$

The region IV is more familiarly known as a Dalitz diagram for a three-body decay process [4]. Within the present context it is relevant to experiments which involve final state resonances [5]. For simplicity we will limit our discussion to the collisions of stable particles, so expression (2.12) will not be satisfied and region IV will not occur as a physical region in Fig.1(b).

2.2. Analytic properties and crossing symmetry

Work on analytic properties of the S matrix was greatly stimulated by the suggestion of Mandelstam [6] that F(s, t) is simultaneously analytic in s and t (and u) in the product of the complex s, t and u planes each cut

along the real axis. Then, in appropriate limits, $F(s,t)$ will be the physical scattering amplitude not only for process (I), but also for process (II) and (III), when the variables are in the corresponding regions of Fig.1(b). The statement that one function $F(s,t)$ can describe several physical processes is known as crossing symmetry. To make it specific, and also for later use, I will state the crossing condition for pion-proton forward scattering ($t = 0$), omitting spin complications.

Let $F_I(E)$ denote the π^+p forward scattering amplitude, and $F_{II}(E)$ denote π^-p forward scattering, where E is the pion laboratory energy

$$F_I(E); \quad \pi^+p \to \pi^+p \tag{2.14}$$

$$F_{II}(E); \quad \pi^-p \to \pi^-p \tag{2.15}$$

$$s = m^2 + \mu^2 + 2mE \tag{2.16}$$

$$F(E) = F(s, t = 0), \text{ for I and II} \tag{2.17}$$

Then $F_I(E)$ and $F_{II}(E)$ are analytic functions in the complex E plane cut along the real axis, on

$$(-\infty < E < -\mu) \text{ and } (\mu < E < \infty) \tag{2.18}$$

In addition F_I, (F_{II}) has a pole at $E = -\mu^2/2m$, $(+\mu^2/2m)$. The poles correspond to the fact that the neutron forms an allowed intermediate state in π^-p scattering. The branch points at $\pm\mu$ occur at the physical threshold for the πp system. In addition there are further branch points along the real axis corresponding to production thresholds [7].

Crossing symmetry is illustrated by analytic continuation from $E + i0$ to $-E - i0$ along the path indicated in Fig.2(a). This gives a relation between the π^+p and π^-p scattering amplitudes

$$F_I(-E - i0) = F_{II}(E + i0) \tag{2.19}$$

These amplitudes are real in $(-\mu, \mu)$ and have the property of Hermitian analyticity, which gives

$$F_I(E + i0) = F_I^*(E - i0) \tag{2.20}$$

Hence crossing symmetry can be stated as

$$F_I(-E + i0) = F_{II}^*(E + i0) \tag{2.21}$$

$$F_{II}(-E + i0) = F_I^*(E + i0) \tag{2.22}$$

Since these functions are analytic, except for real poles and branch cuts, we can express them as Hilbert transforms, i.e. by dispersion relations in the energy E, provided we know or assume their behaviour as $E \to \infty$ in the complex plane.

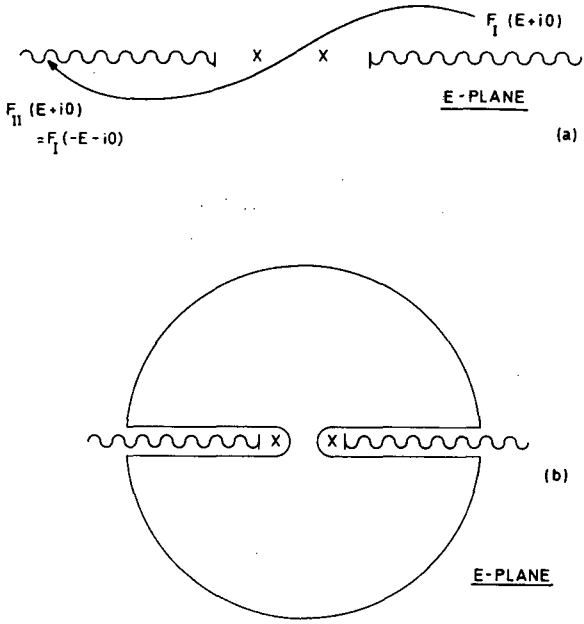


FIG.2. (a) A path of analytic continuation of a forward pion-nucleon scattering amplitude F_I showing its relation under crossing symmetry to the amplitude F_{II}. (b) The contour in the complex E-plane used for deriving a dispersion relation for a scattering amplitude.

2.3. Dispersion relations [8]

It is convenient to consider a symmetric combination of π^+p and π^-p amplitudes, defined by

$$F_S(E) = \tfrac{1}{2}[F_I(E) + F_{II}(E)] \tag{2.23}$$

As we will discuss later, it can be shown that $|F_S(E)|$ increases no faster than $|E \log^2 E|$ as $|E| \to \infty$ in the cut plane. This establishes that a dispersion relation converges if it has two subtractions. It is obtained by writing down the Cauchy integral for $F_S(E)$ using the contour shown in Fig.2(b). Using Eqs (2.19) to (2.22), one obtains the dispersion relation

$$F_S(E) = F_S(\mu) + \frac{8\pi f^2(E^2-\mu^2)}{E^2-(\mu^2/2m)^2}$$

$$+ \frac{4m(E^2-\mu^2)}{\pi}\int_\mu^\infty \frac{dx\, x\, \mathrm{Im}F_s(x)}{2m(x^2-m^2)(x^2-E^2)} \tag{2.24}$$

The first term on the right-hand side of this equation is a subtraction constant and the second term comes from the intermediate neutron pole

that is allowed in $\pi^- p$ scattering. The third term is the dispersion integral that comes from combining the integrals over the right- and left-hand cuts in the energy variable.

2.4. Unitarity and asymptotic behaviour

Unitarity of the S matrix is expressed formally as

$$S S^{\dagger} = S^{\dagger} S = 1 \tag{2.25}$$

It can be combined with other properties to give considerable information about the S matrix. From the definition of total cross-sections and unitarity one obtains the optical theorem, which relates the imaginary part of the forward scattering amplitude to total cross-sections. For example, it gives

$$\mathrm{Im}\ F_S(E) = m(E^2 - m^2)^{\frac{1}{2}} [\sigma_{tot}(\pi^+ p) + \sigma_{tot}(\pi^- p)] \tag{2.26}$$

When this formula is used to substitute for ImF(x) in the dispersion relation (2.24), one obtains a relation that can be tested experimentally. The real part of F can be measured by Coulomb interference in the forward direction and results have been obtained that give good experimental verification of this fundamental analytic property [9].

The analyticity required for Eq.(2.24) has been proved from the axioms of quantum field theory. The required asymptotic behaviour has been established by Martin [10]. In addition to results from quantum field theory, Martin uses unitarity in its form for a partial wave amplitude, namely

$$0 \leqslant |f_\ell(E)| \leqslant \mathrm{Im}\ f_\ell(E) \leqslant 1 \tag{2.27}$$

Using the optical theorem (Eq.(2.26)), Martin's result establishes the Froissart bound [11] on total cross-sections, which is

$$\sigma(\text{total}) < \frac{\pi}{\mu^2} [\log(E/E_0)]^2, \quad \text{as } E \to \infty \tag{2.28}$$

where E_0 gives an arbitrary normalization.

2.5. Partial wave amplitudes

For many applications it is assumed that the partial wave amplitude $f_\ell(E)$ is analytic in the complex E plane, except for real branch cuts and (when not all masses are equal) some complex branch cuts in a finite part of the E plane. This has not been proved from the same rigorous basis as the forward dispersion relation (2.24), but no-one has produced any reason for doubting its validity. However it is also assumed that $f_\ell(E)$ has good asymptotic behaviour as $|E| \to \infty$, and this is a more doubtful assumption. The doubt arises as a consequence of Mandelstam-Regge branch cuts [12],

which indicate that a double dispersion relation for a scattering amplitude would require an infinite number of subtractions.

In Regge theory (which will be considered in more detail in other lectures) it is also necessary to assume that an analytic extension $f(\ell, E)$ of $f_\ell(E)$ exists and has convenient analytic properties as a function of complex ℓ and complex E. Once again, no-one has produced any evidence against this analyticity of $f(\ell, E)$, but there are difficulties and doubts relating to the asymptotic properties in both ℓ and E.

The simplest properties of $f(\ell, E)$ that are normally assumed are that it is regular in ℓ, for real E, except for (Regge) poles and branch cuts. The Regge poles are located in $\mathrm{Im}\,\ell > 0$, when E is above threshold and lie at

$$\ell = \alpha(E) \tag{2.29}$$

The path of the Regge pole $\alpha(E)$ in the complex ℓ plane as E varies through real values is called a Regge trajectory. Using crossing symmetry these poles give information about high energy behaviour in the crossed channel (i.e. process (II), where t, not s (or E), is the energy). Putting $\ell = n$, an integer, Eq.(2.29) may be solved for E and under suitable conditions it gives

$$E = E_n = a_n - ib_n \tag{2.30}$$

This is the well-known Breit-Wigner [13] pole, which is located on the second Riemann sheet as was first noted by Møller [14].

2.6. Symmetries

Lorentz invariance, and the rotational symmetries of certain amplitudes, are closely connected with analytic properties. Their proper relation and formulation may well lead to important advances in our understanding of the collisions of particles with spin and Regge theory over the next few years. However, there are considerable technical complications involved and they will not be considered in this brief review.

Other symmetries, such as isospin invariance and SU(3), are more simple, and I will illustrate their use in an application to high energy scattering in the pion-nucleon system. Using isospin invariance the πn charge exchange amplitude can be expressed in terms of the isospin 3/2 and $\frac{1}{2}$ amplitudes

$$(\pi^- p | F | \pi^0 n) = \frac{\sqrt{2}}{3}[F(\tfrac{3}{2}) - F(\tfrac{1}{2})] \tag{2.31}$$

Hence, if charge exchange scattering tends to zero relative to elastic scattering as the energy tends to infinity, the two isospin amplitudes will become asymptotically equal. From the optical theorem we then deduce that

$$\sigma_{\text{total}}(\pi^- p) \sim \sigma_{\text{total}}(\pi^+ p), \quad \text{as } E \to \infty \tag{2.32}$$

This is a special case of the Pomeranchuk theorem [15], that particle-target and antiparticle-target cross-sections become asymptotically equal as the energy tends to infinity.

2.7. Spin properties

Particles with spin used to be regarded as causing inessential complications in S-matrix theory. They still cause complications but these are now considered to be of great importance both theoretically and in experimental comparison. The spin properties are too extensive for a brief review but I will indicate some aspects that have particular significance.

Scattering amplitudes depend on spin orientation as well as on the invariants (s, t). Thus in addition to the isospin multiplicity, pion-nucleon scattering involves two invariant amplitudes A(s, t) and B(s, t). The scattering amplitude $T_{\alpha\beta}$ can be written

$$T_{\alpha\beta} = A(s,t) - \tfrac{1}{2}i\gamma\cdot(q_1+q_2)B(s,t) \tag{2.33}$$

The amplitudes A and B have essentially the same analytic properties as those of F(s, t) for spinless particles. However, for many collision processes, it is more convenient to use helicity amplitudes F_{++} and F_{+-}, in which the spins have definite orientation with respect to the momentum of the particles. These helicity amplitudes have kinematic singularities due to spin, in addition to those due to physical thresholds etc. The proper treatment of such kinematic singularities (and zeros) is a major task in applications of S-matrix theory, especially in Regge theory [16].

When colliding particles have non-zero spin, it may be possible to choose a sub-amplitude that (under suitable assumptions) can be shown to tend to zero as the energy $|E|$ tends to infinity. Let G(E) denote such an amplitude, and assume that it tends to zero faster than E^{-1} and has only branch cuts on the real E axis. Then, in addition to satisfying a dispersion relation it will satisfy a 'superconvergence relation' or sum rule [17],

$$\int_{-\infty}^{-\mu} \operatorname{Im} G(E)dE + \int_{\mu}^{\infty} \operatorname{Im} G(E)dE = 0 \tag{2.34}$$

This relation follows from our assumptions by using Cauchy's theorem on G(E). Its use as a sum rule involves approximations to Im G by means of unitarity or discontinuity relations, which limit the range of integration to a sum over certain resonances.

3. THEORETICAL FOUNDATIONS

The properties of S-matrix elements have been studied from four different viewpoints, two of which are reasonably fundamental and two are heuristic. These are: (a) axiomatic S-matrix theory; (b) axiomatic quantum field theory; (c) perturbation solutions of quantum field theory, (d) developments from heuristic assumptions, such as the Mandelstam representation and Regge theory.

Most of the successful phenomenology and experimental comparison has followed from the heuristic approach (d), and most of our knowledge of singularities has come from perturbation theory (c). Neither the latter, nor the more rigorous approaches have yet disproved the principal heuristic assumptions, although Mandelstam [12] has cast doubt on the subtraction assumption made in his representation. No result has yet been deduced about the S matrix from axiomatic quantum field theory that is in disagreement with experiment, or with axiomatic S-matrix theory, so either of the rigorous approaches may be regarded as acceptable from a fundamental viewpoint. Philosophical objections are sometimes raised to one or other of these approaches, but it is difficult for a physicist to accept them unless one of the approaches actually makes an incorrect prediction. However, the central problem in strong interactions is how to approximate, and it may be argued that the S matrix is a better vehicle for approximation than quantum field theory. Since axiomatic quantum field theory will be discussed in other lectures, I will be concerned here mainly with an outline of axiomatic S-matrix theory.

3.1. Axiomatic S-matrix theory

The aim of axiomatic S-matrix theory is to derive the properties of S-matrix elements, or collision amplitudes, from a few precisely stated assumptions, which themselves have a clear physical basis. The principal assumptions have been formulated somewhat differently by Stapp [18] and by Olive [19]. These assumptions include:

(a) unitarity;
(b) Lorentz invariance;
(c) cluster decomposition of S-matrix elements;
(d) analyticity;
(e) pole and particle correspondence; residue factorization;
(f) macroscopic causality, or an $i\epsilon$ prescription;
(g) crossing.

A precise statement of the above assumptions requires considerable care and detail which may be found in the literature [18-20]. I will limit myself to a brief qualitative discussion of each assumption where this is possible.

(a) Unitarity follows from the principle of superposition in quantum mechanics and the completeness of states. The analogous formulation in S-matrix theory has been given by Stapp [18] (see also Ref.[20]).

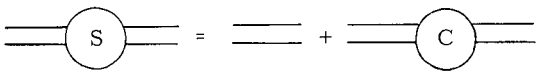
(b) Lorentz invariance tells us that S-matrix elements are functions of the kinematic invariants, s, t, ... It also leads to the expression of S-matrix elements in terms of invariant M functions when the colliding particles have non-zero spin. Alternatively one could use helicity amplitudes, although this has not been done in the axiomatic approach.

(c) Cluster decomposition is related to the short-range character of inter-particle forces. This allows the possibility that in a collision involving several particles, one or more of the particles may have no interaction with the remainder. For two particle scattering, this permits the

S-matrix element to be expressed as a non-interacting part plus an interacting part, namely a δ-function plus a scattering amplitude. Symbolically this is expressed

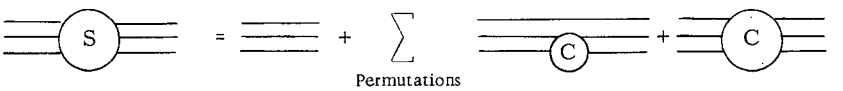
$$S = 1 + iA' \tag{3.1}$$

where A' is proportional to the scattering amplitude. Diagramatically this is written



(3.2)

where the second term on the right is called a connected part, which means that it does not contain the non-interacting term. For a three-body collision, the analogous diagramatic equation is



(3.3)

(d) Analyticity has been expressed as an assumption in various different forms. These all include the assumption that the scattering amplitude A(s, t) is an analytic function of the invariants s and t in some domain, which contains at least a neighbourhood of the physical region. More generally, analytic properties are assumed also for production amplitudes. The following examples indicate the kind of alternative assumptions that can be made:

(i) The singularities of a collision amplitude are the minimum that are consistant with unitarity. This is sometimes called the principle of maximal analyticity [2]. For heuristic purposes it has proved to be a good working principle, although much of the research in this area is concerned with finding out what this principle means in practice.

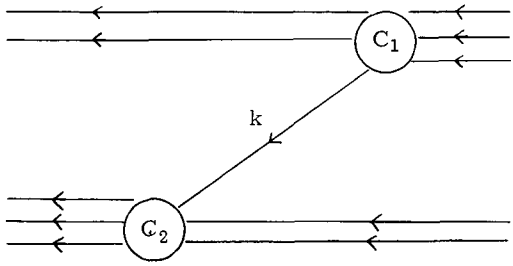
Weaker analyticity assumptions are illustrated by:

(ii) Collision amplitudes are analytic functions of their invariants in a small neighbourhood of the physical regions, except at certain physical singularities, such as normal thresholds for two-body scattering and poles for three- or four-body scattering.

(iii) Collision amplitudes are analytic in small neighbourhoods of the physical regions, but not necessarily the same analytic functions below and above successive normal thresholds. That they are the same functions must then be deduced from some other assumption, for example from causality [18].

(e) Pole and particle correspondence, and residue factorization. The energy-momentum vectors for a multiparticle process may be consistent

with a subprocess that involves a free particle travelling over macroscopic distances. Such a process is illustrated by the diagram



(3.4)

The fact that such a term corresponds to a singularity, when the exchanged particle is on the mass shell, can be deduced from unitarity. Using macroscopic causality, the nature of the singularity may be established to be

$$\frac{i}{k^2 - m^2 + i\epsilon} \tag{3.5}$$

Residue factorization states that when $k^2 = m^2$, the factors corresponding to the subdiagrams C_1 and C_2 are themselves three-body scattering amplitudes. This can be deduced from other assumptions, for example (a) to (d) given above.

(f) Macroscopic causality has been usedby Chandler and Stapp [18] together with weak analyticity, in the form (d) (iii) above, to show that the two-body scattering amplitude is the same analytic function below and above a normal threshold and that the connection is made via the upper half plane in the complex energy variable. This result is essentially the $i\epsilon$ prescription, which shows how to pass a physical singularity whilst remaining on the physical sheet. From a stronger form of causality, namely that only stable particles can carry energy-momentum over macroscopic distances, Chandler and Stapp [18] show that n-body scattering amplitudes are analytic functions (infinitely differentiable) at all physical points except at singularities corresponding to Landau surfaces with positive Feynman parameters. The latter are essentially those expected from perturbation theory in the physical regions. A similar result about the possible physical region singularities has been derived using the $i\epsilon$ prescription and assumptions about nearby analyticity by Bloxham, Olive and Polkinghorne [21]. The basis of these methods is due to Norton [22] who pointed out that a Feynman diagram that corresponds to a singularity with positive Feynman parameters can be interpreted literally as a succession of physically allowed collisions.

(g) Crossing and antiparticles. The pole at $k^2 = m^2$, considered in connection with assumption (e) above, can be regarded as having $k_0 > 0$. With certain assumptions about analytic continuation it can be shown [18, 20]

that the amplitude also has a pole that corresponds to $k_0 < 0$. This pole is interpreted as the singularity due to the existence of an antiparticle. If the exchanged pole is replaced by an exchanged two-body scattering process, within a multi-particle process, an extension of the argument about antiparticle poles leads to the TCP theorem.

Axiomatic S-matrix theory aims to provide a logical basis for the properties that are used for calculations. It is a long way from establishing all the properties that are assumed in heuristic approaches to S-matrix theory, but it does provide an important method for checking some of them.

3.2. Axiomatic quantum field theory

This topic is included in other lectures and is listed here only for completeness. It provides a valuable means for testing properties of S-matrix elements that are assumed or deduced from axiomatic S-matrix theory. It also yields information about analyticity that is additional to the results derived from perturbation theory. This is because the latter is unable to make rigorous use of unitarity. The results about analyticity derived from axiomatic field theory are outlined in a survey by Froissart [23], who gives further references to recent work.

3.3. Perturbation theory

Perturbation theory provides a convenient method for testing hypotheses about analytic properties of S-matrix elements. It is probable that any singularity of a scattering amplitude on the physical sheet will be a singularity of a term in the perturbation series for the amplitude. This is not true for unphysical sheets, where unitarity ensures that certain types of cancellation will occur. For example, a bound state pole is a singularity on the physical sheet, and there is no pole in the same location on an unphysical sheet (in general). Conversely, if a term in perturbation theory is singular on the physical sheet it is safest to assume that the full amplitude will have the same singularity unless one can find an explicit cancellation from another term.

Singularities are found in perturbation theory from the Feynman integrals which make up the series, but it is advisable not to try to do the integrals. The basic method is to study the circumstances in which an integrand has singularities that lie on the contours of integration [7]. This method, and its extensions and applications are described in detail in the literature [1, 24], where further references are given. The results are summarized by the Landau equations [25], but the main work is to evaluate the solutions of these equations.

I will illustrate the study of singularities in perturbation theory by a fourth-order Feynman integral in the scattering amplitude for ϕ^3 theory. This has the form

$$G(s, t) = g^4 \int \frac{d^4k}{\prod_1^4 (q_i^2 - m^2 + i\epsilon)} \tag{3.6}$$

where each q_i depends linearly on k and on the external four momenta. The conditions for G(s, t) to be singular are best obtained by writing it in a form using Feynman parameters α_i

$$G(s,t) = g^4 \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4 \int \frac{\delta\left(1 - \sum \alpha\right) d^4k}{\left\{\sum \alpha_i (q_i^2 - m^2) + i\epsilon\right\}^4} \tag{3.7}$$

This may be singular when

$$\underline{\text{either}}\ q_i^2 = m^2\ ,\ \underline{\text{or}}\ \alpha_i = 0$$

for each value of i. In addition we must have

$$\sum \alpha_i q_i = 0 \tag{3.8}$$

If, for example, we take $\alpha_1 = \alpha_3 = 0$, then we must have

$$q_2^2 = m^2,\ q_4^2 = m^2 \tag{3.9}$$

and

$$\alpha_2 q_2 + \alpha_4 q_4 = 0 \tag{3.10}$$

From these equations we deduce that q_2 and q_4 are either parallel or anti-parallel, and that G(s, t) may be singular when

$$s = (q_2 - q_4)^2 = 4q_2^2 = 4m^2 \tag{3.11}$$

This is the normal threshold at which the centre-of-mass energy $W = s^{\frac{1}{2}}$ is just large enough for q_2 and q_4 to describe physical particles. Attached to the normal threshold in s, is a branch cut from $s = 4m^2$ to ∞. For real t (in $t < 4m^2$) there are no complex singularities of G(s, t) in the s-plane.

More detailed consideration shows that G(s, t) is analytic in complex s and t in the product of two cut planes:

$$\{\text{complex s-plane, cut}(4m^2, \infty)\} \times \{\text{complex t-plane, cut}(4m^2, \infty)\} \tag{3.12}$$

3.4. Heuristic methods, Mandelstam representations, Regge theory

Equation (3.12) illustrates the kind of analyticity property that is utilized in the Mandelstam representation [6]. From this result one can deduce that not only does G(s, t) satisfy a dispersion relation, but so also does Im G(s, t). This gives a double dispersion relation,

$$G(s,t) = \frac{1}{\pi^2} \int_{4m^2}^{\infty} ds' \int_{4m^2}^{\infty} dt' \frac{\rho(s', t')}{(s' - s)(t' - t)} \tag{3.13}$$

The Mandelstam representation is obtained by generalizing Eq.(3.13), (a) by symmetrizing in the variables s, t and u defined in section 2, and (b) by assuming that the representation holds for the full scattering amplitude F(s, t).

The analytic properties assumed for the Mandelstam representation are very reasonable and have been proved in perturbation theory for a large class of terms, subject to certain restrictions on the masses of the interacting particles. The asymptotic properties imply that

$$\left|\frac{F(s,t)}{s^N t^M}\right| \to 0, \text{ as } s \to \infty, \; t \to \infty \tag{3.14}$$

for some fixed N and M. This assumption permits the use of a finite number of subtractions, but Mandelstam [12] has shown that its validity is in doubt. His method is based on Regge theory [26], which is itself heuristic in relativistic interactions.

Regge theory is based on the Sommerfeld-Watson transform of the partial wave series and has been proved for potential scattering by a Yukawa potential [26]. When extrapolated to relativistic interactions using crossing symmetry, Regge theory gives the result [1] that as $s \to \infty$,

$$F(s,t) \sim \frac{b(t)s^{\alpha(t)}\exp[i\pi\{1-\frac{1}{2}\alpha(t)\}]}{\sin[\frac{1}{2}\pi\alpha(t)]\Gamma[\alpha(t)]} \tag{3.15}$$

where $\alpha(t)$ denotes a pole of an analytic extension $f(\ell, s)$ of the partial wave amplitude $f_\ell(s)$. The dominant pole as $s \to \infty$ is the one with the largest real part. For $t = 0$, $F(s, 0)$ is the forward scattering amplitude and one requires $\alpha(0) = 1$ in order to obtain a constant total cross-section.

4. RESULTS AND EXPERIMENTAL TESTS

There are very few results of S-matrix theory that do not depend on poorly understood approximation procedures. However, in certain areas an approximation may work so well as to give numerous comparisons with experiment so that it achieves considerable respectability even though its mathematical basis may not be understood. I believe that Regge theory of high energy collisions will be in this category within the forseeable future. The approximation of dominance by the nearest singularities is extensively used, and may often be valid although, for example, it is manifestly wrong if applied to a simple problem like the charge distribution of a heavy nucleus.

I will note four results that follow from basic properties of S-matrix elements, but I will not attempt to list the numerous results that follow from approximation procedures such as dominance of nearby singularities. It should, however, be noted that the latter often give good agreement with experiment or stimulate experimental research, as happened with the prediction of a ρ meson pole in connection with the nucleon electromagnetic form factors.

4.1. Effective range formulae

A scattering amplitude has a square root singularity in the kinetic energy at threshold. Hence the S-state partial wave phase shift $\delta(k)$ satisfies:

$$k \cot \delta \text{ is analytic near } k = 0 \tag{4.1}$$

For small enough k, this permits an effective range expansion

$$k \cot \delta = \frac{1}{a} + \tfrac{1}{2} rk^2 + \ldots \tag{4.2}$$

This expansion should converge in a circle of finite radius in k^2, of size determined by the nearest physical sheet singularity (stable particle pole, or threshold) or unphysical sheet singularity (resonance pole or virtual state pole). This result satisfies the properties of the S matrix and the approximation is understood.

4.2. High energy bounds

From analytic properties of scattering amplitudes derivable from quantum field theory it has been shown (see Ref. [10]) that the Froissart bound [11] is satisfied by certain total cross-sections for large enough laboratory energy E, (note $s \sim 2mE$)

$$\sigma(\text{total}) \lesssim C[\log(E/E_0)]^2 \tag{4.3}$$

Experimentally cross-sections appear to tend towards constant values, so this rigorous bound is remarkably good. A number of other rigorous bounds have been proved, for example in relation to the forward peak of the differential cross-section, as $s \to \infty$

$$\frac{1}{9}\left[\frac{\sigma^2(\text{total})}{4\pi\sigma(\text{elastic})} - \frac{1}{k^2}\right] \lesssim \frac{d}{dt}\left[\log A_s(s,t)\right]_{t=0} \lesssim C\left(\log \frac{s}{s_0}\right)^2 \tag{4.4}$$

where

$$A_s(s,t) = \text{Im } F(s,t)$$

An especially interesting result is obtained [27] if the Mandelstam representation is assumed, including its assumption about a finite number of subtractions; then at fixed scattering angle θ, as $s \to \infty$

$$\frac{d\sigma}{d\theta} \gtrsim \exp\left[-s^{\frac{1}{2}} (\log s) C(\theta)\right] \tag{4.5}$$

Present indications are that this result is probably not in agreement with experiment, which suggests a faster rate of decrease of the differential cross-section. This would indicate, for example, that if

$$|F(s,t)| \sim s^{\alpha(t)}, \text{ as } s \to \infty \tag{4, 6}$$

then $\alpha(t)$ has no upper bound as t increases to arbitrarily large positive values. Within the context of Regge theory this result could follow either from the existence of resonances with arbitrarily higher angular momentum, or from complications caused by Mandelstam-Regge branch cuts [12].

4.3. Dispersion relations for scattering amplitudes F(s, t)

These have been proved for a range of t near t = 0. Only at t = 0 can they be compared with experiment with any precision. For the symmetric pion nucleon forward scattering amplitude $F_S(E)$, we have (see Eq.(2.24))

$$F_S(E) = F_S(\mu) + \frac{8\pi f^2(E^2 - \mu^2)}{E^2 - (\mu^2/2m)^2}$$

$$+ \frac{4m(E^2 - \mu^2)}{\pi} P \int_{\mu}^{\infty} \frac{dx\, x\sigma_S(x)}{(x^2 - \mu^2)^{\frac{1}{2}}(x^2 - E^2)} \tag{4.7}$$

where

$$\sigma_S = \tfrac{1}{2}[\sigma_{tot}(\pi^+ p) + \sigma_{tot}(\pi^- p)] \tag{4.8}$$

and the optical theorem has the form

$$\text{Im } F_S(E) = 2m\,(E^2 - \mu^2)^{\frac{1}{2}}\sigma_S(E) \tag{4.9}$$

This, and similar dispersion relations have been confirmed experimentally by Lindenbaum and collaborators [9], who measure the real part of $F_S(E)$ by Coulomb interference with near-forward scattering.

4.4. Asymptotic relations between cross-sections

Under a weak additional assumption about the behaviour of the real part of a forward amplitude for large energy, one can prove the Pomeranchuk theorem which states that particle target and antiparticle-target total cross-sections become asymptotically equal. For example

$$\sigma_{tot}(\bar{p}, p) - \sigma_{tot}(p, p) \to 0, \text{ as } E \to \infty \tag{4.10}$$

and

$$\sigma_{tot}(\pi^+, p) - \sigma_{tot}(\pi^-, p) \to 0, \quad \text{as } E \to \infty \tag{4.11}$$

Both these results agree with experiment. As noted in Eq.(2.32), the second of these results follows also from isospin invariance and the experimental fact that

$$\sigma(\pi^- p \to \pi^0 n) \to 0, \quad \text{as } E \to \infty \tag{4.12}$$

5. DISCUSSION OF FURTHER OBJECTIVES

S-matrix theory is directed towards obtaining relations between: (a) the singularities of scattering and collision amplitudes, (b) their zeros, (c) their high energy behaviour.

Singularities are of three types:

(a) threshold singularities and their generalizations, which are most simply obtained by the study of Feynman integrals in perturbation theory,

(b) dynamical singularities, which usually means resonances calculated in a model,

(c) kinematic singularities, introduced because of spin terms, when the colliding particles have non-zero spin.

The first and third type of singularity may be very complicated, but in principle they are understood when the spin is inserted as an 'external' variable. Equally, dynamical singularities cause no serious trouble if they are assumed to be given, and are then combined to give other dynamical singularities that are generalizations of normal thresholds.

The present situation in the phenomenological use of S-matrix theory is well illustrated by the study of elastic scattering and two-body reactions. At low energies it is well described by assuming the dominance of (nearby) resonance poles, and making allowance for inelasticity. It should be noted that the resonances and inelasticity are introduced as parameters and are not derived from basic principles, except isospin invariance and (sometimes) SU(3) invariance. Similarly at high energies Regge theory gives good agreement with many experimental features provided one uses the experimental results to deduce the parameters of the Regge poles and residues. At intermediate energies, phenomenology is less successful, and we have yet to find a model that goes over to a resonance model at low energies and to a Regge model at high energies.

One of the most difficult and challenging problems in the use of S-matrix theory is to find a method for calculating resonance parameters and the related Regge parameters. It is hoped that consistency between the basic properties of analyticity, crossing and unitarity may lead to strong restrictions from which one can calculate, or inter-relate, resonance parameters. The development and application of a method to use consistency in this way and make such calculations is called the bootstrap program, and it will be discussed in another lecture at this symposium.

Finally I should note that many important aspects of S-matrix theory have been omitted from this brief review, including for example its relation to electromagnetic and weak interactions, and the detailed use and derivation of sum rules. Some of these topics will be considered in other lectures in the symposium, others are described in the literature given in the List of References.

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HIGH-ENERGY COLLISIONS OF HADRONS: WHAT ARE THE DYNAMICAL MECHANISMS?

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Abstract

HIGH-ENERGY COLLISIONS OF HADRONS: WHAT ARE THE DYNAMICAL MECHANISMS? A survey is given of the two dominant mechanisms which have been recognized to operate in high-energy collisions of hadrons, namely diffraction scattering and reggeized particle exchange. The main characteristics of these mechanisms are discussed on the basis of the experimental evidence from two-body processes. The complicating features which occur at a higher level of precision are briefly discussed. The lecture is of a descriptive nature and does not enter into the theoretical formalism.

1. INTRODUCTION

Extensive experimental work around large accelerators and vigorous efforts of theoretical interpretation have given over the last 6 or 7 years a first systematic survey of high-energy collisions of mesons and baryons. Great regularities, especially in the energy variation of cross sections, as well as unsuspected structure features in angular distributions, have been discovered experimentally and are hopefully providing the first clues toward the determination of the dominant dynamical mechanisms. Looking at the experimental facts and summarizing the most successful among the many attempts at their theoretical interpretation, one is led to the conclusion that at least two dominant dynamical mechanisms operate in high-energy collisions. They manifest themselves clearly in peripheral collisions of two-body or quasi-two-body type, $A + B \longrightarrow A' + B'$, where A' is a particle or a group of particles going in about the same centre of mass (c.m.) directions as A, and similarly for B, B'. They are the following:

a) A diffractive mechanism controls those collisions which take place without exchange of internal quantum numbers (i.e. A and A' have the same internal quantum numbers, and so have B and B').[1]

b) Reggeized particle exchange dominates those reactions where non-vanishing internal quantum numbers are exchanged between the colliding particles (i.e. A and A' do not have the same internal quantum numbers, and/or similarly for B, B').

Both mechanisms favour small momentum transfers $q = p_{A'} - p_A = p_B - p_{B'}$, and give peaked angular distributions. All processes of the first class, to be called hereafter diffractive collisions, have cross-sections which, for constant values of the momentum transfer, seem to become essentially independent of energy at sufficiently high energy ($E_{lab} \gtrsim 20$ GeV). In contrast, all exchange collisions, taken again at constant momentum transfer, show a rapid decrease of cross-section with increasing energy; the decrease

[1] By internal quantum numbers we mean baryon number, isospin I, strangeness S and G-parity (the latter only for non-strange mesons). These are the quantum numbers exactly conserved by the strong interactions.

appears to follow power laws with exponents depending on the process. Regge theory relates these exponents to the existing particles having the right quantum numbers to be exchanged in the collision.

Beyond these gross features which are rather well established, many issues remain unclear and require extensive further study. The energy variation of diffractive collision cross-sections before they reach their asymptotic limits must undoubtedly be affected by exchange processes and by the contribution of many known or suspected resonances. We have only a preliminary understanding of these apparently very different effects which may well overlap to some degree. On the other hand, reggeized particle exchange only explains in a rough first approximation the power law decreases of exchange collision cross-sections. It becomes increasingly likely that a correct description requires several neighbouring powers of the energy, or a continuum of such powers, and one only begins to guess the dynamical mechanisms which may be responsible for this.

We shall try in this lecture to survey the problems listed above. We shall mention in addition some classes of high-energy phenomena for which our knowledge is even more incomplete and preliminary.

2. THE TWO DOMINANT MECHANISMS

The best evidence for the diffractive mechanism comes from elastic scattering in and near the forward direction. The total cross-section σ_T (which is the imaginary part of the forward elastic scattering amplitude) and the differential cross-section $d\sigma/dt$ taken at small constant t ($t = q_0^2 - |\vec{q}|^2$) have a clear tendency to tend to finite limits as the energy increases. The limits appear to be the same for A+B and $\bar{A}$+B scattering, $\bar{A}$ being the anti-particle of A. The real part of the forward elastic amplitude, as measured through Coulomb interference, approaches slowly zero. These experimental indications are further supported by the fact that one consistently obtains very good agreement between the data and the dispersion relations at constant t, especially in the forward direction (t = 0), when the cross-sections are extrapolated to energy independent values in the asymptotic limit [1].

Several examples of inelastic diffractive collisions of two-body type have also been measured, although with much less accuracy than elastic scattering. Thus, the reactions

$$\pi + p \longrightarrow A_1 + p \qquad \text{Ref. [2]}$$

$$p + p \longrightarrow N^*(I = \tfrac{1}{2}) + p \qquad \text{Ref. [3]}$$

have been observed up to 16 and 28 GeV/c respectively. Within the limited accuracy of the data, such reactions also give, at small constant t, cross-sections $d\sigma/dt$ which seem to become constant at high energy. The cross-section values, however, are appreciably smaller than for elastic scattering. The latter of the two reactions quoted is usefully compared with reactions of type

$$p + p \longrightarrow N^*(I = \tfrac{3}{2}) + p$$

where the experimental data clearly indicate a fast drop of cross-section with increasing energy. Indeed, in order to excite an isospin 3/2 resonance, exchange of a non-vanishing amount of isospin is necessary, so that the process is of exchange type and not diffractive. Basically, diffractive collisions of hadrons are very similar to diffraction phenomena in other parts of physics.

As to exchange collisions, the best evidence concerning their high-energy behaviour comes from the charge exchange reactions

$$\pi^- + p \longrightarrow \pi^0 + n \qquad \text{Ref. [4]} \tag{1}$$

$$\pi^- + p \longrightarrow \eta + n \qquad \text{Ref. [5]} \tag{2}$$

$$K^- + p \longrightarrow K^0 + n \qquad \text{Ref. [6]} \tag{3}$$

near the forward direction, as well as from elastic $\pi^{\pm}p$ scattering near the backward direction [7], the latter reaction being interpreted as a small momentum transfer collision with exchange of baryon number unity between nucleon and pion.[2]

In all these cases, the differential cross-section $d\sigma/dt$ shows at high energy and constant t a power law decrease of type

$$\frac{d\sigma}{dt} \simeq F(t)\, s^{2\alpha(t) - 2} \tag{4}$$

where s is the square of the c.m. energy. The function $\alpha(t)$ is called the Regge trajectory dominating the exchange reaction. In good approximation it is related in a very simple fashion to known particles. The rules appear to be as follows:

i) $\alpha(t)$ is approximately linear in t, $\alpha(t) = \alpha_0 + \alpha' t$, α' being always of order 1 $(\mathrm{GeV}/c)^{-2}$, but α_0 varying from case to case with $\alpha_0 \lesssim \frac{1}{2}$. It has a smooth, approximately linear extrapolation to $t > 0$ satisfying the properties under ii), iii).

ii) Consider the particles E which could be exchanged in the reaction studied (in the sense of the single-particle exchange model, see Fig. 1) without violating any conservation law of strong interactions. For one or a few of these particles [3], the function $\alpha(t)$ becomes equal to the spin J when t is replaced by the square of the mass M, i.e. (see Fig. 2)

$$\alpha(M^2) = J \tag{5}$$

iii) For none of the particles which could be exchanged in the reaction considered (in the same sense as above) is the spin J significantly larger than the quantity $\alpha(M^2)$.

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[2] This interpretation follows if one remembers that, in backward elastic scattering, the outgoing proton carries about the same momentum as the incoming pion.

[3] These particles are the ρ meson for reaction (1), the A meson for (2), both ρ and A for (3), all three functions $\alpha(t)$ being approximately the same. For backward πp scattering they are the nucleon and certain nucleon resonances.

These rules are strikingly well satisfied as far as scattering near t = 0 is concerned. The intercepts α_0 are of order $\frac{1}{2}$ for the charge-exchange reactions (1, 2, 3) and about 0 for backward pion-nucleon scattering. Regarding the t-dependence of $\alpha(t)$ for $t < 0$ in Eq. (4), its manifestation is only clearly seen in $\pi^- + p \longrightarrow \pi^0 + n$ where precise measurements of $d\sigma/dt$ over the interval $-1 \lesssim t \leqslant 0 (\mathrm{GeV}/c)^2$ confirm very well the linear form of $\alpha(t)$ as well as the value of the slope α'. A similar precision has not been reached yet for the other reactions.

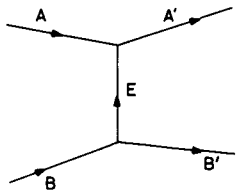

FIG. 1. Single-particle model.

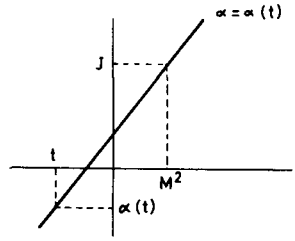

FIG. 2. The $\alpha(t)$ function.

The above rule ii) justifies the denomination of reggeized particle exchange. Indeed, the process is described as single-particle exchange, except for the replacement of the spin J of the exchanged particle by a "continuous spin" $\alpha(t)$ as introduced in Regge theory. It should be recalled that exchange of a spin J particle leads for large energy to

$$\frac{d\sigma}{dt} \simeq F(t)\, s^{2J-2}$$

which is to be compared to Eq. (4).

One can raise the question of such exchange reactions like $\bar{p} + p \longrightarrow \bar{\Sigma}^+ + \Sigma^-$ or $\pi^- + p \longrightarrow K^+ + \Sigma^-$ for which no known particle exists which could be exchanged in the sense described under ii). Although all such reactions are poorly measured, it is striking that their cross-sections are markedly smaller than for analogous processes like $\bar{p} + p \longrightarrow \bar{\Sigma}^0 + \Sigma^0$ or $\pi^- + p \longrightarrow K^0 + \Sigma^0$ where single-particle exchange is possible.

The diffraction and reggeized particle exchange mechanisms have been established experimentally for two-body collisions giving low mass particles in the final state. It is expected that they would also operate for two-body reactions having heavy resonances in the final state, and possibly also for reactions which must be interpreted as possessing more than two particles or resonances in the final state. It must be realized, however, that the distinction between such reactions is not straightforward because of the fact that the heavier resonances are likely to overlap.

One will remark that the constancy of $d\sigma/dt$ in energy for diffractive collisions can be described by a formula of type (4) with $\alpha(t) \simeq 1$. This is why one commonly associates a special Regge trajectory $\alpha(t) \simeq 1$ to diffractive processes. It is referred to as the Pomeranchuk trajectory and written $\alpha_P(t)$. Its slope in t is zero or close to it and therefore definitely smaller than the slope $\sim 1\,(\mathrm{GeV}/c)^{-2}$ of the other known Regge trajectories. By giving it an appropriate non-linear extrapolation for positive t one can force it to pass through one of the two mesons f_0, f_0' of mass 1260 MeV,

1514 MeV and quantum numbers S = I = 0, G = +1. Due to the fact that data are consistent with $\alpha_P(t) = 1$, such an extrapolation appears much more artificial than in the cases discussed above for exchange collisions.

3. ENERGY DEPENDENT EFFECTS IN DIFFRACTIVE COLLISIONS

We described above the two basic mechanisms of high-energy collisions under the most favourable circumstances where they seem to operate in simple form. In many cases, however, additional effects occur which are usually rather complicated. We discuss some of them in the present and following sections.

It is only at sufficiently high energy that the cross-sections for diffractive collisions become energy independent. At the energies studied with present accelerators, many diffractive processes still show an appreciable energy variation of the total and differential cross-sections σ_T and $d\sigma/dt$. Thus, σ_T decreases with energy towards its asymptotic value for $\bar{p}p$, K^-p, π^-p and for π^+p, the decrease being particularly pronounced for the first reaction and becoming successively weaker for the second, third and fourth one. For pp and K^+p, the total cross-section is almost constant beyond 20 and 10 GeV respectively. No total cross-section is found to increase with energy (see Fig. 3).

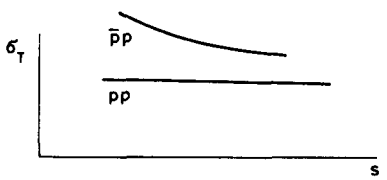

FIG. 3. Total cross-section as a function of s for $\bar{p}p$ and pp.

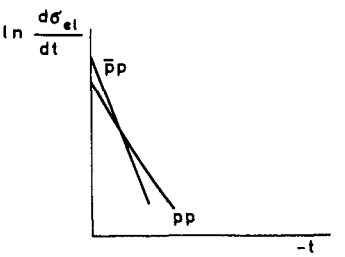

FIG. 4. $\ln\frac{d\sigma}{dt}$ as a function of t for $\bar{p}p$ and pp.

As to the behaviour of $d\sigma/dt$, it is only known with any accuracy for the elastic scattering cross-section $d\sigma_{el}/dt$. Its main energy variation seems to be simply correlated to the behaviour of σ_T: it is rather energy independent when σ_T is, while a decreasing σ_T is accompanied by a decreasing value and a decreasing slope in t of $d\sigma_{el}/dt$ at t = 0. The latter behaviour of $d\sigma_{el}/dt$ is clearly seen only for $\bar{p}p$ where the energy variations are strongest, but it corresponds very well to the normal expectation for diffraction scattering, where a larger total cross-section gives a larger amount of forward scattering and a narrower diffraction peak (see Fig. 4). As the energy increases, the $d\sigma_{el}/dt$ peak shrinks for pp and it expands for $\bar{p}p$. It is expected that the two peaks tend to each other at asymptotic energies.

For a more detailed understanding of the above energy variations of total and elastic cross-sections, one can invoke various types of explanations.

a) Diffraction scattering being the shadow scattering of inelastic processes, σ_T and $d\sigma_{el}/dt$ are affected by all inelastic processes, including those which have an appreciable cross-section at moderate energy and disappear as the energy increases. Such processes are particularly abundant for $\bar{p}p$ (mainly

annihilation which is a baryon exchange process, but also many charge-exchange and strangeness-exchange reactions), rather abundant for K^-p (many charge-exchange and strangeness-exchange processes), again less abundant for π^-p (strangeness exchange has become more difficult than for K^-p) and for π^+p (also charge exchange has now become more difficult). They are rarer for pp and K^+p. All such exchange processes can be expected to decrease as the energy goes up, presumably in about the same way as for the simple exchange collisions discussed in the previous section. This allows to understand qualitatively the energy dependence of σ_T and $d\sigma_{el}/dt$ in terms of the decreasing shadow of inelastic exchange processes as the energy goes up, the inelastic processes surviving at asymptotic energies being presumably all of diffractive type. Note that the inelastic processes need not all be of two-body type.

b) While a diffractive collision like elastic scattering can take place without exchange of internal quantum numbers, it can of course also occur with exchange of some non-vanishing quantum numbers, namely isospin $I \neq 0$ or negative G-parity. Describing such exchange contributions to diffractive collisions by the same Regge trajectories as occur in exchange collisions, one introduces energy dependent terms which decrease and eventually disappear as the energy goes up.

c) There is growing evidence for the existence of many resonances of high mass with internal quantum numbers having the same values as for the familiar, lower mass hadrons. Such resonances will certainly give contributions to σ_T and to elastic scattering when their internal quantum numbers are the ones of the total A + B system and when the energy is in their mass range. Many resonances satisfy the first condition for K^-p and π^-p, quite a few for π^+p, and probably none for K^+p and pp. As to $\bar{p}p$, we know too little about heavy meson resonances to guess how many would contribute. In all cases, the resonance contributions can be expected to disappear for increasing energy because of increasing inelasticity, giving a decrease of σ_T and the accompanying changes of $d\sigma_{el}/dt$.

The approach described under b) is the only one for which a precise study has been carried out. It uses the methods of Regge theory as successfully applied to exchange collisions. Extensive work in this direction has been done and has revealed that, in addition to the Pomeranchuk trajectory and to the Regge trajectories corresponding to exchange of non-vanishing internal quantum numbers, one has to introduce a further Regge trajectory $\alpha_{P'}(t)$ associated, as the Pomeranchuk one $\alpha_P(t) \simeq 1$, to no exchange of quantum numbers. This P' trajectory can be given a linear form $\alpha_0 + \alpha' t$. The value of α_0 must be approximately $\frac{1}{2}$. No good determination of the slope α' is yet possible, but it is compatible with the value 1 $(\mathrm{GeV}/c)^2$ found for quantum-number-carrying trajectories. The P' trajectory could pass, in the sense of Eq. (5), through either of the J = 2 mesons f_0(1260 MeV) and f_0' (1514 MeV). Some authors find it more attractive to introduce two P' trajectories, one passing through each of these mesons.

The main difficulty of the approach just described seems to lie in the so-called cross-over phenomenon. This phenomenon refers to the fact, depicted in Fig. 4, that $(d\sigma_{el}/dt)_{\bar{p}p}$ is larger than $(d\sigma_{el}/dt)_{pp}$ for t in the interval $-0.2 \lesssim t \leqslant 0$ $(\mathrm{GeV}/c)^2$ but becomes smaller for $t \lesssim -0.2$ $(\mathrm{GeV}/c)^2$, and similarly for K^-p versus K^+p, as well as for π^-p versus π^+p. We discuss it briefly for the case $\bar{p}p$ versus pp. The most relevant trajectories are then the Pomeranchuk, the P' and the one associated

with the ω meson ($I = S = 0$, $G = -1$). The cross-over requires that the coupling of the ω trajectory to the proton vanishes at $t \simeq -0.2\,(\mathrm{GeV}/c)^2$. Unless this coupling is different in pp and meson-proton collisions, its vanishing would imply dips at $t \simeq -0.2\,(\mathrm{GeV}/c)^2$ in certain inelastic meson-proton reactions. Such dips are not seen. This complication is an unfortunate feature of the Regge pole description introduced under b), because cross-over is almost immediately understood in diffraction terms. It is indeed nothing but the general diffractive property that, for larger total cross-section, $d\sigma_{el}/dt$ has a higher but narrower forward peak.

4. FURTHER ASPECTS OF ELASTIC SCATTERING AND ITS RELATION TO INELASTIC COLLISIONS

The cross-over phenomenon just discussed is not the only property of elastic scattering which is best understood in terms of diffraction. Recent experiments have revealed marked diffraction minima in $\bar{p}p$ and $\pi^- p$ elastic scattering at 6 GeV/c [8], whereas a weak diffractive structure is seen in pp elastic scattering in the interval 7-20 GeV/c [9]. In the two former reactions, the diffractive structure is expected to become weaker as the energy goes up, thus forming part of the energy dependent effects in elastic scattering. For pp on the contrary, the data rather suggest a diffractive structure which might persist at higher energies. While more measurements are necessary to clarify the picture, it appears likely that the observed structures can best be interpreted in terms of diffraction-theoretical concepts.

Such concepts are traditionally formulated in the framework of optical model calculations in the eikonal approximation. The S-matrix element S_ℓ for angular momentum ℓ is then written as $S_\ell = \exp(-a_\ell)$, a_ℓ being simply related to the optical potential. It is indeed in this framework that most current theoretical investigations are carried out. At asymptotic energies the diffraction pattern can be explained by interference between linear and higher terms in the expansion $1 - S_\ell = a_\ell - \frac{1}{2} a_\ell^2 + \ldots$, the ℓ-dependence of a_ℓ can be associated with the shape of the matter distribution in hadrons, and one therefore hopes to relate it to the electromagnetic form factors [10]. One can in addition introduce energy dependent terms in the optical potential so as to simulate the energy dependent part of elastic scattering. The terms in question can be given a form reproducing the Regge description of elastic scattering mentioned at the end of the previous section [11]. The energy independent part of the optical potential then corresponds to the contributions of the Pomeranchuk trajectory.

Optical model interpretations of diffractive elastic scattering in terms of hadronic matter distribution cannot, however, be the end of the story. One must in addition face the question, already raised in the previous section under a), of understanding shadow scattering in terms of the inelastic processes which are the very cause of the shadow. The problems to be treated here are considerable, because they really amount to developing a theory of inelastic processes sufficiently accurate to allow a successful calculation of the resulting shadow scattering. We are very far from having such a theory, but progress is being made in two directions.

On the one hand, each new step in precision measurement and theoretical fitting of an elastic scattering $A + B \longrightarrow A + B$ gives new information on that

property of inelastic collisions, A + B $\longrightarrow$ inelastic final states, which controls shadow scattering. This property is conveniently expressed by means of the so-called overlap function which, in the simplest case where spins are neglected, is given by the following equivalent forms [12]:

$$F(s,t) = \sum_{\ell=0}^{\infty} (2\ell+1) f_\ell \, P_\ell \left(1+\frac{2t}{s}\right) = \langle \psi_{\text{inel}} \mid \psi'_{\text{inel}} \rangle \tag{6}$$

f_ℓ is related by

$$\sigma^{\ell}_{\text{inel}} = \frac{2\pi}{k^2} (2\ell + 1) f_\ell \,, \quad k = \text{c.m. momentum} \tag{7}$$

to the integrated inelastic cross-section $\sigma^{\ell}_{\text{inel}}$ for angular momentum ℓ. The scalar product $\langle \psi_{\text{inel}} \mid \psi'_{\text{inel}} \rangle$ gives the amount of overlap between the complete inelastic final states ψ_{inel}, ψ'_{inel} of two A + B collisions which occur at the same energy in the same c.m. system, but have an angle θ between the incident directions. θ is related to t by the usual formula

$$\cos\theta = 1 + \frac{2t}{s} + O(s^{-2})$$

The relation between overlap function and elastic scattering is given by

$$S_\ell S^*_\ell = 1 - 2f_\ell$$

where S_ℓ denotes again the S-matrix element for elastic scattering and angular momentum ℓ. For purely imaginary elastic amplitude S_ℓ is real and given by $S_\ell = (1 - 2f_\ell)^{1/2}$.

Henzi [13] has recently analysed the case of $\pi^\pm p$ elastic scattering for $-0.6 \lesssim t \leqslant 0\,(\text{GeV}/c)^2$ by using the accurate data available not only for elastic scattering, but also for the charge-exchange reaction $\pi^- + p \longrightarrow \pi^0 + n$ (the amplitude of the latter is the difference between the two elastic amplitudes, because of isospin invariance). This analysis starts from the data and their theoretical fits as carried out in Regge pole theory. It extracts from them simple expressions for the overlap functions $f^\pm_\ell$ for $\pi^\pm p$ elastic scattering. In good approximation, f^+_ℓ is expressible as an energy independent Gaussian $P \exp(-b^2/B^2)$ in the impact parameter $b = 2\ell/s^{1/2}$. For $\pi^- p$ elastic scattering, the overlap function has the shape $f^-_\ell = f^+_\ell + \delta f_\ell$ where the term δf_ℓ decreases for growing energy and is interpreted as describing the shadow effect of the exchange processes of the type mentioned under a) in section 3.[4] The ℓ-dependence of δf_ℓ is found to be best expressed as

$$\delta f_\ell = (1 - 2f^+_\ell)^{1/2} \Delta f_\ell \tag{8}$$

[4] Henzi notes that, because of charge conservation, inelastic exchange processes are less numerous for $\pi^+ p$ than for $\pi^- p$. He therefore neglects the former (by assuming f^+_ℓ to be energy independent) and studies the shadow effects of the latter.

where Δf_ℓ is the difference of two Gaussians in $b = 2\ell/s^{1/2}$. The fact that Δf_ℓ has a simpler ℓ-dependence than δf_ℓ can be interpreted by saying that δf_ℓ is composed of an uncorrected exchange contribution Δf_ℓ multiplied by the standard correction factor for absorption

$$S_\ell = (1 - 2f_\ell)^{1/2} \tag{9}$$

taken at asymptotic energies. It is natural to expect that simple models for inelastic processes will provide a way of calculating Δf_ℓ.

The general shape of δf_ℓ is given in Fig. 5, and the corresponding contribution $\delta A(t)$ to the imaginary part of the elastic scattering amplitude has the shape illustrated in Fig. 6. The zero of $\delta A(t)$ gives rise to the cross-over point. It may happen that, for larger values of $-t$, $\delta A(t)$ becomes sufficiently negative so as to give a zero in the imaginary part of the complete scattering amplitude $A = A_0 + \delta A$, where $A_0(t)$ is the limiting value at asymptotic energies. This would create a diffraction minimum in $d\sigma/dt \simeq [A(t)]^2$ if the real part of the amplitude is sufficiently small. This mechanism could provide an explanation for energy dependent diffraction minima as observed in $\bar{p}p$ and K^-p elastic scattering.

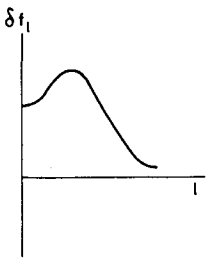

FIG. 5. General slope of δf_ℓ.

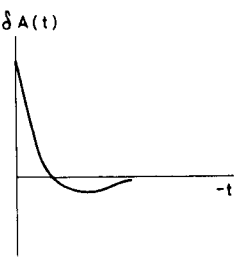

FIG. 6. Shape of $\delta A(t)$.

On the basis of Henzi's results, it may be tempting to write the overlap function at asymptotic energy in a form similar to Eq. (8)

$$f_\ell = S_\ell F_\ell = \sqrt{1 - 2f_\ell}\, F_\ell \tag{10}$$

containing the absorption factor S_ℓ. This would be useful if F_ℓ turns out to have a simpler ℓ-dependence than f_ℓ itself, and it would then be F_ℓ which simple models for inelastic processes must try to reproduce. Diffraction structure in elastic scattering at asymptotic energy may then occur from the non-linear terms in the relation between S_ℓ and F_ℓ, which from Eq. (9) and Eq. (10) reads

$$S_\ell = -F_\ell + \sqrt{1 + (F_\ell)^2} \tag{11}$$

One must hope that experiment will eventually allow to choose the most appropriate, i.e. the simplest function for the description of high-energy elastic scattering, which could be f_ℓ or $S_\ell^{-1} f_\ell = F_\ell$ as just discussed, or

$a_\ell = -\ell \, n S_\ell$ as proposed in optical models, or still other quantities not as yet considered. We note that the series expansions of S_ℓ in powers of a_ℓ and F_ℓ are identical to 2d order included, so that a_ℓ and F_ℓ are expected to be approximately equal.

The second direction of work on inelastic collisions aims at describing the wave function ψ_{inel} of the complete inelastic final state in terms of models sufficiently simple to allow actual calculations, but sufficiently realistic to reproduce the rapidly growing number of experimentally observed quantities and distributions. The most active part of this development concerns an extension of the diffraction and reggeized particle exchange mechanisms (see a and b in section 1) to collisions with three and more particles in the final state [14]. While it is quite unreasonable to expect that this so-called "multi-Regge model" will be sufficient to cover all properties of inelastic collisions, it is likely to give a satisfactory description of such inelastic configurations where pairs of outgoing particles have rather high effective mass. Chan et al. have proposed a very simple way of extending it to the other configurations and have reached considerable success in fitting a variety of channels in πp and Kp collisions [15]. It is clear, however, that the problem of low effective masses for groups of outgoing particles remains essentially unsolved.

Given any model for the inelastic final state ψ_{inel}, it is possible to calculate from it the overlap function F(st), using Eq. (6), and to confront its value with what it should be to agree with the elastic scattering data. With the crudeness of present models for ψ_{inel}, this comparison can be made by using the approximate formula

$$F(s,t) \simeq \left(\frac{d\sigma_{el}}{dt}\right)^{1/2} \tag{12}$$

Michejda [20] has carried out the comparison for the uncorrelated jet model (assuming ψ_{inel} to be a sum of products of real single-particle wave functions) and for the multi-Regge model as extended by Chan et al., both of which give a ψ_{inel} agreeing with the observed single-particle distributions in the inelastic final state. The result is quite interesting: it seems that the only way to obtain an overlap function F(st) as sharply peaked in t as required by (12) is to have the phase $\phi = \psi_{inel} / |\psi_{inel}|$ of ψ_{inel} vary quite rapidly with the outgoing momenta, and the phase factors prescribed by the multi-Regge model appear to have about the right rate of variation. When the phase factor ϕ is replaced by a constant in either model, one obtains an overlap function very flat in t, the slope being 5 to 10 times too small to fit (12). This shows that the overlap function is one of the very few properties of the inelastic final state ψ_{inel} to be strongly sensitive to the dependence of ϕ on outgoing particle momenta.

5. THE POSSIBLE ROLE OF RESONANCES IN HIGH-ENERGY COLLISIONS

The known baryon and meson resonances are very numerous and extend to large mass values (at present up to ~ 4 GeV for baryons and ~ 2.3 GeV for mesons). Although the widths do not increase fast with mass, resonances often overlap, even those with the same internal quantum numbers, and it is

natural to expect that many as yet undetected ones exist. Resonances of the A + B system may therefore affect A + B collisions appreciably, not only at low energy where they give clear maxima in the cross-sections as function of energy, but also at c.m. energies of a few GeV. We briefly discuss their possible role in the latter region.

We first note that resonances of the A + B system are not likely to dominate high-energy A + B collisions, because it would then be difficult to understand the marked similarity observed between e.g. K^+p and K^-p elastic scattering. Indeed, the system K^-p has many resonances whereas K^+p has none or very few. Resonance effects would seem to give at most some additional contributions to the main collision mechanisms in the GeV region, and their effect on total cross-sections and elastic scattering is likely to decrease with increasing energy because of the increasing inelasticity, i.e. the decreasing relative width of heavy A + B resonances to decay in the A + B channel.

Secondly, as mentioned in section 3 (especially point c)), it is remarkable that resonances have the tendency of being more numerous in those systems A + B for which the total cross-section σ_T of A + B collisions shows a more pronounced decrease before reaching its asymptotic value as the energy goes up. This general correlation between number of A + B resonances and decrease of total A + B cross-section is presumably not independent of the consideration in point a) of section 3. Resonances of A + B occur indeed more abundantly for such internal quantum number of A and B which make exchange processes between A and B easy; for example, strangeness considerations lead to no resonances and difficult strangeness exchange in K^+p, but allow both in K^-p.

If resonance contribution to A + B collisions is non-negligible at a few GeV, the Regge pole description of the energy variation of σ_T and $d\sigma_{el}/dt$, mentioned in section 3, must cover also the effect of the resonances. This can only be true, however, in an average sense, i.e. for energy averaged amplitudes, the average being taken over an energy interval ΔE which covers several resonances. Such averages smooth out the fluctuations which the resonances unavoidably create in the energy dependence of amplitudes and cross-sections. These fluctuations, which are undoubtedly very small in σ_T and in $d\sigma_{el}/dt$ near the forward direction, have been clearly seen up to about 4 GeV/c in $d\sigma_{el}/dt$ for backward πp elastic scattering where the resonances interfere with a much smaller background and are therefore more visible.

The Regge pole parametrization of πp elastic scattering, taken in its simplest form containing only the leading power s^{α_i} for each Regge pole α_i, seems to describe well certain energy averaged amplitudes down to energies where resonances are expected to contribute significantly [17]. It seems premature, however, to conclude that this is a general feature, or to state its range of validity.

The analyticity properties of a collision amplitude at fixed t imply that its values at low and high energies are not independent from each other. This allows to relate its asymptotic behaviour to its form in the resonance region through the so-called finite energy sum rules [17, 18], which are a simple but very useful modification of dispersion relations.

The occurrence of resonance effects in hadron collisions and the description of the smooth energy averaged amplitudes by non-resonant mechanisms are analogous to the situation prevailing for nuclear reactions

on complex nuclei in the overlapping resonance region, where the so-called direct reaction mechanisms describe the energy averaged properties.

6. DIFFRACTIVE EFFECTS AND OTHER COMPLICATIONS IN EXCHANGE COLLISIONS

We return to exchange collisions and discuss two types of effects which complicate the picture presented in section 2 for their asymptotic behaviour at large s and small t.

The first one concerns the simple asymptotic power law expressed by Eq. (4). Even in cases where only one particle E exists which can be exchanged in the $A + B \rightarrow A' + B'$ collision (see Fig. 1), it appears that the collision amplitude T must contain at least one additional power of s

$$T \simeq \beta(t)s^{\alpha(t)} + \bar{\beta}(t)s^{\bar{\alpha}(t)} \tag{13}$$

and that the powers must have exponents $\alpha(t)$, $\bar{\alpha}(t)$ which differ by a small number, perhaps $|\alpha - \bar{\alpha}| \lesssim 0.3$. (We choose for T a normalization such that $d\sigma/dt \propto s^{-2}|T|^2$.) This conclusion is derived from the polarization measurements on $\pi^- p \rightarrow \pi^0 n$ [19], a reaction where only the ρ meson has the right quantum numbers to be the exchanged particle E. We do not enter here into the theoretical arguments involved, but stress only that reggeized ρ exchange predicts a single term in (13) and that this implies vanishing polarization for the reaction in question. The measured polarization does not vanish and decreases only slowly as the energy goes up, requiring at least one more term in (13) with $|\alpha - \bar{\alpha}|$ small. It is the smallness of $|\alpha - \bar{\alpha}|$ which explains the success of fitting the measured $d\sigma/dt$ with a single power of s as indicated in Eq. (4).

This raises the question of a possible dynamical mechanism which would generate the further power or powers in Eq. (13) and would be sufficiently concrete not to introduce too many new parameters unrelated to other phenomena. The most attractive candidate seems at present to be <u>diffractive correction effects</u> to reggeized particle exchange, also often called absorptive corrections. The physical idea is that in the exchange collision $A + B \rightarrow A' + B'$, A and B can undergo a diffractive interaction before the exchange process takes place, and similarly A', B' afterwards. There is of course no unique way of expressing these corrections in formulae, but a few possibilities are very natural which explain easily the sign and magnitude of the observed polarization in $\pi^- p \rightarrow \pi^0 n$. Future, more accurate data extending to higher energy will be needed to distinguish between the various possibilities. We note that diffractive corrections give instead of (13) a formula of type

$$T \simeq \beta(t)s^{\alpha(t)} + \int \bar{\beta}(\bar{\alpha}, t)s^{\bar{\alpha}}d\bar{\alpha} \tag{14}$$

the integral covering some range of $\bar{\alpha}$ near $\alpha(0)$.

There are a few exchange reactions like $\gamma + p \rightarrow \pi^+ + n$ [20], $\pi^- + p \rightarrow \rho^0 + n$ [21], $p + n \rightarrow n + p$ [22] where charged pion exchange seems to play a dominant role, because the s dependence of $d\sigma/dt$ corresponds to the pion Regge trajectory (this holds for all three reactions mentioned), or also because the

t dependence has a very sharp peak concentrated at $|t| \lesssim m_\pi^2$ (this holds for the 1st and the 3rd). Normal Regge theory applied to π exchange in such reactions predicts that $d\sigma/dt$ should develop at high energy a pronounced dip in the forward direction ($t \simeq 0$), in opposition to the observations. Agreement with experiment is only restored if the π exchange contribution is supplemented by one or more other terms, giving expressions of type (13) or (14). These terms can again result from diffractive corrections. Whatever their origin, they are characterized in this case by the fact that they correspond at least partly to exchange of natural parity ($P = (-1)^J$), whereas π exchange has of course unnatural parity ($P = -1$, $J = 0$). Thus, a natural parity Regge trajectory $\bar{\alpha}(t)$ could exist with $\bar{\alpha}(0)$ equal to $\alpha(0)$, where $\alpha(t)$ is the π trajectory; the coupling of the two trajectories should develop appropriately related singularities at $t = 0$. The second trajectory is unlikely to correspond to any particle, because if it did, the particle would be a 0^+ meson of low mass, the existence of which is experimentally most doubtful.

7. FURTHER PROBLEMS

Many other aspects of high-energy hadron collisions would deserve to be discussed. It is difficult to treat them all in the framework of a single lecture, and we shall end by listing a few.

i) The hadron classification schemes based on the groups SU(3) and SU(6) as well as the quark model for a composite structure of hadrons are known to be very successful in describing the multiplet structure and many decay properties of hadrons. They can be applied also to high-energy hadron collisions and predict many relations between cross-sections, the large majority of which are verified by the data. This holds in particular for the quark model (which in many cases can be replaced by broken SU(6) symmetry supplemented by appropriate assumptions of "universal coupling") [23].

ii) Considerable progress is made in describing some classes of many-body collisions by an extension of the mechanisms described in section 2. The extension is based on the multiperipheral model as illustrated in Fig. 7, the rule being that E_i ($i = 1, 2, \ldots$) represents reggeized particle exchange if it carries non-vanishing internal quantum numbers, and a diffractive interaction otherwise [14]. This model seems to apply when the effective masses of all pairs of outgoing particles are large.

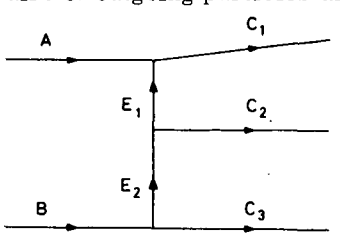

FIG. 7. The multiperipheral model.

iii) Much less is known about other classes of many-body collisions, especially those where several outgoing particles have low effective mass without forming a resonance.

iv) Very little is known about large momentum transfer collisions. The only case well measured is pp elastic scattering around 90° in the c.m. system. The cross-sections are very small and rather smooth in angle. Their drop away from the forward direction is qualitatively described by the 4th power of the proton magnetic form factor [24].

v) Hadron collisions on complex nuclei, light or heavy, turn out to be remarkably simple and instructive at high energy and low momentum transfer [25]. Among other interesting aspects, they allow to determine indirectly scattering cross-sections of unstable particles on nucleons.

vi) Photon-hadron and probably also neutrino-hadron collisions become at high energy very similar to hadron-hadron collisions, the photon acting like a vector meson (ρ^0, ω or φ) and the neutrino probably like a pion-muon pair where the pion initiates the strong interaction part of the collision.

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ASYMPTOTIC ENERGIES*

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Abstract

ASYMPTOTIC ENERGIES. The problem of estimating 'Asymptotic Energies' for at least the gross features of strong interactions is considered in historical perspective. It is shown experimentally that the pion-nucleon forward dispersion relations are valid to beyond 20 GeV/c and that they can serve as a crystal ball to estimate at what energy various asymptotic theorems, such as the Pomeranchuk theorem, will come true.

It is concluded that (a) the experimental data strongly imply the validity of the Pomeranchuk theorem and that it is estimated to come true to within the best foreseeable experimental precision (0.1% cross-section measurements) at beyond 20 000 GeV; (b) (as explained in the text) on the basis of various models for formulating a fundamental length $|\ell|$, if it exists, $|\ell| < 10^{-15}$ cm, and probably $< 10^{-16}$ cm; (c) forward elastic pion-nucleon scattering amplitudes are analytic to the highest energies ($\sim$20 GeV) measured, and that the predictions of unitarity, crossing symmetry, and charge independence are valid for these amplitudes and their behaviour is consistent with the Froissart bound and the Pomeranchuk theorem.

INTRODUCTION

For almost two decades both experimental and theoretical physicists have been concerned with the search for a high enough energy region where at least the gross features of strong interactions would exhibit an asymptotic behaviour with increasing energy.

A few years ago I coined the phrase "Asymptopia .... the theoretically promised land where all asymptotic theorems come true".

Estimates of where the asymptotic region began have changed considerably with time. About two decades ago the Fermi statistical theory [1] seemed to explain most of the gross observable cosmic-ray phenomena at energies greater than several hundred MeV. The cosmic-ray observations of constant or more or less constant total cross-sections for the nuclearly interacting components all fitted one's naive ideas of what asymptotic behaviour should be like. However, the discovery of V-particles or so-called strange particles perturbed this simple picture. Experiments with the post war f.m. cyclotrons soon demonstrated that the pion-nucleon scattering and total cross-sections were rapidly increasing with increasing energy, until $\sim$200 MeV, where the π^-+p cross-sections levelled off. Although this favoured a proposed explanation of these phenomena as an isobar of the type predicted by the strong coupling theory, it was not taken seriously at that time, since the detailed phase shift analysis by computer discovered only non-resonant solutions. However, soon thereafter it became clear from a series of experiments at the 3 GeV cosmotron that the T = J = 3/2 nucleon isobar not only existed, but dominated the pion-nucleon inelastic interaction in the multi-GeV region and in fact, an isobar [2] rather than statistical model was appropriate. For a review of the various experiments

* This work was performed under the auspices of the U.S. Atomic Energy Commission.

and analyses which led to the establishment of the α_{33} resonance solution see Ref. [3].

The next proposed boundary for the asymptotic region was estimated in the early papers on the Pomeranchuk theorem [4], where it was estimated that energies greater than 10 GeV should be sufficient for the validity of the theorem

$$\sigma(\alpha+\beta) = \sigma(\bar{\alpha}+\beta)$$

where $\bar{\alpha}$ is the antiparticle corresponding to α, and σ is the total strong interaction cross-section. Examples of this theorem are:

$$\sigma(\pi^+ + p) = \sigma(\pi^- + p)$$

$$\sigma(K^+ + p) = \sigma(K^- + p)$$

$$\sigma(p+p) = \sigma(\bar{p}+p)$$

Experiments at the AGS and CERN soon demonstrated that this was not the case, and that even at 20 GeV differences of $\gtrsim 5\%$ were observed [5].

One almost universal predicted characteristic of modern asymptotic theories is the validity of the Pomeranchuk theorem. Prediction of where it will become valid within experimental precision and how it is approached is a major concern of these theories. Another major concern is bounds on the forward scattering amplitude such as the Froissart bound, Greenberg-Low bound, etc. Reviews of these matters have been previously given [6, 7].

Several years ago, becoming discouraged with previous attempts (for the most part of an ad hoc nature)[1] to predict asymptotic behaviour, I concluded [6-8] that, based on the most fundamental principles, we did have one precisely quantitative crystal ball that looked all the way to infinity – namely, the measurements of the $\pi^{\pm}$-p forward elastic scattering amplitudes combined with the forward dispersion relations. Just as in optics, the measurement of the real part of the forward scattering amplitude, at an energy ω, gives a value for an integral involving total cross-sections extending to infinite energy.

This paper will to a large degree be concerned with establishing and then using the forward dispersion relations and general behaviour of forward scattering amplitudes to predict asymptotic behaviour.

The survey paper on elementary particle physics by Salam [9] has served as an excellent introduction to the material in this paper, and covers other important aspects of asymptotic physics.

Eden's paper [10] at this conference reviews the theoretical foundations and general properties of scattering amplitudes, and dispersion relations.

Van Hove's paper [11] treats hadron-hadron scattering, and various phenomenological analyses including Regge models.

THE EXPERIMENTAL DETERMINATION OF FORWARD SCATTERING AMPLITUDES AT HIGH ENERGIES

The near forward elastic nuclear scattering amplitude for the pion-nucleon system in a particular charge state can be uniquely completely determined to good precision from differential scattering measurements.

[1] For a further discussion, see Refs [7, 12].

Invariance with respect to rotation and parity, limits the amplitude to two complex functions. Since the spin flip term vanishes as $t \to 0$, only one complex function remains (non-flip). The imaginary part of the forward scattering amplitude can be precisely determined from total cross-section measurements which via the optical theorem (i.e., unitarity or probability conservation) gives directly the imaginary part of the forward amplitude. The real part of the forward scattering amplitude can be directly measured by its interference effects with the known real Coulomb amplitude.

The previously published analyses [8, 12, 13] of an experiment [13] which determined the real part of the pion-nucleon forward scattering amplitude at high energies by observing Coulomb-nuclear interference at very small angles have been improved in stages by further refinements in the electromagnetic corrections which were a major source of uncertainty. One major uncertainty was that the multiple and plural electromagnetic scattering formulae previously available in the literature did not properly take account of $\pi^- e$ collisions and this deficiency was only partly corrected by us in the earlier analyses. We have subsequently taken these effects into reasonable account by redoing the calculations properly for our particular case.

Another source of electromagnetic uncertainty was the electromagnetic real radiative corrections, vacuum polarization effects and the uncertainty in the phase angle [14] introduced by the long-range Coulomb interactions.

All of these electromagnetic effects have subsequently received much more detailed attention by a number of authors [15, 16]. The real radiative and vacuum polarization effects are small, but they were taken into account. The biggest correction is still the phase angle introduced between the nuclear and Coulomb scattering amplitudes by the long-range Coulomb interaction. This phase angle δ appears in the equations for the small angle charged pion-proton scattering as follows

$$\frac{d\sigma}{dt} = \left| \pm \frac{(-\mathscr{F})}{|t|} e^{2i\delta} + F_{N_{re}} + i F_{N_{im}} \right|^2 \tag{1}$$

where the $\pm$ sign is chosen to correspond to the sign of the incident pion; t is the Lorentz invariant four-momentum transfer squared; $\mathscr{F}$ is the equivalent electromagnetic form factor, and F_N is the invariant nuclear scattering amplitude.[2] Invariance with respect to space rotation, and space inversion (P) limits the form of the amplitude to two independent complex functions, an ordinary and a spin flip term. A previous analysis [8] has shown that the spin flip amplitude contribution which contains a $\vec{\sigma} \cdot \vec{k}_i \times \vec{k}_f$ term is negligible at these low $|t|$ values.

The electromagnetic corrections were derived from fundamental quantum electrodynamical considerations and a better basis for understanding their uncertainty was developed [16]. However, it should be noted that due to the fact that sufficient uncertainty was allowed for in our earlier analyses, the major conclusions as to the validity of the pion-nucleon forward dispersion relations, the limits on a fundamental length, and the estimates of where the Pomeranchuk theorem comes true within experimentally attainable precision, have not changed.

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[2] For convenience special units are employed such that $(d\sigma/dt)_{nuclear} = |F_N|^2$.

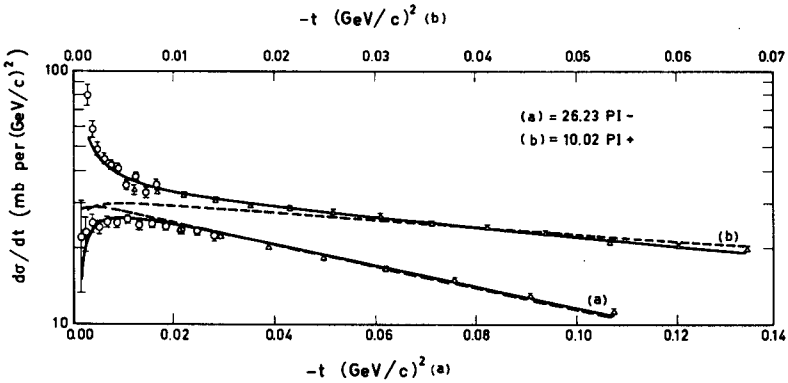

FIG.1. dσ/dt versus -t for small angle elastic $\pi^{\pm}$+p scattering. The Coulomb amplitude squared term has been subtracted out. The solid line is the best fit to the data with α adjustable. The dotted line is the best fit with α considered to be equal to zero.

Figure 1 shows typical experimental results of these small angle scattering measurements. The Coulomb amplitude squared term has been subtracted out, but the nuclear amplitude squared term and the Coulomb-nuclear interference terms ($\sim 1/|t|$) remain. The nuclear amplitude has been fitted by the now well-known parametrization we use:

$$\frac{d\sigma}{dt} = [1 + \alpha^2]\ e^{a+bt+ct^2} \tag{2}$$

where $\alpha = F_{N_{re}}/F_{N_{im}}$ is the ratio of the real to the imaginary part of the nuclear amplitude.[3] In the small $|t|$ region α and b are the important adjustable parameters. The absolute normalization accuracy is within ± 1% and the total cross-section measurements, which via the optical theorem give us a measurement of the imaginary part of the forward scattering amplitude, were determined to an absolute accuracy of 0.3%. These ultra-high precision measurements at very small angles (one to a few milliradians) were only made possible by the new counter hodoscope with on-line computer technique, which we introduced several years ago, and which allowed great increases in both data accumulation rates and accuracy. In spite of this high precision there is in the Coulomb-nuclear interference determination of σ an associated systematic error of 0.02. The dotted line is displaced from the solid line by the systematic error of ± 0.02 which is considered to be predominantly of a scale nature. In addition, estimates have been made of the uncertainty in $\alpha_{\pm}$ introduced by the δ value for the electromagnetic corrections. We use the formula for δ obtained by West and Yennie [16]

$$2\delta = \eta\,[2\ln k\, b'\theta + \gamma \pm \Delta] \tag{3}$$

[3] As discussed in the earlier work, α is found to exhibit no t dependence and the error in extrapolating to t=0 from the small values (down to t=0.001 (GeV/c)^2) observed is calculated to be small. Hence the value of α determined is the appropriate value for forward scattering.

where $\eta = e^2/\hbar c$, $b' = \sqrt{b^2 + r_\pi^2 + r_p^2}$,[4] and Δ represents the uncertainty. From Refs [15, 16] we conclude that $\pm\Delta \approx \pm\gamma$ is a reasonable estimate of the uncertainty. Using this prescription in our fit to the data, this results in $\pm\Delta\alpha = \pm 0.007$, which if compounded with the systematic error 0.02, would only change it slightly. One could presumably hope that this electromagnetic correction uncertainty might further be reduced by future work in the field.

Following the analysis of the experimental data and its justification given in Refs [8, 13] we obtain the results for α shown in Fig. 2. One should recall that until the experimental discovery of sizable real amplitudes at high energies a few years ago [6, 7], it was generally speculated that α would be near zero at high energies $\gtrsim$ 10 GeV. The solid lines in Fig. 2 are the calculated predictions of the pion-nucleon forward dispersion relations using the subsequently given Eqs (6, 7) for $D^\pm$, the experimental determinations for the low energy parameters, and the total cross-sections up to 20 GeV/c, and the Fit I of Table I and Fig. 2 for the total cross-sections at energies above 20 GeV. The selection of this fit will be discussed further in a later section.

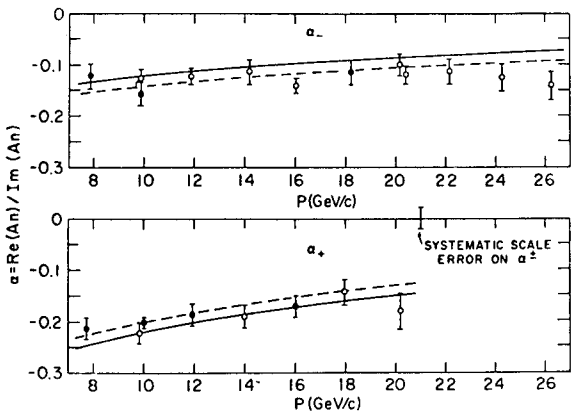

FIG. 2. $\alpha_\pm$ versus momentum. The solid curves are predictions of dispersion relations discussed in the text. The dashed curve is the result of displacing the solid curve by 0.02, the systematic error (predominantly of a scale nature). The solid points represent measurements with lower resolution and demonstrate the insensitivity of the results to resolution.

As one can see from Fig. 3, the agreement between the experimentally determined α and the dispersion relation calculations is excellent within the systematic error for α_+ and quite reasonable for α_-. Although there is a tendency for the highest momentum points in α_- to depart from the calculated curve, this is not considered to be significant (noting the errors), especially since the calculated curve is sensitive to the assumed asymptotic behaviour of the total cross-sections, and could be changed to give even better agreement within its uncertainties. The question of sensitivity of the calculations to asymptotic behaviour of the total cross-sections, and the demonstration of the validity of the pion-nucleon forward dispersion relations reasonably independent of this behaviour, will be considered in the following section. In subsequent sections we shall then use these relations to predict asymptotic behaviour of the forward scattering amplitudes.

[4] $f_N(s,t) = e^{a+b^2t}$, $F_\pi \sim \exp r_\pi^2 t$, $F_{ch} \sim \exp r_p^2 t$.

TABLE I. THE RESULTS OF THREE LEAST SQUARES FITS TO THE $\pi^{\pm}$-p TOTAL CROSS-SECTION DATA (8-29 GeV/c) USING POWER LAWS OF THE MOMENTUM WITH A TOTAL OF FIVE PARAMETERS, AS ILLUSTRATED IN THE TABLE AND DISCUSSED IN THE TEXT.

Fit	$\sigma = A + B/p^C$	A	B	C
I	σ_{π^+}	22.60 ± 0.40	25.9 ± 9.6	1.06 ± 0.24
	σ_{π^-}	22.60 ± 0.40	19.6 ± 1.8	0.67 ± 0.08
II	$\sigma_{\pi^-} + \sigma_{\pi^+}$	44.24 ± 1.88	37.0 ± 8.5	0.69 ± 0.21
	$\sigma_{\pi^-} - \sigma_{\pi^+}$	0	3.85 ± 0.56	0.31 ± 0.06
III	σ_3	22.52 ± 0.51	24.8 ± 10.7	1.02 ± 0.28
	σ_1	22.52 ± 0.51	19.86 ± 1.92	0.58 ± 0.08

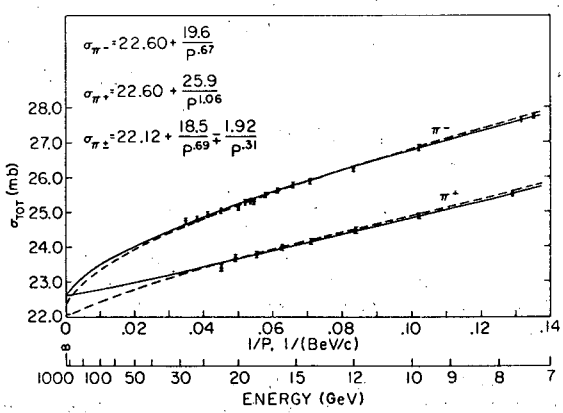

FIG. 3. The predicted values of σ_{total} ($\pi^{\pm}$-p) above 8 GeV/c plotted versus a linear plot of 1/P in the lab system, with a non-linear energy scale (GeV) also shown. The solid lines represent a fit of type I. The dashed lines represent a fit of type II.

THE PION-NUCLEON FORWARD DISPERSION RELATIONS

As is now well known, the required analyticity properties to obtain these relations can be derived on the basis of axiomatic field theory using the L.S.Z. formalism or the Wightman axioms.[5] The microscopic causality condition (i.e. that spacelike points do not interfere) is required to prove

[5] See Refs [7, 8] for a discussion and references to these works.

these relations and they can be considered as an experimental test of special relativity at subnucleon distances. The higher the energy, the smaller the distance probed. This general question will be considered in more detail in the section 'Fundamental Length'.

We use the usual notations for expressing the pion-nucleon forward scattering amplitude[6]

$$F_{\pm}(\omega) = D_{\pm}(\omega) + i A_{\pm}(\omega) \tag{4}$$

with

$$\frac{d\sigma}{d\Omega}(t = 0) = \left|F_{\pm}(\omega)\right|^2 \tag{5}$$

Then, using the least subtracted form of the forward dispersion relations [17][7], [18] which we can expect to converge, we have the well-known pair

$$D^{+}(\omega) = \frac{1}{2}\left[D_{-}(\omega) + D_{+}(\omega)\right] = D^{+}(\mu) + \frac{f^2 k^2}{M\left[1 - \left(\frac{\mu}{2M}\right)^2\right]\left[\omega^2 - \left(\frac{\mu^2}{2M}\right)^2\right]}$$
$$+ \frac{k^2}{4\pi^2} P \int_{+\mu}^{\infty} \frac{d\omega' \, \omega' \left(\sigma_{-}(\omega') + \sigma_{+}(\omega')\right)}{k'(\omega'^2 - \omega^2)} \tag{6}$$

(once subtracted forward dispersion relation)

$$D^{-}(\omega) = \frac{1}{2}\left[D_{-}(\omega) - D_{+}(\omega)\right] = \frac{2f^2\omega}{\left[\omega^2 - \left(\frac{\mu^2}{2M}\right)^2\right]}$$
$$+ \frac{\omega}{4\pi^2} P \int_{+\mu}^{\infty} \frac{d\omega' \, k' \left[\left(\sigma_{-}(\omega') - \sigma_{+}(\omega')\right)\right]}{\omega'^2 - \omega^2} \tag{7}$$

(unsubtracted forward dispersion relation)

where subscripts (±) refer to $\pi^{\pm}$ incident on protons. Laboratory system quantities are employed throughout, and $\hbar = c = 1$, ω = total pion energy = $[\mu^2 + k^2]^{\frac{1}{2}}$, f^2 is the renormalized unrationalized pseudoscalar coupling constant. D^{+} will obviously be a valid dispersion relation and its integral will converge if

$$\sigma_{-} - \sigma_{+} < \text{const}\ \omega^{1-\epsilon} \text{ as } \omega \to \infty \tag{8}$$

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[6] In treating the π-p forward dispersion relations we use the usual definition of the scattering amplitude $d\sigma/d\Omega = |F|^2$ and employ natural units $\hbar = c = \mu = 1$.

[7] Goldberger et al. [17] first obtained essentially this form. For a review and references etc. see Refs [8, 12].

where ϵ is a small but finite fraction. We can reasonably expect this condition to be fulfilled from the observed behaviour of the high energy total cross-sections.

The Froissart bound

$$\sigma_{total} \leq \text{const } (\ell n\ \omega)^2 \tag{9}$$

has been established from analyticity properties proved from axiomatic field theory (L. S. Z.) and polynomial boundedness. $F(s,t) < s^n$ for $s \to \infty$. Clearly, if this bound is satisfied D^+ converges well. For a discussion of this and other bounds see Refs [8-10].

D^- will obviously be a valid dispersion relation and its integral will converge if

$$\sigma_-(\omega') - \sigma_+(\omega') < \text{const } \ell n\ \omega \tag{10}$$

That is, if the Pomeranchuk theorem is valid and the difference of the π^+-p and π^--p cross-sections decreases faster than logarithmically, we can expect the unsubtracted D^- to be a valid dispersion relation.

Hence, in essence the validity of an unsubtracted D^- is a test of the Pomeranchuk theorem. Obviously the dispersion relations for individual charge states can be obtained by adding and subtracting D^+ and D^-. One should note that we have not made use of charge independence but only crossing symmetry in determining these relations.

TOTAL CROSS-SECTION MEASUREMENTS AND PARAMETRIC FITS

To determine the high energy behaviour of the $\pi^\pm$-p total cross-sections and allow evaluation of the pion-nucleon forward dispersion relations, high precision (0.3% absolute error) measurements were made in the incident momentum range of 8-29 GeV/c.

Figure 3 shows those results plotted versus a $1/P_{lab}$ scale which is a convenient way of exhibiting asymptotic behaviour, since infinite energy corresponds to the origin. Table I shows the five parameter power law fits tried. Five parameters were the minimum required and each of these three fits gave a reasonable χ^2. Each fit assumed that the Pomeranchuk theorem is valid and that each cross-section approaches a constant at infinity. Both of these assumptions were tested as follows:

(1) The constant was allowed to be different in π^+-p and π^-+p by the introduction of an additional parameter. However, the solution within the errors gave the same value for both constants.

(2) The constant in Fit I was replaced by a/P^m. The best χ^2 was obtained when m = 0 and limits on m were placed so that $|m| \leq 0.03 - 0.05$. Hence the fits to the data are consistent with the same constant being approached at infinity and in fact imply it.

These fits will be used in evaluation of the dispersion relation integrals. The asymptotic behaviour prediction will be discussed in a later section.

COMPARISON OF EXPERIMENTAL RESULTS WITH THE PREDICTIONS OF THE π-N FORWARD DISPERSION RELATIONS

Fit I was used above 8 GeV in the dispersion relation calculations shown in Figs (2, 4-8), and as can be seen, gave a generally good agreement. Fit III would give very similar results. Fit II, which is similar to a three pole P, P', ρ Regge fit,[8] gave similar good results for D^+ but due to the very slow convergence of the difference gave much too large theoretical values of D^- compared to the experimentally determined values, and therefore was rejected. However, one should note that with two standard deviations[9] on the exponent of the difference in Fit II, it could be brought close to agreement with the experimental values of D^- within the systematic error on D^-.

COMPARISON OF EXPERIMENTAL RESULTS AND D^+ PREDICTIONS

As previously noted, all important systematic errors including the uncertainty in δ tend to make about equal and opposite contributions to α_- and α_+, and hence they must to a large extent cancel in the experimentally determined D^+. Furthermore, the calculated D^+ is quite insensitive to the differences between Fit I, Fit II, and Fit III, while on the other hand, D^- is very sensitive to systematic errors and fit differences. Hence a critical test of the validity of the forward dispersion relation can be made by comparing the experimental results with the D^+ relation. This is done in Fig. 4(a), and we see that the agreement is good. If the total cross-section obeys the Froissart bound (the data are certainly consistent with this), the singly subtracted D^+ dispersion integral converges reasonably well. However, in order to greatly reduce the sensitivity of the integral to the high energy cross-section behaviour beyond the measured momentum range, an additional subtraction was made in D^+ at 20 GeV/c and the result is shown as a dashed line in Fig. 5, which fits the data excellently.

This doubly subtracted D^+ dispersion relation is virtually independent of the high energy behaviour of the total cross-sections. To demonstrate this, Fig. 5 shows the small effects of drastic, and what most of us would agree, are unphysical assumed changes in the behaviour of the high energy total cross-sections. The two assumptions made are that for $P > 35$ GeV/c (1) the total $\pi^{\pm}$-p cross-sections both vanish, and (2) both total cross-sections increase faster than linearly (i.e. as $P^{1.1}$). Since these unreasonable changes introduce only small changes[10] in the predicted values of D^+, it is therefore clear that we are virtually independent of any acceptable changes in the asymptotic cross-section behaviour. Furthermore, variations of the low

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[8] In Regge terminology

$$\alpha_{P'}(0) \approx 1 - 0.69 \pm 0.21 \approx 0.31 \pm 0.21$$

$$\alpha_{\rho}(0) \approx 1 - 0.31 \pm 0.06 \approx 0.69 \pm 0.06$$

[9] For these systematic errors it is our opinion that about two standard deviations represent the allowable range of variation.

[10] The curvature is of no significance, merely reflecting the discontinuous changes introduced in the total cross-section behaviour. The additional subtraction was taken at the top of the error flag of the 20 GeV/c points. In the bottom curve the additional subtraction was taken through the centre of the point.

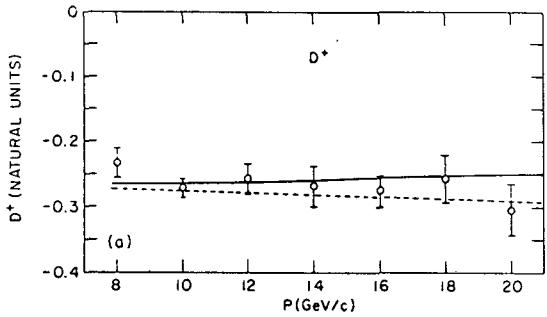

FIG.4(a). D^+ versus momentum. The solid curve is the prediction of the dispersion relations. The dahsed curve is the result of displacing the solid curve by 0.04, the systematic error (predominantly of a scale nature). Natural units have been used with $\hbar = c = \mu = 1$.

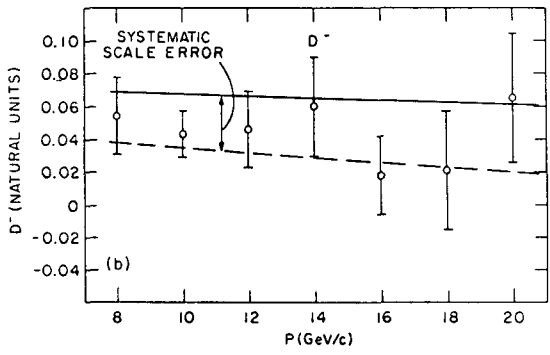

FIG.4(b). D^- versus momentum. The solid curve is the prediction of the dispersion relations. The dashed curve is the result of displacing the solid curve by 0.04, the systematic error.

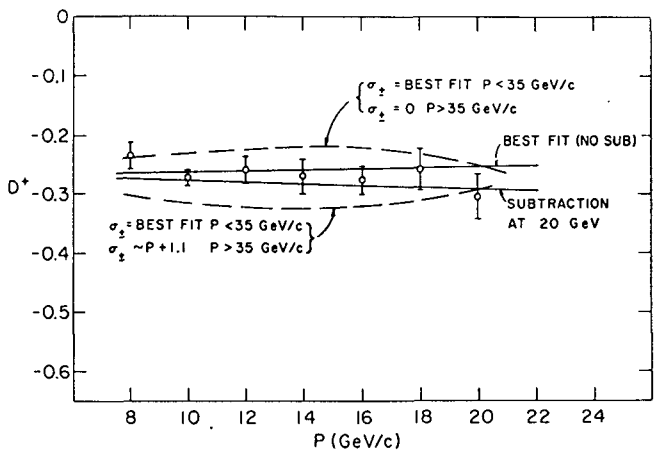

FIG.5. The D^+ dispersion relation with a second subtraction of 20 GeV/c. The small effects of even drastic and unphysical changes in the behaviour of the total cross-sections above 35 GeV/c are illustrated by the dashed line curves.

energy total cross-sections and parameters within their uncertainties make very little difference in the dispersion relation predictions in the high energy range we are interested in. Hence we have critically demonstrated the validity of the doubly subtracted D^+ forward dispersion relation to beyond 20 GeV/c. This constitutes a reasonable experimental proof of the validity of the doubly subtracted D^+ forward dispersion relation to beyond 20 GeV/c. Hence it follows that we have at least proven that $F(s, t \approx 0)$ must be analytic for all s smaller than the value of s corresponding to 20 GeV/c incident pions.

The predictions of the singly subtracted D^+ dispersion relation are in agreement with the data, and hence valid, provided the asymptotic behaviour of the high energy total cross-sections do not change appreciably from the behaviour of the good parametric fits obtained for the data. The sensitivity of the singly subtracted D^+ for asymptotic behaviour of total cross-sections can be deduced from Fig. 6. This figure shows the sum of α_- and α_+ (instead of D^+) since these earlier calculations were available. However, this combination is practically all D^+ with very little D^- mixed in. As would be expected, the sensitivity is mostly to the average level of the total cross-sections at higher energies, and we can conclude that the constant at infinity deduced in our parametric fits gives good agreement and a change of ~20% in its value would lead to disagreements.

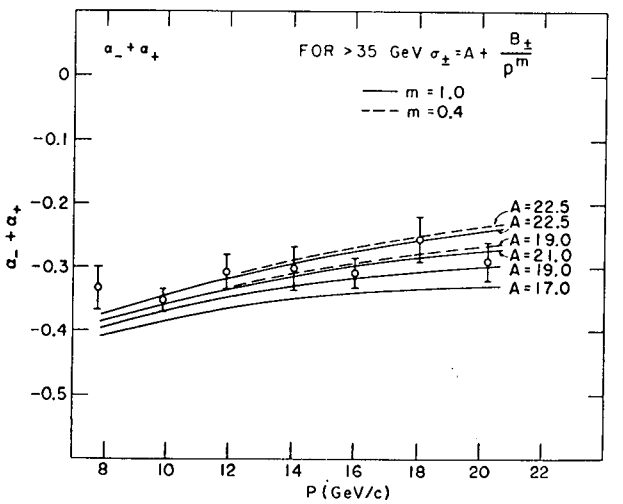

FIG. 6. The predicted ($\alpha_- + \alpha_+$) as a function of assumed asymptotic behaviour. The experimental data is also shown.

The calculated results for D^- for Fit I (simple individual power law fits) are compared to the data from 8 to 20 GeV/c in Fig. 4(b). Since, as previously explained, the systematic scale errors add in D^-, their effect on the comparison is represented by the dashed curve[11] which represents a displacement of the solid curve downward by the systematic error considered as a scale error. This is equivalent to a change of scale of the

[11] There obviously also exists a complementary dashed curve displaced upwards by the systematic scale error which is of less interest since the data do not overlap it.

experimental points for comparison purposes. However, as demonstrated in Fig. 7, D^- is very sensitive to the exact power of convergence of the difference $\sigma^- = (\sigma_- - \sigma_+)/2 = \text{const}/P^n$. For example, Fit II for the total cross-sections, which is similar to the Regge three pole type, has $n = 0.3 \pm 0.06$. As can be seen in the figure, the resultant calculated D^- does not fit the D^- determined from the data. However, 2-3 standard deviations plus the systematic error all taken in the proper direction would lead to an overlap with the data.

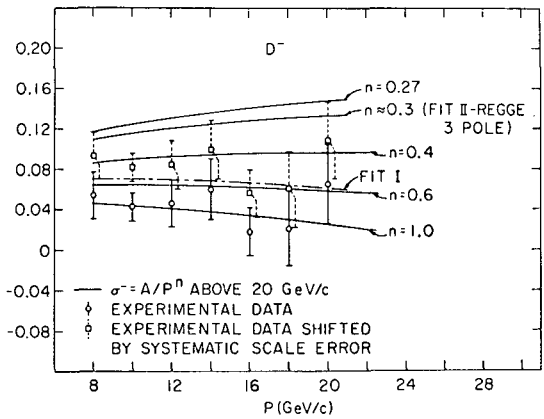

FIG. 7. The calculated values for D^- from the forward dispersion relations are shown as a function of the indicated assumed asymptotic behaviour of σ^-. The open circle points represent the experimental values. The open square points represent the experimental points displaced by the systematic error. It should be obvious from the figure that even two standard deviations on the Regge fit determined parameter $n = 0.306 \pm 0.06$ which then gives $n \approx 0.42$, and maximum displacement by the systematic error on the D^- data, all taken in the best direction to accommodate the Regge fit, still leads to a poor (but acceptable) fit at the high momentum.

It is obvious that an effective power of convergence in the range 0.5 to 1.0 would generally overlap the data. Hence, in spite of the appreciable scale uncertainty, the high sensitivity of the unsubtracted D^- to convergence of the cross-section difference allows us to use it as a crystal ball (assuming the dispersion relations were valid at higher energies) to predict the convergence.

FUNDAMENTAL LENGTH

The dispersion relations employed so far have been derived assuming microscopic causality holds down to infinitesimal distances and space-like separated points cannot interfere with each other (i.e. boson (fermion) field operators at such points commute (anti-commute)). This corresponds to the locality assumption in local field theories. There has been speculation concerned with non-local field theories in which this assumption fails below a fundamental length. It has been shown some time ago [18(a)][12] that if one

[12] Forward dispersion relations for the spin flip amplitudes and their derivatives were obtained by R. Oehme [18(a)] and A. Salam (18(b)].

relaxes this requirement and assumes an acausal region for spacelike points separated by an absolute distance $< |\ell|$ where $|\ell|$ is a so-called fundamental length, then the Cauchy dispersion integrals hold for a new defined function

$$F(\omega) = f(\omega)\, e^{i|\ell|\omega} \tag{11}$$

where as before

$$f(\omega) = D(\omega) + i\,A(\omega) \tag{12}$$

Oehme [18(a)], some time ago evaluated the result for neutral pions and it is straightforward using the crossing symmetry relation between positive and negative frequency cross-sections to derive the result for charged pions. Doing so and subtracting at $\omega = \mu$, and assuming $\ell \ll 1/\mu$, we obtain for D^+

$$D^+(\omega)\cos|\ell|\omega - A^+(\omega)\sin|\ell|\omega = D^+(\mu) + \frac{f^2k^2}{M\left[1-(\mu/2M)^2\right]\left[\omega^2 - \left(\frac{\mu^2}{2M}\right)^2\right]}$$

$$+ \frac{2k^2}{\pi} P\int_0^\infty \frac{\omega'\,d\omega'\,[A^+(\omega')\cos|\ell|\omega' + D^+(\omega')\sin\omega'|\ell|]}{(\omega'^2-\omega^2)(\omega'^2-\mu^2)} \tag{13}$$

When $\omega\ell \to 0$, these relations reduce to the ordinary forward dispersion relations. However, when $\omega\ell$ becomes an appreciable fraction of 1, they change drastically. The approximate calculated value of the singly subtracted D^+ for $|\ell| \approx 10^{-16}$ cm is shown in Fig. 8, and it is clear that even this small distance results in sizable changes in the predicted D^+ values which disagree with the experimental data values, which one should remember are almost free of systematic scale error. It is obvious that unforeseen changes in the asymptotic behaviour could also change the predicted D^+. However, an additional subtraction in D^+ would desensitize us to changes introduced by unforeseen asymptotic behaviour or a fundamental length at higher energies. However, if there were a fundamental length $|\ell|$ such

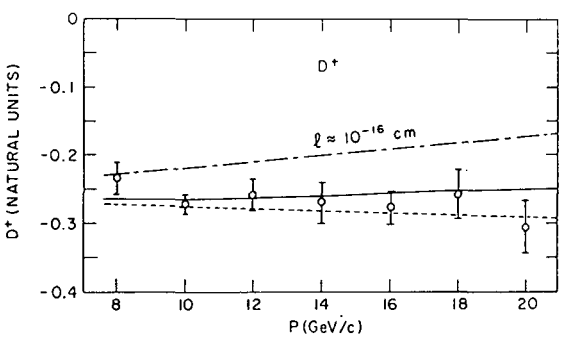

FIG. 8. The singly subtracted D^+ prediction is shown by the solid line. The dashed dot line shows the effect of assuming a fundamental length $\ell \approx 10^{-16}$ cm. The points shown represent our experimental data.

that $\omega_\ell = 1/|\ell|$ occurred within or below our experimental observation range, due to the structure of Eq. (13), especially the left-hand side, we would have drastic changes in both the predicted magnitude and energy behaviour of D^+, and therefore we can certainly conclude that if a fundamental length exists

$$|\ell| < 10^{-15} \text{ cm}$$

There is one uncertainty in the above procedure. Due to the form of the distance between two space-time points, they can correspond to a small space-like separation even when separated by a large spatial distance. It has also been shown that in strictly mathematical models, one cannot have a sudden sharp boundary between local and non-local space-time regions. Nevertheless, in physical models involving a finite transition region, this is possible.[13] The form of the above changes strongly implies that the most important contributions from the acausal region come from the space-like region which is physically close, since the changes depend on the product $\omega|\ell|$, indicating that high energy (i.e. accessibility to short distances) is necessary to observe the effects.

If one attempts to remove this uncertainty, one becomes model dependent. A recent particular model [19] for avoiding this uncertainty expressed the changes in the predicted values for the causal $D_\pm$ due to an acausal region characterized by $|\ell|$ in terms of the values of $D_\pm(\Omega)$, where $\Omega = 1/|\ell|$ is the lab energy corresponding to the fundamental length. Again, using this procedure, we can easily rule out a fundamental length occurring at energies which correspond to those within or below our range of measurement, as it would drastically change the dispersion relations.

If we assume a fundamental length at higher energies and calculate its effects in the present energy range, the procedure is uncertain since we must estimate by extrapolation $D_\pm(\Omega)$ as well as $\sigma_\pm(\Omega)$, and one cannot be sure that the characteristics of these (especially the former) will not change drastically at the energy which corresponds to the fundamental length. If we ignore these uncertainties and assume we can detect ~ 0.04 in D^+, we can conclude from this point of view that $|\ell| \lesssim 3 \times 10^{-16}$ cm.

Similar consideration can be applied to D^-, but due to its extreme sensitivity to asymptotic behaviour and the sensitivity of the D^- data to systematic scale errors, we believe it safer to restrict these analyses to D^+.

Hence, in summary we have been able to show directly that $|\ell| < 10^{-15}$ cm, independent of reasonable uncertainty in asymptotic behaviour. If we assume the characteristics exhibited by the parametric fits of the total cross-section data (at least for the sum) continue at higher energy, we find

$$|\ell| < 3 \times 10^{-16} \text{ cm to } 10^{-16} \text{ cm}$$

depending on the model employed.

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[13] This point was considered further during the discussion by Oehme, the author, and Lehman.

CHARGE INDEPENDENCE

As previously mentioned, so far we have not made use of charge independence in either our experimental analyses or comparisons with dispersion relations. However, we can use the hypothesis of charge independence, and our determination of the forward $\pi^{\pm}$+p scattering amplitudes to predict $d\sigma(t = 0)/dt_{ch.ex.}$. In Fig. 9 a comparison is made with the estimated values of $d\sigma(t = 0)/dt_{ch.ex.}$ obtained in charge exchange experiments. We have, in our predicted values of $d\sigma(t = 0)/dt_{ch.ex.}$, shown both systematic and statistical errors. Statistical errors only are shown in the estimated values from the charge exchange experiments. Stirling et al. [20(a)] estimated a normalization error of 5-10% and Manelli et al. [20(b)] estimated a normalization error of ± 8%.

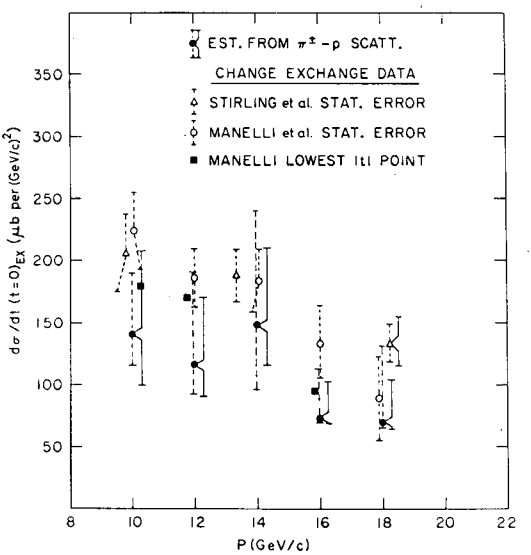

FIG. 9. Assuming charge independence, the predicted values of $d\sigma(t = 0)/dt_{ch.ex.}$ estimated from the $\pi^{\pm}$-p small angle scattering and total cross-section measurements, are compared with the values estimated by the authors from charge exchange scattering experiments. Solid lines are systematic errors; dashed lines are statistical errors. Statistical errors are generally only shown on charge exchange points. The associated systematic errors may be larger.

However, perhaps the most serious uncertainties are those involved in extrapolating the observed $d\sigma/dt_{ch.ex.}$ at finite t, to t = 0. Here as one recalls, the data show a rapidly t dependent shape due to the contribution of a sizable spin flip amplitude at small $|t|$.

In Ref. [20(a)] a parabolic in $|t|$ fit was used to determine $d\sigma/dt$ at t = 0. In Ref. [20(b)] the average of the first three points was taken. We have indicated the lowest $|t|$ point in Ref. [20(b)] which is still expected to be higher than $d\sigma(t = 0)/dt_{ch.ex.}$ to show the importance of uncertainties in these estimates. An analysis of possible electromagnetic corrections should also be made. Nevertheless, in spite of these uncertainties we find an excellent general agreement within our stated systematic errors, and the energy behaviour of the curves is reasonably similar.

Hence, in conclusion I believe we can be quite satisfied that we have a good verification of charge independence within the experimental errors in the range of 8-18 GeV/c. Although the uncertainties in the comparison are considerable since the charge exchange amplitude is $\lesssim$ 10% the individual T = 3/2 and T = 1/2 nuclear amplitudes, we can at least conclude that charge dependent terms in these individual amplitudes seem to be limited to ~1-2%.

FORWARD SPIN INDEPENDENT SCATTERING AMPLITUDES AND DISPERSION RELATIONS FOR p-p AND $\bar{p}$-p

p-p small angle scattering and total cross-section measurements were also made in the 8-26 GeV/c range [21]. Figure 12 shows the new total cross-section results for p-p and previous results for $\bar{p}$-p. In the p-p case, invariance principles limit the scattering amplitude to five independent complex functions. Two of these vanish as $t \to 0$ leaving the ordinary singlet, the ordinary triplet, and one spin flip amplitude. There are three similar amplitudes for $\bar{p}$-p. Therefore the assumption of spin independence of the forward scattering amplitudes is necessary for unique analysis of the experiments and unique predictions for the forward scattering amplitudes. Even with this spin independence assumption, in contrast to the $\pi^{\pm}$-p case, the forward dispersion relations for the p-p, and $\bar{p}$-p pair involve an extensive non-physical region which introduces further considerable uncertainty.

Figure 10 shows our results for α_{p-p}(the ratio of the real to the imaginary part of the nuclear amplitude) and the results of previous investigations [22, 23]. The errors shown at each of our points are those obtained from the least squares fits to the data and do not include the systematic error of ± 0.02, which is mostly of a scale nature. It is clear that there is reasonable agreement with all the previous data [22] except the earlier measurements of Bellettini et al. [23(a)]. Their later remeasurement of their point at 10 GeV/c [23(b)] gave a lower magnitude of α in agreement with our result. If this unexplained difference was assumed to be a scale shift error in their original measurements, and this was applied to their two higher points, it would explain the discrepancy.

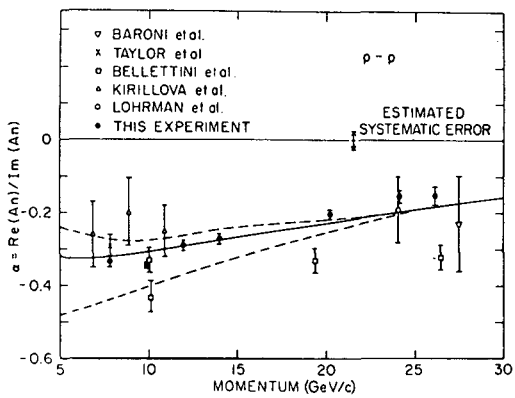

FIG. 10. α, the ratio of the real to imaginary part of the forward p-p scattering amplitude (assuming spin independence) compared to other data, and forward dispersion relation calculations.

The dashed lines [24(a)] in Fig. 10 indicate the range of uncertainty in the forward dispersion relation predictions due to uncertainties in the non-physical region contribution. An additional subtraction has been made at about 25 GeV in this treatment. The solid line is the prediction calculated [24(b)] making definite assumptions (the usual poles) for the non-physical region.

Our data certainly fit the predictions rather well. However, the significance of this is not too clear in view of the spin independence assumption and the uncertainty for the non-physical region contribution in the forward dispersion relation calculations. Nevertheless, it is quite comforting and at least implies that the p-p forward amplitudes are to a large degree spin independent, and that in spite of the extensive non-physical region, the high energy forward dispersion relation predictions are reasonable.

In regard to the spin independence assumption one should note that this gave a good fit to the experimental data. We were unable to obtain a good fit to the data with the assumption of purely imaginary amplitudes with different magnitudes and exponential slopes for ordinary singlet and triplet amplitudes. If we allowed a real part but simply relaxed the constraint that the magnitude of the singlet and triplet amplitudes be equal, we find the best fit when the amplitudes are equal, but the data are consistent with a difference as large as $\sim 30\%$ at which point α was larger by ~ 0.02.

For $\bar{p}$-p at 11.9 GeV/c, we find $\alpha = -0.006 \pm 0.034$ with an additional systematic error of ± 0.06 primarily due to uncertainty in the $\bar{p}$-p total cross-sections. This is in agreement with the dispersion relation prediction of -0.06 within the error. Hence, the forward dispersion relation prediction that $\alpha_{\bar{p}\text{-}p}$ is quite small compared to $\alpha_{p\text{-}p}$ is well verified.

ASYMPTOTIC BEHAVIOUR OF THE $\pi^{\pm}$-p TOTAL CROSS-SECTIONS

Figure 3 shows the predicted asymptotic behaviour of the $\pi^{\pm}$-p total cross-sections for the two parametric Fits I and II[14] versus a 1/P scale, where P is the laboratory momentum of the incident pion.

As noted previously, these good fits all satisfy the Pomeranchuk theorem, and indeed analyses of the data with additional free parameters which allowed violation of the Pomeranchuk theorem, give the result that within the error both total cross-sections became equal at high enough energy and approach the same constant asymptotically. Since we have shown that the π-p forward dispersion relations are valid up to at least 20 GeV/c, and there is no evidence of their failure probably to at least an order of magnitude higher energy, we shall assume that they are valid at all energies. Thus, we can use them as a crystal ball to predict asymptotic behaviours.

Both Fit I and Fit II satisfy the D^+ singly subtracted forward dispersion relation very well. As concluded in the analyses considered previously, varying the constant assumed at infinity by the order of 20% leads to disagreement with D^+. Thus, D^+ tells us that the constant we have determined is about right.

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[14] Fit III is similar to Fit I in general behaviour and therefore is not treated separately.

WHERE WILL THE POMERANCHUK THEOREM COME TRUE?

The best precision we can reasonably expect to attain even in the foreseeable future in measuring the differences of π^++p and π^-+p total cross-sections, is about 0.1%. For Fit I we find this occurs at around 25 000 GeV. Fit I furthermore gives good agreement between the predicted D^- and the experimentally determined D^-. This is a striking confirmation of the Pomeranchuk theorem, since if the Pomeranchuk theorem were not valid, the unsubtracted D^- would diverge and we would hardly expect to find agreement with its predictions. For Fit II (the three pole Regge type) the Pomeranchuk theorem would come true at about 1 600 000 GeV. However, Fit II gives predicted D^- values which disagree badly with experimentally determined values, and hence Fit I is selected, since its D^- predictions also agree with the experimental values.

Figure 11 shows the predicted values of $\alpha_\pm$ at higher momentum for Fit I and Fit II.

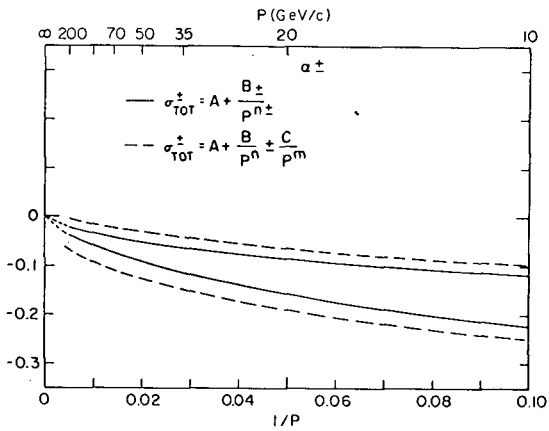

FIG. 11. The predicted values of $\alpha_\pm$ from the forward dispersion relations using Fit I (solid curve) and Fit II (dashed curve). Fit I agrees very well with the present real amplitude measurements, but Fit II (three pole Regge type) does not.

Figure 12 shows a fit, similar to type I, for the p-p and $\bar{p}$-p total cross-sections, which as you recall also fit the forward dispersion relation predictions well, assuming spin independence. In this case, the Pomeranchuk theorem comes true at $\gtrsim$ 35 000 GeV which is a comparable energy to that obtained for the π-p case.

As mentioned before, one clear-cut characteristic of 'Asymptopia' is the validity of the Pomeranchuk theorem. Therefore, a reasonable operational definition of where 'Asymptopia' begins, is that energy beyond which the Pomeranchuk theorem is satisfied to within the best experimental precision. From the analysis above, we see $\gtrsim$ 20 000 GeV is required to reach Asymptopia. Furthermore, it has been experimentally established that the π-p forward dispersion relations are valid to the highest presently attainable energies, and their predictions in the least subtracted convergent form, using extrapolation of presently obtained total cross-section data fits to higher

energies, are correct. This demonstrates that forward scattering amplitudes up to 20 GeV are analytic, and this probably continues to be true to much higher energies, quite possibly to infinity. Furthermore, Lorentz invariance, unitarity, crossing symmetry, and charge independence are all verified experimentally. The experimental results are clearly consistent with the Pomeranchuk theorem and the Froissart bound, the validity of both being strongly supported.

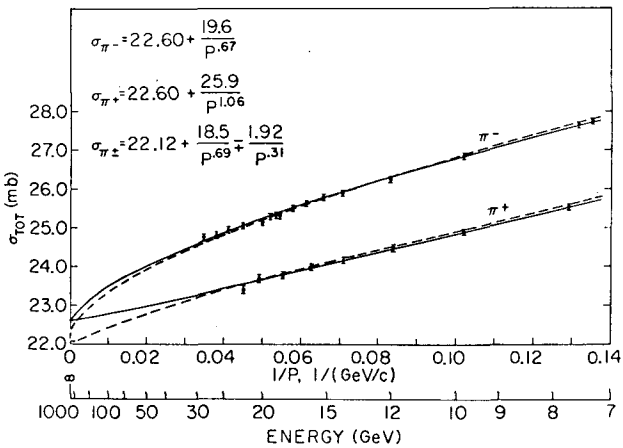

FIG. 12. The predicted values of σ_{total} (p-p) and σ_{total} ($\bar{p}$-p) by a fit of type I to the experimental data, versus a linear plot of 1/P in the lab system with a non-linear energy scale (GeV) also shown.

So far we have pursued asymptotic energy estimates primarily from the point of view of forward scattering amplitudes and forward dispersion relations.

The Regge pole model when first introduced was a promising attempt to budge the gap between 10 GeV and the asymptotic region. However, as Eden [10] has just illustrated in the interesting summary cartoon in his talk, the 1962-1963 high energy differential elastic scattering measurements [25, 26] forced it to adopt quite complex solutions to the elastic scattering measurements and abandon its simplicity.

It is becoming increasingly clear that elastic scattering amplitudes have diffractive characteristics which are not easily accommodated in Regge exchanges which seem to be simpler for some inelastic processes. Furthermore, the Pomeranchuk trajectory, which represents the asymptotic solution and is the most important one, has become flat (within errors) exhibiting no or at best very little Regge-like behaviour as Van Hove discussed this morning.

It now appears clear that the elastic scattering channels cannot in general be described by other than a complex summation of poles (not to mention the neglected cuts) which makes it almost impossible to demonstrate that the asymptotic solution corresponds to a Regge-like vacuum pole. Therefore, the Regge pole model still fails to do what it was originally designed for – to predict the asymptotic behaviour at very high energies.

At present energies we are directly influenced by the complex patterns of hadronic states and excited states observed. As is well known, these

have been quite well representable by the SU_3 and SU_6 symmetry groups (see talks by Ne'eman [27] Gürsey [28] and others on symmetry). A mathematical representation of the symmetry has been presented via the quark hypothesis. Hence, another point of view for estimating asymptotic energies could be the requirement that the equivalent quark threshold (whether fictional or not) be surpassed by enough energy so that threshold and other resonance effects become small.

If the quark mass is assumed as generally believed to be $\gtrsim$ 10 GeV, the threshold lab energy for producing a quark, anti-quark pair is about 250 GeV. If, as in the case of the pion threshold about another order of magnitude in the cm system is required to reach stabilized high energy behaviour, 'Asymptopia' would not be reached also, from this point of view, before 25 000 GeV.

Thus, I conclude that in order to reach 'Asymptopia' a number of bold giant steps in available accelerator energies will have to be taken.

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DYNAMICS OF STRONG INTERACTIONS AND THE S-MATRIX

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Abstract

DYNAMICS OF STRONG INTERACTIONS AND THE S-MATRIX. The physical principles underlying the S-matrix theory of strong interactions are reviewed. In particular, the problem of whether these principles are sufficient to completely determine the S-matrix, i.e. to yield a dynamical theory of strong interactions, is discussed.

The main theoretical successes in understanding particle physics have most often come from a quite elementary use of quantum mechanics. Quantum electrodynamics has taught us that, although we are able to make computations when we know the interaction and the perturbation approach is reliable, the very use of perturbation expansions and of the detailed structure of space-time which is implied by the use of local fields leads to unattractive features of the theory like infinities and renormalization. It seems therefore advisable to gain a better understanding of the notions of quantum mechanics which describe most basically the kind of experiments which we want to describe and explain. The concept which stands first in this respect is the so-called scattering matrix or S-matrix which we are now going to introduce.

A typical experiment in particle physics consists in sending a beam of particles labelled 1 upon a target which for our purposes is a bunch of particles labelled 2, and to observe all particles which are produced in the collision, for instance

$$1 + 2 \rightarrow 3 + 4$$

If we call p_1, p_2, p_3, p_4 the four-momenta of these particles, the conservation of momentum and energy which is well satisfied experimentally implies

$$p_1 + p_2 = p_3 + p_4 \tag{1}$$

Quantum mechanics tells us that the probability for observing the reaction with well-defined values of the momenta is the square of the modulus of a probability amplitude $A(p_1, p_2, p_3, p_4)$ which is linear in the initial state of two free particles $|p_1 p_2\rangle$ and anti-linear in the final state $|p_3 p_4\rangle$. Any such quantity defined on the Hilbert space of states must be the matrix element of an operator which we shall call S, i.e.

$$A(p_1, p_2, p_3, p_4) = \langle p_3 p_4 | S | p_1 p_2 \rangle \tag{2}$$

It is worth mentioning that this concept is quite fundamental and that in fact it does not need the previous introduction of notions such as the microscopic time which appears together with the definition of the Hamiltonian operator. Accordingly the level of the S-matrix concept is more basic than that of local fields and even of the Hamiltonian. For that reason alone, the S-matrix should be investigated in detail and we are now going to review its properties in the order of decreasing generality.

The most fundamental property of the S-matrix is unitarity, i.e. the property

$$S^{\dagger}S = SS^{\dagger} = I \tag{3}$$

which merely expresses the fact that the norm of all final states in a collision, i.e. the probability for getting any kind of final states is equal to 1. Therefore Eq.(2) only expresses the consistency of the probability interpretation of quantum mechanics.

The other properties of the S-matrix, namely analyticity and crossing, have been reviewed in detail in the paper by Eden so that we can be brief on that subject. If we introduce the square s of the energy in the centre-of-mass system and the momentum transfer $t = (p_3 - p_1)^2$ which, for given S, is essentially equivalent to giving the scattering angle, the amplitude $A(p_1, p_2, p_3, p_4)$ is a function of s, t and t, namely, after suppressing a few delta-functions which appear when the initial and final states are identical

$$A(p_1 p_2; p_3 p_4) = \delta_{12}\, \delta_{34}\, \delta^{(4)}(p_1 - p_3)\, \delta^{(4)}(p_2 - p_4) + T(s, t) \tag{4}$$

one gets as the important quantity the function T(s, t).

As it has been explained by Eden, the function T(s, t) is an analytic function of s and t, i.e. it can be extended to complex values of s and t. Much work has been spent on that idea, trying to give a satisfactory basis for justifying analyticity. Let us mention a few of the approaches for situating the problem:

a) It was first believed intuitively that analyticity in s which can be expressed as the existence of a dispersion relation for A(s, t) at a fixed value of t could be derived directly from causality. Indeed we know that causality, expressing that an outgoing signal cannot issue of a linear system before an incoming signal has reached it, allows to prove the dispersion relation of Kramers and Krönig for a host of classical quantities including for instance the refractive index of an optical substance. No satisfactory derivation has yet been given that analyticity in s directly follows from causality.

b) Analyticity in t can be derived directly (i.e. without introducing any unnecessary new concept) from the finite range of strong interactions, i.e. from the fact that the probability for a reaction decreases exponentially with the impact parameter under suitable conditions.

c) Axiomatic quantum field theory implies some analyticity for A(s, t), not quite as much as we should like. This was the most thoroughly investigated approach.

d) Perturbation calculus derived from field theory also gives analyticity of the individual terms of a perturbation expansion of A(s, t) as explained by Eden.

e) Last but not least, dispersion relations for pion-nucleon scattering in the forward direction, i.e. analyticity of $A(s,t)$ for $t=0$, has been checked experimentally as explained in this Symposium by Lindenbaum.

Another important property of the S-matrix is crossing which means essentially that an incoming particle with few-momentum p is equivalent to an outgoing particle with momentum -p. This is an old idea which goes back to the positron theory by Dirac which can be expressed precisely only for an analytic S-matrix. In equations, the crossing relation expresses the fact that the amplitude for $T^{12\to 34}(p_1, p_2, p_3, p_4)$ for the reaction

$$1+2\to 3+4$$

is the analytic continuation of the amplitude $T^{1\bar{3}\to \bar{2}4}(p_1, -p_3, -p_2, p_4)$ of the "crossed" reaction

$$1+\bar{3}\to \bar{2}+4$$

where $\bar{2}$ is the anti-particle of particle 2.

There appears to be a deep interplay of unitarity and crossing in the explicit analyticity properties of an amplitude. For instance, unitarity tells us that the matrix element of the unitary S-matrix in an elastic state of well-defined angular momentum and parity is of the form $e^{2i\delta(s)}$ where $\delta(s)$ is the so-called phase-shift. As long as this scattering is purely elastic, $\delta(s)$ is real, when s is larger than the first inelastic threshold of energy. $|e^{2i\delta(s)}|^2$ is smaller than one to take into account the probability of inelastic reactions so that $\delta(s)$ develops an imaginary part. This suggests that as an analytic function of s, $\delta(s)$ has a singularity when s takes such a value that a reaction channel opens. The same must be true of $A(s,t)$. The unitarity properties of crossed reactions, which correspond to the same function $A(s,t)$ must also give rise to some singularities of $A(s,t)$. It has been suggested and, up to now, well verified, that all the singularities of the scattering amplitude can be derived from the conditions imposed by crossing and unitarity. In this respect, unitarity should fix the Riemann sheet structure of the amplitudes, the position of their singularities as well as the discontinuities across these singularities. As it turns out, these discontinuities can themselves be written in terms of the amplitudes or their analytic continuation.

Let us stop a moment to envision the structure of the analytic S-matrix. We are given the stable particles (at least the ones which are stable under the strong interactions) with their masses and spin. Any reaction between these particles involving an arbitrary number of them depends upon their on-mass-shell momenta (i.e. $p_i^2 = m_i^2$) as analytic functions. The location of the singularities of these analytic functions is given by the mass spectrum, by some self-consistent singularities (resonance poles and associated cuts and so on). Once we know the singularities of the amplitudes and their discontinuities we can compute them by some extension of the Cauchy integral, but the discontinuities are themselves functionals of the amplitude. Finally, we are confronted with an infinite set of non-linear relations relating the infinite set of amplitudes for all possible reactions. We know only how to write explicitly the first few terms in the first few relations. Altogether, they express all the content of unitarity. This pattern leads naturally to a certain number of questions:

1) Is it possible to write explicitly all the relations or at least to give an algorithm for them? No such recipe is known so far, which casts some uncertainty on the next questions.

2) Do these self-consistent relations determine the amplitudes, i.e. do they constitute a dynamics? How many arbitrary parameters (for instance masses, coupling constants...) do they tolerate? As long as question 1) is not answered, question 2) has not much meaning.

3) Can we cut off the infinite number of relations between an infinite number of amplitudes to a finite set of relations involving a reasonable error? This question contains as a prerequisite the next one:

4) How do the amplitudes behave asymptotically when the momenta become large?

The main approaches to these questions up to now have been to investigate some consequences of reduced relations involving a more or less arbitrary cutting-off of amplitudes. Question 4), on the other hand, is rigorously investigated experimentally as you will learn from Van Hove's paper. Before going into the results of S-matrix dynamics, let us consider more carefully if there can be such a dynamics. Strangely enough, this is a question which has not been considered with great attention up to now.

The first question to be considered is obviously: can we reproduce the customary results of the Schrödinger equation formulation in the S-matrix framework? That it is so for the Coulomb problem has been recently answered in the affirmative. More precisely, one can prove that, in a non-relativistic approximation, the only scattering amplitude between charged particles which is compatible with a zero mass of the photon, S-matrix analyticity-unitarity and a few technical assumptions related to asymptotic behaviour is the customary Coulomb amplitude. As a result, we can reproduce the hydrogen atom properties and some more work would presumably give atomic physics as well. Unfortunately, there is some flavour of academism in such problems for them to be pursued actively enough.

One can also reproduce most results of quantum electrodynamics, including the anomalous magnetic moment of the leptons and the Lamb shift by S-matrix techniques without meeting any form of renormalization or infinities. Technically, some important problems remain and, here again, there is not enough motivation for the work to be pushed strongly but, generally, we can assert that S-matrix provides indeed an alternative dynamical formulation to more customary approaches. Sometimes, it is even simpler.

S- matrix is therefore a satisfactory candidate to become a framework for dynamics. In practice its most important successes have been in the phenomenology of strong interactions and they can be listed as follows:

1) Extraction of the pion-pion scattering amplitude from pion production in pion-nucleon collisions.

2) The relation between pion-nucleon phase-shift analysis and the properties of crossed reactions.

3) A better formulation of the meson theory of nuclear forces which would need to be pursued further before being absolutely convincing.

It has also provided a good framework for new ideas such as the introduction of Regge poles. A very interesting speculation is the so-called bootstrap idea, following which the self-consistent relations of S-matrix

theory should not only fix the amplitudes but the very possibility of the particles which enter into the reactions.

In practice, the present situation is so very exciting that it will forbid any conclusion as to the value of S-matrix dynamics. It has been found that the resonances in pion-nucleon scattering can be classified in rising Regge trajectories with a spin J related to the mass M by the strikingly simple linear formula

$$J = A + \frac{M^2}{(1\,\mathrm{GeV})^2}$$

valid up to high values of J ($\geqslant 15/2$). This fact can certainly be expressed inside the framework of S-matrix theory but it must lead to a reconsideration of some earlier approximations. In fact it is simpler than expected. One can therefore wonder whether something simpler than the intricate S-matrix pattern of infinitely many non-linear relations is not at hand, something which could also help to answer the difficult question: why are the amplitudes analytic? Whatever the issue, it can be said that, even now, the S-matrix ideas have been very fruitful, if only to allow a new situation and new questions in strong interaction physics.

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FAMILIES OF REGGE TRAJECTORIES

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Abstract

FAMILIES OF REGGE TRAJECTORIES. It is shown that the factorization property and the analyticity constraints in the Regge pole hypothesis give rise to the appearance of the parity doublets and of a new quantum number M. The necessity and the classification of the families of trajectories is briefly discussed. The relation with the group-theoretical approach is considered.

As Professor Abdus Salam has said in his general lecture, it is possible to put some order into the large (and continuously increasing) number of elementary particles and resonances by means of two independent classifications

a) SU_3 multiplets

b) Regge trajectories.

As has been said in other lectures, these concepts are also useful to classify and to understand the asymptotic terms which appear in the high-energy amplitudes at fixed momentum transfer.

We may hope to obtain a more powerful classification of resonances and a more complete understanding of the forward high-energy scattering, considering families of Regge trajectories connected at vanishing momentum transfer ($t = 0$). These families can probably be described by means of irreducible representations of the homogeneous Lorentz group $O_{3,1}$.

The first approach to this argument has been a group-theoretical study of the Bethe-Salpeter equation. An alternative procedure, which I shall try to discuss partially in the following, is to start from the following two assumptions:

a) Regge pole hypothesis
b) Analytic properties of the amplitude.

By "Regge pole hypothesis", I mean that the amplitude for fixed t and large s can be asymptotically described by means of the contributions of a set of Regge poles with factorized residues and well-defined quantum numbers. I neglect, in the first place, the contributions of other singularities, like Regge cuts.

The basic analytic properties of the amplitude are more easily expressed in terms of the spinor M-function, which depends on the particle spin indices and four-momenta. We consider a region in which t belongs to a neighbourhood of the origin and Re s is larger than a certain fixed value. Our requirement is that the contribution of a set of Regge poles to the M-function in this region is analytic apart from terms which decrease faster than all the Regge pole contributions considered. Note that

the M-function in this region has singularities due to inelastic S-channel thresholds; we have not considered them because they cannot appear in the Regge pole contributions.

Our problem is now to give a complete description of all the possible families of Regge trajectories which satisfy the conditions described above.

Often it happens that the analytic properties are satisfied by the sum of some Regge pole contributions, but not by the contributions of the single poles. This phenomenon is called "conspiracy" between different trajectories and is one of the difficult aspects of our problem. Another difficulty is due to the factorization property of the residues. Each residue can be written as the product of two functions and each of these functions can appear in many different reactions. Therefore, in order to find the properties of these functions, we have to consider the analytic properties of the amplitudes which describe all these reactions.

Due to these difficulties, our problem has not yet been completely solved. It has been treated by means of different techniques

a) The group-theoretical method: by using the matrix elements of the representations of the group $O_{3,1}$ it is possible to build explicitly families of trajectories which have all the required properties. However, it is not clear that all the possible solutions of our problem can be found in this way.

b) The analytic method: it permits one to find some important properties, which every family of trajectories must have, but it is very difficult to obtain by this method explicit formulae for all the trajectories and all the residues.

Combining the results of these two methods, we can understand the situation rather well.

An elementary application of the second method to an oversimplified problem is a simple calculation showing how the factorization property and the analyticity constraints give rise to two important consequences: the appearance of a new quantum number "M" and of parity doublets of Regge trajectories.

We consider the following equal mass scattering process

$$1 + 2 \to 3 + 4, \quad M_1 = M_3, \quad M_2 = M_4 \tag{1}$$

and the corresponding forward s-channel helicity amplitude. As the projection of the spins along the common direction of all the momenta is conserved, we have

$$f^s_{\lambda_1\lambda_2\lambda_3\lambda_4}(s, t = 0) = 0 \text{ if } \lambda_1 - \lambda_3 \neq \lambda_2 - \lambda_4 \tag{2}$$

This is a consequence of the properties of the M-function that we have assumed. At very high energy, the contribution of a Regge pole can be written in the form

$$f^s_{\lambda_1\lambda_2\lambda_3\lambda_4}(s, t = 0) \sim \beta^A_{\lambda_1\lambda_3} \beta^B_{\lambda_2\lambda_4} s^\alpha \tag{3}$$

We have to assume that, at least for some values $\bar{\lambda}_i$ of the helicities, the factorized residues do not vanish

$$\beta^A_{\bar{\lambda}_1\bar{\lambda}_3} \beta^B_{\bar{\lambda}_2\bar{\lambda}_4} \neq 0 \tag{4}$$

From Eq.(2) we have

$$\bar{\lambda}_1 - \bar{\lambda}_3 = \bar{\lambda}_2 - \bar{\lambda}_4 = M \tag{5}$$

This equation defines a quantum number M, which, as we shall see, characterizes the behaviour of the trajectory at $t = 0$. From Eq.(2) it follows also that

$$\beta^A_{\lambda_1\lambda_3}\beta^B_{\lambda_2\bar{\lambda}_4} = 0 \quad \text{if} \quad \lambda_1 - \lambda_3 \neq \bar{\lambda}_2 - \bar{\lambda}_4 = M \tag{6}$$

and from Eqs (4) and (6) we obtain

$$\beta^A_{\lambda_1\lambda_3} = 0 \quad \text{if} \quad \lambda_1 - \lambda_3 \neq M \tag{7}$$

In a similar way we have

$$\beta^B_{\lambda_2\lambda_4} = 0 \quad \text{if} \quad \lambda_2 - \lambda_4 \neq M \tag{8}$$

Equations (7) and (8) describe all the possible different behaviours of the residues at $t = 0$.

We have not yet taken into account that the Regge poles must have a well-defined parity. If we indicate by σ the product of parity and signature (i.e. the "natural parity"), we have the constraints (for elastic scattering)

$$\begin{cases} \beta^A_{-\lambda_1,-\lambda_3} = \sigma\,\beta^A_{\lambda_1\lambda_3} \\ \beta^B_{-\lambda_2,-\lambda_4} = \sigma\,\beta^B_{\lambda_2\lambda_4} \end{cases} \tag{9}$$

It is clear that Eqs (7) to (9) are compatible only if $M = 0$. If we want to obtain asymptotic contributions to amplitudes with non-vanishing "helicity flip", we have to introduce two Regge poles, superposed at $t = 0$, with the same signatures and opposite parities. If we distinguish the residues of these two poles by means of the superscript $\pm$, by means of an extension of the argument used above we obtain

$$\begin{cases} \beta^{A\pm}_{\lambda_1\lambda_3} = 0 \quad \text{if} \quad |\lambda_1 - \lambda_3| \neq M \\ \beta^{B\pm}_{\lambda_2\lambda_4} = 0 \quad \text{if} \quad |\lambda_2 - \lambda_4| \neq M, \quad M > 0 \end{cases} \tag{10}$$

$$\beta^{A+}_{\lambda_1\lambda_3}\beta^{B+}_{\lambda_2\lambda_4} = \beta^{A-}_{\lambda_1\lambda_3}\beta^{B-}_{\lambda_2\lambda_4} \quad \text{if} \quad \lambda_1 - \lambda_3 = \lambda_2 - \lambda_4 \tag{11}$$

This pair of conspiring Regge trajectories is called a "parity doublet".

Summing up, it is possible to classify the families of Regge trajectories by means of

a) The intercept $\alpha(0)$ of the leading trajectories
b) The quantum number M introduced above ($M \geq 0$)
c) The signature τ of the leading trajectories
d) If $M = 0$, there is only one leading pole and one has to specify its natural parity σ.

It is possible to show that, in the general spin case, if we consider the leading poles introduced above and take into account also the terms containing $s^{\alpha-1}$, the identity (2) is not satisfied. Therefore, we have to introduce new "daughter poles" which cancel the unwanted terms.

The structure of the daughter trajectories has been studied by Freedman and Wang in the spinless unequal mass case. The result is that for each "parent" trajectory there is a set of "daughters", labelled by the index $n = 1, 2, 3, \ldots$ with the same natural parity σ and with

$$\alpha_n(t = 0) = \alpha(t = 0) - n, \quad \tau_n = \tau(-1)^n \tag{12}$$

Of course, if there is a doublet of parent trajectories, there are also parity doublets of daughters.

The situation described above is just the result which one obtains also by means of the group-theoretical formalism. It can be shown that the quantum numbers α, M, σ, τ can be interpreted as parameters which label the irreducible representations of the group $O_{3,1}$ (including space and time reflections).

The more detailed versions of the group-theoretical and analytic methods give not only information about the values of the Regge parameters at $t = 0$, but also information about the derivatives of these parameters. Reactions with arbitrary masses and spins can be treated. The classification of the families of trajectories and some other results obtained above hold also in this more general case. For instance, for forward arbitrary mass scattering we have

$$f^s_{\lambda_1\lambda_2\lambda_3\lambda_4}(s, \theta_s = 0) \sim C_{\lambda_1\lambda_2\lambda_3\lambda_4}\, s^{\alpha} + O(s^{\alpha-1}) \tag{13}$$

$$C_{\lambda_1\lambda_2\lambda_3\lambda_4} = 0 \quad \text{if } \lambda_1 - \lambda_3 \neq \lambda_2 - \lambda_4$$

$$\text{or if } |\lambda_1 - \lambda_3| \neq M$$

In the photoproduction process

$$\gamma + p \rightarrow \pi^+ + n \tag{14}$$

it has been shown experimentally that the forward differential cross-section decreases with approximately the same power of s as the differential cross-section at fixed small non-vanishing t. As it is necessarily

$$\lambda_\pi - \lambda_\gamma = \pm 1 \tag{15}$$

from the results given above we can deduce that, if the high energy behaviour of this process is determined mainly by a single family of trajectories, this family must have $M = 1$. One of the leading trajectories of this family could be the pion trajectory.

The total cross-section for unpolarized particles given by the optical theorem is proportional to

$$\mathrm{Im} \sum_{\lambda_1 \lambda_2} f^{s}_{\lambda_1 \lambda_2 \lambda_1 \lambda_2} (s, t = 0) \tag{16}$$

As only amplitudes with vanishing helicity flip appear in this expression, we have that only poles with $M = 0$ can give a contribution to this quantity. In order to avoid cancellation between different terms, we have also to require that $\sigma = 1$. We can conclude that the trajectories P, P', ρ, ω, A_2, which are necessary to interpret the unpolarized total cross-sections, are the leading trajectories of families with the quantum numbers $M = 0$, $\sigma = 1$.

It is clear that fermion trajectories must belong to families with half-integral M, so that parity doublets must necessarily appear.

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LIFE ON THE LIGHT CONE (Behaviour of Regge trajectories near zero energy)

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Abstract

LIFE ON THE LIGHT CONE (BEHAVIOUR OF REGGE TRAJECTORIES NEAR ZERO ENERGY).
1. Introduction; 2. Description of the spin of a particle; 3. Reggeized "particles" and the trouble with the structure of the spin algebra on the light cone; 4. The lesson of field theory; 5. The trajectory formula; 6. Inclusion of internal symmetries - tribes of Regge trajectories; 7. Conclusion.

1. INTRODUCTION

Our purpose is to describe a phenomenon in "Regge theory" which recently received much attention under various names: those names include "conspiracy theory", "daughter trajectories", "Lorentz poles" and probably a few more. Despite the simplicity of the physical ideas involved, there is considerable confusion in the field, mainly as a consequence of the following fundamental theorem, which we quote without proof.

Theorem: Let G be the set of physicists active in group theoretical methods, A the set of physicists working with analytical methods. Then the intersection

$$A \cap G$$

is a set of zero measure.

In view of the theorem just quoted we think that an elementary exposition of the subject, emphasizing the underlying physical ideas rather than technical details, may be useful. We do use both algebraic and analytical arguments, however, only to an extent which, on the one hand, is necessary to expose the ideas clearly, and, on the other, does not require anything but the usual familiarity of the average theoretician with quantum mechanical operators, their commutators, etc.

It should be mentioned that the direction of research which is sketched here is certainly not the only possible line of attack of the problem. Toller describes in his paper an alternative way of looking at things and certainly there are several others, e.g. the one pursued by the Trieste group. It is not my purpose to review the entire field and, correspondingly, the list of references quoted is necessarily incomplete.

2. DESCRIPTION OF THE SPIN OF A PARTICLE

This subject is almost as old as quantum mechanics itself; it received its final formulation in the classical paper by Wigner [1], which is, by the way, one of the most readable papers ever written.

If I have an ordinary particle (with time-like momentum) I say that it has spin S, if in the rest frame the wave function of the particle has $2S+1$ components, and I can find three operators, S_1, S_2, S_3 so that they satisfy the usual angular momentum commutation relations (CR)

$$[S_1, S_2] = iS_3 \text{ (cycl.)}$$

and all the possible states of the particle can be reached by repeated application of the operators S_1, S_2 on any given state.

Starting from this definition it is straightforward to define the spin operators for a state of any given momentum, say $p_\mu = (0, 0, p, \sqrt{p^2 + M^2})$ where M is the mass of the particle.

Indeed, if we denote a state vector in the rest frame by $|Sm\rangle$ (S stands for the spin, m for its projection onto an arbitrarily fixed direction), then the state with momentum p_μ can be generated by a unitary transformation $U(L(p))$. This unitary transformation describes the effect of a Lorentz transformation, $L(p)$, on the state vector; the Lorentz transformation $L(p)$ in turn is specified by the condition that it boosts up the momentum $p_\mu = (0, 0, 0, M)$ into $(\vec{p}, \sqrt{\vec{p}^2 + M^2})$. (This is not sufficient to specify $L(p)$ entirely, since a Lorentz transformation depends in general on six parameters. Three of them are fixed by prescribing, say, the components p_1, p_2, p_3 of the momentum; the remaining arbitrariness means essentially that before boosting we can perform any rotation in the rest frame without affecting the final value of the momentum. This arbitrariness can be used to choose a "convenient" form of $L(p)$, corresponding, e.g. to a "helicity" description of the state.) Whatever the specific form of $L(p)$, it is evident that the operators $S_i(p) = U(L(p))\, S_i U(L(p))^{-1}$ act on the "moving states" $|psm\rangle \equiv U(L(p))|sm\rangle$ in the same way as the S_i did on the states $|sm\rangle$.

Therefore, all that can happen is that the matrix elements $\langle psm'|S_i(p)|psm\rangle$ are linear combinations – depending on the convention we adopt to fix $L(p)$ – of the matrix elements $\langle sm'|S_i|sm\rangle$.

This fact is usually expressed in operator form by stating that the operators S_i have definite index-transformation properties under Lorentz transformation. Let us remember, however, that the statement of index-transformation properties depends on conventions; the physically essential fact is that we have three independent operators to describe the spin. Usually one describes the spin by a four-vector (this is the so-called Pauli-Łubański spin vector, or Wigner spin operator), which is entirely specified by the prescription that its components in the rest frame are $(S_1, S_2, S_3, 0)$. If we go over to a reference frame with momentum, say, $(0, 0, p, \sqrt{p^2 + M^2})$, the components of S_μ become

$$S_\mu(p) = (S_1, S_2, \frac{\omega}{M} S_3, \frac{p}{M} S_3)$$

The fact that only three of the $S_\mu(p)$ are independent can be expressed covariantly by the relation: $p_\mu S_\mu(p) = 0$.

It is convenient to multiply our operators through by the factor M/ω which is just the Lorentz factor coming from the contraction of normalization volume of states. Thus I shall consider the operators

$$W_1(p) = \frac{M}{\omega} S_1$$

$$W_2(p) = \frac{M}{\omega} S_2$$

$$W_3(p) = S_3$$

(and $W_0(p) = (p/\omega)\, S_3$, which I shall not use).

Now the commutation relations are

$$[W_1(p),\ W_2(p)] = i\,\frac{M^2}{\omega^2}\, W_3(p)$$

$$[W_3(p),\ W_1(p)] = iW_2(p)$$

$$[W_2(p),\ W_3(p)] = iW_1(p)$$

These CR have the nice property of allowing a continuation in the mass of the particle. Indeed one can check by direct calculation that if one constructs the corresponding "spin"-operators (in technical terms, the operators of the little group) from Wigner's paper [1], they will satisfy precisely the same CR for time-like, light-like and space-like momenta as those obtained by continuing M in the previous equation (with p fixed) from positive values (M = 0 corresponds to a light-like vector, pure imaginary M to a space-like one).

Now this idea is very similar to the one we used in the old days to obtain "Reggeized" expressions of all kinds of vertices, scattering amplitudes, etc.; it is just the usual technique of analytic continuation of expressions we know in a certain region of the variable. (I do not want to go into details with this problem, such as checking the uniqueness of the continuation, etc.)

3. REGGEIZED "PARTICLES" AND THE TROUBLE WITH THE STRUCTURE OF THE SPIN ALGEBRA ON THE LIGHT CONE

The analytic continuation of the spin algebra we were just talking about becomes an acute problem when we start considering exchanges of Regge poles in a scattering amplitude. In previous works [2] we made it plausible that Reggeized vertices share many properties of ordinary wave functions; I am referring to these works for details. Here I will accept this as a fact and proceed further.

Looking at the CR of the spin algebra we realize that at zero mass (i.e. when the four-momentum becomes light-like) something drastic happens to the algebra – namely two of the spin operators (with our choice of the momentum, W_1 and W_2) commute.

From the geometrical point of view, this corresponds to the well-known fact that a massless particle can have but one spin orientation (two, if one allows for space inversion); all the others freeze in as the mass goes to zero. This seemingly harmless phenomenon leads, however, to unpleasant singularities in scattering amplitudes if a single particle can be exchanged in them. (From this point of view it is immaterial whether the exchanged particle is "Reggeized", i.e. is a composite state, or not.)

Troubles of this kind with single-particle exchange terms were known for quite a time, but it was not until 1967 that the geometrical origin of the difficulty was understood [3] and a universal solution proposed. Instead of presenting the general considerations, we choose to analyse a simple example and abstract the solution from it.

4. THE LESSON OF FIELD THEORY

Suppose Simplicio asked two friends of his, Dr. S. Matrix and Dr. Qu. Field, to solve the following elementary problem:
S: Work out the kinematics of a vector meson coupled to two spinless particles, and let the mass of the meson go to zero in the final expression – how will they act?

Obviously, both of them will realize that letting the mass go to zero can be realized meaningfully only if one views the system from a reference frame, where the meson has a non-vanishing total momentum. From then on, however, they proceed differently.

Dr. Matrix: Well, I know that the meson has spin one, therefore, if I label its polarization states by λ, the helicity, then the vertex must be just proportional to $\frac{W}{\omega}|\vec{p}|Y_1^{\lambda}(\theta,\phi)$ where $|\vec{p}|$, θ, ϕ specify the magnitude and orientation of the relative momentum, and W, ω are the mass and energy of the vector meson, respectively. I can work out the relative momentum in the rest frame of the meson and I find

$$|\vec{p}|_{\text{rest}} = \frac{1}{2W}\sqrt{[W^2 - (m_1+m_2)^2]\,[W^2 - (m_1 - m_2)^2]}$$

where m_1, m_2 are the masses of the scalar particles. Also, for the time-like component p_0, I find

$$p_{0\ \text{rest}} = \frac{m_2^2 - m_1^2}{2W}$$

Let us now transform into the frame where the momentum of the meson is $(0, 0, K, \sqrt{K^2 + W^2})$ so that I have, e.g.

$$p_3 = \frac{\cos\vartheta}{2W}\sqrt{[W^2 - (m_1+m_2)^2]\,[W^2 - (m_1 - m_2)^2]}\left(1 + \frac{K^2}{W^2}\right)^{\frac{1}{2}} + \frac{m_2^2 - m_1^2}{2W}\cdot\frac{K}{W}$$

where ϑ, φ specify the orientation of the momentum in the rest frame.

I see that the vector meson vertex is just proportional to the spherical components p_λ of the relative momentum, $(\lambda = 0, \pm 1)$, so, e.g. the zero helicity component in the limit of small W (keeping K fixed) will be (apart from irrelevant factors) just $\sim \frac{W}{\omega}\cdot\frac{K}{W^2}(m_2^2 - m_1^2)\cos^2\frac{\vartheta}{2}$ whereas for $\lambda = \pm 1$ I get $\sim \frac{\sin\vartheta\ \exp(\pm i\varphi)}{2W}(m_2^2 - m_1^2)\frac{W}{\omega}$.

S: But for $\lambda = 0$ it goes to infinity as W goes to zero ....
Dr. Matrix: - ** ? ... **; *** !!!

I told you years ago that there is some trouble with that unequal mass problem. Of course, it blows up, but I cannot help that; that's how it is ....

At this point his friend interrupts him.

Dr. Field: Now look here. I, of course, believe your kinematics. However, you have to describe your vector meson by a field, say, A_μ. Then I work out according to the Feynman rules the vector meson vertex and find that it is proportional to

$$\sim \frac{W}{\omega} p_\mu e_\mu$$

where now p_μ stands for the relative four-momentum and e_μ is the polarization four-vector of the vector meson. Now I work out the expression of e_μ. In the rest frame of the vector meson I choose

$$e_k^{(\lambda)} = \delta_{k,\lambda} \quad (k = \pm 3), \quad e_0 = 0$$

so that in the moving frame I will have

$$e_\pm^{(\lambda)} = \delta_{\lambda,\pm 1} \qquad \text{(as before)}$$

$$e_3^{(\lambda)} = \delta_{\lambda,0} \frac{\omega}{W}$$

and for the time-like component

$$e_0^{\lambda} = \delta_{\lambda,0} \frac{K}{W}$$

The rest of the kinematics is the same as you did it, and for $\lambda = \pm 1$ I get the same finite result as you do. However, for $\lambda = 0$ the situation is entirely different. Look: the vertex will be proportional to

$$\frac{W}{\omega}\left(p_3 e_3^{(0)} - p_0 e_0^{(0)}\right)$$

I get the following expression for p_0 in the limit of small W:

$$p_0 \sim \frac{K}{W^2}(m_2^2 - m_1^2)\cos^2\frac{\vartheta}{2}$$

or

$$p_0 \sim p_3 \qquad (W \to 0)$$

Also, since $\omega = \sqrt{K^2 + \omega^2} \sim K$, I find that the vertex for zero helicity just goes to zero – as it should – since a massless particle can have $\lambda = \pm 1$ only. My "secret" is, of course, that the time-like component of the four-vector just compensates the bad behaviour of the longitudinal part.

S: Now, you had three components of your e_μ to describe the three spin-states of your particle. Do you ascribe another particle (a scalar) to the fourth component?

Dr. Field: Of course not. I introduced A_μ instead of the three-component spin-"wave function" $\vec{e}$ just to describe Lorentz covariance in a convenient way. I exclude the spin zero component of A_μ with the help of the subsidiary condition $\partial_\mu A_\mu = 0$. By the way, the spin zero component is unphysical anyhow, as you quite well know from your undergraduate studies ....

Now, Dr. Field was absolutely right. He wanted to describe an elementary particle in a convenient way and had the right to impose subsidiary conditions to eliminate the so-called superfluous components of the field. What was a convenient tool in ordinary field theory becomes absolutely obligatory if one describes a composite state. It was shown [3, 4] that in order to avoid conflicts between Lorentz invariance and standard analytic properties of the vertices (or scattering amplitudes), the "wave functions" – i.e. vertices – of massless composite states (or Regge poles, if we choose that description) have to span a representation of the entire homogeneous Lorentz group $\mathscr{L}$ (just as in our example the field A_μ spans the adjoint representation $(1/2\ 1/2)$ of $\mathscr{L}$). There are two essential differences with respect to the elementary particle case, however. The first can be understood in the following way. As is well known, the irreducible representations (IR) of $\mathscr{L}$ (or rather its covering group, SL(2, C)) are characterized by two numbers, (j_1, j_2). The familiar finite-dimensional, non-unitary IR are obtained if both j_1 and j_2 are half-integers. Then the angular momentum contents of this IR is obtained by a Clebsch-Gordan decomposition, i.e. the angular momentum states contained in the IR (j_1, j_2) are

$$j = j_1 + j_2,\ j_1 + j_2 - 1,\ \ldots,\ |j_1 - j_2|$$

I introduce the numbers σ, j_0 by writing

$$j_1 = \frac{1}{2}(\sigma - j_0), \quad j_2 = \frac{1}{2}(\sigma + j_0)$$

then I can write that

$$j = \sigma - \kappa$$

where $\kappa = 0, 1, 2, \ldots, \sigma - |j_0|$. It turns out that one obtains all the IR of SL(2, C) by putting σ formally equal to any complex number, while keeping

j_0 half-integer, as before (it looks like "Reggeizing" j_1 and j_2). The "complex angular momentum contents" are again given by the relation $j = \sigma - \kappa$ ($\kappa = 0, 1, 2, \ldots$). However, κ can increase indefinitely since the IR is infinite dimensional.

At this point there comes the second difference. In a bound state theory we have no reason (or right) whatsoever to impose subsidiary conditions. Hence, all the components with $\kappa = 0, 1, 2, \ldots$ will give rise to a Regge pole in the scattering amplitude. In other words, we find that Regge poles can occur in infinite families only if one is to maintain the good analyticity and invariance properties of scattering amplitudes. This result looks pretty weird at first sight; it turns out, however, that it is quite useful since it helps us to bring some order in the mess that we usually call hadron resonances.

5. THE TRAJECTORY FORMULA

Once we have an infinite family of Regge trajectories, we should like to see how they behave off the point $W = 0$. The problem was solved by means of a power series expansion around $W = 0$ [5]. The resulting formula is extremely simple and its structure can be understood without performing detailed calculations.

Clearly, if one goes off the point $W = 0$, the SL(2, C) classification of Regge trajectories breaks down. However, one has to be careful to break SL(2, C) only, but not rotational invariance. There is, essentially, one such quantity, namely the square of the ordinary angular momentum operator. Now it turns out that the first order term in W is just proportional to $(j + 1/2)$ (of course, taken at $W = 0$, since we expand in powers of W), i.e. $(\sigma - \kappa + 1/2)$. Similarly, the second-order term in W will be a second-order polynomial in $(\sigma - \kappa + 1/2)$, and so on. To take an example, the <u>nucleon</u> trajectories can be expanded as

$$\alpha_\kappa(W) = \sigma - \kappa + A(\sigma - \kappa + \tfrac{1}{2})\, W + [B_1 + A^2(\sigma - \kappa + \tfrac{1}{2}) + B_2(\sigma - \kappa + \tfrac{1}{2})^2]\, W^2 + \ldots$$

The constant σ gives the intercept at $W = 0$ of the leading nucleon trajectory. The magnitude of σ, A, B_1, B_2, etc. depends on dynamics or can be fitted phenomenologically [6]. Lacking a reasonable dynamical theory, we chose the second possibility and fixed the constants by the positions of the following states: $P_{11}(938)$, $P_{11}(1466)$, $S_{11}(1550)$, $F_{15}(1688)$. It turned out that all the known $I = 1/2$ πN resonances can be accommodated beautifully in the scheme. Not only that, but we had to put $S_{11}(1550)$ and $P_{11}(1466)$ on a trajectory with $\kappa = 2$; thus we were able to <u>calculate</u> the first "daughter" trajectory. It turned out to be identical to the trajectory that Regge phenomenologists used to call N_β and N_γ and reproduced the positions of the negative signature resonances with an accuracy of about 5 - 10%.

I do not have the time to go through this analysis in all its details, which can be found in the reference just quoted [6].

However, I would like to emphasize again that this large group apparently <u>does</u> give new predictions and leads to a successful classification of the hadron states.

It is appropriate to mention that very recently two groups of authors (Salam and Strathdee in Trieste, and Jones, Kuo and Teplitz at MIT) gave

new derivations for the trajectory fomula. Salam and Strathdee use the analogy with field theory; Jones, Kuo and Teplitz use consistency conditions imposed on a scattering amplitude where families of Regge trajectories are exchanged.

6. INCLUSION OF INTERNAL SYMMETRIES - TRIBES OF REGGE TRAJECTORIES

This is a rather new development [7] so I just want to make a few remarks about it.

It is clear from the analogy with field theory that the SL(2, C) algebra we were dealing with is generated by a spin tensor. If one believes in some sort of a field theory, the spin tensor is given by the space integral of a local spin density, $S^{\rho}_{\mu\nu}(x)$

$$S_{\mu\nu} = \int d^3x \, S^0_{\mu\nu}(x)$$

Since we are dealing with massless objects, a chiral SU(3) ⊗ SU(3) is a good (approximate) symmetry (as far as SU(3) is good). Denoting the appropriate chiral charges by $F_i^{(\pm)}$ (i = 1, . . . 8), they, of course, satisfy

$$\left[F_i^{(\pm)}, F_j^{(\pm)}\right] = i f_{ijk} F_k^{(\pm)}$$

$$\left[F_i^{(\pm)}, F_j^{(\mp)}\right] = 0$$

while the $S_{\mu\nu}$ satisfy the ordinary CR of the Lorentz group. As long as we consider SU(3) ⊗ SU(3) exact, $S_{\mu\nu}$ will be a pure unitary singlet and so

$$\left[S_{\mu\nu}, F_i^{(\pm)}\right] = 0$$

However, if I break SU(3), the operator $S_{\mu\nu}$ will, in general, acquire an octet part, $S^i_{\mu\nu}$.

It is instructive to see how this can arise. Consider a Lagrangian model, consisting of a quark field Q, interacting with an octet vector meson field A_μ through a Pauli-interaction

$$\mathscr{L} = \mathscr{L}(Q, \bar{Q}) + \mathscr{L}(A^i_\mu) + g\bar{Q} \; \lambda^i \, \sigma_{\mu\nu} Q(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)$$

If I determine in the usual way the spin tensor of the A-field, it will, of course, be a pure unitary singlet. However, if I break SU(3) at the $\bar{Q}QA$ vertex by adding an SU(3) non-invariant form, say

$$g'\bar{Q}\sigma_{\mu\nu}[\lambda^k, \lambda^i]Q(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)$$

(k fixed), then the spin tensor of the A-field will have an octet component, viz.

$$S^k_{\mu\nu} = g' \int d^3x \left\{ \overline{Q}\, \sigma_{\mu 0} [\lambda^k, \lambda^i]\, Q\, A^i_\nu - (\mu \rightleftarrows \nu) \right\}$$

Now the "octet spin tensors" connect the algebras spanned by the $F_i^{(\pm)}$ and the $S_{\mu\nu}$. The seventy operators $S_{\mu\nu}$, $S^k_{\mu\nu}$, $F_i^{(\pm)}$ close to an algebra which is isomorphic to SL(6, C). A physically interesting representation of this algebra can be constructed which contains three SL(2, C) families of Regge poles at W = 0. We call these together a tribe.

For example, the baryon tribe consists of the symmetry limit of an octet with $|j_0| = \frac{1}{2}$, which we denote by $(\sigma, \frac{1}{2};\ 8)$ and two other families, namely $(\sigma + 1, \frac{3}{2};\ 10)$ (which is the ordinary Δ-family) and, finally, a low-lying decuplet family, $(\sigma - 1, \frac{3}{2};\ 10)$. The simplest and, according to preliminary results, quite satisfactory way to break this large algebra is to break SU(3) at W = 0. Since, according to our previous results [6], SL(2, C) is only slightly broken, for low-lying states (both W and κ small), we can neglect SL(2, C) breaking altogether. In this way one arrives at a remarkably simple trajectory formula describing the leading trajectories in the octet and high-lying decuplet family. The formula reads

$$\alpha_{\kappa=0}(\epsilon, W) \approx \sigma + \epsilon + \frac{aY}{\sqrt{3 + \epsilon(\epsilon + 2)}}$$

$$+ b(1 - \epsilon)\left[I(I+1) - \frac{Y^2}{4} - 1 \right]$$

$$+ c\, W^2 (N(\epsilon))^{-\frac{1}{2}}$$

where ϵ is a parameter chosen such that $\epsilon = 0$ for the octet, $\epsilon = 1$ for the decuplet trajectory, $N(\epsilon)$ is the dimension of the corresponding SU(3) representation.

We just mention that this formula (and its more accurate versions) allows one to calculate decuplet masses. For example, for the mass of the non-strange quartet under consideration one finds $M_\Delta^2 \approx 1.3\ \mathrm{GeV}^2$, which, in view of the crudeness of the approximation, can be considered a fair value (the experimental value is $\approx 1.5\ \mathrm{GeV}^2$). Also, one gets "SU(6)-like" mass formulas, e.g.

$$\sqrt{\frac{8}{5}}\ \frac{\Xi^{*2} - N^{*2}}{\Xi^2 - N^2} = 1$$

which is satisfied to about 8%.

7. CONCLUSION

I think that it is fair to say that this new branch of high-energy physics, of which I have tried to give a brief and incomplete sketch, has just begun to evolve. Therefore, instead of trying to gaze into a crystal ball, or trying to draw conclusions which would prove wrong in a matter of a few months, I would just refer you to the appropriate literature [8].

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THEORY AND APPLICATIONS OF UNITARY SPIN *

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Abstract

THEORY AND APPLICATIONS OF UNITARY SPIN. 1. Introduction; 2. The abstract Lie algebra of SU(3); 3. Particle assignments; 4. Unitary symmetry and strong-interaction intensity rules; 5. Breaking SU(3) and the Gell-Mann and Okubo mass formulae; 6. Electromagnetic transitions; 7. Universality of the atonous currents; 8. Non-leptonic atonous interactions; 9. A current-generated algebra and chiral $[U(3) \times U(3)]_{\gamma_5}$; 10. Back to the strong interactions; 11. Origin of SU(3) and its breaking; 12. Observing a strongly-coupled current; 13. Nature of the fixed pole of Abarbanel et al.

INTRODUCTION

This paper consists in a presentation of the ideas and typical results of the theory of unitary spin.[1,2] As described in Professor Salam's introductory lecture, one expects to obtain from this approach

a) A classification of the hadrons

b) Strong-interaction intensity rules (e.g. couplings, branching ratios, relationships and sum rules between amplitudes, high-energy residues, etc.)

c) A mass spectrum, and the corresponding intercepts L(0) for the L(t) trajectories passing through these masses at $L(m^2) = J$

d) Hadron matrix elements coupled to non-strong interactions

e) Exact and approximate conservation laws, with a dynamical role providing the elements of a future dynamical and structural theory. Such a theory will then explain the classification, mass spectrum, etc.

We shall first familiarize ourselves with the abstract matrix algebra of SU(3). We shall then use the particle assignments and discuss SU(3) as a symmetry theory of the strong interactions; it is a broken symmetry and we shall also discuss what is known of the breaking mechanism. We shall then move to electromagnetic and weak interactions, and reformulate the theory of unitary spin: this time we use the universality properties of the weak couplings and start out from the atonous (weak) currents as a more precise foundation.

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[1] I have published a lecture-note volume Algebraic Theory of Particle Physics, W. A. Benjamin (1967), containing what is probably the fullest account to-date of our subject-matter. There are two textbooks, by P. Carruthers and M. Gourdin which deal with the earlier phases of SU(3) theory. Lipkin (Lie Groups for Pedestrians), Frazer (Elementary Particles), Gasiorowitz (Elementary Particles), Pilkuhn (Hadron Interactions) have chapters on Unitary Symmetry.

[2] The main papers relating to SU(3) are contained in The Eightfold Way, a reprint volume edited by M. Gell-Mann and myself, published by W. A. Benjamin, New York (1964). To these should be added H. Goldberg and Y. Ne'eman, Nuovo Cim. 27 1 (1963), which contains the basic idea of a fundamental triplet field with $B = \frac{1}{3}$ (later known as "quarks").

One further preliminary note: now that we know that the SU(2) algebra of I-spin sits in a SU(3) generalized symmetry, there is a tendency to regard this as an obvious result due to first principles, i.e. SU(2)→SU(3) seems natural. However this was really an experimental question, and the algebras of O(4), O(5) ≡ $S_p(4)$ and G_2 (one of the exceptional Lie algebras) could all have been correct answers, fitting the requirement of containing only I_3 and Y (the hypercharge) as additive quantum numbers. The choice of SU(3) at this stage is just a phenomenological identification, which in itself may hopefully lead to a deeper understanding.

THE ABSTRACT LIE ALGEBRA OF SU(3)

The group SU(3) is the set of all unitary unimodular three by three matrices. It is generated by exponentiation of the following set of eight traceless Hermitian matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{1}$$

From (1) by explicit calculation we obtain

$$\mathrm{Tr}\,(\lambda_i \lambda_j) = 2\delta_{ij} \tag{2}$$

$$[\lambda_i, \lambda_j] = 2if_{ijk}\lambda_k \tag{3}$$

Gell-Mann's normalization (2) used in (1) has now become traditional, although it implies remembering that the abstract or physical group generator (for <u>any</u> representation) is then taken as $F_i \sim \frac{1}{2}\lambda_i$, so that the actual set of unitary spin charges satisfies

$$[F_i, F_j] = if_{ijk}F_k \tag{4}$$

The structure constants f_{ijk} are real and totally antisymmetric as can be seen from the equality

$$\mathrm{Tr}\,\lambda_k[\lambda_i, \lambda_j] = 4if_{ijk} \tag{5}$$

which is obtained directly from Eqs (2) and (3).

Beside the commutation relations (3) among the λ's which define the algebra and are independent of the representation, one can also write the anti-commutation relations

$$\{\lambda_i, \lambda_j\} = 2d_{ijk}\lambda_k + \frac{4}{3}\delta_{ij} \tag{6}$$

Here the real constants d_{ijk} are not algebraic coefficients because the anti-commutator does not belong to the Lie algebra. Therefore Eq. (6) is only true for the particular choice (1) of the λ-matrices rather than for any SU(3) representation. The d_{ijk} are symmetric in all three indices on account of the relation

$$\mathrm{Tr}\, \lambda_k \{\lambda_i, \lambda_j\} = 4d_{ijk} \tag{7}$$

obtained from Eqs(2) and (5).

It is of great importance to notice that Eqs (5) and (7) are the only two scalar products which can be formed with three λ-matrices, i.e. from three octet tensors of SU(3). This corresponds to the two possible couplings ("F" and "D") among three octets.

The matrices λ_1, λ_2, λ_3 define the isotopic-spin SU(2) subgroup. In fact one has

$$I_x = \tfrac{1}{2}\lambda_1, \qquad I_y = \tfrac{1}{2}\lambda_2, \qquad I_z = \tfrac{1}{2}\lambda_3 \tag{8}$$

with the diagonal matrix λ_3 standing for the conserved z-component of the isotopic spin. The matrix λ_8 is also diagonal and commutes with the isotopic spin matrices. It represents an observable which is identified with the hypercharge

$$Y = \frac{1}{\sqrt{3}}\lambda_8 = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & -2/3 \end{pmatrix} \tag{9}$$

The electric charges are obtained by substituting in the formula of Gell-Mann and Nishijima

$$Q = e\left(I_z + \frac{1}{2}Y\right) \tag{10}$$

I_z by $\frac{1}{2}\lambda_3$ and Y by $\frac{1}{\sqrt{3}}\lambda_8$. We obtain

$$Q = \frac{1}{2}e\left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right) = e\begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} \tag{11}$$

The baryonic number is given by the unit matrix λ_0 which does not belong to SU(3) but to U(3). Normalizing λ_0 to the same length as the other λ's (see Eq. (2)), we obtain

$$\lambda_0 = \sqrt{\frac{2}{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{12}$$

and the baryonic number turns out to be

$$B = \frac{1}{\sqrt{6}} \lambda_0 = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{pmatrix} \tag{13}$$

where the factor $1/\sqrt{6}$ has been inserted in order to get the correct physical values for the known particles.

Although no known particle belongs to the fundamental representation $\underset{\sim}{3}$ of SU(3) given by (1), (9), (11) and (13), the physical content of the Lie algebra is best presented by introducing three hypothetical particles ("quarks") which may or may not exist (see Footnote 2). They belong to the eigenstates of the matrices Y and Q which have been normalized in a convenient way in order to get the correct physical values for the known particles that are accommodated in higher order representations. Following Gell-Mann, the three quark states will be designated by the symbols u, d, s (up-down and singlet; in Trieste, uno-duo and solo). Their hypercharge y and charge q are then

$$u\left(y=\frac{1}{3},\ q=\frac{2}{3}e\right)\ ,\quad d\left(y=\frac{1}{3},\ q=-\frac{1}{3}e\right)\ ,\quad s\left(y=-\frac{2}{3},\ q=-\frac{1}{3}e\right) \tag{14}$$

Quarks have baryonic number 1/3, i.e. we shall regard the simplest physical baryons as three-quark products. However, if we want to distinguish between a B = 1 state resulting from $\underset{\sim}{3} \otimes \underset{\sim}{3} \otimes \underset{\sim}{3}$ and a similar B = 1 state made of four quarks and one anti-quark $\underset{\sim}{3} \otimes \underset{\sim}{3} \otimes \underset{\sim}{3} \otimes \underset{\sim}{3} \otimes \bar{\underset{\sim}{3}}$ (anti-quarks have B = -1/3 and also opposite I_z and Y), we have to define an additional "quark-counting" quantum number. B counts the number of quarks minus the number of anti-quarks (divided by 3) and this new quantum number

$$D = \frac{1}{\sqrt{6}} \beta\lambda_0 \tag{15}$$

counts their sum (β = 1 for $\underset{\sim}{3}$ and β = -1 for $\bar{\underset{\sim}{3}}$, like the Dirac equation β). Using D amounts to extending the algebra to U(3) × U(1).

It is sometimes convenient to work with the two diagonal λ_3 and λ_8 and the raising and lowering generators

$$I^+ = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad I^- = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad U^+ = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$
$$U^- = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad V^- = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad V^+ = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{16}$$

We use the I-U-V notation introduced by Lipkin.

The action in the octet space is graphically represented in the root diagram of Fig. 1. This diagram shows that there are three SU(2) subgroups whose raising operators are separated by angles of 120°. (We have called

the raising operator of V spin V^- so as to reproduce the effective ΔQ as in I-spin.) They define three spins, i.e. the I-spin, the U-spin and the V-spin. The axis of electric charge is also shown in Fig. 1 at a right angle from the U-axis.

One designates every irreducible representation of SU(3) as $((\lambda, \mu))$ or sometimes simply d, where d is the dimensionality and λ, μ are the coefficients of the highest weight

$$\begin{aligned} \lambda &= 2\, i_z \\ \mu &= 2\, u_z \end{aligned} \tag{17}$$

with

$$d = \frac{1}{2}\,(\lambda+1)(\mu+1)(\lambda+\mu+2) \tag{18}$$

The weight diagram of the representation $((1,0))$ or $\underset{\sim}{3}$ is plotted in Fig. 2. It is represented by one box $\square$ in a Young tableau. Comparing with Fig. 1 we see that I^- will transform $u \to d$, U^- turns $d \to s$, V^- turns $u \to s$. The other fundamental representation $((0,1))$ is the conjugate or *contragradient* $\bar{3}$ and describes anti-quarks. It is defined by taking the transpose of the λ-matrices with minus sign

$$\lambda_i' = -\tilde{\lambda}_i \tag{19}$$

and its weight diagram is given in Fig. 3.

This representation can also be defined as the antisymmetric part of the tensor product of the quark representation by itself, as far as the SU(3) action is concerned (i.e. disregarding B and D). This is due to the fact that the completely antisymmetric part of the product of three quarks is a scalar (the determinant $\epsilon^{ijk} q_i q_j' q_k''$), so that the *antisymmetric product of two quarks* has to cancel the quantum numbers of a quark and thus is isomorphic to an anti-quark. Explicitly

$$3 \otimes 3 = 6 \oplus \bar{3} \tag{20}$$

By the multiplication rules of Young tableaux we find indeed

$$((1,0)) \otimes ((1,0)) = ((2,0)) \oplus ((0,1))$$

$$\underset{\sim}{3} \otimes \underset{\sim}{3} \qquad \underset{\sim}{6} \oplus \bar{\underset{\sim}{3}}$$

Another important product is

$$((0,1)) \qquad ((1,0)) \qquad\qquad ((0,0)) \qquad ((1,1))$$

$$\bar{3} \otimes 3 \qquad = \qquad 1 \oplus 8 \tag{21}$$

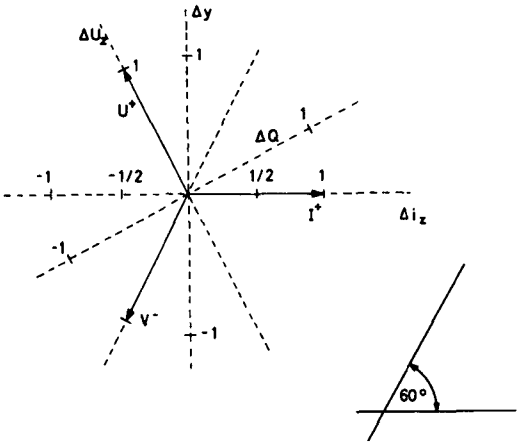

FIG. 1. Root diagram.

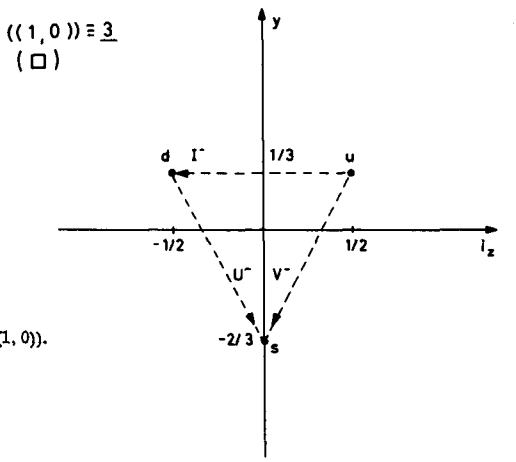

FIG. 2. Weight diagram of the representation ((1, 0)).

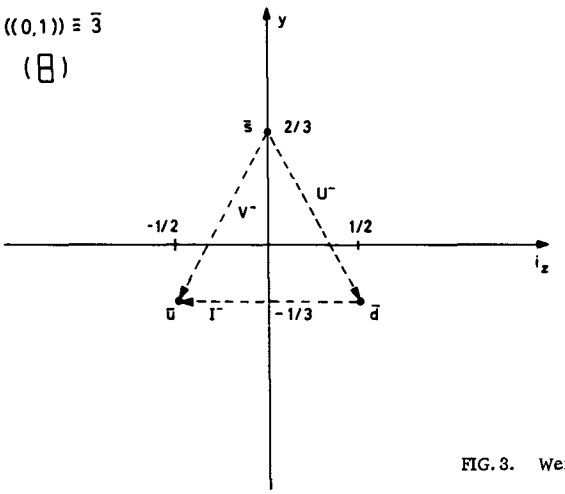

FIG. 3. Weight diagram of the representation ((0, 1)).

The tableau $\equiv ((0,0)) \equiv 1$ is the above-mentioned scalar, i.e. a product of the identity. The remaining $((1,1))$ representation has dimension 8 and is self-conjugate ($\bar{8} \equiv 8$; this is true whenever $\lambda = \mu$). This is the "regular" or "adjoint" representation and can also be obtained by means of the structure constants. To see this, let us rewrite the commutation relations between the λ-matrices

$$[\lambda_i, \lambda_j] = -2F_i{}^{(j}{}_{k)}\,\lambda_k \tag{22}$$

with

$$F_i{}^{(j}{}_{k)} = -\,if_{ijk} \tag{23}$$

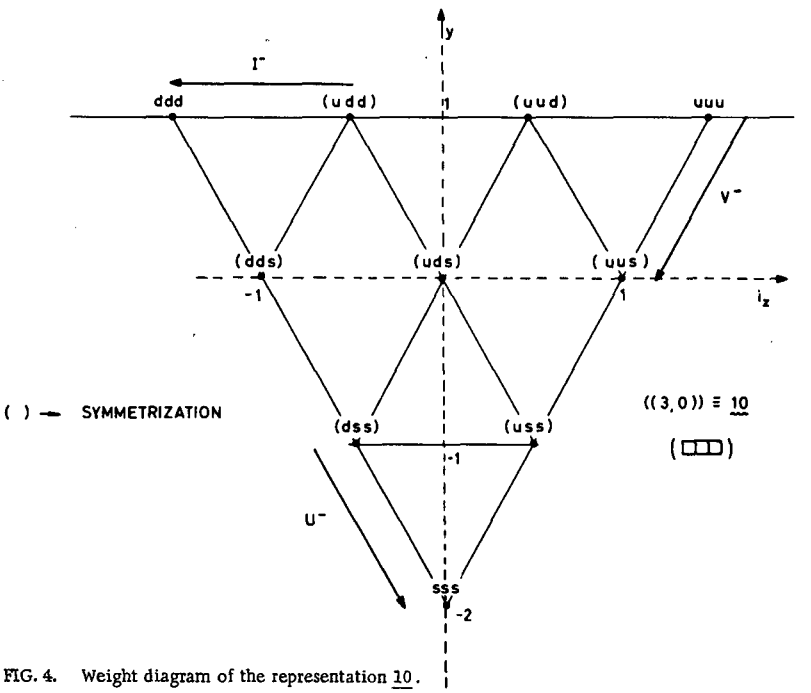

FIG. 4. Weight diagram of the representation 10.

Written in this form it is clear that the structure constants $F_i{}^{(j}{}_{k)}$ can be considered as a set of 8×8 matrices $(F_i)^j{}_k$, where j and k designate the rows and columns of the matrix F_i, operating in the octet space spanned by λ_k vectors. Its weight diagram is a hexagon identical with the root diagram of SU(3) (Fig. 1), including two zero weights (corresponding to the diagonal matrices I_z and Y there). In fact we can always write the vectors in $\underset{\sim}{8}$ space as three by three matrices. After all, such a matrix is indeed the matrix product of a column vector $\underset{\sim}{3}$ and a row vector $\bar{\underset{\sim}{3}}$, which is how we got $\underset{\sim}{8}$ in Eq. (21).

Since $\bar{3} \sim [3 \otimes 3]_{\text{antisym.}}$, we can also construct an octet from the two available products of three quarks

$$3 \otimes 3 \otimes 3 = (\underset{\sim}{6} + \underset{\sim}{\bar{3}}) \otimes \underset{\sim}{3} = \underset{\sim}{1} + \underset{\sim}{8} + \underset{\sim}{8} + \underset{\sim}{10} \tag{24}$$

The representation $\underset{\sim}{10}$ corresponds to $\square\square\square$. It is easy to write the wave functions, remembering the action of T^-, U^-, V^- and starting from the heighest weight "uuu". (See Fig. 4.) Computing the product of two octets we get

$$\underset{\sim}{8} \otimes \underset{\sim}{8} = \underset{\sim}{1} + \underset{\sim}{8}^D + \underset{\sim}{27} + \underset{\sim}{8}^F + \underset{\sim}{10} + \underset{\sim}{\overline{10}} \tag{25}$$

If the two octets are identical, $1+\underset{\sim}{8}^D+\underset{\sim}{27}$ correspond to the symmetric combinations, and $\underset{\sim}{8}^F+\underset{\sim}{10}+\underset{\sim}{\overline{10}}$ to the antisymmetric. The double appearance of 8 in the product is an important feature of SU(3). The reduction coefficients of the antisymmetric product 8^F are just the f_{ijk} of Eq. (3), and the symmetric coefficients for 8^D are the d_{ijk} of Eq. (6), as can be seen from a look at these equations, remembering that a λ_i matrix is a vector component in octet space.

Other useful products are:

$$\underset{\sim}{8} \times \underset{\sim}{10} = \underset{\sim}{8} + \underset{\sim}{10} + \underset{\sim}{27} + \underset{\sim}{35} \tag{26}$$

$$\underset{\sim}{10} \times \underset{\sim}{\overline{10}} = \underset{\sim}{1} + \underset{\sim}{8} + \underset{\sim}{27} + \underset{\sim}{64} \tag{27}$$

PARTICLE ASSIGNMENTS

The basic identification is that of the baryon $J^P = \frac{1}{2}^+$ octet (Fig. 5), which serves also as some kind of "normalization" for the entire set of assignments. It is assigned to $\underset{\sim}{8}$ with B = 1, D = 1. The $J^P = \frac{3}{2}^+$ baryons fit $\underset{\sim}{10}$ with B = D = 1 (Fig. 6). J is the spin, "P" the relative parity. There are already further $\underset{\sim}{8}$ with B = 1 (but D as yet not fixed) at $J = \frac{5}{2}^+$ and perhaps at $J = \frac{1}{2}^-, \frac{9}{2}^+$; $J = \frac{1}{2}^+, \frac{3}{2}^-, \frac{7}{2}^-$ display either $\underset{\sim}{8}$ or $\underset{\sim}{10}$. There are singlet $\underset{\sim}{1}$, B = 1 states at $J = \frac{1}{2}^-, \frac{3}{2}^-$.

For mesons the identified representations are all "nonets", i.e. $\underset{\sim}{1} + \underset{\sim}{8}$. The simplest have $D = \frac{2}{3}$. They exist for $J^{PU} = 0^{-+}, 1^{--}, 1^{++}, 1^{+-}, 2^{++}$ and perhaps for $0^{++}, 2^{-+}$ and higher J. Figure 7 reproduces the 0^{-+} multiplet. The index "U" stands for the charge parity of the eighth component in $\underset{\sim}{8}$.

UNITARY SYMMETRY AND STRONG-INTERACTION INTENSITY RULES

The strong-interaction S-matrix (i.e. the S-matrix after turning-off the electromagnetic and atonous ("weak") interactions) is only approximately invariant under U(3); B, Y and I-spin are exactly conserved, but the remaining four generators ($U^{\pm}$, $V^{\pm}$) are not. This is seen in the mass differences between I, Y multiplets within the same $((\lambda, \mu))$ supermultiplet. We shall return to this question later, but we now observe that this breaking seems relatively "subdued". It appears that actual reactions conserve SU(3) to within some 10%, once we do extract the effects caused directly by the mass-differences. Between 1961 (when the theory was suggested) and 1968, it

seems that many supposed violations vanished when better experiments became available, justifying Dirac's point in his "From a Life of Physics" lecture in this Symposium.

It used to be thought that K couplings violated SU(3) violently, when compared to π couplings. An extremely thorough study by J. K. Kim based on an effective-range analysis was followed by further checks by Chan and Meiere [1] yielding

$$(g_{KN\Lambda})^2 = 13 \pm 3 \tag{28}$$

$$(g_{KN\Sigma})^2 = 0 \pm 1 \tag{29}$$

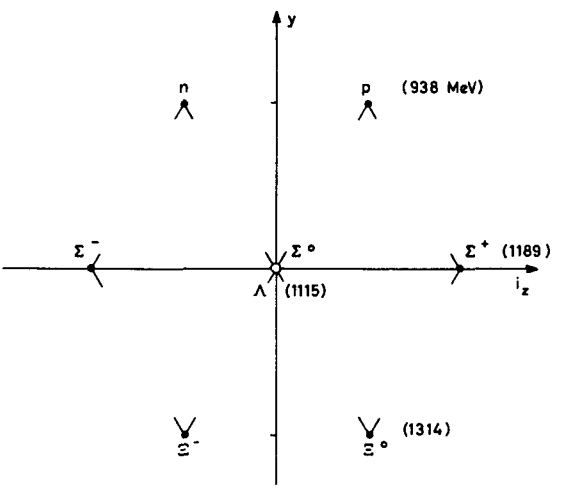

FIG. 5. The $J = \frac{1}{2}^+$ octet.

We have known for more than ten years that

$$(g_{\pi NN})^2 = 14.5 \pm 0.4 \tag{30}$$

SU(3) leaves one free parameter in the $\underset{\sim}{8}^B \otimes \underset{\sim}{8}^P \rightarrow \underset{\sim}{8}^B$ coupling of the baryon octet 8^B to the pseudoscalar $\underset{\sim}{8}^P$. This corresponds to the ratio between F and D couplings in Eq. (25). Since $(g_{\pi NN})^2$ provides an overall scale, we can fix

$$f = F/(F+D) \tag{31}$$

from Eq. (28), using one of the experimental couplings and the appropriate reduction coefficients in the formulae

$$(g_{KN\Lambda})^2 = (g_{\pi NN})^2 (1+2f)^2/3 \tag{32}$$

$$(g_{KN\Sigma})^2 = (g_{\pi NN})^2 (1-2f)^2 \tag{33}$$

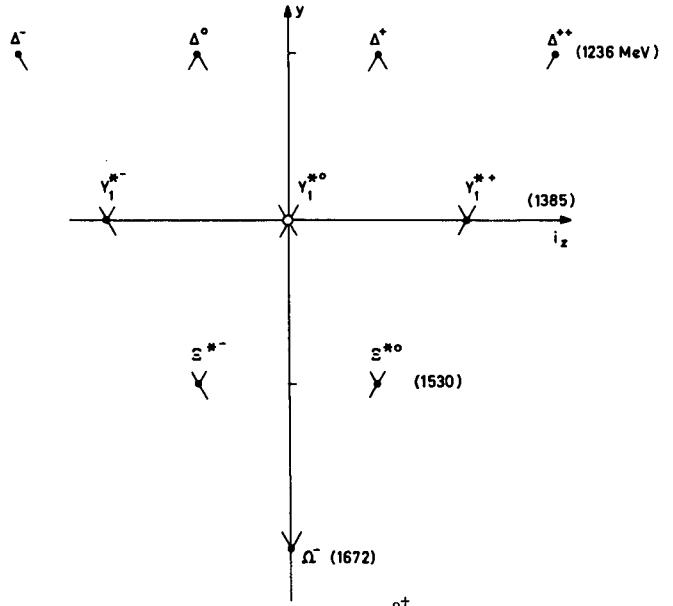

FIG. 6. The $J = \frac{3^+}{2}$ decuplet.

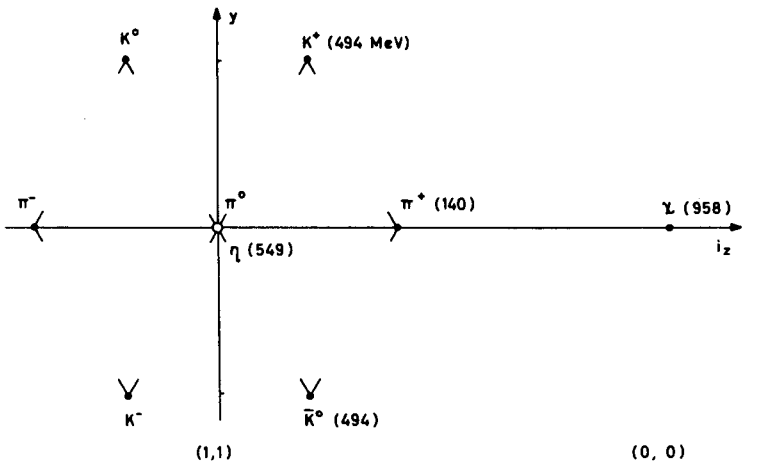

FIG. 7. The 0^- multiplet.

and compute the other coupling. The result is

$$f = 0.39 \pm 0.09 \tag{34}$$

which is compatible with both couplings. Furthermore, the static SU(6) hypothesis of Gürsey and Radicati which will be discussed by Professor Gürsey requires

$$f = 0.4$$

i.e. a ratio of 3 to 2 between D and F. Moreover, we shall see that the P.C.A.C. conjecture which couples the pion universally to a charge which is proportional to the divergence of the axial vector current in atonous (weak) interactions requires a connection between this f and a similar f^W measured in the axial vector currents. From f = 0.39 ± 0.09 one finds

$$f^W = 0.25 \pm 0.08$$

which is consistent with the most recent observed value [2]

$$f^W = 0.33 \pm 0.02$$

corresponding to a ratio of 2:1 for D:F in the atonous axial current. Note that the value of f fixes also the SU(3) predictions for $(g_{\eta NN})^2$, $(g_{\eta\Xi\Xi})^2$, $(g_{\pi\Sigma\Lambda})^2$, $(g_{K\Lambda\Xi})^2$, $(g_{K\Sigma\Xi})^2$, $(g_{\pi\Xi\Xi})^2$, $(g_{\pi\Sigma\Sigma})^2$, $(g_{\eta\Sigma\Sigma})^2$, $(g_{\eta\Lambda\Lambda})^2$. We have rough experimental estimates of all of these, and they are all consistent with SU(3) at this point. In particular we predict $1.5 \geq (g_{\eta NN})^2 \geq 0.5$ and $9 \geq (g_{\pi\Sigma\Lambda})^2 \geq 7$.

Using the results of high-energy scattering, which is entirely dominated by the Regge trajectories passing through the 1^- and 2^+ poles, one can find the factorized residues (or coupling constants) at t = 0 (the forward direction) between these nonets and the baryons and mesons. There again we have one free f parameter for the baryon couplings to each of these octets; for the mesons there is no such freedom owing to charge-conjunction considerations. Once again the results fit SU(3) invariance nicely [3].

It should be noted that the vector mesons ($J = 1^-$) display an almost pure F-type coupling to baryons, even though D is allowed (the coupling to 0^- mesons has to be of F-type because of charge conjugation). This F-type coupling is the one we would get from a phenomenological Lagrangian with a Yang-Mills gauge. In such a model the vector mesons are coupled to the conserved currents of the theory, and the currents are derived from the product

current = canonical momentum × variation of phen. baryon field

The infinitesimal variation from a gauge $\exp(i\,\delta_k\,\lambda^k)$ operating on an octet baryon field B^n is given by (we write the baryon octet as a three by three matrix)

$$\delta_k [\lambda^k, B^n] = i\,\delta_k\, f_{kn\ell} B^\ell \tag{35}$$

so that the interaction Lagrangian is described by

$$f_{nk\ell} \overline{B}^n \gamma_\mu V^k B^\ell \tag{36}$$

displaying F-type coupling. This model determines a direct connection between the V^k and the current-density

$$j^k = f^k_{n\ell} \overline{B}^n \gamma_\mu B^\ell \tag{37}$$

It has been suggested that this "universal" coupling of the V^k be formulated so as to identify the V^k with the hadron density itself

$$V^k \propto j^k \tag{38}$$

This ansatz has been called "field algebra". However, it seems for the moment more physical to regard these results as approximate universality sum rules for strong interactions, derived from a dispersion relation written for the electromagnetic or atonous ("weak") vector current. We shall derive this result further on.

It is possible to get relations between cross-sections using tables of Clebsch-Gordan coefficients for SU(3). A simpler method consists in making use of the I, U, and V-spins. We illustrate the use of U-spin conservation comparing two reactions where a proton is bombarded with a negative π-meson to produce a positive meson and a negative resonance

$$\begin{aligned} &\pi^- + p \longrightarrow \Delta^- + \pi^+ && \text{(a)} \\ &K^- + p \longrightarrow Y_1^{*-} + \pi^+ && \text{(b)} \end{aligned} \tag{39}$$

U:	$\frac{1}{2} \otimes \frac{1}{2}$	$\frac{3}{2} \otimes \frac{1}{2}$
U_{total}	0, 1	1, 2

Conservation of total U-spin leaves only the U = 1 channel open. Therefore the cross-sections for both reactions can be expressed in terms of only one parameter by means of SU(2) Clebsch-Gordan coefficients. The ratio of both cross-sections is therefore parameter independent and is equal to $-\sqrt{3}$, so that we require $\frac{1}{3}\sigma_a = \sigma_b$, where the cross-sections have first to be corrected by extracting the phase space effects due to the mass differences. The result holds nicely.[3]

BREAKING SU(3) AND THE GELL-MANN AND OKUBO MASS FORMULAE

The symmetry breaking interaction responsible for the large mass splittings between I-spin multiplets is assumed to transform like λ_8. Since I and Y are conserved, the breaking for an octet could behave like any of the I = 0, Y = 0 components in the product Eq. (25). We thus assume that the central component of $\underline{27}$, which fits these conditions, does not contribute and that the mass differences are due to an $\mathscr{H}_8$ piece of the Hamiltonian, behaving like the F and D octets' eighth components. This $\mathscr{H}_8$ could contain both U = 0 and U = 1, U_Z = 0 pieces. Since U = 0 would yield m(p) = m(Σ), Δm transforms like the third component of a U-spin vector. It then behaves

[3] H. Harari, in High Energy Physics and Elementary Particles (IAEA, Vienna, 1965), p. 353, has noted all intensity rules which seem broken. However, much of this has been set right by more recent experiments. The remaining broken rules relate to complicated dynamical situations, such as the anomalous peak in $\gamma + p \rightarrow p + \pi$ at $t \rightarrow 0$.

like the operator U_Z giving equal spacing for the Δ^-, Y^{*-}, Ξ^{*-} and Ω^- in the decuplet for instance, in remarkable agreement with the experimental values. For the octet we get equal spacing between the U-spin members of the multiplet formed by the three states

$$\begin{array}{lll} N & U_Z = 1 & (1) \\ -\frac{\sqrt{3}}{2}\Lambda + \frac{1}{2}\Sigma & U_Z = 0 & (2) \\ \Xi & U_Z = -1 & (3) \end{array} \qquad (40)$$

The $U_Z = 0$ state is given as a combination of Λ and Σ. The coefficients can be calculated by means of the algebraic relations, but we already know them because they also appear in the charge matrix $Q = e(\frac{\sqrt{3}}{2}\lambda_\xi + \frac{1}{2}\lambda_8)$ which represents the orthogonal $U = 0$ combination. From Eq. (40) we get the Gell-Mann relation

$$m_N + m_\Xi = \frac{3}{2} m_\Lambda + \frac{1}{2} m_\Xi \qquad (41)$$

in fairly good agreement with experiment.

For mesons we use Eq. (41) for the masses squared and get good agreement with the experimental values. This is because we are in fact taking matrix elements of $\langle \Xi^0 | R^+, U^+] | n \rangle = 0$, where $R^+ = [U^+, H]$ yields m^2 for mesons as can be seen by computing $\partial^\mu j_\mu$ between mesons.

The mixing between the two vector mesons ϕ and ω is another manifestation of this mass-splitting interaction. They are mixtures of the octet $I_Z = 0$, $Y = 0$ state V_8, which is the one satisfying the mass formula (41), and the singlet state V_0. We can express this mixture by means of a parameter α

$$\begin{aligned} |\phi\rangle &= V_8 \cos\alpha + V_0 \sin\alpha \\ |\omega\rangle &= V_0 \cos\alpha - V_8 \sin\alpha \end{aligned} \qquad (42)$$

Introducing the measured values for the ϕ and ω masses and the calculated value for the V_8 mass we obtain the mixing parameter

$$\cos\alpha \sim 0.78$$

In the quark model there is a simple explanation for the equal spacing rule in U-space. It is assumed that the s-quark is heavier than the other two, which are degenerate

$$\begin{aligned} |\phi\rangle &= s\bar{s} \\ |\omega\rangle &= \frac{\sqrt{2}}{2}(u\bar{u} + d\bar{d}) \end{aligned} \qquad (43)$$

This implies, since $|\rho\rangle = \frac{\sqrt{2}}{2}(u\bar{u} - d\bar{d})$,

$$m_\omega = m_\rho \qquad (44)$$

which holds to within 10-15 MeV.

Note that making $\mathscr{H}_8$ linear in m(s) - m(u, d) is not the most general structure allowed for $\mathscr{H}_8$ and would not explain the (Σ, Λ) mass difference since (see Figs 4 and 5) both "contain" just one s-quark, and Σ^0 is in fact identical to Λ except for the coefficients of the various ordered terms. The general mass formula was computed by Okubo [4]

$$m(J, D, B, \lambda, \mu, I, Y) = a(J, D, B, \lambda, \mu) + b(J, D, \lambda, \mu) B \cdot Y + c(J, D, B, \lambda, \mu)[I(I+1) - \frac{Y^2}{4}] \tag{45}$$

The parameter a follows roughly a rule

$$a \sim 900 \text{ MeV} \times D$$

i.e. about 300 MeV "per quark" or anti-quark. This is one of the basic phenomenological features which have been at the root of the success of the quark model. The parameter b is roughly

$$b \sim 150 \text{ MeV}$$

as is obvious from a look at Figs 5 and 6. The parameter c seems to depend strongly on D and B. Higher symmetries such as the static SU(6) and $[U(6) \times U(6)]_\beta$ make the J and D dependence explicit.

Möbius [5] has recently suggested an enlightening analysis of the symmetry breaking in the masses (it could also be done for couplings except that the experimental evidence is known in less detail). He expands the mass in terms of powers of the symmetry breaking and checks the convergence of the actual coefficients. For SU(3), we have two terms linear in λ_8 (the F and D combinations) and one quadratic term (the $\underset{\sim}{27}$)

$$M_0 \operatorname{Tr}(\bar{B}B) + M_1 \operatorname{Tr}(\bar{B}[B, \lambda_8]) + M_1' \operatorname{Tr}(\bar{B}\{B, \lambda_8\}) + M_2 \operatorname{Tr}(\bar{B}\lambda_8 B \lambda_8) \tag{46}$$

for the baryon octet, with

$$M_0 = 1150 \text{ MeV}, \quad M_1 = 144 \text{ MeV}, \quad M_1' = -77 \text{ MeV}, \quad M_2 = 3 \text{ MeV}$$

The coefficients converge nicely. For the pseudoscalar meson octet we find

$$m_0^2 = 0.166 \text{ (GeV)}^2, \quad m_1^2 = 0.125 \text{ (GeV)}^2, \quad m_2^2 = 0.010 \text{ (GeV)}^2$$

(the F coefficient vanishes by unitary parity, i.e. SU(3) $\times$ charge conjugation of the 8th component). The m_0^2 coefficient is of the same order as the m_1^2, a special feature which has recently been explained [6].

ELECTROMAGNETIC TRANSITIONS

The electromagnetic interaction conserves U-spin. (See orthogonality of $U^\pm$ and Q in Fig. 1.) The current transforms like the electric charge operator Q, which belongs to an octet. Therefore it is possible to get

relations among magnetic moments. For instance the proton and Σ^+ should have the same magnetic moment because they are eigenstates of the same U-spin multiplet. We list below the complete set of relations obtained by Coleman and Glashow ($\mu(\Sigma^+) + \mu(\Sigma^-) = 2\mu(\Sigma^0)$ was known previously from I-spin)

$$\begin{aligned}
\mu(\Sigma^+) &= \mu(p) && \text{[present figures: } 2.5 \pm 0.7 \text{ and } 2.79] \\
\mu(\Lambda) &= \tfrac{1}{2}\mu(n) && \text{[present figures: } -0.73 \pm 0.16 \text{ and } -0.95] \\
\mu(\Xi^0) &= \mu(n) && \qquad (47) \\
\mu(\Xi^-) &= \mu(\Sigma^-) = -[\mu(p) + \mu(n)] \\
\mu(\Sigma^0) &= -\tfrac{1}{2}\mu(n)
\end{aligned}$$

The same transformation properties are valid for the electromagnetic mass differences. (These follow Q^2 which is still a U-spin singlet). They are of course superposed upon the splittings given by the Gell-Mann – Okubo mass formula. If we call δ^-, δ^0, δ^+ the electromagnetic contributions to the masses of the three charge states, we get for the baryon octet

p	n	Σ^+	Σ^-	Ξ^0	Ξ^-
$m_N + \delta^+$	$m_N + \delta^0$	$m_\Sigma + \delta^+$	$m_\Sigma + \delta^-$	$m_\Xi + \delta^0$	$m_\Xi + \delta^-$

and by eliminating the three unknowns δ^+, δ^-, δ^0 we obtain the Coleman-Glashow formula

$$m_{\Xi^-} - m_{\Xi^0} = m_{\Sigma^-} - m_{\Sigma^+} + m_p - m_n \qquad (48)$$

We have [6.3 ± 0.7] and [6.66 ± 0.11] for the two sides of Eq. (46) experimentally at present.

Möbius' expansion can be applied to electromagnetic effects. For I-spin it would yield

$$\mathscr{H}(N) = M_0(\bar{N}N) - M_1(\bar{N}\zeta_3 N)$$

$M_0 \sim 938$ MeV, $M_1 \sim 0.65$ MeV (excellent convergence)

$$\mathscr{H}(\Sigma) = \frac{1}{2}M_0 \mathrm{Tr}(\bar{\Sigma}\Sigma) + \frac{1}{2}M_1 \mathrm{Tr}(\bar{\Sigma}\zeta_3\Sigma) + \frac{1}{2}M_2 \mathrm{Tr}(\bar{\Sigma}\zeta_3\Sigma\zeta_3)$$

where Σ is the matrix

$$\begin{pmatrix} \Sigma^0 & \sqrt{2}\,\Sigma^+ \\ \sqrt{2}\,\Sigma^- & -\Sigma^0 \end{pmatrix}$$

yielding

$$M_0 \sim 1193 \text{ MeV} \qquad M_1 \sim 4 \text{ MeV} \qquad M_2 \sim 0.2 \text{ MeV}$$

For pions the I = 1 term vanishes by G-parity

$$\mathscr{H}(\pi) = \frac{1}{2} m_0 \mathrm{Tr}(\pi\pi) - \frac{1}{2} m_2 \mathrm{Tr}(\pi\zeta_3\pi\zeta_3)$$

$$m_0 \sim 137.5 \text{ MeV} \quad m_2 = 2.5 \text{ MeV}$$

At the SU(3) level, we take powers of Q.

$$\mathscr{H}_{em}(B) = \delta m_1 \mathrm{Tr}(\bar{B}[B, Q] + \delta m_1' \mathrm{Tr}(\bar{B}\{B, Q\}) + \delta m_2 \mathrm{Tr}(\bar{B}QBQ)$$

$$\delta m_1 = 4 \text{ MeV} \qquad \delta m_1' = -2.2 \text{ MeV} \qquad \delta m_2 = -1.5 \text{ MeV}$$

which is not so good. The pion's m_2 above is also a $\underset{\sim}{27}$ contribution, and we see that convergence seems slower here.

UNIVERSALITY OF THE ATONOUS CURRENTS

Since 1958 we have known that the atonous (weak) interactions — at least as far as lepton-lepton and lepton-hadron transitions are concerned — are of a V-A nature and can be described by a current-current interaction. These currents are conserved for the vector part, which then explains why the weak couplings are not renormalized by the strong interactions for the hadron currents, a result which was deduced from the quasi-equality of μ and β decay vector couplings. The strangeness-changing current seemed to undergo a strong renormalization, reducing its actual coupling to a few percent of the non-strange case. This looked surprising in an SU(3) framework, since the non-conservation of the $|\Delta S| = 1$ current is due to the Λ-N mass difference, i.e. an SU(3) breaking effect, which should be relatively subdued. Gell-Mann had however pointed out in 1960 (see Footnote 2) that if a second neutrino were found, the answer would lie in the identification of the β-decaying hadron with a "rotated" state in the n-Λ plane.

A coupling is said to be universal if it doesn't get renormalized; it then corresponds at zero momentum-transfer to a charge (because the Fourier transform of the corresponding current then coincides with the space integral of the charge-density) and is then given entirely by the eigenvalue of an algebraic generator. For J = 1/2 states the vector current has only one active form factor at q = 0, coupling together the initial and final spinor

$$\int d^3x \, \langle B_k | j_\ell^0 | B_m \rangle_{q=0} = F_\ell^{(k,m)} \, \bar{u}_k \gamma^0 u_m = f_\ell^{km}$$

Experimentally, e^- and μ^- seem coupled to a neutrino. Muon decay, for instance (we leave out the γ^μ and $\gamma^\mu\gamma^5$) reads (a field ϕ stands for an annihilation, $\bar{\phi}$ for creation)

$$(\overline{e^-}\ \ \nu_e)\,(\overline{\overline{\mu^-}\ \ \nu_\mu}) \qquad [\mu^- \to \nu_\mu + e^- + \bar{\nu}_e]$$

while beta decay is

$$(\overline{e^-}\ \ \nu_e)\,(\overline{\overline{n}\ \ p}) \qquad [n \to p + e^- + \bar{\nu}_e]$$

and the production of muons in the 1962 neutrino experiment is

$$(\overline{\mu^-}\ \ \nu_\mu)\,(\overline{\bar{n}\ \ p}) \qquad [\nu_\mu + n \to p + \mu^-]$$

etc.

One finds experimentally that $(\overline{e^-}\ \nu)$ and $(\overline{\mu^-}\ \nu)$ are coupled with equal strength, and that $(\bar{n}\ p)$ and $(\overline{\mu^-}\ \nu)$ are almost equally coupled. All these terms require at least an SU(2) set of generators, where they would correspond to the action of a lowering operator τ^-. The most general expression for the one neutrino situation arises from the spinors

$$\begin{matrix} a\ \mu^- + b\ e^- & & c\ n + d\ \Lambda \\ \nu & , & p \end{matrix}$$

Experimentally, $a = b$, i.e. $a = 1/\sqrt{2}$; thus $c \simeq d \simeq 1/\sqrt{2}$ since $c \sim a$. This would yield equal strength to $|\Delta S| = 0$ and $|\Delta S| = 1$ decays. However, for two neutrinos we have

$$\begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}, \quad \begin{pmatrix} e^- \\ \nu_e \end{pmatrix}, \quad \begin{pmatrix} c\ n + d\ \Lambda \\ p \end{pmatrix}$$

yielding

$$c \sim 1 \quad , \quad c \gg d$$

The question remained open until in 1963 Cabibbo [7] introduced this form of universality within the framework of SU(3). He suggested that the hadron current be given by

$$j_\mu^A = \cos\theta\ I^+ + \sin\theta\ V^+ \tag{49}$$

This assumption is in excellent agreement with the experimental data if the angle $\theta \approx 0.26$. There are eight observed decays, and they can all be fitted (entirely within the experimental error bars) with one parameter, this θ angle. The two other parameters which seem necessary at first, f for the vector current and for the axial-vector current,are fixed by other

experiments or by theory: $f(v^\mu) = 1$ from universality, and $f(a^\mu)$ is obtained from the strong interactions via P.C.A.C (see our previous discussion of f in pseudoscalar meson couplings).

NON-LEPTONIC ATONOUS INTERACTIONS

This is understood less well. If it is a current-current interaction, it transforms as the product of two octets. CP requires that only symmetrized parts of the product exist, i.e. only representations $\underset{\sim}{1}$, $\underset{\sim}{8}$ and $\underset{\sim}{27}$. The experimentally observed $\Delta I = \frac{1}{2}$ rule (which seems to hold to within 1/600) shows that the contribution of $\underset{\sim}{27}$ is negligible. So far there is no fully satisfactory model for a mechanism through which the strong interactions would enhance the octet part, but this is possible. This situation reminds us of the success of the Gell-Mann – Okubo mass formula where a similar octet dominance is at work. (There too, we do not know if $\underset{\sim}{27}$ is suppressed or $\underset{\sim}{8}$ enhanced.) Alternatively, the $\underset{\sim}{8}$ may be unenhanced, with unobserved neutral currents cancelling out the $\underset{\sim}{27}$ piece. At present the experimental evidence is not conclusive, and we cannot rule out the $\underset{\sim}{27}$ entirely, to fit such a theory. The most hopeful test seems to be the measurement of the ratio of $|\Delta I| = 1/\Delta I = 0$ in the $\Delta S = 0$ atonous interaction, i.e. in nuclear parity-violating transitions. If there is no $\underset{\sim}{27}$, this ratio should be about 15. If $\underset{\sim}{27}$ exists and $\underset{\sim}{8}$ is just enhanced, we expect a ratio of one. (See p. 122 in Footnote 1 or paper 23 in Footnote 2.)

A CURRENT-GENERATED ALGEBRA AND CHIRAL $[U(3) \times U(3)]_{\gamma_5}$

Although SU(3) started out – and still is primarily a symmetry of strong interactions – it can be observed in weak and electromagnetic transitions. Gell-Mann suggested that it be indeed defined through equal-time commutation of weak and electromagnetic currents. Starting from this algebraic structure it is possible to define the algebra and to get correct results even if the underlying symmetry is badly violated. Gell-Mann defined the SU(3) algebra using commutation relations between charges

$$[F_i, F_j] = i f_{ijk} F_k \tag{4}$$

where the

$$F_i = \int F_{i0}\, d^3x \tag{50}$$

are space integrals of the densities, which are local field operators whose matrix elements are correctly measured through perturbation theory via their coupling to the electromagnetic field or the atonous lepton currents.

This algebra accounts only for the conserved vector current component V of the complete weak current. But we know there also exists a non-conserved axial part A. The most interesting property of these two components is the overall atonous universality, i.e. the presence of only one coefficient G in the combined current in the $G\, j_\mu^W (j_\mu^W)^\dagger$ Hamiltonian

$$j_\mu^W = \bar{\psi}(1+\gamma_5)\gamma_\mu\psi \tag{51}$$

which can be written in this form for the leptons, and more generally as $(v_\mu + a_\mu)$ for hadrons.

The axial vector weak current also transforms like an octet. These considerations allow us to enlarge the SU(3) algebra to include both the V and A components (these are all space-integrals)

$$[V_i, V_j] = i f_{ijk} V_k \tag{4'}$$

$$[V_i, A_j] = i f_{ijk} A_k \tag{52}$$

$$[A_i, A_j] = i f_{ijk} V_k \tag{53}$$

Equation (52) expresses the fact that A_j transforms like a vector under SU(3). The last equation, at first simply postulated by Gell-Mann (Salam and Ward had suggested it for the Sakata model), closes the system upon the "chiral" algebra SU(3) × SU(3). Assuming a universal coupling for the entire $j_\mu = v_\mu + a_\mu$ weak current, we have supplied a proof of Eq. (53)(see p. 134 of Footnote 1). This is achieved by examining first the SU(2) subgroup generated by $W_i = \int d^3x\, (v_i^0 + a_i^0)$ and observing that the only solution to $[W_i, W_j] = i\, \epsilon_{ijk}\, V_k$ for i = 1, 2, 3. We then turn to the expansion (taking now i = 1..8)

$$[A_i, A_j] = a\, \underset{\sim}{8} + b\, \underset{\sim}{10} + c\, \underset{\sim}{\overline{10}} \tag{54}$$

Solving, we find that the coefficients b and c are zero. These coefficients can be calculated inside SU(2) because we can turn the Cabibbo SU(3) of W_i into the I-spin SU(2) via a U-spin rotation.

The commutation relation (53) has been checked by Adler and Weisberger, who have been able to compute through it the G_A/G_V ratio, getting a result which is correct to within a few per cent from experiment (see p. 149 of Footnote 1). The application of the methods of dispersion theory to the study of renormalization effects in the non-strong interactions is now known under the title of "current-algebra" and is discussed in these Proceedings by Prof. Fubini. The commutation relations have also been extended to the integrals of space components of v_μ and a_μ, closing upon U(12).

Note: Participants who are less familiar with the present state of particle physics are hereby warned that we are leaving at this point the well-established results and shall herewith deal with more speculative material.

BACK TO THE STRONG INTERACTIONS

The Adler-Weisberger and many other results have shown that we can indeed define the algebraic generators from the non-strong currents. However we then still have to explain why the strong interactions are SU(3), and perhaps SU(3) × SU(3) invariant.

Recent work on phenomenological Lagrangians has suggested that this can be explained by writing a symmetric strong Lagrangian [6]. This is possible in the limit of zero mass pseudoscalar mesons (to achieve chiral invariance) and a zero mass set of vector mesons coupled through a

Yang-Mills gauge. This approach is discussed by Professor Zumino in these Proceedings.

Alternatively one may turn to the "independent-particle" quark model. This assumes that hadron matter has phonon-like excitations with quark quantum numbers and an energy of some 300 MeV. One can then also reproduce the results of the static SU(6) of Gürsey and Radicati etc. (see Professor Gürsey's paper in these Proceedings). It amounts also to the conservation of D, i.e. separate conservation of quarks and anti-quarks ($B + \bar{B} \rightarrow$ mesons is then treated as a rearrangement, for instance). One even has $[U(3) \times U(3)]_B$ symmetry, i.e. separate conservation of each type of quark and anti-quark. This quark model has been used extensively; sometimes it is used as if the quarks were real free particles. However this raises difficulties since they would then either have to follow parastatistics or have an additional multiplicity (nine quarks instead of three).

A third hypothesis reproduces these results in the high-energy limit by assuming a universal coupling of 1^- and 2^+ Regge trajectories to this $[U(3) \times U(3)]_B$. From the coupling of the Pomeranchuk trajectory to D, one finds

$$\frac{\sigma_{\pi p}}{\sigma_{pp}} \longrightarrow \frac{2}{3} \qquad \text{for } s \longrightarrow \infty \tag{55}$$

since D just counts the number of "constituent" quarks in both cases.

Gell-Mann has shown that we may derive low-energy strong-interaction results as approximations based upon the current-generated algebra. This is done by assuming pole-dominance of the matrix elements of these currents, vector-meson poles for the vector currents, pseudoscalar meson poles for the divergences of the axial currents. As an example, take the electromagnetic form factor of a baryon (Fig. 8)

$$\langle B^k | j_\mu^Q | B^\ell \rangle = \frac{1}{2} \bar{u}_f^k \gamma_\mu \Lambda^Q F_1(t)\, u_i^\ell + \text{magnetic term} \tag{56}$$

We insert intermediate states and assume domination of the form factor by the pole term due to a 1^- meson V^Q (the rho or omega). Writing

$$\langle 0 | j_\mu^Q | V_\mu^Q \rangle = m_v^2 (2g_v)^{-1} V_\mu \tag{57}$$

we have

$$\Lambda_{\ell k}^Q F_1(t) = (g_{\ell kQ}^{\text{strong}}/g_v)\; m_v^2 (m_v^2 - t)^{-1} + \int dM^2\, \sigma(M^2)\, M^2 (M^2 - t - i\epsilon)^{-1} \tag{58}$$

and for $t = 0$, discarding the cut contribution we get

$$\Lambda_{\ell k}^Q \simeq g_{\ell kQ}^{\text{strong}}/g_v \qquad \text{or} \qquad g_{\ell kQ}^{\text{strong}} = g_v\, \Lambda_{\ell k}^Q \tag{59}$$

which ensures a universal coupling for vector mesons.

A more systematic version of this approach — and in fact an extension beyond the original currents structure — has been provided by the application of superconvergence equations. This is discussed in these Proceedings by Professors Fubini and Weinberg.

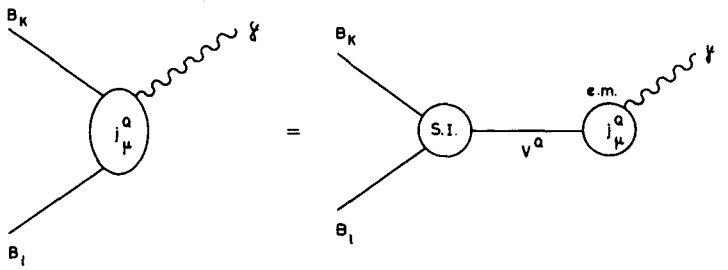

FIG. 8. Diagrammatic form of Eq. (56).

ORIGIN OF SU(3) AND ITS BREAKING

It has been suggested (see p. 297 of Footnote 1) that the emergence of SU(3) as an approximate symmetry of the strong interactions — and therefore as a classification pattern for hadrons — might be due to the interrelation between three interactions: the weak, the electromagnetic and a "fifth" force. The latter would be coupled strongly or semi-strongly to a current (ϕ is an arbitrary real angle)

$$j^V_\mu(x) = \cos\phi\, j^0_\mu(x) + \sin\phi\, j^8_\mu(x) \tag{60}$$

Equal-time commutation of $\int d^3x\, j^V_4(x)$ with space integrals of the 4th components of the atonous vector current given by (49) and of the electromagnetic current (i.e. of the respective charges F^V, $F^{A\pm}$, F^E) yields the algebra of U(3) by closure. Note that $F^{A\pm}$ and F^E would close on U(2) only, were it not for F^V.

Dynamically, it was suggested that the strong interactions consist of an otherwise symmetric bootstrap, with the

$$\mathscr{H}^V = (\gamma^V M^{-2})\, j^V_\mu j^{V\dagger}_\mu \tag{61}$$

fifth interaction Hamiltonian superimposed, the mass M providing a scale. Hadrons are thus non-elementary particles following the Chew-Frautschi hypothesis [8] i.e. lying along Regge trajectories. Their "structure" would be given by a series of bootstrap equations [9] yielding multiplicities, masses, and couplings. Such equations have both symmetric and asymmetric solutions. However, the actual asymmetry observed in nature is picked by $\mathscr{H}^V$ out of this manifold of allowed solutions. In the language in which SU(3) breaking is generally analysed [10-12], this amounts to the insertion of a strong driving term in the eighth direction, yielding the equations (y_i is the solution, e.g. the masses)

$$y_i + d_{ijk}\, y_j\, y_k = c_i \tag{62}$$

with $c_i = 0$ for $i = 1..7$, $c_8 = c$, resulting in $y_i = 0$ for $i = 1..7$,

$$y_8 \sim \pm c_8^{\frac{1}{2}} \tag{63}$$

Notice that this approach has the advantage of supplying a mechanism which fixes the symmetry group itself. The bootstrap equations contain couplings, i.e. reduction coefficients of some Lie group; it is the identification of some of the meson state labels (1^- mesons for SU(3), 0^- for SU(3) $\times$ SU(3)) with poles of Lie algebra densities of the above type which then fixes the rest of the algebra through closure. In the alternative "spontaneous breakdown" approach, SU(3) itself has to be postulated [10-12]; given j^E and $j^{A\pm}$, one then attempts to get a solution in the 8th direction, where the relative direction of $j^{A\pm}$ with respect to j^E is also postulated and defines what we mean by the searched-for "eighth" direction. It turns out that first order calculations, though capable of reproducing [11, 12] something of the sort, still fail to yield the correct answer, since only $\theta = 0$ or $\pi/2$ are allowed. Cabibbo has suggested [13] that it might still be possible to get the physical θ by introducing second order weak interactions with a $\Delta U = 2$ contribution, provided there would occur at the same time some screening of the first order contribution. This seems difficult to realize without raising the question of further higher orders.

In our approach, the question of why the three basic forces should exist at all, and at such angles to each other is again relegated to a more fundamental level.

OBSERVING A STRONGLY-COUPLED CURRENT

To explain the fact that the Δm_V mass splittings are observed to be about twenty to thirty times Δm_E, γ^V was first conjectured to be larger than α by a similar amount. This would have implied

$$1 > \gamma^V \geq 0.2$$

which would still allow a measurement of the matrix elements of $j_\mu^V(x)$ based upon the perturbation expansion, just as suggested by Gell-Mann in the case of $j_\mu^E(x)$ and $j_\mu^{A\pm}(x)$.

However, if $\gamma^V \geq 1$, the picture of $\mathscr{H}^V$ "polarizing" an otherwise symmetric bootstrap is still the same, except that we loose the perturbative definition of $j_\mu^V(x)$ matrix elements. This is then the input corresponding to the strong interaction itself.

Can we observe the matrix elements of $j_\mu^V(x)$ now that $\gamma^V \geq 1$ precludes the use of perturbation theory?

It is convenient at this point to regard $\mathscr{H}^V$ as an effective Hamiltonian derived from a basic interaction between j_μ^V and an intermediate vector meson χ_μ^V

$$\mathscr{H}_{bas}^V = (\gamma^V)^{\frac{1}{2}} j_\mu^V \chi_\mu^V \tag{61'}$$

This is a renormalizable field theory, even for $m_\chi \neq 0$. The j^V_μ current being conserved in strong and electromagnetic interactions, the coupling is conserved (at t = 0). The t-channel exchange of one χ^V_μ dominates at large s, multiple exchanges being reduced by $(\ln s)^{-1}$ at least (this is a cut in the complex J plane, with a flat trajectory for its tip). Moreover, for very large t, Gell-Mann and Zachariasen have shown [14] that the renormalized coupling is identical with the corresponding off-mass-shell form factors. We may thus expect to learn about the matrix elements of j^V_μ from $s \to \infty$, $t \to \infty$ data, provided there exists a mechanism reducing in that region the contributions of strong interactions other than those due directly to $\mathscr{H}^V_{bas}$.

In a recent study of high-energy and large-momentum transfer collision processes [15], Abarbanel et al. have studied such a mechanism. They point out that with Regge dynamics ensuring shrinkage, i.e. decrease in $d\sigma/dt$ (for fixed $t < 0$) at increasing s, fixed-pole contributions would become prominent at large negative t. We would thus indeed observe $(\mathscr{H}^V)^2$ effective matrix elements in $\left.\frac{d\sigma}{dt}\right|_{s,t\to\infty}$.

Abarbanel et al. follow earlier suggestions by Yang and others that p-p scattering at $s \gg m_N^2$ and $-t \gg m_N^2$ should be related to proton structure as probed by electromagnetism and described by $G_{E_p}(t)$ and $G_{M_p}(t)$ form factors. They find a good fit for

$$X = \frac{d\sigma}{dt}\Big/\left(\frac{d\sigma}{dt}\right)_{t=0} \sim G^4_{M_p}(t) \tag{64}$$

which they interpret as a current-current contact interaction (s - independent) with a fixed pole at J = 1 and a coupling $a \approx 0.85 \pm 0.15$. The amplitude is thus constituted of a Regge-type part R(s, t) and the current-current term, so that at large s and t

$$X(s,t) = [a\, G^2_{M_p}(t) + R(s,t)]^2 \tag{65}$$

NATURE OF THE FIXED POLE OF ABARBANELL ET AL. [15]

We would like to suggest that the current-current interaction observed by Abarbanel et al. is indeed $\mathscr{H}^V$, or a contracted $|\mathscr{H}^V_{bas}|^2$ with a very large m_χ. We assume $0 < \phi < 45°$ so that j^V_μ has roughly the quantum numbers of an ω-meson, rather than those of the ϕ, since it couples to nucleons. This also ensures octet-like behaviour for the SU(3)-breaking part of $\mathscr{H}^V$, deriving from the cross-term $\cos\phi \sin\phi \{j^0_\mu, j^8_\mu\}$ which will then dominate any $\underset{\sim}{27}$-like piece in $j^8_\mu j^8_\mu$. Indeed, $\mathscr{H}^V$ is then 32% singlet, 63% octet and 5% in $\underset{\sim}{27}$ (taking the ϕ to be pure $ss^\dagger$, s being the third quark).

Using the Abarbanel et al. result as a positive indication, we know that a J = 1 pole is required in the amplitude, so as to guarantee a surviving contribution at $s \to \infty$. The interpretation of Abarbanel et al. and our χ_μ model correspond both to a $\zeta = -1$ odd signature. As in the case of the photon, a fixed pole does not contribute an imaginary part to the amplitude. However, at small t we loose the Gell-Mann – Zachariasen result and have to consider

higher order contributions to the imaginary part. In a theory [7] in which the Pomeranchuk trajectory has an intercept (the sub-indices stand for U(3) direction and signature)

$$\alpha_{0+}(0) < 1$$

we would then have to hope that those contributions which are invariant under the replacement of one of the incoming particles by its anti-particle dominate over the odd (sign-changing) terms — to ensure positive cross-sections as $s \to \infty$. If $\alpha_{0+}(0) = 1$, this is no more than a lower bound on the residue of $R_{0+}(s,t)$.

We might also consider an alternative $\zeta = +1$ fixed pole. This would create no problem of the above type, since its imaginary part is the same for particle-particle and particle-anti-particle scattering. Following the picture of an algebra of factorized residues we would then couple this pole at each vertex to a scalar density whose Fourier-transform g(t) would behave like

$$g^{(+)}(t) = g(\cos\phi''\beta\lambda''_0 + \sin\phi''\beta\lambda''_8; t) \tag{66}$$

This model of an even pole at J = 1 corresponds to a purely diffractive effect [16] and would provide for an "optical" interpretation of $\mathscr{H}^{(+)}$. The corresponding current-current Hamiltonian is not directly visualizable, though one could perhaps connect it with the $M_{f^0} \to \infty$ limit of a Regge trajectory with a $J = 2^+$ pole like the f^0. However, the appearance of the same $G_{M_p}(t)$ as in electromagnetism would then require a special assumption, perhaps of approximate $[U(1) \times U(1)]_{\beta\lambda_0}$ symmetry (i.e. separate conservation of quark and anti-quark numbers). The above-mentioned auxiliary f^0-like Regge pole at $t \to \infty$ would then play a heuristic role only, the actual pole-dominance of the effective form-factor having to simulate the behaviour of the isoscalar electric current, i.e. with perhaps the physical $f^0(1200)$ playing the role of the $\omega(770)$ here. Note also that various theoretical objections have been raised against a flat Pomeranchuk trajectory [17].

Let us now discuss the SU(3) structure in either case. Our $\mathscr{H}^V$ as given by (61) and (61') is of course not the only possible $\zeta = -1$ current-current Hamiltonian breaking SU(3) in the observed way. We could also have

$$\mathscr{H} = (b\ \delta_{ij} + b' d_{ij8})\, j_i^\mu j_j^{\mu\dagger} \tag{67}$$

with all 8 (or 9) currents participating; alternatively we could have

$$\mathscr{H}_{I=0,1} = c(\delta_{ij} + 2\sqrt{3}\, d_{ij8})\, j_i^\mu j_j^{\mu\dagger} \tag{68}$$

which implies only I = 0, 1 currents, as in a successive-gauges model [18] or

$$\mathscr{H}_{I=0,\frac{1}{2}} = c'(\delta_{ij} - \sqrt{3}\, d_{ij8})\, j_i^\mu j_j^{\mu\dagger} \tag{69}$$

and

$$\mathscr{H}_{I=1,\frac{1}{2}} = c''(\delta_{ij} + \sqrt{3}\, d_{ij8})\, j_i^\mu j_j^{\mu\dagger} \tag{70}$$

TABLE I. PREDICTIONS OF THE VARIOUS MODELS

	$\mathscr{H}^V$	$\mathscr{H}_{0,1}$	$\mathscr{H}_{0,\frac{1}{2}}$	$\mathscr{H}_{0,\frac{1}{2}}$	$\mathscr{H}^+$
No fixed pole in	$(\pi N)_{el,\,ch.\,ex.}$ $(\eta N)_{el,\,ch.\,ex.}$ associated production	$(\eta N)_{el,\,ch.\,ex.}$ associated production	$(\pi N)_{el,\,ch.\,ex.}$ $(\eta N)_{el,\,ch.\,ex.}$	$(\eta N)_{el,\,ch.\,ex.}$	$(\pi N)_{ch.\,ex.}$ $(\eta N)_{ch.\,ex.}$ associated production
$A = \frac{d\sigma}{dt}\Big\vert_{\substack{s\to\infty \\ t\to\infty}}$	(NN) : (KN) = 9 : 1				(NN) : (πN) : (ηN) = 81 : 36 : 4

with the corresponding currents. Suppose the pole were to show up in all amplitudes: we would then be faced with $\mathscr{H}_D$, or equivalently a symmetric $\mathscr{H}_0$ plus either of the three combinations (68), (69), (70). Such a picture – i.e. the very general model of Abarbanel et al. – does not correspond to the idea of a symmetry generated by the bootstrap, since both SU(3) and its breaking are already present in the fixed pole, non-bootstrap piece of the strong interaction. Thus for our approach to be true, we would have to observe either $\mathscr{H}^V$ or at most one of the three broken octet combinations (68) - (70), with one, four, five or seven currents.

All the above possibilities exist for $\mathscr{H}^{(+)}$ as well. However, if indeed $\zeta = +1$ it seems superfluous to introduce an entire set of currents, and we shall stop at our simplest $\mathscr{H}^{(+)}$ structure.

Table I displays the main observational criteria (ηN has been included for the sake of completeness but may be irrelevant since we have as yet no experimental way of observing this reaction even indirectly for that region of s,t). The most interesting test seems to be in πN scattering.

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SU(6) SYMMETRY AND ITS RELATIVISTIC GENERALIZATIONS

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Abstract

SU(6) SYMMETRY AND ITS RELATIVISTIC GENERALIZATIONS. 1. Origins; 2. The heuristic SU(6) symmetry; 3. Vertex symmetry; 4. Applications of the heuristic formulation; 5. Different languages for SU(6); 6. Attempts at formulations of SU(6) within field theory; 7. Comparison with chiral groups. Possible new limit for exact SU(6) symmetry; 8. A specific model; 9. Decoupling of spin from orbital angular momentum; 10. Realization of SL(4,C) in Hilbert space. Current algebra; 11. Outlook.

SU(6) symmetry started four years ago as a generalization of Wigner's [1] supermultiplet theory to particle physics [2, 3]. Since it came from nuclear physics, it is an example of the cross-fertilization of the different branches of our science and, as such, whatever its true value may be, it is a subject not inappropriate to this Symposium. Let me start by tracing the development of the main ideas relevant to this particular symmetry.

1. ORIGINS

1.1. Wigner's supermultiplet theory based on SU(4) (1937)

It is the first example of a group combining space-time properties (spin) and internal charge space (isospin) properties. As shown in Elliott's paper in these Proceedings, the 4-nucleon states equivalent under the group are $P\uparrow$, $P\downarrow$, $N\uparrow$, $N\downarrow$, so that invariance of the Hamiltonian under SU(4) in non-relativistic theory means spin and isospin independence of nuclear forces. The nuclear potential is then a mixture of Wigner and Majorana potentials and the symmetry is broken by spin-dependent Heisenberg forces or tensor forces.

1.2. Møller-Rosenfeld model – Stückelberg's theory of spin and isospin independence (1940)

In order to cancel singular tensor forces, Møller and Rosenfeld [4] chose to generate nuclear forces in the Yukawa fashion from the exchange of 0^- and 1^- mesons coupled to nucleons with symmetrical coefficients. The mesons were assumed to be degenerate in mass.

Soon after, Stückelberg [5] noticed that spin and isospin independent forces would be generated if, in modern language, the exchanged mesons formed the adjoint representation of SU(4). In other words, they should have the quantum numbers of the modern particles $(\vec{\pi}, \vec{\rho}, \vec{\omega})$ and the SU(4) singlet η'. Thus Stückelberg had produced the first field theoretical

description of Wigner's scheme of which, strangely enough, he was totally ignorant. Moreover, he did not realize that he was dealing with SU(4) symmetry.

Rarita and Schwinger [6] realized a year later, again without using the group theory language, that the SU(4) symmetry can be broken by allowing the masses of vector and pseudoscalar mesons to split, but keeping the vertex symmetry. Then, non-singular tensor forces can be generated which give rise to the observed deuteron quadrupole moment for a mass ratio $m_\rho/m_\pi \sim 5$. Today, we are used to the idea that SU(3) and SU(6) symmetries can be violently broken in the mass spectrum and yet remain as very good vertex symmetries. This may be called the principle of the stability of vertex symmetry against mass splittings and was first hinted at in the work of Rarita and Schwinger. Again there was no indication of the connection with Wigner's work.

1.3. Chirality independence of π-N couplings (1960)

A totally different kind of mixing internal and external (space-time) symmetries came from particle physics. This had to do with the fact that, in connection with the V-A form of weak interactions, models could be constructed in which positive and negative chirality parts of the field were decoupled in the limit of zero pion mass and could undergo independent isospin transformations [7, 8]. Allow me to call this phenomenon chirality independence. It is linked to the existence of a chiral group, in this case $SU(2) \otimes SU(2)$. Since chirality is a space-time concept, this group also mixes, to a weaker degree than SU(4), internal and external symmetry properties.

2. THE HEURISTIC SU(6) SYMMETRY (1964)

If we provisionally picture hadrons as bound systems of quarks, the generalization of Wigner's scheme to particle physics becomes obvious. The isotopic spin group is replaced by SU(3) and there are six equivalent states for the three quarks p, n and λ, namely

$$p\uparrow,\ p\downarrow,\ n\uparrow,\ n\downarrow,\ \lambda\uparrow,\ \lambda\downarrow$$

which form the lowest-dimensional non-trivial representation of SU(6). Thus, spin and unitary spin independence of quark forces means invariance with respect to the group SU(6). Since exact spin independence cannot be achieved, we should be prepared for an approximate SU(6) symmetry with large breakings due to spin-dependent forces [2, 3]. From our experience with SU(4), we might also expect the symmetry to be good at the vertex and badly broken in the mass spectrum with, however, multiplets remaining identifiable. The reason why spin cannot be treated like an internal symmetry operator is clear. The total spin of a system of two particles $\vec{S}_1$ and $\vec{S}_2$ is

$$\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{L}_{12}$$

where $\vec{L}_{12}$ is the relative orbital angular momentum of the two particles.

On the other hand, in charge space, their isotopic spins are strictly additive. If the system is in an S-state ($\ell = 0$), then spin behaves very much like isotopic spin and can be combined with internal symmetries.

If we also consider the antiquark states we have twelve states associated with a twelve-dimensional representation of $SU(6) \otimes SU(6)$. When orbital effects are also considered for a two-particle system, we may use the group $SU(6) \otimes SU(6) \otimes 0(3)$. On the other hand, if we have a three-particle system, then we may have an approximate degeneracy for $\ell = 1$ and $\ell = 2$ states. In this case the relevant classification group might be taken as $SU(6) \otimes SU(6) \otimes [SU(3)]_{orb}$, the SU(3) orbital group being Elliott's group in nuclear physics.

Returning to SU(6), a quark-quark spin and unitary spin independent non-relativistic potential will be obtained if we exchange degenerate mesons belonging to the singlet and adjoint representations of SU(6). Hence, we classify the lowest-lying mesons as (1)- and (35)-dimensional multiplets. For baryons we have the three-quark system. The completely symmetrical and antisymmetrical representations are, respectively, (56)- and (20)-dimensional. If quarks are fermions and if they bind in S-state to form baryons then we would expect baryons to belong to a (20)-dimensional representation [3]. Actually the correct assignment [2], which was also the first one proposed, is the (56)-dimensional representation.

Historically the physicists responsible for the introduction of SU(6) to particle physics were as ignorant of Stückelberg's work as Stückelberg was unaware of Wigner's SU(4). This illustrates how logical the development in physics is.

3. VERTEX SYMMETRY

3.1. $SU(6)_W$ symmetry at the baryon-meson vertex

Suppose again that one meson exchange gives spin-, unitary-spin-independent quark-quark or baryon-baryon forces. The question is, what kind of symmetry is needed at the fermion meson vertex? The meson being virtual and having space-like momentum, its spin is not defined in relativistic theory. Therefore, we have no SU(2) group which exchanges the space-like Fourier components of the meson field. On the other hand, we have the fermions on their physical mass shell and spin transformations can be performed on them. For fermions at rest the particle and antiparticle spin operators are, respectively, $\frac{1}{2}(1+\gamma_4)\sigma_i$ and $\frac{1}{2}(1-\gamma_4)\sigma_i$, leading to the group $SU(2) \otimes SU(2)$. This has an SU(2) subgroup generated by the Stech [9] spin operators $\gamma_4\sigma_1$, $\gamma_4\sigma_2$, σ_3. If the third direction is taken as the longitudinal direction, the operators are C-odd transverse spin operators with a C-even longitudinal operator. These can be pictured as the difference of the transverse quark and antiquark spin operators and the sum of the longitudinal quark and antiquark spin operators. This group is called the hybrid or collinear group and is denoted by $SU(2)_W$ because the Stech spin operators are also referred to as W-spin operators [10].

It turns out that the correct spin operators for fermions that must be used in connection with vertex symmetry are the W-spin operators [10, 11].

When fermions transform under $SU(2)_W$, the virtual meson field components are also transformed so that a longitudinal C-even field com-

ponent is transformed into C-odd transverse meson components, while the C-odd scalar component remains invariant. If φ and V_μ denote, respectively, the pseudoscalar and vector mesons, it follows that (φ, V_1, V_2) form an $SU(2)_W$ triplet while V_0 is an $SU(2)_W$ singlet, the space-like momentum q of the virtual meson being oriented along the third direction. The spin group for these virtual mesons is the little group that leaves q invariant. Hence it is the non-compact SU(1, 1). With respect to this group, V_0, V_1 and V_2 form a vector, while φ is a scalar. This situation can be contrasted with physical mesons for which V_1, V_2, V_3 behave like vector components and φ is invariant under the spin group SU(2). Since coupling constants are defined at the fermion vertex when the emitted meson is virtual and has zero energy, the $SU(2)_W$ symmetry at the vertex implies a relation between the coupling constants of φ and V, just like in the Møller-Rosenfeld model. The same holds for the baryon-meson vertex using $SU(6)_W$ symmetry.

3.2. Covariant forms of vertex symmetry

We have seen that the spin is a channel-dependent concept [12, 13] since it is related to the little group that leaves the momentum q invariant and q^2 changes sign when we go to a crossed channel. Now the little group is always a subgroup of SL(2, C), so that if we want to give a channel-independent form to the vertex symmetry, we must replace the spin group by SL(2, C) which is the little group at $q^2 = 0$. The covariant form of $SU(6)_W$ must then be a group which admits SL(6, C) as a subgroup [3, 14].

Another such group is SU(6, 6) which is the largest linear group that leaves $\bar{\psi}\psi$ invariant, ψ denoting the quark field [15, 16]. SL(6, C) is clearly a subgroup of this group. Trilinear meson couplings as well as Yukawa-type baryon meson couplings can be written covariantly [15] using index invariance of wave functions with respect to SU(6, 6). The physical content is the same as $SU(6)_W$ invariance for the vertex.

In spite of this formal progress the symmetry remained at the heuristic level because it could not be extended to the whole Lagrangian, the fermion kinetic term being the stumbling block. However, one had a recipe by means of which $SU(6)_W$ symmetrical vertex functions in momentum space could be immediately written down in a covariant way.

4. APPLICATIONS OF THE HEURISTIC FORMULATION

4.1. Classification scheme for hadrons

4.1.1. Multiplet structure. Spin-unitary spin and parity correlations

From SU(6) one gets spin-unitary spin correlations as well as parity correlations between different SU(3) multiplets. Indeed, we have the following decomposition $[J^P(D)]$ with respect to $SU(2)\otimes SU(3)$ where (D) is the dimension of the SU(3) multiplet

$$(35) = [0^P, (8)] \oplus [1^P, (1) + (8)]$$

$$(56) = \left[\frac{1}{2}^P, (8)\right] \oplus \left[\frac{3}{2}^P, (10)\right]$$

This tells us that (35) mesons are made up of a spinless octet and a vector nonet all with the same parity. Observationally P is negative. Similarly the (56) baryons consist of an octet with $S = \frac{1}{2}$ and a decuplet of the same parity with $S = \frac{3}{2}$

For the low-lying baryons $P = +1$ by convention. There is some possibility for a meson multiplet of dimensionality 405. But the high isospin members of such a multiplet have not been seen. For baryons there is good evidence for a (70) representation with decomposition

$$(70) = \left[\frac{1}{2}^-, (10)\right] \oplus \left[\frac{3}{2}^-, (8)\right] \oplus \left[\frac{1}{2}^-, (8)\right] \oplus \left[\frac{1}{2}^-, (1)\right]$$

We can go further by using the group $SU(6) \otimes SU(6) \otimes O(3)$ based on a quark-antiquark system with orbital excitation. Then the parity of the representation is also fixed. For the low-lying mesons we have

$$(6 \otimes \bar{6};\ \ell = 0) = (1)^- + (35)^-$$

so that we have a singlet 0^- meson in addition to the (35). It can be identified with X^0 or η' (958).

We also expect mesons

$$(6 \otimes \bar{6},\ \ell = 1) \quad [1^+, (1)] \oplus [1^+, (8)] \oplus [1^-, (1) + (8)]$$

$$\oplus [0^+, (1) + (8)] \oplus [2^+, (1) + (8)]$$

Multiplets with such quantum numbers have been definitely established.

For baryons we have $(56 \otimes 1;\ \ell = 0)$ as the usual octet and decuplet. The lowest orbital excitation gives

$$(56 \otimes 1;\ \ell = 1) = \left[\frac{1}{2}^-, (8)\right] + \left[\frac{3}{2}^-, (8)\right]$$

$$+ \left[\frac{1}{2}^-, (10)\right] + \left[\frac{3}{2}^-, (10)\right] + \left[\frac{5}{2}^-, (10)\right]$$

4.1.2. SU(3) mixing angles and selection rules

To give an example of such applications consider η' and η. They have the same J^P, I^C quantum numbers but they belong to different SU(6) multiplets. We should then expect them not to mix. In fact the SU(3) mixing angle is small. Furthermore, SU(4) seems to be a good subgroup of SU(6). Vector mesons should then have definite SU(4) quantum numbers. If φ is taken to be an SU(4) singlet we find the singlet-octet mixing angle for φ and ω to be

$$\sin\lambda = 1/\sqrt{3} = 0.58 \quad (\sin\lambda^{exp} = 0.64)$$

which is in fairly good agreement with the phenomenological value [17]. Also, φ must be less strongly coupled to pion-nucleon channels than ω. Again this is substantiated by experiment.

4.1.3. Mass formulae

The breaking of SU(6) comes from spin-dependent forces induced, say, by the spin orbit coupling. Consider now the SU(6) ⊗ SU(6) multiplet ($6 \otimes \bar{6}$, $\ell = 2$). This has the decomposition

$$[6 \otimes \bar{6}, \ell = 2] = [2^-, (1)] + [2^+, (8)] + [1^-, (1) + (8)]$$
$$+ [2^-, (1) + (8)] + [3^-, (1) + (8)]$$

We see that it contains a vector nonet that can mix with the nonet of [$6 \otimes \bar{6}$, $\ell = 0$], splitting its mass from the pseudoscalar mesons. The orbital angular momentum $\ell = 2$ acts like a spurion contained in a self-conjugate representation belonging to (35) ⊗ (35). Thus, it can be a mixture of (35), (405) and (189). Knowing the transformation property of the perturbation, we can derive mass formulae [2, 18, 19]. For the low-lying hadrons we get sum rules connecting different SU(3) multiplets like

$$\rho - \pi = K^* - K$$

$$\Xi^* - Y^* = \Xi - \Sigma$$

the symbols referring to squared masses for mesons and masses for baryons. Paying some more attention to the possible structure of the perturbation [20], one obtains the Schwinger [21] formula

$$(\omega - \rho)(\phi - \rho) = \frac{4}{5}(K^* - \rho)(\omega + \phi - 2K^*)$$

Such mass formulae can be extended to higher representations provided mixing angles are known.

4.2. Vertex relations

The most spectacular relations are the F/D ratio for the pseudoscalar meson-octet baryon vertex and the μ_P/μ_N ratio of the nucleon magnetic moments. We find F/D = 2/3 [22] or

$$\alpha = F/(F + D) = 0.40$$

The analysis of experimental data by means of K–N dispersion relations [23] yields

$$\alpha^{exp} = 0.39 \pm 0.09$$

The experimental magnetic moment ratio is

$$(\mu_P/\mu_N)_{exp} = -1.46$$

while the SU(6) value [24] is

$$\mu_P/\mu_N = -\frac{3}{2} = -1.50$$

Putting in form factors for the photon-nucleon vertex and requiring $SU(6)_W$ symmetry we also find [12, 25-27]

$$G_N^E(t) = 0$$

$$G_P^E(t)/G_P^M(t) = \text{constant}$$

Thus the electric form factor for the neutron is zero and a scaling law exists for the electric and magnetic form factors. These two predictions, not yet obtainable from other theories, are in spectacular agreement with experiment.

The vector and pseudoscalar meson coupling constants are related by [22, 27]

$$g_\rho/m_\rho = \frac{3}{5}\, g/m$$

where g is the pion nucleon coupling constant given in the nuclear mass. This gives

$$\frac{g_\rho^2}{4\pi} = \frac{9}{25}\,\frac{g^2}{4\pi}\,(m_\rho/m)^2 = 3.4$$

while the experimental value is about 2.8. To compare these values we use universality and an approximately constant ρNN form factor.

Another prediction is

$$g^* = g_{NN^*\pi} = \frac{3\sqrt{2}}{5}\,\frac{g}{m}$$

which is about 30% too high.

Finally, with some assumptions concerning the weak axial vector current [22], one finds

$$G_A/G_V = -5/3 = -1.67$$

while

$$(G_A/G_V)_{exp} = -1.23$$

If one breaks SU(6) symmetry by giving pions zero mass while ρ is massive such that chiral $SU(2)\otimes SU(2)$ symmetry holds and if one neglects the variation of coupling constants for off-mass-shell extrapolation for the ρ, it is possible to lower G_A/G_V and g^* by a factor of $\sqrt{2}$. One then obtains good agreement with experiment [28].

4.3. Relations between total cross-sections

The collinear group is applicable to scattering processes in the forward direction since only one momentum that can define the little group is present in this case. Using the optical theorem one obtains relations between total cross-sections. The first such relation (which has also

been confirmed through other methods) was derived by Johnson and Treiman [29] and now agrees very well with experiment

$$\frac{1}{2}\sigma(K^+P) - \frac{1}{2}\sigma(K^-P) = \sigma(\pi^+P) - \sigma(\pi^-P) = \sigma(K^+N) - \sigma(K^-N)$$

4.4. Peripheral models with symmetric vertices

There are calculations [30] using a one-vector exchange model with, sometimes, unitarity correction for baryon-baryon scattering amplitudes. The general agreement with experiment is impressive.

5. DIFFERENT LANGUAGES FOR SU(6)

Physicists are polyglot. In order to understand a phenomenon from all possible angles, they like to restate or rederive results in quite different forms using different languages. SU(6) results are no exception. The main languages used for their derivation are:

(a) The group theory language, which uses the groups SU(6), $SU(6)_W$, $SU(6)\otimes SU(6)\otimes O(3)$ or the covariant groups SL(6, C) or SU(6, 6).

(b) The quark language [31]. If mathematical quarks are used, then the model just amounts to an intuitive dressing up of the $SU(6)\otimes SU(6)\otimes O(3)$ group. On the other hand, if real quarks are used with non-relativistic potentials and definite masses, then a lot of unverifiable dynamics is put in the model and the results become less convincing, at least until such quarks are discovered.

(c) The current algebra language [32, 33]. When the integrated currents are used, it is group theory all over again. The use of densities goes beyond group theory and establishes a link with field theory. At the moment this is considered the most trustworthy language for stating SU(6) results.

6. ATTEMPTS AT FORMULATIONS OF SU(6) WITHIN FIELD THEORY

There have been various field theoretical approaches which I will review briefly.

6.1. The relativistic quark model

We can write a Bethe-Salpeter equation [34] for a quark-antiquark system and get the relativistic version of the spin-independent potential. There remain, however, many thorny problems. First, annihilation being neglected in the ladder approximation, it is difficult to analyse processes in which quark pairs (mesons) disappear. Second, if we achieve large binding in the quark-antiquark system it is extremely hard to get a similar binding for the three-quark system and no binding for the two-quark system. It is also difficult to explain field theoretically the additivity of quark amplitudes in high-energy scattering [35].

6.2. Current algebra method

This is based on the existence of current operators which satisfy closed commutation relations of the SU(6) type. They can, for example, be abstracted from a quark model Lagrangian with no derivative interactions. The commutation relations are assumed to remain exact for broken SU(6) symmetry and saturated approximately by one-particle states [35, 36]. We have different forms:

(a) Only the observable vector and axial vector currents are used. Their components, when integrated, obey the SU(6)$\otimes$SU(6) algebra in the free quark model. Saturation does not give the magnetic moment ratio.

(b) From free quarks, using all Dirac matrices, we can form expressions that generate the SU(6, 6) algebra or the SU(12) algebra [33]. Unfortunately, in a simple interacting model the Johnson-Low test [37] fails to verify most of the commutation relations in perturbation theory. This algebra involves tensor currents, and SU(6) can be generated using only time components.

(c) Saturation at infinite momentum [33]. In this case only good operators whose matrix elements at $p_3 \to \infty$ do not vanish may be used. Those are just the transverse tensor currents and the longitudinal axial currents together with vector currents. Thus the good algebra is the $SU(6)_W$ subalgebra of the large algebra and by saturation the SU(6) vertex symmetry results can be recovered.

6.3. Spin current algebra

The connection of the abstract algebra with spin can be seen if free-particle spin operators [38] are used. In field theory there are two kinds of covariant spin operators for a free Dirac field, namely

$$W_\mu = \int H_{\mu\lambda}\, d\sigma_\lambda \qquad W_{\mu\nu} = \int H_{\mu\nu\lambda}\, d\sigma_\lambda$$

where

$$\partial_\lambda H_{\mu\lambda} = 0 \qquad \partial_\lambda H_{\mu\nu\lambda} = 0$$

so that W_μ and $W_{\mu\nu}$ are conserved quantities. $H_{\mu\lambda}$ and $H_{\mu\nu\lambda}$ are certain bilinears, also involving derivatives of the Dirac field. Now W_μ is odd under C and $W_{\mu\nu}$ is even. In fact, the transverse part of W_μ, together with the longitudinal part of $W_{\mu\nu}$, form the covariant generalization of the W-spin. Unfortunately, these operators generate an infinite algebra. However, if matrix elements between one-particle states only are considered, one obtains closure and recovers the SU(6) results [38]. A Johnson-Low test in the interacting case also justifies the algebra [39].

6.4. Saturation of SU(3)$\otimes$SU(3) by SU(6) multiplets

Since the SU(6)-type closed algebras are not as firmly established as chiral algebras, one may use only the chiral algebra but saturate it with SU(6) multiplets. Then, many results like the F/D ratio can be recovered, but others like μ_P/μ_N cannot be derived [36].

6.5. S-matrix formulation – bootstrap

It has been found that pseudoscalar and vector mesons can bootstrap themselves if they form a representation [40] of SU(6) ⊗ SU(6). The bootstrap picture can also be extended to the baryons [41]. This provides additional theoretical support for SU(6), subject, however, to the uncertainties inherent in the drastic approximations made for bootstrap calculations.

6.6. Another S-matrix formulation – superconvergence

A quite different possibility for justification of the SU(6) symmetry comes from the superconvergence of certain combinations of scattering amplitudes or combinations of form factors [42]. Using superconvergence relations it is possible to derive SU(6)-type relations between coupling constants and total cross-sections. The reason seems to be the suppression of SU(6)-violating combination of form factors through a more rapid decrease at high energies.

6.7. Lagrangian models based on SU(6, 6) or SL(6, C)

This is in a way the most fundamental approach. If a Lagrangian can be found which exhibits a symmetry under one of the covariant extensions of SU(6) in a definite limit, then:

(a) Relativistic invariance, crossing, unitarity are automatically guaranteed;

(b) A current algebra can be based on the model;

(c) Amplitudes can be expanded around a point for which one knows the symmetry to be exact (like in the soft pion expansions in the case of chiral symmetry);

(d) No-go theorems [43] that forbid the mixing of internal and external space properties can be studied on such a model, seeing what assumptions made for these theorems can break down.

This is why Lagrangian models received attention in the earliest stages of SU(6). The most interesting of these attempts and one of the first ones is the model proposed by Salam, Delbourgo and Strathdee [15]. The Lagrangian exhibits invariance under an SU(6, 6) group acting on the components of meson and baryon field operators, except for the kinetic baryon term. Unfortunately, a zero-momentum baryon limit is not a physical approximation for particle physics. Furthermore, a symmetry of this kind cannot be implemented in the usual Hilbert space. There are also other Lagrangian models [27] for SL(6, C) all suffering from the same defect.

In spite of the incompleteness of the models that have been proposed, this is the direction that seems to offer the highest promise for the justification of SU(6) symmetry in field theory.

7. COMPARISON WITH CHIRAL GROUPS. POSSIBLE NEW LIMIT FOR EXACT SU(6) SYMMETRY

7.1. Chiral groups are not conventional groups

To gain better insight into the kind of mixing of external and internal spaces involved in SU(6), we go back to the only example of such mixing which has worked in field theory, namely, the chiral symmetry [7, 8] which mixes chirality with isotopic or unitary spin.

We first note that a chiral group like $SU(2) \otimes SU(2)$ is not a conventional symmetry group. In the limit of exact symmetry, pions must be massless. Hence the vacuum and one-nucleon states [44] are not unique. We can always go to a representation [7, 45] in which $\bar{\psi}\psi$ is invariant. Then, under a chiral transformation we have

$$|N\rangle \to |N\rangle + \alpha|\pi N\rangle + \ldots$$

so that one-nucleon states are not stable under the group and get mixed with one-nucleon plus one-soft-pion states. On the other hand, nucleon states are stable under isotopic spin transformation.

It follows that in this example of a transformation, mixing space-time with internal symmetry space, the subspace of the Hilbert space that remains invariant consists of one-nucleon state plus any amount of zero mass mesons, in other words, it is an infinite subspace that replaces the one-particle subspace. Such a symmetry we call unconventional.

7.2. If SU(6) is also unconventional, no-go theorems are evaded

Since no-go theorems are only valid for conventional symmetries, we have hope that SU(6) symmetries of the chiral type can be implemented in Lagrangian theories. Indeed if a symmetry is exact for the Lagrangian, and the vacuum or the one-particle states are not stable under the group, then, according to the Goldstone theorem [46], there must exist zero mass bosons in the theory. Following this line of thought we conjecture that an SU(6)-invariant Lagrangian must involve zero mass pseudoscalar and vector mesons and possibly other soft bosons. In such a theory we expect both chiral symmetry and SU(6) symmetry to hold simultaneously. The breaking of the symmetry will occur by keeping invariance under the $SU(2) \otimes SU(2)$ subgroup. Then pseudoscalar mesons will remain massless and vector mesons will acquire a mass giving a first approximation to the observed mass spectrum.

7.3. Superpositions of SU(6) symmetry with chiral symmetry

Some support for the validity of such an approximation comes from phenomenological Lagrangians and a general argument due to Freund [28] which show that the G_A/G_V ratio in such theories might be changed from $-5/3$ to $-5/(3\sqrt{2})$ which agrees with the observed value $-1/23$ to a few per cent.

7.4. Locality, crossing and analyticity lead to SL(6, C) ⊗ SL(6, C)

It can also be seen quite generally that the group must contain SL(6, C) ⊗ SL(6, C):

(a) It must contain SU(3) and the little groups when the momentum can be time-like, space-like or null, since in field theory the particle can be virtual. Then we must use the largest little group SL(2, C) which combines with SU(3) to give SL(6, C).

(b) The amplitudes must be analytic with respect to the meson momentum on mass shell. Then, the little group which leaves a complex momentum invariant is the complex Lorentz group SL(2, C) ⊗ SL(2, C). The combination with SU(3) gives SL(6, C) ⊗ SL(6, C). The same group is obtained if we look for a group which admits both SL(6, C) and the chiral group SU(3) ⊗ SU(3) as subgroups.

7.5. The smallest simple group SL(12, C)

If we require that SU(6, 6) be also contained in the new group, we obtain SL(12, C) which admits SL(6, C) ⊗ SL(6, C) as a subgroup. Finally, we know, as in the case of chiral symmetry with massive fermions, that the massless mesons must transform under a non-linear representation of the group [7].

Having found the general features of the Lagrangian model, we are now ready to construct examples.

8. A SPECIFIC MODEL

8.1. Modification of the mass term and the kinetic term

I shall now review the general features of an SL(12, C) invariant Lagrangian theory [47] proposed recently. In the case of no internal symmetry the group reduces to SL(4, C) which is the largest unimodular linear group that can act on a Dirac field ψ.

The free Lagrangian ψ is

$$\mathscr{L}^0 = -\bar{\psi}\,\gamma_\mu\,\partial_\mu\psi - m\bar{\psi}\,\psi$$

The mass term is invariant under SU(2, 2) but not under chiral transformations. To this end we make the replacement

$$\bar{\psi}\psi \to \bar{\psi}\,M(\phi)\,\psi \quad (M = 1 + \text{meson terms})$$

where $M(\phi)$ is a certain matrix which obeys

$$\overline{M}(\phi) = \gamma_\mu M^\dagger \gamma_4 = M(\phi)$$

Now the kinetic term is not invariant under SL(2, C) or SU(2, 2). We can achieve invariance by making the new replacement

$$\gamma_\mu \to N_\mu(\phi, \phi') = \gamma_\mu + \text{meson terms}$$

where ϕ' symbolizes the additional mesons one might need. M and N_μ must transform linearly under the group, since ψ transforms linearly.

8.2. The model

In the example referred to above we have the realization

$$M = \overline{H} H = \gamma_4 H^\dagger \gamma_4 H \qquad N_\mu = \overline{H} \gamma_\mu H$$

where

$$\mathrm{Det}\, H = 1$$

so that H has the canonical form

$$H = e^{-i\phi'} e^{\phi} \qquad (\mathrm{Tr}\,\phi = \mathrm{Tr}\,\phi' = 0)$$

where

$$\phi = \overline{\phi} = f(i\gamma_5\varphi_5 + i\gamma_5\gamma_\mu\varphi_{5\mu} + i\gamma_\mu\varphi_\mu + \tfrac{1}{2}\sigma_{\mu\nu}\varphi_{\mu\nu})$$

$$\phi' = \overline{\phi}' = f(i\gamma_5\varphi'_5 + i\gamma_5\gamma_\mu\varphi'_{5\mu} + i\gamma_\mu\varphi'_\mu + \tfrac{1}{2}\sigma_{\mu\nu}\varphi'_{\mu\nu})$$

f denoting a length.

We must also add to the Lagrangian a meson term which for $f \to 0$ collapses to a free meson Lagrangian. This Lagrangian must lead to a positive definite Hamiltonian. These conditions are satisfied by

$$\mathscr{L}^{\mathrm{meson}} = \frac{1}{16}\,\mathrm{Tr}\left\{(\partial_\lambda H)(\partial_\lambda H^{-1}) + (\partial_\lambda \overline{H}^{-1})(\partial_\lambda \overline{H}) - \frac{1}{2}\,\partial_\mu(\overline{H}\gamma_\mu H)\,\partial_\nu(\overline{H}\gamma_\nu H)\right\}$$

8.3. Invariance

In this model we have

$$M = e^{2\phi} \qquad N_\mu = e^{\phi} e^{i\phi'} \gamma_\mu e^{-i\phi'} e^{\phi}$$

and the transformation law is

$$\psi \to B^{-1}\psi \qquad \overline{\psi} \to \overline{\psi}\,\overline{B}^{-1}$$

$$H \to B H \qquad \overline{H} \to \overline{H}\,\overline{B}$$

where B is a unimodular constant matrix. The transformation law is therefore non-linear for the fields ϕ and ϕ'.

8.4. Massless limit for the mesons

Meson mass terms destroy the invariance. Then, the currents associated with the group are no longer conserved. We can put symmetry-breaking terms so that, through a generalization of PCAC, the divergences of the currents become proportional to the meson fields. In that case the Lagrangian shows partial invariance.

8.5. Combination with internal symmetries

Let A denote the set of the twelve indices (α, i) where α is a Dirac index and i is the SU(3) index. Since baryons are represented as third-rank tensors of SL(12, C) we can write for the meson-baryon coupling

$$\mathscr{L} = -\bar{\psi}^{ABC} (N^G_{\mu C}\, \partial_\mu + m\, M^G_C)\, \psi_{ABG} + \mathscr{L}^{meson}$$

where the tensor ψ is completely symmetrical in its indices. This Lagrangian can also be used as a phenomenological Lagrangian for the baryon-meson system by using an expansion in f. The Yukawa part of the ψ-ϕ coupling is invariant under the group SU(6, 6) and consequently leads to the familiar SU(6) relations for vertices. We note that π and ρ are part of the fields which appear in the matrix Φ.

9. DECOUPLING OF SPIN FROM ORBITAL ANGULAR MOMENTUM

Consider the kinetic fermion term plus a coupling term which combine into

$$-\bar{\psi}(x)\, e^{\Phi}\, e^{i\Phi'}\, \gamma_\mu\, e^{-i\Phi'} e^{\Phi}\, \partial_\mu\, \psi$$

9.1. Behaviour under the Lorentz transformation

Let Λ be a Lorentz matrix. We have the linear transformations

$$\psi \to \Lambda\psi, \qquad \phi \to \Lambda\phi\Lambda^{-1}, \qquad \phi' \to \Lambda\phi'\Lambda^{-1}$$

Also

$$\gamma_\mu\, \partial_\mu \to \Lambda\gamma_\mu\Lambda^{-1}\, \partial_\mu = \gamma_\mu\, \partial'_\mu$$

where

$$x' = \Lambda^{-1} x$$

As a result we have

$$\psi(x) \to \Lambda\psi(x') = \Lambda\psi(\Lambda^{-1} x)$$

The change in the argument is induced by the orbital angular momentum operator while Λ refers to the spin.

9.2. Evasive solution with $x \to x$

Let $\psi(x) \to \Lambda\psi(x)$ with x not changing. This is no longer a Lorentz transformation, it is an index transformation. The Lagrangian is still invariant under this transformation if

$$\phi(x) \to \Lambda\phi(x)\Lambda^{-1}$$

$$\phi'(x) \to \Lambda\phi'(x)\Lambda^{-1} + \chi(\Lambda, \phi')$$

where the inhomogeneous matrix χ is determined in such a way that

$$e^{\phi} e^{i\phi'} \rightarrow \Lambda e^{\phi} e^{i\phi'} = \Lambda e^{\phi} \Lambda^{-1} (\Lambda e^{i\phi'} \Lambda^{-1}) \Lambda$$

Clearly, this is always possible provided ϕ' transforms non-linearly and is massless. In a chiral transformation it is ϕ which transforms non-linearly. In the evasive case the transformation is formally like an internal symmetry gauge transformation with an inhomogeneous term.

Such a transformation law which decouples the spin from the orbital angular momentum can now be generalized to SL(4, C) on replacing Λ by B^{-1}, where B is a complex unimodular matrix.

Since orbital angular momentum is not involved, it is this pure spin transformation which combines with internal symmetry to give SL(12, C).

10. REALIZATION OF SL(4, C) IN HILBERT SPACE. CURRENT ALGEBRA

10.1. Unconventional character of the symmetry

Define the field

$$\xi = M(\phi)^{\frac{1}{2}} \psi$$

When we apply the generalized chiral transformation

$$\psi \rightarrow e^{\Omega} \psi \qquad (\Omega = \overline{\Omega})$$

we induce on ξ the transformation

$$\xi \rightarrow e^{i\Omega'(\phi, \Omega)} \xi \qquad (\overline{\Omega}' = \Omega')$$

since $\overline{\xi}\xi$ must remain invariant under the transformation. Applying ξ on the vacuum we obtain: one fermion state $\rightarrow$ one fermion + one soft π + one soft ρ + ... Thus one-particle states are unstable, the symmetry is unconventional and no-go theorems are evaded.

10.2. Couplings between ξ and ϕ, ϕ'

If the Lagrangian is written in terms of ξ and the meson fields ϕ and ϕ', we note the following properties:

(a) The coupling involves derivatives of ϕ and ϕ'. Hence, for vector fields $\varphi_{\mu\nu}$ plays the role of Hertz potential;

(b) The fields Φ are coupled to Dirac-type currents that do not involve $\partial_\mu \psi$;

(c) The fields Φ' are coupled to fermion currents of the type $\overline{\psi} \Gamma_A \partial_\mu \psi$ involving derivatives of ψ. The two pseudovector mesons $\varphi'_{5\mu}$ and $\varphi'_{\mu\nu}$ are coupled respectively to the C-odd and C-even spin currents. This indicates that just as the isotopic spin current is dominated by the ρ-meson, the charged spin currents are dominated by the B and A mesons.

10.3. Meson multiplets

If we analyse the forms of the couplings we can deduce the quantum numbers of the mesons involved.

The mesons contained in ϕ have 0^{-+} and 1^{--} just as in an SU(6) multiplet and the representation is doubled. For T = 1 we have $\vec{\pi}, \vec{\rho}$ and also $\vec{\pi}'$ and $\vec{\rho}'$.

The mesons contained in ϕ' have quantum numbers 0^{++} with doubling and 1^{+-} like the B and 1^{++} like A_1. Thus all the quantum numbers of low-lying meson states are reproduced. These mesons form the adjoint representation of the compact part of SL(12, C), namely SU(12).

10.4. Current algebra based on the model

When meson mass terms are added in accordance with PCAC and PCTC (partially conserved tensor currents) we get relations of the form

$$\partial_\mu J_{\mu A} = \frac{m^2}{f} \phi_A \qquad \partial_\mu J'_{\mu A} = \frac{m'^2}{f} \phi'_A$$

for the currents, ensuring smooth extrapolation to the symmetric limit for vanishing meson masses. For such a model the currents are not conserved in general, but the current algebra [48] associated with SL(12, C) continues to hold. This is a generalization of the usual V-A algebra and, through the tensor currents, contains the $SU(6)_W$ group which leads to the observed vertex symmetry.

11. OUTLOOK

The Lagrangian example demonstrates that SU(6) symmetry can be given a form compatible with local field theory. It becomes exact in the limit of zero mass for mesons. PCAC can be generalized to ensure smoothness of this extrapolation. Then we should expect vertex symmetry not only for zero mass virtual mesons but also to a good approximation for mesons on their mass shell. A current algebra can be abstracted from the model and combined with low-energy theorems for all the mesons Φ and Φ' to reproduce SU(6) results as well as the results of chiral symmetry. The group also provides a classification scheme for hadrons.

Now there is a quite different successful classification scheme for hadrons — that of Regge trajectories. It has been realized recently that trajectories based on infinite-dimensional representations of SL(2, C) must be used in order not to conflict with analyticity properties of the amplitude [49]. Through analytic continuation, this group can be extended to the complex Lorentz group SL(2, C) $\otimes$ SL(2, C) whose compact part gives the 0(4) classification [50]. The irreducible representations of this group are associated with exact symmetry at the t = 0 intersect of the trajectories and each irreducible representation contains a Regge trajectory and an infinite number of daughter trajectories. Physical points on this family of trajectories classify particles and resonances.

It seems very remarkable that, starting from a reasonable asymptotic behaviour of amplitudes, we should recover the group SL(2, C) $\otimes$ SL(2, C) which is the minimal extension of the spin group in the Lagrangian formulation of spin independence. Another similarity is that the Toller symmetry holds at t = 0 while spin independence is exact for zero mass mesons.

It seems reasonable to try to use infinite-dimensional irreducible representations of the groups SL(6, C)⊗SL(6, C) or SL(12, C) to classify all particles, including those lying on Regge trajectories. The generalization of the Wigner-type mass formula then appears as a Domokos-type mass formula [51] when we move away from the point t = 0.

These remarks kindle the hope that chirality, spin and unitary spin independence in field theory can be merged with dominance of asymptotic scattering amplitudes by Regge-Toller poles leading to a unification of group theoretical and analyticity methods in particle physics.

As a result it may well turn out that the chaotic appearance of the hadron spectrum conceals a beautiful order connected with the theory of representations of non-compact groups, also initiated by Wigner [52].

In conclusion, I should like to express my sincere thanks to Professor Abdus Salam for his kind hospitality at the International Centre for Theoretical Physics.

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INFINITE MULTIPLETS IN ATOMIC AND HIGH-ENERGY PHYSICS

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Abstract

INFINITE MULTIPLETS IN ATOMIC AND HIGH-ENERGY PHYSICS. 1. Introduction; 2. Hydrogen atom; 3. Relativistic theories.

1. INTRODUCTION

In SU(6) theory there is only a partial synthesis between unitary symmetry and the Lorentz group. Consequently SU(6) is a static theory and has a very limited range of applicability. To put it in another way: there is a glaring discrepancy between SU(6) symmetry and Lorentz invariance, even if the mass differences between the members of the supermultiplets are disregarded. Now, if the unification achieved by SU(6) could be successful, then, according to Professor Dirac, the resolution of the discrepancy should be profitable indeed, especially because this leads to the use of new representations of the Lorentz group. A more complete synthesis was attempted and led to groups even larger than SU(6), e.g. SL(6, C) and SU(6, 6), whose main characteristic is that they are non-compact.

The fact that the S-matrix is unitary suggests that we consider multiplets that transform unitarily with respect to the larger groups. Such multiplets are infinite and hence, by Coleman's law, implausible. (Coleman's law states that if p is the plausibility of a theory that predicts n new particles, then $pn \approx \hbar$.)

Now it is a fact that elementary particle physics has to deal with infinitely many states, though not all with the same mass. The mass differences are a difficulty in unitary symmetry as much as in the larger multiplet schemes. The real challenge, therefore, is to develop a theory that is consistent with the experimental mass spectrum, or better still, predicts it.

Rather than develop the ideas in the framework of elementary particle physics, I shall abide by the spirit of this Symposium, and show some of the intimate connections that are emerging between high-energy theory and non-relativistic quantum mechanics. First I shall show that the problem is common to elementary particles and to quantum mechanics. Next I shall indicate how quantum mechanics has solved the problem, and finally I shall touch briefly on the transfer of the results to the high-energy domain.

2. HYDROGEN ATOM

The spectrum of the non-relativistic hydrogen atom is shown in Fig. 1. The vertical co-ordinate is the energy; each horizontal line represents

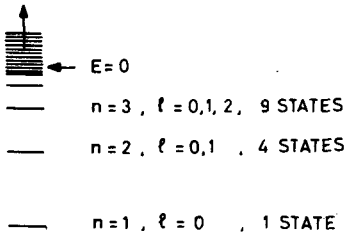

FIG.1. Hydrogen energy levels.

one or more bound states, and the continuum of electron-proton scattering states is shown on top. The degeneracy of each level, and the angular momenta, are also shown. The degeneracy of several states with the same value of angular momentum is, of course, a direct consequence of the rotational invariance of the Hamiltonian

$$[H, L_{ij}] = 0, \qquad L_{ij} = r_i q_j - r_j q_i \tag{1}$$

The degeneracy of states with different angular momenta is referred to as the accidental degeneracy. However, Fock showed that it is a direct consequence of another invariance of the Hamiltonian

$$[H, L_{i4}] = 0, \qquad L_{i4} = (\vec{r}\cdot\vec{q} - i)\, q_i + \tfrac{1}{2} r_i (1 - \vec{q}^{\,2}) \tag{2}$$

The six operators L_{ij} and L_{i4} satisfy the commutation relations of SO(4), the four-dimensional rotation group, and each level of the atom consists of an n^2-dimensional irreducible SO(4) multiplet.

This SO(4) symmetry of the Hamiltonian is actually perturbed by relativistic effects, but the corrections are exceedingly small. The problem is, how do we incorporate SO(4) symmetry in a relativistic framework? Since our symmetry group already contains part of the Galilei group, the spin, the problem is exactly the same as in SU(6) theory. Let us insist on the resolution of the discrepancy. Actually, most physicists do not admit that a discrepancy exists; I am sure that Professor Dirac will agree that this is a normal situation.

The solution is the simplest one imaginable: SO(4) and the Lorentz group (also the Galilei group, if one insists on a non-relativistic framework for the discussion) are embedded in the five-dimensional Lorentz group SO(4, 1). The generators are L_{ij}, L_{i4} and

$$\begin{aligned} L_{i0} &= (\vec{r}\cdot\vec{q} - i) q_i - \tfrac{1}{2}(1 + \vec{q}^{\,2})\, r_i \\ L_{40} &= -\vec{r}\cdot\vec{q} + i \end{aligned} \tag{3}$$

Here we have a non-compact group, analogous to, e.g. SL(6, C) in relativistic SU(6) theory. It is represented on the states of the hydrogen atom by operators that are essentially unitary, uniting an infinite set of states into one irreducible representation.

The economy that is achieved by recasting the Schroedinger theory in group-theoretic language is astonishing. The complete set of states, including bound states and scattering states, forms just one irreducible unitary representation of O(4, 1). Furthermore, it is the simplest representation imaginable – one that has the remarkable property that it can be extended to SO(4, 2) without introducing new states. The extra five operators are

$$\begin{aligned} \Gamma_i &= L_{i5} = r q_i \\ \Gamma_4 &= L_{45} = \tfrac{1}{2} r(\vec{q}^{\,2} - 1) \\ \Gamma_0 &= L_{05} = \tfrac{1}{2} r(\vec{q}^{\,2} + 1) \end{aligned} \tag{4}$$

Of course, none of these new operators, given by (3) and (4), commutes with the Hamiltonian. Instead, they may be considered as a new set of dynamical variables, to replace the canonical variables q_i and r_i. In fact, it is possible to eliminate the latter from the Schroedinger equation, with the simple result

$$\left[\left(E - \frac{1}{2\mu}\right)\Gamma_0 - \left(E + \frac{1}{2\mu}\right)\Gamma_4 + e^2\right]\psi = 0 \tag{5}$$

This equation is similar to the Dirac equation for the electron, and even more similar to the Majorana equation, except that we are so far restricted to one reference frame and to non-relativistic kinematics.

The formula for the inner product in Hilbert space works out to

$$\langle\varphi|\psi\rangle \equiv \int \varphi^*(\vec{r})\psi(\vec{r})\, d^3r = \varphi^\dagger(\Gamma_0 - \Gamma_4)\psi \tag{6}$$

just as one would expect if (5) is interpreted as a Lagrangian equation. The matrix elements of the important quantum mechanical operators become very simple in the new notation. The dipole transition moments are given by

$$\langle\varphi|q_i|\psi\rangle = \varphi^\dagger \Gamma_i\, \psi \tag{7}$$

The currents are thus related to generators, just as in elementary particle symmetry theories. This has some practical utility; the right-hand side is much easier to evaluate than the usual integral formula.

To obtain a simple treatment of phenomena involving finite momentum transfer it is necessary to generalize Eq. (5) so that it will be valid in any reference frame. That is, we must subject Eq. (5) to a Galilei transformation that "boosts" the total momentum from zero to $\vec{p}$, say. That this is not a well-defined problem may be understood by comparison with the Dirac equation. In the rest system it is indistinguishable from the Pauli equation (doubled). Boosted one way it becomes the usual Dirac equation; boosted another way it becomes the Pauli equation. The two forms

are related by the Foldy-Wouthuysen transformation. By far the simplest results are obtained by following Dirac. In this case the Γ-matrices have definite and simple transformation properties with respect to the Galilei group. The boosted form of Eq. (5), in the frame in which the total momentum is $\vec{p}$, turns out to be

$$\left[\left(E-\frac{p^2+1}{2\mu}\right)\Gamma_0 - \left(E-\frac{p^2-1}{2\mu}\right)\Gamma_4 - \frac{1}{\mu}\vec{p}\cdot\vec{\Gamma}+e^2\right]\psi(p) = 0 \tag{8}$$

which is a non-relativistic Majorana-Dirac equation. In this equation p is the centre-of-mass momentum, but $\vec{x}$, the canonical conjugate of $\vec{p}$, is not the co-ordinate of the centre of mass, but the co-ordinate of the electron. If $\vec{p}_1$, $\vec{p}_2$ and $\vec{x}_1$, $\vec{x}_2$ are the momenta and the position co-ordinates of the electron and the proton, then the canonical transformation is

$$\begin{aligned} \vec{p} &= \vec{p}_1 + \vec{p}_2, \qquad \vec{x} = \vec{x}_1 \\ \vec{q} &= -\vec{p}_2, \qquad \vec{r} = \vec{x}_1 - \vec{x}_2 \end{aligned} \tag{9}$$

The "internal" variables $\vec{q}$ and $\vec{r}$ are replaced by an index σ on the wave function $\psi(\vec{p}) = \{\psi_0(\vec{p}), \sigma = 1, 2, \ldots\}$; it will be suppressed whenever possible, just as it is suppressed in the Dirac theory.

Since $\vec{x}$ is the position of the electron, and since an external radiation field sees only the electron, it is not so surprising that the interaction between the atom, represented by $\psi(x)$, and the field, represented by $\vec{A}(x)$, is local. The interaction Lagrangian is simply

$$\begin{aligned} & e\vec{j}(x)\cdot\vec{A}(x) \\ & j_i(x) = \frac{e}{\mu}\psi^\dagger(x)\left[\Gamma_i - \frac{i}{2}\overleftrightarrow{\partial}_i(\Gamma_0 - \Gamma_4)\right]\psi(x) \end{aligned} \tag{10}$$

From now on, to save time, I shall ignore the spin of the photon; then the current is a scalar

$$j(x) = \psi^\dagger(x)\psi(x) \tag{11}$$

and the form factors are given by

$$F_{\psi\varphi}(k^2) = \psi^\dagger(p+k)\varphi(p) \tag{12}$$

To calculate the form factor in a Coulomb field one has to insert a factor $(\Gamma_0 - \Gamma_4)$. Then one obtains the following well-known expression for the ground-state form factor:

$$F_{11}(k^2) = \left[1 + \frac{k^2}{8\mu B}\right]^{-2} \tag{13}$$

Similar results are obtained from relativistic Majorana equations, and for this reason it is worth while to stress some of the important properties of these form factors. First of all they decrease as $k^2 \to \infty$; this is a consequence of using unitary, infinite-dimensional representations of the Lorentz or Galilei groups, and is in contrast with the polynomial form factors obtained from finite-component wave equations. Secondly, the shapes are very similar to those observed by probing the nucleons with electrons. Most important, the internal structure of the system can be discovered by inspecting the charge distribution. This can be done by calculating the Fourier transform of the form factors, or more simply by determining the positions of the singularities.

Because of the similarity between SO(4) and SU(6), it is particularly interesting to examine the SO(4) symmetry of Eq.(8). Of the five Γ-matrices Γ_i, Γ_0, Γ_4, only one linear combination enters into Eq.(8), which may therefore be written as follows:

$$\left[- (-2E/\mu)^{\frac{1}{2}} n + e^2 \right] \psi(p) = 0 \tag{14}$$

Here $\lambda = \lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ is a five-vector of unit length, and

$$n \equiv \sum_{a=0}^{4} \lambda_a \Gamma_a \tag{15}$$

Equation (14) is obviously invariant under the transformations of a subgroup of SO(4, 1), namely that subgroup $G_{p,E}$ that leaves invariant the five-vector λ. This group is the same SO(4) group that was mentioned at the beginning. Actually $G_{p,E}$ is isomorphic to SO(4) only when λ is "time-like", i.e. when $E < 0$; when $E > 0$ it is isomorphic to SO(3, 1). Naturally, n is just the principal quantum number.

To diagonalize (14) is now trivial; one simply takes a representation where n is diagonal. This gives the Green's function

$$G = \frac{1}{-(-2E/\mu)^{\frac{1}{2}} (n - \alpha(E))}, \quad \alpha(E) \equiv \left(\frac{\mu e^4}{-2E}\right)^{\frac{1}{2}} \tag{16}$$

We may use this result to calculate the amplitude for Compton scattering from a bound electron

$$A(E, k^2) = \sum_{n=1}^{\infty} F_{1n} \frac{-\left(\frac{-\mu}{2E}\right)^{\frac{1}{2}} P_{n,4}}{n - \alpha(E)} F_{n1} \tag{17}$$

where $P_{n,4}$ is a four-dimensional spherical function. This sum can be summed exactly, which demonstrates the efficiency of the group theoretical method, for the exact result is here obtained for the first time.

The expansion (17) converges in the region $E < 0$ for which the spectrum of n is discrete, i.e. when the symmetry group is SO(4). For higher ene

energies the amplitude can still be expanded in terms of n, but now the spectrum is pure imaginary. The correct expansion can be obtained by means of a Sommerfeld-Watson transformation

$$A(E, k^2) = \int dn \csc \pi n \, F_{1n} \frac{-\left(\frac{-\mu}{2E}\right)^{\frac{1}{2}} P_{n,4}}{n - \alpha(E)} F_{n1} \tag{18}$$

with the contour of integration shown in Fig. 2. It is necessary to place the contour along the imaginary axis and to pass to the right of the "Regge pole" that comes from the zero of the denominator of the Green's function. To understand that the formal similarity with Regge and Toller theory is not a deception, it is necessary to go over to the relativistic framework.

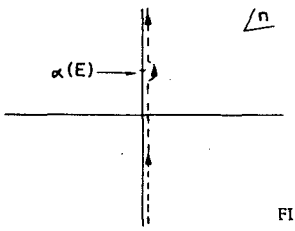

FIG. 2. The n-plane.

3. RELATIVISTIC THEORIES

It is easy to make slight changes in Eq. (8) so as to make it Lorentz invariant. Then it becomes possible to transfer the whole structure into the relativistic domain. However, great care is needed to avoid introducing new and unphysical solutions in the process of making the equation Lorentz invariant. This is not trivial, nor is it impossible.

One of the interesting modifications that are introduced by going over to a relativistic model is that the spectrum of n is discrete in a finite interval of centre-of-mass energy, say in the interval

$$m_-^2 < t < m_+^2, \qquad t \equiv (m_+ + E)^2$$

Here $m_+ = m_1 + m_2$ is the ionization threshold, $t > m_+^2$ corresponds to $E > 0$, and the integral representation (18) for the scattering amplitude is valid. The new feature is the possibility to continue analytically to the neighbourhood of $t = 0$. Physically this corresponds to the process shown in Fig. 3, hydrogen exchange, in backward Compton scattering. Here too, n has a continuous spectrum, and the integral representation (18), rather than the

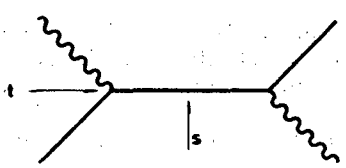

FIG. 3. Hydrogen exchange.

sum (17), is valid. There is a difference, however, between $t > m_+^2$ and $t < m_-^2$. In the case $t < m_-^2$ it is necessary that the Regge pole $\alpha(E)$ or $\alpha(t)$ be real. Therefore, in contrast to the situation shown in Fig. 2, the pole is isolated from the imaginary axis, and one obtains

$$A(s,t) = \int_{-i\infty}^{+i\infty} dn \csc \pi n \, F_{1n} \frac{P_{n,4}}{n - \alpha(t)} F_{n1} + \frac{2\pi i \left| F_{\alpha(t),1} \right|^2}{\sin \pi \alpha(t)} P_{\alpha(t),4} \tag{19}$$

At very high energy, $s \to \infty$, the amplitude is dominated by the pole term. If we expand $P_{\alpha,4}$, using standard addition formulae for four-dimensional spherical functions, then we see that the amplitude is dominated by a family of Regge poles with $\ell = \alpha(t), \alpha(t) - 1, \ldots$. It is easy to verify that, at the point of vanishing momentum transfer, the expansion (19) is not only similar in form, but identical in content, to Toller's expansion. This situation is displayed in a plot of n versus t or ℓ versus t in Fig. 4. Finally, let me emphasize one more point about these expansions. Obviously, not only ℓ and n, but many other quantum numbers, can be Reggeized. For example, one may Reggeize the SU(6) Casimir operator or the quark number. This does not depend on SU(6) being a good symmetry; it only depends on the Casimir operator being a good quantum number. That is, one must suppose that the mixing between different multiplets is small, but it is not necessary that the masses within each multiplet should be equal.

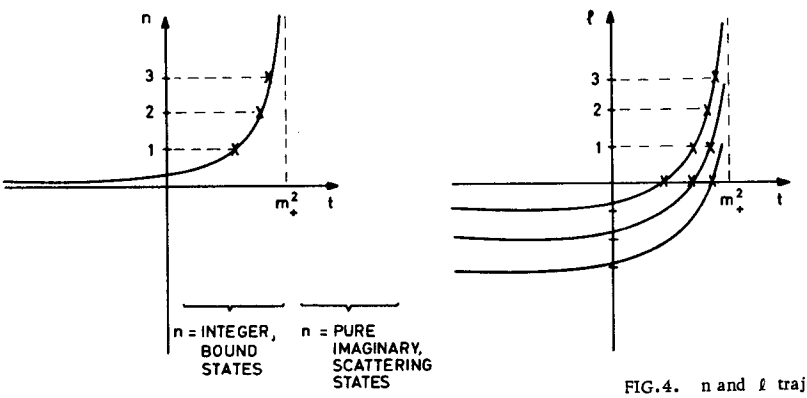

FIG. 4. n and ℓ trajectories.

To end my paper let me state what is, in my opinion, the single most important lesson that can be drawn from this study of the O(4) + Galilei discrepancy: In order to give a meaning to "symmetry" operators that operate between states of different mass, it is necessary to have an off-shell formalism. That is, instead of a pure state-space description

$$L|\text{mass}\rangle = |\text{mass}'\rangle$$

we need a method for off-shell extrapolation. In atomic physics this is accomplished by the Schroedinger wave function. The Majorana theories work in precisely the same way; in fact they seem to be extremely well suited to describe relativistic phenomena.

QUARKS, THE HADRONIC SUB-UNITS?

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Abstract

QUARKS, THE HADRONIC SUB-UNITS? The evidence is reviewed that the spectrum of hadronic states corresponds to a series of SU(6) (spin × unitary spin) supermultiplets $(\underline{a}, L(-1)^L)_n$, with orbital (L, with parity P) and radial (n) excitations, their unitary submultiplets having total spin $\vec{J} = \vec{L} + \vec{S}$. Two mixed supermultiplets $(\underline{35} + \underline{1}, L(-1)^L)_n$ appear rather clearly for the mesons, for n = L = 0 and for n = L = 1 (possibly another for n = L = 2, and an n = 2 radial excitation for L = 0). Below 2 GeV, five baryonic supermultiplets occur, $(\underline{56}, 0+)_n$ with n = 0, 2 and 4, $(\underline{70}, 1-)_1$ and $(\underline{56}, 2+)_2$; the nucleonic member has been identified for each unitary supermultiplet (Y_0^* for unitary singlets), except for two ($\frac{3}{2}$+) decuplets. The hypothesis that hadronic supermultiplets correspond to tightly-bound systems of quark-triplets would account qualitatively for many of their outstanding features; no established states lie outside the supermultiplets then possible, and the L and n excitations would have direct physical interpretation. The low-lying baryonic supermultiplets agree (in quantum numbers and order of excitation energy) with the harmonic shell-model states which Greenberg discussed in 1964 assuming symmetric wavefunctions (parastatistics). With a further additivity assumption, this quark model leads directly to several outstanding features of high-energy collisions. In conclusion, theoretical difficulties faced by this naive quark model are discussed.

1. INTRODUCTION

Over the last few years, the number of established "elementary particles" (as listed, for example, in the annual summary by Rosenfeld et al. [1]) has been increasing quite rapidly. To illustrate this, we may mention that there are at least 19 nucleonic resonances now known below mass 2 GeV, and that there is now good evidence for at least 15 non-strange mesonic resonances with I = 1 and with mass values below 2 GeV. It is clear that none of these particle states merits the description "elementary"; these particle states, both mesonic and baryonic, are conveniently termed the hadrons, a collective term for all the discrete particle states which take part in strong, nuclear interaction processes. It is observed that these hadron states occur grouped in the isospin multiplets characteristic of the isospin SU(2) symmetry (with mass splittings characteristic of the electromagnetic interactions, which depend on charge and thus violate the charge independence expressed by isospin symmetry, typically of the order of ~5 MeV), and that these isospin multiplets themselves appear grouped into SU(3) multiplets, the sub-multiplets having values for the isospin I and the hypercharge Y corresponding to patterns which are characteristic of the representations of the SU(3) group (the mass splittings between these sub-multiplets being due to the moderately-strong interactions which obey SU(2) symmetry but violate SU(3) symmetry, and having values typically of the order of ~200 MeV).

The simplest interpretation of the high multiplicity observed for the hadron states is that the hadrons are themselves built up from hadronic sub-units, so that the hadronic states have internal degrees of freedom.

This parallels the physical situations we are already familiar with in our study of molecular states (where the internal degrees of freedom are concerned with the excitation of relative motion between the constituent atoms), of atomic states (where the internal degrees of freedom describe the motions of the constituent electrons in their motion around the atomic nucleus), and of nuclear states (where the internal degrees of freedom are concerned with the relative motions of the constituent nucleons of the nucleus). In fact, the unitary symmetry SU(3) was first considered by the Sakata school [2], in connection with the Sakaton triplet (P, N, Λ), from which it was then supposed that all the hadronic states observed were constructed. This particular scheme had to be abandoned [3] when it was realized from experiment that the spin ($\frac{1}{2}$+) baryons actually formed an octet representation of the SU(3) group, since the structure of this group is such that the simplest octet state with baryon number $B \neq 0$ which can be constructed from the basic triplet requires the combination of three of these triplet objects.

After some years of work with the octet model [3] (corresponding to representations of the group $SU(3)/Z_3$), Gell-Mann [4] and Zweig [5] returned to the triplet model. They proposed that the basic representations for the SU(3) group should correspond to a triplet of quarks (p, n, λ). Since the observed baryonic states are constructed from three of these basic quarks, these quarks must necessarily have fractional values for their charge, hypercharge and baryon number; in detail, their quantum numbers are as given in Table I. Many other and more complicated triplet schemes have been discussed in the literature (see especially Refs [6-10]), mostly with the aim of achieving a triplet scheme in which all the triplet objects have integral charge values. These schemes all involve (effectively) the introduction of some new additive quantum numbers (e.g. the charm quantum number C, as discussed in Ref.[7]). These possibilities are not at all excluded, but there is no experimental evidence yet which calls for such new quantum numbers (although there are some theoretical problems about the quark model which could be resolved in this event, as we shall discuss later).

Here we shall begin by confining attention to the case of a single quark triplet (q_i). This is the simplest quark model possible, and we shall use it as a guide in discussing the data available on hadronic states. We shall

TABLE I. THE QUANTUM NUMBERS FOR THE QUARK STATES. QUARKS (p, n) FORM AN ISOSPIN DOUBLET, AND λ IS AN ISOSPIN SINGLET

Quark	Charge Q	I_3	Baryon No. B	Hypercharge Y
$q_1 \equiv p$	$+\frac{2}{3}e$	$+\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$
$q_2 \equiv n$	$-\frac{1}{3}e$	$-\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3}$
$q_3 \equiv \lambda$	$-\frac{1}{3}e$	0	$\frac{1}{3}$	$-\frac{2}{3}$

find that there emerges quite a simple picture accounting for the mesonic and baryonic states available to date, although it has a number of aspects which appear puzzling and which are not at all understood.

As pointed out first by Gursey and Radicati [11] and by Sakita [12], the low-lying hadronic states give evidence for the existence of an SU(6) symmetry for the hadronic interactions. This symmetry corresponds to a grouping of SU(3) multiplets, with the same parity but different spin values, into supermultiplets; for example, the baryon octet with spin $(\frac{1}{2}^+)$ and the baryonic decuplet with spin $(\frac{3}{2}^+)$ together correspond to an SU(6) supermultiplet of dimension 56. The existence of these supermultiplets corresponds to an invariance of the hadronic interactions with respect to unitary transformations in the combined space of Pauli spin and of unitary spin. To give these transformations physical meaning, it is necessary that the hadronic sub-units should have non-relativistic motions in the hadron rest frame, since Pauli spin is appropriate only in the non-relativistic regime; as we know well from the example of the Dirac equation, spin and orbital motion become inextricably mixed when the motion becomes relativistic. This situation is quite possible if the quarks are sufficiently heavy; if a denotes the range of the forces between quarks, the condition for non-relativistic motion has the form

$$\hbar/a \ll M_q c \tag{1.1}$$

The fact that quark production has not been observed in accelerator experiments (cf. Section 5) already requires that M_q is greater than 5 GeV, so that this condition can readily be satisfied for a reasonable value for a (say, for $a \approx 10^{-14}$ cm). With such a large mass value, the binding of the quark in mesonic or baryonic states is necessarily very great, being comparable with the total rest mass for the quarks in the system. This requires that the dominant force between quarks or between quark and antiquark should be exceedingly strong, and we shall refer to these binding forces as the superstrong forces. The evidence on SU(6) symmetry for the hadronic states then requires these superstrong forces to satisfy the constraints of SU(6) symmetry, at least in the limit of non-relativistic motion for the constituent quarks. In view of this, SU(6) symmetry is not a universal symmetry, but simply holds approximately for states where the internal motion of the constituent quarks is non-relativistic; hence SU(6) symmetry would not be expected to hold valid for highly-excited hadronic states (with excitation energy $\approx M_q c^2$).

2. MESONIC STATES

The mesonic states are interpreted as bound states of the quark-antiquark systems $\bar{q}_i - q_j$. These systems can have internal orbital angular momentum L and total spin S = 0 or S = 1, which combine to give total spin $J = L \pm 1$, L. There will also be radial vibrations of the system, characterized by a principal quantum number n, with integral values n = 1, 2 ... increasing with increasing excitation energy.

For each $\bar{q} - q$ configuration $(n^{(2S+1)} L_J)$, there are nine independent states (i, j = 1, 2, 3), referred to as a nonet. With respect to SU(3) symmetry, each nonet consists of a unitary octet together with a unitary

singlet state; generally, the SU(3)-breaking forces have matrix elements connecting this singlet state with the (I = 0, Y = 0) member of the octet, giving rise to strong mixing between them and to strong distortion of the physical states from the pattern appropriate to an isolated octet. This situation is in good accord with empirical observation, in that the only mesonic resonances established have (I, Y) values which are appropriate to such nonets; despite considerable search, no resonances have been established for the systems $\pi^+\pi^+$ or $\pi^-\pi^-$(I = 2, Y = 0, characteristic of a {27} representation), $K^+\pi^+$ or $K^-\pi^-$ (I = $\frac{3}{2}$, Y = ± 1, characteristic of {10}, $\overline{\{10\}}$, or {27} representations), K^+K^+ or K^-K^- (I = 1, Y = ± 2, characteristic of a {27} representation), nor for the systems $\rho^+\pi^+$ or $\rho^-\pi^-$ (I = 2, Y = 0, characteristic of a {27} representation).

With SU(6) symmetry, the 36 states (each with different spin component S_3, charge Q and hypercharge Y) with the same (n, L) values separate to form a $\underline{35}$-dimensional SU(6) representation and a singlet SU(6) representation $\underline{1}$, the latter consisting of the unitary singlet state with S = 0. The superstrong forces could be quite different in the $\underline{35}$ and $\underline{1}$ states, so that these representations could be widely separate in mass. Hence, this model leads to the expectation that the singlet and octet states should be approximately degenerate for the S = 1 nonets, the mass splitting being due only to SU(6)- and SU(3)-breaking forces; on the other hand, the S = 0 nonets could consist of widely separate octet and singlet states, with relatively weak mixing.

The (I = 1, Y = 0) member of each nonet will generally appear in the reaction

$$\pi^- + p \rightarrow p + X^- \tag{2.1}$$

where X^- decays to some multi-particle mesonic system. This reaction (2.1) has been studied with particular precision and high statistics for the case where the state X is not directly observed, with the "missing-mass" spectrometer at CERN [1, 3], by studying the momentum distribution of the recoil proton following this reaction. The mass distribution for X^- shows peaks at the π^-, ρ^- masses, then a narrow peak at δ (962), a double A_2 peak around 1300 MeV, then three further peaks in the R-region (1650-1750 MeV), then further narrow peaks which were named S(1930), T(2195) and U(2380). Figure 1 shows $(\text{mass})^2$ for the ρ, upper A_2, R, S, T, U peaks plotted linearly against their order in this sequence, as given by Focacci et al.[13]. These points are quite well fitted by a straight line. This line crosses $(\text{mass})^2 = 0$ at the intercept 0.45; this value is in excellent agreement with the value $\alpha_\rho(0)$ deduced quite independently for the intercept of the ρ-Regge trajectory from the high-energy scattering (especially charge-exchange) data (whose physical domain corresponds with the region $(\text{mass})^2 \leqslant 0$ in Fig.1). This line is evidently that for the ρ-Regge trajectory in the $(\text{mass})^2 > 0$ region. With the $\bar{q}$ - q model, the ρ and A_2 trajectories are expected to coincide, since there are no exchange forces expected to be of importance for this system (the only possibilities involve the exchange of qq systems, which are necessarily very massive and can generate forces only of exceedingly short range). The first two mesons on this line correspond to the configurations $^3S_1$ and $^3P_2$ (see below) and so have J = 1 and J = 2, and it is therefore a natural conjecture that the ordinate in Fig.1 actually denotes the spin J for the corresponding meson, so that

the T-meson, for example, has spin (5^-). This hypothesis leads to a natural interpretation for the unexpected narrowness (typically $\lesssim 20$ MeV) observed for the highest meson states in this sequence, namely that their high initial spin leads to large centrifugal barriers (roughly $L(L+1)/M^* a^2 \approx 15$ GeV for $a \approx 10^{-14}$ cm) for the outgoing decay products, a situation which suppresses their rate of decay. We should mention here that there is independent bubble chamber evidence that there are narrow resonances for high mass values; Alles-Borelli et al. [14] have reported the observation of a narrow peak (width 67 ± 50 MeV) in the state $(\pi^+\pi^-\pi^0)$ at 2207 MeV.

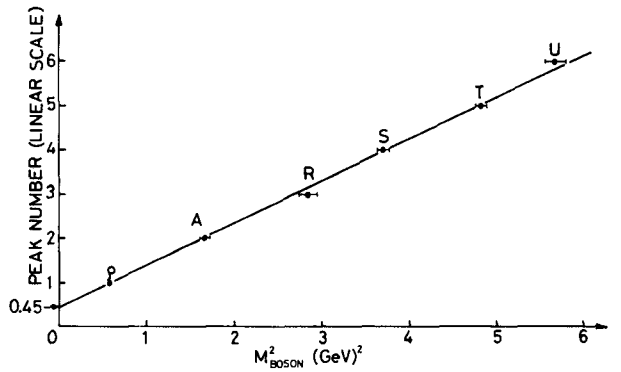

FIG. 1. Plot of (mass)^2 for the series of prominent peaks found by Focacci et al. [13] with the CERN "missing mass" spectrometer to measure recoil protons in the reaction $\pi^- p \to p(MM)^-$, against their order in this spectrum. The presumption is at present that this plot actually shows the (coincident) Regge trajectories associated with the ρ-meson and the A_2-meson. The intercept 0.45 at (mass)$^2 = 0$ is in good agreement with the value required from ρ-exchange processes studied in the physical region (which corresponds to (mass)$^2 \leq 0$).

The sequence of states ρ, A_2, R, S, T, U... is therefore interpreted as a rotational sequence, with (mass)^2 linear in J. This simple behaviour can be reproduced for a simple harmonic potential if a quadratic wave equation is adopted; one equation with this property is the following:

$$(E^2 - 4M^2 - p^2 - 2MV)\psi = 0 \tag{2.2}$$

with $V = V_0 + \frac{1}{2} M\omega^2 r^2$, whose eigenvalues are given by

$$E^2 = (4M^2 + 2MV_0 + \tfrac{3}{2}\hbar\omega M) + \hbar\omega M(2n + L - 2) \tag{2.3}$$

where n is the principal quantum number $n = 1, 2, \ldots$ and L denotes the orbital angular momentum. With this equation, the wave function for the ground-state takes the form

$$\psi(n = 1,\ L = 0) = N \exp(-M\omega r^2/2\hbar) \tag{2.4}$$

We note that the quark mass M always appears in the combination $(M\omega)$; since there is no independent way to deduce ω, the quark mass M cannot be deduced from observations on the bound system.

The $L = 0^+$ states. These correspond to the configurations ($n\,^1S_0$) and ($n\,^3S_1$), pseudoscalar and vector meson nonets, just as the low-lying meson states are observed to be.

The vector meson nonet is well established (cf. Table II). As generally expected for $S = 1$, the ($I = 0$, $Y = 0$) states ω and ϕ are strongly mixed.

The pseudoscalar meson nonet is usually considered to consist of the (π, K, $\bar{K}$, η and η' (969)) isospin multiplets. Here, the state η' is dominantly unitary singlet and stands apart from the octet states (π, K, $\bar{K}$, η), as expected for $S = 0$. However, we should mention that the octet states lie at appreciably lower mass values than the mean mass for the $S = 1$ nonet, contrary to the SU(6) expectation. In fact, the SU(6) singlet state η'(969) lies much closer to the $S = 1$ nonet states than do the $S = 0$ octet states; this suggests that, in first approximation, the $\bar{q}_i - q_j$ superstrong forces are independent of the labels i, j, as well as of the quark spin vectors. In this case, nonets would be expected to occur equally for both $S = 0$ and $S = 1$ states, and the displacement downwards for the pseudoscalar octet would have to be attributed to some SU(6)-breaking interaction of specific form.

One further pseudoscalar state has been reported, the ($I = 0$, $Y = 0$) meson E(1420). This meson must be attributed to the $n = 2$, $L = 0^+$ state with $S = 0$, a radial excitation of the η or η' meson. According to expression (1.4), the $n = 2$, $L = 0^+$ states would be expected to lie near the $n = 1$, $L = 2^+$ states, i.e. in the R-meson region 1650-1750 MeV, so that the state E(1420) lies appreciably lower in mass than this expectation. The $n = 2$ radially excited vector nonet is also expected to occur in the R-meson region but there are no clear candidates for this nonet yet.

The $L = 1^-$ states. Here nonets are expected for the $^3P_2$, $^3P_1$ and $^3P_0$ $n = 1$ configurations; for the $n = 1$ $^1P_1$ configuration, there may be a nonet situation (definitely, if the $\bar{q} - q$ interaction is independent of the quark labels) or a widely separated octet and singlet. For $L \geqslant 1$, spin-orbit interactions can contribute to $(\text{mass})^2$; these are SU(6)-breaking interactions, since the operations of SU(6) transform spin space but leave co-ordinate space unaffected. They separate the $^{(2S+1)}L$ states according to total J. The mass shifts produced by a spin-orbit term proportional to $(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\vec{L}_{12}$ and by a tensor potential proportional to $(\vec{\sigma}_1\cdot\vec{r}_{12}\,\vec{\sigma}_2\cdot\vec{r}_{12} - \frac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2 r_{12}^2)$ are indicated in Table III.

First, let us consider the $I = 1$ states. The A_2 meson [15-17] has now been resolved to show two peaks, at 1315 and 1275 MeV. Morrison [18] has given systematic reasons which argue that the 1275 MeV peak is associated with a spin-parity belonging to the series (0-), (1+), (2-),... (and therefore to some other configuration, with L = even), whereas the 1315 MeV peak corresponds to spin-parity (2+). This conclusion is compatible with the observation that the K_1K_1 decay mode is associated only with the upper peak. The mesons B(1240) and A_1(1080) are now well established and believed to have spin-parity (1+); $G = +1$ holds for the B-meson, since $B \to \pi\omega$, and $G = -1$ for the A_1 meson, since $A_1 \to \rho\pi$. Since we have

$$G = (-1)^{L+S+I} \tag{2.5}$$

for the $\bar{q} - q$ state $^{(2S+1)}L$ with isospin I, the B-meson corresponds to the configuration $^1P_1$, and the A_1 meson to the configuration $^3P_1$. The δ-meson

TABLE II. THE MESON NONETS CORRESPONDING TO THE CONFIGURATION $(n=1)^{(2S+1)}L_J$ FOR L = 0+ AND 1-

(I, Y) =	(1, 0)	(0, 0)	(0, 0)	($\frac{1}{2}$, +1)	($\frac{1}{2}$, -1)
$(JP)^C = (0-)^+$	π (140)	η (548)	η' (970)	K (495)	$\bar{K}$ (495)
$(1-)^-$	ρ (765)	ω(785)	ϕ(1020)	K*(891)	$\bar{K}$*(891)
$(0+)^+$	δ (962)	S (700)	S' (1070)	K*(~1100)?	$\bar{K}$*(~1100)?
$(1+)^+$	A_1(1070)	D(1285)	?	K*(1240)	$\bar{K}$*(1240)
$(1+)^-$	B(1240)	?	??	K*(1320)	$\bar{K}$*(1320)
$(2+)^+$	A_2(1315)	f (1260)	f' (1514)	K*(1420)	$\bar{K}$*(1420)

Note: The parity P is given by $-(-1)^L$, and C denotes the charge conjugation parity for the nonet, related with the configuration by $C=(-1)^{L+S}$. A question-mark ? denotes that no mesonic state has been established with the required quantum numbers; the double query ?? denotes that no mesonic state has been found with the appropriate quantum numbers, but also that the quark model does not strictly require such a mesonic state in this mass region.

TABLE III. EXPECTATION VALUES (IN RELATIVE MAGNITUDE, AS FUNCTION OF J) FOR A SPIN-ORBIT POTENTIAL AND A TENSOR POTENTIAL FOR THE STATES $(2S+1)_{L_J}$

State	$^3L_{L+1}$	^1L_L	^3L_L	$^3L_{L-1}$
Spin-orbit potential	-L	0	+1	L+1
Tensor potential	$\frac{-L}{(2L+3)}$	0	+1	$\frac{-(L+1)}{(2L-1)}$

is consistent with the spin-parity assignment (0+), corresponding to the configuration $^3P_0$, in view of the recent observation [19] of its decay mode $\delta \to \eta\pi$; the identification of $\pi(1016)$ as a $K\bar{K}$ s-wave threshold effect associated with the occurrence of the δ-meson not far below the $K\bar{K}$ threshold is consistent with this assignment. The $(\text{mass})^2$ values for these I = 1 mesons are consistent with a spin-orbit splitting. For the ^3P levels we have $[(1315)^2 - (1080)^2]/[(1080)^2 - (962)^2] = 2.3$ compared with the value 2 expected for L = 1. This ratio completely disagrees with the value -0.4 expected for a tensor interaction; the order of the ^3P levels is given incorrectly by the tensor interaction. The B-meson lies about 50 MeV higher than would be

expected if the $\bar{q}$ - q interaction had no spin or unitary-spin dependence (other than the spin-orbit interaction); in view of the depression of the π-mass relative to the ρ-mass, due to a spin-spin interaction most attractive in the S = 0 state, one would have expected the B-meson mass to have been pulled low by this same interaction, relative to the ^3P mass values.

The mesonic states established for these spin-parity values are listed in Table II. The (2+) nonet is completely known, and there are candidates (although poorly known) for all the states of the (0+) nonet. The $^3P_1$ nonet is not completely known; there is no D' state yet known. No unitary singlet states are known yet for the $^1P_1$ configuration (the H(990) meson which fitted this configuration is now in serious doubt [20]). If these states form an octet, the missing (I = 0, Y = 0) state should have mass value about 1350 MeV, with decay mode $\pi\rho$.

The simplest model for the SU(3)-breaking is the hypothesis that there is a definite addition Δ to the $(\text{mass})^2$, associated with the presence of a strange quark or antiquark in the $\bar{q}$ - q state. This quantity Δ gives rise to the mass difference between the $(I = \frac{1}{2},\ Y = \pm 1)$ states and the $(I = 1,\ Y = 0)$ state, thus

$$\Delta = m^2(I = \tfrac{1}{2},\ Y = \pm 1) - m^2(I = 1,\ Y = 0) \tag{2.6}$$

The values obtained for Δ are given in Table IV, for the mass values listed in Table II. The entries in brackets are uncertain because of the uncertainties concerning the mass values of the corresponding K^* mesons. However, Δ does appear to have approximately the same value for these nonets, as expected on the basis of this simple model for SU(3)-breaking.

We must mention briefly here the other two I = 1 states which have become established in this mass region:

(i) $A_{1.5}(1170)$ has been reported by Ascoli et al.[21] with excellent statistics, in an experiment which has also given excellent statistics on the A_1 and A_2 mesons. It appears to undergo the decay process $A_{1.5} \to \pi\rho$. It is conceivable that this meson could belong to the configuration $n = {}^1S_0$ for n = 2. This would permit its production by diffraction mechanisms, but there is really no evidence to indicate this assignment.

(ii) $A_2(1275)$ is the lower of the two peaks in the A_2 region. The evidence on the angular and energy dependence of its production cross-section suggests that it can be produced diffractively [18]; the direct evidence on the decay process $A_2(1275) \to \pi\rho$ suggests (2-) or possibly (1+). The former assignment would be consistent with the configuration $^1D_2$; there is no place in this model for the second spin-parity possibility.

<u>The L=2+ states.</u> The I = 1 mesons known in the R-meson region are as follows:

(i) $\rho_V(1650) \to 2\pi$, also known as the g-meson. The evidence available suggests spin-parity (3-), mainly on the basis of the narrowness of the forward peak in $\pi\pi$ scattering at this energy; however, the data do not really exclude spin-parity (1-).

(ii) $\pi_A(1640) \to 3\pi$ (probably including πf^0 and some $\pi\rho$).

(iii) $\rho(1700) \to \pi\omega$. Johnston et al.[22] have shown angular distribution evidence which indicates spin-parity (3-) for this state; however, they

TABLE IV. THE VALUE OF Δ (DEFINED BY EQ.(2.6)) FOR THE UNITARY MULTIPLETS CONTAINED IN THE L = 0+ AND L = 1- SUPERMULTIPLETS WITH PRINCIPAL QUANTUM NUMBER n = 1

Configuration (n = 1)	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_2$	$^3P_1$	$^3P_0$
Δ (GeV)^2	0.227	0.210	(0.20)	0.28	(0.37)	(0.28)

identify this state with the g-meson, which does not appear to be necessarily the case.

(iv) $\pi(1690) \to \rho\omega$ has been reported by Danysz et al.[23] in the mode $\omega\pi^+\pi^-$. However, this assignment to I = 1 depends crucially on the assignment of $\pi^+\pi^-$ to ρ^0 decay. This state could have I = 0.

The relationship of the R-meson peaks observed by Focacci et al.[13] with these resonances is quite unclear. The R_1 peak at 1650 MeV could correspond to $\rho_V(1650)$, or $\pi_A(1640)$, or both. The R_2 peak at 1720 MeV could correspond to $\rho(1700)$. The R_3 peak is at 1750 MeV. There is possibly an R_4 peak at 1830 MeV, but this is less certain.

The available configurations are $^1D_2$, for which G = -1, and $^3D_1$, $^3D_2$, $^3D_3$, for all of which G = +1. The state $\rho(1700)$ must be identified with the configuration $^3D_3$, and $\rho_V(1650)$ with the $^3D_1$ configuration. The state $\pi_A(1640)$ can only be identified with the $^1D_2$ configuration; this assignment could account for the prominence of the decay mode πf^0, since the spin-parity (2-) allows s-wave pion emission for this mode, while requiring p-wave pion emission for the mode $\pi\rho$. The configuration $^3D_2$ would then be expected to correspond to a mesonic resonance state between 1650 and 1700 MeV; this would allow 4π decay modes, including $\omega\pi$ and $\rho\rho$, as well as $\bar{K}K^*$ and $\bar{K}^*K$.

We should note again that the first radial excitations of the $^1S_0$ and $^3S_1$ states are expected to lie in the R-meson region and could account for one or more of these I = 1 resonant states. The n = 2 $^3S_1$ state could well mix with the n = 1 $^3D_1$ state, but only through a tensor $\bar{q}$ - q interaction.

The situation in the R-meson region does not add much weight to the evidence in favour of the quark model for the meson states. However, it does indicate the restrictiveness of the possibilities available and the importance of any firm determinations of the spin-parity values associated with the mesonic resonances in this mass region.

3. BARYONIC RESONANCES

The baryonic resonances are interpreted as bound states of the three-quark system qqq. The only unitary multiplets permitted for these configurations are singlet, octet and decuplet; the $\{\overline{10}\}$ and $\{27\}$ representations are permitted only with more complex configurations such as $(qqqq\bar{q})$. In fact no resonances have been established which require assignment to these representations. Despite considerable search, no resonant

states have been established for the K^+p or $\pi^+\Sigma^+$ systems, characteristic of the {27} representation; the situation is less clear for the K^+n and K^0p systems, or for the $\pi^-\Xi^-$ system, characteristic of the $\{\overline{10}\}$ representation.

SU(6) symmetry for the qqq system allows only the representations $\underline{56}$, $\underline{70}$ and $\underline{20}$ for the baryonic resonances. Their structure is given in Table V.

TABLE V. THE PERMUTATION SYMMETRY AND THE (SPIN, UNITARY SPIN) SUPERMULTIPLETS APPROPRIATE TO THE THREE SU(6) REPRESENTATIONS POSSIBLE FOR A qqq SYSTEM

SU(6) representation	Permutation symmetry	(S, {α}) content
$\underline{56}$	S (Symmetric)	$(\frac{3}{2}, \{10\}) + (\frac{1}{2}, \{8\})$
$\underline{70}$	M (Mixed)	$(\frac{3}{2}, \{8\}) + (\frac{1}{2}, \{10\})$ $+ (\frac{1}{2}, \{8\}) + (\frac{1}{2}, \{1\})$
$\underline{20}$	A (Antisymmetric)	$(\frac{3}{2}, \{1\}) + (\frac{1}{2}, \{8\})$

The nucleonic member of the decuplets is observed directly in π^+p total cross-section data. In Fig.2, $(\text{mass})^2$ has been plotted against linear order for the sequence of prominent resonance peaks known from the data, and these points are again well fitted by a straight line. The spin-parity has been determined directly for the first two resonances in this sequence, $(\frac{3}{2}+)$ for $\Delta(1236)$ and $(\frac{7}{2}+)$ for $\Delta(1950)$, and it is natural to interpret this line as the Δ-Regge trajectory, with spin-parities $(\frac{11}{2}+)$, $(\frac{15}{2}+)$ and $(\frac{19}{2}+)$ conjectured for the further states $\Delta(2420)$, $\Delta(2850)$ and $\Delta(3230)$. This situation would appear natural if the wave equation involves a linear relation between E^2 and simple harmonic q-q potentials, as was also the case for the $\bar{q}$-q system.

To each unitary octet, there is one N^* state; to each unitary decuplet there is one Δ^* state. Each unitary singlet consists of one Y_0^* state. In Fig.3, all nucleonic resonances have been plotted, for all of which there are definite spin-parity determinations. Most of these resonances have been obtained by phase shift analysis of πN scattering and polarization data, through the work of Donnachie et al.[24]. To indicate the location of the unitary singlets on this plot, we have somewhat arbitrarily reduced each singlet Y_0^* mass by about 150 MeV, to allow for the general increase in mass with decreasing hypercharge Y for the states of a given unitary multiplet. In this figure, the states with positive and negative parity are plotted separately, and we notice that the states appear to show some grouping into clusters, with an alternating parity with increasing mean mass for the cluster. The interpretation of this feature of the plot will be considered below.

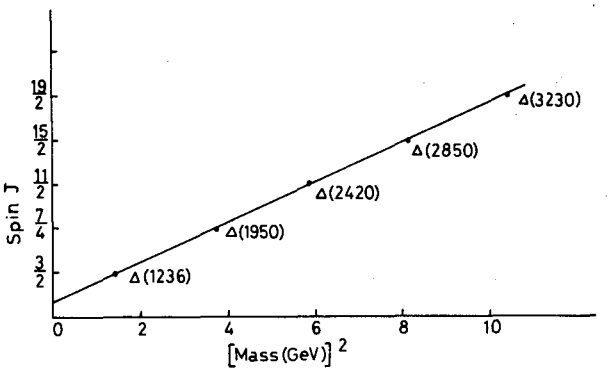

FIG. 2. Plot of (mass)^2 for all the prominent resonances found in the π^+p total cross-section as function of incident pion energy, against the presumed spin-value (measured only for Δ (1236) and Δ (1950)) for each resonance. The presumption at present is that this plot actually shows the Regge trajectory associated with the Δ(1236) resonance.

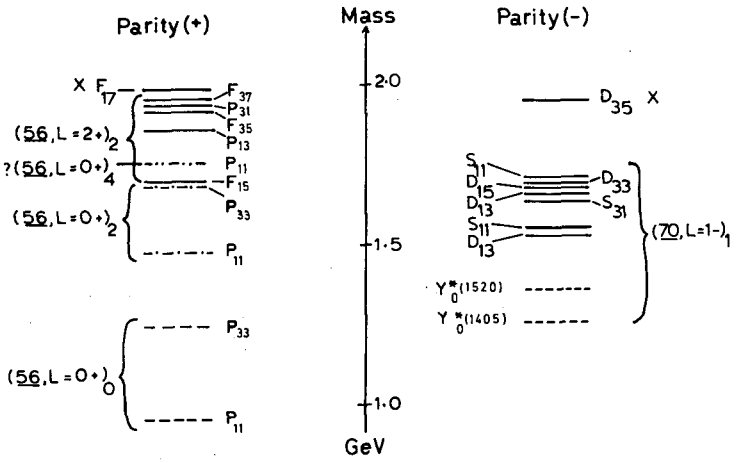

FIG. 3. All known nucleonic resonances below mass GeV, where the notation is $L_{2I,\,2J}$, denoting the orbital angular momentum (S, P, D, F, ..) of the πN channel appropriate to each resonance state, I and J being the isospin and total spin of the resonant state. Each $I = \frac{1}{2}$ resonance corresponds to a baryonic octet, and each $I = \frac{3}{2}$ resonance corresponds to a baryonic decuplet; in order to display the unitary singlet states (Y_0^*), they are plotted with an arbitrary reduction in their mass value by 150 MeV.

It would be natural to suppose that the qqq wavefunction should be totally antisymmetric in the quark labels, corresponding to Fermi statistics for quarks. However, this leads to some difficulties, since the fact is that the lowest SU(6) supermultiplet corresponds to a $\underline{56}$ representation with orbital angular momentum L = 0+, consisting of the ($\frac{1}{2}$+) baryonic rotet and the ($\frac{3}{2}$+) baryonic decuplet. As indicated in Table V, the $\underline{56}$ representation has permutation symmetry S, so that Fermi statistics for quarks requires the ground-state to have an L = 0+ space wavefunction which is totally antisymmetric. This requirement could probably be met if the qq forces were dominated by a space-exchange term, such that they were strongly repulsive

in s-states and attractive in p-states. With shell-model wavefunctions (appropriate with two-body forces of harmonic form), the lowest L = 0+ antisymmetric state comes from a relatively complex configuration given by (1s)(2s)(3s). As a result it is not clear that, with such forces, the antisymmetric space wavefunction with lowest energy would necessarily have L = 0+; an antisymmetric space wavefunction with L = 1+ exists in the simpler configuration $(1s)(1p)^2$. Further, the body form-factor for an L = 0+ antisymmetric space wavefunction necessarily has at least one node. If the radial scale of the qq harmonic potential is adjusted to give the first Regge recurrence of the ($\frac{3}{2}$+) ground-state Δ(1236) as the observed ($\frac{7}{2}$+) state Δ(1950) (i.e. to give the observed slope for the Regge trajectory in Fig.2), this fixes the free parameter in the shell-model wavefunctions and so the position of the node in the body form factor, at $q^2 = 107\ \mathrm{fm}^{-2}$. No node has been found in the electromagnetic form factors for the nucleons. The proton magnetic structure is known best, and the magnetic form factor is well known to be non-zero for momentum transfers up to 500 fm^{-2}; the charge form factor is known only up to about 50 fm^{-2}.

Following Greenberg [25], we therefore consider the opposite possibility, that the qqq wavefunction should be totally symmetric in the quark labels. The ground supermultiplet then corresponds to the lowest shell-model configuration, $(1s)^3$; this configuration will continue to lie lowest with the introduction of ordinary qq forces, attractive in relative s-states. The $(1s)^3$ space wavefunction is totally symmetric, of course, so that the requirement of total symmetry for the complete wavefunction leads to the expectation that the ground-state should correspond to an SU(6) representation $\underline{56}$, as is observed to be the case.

The first excited configuration is then $(1s)^2(1p)$. The symmetric part of this space wavefunction is a spurious state (i.e. it describes only p-state centre-of-mass motion for the $(1s)^3$ configuration), and there is no antisymmetric part. This leads to the prediction that the first excited supermultiplet has L = 1- and M-symmetry. From Table VI, the requirement that the product of the space wavefunction and the SU(6) wavefunction should have S-symmetry then implies M-symmetry for the SU(6) wavefunction, that is, according to Table V, that the first excited supermultiplet corresponds to a $\underline{70}$ representation. The (spin, isospin) multiplets for a (70, L = 1-) supermultiplet are displayed in Fig.4, as they may be split by the SU(6)-breaking interactions possible. We emphasize that the positions shown there for these sub-multiplets are not the result of calculation of the SU(6)-breaking effects, but are simply given by the phenomenological identification of these sub-multiplets from the resonant states indicated by the analysis of the pion-nucleon scattering data (and from the study of Y_0^* resonances, for the case of the unitary singlet states). The sequence of ^2P unitary multiplets is in accord with the expectation for central SU(6)-breaking qq interactions that the octet states should lie midway between the decuplet and singlet states, but the detailed pattern of spin-orbit splittings is not understood at present. In particular, within SU(3) symmetry, the splitting between the two decuplet states cannot result from a qq spin-orbit force, nor from a qq tensor force (unless coupling to more complicated configurations is included); for this reason, Greenberg and Resnikoff [26] have suggested the introduction of a two-body spin-orbit interaction which violates SU(3)-symmetry. However, N^*, Δ^* and Y_0^* states are known which correspond to each of the SU(3)-multiplets expected for

TABLE VI. THE PERMUTATION SYMMETRY FOR THE PRODUCT $\phi \times \psi$ OF TWO FUNCTIONS $\phi(1,2,3)$ AND $\psi(1,2,3)$ FOR ALL POSSIBLE PERMUTATION SYMMETRIES FOR $\phi(1,2,3)$ AND $\psi^{-}(1,2,3)$

$\psi =$	S	M	A
ϕ = S	S	M	A
ϕ = M	M	S + M + A	M
ϕ = A	A	M	S

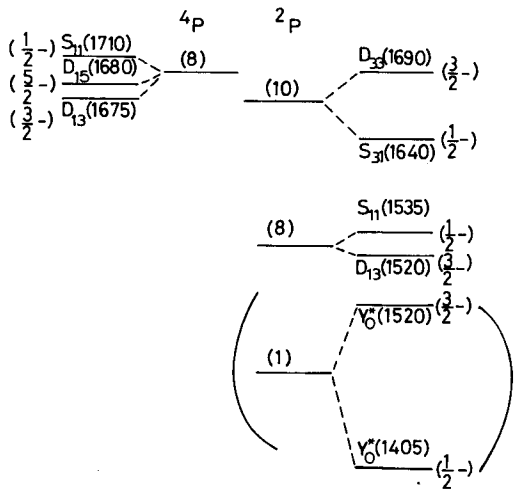

FIG.4. Pattern of unitary multiplets shown for the supermultiplet ($\underline{70}$, L = 1-). The nucleonic member of each multiplet is indicated, and the two Y_0^* states which are SU(3) singlets are shown shifted downwards by about 150 MeV (to take rough account of the Y-dependence of masses within a unitary multiplet) for proper comparison.

the ($\underline{70}$, L = 1-) supermultiplet, and there are no baryonic resonances with negative parity known below 1950 MeV (where there is some evidence for a $D_{35}\Delta^*$ resonance) which do not fit into this supermultiplet.

The N = 2 excited configurations are listed in Table VII, together with the five supermultiplets they contain. With qq forces which are strongly attractive in the s-state, the lowest supermultiplet from a given configuration is expected to be that with the greatest symmetry possible for its space wavefunction, that is the $\underline{56}$ representation, and for given symmetry, that with the lowest orbital angular momentum. Hence we expect the next two supermultiplets to be ($\underline{56}$, L = 0+)$_2$ and ($\underline{56}$, L = 2+)$_2$, where the suffix denotes the value of N. This is in good accord with the facts known. The supermultiplet ($\underline{56}$, L = 2+)$_2$ is shown in Fig.5, with the identification for the nucleonic member of each unitary multiplet; the ^{4}D {10} states lie

close together, so that it is reasonable to expect the missing P_{33} state also to lie in the mass region 1900-1950 MeV. The supermultiplet ($\underline{56}$, $L = 0+)_2$ lies lower, as expected, comparable in mass with the supermultiplet ($\underline{70}$, $L = 1-)_1$, and represents a radial excitation of the ground supermultiplet ($\underline{56}$, $L = 0+)_0$.

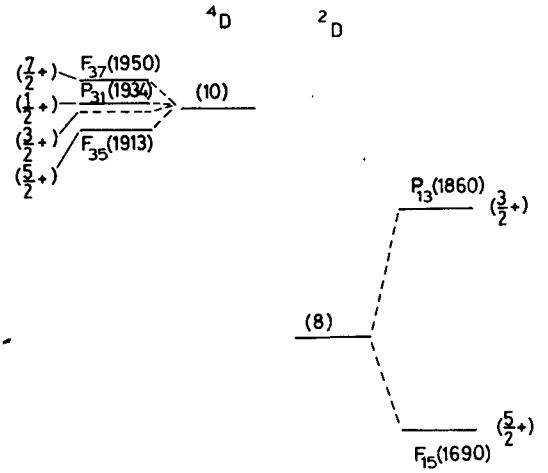

FIG.5. Pattern of unitary multiplets for the supermultiplet ($\underline{56}$, $L = 2+)_2$. The only missing nucleonic state at present is a P_{33} state at about 1900-1950 MeV.

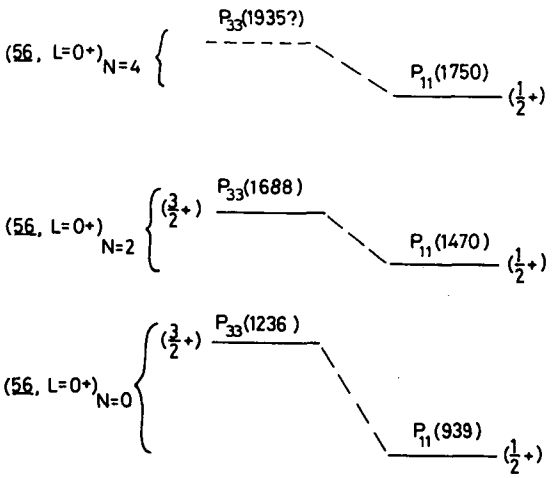

FIG.6. Pattern of unitary multiplets for the series of radially-excited states ($\underline{56}$, $L = 0+)_N$, where N denotes the number of quanta of excitation. The nucleonic member of each unitary multiplet is indicated for N = 0, 2, and 4; the only missing nucleonic state at present is a P_{33} state predicted to be at about 1935 MeV.

TABLE VII. N = 2 EXCITED CONFIGURATIONS AND SUPERMULTIPLETS CONTAINED IN THEM

Quanta of excitation	Shell-model configurations	Supermultiplets ($\underline{a}$, LP) contained
N = 0	$(1s)^3$	($\underline{56}$, 0+)
N = 1	$(1s)^2(1p)$	($\underline{70}$, 1-)
N = 2	$(1s)^2(2s)$	($\underline{56}$, 0+), ($\underline{56}$, 2+),
	$(1s)^2(1d)$	($\underline{70}$, 0+), ($\underline{70}$, 2+), and
	$(1s)(1p)^2$	($\underline{20}$, 1+)
N = 3	$(1s)^2(1f)$	($\underline{56}$, 3-), ($\underline{56}$, 1-),
	$(1s)^2(2p)$	($\underline{70}$, 3-), ($\underline{70}$, 2-), ($\underline{70}$, 1-) twice
	$(1s)(1p)(1d)$	
	$(1s)(1p)(2s)$	($\underline{20}$, 3-), and ($\underline{20}$, 1-)
	$(1p)^3$	

Note: The shell-model configurations and the SU(6) × O(3) supermultiplets to which they give rise are listed for excitation energies up to three quanta. In addition, we note here that there are 17 supermultiplets generated by the N = 4 configurations; these include two supermultiplets $(\underline{56}, 0+)_4$, and one supermultiplet $(\underline{56}, 4+)_4$, whose wavefunctions are given by Karl and Obryk [59].

There is one further state with positive parity in this mass region, the $P_{11}(1750)$ state. Here we are inclined to attribute this to the lowest N = 4 supermultiplet, the $(\underline{56}, L = 0+)_4$, which represents the second radial excitation of the ground supermultiplet; the spacing in $(\text{mass})^2$ between $P_{11}(1750)$ and $P_{11}(1470)$ is comparable with that between $P_{11}(1470)$ and $P_{11}(939)$. The spacing in $(\text{mass})^2$ between $P_{33}(1688)$ and $P_{11}(1470)$ is also comparable with that between $P_{33}(1236)$ and $P_{11}(939)$; assuming a comparable value for the N = 4 states, we are led to expect a further P_{33} resonance at about 1935 MeV. This series of radial excitations for the supermultiplets ($\underline{56}$, L = 0+) are illustrated in Fig.6.

These five supermultiplets account for all nucleonic states known in the mass region closely explored (and for all their known hyperonic counterparts) except for the two levels F_{17} and D_{35} close to 2000 MeV, the upper limit of this region. The F_{17} state could be the lowest state of the next N = 2 supermultiplet $(\underline{70}, L = 2+)_2$, and the D_{35} state could belong to the next supermultiplet with negative parity $(\underline{70}, L = 3-)_3$. There are two positive parity states missing at present, both of them P_{33} states in the mass region 1900-1950 MeV. More detailed πN scattering data should be obtained for this mass region in order to search for these states.

The comparison of these patterns with the data does suggest that the superstrong forces do satisfy SU(6) symmetry in some sense, perhaps in the sense of the non-relativistic quark model described above. From these comparisons we can make some comments concerning the magnitude of the various SU(6)-breaking interactions:

(i) The "spin-spin" and "unitary spin-unitary spin" forces give mass-splitting contributions of order 300 MeV. The latter give rise to the mass splitting of about 150 MeV between decuplet and octet ^2P states; together, they give rise to the decuplet-octet mass splitting of 200-300 MeV for the N = 0 S-states.

(ii) The spin-orbit interactions contribute mass-splittings of order 100 MeV. The mass splitting observed for the ^2P singlet states is 115 MeV; that for the ^2D states is about 170 MeV.

(iii) The SU(3)-breaking interactions contribute mass-splittings of order 100-200 MeV. In a given SU(3) multiplet, the leading mass-splitting term is of the form (-aY), where a is positive. For N = 0 states, $a \approx 150$ MeV holds for the decuplet, and $a \approx 190$ MeV for the octet. For the N = 1 states, $a \approx 135$ MeV is indicated by mass values known for the $(\frac{5}{2}+)$ octet states, whereas $a \approx 80$ MeV is indicated by the $(\frac{7}{2}+)$ decuplet states $\Delta^*(1950)$ and $Y_1^*(2030)$.

We recall that, for the states in the 1000-2000 MeV range, a typical quantum of vibrational energy amounts to about 500 MeV (from the comparison of $P_{11}(1470)$ with $P_{11}(939)$) and a typical quantum of rotational energy amounts to about 400 MeV (from the comparison of $(\underline{56}, L = 2+)_2$ with $(\underline{56}, L = 0+)_0$).

We note that the SU(6)-breaking interactions are all of the same order of magnitude; in particular, the SU(3)-breaking interactions do not appear to be of a smaller order of magnitude than the SU(3)-invariant SU(6)-breaking terms. Further, the SU(6)-breaking interactions appear to be not much smaller than the vibrational energies. This means that quite strong mixing could occur between many sub-states from different configurations with the same parity, which would lead to the possibility of a very complicated physical situation for analysis. Such mixing effects might not disturb the mass patterns for the supermultiplets too seriously, but they could have a strong effect on the branching ratios for the various decay modes of the sub-states for a given supermultiplet or unitary multiplet (e.g. from the mass spectrum, $Y_0^*(1520)$ is clearly a unitary singlet state, yet the branching ratio $\bar{K}N/\pi\Sigma$ for its decay differs significantly from the ratio expected for a pure unitary singlet).

These assignments to supermultiplets may be checked in due course by the study of transitions between the states, of the following kinds:

(i) γ-transitions, corresponding to the photo-excitation processes $\gamma N \to N^*$ or Δ^*, or to radiative decay processes such as $Y_1^* \to \Lambda + \gamma$, or $N^* \to \Delta(1236) + \gamma$. Moorhouse [27] has pointed out one photo-excitation process which is forbidden in the quark model, namely $\gamma p \to N^*(\frac{5}{2}-, 1680)^+$; the experimental data show that this transition does have small cross-section, about several per cent of the cross-section observed for the photo-excitation process $\gamma p \to N^*(\frac{5}{2}+, 1688)^+$.

(ii) Neutrino-excitation processes, such as

$$\bar{\nu} + p \to \mu^+ (\text{or } e^+) + N^{*0} (\text{or } \Delta^{*0}, \text{ or } Y_1^{*0}) \tag{3.1a}$$

$$\nu + p \to \mu^- (\text{or } e^-) + \Delta^{*++} \tag{3.1b}$$

and the corresponding reactions on bound neutrons (in deuterium targets, for example). It is believed that the reaction (3.1b) has been seen for the production of $\Delta(1236)$, but accurate information will await the use of hydrogen targets in neutrino beams.

(iii) Strong decay processes, involving pion (or pseudoscalar meson) emission. The calculation of these transition amplitudes depends on the use of a crude model invoking a phenomenological interaction amplitude $q \to q + \pi$ of the form $G_q \vec{\sigma} \cdot \vec{\rho}_\pi \vec{\tau} \cdot \vec{\phi}_\pi$ (even though the pseudoscalar mesons are considered to be $\bar{q}$ - q bound states). Lipkin [28] has pointed out that most of the amplitudes calculated in this way are connected by Clebsch-Gordan coefficients of the $SU(6)_W$ group (this is not the case for some s-wave amplitudes whose main contribution comes from a quark-recoil correction). Calculations of this kind and their comparison with experiment have been given in a phenomenological spirit (as concerns the space wavefunctions) by Mitra and Ross [29] for the $(70, L = 1-)_1$ states, by Katyal and Mitra [30] for the $(\underline{56}, L = 2+)_2$ states, and specifically for the shell-model wavefunctions described above, by Faiman and Hendry [31] for comparison with all the $N^* \to N\pi$ and $\Delta^* \to N\pi$ partial decay widths known from the πN phase-shift analyses. A qualitatively good fit to the data is obtained in this way (we recall that mixing of states could often cause quite large deviations in the partial widths). For example, the partial width $\Gamma_{\pi N}$ for P_{11} (1470) is calculated by Faiman and Hendry [31] to be 130 MeV if this resonance is assigned to the supermultiplet $(\underline{56}, L = 0+)_2$, 40 MeV if it is assigned to $(\underline{70}, L = 0+)_2$, or 4 MeV if it is assigned to $(\underline{70}, L = 2+)_2$, these being the only possibilities available; the experimental value $\Gamma_{\pi N} = 140$ MeV thus confirms the assignment of $P_{11}(1470)$ to the radially excited supermultiplet $(\underline{56}, L = 0+)_2$.

According to this quark model and to Table VII, the number of baryonic resonance supermultiplets will increase very rapidly as the mass range explored moves upward from that where detailed angular distribution and polarization data are already available. The determination of so many nucleonic resonances from phase-shift analyses will require data which are more accurate and much more detailed (especially with much closer and more complete coverage of scattering angles, as well as at closer energy intervals) than those available in the lower mass region. We cannot expect these higher supermultiplets to be disentangled and identified very quickly.

4. HIGH-ENERGY PHENOMENA

Soon after the first proposal that SU(6) symmetry may hold for hadronic interactions, Johnson and Treiman [32] applied the formalism to derive some relations between forward scattering amplitudes for various meson-baryon collision processes. In view of the optical theorem that the total cross-section is proportional to the imaginary part of the forward scattering amplitude

$$\sigma_{tot} = 4\pi \, \mathrm{Im} \, f_{el}(0°)/k \tag{4.1}$$

these relations are tested most precisely by comparison of the measured total cross-sections, since rather accurate measurements have been made

for these in recent years. The two relations by Johnson and Treiman may be written in the form

$$\tfrac{1}{3}(\sigma(K^- p) + 2\sigma(\pi^+ p)) = \tfrac{1}{3}(\sigma(K^+ p) + 2\sigma(\pi^- p)) \tag{4.2a}$$

$$\tfrac{1}{3}(\sigma(K^- p) + 2\sigma(K^+ n)) = \tfrac{1}{3}(\sigma(K^+ p) + 2\sigma(K^- n)) \tag{4.2b}$$

As shown in Fig.7, the cross-section data available give good agreement with these relations, the equalities being satisfied above 10 GeV/c laboratory momentum to better than 2% accuracy.

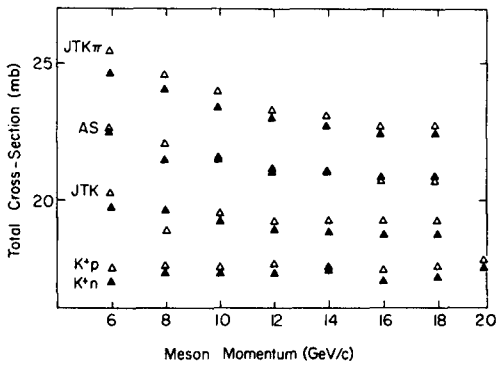

FIG.7. Experimental data on meson-baryon total cross-sections up to 20 GeV/c used for the comparison between the left-hand (open triangle) and right-hand (solid triangle) sides of the equalities predicted for high-energy collisions by the quark model, together with the additivity assumption. The notations are: JTKπ = the Johnson-Treiman relation (4.2a), JTK = the Johnson-Treiman relation (4.2b), and AS = anti-symmetric sum rule (4.3). The K^+p and K^+n cross-sections are also given, for comparison with the equality (4.6).

In view of our present understanding of SU(6) symmetry, such excellent agreement is puzzling, since we do not expect SU(6) symmetry to hold for the superstrong interactions in such relativistic situations. However, it was pointed out by Levin and Frankfurt [33] and by Lipkin and Scheck [35] that these relations could be obtained from the following assumptions, consistent with our present view about the validity of SU(6) symmetry: (i) In their respective centre-of-mass frames, the mesons and baryons are $q\bar{q}$ and qqq bound states respectively, with the structure required by their assignment to the 35- and 56-dimensional representations of SU(6) symmetry, respectively; (ii) The scattering amplitude of interest is obtained by adding the elastic qq and $\bar{q}q$ scattering amplitudes for the interaction of each quark (or antiquark) of the incident particle with each quark (or antiquark) of the target particle, which is the assumption of additivity for the qq and $\bar{q}q$ elastic scattering amplitudes. These assumptions alone already lead to the following "antisymmetric sum rule"

$$\tfrac{1}{3}(\sigma(K^- p) + \sigma(\pi^+ p) + \sigma(K^0 p)) = \tfrac{1}{3}(\sigma(K^+ p) + \sigma(\pi^- p) + \sigma(\bar{K}^0 p)) \tag{4.3}$$

which is very well satisfied above 10 GeV/c (see Fig.7). The addition of SU(3) symmetry for the qq and $\bar{q}q$ amplitudes then leads directly to the relations (4.2) and a "symmetric sum rule"

$$\tfrac{1}{4}(\sigma(\pi^+p)+\sigma(\pi^-p)+\sigma(K^-n)+\sigma(K^+n)) = \tfrac{1}{2}(\sigma(K^-p)+\sigma(K^+p)) \tag{4.4}$$

This relation is satisfied only to an accuracy of typically 15%. For example, at 18 GeV/c, the left-hand side has the value 21.6 ± 0.3 mb, whereas the right-hand side has the value 19.0 ± 0.4 mb. It is not surprising that requirements of SU(3)-symmetry should be violated to this degree. What is surprising is that the Johnson-Treiman relations should be so well satisfied, on the basis of this derivation, which also requires SU(3)-symmetry.

Lipkin [36] has pointed out that these and other high-energy relations between cross-sections can be derived with an additional assumption, without assuming SU(3)-symmetry: the assumption of asymptotic behaviour for the $\bar{q}q$ and qq elastic amplitudes in the Pomeranchuk sense. For the non-strange quarks, this assumption supposes that the energy is sufficiently high, so that all elastic amplitudes (excluding perhaps $(\bar{p}p)$ and $(\bar{n}n)$, which allow an intermediate I = 0 annihilation state) are equal, for example,

$$f(pp) = f(nn) = f(\bar{p}n) = P \tag{4.5}$$

With this assumption (and $f(\lambda p) = f(\lambda n)$, from charge independence), it follows that

$$\sigma(K^+p) = \sigma(K^+n) \tag{4.6}$$

This relation then holds independent of SU(3)-breaking, and it is well satisfied experimentally, to better than 2% accuracy above 10 GeV/c. This gives us some confidence in the appropriateness of the assumption of asymptotic behaviour. For the strange quark, the assumption of asymptotic behaviour adds the further equality

$$f(\lambda p) = f(\bar{\lambda}p) = P + S \tag{4.7}$$

but with a value for the amplitude which will generally differ from Eq.(4.5) owing to SU(3)-breaking effects. With the relations (4.5) and (4.7), the Johnson-Treiman equalities then follow, now without the assumption of SU(3)-symmetry. However, the symmetric sum rule (4.4) does not follow until the assumption of SU(3)-symmetry is made, that S = 0.

The quark model also leads to the prediction of relationships between the cross-sections for quite different processes. For example, the asymptotic assumption leads to the following relation

$$\sigma(pp) = \tfrac{3}{2}\sigma(\pi^+p) \tag{4.8}$$

This equality is not satisfied so well. The cross-section $\sigma(pp)$ is 38.5 ± 0.5 mb at 20 GeV/c, so that Eq.(4.8) predicts 25.7 ± 0.3 mb for $\sigma(\pi^+p)$,

whereas the measured values for $\pi^{\pm}p$ scattering at 20 GeV/c are $\sigma(\pi^+p) = 23.5 \pm 0.2$ mb, and $\sigma(\pi^-p) = 25.0 \pm 0.3$ mb.

With this model for hadronic collisions at high energies, there will be corrections to all the equalities discussed above, arising from multiple interactions between the quarks within the two colliding hadrons. These have been considered for the forward elastic scattering amplitudes by a number of authors, following the eikonal treatment for multiple scattering at high energies which has been developed particularly by Glauber [38]. The second-order corrections to the quantities on the right-hand or left-hand sides of these equalities are calculated to be quite small by most authors, generally of the order of several per cent (see, however, Ref.[39]); this follows from the fact that the qq cross-section implied by the observed NN and πN cross-sections is rather small, given by $\sigma_{tot}(qq) \approx (1/9)\sigma(pp) = 4.3$ mb.

For finite-angle hadron-hadron elastic scattering, the first-order theory (with additivity) leads to a momentum-transfer dependence

$$\sigma(t) = \sigma_{qq}(t)(F_1(t))^2(F_2(t))^2 \tag{4.9}$$

where $\sigma_{qq}(t)$ describes the intrinsic momentum-transfer dependence of the qq elastic scattering cross-section and $F_i(t)$ denotes the form-factor for the quark distribution within the hadron i. As we know well, the differential cross-sections $\sigma(t)$ all fall rapidly with increasing momentum transfer (t) GeV/c; for pp scattering at the highest energies, the cross-section falls initially like exp (-Bt), with $B \approx 10$ $(\text{GeV})^{-2}$. In this situation it will be advantageous, for large momentum transfer, for the total momentum transfer to be shared between two (or more) collisions between the constituents of each hadron. This situation is already well known for nuclear systems, for example in the differential cross-section for the collision of high-energy protons by helium nuclei [40-42] (Fig.8). In this case, the first-order scattering has amplitude $A_1 \exp(-Bt)$, the second-order scattering has (roughly) amplitude $A_2 \exp(-\frac{1}{2}Bt)$ with A_2 much smaller than A_1, the third-order scattering has (roughly) amplitude $A_3 \exp(-\frac{1}{3}Bt)$, and so on, giving total scattering amplitude

$$A(t) = A_1 \exp(-Bt) + A_2 \exp(-\tfrac{1}{2}Bt) + A_3 \exp(-\tfrac{1}{3}Bt) + .. \tag{4.10}$$

up to the greatest multiplicity possible for the successive internal scattering (given by n = 4 for $p-{}^4He$ scattering). This leads to a series of breaks in the slope of the elastic scattering cross-section as function of t, the first break occurring for the momentum transfer at which the second-order amplitude becomes the dominant term of Eq.(4.10), the second break occurring for the momentum transfer at which the third-order amplitude becomes dominant, and so on.

A somewhat similar series of breaks is known in the pp differential cross-section [43-45] (Fig.9) at high energies and it has been pointed out by Harrington and Pagnamenta [46] that such breaks are to be expected from the multiple-scattering mechanism just described if the hadrons are bound states of quarks and antiquarks. In this case the detailed form of the multiple-scattering series is more complicated than the schematic form given in Eq.(4.10), and there are free parameters not otherwise determined, since only $\sigma(t)$ (cf.Eq.(4.9)) is known, but not $\sigma_{qq}(t)$ and $F_i(t)$ separately.

However, for pp scattering, Harrington and Pagnamenta show that the ratio of slopes at the first break is not sensitive to this free parameter, the calculated ratio varying from 0.3 to 0.4 over the complete range, to be compared with the empirical ratio 0.35; similarly the calculated ratio between the differential cross-section at the break and the forward cross-section only varies from 2.8×10^{-3} to 3.6×10^{-3}, where the experimental

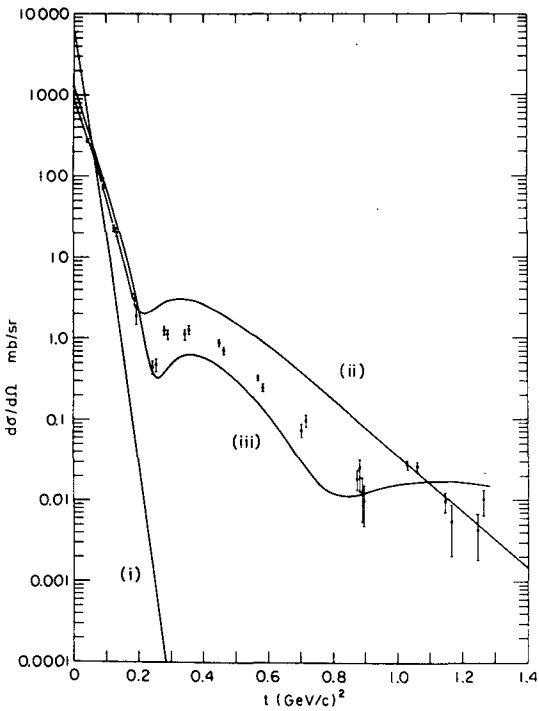

FIG.8. Differential cross-section for p-^{4}He elastic scattering as measured by Palevsky et al. [40] for proton laboratory momentum 1 GeV/c plotted as function of momentum transfer t. The solid curves shown were calculated by Bassel and Wilkin [41] for a representative set of pp and pn scattering parameters. These curves represent the following cases: (i) the first-order scattering, (ii) the cross-section obtained from the sum of the first- and second-order amplitudes, (iii) the cross-section obtained from the sum of the first-, second-, and third-order scattering amplitudes. Curve (iii) shows two minima, analogous to diffraction minima. In the intervals before, between, and beyond these minima, it will be seen that the mean slope has the general character indicated by Eq. (4.10).

ratio is 3×10^{-3}. It is not yet known whether this simple quark model can also account for the higher breaks in the pp differential cross-section, and for the structure found in the πp differential cross-sections. The present calculations do require a rather large real part for the pp forward scattering amplitude A(0), in order to give a smooth transition in slope across the break (rather than a strong diffraction-like minimum), and this does not appear compatible with the pp scattering measurements in the small-angle Coulomb scattering region. However, the qualitative similarity of the structure observed for the p-^{4}He and pp differential cross-sections is

rather suggestive of the existence of substructure for the proton, just as there is nucleonic substructure for ^{4}He, but it must be remembered that quite different interpretations are possible for the structure observed in the pp and πp differential cross-sections [43, 44].

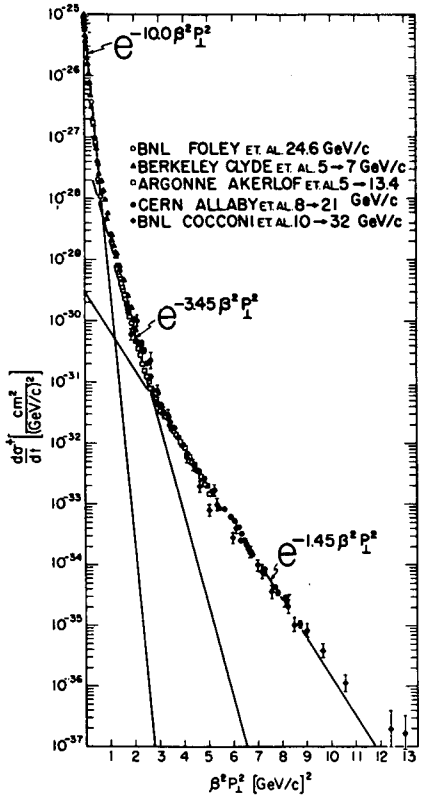

FIG. 9. All differential cross-sections measured for pp elastic scattering, combined on a single plot by Krisch [44], by choice of the ordinate $\beta^2 P_\perp^2$. This parameter has no special significance known and is related to the momentum transfer t by the equation: $\beta^2 P_\perp^2 = t(1 - t^2/4p^2)p^2/E^2$. This curve is well fitted by the sum of three Gaussians. The feature of interest for us here is the appearance of two fairly well-defined breaks in the slope of the curve, at t about 0.6 and 2.7 $(\mathrm{GeV/c})^2$, respectively.

5. PROBLEMS ABOUT THE QUARK MODEL

To conclude, we shall draw attention briefly to a number of problems faced by the quark model for hadronic states.

5.1. Where are free quarks?

Despite considerable search in accelerator experiments, in cosmic ray experiments, and in the analysis of matter, no direct observations have yet been reported which indicate the existence of a free quark. No doubt the quarks are not to be regarded as elementary in any sense; in

consequence of their superstrong interactions they would necessarily have structure. The bound quarks considered above as hadronic sub-units presumably represent some particularly simple mode of excitation of the underlying hadronic stuff. However, if such quark excitations can exist in bound state systems of this hadronic stuff, it would be difficult to understand why there should not also exist a similar excitation of the hadronic stuff outside the bound states. If the bound states behave as if they were composed of quarks, it is the most natural supposition that such quarks should also exist in the free state; to suppose otherwise is not impossible, but it certainly represents some rather contrived thinking.

The experimental situation is summarized in Table VIII, based on Ref.[47]. In due course, the third kind of search listed there promises the greatest sensitivity, since at least the lightest of the quarks must be completely stable [48]. One problem is that the chemistry of quarks is uncertain. Where should one search for quarks? Where should one expect quarks to accumulate? (It is clear that a large fraction of the quarks generated in cosmic ray collisions will end up in the oceans, where the immediate fate of negatively-charged quarks (or antiquarks) will be cap-

TABLE VIII. SUMMARY OF THE QUARK SEARCHES REPORTED BY THE END OF 1967, TAKEN FROM REF.[47]

Technique	Quantity measured	Results available
Accelerators	$\sigma_{prod.}$ (quark) in hadronic collisions up to 30 GeV/c	$\sigma(q) \leqslant 10^{-35}$ cm^2 (corresponds to mass limit $M_q > 5$ GeV, with a conservative estimate for $\sigma(q)$ as function of M_q)
Cosmic rays	Quark flux measured (random in time; at various altitudes) for particles of charge $\left\|\frac{e}{3}\right\|$ or $\left\|\frac{2e}{3}\right\|$	$\phi(q) \leqslant 10^{-10}$ cm^{-2} sr^{-1} sec^{-1} (corresponds to a limit of $\sigma(q) \leqslant 10^{-32}$ cm^2 for $M_1 \approx 10$ GeV, this limit varying roughly as $M_q^{-3.4}$, as discussed in Ref.[48])
Residual charge of matter on earth, sun and meteorites	N(q), number of quarks per nucleon	$<10^{-15}$, in helium, using charge and mass spectrometer $<5 \times 10^{-19}$, in graphite $<10^{-17}$, in iron meteorites $<5 \times 10^{-27}$, in air[a] $<5 \times 10^{-29}$, in sea water[a] $<10^{-10}$, in solar atmosphere

[a]These two limits depend completely on uncertain assumptions about surface chemistry in relation to quarks.

ture into the oxygen atoms of the sea water.) By what means can one try to concentrate the quarks, that is to enrich the quark content of a particular piece of matter by a known factor?

Otherwise, the most hopeful experiments in the near future are those with cosmic rays, especially the searches for quarks in coincidence (or with some characteristic time delay) with the arrival of very large air showers. If the quark mass M_q is less than 20 GeV, then the greatest hope for a clean and penetrating search for quark production is likely to be in proton-proton interactions in the Intersecting Storage Rings now under construction at CERN. However, we must be prepared for the possibility that the quark production cross-sections may be rather low. Statistical model calculations by Chernavsky et al. [49] and by Hagedorn [50] certainly indicate production cross-sections of the order 10^{-36} cm^2 for $M_q = 2$ GeV, and falling very rapidly with increasing mass M_q (by a factor of 10^{-5} for each additional GeV of mass). On the other hand, if quarks are already present in protons and mesons, one might expect that these could be released much more readily by knock-out processes when the initial energy is sufficiently far above the threshold for disintegration of the hadron into quarks.

5.2. Quark statistics

The assumption that the qqq wavefunction for baryonic states is symmetric in the quark labels leads very directly to a successful account of the baryonic supermultiplets, both regarding their configurations and their order of occurrence with increasing mass. The simplest interpretation, namely that quarks obey Bose statistics, is certainly excluded, because (i) the spin-statistics theorem, one of the deepest theorems in quantum field theory, forbids the possibility of Bose statistics for any particle with half-integral spin, and (ii) the exchange of two protons corresponds to the interchange of each quark of one proton with a corresponding quark of the other proton. Bose statistics for the quarks would then imply Bose statistics for protons, which is certainly not the case.

This situation appears to require that there should exist some other degree of freedom in the specification of a quark state. This additional variable can be introduced either (i) implicitly, as in the parastatistics scheme [52], in such a way that it cannot be directly observed, or (ii) explicitly, together with some constraint on the Hamiltonian, such that this additional variable would only become apparent for states of high excitation energy, beyond the mass region explored in detail to date. The scheme of Han and Nambu [10] provides an example of this latter possibility.

The parafermi statistics scheme (originally proposed by Green [51] and then applied to quarks by Greenberg [52]) is equivalent to describing the quarks by creation operators

$$q^{\dagger}_{\alpha} = \sum_{i=1}^{p} b^{i\dagger}_{\alpha} \tag{5.1}$$

where p is called the order of the parafermi statistics. Each field operator b^{i}_{α} obeys anticommutation relations with respect to field variables with the

same i, but obeys commutation relation with respect to field variables with $j \neq i$. It is postulated that all interactions involve the quark fields only in the combination $q^\dagger$(or q); this ensures that the p quark states for $i = 1, 2, \ldots, p$ are physically indistinguishable, although not identical. Wavefunctions for the qqq system symmetric in space, spin and unitary-spin variables then become possible because it is possible to construct a totally antisymmetric function in the new, discrete variable i; indeed, to ensure that there is just one such function for the description of a proton, we must choose $p = 3$. With this scheme, the qqq proton states will obey Fermi statistics (which is the case $p = 1$, of course), as required. We should emphasize that it is also possible to construct functions of the implicit variable i which are symmetric or of mixed symmetry, and that these will go with (space, spin, unitary spin) wavefunctions which are antisymmetric or of mixed symmetry; we must suppose that the forces are such that these configurations lie high in mass. This situation is discussed further in Ref. [53].

The Han-Nambu scheme proposes the existence of three quark triplets, q^i_α for $i = 1, 2, 3$. These can be arranged so that all quarks have integral charge, if desired. Han and Nambu proposed that an SU(3)-invariance should hold with respect to the upper suffix as well as for the lower suffix, giving an over-all symmetry $SU(3)' \times SU(3)$. For comparison with the experimental situation, they postulated that the strong interactions should be such that the SU(3)'-singlet states always lie low in mass relative to other states. For the qqq system, the SU(3)'-singlet state has the form

$$\sum_{\alpha\beta\gamma} \sum_{ijk} \epsilon_{ijk} q^{i\dagger}_\alpha(1)\, q^{j\dagger}_\beta(2)\, q^{k\dagger}_\gamma(3)\, \psi^{\alpha\beta\gamma}(1, 2, 3) \tag{5.2}$$

As a result of the antisymmetry of the factor ϵ_{ijk}, the wavefunction ψ will be symmetric in the quark labels 1, 2, 3, as required by the empirical situation.

This scheme requires additional quantum numbers, which must become evident in the study of the non-singlet SU(3)' states which will occur for some sufficiently high energy of excitation [54]. At present there are no indications for such additional additive quantum numbers in strong interaction physics. We emphasize that here the variable i is observable; for example, in the scheme which gives the quark integral charges (as presented by Han and Nambu), the quark q^1_1 is readily distinguished from q^2_1 and q^3_1, since the former has charge +e, whereas the latter have charge zero.

5.3. Saturation

The problem may be illustrated by asking the question "why is there no tightly-bound system of four quarks?", that is, "why does saturation set in at $N_q = 3$?". If there are superstrong attractive qq forces, forming qq bonds so strong that the binding energy in the qqq system is of order $3\, M_q$, then the addition of a further quark adds three further such bonds and the natural expectation is that the qqqq system should also have very low mass.

To avoid this conclusion, some strongly repulsive effect must come into play when the fourth quark approaches the qqq system. The following possibilities can be considered [55-58].

(i) With the introduction of an additional three-valued quantum number, implicit or explicit, only three quarks can be placed in the (1s) shell. A fourth quark must then be placed in the (1p) shell, which is energetically less favourable. However, from our identification of the $(1s)^2(1p)$ configuration with the ($\underline{70}$, L = 1-) supermultiplet, we see that the (1s)-(1p) energy difference is only of order 500 MeV, which is much too small an effect.

(ii) If the qq interaction is dominantly of space-exchange character, so that it is strongly repulsive in the p-wave, the requirement that the fourth quark be in a (1p)-state would give such a strong repulsion that the 4q state might well not bind. But the fourth quark could then be put into the (1d)-state, or the (2s)-state, where there would still be strong attraction. Also, this suggestion would be in conflict with the mass value known for the $(1s)^2(1p)$ configuration. In fact, this suggestion might well be more appropriate to the situation where quarks obey Fermi statistics, and where the qq forces have a strong space-exchange character but are attractive in p-states. However, the relative locations of the low-lying levels have not yet been worked out for this situation, and it is not clear to what extent a qqqq state would then be unfavourable.

(iii) There could be strongly repulsive three-body forces in multiquark systems [56]. These forces need not have a large effect on the properties of the qqq systems, but their repulsive effect could be very strong already in the qqqq system, where there are four triple (qqq) bonds.

(iv) There have been speculations about special relativistic effects which might come in when a fourth quark is added, which might lead to a weakening of its attraction to the other quarks, but this does appear to be special pleading: this does not lead naturally to saturation just at $N_q = 3$.

The fact is that we do not know any mechanism which would lead naturally to this saturation property. The mechanisms which we know to cause saturation at N = 4 for a system of nucleons are appropriate and effective only for a situation of low binding energy. Further discussion of various aspects of this problem have been given by Greenberg and Zwanziger [55] and by Morpurgo [57, 58].

5.4. Nature of superstrong forces

The nature of the superstrong forces is not known. How are they transmitted? Are there some very heavy mesons which have superstrong couplings with hadrons, whose effect on hadron-hadron interactions is masked by the repulsive cores observed to be effective at very close approach? Why does it happen that SU(6)-symmetry should hold so well just in the non-relativistic qq interaction? This appears entirely accidental, in our present view. If the primary interaction had only weak dependence on quark spin in the non-relativistic regime (as for vector meson exchange with γ_μ coupling) and did not depend on quark type, the resulting force would necessarily be in agreement with SU(6), being independent of spin and unitary spin; this force would have no space-exchange component. What is the relation between the $\bar{q}q$ and qq interactions? The former gives

a bond with a binding of almost $2M_q$ in the $\bar{q}q$ system, whereas the latter appears rather weaker, not leading to any low-lying qq bound state and giving binding of only $(M_q - \frac{1}{3})$ GeV per qq bond in the qqq state. In each system, the forces are just such as to give the ground-state at essentially zero mass (relative to mass M_q), which appears to be remarkably coincidental.[1]

How can the additivity assumption hold good? Such an assumption could well be appropriate for a situation of weak binding, where the constituents are spatially separate for a large fraction of the time, but it does not seem reasonable for a tight-binding situation. With tight-binding, all the quarks lie within the superstrong force field of the other quarks practically all the time. In this situation, the assumption that any incident quark interacts with only one target quark at a time (which underlies the hypothesis of additivity) would appear rather unlikely to be a good approximation, except perhaps roughly so for peripheral collisions (which are likely to be dominant for scattering with small momentum transfer, but certainly not for so large momentum transfer processes, with $\sqrt{t} \geqslant 1\ (\mathrm{GeV})^{-1}$).

Thus it appears that the quark model for hadronic states, as discussed in this survey, faces a large number of serious theoretical questions. It seems that the model can scarcely be justifiable in this naive form, in which the quarks are treated essentially as rigid and separate sub-units, although perhaps with their own internal structure. It is more likely that the model represents some first approximation to a much more complex theory of the excitations of some hadronic stuff. At the present stage, the model is rather phenomenological in that it is being used to provide a framework within which to analyse and interpret the empirical data on hadronic interactions. We must first determine what features a model should have to account for the outstanding features of the data. It is better for us to consider these simple models and to take them seriously as a semi-phenomenological approach, calculating their predictions in detail and pushing forward their comparison with experiment as far as possible, rather than to dwell on their short-comings as seen from the standpoint of theoretical physics today.

The history of the shell-model in nuclear physics provides a relevant and salutary lesson on this point. Thirty years ago, it was repeatedly asserted by theoreticians that no shell-model approach could possibly be valid for nuclear systems, in view of the short range of the nuclear forces and the lack of any dominating central field of force for the nucleons, and they patiently explained again and again the great differences between the situation for multi-electron atoms and for nuclei, which made worthless, a priori, any such shell-model hypothesis for nuclei. Fortunately, some physicists were not diverted by this criticism and not discouraged by their inability to counter all arguments against such a possibility, but spent their time looking carefully at the nuclear data as they became available from the experimental workers, until the empirical evidence for a shell-model structure for nuclei became overwhelming. It is only in the last decade that we have begun to understand the factors in the physical situation which

[1] Dr. C. Wilkin has pointed out to me that Glauber [38, 60] has shown that the additivity assumption can be justified for overlapping strong interactions, provided that the energy transfer is small and that the interactions are spin-independent and sufficiently smooth.

lead to the validity of the shell model for nuclear states, and it is only in the last few years that we have begun to relate the effective nuclear forces acting between nucleons in shell-model states with the nuclear forces deduced from the laboratory study of nucleon-nucleon scattering.

Similarly, we may hope that, in due course, we may learn how to understand the essential features of the quark model, an intermediate and no doubt much over-simplified model, in terms of some deeper theory of hadronic stuff still lying far beyond us.

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WAVE EQUATIONS FOR INFINITE-COMPONENT FIELDS

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Abstract

WAVE EQUATIONS FOR INFINITE-COMPONENT FIELDS. It is shown that when the internal motion can be described by algebraic methods, the proposed equation for the motion of the system as a whole is equivalent to a Majorana type equation.

It is generally accepted that wave equations for infinite-component fields, like the one of Majorana, should be able to represent the centre-of-mass motion of classical composite systems like the hydrogen atom. The various models proposed have some promising features but also known diseases, i.e. they have solutions for space-like total momenta but do not reproduce the exact mass spectrum.

Certainly the possibility of representing known soluble systems is an important check for the validity of these theories and constitutes a sine qua non for generalizing them to probably more complicated and certainly less known systems like the so-called elementary particles. On this line of thought it is reasonable to try to construct a relativistic Majorana equation starting from the hydrogen atom equation. Since we wish to describe the hydrogen atom with its internal O (4) symmetry, we have to keep the non-relativistic description for the internal motion and a relativistic one for its centre-of-mass motion. We have thus to give a generalization of the Schrödinger equation for the hydrogen atom as seen from a moving observer.

The equation is easily found to be

$$\left[|P| - m_1 - m_2 + \frac{(p^T)^2}{2\mu}\right] \varphi(p^T) = \frac{e^2 |P|}{2\pi^2} \int \frac{\varphi(\ell)\, \delta(\ell P)}{(p^T - \ell)^2} d^4\ell \qquad (1)$$

where P_μ is the total momentum, $|P| = +\sqrt{P^2}$, m_1, m_2 are the masses of the constituent particles, p^T is the transverse part of the relative momentum defined by $p_\mu^T = p_\mu - p_\nu P^\nu P_\mu / P^2$ and μ is the reduced mass.

To show the O(4) symmetry one can introduce a generalization of the Fock transformation by projecting out the p space on a five-dimensional hyperboloid

$$\eta_\mu \eta^\mu - \xi_\mu \xi^\mu = -1 \qquad (2)$$

where

$$\eta_\mu = \frac{2ap_\mu}{a^2 - p^2}, \quad \xi_\mu = \frac{a^2 + p^2}{a^2 - p^2}, \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

and

$$a = +\sqrt{2\mu\,(m_1 + m_2 - |P|)}$$

and one arrives at the relativistic generalization of the well-known Fock integral equation

$$\frac{a}{\mu}\,\psi_p(\xi^T) = -|P|\,\frac{e^2}{2\pi}\int \frac{1}{(\xi - \eta)^2}\,\psi_I\,\delta(\eta\cdot P)\,\frac{d^4\eta}{\eta_\mu} \tag{3}$$

where

$$\psi_I(\xi^T) = (1 - \xi_\mu)^{-2}\,\varphi(p^T)$$

The SO(4) symmetry is represented by the stability group of P in the five-dimensional ξ space or by the group of motion on the sphere $\xi^T = 1$ (intersection of the hyperboloid with a plane orthogonal to P_μ). Bringing Eq.(3) into differential form and substituting for a its value given above one arrives easily at the Klein-Gordon-like equation

$$\left\{P^2 - \left[m_1 + m_2 - \frac{\mu e^2}{2N^2}\right]\right\}\psi_p(\xi^2) \tag{4}$$

where $N^2 - 1$ is, in the ξ space, the angular part of the four-dimensional Laplace operator in the hypersphere $\xi^{T^2} = 1$. It can easily be shown that N^2 is a Poincaré invariant operator with eigenvalues = 1, 4, 16 ..... n^2 ..... where n is the Balmer quantum number.

One can now look for an equation of the Majorana type which, when squared, gives Eq.(4) which guarantees the exact Balmer spectrum for the system.

Further, by analogy with the Majorana equation, one will require the ψ_p to build up the basis for the most degenerate infinite dimensional representation of SO(4, 1). There are two possibilities: either one extends the momentum space with a fifth dimension in which P has an arbitrary component P_n thus obtaining the equation

$$\left[P_\mu\Gamma^\mu - P_n\Gamma^n - (m_1 + m_2)\,N + \frac{e^n_\mu}{2N}\right]\psi_p = 0 \tag{5}$$

and the Γ_α together with the generators of SO(4, 1) build up an SO(n, 2) algebra. Balmer's solutions are obtained in this case by setting at the end the arbitrary parameter $P_4 = 0$. Or one writes directly

$$\left[P_\mu\Gamma^\mu - \sqrt{P^2 - (m_1 + m_2)^2 + (m_1 + m_2)\,|P|}\;\Gamma^4 - e^2\sqrt{\frac{\mu(m_1 + m_2)}{2}}\right]\psi_p = 0$$

In both cases there are no solutions for P space-like and, since the equation can be deduced from the Bethe-Salpeter equation via covariant steps, one can obtain charge conjugation by projecting out in the original equation anti-particle states instead of particle states.

This exposition is essentially a synthesis of the following work: BISIACCHI, G., BUDINI, P., CALUCCI, G., Phys. Rev. 172 (1968) 1502.

CURRENT ALGEBRA

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Abstract

CURRENT ALGEBRA. The latest developments of current algebra are discussed, particularly those concerning the contributions from the semi-disconnected graphs to the sum rules. A unified treatment of low-energy theorems and of the dispersive formulation of the sum rules is also presented.

In this paper I would like to discuss the main physical ideas underlying the development of current algebra and to indicate some of the most important results.

One of the fundamental questions which arose in constructing quantum mechanics was to find a general way to introduce quantization. This was provided by the well-known commutation relations between canonically conjugate variables

$$[\, p_\nu , q_\gamma \,] = -i\, \delta_{\nu\gamma} \tag{1}$$

Similarly, in formulating a general scheme for elementary-particle physics, one wishes to identify the new "co-ordinates" for which we want to assume fundamental commutation relations.

Current algebra starts from the point of view suggested by Gell-Mann that such "co-ordinates" must in some way be observable. It was therefore suggested that those fundamental operators must be identified with the currents appearing in weak and electromagnetic interactions. Because of the relative weakness of those interactions, the matrix elements of those currents between hadronic states can be measured, at least in principle, with good accuracy. This is, of course, because we are allowed to use lowest Born approximations in those interactions so that, for example, the matrix elements

$$\langle p | j_\mu^{e.m.} | p \rangle, \quad \langle p | j_\mu | n \rangle, \quad \langle \pi | j_\mu | \nu \rangle$$

can be directly measured by means of e p scattering $\nu + p \rightarrow e + N$ and $\pi \rightarrow e + \nu$, respectively.

It is important to note that "weak" and electromagnetic interactions are recognized to be weak only for the present experimental energies of a few GeV. At higher energy, say of 1000 GeV, it is indeed possible that "weak" interactions will become rather strong so that the problem of higher-order perturbation terms in weak interactions will become crucial. So in some ways, current algebra has been developed by "putting under the rug" this fundamental and still completely unsolved question.

In this respect it is to be noted that it is always a wise policy to select those problems which can be understood now and leave other unsolved

questions to future investigations. We should like to remark that consideration of weak and electromagnetic currents as the new co-ordinates has been a very fruitful idea and has led to much improvement in our understanding of elementary-particle physics.

First of all we are led to consider the analogy between electromagnetic and weak interactions.

It is well known that an electromagnetic current is not invariant under isospin rotation but splits into an isoscalar part j_μ^s and an isovector part j_μ^3, which is the third component of a vector j_μ^α in the isotopic spin space. It is therefore important to ask whether nature does make use of the other two components $j_\mu^\pm$ of the vector current. One is thus led to the idea that the charged vector currents $j_\mu^\pm$ are indeed those which appear in weak strangeness-conserving currents. This point of view has led to many beautiful predictions in wonderful agreement with experiment.

The next step was to consider weak strangeness-non-conserving currents. This was possible by means of SU_3; in this way one was led to classify electromagnetic and weak vector currents by means of a single SU_3 octet. The form of the weak current is given in the fundamental form suggested by Cabibbo.

One has finally considered axial weak currents which are classified by means of an "odd" SU_3 octet connected to the "even" octet by the chiral transformation $j_\mu \rightarrow \gamma_5 j_\mu$.

Electromagnetic interactions exhibit the fundamental universality property, i.e. the electromagnetic coupling constants of the proton π^+, positron, etc., are all equal.

Exact current conservation

$$\frac{\partial j_\mu^{e.m.}}{\partial x_\mu} = 0 \tag{2}$$

which leads to

$$Q^{e.m.} = \int j_0^{e.m.}\, d^3x \tag{3}$$

ensures that the equality between un-renormalized coupling constants is translated in the exact equality between the renormalized coupling constants.

In close analogy, weak interactions exhibit striking universality properties which can again be translated in terms of experimental coupling constants because, in analogy with electromagnetism, all those currents exhibit an approximate form of conservation law, so that the weak charges

$$Q_\alpha = \int j_{0\alpha}\, d^3x \tag{4}$$

are all approximate constants of the motion. Not all weak charge conservation laws are on the same ground. For the vector S conserving charge, exact conservation takes place once electromagnetic interactions are

switched off, whereas for the vector strange currents one requires exact SU_3. Finally, as we shall see later, we believe that axial charge conservation takes place in the limit in which the masses of the bosons in the pion octet all tend to zero.

In order to discuss more precisely the consequences of partial conservation, we are led to the central point of this paper, namely the fundamental commutators between currents.

Let us list these relations on the basis of their reliability. First of all, the relations involving at least one charge[1]

$$[Q_\alpha(t),\ Q_\beta(t)] = f_{\alpha\beta\gamma} Q_\gamma(t) \tag{5}$$

$$[Q_\alpha(t),\ j_{\mu\beta}(x,t)] = f_{\alpha\beta\gamma} j_{\mu\gamma}(x,t) \tag{6}$$

These relations simply state that the charges Q_α are the generators of the group $SU_3 \times SU_3$. Since, as we shall see in more detail in the following discussion, current algebra is particularly powerful when combined with partial conservation, Eqs (5) and (6) have been widely used leading to many beautiful results in good agreement with experiment.

The next relations are those between the time components of the currents. They are postulated to have the simplest form compatible with $SU_3 \times SU_3$ and locality

$$[j_{0\alpha},\ j_{0\beta}] = f_{\alpha\beta\gamma}\ j_{0\gamma}\ \delta^3(x) \tag{7}$$

Equation (7) has led to several sum rules, some of them in reasonable agreement with experiment. The general program of studying the general saturation of Eq.(7) in the infinite momentum frame is being carried out by Gell-Mann and his co-workers. Finally, we are left with the commutators also involving space components of the currents. These commutators are much more model-dependent than the others. Their simplest and more appealing form is that following from the "field algebra" model

$$[j_{0\alpha}, j_{i\beta}] = f_{\alpha\beta\gamma}\ j_{i\gamma}\ \delta^{(3)}(x) + S^{\alpha\beta}_{i\ell} \frac{\partial}{\partial x} \delta^{(3)}(x) \tag{8}$$

$$[j_{i\alpha},\ j_{i\beta}] = 0 \tag{9}$$

The problem of the commutators of the form (8), (9) and of the higher commutators involving time derivatives of the current is still rather open. A beautiful indication comes, however, from the spectral function sum rules derived by Weinberg, closely related to those commutators.

In this short survey I shall limit myself to discussing some of the consequences of the commutators in Eqs (5) and (6). In order to illustrate

[1] It is understood that those relations involve both vector and axial vector currents according to the well-known rule

$$[V,V] = V,\ [V,A] = A,\ [A,A] = V$$

the importance of the combined use of equal time commutators and partial current conservation, let us start by considering the ideal case of exactly conserved charges

$$[Q_\alpha H] = 0 \tag{10}$$

Let us consider the matrix elements of Q_α between two states $\langle A|$ and $\langle B|$

$$\langle A|Q_\alpha|B\rangle = \langle A|j_{0\alpha}|B\rangle\, \delta^3(\vec{p}_A - \vec{p}_B) \tag{11}$$

using Eq.(10) we get

$$(E_\alpha - E_B)\langle A|Q_\alpha|B\rangle = 0 \tag{12}$$

Equations (11) and (12) tell us that the only non-vanishing matrix elements of Q_α are between states of equal four momentum, i.e. of equal masses. Let us now sandwich Eq.(5) between one-particle states

$$\langle p|[Q_\alpha, Q_\beta]|p\rangle = f_{\alpha\beta\gamma}\langle p|Q_\gamma|p\rangle \tag{13}$$

and use completeness in the intermediate states. According to the previous discussion, only intermediate states with four-momentum p_μ can contribute. Equation (13) requires the existence of a multiplet of particles of equal masses, differing in internal quantum numbers.

Equation (13) simply becomes

$$\sum \langle L|Q_\alpha|\gamma\rangle\langle\gamma|Q_\beta|L'\rangle = f_{\alpha\beta\gamma}\langle L|Q_\gamma|L'\rangle \tag{14}$$

where the states all belong to the multiplet. The solution of Eq.(14) is simply

$$\langle L|Q_\alpha|\gamma\rangle = r\, C^\alpha_{L\gamma} \tag{15}$$

where $C^\alpha_{L\gamma}$ is the appropriate Clebsch-Gordan coefficient and the "renormalization ratio" r is equal to one

$$r = 1 \tag{16}$$

We thus see that the equal time commutator (5) plus exact charge conservation tell us that the matrix element $\langle L|Q_\alpha|\gamma\rangle$ must obey the Wigner-Eckart theorem and that there is complete absence of renormalization. The same argument applied to the commutator (6) leads to the re-

sult that the matrix elements of the current j_μ^α must again obey the Wigner-Eckart theorem.

All this is very nice and in some way trivial. What is not at all trivial is the important fact implied by the famous Goldstone theorem that the existence of zero-mass bosons with the same quantum number as Q_α invalidates all previous conclusions. We wish to show that in the presence of such bosons, the combined use of Eqs (5), (6) and (10) does not necessarily lead to absence of renormalization or to the Wigner-Eckart theorem for the matrix elements.

The mechanism by which "this spontaneous breakdown of symmetry" takes place is very simple. If a zero-mass boson exists we can construct a matrix element between equal four-momentum states by writing

$$\langle p | Q_\alpha | p, b \rangle \tag{17}$$

where the boson b has zero four-momentum (soft boson). In this case, even if the charge is conserved, the matrix element (17) can be non-vanishing and make an extra contribution to the completeness sum.

One might feel that the contribution (17) to the completeness sum is indeed vanishing, since only one state out of the continuum is allowed to give a non-zero contribution. We note however that the matrix element

$$\langle p_1 | Q_\alpha | p_2 b \rangle$$

exhibits a disconnected term

$$\delta^3 (p_1 - p_2) \langle 0 | Q_\alpha | b \rangle \tag{18}$$

whose contribution is certainly non-vanishing.

The presence of contributions (18) (related to the so-called degeneracy of the vacuum) in the completeness sums invalidates the previous conclusions about absence of renormalization and the Wigner-Eckart theorem.

Let us now pass from the ideal world of conserved currents and zero-mass particles to the real hadrons where no current (except the electromagnetic one) is exactly conserved and no zero-mass particles exist.

Even in this more complicated case the previous argument leads to a very important new insight into the physical situation. Let us consider in some detail the situation of the axial charges and concentrate for simplicity on the strangeness conserving one. If the charge were an approximate constant of the motion, one would be led to conclude that good parity doublets must exist with small spacing and simple relations between the matrix elements of the currents. Since such parity doubling is not seen in any clear way, at least for what concerns low-lying states, one could be led to conclude that axial charges are very bad constants of the motion.

However, as pointed out by Nambu and Gürsey, the presence of a small mass boson, the pion, allows an almost conserved axial current together with little evidence for a simple doublet scheme. This leads to the very important point of view (which is at the root of soft pion theory) that the divergence of the axial current vanishes in the limit of zero pion

mass. In the case of vector currents, no scalar particles of small mass exist, so if the charges are reasonably good constants of the motion, we have no excuse for not finding SU_3 multiplets.

The most important applications of current algebra are of two kinds. First of all one can derive dispersion sum rules by summing on intermediate states by means of completeness; an example is the well-known Adler-Weisberger sum rule expressing the renormalization ratio g_A/g_V in terms of an integral on the total π nucleon cross-sections. Secondly, one can use the exact conservation of the axial charge in the limit of zero-mass pion in order to derive an exact low-energy theorem valid for soft pions of zero four-momentum. This exact formula is then translated by means of a mass extrapolation in an approximate expression valid for a physical pion.

In this report I wish to sketch a unified treatment of both applications which has the advantage of leading directly to a low-energy theorem for physical pions in which the form of the correction terms is explicitly exhibited. Our procedure starts from the very important observation that the commutator between two charges

$$[Q_\alpha(t),\ Q_\beta(t)] = f_{\alpha\beta\gamma} Q_\gamma(t) \tag{19}$$

is not explicitly covariant since equal time refers to a well-defined frame of reference. Relativistic invariance is recovered by asking Eq.(19) to be valid in any inertial frame of reference.

This means that in correspondence to a single choice of values of α and β we shall still have an infinity of commutation relations, one corresponding to each frame of reference. Thus, it is not surprising that, when we derive sum rules by taking the expectation value of Eq.(19) between one-particle states, we shall obtain an infinity of sum rules in correspondence to all possible values of the momentum p of the "target".

It is to be noted that this happens only when we have to deal with operators which are not constants of the motion. If the Q_α are exactly conserved, Eq.(19) becomes independent of time and all sum rules do coincide.

Let us now discuss in detail the commutator between two axial charges

$$[Q_A^+(t),\ Q_A^-(t)] = 2\, Q_V^3(t) \tag{20}$$

The one-particle matrix element of Eq.(20) can be written in a particularly convenient form by introducing the divergence of the axial current

$$D^\pm = \frac{\partial}{\partial x_\mu} j_\mu^\pm \tag{21}$$

If we define

$$w^\pm(q_\mu, p_\mu) = \int \langle p | D^\pm\left(-\frac{x}{2}\right) D^\mp\left(\frac{x}{2}\right) | p \rangle\, e^{iqx}\, d^4x \tag{22}$$

we obtain

$$\frac{1}{p_0}\,\frac{1}{\pi}\int \frac{w^+(q_0\,,\,\vec{q}=0)-w^-(q_0,\,\vec{q}=0)}{q_0^2}\,dq_0 = 1 \tag{23}$$

Equation (23) can be rewritten in a more elegant and convenient form by using the fact that w is a function only of the two invariants

$$\nu = qp,\quad u = q^2 \tag{24}$$

We obtain

$$\iint \frac{w^a(\nu,u)}{\nu^2}\,\delta\left(\frac{\nu^2}{p_0^2}-u\right)d\nu\,du = 1 \tag{25}$$

where

$$w^a = w^+ - w^-,\quad p_0 = \sqrt{p^2+M^2}$$

Equation (25) exhibits very clearly the frame dependence of our sum rules. For each choice of p_0, Eq.(25) connects the values of $w^a(\nu,u)$ along a different parabola whose curvature depends on the value of the momentum of the target. In the particularly interesting case of $p\to\infty$ the parabola reduces to the straight line $u = 0$.

Let us now list the different contributions to the integral (25). A detailed discussion of those terms can be found in Ref.[1]. Here we shall simply state that we shall have a contribution when one of the variables u, $s = u + 2\nu + M^2$, $\bar{s} = u - 2\nu + M^2$ has the value of the mass of a physical state. Our contributions (see Fig. 1) can thus be obtained as the intersections of our parabola with the lines (see Fig. 1) (a) $u = m^2$, (b) $u + 2\nu + M^2 = M'^2$, (c) $u - 2\nu + M^2 = M'^2$.

It is easy to see that the intersections of the line (a) correspond to the graphs listed in Fig. 2, whereas the two intersections of the line (b) correspond to the direct and Z graphs shown in Fig. 3.

In the limiting case of $p\to\infty$, corresponding to the straight line $u = 0$, the contributions from the graphs in Figs 2 and 3b disappear since the corresponding intersections move to infinity and we are left with only the simple direct graphs of Fig. 3a.

In this limit we obtain the famous Adler-Weisberger relation (we have explicitly exhibited the one-nucleon contribution to the sum rule)

$$\left(\frac{g_A}{g_V}\right)^2 + \frac{1}{\pi}\int \frac{w^a(\nu,0)}{\nu^2}\,d\nu = 1 \tag{26}$$

where $w^a(\nu,0)$ can be related, by means of P.C.A.C., to the difference between π^+ and π^- total cross-sections.

All this is well known. The amusing fact is that if we consider the same Eq.(25) in the target rest frame, we obtain a sum rule which is directly related to the well-known low-energy limit for pion-nucleon scattering.

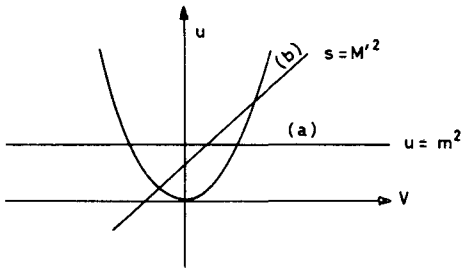

FIG. 1. Contributions to Eq. (25).

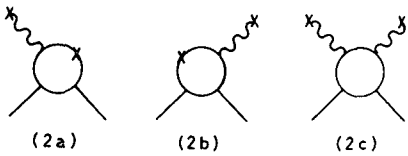

FIG. 2. Graphs corresponding to line (a) of Fig. 1.

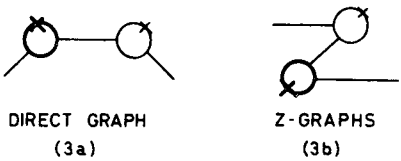

FIG. 3. Graphs corresponding to line (b) of Fig. 1.

This can easily be seen by noting that $w^a(\nu, u)$ has a pion contribution at $u = m_\pi^2$ of the form

$$(w, u) = f_\pi^2 \cdot m_\pi^2 \, T_{\pi N} \; (\nu) \, \delta'(u - m_\pi^2) \tag{27}$$

where f_π is the decay rate of the pion and $T_{\pi N}(\nu)$ is the forward pion-nucleon scattering amplitude. In correspondence to the $p = 0$ parabola

$$\frac{\nu^2}{M^2} - u = 0$$

$T_{\pi N}$ will be evaluated at the physical threshold $\nu = \nu_0 = M m_\pi$.

The derivative of the δ function comes from the fact that both "pions" become real at the same point $u = m_\pi^2$ (see Fig. 2c).

In order to deal in a simple way with this δ' function, we shall take P.C.A.C. seriously and assume for the divergences D^α the same equal-time commutation relation of the pion fields

$$[D^\alpha, D^\beta]_{\text{equal time}} = 0 \tag{28}$$

leading to the sum rule

$$\iint w^a(\nu, u) \, \delta\left(\frac{\nu^2}{M^2} - u\right) d\nu \, du = 0 \tag{29}$$

If we combine Eqs (25) and (29) we get

$$\iint \frac{w(\nu, u)}{\nu^2} \left(1 - \frac{u}{m_\pi^2}\right) \delta\left(\frac{\nu^2}{M^2} - u\right) d\nu\, du = 1 \tag{30}$$

We can now exhibit explicitly the pion contribution and rewrite Eq.(30) in the form

$$\frac{f_\pi^2}{m_\pi M}\left[T_{\pi N}^0 - m_\pi^2 \iint \frac{F(\nu, u)\, \delta\left(\frac{\nu^2}{M^2} - u\right)}{\nu^2 (u - m_\pi^2)}\, du\, d\nu\right] = 1 \tag{31}$$

where

$$F(\nu, u) = \left(1 - \frac{u}{m_\pi^2}\right)^2 (w, u)$$

$$F^+(\nu, u) = \int \langle p | \chi^+\left(-\frac{x}{2}\right) \cdot \chi^-\left(\frac{x}{2}\right) p \rangle e^{iqx}\, d^4x \tag{32}$$

and χ is defined as

$$(\Box + m_\pi^2) D = m_\pi^2 \chi \tag{33}$$

We see that if we keep only the pion contribution in the left-hand side of Eq.(31), we obtain the well-known low-energy limit for pion-nucleon scattering. It is important to note that such expansion involves the amplitude for physical pion scattering at the physical scattering threshold. The integral on $F(\nu, u)$ represents therefore the correction which one has to introduce in order to transform the usual exact expression of a low-energy limit for mathematical zero-mass pions into an approximate but more useful expression valid for physical pions. The main contribution to the correction integral is expected to come from the direct graph contributions shown in Fig.3a. A very important feature is that, since we are working with integrated axial charges in the zero-momentum frame, the only contributions will come from states with the same spin of the target and opposite parity, so that the correction will be multiplied by a factor $(m_\pi/\Delta M)^2$, ΔM being the mass difference between the parity doublet. This means that the soft pion formula is expected to be particularly reliable in cases, like the nucleon, where the opposite parity states have a rather high mass.

The same treatment can be applied to electroproduction of pions. One saturates the commutators between Q_A and the electromagnetic current in the Breit frame of reference. One obtains a prediction for the physical electroproduction amplitude at the physical threshold in the Breit system. The leading term depends only on f_π and on the vector and axial vector form factor of the nucleon.

To conclude, we have seen that the saturation of charge commutators leads to very interesting results both at infinite momentum and zero momentum. It is to be noted that in the limit $m_\pi \to 0$ the Adler-Weisberger

relation is completely equivalent to the low-energy soft pion formula. In that limit, therefore, the choice of frame does not matter. This confirms the idea that in that limit the axial charge is indeed exactly conserved.

Looking at the results obtained in the last years by means of current algebra, one sees that they are not in the form of absolute theoretical predictions but rather of successful correlations between experimental data. The Adler-Weisberger relation allows the famous g_A/g_V ratio to be obtained in terms of experimental pion-nucleon cross-sections.

In a similar way, the success of the soft pion formulae follows on the empirical facts of the smallness of the pion mass and the absence of closely spaced parity doublet.

Similarly, the reasonable agreement of the SU_3 formulae is due to the absence of small mass scalar bosons.

Of course, a fundamental explanation, for example, of why the pion mass is small and the scalar boson's mass is large lies outside the range of current algebra (which uses only a few general theoretical ingredients like equal-time commutators, Lorentz invariance and completeness) but requires a more specific theory like the quark model or may be a general bootstrap scheme.

However, waiting for a fully successful fundamental approach, we have to be satisfied that so many successful correlations between experimental data have been obtained in a model-independent way.

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FROM CHIRAL DYNAMICS TO CHIRAL ALGEBRA

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Abstract

FROM CHIRAL DYNAMICS TO CHIRAL ALGEBRA. An attempt is made to reconcile two views on chiral symmetry. The author starts with chirality as a dynamical symmetry and derives its algebraic aspects by using dispersion theory or the phenomenological Lagrangian method.

Current algebra is a technique for deriving the physical consequences of chiral symmetry. Its practitioners have largely become divided into two jarring schools, reflecting two different views as to what kind of symmetry chirality is. One school sees it as a badly broken symmetry of the ordinary algebraic sort, like isospin, which groups particles into large chiral multiplets and yields relations among processes involving a fixed number of particles belonging to specified multiplets. The method used by Fubini, Furlan, Dashen, Gell-Mann et al. to obtain these algebraic relations is to saturate the current commutation rules at infinite momentum with a finite or denumerable number of one-particle states. The other school, essentially founded on the work of Nambu, sees chirality as a dynamical symmetry which manifests itself only in soft-pion theorems, just as general covariance yields soft-graviton theorems (like the Principle of Equivalence), and as local gauge invariance yields soft-photon theorems. In this view, chirality is only weakly broken by the pion mass. I refer to the analogy between chirality and general covariance only to emphasize the huge difference between a dynamical symmetry like general covariance and an algebraic symmetry like Lorentz invariance or isospin.

The purpose of this paper is, as of the paper by Fubini in these Proceedings, to reconcile these two views of chiral symmetry. However, I will start with chirality as a dynamical symmetry and derive its algebraic aspects by using either dispersion theory or the phenomenological Lagrangian method. These two approaches give precisely the same results, but I feel that the Lagrangian approach, though perhaps less familiar and convincing, is more illuminating physically and I will therefore discuss it first.

As shown in Zumino's paper in these Proceedings, a Lagrangian which conserves isospin will also be chiral-invariant (in the same dynamical sense that Einstein's Lagrangian is generally covariant), if constructed out of general hadron fields ψ, their covariant derivatives

$$D_\mu \psi \equiv [\partial_\mu + 2i F_\pi^{-2}(1 + F_\pi^{-2}\vec{\pi}^2)^{-1} (\vec{\pi} \times \partial_\mu \vec{\pi}) \cdot \vec{t}]\psi \qquad (1)$$

and the covariant derivative of the pion field

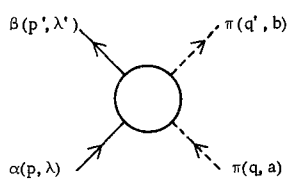
$$D_\mu \vec{\pi} \equiv (1 + F_\pi^{-2} \vec{\pi}^2)^{-1} \partial_\mu \vec{\pi} \tag{2}$$

Here $\vec{t}$ is the isospin matrix of ψ and $F_\pi \simeq 190$ MeV is the pion decay amplitude. We will ignore chiral symmetry breaking and set the pion mass equal to zero throughout.

A Lagrangian constructed according to these rules will be chiral-invariant without any need to group particles into chiral multiplets. How then can we ever hope to derive algebraic results from such theories? Our answer comes from inspection of Eqs (1) and (2); any Lagrangian built out of ingredients like $D_\mu \psi$ and $D_\mu \vec{\pi}$ will be highly non-linear and replete with derivative interactions, so the S-matrix in such a theory will generally behave badly at high energy. The algebraic aspects of chiral symmetry arise from the necessity of detailed cancellations of terms in the S-matrix which grow too fast with increasing energy.

Of course, we are not going to use the chiral invariant Lagrangian to compute exact S-matrix elements, but will instead use it in the tree approximation, taking higher order diagrams into account only by including a number of resonances among the virtual particles in our trees. We know this is a good approximation at very low energies, and we hope it remains useful in the medium-energy resonance region, but can we use it to say anything at all about the S-matrix at high energy? Obviously not, but we can insist that the tree graphs do not behave worse at high energy than would be expected, say, from Regge pole theory. That is, if individual tree graphs do grow too fast at high energy, then we shall require that the rapidly growing contributions of these tree graphs cancel, not with the continuum, but among themselves.[1] (As we shall see, this assumption gives the same results as the saturation of dispersion relations by single-particle states, in those cases where unsubtracted dispersion relations are available.)

To become more specific, we shall consider the elastic or inelastic forward scattering process



Here α and β label the initial and final target particles, p and p' are their four-momenta, and λ and λ' are their helicities while q and q' are the four-momenta of the initial and final massless pions, and a and b are their isovector indices. It is extremely convenient to adopt a co-linear coordinate system with

$$\vec{q} = \vec{n}\omega, \quad \vec{p} = -\vec{n}p, \quad \vec{q}\,' = \vec{n}\omega', \quad \vec{p}\,' = -\vec{n}p' \tag{3}$$

where $\vec{n}$ is a fixed unit vector. This automatically satisfies the condition

[1] The consequences of this assumption have been worked out in special cases by Wess and Zumino; it is experimentally verified in πN forward scattering.

$t = 0$ for forward scattering, while energy and momentum conservation are satisfied if

$$p + (p^2 + m_\alpha^2)^{1/2} = p' + (p'^2 + m_\beta^2)^{1/2} \equiv \mathscr{E} \tag{4}$$

and

$$\omega' = \omega + (m_\alpha^2 - m_\beta^2)/2\mathscr{E} \tag{5}$$

In this system helicity is conserved, so $\lambda = \lambda'$. We will hold $\vec{p}$ and $\vec{p}'$ fixed, and examine the behaviour as of the Feynman amplitude $M_{\beta b,\alpha a}(\omega,\lambda)$ regarded as a function of ω alone. In particular, we will study the linear combinations

$$M^{(+)}_{\beta b,\alpha a}(\omega,\lambda) \equiv \frac{1}{2}[M_{\beta b,\alpha a}(\omega,\lambda) + M_{\beta b,\alpha a}(-\omega',\lambda)] \tag{6}$$

$$M^{(-)}_{\beta b,\alpha a}(\omega,\lambda) \equiv \frac{1}{\omega+\omega'}[M_{\beta b,\alpha a}(\omega,\lambda) - M_{\beta b,\alpha a}(-\omega',\lambda)] \tag{7}$$

Crossing symmetry gives

$$M_{\beta b,\alpha a}(\omega,\lambda) = M_{\beta a,\alpha b}(-\omega',\lambda) \tag{8}$$

so $M^{(\pm)}$ are even/odd in isospins

$$M^{(\pm)}_{\beta b,\alpha a}(\omega,\lambda) = \pm M^{(\pm)}_{\beta a,\alpha b}(\omega,\lambda) \tag{9}$$

Regge-pole theory would give $M^{(-)}$ the asymptotic behaviour $\omega^{\alpha_1(0)-1}$, where $\alpha_1(0)$ is the leading $T = 1$ Regge trajectory as $t = 0$, so with $\alpha_1(0) = \alpha_p(0) \approx 0.5$ we expect that

$$M^{(-)}_{\beta b,\alpha a}(\omega,\lambda) \to 0 \quad \text{as} \quad \omega \to \infty \tag{10}$$

Also, the part of $M^{(+)}$ with $T = 2$ in the t-channel should go like $\omega^{\alpha_2(0)}$, where $\alpha_2(0)$ is the leading $T = 2$ trajectory at $t = 0$, so if $\alpha_2(0) < 0$ (as suggested particularly by Gilman and Harari) then for $\omega \to \infty$ only the $T = 0$ part of $M^{(+)}$ should survive, and so

$$M^{(+)}_{\beta b,\alpha a}(\omega,\lambda) \propto \delta_{ab} \quad \text{as} \quad \omega \to \infty \tag{11}$$

Eqs (10) and (11) are the two demands we shall make on the asymptotic behaviour of the tree graphs.

In general, the Feynman amplitude in the tree approximation will be of the form

$$\begin{aligned} M_{\beta b,\alpha a}(\omega,\lambda) &= P_{\beta b,\alpha a}(\omega,\lambda) \\ &+ \sum_\gamma (m_\alpha^2 - m_\gamma^2 + 2\omega\mathscr{E})^{-1}\, Q^{(\gamma)}_{\beta b,\alpha a}(\omega,\lambda) \\ &+ \sum_\gamma (m_\alpha^2 - m_\gamma^2 - 2\omega'\mathscr{E})^{-1}\, Q^{(\gamma)}_{\beta a,\alpha b}(-\omega',\lambda) \end{aligned}$$

The first term is a polynomial in ω arising from graphs of the form

while the second and third terms come from the graphs

with $Q^{(\,)}$ a polynomial coming from the numerator of the γ-propagator and any derivatives acting at the pion vertices. Chiral symmetry tells us that each pion field can enter interactions only as the covariant derivative $D_\mu \vec{\pi}$ except that Eq.(1) implies a direct interaction of $\vec{\pi} \times \partial_\mu \vec{\pi}$ with the vector current. Thus chirality imposes on Eq.(12) the additional information

$$P_{\beta b,\,\alpha a}(\omega,\lambda) = \omega\omega' \widetilde{P}_{\beta b,\,\alpha a}(\omega,\lambda) + 4\,i\,F_\pi^{-2}\,\mathscr{E}(\omega+\omega')\,\epsilon_{abc}\,(T_c)_{\beta\alpha} \tag{13}$$

$$Q_{\beta b,\,\alpha a}(\omega,\lambda) = \omega\omega' \widetilde{Q}_{\beta b,\,\alpha a}(\omega,\lambda) \tag{14}$$

where $\widetilde{P}$ and $\widetilde{Q}$ are again unknown polynomials.

In general $M^{(\pm)}$ contain terms of high order in ω as $\omega \to \infty$ with unknown coefficients, and there is not much to be learned by demanding that these coefficients vanish. Matters are different, however, for the coefficient of ω^0 in an asymptotic expansion of $M^{(\pm)}$, which can be isolated by a contour integration

$$C^{(\pm)}_{\beta b,\,\alpha a}(\lambda) \equiv \frac{1}{2\pi i}\oint M^{(\pm)}_{\beta b,\,\alpha a}(\omega,\lambda)\,\omega^{-1}\,d\omega \tag{15}$$

the contour being taken as a large circle in the complex ω-plane. Because of the factor $\omega\omega'$ in Eqs (13) and (14), the only singularities which contribute to the integral are the poles on the γ-particle mass shell in Eq.(12), and the pole at $\omega = 0$ arising from the second term of Eq.(13). A straightforward calculation then gives for the coefficients of ω^0 the values:

$$C^{(-)}_{\beta b,\,\alpha a}(\lambda) = 8\,F_\pi^{-2}\,\mathscr{E}\,\{i\,\epsilon_{abc}\,T_c - [X_a(\lambda), X_b(\lambda)]\}_{\beta\alpha} \tag{16}$$

$$C^{(+)}_{\beta b,\,\alpha a}(\lambda) = 4\,F_\pi^{-2}\,\{[X_b(\lambda), [X_a(\lambda), m^2]] + [X_a(\lambda), [X_b(\lambda), m^2]]\}_{\beta\alpha} \tag{17}$$

where m^2 is the mass matrix

$$(m^2)_{\beta\alpha} \equiv m_\alpha^2\,\delta_{\beta\alpha} \tag{18}$$

and $X_a(\lambda)$ is a dimensionless Hermitian matrix defined so that the Feynman amplitude for the co-linear decay $\alpha \to \beta + \pi$ with helicity λ is $2\,F_\pi^{-1}(m_\alpha^2 - m_\beta^2)\,[X_a(\lambda)]_{\beta\alpha}$. The rate for the decay $\alpha \to \beta + \pi$ with α at rest and unpolarized is then[2]

$$\Gamma(\alpha \to \beta + \pi;\lambda) = \frac{(m_\alpha^2 - m_\beta^2)^3\,|(X_a(\lambda))_{\beta\alpha}|^2}{4\pi m_\alpha^3 F_\pi^2\,(2\,J_\alpha + 1)} \tag{19}$$

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[2] This is for massless pions; to calculate actual decay rates we must of course take into account the effect of the finite pion mass on phase space and centrifugal barrier factors.

(The Goldberger-Treiman relation tells us that $(\vec{X}(\lambda))_{\beta\alpha}$ is the axial vector coupling matrix; this definition can be used even if $m_\alpha = m_\beta$.)

Our demand that $M^{(-)}$ vanish as $\omega \to \infty$ gives $C^{(-)} = 0$, or

$$[X_a(\lambda), X_b(\lambda)] = i\,\epsilon_{abc}\,T_c \tag{20}$$

Isospin conservation tells us further that

$$[T_a, X_b(\lambda)] = i\epsilon_{abc}\,X_c(\lambda) \tag{21}$$

and of course

$$[T_a, T_b] = i\epsilon_{abc}T_c \tag{22}$$

Hence the elementary particles must for each helicity be grouped into multiplets which furnish representations of the chiral algebra (20), (21), and (22).

Unfortunately these representations are in general reducible, because the mass matrix m^2 generally does not commute with $\vec{X}(\lambda)$. Our demand that the part of $M^{(+)}$ with $T = 2$ in the t-channel vanish as $\omega \to \infty$ gives

$$C^{(+)}_{\beta b,\alpha a}(\lambda) \propto \delta_{ba} \tag{23}$$

It is a straightforward matter to show that Eqs (17) and (23) require the mass matrix to have the form

$$m^2 = m_0^2(\lambda) + m_4^2(\lambda) \tag{24}$$

where

$$[X_a(\lambda), m_0^2(\lambda)] = 0 \tag{25}$$

and

$$[X_a(\lambda), m_b^2(\lambda)] = i m_4^2(\lambda)\,\delta_{ab} \tag{26}$$

$$[X_a(\lambda), m_4^2(\lambda)] \equiv i m_a^2(\lambda) \tag{27}$$

That is, the mass matrix is the sum of a chiral-scalar and the fourth-component of a chiral-four-vector. Note that this is not an approximation based on some assumption of weak chiral symmetry breaking, but is an exact consequence of our assumptions about the asymptotic behaviour of tree graphs. Note also that Eqs (17) and (24) to (27) give

$$C^{(+)}_{\beta b,\alpha a}(\lambda) = -8\,F_\pi^{-2}(m_4^2(\lambda))_{\beta\alpha}\delta_{ab} \tag{28}$$

Were it not for trajectories with vacuum quantum numbers and $\alpha(0) \geq 0$, we would require that $M^{(+)}$ vanish as $\omega \to \infty$ and hence that $m_4^2 = 0$. In only this sense, algebraic chiral symmetry is broken by the vacuum.

Before going into the application of these results, I would like to indicate briefly how they can be derived from dispersion theory. Here it is necessary

to distinguish the cases of elastic and inelastic forward pion scattering. For $m_\alpha = m_\beta$ we can use current algebra to derive the limits of $M^{(\pm)}$ as $\omega = \omega' \to 0$ and can then convert these to generalized Adler-Weisberger relations by using the unsubtracted dispersion relations for $M^{(-)}$ and the $T = 2$ part of $M^{(+)}$. For $m_\alpha \neq m_\beta$ we can use the Adler "self-consistency" requirement to show that the $M^{(\pm)}$ vanish as $\omega \to 0$ (and $\omega' \nrightarrow 0$) and can then use the unsubtracted dispersion relations to derive inelastic sum rules analogous to those of Adler-Weisberger. When all these sum rules, elastic and inelastic, are saturated with single-particle states, we again obtain Eqs (20) and (23) to (26).

Now to applications. The most fundamental representation is that containing the pion itself. It is a simple matter to show that for $\lambda = 0$ the matrix $X_a(\lambda)$ commutes with the sign factor $G\Pi(-)^J$ (where G is G-parity, Π is ordinary parity, and J is the spin), so we can expect the chiral partners of the pion to be the $\lambda = 0$ states of mesons with $G\Pi(-)^J = +1$, i.e. the ρ, σ, A1, f, f', g, ... The simplest representation that can account for an appreciable number of these mesons is the direct sum of the chiral four-vector V_A and the antisymmetric tensor t_{AB}. (Here A, B, ... are vector indices running over 1, 2, 3, 4). The G parities of V_4 and t_{ab} are equal, and opposite to the G parities of V_a and t_{4a} (if not, just replace t_{AB} with its dual) so we identify the $\lambda = 0$ meson states of definite mass as

$$\pi_a = t_{4a} \cos\theta + V_a \sin\theta$$
$$A1_a = -t_{4a} \sin\theta + V_a \cos\theta$$
$$\sigma = V_4$$
$$\rho_a = \tfrac{1}{2}\epsilon_{abc} V_{bc}$$

The pion-decay matrix $X_a(0)$ acts on V_A and t_{AB} as chiral "boosts", so we can use ordinary SO(4) tensor analysis to compute its physical elements

$$(\pi_b | X_a(0) | \rho_c) = -i\epsilon_{abc} \cos\theta$$
$$(\pi_b | X_a(0) | \sigma) = i\delta_{ab} \sin\theta$$
$$(\rho_b | X_a(0) | A1_c) = i\epsilon_{abc} \sin\theta$$
$$(\sigma | X_a(0) | A1_b) = -i\delta_{ab} \cos\theta$$

The observed ρ-width gives $\theta = 45°$ to $50°$ (mod $\pi/2$).

In this example, the chiral-scalar part m^2 of the mass matrix is a linear combination of $V_A V_A$ and $t_{AB} t_{AB}$, while its chiral form-vector part m_4^2 is proportional to $t_{4A} V_A$. By imposing the condition that $m_0^2 + m_4^2$ be diagonal and that the pion mass be zero we then find

$$m_{A1}^2 = m_\rho^2 + m_\sigma^2$$
$$m_\rho^2/m_\sigma^2 = \tan^2\theta$$

With $\theta = 45°$ we obtain the appealing results $m_\sigma = m_\rho$ and $m_{A1} = \sqrt{2}\, m_\rho$.

So far we have done no more than re-derive the results of Gilman and Harari in a more general formalism, except that we have also been able to obtain the relative signs of some $\lambda = 0$ decay amplitudes. (This sign information is important though, because the two decay chains $A1 \to \rho + \pi \to 3\pi$ and $A1 \to \sigma + \pi \to 3\pi$ can interfere.) However, our use of inelastic as well as elastic sum rules allows us to see a possible reason why θ should have

the very desirable value 45°. Although $m_0^2 + m_4^2$ is diagonal by definition, the chiral-vector part m_4^2 will in general have an off-diagonal π-A1 element proportional to cos 2θ and this means according to Eq. (28) that the tree graphs contribute to the forward scattering amplitude for the process $\pi + \pi \to \pi$ + A1 a term proportional to cos 2θ which goes as ω^0 for $\omega \to \infty$. If we conjecture that the vacuum trajectories with $\alpha(0) \geqq 0$ cannot change a π into an A1, then we must require that this term vanish, and hence that $\theta = 45°$ mod $\pi/2$.[3]

This is all very nice, but it raises more problems than it solves. The amplitudes predicted for A1$\to \rho + \pi$ with $\lambda = 0$ (and also $\lambda = \pm 1$) are in definite disagreement with those obtained from the phenomenological gauge invariant Lagrangians of Schwinger, Wess and Zumino, or from the equivalent "hard-pion" results of Schnitzer and myself. Either we must include more meson states in the algebraic approach or more complicated proper three-point functions in the hard-pion approach, or both. The unification of these different approaches is an open problem of the highest importance.

We could continue in this way to constant matrices $\vec{X}(\lambda)$ and m^2 for each helicity λ. However, the solutions for different helicities are obviously not independent. For one thing, m^2 must obviously be helicity independent, and perhaps (if the P and P' residues are helicity independent) the matrices m_0^2 and m_4^2 are separately independent of λ. For another, the transition $\alpha \to \beta + \pi$ must involve only p-wave pions if α and β are nearly degenerate particles of equal parity; this imposes on $\vec{X}(\lambda)$ a helicity dependence given by the Clebsch-Gordan coefficient $C_{J_\beta 1}(J_\alpha \lambda ; \lambda 0)$. It would be very exciting if these clues could be used to put together all helicity states of the hadrons in a representation of the commutation relations (20) to (22) and (24) to (27) valid for all λ.

As a first step in this direction, let us see what would follow if we supposed that <u>all</u> pion transitions involve only p-wave pions. The most convenient way to express this assumption is to write $X_a(\lambda)$ as the third-component of a three-vector:

$$[X_a(\lambda)]_{\beta\alpha} \equiv (\beta\lambda | D_{a3} | \alpha\lambda)$$

where

$$[J_i, D_{aj}] = i\epsilon_{ijk} D_{ak}$$

$$[T_a, D_{bj}] = i\epsilon_{abc} D_{cj}$$

Here $\vec{J}$ is the usual angular-momentum matrix, but defined to act on helicity indices only. We assume that the commutations of the D_{ai} do not contain terms with T = 2, so as not to find particles with more than a few low isospins in our representations. The most general form for these commutators is then

$$[D_{ai}, D_{bj}] = i\epsilon_{abc} A_{cij} + i\epsilon_{ijk} \delta_{ab} B_k$$

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[3] Van Hove mentions in his paper in these Proceedings that the cross-section $d\sigma/dt$ near t = 0 for the process $\pi + p \to$ A1 + p becomes constant in s as $s \to \infty$, but this value is very much smaller than the corresponding elastic cross-section for $\pi + p \to \pi + p$. If interpreted in terms of Pomeron exchange, this means that the Pomeron "coupling constant" for $\pi \to$ A1 is non-zero but much smaller than that for $\pi \to \pi$.

where A_{cij} is an isovector symmetric three-tensor and B_k is an isoscalar three-vector. The fundamental chiral commutation relation (20) now reads

$$A_{c33} = T_c$$

The Jacobi identity for $[D_{ai}, [D_{bj}, D_{ik}]]$ together with the known properties of T_a and J_k then require that

$$A_{cij} = T_c \delta_{ij}, \quad B_k = J_k$$

Thus D_{ai}, T_c and J_i together constitute the algebra of SU(4). This explains why we can consistently saturate all sum rules for $\pi N \to \pi N$, $\pi N \to \pi \Delta$ and $\pi \Delta \to \pi \Delta$ with just the N and Δ, and why we then find that $g_A/g_V = 5/3$.

By further manipulating Jacobi identities we can show that if m_0^2 and m_4^2 both commute with $\vec{J}$ then m_0^2 is an SU(4) invariant and m_4^2 belongs to a 20-dimensional representation, which in terms of the isomorphic algebra SO(6) is just the antisymmetric tensor of rank three. Thus our assumption of pure p-wave pion transitions has led us to SU(4), in the sense that it is the underlying algebra of the transition matrices, but not in the sense of a general symmetry of nature, for the hadron masses "break" SU(4) in a well-defined way.

Of course, this example has been quite academic; there are many important pion transitions which do not involve p-wave pions. To be a little more realistic, we might suppose instead that all pion decays involve only p-wave or s-wave pions. This requirement may be met by writing

$$[X_a(\lambda)]_{\beta\alpha} = (\beta\lambda | \{S_a \sin\theta + D_{a3} \cos\theta\} | \alpha\lambda)$$

with S_a an isovector three-scalar and D_{a3} as before. The fundamental commutation relation (20) then implies that S_a, D_{ai}, J_i, and T_a all belong to the Lie algebra of SO(7). It is possible that we may be able to fit the more important hadrons into irreducible representations of SO(7), which of course will be reducible under SO(6) $\equiv$ SU(4) and even more reducible for each λ under SU(2) $\otimes$ SU(2). For instance, the antisymmetric SO(7) tensor of rank three contains the π, ρ, σ, A1, ω, B and a mysterious thirty-fifth O^+ meson with T = 0 and G = -1. The mesons with $G\,\Pi\,(-)^J = +1$ are just those whose $\lambda = 0$ states have been discussed above, so for these states SO(7) will reproduce the successful results already derived. A good deal more work must be done to explore the representations of SO(7) and to extend this approach to include d-wave pions.

I said at the beginning that my hope is for a reunification of current algebra. With this end in view I should close by saying a few words about the relation between the present work and the method of saturating current commutation relations at infinite momentum. The $p = \infty$ method was used some time ago to show that pion transition matrices satisfy the chiral commutation relations (20), but to my knowledge it has not been used to obtain the chiral transformation properties of the mass matrix. Perhaps a more significant difference is that the $p = \infty$ method rests on the vanishing of the so-called "pair state" or "Z-graph" contributions, while the methods I have described, both dispersive and perturbative, depend upon assumptions about the asymptotic behaviour of physical scattering amplitudes. It is believed

that these different assumptions are equivalent, and certainly this belief is supported by the equivalence of some of their consequences, but in my opinion there is not yet any complete understanding of the relation between the disappearance of pair states at $p = \infty$ and the vanishing of physical Feynman amplitudes at infinite energy. Lacking such an understanding, it is clearly better to rely on asymptotic assumptions which are verifiable rather than those which are purely formal.

On the other hand, there are problems which are perhaps best approached by the method of infinite momentum. These are the problems which depend on those detailed properties of the currents which are not mere reflections of the underlying chiral symmetry of hadron physics. For instance, I have shown how to derive higher symmetries than chirality by making assumptions about the partial waves which enter in pion transitions, but I would not know how to use the space-space current commutation relations to pick a particular higher symmetry or multipole structure. Similarly, I would not have any idea how to use dispersion theory or phenomenological Lagrangians to pursue the "Gell-Mann program" of exploiting the detailed locality properties of the current commutation relations, and, finally, in what I have described, the quark model is not even useful heuristically. I personally do not feel that these are such important points, because I do not think the currents themselves are as interesting as the symmetry they express.

Lastly, we have before us the unexplored problem of obtaining the algebraic consequences of the requirement that <u>multipion</u> processes have reasonable asymptotic behaviour. Here dispersion relations are unavailable and the $p = \infty$ method is probably inapplicable, so I see no alternative to the further use of chiral-invariant Lagrangians in the tree approximation.

PHENOMENOLOGICAL LAGRANGIANS

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Abstract

PHENOMENOLOGICAL LAGRANGIANS. 1. Introduction; 2. Non-linear chiral symmetry; 3. Beyond non-linear chiral symmetry; 4. Standard forms of non-linear realizations.

1. INTRODUCTION

In this note we describe the group-theoretic structure of phenomenological Lagrangian theories. These theories are expressed directly in terms of the phenomenological fields associated with the actual particles or resonances rather than in terms of fundamental fields (which could be taken to be certain spinor fields). Clearly, from a more basic point of view, the phenomenological fields should be expressed as (complicated) functions of the fundamental fields. In phenomenological Lagrangian theory the question of the connection between the phenomenological and the fundamental level is not considered, and one only attempts to find regularities in the interactions among the phenomenological fields with each other. These regularities may be valid only in a limited domain of energy or momentum transfer (usually small energies) and are usually expressed in group-theoretic language.

We shall attempt to describe the properties of phenomenological Lagrangian theories from the point of view of general structure. Concrete applications are given here only as illustrative examples. Many more applications can be found in the existing literature; we quote only some of the relevant papers. Additional references can be found in Ref. [1].

2. NON-LINEAR CHIRAL SYMMETRY

In most phenomenological field theories the Lagrange density is not an arbitrary function of the fields. The fields transform in some way under an internal symmetry group (such as chiral $SU(2) \times SU(2)$ or chiral $SU(3) \times SU(3)$) and the Lagrange density consists of a main part which is invariant under this group and a symmetry-breaking part which is usually assumed to have simple transformation properties under the group. In general the fields transform non-linearly under the group and their transformation laws become linear only when restricted to a certain subgroup of the full group (in the case of chiral groups this is the subgroup of the parity conserving transformations). For example, in the case of $SU(2) \times SU(2)$, the transformation properties of the pion field $\vec{\phi}$ and of the nucleon field

$$N = \begin{pmatrix} p \\ n \end{pmatrix}$$

under an infinitesimal chiral transformation (a transformation generated by the axial vector charge) are

$$\delta\vec{\xi} = \vec{\alpha}(1 - \vec{\xi}^2) + 2\vec{\xi}\,\vec{\alpha}\cdot\vec{\xi} \tag{1}$$

and

$$\delta N = i(\vec{\xi}\times\vec{\alpha})\cdot\vec{\tau}\,N \tag{2}$$

Here $\vec{\alpha}$ is the group parameter and $\vec{\xi} = a\,\vec{\phi}$ is the pion field up to a normalization factor which turns out to be given by

$$a = \frac{1}{F_\pi} \tag{3}$$

where F_π is the pion decay constant. Under a transformation of the parity-conserving isospin subgroup, the fields $\vec{\phi}$ and N transform linearly in the obvious way indicated by the fact that they are, respectively, an isovector and an isospinor

$$\delta\vec{\xi} = -2\vec{\beta}\times\vec{\xi} \tag{4}$$

$$\delta N = i\vec{\beta}\cdot\vec{\tau}N \tag{5}$$

The above non-linear chiral transformations should be compared with those assumed, e.g. in the τ - model, where the nucleon and the pion fields transform linearly [2]. In the τ-model, by a chiral transformation, the nucleon field, which we call now ψ, transforms as

$$\delta\psi = i\vec{\alpha}\cdot\vec{\tau}\gamma_5\psi \tag{6}$$

while the pion field $\vec{\pi}$, together with a scalar isoscalar field σ, transforms as

$$\begin{aligned}\delta\vec{\pi} &= -2\vec{\alpha}\sigma\\ \delta\sigma &= 2\vec{\alpha}\cdot\vec{\sigma}\end{aligned} \tag{7}$$

The need to introduce the field σ stems from the fact that the group $SU(2)\times SU(2)$ (plus parity) does not have an irreducible three-dimensional representation; the smallest representation in which one can accommodate the pion is the four-dimensional representation, together with the σ. In contrast, the non-linear realization (1) operates on the same three-component object on which the isospin transformation (3) operates. In general, as we shall see, if the isospin transformation of a field χ is

$$\delta\chi = 2i\vec{\beta}\cdot\vec{T}\chi \tag{8}$$

then the chiral non-linear transformation of that field is

$$\delta\chi = i(\vec{\xi}\times\vec{\alpha})\cdot\vec{T}\chi \tag{9}$$

The only exception is the field ξ itself which transforms as in relation (1), the basic difference between the expressions (1) and (2), or (9) being that

relation (1) has no fixed point while $N = 0$ or $\chi = 0$ are fixed points for expressions (2) and (9). From the transformation (9) we see that, in the non-linear method, there is no increase in the dimensionality of the multiplets when one enlarges the group from the parity-conserving subgroup to the full chiral group. A symmetry realized non-linearly as described here has completely different physical consequences than one which is realized linearly. Instead of the usual relations between different particles of a linear multiplet, one finds here connections between processes involving different numbers of pions. These relations are the same as those obtained from the soft pion method and the non-linear Lagrangian technique can be considered as an alternative way to derive soft pion results, which is perhaps simpler than that based on a direct use of current algebra and conservation or partial conservation equations for the currents. An interesting feature of non-linear transformations like (5) is that the expression $\bar{N}N$ is an invariant (while $\psi\psi$ is not invariant under the transformation (6)). This fact permits mass terms in the Lagrangian to be introduced without spoiling the symmetry, even in a formal way, for all fields other than the pion field. A mass term for the pion field will break the symmetry.

To give a concrete example we shall, first, limit ourselves to a system consisting of nucleons and pions only. A Lagrangian invariant under transformations (1) and (2) (and also, of course, (4) and (5)) is

$$L = -\frac{1}{2}\frac{(\partial_\mu \phi)^2}{(1 + a^2\phi^2)^2} + i\bar{N}(\gamma_\mu\partial_\mu + m_N)N$$
$$- \frac{1}{1 + a^2\phi^2}\left\{\frac{f}{m_\pi}\bar{N}\gamma_\mu\gamma_5\tau N\,\partial_\mu\phi + a^2\bar{N}\gamma_\mu\tau N(\phi\times\partial_\mu\phi)\right\} \qquad (10)$$

We shall describe below the method used to construct invariant Lagrangians. Observe that Lagrangian (10), when expanded in the field ϕ, generates multi-pion vertices. Vertices with different numbers of pions are related to each other in a well defined way involving the parameter a. The pseudovector coupling constant $f(f^2/4\pi = 0.08)$ and the constant a (which is determined from low-energy S-wave pion-nucleon scattering to be $a = 0.8/m_\pi$) are not related by the requirement of chiral invariance.

To L one must add a symmetry-breaking term which, at the same time, gives the pion a mass

$$L'(\phi^2) = -\frac{1}{2}m_\pi^2\phi^2 + \ldots \qquad (11)$$

where the dots denote higher powers of ϕ. Different choices for the function $L'(\phi^2)$ give rise to different pion-pion interactions. One may determine this symmetry-breaking term by the requirement that it transform in a particular way under chiral transformations. For instance the choice

$$L' = -\frac{1}{2}m_\pi^2\frac{\phi^2}{1 + a^2\phi^2} \qquad (12)$$

is the sum of a chiral invariant and of the fourth component of a chiral four vector and seems to be favoured by the experiments.

From the Lagrangian L and the known transformation laws (1), (2), (4) and (5) of the fields one can generate the currents in a well-known way. They are complicated non-linear functions of the fields. The first terms in an expansion in ϕ turn out to be, for the vector current,

$$V_\mu = \bar{N}\gamma_\mu \frac{\tau}{2} N - \phi \times \partial_\mu \phi - \frac{2f}{m_\pi} \phi \times \bar{N}\gamma_\mu \gamma_5 \frac{\tau}{2} N + \ldots \tag{13}$$

and for the axial vector current

$$A_\mu = \frac{f}{am_\pi} \bar{N}\gamma_\mu \gamma_5 \frac{\tau}{2} N + \frac{1}{2a} \partial_\mu \phi - \frac{2a^2 m_\pi}{f} \phi \times \bar{N}\gamma \frac{\tau}{2} N + \ldots \tag{14}$$

This expression for the axial vector current implies the relation (3) and it also gives

$$-\frac{G_A}{G_V} = \frac{f}{am_\pi} \tag{15}$$

This relation connects G_A/G_V to the strong interaction parameters f and a and can be interpreted as a form of the Adler-Weissberger relation. Combined with Eq.(3), Eq. (15) gives the Goldberger-Treiman relation expressed in terms of the pseudo-vector coupling constant f. Observe that, since $f \simeq 1.0$, we obtain $-G_A/G_V \cong 1.0/0.8 \cong 1.2$

For a more complete description of low-energy pion-nucleon scattering one must introduce into the theory also the N* resonance. It is not difficult to include it in a chiral invariant Lagrangian, since its transformation properties are given by equations like Eqs (8) and (9) with the appropriate matrix $\vec{T}$. To describe the space-time properties one can use, e.g., a Rarita-Schwinger field χ_μ. Pion-nucleon scattering at low energy can be described by the sum of the graphs (crossed graphs must also be included) shown in Fig. 1.

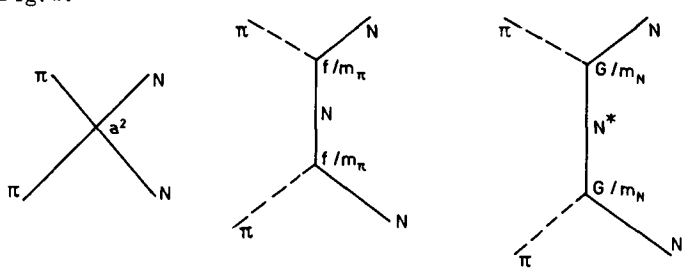

FIG. 1. Graphs of pion-nucleon scattering at low energies.

The $NN^*\pi$ coupling constant G appears in the interaction

$$\frac{G}{m_N} (\partial_\mu \phi) \bar{N}\chi_\mu + \quad \text{Hermitian conjugate}$$

In using phenomenological Lagrangians one follows the prescription that, for each process (given external lines), one includes only all tree diagrams (no internal loops) which contribute to it. We call this the "phenomenological approximation". Figure 1 shows all the tree diagrams contributing to pion-nucleon scattering within the framework of a model including only pions, nucleons and the N*.

3. BEYOND NON-LINEAR CHIRAL SYMMETRY

Chiral symmetry does not determine the ratio f/am_π and therefore it leaves the value of G_A/G_V undetermined. To obtain further connections among the coupling constants one may superimpose on the theory other symmetries, such as some version of SU(6) symmetry [3]. However, since SU(6) is not understood as a Lagrangian symmetry, we do not favour this procedure. An alternative approach will be discussed in the following.

In view of the high non-linearities and derivative couplings, the scattering amplitudes calculated in the phenomenological approximation using a Lagrangian such as expression (10) have a very bad behaviour at very high energies. The requirement that this behaviour be improved as a consequence of cancellations among the tree diagrams will give rise to connections among the coupling constants of the theory. Consider, for instance, the tree diagrams given above for pion-nucleon scattering. For $t = 0$ (forward scattering) and $s \to \infty$, the isospin antisymmetric part of the scattering amplitude has a term which behaves asymptotically like a constant. If we require that this term vanish we obtain the relation

$$\left(\frac{f}{m_\pi}\right)^2 = a^2 + \frac{(m_N + m_{N^*})}{18\, m^2/N^*}\left(\frac{G}{m_N}\right)^2 \tag{16}$$

Remembering relation (15) and introducing the equivalent pseudo-scalar nucleon-pion coupling constant G_{PS} by

$$\frac{G_{PS}}{2m_N} = \frac{f}{m_\pi}$$

we obtain

$$\left(\frac{G_V}{G_A}\right)^2 = 1 - \frac{4m_N(m_N + m_{N^*})}{9m_{N^*}^2}\left(\frac{G}{G_{PS}}\right)^2 \tag{17}$$

For $m_N = m_{N^*}$ this equation is satisfied by the SU(6) values

$$-\frac{G_V}{G_A} = \frac{3}{5} \qquad \frac{G}{G_{PS}} = \frac{\sqrt{2}}{5}\,3 \tag{18}$$

To improve on these values one must introduce other pion-nucleon resonances.

This way of obtaining SU(6) type results by superimposing on a chiral symmetric theory requirements on its high-energy behaviour seems very promising. Until now the following general situation has been studied [4]. One considers the (possibly inelastic) scattering $\pi + X \to \pi + Y$ (Fig. 2)

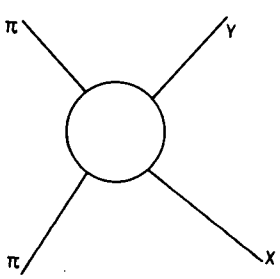

FIG. 2. Graph of $\pi + X \to \pi + Y$ scattering.

where X and Y are any two particles or resonances, and calculates the scattering amplitude in the phenomenological approximation by using as intermediate states any particles or resonances that may occur. One then imposes appropriate requirements for the behaviour of the amplitude for $t = 0$ and $s \to \infty$. Precisely, one requires that the part of the amplitude which is antisymmetric in the isospin indices a and b of the pions should vanish at infinite s and that the part which is symmetric should tend to a constant proportional to δ_{ab}. These asymptotic requirements correspond to the assumption that the amplitude calculated with tree diagrams should not behave worse at high energies than is expected of the physical amplitude, for which we accept the behaviour suggested by Regge-pole theory. They give rise to a number of relations between masses and coupling constants. In principle, one should use all intermediate states which may occur, and these are in general an infinite number. However, in practice, one restricts oneself to a finite number of intermediate states, as in the above example of pion-nucleon scattering.

In carrying out this programme one finds that it is necessary to complete the linear multiplets of $SU(2) \times SU(2)$. Thus, while a non-linear chiral theory requires only the pion field, all the convergence requirements at infinity cannot be satisfied without again introducing the scalar isoscalar field σ into the theory which is to be introduced as a chiral invariant, with its own coupling constants, independent of those of the pion. The high-energy convergence requirements then give relations between coupling constants and masses.

4. STANDARD FORMS OF NON-LINEAR REALIZATIONS

We make first some useful observations. Assume that a non-linear Lagrangian is known in terms of a set of field variables ϕ

$$L[\phi] = L_0[\phi] + L_1[\phi] \tag{19}$$

where $L_0[\phi]$ is the Lagrangian of free fields, and $L_1[\phi]$ the interaction. Now express the field variables ϕ as non-linear but local functions of another set of field variables χ

$$\phi = \chi F[\chi], \quad F[0] = 1 \tag{20}$$

The resulting Lagrangian can always be separated into a free-field Lagrangian and an interaction

$$L[\chi F[\chi]] \equiv L_0[\chi] + L_2[\chi] \tag{21}$$

The results obtained from the Lagrangian $L[\phi]$ (for example, by means of suitable Feynman rules) can be compared with those obtained from

$$\hat{L}[\phi] = L_0[\phi] + L_2[\phi] \tag{22}$$

A theorem of relativistic field theory states that, although the many-particle propagators will in general differ in the two cases when the momenta of the external lines are off the mass shell, the on-mass shell S-matrices calculated with $L[\phi]$ and $\hat{L}[\phi]$ are identical (it may be necessary

to introduce appropriate wave-function renormalizations). This result has an analogue in axiomatic field theory [5] (irrelevance of the choice of a particular set of local interpolating fields). Stated in the simplest way the reason for its validity is that the terms higher than linear in the expansion of the function $\chi F[\chi]$ do not contribute on the mass shell because they do not contain one-particle singularities. We have stated the above theorem as a property of the exact solution, but it is not difficult to show that it is also valid in the phenomenological approximation of tree diagrams (actually the result is also true if one includes diagrams with only one internal integration or with only two internal integrations etc.; for tree diagrams, of course, no wave-function renormalizations are necessary).

If a set of fields ϕ transforms according to a certain non-linear realization of a group, another set of fields χ which are local non-linear functions of ϕ as in expression (20), will also transform according to a non-linear realization of that group. In view of the dynamical equivalence of ϕ and χ we shall define two such realizations of the group (obtained from one another by a non-linear transformation of the variables in the field manifold) as equivalent. Observe that, for $\chi = 0$ also $\phi = 0$. Therefore, in the field manifold there is a special point, the origin, and only such changes of variables are allowed which leave the origin unchanged.

The internal symmetry group G operates on the field manifold and there may be a subgroup H which leaves the origin fixed. It is possible to show that the co-ordinates on the field manifold can be chosen such that a group which leaves the origin fixed operates linearly. In addition, the co-ordinates can be further chosen so that the group transformations take a standard form, which will be given below. These considerations explain why we are led to the general problem of finding all the non-linear realizations of the internal symmetry group G which become linear when restricted to a given subgroup H. The solution of this problem is given by the standard forms of non-linear realizations given below [6].

Let G be a compact, connected, semi-simple Lie group with n parameters and H a continuous subgroup of G. We denote with V_i ($i = 1, 2, \ldots n-d$) the generators of H and with A_ℓ ($\ell = 1, 2, \ldots d$) the remaining generators chosen so that V_i and A_ℓ together form a complete set of generators of G orthonormal with respect to the Cartan inner product (for $SU(3) \times SU(3)$ one may think of $V_i = -i\lambda_i$, $A_\ell = -i\gamma_5 \lambda_\ell$ where the λ are Gell-Mann matrices). Every element $g \in G$ can be decomposed, in some neighbourhood of the identity into a product of the form[1]

$$g = e^{\xi \cdot A} e^{uV} \tag{23}$$

[1] This decomposition corresponds to a particular parametrization of the left cosets G/H by means of the parameters ξ_ℓ. Any other parametrization would give rise to a completely equivalent treatment. However, the particular parametrization used here is very convenient, in general, because of the simple and well-known properties of exponentials. Observe that in section 2 the parameter $\vec{\xi}$ has a different, although equivalent, meaning. There we were working with the group $SU(2) \times SU(2)$. For this group it is very convenient to use the Cayley parametrization of the cosets in which one writes, instead of the exponential

$$e^{\xi \cdot A}$$

the Cayley form

$$\frac{1 + \frac{1}{2}\xi \cdot A}{1 - \frac{1}{2}\xi \cdot A}$$

For $SU(2) \times SU(2)$ this Cayley parametrization has been used extensively in applications, but for a general group, including $SU(3) \times SU(3)$, the Cayley parametrization presents some formal complications and the exponential parametrization is more convenient.

where

$$\xi \cdot A = \sum_{\ell} \xi_{\ell} A_{\ell} \quad u \cdot V = \sum_{i} u_i V_i$$

and ξ_ℓ and u_i are real parameters. Therefore, for any element $g_0 \in G$ one can write

$$g_0 e^{\xi \cdot A} = e^{\xi' \cdot A} \cdot e^{u' \cdot V} \tag{24}$$

where

$$\xi' = \xi'(\xi, g_0), \quad u' = u'(\xi, g_0)$$

are functions of the indicated variables which are determined by the structure of the group. Let further

$$\psi \to D(h)\psi \tag{25}$$

with $h \in H$ be a linear representation of the subgroup. It is easy to verify that the transformations

$$g_0 : \quad \xi \to \xi', \quad \psi \to D(e^{u' \cdot V})\psi = e^{u' \cdot T_i}\psi \tag{26}$$

give a (non-linear) realization of G. Here T_i are the matrices which represent the generators V_i in the representation $D(h)$. Clearly the transformation on ξ can be considered a group realization by itself. The transformation on ψ, on the other hand, is meaningful only together with that on ξ since u' is a function of ξ and therefore also $D(e^{u' \cdot V})$. If it is reducible, the field ψ decomposes, by a suitable choice of basis, into a set of fields $\psi^{(a)}$ with a smaller number of components. These fields do not mix. We shall refer to expression (26), with D written as a fully decomposed representation as to the fundamental standard form for a realization. As explained above, expression (26) is a standard form in the sense that any realization of G by transformations induced on the field manifold, which become linear when restricted to the subgroup H, can be put into the form (26) by an allowed change of co-ordinates on the field manifold.

It is easy to verify that, if g_0 belongs to the subgroup H, ($g_0 \subset h$), the transformation (26) becomes linear

$$\xi \to \xi' = D^{(b)}(h)\xi, \quad \psi \to D(h)\psi \tag{27}$$

where $D^{(b)}$ is the linear representation given by

$$e^{\xi' \cdot A} = h e^{\xi \cdot A} h^{-1} \tag{28}$$

and is uniquely determined by the structure of G.

Observe that, if a field Ξ transforms according to a linear irreducible representation $\mathscr{D}(g)$ of the group G

$$\Xi \to \Xi' = D(g)\Xi \tag{29}$$

the field Ψ defined by

$$\Psi = D(e^{-\xi\cdot A})\,\Xi \tag{30}$$

transforms as

$$\Psi \to \Psi' = D(e^{u'\cdot V})\,\Psi \tag{31}$$

which is the same as the standard form (26). Now $\mathscr{D}(h)$ may be reducible, in which case the field Ψ breaks up into a set of fields $\Psi^{(a)}$ with a smaller number of components. In this case the apparent physical connection among the larger number of components of the field Ξ must be illusory since the physical consequences of the theory must be the same whether one uses the field Ξ or the fields $\overline{\Psi}^{(a)}$. In a Lagrangian theory employing the field Ξ this can be seen from the possibility of constructing a sufficient number of additional invariants which, when added to the Lagrangian, destroy the spurious relations among the amplitudes. In any case, the dimensionality of the possible multiplets is that of the irreducible linear representations of the subgroup H and not that of the irreducible linear representations of G. So, for $SU(3) \times SU(3)$ one realizes the symmetry with eight pseudo-scalars, described up to a factor by the parameters ξ_ℓ (not nine pseudo-scalars or eighteen scalars plus pseudo-scalars), eight baryons, ten baryon resonances, etc. Naturally, in a Lagrangian theory one may well choose to use the linearly transforming field Ξ and not to include in the Lagrangian the additional invariants which purely group-theoretic considerations would allow. In this case one will obtain stronger results, but one should be clear that they do not follow from group invariance alone.

5. CONSTRUCTION OF INVARIANT LAGRANGIANS

This proceeds through the introduction of "covariant" derivatives, which are standard forms for the field gradients, in the sense that they transform just like the fields, while the simple field gradients do not [7]. The covariant derivatives $D_\mu \xi$ and $D_\mu \psi$ of the pseudo-scalar field ξ and of any other field ψ are defined as

$$\begin{aligned} D_\mu \xi &= p_\mu \\ D_\mu \psi &= \partial_\mu \psi + v_\mu T \psi \end{aligned} \tag{32}$$

where p_μ and v_μ are given by

$$e^{-\xi\cdot A}\,\partial_\mu e^{\xi\cdot A} = v_\mu\cdot V + p_\mu\cdot A \tag{33}$$

The matrices T_i are defined in expression (26). It is easy to see that the covariant derivatives transform as

$$\begin{aligned} (D_\mu \xi)' &= D^{(b)}(e^{u'\cdot V})\,(D_\mu \xi) \\ (D_\mu \psi)' &= D(e^{u'\cdot V})\,(D_\mu \psi) \end{aligned} \tag{34}$$

Equation (33) can be used to compute p_μ and ν_μ as functions of ξ and $\partial_\mu \xi$. Clearly

$$p_\mu = \partial_\mu \xi + \ldots$$

where the dots denote non-linear terms. From the transformation properties (26) and (34) it follows that any (local) function of ψ, $D_\mu \psi$ and $D_\mu \xi$ only

$$L(\psi, D_\mu\psi, D_\mu\xi)$$

which is invariant under H will automatically be invariant under the full group.G. It is not difficult to show that all invariant Lagrangians can be obtained in this way. By this method, one can obtain [8], e.g. the Lagrangian (10) for SU(2) × SU(2).

It is not difficult to generalize these considerations to the case in which the elements of G become co-ordinate-dependent (gauge groups). One introduces a set of gauge fields $\rho_{\mu i}$ and $a_{\mu \ell}$ associated, respectively, with the generators V_i and A_ℓ. Let their transformation law be given by

$$\rho'_\mu \cdot V + a'_\mu \cdot A = g(\rho_\mu \lambda \cdot V + a_\mu A) g^{-1} - \frac{1}{\lambda} (\partial_\mu g) g^{-1} \tag{35}$$

where λ is a constant which, as it turns out, gives the strength of the universal coupling of the gauge field to all other fields. Instead of defining $v_{\mu i}$ and $p_{\mu \ell}$ by (33) we define them now by

$$e^{-\xi \cdot A} \{\partial_\mu + \lambda (\rho_\mu \cdot V + a_\mu \cdot A)\} e^{\xi \cdot A} = v_\mu \cdot V + p_\mu \cdot A \tag{36}$$

and take for the covariant derivatives the expressions (32) but with the new meaning of v_μ and p_μ. It is easy to verify that relations (34) are now valid even if the group element is co-ordinate-dependent. The most general invariant Lagrangians can be obtained by adding to the generalized Yang-Mills Lagrangian [9] for the gauge fields ρ_μ and a_μ any local function of ψ, $D_\mu \psi$ and $D_\mu \xi$ which is invariant under H. A mass term for the gauge fields reduces again the invariance under the gauge to that under the co-ordinate-independent group. Observe that the gauge fields transform linearly as given by the Law (35). This linear transformation property is chosen because of the close connection between the gauge fields and the currents of the weak and electromagnetic interactions, a connection which, in the algebra of fields approach [10], is a direct proportionality.

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WEAK INTERACTIONS AND CPT ASYMMETRIES

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Abstract

WEAK INTERACTIONS AND CPT ASYMMETRIES. 1. Introduction; 2. General symmetry properties of weak interactions; 3. CP asymmetry; 4. Phenomenological Lagrangian and its selection rules; 5. Limitation of Fermi's theory and higher-order weak interactions.

1. INTRODUCTION

When Abdus Salam asked me to review in this Symposium the present status of weak interactions and CPT asymmetries, I thought this would be an easy task. It is well known that the status of this year does not differ too much from that of last year, which in turn is about the same as that of the year before last, etc. Therefore, I could simply refer you to the annual report of the 1967 International Conference on High Energy Physics, which is also quite similar to the 1966 annual report. However, such a description would not do justice to the enormous efforts during the past few years. Indeed, a number of new experimental results have been reported. The apparent slowness of progress lies in the curious, but perhaps profound, difficulty that in spite of the small probable errors quoted in some of these publications, very similar experiments performed by different groups, and sometimes even by the same group but at different times, may give contradictory results.

It should be emphasized that such a phenomenon of mutual contradiction has been an almost integral part of weak interaction research for quite some time. A classic example is the Michel ρ-parameter [1] in μ-decay

$$\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$$\mu^+ \longrightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

The normalized momentum spectrum of $e^\pm$ is given by

$$N(x) = 6x^2\left[2(1-x) + \frac{4}{9}(-3+4x)\rho\right]$$

where

$$\int_0^1 N(x)\,dx = 1$$

and x is related to the magnitude of the electron momentum p_e and the muon mass m_μ ($\hbar = c = 1$) by

$$x = \frac{p_e}{\frac{1}{2}m_\mu}$$

As one can see from Fig.1, different ρ-values give very different shapes of the electron spectrum. The theoretical value based on the two-component neutrino theory [2] gives

$$\rho_{th} = \frac{3}{4}$$

which, as shown in Fig. 2, is in excellent agreement with the present experimental value [3]

$$\rho_{exp} = 0.747 \pm 0.005$$

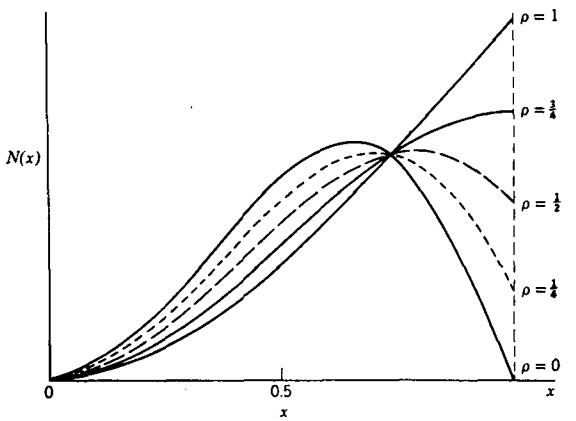

FIG. 1. Shapes of the electron spectrum for various values of ρ.

The theoretical value was calculated near the end of 1956. Figure 3 shows that before 1956 some rather different experimental values have been reported. (The first correct ρ-value – within one standard deviation – was determined by Sargent et al. [4].) Similar fluctuations and changes have also been observed in many other experiments, for example the measurements on the π_{e2} decay rate, the experiments on angular correlations between the β-particle and the recoiled ^{6}He nucleons, the more recent results on the $2\pi^0$ decay of the long-lived K-meson, etc.

Such evolutionary changes certainly make the task of a reviewer difficult. In this summary paper, an effort will be made to distinguish the different degrees of reliability of various experiments and theories. I will, now and then, follow the admirable example of the Guide Michelin, and use the rating symbols of *, ? and X. These ratings are quite subjective; their yearly changes might give a more truthful representation of our understanding of the subject.

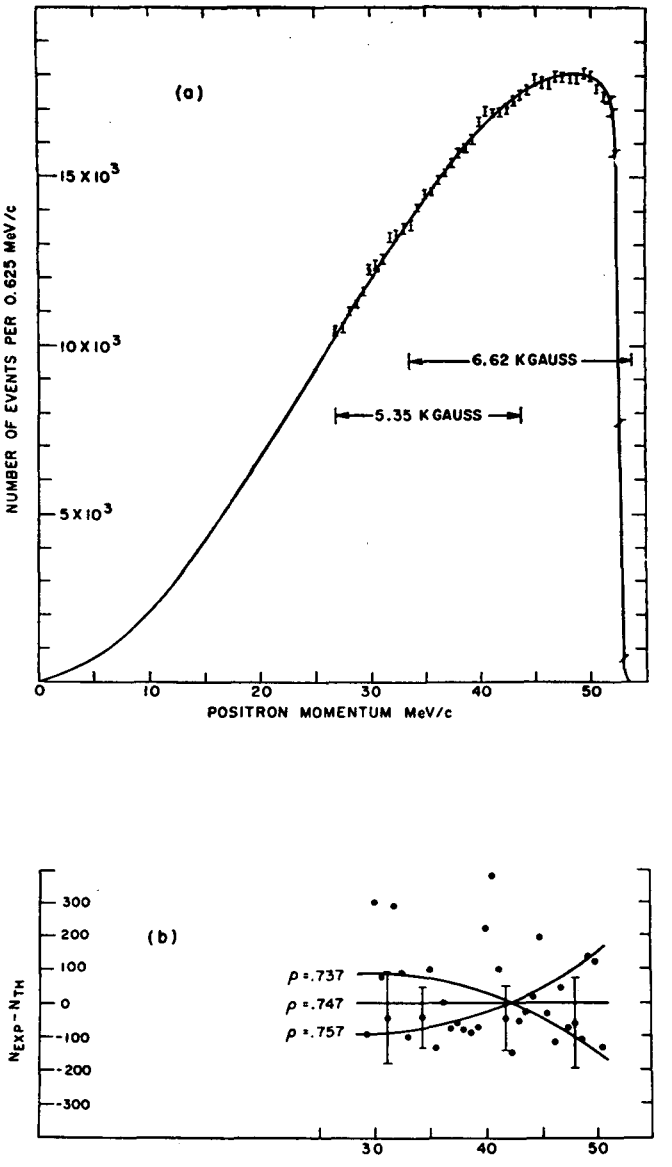

FIG. 2. a) Number of events per 0.625 MeV/c ; b) experimental value of ρ.

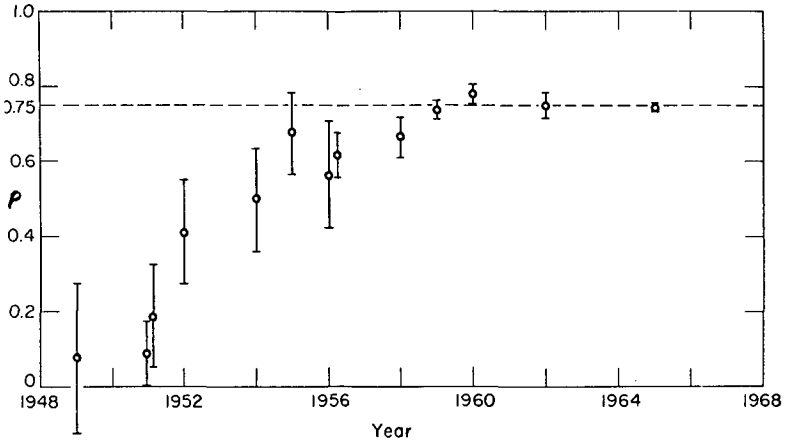

FIG. 3. Experimental values of ρ reported since 1948.

2. GENERAL SYMMETRY PROPERTIES OF WEAK INTERACTIONS

To illustrate the use of such a rating system, we may first discuss those general properties of the weak interaction that are expected to hold to all orders of the weak interaction (for details see, for example, the review article by Lee and Wu [5]).

***	Lorentz invariance Charge conservation Baryon number conservation
**	CPT symmetry Two-component theory of the neutrino
*	Conservation of lepton numbers, L_e and L_μ μ-e symmetry
?	$U_2 \times U_2$ symmetry among leptons

All these symmetry properties are consistent with our present knowledge of the weak interaction. The best evidence of CPT symmetry comes from the mass equality[1]

$$\langle K^0 | H | K^0 \rangle = \langle \overline{K}^0 | H | \overline{K}^0 \rangle$$

[1] The K_1^0 and K_2^0 mass difference has been measured by many groups, see Ref. [6].

which is accurate to 1 part in 10^{14}, where H is the total Hamiltonian; in addition, there are the lifetime measurements [7]

$$\left|\frac{\Delta\tau}{\tau}\right| < \begin{cases} 10^{-3} & \text{for } K^{\pm} \\ 10^{-3} & \text{for } \mu^{\pm} \\ 6 \times 10^{-3} & \text{for } \pi^{\pm} \end{cases}$$

where $\Delta\tau$ is the difference between the lifetime τ of the particle and that of its antiparticle. One notes that the CPT operation used in quantum field theory is by no means an obvious one. The requirement of CPT invariance states that when one changes particles to antiparticles, but keeping their momenta fixed and helicities reversed, all matrix elements of the interaction should become their respective complex conjugates. None of these experiments has any bearing on the particular anti-unitary nature of CPT symmetry; this accounts for its ** rating in the above list, instead of ***.

Before 1956, the space inversion P and the particle-antiparticle conjugation C would probably have belonged to the *** rank. Now both are removed from the list. Before 1964, CP would probably have been rated as, perhaps, a * symmetry; it is now also removed from the list.

Throughout the lecture, C, P and the time reversal operator T denote, respectively, the usual C_{st}, P_{st} and T_{st}, determined by the strong interaction.

3. CP ASYMMETRY

The first experiment observing CP asymmetry was reported by Christenson et al. [8], who found that the long-lived component K_2^0 of the neutral K-meson has a 2π decay mode

$$K_2^0 \rightarrow \pi^+ + \pi^-$$

Since both the particle-antiparticle conjugation operator C and the space inversion P (in the centre-of-mass system) interchange π^+ and π^-, the final (2π) system in this decay must be of CP = +1. On the other hand, the same long-lived component K_2^0 also decays into 3π,

$$K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0$$

in which the π^+ and π^- are observed to be predominantly in the S-state. Thus, the final 3π-state is (or, at least, predominantly is) of CP = -1.

Now, K_2^0 is, by definition, one with a definite lifetime. The fact that it can decay into different states with opposite CP values shows conclusively that CP is not conserved in the K_2^0 decays. The CP violation is, therefore, established independently of the detailed theory of the K_1^0, K_2^0 system.

More recently, CP violation has also been observed in

$$K_2^0 \longrightarrow 2\pi^0$$

$$K_2^0 \longrightarrow \begin{cases} \pi^- + e^+ + \nu_e \\ \pi^+ + e^- + \bar{\nu}_e \end{cases}$$

and

$$K_2^0 \longrightarrow \begin{cases} \pi^- + \mu^+ + \nu_\mu \\ \pi^+ + \mu^- + \bar{\nu}_\mu \end{cases}$$

So far, there is no direct observation of time reversal asymmetry. If CPT invariance is assumed, then CP asymmetry implies T asymmetry.

The amplitudes of CP violation are uniformly small. For the 2π decay modes of K_2^0, it is useful to define

$$\eta_{+-} = \frac{\text{Amplitude } (K_2^0 \rightarrow \pi^+\pi^-)}{\text{Amplitude } (K_1^0 \rightarrow \pi^+\pi^-)}$$

and

$$\eta_{00} = \frac{\text{Amplitude } (K_2^0 \rightarrow 2\pi^0)}{\text{Amplitude } (K_1^0 \rightarrow 2\pi^0)}$$

The values $|\eta_{+-}|^2$ and $|\eta_{00}|^2$ are directly obtainable from the observed decay rates. The present experimental values are

$$*** \quad |\eta_{+-}|^2 = (3.6 \pm 0.2) \times 10^{-6}$$

and

$$|\eta_{00}|^2 = (24 \pm 5) \times 10^{-6} \quad \text{(Ref. [9])} .$$

$$(19 {}^{+9.5}_{-6.9}) \times 10^{-6} \quad \text{(Ref. [10])}$$

$$(-2 \pm 7) \times 10^{-6} \quad \text{(Ref. [11])}$$

The last value is obtained through a subtraction of 102 expected background events from a total of 100 possible candidates. One may mention that the earlier experiment on $|\eta_{00}|^2$ has just lost its ** rating; the authors are encouraged to improve the experiment and to return to its former high standing.

In addition, the phase of η_{+-} has been measured. Its present value [12] is reported to be

$$\text{phase } (\eta_{+-}) = (51.2 \pm 11)^\circ$$

For the leptonic three-body decay modes, the CP violation is usually expressed in terms of the parameter

$$4 \operatorname{Re} \epsilon = \frac{\text{Rate } (K_2^0 \to \pi^- \ell^+ \nu_\ell)}{\text{Rate } (K_2^0 \to \pi^+ \ell^- \bar{\nu}_\ell} - 1$$

The present experimental value [13, 14] of the right-hand side is

$$(8.1 \pm 2.7) \times 10^{-3} \qquad \text{for } \ell = \mu$$

and

$$(4.48 \pm 0.72) \times 10^{-3} \qquad \text{for } \ell = e$$

At present, there are certain doubts as to the precise nature of the CP violating interaction H_-, defined by

$$H_- = \tfrac{1}{2}[H - (CP)H(CP)^{-1}]$$

where H is the total Hamiltonian. Table I summarizes the present theoretical status.

TABLE I. SUMMARY OF PRESENT STATUS OF CP VIOLATION

	Coupling constant of H_-	Selection rule
? Super-weak [15]	$\sim 10^{-9}\, G_{wk}$	$\Delta S = \pm 2$ allowed
? Milli-weak [16] [a]	$\sim 10^{-3}\, G_{wk}$	$\Delta S = \pm 1$ allowed $\Delta S \neq 2$
? Electromagnetic [17] ? Milli-strong [18]	e $\sim 10^{3}\, G_{wk}$	$\Delta S = 0$ only P conserved C not conserved

[a] The possibility that the CP violation is due to the weak interaction is an obvious one after the experimental discovery of $K_2^0 \to \pi^+\pi^-$. The precise mechanism of such a weak CP violation has been the subject of many theoretical papers. Ref. [16] gives only an incomplete list of these papers.

If H_- is the super-weak interaction, then neglecting effects $O(10^{-9})$, only in the decay of K_1^0 and K_2^0 can one detect any CP violations; furthermore, all such CP violations are determined by the state-vectors of K_1^0 and K_2^0. If one assumes CPT invariance, the K_1^0 and K_2^0 states are related to the states K^0 and $\bar{K}^0$ by

$$K_1^0 = [2(1 + |\epsilon|^2)]^{-\frac{1}{2}} [(1+\epsilon)K^0 + (1-\epsilon)\bar{K}^0]$$

and

$$K_2^0 = [2(1 + |\epsilon|^2)]^{-\frac{1}{2}} [(1+\epsilon)K^0 - (1-\epsilon)\bar{K}^0]$$

where

$$\overline{K}^0 = (CPT)K^0$$

In the super-weak case

$$\epsilon = \eta_{+-} = \eta_{00}$$

in addition [19]

$$\text{phase } \epsilon = \tan^{-1} \frac{2(m_2 - m_1)}{\gamma_2 - \gamma_1} \cong 44°, \text{ or } 224°$$

where m_i and γ_i are, respectively, the mass and the width of K_i^0 (i=1, 2). These values are quite consistent with the present experimental situation. In this case, for all practical purposes, we have already determined all that could be measured concerning CP violating parameters. The super-weak interaction has a certain appeal, because, then, we do not have to think any more.

If the CP violations are due to a milli-weak interaction, then there should be other weak processes besides K_1^0 and K_2^0 decays that exhibit some milli-CP-violating amplitudes. If the CP non-conservation interaction belongs to the milli-strong case, then, in addition, there should be some strong processes that exhibit some small C or T violations. If the CP non-conserving interaction is the electromagnetic interaction, then, in addition, there should be some electromagnetic processes that have sizable C, T violating amplitudes.

For example, the decay

$$\eta^0 \rightarrow \pi^+ \pi^- \pi^0$$

is forbidden by the known strong interaction, but allowed by the known electromagnetic interaction. If the CP violating interaction belongs either to the electromagnetic case or to the milli-strong case, then in this η^0 decay there could be a small $\pi^\pm$ asymmetry $\sim 1\%$ to 0.1%, depending on the isospin selection rule of the CP violating interaction. The asymmetry parameter A is defined to be

$$A = \frac{N(E_{\pi^+} > E_{\pi^-}) - N(E_{\pi^+} < E_{\pi^-})}{N(E_{\pi^+} > E_{\pi^-}) + N(E_{\pi^+} < E_{\pi^-})}$$

The recent experiment [20] of 36 500 η^0 events gives

$$A = (1.5 \pm 0.5) \times 10^{-2}$$

as compared to the previous values [21]

??? $\quad A = (7.2 \pm 2.8) \times 10^{-2}$ (1351 events)

and [22]

$$A = (0.3 \pm 1.0) \times 10^{-2} \qquad (10\,665 \text{ events})$$

Through interactions which violate P and T, the neutron can acquire an electric dipole moment. The present experimental value is

$$\mu(n) = (-2 \pm 3) \times 10^{-22} \text{e cm}$$

reported by Miller et al. [23], and

$$\mu(n) = (+2.4 \pm 3.9) \times 10^{-22} \text{ e cm}$$

reported by Shull and Nathans [24]. While these results are still consistent with all four general cases of CP violating interactions, they have ruled out many specific theoretical models of CP violation.

Needless to say, all of these experiments are exceedingly difficult; in my opinion, many certainly deserve to be in the * class.

4. PHENOMENOLOGICAL LAGRANGIAN AND ITS SELECTION RULES

We now turn to other aspects of weak interactions. All known weak processes can be classified phenomenologically into three classes: purely leptonic, semi-leptonic and non-leptonic, described respectively by the following three terms of the effective Lagrangian

$$\mathscr{L}_{wk} = \mathscr{L}_{\ell\ell} + \mathscr{L}_{\ell h} + \mathscr{L}_{hh}$$

where the subscript ℓ denotes leptons and h hadrons. Among these, the μ-decay part of $\mathscr{L}_{\ell\ell}$ [5] is known explicitly:

$$** \quad (\mathscr{L}_{\ell\ell})_{\mu\text{-decay}} = \frac{G_{wk}}{\sqrt{2}} (j_\lambda)^*_\mu (j_\lambda)_e + \text{h.c.}$$

where G_{wk} is the Fermi constant $\cong 10^{-5}\, m_N^{-2}$,

$$(j_\lambda)_\ell = i\psi^\dagger_\ell \gamma_4 \gamma_\lambda (1+\gamma_5) \psi_{\nu_\ell}$$

$(j_\lambda)^*$ is the Hermitian conjugate of (j_λ) times +1, or -1, depending on whether $\lambda \neq 4$ or $\lambda = 4$, ℓ = e or μ, ψ_ℓ describes the lepton field and γ_1, γ_2 .. γ_5 are the Dirac matrices. The semi-leptonic reaction is well described by the V-A interaction form [25, 26]

$$* \quad \mathscr{L}_{\ell h} = \frac{G_{wk}}{\sqrt{2}} (J_\lambda + S_\lambda)^* j_\lambda + \text{h.c.}$$

where

$$j_\lambda = (j_\lambda)_\mu + (j_\lambda)_e$$

J_λ and S_λ are, respectively, the strangeness conserving and the strangeness non-conserving part of the hadron current, which can be further decomposed into a vector and an axial-vector part

$$J_\lambda = V_\lambda + A_\lambda$$

and

$$S_\lambda = V_\lambda^S + A_\lambda^S$$

The explicit form of $\mathscr{L}_{hh}$ is still not known. An attractive idea is to express the entire weak interaction Lagrangian [26] as

$$? \quad \mathscr{L}_{wk} = \frac{G_{wk}}{\sqrt{2}} \mathscr{J}_\lambda^* \mathscr{J}_\lambda$$

where

$$\mathscr{J}_\lambda = J_\lambda + S_\lambda + j_\lambda$$

Table II presents the various selection rules satisfied by the $\mathscr{L}_{wk}$. (The references for the various selection rules are quite numerous. We give only a partial list of the theoretical papers [27] .)

TABLE II. SELECTION RULES SATISFIED BY THE $\mathscr{L}_{wk}$

* $\Delta S \neq 2$ rule for $\mathscr{L}_{hh}$ Isotriplet: V_λ, $(J_\lambda^{el})_{I=1}$, V_λ^*
$\Delta S \neq 2$ rule for $\mathscr{L}_{\ell h}$ $SU_2 \times SU_2$ algebra for $Q_V = \int V_4 \, d^3r$, $Q_A = \int A_4 \, d^3r$, etc. $\lvert \Delta \vec{I} \rvert = \frac{1}{2}$ rule for S_λ (i.e. for $\mathscr{L}_{\ell h}$) $\lvert \Delta \vec{I} \rvert = \frac{1}{2}$ rule for $\mathscr{L}_{hh}$, or $? \, \mathscr{L}_{hh} = \frac{G_{wk}}{\sqrt{2}} (J_\lambda + S_\lambda)^* (J_\lambda + S_\lambda)$ in violation of $\lvert \Delta \vec{I} \rvert = \frac{1}{2}$ $\mathscr{L}_{\ell h} \sim SU_3$ octet
? $\Delta Q = \Delta S$ rule $\mathscr{L}_{hh} \sim SU_3$ octet $SU_3 \times SU_3$ algebra for $\int S_4 \, d^3r$, etc. algebra of the local currents

5. LIMITATION OF FERMI'S THEORY AND HIGHER-ORDER WEAK INTERACTIONS

The use of the phenomenological Lagrangian was originally introduced by Fermi [28] to describe β-decay, in a spirit quite similar to that of his pseudo-potential in nuclear physics. The lowest-order matrix elements of the phenomenological Lagrangian $\mathscr{L}_{wk}$ give directly the observed reaction amplitudes; the higher-order elements are, in general, divergent and should be discarded. To search for the underlying equations of the phenomenological Lagrangian is the basic problem of weak interactions.

We note that even as a phenomenological Lagrangian, the usual (current $\times$ current) form has to be modified; the rule of using only the lowest-order matrix elements of $\mathscr{L}_{wk}$ must be changed at high energy. For example, although the low-energy μ-decay process is well described by the phenomenological Lagrangian

$$(\mathscr{L}_{\ell\ell})_{\mu\text{-decay}} = \frac{G_{wk}}{\sqrt{2}} (j_\lambda)^*_\mu \, (j_\lambda)_e$$

the same phenomenological Lagrangian implies

$$\nu_\mu + e^- \rightarrow \nu_e + \mu^-$$

occurs only in S-wave, and its cross-section [29] is

$$\sigma = \frac{4}{\pi}(G_{wk} p_\nu)^2$$

where p_ν is the momentum of ν_μ in the centre-of-mass system. Since from unitarity

$$\sigma < \frac{\pi}{2}\left(\frac{1}{p_\nu}\right)^2$$

the above expression <u>cannot</u> be valid if

$$p_\nu > 300 \text{ GeV}$$

Thus, modification must come in at a momentum Λ, beyond which the above phenomenological Lagrangian should be modified. The unitarity limit gives

$$\Lambda < 300 \text{ GeV}$$

It has been speculated that perhaps weak interactions are transmitted by a spin-one charged boson $W^\pm$, and the above phenomenological Lagrangian is really the second-order effect through emissions and absorptions of $W^\pm$. In such a case, one may identify

$$? \quad \Lambda = m_W$$

The present experimental results from high-energy neutrino reactions require

$$m_W > 2.5 \text{ GeV}$$

The charged vector boson theory is unrenormalizable. The higher-order weak interaction effects remain divergent. Yet, the mass difference Δm_K between K_1^0 and K_2^0 appears in every way to be a higher-order weak interaction effect $\propto G_{wk}^2$. From what is essentially a dimensional analysis, one may estimate

$$\frac{|\Delta m_K|}{m_K} \sim \left(\frac{G_{wk}}{4\pi}\right)^2 . \theta^2 (\Lambda')^4$$

where θ is the Cabibbo angle, and Λ' is some cut-off energy parameter. By comparing with the experimental value of $\Delta m_K/m_K$, one finds

$$\Lambda' \sim O(m_N)$$

A direct perturbation calculation using either $W^{\pm}$-theory or Fermi theory gives the ridiculous answer $\Lambda' = \infty$. The usual assumption of current-algebra makes it unlikely that $\Lambda' \sim O(m_N)$ can be due to the strong interaction. It seems quite likely that this Λ' parameter is connected with the previous limit Λ of the phenomenological Lagrangian. At present, there are many theoretical attempts trying to understand the higher-order weak interaction effect; as yet, none is a * idea.

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QUANTUM ELECTRODYNAMICS

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Abstract

QUANTUM ELECTRODYNAMICS. After a brief review of the experimental situation, the role of the weak interactions in providing a possible cut-off for conventional quantum electrodynamics is discussed.

When Abdus Salam asked me to give a lecture on quantum electrodynamics, the first question I asked myself was how to define the subject; after experiencing some difficulty in arriving at a suitable definition, I decided to try consensus science, and poll my colleagues. I did so, and can report to you a consensus definition, based on the usual reliable statistical sample. I should warn you, before I report the result of the poll, that, as has become common in such cases, I shall not stay within the consensus very long, but will shortly go my own way, and turn to more controversial questions.

I

The consensus is the following: quantum electrodynamics is a theory which attempts to describe the behaviour of electrons, photons, and muons in "external" electromagnetic fields by means of Maxwell's equations, suitably quantized, and the Dirac equation, suitably modified by hole theory. One should remark that, since no one has succeeded in solving these equations, or even in showing that a solution exists, the bare theory described above must be supplemented by a few working rules and some folk-lore, to wit: one calculates by means of a power series in the fine structure constant, renormalized according to Dyson's prescription; it is hoped that this series gives a good approximation to the correct solution (assuming such a thing exists), except for such quantities as logarithmically divergent masses and charges, which one does not attempt to calculate, and also except for long-time processes, such as bound states and narrow width resonances, to which one may apply ad hoc methods (damping theory, Schrödinger equation, etc.).

One presumes to test this theory by its success in predicting atomic energy levels, magnetic moments, and collision phenomena involving electrons, muons, photons and atoms. The nucleus of the atom is for most purposes described as the fixed source of an electric and magnetic field. In particular, the most sensitive tests have been the fine structure of the n = 2 states of hydrogen and deuterium; the ground-state hyperfine structure of hydrogen, positronium and muonium (the atom consisting of a negative electron bound to a positive muon); the electron and muon magnetic moments; production of electron and muon pairs from atoms by photons, electrons or muons; wide angle bremsstrahlung; comparison of electron, positron and muon scattering by atoms; and stored electron-electron and electron-positron scattering.

I will very briefly summarize the results of the confrontation between theory and experiment. (Those who would like to see a detailed discussion, I refer to the article by Gatto in High Energy Physics (Burhop, E.H.S., Ed.), Academic Press, New York (1967), or the review talk by Yennie at Stanford in autumn 1966.)

The high-energy data are the most straightforward to discuss. They contain no serious hint of any disagreement with the theory up to virtual masses of the order of 1 BeV, or down to distances of 10^{-14} cm. This is an extrapolation of Maxwell's original theory over a frequency range of $> 10^{22}$.

The fine and hyperfine structure data test the low-energy aspects of the theory to about one part in a hundred thousand. Unfortunately, there have tended to be discrepancies of two or three times that amount between different experiments, as well as between some of the experiments and theory. The long-term trend, however, definitely seems to be that theory and experiment in this area are approaching each other. The data should be watched and theory studied further, of course, but for the present there is substantial agreement.

The electron and muon magnetic moment measurements are not at the moment in agreement with theory. The muon g-2 differs by about one part in a thousand from the expected value, with a claimed three standard deviation accuracy, according to the most recent result of Farley and collaborators. The electron g-2 differs from its expected value by about fifty parts per million to two standard deviations, according to a re-evaluation of the data of Crane and Wilkinson by Rich. It is the case in most models that would give rise to anomalies that the correction to g-2 is expected to vary proportionally to the square of the lepton mass; with this criterion, it can be seen that the electron anomaly is enormous, roughly equivalent to one part in five in the muon g-2 value. With a model in which the anomaly varied with the lepton mass, the two discrepancies are comparable; however, they are in opposite directions, so that it is unlikely that the same model could account for both effects. Clearly, this is also a situation that calls for more accurate measurements. An experimental observation due to T.D. Lee is perhaps relevant at this point. He points out that a three standard deviation discrepancy has a fifty percent probability of being right.

In summary, we must say that quantum electrodynamics has continued to hold its own amazingly well, considering the complicated world in which it must make its way. It is certainly a triumph of heredity over environment.

II

I would like to turn next to questions which involve the effect of the environment (that is to say the weak interactions of the leptons with each other, with hadrons and with possible intermediate bosons) on electromagnetism as well as the interactions of photons with hadrons and with possible intermediate bosons. I should emphasize that the following will be highly qualitative; there are no firm calculations to go with it.

What is in question is the separability of the different levels of interaction: strong, electromagnetic, weak and gravitational. Evidently the phenomena we have just finished discussing achieve this separation well, in view of the excellent agreement between theory and experiment. The position taken by the organizers of this symposium is clearly that the separation can be achieved; in fact, they appear to have achieved it.

How do we express our apparent conviction as to the separability of the interaction levels? I submit we have in mind something like the following: a quantum theory with a Lagrangian function which can be written as a sum of a strong Lagrangian (including a free Lagrangian for all non-strongly interacting particles); an electromagnetic interaction Lagrangian, which is essentially of the form $L_{em} = ej_\mu A_\mu$, where e is the unit electric charge, j_μ the electromagnetic current density (divided by e, of course) and A_μ is the electromagnetic vector potential; a weak interaction Lagrangian which is proportional to a small constant g; and a gravitational interaction Lagrangian which is proportional to a much smaller constant GM^2, with M an almost arbitrary particle mass. I fear that my Lagrangian will have alienated my readers: S-matrix theorists, who reject the strong Lagrangian, but perhaps tolerate the others; general relativists, who must certainly dislike the separation of the gravitational Lagrangian into a free and an interacting part; axiomatists, who don't believe <u>any</u> of the Lagrangians exist; and the rest for other reasons. Nevertheless, without any fanatical belief in the final triumph of the Lagrangian language, I use it since it is one in which we have all been schooled, and which we can use to communicate. We note that GM^2 is roughly 10^{-40}, g^2 roughly 10^{-5}, e^2 roughly 10^{-2}, so that we can consider setting G, g and e successively equal to zero, and thereby consider in turn three imaginary worlds (in none of which, of course, would there be any people). In the first, with G = 0, planets, stars, solar systems, etc. would cease to exist. In the second, with g also set equal to zero, weak decays would not take place, so that, for example, the neutron would be stable, neutrinos would not interact, etc. With e set equal to zero, atoms and molecules would not exist, and photons, electrons and muons would not interact. The assumption under discussion is that each of these imaginary worlds is a possible one, with properties almost like the real world, except as noted above.[1]

Before continuing, we observe that the theory we were discussing in Section I applied to the case G = 0, g = 0. The strong interactions were of course not set equal to zero, but could be hopefully ignored since their effect (other than as a source of a given electromagnetic field) involves a hadronic polarizability. For example, one class of Feynman diagrams (Fig. 1a) for the electron pair production on protons involves the polarizability of the proton. It is of the same order in e as the two diagrams (Fig. 1b) which involve only the current density of the proton. Similarly one class of diagrams (Fig. 2) in the fourth-order magnetic moment of the electron (or muon)

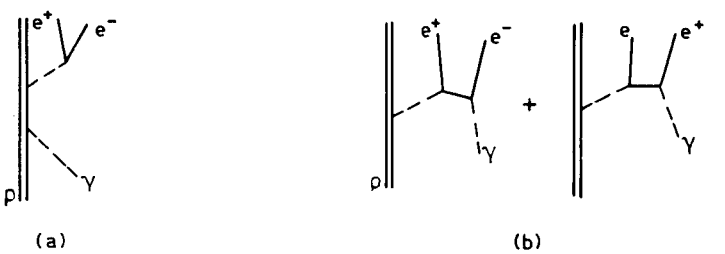

FIG. 1 (a) A class of Feynman diagrams for the electron pair production on protons;
(b) Two Feynman diagrams for electron pair production involving the current density of the proton.

[1] The Bible should accordingly be modified to start: God said "Let there be strong interactions". <u>Then</u> God said "Let there be light".

involves the hadronic vacuum polarization. The effect of these diagrams is inversely proportional to hadronic masses, whose scale is much larger than the leptonic masses. These effects cannot be estimated with precision, but it is not unreasonable to suppose them to be small.

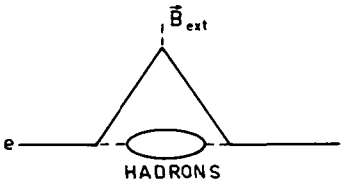

FIG. 2 A class of Feynman diagrams in the fourth-order magnetic moment of the electron.

III

Within the framework of the model under discussion (i.e. G and g both set equal to zero) there is, of course, no question of ignoring the strong interactions as a matter of principle; it is only hoped in practice that their effect may be simulated by a small number of phenomenological functions, such as charge and current densities of the proton. There is a question of principle, however, in whether one can set g and G equal to zero and still have a theory which approximates the real world. It is this question which I wish to discuss next. In what follows, I will never consider the case G not equal to zero, although it may be that sooner or later we will have to take into account the effect of gravitational interactions.

First a few remarks about the weak interactions. The vector nature of the weak interactions strongly suggests that they are mediated by the exchange of a charged spin-one boson, the intermediate vector boson (I.V.B.). If the mass of the boson is M, then the low-energy weak interactions determine the value of g, which is given by

$$g^2 / M^2 = \frac{10^{-5}}{\sqrt{2}} / M_n^2 \qquad (1)$$

where M_n is the mass of the nucleon. If now, taking the weakness of the coupling constant 10^{-5} as a guide, we calculate in lowest order perturbation theory the amplitude for the production of a pair of bosons at very high energy by, say, a pair of neutrinos, we find only one partial wave, J = 1, and its value is roughly (not counting π's, $\sqrt{3}$, etc.)

$$F^1 \sim (g^2 / M^2)\, q \qquad (2)$$

where q is the centre-of-mass momentum. We recall that unitarity limits any partial wave to 1/q or less; hence unitarity, if correct, invalidates the weak coupling theory below an energy q such that $q^2_u \simeq 10^5 M^2_n$, or $q_u \sim 300$ BeV. Thus at energies of the order of q_u the emission of the I.V.B. is a strong process, and all interacting particles will carry clouds of vector bosons. Next, taking the weakness of the finestructure constant as a guide, let us calculate the analogous photoproduction process, i.e. two photon production of a pair of I.V.B.'s or, equally well, the scattering of one photon by one I.V.B. Here we obtain a result analogous to Eq.(1),

which implies a maximum photon energy for weak coupling to be valid of the order

$$k_u \sim M/e \sim 10M \tag{3}$$

Thus at energies over 10M photons are strongly coupled to I.V.B.'s. Clearly, at the maximum of (q_u, k_u) photons will be strongly coupled to all particles which participate in the weak interactions, i.e. to all known particles. For reasons which I shall not discuss here, it seems quite likely that in fact the mass of the vector particle is in the range 2-10 BeV, so that the strong coupling of photons to all other particles would set in below several hundred BeV.

This type of behaviour is to be contrasted with the behaviour of a weakly coupled renormalizable theory, such as quantum electrodynamics; there the equivalent of k_u, if it exists at all, comes at an enormously higher energy, like m exp $(3\pi \times 137)$, where m is a characteristic mass of the theory. Thus the natural cut-off of spinor electrodynamics is enormously higher than that of the electrodynamics of the I.V.B.

We see that with the weak cut-off under discussion, all self-energies and coupling constant renormalizations may not only be finite but relatively small (of order $1/137 \log q_u/M_n$). There is therefore no possibility of imposing the requirement that they be finite within the framework of the original theory. It is clear that in a renormalizable theory, only the so-called divergent quantities will be sensitive to this cut-off. Therefore, the plan when the separation between electromagnetic and weak interactions can best be checked is in the calculation of masses and coupling constants.

IV

Let us see how this applies to various aspects of electrodynamics. First, the consensus theory: It has long been considered a matter of considerable interest whether the correct solution of the conventional theory was finite. For example, Johnson, Baker and Wiley have argued that if the charge renormalization constant Z_3 is finite, then so is the mass renormalization $m - m_0$, provided m_0 is chosen to be zero. Whether or not Z_3 is finite is, I believe, a totally open question at present. However, it is a question which may have very little physical relevance, since in any case the weak processes we have been discussing may come in strongly at a much lower energy than the non-perturbative processes associated with pure spinor electrodynamics.

Next, hadron physics. Here, as contrasted with the case of electron physics, there has appeared the tantalizing possibility of calculating the mass differences within a multiplet. Let me again review the assumptions underlying that possibility.

(1) The theory with $e = 0$ is assumed to exist, make sense, lead to finite mass ratios, <u>and</u> conserve isotopic spin; that is to say, in this limit the particles will occur in multiplets with exactly degenerate masses, approximately equal to their observed masses.

(2) The only interaction (in the sense of terms in a Lagrangian) that splits the multiplets is the electromagnetic interaction. (This is again repeating the assumption that the weak interaction is irrelevant.)

(3) The second order (in e) self-mass is finite (and presumably so are the higher orders, although smaller). The divergence is to be removed by the softening effect of the strong interactions. One example of such a softening effect is the rapid fall-off with momentum transfer of the proton form factors. A reason for the softening might be found in the suggested bootstrap nature of the hadrons. Thus, if the hadrons are essentially compound particles, then their electromagnetic mass shifts should be finite, just as the shifts of nuclear energy levels due to the proton-proton Coulomb interaction are finite. Dashen and Frautschi and others have attempted to exploit this idea in calculating the proton-neutron mass difference; however, I think at present one must view such attempts as not yet fully convincing.

How can one attempt to verify these hypotheses? It is quite hard, as we shall see. Suppose one makes the usual assumption as to the relation between charge and isotopic spin: $Q = A + BI_3$. Then the second order self-mass will appear as a sum of three terms: $\delta m = C + DI_3 + EI^2_3$. Can we verify this formula for the splitting of an isotopic multiplet? Clearly, we need a multiplet with I greater than one to avoid an identity. Such multiplets have been identified in nuclei, and indeed agreement has been found. However, if one looks at the elementary processes underlying these nuclear level differences, one sees that they are dominated by the nucleon and pion mass differences, the nucleon-nucleon Coulomb interaction, and coupling constant charge dependence: none of these has the vector-model strength to violate the three-term mass formula above. Thus the nuclei do not provide much help. The N*(1238) has $I = 3/2$, and thus has the possibility of violating the three-term formula. However, the instability of the state makes a precise measurement of its mass extremely difficult; in addition, the same reservations as above apply to a certain extent.

It is in any case not clear how significant a check of the three-term mass formula would be. If, for example, an I_3 dependent logarithmic cut-off were required to make δm finite, but the isotopic dependence did not change the order of magnitude of the cut-off (as would be the case for the weak interaction cut-off we have discussed previously), then, if the cut-off is large compared to the hadronic masses, the peculiarly insensitive nature of the logarithm would make the three-term formula hold to high accuracy. Of course, a clear violation of the three-term formula would be very significant.

During the past few years the feeling of many physicists with respect to this question (whether the multiplet mass differences can be calculated without a cut-off) has oscillated, from "no" to "yes" to "no"; probably "no" is again in the ascendency. The relevant new observation is due to Bjorken, who showed that the logarithmic divergence was proportional to the one-particle matrix element of the commutator

$$C = [\, j_i(\vec{x}),\ j_i(\vec{y})]$$

where $\vec{j}$ is the electromagnetic current density operator. In most but not all models, C fails to vanish, fails to be a c-number, and fails to be I-spin independent.

If the Bjorken commutator fails to be a c-number, then the bootstrap argument as to the finiteness of electromagnetic corrections is wrong. In fact, it is very hard to see how a bootstrap theory can produce local currents of the type we have been using for this discussion. We can thus list a number of reasonable possibilities.

(1) The Bjorken commutator is irrelevant by virtue of the bootstrap nature of the hadrons; local currents and their time derivatives are not definable, and self-masses are finite.

(2) The Bjorken commutator is relevant, but is independent of the isotopic spin by virtue of special equations of motion. Multiplet mass differences are finite. A model with these properties has been constructed by T.D. Lee. This is not a bootstrap theory, however, since the divergence persists, but has simply been made isotopic spin independent.

(3) The Bjorken commutator is relevant, non-zero, and isotopic spin dependent. This is the case in most simple models of the strong interactions. The logarithm turns out to be about 10, a not unreasonable number, if we use the weak cut-off. Within category (3) we can still distinguish two cases:

(a) All three coefficients in the mass formula are divergent, and must be cut off; or

(b) the coefficient of $I^2{}_3$ in the mass formula is finite, so that only the $I = 1$ part of the mass splitting involves the cut-off (the isotopic decomposition of the mass formula could still hold in this case in spite of the I-spin dependence of the cut-off by virtue of our previous arguments about the nature of the logarithm). As Bjorken pointed out, many models of the strong interactions have this property; in particular, a model in which the currents are constructed from $I = 1/2$ particles which interact with $I = 0$ fields (like quarks, for example) cannot produce a commutator with an $I^2{}_3$ dependence. In such a model the pion mass difference would be cut-off independent, as would the $I = 2$ part of the Σ mass difference, $\frac{\Sigma_+ + \Sigma_-}{2} - \Sigma$. Only the $I = 1$ mass differences would require a cut-off.

Neither case (a) nor (b) precludes the possibility of calculation of the mass differences; however, both require, in addition to a solution of the strong interaction problem, a model of the cut-off mechanism. If the one we have suggested, i.e. the weak interaction cut-off, is operative, it could well be that only one added parameter is needed in order to calculate all hadronic mass differences. In the case of model (b) this parameter would be very much like the tadpole of Coleman and Glashow.

I will leave open the question as to which of these ways nature has chosen to make the observed finite mass differences finite. I need not remind you that she may also have chosen another one.

NUCLEAR PHYSICS

THE NUCLEAR SHELL MODEL

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Abstract

THE NUCLEAR SHELL MODEL. Some of the basic problems in nuclear physics are discussed. They include the extent to which nuclei can be described by using the nucleon co-ordinates only, the possible meaning of the success of the shell model in interpreting some of the data on energies of nuclei and their electromagnetic properties, recent studies on the Hartree-Fock approximation, and the possible conclusions to be drawn from two-body correlations in nuclei.

The history of the development of the nuclear shell model is too well known to be repeated here. Let me therefore start right away with a formulation of some of the conjectures of the shell model, the problems which they raise, the extent to which they are understood, and their possible relevance to more fundamental problems of present-day physics.

Nuclei constitute the only system of strongly-interacting particles for which we have available today extensive experimental information on many very well defined bound and quasi-bound states. Such information has to do both with directly measurable quantities like spins, energies, moments, form-factors etc., as well as less direct evidence on the structure of these states through their involvement as initial or final states in various reactions. Our attempts to elucidate the laws that govern the physical behaviour of the microcosmos have had spectacular success whenever the dominant interaction was of an electromagnetic origin. It is, therefore, necessary to inquire to what extent the same, or similar, laws govern also systems in which the strong interactions play the most dominant role.

The first and most fundamental problem in nuclear physics is the extent to which a nucleus can be described as a system composed of individual nucleons. In order for a system to be describable as a composition of more elementary entities it is necessary that the energy required for its excitations be small compared with the intrinsic excitation energy of the latter. That this, indeed, is the case in nuclei is due, basically, to the very fortunate circumstances of the existence of a hadron of an exceptionally low mass – the pi-meson. In a system well approximated by a set of individual nucleons, the single-particle excitations are of order

$$\Delta E_{s.p} = \frac{1}{2M_n}\left(\frac{\hbar}{R}\right)^2$$

where M_n is the nucleon's mass, and R – the dimension of the system – is of the order of the range of the forces, i.e.

$$R \approx \frac{\hbar}{m_\pi c}$$

Hence

$$\Delta E_{s\text{-}p} \approx \frac{1}{2} m_\pi c^2 \frac{m_\pi}{M_n} \approx 10 \text{ MeV}$$

This should be compared with an intrinsic excitation of the order of 300 MeV or more for the nucleons themselves. It is thus enough for the pion to have been five times heavier to wipe off the whole of nuclear physics as formulated today.

The expected validity of the description of the nucleus in terms of the co-ordinates of the nucleons only has been verified in many ways. The most striking evidence in this direction is perhaps the occurrence of "magic numbers" at the same numbers for protons and for neutrons. The "proton liquid" and the "neutron liquid" in the nucleus do not mix up to form a unified "nucleon liquid" inside the nucleus. However, it is still not clear to what extent the nucleons inside the nucleus retain their free properties: is the interaction among nucleons inside the nucleus the same as that between free nucleons? Are their intrinsic magnetic moments, or more generally their electric and magnetic form factors, the same when immersed inside the nucleus, as for the free nucleon?

That these questions are not trivial can, perhaps, be inferred from the claimed reduction in the half-life of a lambda-hyperon bound in ${}^{3}_{\Lambda}H$ (see Block's comments in the discussion following Davis and Sacton's contribution to the Second International Conference on High Energy Physics and Nuclear Structure [1]). To the extent that the effect is real we may have here a direct evidence for a modification in the intrinsic property of a hadron bound in a medium with which it interacts strongly[1]. Similar effects on some of the intrinsic properties of the bound nucleons are not excluded, and the extent to which we shall be able to understand them in the realm of nuclear physics may influence our general understanding of strong-interaction physics.

The study of such fundamental problems is, however, made difficult by the complexity of the many-body problem. It is for this reason that the development of nuclear physics has gone along two rather distinct paths which are only starting to merge together:

One approach was stimulated by various regularities observed in the low-energy data, and consisted of parametrizing such data in the framework of one model or another. The most successful of these models were the collective model of Bohr-Mottelson, with all its improvements and modifications, and the jj-coupling shell model of Mayer and Jensen. The two models are supposed to be applicable in mutually non-overlapping portions of the periodic table; whereas the collective model does, on the whole, correlate more data, doing so surprisingly accurately, the shell model, which confines itself to fewer nuclei, is probably more directly connected to a "fundamental theory".

A different approach to nuclear structure starts from the formulation of the many-body problem with given forces and attempts to approximate its solutions as well as possible. In inventing all sorts of approximations one is naturally guided by the success of some of the phenomenological approaches, but the real success of this whole program

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[1] Very recent measurements seem to cast doubt on the reality of this effect (GAL, A., private communication).

will only be achieved if and when a mathematically valid approximation to the many-body problem will be shown to reproduce the empirical values of the various parameters used in the phenomenological approaches of the shell model and the collective model.

Since the collective model as such, as well as the details of various many-body calculations, are discussed in separate lectures, I shall confine myself to the phenomenological analysis of the shell model and to some special features of the many-body problem as applied to nuclei, i.e. to small, finite, systems.

The phenomenological approach to the shell model starts from the assumption that the nucleus, to a good approximation, can be represented as a Fermi gas of non-interacting protons and neutrons, and proceeds to correct this crude model in a perturbative way. The general validity of this approximation stems, as we know today, from the combination of two effects: the nucleon-nucleon force is basically a weak force, in the sense that it allows only a single bound state in the two-nucleon system. Furthermore, the Pauli principle reduces the effects of a two-nucleon scattering process inside the nucleus, since many of the possible final states are occupied. The result of these two factors is to produce a two-body density matrix inside the nucleus which looks very much like a free Fermi-gas two-body density matrix at the same overall nuclear density, except at very small relative distances $|\vec{r}_1 - \vec{r}_2|$. The average distance between two nucleons in a nucleus is $r_0 \cong 1.2$ fm, and the healing distance, i.e. the distance beyond which their relative wave function is expected to go over into that of the non-interacting Fermi gas, is of the order of $h = 0.6\text{-}0.7$ fm. Since the effects of deviations from the free-gas approximation are expected to go like $(h/r_0)^3$, we see that independent particle models for nuclei should, indeed, represent a fairly good approximation.

The derivation of the observed nuclear density from "first principles" is one of the first aims of the more fundamental approaches to the study of nuclear structure. However, if one takes the observed density as given, a number of features can be explained within the framework of a remaining independent particle model. Thus, at the observed nuclear density $\rho = 1.72 \times 10^{38}$ particles/cm^3 the Fermi momentum is $k_f = 1.36\,f^{-1}$ leading to an average kinetic energy per particle of $\langle T \rangle = 23$ MeV. From nucleon-nucleon scattering we obtain an effective square well potential, representing their mutual interaction, of depth $U \cong -40$ MeV. Combining the two numbers we find an average binding energy per particle in nuclear matter of about -20 MeV, in qualitative agreement with the Weizsäcker mass formula.

The symmetry energy, or more precisely the coefficient $a_4 = 23.7$ MeV of the term $a_4(Z - N)^2/A$ in the Weizsäcker mass formula, can also be easily evaluated from an independent Fermi-gas model once the Fermi momentum is given. All one has to do is to figure out the energy required to transform a neutron into a proton in a mixed Fermi gas of a given density, assuming that all the lowest states are occupied. A value of $a_4 = 19$ MeV is thereby obtained in fair agreement with the experimental value of 23.7 MeV.

One can try to estimate on the same basis also other gross features of nuclei, such as the surface energy, and even surface thickness, with a fair degree of success. But perhaps the most interesting indications

of the validity of the independent particle approximations are given by calculations of Hartree-Fock self-consistent potentials with an imposed given density as an additional condition. Since nuclear equilibrium results from large cancellations between positive kinetic energies and negative potential energies, both big, in magnitude, compared with the resulting binding energies, we cannot expect a reliable estimate of the equilibrium density to come out of relatively crude calculations starting from first principles. However, imposing the observed density on such calculations enables us to have some idea as to the extent to which a fundamental theory explains the rest of the data.

In one such attempt, Krieger et al. [2] impose the observed density on the system by adding to the Hamiltonian a term of the form

$$\frac{c_0}{\rho_0^2} \sum \rho^2(r_i) \quad \text{with} \quad \rho(r) = \sum_{k<k_f} \phi_k(r)\, \phi_k^*(r)$$

ρ_0 is the observed nuclear density and $\phi_k(r)$ are the single-particle states in a self-consistent Hartree-Fock solution to the Hamiltonian H'

$$H' = H + \frac{c_0}{\rho_0^2} \sum \rho^2(r_i), \quad H = \sum T_i + \sum_{i<j} v_{ij}$$

The coefficient c_0 is adjusted so as to reproduce the observed density, and the interaction v_{ij}, in this particular study, was taken to be a spin-dependent and velocity-dependent relative s-state interaction of a Gaussian shape, which reproduces the low-energy nucleon-nucleon scattering data.

Several results can be obtained from this semi-phenomenological calculation. For instance the binding energy per particle in ^{16}O is obtained as -6.4 MeV as compared with an experimental value of -8.72 MeV. In ^{40}Ca a binding energy per particle of -8.4 MeV is computed in this way, compared to an experimental value of -10.27 MeV.

The detailed r-dependence of the nuclear density can be obtained as well, as shown in Fig.1. It leads to a root mean square radius r_{rms} = 2.4 fm for ^{16}O and r_{rms} = 3.1 fm for ^{40}Ca, as compared with experimental values of 2.64 fm and 3.52 fm respectively [3].

Furthermore, the single-particle energies can be computed and compared with the measured positions of the 1s and 1p levels in (e, e'p) and (p, 2p) experiments. Figure 2, reproduced from Ref.[4], shows a comparison of measured positions of such levels in various nuclei with various Hartree-Fock calculations [5]. One sees from it that the obvious non-local properties of the average nuclear potential which reflect themselves in the large A-dependence of the single observed single-particle energies, are fairly well reproduced by the semi-phenomenological Hartree-Fock calculations.

The phenomenological approach can be tested even at an earlier stage if one is willing to start from an independent particle model and consider only first-order perturbation. Indeed, a spherical potential leads to an exact degeneracy of the single-particle levels with respect to the magnetic quantum numbers, and to an approximate degeneracy, or more precisely clustering, of different groups of single-particle levels. The latter

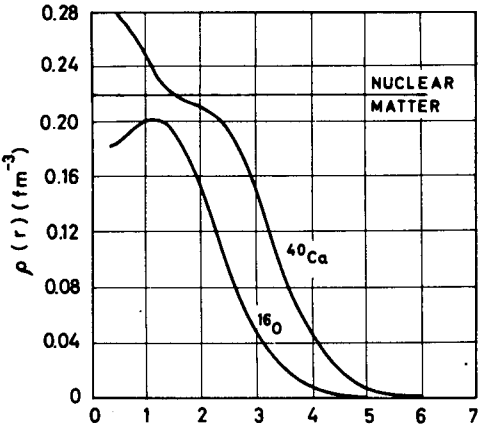

FIG. 1. The nuclear density of ^{16}O and ^{40}Ca. Reproduced from Ref. [2].

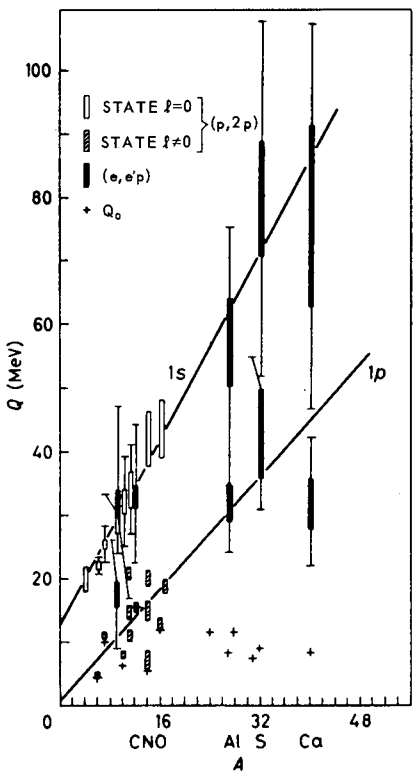

FIG. 2. Hartree-Fock single-particle energies against the mass number. Reproduced from Ref. [4].

transforms into the well-known exact SU(3) degeneracy in the case of the harmonic-oscillator potential. A two-body residual interaction can then be introduced in first order to remove part of these degeneracies, and its predictions can be compared with the vast amount of experimental data available on the structure of various low-lying configurations in nuclei near closed shells. Such calculations can be carried out at a rather

high degree of generality by using the techniques developed by Racah, as subsequently extensively used by his collaborators and students. The basic idea behind these calculations is rather simple and straightforward:

If the residual interaction among the nucleons is assumed to be a two-body interaction, then we can calculate the matrix elements of this interaction within a given two-body configuration, or finite set of such configurations. When we now allow n particles to occupy the same configuration or the same finite set of configurations, their energies are completely determined by the matrix elements of the interaction in the two-body configurations. Thus, if E(n;k) represents the energy of the k-th state in the n-particle configuration, it is possible to establish algebraic relations of the type

$$E(n;k) = F_{nk}(V_\alpha) \tag{1}$$

relating the energies in the n-particle configuration to matrix elements V_α in the two-particle configurations. The structure of the algebraic functions F is independent of the interaction and is essentially determined by the symmetry groups of the single-particle states. In simple cases, such as for pure configurations, Eq.(1) reduces to a linear relation

$$E(n;k) = \sum \langle nk;\alpha \rangle \, E(2;\alpha) \tag{1a}$$

In real cases, especially if one feels justified in confining oneself to the study of a pure j^n or $j_1^{n_1} j_2^{n_2}$ configurations, there are often more levels E(n;k) than there are two-particle matrix elements V_α. Equation (1a) then implies the existence of linear relations among the energies E(n;k) of the type

$$\sum_k c_k^{(n)} \, E(n;k) = 0 \tag{2}$$

where the coefficients $c_k^{(n)}$ are independent of the two-body interaction matrix elements V_α.

In atomic spectroscopy the earlier relations of type (2) were known as "interval rules". In elementary particle physics they are known as "mass formulae", and in nuclear physics they are usually referred to as "predictions of the shell model". Inasmuch as they are derived from general symmetry considerations it is perhaps most appropriate to call them all "Racah relations".

An example of a most extensive study of this type is given by Federman and Talmi [6], who studied the energy levels of all the Ca isotopes from ^{42}Ca to ^{50}Ca, allowing the neutrons to occupy the $f_{7/2}$ levels, and the $p_{3/2}$ levels, and including also the possibility of the system being excited to a rotational band of deformed states whose characteristic features are taken from an observed spectrum in ^{40}Ca. The proton configuration is assumed to form a closed shell in agreement with other experimental evidence. There are 30 pieces of experimental data which, in this analysis, should all be given in terms of 14 two-body matrix elements. A least-square fit of the data treating these 14 matrix elements as variable parameters results in the numbers shown in Table I and is seen to interrelate the data very well. Such fits yield, in addition,

TABLE I. EXPERIMENTAL AND CALCULATED ENERGIES (IN MeV) OF CALCIUM NUCLEI[a]

Nucleus	Spin	Experimental	Calculated
^{42}Ca	0^+	19.84, 18.00	19.79, 17.98
	2^+	18.32, 17.42	18.34, 17.43, 15.37
	4^+	17.09	17.10, 16.17, 15.68
	6^+	16.65	16.51
^{43}Ca	$3/2^-$	27.17, 25.71	27.23, 25.73, 24.75
	$5/2^-$	27.39	27.53, 25.28
	$7/2^-$	27.76	27.83, 25.73, 23.98
	$9/2^-$	25.66	25.75, 24.41
	$11/2^-$	26.08	25.90, 24.46
^{44}Ca	0^+	38.90, 37.02	38.80, 37.03, 33.53
	2^+	37.74, 36.24, (35.55)	37.45, 36.01, 35.21, 34.36
	4^+	36.62, (36.04)	36.74, 36.01, 35.23, 33.58
	6^+	35.60	35.64, 34.03
^{45}Ca	$3/2^-$	(44.88), (44.41), (44.06)	44.93, 44.41, 43.47
	$5/2^-$	46.13	46.14, 44.13, 43.46
	$7/2^-$	46.31	46.34, 45.28, 43.98, 42.83
^{46}Ca	0^+	56.72	56.80, 55.02, 52.34
	2^+	55.37, 53.70	55.32, 53.80, 52.97, 52.20
^{47}Ca	$3/2^-$	61.99	61.96, 61.05
	$7/2^-$	64.00	64.02, 62.14
^{48}Ca	0^+	73.94, 69.65	73.97, 69.65
	2^+	70.11	70.12, 67.86
^{49}Ca	$3/2^-$	79.08	79.15
^{50}Ca	0^+	85.43	85.46

[a] The total energy of each level is given minus the binding energy of ^{40}Ca. Underlined experimental energies were not included in the fitting procedure. Reproduced from Ref. [6].

TABLE II. THE QUADRUPOLE OPERATOR IN ^{51}V

Quantity	B(E2, 7/2→ 3/2)	B(E2, 7/2→ 5/2)	B(E2, 7/2→ 9/2)	B(E2, 7/2→ 11/2)	Q(7/2)
Experimental value	0.27	0.92	0.22	0.90	-0.052 ± 0.010
Calculated value	0.22	0.96	0.27	0.86	-0.058

B(E2)s are given in units of $e^2 \times 10^{-50}$ cm^4; Q - in units of 10^{-24} cm^2. Reproduced from Ref. [7].

TABLE III. COMPARISON OF MATRIX ELEMENTS $\langle a\,b\,J\,|V|\,c\,d\,J\rangle$ with V

j_a	j_b	j_c	j_d	J	(nn) or (pp) interaction G	$G + G_{p-h}$	$G + G_{phonon}$	Emp.	(np) interaction G	$G + G_{p-h}$	$G + G_{phonon}$	Emp.
$0p_{\frac{1}{2}}$	$0d_{\frac{1}{2}}$	$0p_{\frac{1}{2}}$	$0d_{\frac{1}{2}}$	2	-0.59	0.89	0.67	0.49	-2.78	-1.58	-1.99	-1.75
				3	-1.64	-0.67	-0.95	-0.07	-2.56	-1.55	-1.83	-1.60
$0p_{\frac{1}{2}}$	$1s_{\frac{1}{2}}$	$0p_{\frac{1}{2}}$	$1s_{\frac{1}{2}}$	0	-0.90	-0.90	0.35	0.84	-1.90	-1.10	-1.42	-1.12
				1	-0.15	0.73	0.47	0.17	-1.51	-1.02	-1.27	-0.97

G, $G+G_{p-h}$ and $G+G_{phonon}$ are calculated with the Hamada-Johnston potential with $\hbar\omega = 14$ MeV. Reproduced from Ref. [11].

an empirical determination of the matrix element V_α of the two-body interaction; these should eventually be calculable in terms of a more fundamental approach.

Energies are not the only parameters that can be interrelated in this way. Matrix elements of various single-particle operators have been computed in many n-particle states and related, thereby, to a few single-particle matrix elements of the same operator. Thus, in a recent calculation [7], Talmi has studied the electric quadrupole operator in the low-lying states of ^{51}V. The energies of these levels were successfully explained [8] in terms of the proton $f^3_{7/2}$ configuration, the neutron configuration being taken to represent a closed shell. Four E2-transition probabilities and one static quadrupole moment have been measured in this nucleus, all involving different states ascribed to the same configuration $f^3_{7/2}$. Thus, all these five quantities should be given in terms of a single matrix element of the quadrupole operator in the state $7/2^+$. Indeed, Talmi finds that a reduced matrix element of $r^2 Y_{2\chi}(\theta, \phi)$ given by

$$|\langle 7/2 \,|e_0^2 r^2 Y_2 \,|\, 7/2\rangle|^2 = 5.19\, e^2 \times 10^{-50}\ \mathrm{cm}^4 \tag{3}$$

gives the best fit to the data as shown in Table II, which is reproduced from Ref.[7].

In Eq.(3) e^2 is the proton charge, whereas e_0 is the charge associated with the quadrupole operator. The natural inclination is to put $e_0 = e$, since the quadrupole moment is generated by the protons of ^{51}V (Table II). This, however, leads to a r.m.s. radius for the protons which is about 60% bigger than any reasonable estimate based on known nuclear sizes. The situation is characteristic of what is found in great many of the calculations in the nuclear shell model: a good fit to experimental data is obtained through the association of low-lying nuclear states with relatively simple configurations. However, the parameters that have to be used to obtain these fits are different from those expected on the basis of a naive interpretation. Rather, it seems that we have to consider these parameters as renormalized charges, or interactions, and the conclusion that we can really draw from such fits is that with renormalized quantities it is possible to interrelate the data pertaining to low-energy data assuming, for these states, simple wave functions. In the particular case of ^{51}V the renormalized, or as it is often called effective, charge e_0 of the proton, turns out to be [7]

$$e_0 = 1.61\ e$$

This is not an unreasonable value for a renormalized charge for the quadrupole operator, especially since it was shown a long time ago [9] that various contributions to the quadrupole operator arising from polarization of the closed shells tend to add up coherently.

Several interesting attempts have been made recently to understand the observed values of renormalized quantities in terms of the "bare" quantities and the effect on them of the polarization of the core of closed shells. Brown and Kuo [10], and Kuo [11] were able to show that the renormalized matrix elements of the two-body interactions in many

nuclei come close to those derived from the phenomenological analysis of the experimental data such as given in Table I. Characteristic values for such renormalized matrix elements and their comparison with empirical [12] matrix elements of the effective interaction for the oxygen isotopes are given in Table III, reproduced from Kuo [11]. Here G is the G-matrix in zeroth order, while G_{p-h} and G_{phonon} are two types of polarization correction to it, consisting respectively of particle-hole excitation, and phonon excitation (coherent multi-bubble exchange). The importance of core-polarization for the improvement of the agreement between theory and experiment is to be noted. It is, however, clear from more detailed inspection of the data that although many gross features of the empirical, effective, interaction matrix elements are reproduced by the inclusion of core-polarization (see in particular Brown and Kuo, loc. cit.), there still remain important systematic differences between theory and experiment that await their interpretation.

On the more fundamental level efforts are concentrated, for the time being, mostly on the derivation of the ground-state properties. The Hartree-Fock approximation plays the dominant role in all such studies and a variety of interesting results have emerged so far.

The Hartree-Fock method picks on the "best" single-particle determinantal wave function for the many-nucleon system. Since it involves a variational principle it produces the lowest possible energies for the occupied states. Thus a natural feature of the Hartree-Fock method is an energy gap between occupied and unoccupied levels [13]. The Hartree-Fock (H.F.) potentials for different nuclei therefore differ from each other markedly – a fact which inhibits greatly the use of H.F. wave functions for nuclear reactions.

Another important feature of the H.F. wave functions has to do with their being manifestly independent-particle wave functions. This aspect of the H.F. approximation is, of course, at the root of its whole "philosophy" and dominates every facet of the calculation. It was pointed out a long time ago by Lipkin [14] that almost all soluble problems in physics are brought eventually into a form involving a sum of independent single-particle operators. On the other hand the symmetry of a problem results in a conservation theorem, which, in its turn, implies correlation between particles; in fact, in order to conserve a total quantity, any change in the state of one particle affecting this quantity should be compensated by an appropriate change in the state of another particle. There is, therefore, a basic "internal conflict" between the conservation theorems resulting from the symmetry of a many-particle system and the attempt to describe it by means of the H.F. approximation.

It should not surprise us, therefore, that the best H.F. wave functions violate several conservation theorems. The best known example is, of course, that of the total linear momentum which is violated from the beginning through the introduction of a preferred point in space – the origin of the H.F. single-particle potential. The conservation of total angular momentum is violated through the introduction of deformed H.F. potentials. In fact, the spherical potential is self-consistent only for nuclei with no particles outside closed shells, and even for consistency alone one has to introduce axially symmetric deformed potentials when dealing with non-closed shells nuclei. In some treatments of the many-body problem even the conservation of the number of particles is given

up through the introduction of the Bogolyubov transformation and the resulting Hartree-Fock-Bogolyubov approximation.

The violation of the various symmetries in the H.F. method requires some care in the interpretation of the data. Spurious states and modified energies come out of the use of symmetry-violating wave functions, and these have to be corrected. In the case of the linear momentum the problem is relatively simple due to the exact separability of the centre-of-mass motion. Thus, the Hamiltonian

$$H' = H - \frac{p^2}{2AM} \tag{4}$$

where P is the total linear momentum and AM is the total mass of the nucleus, leads to infinitely degenerate states: states corresponding to the same intrinsic structure but different centre-of-mass energy are all degenerate in H'. Thus, Eq.(4) allows also eigenstates which are localized in the centre-of-mass co-ordinate and these can be legitimately approximated by the H.F. method. Since P^2 is a two-body operator we see that formally the transition from H to H' involves a change in the interaction

$$v_{ij} \rightarrow v_{ij} - \frac{\vec{p}_i \cdot \vec{p}_j}{AM}$$

and in the single nucleon mass

$$\frac{1}{M} \rightarrow \frac{1}{M}\left(1 - \frac{1}{A}\right)$$

The situation with the angular momentum violating wave functions is more complex. Here one uses the empirical fact that for a large class of nuclei, the so-called deformed nuclei, the energies of the various states seem to satisfy a relation of the type

$$E(\alpha, J) = E(\alpha) + \frac{\hbar^2}{2\mathcal{J}}[J(J+1) - K(K+1)] \tag{5}$$

where $E(\alpha)$ is the energy of the intrinsic states α which leads to a rotational band with lowest angular momentum J=K. It is seen from Eq.(5) that, in analogy with Eq.(14), the Hamiltonian

$$H' = H - \frac{\hbar^2}{2\mathcal{J}} J_{op}^2 \tag{6}$$

where $\vec{J}_{op}$ is the operator for the total angular momentum, leads to a degenerate set of states: all the members of the same rotational band in H are degenerate in H'. Since these states belong to different angular momenta, it is clear that the eigenstates of H' can be taken to be oriented in space, thus making a deformed potential an appropriate first stage in the approximate solution of H'. The lowest energy of H

is obtained then, through the H.F. deformed ground-state of H', by taking

$$E = E' + \frac{\hbar^2}{2\mathcal{J}}\langle J^2 \rangle \tag{7}$$

In Eq.(7) the expectation value of J^2 is taken with the deformed ground-state H.F. wave function of H', and the moment of inertia $\mathcal{J}$ is taken from the empirical data on the rotational states. In principle $\mathcal{J}$ could have been calculated as well, however the range of validity of the various formulae for its derivation is not so clear as yet.

In an actual H.F. calculation in which all the nucleons in the nucleus are treated on an equal footing [15], one proceeds as follows: The single-particle wave functions $|\lambda\rangle$ are expanded in a set of harmonic-oscillator wave functions $|n\ell jm\tau_z\rangle$

$$|\lambda\rangle = \sum_{n\ell jm\tau_z} C^{(\lambda)}_{n\ell jm\tau_z} |n\ell jm\tau_z\rangle \qquad \sum_{n\ell jm\tau_z} C^{(\lambda)*}_{n\ell jm\tau_z} C^{(\lambda')}_{n\ell jm\tau_z} = \delta_{\lambda\lambda'} \tag{8}$$

The spring constant of the harmonic-oscillator potential which generates the functions $|n\ell jm\tau_z\rangle$ could be chosen arbitrarily if the summation in Eq.(8) were to extend over an infinite set of states. In practice the summation is limited to a finite sub-set of states: all the occupied shell model levels plus the closest unoccupied levels in the shell model description of the particular nucleus considered. Consequently the spring constant is chosen closest to the one which gives the best harmonic-oscillator description of the ground-state of the nucleus in question. The C's in Eq.(8) are then taken as the variational parameters and the Hartree-Fock procedure carried out. In fact, denoting for short $|k\rangle \equiv |n\ell jm\tau_z\rangle$, a given choice of the C's yields a single-particle potential

$$\langle k|U|k'\rangle = \sum_{\lambda,\ell,\ell'} C^{(\lambda)*}_{\ell} C^{(\lambda)}_{\ell'} \langle k\ell|U|k'\ell'\rangle \tag{9}$$

where the summation extends over all occupied states λ. With the matrix elements (Eq.(9)) of U, computed for all the finite set of harmonic-oscillator states to which we choose to limit ourselves, one goes into the eigenstate equation

$$\sum_{k'} \langle k|T + U|k'\rangle C^{(\lambda)}_{k'} = \epsilon_\lambda C^{(\lambda)}_{k} \tag{10}$$

obtaining a new set of orthogonal C's. These are then fed back into Eq.(9), repeating the sequence of calculations until self-consistency is obtained.

If in Eq.(8) one limits the summation, for any given λ, only to states of the same values of ℓjm and τ_z (i.e. one sums over n only), one obtains the spherical Hartree-Fock approximation which, as we have said, is self-consistent only for closed shells.

Allowing for the summation over different values of m as well, we automatically introduce a deformed axially symmetric potential which retains invariance under time reversal if we stipulate further that [10]

$$C^{(\lambda)}_{n\ell j\,-\,m\tau_z} = (-1)^{l+j-m}\, C^{(\lambda)*}_{n\ell jm\tau_z} \tag{11}$$

If we allow summations over values of ℓ of the same parity only, the states $|\lambda\rangle$ will have a definite parity too and the H.F. many-particle wave function will consequently be an eigenstate of the total parity operator.

Attempts to use wave functions of mixed parity, i.e. allowing a summation over values of ℓ of mixed parity in Eq.(8), were made by Bassichis and Svenne [16]. They found that for reasonable strength of the tensor force in the nucleon-nucleon force, the Hartree-Fock wave function comes out automatically to be of a definite parity. Only if one takes tensor forces of twice the accepted strength, or more, does one gain by going over to states of mixed parity.

Bassichis and Svenne [16] also found that if C_m and C_{-m} were taken to be unrelated initially, the minimal solution was obtained for values of C which satisfied the time reversal requirement (Eq.(11)).

It is interesting to see some of the actual numerical results of the H.F. calculations to appreciate the present state of the art. The calculations I shall refer to have all been carried out with the Tabakin two-body force [17], which is given by a separable potential of the form

$$V(\vec{r}, \vec{r}\,') = \frac{h^2}{\mu} \sum_{\substack{JTS \\ M\ell\ell'}} \left\{ -g^{JTS}_{\ell}(r)\, g^{JTS}_{\ell'}(r') + h^{JTS}_{\ell}(r)\, h^{JTS}_{\ell'}(r') \right\} \times p_T \times y^{M*}_{\ell SJ}(\vec{r}) \cdot y^{M}_{\ell' SJ}(\vec{r}\,') \tag{12}$$

where μ is the nucleon mass, P_T the isospin projection operator, and $y^M_{\ell SJ}(\vec{r})$ the vector spherical harmonics involving the spin wave functions. The radial functions g(r) and h(r) are of a modified Yukawa type which are made to fit to two-body data when one limits oneself to $\ell, \ell' = 0$, 1 and 2. There are, therefore, no free parameters in $V(\vec{r}, \vec{r}\,')$ as applied to the many-body problem.

Figure 3, reproduced from Kerman et al. [18] shows the behaviour of the ground-state Hartree-Fock energy of ^{16}O using the Tabakin potential. Since ^{16}O is a closed-shell nucleus, only spherical wave functions (i.e. well defined values of ℓjm and τ_z) were used, and the calculations were carried out using a different number N or radially different wave functions in the summation (Eq.(8)). It was found earlier [19] in a somewhat simpler calculation, that very little is gained in nuclear matter calculations by going beyond N=4. Figure 3 indeed shows that the sensitivity of the results to the spring constant of the harmonic oscillator diminishes drastically as the number of terms in Eq.(8) is increased. For $N=\infty$ the H.F. ground-state energy should, of course, be independent of γ.

For non-closed shell nuclei, Bassichis et al. [15] computed the H.F. energies, for a fixed value of the oscillator spring constant ($\hbar/M\omega = 2.6\ f^2$), with the subsidiary condition that the system has a definite quadrupole moment Q. This was done to ease the search for the deformation leading to the lowest energy. Figure 4 shows the resulting H.F. ground-state energy for ^{12}C as a function of Q which indicates clearly the expected negative intrinsic quadrupole moment for the intrinsic ground-state of ^{12}C. The calculation was carried out in the

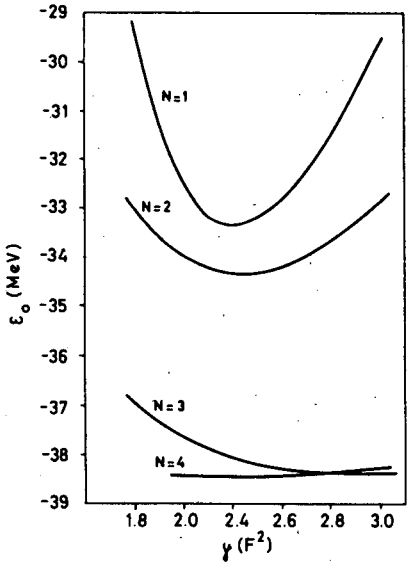

FIG.3. Ground-state energy of ^{16}O with H.F. wave functions as a function of the oscillator parameter $\gamma = \hbar/M\omega$, taken for different number of terms N. Reproduced from Ref. [18].

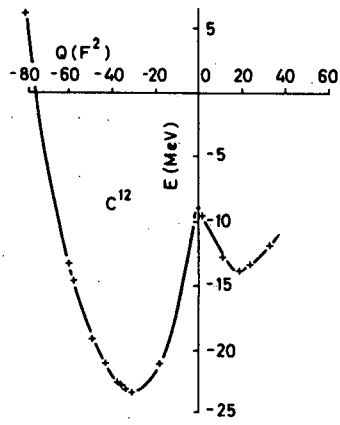

FIG.4. The Hartree-Fock energy of ^{12}C as a function of the deformation. This curve was obtained by adding μQ_{op} to the Hamiltonian and varying μ. A similar behaviour appears for all the closed-sub-shell nuclei. Reproduced from Ref. [15].

sub-space of the harmonic oscillator levels 1s, 1p, 2s, 1d, 2p and 1f (these levels are filled in the shell model description of ^{40}Ca).

Table IV shows the resulting wave functions of the Hartree-Fock approximation for several nuclei with their amplitudes in the various harmonic-oscillator wave functions. The validity of the pure shell model description of these nuclei can be inferred from this table rather easily.

The H.F. ground-state energies as computed by these methods still fall short of the experimental binding energies of the corresponding

nuclei. Bassichis et al. [15] have therefore computed the second-order correction to the binding energy (the first-order vanishes, of course) using the H.F. wave functions, obtaining very interesting results: although the second-order correction to the potential energy is only about 17% of the total potential energy, it represents a very important correction to the total binding energy. This is dramatically shown in Table V, where ΔE_2 is the second-order correction, $E^{J=0}$ is the H.F. ground-state energy corrected for the rotational energy in the deformed potential (see above) and E_{tot} is the total binding energy of these nuclei.

Summing up these results it seems that H.F. calculations involving finite nuclei have reached now a stage where it may start to be meaningful to study the fit obtained to energies of complex nuclei, by different forces which fit the two-body data equally well, and perhaps the day is not far when we shall be able to obtain some indication as to whether the two-nucleon interaction is, or is not, appreciably changed for nucleons immersed in nuclear matter. A fundamental question referred to earlier will thus find its solution.

It is conceivable that future work in the general domain will include attempts to compare nuclear energies derived from potentials which coincide on the energy shell but deviate from each other markedly off the energy shell. It will also be interesting to compare electromagnetic properties of nuclei, using H.F. wave functions, probably corrected to first-order, with the corresponding empirical data. It can be expected that these properties will be more sensitive to the size of the truncated harmonic-oscillator space in which the H.F. wave functions are expanded. Indeed, it will be particularly interesting to see how the importance of first-order corrections is reduced as this truncated space increases in size. The great progress in machine computations may make all these undertakings realizable in the near future, and we shall probably learn many interesting things about strong-interaction physics when these calculations are carried out.

The extent to which the two-nucleon interaction is, or is not, modified inside the nucleus can be possibly inferred also from a deeper study of the two-nucleon correlations in nuclei. These present severe experimental difficulties, and there is actually some confusion as to what one actually means by the two-nucleon correlations. The only quantity which is unambiguously defined is the deviation of the two-body density matrix from the single-particle density matrix, i.e.

$$K(12) = \rho(1,2) - \rho(1)\,\rho(2) \tag{12a}$$

This quantity can be measured with an outside probe only via a three-body force involving this probe and two of the nucleons: either a real three-body force such as that which possibly exists between the lambda particle and the nucleons in hyper-nuclei [20], or an effective three-body force which results from a double scattering of a strongly-interacting probe shot at complex nuclei.

With some simplifying assumptions one can infer the existence of two-body correlations from the single-particle density matrix. Thus if R is the dimension of the nucleus, one expects to find momenta $k \gg R^{-1}$ in the single-particle density matrix if there is a strong pair correlation over

TABLE IV. THE COEFFICIENTS OF THE EXPANSION OF THE OCCUPIED ORBITALS IN THE HARMONIC-OSCILLATOR BASIS[a]

Nucleus	(in π) \ (nℓj)	$1s_{1/2}$	$1p_{3/2}$	$1p_{1/2}$	$1d_{5/2}$	$2s_{1/2}$	$1d_{3/2}$	$1f_{7/2}$	$2p_{3/2}$	$1f_{6/2}$	$2p_{1/2}$
^{8}Be	$\frac{1}{2}$ +	-0.9633			-0.1987	-0.0177	+0.1795				
	$\frac{1}{2}$ -		+0.8463	-0.5046				+0.1315	-0.0055	-0.1050	+0.0272
^{12}C	$\frac{1}{2}$ +	+0.9848			-0.1231	+0.0544	+0.1097				
	$\frac{3}{2}$ -		+0.9936					-0.0825	+0.0379	+0.0665	
	$\frac{1}{2}$ -		-0.7142	-0.6922				+0.0820	-0.0380	+0.0499	-0.0067
^{16}O	$\frac{1}{2}$ +	+0.9983				0.0583					
	$\frac{1}{2}$ -		-0.9967						-0.0811		
	$\frac{3}{2}$ -		+0.9967						+0.0811		
	$\frac{1}{2}$ -			-0.9999							-0.0020
^{20}Ne	$\frac{1}{2}$ +	-0.9948			-0.0480	-0.0768	+0.0462				
	$\frac{1}{2}$ -		+0.9390	-0.3076				+0.1099	+0.0464	-0.0937	+0.0268
	$\frac{3}{2}$ -		-0.9916					-0.0318	-0.1227	+0.0260	
	$\frac{1}{2}$ -		+0.3155	+0.9436				-0.0262	+0.0526	+0.0705	+0.0399
	$\frac{1}{2}$ +	+0.0250			-0.8614	+0.4010	+0.3106				

[a] Only the +m, $\tau_z = +1/2$ orbitals are listed. The $\tau_z = (1/2)$ C are identical with those listed. The C for -m can be obtained by using $C^{-m\tau_z}_{n\ell j} = (-1)^{j-m} C^{+m\tau_z}_{n\ell j}$. Reproduced from Ref. [15].

TABLE V. THE HARTREE-FOCK ENERGY WITH SECOND-ORDER CORRECTIONS[a]

Nucleus	ΔE_2	E^{j-0}	E_{tot}	E_{obs}	E_{tot}/A	E_{obs}/A
^{4}He	-7.04	-12.78	-19.82	-28.5	-4.96	-7.13
^{5}Be	-17.57	-17.57	-35.14	-59.0	-4.29	-7.38
^{12}C	-37.57	-29.70	-67.27	-98.6	-5.61	-8.21
^{14}O	-68.46	-46.99	-115.45	-139.5	-7.22	-8.72
^{20}Ne	-94.43	-56.05	-150.48	-179.2	-7.52	-8.96
^{24}Mg	-126.47	-68.31	-694.78	-224.7	-8.12	-9.36
^{25}Si	-159.11	-91.23	-250.34	-272.0	-8.94	-9.71
^{31}S	-197.33	-109.27	-306.60	-317.3	-9.58	-9.92
^{36}Ar	-239.05	-131.92	-370.97	-363.4	-10.35	-10.10
^{40}Ca	-282.99	-154.70	-437.69	-410.8	-10.94	-10.27

[a] E^{j-0} is the usual Hartree-Fock result, corrected for rotational energy for the deformed nuclei. E_{total} is the result including second-order contributions. That the second-order contribution is large should not be taken as an indication that the perturbation series does not converge. The second-order contribution is in fact only about 17% of the Hartree-Fock potential energy, indicating convergence. Reproduced from Ref. [15].

distances of order $k^{-1} \ll R$. In this sense a measurement of the single-particle density matrix can confirm expectations based on a correlated two-body density matrix. However, the reverse procedure is not unique: it is not possible to conclude rigorously from the study of a single-particle density matrix to what extent the two-body density matrix is correlated in the sense that in Eq.(12) we have $K(12) \neq 0$.

An example can perhaps clarify the present situation: the nucleus ^{4}He has now been studied using both electron scattering [21] and high proton scattering [22]. The electron scattering measures, in first-order Born approximation, the Fourier transform of the charge distribution. To the extent that we wish to take for the description of ^{4}He harmonic-oscillator wave functions, we have a Gaussian charge distribution with a corresponding Gaussian form factor. The experimental form factor [21] shows a dip at $q^2 = 10\ f^{-2}$ thus indicating a clear deviation from a Gaussian charge distribution. Frosch et al. [21] could reproduce the data with the empirical charge distribution shown at the insert of Fig.5 and given analytically by

$$\rho(r) = \frac{r_0\left[1 + w\left(\frac{r}{c}\right)^2\right]}{\left[1 + e^{\frac{r-c}{z}}\right]} \tag{13}$$

where w, c and z are numerical parameters. The same experimental

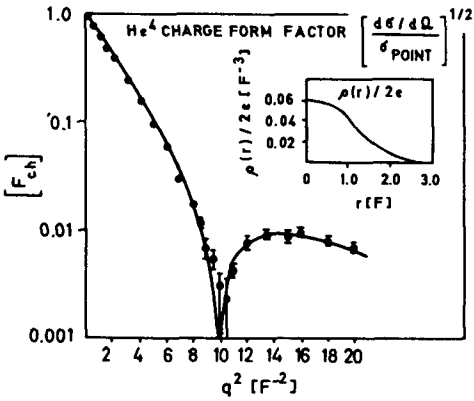

FIG. 5. Experimental values of the ^{4}He charge form factor (points) and values calculated from the charge distribution shown in the inset (solid curve). Reproduced from Ref. [21].

data were fit by Bassel and Wilkin [23] to the same accuracy by a double Gaussian charge density modified by correlations

$$\rho(r_1, r_2, r_3, r_4) = \prod_{i,\, j>i} e^{-\alpha^2 r_i^2} \left[1 - D e^{-\frac{\alpha^2 r_i^2}{\gamma^2}} \right] \left[1 - C^{-\delta^2 r_{ij}^2} \right] \tag{14}$$

(see Fig.6) where α, γ, δ, C and D are numerical parameters. It is obvious that this experiment cannot tell the difference between a correlated and a non-correlated wave function.

With a strongly-interacting projectile we expect to see multiple scattering at high momentum transfers and a priori we could expect some effects of the two-body correlations there. The situation, however, is obscured by the uncertainties in the theoretical analysis of such multiple scatterings. The best theory available today for such processes is probably that developed by Glauber [24]. Its main assumptions are those of the optical limit $\lambda \ll d$ where d is the distance between two scatterers, the straight-line approximation for the path of the projectile in the nucleus, the additivity of the phase-shifts in subsequent collisions, and the neglect of off-energy shell effects between successive collisions. In practical applications [25] one also neglects the spin dependence and charge exchange part of the elementary nucleon-nucleon scattering amplitude.

Figure 7 shows a fit [23] to the p-^{4}He data [22] using a Gaussian distribution for the nucleons in ^{4}He. Curve 1 shows the contribution of single scattering; curve 2 that of single plus double scattering; curve 3 - single + double + triple; etc. The role played by multiple scatterings at high momentum transfers is quite obvious. A considerably improved fit to the data is obtained if one uses (Ref. [23]) for the nucleon density the expression (14) whose parameters were made to fit the electron data. This is shown in Fig.8. Unfortunately, however, it turns out that the fit in this case is very insensitive to the correlation function, and a single-particle density with no correlations, that leads to a good agreement with the electron data, seems to fit the proton data as well as the curve in Fig.8.

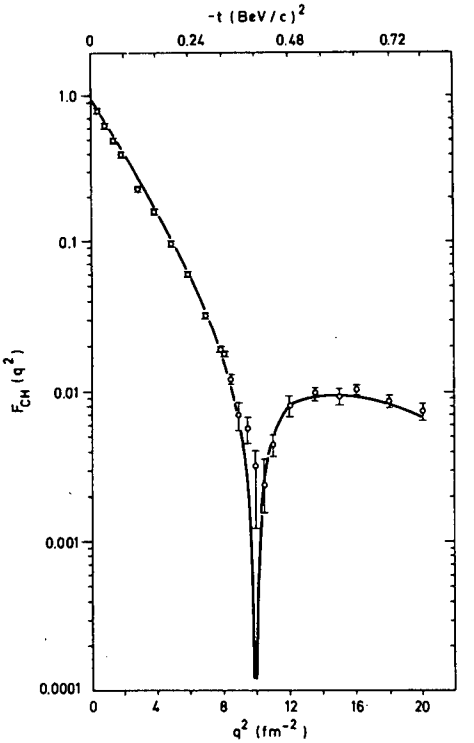

FIG. 6. e-^{4}He elastic form factor with a fit to a double Gaussian charge density modified by correlations. Reproduced from Ref. [23].

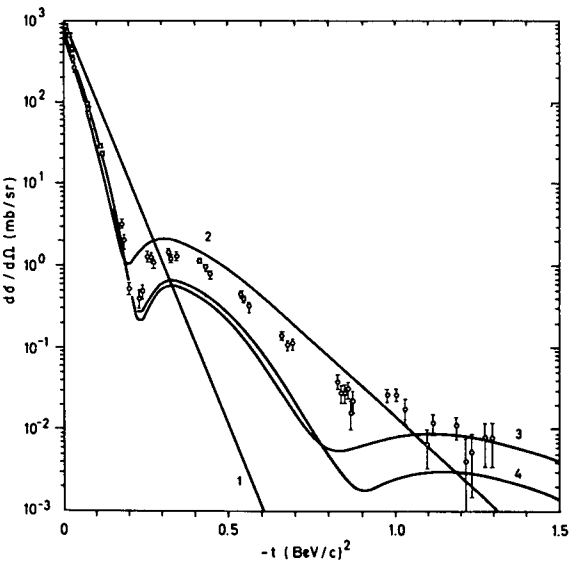

FIG. 7. p- ^{4}He scattering at 1 GeV, with theoretical predictions involving 1, 2, 3 and 4 successive scatterings. Reproduced from Ref. [23].

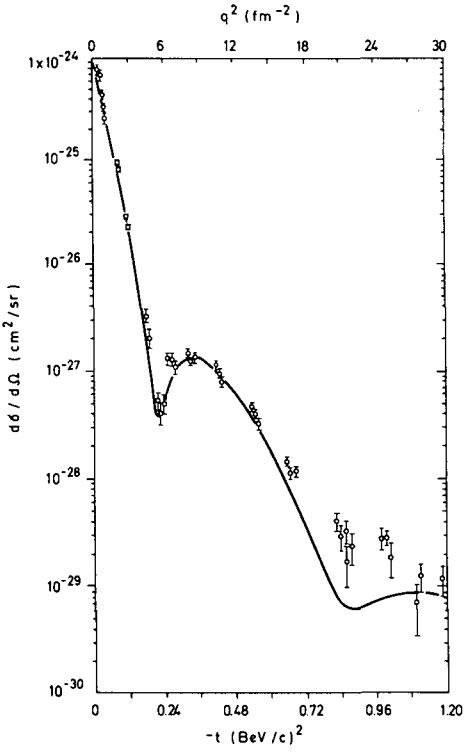

FIG. 8. p- ^{4}He scattering at 1 GeV, with the theoretical prediction using Glauber's theory and a double Gaussian density modified by correlations. Reproduced from Ref. [23].

The discrepancies between theory and experiment at high momentum transfers may reflect either real effects or a breakdown of the Glauber approximation which, by its own nature, is a small-angle approximation. Neither the data nor the theory is in a good enough shape to draw definite conclusions at this stage. However, we should remember that in fitting the data in Fig.8 there have been no free parameters whatsoever: the nuclear density being determined from electron scattering and the elementary nucleon-nucleon scattering amplitude being taken from the two-body data. The results are encouraging enough to lead us to believe that we have a fairly good approximation method at hand whose exact range of validity should be further investigated. The subject is of particular interest not only because of its potentialities in elucidating problems of nuclear structure, but also because of its possible uses in the determination of scattering amplitudes of very short-lived particles off nucleons. A study of the information that can be obtained in this way from the coherent photo-production of ρ^0 was given by Drell and Trefil [26], but it is obvious that a large class of reactions can be analysed in a similar fashion once the nature of the approximations is better understood.

I have touched on only very few aspects of the nuclear shell model. Space and time do not permit me to do justice to the whole subject. I do hope, however, that in presenting this brief review I have succeeded in

indicating some of the basic problems in strong-interaction physics to whose solution we may come closer by further detailed study of nuclear physics, and that some of us may find further applications of the methods used in the studies of the nuclear shell model. The versatility of nuclei on the one hand, and their being manifestly finite on the other, makes them a unique system for the study of many features in modern physics.

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NUCLEAR PHYSICS AND THE MANY-BODY PROBLEM

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Abstract

NUCLEAR PHYSICS AND THE MANY-BODY PROBLEM. A review is given of the use that has been made of stationary and time-dependent Hartree-Fock theory in nuclear physics. There is also a discussion of the oscillations in charge density expected from the shell model of nuclei and their bearing on the properties of nuclear matter.

I shall discuss mainly the kind of understanding that we can get of quite a large variety of things in nuclear physics by the use of rather simple ideas based on self-consistent field theory. Of course, once the shell model of nuclear structure is accepted, the idea of using self-consistent field theory is obvious. Shell-model particles are moving in a certain potential and this potential is in some sense due to the average effect of all the nucleons in the system. So the Hartree-Fock theory is an obvious tool to use in nuclear physics and it has been used for a very long time indeed, more or less since nuclear physics began, or at least since the neutron was discovered. On the other hand, this approach has very many limitations. I am not going to emphasize these; I want to take the other attitude and point out how many things we can understand qualitatively in terms of the Hartree-Fock theory with its various generalizations.

If we start asking detailed questions, about exactly what interaction should be used in the Hartree-Fock theory, we get into very considerable difficulties. One difficulty is that it is generally believed that the nucleon-nucleon interaction has a hard core. Once we start taking determinants of wave functions which are spread out over the whole nucleus, the matrix elements of a potential with a hard core are necessarily infinite. It doesn't help very much to use the slightly softer cores which have been advocated recently, because, although they are called soft, they are not all that soft, and so matrix elements are still rather large and positive. I shall therefore ignore the question of what is a suitable form of the interaction.

I wish to point out a few of the differences between atoms and nuclei; it is important to understand these because there have been various attempts to apply the methods that nuclear physicists have been advocating to atomic systems, and some of these attempts have met with a considerable amount of criticism which I think has mostly been justified. The difference between the nucleons in the nucleus and the electrons in an atom is that, despite the repulsive core, the nucleon-nucleon force is predominantly attractive, whereas the electron-electron interaction is completely repulsive. This has various important effects.

The first effect can be seen in its simplest form if a single valence nucleon is added to a doubly closed shell nucleus like ^{16}O or ^{40}Ca. If Hartree-Fock theory is taken seriously, the extra particle in, for example, a d-state produces a potential which is not spherically symmetric, and so the system no longer has spherical symmetry. I must emphasize that nobody really has solved the Hartree-Fock equations properly, in the way they have been solved for spherically symmetric atoms, although recently, particularly in the M.I.T. group, there has been a lot of work which almost solved them. Of course, the resulting shape depends on which particular d-orbital we choose to fill. Put the nucleon in an equatorial orbit, and it has one shape; put it in an orbit which has its maxima at the poles, and it has a different shape. The equatorial orbit is one with maximum projection of angular momentum in the z-direction, the polar orbit is one with $m = 0$. If there is, say, an extra nucleon in an equatorial orbit, because the force is predominantly attractive, the closed shell part, the core, will be distorted, and it will be distorted in such a way that there is overlap between the core and the extra particle. If an extra nucleon is added in an equatorial orbit, the core will be distorted into an oblate shape, and if the orbit has its maxima at the poles, the core will be distorted into a prolate shape. If we go on taking the Hartree-Fock theory seriously, we can in this way, by adding more than one particle, build up quite large distortions. This is a cumulative effect because the nucleons try to come as close together as possible. It is quite opposite to the effect in atoms where the effect of adding a d-electron with $m = 2$ will be, if the Hartree-Fock equations are taken seriously, to distort the core in the opposite direction and keep the core electrons as far away from the extra electron as possible. Since the valence electrons also try to keep away from one another, there is no big distortion produced when several valence electrons are added to an atom. We tend to get large distortions in nuclei, whereas in atoms, in the Hartree-Fock theory, the distortions tend to cancel out and it makes more sense to ignore them. In nuclei they simply cannot be ignored, because there are these cases of nuclei which we know to be very strongly deformed and which display rotational spectra.

Another important effect can be seen by considering two valence nucleons – we forget about their effect on the core but just consider the coupling between them. In atoms the coupling scheme between electrons is given by Hund's rules which essentially mean that the orbital and spin angular momenta separately are as large as they can possibly be. With nuclei the situation is quite opposite: if two like nucleons are added, two neutrons or two protons, they couple invariably to angular momentum zero. There is one consequence of this which should be considered in terms of the Hartree-Fock theory and this is that an angular momentum zero state for two particles is a superposition of a large number of different states, in terms of the particle orbitals with a given angular momentum and z-component of angular momentum. The opposite case, the Hund's rules case, is rather different, because if the maximum possible projection of angular momentum in the z-direction is taken, this state can be written as a single determinant, whereas the state of angular momentum zero which occurs in nuclear physics cannot. However, a very important advance was made by Bohr, Mottelson and Pines who pointed out that the states which occur in nuclear physics with particles paired to angular momentum zero can be treated using the theory of superconductivity as suggested by Bardeen, Cooper and Schrieffer. This is known

to be a generalization of Hartree-Fock theory, a generalization in which the conservation of the total number of particles is violated. This is a very nice idea because it means that we can extend Hartree-Fock theory in this way to take account of the pairing of nucleons, the coupling to states of angular momentum zero.

When we have nuclei close to closed shell nuclei, there is pairing and there is no distortion. The pairing works against the distortion of nuclei to a spheroidal shape, and when there are more particles, we eventually get a transition to a spheroidal shape and can think of this as something like a phase transition, a phase transition from the spherical phase to the deformed phase. Of course, the only parameter at our disposal, if considering the ground-state, is the number of particles, and this is not a continuous variable, so we must not take this notion about phase transitions too seriously. Also, of course, we know that phase transitions in the mathematical sense do not occur in finite systems. There are no singularities of the energy as a function of coupling constant or whatever. Such singularities are characteristic of infinite systems. These singularities do, however, occur in the Hartree-Fock theory because, as there is a change in symmetry, there will necessarily be non-analytic behaviour. In passing from the range of nuclei for which the solution of the generalized Hartree-Fock equations is strictly spherical to those for which the solution is spheroidal, clearly something is behaving non-analytically and the energy must reflect this non-analytic behaviour.

However, I will forget about this difficulty of finite systems for the moment and point out a very old argument in the theory of phase transitions. In one of his fundamental papers on the theory of second-order phase transitions, published in 1937, Landau argues that the transition from the solid state to the fluid state is necessarily a first-order phase transition and cannot, except accidentally, be a second-order phase transition; that is, it will not be a second-order phase transition over any range of the values of pressure or temperature although it may be so at an isolated point. We can make the same argument about the transition from the spherical state to the spheroidal state. The argument is simply that we have some parameter, the deformation in this case, which describes the change in symmetry, and we consider what is the energy as a function of this parameter. Although the energy always has a stationary point when this parameter is zero, so that the spherical state gives a stationary point of the energy, it does not have to be a minimum; it may be a maximum. We know that the first derivative of the energy is necessarily zero and we ask what does the second derivative do and what do the third and fourth derivatives do. In those transitions which are necessarily second order, the third derivative vanishes, because if it does not vanish, an asymmetric curve results like the one shown in Fig. 1. At zero deformation there is a stationary point, but with a non-vanishing third derivative, this kind of asymmetric shape results and the second minimum appears and goes below the original minimum for some value of the particle number. This will occur before the second derivative at the origin vanishes, so that we still have a minimum at the origin, but a further minimum later on. We will forget about further derivatives, but we have to consider at least four terms in the Taylor series expansion and, unless the third term vanishes, we get the asymmetric shape with a distinct minimum, and this is characteristic of a first-order phase transition.

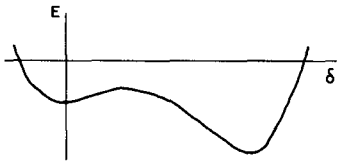

FIG. 1. Energy E as a function of deformation δ for a deformed nucleus.

Whether or not the third derivative must vanish depends on the nature of the symmetry change. When one goes from a spherical nucleus to a spheroidal nucleus, the symmetry change is characterized by a quadrupole deformation of the nucleus, something with angular momentum 2 and even parity. The second derivative is calculated by coupling together two of these deformations; we have to couple them together in a spherically symmetric way, which is easy – just a Clebsch-Gordan coefficient for the symmetrical coupling of two things with angular momentum 2 and even parity to form something with angular momentum zero and even parity. For the third derivative we have to consider how to couple together three of these to get something with angular momentum zero and even parity, and this, of course, we can do. We can couple together in a symmetric way three things with angular momentum 2 and even parity to get something with angular momentum zero. By using Landau's argument, we can show that there is no symmetry reason for the third derivative to vanish. It might vanish, of course, but there is no necessity for it to vanish, so we must expect the energy as a function of deformation to go in the way I have shown. Therefore, we would expect the transition to be discontinuous so that, as soon as spheroidal nuclei appear, the deformation should not come in gradually but should go immediately to a finite value.

For reasons I have given before, this argument is, of course, really applicable only to the infinite system and must not be taken too seriously for finite systems. We have to ask just how seriously can we take any such argument based on the idea that the nucleus is sufficiently large that it can have sharp transitions of this kind. Obviously, when we go through this argument, we have to consider the effect of fluctuations. The curve in Fig. 1 has two minima and it must be possible to have fluctuations in which the system goes from one minimum to the other. These are not thermal fluctuations but fluctuations due to quantum zero point motion. We can regard this either as tunnelling from one minimum to the other or, if the minima are rather different in energy, as a virtual transition. At any rate, there will be a finite probability, even if one minimum lies significantly lower than the other, of finding the system at the other minimum. In the last few years some interesting cases have come up where this idea of two minima seems to be relevant. The best-known case is ^{16}O, a nice doubly magic closed shell nucleus which, however, appears to have an excited state on which a rotational band is built. There is a sequence of levels 0^+, 2^+, 4^+ and 6^+, spaced more or less like the rotational band in a strongly deformed nucleus, which are excited by the sort of reactions we would expect to excite rotational levels – α-particles on ^{12}C for example – and so in this case we do seem to have the coexistence of a spherical state and a spheroidal state. However, there may be some mixing between them for the reasons I have given, just quantum mechanical zero point motion. Unless the two states have a real difference in symmetry, there will be mixing between them and we shall not expect either the ground-

state or the first excited state, the one at the bottom of this rotational band, to be pure spherical or pure spheroidal. We have an analogous situation in ^{20}Ne where the low-lying energy levels are characteristic of a deformed nucleus, but it may be that excited states form which are characteristic of a spherical nucleus.

In this connection it is worth pointing out what has been obvious for a long time, that the description of states in terms of single particle motion or in terms of some different type of correlation between particles, as in the α-particle model, is rather blurred. When we try to calculate, for example, the properties of light nuclei, using an α-particle model, we get wave functions – at least for the ground-state – which overlap strongly with the wave functions which we would calculate using the shell model. This would not be so if ordinary α-particle dimensions were used, but it seems that the size of the α-particle is rather different when it is bound in a nucleus. Of course, the description of excited states may be much simpler in terms of a different model. We may get states which are very simply described in the shell model but which are rather complicated when using a deformed shell model or an α-particle model, and we may get other states which are very simple in an α-particle model or in a deformed nucleus model but which are a rather complicated superposition of many configurations when we use the spherical shell model.

I shall now discuss the description of excited states by using an equation of motion based on the Hartree-Fock theory. The equations of time-dependent Hartree-Fock theory are

$$
\begin{aligned}
&-\frac{\hbar^2}{2m}\nabla^2\phi_i + V(\vec{r})\,\phi_i(\vec{r}) = i\hbar\frac{\partial\phi_i}{\partial t} \\
&V(\vec{r}) = \sum_{j=1}^{A}\int v(\vec{r}-\vec{r}\,')\,|\phi_j(\vec{r}\,')|^2\,d^3\vec{r}\,'
\end{aligned}
\tag{1}
$$

I have put them in a very simplified form, in that I have ignored the Fock part of it and have written down a self-consistent potential which is just of the Hartree form; there is no exchange in it. This is not because I want to ignore exchange, but it is just because I want to write things down simply. We cannot ignore exchange in nuclear physics when we have forces which are explicitly exchange forces – charge exchange or spin exchange – but it does make things look much simpler if we ignore these and just take a local potential V(r) and write it down in this way. We have the usual sum over all occupied states. ϕ_j is the wave function for an occupied state, $v(\vec{r}-\vec{r}\,')$ is the interaction between two particles, and in the top line we have what looks like the Schrödinger equation, but it is not, because the potential V depends on the solutions of the equation in the usual manner of self-consistent theory. I find the equation, which was due to Dirac in 1930, very beautiful. It was not originally in quite this form, as it was written in terms of the density matrix, but it comes to the same thing. The wave function is taken to be a determinant of the lowest of these solutions of the equation.

A special case of a solution of this equation is the solution of the ordinary Hartree-Fock equation. This is a stationary solution of the equation in the

sense that all wave functions have just a varying phase. The dependence on time is through a factor $\exp(-i\epsilon_i t)$, where ϵ_i is the Hartree-Fock energy of the particular state. The determinant varies with an overall phase, but the important thing is that the square modulus of ϕ_j is independent of time for this stationary solution, so the potential is independent of time and the solution is stationary in the usual sense.

This is one special solution of these equations and can be taken to represent the ground-state of the nucleus. The potential $V(\vec{r})$ will not have the full invariance of the two-particle interactions; in particular, it cannot be translation-invariant for a finite nucleus. If it were translation-invariant, the ϕ's would have to be plane waves and the nucleus would be spread out all over space. So the potential $V(\vec{r})$ necessarily lacks some of the symmetry of the original Hamiltonian. However, the equations taken as a whole still have this symmetry, that is, we have translation-invariance in the sense that if we have a solution located at one point in space, we can get an equally good solution to the same set of equations which is located at a different point in space; we have Galilean invariance. If we have a solution which is stationary with the centre-of-mass fixed at one point in space, we can construct solutions which are moving uniformly through space without any distortion of the nucleus, and this is the Galilean invariance. With deformed nuclei, we have another possibility; we get solutions in which the axis is pointing in any direction in space. But we can also construct solutions which are rotating uniformly, although this is not trivial. With asymmetric distortion, we get a motion which is closely analogous to – I was going to say rigid body rotation, but it is more general than that – something like the rotation of a fluid drop. It is not trivial, and we cannot see immediately what must be the solutions for the rotation in the way we can see what must be the solution for uniform translational motion. At least, if the angular velocity is small, we can write down quite reasonable equations which will enable us to see the characterization of the rotation and in particular to calculate the moment of inertia. This is something which has been done for the complete Hartree-Fock equations, or at least with the idea of solving these equations as well as possible, originally by a student of Peierls, Husain. The same has been done recently and much better by various people who have been looking more seriously at the Hartree-Fock equations, by Bar-Touv and Kelson in particular. This classical rotational motion must be quantized. Several methods of doing this are known, but none is particularly simple or elegant.

I shall not discuss the rotational motion and how to calculate this, but consider the possibility of small oscillations about the solution of the time-independent Hartree-Fock equations, the stationary solution of this equation. If we take this solution and then write the wave function ϕ_i as $\phi_i^0 + \delta\phi_i$, we must consider the effect of small changes in ϕ, in both lines of Eq.(1). There will be a δV, and all this is written as follows:

$$-\frac{\hbar^2}{2m}\nabla^2\delta\phi_i(\vec{r}) + V_0\,\delta\phi_i(\vec{r}) + \sum_{j=1}^{A}\int d^3r'\, v(\vec{r}-\vec{r}\,')\,\phi_j^{0*}(\vec{r}\,')\,\phi_i^0(\vec{r})\,\delta\phi_j(\vec{r}\,')$$
$$+\sum_{j=1}^{A}\int d^3r'\, v(\vec{r}-\vec{r}\,')\,\phi_j^0(\vec{r}\,')\,\phi_i^0(\vec{r})\,\delta\phi_j^*(\vec{r}\,') = i\hbar\frac{\partial\,\delta\phi_i}{\partial t} \tag{2}$$

The first two terms come from just the $\delta\phi_i$ in the first part of Eq.(1) and the third and fourth terms come from the change in the self-consistent potential. Small changes are made in the system, and the system described by the time-dependent Hartree-Fock equations behaves very much as a classical system – when we make small changes in it, it oscillates. Now we know, and this has been worked out in a good deal of detail, that $\hbar$ times the frequency of oscillation corresponds in a quite definite way to an excitation energy of the system. So we now have this linear set of equations for $\delta\phi_i$, assuming that the original ϕ_i^0's are known, and we can look for normal mode solutions. They are not quite in the usual form because at one point there is the complex conjugate instead of the original $\delta\phi$. This has some important consequences which I shall mention soon.

One thing, mentioned by De-Shalit, which must be said about these equations is that we know that the equation of small oscillations must be stable and can show that the normal modes have real frequencies. We cannot tell this just from the structure of these equations – they might have complex frequencies, and in fact they might have pure imaginary frequencies. But we can show that if this system of equations has pure imaginary frequencies, it is a consequence of the fact that the solution of the stationary Hartree-Fock equations does not give a real minimum of the energy but only a saddle point of the energy. This may happen, for example, if we choose a spherical solution of the Hartree-Fock equations which is a real solution but, nevertheless, there exists a spheroidal solution with a lower energy. In that case it may be that these equations will have an imaginary solution, and when we go to this spheroidal solution of the Hartree-Fock equations and construct the corresponding equations in that case, then we will get only stable oscillations and there will be no imaginary frequencies.

Now I want to spend some time on interpreting the various terms in this equation. I have not actually written the normal mode equation, but it can easily be converted into a normal mode equation. I want to point out how these terms are related to what nuclear physicists do naturally. We must have a $\delta\phi$ which is orthogonal to the occupied states. If we take only the first two terms of Eq.(2), together with the right-hand side, we get frequencies which are just the differences between single-particle energies in the stationary Hartree-Fock potential, that is, they are the ordinary unperturbed shell-model particle-hole excitation energies. Many books discussing the Hartree-Fock theory quote a theorem that the energies of excited states are just the differences between energy levels above and below the Fermi surface in the Hartree-Fock potential, plus a correction which is stated to be small; in fact, for large systems as in solid-state physics, where this theorem is most generally useful, these correction terms are small except for a very small class of states.

However, these small corrections do in some cases work in a coherent manner which produces large energy shifts. The corrections are due to the third and fourth terms. In the third term there is just a $\delta\phi$ without any complex conjugation. The effect of this is to take into account the interaction between the particle-hole states. If we solve the equations as I have written them, ignoring the fourth term, we get the same problem as if we construct all the particle-hole states, calculate the matrix elements of the interaction between these particle-hole states, and then diagonalize the Hamiltonian between them. This is adequate for many purposes in nuclear physics. For example, one important case of this is the giant

dipole resonance in nuclei, the 1^- excitation which lies above the ordinary particle-hole levels. The energy of this for ^{16}O was calculated by Elliott and Flowers about twelve years ago, and it can be found by a configuration mixing calculation where we diagonalize between these various particle-hole configurations. The matrix elements of the interaction act in a coherent manner, so that the energy level is raised by quite a large amount, despite the fact that individually the extra terms are quite small.

The fourth term is rather special. Because it contains the complex conjugate of $\delta\phi$, it is out of phase with the other terms for most states and does not make a large contribution. However, if the shift from the unperturbed particle-hole energy is very large, the fact that this term is out of phase is more or less irrelevant and it makes as important a contribution as anything else. One very well-known example of this is the case of plasma oscillations in Fermi systems where, if this term were ignored, we would simply get an infinite result for the plasma frequency, since this term shifts the plasma frequency down to the proper value. If we ignore this term, we get a silly answer, and so it is quite vital in that particular problem. In nuclear physics there are the cases of low-lying vibrational modes – the octupole state in all nuclei and the quadrupole state in nuclei other than closed-shell nuclei. If we ignore the fourth term in those cases, we do not get such a large energy-shift as if it is taken into account. There is no doubt, for strongly collective states, this term ought to be taken into account. It should be remembered that in these cases we should use generalized Hartree-Fock theory, taking into account pairing. A more complicated set of equations results – there are twice as many equations – so these have not been written down here, but in principle there is no difficulty.

The frequencies of normal modes obtained from Eq.(2) are equal to the energies of excited states in the random phase approximation, divided by $\hbar$. One way of understanding the random phase approximation is to think of it as the result of summing an infinite sequence of Feynman diagrams. Figure 2(a) shows one of the diagrams which, if we like to think in terms of adding up an infinite set of diagrams, must be taken into account when calculating just the configuration mixing of particle-hole states. There is a sequence of something like this, with a single chain going through with many bubbles in it. The particle-hole pair interacts and forms another particle-hole pair, which interacts again and forms another particle-hole pair, and so on. This, happening to any order, is what we add up when we perform the diagonalization of the Hamiltonian between particle-hole states. Figure 2(b) shows the sort of thing we take into account when we put in the fourth term of Eq.(2). The time axis is vertical, so the chain goes backwards in time as well as forwards, and if we interpret the Feynman diagrams, it is clear that we have initially the simultaneous creation of two particle-hole states. These must have the same symmetry as the state being considered, the same angular momentum, for example, so that, if the state were an octupole state, these particle-hole states would have to be coupled to angular momentum 3 and have odd parity. There is the creation from the vacuum – that is from the ground-state – of a pair of these particle-hole excitations and subsequently one interacts with the original particle-hole state and the other goes forward to the final state. This explanation in terms of perturbation theory shows that what we are taking into account is the virtual creation of these excitations in

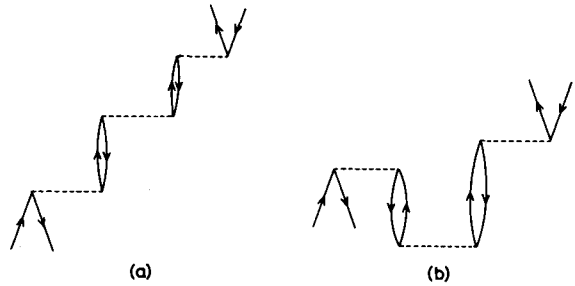

FIG. 2. Typical Feynman diagrams included (a) in a configuration mixing calculation of particle-hole state energies and in the random phase approximation, and (b) in the random phase approximation only.

the zero-point motion of the ground-state. In the ground-state, if there is a low-lying vibrational mode, this mode will have zero-point motion and that must be taken into account to get a sensible calculation.

I shall not discuss how these calculations are done and what kind of interaction is put in. I shall simply point out that there are two general philosophies on this. There is the Migdal philosophy that this is, after all, something very like Fermi liquid theory, as developed by Landau for liquid ^{3}He; there are parameters characteristic of nuclear matter and of the nuclear surface, and we have to fix up these parameters to get agreement with experiment. I think my only criticism of this philosophy is that we have to make assumptions about the special form of these parameters, and the behaviour in the surface may not be real, it may just be something which accidentally fits the data. It is worth taking the opposite point of view, which is that we can really calculate these things from first principles, and this is what Gerry Brown's group has been most involved in for the last several years. Their belief is that we can really calculate these parameters, such as the interaction strength, from first principles and then do further calculations to get the excited states.

There is one further topic I wish to mention briefly; it is a question I have been interested in for a long time and still cannot answer: to what extent can one tell how sharp the Fermi surface is. For example, for nuclear matter, we can calculate in principle the occupation number of states of various momentum, the actual probability of there being a particle with a particular momentum. For a finite nucleus there is a corresponding problem. We can ask what is the Dirac density matrix and then, when we diagonalize it, what are the values of the diagonal matrix elements. The first question is just a special case of this. For a non-interacting system there is just a sharp cut-off at the Fermi surface. It is generally supposed that when we ask the number of states with a given value of the momentum, for an interacting system we expect to get something which looks like Fig. 3. We do not know how large the gap denoted by ζ is. In fact there are people who will argue, I think quite reasonably, that it is not really there at all. Still, something rather abrupt must happen near the Fermi surface, as we know from many effects in solid-state physics. It was shown by Migdal a long time ago that the magnitude ζ of the gap is related to the pole of the one-particle Green's function at the Fermi surface. Therefore it is equal to the square of the overlap between a particle state and a quasi-particle state. The overlap between a bare particle and a physical interacting particle seems to be very definite, and it would be nice to have some way of

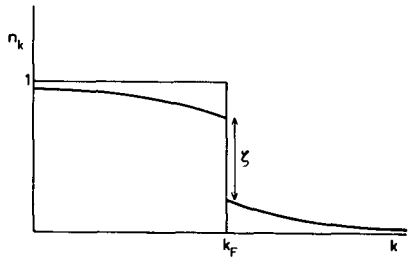

FIG. 3. Occupation number n_k as a function of momentum k for an interacting Fermi system.

finding out how large it is. For electrons in a metal, calculations of this quantity have shown that it is something rather near to unity and so, although there are various experiments which could in principle measure it, the measurement is not easy. In nuclear physics it has been estimated, by Brueckner and Bethe and others, to be something in the region of 0.8, but it might be less, and also it might be less in liquid ^{3}He.

I have been intrigued by the question of how to measure ζ for a long time, and I am not the only person who has had the following idea. We talk quite freely about strength functions in this Green's function language as giving the magnitude of the overlap between particles and quasi-particles, and also nuclear physicists talk about strength functions; these two strength functions might be related. Using strength functions in neutron elastic scattering, for example, ζ could be measured, or at least it could be if neutron scattering could be done with neutrons whose momentum in nuclear matter is equal to the Fermi momentum. The energy of such neutrons is less than zero, but we can perhaps extrapolate to this energy. I am convinced that the strength functions derived from the experimental results have nothing whatever to do with ζ, and in the case of elastic scattering I can show that the same elastic scattering data can be produced in ways that give completely different values of ζ. This seems to me to be the most straightforward way of showing that these two have nothing whatever to do with one another. Some consider this is true. Yet when one does other calculations of direct reactions, e.g. pickup and stripping reactions, in some way one measures the overlap between the actual particle state and the ideal shell-model state. I do not think this is true, because these calculations always depend on the form of interaction, and probably we can modify the form of interaction in such a way as to reproduce the same result with different amounts of overlap. This has been shown in considerable detail for problems in solid-state physics, particularly by Nozières. For the analogous question it has been shown quite conclusively that low-energy experiments, which do not involve a rather large transfer of energy, do not show up the overlap between physical interacting particles and bare particles. So I think what I have stated will be quite generally accepted by the solid-state physicists and I would arouse a good deal of opposition if I said it were not so.

There remains the question of what experiments are suitable to determine this quantity. I have recently studied an effect of the Fermi surface which has been discussed in detail by Kohn and Sham, namely, that if we introduce a perturbation into a solid, we get an oscillating electron density whose wavelength depends on the Fermi momentum, and this is a result of the abrupt change in occupation numbers across the Fermi surface. One form in which this effect is seen is in the Friedel oscillations of the

potential due to an impurity. Another form is in the Ruderman-Kittel interaction between spins, which has an oscillatory form resulting from the sharpness of the Fermi surface. Yet another form of this which has been discussed by Kohn and Sham is due to the effect of a surface on the system. We would expect oscillations not only near a local impurity potential but we would also expect oscillations of particle density near the physical surface. Figure 4 shows the potential energy near the surface of semi-infinite nuclear matter, with the Fermi energy marked on it, and the density of particles is also plotted. Particles are bound to the left-hand side and do not penetrate to the right. There are oscillations which decay as we go away from the surface. The wavelength is π/k_F and the magnitude of the peak is about 8% above the mean value of the density. Presumably, if these particles interact, the strength of these oscillations is damped down and this gives at least a very indirect measure of the amount to which the Fermi surface is truly sharp, in a problem where there is a physical surface.

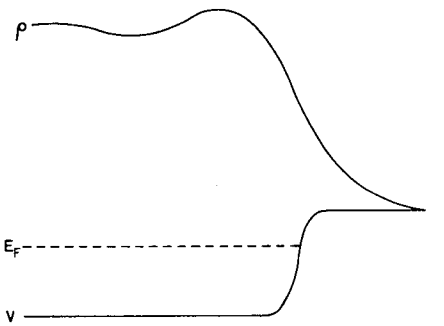

FIG.4. Density ρ of semi-infinite nuclear matter with Fermi energy E_F. The interactions between nucleons are only taken into account through the supposedly self-consistent potential well V.

Of course, in nuclear physics, we do not have a semi-infinite system such as is shown in Fig.4, we only have finite nuclei. A student at Birmingham, M.A. Thorpe, has recently studied the shell-model calculations of nuclear densities and investigated to what extent they have this property near the nuclear surface. Figure 5 shows the proton density for tin and for lead, with the wave functions calculated in a Woods-Saxon potential, compared with the Thomas-Fermi approximation to the density. The oscillations in density show up clearly, and the distance between the peaks gives a reasonable value for π/k_F. Looking through nuclei we see that these oscillations do occur in the shell model, unless we use a harmonic potential. They appear as a result of the shell structure with discrete angular momentum states, but they can be regarded in this other way as a result of there being a Fermi surface and something like a plane wave representation being relevant.

There is another feature here which we do not get in nuclear matter. It is clearly important in all finite nuclei and we can see its origin well enough. There is a bump at the centre or, in some cases, a dip. This must obviously be there — the density at the centre depends solely on the number of s-protons and of course we do not fill s-shells very often, only once in every other major shell. Whether this density at the centre is larger or smaller than average depends on whether there are rather more

s-protons than would be expected on the average or rather less. Since there are very few s-shells filled in real nuclei, this effect must be quite large. When we have just filled an s-shell, as in Ca, for example, there is a bump at the centre. When we go over to Sn, filling up the f-, p- and g-orbits, there is by that time a dip at the centre. It would be nice to know what the effect of interaction on this distribution is, and this is one of the things we are studying. Probably, long-range interactions do not influence very much these bumps at the surface. I would hope, and Bethe would certainly hope, that the bumps at the centre can be smoothed out, because

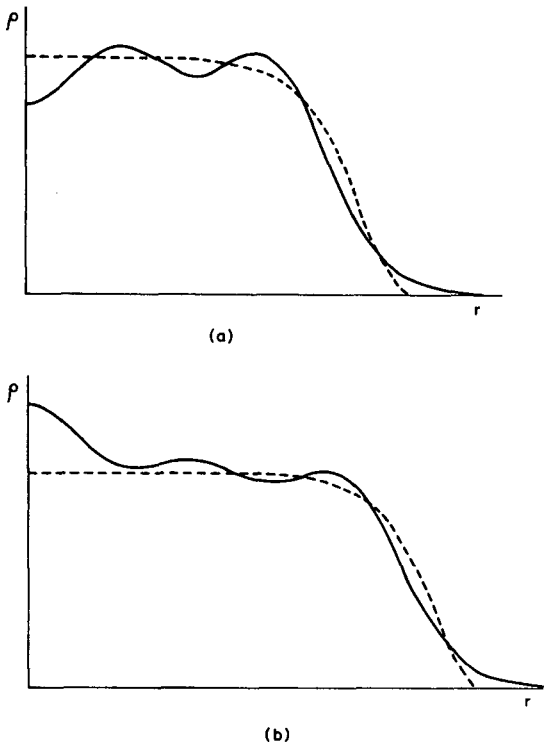

FIG.5. Density of non-interacting protons in a Woods-Saxon potential well as a function of the distance r from the centre for (a) tin and (b) lead (copied from M.A. Thorpe, unpublished M.Sc. thesis, Univ. Birmingham).

it is rather easy to get rid of a few s-protons, since they get virtually excited to low-lying states which are not s-states. I doubt if the bump at the surface does, except by interactions to rather high energies, because it is a characteristic of this sharpness of the Fermi surface. Low excitations are going to be characterized by rather long wavelengths and they will not very much influence the wiggles near the surface.

MICROSCOPIC THEORY OF EFFECTIVE OPERATORS OF ELECTROMAGNETIC INTERACTIONS IN NUCLEI*

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Abstract

MICROSCOPIC THEORY OF EFFECTIVE OPERATORS OF ELECTROMAGNETIC INTERACTIONS IN NUCLEI. The use of realistic two-nucleon potentials in microscopic theories of nuclear structure makes it possible to calculate effective electromagnetic operators. E2- and M1-operators for ^{116}Sn are calculated from the Yale-Shakin potential in terms of particle-hole excitations and are applied to the calculation of observable quantities.

Very little has been said in the session about the electromagnetic properties of nuclei. The many-body nuclear problem introduces an enormous complexity and leads to prohibitive dimensions in the complete shell-model description of the nuclear spectra and structure. The mathematical situation forces us to eliminate from our explicit treatment the so-called core, the supposedly "inert" part of the nucleus. As for the electromagnetic operator in nuclei one has usually resorted to the purely phenomenological concept of "renormalized" effective charges of the valence protons and neutrons which in practice were only ad hoc adjustable parameters.

This situation is a sacrifice which must not be tolerated in a microscopic theory, particularly if we pretend to have a description based on the "first principles", i.e. on the realistic two-nucleon potentials.

It turns out that a fully microscopic description is possible in terms of effective operators theoretically derived and justified in terms of virtual excitations and de-excitations of core nucleons.

Consider a 2^λ-pole transition from a nuclear state of energy E to one energy E' both described microscopically in an explicit way in terms of valence nucleons only. The many-body perturbation theory leads to the following frequency ($\Delta E = E' - E$)-dependent effective operator for this electromagnetic process defined in terms of its reduced single-particle (valence nucleon) $n \to n'$ matrix elements:

$$\begin{aligned}\langle n' \| \hat{O}_\lambda^{(eff)} \| n \rangle = \langle n' \| \hat{O}_\lambda \| n \rangle &+ 2 \sum_{ph}^{(neutrons)} \Big[F_{VN}(n'nph, \lambda) \frac{q_\nu(ph)}{E_p^0 - E_h^0 - \Delta E} \\ &\times \langle p \| \hat{O}_\lambda^{(\nu)} \| h \rangle + \langle h \| \hat{O}_\lambda^{(\nu)} \| p \rangle \frac{q_\nu(ph)}{E_p^0 - E_h^0 + \Delta E} F_{VN}(n'nhp, \lambda) \Big] \\ &+ 2 \sum_{ph}^{(protons)} \Big[F_{VP}(n'nph, \lambda) \frac{q_\pi(ph)}{E_p^0 - E_h^0 - \Delta E} \langle p \| \hat{O}_\lambda^{(\pi)} \| h \rangle \\ &+ \langle h \| \hat{O}_\lambda^{(\pi)} \| p \rangle \frac{q_\pi(ph)}{E_p^0 - E_h^0 + \Delta E} F_{VP}(n'nhp, \lambda) \Big] \end{aligned} \tag{1}$$

* Based on work done by M. Gmitro, A. Rimini, J. Sawicki and T. Weber.

Here the first term is the bare matrix element of the $\hat{O}_\lambda$-operator (it may be zero), $q_{\nu(\pi)}(ph)$ are the neutron (proton) projectors for the respective particle-hole (ph) pairs taking into account also the effect of the Pauli exclusion principle for transitions where a valence (partly occupied) sub-shell is involved in (ph); F_{VN}, F_{VP} (abcd, J) are reduced particle-hole-coupled elements of the two-body reaction matrix operators K_{VN}, K_{VP}, respectively, of a neutron (N) or a proton (P) with a valence nucleon (V); for identical charge nucleons the F-elements are anti-symmetrized.

While the second-order terms are most important, similar third- and higher-order terms obtained by iteration of the diagrams involved lead to only negligible corrections, in general.

The present theory has been successfully applied to study some single closed-shell nuclei of the so-called vibrational region.

TABLE I. THE EFFECTIVE QUADRUPOLE CHARGE MATRIX $e_2(n, n')$ FOR ^{116}Sn

	$3s\frac{1}{2}$	$2d\frac{3}{2}$	$2d\frac{5}{2}$	$1g\frac{7}{2}$	$1h\frac{11}{2}$
$3s\frac{1}{2}$	-	0.6154	0.6096	-	-
$2d\frac{3}{2}$	-	0.6459	0.6398	1.0516	-
$2d\frac{5}{2}$			0.6521	1.1054	-
$1g\frac{7}{2}$				1.0844	-
$1h\frac{11}{2}$					0.6535

One example is that of the effective E2-operator in the tin isotopes [1, 2]. The proton core is closed shell (Z=50) and the valence nucleons are only neutrons occupying the five sub-shells between N = 50 and 82. The Yale-Shakin realistic nuclear force is applied and a Woods-Saxon single-particle basis. In Table I we give the effective quadrupole charge matrix $e_2(n, n')$ for ^{116}Sn for the $\Delta E = 0$ matrix elements; here $e_2(n, n') \equiv \langle n' \| \hat{O}_2^{(eff)}(0) \| n \rangle / \langle n' \| \hat{O}_2 \| n \rangle_{ref}$ where $\langle\ \rangle_{ref}$ is the "reference matrix" defined for "direct" $n \to n'$ transitions and $e_{eff}^{(2)} = 1$. The effective E2-operators lead to the values of $B(E2, 2_1^+ \to 0_1^+)$ and of the quadrupole moment $Q(2_1^+)$ compatible with the existing data [1, 2].

Similarly, the effective M1-operator leads to a nice agreement with the observed value of the gyromagnetic factor $g(5_1^-)$ of the 5_1^- state of ^{116}Sn: $g(5_1^-) = -0.065 \pm 0.005$. While the bare M1-operator and $|5_1^-\rangle$ vector of Refs [2, 3] lead to $g(5_1^-) = -0.1504$, a reduction due to the second-order terms in the corresponding effective M1-operator leads to $g(5_1^-) = -0.0611$.

The inelastic $(0_1^+ \to 2_1^+)$ electron scattering form factor for ^{116}Sn calculated with the appropriate E2-effective operator and the eigenvectors

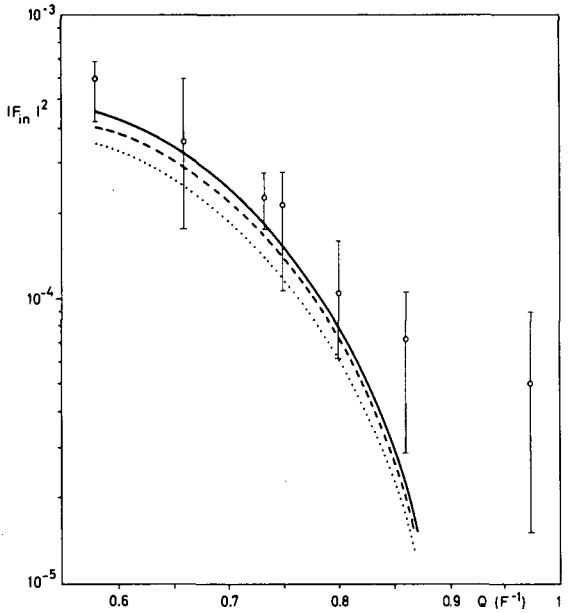

FIG.1. Inelastic electron scattering form factor for the reaction $^{116}Sn(e,e')^{116}Sn(2_1^+)$ at 150 MeV; the three theoretical curves correspond to the vectors $|0_1^+>$ and $|2_1^+>$ of the three respective variants of the quasiparticle theory of Ref.[3]; the experimental data are of Ref.[4]. (This figure is taken from Ref.[2].)

of Refs [2, 3] is given in Fig. 1. The incident electron energy is 150 MeV and the data correspond to the most recent results of the Saclay group [4].

The present calculations are most encouraging. On the other hand, given the shell model situation in the so-called vibrational nuclei, the theory of such effective operators is probably the only possible fully microscopic description of the electromagnetic properties of such nuclei.

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EXCITATIONS IN NUCLEI

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Abstract

EXCITATIONS IN NUCLEI. 1. Spectrum of elementary excitations; 2. Static deformations. Rotational spectra; 3. Relationship between members of a collective family; 4. Analysis of collective parameters; 5. Spurious degrees of freedom; 6. Pairing modes.

The nucleus consists of a handful of particles with fairly well established interactions, and so one might think that the only proper approach to the subject would be to start from the Schrödinger equation for the A nucleons and proceed by developing appropriate and well-defined approximation methods. However, even systems of few particles and a hundred turns out to be quite a handful, exhibit an enormously rich variety of structural facets and, as in other systems where many degrees of freedom are involved, the central problem becomes that of establishing the relevant degrees of freedom and of developing the appropriate concepts that govern their interplay. Progress in this direction has been achieved by a combination of approaches, at many levels of phenomenology. This makes it difficult to give anything like a systematic presentation of the subject, and we have selected a few problems of a fairly general nature that have puzzled nuclear physicists, and that might be suited to elicit response from the broader community of quantal physicists.

1. SPECTRUM OF ELEMENTARY EXCITATIONS

To introduce the quanta of nuclear physics, let us consider, as an example, the spectrum of elementary excitations based on the ground state of ^{208}Pb. This "vacuum" state has especially simple properties (has a minimum of degeneracy) on account of the closed-shell configuration of 82 protons and 126 neutrons.

The elementary modes of excitation of fermion type are shown in Fig. 1. The quantum numbers and approximate energies of the excitations within the first few MeV can be simply interpreted on the basis of single-particle motion in a central potential with spin-orbit coupling. This interpretation of the excitations is strikingly borne out by the single-nucleon transfer reactions (stripping and pick-up reactions, such as (d, p), (d, t), (t, α) etc.), which directly measure the matrix elements of the nucleon creation and destruction operators (the nucleon field operators), and which are found to populate just these levels with large cross-sections.

Other properties of the quasi-particles (single-particle and single-hole excitations), such as electromagnetic and β-decay transition moments, confirm the interpretation, but at the same time show renormalization effects due to the coupling to the underlying closed-shell structure. In the present example, the couplings are weak in the sense that each individual particle is

only little perturbed, but the renormalization effects may become large due to the accumulative response of all the particles embedded in the vacuum. For example, a neutron carries an effective charge of about one unit, and β-decay moments are reduced, sometimes by an order of magnitude.

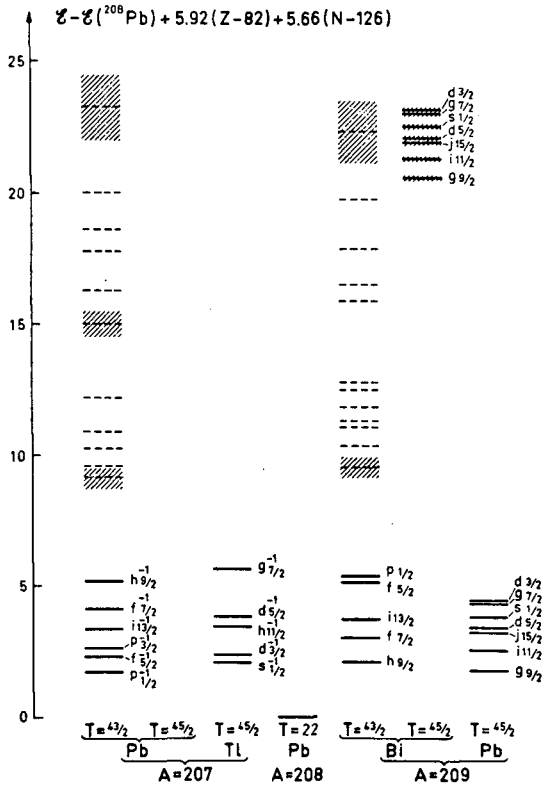


FIG. 1 Quasiparticle excitations based on ^{208}Pb. The dashed levels only illustrate schematically the spectrum at higher excitations.

The figure also indicates the pattern of higher-lying single-particle and single-hole states, which are still only very incompletely explored. Quite new problems arise because these states occur at excitations where the total level density is very high. Some of these problems will be taken up at a later session and we shall only mention the basic concept of strength function, which refers to the manner in which the properties of the elementary mode may become shared by a large number of neighbouring levels. The strength function is a generalization of the line structure of a decaying particle, and incorporates the consequences of the finiteness of the nuclear system and the associated lack of irreversibility of the decay process. A remarkable feature is the occurrence of quite sharp levels (the isobaric analogue states), which owe their stability to the approximate validity of the isospin quantum number, which persists in spite of the strength of the symmetry-violating Coulomb forces responsible for the large splitting between members of a T-multiplet.

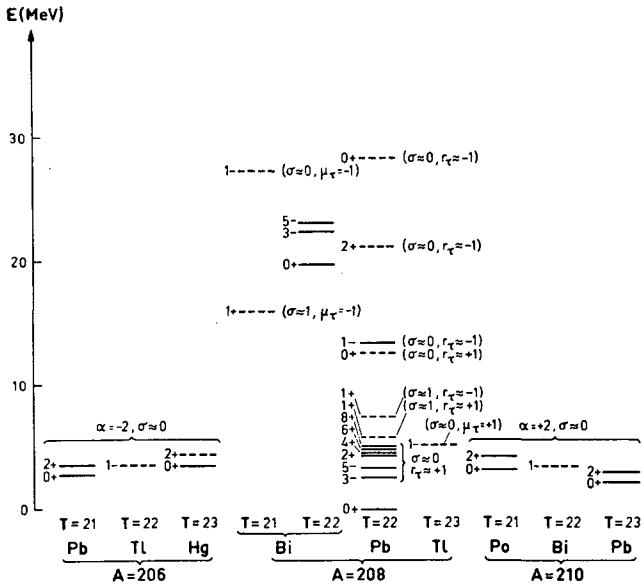


FIG. 2 Boson excitations based on ^{208}Pb. The dashed levels represent theoretical estimates.

The boson excitations in ^{208}Pb are illustrated in Fig. 2, and show a rich spectrum, which is rapidly being supplemented by new discoveries. The excitations in the figure are collective in the sense that they have large matrix elements (in single-particle units), and thus involve many degrees of freedom of the nucleonic motion. Additional excitations are observed, which can be approximately described in terms of two elementary fermion excitations; on account of the finite geometry, the distinction between collective and two-particle excitations is, of course, not a sharp one.

The collective bosons can be associated with different types of deformations in the nucleon density and in the average potential. For example, the lowest excited states in ^{208}Pb correspond to shape oscillations of octupole and diatriacontrapole type. In addition to the quantum numbers λ and π (angular momentum and parity), one can characterize the excitations by additional, approximate-symmetry quantum numbers. Thus, the octupole mode may be assigned the spin quantum number $\sigma \approx 0$, since it is found that the oscillating nuclear potential associated with this mode has only a weak spin dependence. This symmetry belongs naturally to the macroscopic picture of a shape oscillation, but is remarkable because the strong spin-orbit coupling, which decisively influences the nuclear shell structure, violates the invariance with respect to rotations in spin space. Thus, in the microscopic description of the octupole mode, in terms of superpositions of excitations of individual nucleons, the σ-symmetry is completely broken. It appears on the macroscopic level, when we consider the long wave-length components of the oscillations in the nucleon density and potential, involving the superposition of many individual particle excitations. In a similar manner, one can characterize the collective properties of the excitations by a charge symmetry quantum number, r_τ; the octupole mode involves approximately charge symmetric collective density variations ($r_\tau = +1$), though the individual

neutron and proton excitations involve different single-particle orbits, on account of the large neutron excess.

The charge antisymmetric mode ($r_\tau = -1$) with $\lambda\pi = 1^-$ and $\sigma \approx 0$ is the oldest established nuclear vibration, the giant dipole resonance. For the $\sigma = 1$ spin vibration modes indicated in the figure, there is so far only indirect experimental evidence from the renormalization of the spin magnetic moments of the fermion excitations.

As already mentioned, the total nuclear isospin T is an approximate constant of the motion, but for the excitations, the isospin quantum number τ may be badly broken, because of the strong coupling to the isospin T_0 of the vacuum state (the neutron excess). For example, the dipole mode in a nucleus with small T_0 has $\tau = 1$, and gives rise to excitations with $T = T_0$, $T_0 \pm 1$ forming a triplet with closely related properties. With increasing neutron excess, however, the $T_0 + 1$ component is expected to decrease in energy and to gradually lose its strength, which is shifted on to the $T_0 - 1$ component, which moves to higher energies. This pattern remains to be tested experimentally.

SU_4 symmetry, representing the product of the SU_2 spin and isospin symmetries, was considered at an early stage of nuclear-structure physics. It has been found that the symmetry is strongly violated especially by the interactions coupling spin and orbit. Thus, at the microscopic level, the SU_4 quantum numbers, the partitions, have lost their significance except in certain light nuclei. However, as for the spin symmetry, the SU_4 symmetry may reappear at the macroscopic level. For example, the pattern of 0+ and 1+ collective modes with T = 22 and 21, respectively, are expected to have relations corresponding qualitatively to a super-multiplet of a high dimensionality ([f] = [22, 22, 0, 0]).

The modes with nucleon transfer number $\alpha = \pm 2$ may be viewed as oscillations in the nuclear pairing field, which represents the creation and annihilation of two nucleons at the same point in space or close together. The pairing effect, responsible for the difference in binding between even and odd nuclei, has been known from the early days of nuclear physics, but its many-particle structure was only understood after the advent of the microscopic theory of superconductivity. A basic manifestation of the correlation effect is that it gives rise to collective transitions between nuclei differring by two particles. These transitions are now being studied by reactions such as (t, p), (^{3}He, p), etc., by which two nucleons are simultaneously exchanged between projectile and target, and the $\alpha = \pm 2$ modes in Fig. 2 are found to be excited with strongly enhanced matrix elements.

2. STATIC DEFORMATIONS. ROTATIONAL SPECTRA

The boson spectrum considered above has a vibrational character, i.e. it corresponds to oscillations of the various density components about the equilibrium represented by the vacuum state (the ground state of ^{208}Pb). However, for nuclei with many particles in unfilled shells, the independent-particle motion gives rise to a highly degenerate ground state, and the system lifts the degeneracy by reducing the symmetry, i.e. by deforming itself (a well-known feature of quantal systems, referred to in molecules as the Jahn-Teller effect). In nuclei, the most important stable deformations occur in the $\lambda\pi = 2^+$, $\alpha = 0$ channel (quadrupole shape deformations) and in the $\lambda\pi = 0^+$, $\alpha = \pm 2$ channel (monopole pairing deformation (superfluidity).

Stable deformations lead to the occurrence of families of collective excitations with a rotational rather than vibrational relationship, and with degrees of freedom depending on the symmetry violation in the intrinsic motion caused by the deformation. Some examples will illustrate the variety of patterns that may arise.

Let us first consider the breaking of invariance with respect to rotations $\mathcal{R}$ in ordinary three-dimensional space. The separation between collective and intrinsic motion corresponds to a Hamiltonian of the form

$$H = H_{intr} + H_{coll} \tag{1}$$

where H_{coll} depends on the collective quantum numbers, such as the angular momentum I while H_{intr} describes motion relative to the intrinsic frame specified by the collective orientation angles, the conjugates of the angular momentum variables. Thus, the total wave function has the form

$$\Psi = \Phi_{int}(q)\, \mathcal{D}_{coll}(\theta) \tag{2}$$

where q are intrinsic variables (rotational scalars) while θ represents the collective angles. In the extreme case of symmetry violation, the intrinsic deformed system does not go into itself by any rotation of the co-ordinate system. The family of collective states then has the full degrees of freedom of the asymmetric rotor, i.e. there are three collective orientation angles (the Euler angles) and the angular momentum variables I, M, and K (the projection of I on an intrinsic axis), of which K will not in general be a constant of the motion. Thus, the collective spectrum comprises (2I + 1) states for each value of I and M, and therefore forms a two-dimensional system of trajectories in angular momentum space. This situation occurs in molecules, but in nuclei the observed deformations preserve part of the rotational invariance, with a resulting reduction in the collective degrees of freedom. Thus, if the intrinsic Hamiltonian is invariant with respect to a rotation $\mathcal{R}$, this operation is an instrinsic degree of freedom, $\mathcal{R}_i$, and the collective motion must satisfy the constraint that $\mathcal{R}_e$ (the rotation $\mathcal{R}$ considered as an external operation acting on the orientation angles) equals $\mathcal{R}_i$. For example, the well-established nuclear deformations have axial symmetry; the component K of angular momentum with respect to the symmetry axis is therefore an intrinsic property, and the rotational band is reduced to a single trajectory with I = K, K + 1, $\cdots$. A further reduction of the rotational degrees of freedom occurs if H_{int} is invariant with respect to finite rotations, such as a rotation $\mathcal{R}_2(\pi)$ of π about an axis (2) perpendicular to the symmetry axis. For K = 0, we then have bands with I = 0, 2, $\cdots$, well known for the ground-state configurations of even-even nuclei.

New patterns arise if the deformations are linked to the violation of other symmetries; thus, a breaking of $\mathcal{P}$-symmetry (space reflection) gives rise to parity doublets, but if the deformation retains the combined symmetry $\mathcal{R}_2(\pi)\,.\,\mathcal{P}$, we obtain, for K = 0, a band with I = 0^+, 1^-, $2^+ \cdots$. A link between rotational and isospin symmetry breaking is, as is well known, a possible feature in the structure of the nucleon itself. For example, in the strong coupling model of nucleon-pion interaction, the deformed meson field is invariant with respect to simultaneous rotations in spin-orbital and isospin space and gives rise to a rotational band with I = T.

The pairing deformations provide a new dimension to the collective families. These deformations violate the gauge invariance associated with nucleon number conservation, and give collective branches with baryon numbers A, A ± 2, A ± 4, ··· representing rotational motion in gauge space. The deformations conserve the distinction between even and odd nuclei (are invariant with respect to gauge transformations $\mathscr{G}(\pi)$).

One notes the close similarity to the description of the superfluid system, which Anderson has discussed; the state with finite expectation value of the pairing field is here the intrinsic state, and the phase ϕ is the collective coordinate, corresponding to the orientation of a deformed system. In the nucleus, the main effects are associated with the part of the phase field that is a constant over the nucleus. The overall number conservation is explicit in the formalism; the symmetry is broken at the intrinsic level, and the pairing field is expressed in terms of intrinsic variables

$$\psi_{\text{pairing}} \sim a^{\dagger}(r, m_s)\, a^{\dagger}(\overline{r, m_s})\ \exp\{-2i\,\phi_{m_{\text{coll}}}\} \tag{3}$$

which commutes with the number operator. The direct manifestation of the coherence is in the collective "rotational" spectrum, the numbers of which are connected with large matrix elements for two-nucleon transfer.

A richer spectrum of pair addition modes occurs, when we include pair correlations between neutrons and protons. The main pairing effect occurs in the ^1S channel with isospin $\tau = 1$, and the pairing field therefore involves a deformation in isospace (the gap parameter Δ is an isovector). The favoured pairing field has axial symmetry and invariance with respect to the rotation $\mathscr{R}_2(\pi)$ in isospace multiplied by a gauge transformation $\mathscr{G}(\pi/2)$, and the collective rotational spectrum, for even A and $K_\tau = 0$, contains the states with even values of $T + A/2$. The ground states of the even-even nuclei, together with their isobaric analogue states, form a family of this type. (The problem is related to superfluidity associated with a condensate in states with finite angular momentum in the relative motion of the paired particles).

Another manifestation of the permanent distortions is in terms of the modified particle degrees of freedom. As already mentioned, the quasiparticle excitations can be directly studied in the one-particle transfer reactions. For example, if one considers an even-even deformed nucleus and transfers a particle into a given orbit ν in the deformed field, the intensity pattern of the rotational band associated with this orbit gives a fingerprint of the orbit, from which one can deduce the probabilities $c_\nu^2(j)$ that the particle possesses an angular momentum j, while the relative intensities for producing the states by adding a particle or removing a particle give the probability $v^2(\nu)$ that the single-particle orbit is occupied in the pair correlated phase.

3. RELATIONSHIP BETWEEN MEMBERS OF A COLLECTIVE FAMILY

The relationship between members of a collective family is expressed by the dependence of the different matrix elements on the collective quantum numbers. The problem has been especially studied for the rotational bands in I-space, where the empirical information is very rich. For not too large values of the angular momentum I the various matrix elements can be expressed in a power series in I. The form of this series follows from the

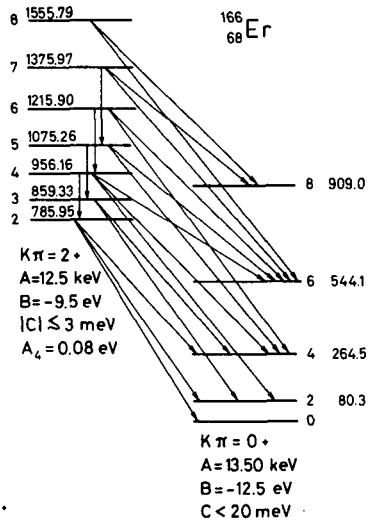


FIG. 3 E2-transitions in ^{166}Er.

symmetry of the intrinsic Hamiltonian, which, as mentioned above, governs the degrees of freedom of the collective motion.

The level of sophistication reached by the experimental studies may be illustrated by the E2-transitions between the K = 2^+ band and the K = 0^+ band of the nucleus ^{166}Er (see Fig. 3), which possesses a shape deformation with axial symmetry and $\mathscr{R}_2(\pi)$ invariance. The expansion of the E2-transition probability is obtained by transforming the E2-moment to the intrinsic frame with orientation ω,

$$\mathscr{M}(E2,\mu) = \sum_{\nu} \mathscr{M}'(E2,\nu)\, \mathscr{D}^2_{\mu\nu}(\omega) \tag{4}$$

The components $\mathscr{M}'(E2,\nu)$ are scalars and can be expanded in powers of the angular momentum components with respect to the intrinsic frame, and one obtains, for the reduced transition probability

$$\begin{aligned} B(E2;\, I_i K = 2 \to I_f K = 0) &= (2I_i + 1)^{-1} \langle I_f \| \mathscr{M}(E2) \| I_i \rangle \\ &= \langle I_i\, 2, 2 - 2 | I_f 0 \rangle^2 (M_1 + M_2 (I_i(I_i + 1) - I_f(I_f + 1)) + \ldots)^2 \end{aligned} \tag{5}$$

The experimental data appear to be well described by the first two terms in this expansion, corresponding to a straight line in Fig. 4. The leading-order term M_1 is related to the transition probability in a nucleus with fixed orientation, while M_2 expresses the perturbations caused by the Coriolis forces associated with the rotational motion. The establishment of such intensity relations provides a decisive test of the rotational coupling scheme. A large body of data on energies and matrix elements for weak, electromagnetic, and strong interaction transition moments is available and has been analysed to yield informations on the intrinsic structure and its coupling to the rotational motion.

For large quantum numbers I, the rotational perturbations may become so large that the power series expansion breaks down. In heavy nuclei, this occurs for I ~ 10-20 for the energy expansion. However, the family relationship may still be retained, manifesting itself in a smooth dependence of the

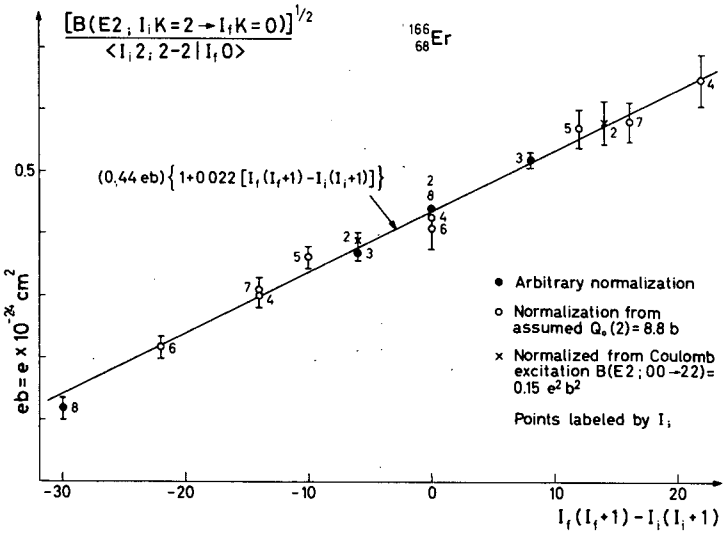


FIG. 4 Rotational intensity relation for E2-transitions in ^{166}Er.

properties on the collective quantum number. For example, for the collective family associated with the pair addition mode, the nuclear properties can be expanded in the collective quantum number A but not around A = 0, only in a local region around $A = A_0$.

Many still unanswered questions arise in connection with the possibility of discontinuities in the trajectories. Such discontinuities may occur as a result of the shell structure, i.e. of specific quantal features in the underlying intrinsic dynamics. For example, the pair addition mode is found to have a discontinuity at major shell closings and has been observed to fractionate at abrupt transitions between spherical and deformed nuclei.

Challenging problems are associated with the structure of rotational bands for very large values of I; a number of different effects may come into play, such as a transition from a superfluid to a normal Fermi system, major centrifugal distortions leading to asymmetries in the nuclear shape, as in the rotating stars, or to fission in the very heavy elements, and a discontinuity may occur when the angular momenta of the particles in the unfilled shells have achieved maximal alignment. In the heavy nuclei, it appears that the pairing is especially vulnerable to the effect of the rotation and, at the highest I-values studied, we are beginning to see effects indicative of the disappearance of the stable pairing field. The effect is quite similar to the disappearance of superconductivity at the critical magnetic field. The phenomenon may afford the opportunity for studying such a "phase transition" under favourable conditions due to the special simplicity of the pairing field, which, in first approximation, couples only the single-particle states that are conjugate under time reversal.

Recently, Anderson objected to talking about phase transitions in the small particle superconductors, and it is of course true that we do not need non-analytic functions to describe the transitions in the finite systems. On the other hand, the phenomena are closely related and, in a system like the nucleus, we have the possibility of studying the transition in terms of the

spectrum of individual quantum states. Thus, the transition from pair-correlated to normal system with increasing angular momentum involves the coupling between the bands associated with the ground state and the excited bands representing fluctuations in the pairing field.

4. ANALYSIS OF COLLECTIVE PARAMETERS

As in other domains of quantal physics, the analysis of nuclear excitations involves a series of different levels of phenomenology. First, there is the general classification, in terms of symmetries, particle configurations, and collective families, on the basis of which the empirical data are expressed in terms of intrinsic parameters, such as the average nuclear potential, effective particle moments and interactions, parameters in the rotational expansions, or, in the case of vibrations, the restoring forces, mass parameters, anharmonicity parameters, etc.

On the next level, one may attempt to describe the collective nuclear properties in terms of macroscopic properties of nuclear matter, such as surface tension, compressibility, polarizability (or more general response functions) and the properties of the collective flow. Among the notable successes of this approach has been the determination of the eigenfrequency for the dipole mode on the basis of a two-fluid hydrodynamical model in which the coupling of the fluids can be related to the symmetry energy in the nuclear masses. However, such an approach is of limited scope owing to the important quantal effects associated with the shell structure, which deeply affect the low-energy collective nuclear properties. The effect of the shell structure on vibrational eigenfrequencies is illustrated in Fig. 5 and is seen to be especially pronounced for the quadrupole mode, where indeed it leads to instability of the spherical shape in certain regions of nuclei with a resulting splitting of all the modes.

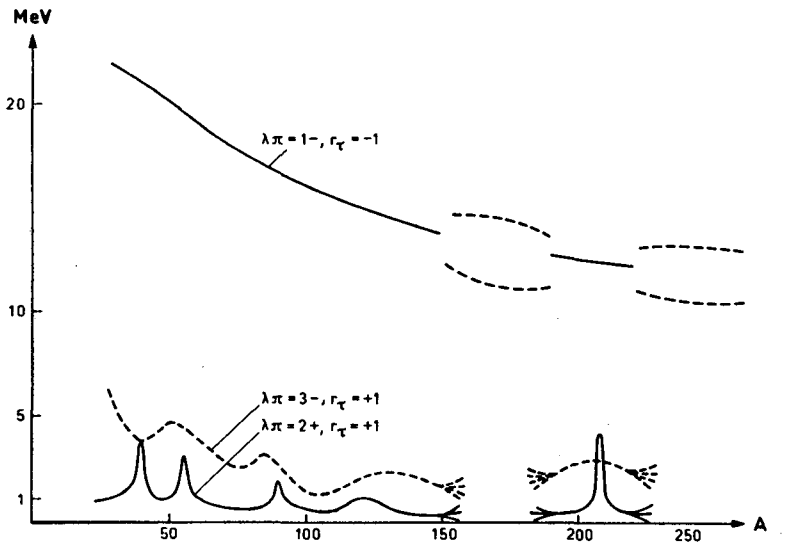
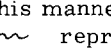
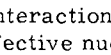


FIG. 5 Energies $\hbar\omega$ of the giant dipole resonance and the low-energy quadrupole and octupole shape oscillations.

The shell structure expresses the quantization of the nucleonic motion in the nucleus as a whole and therefore implies an essential non-locality in the collective nuclear properties. The inclusion of these effects involves going to a more microscopic level, which treats explicitly the degrees of freedom of the individual particles and the interactions which generate the correlated nucleonic motion. One may attempt directly to introduce the two-particle interactions, as determined from the two-nucleon scattering experiments, but the problem is a difficult one, because of the many correlation effects which must be considered; thus, the effective forces between nucleons in a nucleus are rather different from those of free nucleons.

In the analysis of the nuclear excitations, considerable success has been achieved by imposing another level of phenomenology, which can be formulated in various ways. We know from the existence of the nuclear shell structure that the interactions can be represented in first approximation by an average field, and, thus, we may attempt to describe the coupling between the particles and the collective motion in terms of oscillations in the nuclear field associated with collective deformations. In this manner, the interactions are described by a three-field coupling  representing the coupling of the particles to the nuclear deformation, rather than the four-field coupling  representing the nucleonic interactions. The approach is sometimes formulated in terms of simplified effective nucleonic interactions of separable type, such as the quadrupole force (representing the product of the quadrupole moments of the interacting particles) but these interactions are merely constructions designed so as to yield the three-field couplings, when treated in lowest order. The determination of the eigenfrequencies of the collective motion is then a self-consistency problem similar to that involved, for example, in the determination of the plasma frequency or zero sound, as described by Pines. In both cases, the interactions are introduced in terms of the relation between variations in density and potential.

This relation, between the nuclear deformations and the potentials they generate, involves coupling constants for the vertices. These are treated as phenomenological parameters, but, for some of the important types of deformations, the approximate values of the coupling constants can be deduced from the known properties of the average potential in nuclei. Thus, for a shape deformation, the assumption that equipotential and equidensity surfaces have the same shape fixes the coupling. On this basis, it has been possible to account for many features in the nuclear collective properties, such as the main trends in the variations in the vibrational frequencies illustrated in Fig. 5. Among other major achievements of the microscopic theory may be mentioned the calculation of nuclear equilibrium shapes, of the dependence of the pairing parameter Δ(the gap parameter) on the shell structure, and of the nuclear moments of inertia, which would have the rigid-body values in the absence of pairing, but which are typically two or three times smaller by essentially the same mechanism that gives rise to diamagnetism of the superconductors. Many new collective modes are also predicted, only part of which have yet been established. The condition for occurrence of well-defined collective modes, which was discussed by Pines for the infinite media, are affected in an important manner by the shell structure. Thus, the frequency spectrum of two-quasiparticle excitations of a given symmetry has considerable structure, and collective modes may occur either below the lowest intrinsic frequencies or above the highest, or whenever there is an appreciable gap in the intrinsic excitation spectrum.

In spite of the progress achieved, it must be emphasized that the microscopic analysis of the collective nuclear properties is still in a rather preliminary stage. It is important that the theory makes detailed predictions concerning the microscopic structure of the collective motion. With the tools available, this can be tested in great detail by a variety of reactions (see Fig. 6), such as (a) the production of phonons by transfer of a particle to a target with one quasi-particle, (b) the decay of phonons into two quasi-particles (if the phonons are energetically stable, one may consider the isobaric analogue states, which are unstable with respect to proton emission), (c) inelastic nucleonic scattering with the excitation of a phonon, (d) production of states with quasi-particle and a phonon, (e) energy splittings within the multiplet formed by a quasi-particle and a phonon, as well as a variety of higher-order effects, such as (f) the scattering of two phonons (anharmonicity in the vibrational motion).

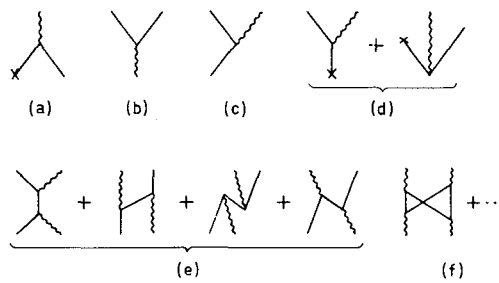


FIG. 6 Diagrams for various processes probing the structure of the phonons (see explanation in the text).

The field is a huge one and concerns the whole problem of the interaction between the elementary excitations, which is inseparable from the structure of the excitons themselves. It is an area of vigorous activity, and it is impossible on this occasion to mention the many new ideas and methods which are being pursued and explored. The perturbation expansion can only be directly applied for the modes involving a relatively weak coupling to the particle motion; however, in the case of special importance for the low-energy quadrupole mode, the coupling is strong or intermediate; the dimensionless coupling constant is for most nuclei greater than unity. In some situations, we have a genuine strong coupling phenomenon resulting in a stable deformation and associated rotational spectra, but for a large class of nuclei, we are dealing with a transitional type of spectra intermediate between rotational and harmonic vibrational. The strength of the coupling at the same time implies large anharmonicities in the vibrational spectra of even nuclei, i.e. interactions between the quanta comparable with their own frequency. These anharmonicity phenomena have for long presented a major challenge. Some insight has been gained by many different approaches, based on power series expansions of the Hamiltonian, analyses of the potential energy surfaces of deformation and of the collective flow, and attempts to isolate and specifically include the particle degrees of freedom mainly responsible for the anharmonicity. However, we are dealing with a delicate instability phenomenon, as can be seen from the fact that a quadrupole moment added to the system typically induces a moment 20-30 times larger. Peculiar phenomena may therefore occur and surprises may still be in stall.

5. SPURIOUS DEGREES OF FREEDOM

In the description of the nuclear dynamics in terms of collective and individual particle degrees of freedom, there arises the problem of the overcompleteness of the degrees of freedom, since the collective bosons are composites of the fermions. This is a general problem of many-body physics, but has been felt especially acute in the nucleus because of the finite geometry and the resulting discrete degrees of freedom. In a sense, the lesson has been that there is no cause for special worries, in that the elimination of the spurious degrees of freedom comes as part of the systematic analysis of the interaction effects. For example, the effect of the exclusion principle acting between a particle and a boson is contained in the interaction (e) in Fig. 6, and the fact that bosons consisting of fermions cannot be freely superposed, as a result of the exclusion principle for the fermions, is expressed as part of the interaction (f) in Fig. 6.

One may attempt to avoid the explicit use of collective variables by the so-called projection method. Thus, for a deformed nucleus, one may project states of specified angular momenta IM from the intrinsic state $\Phi_{\alpha,k}$, by superposing states with different orientation ω

$$\Psi_{\alpha,IKM} = \int d\omega \; \Phi_{\alpha,K}(\omega)\mathscr{D}^{I}_{MK}(\omega) \tag{6}$$

and in this manner represent the rotational band. However, this approach ignores the dynamic correlations by which the nucleus itself generates the family of rotational states and therefore does not give the separation of the motion expressed by the wave function (2).

6. PAIRING MODES

As a last topic, I should like to come back to the structure of the collective modes in the pairing channel. The development of this subject has been stimulated by recent studies of double transfer reactions.

We shall consider medium heavy nuclei, where the neutron excess is fairly small, and shall view the situation with the nucleus ^{56}Ni as a basis. This nucleus has N = Z = 28 and closed shells of neutrons and protons (like a solid with filled conduction bands). The excitations of the pairing field are therefore of vibrational type (Cooper pairs); there are two types of quanta corresponding to the addition of pairs into the shells above 28 (α = +2, $\lambda\pi = 0^+$) and removal of pairs from the shells below 28 (α = -2, $\lambda\pi = 0^+$): each pair has unit isospin, so in all there are six modes. The one-quantum excitations represent the ground states of ^{58}Ni and ^{54}Fe and their isobaric analogues. There is considerable coupling between the particles in the shells above and below 28, but this is included as a renormalization of the quanta. The excitations with two quanta are shown in Fig. 7. The full-drawn lines correspond to ground states, and the dashed ones to excited states, recently found in double-transfer reactions. On the basis of the observed interactions between the quanta, one can attempt to construct the pattern of states with more quanta, as is illustrated schematically in Fig. 8. Of the states with quanta of both types, only the few lowest are included in the figure.

This spectrum of excitations comprises all the ground states of even-even nuclei in this domain and, in addition, a host of other 0^+ states, which

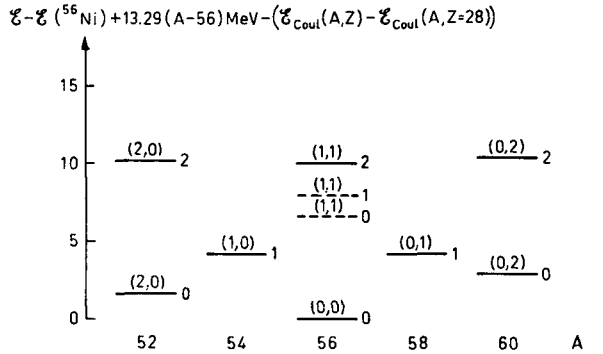


FIG. 7 Pairing vibrational excitations with one and two quanta added to ^{56}Ni. The levels have $I\pi = 0^+$, and the quantum numbers (n_1, n_2) give the numbers of quanta with $\alpha = -2$ and $\alpha = +2$, respectively. The levels are also labelled by the total isospin T. Full-drawn lines represent ground states, dashed lines excited states.

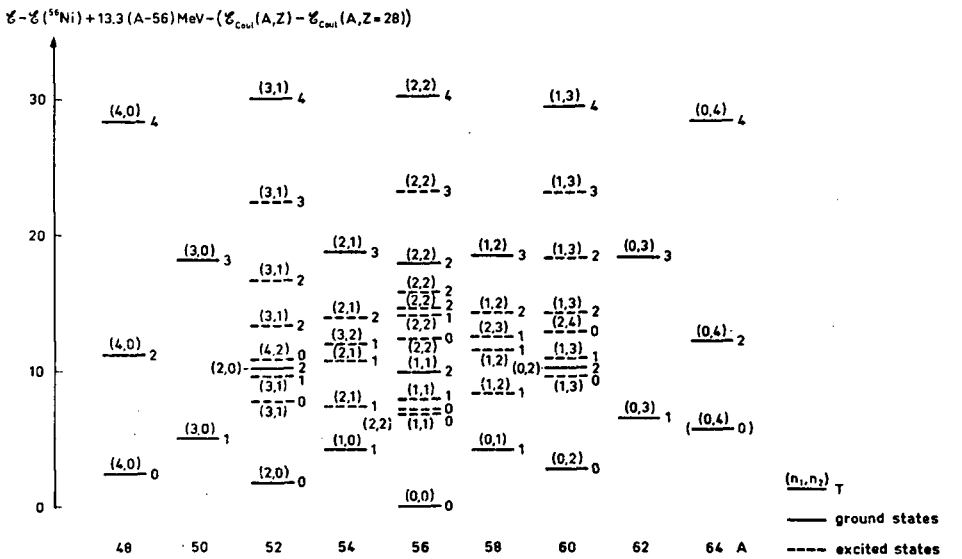


FIG. 8 Schematic illustration of many-pair excitations based on ^{56}Ni.

should all be viewed together, as a single family. A number of the additional members have been found, but the evidence is still very incomplete. (For a review of the experimental data, see O. Nathan, in Nuclear Structure, Proc. Int. Symp., Dubna, (1968) 191. In trying to analyse the interaction between the quanta and developing a quantitative treatment of the spectra, at least three major effects must be considered.

(a) The first - the most conspicious in the figure - is the favouring of states with low isospin. The nucleonic interactions contain an important component with this property, which can be deduced from the symmetry term in the nuclear masses. It appears that this interaction mainly contributes

to the isovector component in the nuclear monopole field (the spherically symmetric average potential), which is readily taken into account.

(b) Next, there is the pair field coupling, which implies interactions between the quanta, associated with the exclusion principle. For few quanta, these effects can be treated in terms of interactions between pairs of quanta (see Fig. 6 (f)). When many quanta are present, large anharmonicities may develop, but one can now treat the system in terms of a static pairing field, i.e. as a superfluid system. The states then organize themselves into rotational bands in isospace and in nucleon-number space, such as mentioned above. Owing to the simplicity of the pairing field, the vibration-rotation transition can be followed much more easily than for the quadrupole modes mentioned earlier and forms an instructive example of such a transition.

As regards the accuracy of such calculations, there are several points to consider. Schrieffer yesterday quoted a very comforting number for the superconductor, but in the nucleus the number of particles participating in the correlation effect is not very large. The number of doubly degenerate levels over which the diffuseness of the Fermi surface extends is about 10 in favourable cases, only 3 in less favourable cases. In a quantitative calculation, it may therefore be necessary to go beyond the leading approximation of a static pairing field and include fluctuation effects.

(c) In addition, we must consider interactions in other channels; these will modify the structure of the pairing condensate only if they are strong enough to cause instability or near-instability of the system, but this is expected to happen in the channel of quadrupole shape oscillation ($\alpha = 0$, $\lambda = 2$). Indeed, the spectra of the nuclei in the region considered are known to have large amplitudes of quadrupole motion, and it is even likely that some members of the family have static quadrupole deformations with associated rotational spectra in I-space (corresponding to a new phase modification, like ferromagentism, added to the superfluidity). The analysis therefore requires that we view the 0^+ states in the figure together with the quadrupole excitations in the various nuclei as a single family. If the structure in the $\alpha = 0$ channel changes smoothly with A and T, the 0^+ states in Fig. 8 will form a well-defined sub-family, though the quantitative pattern of energies and transition probabilities is affected. If abrupt changes take place, new generic relations develop.

The unravelling of these patterns, which are gradually emerging as a synthesis of evidence from reactions in a variety of different channels, may be expected to present many challenging new problems concerning the structure of excitations in quantal systems.

NUCLEAR EXCITATIONS OF SPECIAL SYMMETRY

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Abstract

NUCLEAR EXCITATIONS OF SPECIAL SYMMETRY. 1. Introduction; 2. Formation of higher isospin states; 3. Coulomb mixing; 4. T-forbidden resonances; 5. Conclusions.

1. INTRODUCTION

Unlike elementary particle physics, nuclear physics has few examples of slightly broken symmetries. Isobaric spin symmetry is a case in point; moreover, the cause of the symmetry breaking is here presumed to be understood, namely it is due to the Coulomb interaction between protons. This interaction is weak compared with nuclear interactions and, therefore, once again, we have hope of being able to get away with a perturbation approach to calculating violations of this symmetry. The weaker interactions such as are involved in beta decay, electromagnetic decay properties, electron scattering, Coulomb excitation, internal conversion, help us to avoid facing the strong interactions directly.

Among the millions of complicated (many-body) excited states of nuclei, most of which will undoubtedly elude theoretical description except on a statistical basis, certain states are especially favoured in regions of excitation whose location is uniquely correlated with low-lying states of neighbouring nuclei, thanks to the (near-) equality of neutron-neutron, proton-proton and neutron-proton forces.

In this brief review, fully conscious of my conspicuous role as a low-energy, experimental nuclear physicist in a lion's den, I want to convey a little of the excitement which exists in nuclear physics today, using the special illustration of isospin symmetry. As we have already seen from some of the slides of Professor Bohr's talk, values of the isospin-quantum number T as high as 22 are commonplace today. As recently as 1961, the highest value of this quantum number that one encountered was 1, or 3/2 at most; a major explosion occurred with the discovery by Anderson and collaborators [1] that isospin was a meaningful concept up to values of at least 12 or 15; in other words, heavy nuclei showed features of isospin conservation.

2. FORMATION OF HIGHER ISOSPIN STATES

Before giving specific illustrations of these relatively recent breakthroughs, we summarize in Fig. 1 the six working rules of the isospin game as played by nuclear physicists, defining the relation between T and

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T IN NUCLEAR PHYSICS

(1) $T_z = \frac{N-Z}{2}$

(2) $T \geq T_z$

(3) $T = T_z$ FOR MOST LOW-LYING STATES

(4) $T = -T, -T+1, \ldots\ldots +T$ (2T+1 MEMBERS)

(5) $T^- |T_0 T_0\rangle = \sqrt{2T_0}\, |T_0\, T_0 - 1\rangle$

(6) $\sum_i T_i$ (BEFORE) $= \sum_j T_j$ (AFTER) $\longrightarrow$ CHARGE INDEPENDENCE

$\sum_i T_z^i$ (BEFORE) $= \sum_i T_z^j$ (AFTER) $\longrightarrow$ CHARGE CONSERVATION

FIG. 1. Rules of the isospin game in nuclear physics.

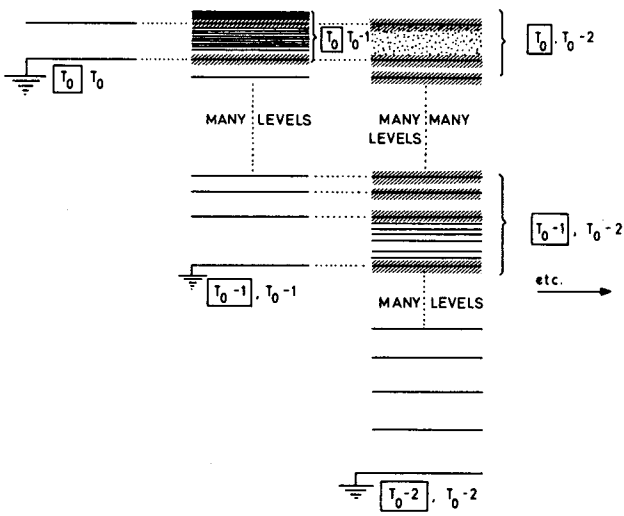

FIG. 2. Relationships of analogue states in neighbouring isobars. Ground states are denoted by "electrical" symbol.

its projection, the multiplet structure of T, the usual $T = T_z$ as found empirically for low-lying states of nuclei, the isospin-lowering operator T^- which is encountered experimentally, and, finally, the isospin-conservation law stated explicitly for cases where objects of various isospin values collide and form new products. The next slide (Fig. 2) shows the actual escalation of isospin in neighbouring nuclear species of the same baryon number A. One must imagine 2T + 1 such level diagrams, of which only three are illustrated. We see the onsets of higher isospin values whenever a new "treshold" is reached in a neighbouring nucleus, while the lower-valued T states continue in ever increasing level density.

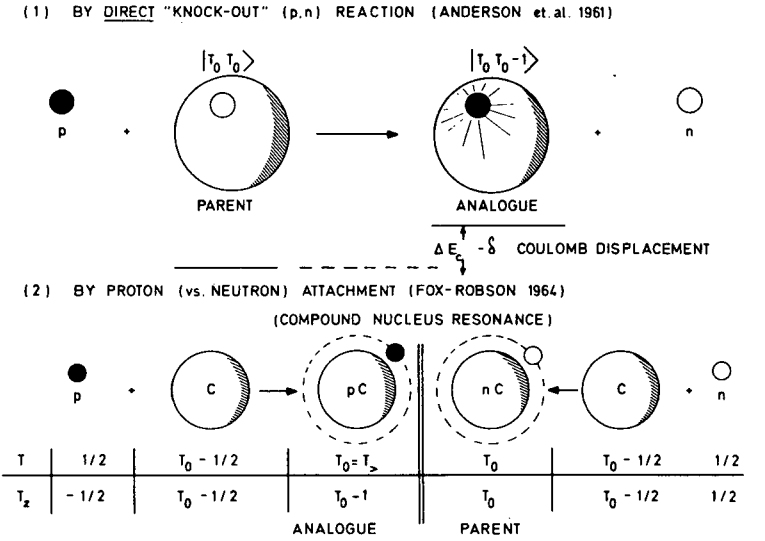

FIG.3. Analogue state formation processes.

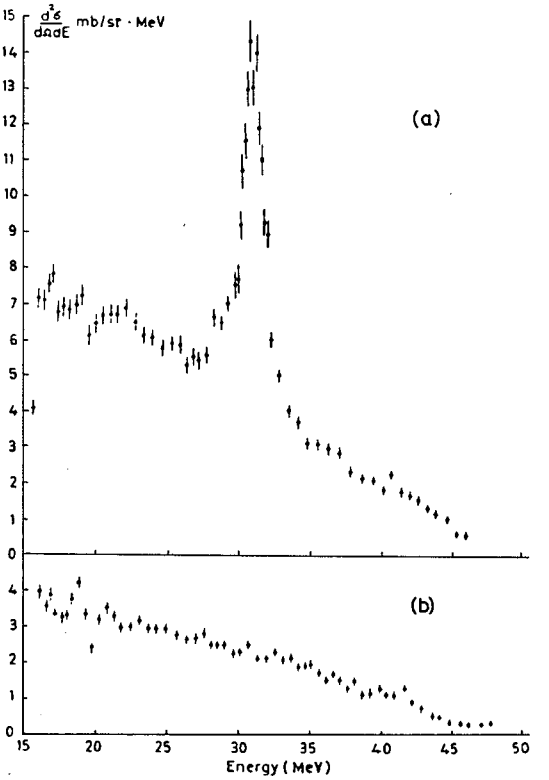

FIG.4. (p, n) reaction with 50 MeV protons on (a) platinum and (b) zirconium (from Ref.[3]).

It is these higher T states, imbedded in their lower-T surroundings, which we wish to form and discuss here. Note that by their very relationship to ground and low-lying states in neighbouring species, these higher T states have greater "simplicity" of structure than the complicated continuum states surrounding them; they, in fact, mix with the latter via the Coulomb interaction and share some of their "simplicity" with a few million states lying within their width (in a heavy nucleus such as lead).

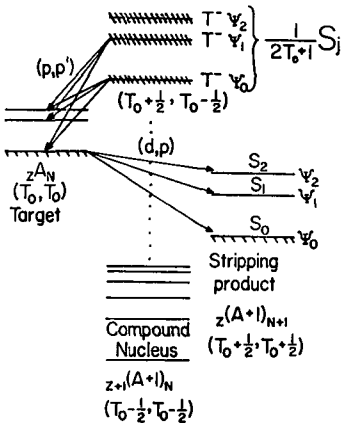

FIG. 5. Relation between analogue resonance formation and stripping on the same target nucleus, leading to the "parent" states.

Figure 3 illustrates the two most popular methods of forming these states: the direct proton-neutron substitution by knock-out [1] and the proton attachment into a compound resonance of higher T [2], as compared with neutron addition into bound analogue states in the neighbour. Incidentally, neutron removal is just as good, but cannot be done resonantly. The next slide (Fig. 4) shows a particularly spectacular example [3] of the first type of reaction for the case of 50-MeV protons on platinum; the energies of outgoing neutrons are shown, corresponding to residual nuclear excitation running from right to left. We see a pronounced peak about 15 MeV above the ground-state neutrons (at the far right end of the spectrum) corresponding to the isobaric analogue state of the target platinum nucleus in the corresponding gold nucleus of the same baryon number. The Q value for this group is exactly equal to the Coulomb displacement energy between the two isobars, i.e. the additional energy due to the proton charge.

The second method, which has proved even more popular than the first if one is to judge by the over 300 publications since its first discovery [2], is related in Fig. 5 to the analogue stripping reaction, showing the relationships which exist between the wave functions, spectroscopic factors S and level positions.

One of the most studied nuclei is lead-208, which has closed shells of both neutrons and protons and which has already been cited by Professor Bohr in the preceding talk. The four curves of Fig. 6 show excitation functions [4] for proton bombarding energies between about 14 and 18 MeV (one must add about 3.8 MeV to obtain excitation energies above the ground

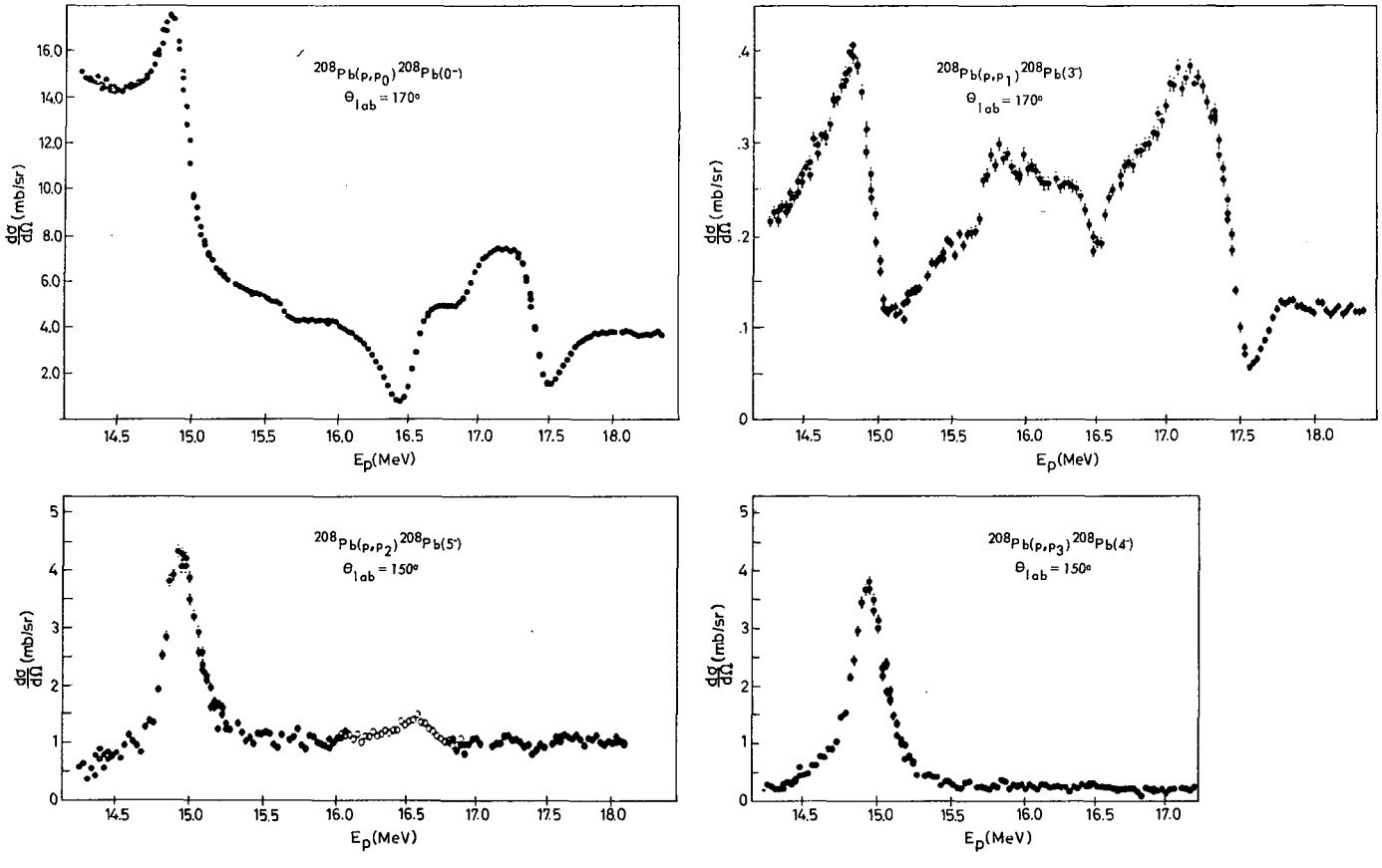

FIG. 6. Proton excitation curves for elastic and inelastic scattering on doubly-magic ^{208}Pb (from Ref.[4]).

state of bismuth-209), for elastic scattering (0^+), inelastic scattering to the collective octupole (3^-) state at 2.62 MeV in the target ^{208}Pb, and the 5^- and 4^- states at 3.20 and 3.48 MeV, respectively. The wiggles in the curves correspond to various compound resonances in the nucleus ^{209}Bi, analogues of the single-particle states in ^{209}Pb known as $g_{9/2}$, $d_{5/2}$, $s_{1/2}$, $g_{7/2}$, $d_{3/2}$ .... One notes that the elastic and (3^-) scattering show most of these resonances, while the 4^- and 5^- states only respond at the $g_{9/2}$ resonance. This is a consequence of the structure of these states, the last two of which are thereby dramatically shown to be mainly of the type $g_{9/2}$ (particle) x $p_{1/2}$ (hole). By contrast, the 3^- collective state is built out of many particle-hole excitations and, therefore, resonates at most single-particle compound states. Who would have expected, only a few short years ago, that such resonances would be observed in the heaviest stable nucleus, using bombarding energies below the Coulomb barrier (about 15 MeV)?

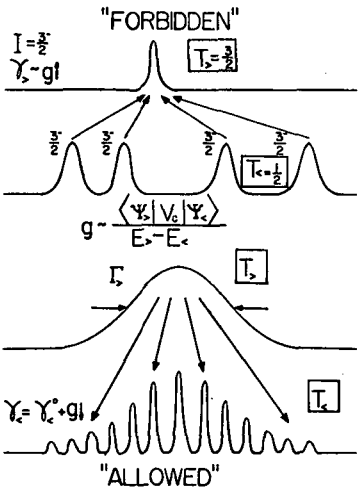

FIG.7. Coulomb mixing for forbidden and allowed analogue resonances.

3. COULOMB MIXING

We review the general problem of Coulomb mixing in Fig. 7. The lower part of the figure illustrates the situation for a heavy nucleus, where a state of higher isospin $T_>$, having a large reduced proton width, is shown to share its strength with the many underlying states of the same spin and parity, raising them up from the general continuum background via the Coulomb interaction (downward arrows). The mixing is governed by the perturbation coefficient given in the centre, whose symbolism is self-evident. For the remainder of this talk we shall be concerned with a more special kind of analogue resonance, namely one that cannot be formed without violating the isospin-selection rules to a greater or lesser extent. The appropriate situation is illustrated in the upper half of Fig. 7, where the few surrounding lower-isospin states lend strength to the isolated,

(1) EXAMPLE

$p + {}^{12}_{6}C_{6} \longrightarrow {}^{13}_{7}N^{*}_{6}$

DIRECT	T	1/2	0	(ΔT=1)	3/2
VIA IMPURITY	T'	1/2	[1]		3/2
VIA IMPURITY	T'	1/2	0		[1/2]
	T_z	-1/2	0		-1/2

(2) EXAMPLE

$p + {}^{23}_{11}Na_{12} \longrightarrow {}^{24}_{12}Mg^{*}_{12}$

T	1/2	1/2	(ΔT=1)	2
T_z	-1/2	+1/2		0

(ΔT=2)

${}^{20}_{10}Ne_{10} + \alpha$

T	0	0
T_z	0	0

FIG. 8. Examples of T-forbidden analogue resonance formation.

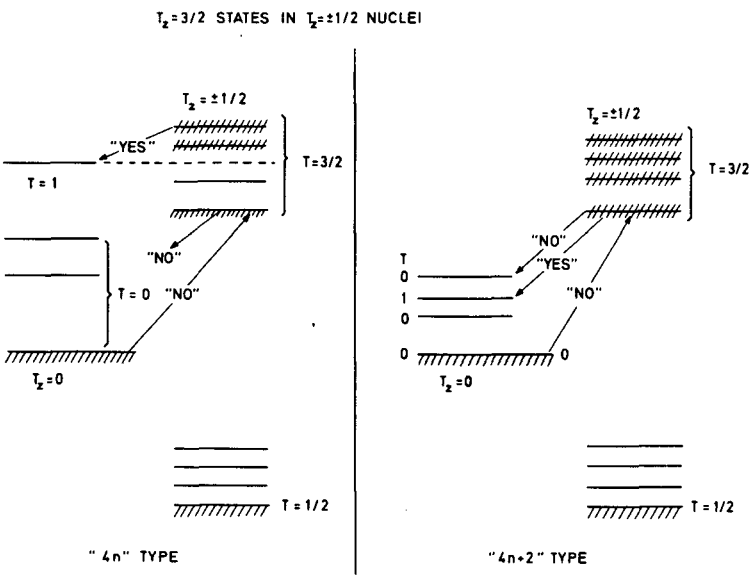

FIG. 9. Energetics of T = 3/2 forbidden analogue resonances.

narrow $T_>$ resonance which has zero width in the perfect isospin situation (upward arrows), the mixing again being governed by the same coefficient g. There is no "fine structure" here as in the more conventional "allowed" analogue resonance case. The existence of these exceedingly narrow states at rather high excitation energy is an interesting recent discovery [5] and vividly illustrates the isospin-selection rules at work.

4. T-FORBIDDEN RESONANCES

Figure 8 shows two specific cases of isospin violation in the formation of compound resonances, one involving the formation of $T = 3/2$, $T_z = -1/2$ states in ^{13}N [5], the other the formation of $T = 2$, $T_z = 0$ states in ^{24}Mg [6]. The possible isospin impurities through which the formation (and decay) can take place are listed. We clearly see "twice" forbidden and even third-forbidden paths occurring here. A word is in order to explain the extreme sharpness of these resonances. Figure 9 shows the energetics for most of the low-lying analogue states for the case of nuclei built of integral numbers of alpha particles (4n-type). The non-availability of $T = 1$ states until high target-excitation energies are reached, insures the narrow nature of the forbidden resonances; note that for odd-odd nuclei, illustrated in the right-hand part of the figure, there are low-lying $T = 1$ states available; even though the formation of $T = 3/2$ states is forbidden, once formed there is no inhibition to their decay via the $T = 1$ states by inelastic scattering, and the $T = 3/2$ states are as wide as "ordinary" $T = 1/2$ resonances.

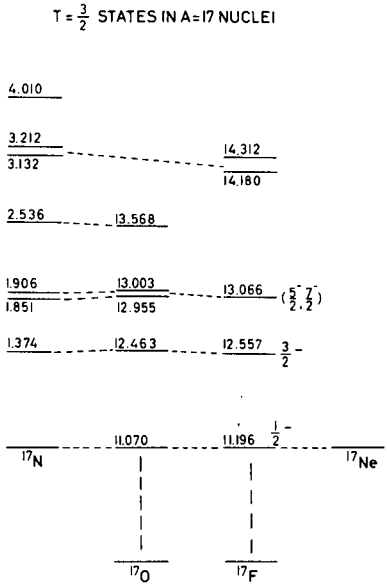

FIG. 10. T = 3/2 quadruplet for A = 17 (from Ref. [9]).

As an example, we show the A = 17 quadruplet (Fig. 10), the levels having been adjusted so as to be aligned. This removes the actual parabolic variation of masses versus T_z within the multiplet which is of great intrinsic interest, but will not be discussed further here [7]. A very important corollary piece of experimental information comes from isospin-allowed two-nucleon transfer reactions, of which an example is shown in Fig. 11. The $T = 3/2$ states in $^{13}N(T_z = -1/2)$ and $^{13}C(T_z = 1/2)$ are shown to be formed as residual (rather than compound) states by (p, t) and (p, ^{3}He) reaction, respectively [8]. The states are seen to be very narrow

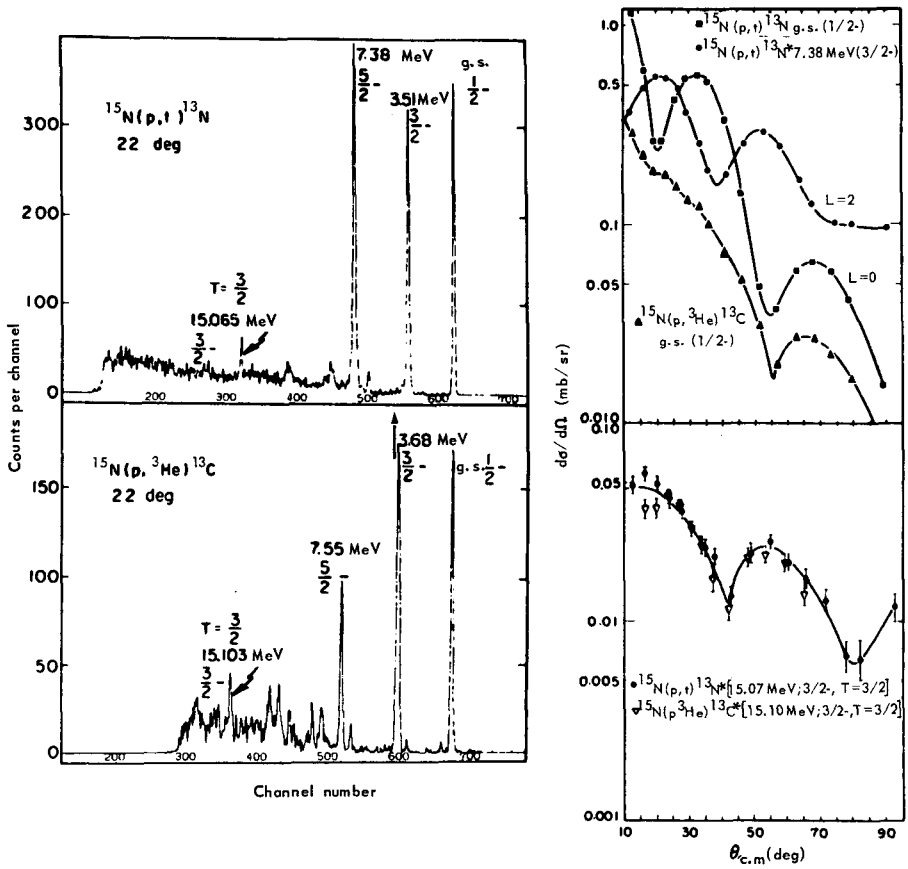

FIG.11. Isospin-allowed two-nucleon transfer reactions leading to T = 3/2 residual states in ^{13}C and ^{13}N (from Ref.[8]).

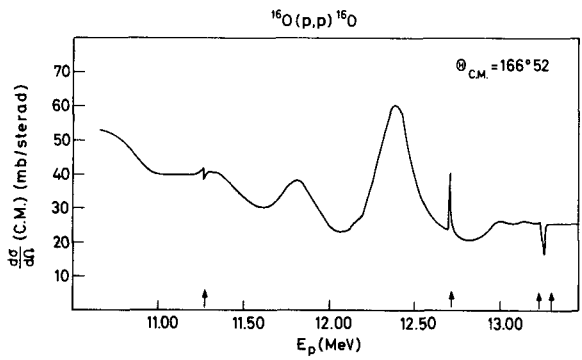

FIG.12. Proton elastic scattering excitation curve for ^{16}O, showing the first three analogue states of ^{17}N in ^{17}F (arrows) (from Ref.[5]).

(< 80 keV), while we showed the former to have an actual width of about 1.5 keV [9]. Only compound resonance formation allows the necessary precision to make these determinations. The lower right-hand figure shows the complete agreement of the angular distributions leading to these higher-order mirror states, a fact which is entirely due to the exclusively T = 1 nature of the two-nucleon transfer, a circumstance which does not apply to the T = 1/2 mirror states(such as the $1/2^-$ ground states) which are seen to have entirely different shapes. Moreover, a normalization factor of 2 was required to bring the curves into coincidence, this factor being the appropriate Clebsch-Gordan coefficient in the isospin coupling involved here. This type of result provides the strongest evidence for the T = 3/2 nature of these states.

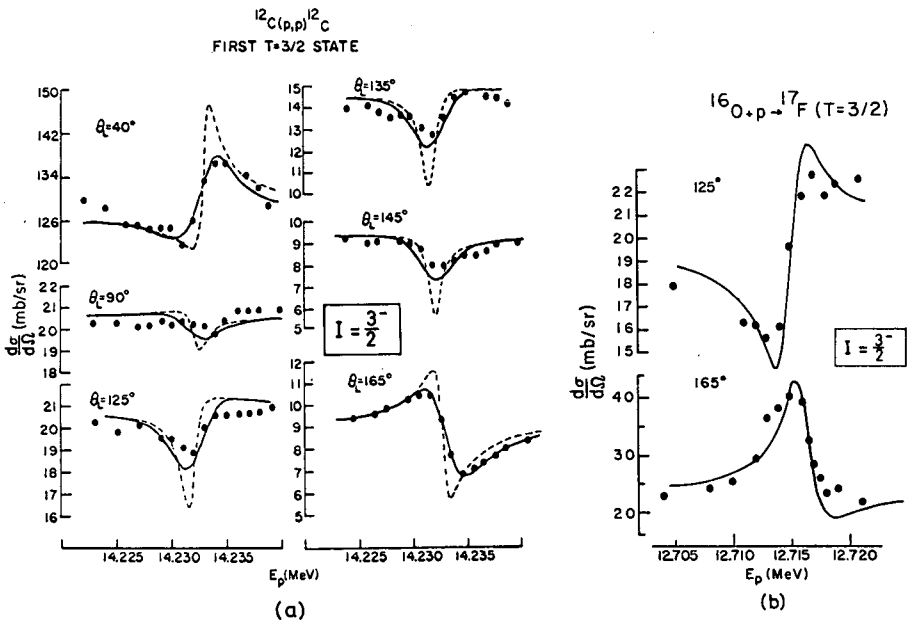

FIG. 13. Examples of forbidden analogue resonances in ^{13}N and ^{17}F with theoretically predicted shapes (dashed curves) and taking into account finite experimental resolution (solid curves) (from Ref.[9]). a) $^{12}C_{(p,p)}{}^{12}C$; b) $^{16}O+p \rightarrow {}^{17}F$ (T = 3/2).

The next slide (Fig. 12) shows the schematic excitation curve for the elastic scattering of protons on ^{16}O, leading to $T_z = -1/2$ states of the A = 17 quadruplet in ^{17}F. We find three resonances corresponding to the first three states of ^{17}N. After a detailed analysis, including the effects of finite resolution [9], illustrated in Fig. 13 for both A = 13 and A = 17, we find the three resonances for the latter nucleus to have widths of 500, 1500, and 2000 eV, respectively, while the former has a width of 1500 eV. These are the narrowest states ever seen at such high excitation energies.

Finally, (Fig.14), we illustrate the second example mentioned in Fig. 8, namely the formation of a forbidden T = 2 resonance in a normally T = 0

nucleus, which was studied via its gamma decay by double cascade through some T = 1 intermediate states [6]. The formation is via proton bombardment of ^{23}Na (right-hand side); the alpha decay ($\Delta T = 2$!) has been reported [10], indicating definite tensor Coulomb admixtures. The possibility of looking for gamma transitions connecting states differing by two units of isospin such as the dipole transition shown dashed, certainly opens interesting prospects.

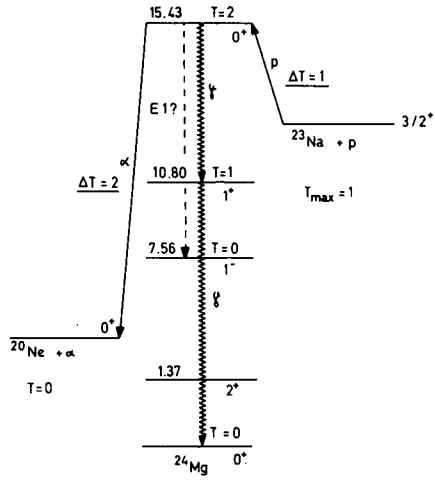

FIG.14. Decay scheme and formation of T = 2 state in ^{24}Mg (from Ref.[6]).

5. CONCLUSIONS

The systematic exploration of special symmetry states at high excitation energies (which, however, are not really "highly excited") is fruitful in that it provides links between nuclear species which, at first sight, seem unrelated. Not only do we obtain spectroscopic information, but light is shed on questions of isospin purity, the sources and types of admixtures, and the Coulomb displacement energies between nuclei, as well as the energetics of nuclei far from the valley of stability. Moreover, analogue states allow us to study the nuclear many-body continuum by injecting "test" states of known properties into its midst and following the effects produced thereby. Most of the problems raised by these experiments are far from having received satisfactory theoretical answers. The challenge to the experimentalist is undiminished, with several years of projected experiments clearly ahead as of now.

If nuclear physics is "dead", then this speaker, for one, could clearly not stand the pace of a "live" field if he is to judge from the stimulation and excitement provided by the nucleus in the past two years.

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MICROSTRUCTURE IN NUCLEAR SPECTRA AT HIGH EXCITATION

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Abstract

MICROSTRUCTURE IN NUCLEAR SPECTRA AT HIGH EXCITATION. The statistical microstructure of highly excited systems is illustrated by the distribution and fluctuations of levels, widths and cross-sections of nuclei both for the case of sharp resonances and the continuum case. The coexistence of simple modes of excitation with statistical effects in terms of strength functions is illustrated by isobaric analogue states. The analogy is made with similar phenomena for coherent light, is solid-state physics and high-energy physics.

On hearing the discussions on the theory of condensed matter, it is interesting to reflect on the fact that the study of the elementary excitations in this field has mostly been done by inelastic scattering of neutrons and X-rays. The energy in such investigations is large compared with the energy loss ΔE in the scattering. If we consider the corresponding situation in nuclei, the characteristic energy of the nucleus is determined by the internucleon spacing of the order $\hbar/m_\pi c \approx 1.4$ fermis where m_π is the pion mass. Thus $\Delta E_{nuclear} \sim m_\pi^2 c^2/2M \cong 10$ MeV where M is the nuclear mass. Most accelerators used for studies of nuclei are in the range of a few times this energy, so that energy losses tend to be of the same order as the energy of the probe particles. Very few investigations of nuclei by particles in the GeV range have been carried out. However, some important experiments of this kind have been done in recent years. An example of observations is shown in Fig. 1, taken from Kullander et al., which represents the nuclear analogy of the Frank-Hertz experiment of atomic ionization. An incident proton of 460 MeV knocks out a nuclear proton by a quasi-elastic collision on ^{12}C and produces a "nuclear ionization". Two well-defined peaks are observed separated by ~ 17 MeV. These can be identified by angular distributions with the $1s_{\frac{1}{2}}$ and $1p_{\frac{1}{2}}$ shell model states in ^{12}C. Results of this kind represent our most vivid and tangible evidence for the validity of the nuclear shell model in deep-lying orbits and this situation is typical for many elements, also for considerably heavier ones.

The point of particular interest at present is that we here see an extremely simple nuclear spectrum with few states. On a nuclear scale the excitation energies in question are quite high, being commonly 15 MeV and even much more. If we now turn our attention from this solid-state aspect of the nucleus to the more conventional way of looking at the nucleus by low-energy accelerators, we should inquire: which picture do such probes give of the nucleus?

If we start near the nuclear ground state we see there a relatively small number of states corresponding to a rich variety of modes of excitations of the nuclear degrees of freedom as described by A. Bohr. With increasing energy the number of states increases very rapidly in an essentially exponential manner so that at about 6 MeV there are of the order of 10^4 states in medium-weight elements and beyond 10^6 states in

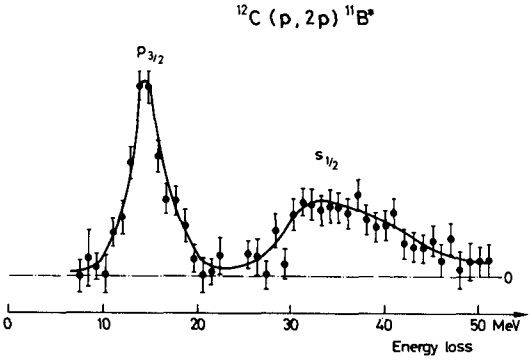


FIG.1. Energy loss observed in quasi-elastic proton knockout for the reaction $^{12}C(p, 2p)\,^{11}B^*$ (from Tyrén et al., Ref. [1]).

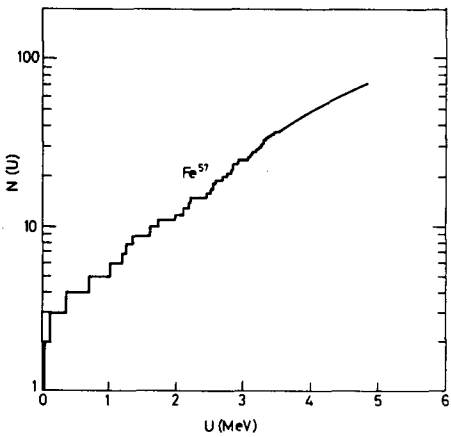


FIG.2. Variation of the number of states N(U) with excitation energy U for ^{57}Fe (from Ericson, Ref.[2]).

heavy elements, with the number of states still rapidly increasing. This rapid increase of the number of states is qualitatively illustrated in Fig. 2 for ^{57}Fe by plotting the total number of states below the energy U on a log-scale versus the energy U. The rapid increase of states is typical of the numerous degrees of freedom available in a many-body system as can already be seen in an extremely crude way in the excitation spectrum of an ordinary degenerate Fermi gas. It is therefore instantly clear that this way of looking at the nucleus reveals an infinitely more complex aspect of the nucleus than the previous one and that superficially the two are utterly different: in the region of the deceptively simple-looking single-particle states there must be literally millions of levels. In a certain sense the high-energy probe saw a global property of these states. We therefore started by seeing the whole forest so to speak; we will now enter into it to look at the individual trees, so as to obtain a better understanding of this dual property of the nucleus. It is therefore of great interest to investigate:

1) What is the nature of the wave functions of the numerous states in the fine structure and how are their eigenvalues and other properties distributed?

2) How do these states conspire to give the appearance of simple-looking states?

3) Are there aspects of the microstructure of such systems which go beyond the properties of resonant states?

Certain of these properties can be studied in detail after some very specific assumptions on the basis of the soluble random-matrix model to be discussed by Rosenzweig. Here we shall mainly discuss these properties from a physical viewpoint.

Let us now consider a specific nucleus at high excitation with a very large number of densely spaced states; the typical spacing is D. If we were to view these states in terms of shell model configurations, collective states or other simple modes of excitation which are useful for description at low excitation energies, we can, in general, not expect to find such a basis for the description to be very useful. There will certainly be residual interactions in such a picture and the residual matrix elements will cause an intimate mixing of states. This clearly implicitly assumes that all such mixing matrix elements between the states are of similar importance. This assumption will have to be tested in practical cases, and there are even certain cases when it is invalid; however, it is the simplest as well as a natural first assumption so that we shall explore it. In this situation it becomes obvious that we do not wish to describe the exact properties of an individual state. The appropriate phrasing about the properties of the system is instead of a statistical nature. We have to inquire into the averages and fluctuations of the system in order to display those properties which are general for large classes of interactions.

At this point we face a dilemma. If we want to discuss the statistical properties of the nucleus, we should clearly start from an ensemble of forces. Therefore, we should like to sample some property of the nucleus for different forces selected from the ensemble. This is clearly impossible. The nucleus is a dynamical system of given forces. If we know these forces precisely enough we do not doubt that we can calculate the nuclear properties at high excitation exactly, even if only in principle. Further, if we perform a nuclear-physics experiment we know that the outcome is reproducible so that if the experiment is done over again tomorrow we shall find the same results. It is, therefore, perfectly obvious that in this sense there is nothing statistical or random about the nucleus. One might think that it would be possible to change the interaction by applying external perturbations as in classical statistical physics in which a new member of the ensemble is easily produced by applying small perturbations. Even this does not work for the nucleus in practice, since the nucleus is remarkably insensitive to outside perturbations. In practice, one requires perturbation matrix-elements to be of the order of 10^{-4} or so of typical nuclear-matrix elements in order to shift a resonance position by one resonance spacing in a heavy nucleus. The answer to our problem is that the nucleus is not statistical if looked at in its totality. However, many sub-properties of the system can be studied, statistically. Examples is the widths of the numerous nuclear resonances for which both distributions and correlations can easily be studied although they all are reproducible. Looked at in this

way the nucleus somewhat resembles a book of random numbers. There is nothing random about such a book once it has been printed and stands on a shelf in the library since we can look up the numbers again and again in a way. In a similar sense we shall proceed to sample the nucleus for statistical properties, neglecting correlations or at least minimizing correlation effects inside the system.

With this word of caution let us now look at the expected properties of the wave function. For concreteness we shall consider the specific case of slow neutrons producing sharp nuclear resonances which are characterized by their widths. The incident wave function of the neutron enters into the nucleus: its probability of forming a particular resonance is governed by the squared overlap function γ_i^2 of the incident wave with that of the nuclear state $|i\rangle$ at the surface. This therefore means that the nuclear state is projected onto a fixed direction in the Hilbert space. If we now insist on very strong mixing interactions in the nuclear states, the principle which governs the distribution of γ_i is this: the nuclear state should have so little to do with the incident wave that it is not even orthogonal to it.

To put this somewhat more mathematically: the real (time reversal) projection γ_i should be invariant under rotations of the Hilbert space of the resonant states $|i\rangle$, and has therefore a normal distribution:

$$P\left(\gamma = \left(\frac{\Gamma_n^0}{D}\right)^{\frac{1}{2}}\right) d(\gamma) \propto \exp\left\{-\frac{\gamma^2}{2\overline{\gamma^2}}\right\} d(\gamma)$$

where Γ_n^0, is the reduced neutron width. Figure 3 shows the experimental distribution of this quantity for ^{232}Th s-wave neutron resonances. The curve labelled $\nu = 2$ corresponds to the distribution which would have been expected from the same assumptions if γ were a complex number with no preference for its real and imaginary part.

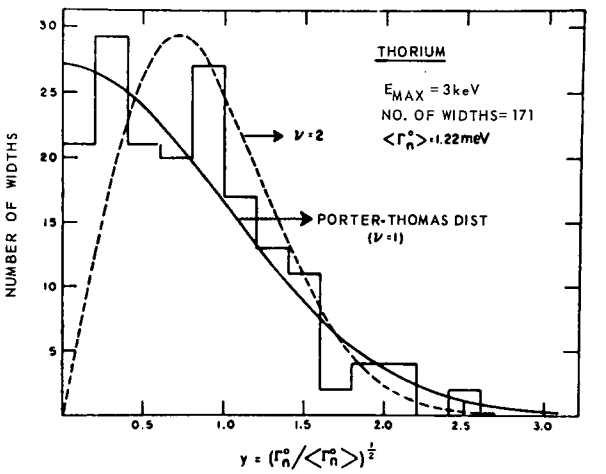


FIG.3. The distribution of reduced slow-neutron widths for ^{232}Th as a function of $(\Gamma_n^0/\langle\Gamma_n^0\rangle)^{\frac{1}{2}}$ (from Garg et al., Ref. [3]). The dashed curve ($\nu = 2$) refers to the theoretical curve for 2 instead of 1 degree of freedom.

The next question is now how this statistical feature is consistent with the gross structure. Consider again the neutron being brought up to the nuclear surface. If there were no mixing interactions it would, on entering the nucleus, feel an interaction region of the nuclear size in which it can be reflected back and forth so that, at certain frequencies, size resonances are produced. Thus the incident neutron brings a wave packet of specific properties into the nucleus. The mixing forces will now pull on this wave packet and spread out its probability amplitude on all the resonant states in an energy region $\Gamma_0 \simeq 2\pi v^2 D$ by the golden rule where v is the residual interaction. The residual interaction therefore represents a nuclear relaxation phenomenon describing the dissipation of the wave packet among the many resonances as is easily seen in a time description of the process. There are basically three different situations depending on the strenght of the mixing interaction (Fig. 4).

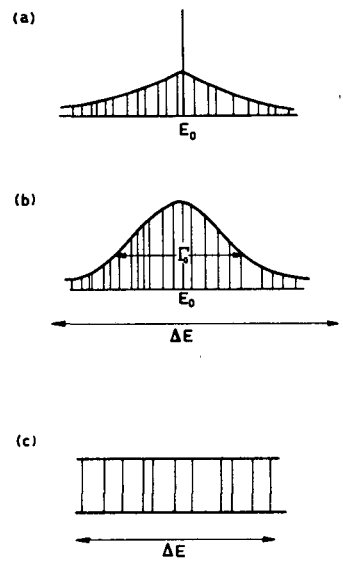


FIG. 4. a) Weak case $\Gamma_0 < D$. In this case there will be one very strong resonance E_0 with probability amplitudes

$$a(E_i) \propto \frac{1}{E_i - E_0}$$

for the remaining ones;

b) Intermediate case $\Gamma_0 < \Delta E$ where ΔE is the energy region around E_0 for which v contributes. This is a most important case in nuclear physics. The probability

$$|a(E_i)|^2 \propto \frac{1}{(E_i - E_0)^2 + (\Gamma_0/2)^2}$$

has an approximately Lorentzian form;

c) Strong interaction $\Gamma_0 \gg \Delta E$. The strength of the incident wave packet is uniformly mixed over the entire energy interval corresponding to an infinitely short relaxation time for the wave packet in the nucleus (uniform model).

In the picture as it appears in b) we can now understand the appearance of broadened states of simple nature. Inside these states there can literally be 10^4 to 10^6 or more levels each contributing a tiny bit to the envelope producing the peak. Since the total probability has to add up to unity this means that the probability of the simple configuration in any one of the states is extremely small, typically of order 10^{-4} to 10^{-6}, i.e., a probability amplitude of 10^{-2} to 10^{-3}. As a consequence this small amplitude has practically no effect on the state, which spends nearly all its time on other occupations. The dissipation of the incident wave packet by a relaxation phenomenon among the many resonances with a typical width has another consequence. This corresponds effectively to an absorption of the incoming wave into the compound states and motivates the introduction of an absorptive potential for the description of elastic slow-neutron scattering in nuclei, for example. In this way, the broadened size resonances can easily be described phenomenologically by optical model potentials.

The energy region Γ_0 over which the initially sharp state has its strength distributed among the other nuclear states is intimately linked to the spreading interaction v. A detailed study of the nuclear relaxation requires, therefore, that this interaction be known. Unfortunately, the spreading interaction is known only crudely and imperfectly in most cases in nuclear physics. A notable exception to this is given by the so-called "isobaric analogue states" discussed by Temmer, for which the spreading interaction is simply the Coulomb interaction between nucleons. In the absence of Coulomb forces, the assumption of charge independence implies that isobaric or isotopic spin is a good quantum number. Then we have isobaric spin multiplets, so that we can generate "analogous" states in another nucleus simply by switching neutrons into protons, etc. However, nuclear Coulomb energies are very important. For example in lead, the Coulomb energy is about 1 GeV which is an enormous energy on the nuclear scale. It was therefore thought for a long time that isobaric spin was a useless concept in heavy elements. To the great surprise of nuclear physicists it was discovered a few years ago that there existed rather sharp resonances in nuclei at about 10 MeV excitation which could clearly be identified with states produced by switching a neutron into a proton. The excitation energy corresponds essentially to the Coulomb energy B_C it takes to transform a neutron into a proton. The occurrence of these resonances at very high excitation in nuclei was as surprising to the nuclear physicist as it would be to the high energy physicist to find resonances at 50 GeV centre-of-mass energy.

In the absence of mixing, the resonances have isospin $(T_0 + \frac{1}{2})$ where T_0 is the isospin of the target nucleus C in the particular case of nucleon + C scattering. These states are widely separated but are embedded among a vast number of states of isospin $(T_0 - \frac{1}{2})$ (see Fig. 5).

The explanation of the occurrence of these highly excited states of simple nature is essentially that the Coulomb interaction is a nearly diagonal interaction because of its long range and smooth behaviour. Consequently, the main effect of it is simply to change the zero-point energy of the state by the Coulomb energy of a single proton. The off-diagonal matrix elements are very much weaker. It is these that violate isospin and cause the mixing with the numerous background states of isospin $(T_0 - \frac{1}{2})$. Here we have, therefore, a typical "strength-function" phenomenon with a known interaction. As an example of the apparent structure of such a state, the upper

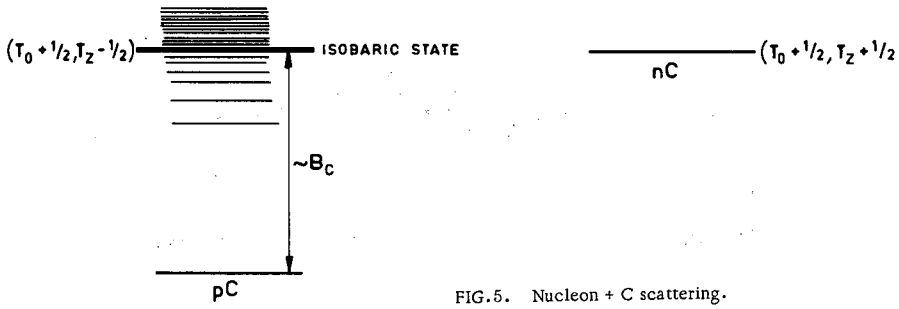


FIG.5. Nucleon + C scattering.

curve in Fig. 6 shows elastic proton scattering on ^{22}Mo studied with high energy resolution in the region of an analogue resonance by Fox et al. One observes a cross-section which varies very rapidly with energy and there is no obvious resonance of outstanding character to be seen. The lower curve shows a larger energy resolution. In this case we see a beautiful Breit-Wigner resonance interfering with a potential scattering background. Clearly the individual resonances carry the strength of "isobaric analogue state" which shows up in a simple way only as a sum rule.

In this specific case it is possible to achieve a solution close to an analytic solution of the mixing. Denote the original isospin $(T_0 + \frac{1}{2})$ in the pC system by 0 with eigenvalue ϵ_0 and the background isospin $(T_0 - \frac{1}{2})$ states by $\lambda \neq 0$ with eigenvalues ϵ_λ. The mixing interaction which violates isospin has only matrix elements between states of different isospin so that only $v_{0\lambda}$ and $v_{\lambda 0}$ are non-vanishing. The Hamiltonian is then:

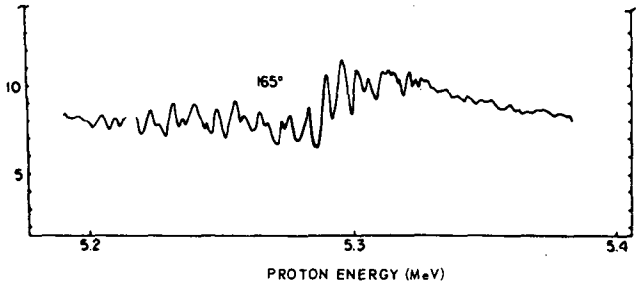
$$\begin{pmatrix} \ddots & & 0 & \vdots & & & 0 \\ & \epsilon_{-2} & & v_{-20} & & & \\ 0 & & \epsilon_{-1} & v_{-10} & & & \\ \cdots & v_{0-2} & v_{0-1} & \epsilon_0 & v_{01} & v_{02} & \cdots \\ & & & v_{10} & \epsilon_1 & & 0 \\ & & & v_{20} & & \epsilon_2 & \\ 0 & & & \vdots & 0 & & \ddots \end{pmatrix}$$

The eigenvalue equation is hence

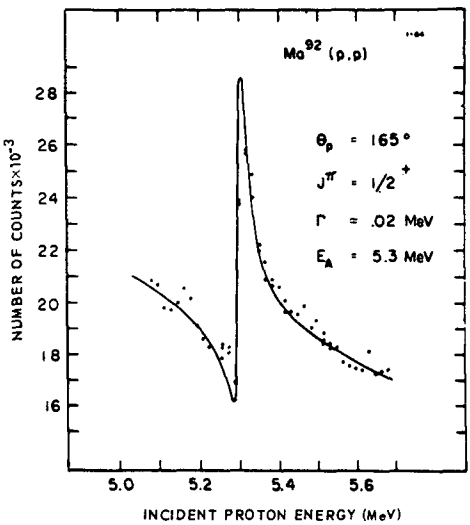
$$\sum_{\lambda \neq 0} \frac{v_{0\lambda}^2}{\epsilon - \epsilon_\lambda} = \epsilon - \epsilon_0$$

with the amplitude of admixture C_λ of the state 0 into the exact state λ:

$$C_\lambda = \frac{v_{\lambda 0}}{(\epsilon - \epsilon_\lambda)} \left[1 + \sum_{\mu \neq 0} \frac{v_{\mu 0}^2}{(\epsilon - \epsilon_\mu)^2} \right]^{-\frac{1}{2}}$$



(a)



(b)

FIG. 6. Excitation function for elastic proton scattering on ^{92}Mo at 165° as seen with (a) good energy resolution, (b) poor energy resolution (according to Richard et al., Ref. [4]).

This can be solved in exactly closed form in a highly oversimplified version if the states λ are equidistantly spaced $\epsilon_\lambda = \lambda D$ about $\epsilon_0 = 0$ and if $v_{0\lambda} = \pm v = \text{const}$.

$$C_\lambda^2 = \frac{v^2}{(\epsilon_\lambda - \epsilon_0)^2 + \left(v^2 + \frac{\pi^2 v^4}{D^2}\right)}$$

which explicitly verifies our previous discussion for this particular case with $\Gamma_0 = 2\pi v^2/D$ as expected for $\pi v/D > 1$. The explicit evaluation of the Coulomb interaction gives the correct magnitude of the small widths observed, although the detailed way in which the best evaluation of the Coulomb terms should be made is still under some discussion.

The reason why the question has not yet been fully settled is that our discussion is only half the problem. There are two different ways by which the mixing effects are introduced physically: 1) by mixing within the range of the nuclear forces by the non-diagonal matrix-elements we have just discussed, 2) by the long range Coulomb field outside the nucleus since in the outer region a proton knows beyond doubt that it is a proton and a neutron that it is a neutron, each with its characteristic asymptotic wave function. In this second case no pretence of pseudo-identity of neutrons and protons is possible, for now a spade is a spade. Both of these effects have to be fully and correctly accounted for simultaneously which presently is being done.

The lesson from the various strength-function phenomena in nuclear physics is clearly that the appearance of simple-looking excitations is no guarantee that such excitations may not have a secret life with a very considerable fine structure. In solid-state physics billions of resonances must hide inside the simple structures observed.

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DISTRIBUTION OF ENERGY LEVEL SPACINGS AND CONSERVATION LAWS*

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Abstract

DISTRIBUTION OF ENERGY LEVEL SPACINGS AND CONSERVATION LAWS. 1. Introduction; 2. The random-matrix model; 3. Perturbation of spacing distribution by a small term in the Hamiltonian odd under time-reversal operation; 4. Atomic spectra: dependence of the spacing distributions on relative magnitudes of spin-orbit and central forces; 5. Thermodynamics of small metallic particles; 6. Concluding remarks.

1. INTRODUCTION

My aim is to give an introduction to the random-matrix model proposed by Wigner some ten years ago. The model has provided a basis for deriving the statistical laws which govern the fluctuations of the energy levels and decay widths of highly excited nuclei. During the last ten years the model has been developed very greatly from the analytical point of view, and also the range of its applications has been widened considerably and at the present time includes such diverse topics as nuclear spectra, atomic spectra, and the thermodynamics of small metallic particles at low temperature. The bibliography at the end contains references to several reviews [1-3] and a recent book by M.L. Mehta [4] which deals systematically with the analytical theory.

This presentation will emphasize particularly one aspect of the situation, namely the dependence of the statistical properties on the symmetries of the quantum mechanical system under consideration.

To set the stage for these discussions, we shall describe a typical situation with which the model attempts to deal. The example is the historical one which provided the original motivation, and it is also the one which has been studied the most, both experimentally and theoretically. Let us consider the reaction ^{238}U + neutron (slow) leading to the compound nucleus ^{239}U, and, in particular, the total cross-section as a function of the kinetic energy of the incident neutron. The experimenters have available a virtually monochromatic beam, the energy of which may be varied continuously from 0 to about 4 keV. In this energy interval, there occur approximately two hundred resonances which are, generally speaking, well isolated and, as a rule, may be fitted extremely well - one by one - by a single level Breit-Wigner formula, and in this way the values of the resonance parameters which occur in the formula may be deduced for each of the resonance levels.

At these energies (0 to 4 keV) the only modes of decay of the compound nucleus which are energetically possible are re-emission of the

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neutron or the emission of a gamma ray. Thus, by a careful analysis, one obtains the position of the resonance level, the neutron width and the total radiation width. Additional experimentation often also leads to information about the partial radiation widths to the various excited states of the compound nucleus.

Some of these quantities, for example the neutron widths and also the spacing between successive resonances, fluctuate greatly from level to level. Nevertheless, these quantities have well-defined average values, which may be deduced from the experiment indicated above simply by forming the arithmetic mean of the 200 values. For the sake of maintaining an appropriate perspective, it should be recalled at this point that the energy dependence of these average values (level density, miscellaneous "strength functions", etc.) strongly reflect the structure of the nuclei, and this has provided the motivation for significant theoretical developments during the past decade (see, for example, the contributions of Feshbach and Ericson to these Proceedings).

However, we shall not be dealing with this "gross structure" but rather attention will be focussed on the fluctuations of these quantities in an energy interval in which the averages are essentially constant but which contains nevertheless many resonances. And since time is limited, the discussion will be only about one of these quantities, namely the spacing between successive energy levels. Figure 1 contains a schematic diagram of the levels of ^{239}U. The following points should be noted.

The first level E_1 which is observed, is of the order of a million levels above the ground state. The excitation energy equals the neutron binding energy (4.78 MeV) plus the kinetic energy E_1 of neutron (a few electron volts). These resonances, generally speaking, represent exceedingly complicated many-particle excitations. Even if one knew the Hamiltonian of ^{239}U exactly, which is certainly not the case, it would be impossible to solve the eigenvalue problem with sufficient precision to reproduce in detail the particular arrangement of energy levels illustrated schematically in Fig.1.

Secondly, it should be noted that all the levels observed in the reaction are $1/2^+$ levels because they are formed by s-wave neutrons incident on a target of zero spin.

The average value of the spacing, denoted by D, has a magnitude of 17.2 eV, and in the interval of a few keV above the neutron binding energy the average value D remains essentially constant.

Next, let us define a set of dimensionless numbers, the successive spacings divided by the mean value,

$$x_1 = \frac{S_1}{D}, \quad x_2 = \frac{S_2}{D}, \ldots$$

From this set one can construct a histogram which will represent the empirical probability distribution of the spacing x. Let P(x) denote the probability distribution of the spacing.

About ten years ago, Wigner surmised that P(x) should be independent of the detailed form of the residual interaction between the nucleons[1].

[1] The random matrix model was introduced by Wigner through remarks at various conferences around 1957 and some of these conference proceedings are reprinted in Ref. [2].

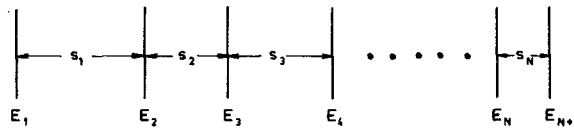

FIG.1. Schematic diagram of positions of resonance levels in ^{239}U. All the resonances are $1/2^+$ levels. Successive spacings are denoted by S_1, S_2, . . . S_N.

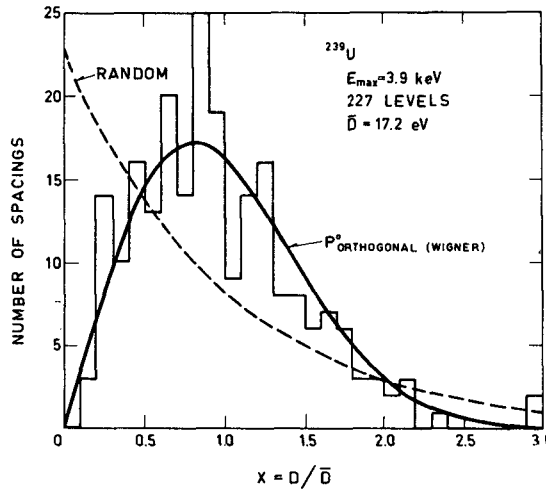

FIG.2. Histogram of the observed distribution of spacings for ^{239}U. From GARG, PETERSEN, RAINWATER, HAVENS, JR. Phys. Rev. 134B (1964) 985. The solid curve represents the Wigner distribution, Eq.(1).

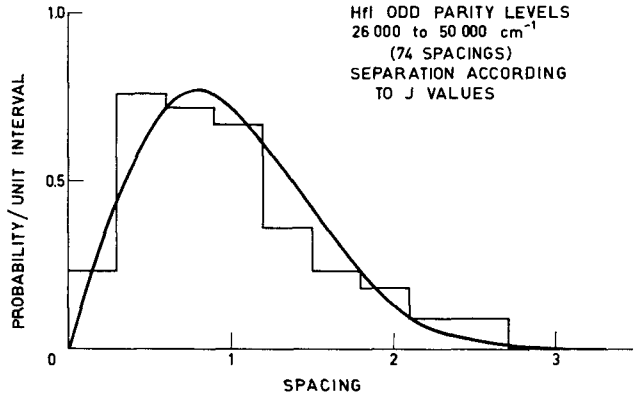

FIG.3. Plot of the empirical distribution of nearest-neighbour spacings for the odd-parity atomic energy levels of neutral Hf. The Wigner distribution, represented by the continuous curve (Eq.(1)) is in excellent qualitative agreement with the empirical distribution (see Ref. [10]).

An indication of the correctness of this surmise may be obtained by comparing the histograms of Figs 2 and 3. The histogram of Fig.2 is based on the neutron resonance levels of ^{239}U, whereas Fig.3 is based on the atomic spectrum of neutral Hf (excitation energy of only a few

eV). Thus, these very much different systems have similar distributions of the spacing between successive energy levels. The solid curve represents the simple form

$$P(x) = \frac{\pi}{2} x e^{-\frac{\pi}{4} x^2} \tag{1}$$

This simple formula is sufficiently close to the solution which follows from the random-matrix model, to be discussed next, that it is convenient to use it.

Let us make one final remark. The atomic system Hf is really a very much simpler system than the nucleus ^{239}U, and it is, in fact, possible to obtain a detailed theoretical description of the atomic spectrum of Hf. Nevertheless, Hf seems to be already sufficiently complex for the statistical properties to show up.

2. THE RANDOM MATRIX MODEL

The features of the problem which are relevant and which must find expression in the theoretical model will now be briefly discussed.

The energy levels and wave functions are the solutions of a quantum-mechanical problem. This is expressed in the model by representing the system by a Hermitian matrix H.

The system is assumed to be strictly time-reversal invariant, and this is probably a very good approximation. Spin and parity are assumed to be strictly conversed. These conservation laws are expressed by conceiving of the matrix as being constructed in a basis in which the Hermition matrix is real and diagonal with respect to spin and parity. Thus, the matrix is real and symmetric and diagonal with respect to spin and parity.

It is assumed that no other quantity which may be relevant for the properties to be discussed is even approximately conserved. This assumption is implemented by treating all of the matrix elements as statistically equivalent.

A simple matrix ensemble which meets all of the above requirements is given by the following joint probability density of matrices

$$P(R) \propto e^{-\sum_{i,j} R_{ij}^2} = e^{-\operatorname{tr} R^2} \tag{2}$$

with respect to the volume element

$$d[R] = \prod_{i \leq j} d R_{ij} \tag{2a}$$

While the ensemble (2) is not the only one which meets the above requirements, it does fulfill the criterion of beauty stressed by Dirac at this symposium. The ensemble is invariant under orthogonal transformations, i.e. the ensemble is the same in all of the bases which, under our assumptions, should be statistically equivalent. The ensemble is

beautiful also for another reason: it factors into a product of Gaussian distributions.

While we have no intention of presenting too many analytical details, we do want to take the ensemble one easy step further, as this will enable us to note several important features. The step consists of transforming the matrix ensemble to the space of eigenvalues. The eigenvalue equations read:

$$R = X^T \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & 0 \\ 0 & & \ddots & \\ & & & \lambda_N \end{pmatrix} X, \qquad X X^T = I \tag{3}$$

The orthogonal matrix which transforms R into diagonal form is denoted by X. The eigenvalues are denoted by $\lambda_1 \leq \lambda_2 \leq \ldots \lambda_N$. The volume element in the matrix space (2a) becomes

$$d[R] \to \prod_{i<j} |\lambda_i - \lambda_j| \, d\lambda_1 \, d\lambda_2 \ldots d\lambda_N \, d[X] \tag{4}$$

in which d[X] denotes the invariant measure on the group of orthogonal matrices in N dimensions and provides the statistical model of the N eigenvectors, which, however, will not concern us here. The joint probability distribution of the N eigenvalues is given by:

$$P(\lambda_1, \lambda_2, \ldots \lambda_N) \propto e^{-\sum_1^N \lambda_i^2} \prod_{i<j} |\lambda_i - \lambda_j| \tag{5}$$

The joint probability density of eigenvalues contains the famous difference product $\prod_{i<j} |\lambda_i - \lambda_j|$. By inspection we conclude that

$$P(x) \sim x, \quad x \ll 1 \tag{6}$$

To give an indication of the kind of problem which has, in fact, been treated successfully by the analytical theory of random matrices, we indicate what needs to be done in order to obtain the P(x) for all values of x. One needs to calculate

$$A_N(x) = \int_{(\text{out } x)} e^{-\sum_1^N \lambda_i^2} \prod_{i<j} |\lambda_i - \lambda_j| \, d\lambda_i \ldots d\lambda_N \tag{7}$$

$$P_N(x) = \frac{d^2}{dX^2} A_N(x)$$

The domain of integration denoted by (out x) consists of those matrices which do not have any eigenvalues in the interval (-x/2, x/2). This formidable problem was beautifully solved by Gaudin and Metha [4] who showed that as $N \to \infty$ the spacing distribution P(x) is given by a rapidly converging infinite product involving the eigenvalues of a limiting integral equation. Furthermore, it turns out that the solution is very

well approximated by the Wigner distribution (1). The exact solution and the Wigner distribution are so nearly the same that the former, because of its simple form, is almost always used in practice.

3. PERTURBATION OF SPACING DISTRIBUTION BY A SMALL TERM IN THE HAMILTONIAN ODD UNDER TIME-REVERSAL OPERATION [5,6]

There has been some interest lately in the possibility that the nuclear Hamiltonian may contain a small part which is odd under the operation of time reversal. Since the energy levels of highly excited nuclei are rather close together, perhaps relatively small matrix elements arising from the odd part of the Hamiltonian will produce visible shifts in the statistical distributions. It is only necessary that these matrix elements shall not be too small compared to the spacing between successive levels, which in the case of ^{239}U is less than 20 eV. We shall give a brief theoretical discussion of the matter.

It is postulated that the properties of the state under consideration may be described by a Hermitian matrix of high dimensionality N as follows

$$H = E + R + iS \tag{9}$$

In the above, E is a multiple of the unit matrix and defines the energy region under consideration. The matrix is constructed in the same basis as before, namely the basis in which the time-reversal even part of the residual interaction gives rise to the real symmetric matrix $R = [r_{jk}]$, whereas the odd part results in the anti-symmetric matrix $S = [s_{jk}]$.

The statistical model, that is the probability distribution of matrices H, will be defined with respect to the volume element

$$d[H] = \prod_{1 \le j \le k \le N} dr_{jk} \prod_{1 < j < k < N} ds_{jk} \tag{10}$$

The volume element (10) when transformed to the space of eigenvalues and eigenvectors becomes

$$d[H] \propto \prod_{i < j} (\lambda_i - \lambda_j)^2 \, d\lambda_1 \ldots \lambda_N \, d[U] \tag{11}$$

In the above, d[U] represents the invariant measure on the group of unitary matrices U in N dimensions; it is the volume element for the statistical model for the N eigenvectors which, however, we will have no occasion to discuss further in this talk.

The important difference between (11) and the analogous expression (4) discussed earlier, insofar as the statistical properties of the eigenvalues are concerned, lies in the presence of the additional factor $|\lambda_i - \lambda_j|$, from which we infer that for small values of the spacing x, the probability density will now be quadratic in x rather than linear. Thus, we have

strict time-reversal invariance	$P(x) \propto x,\ x \ll 1$	(12)
violation of time-reversal invariance	$P(x) \propto x^2,\ x \ll 1$	

This is also illustrated in Fig.4.

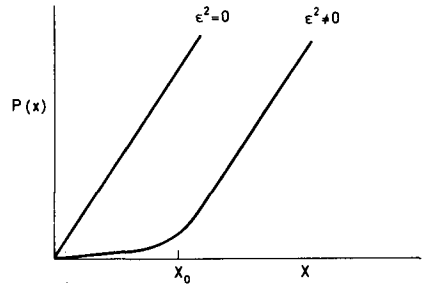

FIG.4. Schematic diagram of effect on the spacing distribution of the presence of a small term in the Hamiltonian which is odd under the operation of time-reversal. Only the behaviour for small values of x ($x \ll 1$) is indicated. The curve labeled $\epsilon^2 = 0$ corresponds to the Gaudin-Mehta distribution (strict time-reversal invariance). The curve labelled $\epsilon^2 \neq 0$ corresponds to the presence of a small time-reversal odd term. X_0 marks the point at which the quadratic behaviour ends and the approximately linear behaviour begins. Monte-Carlo calculations indicate that $X_0 \approx \epsilon$.

To make a definite calculation one needs, in addition to the volume element discussed above, a probability distribution of matrices. To arrive at an ensemble we shall make the physical assumption that the only quantities which are relevant for the properties to be discussed are the relative magnitudes of the even and odd parts of the Hamiltonian and we shall express this as follows:

$$\begin{aligned} \langle \mathrm{tr}\, S^2 \rangle &= \epsilon^2 \\ \langle \mathrm{tr}\, R^2 \rangle &= 1 - \epsilon^2 \\ \langle 1 \rangle &= 1 \end{aligned} \tag{13}$$

The bracket $\langle\ \rangle$ denotes expectation values with respect to the ensemble $P(H)\, d[H]$. We shall now obtain $P(H)$ by means of the principles of information recently discussed systematically in this context by Balian [7]. One defines the amount of "missing information" S as follows:

$$S \equiv -\int P(H) \log P(H)\, d[H] \tag{14}$$

One seeks the ensemble $P(H)$ which maximizes S subject to the constraints (13). This leads to the probability distribution of matrices

$$F_N(\epsilon^2, H) \propto \exp \left[\frac{-\mathrm{tr}\, R^2}{1 - \epsilon^2} + \frac{\mathrm{tr}\, S^2}{\epsilon^2} \right] \tag{15}$$

Although the above ensemble is very simple in the matrix space – it is just a product of Gaussian distributions – it is very difficult in the eigenvalue space. The methods of Gaudin, Mehta, and Dyson do not

seem to be applicable. Therefore, we did a Monte-Carlo calculation up to dimension 100. Insofar as the small spacings – which we have been emphasizing in this talk – are concerned, there are two main conclusions: (1) The value of the spacing for which the probability distribution P(x) changes from the quadratic form to the linear form is approximately equal to ϵ. (see Fig.4). (2) The linear parts seem to be approximately parallel to the line $\pi^2 x/6$ the form which was obtained by Gaudin and Mehta for the case of strict time reversal invariance.

According to a private communication, Wigner made a perturbation calculation in ϵ^2, and with his results we shall have more precise analytical knowledge of the function P(x).

Only one brief remark will be made about the important question as to whether one can say anything about the value of ϵ^2 by an analysis of the experimental data. We were able to conclude tentatively on the basis of the single-channel width distributions (not on the basis of the spacings!) that $\epsilon^2 \leqslant 0.01$.

4. ATOMIC SPECTRA: DEPENDENCE OF THE SPACING DISTRIBUTIONS ON RELATIVE MAGNITUDES OF SPIN-ORBIT AND CENTRAL FORCES [8]

When the spacing distributions of the atomic spectra in the three long periods were studied in 1960, it soon became clear that the Gaussian ensemble of real symmetric matrices (2) was sometimes completely inadequate. Although the levels of the third long period follow the Wigner distribution, and those of the first long period can also be reduced to this case when the additional quantum numbers S and L are taken into account, a different regime prevails for the spectra of the second long period. In the second long period L and S are approximate constants of motion, and this necessitates a statistical distinction between the average magnitudes of the Coulomb and spin-orbit interactions. A reasonable statistical model may be obtained as follows.

Let us consider the representation of states in which the Coulomb interaction between the electrons is diagonal. In this representation, the off-diagonal matrix elements of the entire Hamiltonian refer to the spin-orbit interaction only, whereas the diagonal elements refer, at least for the most part, to the central forces. We will describe our incomplete knowledge about the system, again with the help of an ensemble P(R), but we will specify that the magnitude of the two kinds of matrix elements may be different. This is done, as before, by prescribing the expectation values of the square of the matrix elements:

$$\begin{aligned} \langle R_{ii}^2 \rangle &= 1 - \epsilon^2, && \text{(residual electrostatic interaction)} \\ \langle R_{ij}^2 + R_{ji}^2 \rangle &= \epsilon^2,\ i \neq j && \text{(spin-orbit interaction)} \qquad (16) \\ \langle 1 \rangle &= 1 \end{aligned}$$

The quantity $N\epsilon^2/(1 - \epsilon^2)$, where N denotes the dimension of the matrix, is a measure of the relative magnitudes of the residual electrostatic and spin-orbit interactions.

If one maximizes the missing information subject to the constraints (16), one obtains the ensemble

$$\exp\left[-\frac{\sum R_{ii}^{2}}{2(1-\epsilon^{2})}-\sum_{i\neq j}\frac{R_{ij}^{2}}{2\epsilon^{2}}\right] \tag{17}$$

With the help of this ensemble the transition from the exponential distribution ($\epsilon^2 = 0$) to the Wigner distribution ($N\epsilon^2/(1 - \epsilon^2) \to \infty$) could be elucidated and a satisfactory discription of the spacing distribution of the three long periods was obtained. As for the case of section 3, the analysis was based on a Monte-Carlo calculation.

5. THERMODYNAMICS OF SMALL METALLIC PARTICLES [9]

Gorkov and Eliashberg [11] consider a collection of small metallic particles at sufficiently low temperature so that the average spacing Δ between electronic levels near the Fermi energy is large compared to kT. In this case, the particular arrangement of the electronic levels matters in the determination of such properties as the specific heat, magnetic susceptibility, etc. The shapes of the individual particles is necessarily somewhat irregular, and therefore the positions of the electronic energy levels also will vary from one particle to the next. The spectra of the collection of particles must therefore be described by a distribution. The form of the distribution again depends upon the conservation laws.

For example, there are two situations in which our ensemble (15) is assumed to be relevant:

$\epsilon^2 = 0$: small spin-orbit coupling, spin is a conserved quantity

$\epsilon^2 = 1/2$: large spin-orbit coupling, $\mu H \gg \Delta$ (H is the external magnetic field).

One of the amusing results is that the temperature dependence of the specific heat of the ensemble of particles, at sufficiently low temperatures, will depend on the value of ϵ^2.

6. CONCLUDING REMARKS

We have briefly discussed three different physical situations (nuclear spectra, atomic spectra, thermodynamic properties of small metallic particles) in which the distribution of energy levels and average properties derived from them depends on the degree to which the relevant quantities are conserved.

The random-matrix model is the theory which is appropriate for the treatment of these fluctuation phenomena. Both from the physical and technical points of view, one may distinguish two cases: (1) the so-called "simple" systems, and (2) those systems which are not simple.

A "simple" system is one in which a few quantum numbers, such as spin, parity, etc., are conserved exactly and no other relevant quantum numbers are conserved even approximately. For these simple systems, not only are the relevant matrix ensembles readily defined, but there also exists an extensive analytical theory for the development of the consequences. Mehta's book [4] deals essentially with "simple" systems.

A system which is not simple is characterized by "approximately conserved" quantities, i.e. there are relevant quantities in the problem which are neither exactly conserved nor completely washed out. There seems to be no difficulty in postulating matrix ensembles which should be suitable statistical models for these cases. However, the analytical theory for these non-simple systems is only in its infancy, and such results as exist today have been obtained for the most part by Monte-Carlo computations.

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NUCLEAR SYMMETRIES

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Abstract

NUCLEAR SYMMETRIES. A variety of symmetries in nuclei is discussed; although none of them are pure many properties connected with symmetry are found in low-lying states.

A system has symmetry if the Hamiltonian is invariant under a set of operations g. Algebraically this means that $[H, g] = 0$ and the set g forms a group G.

The consequences of such symmetry are:

1) a labelling of the energy levels according to their transformation properties under G;
2) degeneracies in energy;
3) selection rules for transitions;
4) relations between matrix elements.

The importance of symmetry is that these consequences are independent of the detailed properties of H and follow entirely from the fact that H is invariant under the operations of G. This means that one may investigate symmetries and make accurate predictions for an assumed symmetry without detailed assumptions about the Hamiltonian. Such a technique is clearly of value when the interactions are uncertain and the many-body problem is insoluble.

These general remarks about symmetries are valid whether we are dealing with crystals, molecules, atoms, nuclei or elementary particles. The only difference is that the operators g will differ from one system to another, not only in detail but also in the co-ordinates or degrees of freedom on which they operate. To illustrate the consequences of symmetry let us take the very simple example of one particle moving in a fixed spherically symmetric potential $V(r)$. It is well known that the eigenfunctions have the form

$$\psi_{n\ell m} = R_{n\ell}(r)\, Y_{\ell m}(\theta,\varphi)$$

with energies $E_{n\ell}$, independent of m. Here the operations g are the rotations in three dimensions, a group denoted by R_3. The energy levels are labelled by ℓ with degeneracy described by $m = \ell, \ell-1, \ldots -\ell$. The actual energies and the label n will depend on the form of $V(r)$ but the labelling by ℓ and the characteristic $(2\ell+1)$ — fold degeneracies follows simply from the R_3 invariance illustrating points 1) and 2). For a transition governed by a particular physical process, with corresponding operator Q, the probability is proportional to the square of the matrix element $\langle f | Q | i \rangle$ between initial and final states. We speak of a selection rule for that process if, for some initial state i, there is only a selection of possible final states f for which the matrix element is non-zero and the transition may take place.

In this illustration for the electric dipole decay process the selection rule is that $\ell_f = \ell_i$ or $\ell_i \pm 1$. This is a typical result, that the selection rule is expressed in terms of the symmetry labels ℓ and is independent of the details such as the n-label. To illustrate the last feature 4) consider adding to the Hamiltonian a small perturbation H_1 which does not have R_3 invariance but transforms like the z-component of a vector. The energy shift for a level $|n\ell m\rangle$ is then given by

$$\langle n\ell m | H_1 | n\ell m \rangle = m D_{n\ell}$$

where the constant $D_{n\ell}$ is independent of m. In the same way, for a quadrupole perturbation H_2, the shift is given by

$$\langle n\ell m | H_2 | n\ell m \rangle = (\ell(\ell + 1) - 3m^2) F_{n\ell}$$

In each case we see that the m-dependence of the shift is entirely determined by the symmetry properties. In other words, we see that the symmetry breaking perturbations split the original degeneracy in a manner determined entirely by the symmetry. In this example with the group R_3 the symmetry label ℓ and the degeneracy label m were both integers, but when half-integer spin is introduced for the particles in a system, the labels take on their most general form for this group, namely integers or half-integers. For larger groups the corresponding labels become more elaborate and usually take the form of a finite set of integers or half-integers. This is illustrated in Table I where the conventional labels for some of the groups used in nuclear structure are collected together.

TABLE I. CONVENTIONAL SYMMETRY LABELS USED IN NUCLEAR STRUCTURE

Symmetry label	Degeneracy label	Invariance property	Group G
ℓ	m	orbital spherical symmetry (one particle)	R_3
L	M_L	orbital spherical symmetry (many particles)	R_3
S	M_S	spin spherical symmetry (many particles)	R_3
J	M	overall spherical symmetry	R_3
T	$M_T = (Z-N)/2$	charge independence	$R_3 \equiv SU_2$
(P P' P")	$T S M_T M_S \gamma$	charge-spin independence	SU_4
$(\lambda \mu)$	K L M $\epsilon \Lambda \nu$	oscillator invariance	SU_3

Before coming on to nuclei, let us spend a few minutes recalling the symmetry properties of atoms, which are basically very similar to nuclei. If we neglect the spin-orbit corrections, the Hamiltonian governing the electronic motion is spin independent and therefore invariant under the most general spin transformation which preserves the orthonormality and antisymmetry of the wave function. Since there are just two degrees of freedom in the spin state of a spin $\frac{1}{2}$ particle these operations form the group SU_2 of special unitary transformations in two dimensions. They are unitary to preserve orthonormality and the special (or unimodular) restriction eliminates trivial changes of phase. In fact, these transformations are identical with those induced by rotations of the spin variables. Consequently, in addition to the labels $L M_L$ due to the orbital R_3 invariance we have analogous labels $S M_S$ due to the spin invariance. This is the familiar Russell-Saunders coupling (or LS coupling).

It is observed that states with highest spin S lie lowest in energy, a result known as Hund's rule. There is a simple symmetry explanation for this which has an analogue in nuclear structure. We notice that with two spin $\frac{1}{2}$ particles the $S = 1$ state is symmetric while the $S = 0$ state is antisymmetric. In fact this result generalizes to give a unique link between the S-value and the permutation symmetry of the spin function, the high S-values corresponding to greater symmetry. Now, since the electron-electron force is repulsive, the lowest energy corresponds to antisymmetry in the orbital wave function. Through the Pauli principle, this implies symmetry in the spin wave function and therefore a high S-value, which is Hund's rule.

Let us now turn our attention to nuclei which we regard as non-relativistic assemblies of neutrons and protons interacting through the strong attractive nuclear force and the electromagnetic Coulomb repulsion between protons. If we neglect the Coulomb force the nuclear forces themselves have been found to be charge independent to a high degree. Then, regarding the neutron and proton as two states of the same particle, the nucleon, we have a situation exactly analogous to the spin-independence in the atom. The resulting labels $T M_T$ are called the isospin and $M_T = \frac{1}{2}(Z - N)$ where Z and N denote the numbers of protons and neutrons, respectively. The nuclear forces being attractive, we have Hund's rule in reverse, the small T coming lowest in energy. For chosen Z and N the ground state therefore has $T = \frac{1}{2}(Z - N)$ and the M_T degeneracy corresponds to different nuclei with the same A but different N and z. Thus, for example, the structure of the nuclei ^{25}Mg and ^{25}Al which have $T = \frac{1}{2}$ with $M_T = \pm\frac{1}{2}$, respectively, should be identical. Figure 1 gives the energy level diagrams of these two nuclei and shows clearly the near-degeneracy of corresponding levels. There are dramatic selection rules in isospin, as, for example, the forbidding of $T = 0 \rightarrow T = 0$ transitions by electric dipole radiation.

It is an interesting result that, although the contribution of the Coulomb force to the total binding energy of a nucleus is appreciable, the mixing of the isospin label is believed not to exceed a few per cent. For example, in a medium weight nucleus of mass $A = 100$ with a binding energy of about 800 MeV the total Coulomb energy is about 300 MeV.

Apart from the almost universal symmetries of overall rotations and parity, the isospin symmetry is the only near-exact symmetry known in nuclei. There are, however, several very approximate symmetries which

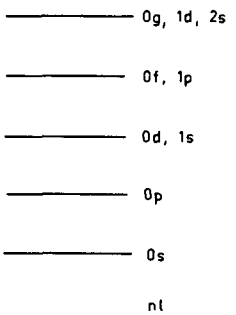

FIG. 1. Energy-level diagrams for mirror nuclei ^{25}Mg (Z = 12, N = 13) and ^{25}Al (Z = 13, N = 12).

have been used to explain qualitative features and to provide a systematic basis in which to carry out quantitative calculations.

The first of these is the SU_4 (supermultiplet) symmetry of Wigner. Let us suppose that the Hamiltonian is invariant under the most general transformation of charge and spin co-ordinates, in other words, that it is charge-spin independent. This includes, for example, the operation of flipping the spins of neutrons but not protons. There are four degrees of freedom in the charge-spin state of a nucleon $n\uparrow$, $n\downarrow$, $p\uparrow$, $p\downarrow$, so the group is SU_4. A study of the mathematics of this group shows that the symmetry label needs three numbers to specify it, conventionally denoted by $(P\,P'\,P'')$. In general, one needs six numbers to specify the degeneracy label uniquely but in many cases four are sufficient. There is a variety of choice here but the most useful set is $T\,S\,M_T\,M_S$ which describe the properties of the function under the more restricted set of separate transformations in spin and isospin $SU_2 \times SU_2$. We are all familiar with the rule that, given ℓ, the range of m values is $m = \ell, \ell - 1, \ldots, -\ell$. The corresponding rule for the SU_4 group is less familiar but equally definite. It is illustrated in Table II. It is usual to refer to the $(2S + 1)$ states with given S and $M_S = S, S - 1, \ldots, -S$ as a multiplet. The word supermultiplet is used to describe the set with given $(P\,P'\,P'')$ and all allowable $T\,S\,M_T\,M_S$. The members of a supermultiplet will all be degenerate if the Hamiltonian really possesses the charge-spin invariance and an argument similar to that leading to Hund's rule says that supermultiplets with small values of $(P\,P'\,P'')$ lie lowest in energy. There is no such degeneracy in the observed nuclear energies but detailed calculations on light nuclei show that the ground state is dominated by the lowest supermultiplet with impurities of the order of 10 - 30%. The retardation of many allowed β-decays is due to this symmetry, the transitions taking place through the impurities.

The nuclear potential is an extremely complicated object making it difficult to discuss its approximate symmetries. We may, however, get a good picture by looking at its strength in the states of low relative angular momentum which dominate most processes since the force has short range. We measure the strength by taking the matrix element in the nodeless oscillator radial wave function of the interparticle distance. The values are shown in Table III using the usual spectroscopic notation. For the triplet states we give the mean value and the greatest difference between members of the triplet. One sees that there is little spin dependence in the important S-states. There is, however, a large spin dependence in the P-states and, not shown in the Table, an appreciable coupling between S and D. It is therefore the strong S-state forces which generate the

TABLE II. SPIN-ISOSPIN STRUCTURE OF SOME SIMPLE SUPER-MULTIPLETS

n	(P P' P")	T S
2	(1 0 0)	1 0
		0 1
	(1 1 1)	1 1
		0 0
3	($\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$)	$\frac{1}{2}$ $\frac{1}{2}$
	($\frac{3}{2}$ $\frac{1}{2}$ $\frac{1}{2}$)	$\frac{3}{2}$ $\frac{1}{2}$
		$\frac{1}{2}$ $\frac{3}{2}$
		$\frac{1}{2}$ $\frac{1}{2}$
	($\frac{3}{2}$ $\frac{3}{2}$ $\frac{1}{2}$)	$\frac{3}{2}$ $\frac{3}{2}$
		$\frac{1}{2}$ $\frac{1}{2}$
4	(0 0 0)	0 0
	(1 1 0)	1 1
		1 0
		0 1
	(2 0 0)	2 0
		0 2
		1 1
	(2 1 1)	2 1
		1 2
		1 1
		1 0
		0 1
	(2 2 0)	2 2
		1 1
		0 0

supermultiplet symmetry and the non-central forces, spin-orbit and tensor which tend to break the symmetry.

There is one more symmetry of a still more approximate kind which should perhaps be mentioned since, by chance, it involves the same abstract group as that which has enjoyed success in elementary particle physics. I refer to the group SU_3. It should be made immediately clear that the physical operations here are completely different from those used in particle physics. In nuclei, the group has been used to describe the spatial dependence of the force and is quite separate from the Wigner supermultiplet sym-

metry which has to do with the charge-spin dependence. In fact, this SU_3 symmetry may be used as a supplement to the supermultiplet scheme.

We introduce this SU_3 symmetry by first recalling the spectrum for a single particle moving in a fixed spherically symmetric oscillator potential. The familiar degeneracies, shown in Fig. 2, are not accidental but due to the invariance of the Hamiltonian under the group SU_3. In simple terms, this group flips the oscillator quanta from one direction to another

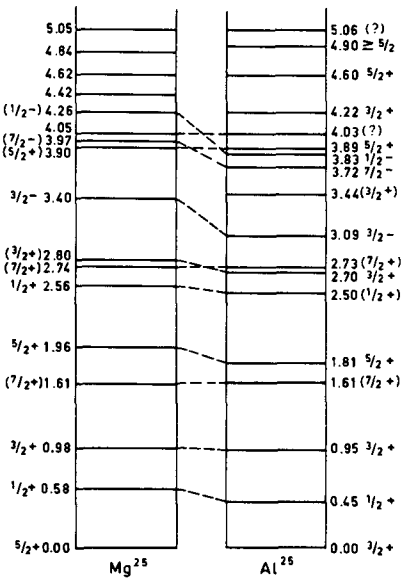

FIG. 2. Energy-level diagram for spherically symmetric harmonic oscillator potential.

and includes the rotations. An oscillator is quite a good approximation to the average field in which nucleons move in the shell model. Thus, we may classify the many-particle wave functions by the symmetry labels $(\lambda\mu)$ of SU_3. Such wave functions will be eigenfunctions only if the nuclear Hamiltonian satisfies the oscillator invariance. We can get a rough test of this by looking at matrix elements as in Table III. The relevant means and spreads are given in Table IV for the singlet spin state. There are large spreads which mean that there will again be no degeneracies. However, the os state of relative motion lies clearly below the rest. The lowest SU_3 state will be the one with greatest probability of pairs of particles lying in an os relative state. This induces a correlation in the positions of particles tending to pack them together along some arbitrary direction. Naturally, this produces large quadrupole moments in the ground states. Furthermore, the arbitrary choice of direction leads naturally to the characteristic SU_3 degeneracies. In practice, the substantial symmetry-breaking terms will split this degeneracy through retaining the R_3 symmetry. The splitting produces the familiar features of a rotational spectrum with quantum numbers KLM. Calculations for light nuclei in the region $16 < A < 40$ have shown that the lowest SU_3 states produced in this way do indeed dominate the ground state but impurities of the order of 30% must be expected.

To summarize, we see in nuclei a variety of symmetries, none of which is exact and some of which are very approximate. Nevertheless,

TABLE III. MATRIX ELEMENTS OF NUCLEON-NUCLEON POTENTIAL IN OSCILLATOR WAVE FUNCTIONS WITH n = 0, b = 1.6 fm.

State	Mean (MeV)	Spread (MeV)
^1S	-6.0	-
^3S	-7.0	-
^1P	1.4	-
^3P	-0.2	4.0
^1D	-0.5	-
^3D	-0.6	5.9

TABLE IV. MEANS AND SPREADS FOR SINGLET SPIN STATE

Oscillator degeneracy	Mean (MeV)	Spread (MeV)
2s, 1d, 0g	-0.5	2.6
1s, 0d	-1.1	3.9
0s	-6.0	-

the low-lying states are found to possess many of the qualitative and even quantitative features to be expected if the symmetry were pure or only weakly perturbated from pure symmetry.

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THE S-MATRIX IN NUCLEAR PHYSICS

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Abstract

THE S-MATRIX IN NUCLEAR PHYSICS. 1. Introduction; 2. Reaction types; 3. Reaction mechanisms; 4. Characteristics of nuclear reaction theory; 5. Prompt reactions; 6. Optical model; 7. Time delays; 8. Case of three time scales; 9. Reaction theory and experimental data; 10. "Frozen" approximation; 11. Lower-energy problems; 12. Difficulties and unsolved problems.

1. INTRODUCTION

Theoretical investigations into nuclear reactions are directed towards the resolution of three kinds of questions. The first asks for a description of the phenomenon in question in terms of a minimal number of parameters. A particular description is useful to the extent that these parameters are intrinsic dynamic properties of the nuclear system. Such a parametrization is illustrated by the well-known expression for the S-matrix in the neighbourhood of a resonance

$$S = \exp(2i\phi)\,\frac{E - E_r - i\Gamma_r/2}{E - E_r + i\Gamma_r/2} \tag{1}$$

In this expression E_r and Γ_r are two parameters which locate the energy at which the resonance occurs and its width while $\exp(2i\phi)$ is the S-matrix for the non-resonating background. A second question asks for the formal connection between these parameters, the wave function of the system and the responsible interaction. In Eq.(1) the matrix element of what interaction energy between what states gives Γ_r and E_r? The third problem involves calculation of the parameters. In the early development of the theory this calculation may be in terms of models but the eventual aim is to calculate it from first principles.

It is, of course, these first principles which distinguish the study of the nucleus from other disciplines. We assume that the nuclear system can be described by a non-relativistic many-body Hamiltonian in which the interaction is described by the nucleon-nucleon forces. The fundamental problem of nuclear reaction theory is the determination of the intrinsic parameters of the S-matrix obtained from the analysis of experimental data from the above Hamiltonian.

Since nuclear forces are strong forces, perturbation theory will not suffice. Both the representation of the experimental results and its explanation in terms of the nuclear Hamiltonian must be valid in the presence of strong interactions.

Nuclear reactions have been observed from close-to-zero energy of the incident particle to as much as 20-GeV protons. Particle species have

included not only the traditional neutrons and protons but also composite particles varying from the deuteron, ^3H, ^3He, ^4He, to nuclei as heavy as ^{32}S. Extensive experiments have been performed with electrons as well as γ-rays up to several GeV. Muons, as well as the bosons, pions and kaons have also served as projectiles. Target nuclei include all the stable nuclei. With this tremendous range in experimental conditions it is no surprise that a rich variety of phenomena have already been observed. It should be added that the reaction phenomena which are observed in nuclear physics are very similar in character to those observed in atomic physics as well as in elementary particle physics.

2. REACTION TYPES

It is, of course, not possible to give a very complete description of all of the various nuclear reactions and we shall only present a broad impressionistic picture here. We can classify the reactions which take place in terms of the quantities which are transferred between the interacting particles. In elastic scattering, the only quantities transferred are the momentum transfer $\vec{q}$ and helicity if the particles have spin, or if the angular-momentum partial-wave decomposition is more appropriate the label may be the transferred angular momentum and parity. If the scattering is inelastic the energy transfer ω must be included, as well as other quantities which characterize the excitation. As the earlier speakers have surely discussed, these excitations can be collective, i. e. the target nucleus as a whole is involved, rotational and vibrational levels can be excited. These excitations can be of the shell-model type in which individual nucleons in the target are excited to new shell-model levels. The emphasis so far is on the low-lying states whose nature can by and large be described in this way. As the excitation energy increases the spectrum loses its discrete character, and a broad continuum is observed. At still moderate incident energies (some tens of MeV) this distribution can often be described in terms of an "evaporation" spectrum as if the nucleus were heated to some temperature and the observed particles left the nucleus by "evaporation". At higher energies, part of the continuum will include quasi-elastic processes in which the incident particle ejects a target nucleon by direct collision.

Charge exchange processes such as (p, n) (d, 2n) reactions involve the transfer of a unit isotopic spin to the target. With incident pions one can achieve a transfer of two units of isospin in the $\pi^{\mp} \rightarrow \pi^{\pm}$ transition.

Exchange of strangeness or hypercharge will lead to the formation of hypernuclei. For the most part these have been obtained by the absorption of negative kaons. But, in principle, these can also be found in a (K, π)-reaction.

Particle exchange processes in which the baryon number B of the incident particle and therefore of the target nucleus change are of particular importance in nuclear physics. The so-called stripping and its increase, pick-up, are examples of this kind (see Fig. 1). A wide variety of the particle transfer reactions have been observed. Let me cite such examples of two-particle transfers as the $(^3H, p)$ reaction, in which two neutrons are transferred, and (n, α) experiments in which $\Delta B = 3$. In the collision between heavy ions considerably larger nucleon transfers have been observed.

In the above particle-transfer reactions the final emergent particle although composite, may be thought of as a single particle because it is in a bound state. However, when the energy is adequate the composite system need not be bound so that the final system no longer consists of two particles, the emergent particle and the residual nucleus. We have already mentioned one such process, the (d, 2n) reaction. In the evaporation regime (n, 2n) reactions occur. In the high-energy (hundreds of MeV) domain, reactions which are expected to yield important nuclear structure information include the (p, 2N), (γ, 2N), (e, e'N), reactions. Meson as well as photon absorption generally results in the emission of two nucleons (π, 2N).

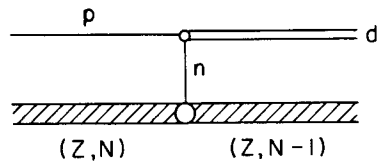

FIG. 1. Diagram for the pick-up (p,d) reaction.

A list of reaction types must include the catastrophic fission process. Historically, fission was first observed to follow the absorption of neutrons by nuclei such as ^{235}U and ^{238}U. Other particles, e.g. the proton, the alpha particle, as well as the photon have been employed. Of course, the very energetic protons produced by the high-energy accelerators or gamma rays from SLAC will lead to fission or to spallation as it is more generally referred to if there are several fragments. The inverse of binary fission can occur in the collision between two heavy ions but the nuclei involved so far have been comparatively light. However, even here some interesting phenomena have been uncovered.

3. REACTION MECHANISMS

From the reaction types listed above we turn to the various reaction mechanisms. It should be emphasized that each of the reactions can proceed simultaneously via several differing mechanisms and of course interference between the corresponding amplitudes can occur and is informative.

The reaction mechanisms can be classed according to the time delay between the entrance of the incident particle into the target nucleus and the emission of the emergent particle. The scattering of a particle by an external potential, potential scattering, is a familiar example of a "prompt" process. The time delay here is of the order of magnitude of the transit time. It is characteristic of the prompt processes that the emergent beam carries the "memory" of incident beam direction. At "high" energy, defined as the energy for which the transit time through the nucleus is small compared to the periods of the motion of the nucleons in the nucleus, much of what occurs is a prompt nature. There are the various reactions in which the largest nucleus is not disrupted, recoiling as a whole unit, possibly in an excited state. The target nucleus acts as a unit, and we would like to think of these reactions as examples of collective nuclear motion. The

simplest example is elastic scattering which is peaked sharply in the forward direction ($\theta \sim (1/kR)$). Inelastic scattering in which collective excitations, e. g. vibrations and rotations, or "single particle" excitations may be thought of as prompt "nearly" elastic processes, the time delay in both the elastic and nearly elastic processes being given by the time required to distribute the recoil and excitation energy among the A target nucleons. This time is of the order of magnitude of R/v_F where v_F is the velocity of the nucleons at the "top" of the Fermi sea describing the target nucleus.

The nearly elastic and elastic scattering described above occur for distant collisions between the incident and target nucleons. When the collision is more nearly "head on", the most likely process involves the ejection of a target nucleon. These processes at high energy are describable in terms of quasi-elastic scattering. In other words, the incident particle and the target nucleon have a more or less "free-space" collision with free-space kinematics, the target nucleon and incident particle continuing on through the residual nucleus. Many quasi-elastic processes such as (p, 2p), (e, e' p) have been observed. The quasi-elastic process is an "on-the-energy-shell" process. Again we do not attempt to distinguish between exactly on and nearly on the energy shell. Off-the-energy-shell processes in which free-space kinematics are not satisfied are important for the determination of the correlation between target nucleons. An alternative description of these off-the-energy-shell events considers them to be a collision with two target nucleons. Processes such as (p, pd) or gamma and slow pion and kaon absorption in which much energy but little momentum is transferred have considerable components which are describable as such a two target nucleon collision. These remarks can be obviously generalized to collisions with larger clusters of target nucleons. In the more drastic processes the residual nucleus is often left in a highly excited state which decays by nucleon evaporation generating a low energy nucleon continuum thereby.

A great many of the qualitative aspects of high energy reactions are preserved as the energy is lowered from the GeV range to the MeV range. In particular, prompt processes remain significant. However there is an essential difference when the incident particle is composite, e. g. $2 \leq A \leq 4$. These particles generally do not penetrate deeply into the nucleus. The reaction is typically peripheral, most of the interaction occurring within the nuclear surface. For example in the stripping (d, p) reaction the neutron in the incident deuteron is transferred to the target nucleus usually ending up as a valence neutron in some single particle shell model orbit, the target nucleus remaining unexcited. In principle it is possible for the target nucleus also to be excited but this requires a two step process with, in most cases, a correspondingly reduced probability. Thus the (d, p) reaction is a very useful tool for the investigation of single particle neutron levels. On the other hand, incident nucleons which can penetrate more readily into the target nuclear interior may be expected to excite all states with comparable probability. This proves to be the case and is illustrated in Fig. 2 where the cross-sections are given for processes in which the same residual nucleus is excited by stripping and by proton bombardment. In the stripping case a number of the low-lying levels evident in the proton case are absent.

To repeat, composite incident particles interact most vigorously with the surface of the target nucleus. In particle transfer cases, of which stripping is an example, the transferred particle or particles generally

end up as valence particles in the residual nucleus. Inelastic scattering of composite particle, e.g. the (d, d'), (α, α') etc. reactions, will induce characteristic motions of the surface of the target nucleus, particularly various kinds of vibrations. Each of these surface reactions will have anisotropic angular distribution, whose shape is characteristic of the orbital angular momentum and/or parity transferred in the process. As expected,

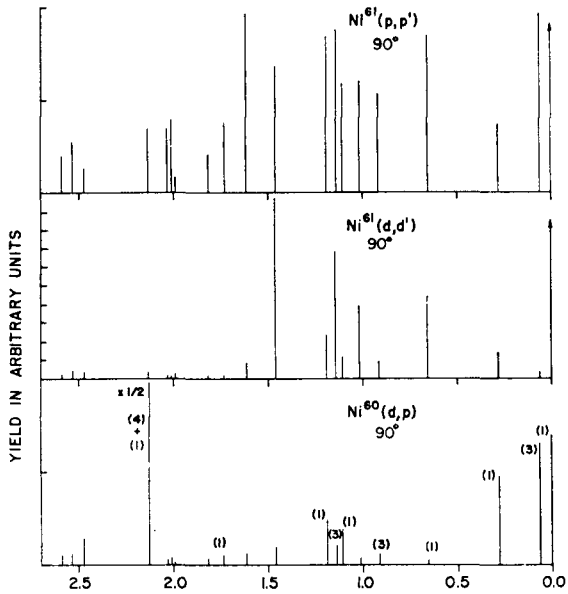

FIG. 2. Excitation energy in ^{61}Ni (MeV).

as this angular momentum increases the first maximum of the angular distribution occurs at correspondingly larger angles. Not all peripheral interactions must be peaked in the forward direction. Several examples are given in Figs 3 and 4. In Fig. 3 the angular distributions [1] of the protons produced in the stripping reactions ^{50}Zr(d, p) to various final states are presented. Each curve is labelled by its Q value for the transition as well as the transferred values of ℓ, and j the orbital and total angular momentum carried by the neutron. Note that there is a characteristic angular distribution for each value of transferred ℓ, the distributions for ℓ = 0 being similar to each other but differing in a marked way from the angular distribution ℓ = 2 transfer reaction which in turn differs from ℓ = 4. Figure 4 shows the angular distribution [2] for the scattering, both elastic and inelastic, of 43 MeV alpha particle by ^{58}Ni. Observe that the oscillations in the angular distribution for processes, which excites levels in the residual nucleus with ℓ = 2 and ℓ = 4 are in phase, those of ℓ = 3 and elastic scattering are in phase. The groups ℓ = 2.4 and ℓ = 0.3 are exactly out of phase. It is obvious that these reactions are most important for nuclear structure for they can help determine the angular momentum and parity of the residual nuclear levels populated by the reaction.

Reactions involving nucleons, photons, electrons, etc., tend to be volume reactions. As long as the reaction is prompt, the angular distributions are anisotropic. However, their shape and the nature of the state of the residual nucleus are not as easily correlated. Prompt reactions are referred to as "direct reactions".

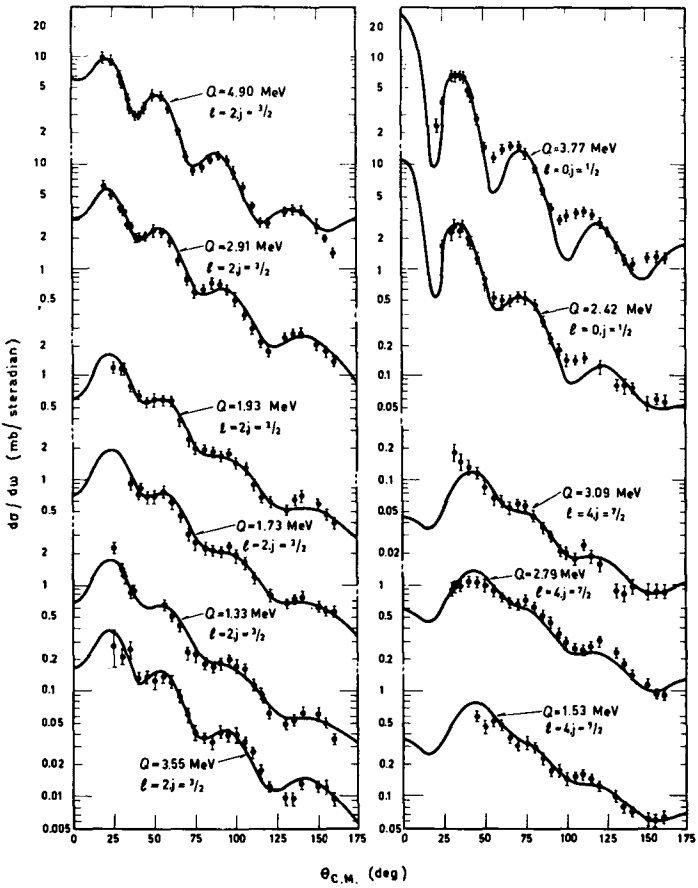

FIG. 3. Angular distributions for ℓ = 0, 2, 4 transitions in the ^{90}Zr(d,p) reaction for 12 MeV deuterons.

As these lower-energy resonance reactions become an important feature of the observed cross-sections (see Fig. 5) [4], the width of these resonances can be as little as one meV for very slow particles incident on heavy nuclei while it can rise to several tens of keV for more energetic particles incident on the lighter nuclei. These resonances are interpreted as unstable compound states with definite quantum numbers with a life-time given by $\hbar/\Gamma$ (where Γ is the full width at half maximum). This life-time is several orders of magnitude longer than the time it takes the incident particle to pass through the target nucleus. Resonance reactions are thus reactions in which the time delay is very large as contrasted with the prompt reactions discussed just above.

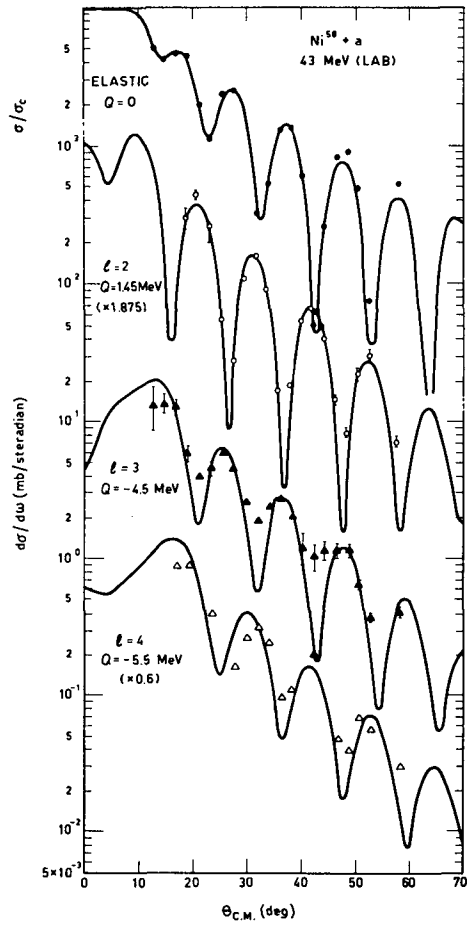

FIG. 4. Angular distribution for the elastic scattering and ℓ = 2, 3, 4 transitions in the ^{58}Ni(α,α) reaction for 43 MeV alphas.

Reactions proceed both via compound nucleus formation or by the prompt processes. In elastic scattering this is a familiar effect resulting in the interference between the "potential" scattering as the direct process is often referred to, and the compound nuclear scattering. In Eq. (1) the phase ϕ gives the potential scattering. In Fig. 5a the elastic scattering ^{27}Al(p, p) and ^{24}Mg(α,α) illustrate this interference phenomena whereas it is not visible in the reactions ^{27}Al(p,α)^{24}Mg [3]. This means that these particle transfer reactions in this case do not have very strong prompt amplitudes. Generally, however, there will be interference effects between direct-reaction and compound nuclear amplitudes. In fact, the unique determination of the nature of the nuclear excitation which is provided by the direct excitation may occasionally be seriously compromised if compound nuclear effects are large. Of course, if the energy resolution is sufficient and if the resonances are well separated in energy, the interfering amplitudes

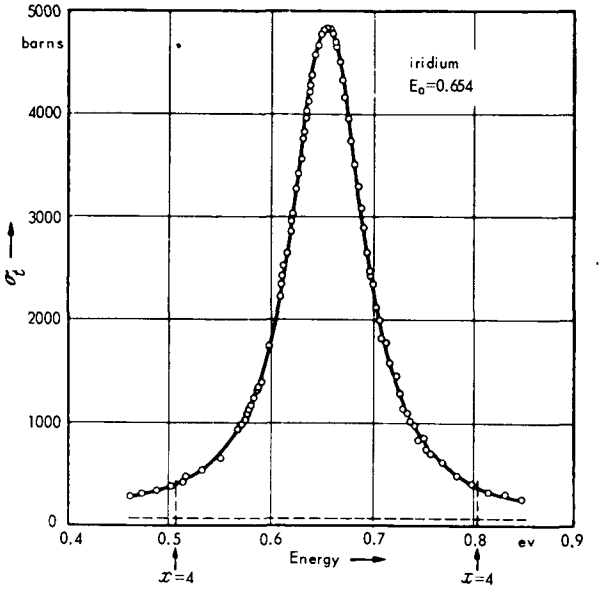

FIG. 5. The measured total cross-section for a neutron incident on iridium. Both Doppler and possible resolution effects are present.

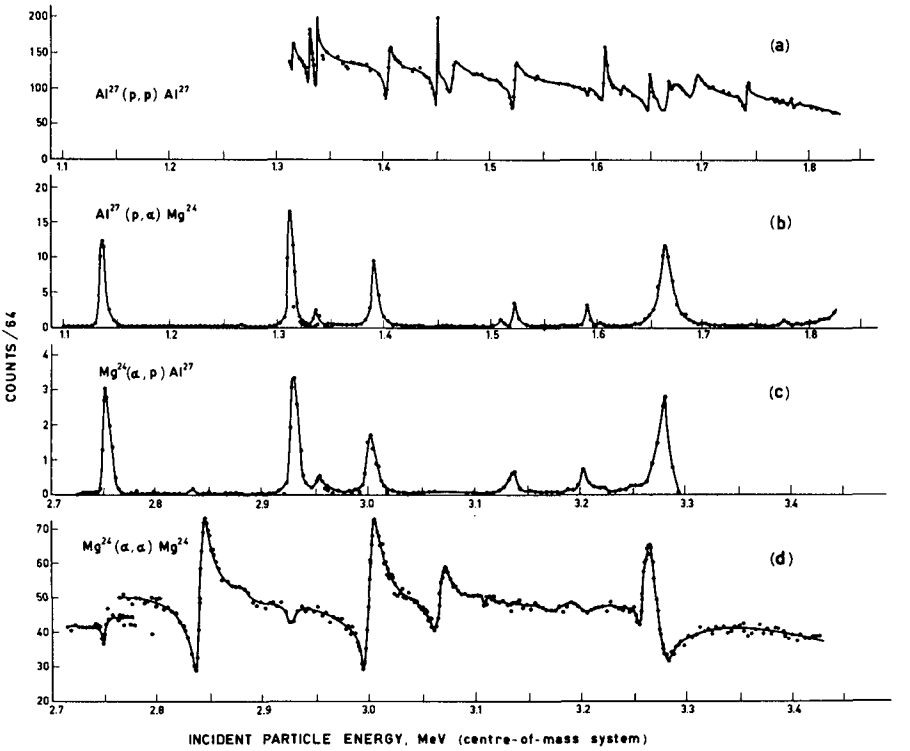

FIG. 5a. Excitation curves for the ^{27}Al + p and ^{24}Mg + α systems displaying several resonances in the compound nucleus ^{28}Si.

can be disentangled. The direct-reaction amplitude varies slowly with energy, the compound nuclear varies rapidly. In addition, the identification of the direct processes depends on (a) the strong anisotropy of its angular distribution so that in the forward cone it dominates the compound nuclear reaction, and (b) the large number of open channels to which the compound nucleus can decay so that its contribution to a particular reaction mode is limited by unitarity.

The determination of the parameters describing a resonance is straightforward as long as the potential phase ϕ does not vary appreciably over the resonance width (in some parameterizations this energy dependence occurs as an energy dependence of the resonance energy and the width. This dependence is above and beyond that which is present because of threshold and penetrability effects.) "Appreciably" here is measured by the products $\Gamma\partial\phi/\partial E$ and $\Gamma^2\partial^2\phi/\partial E^2$ which should be much less than unity if the simple resonance description with constant parameters is to apply. What does it mean if the energy dependence is appreciable? Only that there is a nearby resonance or resonances which brings us to the case of overlapping resonance. The relevant parameter here is Γ_r/D_r where D_r is the spacing between resonances. If Γ_r/D_r is large overlapping is serious and the simple representation (1) is inadequate. We shall return to the solution of this problem later. It should be mentioned here that as the excitation energy increases the density of levels increases exponentially so that the levels rapidly become overlapping.

So far we have had examples of prompt direct reactions and the relatively slow time-delayed compound nuclear reactions. In terms of the compound system (note the difference from compound nucleus) the direct reactions involve relatively simple types of excitation involving a minimal number of degrees of freedom whereas the compound nucleus is thought of as being very complex in nature. Of course, this remark is representation-dependent (a vibration is not so simple when looked at from the shell-model point of view), and for this reason the time-delay description seems to be a more satisfactory characterization.

The following question immediately suggests itself: Do reactions exist whose time delay is longer than the direct reaction time delay but shorter than that for compound nucleus reactions? The corresponding resonant structure in the cross-section would have widths which are intermediate between those of the compound nucleus and of the single particle resonances. The nature of the state of the compound system which is excited should be somewhat more complex than the direct type but still considerably less complex than a compound nuclear state. Generally these states occur in an energy region in which there are many compound nuclear levels. Except for minor miracles the intermediate states will couple to the compound nuclear levels and one will with infinitely good resolution find only many relatively narrow resonances or peaks. The intermediate state will make itself felt by a modulation of the cross-section, the widths of resonances being on the average larger in the intermediate state resonance region. The actual intermediate resonance can be recovered if an energy average over the fine structure is taken. Of course, the width of this intermediate resonance is larger than would prevail in the absence of coupling to the fine structure. If the additional spreading is too large it may not be observable. Ideally one would like an averaging interval large compared with D_r but small compared with Γ_d where Γ_d is the width of the intermediate states. Examples

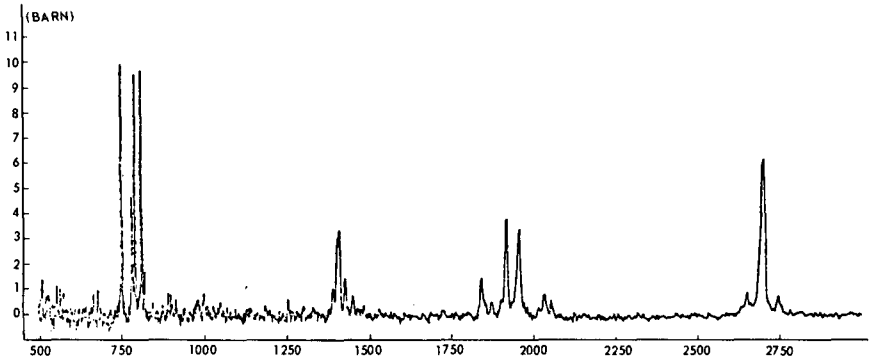

FIG. 6. Fission cross-section for neutrons incident on ^{240}Pu.

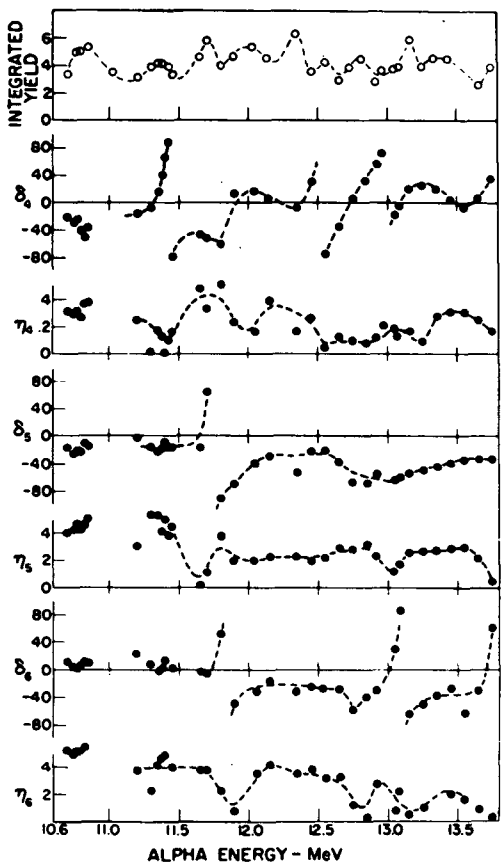

FIG. 7. Phase shifts δ_ℓ and damping amplitude $\eta_\ell \equiv |s_\ell|$ for $\ell = 4, 5, 6$ as a function of alpha-particle energy. The topmost curve gives the sum of the ratio of the differential cross-section to the Rutherford cross-section over the measured angular range.

of such resonances have been shown in the isobar analogue reaction. We expect it to be a more general phenomenon and to demonstrate that point I shall now show three other intermediate-structure examples. In the first example, Fig. 6, the fission cross-section for neutrons incident on ^{240}Pu obtained by Magneco and Theobald [5] is presented. We note the presence of groups of strongly excited fine-structure resonances. The width of these groups is about 70 eV, their separation in energy about 700 eV.

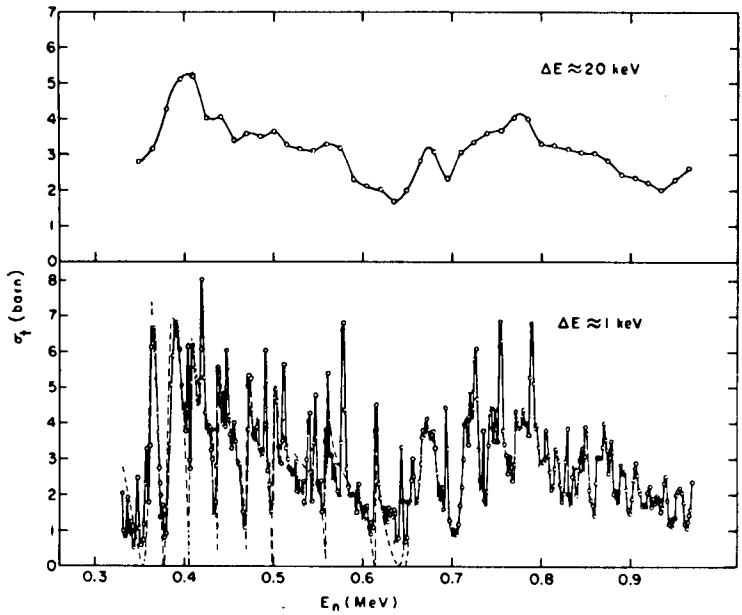

FIG. 8. Total neutron cross-section for Fe. Upper curve gives the integrated differential cross-section. Lower curve (points and solid curve) high-resolution data.

This phenomenon has recently been observed in a few fissile nuclei. In the second example, Fig. 7, the phase shift analysis of alpha particle scattering by ^{26}Mg is shown [6]. We observe that the phase shifts pass through $\pi/2$ several times for $\ell = 4, 5$ and 6 partial waves indicating resonances. When the energy resolution is increased, fluctuations appear indicating the presence of underlying fine-structure resonances. This phenomenon has been shown to exist for a number of nuclei in this mass region and for a number of orbital angular momenta extending from $\ell = 1$ to $\ell = 6$ [7]. In Fig. 8, we present the total neutron cross-section for neutrons incident on Fe [8]. The lower curve gives the results obtained when neutrons with 1 keV energy resolution are employed. The upper curve gives the results for a 20 keV energy resolution. We observe the presence of two intermediate resonances each with an energy width of about 150 keV.

This completes the general outline in which it was intended to describe in rather broad terms the nature of the phenomena which nuclear reaction theory is required to analyse and explain.

4. CHARACTERISTICS OF NUCLEAR REACTION THEORY

We enumerate first the gross characteristics which such a theory must include in order to adequately represent the observed phenomenon. It must obtain in a unified fashion the amplitudes corresponding to prompt direct reactions, to delayed compound nuclear reactions and to the intermediate resonance reactions. Since these various components can interfere, what we are asking for is a decomposition of the transition amplitude into these three components (or more if there are more ranges of time delay which need to be included). The theory must be able to describe elastic and inelastic scattering, particle transfer reactions, photon or for that matter meson production or absorption. It must be able to describe these phenomena at both low and high energies. Another problem arises from the great variety of reactions which can occur at even a moderate excitation energy. To calculate and describe each and every threshold, as well as the multitudinous resonances would undoubtedly be a waste of time. What we should like to do is to extract the simpler physical content.

My discussion of the solution to these problems will be necessarily idiosyncratic [9]. There are other attitudes [10] but there is not sufficient time to properly describe these as well.

5. PROMPT REACTIONS

Let us first consider the prompt reactions. It is central to our discussion that the interaction which leads to prompt reactions can be represented by a potential or potential matrix. For example, prompt elastic scattering is predicted from an equation of the form

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V(\vec{r}, \vec{S}_P\vec{S}_T, \vec{T}_P\vec{T}_T)\psi = E\psi \tag{2}$$

Here $\vec{r}$ is the co-ordinate of the incident particle taken with respect to the centre of mass of the target nucleus, $\vec{S}_P$, $\vec{S}_T$, $\vec{T}_P$, $\vec{T}_T$ the spin and isotopic spin operators for the projectile and target. If other channels beside the elastic one are prompt then one must deal with a coupled set of equations. A two channel case would then be described by

$$\begin{aligned} -\frac{\hbar^2}{2\mu_1}\nabla^2\psi_1 + V_{11}\psi_1 - E_1\psi_1 &= V_{12}\psi_2 \\ -\frac{\hbar^2}{2\mu_2}\nabla^2\psi_2 + V_{22}\psi_2 - E_2\psi_2 &= V_{21}\psi_1 \end{aligned} \tag{3}$$

The first channel describes the elastic scattering while the other could describe a particular type of inelasticity such as the excitation of a rotational state or of a particle transfer reaction. It is a trivial matter to generalize to many channels.

The presence of the time-delayed compound nuclear resonance mechanism which can also lead to the identical scattering and reactions has an effect on the description of the prompt processes via Eqs (2) and (3). In a complete description which we shall give later on (see Eq.(19)) we need to need to add the time-delayed amplitude to the prompt amplitude to obtain the total amplitude. Since some of the incident beam participates in the time-delayed processes, the S-matrix formed from the prompt amplitudes alone cannot satisfy the unitarity condition. Thus, the integrated intensity of the prompt amplitude will be less than the incident intensity so that if we consider the prompt amplitude only we would conclude that an absorption has occurred. The potential in Eq.(2) or the potential matrix in Eq.(3) must be complex (non-Hermitian) in order to describe this effective absorption. In this way we are led to the complex potential model or, with a less precise terminology, to the "optical" model.

6. OPTICAL MODEL

Let us describe this origin of the optical model in a more explicit fashion. To discuss time delay it is necessary to construct a wave packet. The wave-packet wave function ψ is thus an energy average of the exact wave function Ψ over some ΔE producing a wave packet whose width in time is $\hbar/\Delta E$. We therefore have

$$\psi(E_0) = \frac{1}{2\Delta E} \int_{E_0 - \Delta E}^{E_0 + \Delta E} \Psi(E)\, dE = \langle \Psi \rangle \tag{4}$$

ΔE must satisfy the condition

$$\Delta E \gg \Gamma_r \tag{5}$$

Of course, other ways of averaging are possible but we shall not enter into a discussion of this point. Pictorially we can describe the process as follows. The incident wave packet strikes the target nucleus. A prompt wave packet identical in shape but with reduced amplitude is produced after the interaction. Later, at a time of the order $\hbar/\Gamma_r$, a delayed wave packet will be emitted. The compound nuclear system decays. By making the potentials V complex, we obtain a description of $\langle \Psi \rangle$. The transition amplitude corresponding to $\langle \Psi \rangle$ is similarly an average

$$t = \langle T \rangle \tag{6}$$

It has not been possible practically to construct such a wave packet so that the cross-section corresponding to Eq.(6), $|\langle T \rangle|^2$, is not measurable. The measured cross-section includes both delayed and prompt contributions. However, for processes which are dominated by the prompt cross-section the correction coming from compound nuclear formation is not important.

Coupled equations or approximations to them form the basis for the treatment of direct reactions.[1]

The question arises as to how many channels we should couple together. As the excitation energy increases the number of open channels grows exponentially, and the use of the coupled channel description becomes rapidly impractical. It is however, possible to restrict the number of channels to the elastic one and to those which require special attention. The imaginary part of the potential or potential matrix then contains not only the time delayed channels but also some "real" absorption due to open channels not explicitly included as one of the coupled channels. Since the potentials used are generally smooth functions of the energy and momentum the error introduced by this procedure might be particularly important near the thresholds for the various omitted open channels where the energy dependence of the transition amplitude is relatively rapid. This reflects itself in the included channels in discontinuities in the energy slope of the cross-section, i.e. to various kinds of cusps. However these special features show up over a limited energy range. Again if we energy average as indicated in Eq.(6) their effect will be relatively unimportant and the complex potential remains as a good representation of the interaction as far as the prompt processes are concerned. The use of the complex potential is not restricted to nuclear physics having been adopted for example, by the atomic physicists in their discussion of electron atom scattering. We believe that it has an appropriate place in the discussion of high-energy elementary-particle reactions, and we are in the process of making such an attempt using the Bethe-Salpeter rather than the non-relativistic Schroedinger equation.

The generalized optical model provides us with a form for the description of prompt processes. We still must obtain a description of resonance phenomena which is valid in the presence of prompt processes, and from this determine the relation between the parameters of the optical model which averages over the resonance structure and those describing resonance structure. We must relate the interactions employed to the nucleon-nucleon forces. Let us turn our attention to these problems.

It will be convenient to separate the prompt and time-delayed components of the reaction amplitude. Towards this end we presume that the existence of a projection operator P which projects out of the exact wave functions those channels which have prompt components and for that matter any other channels which may be of particular importance for these processes.[2] The remaining states

$$Q\Psi = (1 - P)\Psi \tag{7}$$

are the time-delaying states. If there were no coupling of $P\Psi$ to a time-delaying mechanism, i. e. if $P\Psi$ were not coupled to $Q\Psi$, $P\Psi$ would asymptotically contain only prompt components and would satisfy equations of

[1] Even when compound nuclear processes are important, procedures can be devised which permit the calculation of the energy average cross-section $\langle |T^2| \rangle$.

[2] It is not necessary to project out all the prompt channels. Generally all the open channels will have prompt components. But on some occasions it is not desirable or practical to explicitly consider all these open channels but to restrict attention to those open channels which, as we know, either a priori or a posteriori have a decisive effect on these prompt reactions.

the form of Eq.(3) with real V_{ij}. It is the coupling which introduces the time delay as we shall shortly show. The efficacy of a particular choice of P is determined in part by the success in separating off the prompt processes. In principle, this can be verified for a particular calculation by examining the energy dependence of the results as given later in Eq.(19). Prompt reaction cross-sections should vary slowly with the energy. The parameters describing cross-section fluctuations such as resonances should vary slowly over the width of the fluctuation. The fluctuations themselves will vary rapidly with energy. By choosing P properly the slow and rapid energy dependence of the transition amplitude may be separated.

Consider a simple example. Suppose that elastic and inelastic scattering to the first excited state of the target nucleus by an incident nucleon were the only interesting channels. Then $P\Psi$ would have the form

$$P\Psi = \mathscr{A}[u_0\phi_0 + u_1\phi_1] + [\ldots\ldots] \tag{8}$$

Here ϕ_0 is the wave function of the target nucleus in the ground state while ϕ_1 is the wave function for the first excited state. The coefficients u_0 and u_1 are functions of the relative particle-nucleus co-ordinate. $\mathscr{A}$ is the antisymmetrization operator. From the asymptotic properties of u_1 and u_0 one can obtain the elastic and inelastic scattering amplitudes. The open bracket in Eq.(8) refers to any state or states which appear to be essential for the description of the prompt channels. The selection of the terms to be included in the open bracket determine the nature of the reaction model. This flexibility in the choice of P is of course desirable.

7. TIME DELAYS

We shall now show that the coupling between $P\Psi$ and $Q\Psi$ introduces a time delay and then go on to derive the transition amplitude which consists of a prompt and a time-delayed part.

The wave function has the form

$$\Psi = P\Psi + Q\Psi \tag{9}$$

We now determine the coupled equations for $P\Psi$ and $Q\Psi$. These are

$$(E - H_{PP})\, P\Psi = H_{PQ} Q\Psi \tag{10}$$

$$(E - H_{QQ})\, Q\Psi = H_{QP} P\Psi \tag{11}$$

$$H_{PP} \equiv PHP \quad H_{PQ} \equiv PHQ \text{ etc.}$$

The equation for the prompt channels of interest is obtained by eliminating $Q\Psi$:

$$\left(E - H_{PP} - H_{PQ} \frac{1}{E^+ - H_{QQ}} H_{QP}\right) P\Psi = 0 \tag{12}$$

$$E^+ \equiv E + i\eta, \quad \eta \to 0^+$$

This is an exact equation. We notice the appearance of the explicit energy dependence which is carried by the propagator $[1/(E^+ - H_{QQ})]$. The presence of this propagator explicitly demonstrates the presence of a time delay in the Q channels.

It is interesting to note that the effective potential

$$H_{eff}^{(E^+)} = H_{PP} + H_{PQ} \frac{1}{E^+ - H_{QQ}} H_{QP} \tag{13}$$

satisfies a dispersion relation:

$$\mathrm{Re}\, H_{eff}^{(E)} = H_{PP} + P \sum_{\lambda} \frac{H |\Phi_\lambda\rangle\langle\Phi_\lambda| H}{E - E_\lambda} P - \frac{1}{\pi} \mathscr{P} \int_{\mathscr{E}_0}^{\infty} \frac{d\mathscr{E}\, \mathrm{Im}\, H_{eff}(\mathscr{E})}{E - \mathscr{E}} \tag{14}$$

where $\mathscr{E}_0$ is the threshold for the continuum of the spectrum of H_{QQ}. E_λ are the discrete eigenvalue and Φ_λ the corresponding eigenfunctions. When E is near a discrete eigenvalue of H_{QQ} the propagator varies rapidly with energy and so will the effective Hamiltonian in Eq.(12). This behaviour is also revealed by the transition matrix T. From Eq.(10) we see that

$$T_{fi} = (T_P)_{fi} + \langle \psi_f^{(-)} H_{PQ} Q \Psi \rangle \tag{15}$$

where $\psi_f^{(-)}$ satisfies the equation:

$$(E - H_{PP})\, \psi_f^{(-)} = 0 \tag{16}$$

with the usual incoming-wave boundary conditions. The first term $(T_P)_{fi}$ on the right-hand side of Eq.(15) represents the prompt amplitude as computed from H_{PP}. We can compute $Q\Psi$ from the equation it satisfies:

$$[E - H_{QQ} - W_{QQ}]\, Q\Psi = H_{QP} \Psi_i^{(+)} \tag{17}$$

where

$$W_{QQ} \equiv H_{QP} \frac{1}{E^+ - H_{PP}} H_{PQ} \tag{18}$$

W_{QQ} represents the contribution to the Q channels arising from the coupling to the prompt channels. Substituting Eq.(17) into Eq.(15) we obtain

$$\begin{aligned} T_{fi} &= (T_P)_{fi} + \langle \psi_f^{(-)} H_{PQ} \frac{1}{E^+ - H_{QQ} - W_{QQ}} H_{QP} \psi_1^{(+)} \rangle \\ &= (T_P)_{fi} + (T_R)_{fi} \end{aligned} \tag{19}$$

If we have chosen P properly the second term T_R on the right-hand side contains the time-delayed term. Indeed, if we restrict the energy domain ΔE under scrutiny to one containing a finite number of resonances, we can in principle choose Q to subtend a corresponding finite vector space so that T_P has a slow energy dependence in the interval ΔE while the resonances in ΔE all come from the T_R term. For finite energy ΔE a perfect separation is possible and in Ref.[9] a systematic way of achieving this is given. It should be noted that the success of a choice of P and Q can be tested a posteriori, i.e. by examining the energy dependence of T_P and T_R.

For example, let us choose ΔE so that only a single "isolated" resonance occurs in ΔE. Then the choice for Q-space indicated is such that it contains only one state Φ_s which is an eigenstate of $H_{QQ} + W_{QQ}$:

$$(H_{QQ} + W_{QQ})\,\Phi_s = \left(E_s - \frac{i\Gamma_s}{2}\right)\Phi_s \tag{20}$$

where

$$\Gamma_s = 2\pi \langle \Phi_s\, H\, \delta(E - H_{PP})\, H\, \Phi_s \rangle \tag{21}$$

Equation (17) becomes

$$T_{fi} = (T_P)_{fi} + \frac{\langle \psi_f^{(-)} H_{PQ} \Phi_s \rangle \langle \Phi_s H_{QP} \psi_i^{(+)} \rangle}{E - E_s + i\,\Gamma_s/2} \tag{22}$$

the Breit-Wigner resonance amplitude to which the effects of the prompt direct reactions are added. If there are several resonances in ΔE several terms corresponding to them exist. Note that any energy dependence in T_P will reflect itself in the matrix elements of the numeration through $\psi_{fi}^{(\pm)}$. In the energy domain in which the prompt reaction resonates these matrix elements and therefore the corresponding widths will also be larger on the average. Thus the behaviour of the prompt reactions implies a correlation among the matrix elements describing the compound nuclear reaction.

Since the density of resonances increases as E increases it is clear that different operators P and Q will be used for differing E and ΔE if we require a clean-cut separation between delayed and prompt reaction components. In other words, the V_{ij} in Eq.(3) or V in Eq.(2) are slightly energy-dependent. It is possible to use a fixed P and Q, and there are several formalisms doing exactly that, and, thus, leading to parameters which are more independent of energy. However, in so doing one blurs the separation of time-delayed and prompt reactions. The T_R term in Eq.(19) will now contain prompt reactions and one must sum over "distant" levels in order to obtain the direct prompt reactions. The above discussion simply recognizes this feature and includes it explicitly from the beginning. The experimentalist does the same for in analysing his data he will presume the form of Eq.(22) for the case of a single resonance (or its generalization in the presence of several resonances). Experimentally, in the case of isolated resonances where this analysis is most profitable he finds that $(T_P)_{fi}$ is a slowly varying function of energy.

Although it is possible to extend Eq.(19) very easily to the case of many levels, the resulting expression has the one important drawback, of not showing unitarity in a manifest fashion. And, in fact, the formalism readily gives relations between the parameters which subsequently can be shown to guarantee unitarity. The procedure is not incorrect, but inconvenient. Unitarity becomes manifest if we employ the reaction-matrix representation of S:

$$S_{fi} = \langle \psi_f^{(-)}, \frac{1 - i\pi K}{1 + i\pi K} \psi_i^{(+)} \rangle \tag{23}$$

where K, the reaction matrix is Hermitian. K is readily determined in terms of H. When all the Q channels are closed we find [11]

$$\begin{aligned} K &\equiv H_{OQ} [E - H_{QQ} - R_{QQ}]^{-1} H_{QO} \\ R_{QQ} &= H_{QP} \frac{\mathscr{P}}{E - H_{PP}} H_{PQ} \end{aligned} \tag{24}$$

Here O is an operator which selects those states in P space which are on the energy shell. It is no more or less difficult to treat K than T but determination of T requires a second step in treating Eq.(23) and represents the price paid in order to obtain explicit unitarity. We shall not describe this process in any detail but content ourselves with quoting the result obtained for pure elastic scattering (the only open channel is elastic). Then

$$T = (T_P)_{fi} + e^{2i\delta} \frac{\sum_i [\gamma_i/(E - \epsilon_i)]}{1 + i\pi \sum_i [\gamma_i/(E - \epsilon_i)]} \tag{25}$$

where the γ_i are positive constants. The constants ϵ_i are eigenvalues of $H_{QQ} + R_{QQ}$. The number of terms in the sum corresponds to the dimensions of the Q space and therefore, to the number of resonances in the energy interval of interest. We note that

$$\langle \psi^{(+)} K \psi^{(+)} \rangle = \sum_i \frac{\gamma_i}{E - \epsilon_i} \tag{26}$$

has the familiar property of negative energy slope where it is assumed that the parameters have at most a slow energy dependence. More generally, with the same assumption the energy derivative of operator K Eq.(24) is semi-positive definite.

8. CASE OF THREE TIME SCALES

The procedure to be used when three time scales are present: prompt (direct reactions), large time delay (compound-nucleus resonance reaction), intermediate time delay should now be evident [12]. We divide our Hilbert

space into a corresponding three components given by the projection operators P, q, d where

$$P + q + d = 1 \qquad Pq = Pd = qd = 0$$

The equations corresponding to Eqs (10) and (11) are

$$\begin{aligned}
(E - H_{PP})\, P\Psi &= H_{Pd}(d\Psi) + H_{pq}(q\Psi) \\
(E - H_{dd})(d\Psi) &= H_{dP}(P\Psi) + H_{dq}(q\Psi) \\
(E - H_{qq})\,(q\Psi) &= H_{qP}(P\Psi) + H_{qd}(d\Psi)
\end{aligned} \tag{27}$$

It is straightforward to solve these equations formally and obtain a general transition matrix which exhibits the separation into direct intermediate resonance and compound nuclear resonances. We shall simplify the presentation by introducing the doorway-state hypothesis.

$$H_{Pq} = H_{qP} = 0 \tag{28}$$

These equations state that it is not possible to progress in time from a prompt amplitude to a compound nuclear amplitude without first generating some of the intermediate amplitude, the intermediate states acting as "doorways" between the very prompt and the very delayed reaction amplitudes. With this approximation one obtains:

$$T_{fi} = (T_P)_{fi} + (T_d)_{fi} + (T_R)_{fi} \tag{29}$$

$$(T_d)_{fi} = \langle \psi_f^{(-)}\, H_{Pd} \frac{1}{E - H_{dd} - W_{dd}} H_{dP}\, \psi_i^{(+)} \rangle \tag{30}$$

$$(T_R)_{fi} = \langle \omega_f^{(-)} H_{dq} \frac{1}{E - H_{qq} - \widetilde{W}_{qq}} H_{qd} \omega_i^{(+)} \rangle \tag{31}$$

In these equations

$$W_{dd} \equiv H_{dP} \frac{1}{E^+ - H_{PP}} H_{Pd} \tag{32}$$

$$\widetilde{W}_{qq} \equiv H_{qd} \frac{1}{E^+ - H_{dd} - W_{dd}} H_{dq} \tag{33}$$

$$\omega_{fi}^{(\pm)} = \frac{1}{E^+ - H_{dd} - W_{dd}} H_{dP}\, \psi_{fi}^{(\pm)} \tag{34}$$

We illustrate these formulas in the case where we have but one doorway state, i. e. the d part of Hilbert space involves only one state but there

are many compound nuclear resonances which for simplicity of presentation are assumed to be not overlapping. Then

$$T_{fi} = (T_P)_{fi} + \frac{\langle \psi_f^{(-)} H_{Pd} \Phi_d \rangle \langle \Phi_d H_{dP} \psi_i^{(+)} \rangle}{E - E_d + \frac{i}{2} \Gamma_d}$$

$$+ \sum_q \frac{\langle \omega_f^{(-)} H_{dq} \Phi_q \rangle \langle \Phi_q H_{qd} \omega_i^{(+)} \rangle}{E - E_q + \frac{i}{2} \Gamma_q} \tag{35}$$

If $\Gamma_q \gg \Gamma_q$ we obtain the intermediate-state resonance structure with a "broad" resonance superimposed upon many narrow ones. Moreover as one can see by examining the expression for Γ_q in the region of the broad resonance the widths Γ_q will be larger as expected. It is, of course, possible to re-express the second sum in terms of a corresponding K-matrix and this is, in fact, most useful for analysing the data.

It should be apparent from this discussion that a reaction formalism simply provides a framework which can be used to analyse the data and into which it is possible to insert whatever physical insight one might have with regard to the process in question. On the other hand, the required fundamental calculations if the results are to be understood on the basis of a many body Hamiltonian are also indicated.

9. REACTION THEORY AND EXPERIMENTAL DATA

We shall return to this matter later but for the present continue to discuss how reaction theory may be used to understand and collate experimental data. It is clear that generally it would not be very informative to list the width and resonance energy of each individual resonance even if this were possible. In fact, it is not possible to do this because the energy resolution rapidly becomes inadequate and, which is most important, the resonances become rapidly overlapping as the excitation energy increases. The approach of nuclear-reaction theory has been statistical, i.e. it aims at obtaining average values of the auto- and cross-correlations of the cross-sections. My comments on this subject will be restricted to two aspects, the statistical theory of nuclear reactions and the optical model.

The optical model as we mentioned earlier attempts to describe the average amplitude. From this the average prompt cross-section can be calculated. But it is still necessary to add on the average effects of the delayed amplitudes before comparison with the average cross-section becomes possible. An approximation to this contribution can be obtained from statistical reaction theory.

The complex potential matrix of the optical model can be derived directly from Eq.(19) for T. First the energy average of T over the second term is calculated. This is based on the assumption that in the domain ΔE over which the average is carried out both T_P and the wave functions $\psi_{fi}^{(\pm)}$ do not vary, which, as may be recalled, was the criterion used for the best choice of P and Q. In the second step we ask what complex potential matrix will

give this average amplitude. This can be carried out without much difficulty. We shall not give the final expression but only exhibit the final form. The original equation

$$\left(E - H_{PP} - H_{PQ} \frac{1}{E^{+} - H_{QQ}} H_{QP}\right)(P\Psi) = 0$$

is replaced by

$$(E - H_{PP} - H_{PQ}\, O_{QQ} H_{QP})\Psi_{opt} = 0 \tag{36}$$

The effect of averaging is thus to replace the propagator by some average operator: O is, of course, not Hermitian; it is slowly energy-dependent and non-local. If P contains several channels then Eq.(36) represents a set of coupled equations such as Eq.(3). It is noteworthy that both the diagonal and coupling potentials are complex. The consequent absorption arises from (a) emission into open channels not included in P, and (b) coupling to time-delaying channels. For example, the complex coupling potential between channels a and b will include, inter alia, absorption terms arising from the coupling of channel a to the time-delaying channel Q which in turn are coupled to channel b.

We have already alluded to the case for which P does not include all the open channels. This will be particularly the case at high energies where there are so very many open channels that it would be impossible to deal with each of them individually. The P space is then often restricted to the elastic channel only. In that event absorption also includes the real processes in which the departure of incident particle from the elastic channel is permanent. In the high-energy limit these real processes are expected to dominate the reaction.

The actual evaluation of the operator O_{QQ} in Eq.(36) is greatly simplified if one makes the doorway-state hypothesis of Eq.(28). One can again proceed with energy averaging working now from Eq.(29). We give the result in two forms in the case of isolated doorway states:

$$\left(E - H_{PP} - \sum_{d} \frac{\langle H_{Pd}\,\omega_d\rangle\langle\omega_d\, H_{dP}\rangle}{E - E_d + \frac{1}{2} i\Gamma_d^{\downarrow}}\right)\psi_{opt} = 0 \tag{37}$$

We see a typical dispersive-type potential. The width $\Gamma_d^{\downarrow}$ represents the "absorption" generated by coupling between the d and q channels; i.e. between the intermediate and the time-delaying channels. The width of the resonance predicted by Eq.(37) will, of course, exceed $\Gamma_d^{\downarrow}$ by roughly the width the level would have if it were not coupled to the time-delaying channels. A second representation is to rewrite Eq.(37) as a pair of coupled equations

$$\begin{aligned} (E - H_{PP})\psi_{opt} &= H_{Pd}\,\omega_d \\ (E - H_{dd} - \hat{H}_{dd})\,\omega_d &= H_{dP}\,\psi_{opt} \end{aligned} \tag{38}$$

In this equation $\hat{H}_{dd}$ is just the non-Hermitian term which arises from the averaging over the delayed amplitudes generated by the coupling to q space. If the doorway-state hypothesis is not made similar terms must be added to H_{PP}, H_{Pd} and H_{dP}. The consequent modifications are of interest and observed in the isobar analogue case.

Since the cross-sections are proportional to $|T|^2$, the optical model does not give the energy average of the cross-sections. One must also compute the fluctuation cross-section

$$\sigma_{f\ell} \sim \langle |T|^2 \rangle - |\langle T \rangle|^2$$

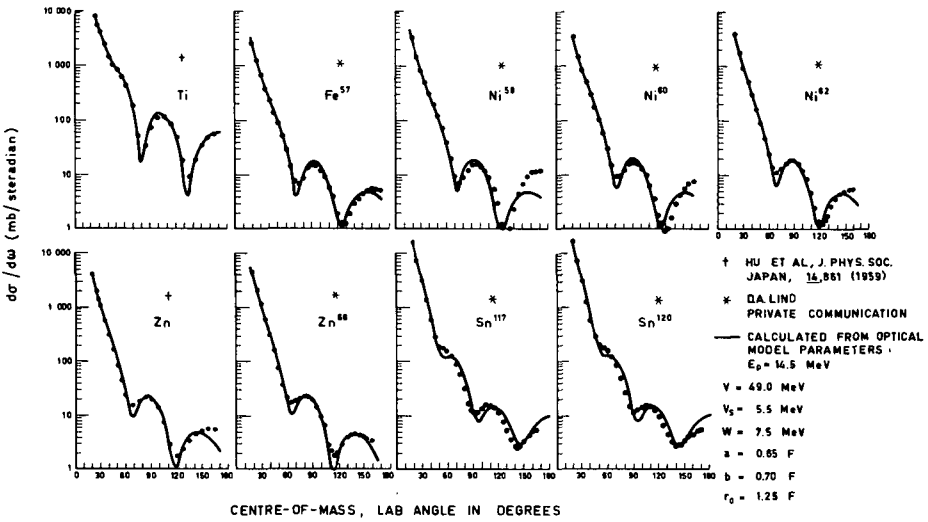

FIG. 9. Differential elastic cross-sections for 14.5 MeV protons.

The statistical theory of nuclear reactions addresses itself to this problem. It energy-averages the cross-sections using, first, the assumption that the compound nuclear level density is large and, second, the random phase approximation which states that the interference between amplitudes involving different compound nuclear states will disappear when the energy average is performed. Third, it is assumed that in the energy interval over which the average is taken the final parameters which appear, e.g. the widths and level spacing, can be considered to have been selected from appropriate probability distributions. We shall not go into the question of what these are but refer the reader to the volume by C.F. Porter for the details. Suffice it to say that once these assumptions are made a very definite prescription for the calculation of the fluctuation cross-section emerges. A still further simplification results if an energy average over the final states is made. One then obtains the evaporation-theory results. Some additional assumptions are required but require too detailed a description to make their discussion worthwhile here.

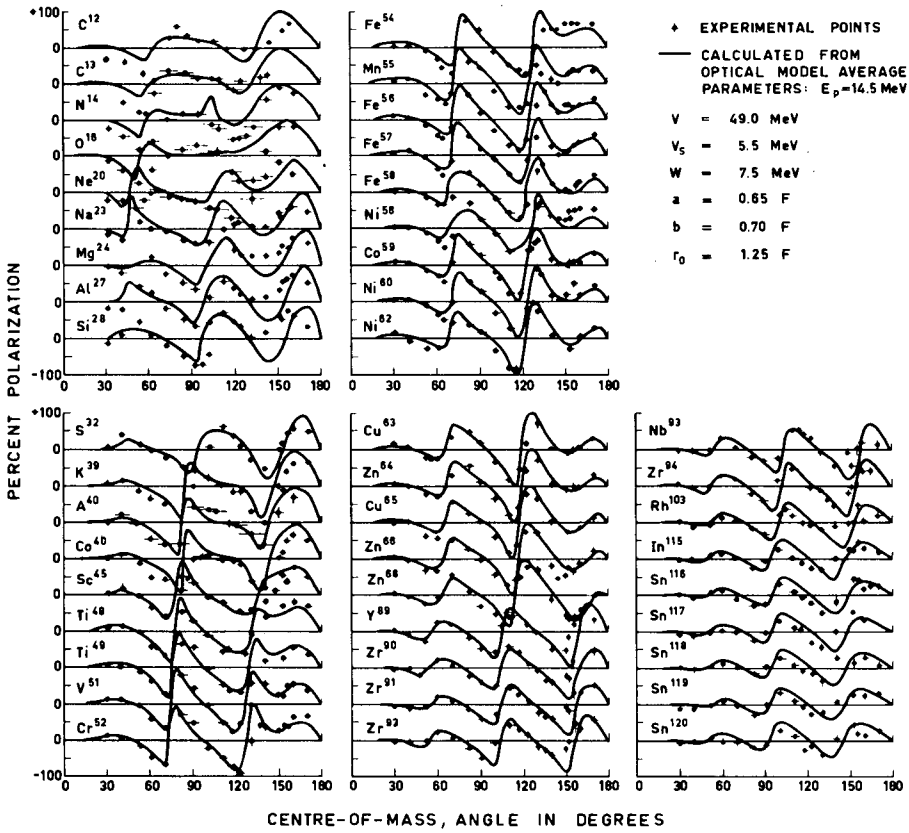

FIG. 10. Angular dependence of the polarization on the elastic scattering of 14.5 MeV protons.

These formulas have proven to be very useful as they yield reasonable values for total cross-sections and angular distributions for both elastic and inelastic scattering. Examples of optical-model fits are given in Fig. 9 and Fig. 10 in which angular distributions for the elastic scattering and polarization for protons of 14.5 MeV incident on a variety of target nuclei are shown [13]. There is no question, however, that the statistical calculations are rough and recent years have shown a more precise experimental testing of its results and a theoretical re-examination of the principal assumptions. The most suspect of these, that of randomness, is gradually being revised. It is, for example, violated in the neighbourhood of intermediate resonances. This program is still in its initial states and I shall not attempt to describe it here.

We can briefly describe the situation as follows. Prompt reactions describe quite well what happens for small inelasticities. Large inelasticities are dominated by the compound nucleus contribution to which the statistical theory gives a reasonable first estimate. The in-between region is just beginning to be investigated.

The division of the amplitude into prompt and delayed components is appropriate for "low"-energy reactions for which consideration of compound nucleus processes is essential. For higher-energy incident particles (e.g. nucleons with several hundred MeV) compound nuclear processes are not of primary concern. The more significant separation now depends upon the number of collisions the incident particle makes with the nucleons making up the target nucleus. For example, $P\Psi$ would be the elastic channel, $q_1\Psi$ would contain those states generated by a single scattering which are orthogonal to the elastic channel, $q_2\Psi$ those states generated by a double scattering orthogonal to the states in $P\Psi$ and $q_1\Psi$ and so on.

The nature of the phenomenological analysis used for high-energy data can be illustrated using elastic scattering as example. In this case the attempt is made to extract the dependence of the complex phase shift δ on the impact parameter ρ which is related to the orbital angular momentum ℓ by

$$k\rho = \ell + \tfrac{1}{2}$$

where the momentum of the incident particle in the centre of mass system is $\hbar k$. For large k, it is, in fact, a good approximation to consider ℓ as a continuous variable. The procedure used is to assume a form for $\delta(k\rho)$ and adjust the parameters of that form until the data is fitted. A typical form is illustrated in Fig. (10a). The most prominent features of elastic scattering at high energies include the forward diffraction peak and the various secondary maxima. These are, for the most part, determined by the radius of the scattering system. But outside the forward peak the simple geometrical analysis fails above a critical angle, and the cross-section will now depend not only on the radius but also on other parameters such as the "surface width".

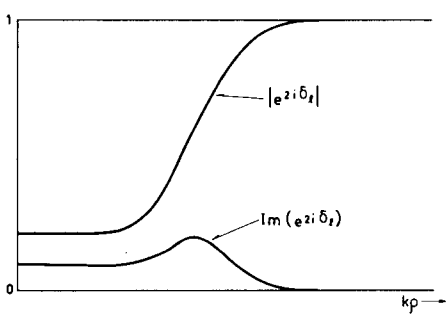

FIG. 10a. Schematic dependence of $\delta(k\rho)$.

The above analysis and indeed all of the reaction theory described earlier has been directed toward the problem of finding a theoretically motivated parametric representation of the data. We now attempt to exploit the same formalism to obtain a connection between these parameters and nucleon-nucleon forces. This is, of course, a formidable problem and we can only describe here some of the attacks on this problem which are now being made.

10. "FROZEN" APPROXIMATION

At high energies the frozen approximation becomes possible; i.e. it is assumed that the incident particle travels through the target at a velocity very much greater than the velocity of the individual nucleons making up the target so that the latter may be regarded to be at rest. The H_{PP} term corresponds to single scattering, i. e. to the situation in which the scattered wave is additively made up of the waves scattered from each individual nucleon. The second term

$$H_{PQ} \frac{1}{E^+ - H_{QQ}} H_{QP}$$

involves obviously two scatterings at least, one in which the target nucleus charges its state ($P \rightarrow Q$), and after propagation scatters back into the elastic channel. In a very abbreviated form the frozen approximation is introduced by replacing H_{QQ} by an average excitation potential and using completeness to sum over the intermediate target nucleus states. As indicated by the formulas the result is now averaged over the ground state of the target nucleus. This furnishes us with an optical potential to be inserted into the Schroedinger equation which iterates these processes. Triple scattering can be included by dividing the Q-space into those target nuclear states with one target nucleon and with two target nucleons excited. If only the first two terms are kept only the target nucleus density and the two-particle correlation functions will enter. At small angles the target nucleus density will be most significant since as we recall that peak depends principally on the size of the scattering system. The effect of correlations should become more apparent at the larger scattering angles. The extension of these techniques to reactions is straightforward and for example it can be readily seen that reactions of the sort (p, pd) are sensitive to two-body correlations both for the ground state and continuum states of the target nucleus. These high-energy calculations as, in particular, the effect of double and multiple scattering are greatly simplified if the semi-classical multiple-scattering approximation is used. We refer the reader to a recent review by Glauber [15] where this method is described and further references are given.

11. LOWER-ENERGY PROBLEMS

At lower energies this fundamental problem is more difficult. Linked cluster expansions have been obtained which yield the effective Hamiltonian of Eq.(13) for incident nucleons and more recently for composite particles incident on the target nucleus. The "vacuum" for these calculations is given by the Hartree-Fock wave functions. These expressions are perturbation expansions so that the range of their applicability may be limited. A second procedure which has had some measure of success employs the models which have been developed to describe low-lying nuclear levels. For example, one may be able to decide in advance or because of the presence of intermediate resonances that certain doorway states couple very strongly to the incident channel (or to the exit channel). These states are usually

obtained in terms of certain models and corresponding residual interactions. It is also necessary to make some assumptions as to the spreading width of the doorway states. These may be made in terms of the model or left as an empirical parameter. It then becomes possible to solve the coupled equations (Eq.(38)) and compare with experiment. For the light nuclei one may make use of the shell model. In one technique all the open channels are put in P space and the Q space is limited to a few low-lying bound states. Again using the corresponding residual potential it becomes possible to determine the eigenvalues and eigenvectors of $H_{QQ} + W_{QQ}$ (see Eq.(17)) and thereby obtain the transition matrix Eq.(19). An equivalent approach uses the target state and the basis functions of Q space to obtain a set of coupled equations. These calculations have had some success. Finally, there is the approach in which one attempts a direct shell-model solution of the (A + 1) body problem not using directly the information available on the A-particle problem. The shell model secular determinant is now replaced by an integral equation which is solved numerically. There is not much point into going into the details of each of these approaches. This type of investigation whose goal is to determine the relation between the reaction and nuclear forces is of fairly recent origin and much remains to be done before definitive results can become available.

12. DIFFICULTIES AND UNSOLVED PROBLEMS

In this review of reaction theory we have not pointed out the difficulties and unsolved problems. It is true that the proper phenomenological description of many of the presently known reaction types has been uncovered and in some cases a quantitative understanding has been achieved. But many mysteries of all sorts remain. We shall discuss a few of these.

We shall first outline some formal difficulties which are important for the calculation of reaction amplitudes directly from nucleon-nucleon scattering. The first has to do with the Pauli exclusion principle. The problem raised here can be phrased as a technical problem. If the wave function for the system is expanded in terms of the various states of the target nucleus

$$\Psi = \mathscr{A} \sum u_t(\vec{r}_0)\, \phi_t(\vec{r}_1 \ldots \vec{r}_s)$$

the coefficients $u_t(\vec{r}_0)$ are not independent. In the language of second quantization the set $a_k^\dagger|t\rangle$, where $|t\rangle$ represents a possible state of A particles and where $a_k^\dagger$ is the creation operator, does not form a linearly independent basis set. The set $a_k^\dagger|t\rangle$ is said to be overcomplete. This difficulty arises because the state obtained by exchanging a particle in the target with a particle in the projectile may be present in the above series expressed in terms of continuum wave functions of the target nucleus. The state so obtained is, of course, identical to within a phase factor to the state from which it was obtained. Hence the lack of independence. The resolution of this difficulty can be achieved if the technical problem of determining the eigenvalues of the density matrix associated with the states in P space is solved. This is readily accomplished if these states are expressible as a finite sum of Slater determinants. It is tri-

vially solved for one. However, not all nuclear models can be expressed in this way. Problems of antisymmetrization are much more difficult for incident or emerging composite projectiles. Although, in principle, they can be discussed in a manner similar to that employed for incident nucleons the mathematical difficulties are severe.

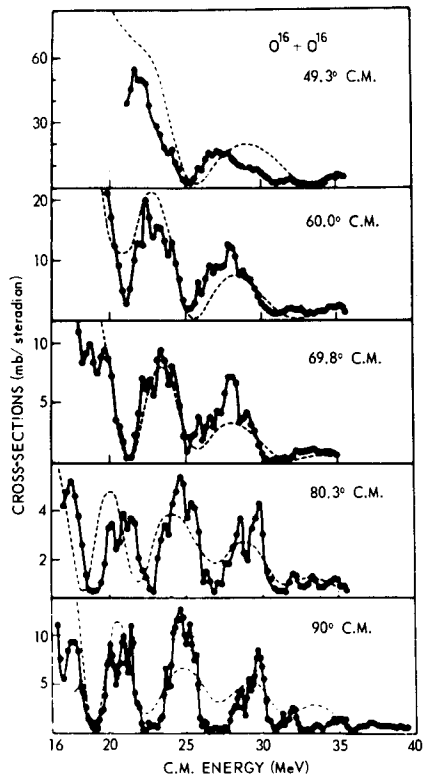

FIG. 11. $^{16}O + {}^{16}O$ elastic-scattering excitation function measured at five different angles.

A second problem is the so-called recoil problem. It is obvious that linear momentum must be conserved. In other words, the wave functions describing the target and residual nuclei must be momentum eigenstates. Unfortunately, most nuclear models such as a shell model do not provide such wave functions. Starting with a shell model, for example, it is, therefore, necessary to introduce correlations in the motion of the target nucleons so that the target centre-of-mass moves with constant momentum. These correlations must be consistent with the Pauli exclusion principle and, in fact, complicate the treatment mentioned above of the Pauli principle by introducing a non-trivial modification of the target nucleus wave function. Solving this double problem of recoil and Pauli principle has been recently accomplished in the case that the target nucleus can be described by random-phase-approximation (RPA) wave functions. The RPA permits the generation of constant momentum eigenfunctions from the Hartree-Fock wave function, i.e. it inserts the required correlations.

The effects of the Pauli principle and recoil are included only implicitly in the phenomenological treatments such as the optical model through the use of effective empirical potentials. It is in relating these potentials to the structure of the colliding systems that the above effects need to be dealt with explicitly.

Another problem which is often pointed to as a difficulty in the reaction formalism is the analysis of reactions leading to three-body (or more) final states. In the description given above this leads to no difficulty as long as the interacting three-body states are used in the description of P space. There is a difficulty, of course, as there is for any reaction formalism of determining what these three-body wave functions should be. Solving the three-body problem remains a formidable task so that we cannot expect to be using a very precise wave function. However, for many purposes such precision is not needed.

Leaving these very general formal questions we can ask how well are the reaction types understood. Has the proper phenomenological framework been found, what is being coupled to what, and how are the corresponding interaction energies best represented? A few current problems of this kind should be mentioned. The story of the collision of heavy ions is far from finished. Very distant collisions involving the effects of the electromagnetic interaction are well understood. But the closer collisions involve some new phenomena such as the resonance structure in Fig. 11. These give the elastic ^{16}O- ^{16}O scattering as a function of energy for several scattering angles. For this we do not have a very satisfactory explanation. Another recently opened area of investigation is the scattering of high-energy particles by nuclei. Although the nuclear size effects are well understood the significance of the nucleon correlations obtained from experiments is still an open question in my opinion. These are but two examples. We could easily list several dozens of such problems of varying degrees of significance and subtlety. Even when a quantitative analysis of experiment is available the understanding of the results in terms of the structure of the colliding systems involves many unresolved issues and we include here as old such subjects as stripping theory and the optical model.

However, it may be expected that we are entering a period of rapid development in this area. It is one of the strengths of modern nuclear physics that there are many ways of experimentally probing a given nucleus. Each of these gives somewhat different information, reflecting differing aspects of the nuclear wave function. The theory and model which predicts the wave function is thus severely tested. It should be understood that it is only recently that these investigative possibilities have become available, and the information they will yield are in the future. In the early history of nuclear physics, reactions were used to determine overall features of nuclear states such as energies which followed simply from the application of conservation principles to the reaction. With the growth of understanding and experimental capability it is the wave function which is under scrutiny.

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FOUNDATIONS OF QUANTUM MECHANICS AND
FUNDAMENTALS OF FIELD THEORY

EPISTEMOLOGY OF QUANTUM MECHANICS

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Abstract

EPISTEMOLOGY OF QUANTUM MECHANICS. The general consequences of quantum mechanics are discussed and the open questions are outlined, with some of the author's views.

In these Proceedings Bohm, and to some extent Bastin, will discuss the possible structures of possible new theories. The rest of us will discuss the epistemological consequences of the present quantum mechanics. I shall deal first with those consequences which are part of the body of quantum mechanics and should be contained even in textbooks. Second, I shall then outline the open questions and put my own views forward in the last section.

1. If we describe states, that is, physical situations, by a state vector, the temporal change of this state vector is uniquely determined (by the time-dependent Schrödinger equation). Thus, in terms of the state vector, quantum mechanics is a causal theory.

2. We do not experience the state vector directly in the same way as classical theory envisages that we perceive the positions of particles directly. Hence, the temporal change of the state vector has no immediate consequences. Our perceptions, which are what quantum mechanics wants to explain ultimately, are the results of observations which we make. The observations are also called measurements, and the two words are used interchangeably in such discussions as we are carrying out now. The results of the observations are not uniquely determined by the state vector; the same observation on two identical state vectors may give different results, may lead to different perceptions. Several perceptions are possible as a rule; the probabilities with which one or the other perception will occur can be calculated on the basis of the knowledge of the state vector, but only probabilities can be calculated. The outcome of a particular observation cannot be foreseen unless the probability of a single result happens to be 1.

3. The state vector of the object after the observation has been the subject of some discussion. It has been argued that the state vector after the observation cannot be given, that the object is lost as a result of the observation made thereon. This is unquestionably the case with some observations, as for instance if one observes the presence of a light quantum at a certain silver chloride crystal (contained in a photographic plate) by letting it be absorbed by that crystal. There are, however, other observations which do not destroy the object; the observation of the direction of the spin of an electron by means of the Stern-

Gerlach experiment is of this latter nature. The distinction between the two types of measurements has been made already by Pauli. One avoids many difficult problems if one considers only object-destroying measurements. However, if one disregards the object-preserving measurements, or refuses to make any statement concerning the state vector after the measurement, one gives up what appears to most of us the most important objective of quantum mechanics: to furnish probability connections between subsequent observations on the same object. It is true that one can bring objects into definite states, with definite state vectors, by means other than observations. One can then follow the changes of this state vector by means of Schrödinger's equation (or its generalizations). One can then carry out an observation on the system and ascertain that the various possible results of this observation appear with the frequency predicted by quantum mechanics measurement theory. However, if this were the only function of quantum mechanics, it would serve only as its own confirmation and not an image by which one can follow the changes of an object at least by statistical laws.

For this reason, I shall assume the existence of object-preserving observations and in fact will deal only with observations of this nature. This is a somewhat arbitrary restriction, on a somewhat controversial question.

4. There are two attitudes concerning the state vector. One is that it "represents reality" and the observations only serve to ascertain that reality. The other view holds that the state vector is only a tool for calculating the probabilities of the various possible outcomes of measurements. The reality is only the perception, the result of the measurements. Emotionally, surely, the two attitudes are different. However, no observable conclusion depends on which attitude one adopts and it is not necessary to adopt either.

5. The process of measurement is a physical process and physical theory should be able to describe it. When it does, it regards the measuring instrument, plus the object on which this carries out the measurement, as a single joint system the temporal changes of which describe the measurement. There are two differing opinions on the way the temporal changes should be considered:

(a) According to the first view "measuring instruments must be described classically".
(b) Quantum mechanics is necessary for the description of the measuring instrument also.

The quotation in (a) is from V. Fock but may represent also Bohr's view. von Neumann's discussion of the measuring process reflects the other opinion (b).

von Neumann gave a model for the measuring process. If one denotes the state vector of the initial state of the apparatus by a_0, the state in which its pointer indicates the result μ of the measurement by a_μ, and the corresponding state of the object by σ_μ, the interaction between instrument and object leads from the state $a_0 \times \sigma_\mu$ to

$$a_0 \times \sigma_\mu \rightarrow a_\mu \times \sigma_\mu \tag{1}$$

σ_μ corresponds to the state of the object for which the measurement gives the result μ with certainty. Equation (1) assumes that, if the object happens to be in such a state, its state σ_μ is not changed by the measurement. This is not a consequence of the quantum mechanical equations of motion. In fact, it is not difficult to devise interactions such that

$$a_\mu \times \sigma_\mu \rightarrow a_\mu \times \sigma_0 \tag{2}$$

i.e. that the state of the object becomes after the measurement independent of its initial state μ. However, the rest of the discussion is restricted to such interactions in which this is not the case and only interactions which result in states such as the right side of (1) will be called measurements. These are, then, object-preserving measurements of a special nature: they are measurements which do not change the state σ_μ of the object if this is a state for which the measurement will give a definite result (the result μ) with certainty. It may be well to remark, though, that, as far as can be seen, none of the conclusions which follow is altered if, instead of (1)

$$a_0 \times \sigma_\mu \rightarrow a_\mu \times f_\mu \tag{3}$$

is assumed as long as the f_μ form an orthonormal set.

6. It now follows from the linearity of the equation which describes the temporal development in quantum mechanics that if one starts with the object being in the state $\Sigma\alpha_\mu \sigma_\mu$ the final state will be

$$a_0 \times \sum \alpha_\mu \sigma_\mu \rightarrow \sum \alpha_\mu (a_\mu \times \sigma_\mu) \tag{4}$$

If the various a_μ correspond to states which have a classical (non-quantum) description, the state on the right side of (4) involves a linear combination of such states and has no classical description. This conclusion remains valid even if one considers interactions of the type of (2) or (3). This simple fact is the reason that I am unable to accept the point of view that the measuring instrument must be described by means of classical mechanics. It seems to me that this assumption is in conflict with the linearity of the quantum theoretical equations of motion. Naturally, if one is willing to alter these equations, the objection becomes invalid.

The preceding discussion is predicated on the assumption that the initial state of the apparatus, a_0, was uniquely determined. It has been recognized already by von Neumann that this is not a necessary assumption and that the apparatus' initial state may not be unique; it may be one of several states, each with a certain probability. However, as d'Espagnat will demonstrate in these Proceedings, this does not alter the conclusion we arrived at.

7. It may be well to observe here that the orthogonality of the σ_μ, in (1) or (2) or (3) can be deduced from the unitary character of the transition represented by the arrow in these equations. In fact, the right sides are orthogonal for $\mu \neq \nu$

$$(a_\mu \times \sigma_\mu , a_\nu \times \sigma_\nu) = (a_\mu , a_\nu)(\sigma_\mu \times \sigma_\nu) \tag{5}$$

and this is 0 because the states a_μ and a_ν are even macroscopically distinguishable, hence orthogonal, $(a_\mu, a_\nu) = 0$. It then follows from the unitary nature of the arrow that

$$(a_0 \times \sigma_\mu, a_0 \times \sigma_\nu) = (a_0, a_0)(\sigma_\mu \times \sigma_\nu) = (\sigma_\mu \times \sigma_\nu) = 0 \tag{6}$$

The same conclusion is arrived at if one uses, instead of (1), either (2) or (3). If one defines the quantity Q which is being measured so that it satisfies

$$Q\,\sigma_\mu = q_\mu \sigma_\mu \tag{7}$$

and labels the various states σ_μ by real q_μ, (6) guarantees the self-adjoint nature of Q.

8. Naturally, one can escape the preceding conclusions by postulating that quantum mechanics has no validity for macroscopic objects and that all measuring instruments are macroscopic. This point of view was advocated, explicitly, by Ludwig but it may well have been also part of N. Bohr's philosophy. It is not easy to discuss this suggestion, not even if one postulates a definite theory, the classical one, for macroscopic bodies. One reason is that the dividing line between macroscopic and microscopic bodies is unsharp, whereas the separation between quantum and classical theories, with the infinitely greater variety of possible states in quantum theory, is sharp. It is difficult to discuss it also because there are so many quantum phenomena – superconductivity, laser action, among them – which have a quantum origin yet are manifested by macroscopic systems. However, since the present discourse is intended to deal with the epistemology of quantum mechanics, it can forego this discussion.

9. If we return to Eq. (4), we see that the quantum mechanical description of observation only describes the establishment of a statistical correlation between apparatus and object. This correlation consists in the appearance of terms $a_\mu \times \sigma_\mu$ only, i.e. the absence of $a_\mu \times \sigma_\nu$ terms in (4), with $\mu \neq \nu$. If we adopt the usual theory of measurement to observations on the joint system of apparatus plus object – the theory which we discussed for the measurements on the object alone – it follows that the renewed observation of the σ_μ system on the object, coupled with an observation of the a_μ system on the apparatus, always gives concordant results: if the apparatus is found in the state a_μ, the object will be found to be in the state σ_μ (and the probability of this is $|\alpha_\mu|^2$) so that we can obtain the state of the object by ascertaining in which of the states a_μ the apparatus is, i.e. by reading the pointer position of the apparatus.

On the other hand, the quantum description implied by the arrow in (4) does not go further than the establishment of a statistical correlation. If a measurement is now undertaken on the apparatus, to read its pointer position – and one describes this second measurement in a similar way – one merely obtains a state

$$\sum \alpha_\mu (b_\mu \times a_\mu \times \sigma_\mu) \tag{8}$$

where b_μ refers to the second measuring instrument. There is now a correlation between the states of the two apparati and of the original object.

The procedure can be continued by introducing a third apparatus to measure the state of the second, and so on. However, the resulting chain does not alter the fundamental fact: the quantum mechanical equations do not give a description of the completion of the process of observation from which a single one of the terms $a_\mu \times \sigma_\mu$ (or $b_\mu \times a_\mu \times \sigma_\mu$) emerges. This should not be surprising since, as was pointed out at the outset, the temporal development described by quantum mechanics is deterministic, the result of the measuring process subject to statistical laws, i.e. not deterministic. None of this argument is new and all of it is given, for example, in von Neumann's book.

In order to complete the measurement, it is necessary to assume that the observer has, in the words of London and Bauer, an immediate knowledge of the state of the apparatus, that is he directly perceives the state a_μ of the apparatus of (4), or the state b_μ of the second apparatus of (8), or of a subsequent apparatus. If he perceives a_μ, the future behaviour of the state of the object is as if its present state vector were σ_μ, if he perceives b_μ, the future behaviour of object and first apparatus is as if their state vector were $a_\mu \times \sigma_\mu$. These facts are often described more pictorially by the statements that the state vectors "collapse" to σ_μ in the first, to $a_\mu \times \sigma_\mu$ in the second case. Many physicists do not like to accept the postulate that the state vector changes in two ways: in the course of time, continuously, according to the Schrödinger equation, and as a result of observations, abruptly. It is possible to avoid this by considering the state vector to be only a calculational tool, a tool for calculating the probability that an observation which one may undertake will give one result or another (one of the μ of (4) or (8), or another). The two underlying points of view were discussed already in Section 4; there is no difference between their conclusions. The second is, in some ways, more honest and less artificial; it is easier to think in terms of the first one. von Neumann used the first one.

10. We now come to the discussion of the difficulties of the epistemology of quantum mechanics which was presented in the preceding sections. The most obvious of these is that, particularly if the second of the points of view just discussed is accepted, quantum mechanics does not describe reality but only gives statistical correlations between subsequent observations. It does not even commit itself to a "reality" that may be behind these observations in the way classical theory implies this. I am afraid this cannot be helped and only the future will tell us whether our fellow physicists will get used to this as they got used to the idea that there is no absolute time. The jump is much greater.

11. A few more mathematical questions arise. The discussion of these which follows represents, in part, my own views.

The first question which naturally arises is whether it is reasonable to assume an interaction such as implied by (1) or (4). More concretely, for what sets of σ_μ is it reasonable to assume that an instrument can be found, in a proper state a_0, such that the interaction with the object produces what is implied by the arrow? This question was analysed, for

most sets σ_μ, by Araki and Yanase, who demonstrated that the instrument must be, in a definite sense fully explained by them, large. Even apart from their point, it appears that the assumption of the existence of an apparatus to implement the arrow in (1) or (4) implies, except for certain sets σ_μ, a good deal of wishful thinking – perhaps as much as exhibited by biologists.

One may inquire, second, whether the picture of observations giving information about the outside world, quantum theory providing a link between these observations, is an adequate picture of our acquiring and using information. The answer must be, surely, in the negative even if we consider observations of a macroscopic nature in the sense that they are transmitted by many quanta. One reason for the inadequacy of the picture is that it does not explain how we acquired knowledge concerning the nature of the various apparati we use. The answer of the quantum theorist to the question how we know which apparatus measures what is that we undertook observations in order to ascertain the nature of the apparati we use. However, this only starts another chain because the observation which provided us with information about the nature of one apparatus had to use another apparatus and we had to have information on the nature of this second apparatus. This leads to a chain similar to that encountered in the preceding section, which has an end as little as that chain had, without being resolvable, however, by an epistemological renunciation. More generally, the picture of subsequent impressions, with statistical relations between them, is an almost ridiculously simplified picture of our consciousness. It assumes that we are simple automatons – storing and registering information – an oversimplification of a fantastic crudity.

A third question which appears well worth mentioning concerns the reason for all our direct observations to be of macroscopic nature in the sense that they give positions. Quantum mechanics does not give preference in any obvious way to observations of position co-ordinates, not even to those of macroscopic bodies. It is not clear, therefore, why the chain of observations described in Section 9 (the second element of which is illustrated by (8)) has to go through the observation of a position operator. C. Townes has, in a private conversation, revived this problem which is, to my mind, closely connected with the problem last discussed and which probably will not be resolved without resolving that more general problem.

12. Much, if not most, of our knowledge is not acquired directly but is communicated to us by others and many, if not most, results of observations are also acquired second hand. How can such observations be fitted into the general picture sketched in Section 9? The answer of the fully consistent quantum theorist is that a person from whom we obtain information is simply a link in the chain described in Section 9. His observation establishes a statistical correlation between his own state and the state of the object observed (directly or through a chain) and his communication to us is simply an observation of ours (again presumably through a chain) which informs us of his state. The correlation between this and the state of the object then permits us to infer the state of the latter.

There is no logical flaw in the argument of the "consistent quantum theorist". Nevertheless, his conclusion can hardly be accepted. The observer who has made a direct observation on the system is a person similar to myself, with a consciousness just like my own, and he has made a choice of the various $\ldots \times b_\mu \times a_\mu \times \sigma_\mu$ (which appear in (8) or its generalizations) just as I would make one. The assumption that his state was, before he made his communications to me, a linear superposition of various observation results is not credible. I am therefore led to the view that the state of the one who made the communication to me cannot be described by the usual description of quantum mechanics, that this theory is not adequate for the description of life, including consciousness.

This last conclusion should not be a surprising one. Whenever science was extended to new sets of phenomena, it had to introduce new concepts, new descriptions, new ideas. Life and consciousness certainly constitute a new area, divorced from the area of phenomena with which physics is dealing now.

The question naturally arises whether there is a significant difference between the point of view just sketched, which denies the validity of quantum mechanics for the phenomena of life and consciousness, and the point of view mentioned in Section 8, which denies it for all macroscopic bodies. At first sight, the difference appears to be small. In fact, it may be quite large. The point of view mentioned in Section 8 suggests a return to classical theory for all macroscopic bodies, the point of view of the present section suggests the development of a new theory of life. Such a new theory, if it can be developed, will have almost surely profound effects on our epistemology. However, as far as the present state of physics, and particularly of quantum mechanics, is concerned, it is my belief that the epistemology sketched in the first part of this discussion expresses their spirit in a natural way.

"HIDDEN VARIABLE" THEORIES AS A STEP TOWARDS A NEW LANGUAGE STRUCTURE FOR PHYSICS

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Abstract

"HIDDEN VARIABLE" THEORIES AS A STEP TOWARDS A NEW LANGUAGE STRUCTURE FOR PHYSICS. The author stresses the need for considering new language structures in an attempt to penetrate new domains.

In this paper, I would like to emphasize the importance of theories, not only as a source of inferences about results of experiments, but also as a means of description, or in short, as a part of the language of science.

One of the first significant points about any particular language structure is that in it (either tacitly or explicity), certain combinations of words and sentences are ruled to be relevant to the subject under discussion (and therefore meaningful in the context of this subject), while others are ruled to be irrelevant to this subject (and therefore meaningless within this context). It has perhaps not been emphasized sufficiently that physical theories contain such relevance rulings, and that indeed to change these rulings is to change the language structure in a basic way. Thus, Newtonian mechanics rules absolute time and space to be relevant, while Einstein's relativity theory rules them to be irrelevant. Classical physics rules well-defined orbits to be relevant, while quantum physics rules them to be irrelevant.

What is most characteristic of the mathematical aspects of the "quantum" language is the feature of linear superposition (of wave functions or of vectors in Hilbert space). Along with all the other features of the "quantum language" this implies that in a series of measures of non-commuting "observables" (e.g. measurements of the "spin" in a series of different directions), the statistical distribution of the results of the $(n+1)^{st}$ measurement is independent of the results of the first n measurements. This feature may be called "the requirement of complete randomness". It means that quantum mechanics rules out the order of successive measurements of this kind as not being relevant to the laws of physics. It has to be emphasized, that this ruling is essential to the consistency of the quantum theory. Indeed, without it, we would not have quantum theory, but rather, an incoherent set of mathematical and physical statements.

Now, there could be no possible objection to quantum theory, if all it did were to make rulings of this kind which were regarded as valid in some similar domain. However, it has been very widely agreed that quantum theory should provide an in principle complete description of all that there is to be described. This means for example that linear

superposition and "the requirement of complete randomness" in successive measurements of non-commuting observables ought to hold, no matter what field of human knowledge one may be considering.

What I want to question here is whether the notion of such "completeness" is actually relevant, to any theory, physical or otherwise. Rather, I would like to suggest another view, i.e. that all theories are inherently incomplete, in the sense that they have limited domains, areas, or fields, in which they are relevant. It is only as a result of certain fortuitous features of its historical development that physics came to be regarded as a "fundamental" subject that in principle covered the totality of all that exists or that could possibly exist. But in my view, experience is showing with greater and greater force that physics is not different in this respect from all other forms of human knowledge. Thus, it is "open ended" rather than built out of a closed language structure, which would in principle provide a complete description of everything.

Professor Wigner has, in a way, tacitly agreed with the above point of view, by suggesting that quantum theory is not relevant to the description of what is essential to life (and especially to intelligent and sentient beings). But then, I am led to ask: "Why assume that quantum theory is relevant to the totality of all that is not alive?" Does it not seem arbitrary to make such a sharp distinction between life and non-life, when life evolves from inanimate matter, is nourished by inanimate matter, and returns to inanimate matter (in death)?

I have the impression that the full effects of the assumption of completeness are not generally realized. Firstly, let me say that it is only the desire for such completeness that brings about the "measurement problem" in the first place. One wishes to "complete" the description given by Schrödinger's equation (or by operators), so as to describe even the "process of measurement" in terms of them. But Bohr, who did the original work in this field, explicitly asserted that this could not be done in a consistent way and that the notion of doing it should therefore not even be entertained [1,2]. Rather, he emphasized the unanalysable wholeness of the description of the experimental conditions and of the results of our observations. This means that there is no "observed system" that is distinct from the "measuring apparatus". Nor is there a "quantum state", separate from the total conditions of observation. That is to say, the notion that we observe is one thing, while the "wave function" or the "quantum state" is something else that exists separately (as if it were some kind of "object"), is in Bohr's view not compatible with what is meant by the term "quantum". Rather, the two interpenetrate, in much the same way as do (for example) a radiowave and the signal with which it is modulated. To suppose that the "quantum state" is some kind of "objective reality" distinct from our description of the whole set of conditions of observation and results of observation is, in Bohr's view, something like saying that the modulation of a radiowave has a separate existence, distinct from that of the radiowave itself.

Much of the discussion this afternoon has been based on the notion that the "quantum state" is something separate and distinct from whatever it is that may be directly observable or measurable at the macroscopic level. This notion then leads, of course, to the question of how we can "know" this "quantum state". So there arises a "problem of measurement" which contains such unfathomable and undecidable questions

as to whether the mind of man, which has the quantum theory as part of the contents of its thoughts, is also a "dynamical system" that follows the very theory that is part of its contents. And if, with Professor Wigner, we suggest that quantum theory is probably irrelevant to the mind of man (and perhaps to certain aspects of his biological function as well), then we are led to the insoluble problem of how life and intelligent perception, which is now supposed to be so utterly different from "quantum theoretical systems", can nevertheless be in continual and mutual interaction with them. Will the attempt to couple two such incommensurable kinds of "systems" not lead to all sorts of incoherence, especially in the "problem of measurement" where this interaction is crucial?

In Bohr's view, all these questions would have to be ruled out as irrelevant and meaningless (as would, of course, a great deal of what appears in most quantum theory text books). Indeed, Bohr's language has no place in it at all for a "measurement problem".

In addition to the school that more or less follows von Neumann (and thus rejects Bohr) in the assumption of a separation between "the measurement process" and "the quantum state whose properties are measured" (or equivalently between "classical" and "quantum" observables), there are yet other workers who assert their adherence to Bohr's point of view, but still accept as relevant the problem of trying to describe the "process of measurement" in terms of such a separation. So, all in all, it seems that there is a great deal of unclarity in the foundations of quantum theory. Not only do different schools disagree in rather fundamental ways, but in Bohr's language it is impossible and meaningless even to talk about what the other schools are trying to do "to solve the measurement problem". So no contact is possible between Bohr and these schools. Nor is it clear how Bohr could meaningfully discuss with those who express their acceptance of his views, but also accept the relevance of the "measurement problem".

Of course, most physicists do not regard all these difficulties as being very serious, because there is a widespread belief that the "essence" of the theory is in the mathematical formalism, which has, after all, successfully treated a wide range of experimentally observed phenomena. In my view, however, this view neglects the fact that our thinking is one indivisible whole, in which informal or "philosophical" assumptions are inseparably intertwined with assumptions about the formalism. Confusion in the informal assumptions must ultimately lead to a corresponding confusion in inferences from the formal parts of the theory. In domains that are familiar, one can become skilled at "adapting" his reasoning, so as to avoid the effects of such confusion. But as physics enters new dimensions, one no longer knows how to do this. And (as I have found by discussions at this Symposium and in other places), a great many physicists agree that physics is now facing a kind of "crisis", suggesting that current fields of enquiry are approaching exhaustion of their possibilities, so that there is a need to enter new domains. I suggest that confusion in our basic informal premises concerning the quantum theory may be "mixing us up" so much that we are unable to enter these new domains, without getting "lost" in the first few steps. So it may well be useful to look at these basic premises in a new light.

In my view, one of the reasons why the basic premises of quantum theory are not clear is in its treatment of the question of necessity and

contingency. Now, what is necessary is that which cannot be otherwise. What is contingent "just happens" to be as it is, but could be otherwise. Each field that is abstracted for observation or enquiry contains both necessity and contingency. Thus, in classical physics, the laws of motion are taken to be necessities, while the boundary conditions are taken to be contingencies. But of course, what were contingencies in one field reveal themselves as necessary in another. Thus, in fields broader than those of dynamics, the initial conditions are seen as necessities, but in turn, these fields have their own kinds of contingencies (e.g. the initial conditions of a dynamical system may be made necessary by a human being, who set them, but in turn, his decision depends on further contingencies, etc.).

It is very common for contingencies to be subjected to a statistical distribution. Thus, the relative frequencies of automobile accidents are described in insurance statistics, and with the aid of the theory of probability, one can draw inferences about rates of similar accidents in the future. In this limited field of description, each accident is a contingency. But in a broader field, it reveals itself as a necessity. Thus, in an individual car, it may be the result of failure of mechanical parts, or of inattention by the driver, or of other causes. Each of these causes is itself a contingency, dependent on further causes in yet broader fields. And so it goes without limit.

The notion of necessity and contingency outlined above implies that every description is inherently incomplete, because what happens in the field to which it applies is contingent on what is outside the field in question. But, of course, the assumption of the completeness of description afforded by the quantum theory denies such a notion of necessity and contingency. Rather, it implies what may be called "absolute contingency", i.e. that the results of successive measurements follow on each other in a "completely random" way, so that their contingent order of "fluctuation" could never be revealed as a new kind of necessity, operating in a broader field. This leads to a kind of notion of "complete disorder" in results of successive measurements, which are nevertheless assumed to be "ordered statistically" in some way according to the laws of probability.

In my view, this idea of "order of disorder" is an inherently unclear one. Indeed, attempts to define "disorder" have failed, just because every description is some kind of order, and therefore denies what is intended to be meant by the word "disorder". Thus, the word "disorder" can have no meaning at all. What is usually called a "random" sequence has some kind of order, but one that is not relevant to the laws of the limited field that is under investigation. Nevertheless, it is relevant in a broader field where it is revealed as necessary (i.e. given by law) rather than contingent. In this way, one comes to a more clear formulation of what is to be meant by "randomness". Without some step of this general nature, there is always room for inherent incoherence in our formulations of physical law, which incoherence can (as indicated earlier) become relevant in the new domains that we probably have to enter, to resolve the present state of "crisis" in physics.

So one of the main criticisms of the quantum theory is that it has incoporated a language structure in which the notion of the irreparably interwoven character of contingency and necessity is, in effect, ruled out as irrelevant and meaningless. It seemed to me, therefore, that it

could be useful to develop a new form of theory, in which the notion would be relevant. But, of course, this requires that instead, the notion of completeness of description be ruled out as irrelevant and meaningless.

Recently, Bub and I developed a theory of "hidden variables" which serves to illustrate the relevance of contingency and necessity in that domain which is now treated in terms of the quantum theory [3]. This was not intended as any kind of definitive theory, but rather as an example, which could help to bring out the key points in the discussion in a relatively specific way.

Our first step was to define what could be called the "non-hidden" variables; i.e. parameters that can be regarded as directly manifest in observation. Since an observation is in the current quantum theory said to determine the "quantum state" (i.e. the wave function), we took the directly manifest parameters to be the coefficients, c_r, in our orthonormal expression of the wave function

$$\psi = \sum_r c_r \psi_r \tag{1}$$

The "hidden variables" were taken to be the coefficients in an orthonormal expression of a <u>dual</u> function

$$\phi = \sum_r b_r \phi_r \tag{2}$$

It is necessary to emphasize that ψ and ϕ are <u>different</u> in certain essential ways. First of all, they are in different Hilbert spaces. Then, whereas ψ will satisfy certain equations that can be described within the context of the theory, ϕ will not. That is to say, ϕ will represent what is <u>contingent</u> in this context. For this reason, we shall call the ϕ <u>contingent parameters</u> (this is a better name than "hidden variables" which arose historically to suggest that something is "covering them up", whereas in fact, as will be seen, that which "hides" them may well be nothing but our failure to pay adequate attention to them).

With regard to ψ, we propose that it satisfies an equation similar to Schrödinger's in certain ways, but different in others. The differences from Schrödinger's equation are however crucial, because, as will be seen, they imply a new language structure in our theory, different from that of the current quantum theory.

The equation that we suggest is

$$\frac{\partial c_r}{\partial t} = \frac{i}{\hbar} \sum_s H_{rs}\, c_s + \lambda\, c_r \sum_s J_s \left(\frac{J_r}{K_1} - \frac{J_s}{K_s} \right) \tag{3}$$

where

$$J_r = c_r^* c_r, \quad J_s = b_s^* b_s$$

Evidently the first term on the right-hand side of the above represents the usual Schrödinger equation. The second is, however, <u>non-linear</u> (through the term $J_r = c_r^* c_r$) and dependent on the contingencies b_r. In addition, it depends on the parameter λ. The precise form of λ (as

a function of time) is not significant for the questions that we are discussing here. Only its general behaviour is relevant.

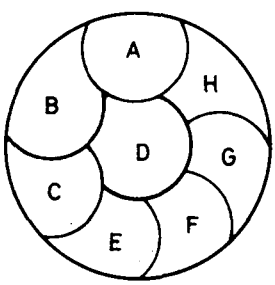
Our proposal is that in a certain way λ takes into account what is outside the context under discussion (or what can be called a kind of "general background" or sometimes "the overall environment"). In this regard, our view is similar to that of Mach, who suggested that the inertial properties of objects reflect a general context (or "background") which includes even the distant stars. We are proposing, however, that all "properties" (and not merely the "inertia") may reflect the general context (or "background") in a similar way. But in addition, we suggest that this dependence on context need not be restricted to the distant stars. Rather, there may be a dependence of each "property" on any feature of the general context. And in particular, we propose that this context may include what is usually called "measuring apparatus".

Where the context is such that in the language of current quantum theory a "measurement" is said to be taking place, λ will be large. On the other hand, between such "measurements" λ will be negligible, so that Schrödinger's equation is satisfied. So what is, in the language of current theory, described as a "measurement process" is now replaced by the description: λ is large, for a certain period of time, in which the directly manifest parameters, c_r, are coupled to the contingent parameters c_s.

Thus, in our language, the term "measurement" is not relevant. What has been called a "measurement process" in the older language is, in our language, only a particular case of the general law for the determination of the movement of the c_r, in which λ happens to be large. Indeed, in our language, we are free to consider more general situations in which λ might be large, in some naturally occurring situation, and not merely in situations created by man (e.g. laboratory experiments).

So there is no occasion to single out "measurements" as being basically different from anything else. For this reason, there is no special "measurement problem" in our theory. In this respect, our point of view is similar to that of Bohr, in which there is also no "problem" of measurement (though, of course, in other respects there are crucial differences between the two points of view).

When Eq.(3) is solved, it is found that the c_r vector in Hilbert space goes into one out of a set of limiting possibilities, depending on the region in which the c_r vector was initially. If we describe all the possibilities symbolically by a circle, then each of the regions indicated in the diagram goes into a corresponding final vector, in which $|c_1|^2 = 1$ for some particular value, $r = s$, and $|c_r|^2 = 0$ for all $r \neq s$.



In other words, the c_r vector is, in effect, "projected" into one of the axes of Hilbert space.

Consider, for example, the case of "spin", for which there are two axes in Hilbert space. Thus, we have c_1, c_2 and b_1, b_2 for the directly manifest parameters and the contingent parameters respectively. When the corresponding equations are solved, we find that

$$\text{(A)} \quad \text{if } \frac{J_1}{K_1} > \frac{J_2}{K_2}, \text{ then } |c_1|^2 \to 1 \text{ and } |c_2|^2 \to 0$$

while

$$\text{(B)} \quad \text{if } \frac{J_2}{K_2} > \frac{J_1}{K_1}, \text{ then } |c_2|^2 \to 1 \text{ and } |c_1|^2 \to 0$$

These two possibilities cover all the initial conditions (except for a set of measure zero which can be disregarded in this treatment).

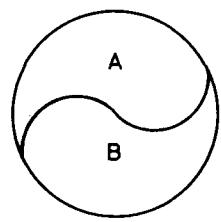
If we assume a uniform probability of the contingent parameters, b_r, on a "shell" of Hilbert space, $|b_1|^2 + |b_2|^2 = 1$ then we find by straightforward integration of the measure corresponding to each possible result that

$$P_A = |c_1|^2 \text{ and } P_B = |c_2|^2$$

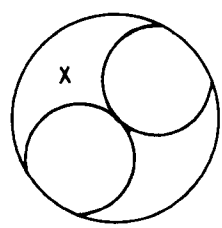
This is the usual quantum theoretical formula for probabilities.

But, of course, if the probability distribution of the contingent parameters is not uniform, we can obtain new probabilities of results, not agreeing with those given by the quantum theory. And in particular, a delta function distribution is, in principle, possible, that determines every result uniquely.

How can we obtain distributions that differ from the "normal" uniform one? This can be done with the aid of a series of "measurements" of what in the language of current quantum theory are called "non-commuting observables". Consider, for example, the "measurement" of spin in a series of different directions. The first "measurement" divides up the <u>contingent</u> parameters, as symbolized in the following diagram:



Suppose the result, A, has been obtained. Then the contingent parameters no longer have a uniform distribution, but instead are restricted to the region corresponding to B. The next "measurement" then divides up the contingent parameters again as follows:



with the result, for example, that the contingent parameters are restricted to the region X.

By means of a series of such "measurements" the contingent variables can be so restricted that their distribution approximates a delta function. Thus, they cease to be unknown (or "hidden") and indeed, they now reveal themselves in new statistical distributions of the directly manifest results. Therefore, the $(n + 1)^{st}$ result no longer has a "completely random" relationship to the results that came before. That is to say, the order of successive results is now physically relevant, whereas the usual quantum mechanical language requires that, as has been seen, this order has no meaning for the laws of physics.

One may ask why such orders have not yet been observed. One possible answer is that the contingent parameters, b_r, undergo a stochastic process, with a relaxation time, τ, that is shorter then any interval that has thus far been investigated in actual physical observations. So in all observations to date, there has been a "uniform" probability distribution of the contingent parameters, leading to a distribution of results in agreement with that given by the current quantum theory. But if measurements are made involving shorter intervals of time, then something new may be discovered, i.e. that the order of results is physically relevant, and not "completely random", as required by the current quantum theory.

When one goes into the actual experimental details, one discovers that existing experiments that might be significant for this question have involved rather long intervals of time between successive "measurements". Recently, Papaliolios [4] has studied somewhat shorter intervals, in observations of the polarization properties of light, obtaining agreement with the usual quantum theoretical results. But aside from the fact that the intervals he studied were still fairly long, there is also the question of whether a photon is not to be regarded as inherently a part, of a field, with an infinite number of degrees of freedom. (As yet, we have not developed a theory for systems that cannot be described as having some finite number of degrees of freedom.) So for these reasons, it is not clear whether the test has yet been relevant to the questions raised by the theory.

In addition, of course, there could be entirely different ways in which the contingent parameters, b_s, could depend on what was outside the context of the theory (i.e. different from a stochastic process). Some of these are being investigated now. And, moreover, there is a wide range of further kinds of experiments, some of which are also now being considered, which could also provide tests of our theory.

At this stage, it would therefore be premature to try to decide whether observations tend to confirm our theory or not. In any case, as indicated elsewhere in this talk, it was not our main intention to produce a more or less well defined theory of this kind. Rather, we wished mainly to use the theory as a means of giving some relatively definite content to the notion that physical phenomena may require a new language for their description.

The language that has been proposed here contains several novel features. Firstly, as has been seen, whatever may be the field covered in our theoretical discourse, we now recognize the inherent incompleteness of what can be discussed in terms of this field. Thus, the parameters

c_r, that are directly manifest in the field discussed in our theory, depend on contingent parameters outside this field, as well as on the coefficient, λ, which in some way takes into account the general context. In addition, we leave the theory open, to allow for further dependence on contingencies and on the general context in ways that can be specified in more detail on the theory as developed. So we regard our descriptions as "open-ended", rather than as "closed" and in principle "complete".

An important part in making possible such an "open-ended" description was played by the recognition that each field depends on contingencies, which can reveal themselves as necessities, in broader fields. Thus, as has been seen, the contingent variables, b_s, can be seen, in the broader field of a series of "measurements", to follow a necessary order. This order cannot even be described in terms of the usual quantum theoretical language.

I would like to emphasize the importance of new language structures for helping us to see what it means to test the basic principles of currently accepted theories. For example, Professor Wigner has conceded that testing for the validity of quantum statistical distributions in a series of spin measurements would be worth doing. But in making this concession, he has by implication suggested that the need for such a test is obvious and that no new language structures are required to help carry it out. The difficulty with this proposal is that without a new language structure, there is no way to know what is a relevant kind of test. Thus, for example, if one just decides to test for the validity of quantum statistical predictions in successive measurements, the conditions of the test will be largely fortuitous. On the other hand, particular examples of theories with new language structures can suggest relevant tests. Thus, the proposal that there may be a stochastic process with relaxation time, τ, emphasizes the importance of making "measurements" that follow on each other in very short time intervals. Other proposals for the contingent variables (which are now under investigation) lead to yet other indications of relevant tests. To try to test the basic principles of the quantum theory without the possession of alternative language structure could therefore be very misleading. Indeed, one may get "agreement" in some fortuitously chosen context, and take this to be a relevant test for such principles. In this way, one could be "trapped" all the more firmly into believing in the "complete applicability" of the "quantum" language, because apparently it has even been "tested experimentally".

The notion that different language structures should be considered before there is strong experimental evidence suggesting that they are needed is not really new. Thus, Einstein proposed general relativity to a large extent on such a basis (i.e. that it gave a more coherent account of certain physical phenomena, i.e. gravitation and inertia, than did previous descriptions). Such new theoretical structures of ideas then suggested relevant experimental tests, to indicate the limits of applicability of older ideas. Similarly, the theory of "contingent parameters" provides, in my view, a more coherent account of the role of contingency and necessity than does the current quantum theory. In doing this, it avoids a number of obscure and unclear features, which reveal themselves in current theory in the "measurement problem".

Of course, I do not wish to insist that such an approach is the only valid one. It is also important to keep in mind the contradictory features

of current theories and the various unsolved problems in them (e.g. infinities in field theory, difficulties of generalization of general relativity, explanations of symmetries and other properties of particles, etc.). Indeed, work along these lines is also being done.

But I would like to emphasize the need for considering new language structures (not necessarily those proposed here) in an attempt to penetrate into new domains. The old language structure limits us in all sorts of ways, most of them necessarily tacit and unknown. The best means of becoming aware of such limitations is to try out new structures, in a fairly free manner, being guided largely by the need to avoid insoluble "problems" and persistently unclear features of the formulation of current theories. As happened in previous periods of fundamental change in science, a basic new step is often made possible by the perception of the irrelevance of certain aspects of the old language, which perception leaves room for our thinking to move in new ways, and to express itself in terms of new language structures.

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REMARKS ON THE NOTION OF CLASSICAL SYSTEMS

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Abstract

REMARKS ON THE NOTION OF CLASSICAL SYSTEMS. The author discusses the theory of measurement in quantum physics and indicates that general quantum laws are not as universal as one might think.

Let S be a quantum system, A a measuring instrument. In the simplified description in which both systems are initially described by kets it is, as we know, in general impossible to ascribe a definite state vector to each system after the interaction has taken place and this leads to the major difficulty of the measurement theory that sometimes, at least, the instrument A does not seem to fall into a definite state. A more realistic procedure is however to describe A initially with the help of a density matrix which is not necessarily a projection operator. An even more realistic approach should also take into account the fact that a measurement may – and in general does – induce a perturbation on the measured system. Finally it should be observed that, after the interaction has taken place, the pointer of the instrument need not be rigorously in a state that has a non-zero amplitude only inside one definite graduation interval: a state for which the amplitude corresponding to one such interval is very large and all the other amplitudes are non-zero but extremely small is of course also acceptable.

It might a priori be hoped that, when all these three remarks are taken into consideration the essential difficulty of the measurement theory can be overcome. It has been shown however that within the framework of ordinary quantum mechanics (a large but finite number of degrees of freedom and no superselection rules) this is in fact not possible [1, 2].

One possible thread of arguments leading to this result may be very briefly sketched as follows. Let $\psi_1 \ldots \psi_n \ldots$ be the eigenfunctions of the quantity L to be measured on S and let m_0^A be the density matrix describing A before the interaction. Let $F_0^{(q)}$ be the eigenfunctions of m_0^A. It is then legitimate for our purpose to describe the initial state of the compound system S+A as a component of a mixture of states

$$\psi F_0^{(q)} \tag{1}$$

where

$$\psi = \sum a_n \psi_n \tag{2}$$

describes the initial state of the system S. After the interaction has taken place that mixture is described by the most general state vectors that can

be obtained by transforming expression (1) with the help of the time-evolution operator (account being taken of the fact that A is well suited for measuring L), or alternatively by the corresponding density matrix M_f. It can then easily be shown [1] that if ψ has the general form of Eq. (2) no ensemble L exists which corresponds to the pointer lying, for each individual instrument, in one definite graduation interval and which, at the same time, has M_f as its density matrix. Moreover this impossibility remains when, instead of requiring that the pointer of each individual instrument should lie rigorously in one interval only, one simply requires that the amplitude for it to be found in one interval should always dominate all the other amplitudes, i.e. that the pointer should never be with comparable amplitudes in a quantum superposition of macroscopically distinct states.

This latter condition however is but a special case of the general statement that macroscopic objects cannot occupy several macroscopically distinct positions at the same time. Now in our usual philosophy about Nature this statement is considered as bearing on the objects themselves and not on our abilities at getting informations about them. It then follows that if the statement is true all its consequences should be true that have a physical meaning (i.e. that are in principle observable), and not merely those that can be tested in practice. To stress even more the point at issue, this is because, in the "common sense" opinion discussed here, such practical limitations in our abilities have nothing to do with the fact – much prior to them – that macroscopic objects occupy macroscopically defined positions. Now in our elementary case (no superselection rule, a finite number of degrees of freedom) two mixtures that would have different density matrix could in principle be distinguished. We are thus necessarily led to require that the density matrix M_f obtained by applying the time-evolution operator should indeed be identical with a density matrix constructed with the help of states of the pointer satisfying the condition described above. Since this, as already mentioned, is not possible, we must conclude that the concept of "classical system" usually introduced at this stage of the theory either defines an empty class or is partly subjective, contrary to common feeling.

This difficulty of quantum mechanics is of a quite special nature since it is not related to any conflict with experiment. It is non-existent therefore for a mind whose only purpose is to predict future observations from a knowledge of previous ones. It is real however for anybody who wants his scientific notions not to come into conflict with his other ideas, in particular when among the latter is the common and useful one that macroscopic objects always have definite positions in space, at least approximately. We think it constitutes a definite indication that this opinion – concerning macroscopic objects – is false and/or that the general quantum laws are not as universal as one thought.

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THE CONTINUITY PROBLEM: A WAY OUT?

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Abstract

THE CONTINUITY PROBLEM: A WAY OUT? A case is presented for regarding quantum theory as a theory of limited applicability and an attempt is made to estimate its success in its proper field of application seeing it from outside. It is concluded that quantum theory does not provide an explanation of atomicity (or discreteness). This failure is attributed to the appeal inherent in all current forms of quantum theory to ideas of continuum physics which - unless they can be otherwise formulated - are incompatible with the assumption of discreteness. The Bohr philosophy is seen to be a way of working in the presence of this incompatibility, but not a solution of it. A theory is described in general terms in which the continua of space and time are constructed by progressive interpolation of points, starting with algebraic structures which incorporate the scale-constants (dimensionless constants) of physics. Besides showing that it is possible to solve the discrete/continuous incompatibility, this approach assigns to the observing system the necessary place in physical theory that quantum theory claims for it.

Several authors in these Proceedings have raised the question whether it is possible to discuss the adequacy of quantum theory from a foundational point of view while working within the general theoretical framework of that theory. Bohm argues from a general point of view against the expectation that a theory like quantum theory can be complete, in the sense of affording a description of all possible phenomena, and in this way he recognizes the existence of phenomena which are best explained by theories that are not part of the recognized structure of quantum theory. Wigner, and also Jauch, on the other hand, prefer to view the possible inadequacies of quantum theory only in its own terms and concentrate their scrutiny on the theory of measurement – a part, or at least a putative part – of quantum theory.

My approach is heavily in favour of seeing quantum theory as we know it, as one among different possible theories. I think we must make an effort to assess its successes and failures from a point of view which does not implicitly assume its essential correctness, while yet trying to give proper weight to the different reasons people have for thinking that the general picture presented by quantum theory is the only possible one.

Quantum theory was proposed to take into account a certain class of experimental facts, namely those facts which forced on our attention that there exist discrete attributes of the physical world which cannot be incorporated within an essentially continuous classical theory. It is proper and sensible to ask how far quantum theory has succeeded in this task. Of course, the early forms of the theory never attempted to explain discreteness in the sense that they could be said to have incorporated both the discrete and the continuous within one theoretical structure. They simply imposed discreteness as a mathematical constraint on the range of values available as allowable experimental results of the measurement of certain physical quantities. (This description is directly applicable to the energies of atomic structures and other "scale-fixed" quantities and covers measurements involving free particles if we take the familiar probabilistic interpretation of the function of the constant $\hbar$ in the un-

certainty relation between simultaneous momentum and position measurements.)

As quantum theory developed, however, people's attitude towards the problem of explaining the discrete values seems gradually to have changed. Now, probably, a majority of physicists regard the modern form of quantum theory as a coherent intellectual structure within which both discrete and continuous quantities properly appear on terms of equality. They would say perhaps that modern quantum theory gives us an understanding of the intrusion of discreteness within continuum physics.

There is, however, one problem, in the context of the explanation of the discrete, which no physicist would claim to have solved. This is the calculation of the values of the coupling constants or interaction constants which specify the relative strengths of the fundamental fields of physics, and which can be regarded as parameters which determine the scale of the microscopic structures in terms of the cosmological ones. These constants, being dimensionless, have for a long time been a source of interest to those who would like a firm starting point for a physics which comprehends the fact of atomicity and who are dissatisfied with the coherence of current quantum theory. The best known contribution to the history of thinking about these constants is Eddington's conjecture [1] that they are more fundamental than the dimensional constants ($\hbar$, c, and the like) of which they are usually written as ratios, and that they may well originate in hitherto obscure algebraic relationships like group structures. A quite different position was taken by Dirac [2], who argued that no importance need be attached to the values of the interaction constants since they might change with time, in which case these values at any particular epoch would not be significant.

The difference in outlook which underlies such different evaluations of the significance of the values of the interaction constants can again be reduced to a difference of opinion as to whether quantum theory really explains the existence of discrete magnitudes. If it does explain them, then there remains a problem which may or may not be soluble within quantum theory itself, namely the calculation of the actual numerical values of the constants, but there can be different opinions as to the seriousness of this problem's being left unsolved. If, on the other hand, quantum theory has not given an adequate explanation of discreteness itself, then the constants constitute a much more pressing problem, for one has – if one is concerned at all with generality in physical theory – to construct first an existence theorem, namely that definite interaction constants exist (i.e. that they have some value rather than no value), and only then separately to calculate the values they actually have. Accordingly, an argument like Dirac's is only cogent at all if one is already independently assured theoretically of the existence of atomicity in the world.

The motivation of the present project of Heisenberg[1] and his associates is pertinent to my argument at this point. Heisenberg considers that it should be possible to start with the relation of discreteness to the continuum, as that relation is understood in current quantum theory, and to derive values for the interaction constants and possibly other constants like mass ratios. He indeed claims to have obtained values

[1] For a general description of this project see Ref. [3].

for the electromagnetic constant that is of the right order of magnitude, though the vital, enormous ratio of this constant to the gravitational one has no counterpart in his theory as yet. The ultimate origin of any numbers that arise within this theory is to be sought in the group structures which are assumed at the outset, though the mathematics is not merely combinatorial in nature but depends strongly upon dynamical concepts which appear in a non-linear wave-equation. I shall not attempt to judge the correctness of Heisenberg's work. It is acknowledged to contain arbitrary features and it is very complex.

If, however, Heisenberg's philosophy is right, then it follows that the only way to prove the existence theorem for the constants is to derive their values. I think that this is indeed the correct conclusion and anyone attempting to say that quantum mechanics affords an adequate understanding of atomicity without having obtained values for these constants is misguided. This being the case, our situation remains as it originally was: quantum theory is still essentially a continuum theory, with discreteness imposed as a mathematical constraint.

To return now to my original picture of quantum theory as one theory among other possible ones, having a definite set of facts to explain, I shall assume – to put a complex position in a sharp, unambiguous, if crude way – that quantum theory has its main area of undisputed success in the theory of atomic spectra and in solving problems (such as those of the theory of solids) which arise as fairly natural extensions of that theory (in short, in the area which - following Eddington [4] - it is convenient to call "scale-fixed" physics). I argue, moreover, that in this area the discoveries made by quantum theory have been discoveries of combinatorial relations and of predictive schemes expressed in terms of such relations. Where there has been dynamics, in the strict sense, it has been imported from classical ways of thinking, and a way of working has been established in which a rather uneasy association of combinatorial scheme with classical type dynamics has become the rule, with a generalized faith in the unitary nature of physical explanation to serve in place of any real synthesis of these two fundamentally different kinds of thinking. Hence we think in classical concepts which presuppose indefinite divisibility of material, and express the presupposition in the mathematical representation of the space and time continua, and yet we work formally with a discrete theory. Moreover we can't afford to be cavalier about logical consistency. The experimentalist may be tempted to use the pragmatic argument that if the theory gives results it must be substantially right, but consider the man who says he can make his theory work if he is always allowed to put 2=1, and asks why, if the experiments support him, should the logical purists object? He is, after all, working to a perfectly clear rule.

From now on, any argument hinges on two closely related questions: must a proper physical explanation of discreteness and atomicity be a dynamical one, and what is it that is wrapped up in this idea of "dynamical"?[2] To the first question I have already implicitly given the

[2] I think that the distinction between a combinatorial and a dynamical explanation is one that is in practice clear to the physicist. For example, the reason why the back wheel of a bicycle goes round - say - 2.4 times each time the pedals are rotated can be stated entirely in combinatorial terms, whereas to decide the speed or angle to the vertical at which it is safe to bicycle round a given track under given surface conditions will require extensive dynamical considerations.

definite answer yes, by concurring with physicists' universal assumption that a merely mathematical imposition of discreteness is no final explanation whether or not quantum theory has already done better. The second question therefore becomes the substantive one and is much harder to answer. The Copenhagen School answers that (a) all physical thinking is ultimately interpreted in classical and therefore in dynamical terms, and that (b) there are reasons why this has to be the case. Of this position, I should first say that it is the only attempt within the canon of quantum theory which recognizes the conceptual difficulties of the subject. Secondly, though, the Copenhagen position seems to be negative in its application. Thus it throws no light on how to apply classically interpretable concepts to the two very profound difficulties of defining locality and of understanding the polarization correlation experiments (see, for example, Refs [5] and [6]). (Notice that I do not produce the well-known infinity difficulties of quantum field theory as evidence of the conceptual unsatisfactoriness of quantum theory for just the same reason that I did not consider the program of quantum field techniques as an admissible solution to the discrete/continuous incompatibility, namely, quantum field theory takes over the quantum theory formalism entire, and what is now at stake is precisely the conceptual basis of that formalism. In any case, the perplexities about infinities really arise because there is no agreement on how to use that formalism, and this kind of difficulty again leads us behind that formalism.)

The problem of defining locality is operational. One may have an experiment in which the position of an elementary particle is pretty well known. There will be a quantum state vector which specifies the extent of this knowledge. Now suppose we inquire into the definition of the concepts in terms of which this knowledge is expressed. Position, for instance, or momentum, presuppose an ordered set of other similar pieces of knowledge which would need observations of similar "nearby" particles. Such knowledge is very hard to come by and requires vastly complicated experimental methods. Therefore, the operational situation presupposed by the dynamical concepts that are used in describing the experiments is wildly at variance with that which usually exists. Moreover, the fact that it is easy to write a quantum state corresponding to enough particle measurements to give an adequate representation of the dynamical concepts is neither here nor there, for the spatial or temporal order assumed for them is determined merely by mathematical analogy with classical physics and corresponds to no experimentally attainable reality in this new situation.

Conclusions from the polarization experiments indicate an opposite kind of breakdown of the classical intuitive continuum. In this case there is more structure in the experimental situation than the classical-type space concepts would allow for or indeed permit. Certain products of a disintegration process – namely polarized photons – which are of course out of reach of any possibility of mutual signalling, exhibit linked polarization, and therefore there seems no alternative but to regard them as parts of a linked structure.

A different sort of example of the way in which the classical language and the classical set of concepts prove inadequate as the interpretative medium for the quantum formalism is the controversy over measurement.

The various quantum theories of the measurement process necessarily stylize that process. Then disagreement arises in deciding to what extent this stylized definition of measurement covers all the possible ways in which information can be gained about a given experimental situation. The interesting thing is that there seems no way to settle this question once for all: there just have to be differences of opinion. I myself cannot see that this matters very much in this particular case, but there are those who think it does. Some feel that an approach like that of Loinger and Prosperi [7] might fit the last missing piece into quantum theory as the complete unitary description of nature.

Others – Wigner [8] being their main spokesman – seem to agree with these former in seeing the world very much through the quantum formalism spectacles, but to maintain (in disagreement with them) that a proper understanding of quantum theory consists in seeing that an appeal to something beyond or outside that theory is inevitable at the interface between observing system and world that we call measurement. In this situation the fight over the definition of "measurement" has assumed a last ditch character. The two sides in this dispute both obtain their – different – concepts of measurement, however, from a physical intuition which originates in their classical training. Those who hold the Copenhagen position would regard this as normal, but from my position, both sides fail to get the unambiguous answer that they require, just because this kind of appeal to classical training has not been formulated.

It is not much good analysing the difficulties which arise because we have never formulated what it is we appeal to when we use classical dynamical thinking, nor does it help much to stress the inadequacy of the continuous space and time concepts, unless one has an alternative path to advocate. My first undertaking in this paper was to suggest a fairly simple-minded and direct approach which recognizes the irreducibly discrete nature of the subject matter of quantum theory - especially in the origin of the scale-constants – and I took this discreteness as a starting point. It was then necessary for me to show that the subsequent history of quantum theory had not fundamentally altered this simple picture, even though it is generally supposed to have done so. On the contrary, I contend that the field is still very much open for innovation, and a high value should be set on new insight into the problems of the relations of the discrete and the continuous. I think that the importance of the scale constants (and perhaps other dimensionless numbers) is that because they stand obstinately outside the structure of current physics, they force us to consider schemes, systems or theories in which they might play a part and which could be valid independently of our deeply instilled classical intuitions, including those of the continua of space and time. (For, as we have seen, we are unable to rely on these intuitions in our dealings with the relations of the discrete and the continuous.)

In any such scheme or theory, it will obviously have to be the case that the same principles that are used to incorporate the scale-constants shall by a natural further development generate a sufficiently numerous set of points, with appropriate ordering and topology, adequately to represent the classical concepts. This requirement is an obvious analogue to the correspondence principle in the generalized form in which that principle has been enunciated by some authors. However, it avoids the

circularity implicit in the correspondence principle, namely, that the correspondence principle requires macroscopic form to be constructed out of microscopic form and yet ascribes to the latter the character of being the necessary form for perception.

This requirement that one set of principles generate both the constants and the continua can clearly be satisfied if each of the continuous variables of classical physics is constructed by progressive interpolation of points. In other words, the dynamical continuum should be defined as the field of progressively determined points, in the sense that a new point can always be interpolated between existing ones as this becomes necessary. This view of the continuum holds a place in writings on the foundations of mathematics, for it has been promulgated by Brouwer [9], in the context of his idea of free choice sequences. For my purpose, the important point to notice is the way that the progressive interpolation idea (which can easily be related to the free choice idea) fits in with what high-energy particle physics requires operationally, in the sense that what the mathematician imagines is like what the physicist has to do experimentally. (I have already mentioned the operational dilemma of the particle physicist in connexion with the definition of locality.)

There is, of course, no need to assume that the progressive interpolation view of the continuum is the only way of imagining the relation of the discrete to the continuous, starting from the discrete. For example, Finkelstein [10] imagines a code which maps a discrete structure onto a continuous space. However, the progressive interpolation idea has been pursued by some thinkers (see, for example, Ref. [11]) and it is my own way of working [12]. I shall end by describing the outlines of my approach, not with the idea that I could give enough detail to enable judgement to be passed on it, but only so that my motivation and method could be seen as a reasonable way of advance, given the substantial correctness of the path which I have tried to trace in this paper through the conceptual perplexities of quantum theory.

I have tried to make up an abstract system to represent as simply as possible what algebraic structure results when

(a) each of a set of entities has the power to discriminate among the remaining members of the set in the sense that a decision is reached as to whether a given other member is identical with or different from the member which begins the process[3];
(b) the discrimination process is added to the set, being regarded as an entity of the same kind; and

[3] In their survey of the evaluation of attempts to solve the measurement problem of quantum theory, Wigner, Jauch and Yanase (Nuovo Cimento, 1968) come to the conclusion that however you look at things, an epistemological problem remains for the theorist. Some physical effect has to be ascribed to the acquisition of knowledge as distinct from any process described by that knowledge. In the formal structure which I am describing, this problem at least is solved. Every element of the structure is defined in terms of its potentiality for communication with other elements. Thus I am not forced into impossible assumptions such as that the brain suddenly engineers the collapse of the wave function (which must be engineered somehow if our appeal to that collapse is not to be a mere re-statement of our problem), nor - at the other extreme - to a general idealist philosophy to justify a specific scientific assumption, nor into any of the intermediate positions which have to be made to work by some essentially ad hoc principle (like the collapse of the wave function).

(c) entities are ordered sets of binary units conveniently written 0.1. These are the primitive descriptors from which all more complex entities are constructed in a hierarchy of stages.

Parker-Rhodes discovered a way of constructing such a system by operating on ordered sets of "vectors" by matrix multiplication and regarding the matrices as new vectors of order j^2, where j was the order of the original vectors. In this mathematics, the discrimination operation upon the primitive discriminators is the symmetric difference operation, and when the whole process was subjected to critical scrutiny and finally based on logically secure foundations, the matrix transforms turned out to be the only way of extending the generating operation which does not make special assumptions.

The reason for constructing hierarchies of the foregoing type was precisely to generate large sets of points capable of approximating to the physical continuum using the same mathematical apparatus as had generated small sets whose orders could be identified with discrete constants. However, we were much surprised to find that the mapping operation introduced by Parker-Rhodes gave the maximum values for the numbers of discriminable entities at two successive stages in the construction of the hierarchy as the integer 137 and an integer of the order 10^{38}. These integers have roughly the right values to define the relative scales of the electromagnetic and the gravitational fields. Moreover, the construction cannot be carried past this level and this would amount to saying that the gravitational is the weakest field. Given finite values for the scale-constants and therefore also for their constituent factors (constants like $\hbar$ and c), and given also the principle of the progressive determination of space and time points, it at once follows that there will be some sort of limit on the possibility of indefinitely refining measurements. That this limit will take the form of the Heisenberg uncertainty relation does not, of course, follow without a detailed argument which I shall not here provide.

In this work we have suffered a good deal from the results of the theory being too good to be true. However, I think that at least a good proportion of this reaction is due to the fact that people associate the scale-constants with extremely factual, detailed and tangible experimental matters about which calculations of a general sort can only be in the nature of the miraculous. It will bring me back to my earlier themes and make a suitable conclusion if I briefly comment on this opinion. It draws a lot of its strength from the extreme realism which most of us have about the space and time continua in our background thinking (however much or little we may, as an intellectual matter, acknowledge their limitations). If we are prepared to subject all our thinking to a rigorous scrutiny in which our dependence on these concepts is limited to their range of experimental applicability, we shall be able to see questions concerning the values of scale-constants in a new and much more light-hearted way. It is the old idea that matter is immutably "out there" in space whatever we, as theorists, may concoct, which gives the scale-constants their awesome mien. That idea put in its place, we are free to be enterprising in looking for ways to fit them and their origins into theories. And we need to be.

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QUANTUM FIELD THEORY
Introductory remarks

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Abstract

QUANTUM FIELD THEORY: INTRODUCTORY REMARKS. A brief account is given of the major changes in the understanding of the general principles of quantum physics, invariance under Lorentz transformations and translations in space-time, locality and causality, and stability during the past decade.

Scientists engaged in work on quantum field theory are often classified as "axiomatists". I have always felt that this is a very unfortunate terminology. Since it is one of the objectives of this Symposium to stimulate the building of bridges between different subjects and between different approaches to one subject I think it is appropriate to spend a few minutes trying to define the objectives envisaged in the so-called "axiomatic approach" and to dispel some misunderstandings.

In the early 1950s when we were confronted on the one hand with unquestionable successes of quantum electrodynamics (unexpected by many sceptics) and, on the other hand, plagued by a strong disaffection with the mathematically and conceptually ill-defined parts in the formulation of this theory, it appeared a rather natural thing to abstract from the theory those features which could be clearly formulated without reference to perturbation expansions and without mathematical ambiguities. Doing this one obtains not a theory, but a framework. One describes a whole class of theories which share certain properties. Wightman gave a very detailed and mathematically precise description of those properties which he considered as essential around 1955. He called them "axioms" and proceeded to study their consequences. It is, of course, not warranted to attach any religious feelings to these or other "axioms" and to mistake them for dogmas. The work "axiomatic" is used only in the sense that one does not have a detailed specification of equations of motion etc., but instead a list of requirements, the strength of which is not a priori known. Originally it was believed that these requirements are so mild and general that they allow many detailed theories within the given framework. A closer study has shown that the requirements imposed are extremely restrictive and one may fear that they do not allow any interesting theory at all. For this reason it has become one of the crucial tasks in this approach to construct at least one nontrivial model theory within this framework. Some recent work on this question will be reported by Jaffe in these Proceedings. It may be noted in passing that all these comments apply in essentially the same way to "axiomatic S-matrix theory".

What then are the axioms or, as I prefer to call them, "postulates" of quantum field theory? I shall list them in somewhat general, nontechnical terms in order to bring out their intuitive physical meaning.

They are:

(1) The general principles of quantum physics.
(2) Invariance under Lorentz transformations and translations in space-time.
(3) Locality and causality.
(4) Stability.

This list has practically remained unchanged during the past decade. What has changed is our judgement of what constitutes the most relevant mathematical formulation of these principles. As a result of such changes in emphasis new problems concerning the understanding of the structure of the theory have appeared. I shall give a few examples of this. The earlier formulation denoted by (a) is confronted with a later formulation (b). Only principles (1) and (3) shall be discussed in this way:

(1(a)) One deals with a Hilbert space. The observables are represented by operators in this space, the states by vectors. Probabilities for events are calculated in the well-known way described for instance in the books of Dirac and von Neumann. It was pointed out by Wigner that there are some limitations to the validity of the superposition principle between the state vectors. Such restrictions are taken care of by imposing so-called "superselection rules".

(1(b)) One considers the algebra generated by the observables as an abstract algebra $\mathfrak{A}$. The scheme described in paragraph (1(a)) is then one possible representation of this algebra by operators in a Hilbert space. But there are many other such representations which are mutually inequivalent. It can be shown that the physical content is determined completely by the algebraic structure of $\mathfrak{A}$ (provided that this structure has been specified in a maximal way so that $\mathfrak{A}$ is a "simple" algebra).

In other words, the calculation of probabilities for events can be formulated in a way independent of Hilbert space representations. The way in which "states" are described in this algebraic approach will be described briefly in Ruelle's paper. It will be apparent then that one has a much larger collection of states than those appearing in the scheme (1(a)) and that it is very convenient to make use of this larger set in discussing questions like spontaneous breakdown of symmetries; analogy between the vacuum state in quantum field theory and equilibrium states in quantum statistical mechanics; and structure of superselection rules.

(3(a)) The basic objects, from which the algebra of observables is constructed, are a system of finite component covariant field operators $\psi_\alpha(x)$, $\phi_\lambda(x)$. There should be some dynamical law which determines the fields at an arbitrary space-time point in terms of those in a small (possibly infinitesimal) time interval $|t| < \epsilon$ (primitive causality). It is also assumed that the fields either commute or anti-commute if the relative distance is space-like (locality, relativistic causality)

$$[\psi_\alpha(x), \phi_\lambda(y)]_I = 0 \quad \text{for space-like } x-y$$

Originally it was believed that the different basic fields should be associated with different elementary particles. In particular the transformation character of a field was taken to be given by the spin of the "corresponding" particle.

(3(b)) The notions of locality and causality are introduced by giving the algebra of observables a structure parallel to that of Minkowski space. To each finitely extended region $\mathcal{O}$ of space-time there should be a sub-algebra of $\mathfrak{A}$ called $\mathfrak{A}(\mathcal{O})$. The physical interpretation of the theory is completely based on this correspondence between geometric regions and sub-algebras by demanding that $\mathfrak{A}(\mathcal{O})$ should be generated by all these observables which can be measured within $\mathcal{O}$. It is very satisfactory indeed to find that this purely geometric description of the physical interpretation of the mathematical objects in the theory is sufficient. In other words one does not need in the formulation of the theory any reference to particles and corresponding fields. Those relations between different sub-algebras $\mathfrak{A}(\mathcal{O})$ which express primitive and relativistic causality and Poincaré invariance are trivial to write down. I shall omit them here.

Some comments should be added, however, concerning the relation between the frames described in paragraphs (1(a)), (3(a)) and (1(b)), (3(b)).

(i) In paragraph (3(b)) there is no mentioning of fields. It may be that a certain set of finite component fields can be used to generate the local algebras $\mathfrak{A}(\mathcal{O})$. We do not know on either mathematical or physical grounds whether such a special structure of the local algebras is desirable. If such a structure exists then the parametrization in terms of fields is certainly non-unique.

(ii) Observables have to commute, not anti-commute, at space-like distances. Thus commutativity is assumed between the algebras of space-like separated regions. The appearance of Fermi statistics (or parastatistics) for certain particles and the connection between spin and statistics becomes therefore a much deeper problem in the frame described in paragraph (3(b)). It is tied to the problem of the structure of superselection rules, i.e. to the structure of the "physical spectrum" of $\mathfrak{A}$.[1] Only a few results are known so far in such a structure analysis, but it is a fascinating subject, involving among other things a determination of the gauge groups from the algebra of observables $\mathfrak{A}$ and its local substructure.

Let me close with a few remarks on the question whether the principles (1) through (4) appear reasonable to-day or whether we should expect to be forced into major modifications of these principles in elementary-particle physics. Again I shall comment only on the principles (1) and (3).

It has been felt for a long time that microscopic locality is a very stringent and possibly bad extrapolation from our experience. No major conceptual change would result from a relaxation of this requirement, but one would lose a lot of structure. One would also lose the understanding of a few rules which seem to depend very crucially on strict locality (connection of spin and statistics, existence of anti-particles and TCP theorem). We have heard of the experimental evidence against a failure of locality up to distances of order 10^{-14} cm. Moreover, at the present time we would not gain from such a relaxation of (3), since nobody has yet succeeded in constructing a model of a macroscopically (but not microscopically) local theory which is Lorentz invariant and

[1] The spectrum of an algebra consists of the (equivalence classes of) irreducible representations. The word "physical" shall indicate that we want to consider only such states which become at large distances from the origin indistinguishable from the vacuum state.

allows creation of particles. Thus, Pauli's comment on non-local theories, given around 1953, still seems to apply: "These theories help where no help is needed and they do not help where help is needed".

Concerning the assumptions involved in (1) the situation is less tractable. Many radical conceptual changes are conceivable. But without indication of some conflict between the assumption (1) and experience (such as e.g. a failure of the superposition principle between states of the same charge quantum numbers) it is difficult to pursue alternatives to (1) in a serious way.

PROGRESS IN CONSTRUCTIVE FIELD THEORY

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Abstract

PROGRESS IN CONSTRUCTIVE FIELD THEORY. 1. Introduction; 2. The $(\phi^4)_2$ theory; 3. Time translation without cut-offs for $(\phi^4)_2$; 4. The vacuum state and Hamiltonian for $(\phi^4)_2$; 5. The $(\bar{\psi}\psi\phi)_2$ theory; 6. The $(\phi^4)_3$ theory.

1. INTRODUCTION

I shall report on some progress made during the past year on the existence problem for local quantum field theory. The basic question is an old one: "Are locality, Lorentz covariance, and positive energy consistent with a non-trivial scattering matrix?" One way to answer "yes" to the question is to follow a constructive approach, namely, start with a Lagrangian, construct the corresponding Hamiltonian, and use that Hamiltonian to establish on a rigorous mathematical basis the existence of a local quantum field.

In following the constructive approach it is convenient to work in Fock space, which is not correct [1]. We must pay for this by encountering infinite volume divergences and ultra-violet divergences, which are removed by volume and momentum space cut-offs. Since these cut-offs destroy covariance and locality, the program is to first solve the cut-off theory, and then to take limits of these cut-off solutions as the cut-offs are removed [2]. The limiting theory will be realized on the Hilbert space of physical states.

So far, two interactions have been intensively studied, the scalar ϕ^4 theory and the scalar Yukawa coupling $\bar{\psi}\psi\phi$. The strongest results have been obtained in space-time of two dimensions. One can only hope that in the near future a four-dimensional problem will be solved.

2. THE $(\phi^4)_2$ THEORY

The best results are for the ϕ^4 theory in two-dimensional space-time. For this model, it is known that there does exist a local, Heisenberg picture field.

Let us discuss this model in more detail. In perturbation theory, the only ultra-violet divergences are removed by Wick ordering the interaction Hamiltonian. As is expected in any model with vacuum polarization, there is a linear volume divergence in the vacuum self-energy in every order of perturbation theory. While the volume divergence does not occur in the perturbation series for the Green's functions, which are finite in every order, it is known that the perturbation series for the Green's functions diverges when summed to all orders [3]. Thus the perturbation series will be useful for constructing a solution only if it is formally resummed. In the case of Euclidean quantum field theory, Symanzik [4] has an ambitious program

which has succeeded in using such resummation methods to study a theory in infinite volume but with other cut-offs.

We will not discuss this work further, but return to the problem in Minkowski space-time and the program outlined above. With both a momentum space cut-off K and a finite space volume V, it is known that the Hamiltonian $H_{K,V}$ is self-adjoint, has a unique ground-state $\psi_0(K, V)$ and gives rise to Heisenberg picture fields [5]

$$\phi(x) = e^{iH_{K,V}t}\ \phi(\vec{x}, 0)\, e^{-iH_{K,V}t}$$

with vacuum expectation values

$$(\psi_0(K, V),\ \phi(x_1) \ldots \phi(x_n)\psi_0(K, V))$$

The time translation automorphism determined by the Hamiltonian $H_{K,V}$ is for any operator A

$$A \rightarrow A(t) = \sigma_t^{K,V}(A) = \exp(iH_{K,V}t)\, A \exp(-iH_{K,V}t)$$

3. TIME TRANSLATION WITHOUT CUT-OFFS FOR $(\phi^4)_2$ [6]

We now see how it is possible to construct a time translation σ_t for the $(\phi^4)_2$ theory without cut-offs. This σ_t will preserve locality. We study the Hamiltonian on Fock space [7]

$$H_\ell = H_0 + \lambda \int :\phi^4(\vec{x}):\ g(\vec{x}/\ell)\, dx = H_0 + H_{I\ell}$$

where $g(\vec{x})$ is a positive, infinitely differentiable function which equals one for $|x| \leq 1$ and which vanishes for $|x| \geq 2$. The function $g(\vec{x})$ provides a volume cut-off, and no ultra-violet cut-off is necessary.

We have already alluded to the fact that $H_{I\ell}$ is not a small perturbation of H_0, which is clear from the fact that H_0 is quadratic in the fields while $H_{I\ell}$ is quartic. Thus we cannot use the perturbation theory developed by Kato [8], which applies to regular perturbations of self-adjoint operators. Instead, it has been necessary to develop a new form of non-analytic perturbation theory [9] to deal with H_ℓ. Although Wick ordering destroys the formal positivity of $H_{I\ell}$, the negative part of $H_{I\ell}$ can be estimated in terms of the unperturbed operator H_0, and it is on this fact that the singular perturbation theory rests. The two crucial estimates for $H_{I\ell}$ are that, given $\epsilon > 0$, there exists a constant $b(\epsilon, \ell)$ such that

$$-H_{I\ell} \leq \epsilon H_0 + b \qquad [10]$$

and

$$[H_0^{\frac{1}{2}}, [H_0^{\frac{1}{2}}, -H_{I\ell}]] \leq \epsilon H_0^2 + b \qquad [6]$$

These estimates, along with the fourth-order character of $H_{I\ell}$, allow us to establish the result:

Theorem [6]. The Hamiltonian H_ℓ is a self-adjoint operator on the domain $D(H_0) \cap D(H_{I\ell})$. It is essentially self-adjoint on the domain $D_0 = \bigcap_{n=0}^{\infty} D(H_0^n)$.

Thus we have established the existence of σ_t^ℓ. The next step is to show that for any bounded space-time region, and for ℓ sufficiently large, σ_t^ℓ actually gives the correct time translation for the theory without cut-offs, and hence it preserves locality.

It is convenient to introduce the polynomial algebra P(O) generated by the bounded functions $U(f) = \exp(i\phi(f))$ and $V(f) = \exp(i\pi(f))$ of the $t = 0$ canonical fields averaged with test functions f which vanish outside the bounded region 0. The von Neumann algebra $\mathfrak{A}(O)$ is the set of weak limits of operators in P(O).

The property of σ_t^ℓ which is important is that it propagates influence not faster than the speed of light. In other words, σ_t^ℓ transforms any operator in $\mathfrak{A}(O)$ into an operator in $\mathfrak{A}(O_t)$, where O_t is the region O expanded by $|t|$ and the velocity of light is taken to be one. Furthermore, if O is contained in the interval $|\vec{x}| \leq R$, then the action of σ_t^ℓ on $\mathfrak{A}(O)$ is independent of ℓ whenever $\ell > R + |t|$. Clearly this is the case if $\lambda = 0$.

The formal basis for the fact that σ_t^ℓ propagates at the speed of light was noted in perturbation theory by Guenin [11]. In the case that the total Hamiltonian is essentially self-adjoint, the result was proposed by Segal [12]. A proof of this theorem relies on the Trotter [13] product formula

$$\begin{aligned} \exp(iH_\ell t) &= \operatorname*{st.lim}_{n\to\infty} (\exp(iH_0 t/n) \exp(i\, H_{I\ell}\, t/n))^n \\ &= \operatorname*{st.lim}_{n\to\infty} \quad U_\ell(t/n)^n \end{aligned}$$

which is a mathematically precise version of Feynman's path integral method [14].

Thus for A in $\mathfrak{A}(O)$

$$\sigma_t^\ell(A) = \operatorname*{st.lim}_{n\to\infty} \; U_\ell(t/n)^n A\, U_\ell(t/n)^{-n}$$

We consider the action of each transformation

$$B \longrightarrow U_\ell(t/n)\, B\, U_\ell(t/n)^{-1}$$

which is repeated n times. We assume that B is in $\mathfrak{A}(O_a)$, for $a = |t|r/n$, and $r = 0, 1, \ldots, n-1$. We can break $H_{I\ell}$ into two parts, namely $H_{I\ell}(g) = H_{I\ell}(g_1) + H_{I\ell}(g_2)$, where $g_1(x/\ell)$ vanishes outside $O_{a+\epsilon}$, and g_2 vanishes on $O_{a+\epsilon/2}$. Since $H_{I\ell}(g_2)$ commutes with $\mathfrak{A}(O_a)$, $\exp(iH_{I\ell}\, t/n)\, B \exp(-H_{I\ell}\, t/n)$ is in $\mathfrak{A}(O_{a+\epsilon})$, and $U_\ell(t/n) B\, U_\ell(t/n)^{-1}$ is in $O_{a+t/n+\epsilon}$. Hence $\sigma_t^\ell(A)$ is in $\mathfrak{A}(O_{|t|+\epsilon})$ for any $\epsilon > 0$.

Furthermore, if $\ell > R + |t|$, then the g_1's are all equal to one on $O_{a+\epsilon/2}$, so that $\sigma_t^\ell(A)$ does not depend on ℓ. Since the algebras $\mathfrak{A}(O)$ are local, σ_t preserves locality. Thus we have established the following theorem.

Theorem. There exists a Haag theory of bounded local observables corresponding to the $(\phi^4)_2$ theory.

Of course the operator $\lim_{\ell\to\infty} H_\ell$ does not exist, so it is necessary to discover if there is a representation of the algebra of bounded observables such that a vacuum state and a global Hamiltonian do exist.

4. THE VACUUM STATE AND HAMILTONIAN FOR $(\phi^4)_2$

Let us return to the Hamiltonian $H_{K,V}$, where V denotes a periodic box Λ of volume V. We know that $H_{K,V}$ has a unique vacuum vector $\psi_0(K, V)$, which determines a state $\omega_{K,V}$ on the algebra P(O) for $O \subset \Lambda$. This state is regular on P(O) in the sense that

$$\omega_{K,V}\,(\alpha^*(f)\,\alpha(f) + \beta^*(f)\,\beta(f)) = |f|_s^2$$

where

$$\alpha(f) = U(f) - I, \quad \beta(f) = V(f) - I$$

and $|f|_s$ is a suitable norm of f. This regularity ensures that $\omega_{K,V}$ varies smoothly as f does.

We can ask when these approximate vacua have a regular, translation invariant limit as one lets K or V tend to infinity. In the case $V \to \infty$, $K < \infty$, it has been established that a subsequence of the $\omega_{K,V}$ converges [15], while for $V < \infty$, $K \to \infty$, the $\omega_{K,V}$ are known to converge [6, 15].

More generally, such a regular vacuum ω is known to exist whenever the average energy density is bounded below, independent of the cut-offs

$$\frac{H_{K,V}}{V} > -M$$

where $H_{K,V}$ has the expectation value zero in the no-particle state. This condition is satisfied in perturbation theory by the $(\phi^4)_2$ theory, or any theory with no ultra-violet vacuum-energy divergence. Furthermore, this condition appears true independently of perturbation theory [16]. A weaker, yet sufficient condition for the existence of ω is that the local particle density be uniformly bounded in the ground-state

$$\left\langle \frac{N_V}{V} \right\rangle_{\psi_0(K,V)} \leqslant M_0$$

Here N_V is the number operator in the volume V. This condition could be said to express that the interaction is "locally Fock".

It turns out that as a consequence of the same bound on the average energy density or the average bare particle density there exist matrix elements of a global Hamiltonian. In fact, one can establish that for states on P corresponding to expectation values in the vectors $P(O)\psi_0(K, V)$, a subsequence of expectation values of the Hamiltonians $\hat{H}_{K,V}$ converges as K and $V \to \infty$. Here the $\wedge$ denotes that $H_{K,V}$ is replaced by $H_{K,V} - E_{K,V}$, where $E_{K,V}$ is the energy of the state $\psi_0(K, V)$.

Important questions are: Is the state ω constructed in this manner invariant under the automorphism σ_t of $\mathfrak{A}$ defined above? If so, does the ω correspond to a limit of the ground-states of the H_ℓ, which are translation invariant only in the limit $\ell \to \infty$? Are the states which define matrix elements of the non-cut-off Hamiltonian dense in the space of physical states? Is Lorentz covariance valid?

5. THE $(\bar{\psi}\psi\phi)_2$ THEORY

The total cut-off Hamiltonians $H_{K,V}$ and $H_{K,\ell}$ were studied by Lanford [17], who showed that Heisenberg picture fields and their vacuum expectation values exist. Lanford's methods use perturbation theory but break down as either cut-off is removed. The time translation automorphism σ_t^K is now known to exist for a theory in an infinite volume [18]. Furthermore, with $K < \infty$, the average energy density $H_{K,V}/V$ is semibounded so that the above method [15] can be used to construct the physical representation for that theory.

In the case of the Hamiltonian $H_{K,\ell}$, for $\ell < \infty$ it is known that $\lim_{K\to\infty} H_{K,\ell}$ exists as a closable semi-bounded form [19, 10], and hence it has a self-adjoint Friedrichs extension. This involves the addition of two counter-terms, namely a mass renormalization

$$\delta m^2(K, \lambda) \int : \phi_K(\vec{x})^2 : g^2(\vec{x}/\ell)\, d\vec{x}$$

and a vacuum self-energy renormalization $E_0(K, \lambda)$. Both counterterms are logarithmically divergent in K, as is indicated by the primitively divergent diagrams in perturbation theory

and

both of which diverge logarithmically.

The method by which the renormalization is carried out is to write down a dressing transformation which connects $H_{K,V}$ and H_0. Formally, this is just the wave operator

$$W_{K,\ell} = \lim_{t\to-\infty} \exp(+iHt)\exp(-iH_0 t)$$

for which

$$H_{K,\ell}\, W_{K,\ell} = W_{K,\ell} H_0$$

The idea is to study the limit of this equation on nice vectors in Fock space as $K \to \infty$, keeping $\ell < \infty$. The operator $W_{K,\ell}$ has a formal perturbation expansion in a Wick ordered series [20]

$$W = : \exp(-\Gamma Q):$$

where Γ is an operation defined by

$$[H_0, \Gamma Q] = Q$$

so that ignoring boundary terms

$$Q \mp -i \int_{-\infty}^{0} Q(t)\, dt$$

where

$$Q(t) = \exp(iH_0 t)\, Q \exp(-iH_0 t)$$

The formal expansion for Q is [20]

$$Q = H_I - (H_I \Gamma H_I - : H_I \Gamma H_I :) + \text{higher-order terms}$$

In practice, $W_{K,\ell}$ is much too complicated to study. In fact an approximate dressing transformation $T_{K,\ell}$ is sufficient to do the renormalization. Glimm [19] retained in Q only a part of the first- and second-order terms, namely the high-energy contributions to the fermion pair creation terms which occur in Q up to second order. The resulting dressing transformation renormalizes the Hamiltonian, as it is the pair creation terms which give rise to infinities. In addition, it is convenient to use an approximate Γ operator to avoid singular integrals in defining ΓQ. Then

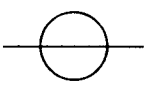
$$\lim_{K\to\infty} (T_{K,\ell}\,\Omega,\ H_{K,\ell}\, T_{K,\ell}\,\Omega) = (\psi,\ H_{ren}\,\psi)$$

exists as a closable, semi-bounded form for any Ω with a finite number of particles and wave functions in $\mathscr{D}$.

The vectors ψ in the range of T_ℓ are in the domain of $N^{\frac{1}{2}}$, but not of H_0. In other words, the free and the interaction parts of H_{ren} are not separately defined, so the total Hamiltonian cannot be essentially self-adjoint. Thus the Trotter formula cannot be applied directly, and it is an open question whether a local $(\bar{\psi}\psi\phi)_2$ Haag theory exists.

6. THE $(\phi^4)_3$ THEORY [21]

In addition to mass renormalization and vacuum self-energy renormalization, a field strength (wave function) renormalization is necessary in the $(\phi^4)_3$ theory. The primitively divergent diagrams are



and

which are respectively logarithmically, linearly, and logarithmically divergent.

In constructing a dressing transformation to renormalize this Hamiltonian, two new problems must be dealt with. First, as the ultra-violet cut-off is removed, the range of T_ℓ no longer lies in Fock space, as a consequence of the linear, rather than logarithmic, divergence of the vacuum energy. It is thus necessary to construct a new space $\bar{H}_{ren}$ with the scalar product given by

$$||T_\ell\,\Omega||^2_{ren} = \lim_{K\to\infty} ||T_{K,\ell}\,\Omega||^2 / ||T_{K,\ell}\,\Omega_0||^2$$

where $||T_{K,\ell}\,\Omega_0||$ is logarithmically divergent.

A second, technical difficulty is present even for the cut-off $T_{K,\ell}$. It arises from the divergence of the exponential expansion for $T_{K,\ell}$. T, supressing K and ℓ, is written as a product of two parts, $T = T_1 T_2$, where $T_1 = \exp(-\Gamma H_{I,C})$ and T_2 is recursively defined. The term $H_{I,C}$ is the four-particle creation part of H_I, and thus creates too many bosons for even the cut-off exponential to converge. This difficulty is overcome by writing

$$H_{I,C,K,\ell} = \sum_{j=1}^{K} H_{I,C,K_j,\ell}$$

corresponding to different momentum cut-offs K_j, and by retaining in the series expansion of T_1 only powers of $\Gamma H_{I,C,K_j,\ell}$ of degree less than j. A similar procedure is carried out for T_2.

The result is that by including mass and vacuum counter-terms in $H_{I,K,\ell}$, the renormalized Hamiltonian

$$H_{ren,\ell} = \lim_{K\to\infty} H_{K,\ell}$$

exists as a densely defined operator on the new Hilbert space $\overline{H}_{ren}$. It is not known whether $H_{ren,\ell}$ can be made positive by a finite renormalization, and so a Friedrichs extension is not known to exist.

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FIELD THEORY AND STATISTICAL MECHANICS

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Abstract

FIELD THEORY AND STATISTICAL MECHANICS. 1. Introduction: systems with an infinite number of degrees of freedom (SINDFs); 2. Is a study of SINDFs by rigorous mathematical techniques worth while?; 3. The states of SINDFs; 4. Equilibrium states of infinite systems in statistical mechanics; 5. Existence problem in local field theory; 6. Phenomenological analogies between SINDFs; 7. Breakdown of symmetry; 8. Thom's theory of morphogenesis.

1. INTRODUCTION: SYSTEMS WITH AN INFINITE NUMBER OF DEGREES OF FREEDOM (SINDFs)

Mathematical physics, as it exists today, consists mainly in the study of systems with an infinite number of degrees of freedom. This is true of relativistic quantum mechanics, of statistical mechanics, of general relativity and of the mechanics of continuous media. To say a priori that the number of degrees of freedom of a system is not finite is a negative statement and it is not clear that it has useful consequences. However, if one considers concrete situations, one is struck by analogies which exist in the study of very different systems. One can distinguish between methodological and phenomenological analogies, and I shall divide this paper roughly into two parts discussing successively methodological and phenomenological aspects of the study of SINDFs.

2. IS A STUDY OF SINDFs BY RIGOROUS MATHEMATICAL TECHNIQUES WORTH WHILE?

One cannot claim to understand a physical property of a system unless one can construct a mathematically consistent model of the system which exhibits this property. However, in many branches of physics, this mathematical existence problem is not crucial. In the study of SINDFs with interactions, the existence problem is both difficult and important. The best illustration of this difficulty and importance is given by relativistic quantum mechanics where there has been much heated discussion about the possibility of describing particles by local fields. The problem of constructing non-trivial local fields is unfortunately very hard and has not been solved. This has been taken by some as an indication that there exist only trivial local fields (i.e. without scattering). By comparison, in statistical mechanics the existence problem (for equilibrium states of infinite systems) is also quite difficult, but nobody ever seriously doubted the existence of solutions, and hosts of solutions have eventually been found.[1]

[1] See, for instance, RUELLE, D., Statistical Mechanics, Benjamin, to be published.

I shall come back shortly to the existence problem for local field theories.

3. THE STATES OF SINDFs

A state of a SINDF is defined as an expectation value functional $\rho(\bullet) = \langle \bullet \rangle$

$$A \to \rho(A) = \langle A \rangle$$

where we assume
(a) linearity: $\rho(\lambda_1 A_1 + \lambda_2 A_2) = \lambda_1 \rho(A_1) + \lambda_2 (A_2)$
(b) positivity: if $A \geqq 0$, then $\rho(A) \geqq 0$
(c) normalization: $\rho(1) = 1$

To be logical, we should have said first what the A's are. Depending on the case, they will be interpreted as products of "field operators" or objects generated by the "observables of the theory" or something of that sort. It is assumed usually that the A's can be added, multiplied, and multiplied by complex numbers, more precisely, it is assumed that they form an algebra over the complex numbers[2].

We think thus of a SINDF as described by an algebra $\mathfrak{A}$, its possible states being given by expectation value functionals ρ. The situation is usually a little more complicated in the sense that there is additional structure associated with the algebra $\mathfrak{A}$. The most important example is that of a physical invariance group G acting on $\mathfrak{A}$ by automorphisms (transformations of the algebra which preserve the algebraic operations):

$$A \to \tau_g A \qquad A \in \mathfrak{A} \qquad g \in G$$

The invariant states will be of particular interest, these are the states ρ, such that

$$\rho(A) = \rho(\tau_g A) \text{ for all } A \in \mathfrak{A}, \qquad g \in G$$

When a state ρ is given on the algebra $\mathfrak{A}$, it is possible to construct a Hilbert space $\mathfrak{H}$, containing a vector Ω, and a representation[3] π of $\mathfrak{A}$ in $\mathfrak{H}$ such that the state ρ is the expectation value for the vector Ω:

$$\rho(A) = (\Omega, \pi(A)\Omega)$$

This is the Gelfand-Segal construction for C^*-algebras, or the Wightman construction for fields[4]. If ρ is invariant under a group G, there is also a unitary representation U of G in $\mathfrak{H}$ such that

$$U(g)\,\pi(A)\,U(g^{-1}) = \pi(\tau_g A), \qquad U(g)\,\Omega = \Omega$$

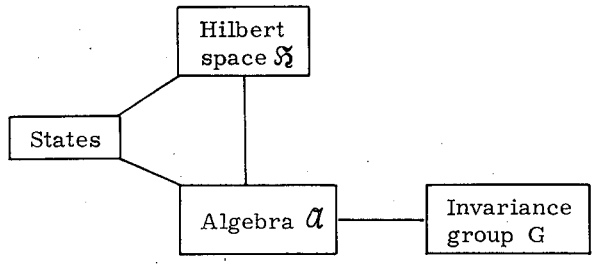
---

[2] It is mathematically very convenient to assume that $\mathfrak{A}$ is a C^*-algebra. The use of C^*-algebras has been advocated particularly by I. Segal, R. Haag and D. Kastler. Nevertheless, algebras of unbounded objects ("fields") are still mostly used.

[3] i.e. π is linear and multiplicative from $\mathfrak{A}$ to operators in $\mathfrak{H}$.

[4] To make the construction unique, it is necessary to say that $\pi(\mathfrak{A})\Omega$ is dense in $\mathfrak{H}$.

The situation just described is summarized in the following diagram



We shall see why it is not advisable to describe states in terms of vectors in a Hilbert space right from the beginning, as one would do for quantum systems with a finite number of degrees of freedom.

4. EQUILIBRIUM STATES OF INFINITE SYSTEMS IN STATISTICAL MECHANICS

We consider quantum statistical mechanics. In that case $\mathfrak{A}$ is the algebra of canonical commutation or anticommutation relations (for bosons and fermions, respectively). The invariance group G may be taken to be the Euclidean group (translations and rotations in three dimensions). A state ρ is described by so-called "reduced density matrices" which are expectation values of products of creation and annihilation operators. We shall assume that an equilibrium state ρ is invariant under the group G. We can, as above, construct from ρ a Hilbert space $\mathfrak{H}$ and a representation π of $\mathfrak{A}$; π is a representation of the canonical commutation or anticommutation relations.

Let ρ_1, ρ_2 be different equilibrium states (corresponding, say, to different interactions or different temperatures). It can be shown that, in general, the corresponding representations of the canonical commutation or anticommutation relations are not unitarily equivalent (or quasi-equivalent). This implies that the Hilbert spaces $\mathfrak{H}_1$ and $\mathfrak{H}_2$ cannot be identified in any useful way (in contradistinction with the case of a finite number of degrees of freedom).

To summarize the situation, in statistical mechanics the algebra $\mathfrak{A}$ is independent of the interaction, the state ρ depends on the interaction and so does the representation π of $\mathfrak{A}$ associated with it.

I shall not describe the construction of an equilibrium state of an infinite system; it is the limit, in a suitable sense, of the equilibrium state of a system contained in a "finite box" as the box tends to infinity. The equilibrium state for a finite box corresponds – essentially – only to a finite number of degrees of freedom.

5. EXISTENCE PROBLEM IN LOCAL FIELD THEORY

This problem is not solved, but comments about its difficulty can be made. A formal local Hamiltonian like

$$H = \text{free Hamiltonian} + \lambda \int dx\, \phi(x)^4$$

has a priori no meaning. A first step in trying to give it a meaning consists in restricting the system to a "finite box" as in statistical mechanics; this is obtained by introducing a "space cut-off" in the above integral. Another cut-off, the "momentum cut-off" is necessary to make the above Hamiltonian meaningful, and to remove this momentum cut-off an infinite renormalization is at the same time needed. The removal of momentum cut-off and infinite renormalization have been performed by J. Glimm on the above Hamiltonian in the case of three space-time dimensions. This is a remarkable achievement; unfortunately, while this gives the Hamiltonian as symmetric operator in a Hilbert space, it says nothing about the existence of a vacuum state. Glimm keeps the space cut off; on the other hand, K. Symanzik has done work which amounts to removing the space cut-off while retaining the momentum cut-off. Symanzik's method is inspired by statistical mechanics.

Going back to our description of SINDFs by states on an algebra $\mathfrak{A}$, we remark that the structure of $\mathfrak{A}$ may be modified in the course of renormalization. More precisely, this appears to be the case if there is an infinite wave function renormalization. In this case therefore, the interaction determines not only the state, the Hilbert space structure, but also the algebra $\mathfrak{A}$. The state of main interest in field theory is the vacuum state, it corresponds to the ground-state of statistical mechanics (zero temperature state).

6. PHENOMENOLOGICAL ANALOGIES BETWEEN SINDFs

The following behaviour is typical of many SINDFs. If an equilibrium state depends on several parameters then, as these parameters are varied, the state usually changes smoothly but occasionally undergoes a catastrophic transformation. A catastrophic transformation may in particular change abruptly the symmetry group of the state.

An example of this situation is provided by statistical mechanics at temperature $T \neq 0$: an equilibrium state will depend smoothly on the temperature and density[5] with occasional catastrophic changes called phase transitions. Another example is the transition from laminar to turbulent flow in fluid mechanics as the Reynolds number is increased. In these examples both the nature of the catastrophic changes and the smooth behaviour outside require understanding.

7. BREAKDOWN OF SYMMETRY

The abrupt change of the symmetry group associated with a catastrophic transformation, as described above, is not always present (it is not there in the standard gas-liquid phase transition). It may thus be only a secondary phenomenon, but it is an interesting one, and which is only superficially understood.

To describe it, we go back to the language of states over an algebra $\mathfrak{A}$ with a natural invariance group G. We assume that an equilibrium state is given by a state ρ invariant under G. It happens, however, that

[5] The smoothness can at least be proved for low density.

ρ can be decomposed in a natural manner into a superposition of states ρ_α invariant only under a subgroup H of G. As an example take G the Euclidean group, H a group of lattice translations; ρ may describe a crystal state averaged over translations and rotations; there is a natural decomposition of ρ into states ρ_α corresponding to a given orientation and position of the crystal. The situation just described is called breakdown of the symmetry group G. A phase transition is often accompanied by symmetry breakdown or a sudden change in the type of symmetry breakdown.

Unfortunately, it is often not easy to make precise what is meant by natural decomposition of ρ into states ρ_α with lower symmetry. In fluid mechanics a decomposition into pure states is probably what is needed, but that would be inadequate in statistical mechanics.

8. THOM'S THEORY OF MORPHOGENESIS

In conclusion, I want to present a model which exhibits the sort of catastrophic transformations discussed before. In this model, there is at each point $\omega \in \mathbb{R}^4$ (space-time) a system described by a point $x_\omega \in \mathbb{R}^n$ (n is some integer, which may be large). It is assumed that, for each ω, there is a smooth potential V_ω in $\mathbb{R}^n$, that x_ω is obtained by taking the minimum of V_ω and that V_ω depends smoothly on ω. As ω and V_ω vary continuously, x_ω may jump suddenly (think of a potential V_ω with two minima, one being lower in some region, the other lower in another region). The ways in which these jumps may occur belong to a limited number of types (seven essentially), which R. Thom has classified[6]. Thom's motivation was provided by the phenomena of biological tissue differentiation (morphogenesis), which he thinks can be explained in terms of the model just described. Whether or not this is true[7], the theory provides the simplest examples of the catastrophic transformations typical of SINDFs.

[6] See THOM, R., Stabilité Structurelle et Morphogénèse, Benjamin, New York, to be published.

[7] One objection could be that tissues have a discrete cellular and, at another level, molecular structure instead of being smooth.

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