PHYSICS OF FAST AND INTERMEDIATE REACTORS

III

Proceedings of a Seminar, Vienna, 3-11 August 1961

INTERNATIONAL ATOMIC ENERGY AGENCY · VIENNA 1962

PHYSICS OF FAST AND INTERMEDIATE REACTORS

VOL. III

The following States are Members of the International Atomic Energy Agency:

AFGHANISTAN
ALBANIA
ARGENTINA
AUSTRALIA
AUSTRIA
BELGIUM
BRAZIL
BULGARIA
BURMA
BYELORUSSIAN SOVIET SOCIALIST REPUBLIC
CAMBODIA
CANADA
CEYLON
CHILE
CHINA
COLOMBIA
CONGO (LÉOPOLDVILLE)
CUBA
CZECHOSLOVAK SOCIALIST REPUBLIC
DENMARK
DOMINICAN REPUBLIC
ECUADOR
EL SALVADOR
ETHIOPIA
FINLAND
FRANCE
FEDERAL REPUBLIC OF GERMANY
GHANA
GREECE
GUATEMALA
HAITI
HOLY SEE
HONDURAS
HUNGARY
ICELAND
INDIA
INDONESIA
IRAN
IRAQ
ISRAEL
ITALY
JAPAN
REPUBLIC OF KOREA
LEBANON
LUXEMBOURG
MALI
MEXICO
MONACO
MOROCCO
NETHERLANDS
NEW ZEALAND
NICARAGUA
NORWAY
PAKISTAN
PARAGUAY
PERU
PHILIPPINES
POLAND
PORTUGAL
ROMANIA
SENEGAL
SOUTH AFRICA
SPAIN
SUDAN
SWEDEN
SWITZERLAND
THAILAND
TUNISIA
TURKEY
UKRAINIAN SOVIET SOCIALIST REPUBLIC
UNION OF SOVIET SOCIALIST REPUBLICS
UNITED ARAB REPUBLIC
UNITED KINGDOM OF GREAT BRITAIN AND NORTHERN IRELAND
UNITED STATES OF AMERICA
VENEZUELA
VIET-NAM
YUGOSLAVIA

The Agency's Statute was approved on 26 October 1956 at an international conference held at United Nations headquarters, New York, and the Agency came into being when the Statute entered into force on 29 July 1957. The first session of the General Conference was held in Vienna, Austria, the permanent seat of the Agency, in October, 1957.

The main objective of the Agency is "to accelerate and enlarge the contribution of atomic energy to peace, health and prosperity throughout the world".

Printed in Austria by Globus, Druck- und Verlagsanstalt, Vienna
March 1962

PROCEEDINGS SERIES

PHYSICS OF FAST AND INTERMEDIATE REACTORS

III

PROCEEDINGS OF THE SEMINAR
ON THE PHYSICS OF FAST AND INTERMEDIATE REACTORS
SPONSORED BY
THE INTERNATIONAL ATOMIC ENERGY AGENCY
AND HELD IN VIENNA, 3 — 11 AUGUST 1961

INTERNATIONAL ATOMIC ENERGY AGENCY
VIENNA 1962

PHYSICS OF FAST AND INTERMEDIATE REACTORS, IAEA, VIENNA, 1962
STI/PUB/49

FOREWORD

It is generally agreed that the ultimate economic advantage of power produced by nuclear fission over that produced by conventional sources depends on the ability of a certain type of reactor to breed precious nuclear fuel out of the plentiful but not readily fissionable isotope of uranium. This fact is mainly responsible for the importance attached to the development of fast power reactors, but many other interesting properties of unmoderated or weakly moderated reactor systems have also been brought to light by reactor physicists.

In August 1961 the Agency organized in Vienna a Seminar on the Physics of Fast and Intermediate Reactors, at which all the topics relating to this important branch of reactor science were discussed. The main feature of this meeting was extensive discussion of the 66 written contributions, which set the stage for a wide exchange of experience and ideas throughout 13 half-day sessions. The Seminar was attended by 132 scientists from 22 Member States and two international organizations.

It is hoped that these Proceedings of the Seminar, which include both the papers presented and a record of the discussions, will be useful as a reference work both to research workers in the field and to newcomers to it for many years to come. The Agency's thanks are due to all the participating scientists for their written or oral contributions and especially to those among them who, as session chairmen, led the discussions and contributed greatly to the success of the meeting.

March 1962

Scientific Secretary
Seminar on the Physics of Fast
and Intermediate Reactors

EDITORIAL NOTE

The papers and discussions incorporated in proceedings published by the International Atomic Energy Agency are checked for scientific accuracy by the Agency's experts in the subjects concerned and edited by the Agency's editorial staff to the extent considered necessary for the reader's assistance. The views expressed and the general style adopted remain, however, the responsibility of the named authors or participants.

The units and symbols employed are to the fullest practicable extent those standardized or recommended by the competent international scientific bodies.

The affiliations of authors are those given at the time of nomination.

The use in these Proceedings of particular designations of countries or territories does not imply any judgement by the Agency as to the legal status of such countries or territories, of their authorities and institutions or of the delimitation of their boundaries.

CONTENTS OF VOLUME III

IV. REACTOR DYNAMICS

IV.1. Kinetics

IV.2. Stability

IV.3. Doppler effect

IV.4. Safety problems

V. PHYSICS OF SPECIFIC REACTORS

PROCEEDINGS

IV. REACTOR DYNAMICS

1. KINETICS

COURBES D'INFLUENCE POUR LE CALCUL DE L'EFFET DES DISTORSIONS DE LA STRUCTURE SUR LA RÉACTIVITÉ*

F. STORRER**
BELGONUCLÉAIRE, BRUXELLES,
BELGIQUE

Abstract — Résumé — Аннотация — Resumen

Influence curves for calculation of the structural distortion effect on reactivity. Thermal structural expansion and distortion play a basic role in the feedback loop of fast reactors. They are pronounced because of the high figures of temperature increase, and the larger the portion of neutrons which escape from a given unit volume of the core, the greater is their effect on reactivity.

The calculation of the structural distortion effect on reactivity is especially complicated in the case of transients, because the spatial temperature distribution varies with time and the structure is generally hyperstatic to a very great extent. The calculation is far more simple if influence curves are first calculated showing the reactivity corresponding to distortion caused by unit point perturbation, either of the temperature or the radial temperature gradient. These influence curves are analogous, in respect of overall structural distortions, to the curves giving local temperature coefficients. On the basis of these curves, the reactivity corresponding to an arbitrary temperature distribution may be obtained by means of simple integration. Examples are given showing these curves and the method used for obtaining them.

The determination of temperature distribution has been treated by the author in document APDA-132.

Courbes d'influence pour le calcul de l'effet des distorsions de la structure sur la réactivité. Les dilatations et distorsions thermiques de la structure jouent un rôle primordial dans la boucle de rétroaction des réacteurs à neutrons rapides. En effet, elles sont fortes par suite des valeurs élevées de l'accroissement de température, et leur action sur la réactivité est d'autant plus grande que la fraction des neutrons qui s'échappe d'un volume unitaire du cœur est élevée.

Le calcul de l'effet des distorsions de la structure sur la réactivité est compliqué surtout lors de transitoires, car la distribution spatiale des températures varie avec le temps et la structure est en général hyperstatique à un très haut degré. Ce calcul est fortement simplifié si on calcule au préalable des courbes d'influence indiquant la réactivité correspondant à la distorsion provoquée par une perturbation ponctuelle unitaire, soit de la température soit du gradient radial de température. Ces courbes d'influence sont analogues pour les distorsions d'ensemble de la structure aux courbes donnant les coefficients locaux de température. A partir de ces courbes, la réactivité correspondant à une distribution arbitraire de la température peut être obtenue par simple intégration. Des exemples sont donnés, montrant de telles courbes, ainsi que la méthode employée pour les obtenir.

Le problème de la détermination de la distribution des températures a été traité par l'auteur dans le document APDA—132.

---

* Travail effectué à l'Atomic Power Development Associates, Detroit, Mich. (Etats-Unis).

** Détaché par Euratom au CEA.

Кривые влияния для расчета действия искажения структуры на реактивность. Тепловые расширения и искажения структуры играют первостепенную роль в контуре обратной связи реакторов на быстрых нейтронах. Действительно, они велики в связи с высокими значениями роста температуры и их действие на реактивность тем сильнее, чем больше доля нейтронов, вылетающих с единицы объема активной зоны.

Расчет действия искажений структуры на реактивность является сложным делом, особенно во время переходных режимов, так как пространственное распределение температур изменяется со временем, а структура, как правило, отличается очень высокой гиперстатичностью. Этот расчет значительно упрощается, если предварительно вычисляются кривые влияния, указывающие реактивность, соответствующую искажению, вызываемому точечным возмущением либо температуры, либо радиального температурного градиента. Кривая влияния искажения всей структуры подобна кривым, дающим местные коэффициенты температуры. Исходя из этих кривых, реактивность, соответствующая произвольному распределению температуры, можно определить при помощи простого интегрирования. Даются примеры таких кривых, а также методы их получения.

Напомним также, что проблема определения распределения температур обсуждалась автором в документе APDA-123.

Curvas de influencia para calcular el efecto de la distorsión de la estructura sobre la reactividad. Las dilataciones y distorsiones térmicas de la estructura ejercen una influencia primordial sobre el circuito de retroacción de los reactores rápidos. En efecto, se trata de fuertes deformaciones debido a los grandes aumentos de temperatura y su influencia sobre la reactividad es tanto mayor cuanto más elevada es la fracción de neutrones que escapa de un volumen unitario del cuerpo del reactor. El cálculo del efecto producido por las distorsiones de la estructura sobre la reactividad es complicado, especialmente durante el funcionamiento en régimen transitorio, pues la distribución espacial de las temperaturas varía en función del tiempo y en general la estructura es considerablemente hiperestática. El cálculo se puede simplificar apreciablemente si se establecen de antemano las curvas de influencia que indican la reactividad correspondiente a la distorsión provocada por una perturbación puntual unitaria, sea de la temperatura, sea del gradiente radial de ésta. Estas curvas de influencia son análogas, para las distorsiones del conjunto de la estructura, a las curvas que dan los coeficientes locales de temperatura. A partir de estas últimas curvas, la reactividad correspondiente a una distribución arbitraria de la temperatura puede determinarse por integración simple. El autor cita ejemplos de estas curvas y del método empleado para obtenerlas.

El problema de la determinación de la distribución de las temperaturas ha sido tratado en el documento APDA-132.

Introduction

Les dilatations et les distorsions des divers constituants et des éléments structuraux d'un réacteur rapide fournissent la plus grande partie de la contre-réaction de réactivité. Ceci provient, d'une part, de la faible valeur des autres sources de contre-réaction, et, d'autre part, de la grande influence des détails de la géométrie sur la réactivité dans un cœur où les fuites de neutrons sont élevées.

Ce dernier point peut être illustré par un exemple simple. On montre facilement [1] que, dans un réacteur nu, la variation de réactivité résultant d'une dilatation uniforme du réacteur est donnée par:

$$\frac{\triangle k}{k} = -4\frac{\triangle L}{L}\frac{K_\infty - 1}{k_\infty} \tag{1}$$

où $\Delta L/L$ est la variation relative des dimensions linéaires du réacteur. Dans

un réacteur critique $(k\infty^{-1})/k\infty$ a n'est autre que la fraction des neutrons s'échappant du réacteur. $k\infty$ et par conséquent $(k\infty/-1)/k\infty$ sont particulièrement élevés dans un réacteur rapide, même de grandes dimensions. $\Delta L/L = \alpha \Delta T$ est également élevé dans un réacteur rapide, par suite de la grande valeur des accroissements de température ΔT.

Dans ce rapport nous présentons une méthode qui permet de calculer la contre-réaction de réactivité résultant de ces dilatations et distorsions. L'emploi de cette méthode est utile en régime stationnaire, tandis qu'il est impératif dans le cas de transitoires.

Pour les phénomènes usuels entraînant une contre-réaction de réactivité, tels que l'effet Doppler et l'expansion du réfrigérant, la contre-réaction de réactivité $R(t)$, tant en régime stationnaire qu'au cours d'un transitoire, se calcule en général par une ou plusieurs intégrales du type:

$$R(t) = \int^{v} T(\mathbf{r}', t)\, E(\mathbf{r}')\, \mathrm{d}\,\mathbf{r}' \tag{2}$$

où $T(\mathbf{r}', t)$ est la température au point $\mathbf{r}'$ et au temps t, mesurée à partir d'une certaine référence, et où $E(\mathbf{r}')$ est le coefficient local de température. Cette intégrale suppose qu'il existe au moins un certain domaine de valeurs où la réactivité dépend linéairement des températures.

En pratique, on écrit des intégrales séparées pour chaque constituant du réacteur, et même souvent pour chaque phénomène amenant une contre-réaction. Notons que, dans la plupart des cas, le calcul du coefficient de température $E(\mathbf{r}')$ est basé sur des valeurs locales de certaines propriétés du matériau envisagé, telles que le coefficient de danger.

Déjà, dans le calcul de l'effet de la dilatation axiale des éléments combustibles du cœur, le calcul de $E(\mathbf{r}')$ fait appel non seulement à la valeur locale du coefficient de danger mais aussi à sa valeur à l'extrémité de l'élément. Dans le cas des dilatations ou distorsions d'ensemble de la structure et des assemblages d'éléments combustibles, provoquées par des pointes de température ou par des gradients de température, le calcul de la fonction $E(\mathbf{r}')$ est plus compliqué. En effet, une perturbation locale de température au point $\mathbf{r}'$ peut provoquer des distorsions et des déplacements de matériaux dans une région étendue ou même dans tout le réacteur. Le calcul de $E(\mathbf{r}')$ à ce point $\mathbf{r}'$ implique donc dans ce cas les déplacements et les coefficients de danger dans tout le volume affecté par une perturbation de température en $\mathbf{r}'$. On peut voir que $E(\mathbf{r}')$ est la contre-réaction de réactivité pour une température qui est une fonction de Dirac à trois dimensions au point $\mathbf{r}'$. Le calcul est souvent compliqué, car la structure du réacteur est en général hyperstatique à un haut degré.

Nous présenterons tout d'abord la formulation générale liant la contre-réaction de réactivité au champ vectoriel des déplacements et au champ scalaire des températures. Cette formulation générale est appliquée à titre d'exemple à des cas simples et bien connus — tels que l'expansion du réfrigérant et la dilatation axiale des éléments combustibles — et ensuite à des cas plus compliqués, tels que l'arcage des assemblages provoqué par des gradients transversaux de température.

Notons que la formulation générale se simplifie souvent dans des cas particuliers. C'est ainsi que la variable $\mathbf{r}'$ de l'intégrale peut souvent être remplacée par une seule coordonnée, ce qui réduit l'intégrale à trois dimensions à une intégrale à une dimension.

Nous verrons aussi que, dans certains cas, il est préférable de prendre un gradient de température comme variable indépendante, plutôt que la température. Au coefficient de température $E(\mathbf{r}')$ vient alors se substituer un coefficient de gradient.

En général, la fonction $E(\mathbf{r}')$ portée en graphique en fonction de $\mathbf{r}'$ ou d'une de ses coordonnées est une courbe d'influence indiquant la contre-réaction de réactivité pour une perturbation en fonction de Dirac — nous dirons «perturbation unitaire» — de la température ou du gradient de température au point $\mathbf{r}'$.

Il est important de noter que, pour ces distorsions d'ensemble de la structure, la fonction $E(\mathbf{r}')$ peut atteindre des valeurs élevées pour des points $\mathbf{r}'$ où les coefficients de danger des matériaux sont pratiquement nuls, par exemple à un point de la structure de support aval des assemblages. Ceci est particulièrement important par suite à la fois des hautes températures prévalant dans ces régions et des grands retards affectant ces températures.

1. Formulation générale

Les formules analytiques liant l'effet de réactivité d'une distorsion ou dilatation de la structure aux champs des déplacements et des températures sont présentées ici dans leur forme la plus générale. Elles peuvent évidemment se simplifier dans des cas particuliers.

1.1. Calcul des déplacements $\boldsymbol{\delta}(\mathbf{r}, t)$ en fonction des températures $T(\mathbf{r}', t)$

Si la structure du réacteur est élastique, c'est-à-dire si elle répond partout à la loi de Hooke, si les supports sont élastiques et s'il n'y a pas de discontinuité dans les contacts entre les éléments eux-mêmes, ou entre ceux-ci et leurs supports, et si le coefficient de dilatation thermique est également constant, on peut appliquer le principe de superposition, pour les petites déformations, même si la structure est fortement hyperstatique:

$$\boldsymbol{\delta}(\mathbf{r}, t) = \int^{v} T(\mathbf{r}', t)\, \mathbf{d}(\mathbf{r}', \mathbf{r})\, \mathrm{d}\mathbf{r}'. \tag{3}$$

Ici le vecteur $\boldsymbol{\delta}$ indique à la fois l'amplitude et la direction du déplacement du point $\mathbf{r}$ de la structure, et T est la température. On peut vérifier facilement que le noyau vectoriel de l'intégrale $\mathbf{d}(\mathbf{r}', \mathbf{r})$ donne le déplacement du point $\mathbf{r}$ pour une température qui soit une fonction de Dirac en trois dimensions au point $\mathbf{r}'$. Dans certains cas, lorsque la distribution de température dans des éléments de grand allongement peut s'écrire:

$$T(z, y) = T(z) + y\, G(z)$$

où $T(z)$ est la température moyenne à la position axiale z et $G(z)$ est le gradient de température dans la direction transversale y, il est plus simple de scinder l'intégrale (3) en deux parties:

$$\boldsymbol{\delta}_1(\mathbf{r}) = \int T(z')\, \mathbf{d}_1(z', \mathbf{r})\, \mathrm{d}z' \tag{4}$$

et

$$\boldsymbol{\delta}_2(\mathbf{r}) = \int G(z')\, \mathbf{d}_2(z', \mathbf{r})\, \mathrm{d}z' \tag{5}$$

où $\mathbf{d}_2(z', \mathbf{r})$ est le déplacement en $\mathbf{r}$ résultant d'un gradient unitaire en z'.

Dans la suite de cet exposé nous emploierons la notation (3), étant bien entendu que T peut représenter soit une température soit un gradient de température, et que l'intégrale à trois dimensions peut être remplacée par une intégrale à une ou deux dimensions.

L'intégrale est étendue à tout le volume du réacteur, mais il est évident que le noyau $\mathbf{d}(\mathbf{r}', \mathbf{r})$ s'annule lorsque la température au point $\mathbf{r}'$ n'influe pas sur le déplacement au point $\mathbf{r}$.

Il est à noter que l'expression (3) suppose qu'il n'y a aucun retard entre les températures et les déplacements qui en résultent. Elle ne peut donc être employée telle quelle que pour des transitoires suffisamment lents pour que l'inertie de la structure puisse être négligée. C'est le cas de tous les transitoires envisagés lors de la conduite normale du réacteur et lors d'études de stabilité, le retard entre la puissance et la contre-réaction de réactivité provenant presque exclusivement des délais inhérents à l'échange thermique.

1. 2. Calcul de la réactivité $R(t)$ en fonction des déplacements $\boldsymbol{\delta}(\mathbf{r}, t)$

Dans l'hypothèse où tous les déplacements sont suffisamment faibles pour qu'on puisse calculer leur effet de réactivité par une méthode de perturbation, on peut, à nouveau, invoquer le principe de superposition dans le calcul de la contre-réaction de réactivité. Si le réacteur ne comportait qu'un seul constituant homogène dont le coefficient de danger par unité de volume serait $\varrho(\mathbf{r})$, la contre-réaction de réactivité résultant de distorsions ou de dilatations pourrait se calculer par une quelconque des deux expressions suivantes:

$$R(t) = \int^{v} \boldsymbol{\delta}(\mathbf{r}, t) \cdot \operatorname{grad} \varrho(\mathbf{r})\, \mathrm{d}\mathbf{r} \tag{6. a}$$

ou

$$R(t) = -\int^{v} \varrho(\mathbf{r}) \operatorname{div} \boldsymbol{\delta}(\mathbf{r}, t)\, \mathrm{d}\mathbf{r}. \tag{6. b}$$

Ces deux expressions sont équivalentes, car leur différence

$$\int^{v} (\boldsymbol{\delta} \cdot \operatorname{grad} \varrho + \varrho \operatorname{div} \boldsymbol{\delta})\, \mathrm{d}\mathbf{r} = \int^{v} \operatorname{div} (\varrho \boldsymbol{\delta})\, \mathrm{d}\mathbf{r} = \int^{s} \varrho\, \boldsymbol{\delta} \cdot \mathrm{d}\boldsymbol{\sigma} \tag{7}$$

Cette dernière expression est nulle si la surface S est prise suffisamment grande pour que le vecteur $\varrho\boldsymbol{\delta}$ s'y annule.

Dans un réacteur hétérogène où les fonctions figurant dans l'intégrale présentent des discontinuités, il faut employer l'une ou l'autre des expressions (6.a) ou (6.b), selon le cas: pour l'expansion du réfrigérant, il faut employer l'expression (6.b), tandis que pour les distorsions il faut employer l'expression (6.a).

1. 3. Calcul de la réactivité en fonction des températures $T(\mathbf{r}' t)$

En substituant l'expression (3) dans l'expression (6.a) ou (6.b) on obtient respectivement:

$$R(t) = \int^{v}\int^{v} T(\mathbf{r}', t)\, \mathbf{d}(\mathbf{r}', \mathbf{r}) \cdot \operatorname{grad} \varrho(\mathbf{r})\, \mathrm{d}\mathbf{r}\, \mathrm{d}\mathbf{r}' \tag{8. a}$$

ou

$$R(t) = -\int^{v}\int^{v} T(\mathbf{r}', t)\, \varrho(\bar{\mathbf{r}}) \operatorname{div}_{\mathbf{r}} \mathbf{d}(\mathbf{r}', \mathbf{r})\, \mathrm{d}\mathbf{r}\, \mathrm{d}\mathbf{r}'. \tag{8. b}$$

La notation $\mathrm{div}_{\mathbf{r}}$ indique que la divergence doit être prise par rapport à la variable $\mathbf{r}$.

En posant:

$$E(\mathbf{r}') = \int^{\nu} \mathbf{d}(\mathbf{r}', \mathbf{r}) \cdot \mathrm{grad}\, \varrho(\mathbf{r})\, \mathrm{d}\mathbf{r} \tag{9. a}$$

ou:

$$E(\mathbf{r}') = -\int^{\nu} \varrho(\mathbf{r})\, \mathrm{div}_{\mathbf{r}}\, \mathbf{d}(\mathbf{r}', \mathbf{r})\, \mathrm{d}\mathbf{r} \tag{9. b}$$

les expressions deviennent:

$$R(t) = \int^{\nu} T(\mathbf{r}', t)\, E(\mathbf{r}')\, \mathrm{d}\mathbf{r}'. \tag{10}$$

Le noyau scalaire $E(\mathbf{r}')$ de l'intégrale représente la contre-réaction de réactivité pour une température qui soit une fonction de Dirac en $\mathbf{r}'$.

Rappelons qu'ici aussi on peut dans certains cas préférer prendre comme variable un gradient de température plutôt que la température:

$$R(t) = \int G(z', t)\, E_G(z')\, \mathrm{d}z'. \tag{11}$$

Les fonctions $E(\mathbf{r}')$ et $E_G(z')$ doivent être déterminées au préalable, à partir des valeurs des coefficients de danger et des distorsions résultant de perturbations locales de température.

Les fonctions $T(\mathbf{r}', t)$ et $G(z', t)$ peuvent être obtenues par des méthodes analytiques, digitales ou analogiques.

La contre-réaction de réactivité peut dès lors être obtenue par simple intégration ou sommation.

Les fonctions $E(\mathbf{r}')$ et $E_G(z')$ ne sont autres que les coefficients locaux de température ou de gradient. Elles peuvent être définies pour tous les phénomènes amenant une contre-réaction de réactivité, mais dans ce rapport nous insistons plus particulièrement sur leur application aux distorsions ou dilatations de la structure ou du combustible.

2. Exemples de calcul

Nous appliquons tout d'abord la formulation générale à deux exemples triviaux et ensuite nous traitons divers cas de distorsion par arcage des assemblages d'éléments combustibles.

2. 1. Expansion du réfrigérant

Dans ce cas:

$$\mathrm{div}_{\mathbf{r}}\, \mathbf{d}(\mathbf{r}', \mathbf{r}) = 3\alpha\, \delta(\mathbf{r} - \mathbf{r}') \tag{12}$$

où 3α est le coefficient volumétrique d'expansion thermique, et par (9.b):

$$\begin{aligned} E(\mathbf{r}') &= -3\alpha \int^{\nu} \varrho(\mathbf{r})\, \delta(\mathbf{r} - \mathbf{r}')\, \mathrm{d}\mathbf{r} \\ &= -3\alpha\, \varrho(\mathbf{r}') \end{aligned} \tag{13}$$

où ϱ est le coefficient de danger du réfrigérant par unité de volume.

$$R(t) = -3\alpha \int^{\nu} T(\mathbf{r}', t)\, \varrho(\mathbf{r}')\, \mathrm{d}\mathbf{r}' \tag{14}$$

les intégrales s'étendant évidemment au volume du réfrigérant uniquement.

On peut remplacer cette intégrale par une intégrale s'étendant à tout le volume du cœur:

$$R(t) = -3\alpha V_R \int^{\nu} T(\mathbf{r}', t)\, \varrho(\mathbf{r}')\, \mathrm{d}\mathbf{r}' \tag{14'}$$

où V_R est la fraction volumétrique du réfrigérant dans le cœur. Il est évident que la fonction $T(\mathbf{r}', t)$ doit être rendue artificiellement continue.

Dans ce cas très simple où un accroissement de température n'a qu'un effet purement local, la fonction $E(\mathbf{r}')$ n'est donc autre que le coefficient de danger.

2. 2. Dilatation axiale des éléments combustibles du cœur

Prenons, par exemple, le cas où les éléments combustibles du cœur sont attachés à leur base $z=0$, et où ils sont libres de coulisser axialement par rapport à la structure de l'assemblage. Leur déplacement axial ne sera fonction que de la température moyenne à chaque section transversale z', température que nous appellerons $T_i(z')$ pour l'élément i.

Pour cet élément, le déplacement $\mathbf{d}_i(z', z)$ à la section z pour une variation unitaire de la température moyenne en z' est donné par:

$$\left.\begin{aligned} \mathbf{d}_i(z', z) &= \alpha\, \mathbf{1}_z && \text{pour } z > z' \\ &= 0 && \text{pour } z < z' \end{aligned}\right\} \tag{15}$$

où $\mathbf{1}_z$ est le vecteur unitaire dans la direction z.

Pour cet élément on aura donc, si la variation radiale du coefficient de danger à l'intérieur de chaque élément peut être négligée:

$$\begin{aligned} E_i(z') &= A \int_0^H \mathbf{d}_i(z', z) \cdot \operatorname{grad} \varrho_i(z)\, \mathrm{d}z \\ &= A\alpha \int_{z'}^{H} \frac{\mathrm{d}\varrho_i(z)}{\mathrm{d}z}\, \mathrm{d}z = A\alpha\, [\varrho_i(H) - \varrho_i(z')] \end{aligned} \tag{16}$$

où A est l'aire de l'élément combustible.

On obtient ensuite:

$$R_i(t) = A\alpha \int_0^H T_i(z', t)\, [\varrho_i(H) - \varrho_i(z')]\, \mathrm{d}z'. \tag{17}$$

La contre-réaction de réactivité provenant de la dilatation axiale de tous les éléments du cœur peut s'obtenir en prenant la somme des $R_i(t)$ pour tous les éléments i. Cette somme peut être remplacée approximativement par l'intégrale:

$$R(t) = 2\pi\alpha V_c \int_0^H \int_0^R r'\, T(r', z', t)\, [\varrho(r', H) - \varrho(r', z')]\, \mathrm{d}z'\, \mathrm{d}r' \tag{18}$$

où V_c est la fraction volumétrique du combustible dans le cœur. Notons que, par suite de l'emploi du facteur V_c, l'intégrale s'étend à tout le volume du cœur. $T(r', z', t)$ est une fonction rendue artificiellement continue dont la valeur au centre de chaque élément combustible représente la température moyenne de cet élément à la cote z'.

2. 3. Arcage des assemblages — Cas isostatique

Prenons le cas où les assemblages sont encastrés à leur base et où le gradient radial de température dans les assemblages arque ceux-ci vers l'extérieur, entraînant avec eux le combustible. Nous supposons que les assemblages n'entrent jamais en contact entre eux. Le comportement de chaque assemblage est donc indépendant.

Nous prenons le gradient radial comme variable indépendante, soit $G_i\,(z', t)$ pour l'assemblage i.

$\mathbf{d}_i\,(z', z)$ est le déplacement de l'assemblage au niveau z, causé par un gradient en fonction de Dirac à une dimension au niveau z':

$$G_i\,(z) = \delta\,(z - z'). \tag{19}$$

On trouve facilement qu'un tel gradient produit une cassure d'angle α au niveau z', tandis que pour $z \neq z'$ l'assemblage reste rectiligne. α est le coefficient de dilatation thermique.

Dès lors:

$$\left.\begin{aligned} \mathbf{d}_i\,(z', z) &= \alpha\,(z - z')\,\mathbf{1}_r && \text{pour } z > z' \\ &= 0 && \text{pour } z < z' \end{aligned}\right\} \tag{20}$$

où $\mathbf{1}_r$ est le vecteur unitaire dans le sens radial. On obtient alors:

$$\begin{aligned} E_i\,(z') &= A \int_0^H \mathbf{d}_i\,(z', z) \cdot \operatorname{grad} \varrho_i\,(r, z)\,\mathrm{d}z \\ &= \alpha\, A \int_{z'}^H (z - z')\,\frac{\delta\,\varrho_i\,(r, z)}{\delta\, r}\,\mathrm{d}z \end{aligned} \tag{21}$$

où A et ϱ sont l'aire et le coefficient de danger du combustible dans l'assemblage. Les variations de réactivité provenant du déplacement des matériaux structuraux et du réfrigérant sont beaucoup plus faibles, et on peut éventuellement en tenir compte en ajustant la fonction $A\,\varrho$.

On a donc pour l'assemblage i:

$$R_i\,(t) = \alpha\, A \int_0^H \mathrm{d}z'\; G_i\,(z', t) \int_{z'}^H (z - z')\,\frac{\delta\,\varrho_i\,(r, z)}{\delta\, r}\,\mathrm{d}z. \tag{22}$$

Pour l'ensemble des assemblages on peut à nouveau remplacer la sommation par une intégrale, et on obtient:

$$R\,(t) = 2\pi\,\alpha\, V_c \int_0^R \int_0^H \mathrm{d}z'\,\mathrm{d}r'\,r'\,G\,(r', z', t) \int_{z'}^H (z - z')\,\frac{\delta\,\varrho\,(r', z)}{\delta\, r'}\,\mathrm{d}z \tag{23}$$

où V_c est la fraction volumétrique du combustible dans le cœur.

2. 4. Arcage des assemblages — Cas hyperstatique simple

Nous considérons ici le cas où chaque assemblage est hyperstatique, mais où les assemblages n'ont aucun contact entre eux. Ce serait le cas, par exemple, si les assemblages étaient encastrés à leur base et reliés en plus à une plaque de tête par une attache rigide ou élastique. Le principe de superposition est toujours applicable, et les formules sont similaires à celles du paragraphe pré-

cédent exception faite pour la déflection $\mathbf{d}_i(z', z)$, qui doit être calculée par un calcul hyperstatique et n'est pas nulle pour $z < z'$.

On a donc:

$$E(z') = A\int_0^H d_i(z', z)\,\frac{\delta\,\varrho_i(r, z)}{\delta\, r}\,\mathrm{d}z \tag{24}$$

$$R_i(t) = A\int_0^{H_T}\mathrm{d}z'\,G_i(z', t)\int_0^H d_i(z', z)\,\frac{\delta\,\varrho_i(r, z)}{\delta\, r}\,\mathrm{d}z \tag{25}$$

$$R(t) = 2\pi\, V_c\int_0^{RH_T}\int_0^{}\mathrm{d}z'\,\mathrm{d}r'\,r'\,G(r', z', t)\int_0^H d_i(z'\,z)\,\frac{\delta\varrho\,(r', z)}{\delta r'}\,\mathrm{d}z. \tag{26}$$

$z = H$ représente la surface supérieure du cœur, ou celle de la couverture si on tient compte des effets de réactivité provenant de la couverture, tandis que $z = H_T$ représente le sommet de l'assemblage, même si celui-ci est hors de la partie réactive du réacteur. En effet, dans le cas hyperstatique, un gradient dans cette partie de l'assemblage peut amener des distorsions appréciables dans le cœur.

3. Séparation des variables

Dans les exemples simples qui précèdent, la contre-réaction de réactivité peut être calculée même si les fonctions T, G et ϱ ne sont pas séparables en une fonction de z et une fonction de r. Si on peut faire l'hypothèse de la séparabilité de ces fonctions sans commettre une approximation trop grossière, le calcul est fortement simplifié. Pour les cas plus complexes, comme dans l'exemple 4 qui suit, le calcul deviendrait excessivement compliqué si on ne pouvait faire l'hypothèse de la séparabilité.

Notons que, pour une température d'entrée constante, les fonctions T et G sont séparables si la densité de puissance est séparable et si la géométrie et le débit sont les mêmes pour tous les chenaux parallèles. Le cas où les températures d'entrée sont variables peut être traité séparément, car leur propagation dans le cœur est la même pour tous les chenaux.

Examinons d'abord les exemples précédents dans le cas où les fonctions sont séparables.

3. 1. Expansion du réfrigérant

Si
$$T(\mathbf{r}, t) = f(r)\,T(z, t) \tag{27}$$

et
$$\varrho(\bar{r}) = \varrho_r(r)\,\varrho_z(z) \tag{28}$$

l'expression (14′) devient:

$$\begin{aligned} R(t) &= 3\alpha\, V_R\int_0^H\int_0^R 2\pi\, r'\, f(r')\,T(z', t)\,\varrho_r(r')\,\varrho_z(z')\,\mathrm{d}r'\,\mathrm{d}z' \\ &= -6\pi\alpha\, V_R\int_0^R r'\, f(r')\,\varrho_r(r')\,\mathrm{d}r' \times \int_0^H T(z', t)\,\varrho_z(z')\,\mathrm{d}z' \\ &= \int_0^H T(z', t)\,E_1(z')\,\mathrm{d}z' \end{aligned} \tag{29}$$

où

$$E_1(z') = -6\pi\alpha V_R \varrho_z(z') \int_0^R r' f(r') \varrho_r(r') \, dr. \tag{30}$$

La fonction $E_1(z')$ peut être calculée une fois pour toutes, et seule l'intégrale en z' est fonction du transitoire envisagé.

3.2. Dilatation axiale des éléments combustibles du cœur

En faisant les mêmes hypothèses, l'expression (18) devient:

$$R(t) = 2\pi\alpha V_c \int_0^R r' f(r') \varrho_r(r') \, dr' \times \int_0^H T(z', t) [\varrho_z(H) - \varrho_z(z')] \, dz'$$

$$= \int_0^H T(z', t) E_2(z') \, dz' \tag{31}$$

où

$$E_2(z') = 2\pi\alpha V_c [\varrho_z(H) - \varrho_z(z')] \int_0^R r' f(r') \varrho_r(r') \, dr. \tag{32}$$

3.3. Arcage des assemblages — Cas isostatique

A l'aide des relations (27) et (28) on trouve:

$$G(\mathbf{r}, t) = \frac{\delta T(\mathbf{r}, t)}{\delta r} = f'(r) T(z, t) \tag{33}$$

$$\text{et: } \frac{\delta \varrho(r', z)}{\delta r'} = \varrho_r'(r') \varrho_z(z)$$

et l'expression (23) devient:

$$R(t) = 2\pi\alpha V_c \int_0^R r' f'(r') \varrho_r'(r') \, dr' \times \int_0^H dz' \, T(z', t) \int_{z'}^H (z - z') \varrho_z(z) \, dz$$

$$= \int_0^H T(z', t) E_3(z') \, dz' \tag{34}$$

où:

$$E_3(z') = 2\pi\alpha V_c \int_0^R r' f'(r') \varrho_r'(r') \, dr' \times \int_{z'}^H (z - z') \varrho_z(z) \, dz. \tag{35}$$

3.4. Arcage des assemblages — Cas hyperstatique simple

$$R(t) = 2\pi V_c \int_0^R r' f'(r') \varrho_r'(r') \, dr' \times \int_0^{H_T} dz' \, T(z', t) \int_0^H d_i(z', z) \varrho_z(z) \, dz$$

$$= \int_0^{H_T} T(z', t) E_4(z') \, dz' \tag{36}$$

où :

$$E_4(z') = 2\pi V_c \int_0^R r' f'(r') \varrho'_r(r') \, dr' \times \int_0^H d_i(z', z) \varrho_z(z) \, dz. \tag{37}$$

4. Arcage des assemblages — Cas hyperstatique général

Dans ce cas, chaque assemblage est hyperstatique en lui-même et, de plus, il est en contact permanent avec ses voisins en un ou plusieurs points. A l'hyperstaticité axiale vient donc s'ajouter une hyperstaticité radiale, car les réactions agissant sur chaque assemblage sont fonctions des déflections et de la rigidité de l'ensemble. Ainsi, par exemple, dans le réacteur Enrico Fermi, chaque assemblage est encastré à sa base, et son sommet est relié à la plaque de tête par un support élastique («hold-down finger»). De plus, chaque assemblage est en contact permanent avec ses voisins à hauteur des plaquettes, un peu au-dessus du plan médian du cœur.

La méthode générale exposée dans ce rapport est toujours applicable, mais, vu la complexité du problème, son application serait d'une difficulté prohibitive si les variables ne pouvaient être séparées.

Dans le calcul de l'effet d'arcage dans le réacteur Fermi, nous avons donc supposé que les variables sont séparables. Le détail de ce calcul est présenté dans un mémorandum interne APDA [2]. Nous en présentons ici les grandes lignes.

On suppose donc que $T(\mathbf{r}, t) = f(r)\, T(z, t)$ où

$$\frac{2}{R^2} \int_0^R r f(r) \, dr = 1$$

de sorte que $T(z, t)$ représente la température moyenne au niveau z. On a donc :

$$G(\mathbf{r}, t) = f'(r)\, T(z, t).$$

Dans le calcul, on tient compte du fait que le support en tête de l'assemblage est flexible, et que les assemblages sont légèrement compressibles dans le sens transversal à hauteur des plaquettes. Le calcul, tant pour un $T(\mathrm{z}, \mathrm{t})$ arbitraire, que pour un $T(z, t)$ en fonction de Dirac, est divisé en trois parties.

On calcule : *a*) les déformations et la réactivité pour le cas des assemblages isostatiques encastrés à leur base, *b*) les déformations et la réactivité introduites par les liaisons à la plaque de tête, et *c*) les déformations et la réactivité introduites par les liaisons au niveau des plaquettes.

Les formules intervenant dans les parties *b*) et *c*) de ces calculs peuvent également servir à calculer les déformations et la réactivité introduites par la dilatation thermique de la plaque de tête et par celle des boîtiers des assemblages au niveau des plaquettes.

Comme pour le cas hyperstatique simple (voir équation 36), nous avons pu exprimer la contre-réaction de réactivité provoquée par l'arcage des assemblages par une intégrale :

$$R(t) = \int_0^{H_T} T(z', t)\, E(z') \, dz' \tag{38}$$

où, comme nous l'avons défini plus haut, $T(z', t)$ est une mesure des gradients au niveau z'.

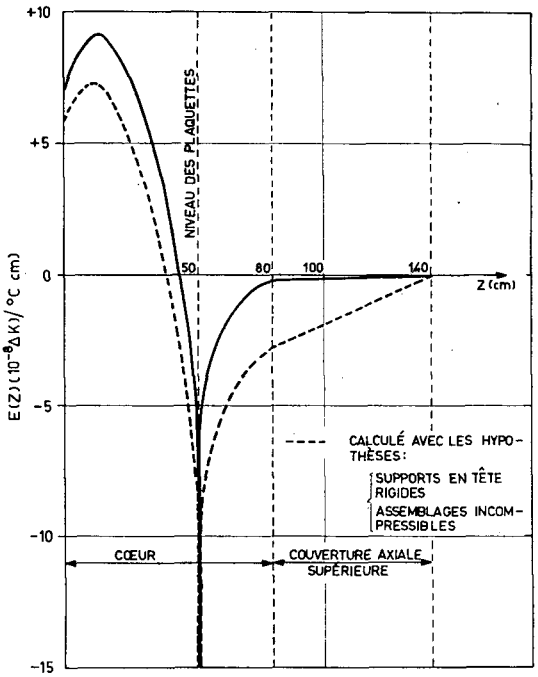


Figure 1
Fonction $E_{(z')}$.

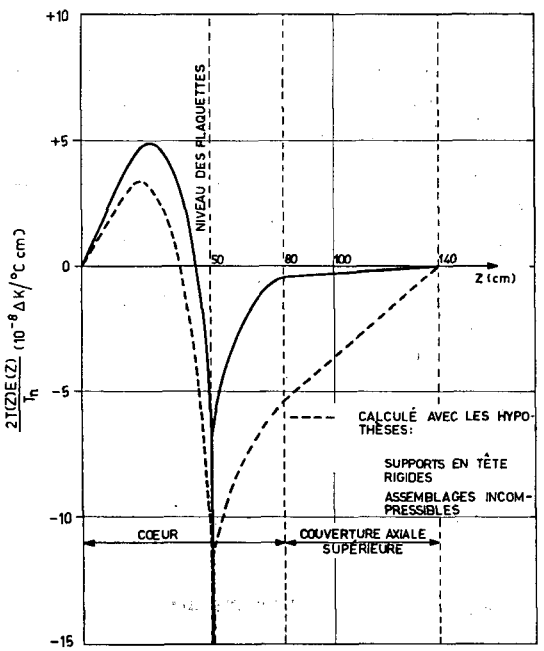


Figure 2
Fonction $T_{(z')}\,E_{(z')}$ pour une fonction $T_{(z')}$.

La figure 1 montre la fonction $E(z')$ depuis la base du cœur jusqu'au sommet de la couverture axiale supérieure. La courbe n'a pas été dessinée pour la couverture axiale inférieure, car les gradients y sont négligeables. Notons aussi que les effets de réactivité ont été calculés pour les déplacements dans le cœur seulement. On voit que pour un gradient à la base du cœur la contre-réaction est positive, tandis qu'elle est négative dans la partie supérieure du cœur et dans la couverture.

Nous avons également représenté en pointillés cette même fonction $E(z')$ calculée dans l'hypothèse où les doigts supportant la tête des assemblages seraient parfaitement rigides et où les assemblages ne seraient pas du tout compressibles dans le sens radial au niveau des plaquettes. Il est clair que cette absence de compressibilité rendrait le coefficient $E(z')$ plus négatif.

Sur la figure 2 nous avons représenté la fonction $T(z')\,E(z')$ pour une fonction $T(z')$,qui est une bonne approximation de la distribution de température en régime stationnaire, soit $T(z')$ variant linéairement dans le cœur depuis zéro jusqu'à sa valeur maximale T_m, et restant ensuite constante dans la couverture. Sur la figure, cette fonction a été normalisée comme suit: $2\,T(z')\,E(z')/T_m$ de sorte que l'intégrale de cette courbe donne la contre-réaction de réactivité pour une variation d'un degré de la température moyenne du réfrigérant dans le cœur. La courbe en pointillés correspond à la courbe en pointillés de la figure 1.

L'intégrale de la courbe de la figure 2 donne donc le même résultat que celui que l'on obtient en général directement pour la configuration stationnaire des gradients de température.

La courbe de la figure 1 permet également de calculer ce cas particulier, mais, en plus, elle permet de calculer la contre-réaction pour une configuration quelconque des gradients. Si l'on suppose, par exemple, un mélange parfait du réfrigérant à la sortie du cœur, le gradient — ainsi que la fonction $T(z', t)$ qui en est une mesure — s'annule dans la couverture. On peut déduire immédiatement de la figure 1 ce que sera dans ce cas la contre-réaction de réactivité, par une simple intégration. Même avant tout calcul, on remarque que la contre-réaction sera altérée dans un sens positif.

Mais le principal intérêt de cette courbe d'influence réside évidemment dans les calculs de transitoires, car dans ce cas la configuration des gradients de température varie de façon continue. A chaque instant, la contre-réaction de réactivité est donnée par l'expression (38).

En pratique, $T(z, t)$ est calculé soit par une machine digitale, soit par simulation. Dans ce dernier cas, on obtient les valeurs de $T(z, t)$ pour un nombre discret de sections axiales. L'intégrale est alors remplacée par la somme:

$$R(t) = \sum_{i=1}^{n} T_i(t)\,E_i \tag{39}$$

où

$$E_i = \int^{\text{section } i} E(z)\,\mathrm{d}z \tag{40}$$

et $T_i(t)$ est la valeur moyenne de $T(z, t)$ pour la section i, telle qu'elle est obtenue par simulation.

Remarquons que l'on peut tirer des conclusions qualitatives par simple inspection de la courbe de la figure 1. Lors d'un transitoire, la réponse des tempé-

ratures est d'autant plus rapide que z est plus petit. Comme par ailleurs $E(z)$ est positif pour les faibles valeurs de z, on peut conclure que la contribution de l'arcage à la contre-réaction de réactivité sera moins négative lors d'un transitoire et pourrait même devenir positive pour un transitoire suffisamment rapide.

5. Remarques diverses

5.1. Calcul des températures et de la contre-réaction en fonction de la puissance

En restant toujours dans le domaine linéaire, on peut écrire:

$$T(\mathbf{r}', t) = \int_0^{\infty} \int^{\nu} Q(\mathbf{r}'', t - t') L(\mathbf{r}'', \mathbf{r}', t') \, dt' \, d\mathbf{r}'' + \int_0^{\infty} T_e(t - t') M(\mathbf{r}', t') \, dt' \qquad (41)$$

où $Q(\mathbf{r}, t)$ est la densité de puissance et T_e est la température d'entrée du réfrigérant.

En général, il suffit de considérer le mode fondamental de la densité de puissance et on peut écrire:

$$Q(\mathbf{r}, t) = q(\mathbf{r}) P(t) \qquad (42)$$

et (41) peut donc s'écrire:

$$T(\mathbf{r}', t') = \int_0^{\infty} P(t - t') N(\mathbf{r}', t') \, dt' + \int_0^{\infty} T_e(t - t') M(\mathbf{r}', t') \, dt' \qquad (43)$$

où:

$$N(\mathbf{r}', t') = \int^{\nu} q(\mathbf{r}'') L(\mathbf{r}'', \mathbf{r}', t') \, d\mathbf{r}'' . \qquad (44)$$

En substituant cette expression (43) dans (2) on obtient:

$$R(t) = \int_0^{\infty} \int^{\nu} P(t - t') N(\mathbf{r}', t') E(\mathbf{r}') \, d\mathbf{r}' \, dt' + \int_0^{\infty} \int^{\nu} T_e(t - t') M(\mathbf{r}', t') E(\mathbf{r}') \, d\mathbf{r}' \, dt$$

$$= \int_0^{\infty} P(t - t') k(t') \, dt' + \int_0^{\infty} T_e(t - t') I(t') \, dt' \qquad (45)$$

$$\text{où: } k(t') = \int^{\nu} N(\mathbf{r}', t') E(\mathbf{r}') \, d\mathbf{r}' \qquad (46)$$

$$I(t') = \int^{\nu} M(\mathbf{r}', t') E(\mathbf{r}') \, d\mathbf{r}'. \qquad (47)$$

5.2. Cas où l'inertie est importante

On ne peut négliger l'inertie si la structure est telle que le rapport inertie sur rigidité est suffisamment élevé pour amener entre les températures et les déformations qui en résultent des retards qui sont du même ordre que les retards inhérents à l'échange thermique. Il faut également tenir compte de l'inertie lors de transitoires extrêmement rapides.

Dans ce cas, l'expression (3) doit être remplacée par:

$$\delta(\mathbf{r}, t) = \int_0^{\nu} \int_0^{\infty} T(\mathbf{r}', t - t')\, \mathrm{d}(\mathbf{r}', \mathbf{r}, t')\, \mathrm{d}t'\, \mathrm{d}\mathbf{r}' \tag{48}$$

et les expressions (10) et (9.a) sont remplacées par:

$$R(t) = \int_0^{\nu} \int_0^{\infty} T(\mathbf{r}', t - t')\, E(\mathbf{r}', t')\, \mathrm{d}t'\, \mathrm{d}\mathbf{r}' \tag{49}$$

$$E(\mathbf{r}', t') = \int_{\nu} \mathbf{d}(\mathbf{r}', \mathbf{r}, t') \cdot \operatorname{grad} \varrho(\mathbf{r})\, \mathrm{d}\mathbf{r}\,. \tag{50}$$

Il est à noter que le rôle de l'inertie est plus important dans le cas de l'arcage des assemblages que dans celui de la dilatation axiale du combustible.

5. 3. Notation de Laplace

Dans toutes les expressions de ce rapport, sauf celles du chapitre 5, le temps n'apparaît que comme paramètre. Les expressions peuvent donc s'écrire immédiatement en notation de Laplace. Ainsi l'expression (10) peut s'écrire:

$$\tilde{R}(s) = \int_{\nu} \tilde{T}(\mathbf{r}', s)\, E(\mathbf{r}')\, \mathrm{d}\mathbf{r}' \tag{51}$$

où $\tilde{R}$ et $\tilde{T}$ sont les transformées de Laplace de R et T.

Dans certaines expressions des paragraphes 5.1 et 5.2, le temps apparaît explicitement, et, par exemple, les expressions (45) et (49) deviennent en notation de Laplace:

$$\tilde{R}(s) = \tilde{P}(s)\, \tilde{k}(s) + \tilde{T}_{\mathrm{e}}(s)\, \tilde{I}(s) \tag{52}$$

$$\tilde{R}(s) = \int_{\nu} \tilde{T}(\mathbf{r}', s)\, \tilde{E}(\mathbf{r}', s)\, \mathrm{d}\mathbf{r}'\,. \tag{53}$$

Pour $\tilde{k}(s)$, le coefficient dynamique de puissance-réactivité, et pour $\tilde{I}(s)$, le coefficient dynamique de température-réactivité, les expressions (46) et (47) donnent:

$$\tilde{k}(s) = \int_{\nu} \tilde{N}(\mathbf{r}', s)\, E(\mathbf{r}')\, \mathrm{d}\mathbf{r}' \tag{54}$$

$$\text{et:}\quad \tilde{I}(s) = \int_{\nu} \tilde{M}(\mathbf{r}', s)\, E(\mathbf{r}')\, \mathrm{d}\mathbf{r}'\,. \tag{55}$$

Le calcul analytique des expressions $\tilde{N}(\mathbf{r}, s)$ et $\tilde{M}(\mathbf{r}, s)$ est donné dans la référence [3].

RÉFÉRENCES

[1] DIETRICH, J. R., *Brit. J. appl. Phys.* Suppl. **5** (1956) S9—S26.
[2] STORRER, F. et DOYLE, T. A., Technical memorandum 31, APDA (1961).
[3] STORRER, F., APDA-132 (1959).

THE EFFECTIVE LIFETIME AND TEMPERATURE COEFFICIENT IN A COUPLED FAST-THERMAL REACTOR

W. HÄFELE
KERNFORSCHUNGSZENTRUM KARLSRUHE,
FEDERAL REPUBLIC OF GERMANY

Abstract — Résumé — Аннотация — Resumen

The effective lifetime and temperature coefficient in a coupled fast-thermal reactor. The theory of coupled systems was extensively developed by Avery and co-workers at the Argonne National Laboratory. One of the main points of interest in a coupled system is the larger effective lifetime of neutrons. The effect of the thermal component acts as a sort of neutron-delayer. As in the theory of delayed neutrons the delaying effect disappears if the reactivity worth is high enough to make the fast component critical by itself. In the study a coupled reactor is considered where the fast component suffers a sudden reactivity step a_0. Because of the increasing power-level the temperature rises and two temperature coefficients start to work: the temperature coefficient of the fast component and the temperature coefficient of the thermal component. The problem is considered with one group of delayed neutrons (in the ordinary meaning). A formalism is given to express the effective lifetime and temperature coefficient during the different stages of the excursion. Excursions for different a_0 are given so that the limit of fast-reactor kinetics is reached.

Temps de vie effectif et coefficient de température dans un réacteur à couplage neutrons rapides - neutrons thermiques. La théorie des systèmes à couplage a été mise au point par Avery et ses collaborateurs au Laboratoire national d'Argonne. L'une des caractéristique les plus intéressantes d'un système à couplage est que le temps de vie effectif des neutrons est plus long. L'effet de la partie thermique contribue en quelque sorte à retarder les neutrons. Comme dans la théorie des neutrons retardés, l'effet de retardement disparaît lorsque la réactivité a une valeur suffisamment élevée pour rendre la partie rapide critique par elle-même. L'auteur du mémoire considère un réacteur à couplage dont la partie rapide subit un saut instantané de réactivité, a_0. La température s'élève à cause de l'augmentation de puissance et deux coefficients de température commencent à s'appliquer: le coefficient de température de la partie rapide et le coefficient de température de la partie thermique. Le problème est étudié avec un groupe de neutrons retardés (au sens habituel). Un formalisme exprime le temps de vie effectif et le coefficient de température aux différents stades de la saute de puissance. L'auteur indique les sautes de puissance pour différentes valeurs de a_0 jusqu'à ce que soit atteinte la limite de la cinétique du réacteur à neutrons rapides.

Эффективный срок жизни и температурный коэффициент нейтронов в двояком реакторе на быстрых и тепловых нейтронах. Теория двояких систем была подробно разработана Р. Эйвери и сотрудниками в Аргонской национальной лаборатории. Одним из основных интересных моментов в спаренной системе является больший эффективный срок жизни нейтронов. Тепловой компонент действует как своего рода замедлитель времени жизни нейтронов. Как и в теории запаздывающих нейтронов, эффект запаздывания исчезает, если реактивность достаточно высока, чтобы быстрый компонент стал критичным сам по себе. В исследовании рассматривался спаренный реактор, в котором быстрый компонент подвергается действию внезапного ступенчатого скачка реактивности a_0. Из-за возрастающего

уровня энергии температура поднимается и начинают работать два температурных коэффициента: температурный коэффициент быстрого компонента и температурный коэффициент теплового компонента. Эта проблема рассматривается с одной группой запаздывающих нейтронов (в обычном значении). Приводится формализм для выражения эффективного срока жизни и температурного коэффициента во время различных стадий исследования. Даны такие отклонения для различных значений a_0, при которых достигается предел кинетики реакторов на быстрых нейтронах.

Vida efectiva y coeficiente de temperatura en un reactor con acoplamiento rápido-térmico. La teoría de los sistemas acoplados fue ampliamente desarrollada por Avery y sus colaboradores en el Argonne National Laboratory. Una de las características más interesantes de los sistemas acoplados es la prolongación de la vida efectiva de los neutrones. La componente térmica actúa como una especie de retardador neutrónico. Como en la teoría de los neutrones retardados, el efecto retardador desaparece cuando la reactividad adquiere un valor suficientemente elevado para que la componente rápida alcance la criticidad independientemente. El autor examina un reactor con acoplamiento en el que la componente rápida sufre un salto instantáneo de reactividad a_0. La temperatura aumenta como consecuencia del incremento del nivel de potencia y comienzan a actuar dos coeficiente térmicos: el que corresponde a la componente rápida y el coeficiente de temperatura de la componente térmica. El problema se estudia en relación con un grupo de neutrones retardados (en el sentido corriente del término). El autor presenta una serie de fórmulas que expresan la vida efectiva de los neutrones y el coeficiente de temperatura en las diferentes etapas del salto de reactividad. El autor indica esos saltos para distintos valores de a_0, hasta alcanzar el límite correspondiente a la cinética de los reactores de neutrones rápidos.

Introduction

The concept of a coupled fast-thermal reactor was developed by Avery [1, 2] at the Argonne National Laboratory. One of the main points of interest in a coupled system is the increased neutron lifetime in comparison with the neutron lifetime in a pure fast reactor. The effect of the thermal component acts as a sort of neutron-delayer. As in the theory of delayed neutrons the delaying effect disappears if the reactivity worth is high enough to make the fast component critical by itself. The similarity of the kinetic behaviour of a coupled system to the behaviour of a pure fast reactor is of interest for safety considerations.

The more generalized aspect of safety is the background of this paper. One safety disadvantage of a pure fast system is the possibility of a melt-down with a subsequent second power excursion in the ball of molten core material which can be strongly supercritical because of the absence of cooling channels.

If the fast component of a coupled reactor is so small that even in the absence of cooling channels (after a melt-down) the system is sub-critical, the second type of power excursion cannot occur and one can argue that even a positive Doppler coefficient may be acceptable in this case. If allowance for a positive Doppler coefficient is made, the acceptable enrichment in the fast component may be higher and the critical mass therefore lower. The thermal component should have a conversion or breeding ratio high enough to keep the overall breeding ratio larger than one. The breeding ratio of a pure U^{233}—Th system is rather insensitive to spectrum effects; thermal as well as fast systems have a breeding ratio slightly higher than one. If therefore the thermal component of a coupled system is a U^{233}—Th one, one expects breeding ratios rather higher than one and the inherent

safety of such a system is similar to that of a pure thermal system, especially with respect to a possible second runaway in the molten core material. This concept obviously suggests the idea of steam-superheating in the fast component.

Theory

The theoretical treatment of this paper is given in terms of Avery's [2] integral theory of coupling several reactor components. In the steady state and in the case of only two components, one of these being fast (the first) and the other thermal (the second), the fission rate S_1 has two sources: neutrons born in the first component and neutrons born in the second. The same is true for S_2. Therefore we have

$$S_1 = k_{11} S_1 + k_{12} S_2 \tag{1}$$

$$S_2 = k_{21} S_1 + k_{22} S_2 \,. \tag{2}$$

k_{ij} is the coupling factor, which gives the neutrons from j to i. From (1) and (2) follows.

$$(1 - k_{11})\,(1 - k_{22}) = k_{12} k_{21} \tag{3}$$

and

$$\frac{S_2}{S_1} = \frac{1 - k_{11}}{k_{12}} \,. \tag{4}$$

Equation (3) is a criticality condition so that not all k_{ij} are free parameters. S_i is the total fission rate in i and involves neutrons of all energies, the k_{ij} following principally from the solved relevant multi-group problem.

One can read Eqs. (1) and (2) in a different way:

$$S_1 = S_{11} + S_{12} \tag{5}$$

$$S_2 = S_{21} + S_{22} \,. \tag{6}$$

S_{ij} is that part of S_i which is originated in j. This is the way to define k_{ij}:

$$k_{ij} = \frac{S_{ij}}{S_j} \,. \tag{7}$$

From this point of view Avery derives the time-dependent equations. He considers only the case of a kinetic behaviour close to criticality and gives a linearized equation for the reciprocal time period ω. In a unique system the inhour equation without delayed neutrons has the following form

$$k - 1 = \omega l \tag{8}$$

where k is the multiplication factor, l the lifetime and ω the reciprocal time period. For values of $\omega \gg 1/l$ (8) has to be replaced by (see [3])

$$\ln k = \omega l \,. \tag{9}$$

Usually it is not necessary to consider such large values of ω because a unique reactor is never supercritical to such a degree.

But in the case of a coupled reactor, where the fast component is considered to become critical by itself, it happens that $\omega \gg 1/l_{22}$ if l_{22} is the lifetime of the neutrons in the thermal component. In order to have the most general case and the most appropriate starting point we sharpen (7) to the non-stationary case

$$k_{ij} = \frac{S_{ij}(t)}{S_j(t - l_{ij})} \tag{10}$$

where l_{ij} is the lifetime of neutrons going from j to i. Neutrons arriving in i at the time t are born in j at the time $t—l_{ij}$. If we consider one representative group of delayed neutrons of fraction β and decay constant λ and if we define that m_i is the multiplication factor in i with $m_i=1$ in the stationary case, we obtain:

$$k_{11}\, m_1\,(l—\beta)\, S_1\,(t—l_{11}) + k_{11}\,\lambda_1\, C_1\,(t—l_{11}) + k_{12}\, m_2\,(1—\beta)\, S_2\,(t—l_{12}) + \\ + k_{12}\,\lambda_2\, C_2\,(t—l_{12}) = S_1\,(t) \tag{11}$$

$$k_{21}\, m_1\,(1—\beta)\, S_1\,(t—l_{21}) + k_{21}\,\lambda_1\, C_1\,(t—l_{21}) + \\ + k_{22}\, m_2\,(1—\beta)\, S_2\,(t—l_{22}) + k_{22}\,\lambda_2\, C_2\,(t—l_{22}) = S_2\,(t) \tag{12}$$

$$\frac{\partial C_1}{\partial t} = —\lambda\, C_1 + \beta\, m_1\, S_1 \tag{13}$$

$$\frac{\partial C_2}{\partial t} = —\lambda\, C_2 + \beta\, m_2\, S_2\,. \tag{14}$$

Equations (13) and (14) are correct for any point on the time axis. C_i are the precursors in i, originated by S_i.

We now assume proportionality of all time-dependent quantities to $e^{\omega t}$.

$$S = S_0 e^{\omega t}$$

$$C = C_0 e^{\omega t}$$

$$k_{11}\, m_1\, e^{-\omega l_{11}} \left(1 — \beta \frac{\omega}{\omega + \lambda}\right) (S_1)_0 + k_{12}\, m_2\, e^{-\omega l_{12}} \left(1 — \beta \frac{\omega}{\omega + \lambda}\right) (S_2)_0 = (S_1)_0 \tag{15}$$

$$k_{21}\, m_1\, e^{-\omega l_{21}} \left(1 — \beta \frac{\omega}{\omega + \lambda}\right) (S_1)_0 + k_{22}\, m_2\, e^{-\omega l_{22}} \left(1 — \beta \frac{\omega}{\omega + \lambda}\right) (S_2)_0 = (S_2)_0\,. \tag{16}$$

If one uses the abbreviation

$$\gamma = \left(1 — \beta \frac{\omega}{\omega + \lambda}\right) \tag{17}$$

one arrives at the following inhour equation:

$$(k_{11}\, m_1\, \gamma — e^{\omega l_{11}})\,(k_{22}\, m_2\, \gamma — e^{\omega l_{22}}) — k_{12}\, k_{21}\, m_1 \times m_2 \times \gamma^2\, e^{\omega (l_{11} - l_{12})} \times \\ \times e^{\omega (l_{22} - l_{21})} = 0\,. \tag{18}$$

Within the framework of the integral theory of coupling two reactor components, (18) is the correct inhour equation. In reality the k_{ij} will change for large values of ω but we must neglect this effect here. We now follow Avery, if we put for reasons of simplicity

$$l_{11} = l_{12} \tag{19}$$

$$l_{22} = l_{21}\,.$$

Since the fast component will never be very supercritical by itself, we put

$$e^{\omega l_{11}} = 1 + \omega\, l_{11} \tag{20}$$

but we are not allowed to expand $e^{\omega l_{22}}$. Eqs. (19) and (20) lead to the following final expression:

$$F=(k_{11}\,m_1\,\gamma-1-\omega l_{11})\,(k_{22}\,m_2\,\gamma-\mathrm{e}^{\omega l_{22}})-k_{12}\,k_{21}\,m_1\,m_2\,\gamma^2=0\,. \tag{21}$$

Let us examine (21) briefly:

(1) For $\omega \ll l_{11}$ and $\omega \ll l_{22}$ and $m_1=m_2$ from (21) follows

$$m-1=m\beta\,\frac{\omega}{\omega+\lambda}$$

that is, the usual inhour equation of a unique reactor in the region of delayed criticality. If $m_1=m_2$, our coupled system is unique because both components have the same type of delayed neutron.

(2) Let us neglect second and higher effects

$$(m_i-1)^p\times\omega^q \text{ with } p+q\geqq 2 \text{ and } i=1.2\,.$$

Then we get the following:

$$\alpha_1\,m_1+\alpha_2\,m_2-1=\omega\,(\alpha_1\,l_{11}+\alpha_2\,l_{22})+\beta\,\frac{\omega}{\omega+\lambda}\,(m_1\,\alpha_1+m_2\,\alpha_2)$$

and α_i is the reactivity partition as defined by Avery:

$$\alpha_1=\frac{\dfrac{1}{1-k_{11}}}{\dfrac{1}{1-k_{11}}+\dfrac{1}{1-k_{22}}} \tag{23}$$

$$\alpha_2=\frac{\dfrac{1}{1-k_{22}}}{\dfrac{1}{1-k_{11}}+\dfrac{1}{1-k_{22}}}\,. \tag{24}$$

Up to this stage Avery had developed his considerations.

(3) Suppose $k_{11}\,m_1\,\gamma\gg 1$, so that $\omega l_{22}\gg 1$. Then we get:

$$k_{11}\,m_1\,(1-\beta)-1=\omega l_{11}\,. \tag{25}$$

We now have the behaviour of a decoupled fast assembly.

In order to obtain an effective lifetime and temperature coefficient at each point (m_1, m_2, ω), we proceed as follows:

$$\frac{\partial F}{\partial m_1}\,\delta m_1+\frac{\partial F}{\partial m_2}\,\delta m_2+\frac{\partial F}{\partial \omega}\,\delta\omega=0 \tag{26}$$

$$l_{\mathrm{eff}}=-\frac{\dfrac{\partial F}{\partial \omega}}{\dfrac{\partial F}{\partial m_1}+\dfrac{\partial F}{\partial m_2}}\,. \tag{27}$$

By defining l_{eff} in such a way, we have already used implicitly two generalized expressions for α_1 and α_2

$$\alpha_1=\frac{\dfrac{\partial F}{\partial m_1}}{\dfrac{\partial F}{\partial m_1}+\dfrac{\partial F}{\partial m_2}} \tag{28}$$

$$\alpha_2 = \frac{\frac{\partial F}{\partial m_2}}{\frac{\partial F}{\partial m_1} + \frac{\partial F}{\partial m_2}} \quad . \tag{29}$$

This implies

$$m_{\text{eff}} = m_1\,\alpha_1 + m_2\,\alpha_2 \; . \tag{30}$$

Using Eqs. (18) and (3) one obtains explicitly the α's:

$$\alpha_1 = \frac{\frac{1+\omega l_{11}}{m_1}\,(e^{\omega l_{22}} - k_{22}\gamma m_2)}{\frac{1+\omega l_{11}}{m_1}\,(e^{\omega l_{22}} - k_{22}\gamma m_2) + \frac{e^{\omega l_{22}}}{m_2}\,\frac{(1-k_{11})\,(1-k_{22})\,m_1 m_2 \gamma^2}{e^{\omega l_{22}} - k_{22} m_2 \gamma}} \tag{31}$$

$$\alpha_2 = \frac{\frac{e^{\omega l_{22}}}{m_2}\,\frac{(1-k_{11})\,(1-k_{22}) m_1 m_2 \gamma^2}{e^{\omega l_{22}} - k_{22} m_2 \gamma}}{\frac{1+\omega l_{11}}{m_1}\,(e^{\omega l_{22}} - k_{22}\gamma m_2) + \frac{e^{\omega l_{22}}}{m_2}\,\frac{(1-k_{11})\,(1-k_{22})\,m_1 m_2 \gamma^2}{e^{\omega l_{22}} - k_{22} m_2 \gamma}} \; . \tag{32}$$

For small values of ω the α_i tend towards Eqs. (28) and (29). For large values of ω $\left(\omega \gg \frac{1}{l_{22}}\right)$ the retardation effect described by $e^{-\omega l_{22}}$ delays the neutron from the thermal component and we obtain:

$$\alpha_1 \to 1 \tag{33}$$

$$\text{for } \omega l_2 \gg 1 \; .$$

$$\alpha_2 \to 0 \tag{34}$$

For l we have

$$l_{\text{eff}} = \alpha_1 \frac{m_1}{1+\omega l_{11}}\, l_{11} + \alpha_2 m_2 l_{22} - \frac{\gamma'}{\gamma}\,(\alpha_1 m_1 + \alpha_2 m_2) \; . \tag{35}$$

Please note that $-(\gamma'/\gamma) > 0$; l_{eff} is somewhat asymmetric with respect to l_{11} and l_{22}, but we expanded $e^{\omega l_{11}}$ to $1+\omega l_{11}$ and we did not expand $e^{\omega l_{22}}$.

If $\alpha_1 \to 1$ the following relation holds:

$$\frac{m_1}{1+\omega l_{11}} \to \frac{1}{k_{11}(1-\beta)} \tag{36}$$

we have

$$l_{\text{eff}} = \frac{l_{11}}{k_{11}(1-\beta)} \; . \tag{37}$$

One can read the result (37) immediately from (21). If $e^{\omega l_{22}}$ becomes very large the first bracket has to be zero and the fast component behaves like a decoupled reactor. If we redefine the multiplication factor in the following way:

$$\tilde{m}_1 = k_{11}\,(1-\beta)\,m_1 \tag{38}$$

and we have

$$\tilde{l}_{\text{eff}} = l_{11} \; . \tag{39}$$

Numerical evaluation

It is possible to describe an excursion without solving all the time-dependent equations if the *explicit* time-dependence is ignored. ϑ_i shall be the temperature excess against the stationary case in the component i. B_i shall be the *negative*

temperature coefficient in i and a_0 a sudden reactivity step in the fast component. Then we have:

$$m_1 = 1 + a_0 - B_1 \vartheta_1 \tag{40}$$

$$m_2 = 1 - B_2 \vartheta_2 . \tag{41}$$

For the ratio ϑ_2/ϑ_1 we have

$$\frac{\vartheta_2}{\vartheta_1} = \frac{S_2}{S_1} = \frac{1-k_{11}}{k_{12}} . \tag{42}$$

Eq. (42) assumes again that the k_{ij} are constant during the excursion and the cooling flows in 1 and 2 are the same. Using (40), (41) and (42) Eq. (21) establishes a function

$$\left.\begin{aligned} \vartheta_1 &= \vartheta_1(\omega) \\ \omega &= \omega(\vartheta_1) . \end{aligned}\right\} \tag{43}$$

or

According to (30) the effective temperature coefficient is the following

$$B_{\text{eff}} = \alpha_1 B_1 + \alpha_2 B_2 \frac{1-k_{11}}{k_{12}} . \tag{44}$$

If B_{eff} is positive — which means that the overall temperature coefficient is negative because of (40) and (41)—(43) gives for $\vartheta_1 = 0$ the largest value of ω and ω becomes zero for $\vartheta_1 = \vartheta_{1\max}$. If B_{eff} is negative, Eq. (43) starts for $\vartheta_1 = 0$ with a certain value of ω and ω becomes larger if ϑ_1 increases. The relations (40—43) make the following quantities functions of ω (or ϑ_1):

$$l_{\text{eff}},\ B_{\text{eff}},\ \alpha_1,\ \alpha_2 .$$

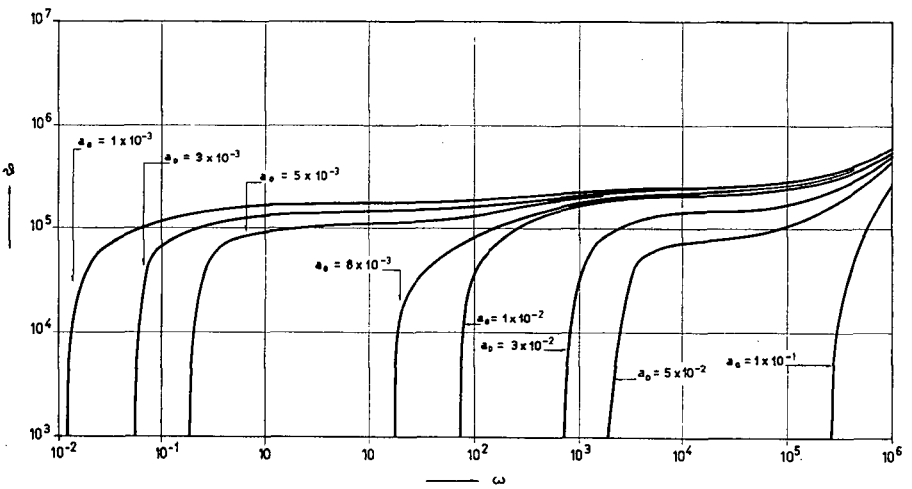


Fig. 1
Temperature excess ϑ as a function of reciprocal period ω, initial reactivity-step a_0 parameter.

$k_{11} = 0.94$ $k_{22} = 0.23$ $k_{12} = 0.462$ $B_1 = -3 \times 10^{-7}$ $B_2 = 3 \times 10^{-5}$

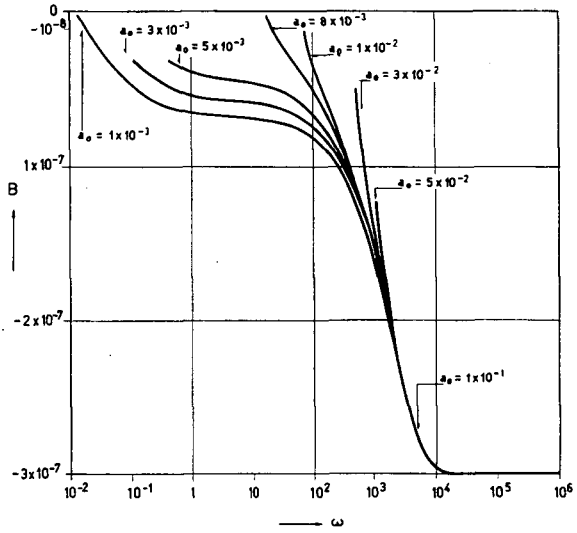


Fig. 2
Effective temperature coefficient B as a function of reciprocal period ω, initial reactivity-step a_0 parameter.
$k_{11} = 0.94$ $k_{22} = 0.23$ $k_{12} = 0.462$ $B_1 = -3 \times 10^{-7}$ $B_2 = 3 \times 10^{-5}$

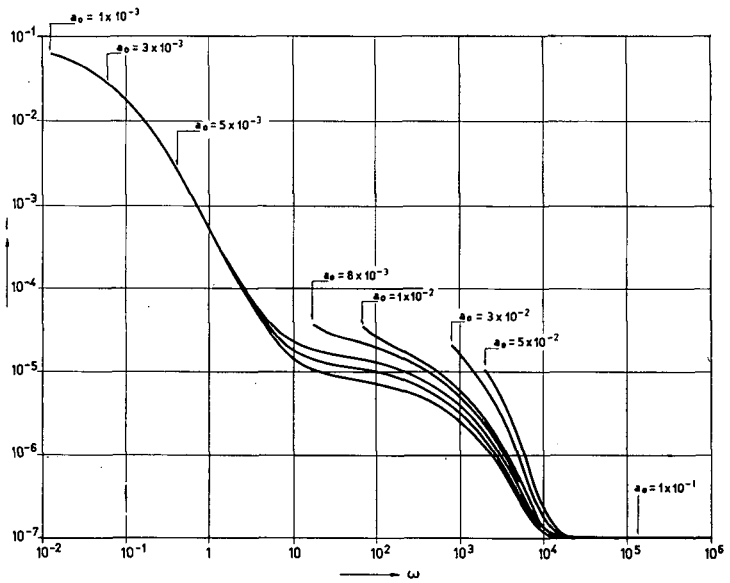


Fig. 3
Effective lifetime l as a function of reciprocal period ω, initial reactivity-step a_0 parameter.
$k_{11} = 0.94$ $k_{22} = 0.23$ $k_{12} = 0.462$ $B_1 = -3 \times 10^{-7}$ $B_2 = 3 \times 10^{-5}$

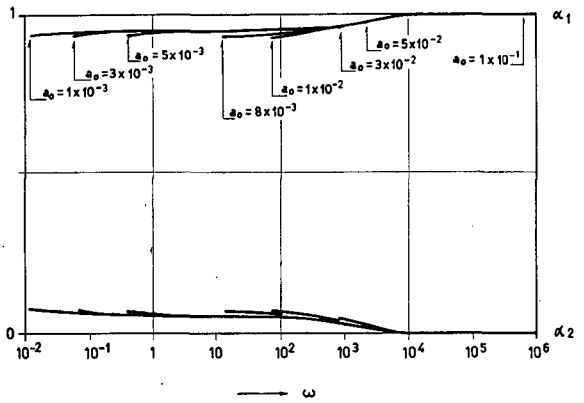


Fig. 4
Reactivity partition α_i as a function of reciprocal period ω, initial reactivity-step a_0 parameter.
$k_{11} = 0.94$ $k_{22} = 0.23$ $k_{12} = 0.462$ $B_1 = -3 \times 10^{-7}$ $B_2 = 3 \times 10^{-5}$

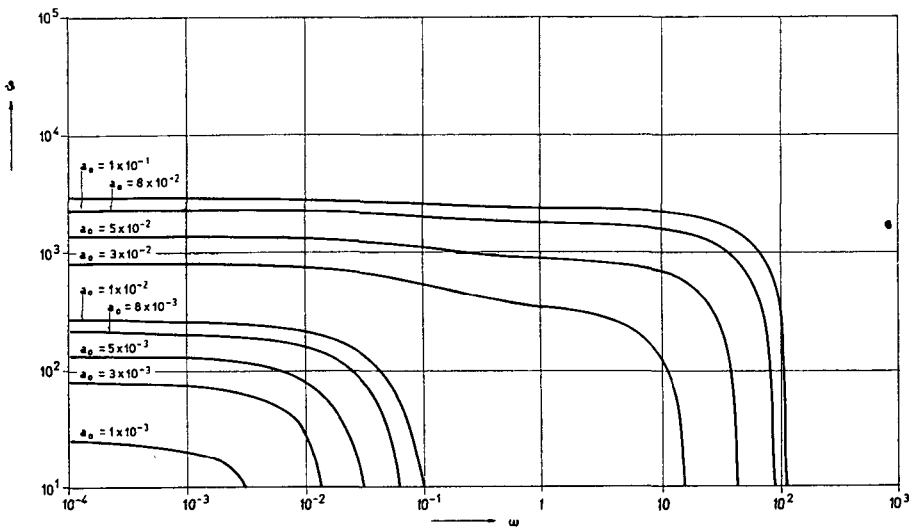


Fig. 5
Temperature excess ϑ as a function of reciprocal period ω, initial reactivity-step a_0 parameter.
$k_{11} = 0.6$ $k_{22} = 0.75$ $k_{12} = 0.5$ $B_1 = -3 \times 10^{-7}$ $B_2 = 3 \times 10^{-5}$

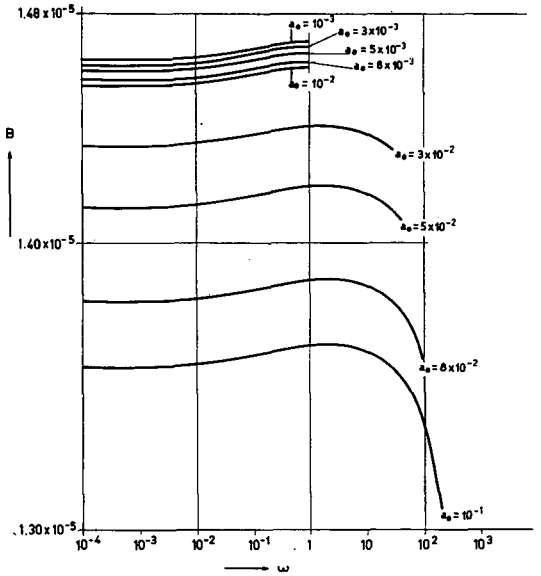


Fig. 6
Effective temperature coefficient B as a function of reciprocal period ω, initial reactivity-step a_0 parameter.
$k_{11} = 0.6$ $k_{22} = 0.75$ $k_{12} = 0.5$ $B_1 = -3 \times 10^{-7}$ $B_2 = 3 \times 10^{-5}$

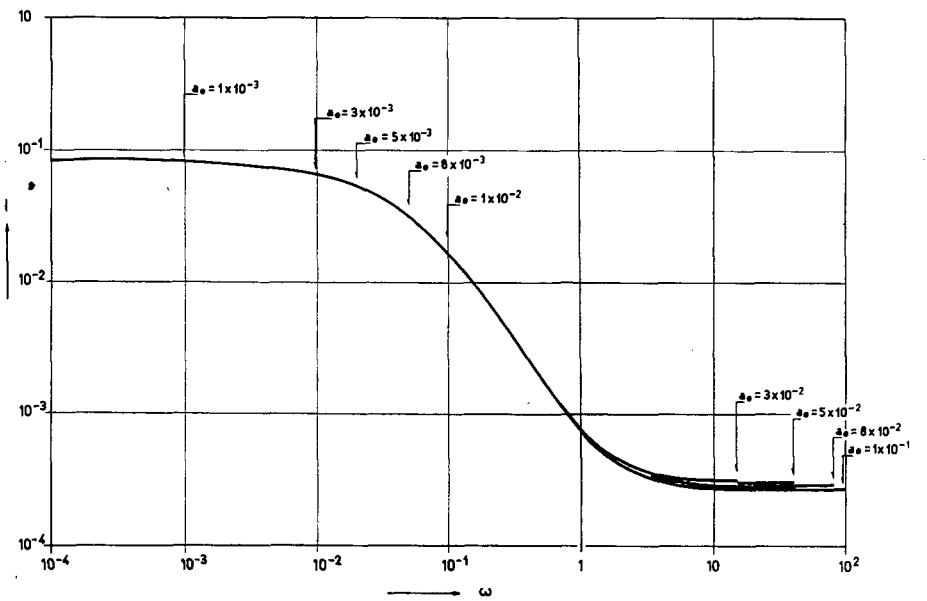


Fig. 7
Effective lifetime l as a function of reciprocal period ω, initial reactivity-step a_0 parameter.
$k_{11} = 0.6$ $k_{22} = 0.75$ $k_{12} = 0.5$ $B_1 = -3 \times 10^{-7}$ $B_2 = 3 \times 10^{-5}$

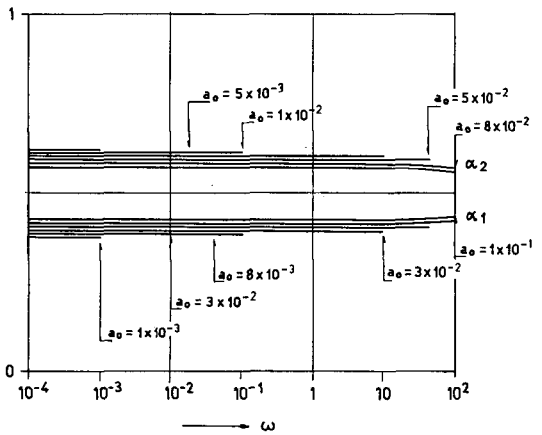


Fig. 8
Reactivity partition α as a function of reciprocal period ω, initial reactivity-step a_0 parameter.
$k_{11} = 0.6$ $k_{22} = 0.75$ $k_{12} = 0.5$ $B_1 = -3 \times 10^{-7}$ $B_2 = 3 \times 10^{-5}$

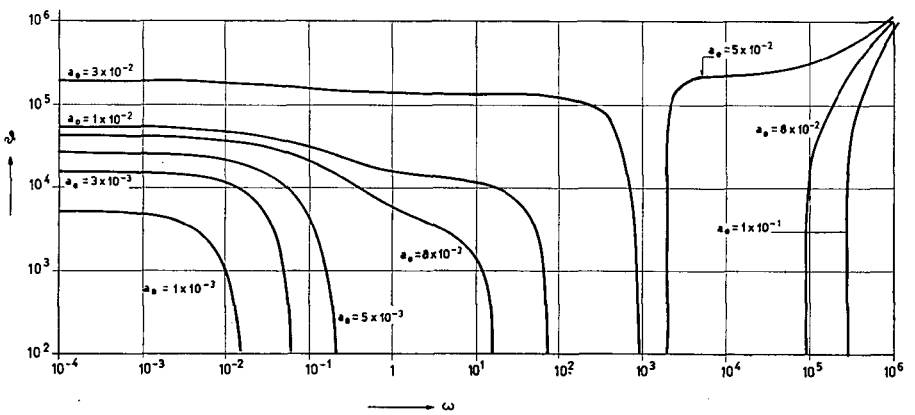


Fig. 9
Temperature excess ϑ as a function of reciprocal period ω, initial reactivity-step a_0 parameter.
$k_{11} = 0.94$ $k_{22} = 0.23$ $k_{12} = 0.462$ $B_1 = -10^{-7}$ $B_2 = 3 \times 10^{-5}$

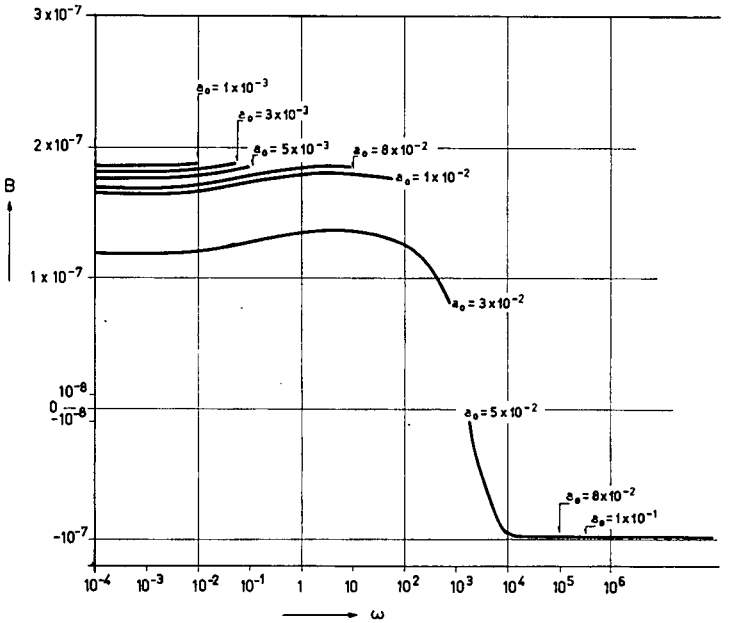


Fig. 10

Effective temperature coefficient B as a function of reciprocal period ω, initial reactivity-step a_0 parameter.

$k_{11}=0.94$ $k_{22}=0.23$ $k_{12}=0.462$ $B_1=-10^{-7}$ $B_2=3\times10^{-5}$

Three examples are given. For all three examples we have

$l_{11}=1\times10^{-7}$ (s) $\lambda=8\times10^{-2}$ (s^{-1})

$l_{22}=5\times10^{-4}$ (s) $\beta=6.8\times10^{-3}$

a_0 varies between 1×10^{-3} and 1×10^{-1}.

(1) The first example uses the following numbers

$k_{11}=0.94$
$k_{22}=0.23$ } weak coupling as in Avery's paper [1].
$k_{12}=0.462$

$B_1=-3\times10^{-7}$; $B_2=3\times10^{-5}$.

The small coupling of the thermal component with the comparatively large negative temperature coefficient keeps the overall temperature coefficient positive, and we have a run-away. Figs. 1—4 show the behaviour of ϑ, B, l and α_i as a function of ω. B tends towards B_1, l to l_{11} $1/k_{11}$ $(1-\beta)$ and $\alpha_1\to1$, $\alpha_2\to0$. Somewhat remarkable are the steps in the curve $l_{\text{eff}}=l_{\text{eff}}(\omega)$. For different regions of ω, l_{eff} is almost constant, the changes being sharp ones because the values of l_{11} and l_{22} differ greatly.

(2) The second example (Figs. 5—8) describes the same case, only the coupling is stronger:

$k_{11}=0.6$
$k_{22}=0.75$ } strong coupling, the other figures as in example (1).
$k_{12}=0.5$

We have a stable behaviour: ω decreases if ϑ increases.

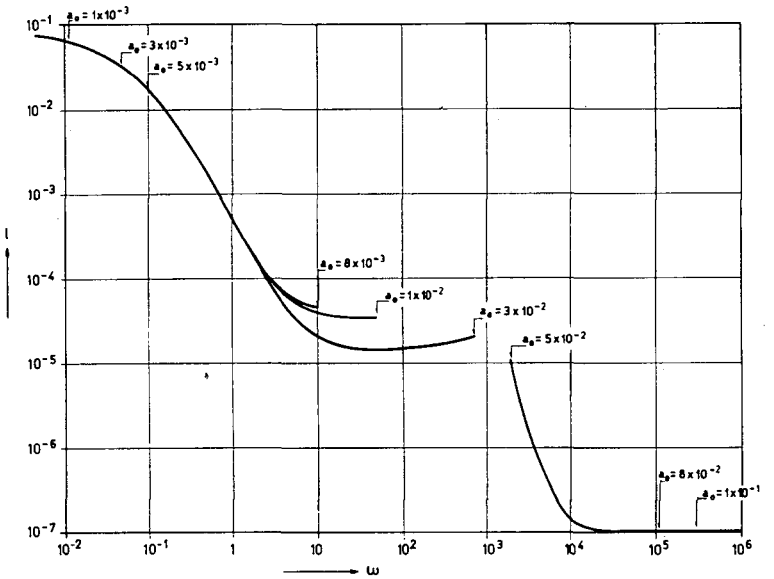


Fig. 11
Effective lifetime l as a function of reciprocal period ω, initial reactivity-step a_0 parameter.
$k_{11} = 0.94$ $k_{22} = 0.23$ $k_{12} = 0.462$ $B_1 = -10^{-7}$ $B_2 = 3 \times 10^{-}$

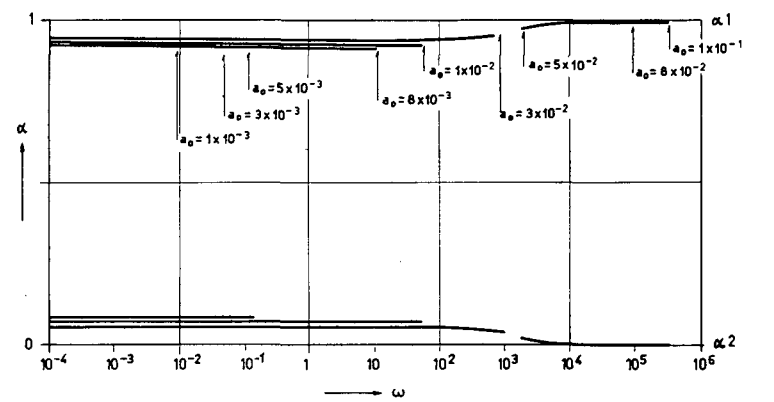


Fig. 12
Reactivity partition α_i as a function of reciprocal period ω, initial reactivity-step a_0 paramater.
$k_{11} = 0.6$ $k_{22} = 0.23$ $k_{12} = 0.462$ $B_1 = -10^{-7}$ $B_2 = 3 \times 10^{-5}$

(3) The third example (Figs. 9—12) has again the weak coupling of the first case.

$$\left.\begin{aligned} k_{11}&=0.94\\ k_{22}&=0.23\\ k_{12}&=0.462 \end{aligned}\right\} \text{ weak coupling as in [1].}$$

but for the B_i we have

$$B_1=-1\times10^{-7};\quad B_2=3\times10^{-5}.$$

If the initial reactivity step a_0 is small enough to keep α_1 low, the overall temperature coefficient is negative and the reactor is stable, but if a_0 is large, the corresponding ω is high enough to make α_1 so large that the overall temperature coefficient is positive and we have a run-away.

Thus, the proposed method gives quick answers, whether or not a coupled reactor is stable against sudden reactivity steps a_0 which need not necessarily be small.

ACKNOWLEDGEMENT

The author wishes to thank W. Münzner for the numerical evaluations.

REFERENCES

[1] AVERY, R., *Nucl. Sci. Engng.* **3** (1958) 129—144.
[2] AVERY, R., Proc. 2nd UN Int. Conf. PUAE **12** (1958) 182.
[3] WEINBERG, A. M. and WIGNER, E. P., The Physical Theory of Neutron Chain Reactors, The University of Chicago Press (1958) 169.

SM-18/87

ИЗУЧЕНИЕ КИНЕТИКИ ФИЗИЧЕСКОГО РЕАКТОРА ВЕРОЯТНОСТНЫМ МЕТОДОМ

А. И. МОГИЛЬНЕР
АКАДЕМИЯ НАУК СССР, МОСКВА,
СССР

Abstract — Résumé — Аннотация — Resumen

Study of the kinetics of a research reactor by the probability method. Among the statistical methods used to determine kinetic parameters of critical and near-critical assemblies, the most suitable for the intermediate energy range is the Feynman alpha method, which is based on measurement of the relationship between relative dispersion $(\overline{n^2}-\overline{n}^2)/\overline{n}=1+\psi(t)$ in the number of counts n for a given interval of time, and the duration of the interval (t).

According to the papers presented by a number of American scientists at the 1958 Geneva Conference, their dispersion measurements were carried out by direct recording of the number of pulses in the intervals, and involved subsequently a laborious process of squaring the numerous data obtained. In order to obtain the value of $\alpha=(1-k_p)/l$, Lukov processed data for $\sim 10^4$ intervals.

By the method described in this paper, dispersion values are obtained indirectly by applying the law of distribution for the number of counts, as distinct from Poisson's law. In this way the laborious operations needed to obtain the data can be performed automatically in the process of measurement. Further, a five-channel statistical analyser has been developed and is described in the paper. This device, which is constructed of the simplest elements (triggers), can carry out the operations referred to above simultaneously for five different intervals. Thus, even in the case of a thermal reactor, processed data for $\sim 3\times 10^5$ intervals can generally be accumulated in five minutes, a fact which guarantees a high degree of statistical precision and permits a better utilization of the information obtained. In the intermediate-energy range, data can be processed for more than 10^6 intervals in the same period of time.

The paper gives examples of the application of statistical analysis, and the results of measurements made with a multi-channel statistical analyser.

Etude de la cinétique d'un réacteur physique par la méthode de probabilité. Parmi les diverses méthodes statistiques qui permettent de déterminer les paramètres de cinétique des ensembles critiques et installations analogues, la plus utile dans la région des énergies moyennes est la méthode de Feynman-α fondée sur la mesure de la dépendance de la dispersion relative $(\overline{n^2}-\overline{n}^2)/n=1+\psi(t)$ du nombre de coups n pour un intervalle de temps donné, par rapport à la durée de cet intervalle t.

Dans les études d'un certain nombre d'auteurs américains qui ont été présentées à la Conférence de Genève de 1958, la dispersion est mesurée par enregistrement direct du nombre d'impulsions dans les intervalles, suivi d'un dépouillement complexe, par l'intermédiaire d'un grand nombre d'élévations au carré. Pour obtenir la grandeur $\alpha=(1-k_p)/l$, V. Loukov a dépouillé les données dans $\sim 10^4$ intervalles.

Le présent mémoire détermine la valeur de la dispersion d'une façon indirecte, en appliquant la loi de distribution du nombre de coups — loi qui diffère de celle de Poisson — ce qui permet d'effectuer automatiquement, au cours des mesures, les opérations les plus complexes, indispensables pour l'obtention de α. De plus, l'analyseur statistique à cinq canaux décrit dans le mémoire, qui se compose des éléments les plus simples (déclencheurs), effectue ces opérations simultanément dans cinq intervalles différents. Par conséquent, il se produit en 5 minutes, même pour un réacteur thermique, une accumulation dans $\sim 3\cdot 10^5$ intervalles de données complètement

exploitées, ce qui assure une grande précision statistique grâce à une meilleure utilisation des renseignements. Dans la région des énergies moyennes, le traitement des données exige le même laps de temps dans plus de 10^6 intervalles.

Le mémoire cite des exemples d'application d'une analyse statistique, ainsi que les résultats des mesures effectuées à l'aide d'un analyseur statistique multicanaux.

Изучение кинетики физического реактора вероятностным методом. Среди применяемых статистических методов определения параметров кинетики критических и близких к ним сборок наибольшей пригодностью в промежуточной области энергий обладает метод Фейнмана-альфа, основанный на измерении зависимости относительной дисперсии $(\overline{n^2}-\overline{n}^2)/n=1+\psi(t)$ в числе отсчетов n за некоторый интервал времени от длительности этого интервала t.

В работах ряда американских авторов, представленных на Женевскую конференцию 1958 года, измерения дисперсии выполняются путем непосредственной записи числа импульсов в интервалах с последующей трудоемкой обработкой посредством многочисленных возведений в квадрат. Для получения величины $\alpha=(1-k_p)/l$, Луков обрабатывал данные по $\sim 10^4$ интервалам.

В настоящей работе значение дисперсии определяется косвенно на основе использования закона распределения для числа отсчетов, отличного от закона Пуассона, что позволяет наиболее трудоемкие операции, необходимые для получения α, выполнять автоматически в процессе измерения. Более того, разработанный и описанный в докладе 5-канальный статистический анализатор, построенный из простейших элементов (триггеров), выполняет эти операции одновременно для 5 различных интервалов. В результате даже для теплового реактора за 5 минут накапливаются в основном обработанные данные по $\sim 3\cdot 10^5$ интервалам, что обеспечивает высокую статистическую точность за счет лучшего использования информации. В промежуточной области энергий за это же время обрабатываются данные более чем за 10^6 интервалов.

В докладе приводятся примеры применения статистического анализа и результаты измерений, выполненных с помощью многоканального статистического анализатора.

Estudio de la cinética de un reactor de investigación mediante el cálculo de probabilidades. Entre los métodos estadísticos utilizados para determinar los parámetros cinéticos de los conjuntos críticos y cuasi-críticos el que mayor utilidad tiene en el terreno de las energías intermedias es el alfa de Feynman, que se basa en la determinación de la relación de dependencia existente entre la dispersión relativa $(\overline{n^2}-\overline{n}^2)/n=1+\psi(t)$, para un número n de lecturas en un intervalo de tiempo dado t y la duración de este último.

En los trabajos de una serie de autores estadounidenses, presentados a la Conferencia de Ginebra del año 1958, la determinación de la dispersión se efectuaba por registro directo del número de impulsos en los intervalos correspondientes, seguida por un laborioso cálculo de cuadrados. Para obtener el valor de $\alpha=(1-k_p)/l$, V. Lukov trabajó con datos de hasta 10^4 intervalos, aproximadamente.

En cambio, en el presente trabajo los valores de dispersión se determinan indirectamente aplicando la ley de distribución del número de impulsos que, a diferencia de la ley de Poisson, permite efectuar automáticamente, durante el proceso de medición, las laboriosas operaciones necesarias para obtener el valor de alfa. Además, un analizador estadístico de cinco canales ideado especialmente y descrito en la memoria, que se construyó con los elementos más sencillos (disparadores), efectúa dichas operaciones simultáneamente para cinco intervalos diferentes. Gracias a este método, aun en el caso de los reactores de neutrones térmicos, es posible obtener en cinco minutos datos elaborados que correspondan a $3\cdot 10^5$ intervalos, aproximadamente, lo que permite alcanzar una elevada exactitud estadística y aprovechar mejor la información así obtenida. En el caso de las energías intermedias, es posible elaborar en el tiempo señalado los datos correspondientes a más de 10^6 intervalos.

El autor presenta ejemplos de la aplicación del análisis estadístico y resultados de las determinaciones efectuadas por medio del analizador estadístico multicanal.

1. Существующие методы исследования параметров кинетики реактора

Статистические методы исследования параметров кинетики реактора прочно входят в практику лабораторий, занимающихся изучением критических сборок. Широко применяется при изучении быстрых и жестких промежуточных систем метод Росси-альфа [1], в реакторах с более мягким спектром хорошо зарекомендовал себя метод Фейнмана-альфа [2]. В последние годы в этой области спектра успешно апробирован предложенный Ч. Коном [3] метод определения величины $\alpha_0 = \beta_{эфф}/l$ *где* l-среднее время жизни мгновенных нейтронов, $\beta_{эфф}$ — эффективный выход запаздывающих нейтронов, основанный на изучении распределения энергии в спектре шумов критического реактора. Частотный (спектральный) метод тесно связан с общей теорией шумов, разрабатываемой в применении к реакторам Муром [4].

Частотный метод, в котором изучаются флуктуации тока ионизационной камеры, применим лишь для критических реакторов. Успешно применяемый в промежуточной области энергий метод Фейнмана пригоден для подкритического состояния реактора и практически не пригоден для критического котла ввиду трудности в обеспечении точной стационарности.

Как известно, метод Фейнмана основан на зависимости дисперсии в числе отсчетов от параметров кинетики [5]:

$$\frac{\overline{n^2}-\overline{n}^2}{\overline{n}} = 1 + \psi(t) \qquad \psi(t) = \frac{\varepsilon \overline{\nu(\nu-1)} k_p^2}{(1-k_p)^2 \overline{\nu}^2}\left(1 - \frac{1-e-\alpha t}{\alpha t}\right) \qquad (1)$$

где n — число отсчетов нейтронного детектора, помещенного в реактор, за интервал времени t, ν — выход нейтронов в процессе деления, k_p — коэффициент размножения на мгновенных нейтронах, l — среднее время жизни мгновенных нейтронов, $\alpha = (1-k_p)/l$, ε — эффективность детектора в отсчетах на деление в реакторе. Местоположение детектора выбирается с учетом желательности максимального значения ε для того чтобы величина $\psi(t)$ вносила заметный вклад в дисперсию.

Метод Фейнмана успешно применялся Луковым и Черчиллем для измерений среднего времени жизни мгновенных нейтронов в реакторах *ZPR*-IV и *ZPR*-V.

При этом числа отсчетов n для определенного значения t фиксировались на ленте самописца и затем вручную составлялись величины $\frac{\overline{n^2}-\overline{n}^2}{\overline{n}} - 1 = \psi(t)$. Из набора $\psi(t)$ путем дальнейшей обработки методом наименьших квадратов получались параметры

$$\alpha = (1-k_p)/l \quad \text{и} \quad Z = \frac{\varepsilon \overline{\nu(\nu-1)} k_p^2}{(1-k_p)^2 \overline{\nu}^2}$$

Простота экспериментальной техники является привлекательной чертой метода. Однако, трудоемкая и длительная процедура обработки $\sim 10^4$ значений n после проведения эксперимента значительно снижает его преимущества. Путем усложнения аппаратуры (построив устройство, подобное применяемым в корреляционном методе, например, [6]) можно автоматизировать процесс обработки для получения значений $\bar{n}$ и $\overline{n^2}$. Тем не менее и в этом случае требуемый объем „памяти", необходимой для возведения в квадрат, сильно ограничивает применимость этой техники.

2. Вероятностный метод изучения кинетики реактора

Применение вероятностного метода [7, 8] также основано на выражении (1) для относительной дисперсии. В отличие от других в этом методе экспериментальное значение ψ получается косвенно путем использования закона распределения для числа отсчетов от нейтронного детектора.

Производящая функция для этого распределения

$$\Pi(\varrho) \equiv P_0 + P_1\varrho + P_2\varrho^2 + \ldots + P_k\varrho^k + \ldots \qquad (2)$$

где P_k — вероятность регистрации k отсчетов в данном интервале, имеет вид (отрицательно-биномиальное распределение)

$$\Pi(\varrho) = \left[1 + \psi(1 - \varrho)\right]^{-\bar{n}/\Psi}.$$

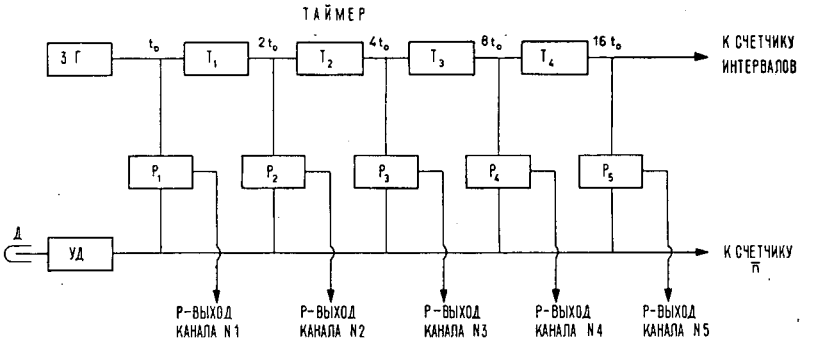
Это распределение — двухпараметрическое: $\bar{n}$ есть среднее значение числа отсчетов в интервале, $\psi = \frac{\overline{n^2} - \bar{n}^2}{\bar{n}} - 1$. Вероятность получения k отсчетов можно получить из производящей функции (2) с помощью соотношения $P_k = \frac{1}{k!} \left.\frac{d^k \Pi(\varrho)}{d\varrho^k}\right|_{\varrho = 0}$. Распределение полностью определено, если найдены две независимые характеристики его, например, среднее число отсчетов в интервале и какая-либо из вероятностей P_k. В частности, при малых загрузках на интервал достаточно измерить $P_0 = \Pi(0) = (1+\psi)^{-\bar{n}/\Psi}$ вероятность отсутствия отсчетов в интервале длительностью t. В этом случае полученное из эксперимента значение $Q = \frac{1}{\bar{n}} ln\,('/p_0)$ (3). дает возможность определить величину Ψ из соотношения

$$Q = \frac{ln\,(1+\psi)}{\varphi}. \qquad (3а)$$

3. Описание 5-канального статистического анализатора, использовавшегося для косвенного определения дисперсии

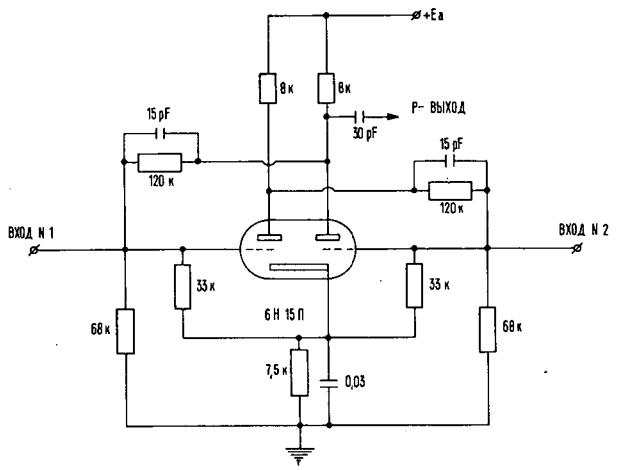
Для измерения зависимости $\psi(t)$ был построен 5-канальный статистический анализатор (см. фиг. 1), позволяющий получить значения $\psi(t)$ посредством одновременного измерения $\bar{n}(t)$ и $P_0(t)$ для 5 различных интервалов времени t.

Основным элементом анализатора является так называемый p — элемент, предназначенный для определения величины P_0. p — элемент (фиг. 2) представляет собой триггер с двумя устойчивыми состояниями с управлением по обеим сеткам. На вход № 2 поступают периодические хронирующие импульсы отрицательной полярности от таймера, период повторения которых задает длительность интервала t. После хронирующего импульса правый триод остается в закрытом состоянии, левый — в открытом. На вход № 1 поступают импульсы от детектора той же (в данном случае отрицательной) полярности. Если в течение данного интервала от детектора поступает один или более одного импульса, состояние триггера изменится на обратное, так что очередной хронирующий импульс, восстанавливая первоначальное состояние, выдает положительный импульс на выходе. При отсутствии отсчетов в данном интервале состояние схемы не изменится,



Фиг. 1

Блок-схема статистического анализатора. *ЗГ* — звуковой генератор, *Т* — триггер, P_k — p = элемент, k — *го* канала, *д* — детектор, *УД* — усилитель и дискриминатор.



Фиг. 2

Принципиальная схема p — элемента.

и очередной хронирующий импульс не будет зафиксирован на выходе p — элемента.

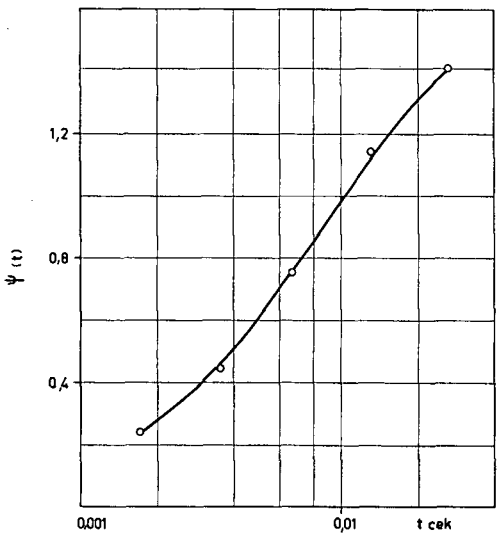
Таким образом, число импульсов на выходе p — элемента равно числу интервалов, в которых был отмечен по крайней мере один отсчет. Знание полного числа интервалов (число хронирующих импульсов) позволяет определить вероятность P регистрации не менее одного отсчета в интервале заданной продолжительности, которая связана с искомой вероятностью отсутствия отсчета P_0 посредством простого соотношения $P_0(t) = 1 - P(t)$. Эта величина вместе с регистрируемой средней загрузкой на интервал n определяет величину $\psi(t)$ из равенств (3) и (3[а]).

Таймер состоит из звукового генератора с пересчетным устройством, состоящим из четырех триггеров. Таймер одновременно выдает 5 серий хронирующих импульсов с периодичностями, отличающимися на фактор 2. Каждая из серий подается на вход № 2 p — элемента своего канала. Импульсы детектора поступают на вход № 1 каждого канала.

Весь 5-канальный статистический анализатор содержит 18 двойных триодов, работающих либо в режиме триггера, либо в режиме катодного повторителя.

Таким образом, в результате одного измерения получается набор из 5 значений $\psi(t)$, который позволяет получить искомые параметры кинетики.

На фиг. 3 показана одна из зависимостей $\psi(t)$. Кривая проведена методом наименьших квадратов.



Фиг. 3

Зависимость $\psi(t)$ для критического реактора ПФ-2а. Кривая проведена методом наименьших квадратов.

Точность измерения α зависит от эффективности детектора (точнее, величин ψ), количества обработанных интервалов, правильности выбора длины интервалов и др. (см. [8]). Оптимальным для p — метода является выбор длительности среднего (третьего) интервала d из условия $\alpha d \sim 1$, средней загрузки на интервал в пределах $0{,}1 < \bar{n} < 3$ и выбор как можно большей эффективности ε. Практически при $\alpha \backsim 100$ сек$^{-1}$ в критическом реакторе точность измерения α составляла $\sim 2\%$ за 5 минут при $\psi_{max} \sim 1$. За это время полное число обработанных интервалов имело порядок $3 \cdot 10^5$.

Метод и аппаратура допускают расширение пределов применимости в сторону больших скоростей счета (соответственно большие значения $\bar{n}$) с измерением P_K или $q_K = P_K + P_{K+1} + \ldots$ при $k > 1$.

4. Результаты измерений, выполненных с помощью статистического анализатора

Ниже описаны некоторые результаты измерений, выполненных с помощью статистического анализатора на одном из вариантов реактора ПФ-4 и реакторе ПФ-2а. Детектором нейтронов в этих опытах служил пропорциональный счетчик СНМ-5 с BF_3 -заполнением. Вместе с детектором

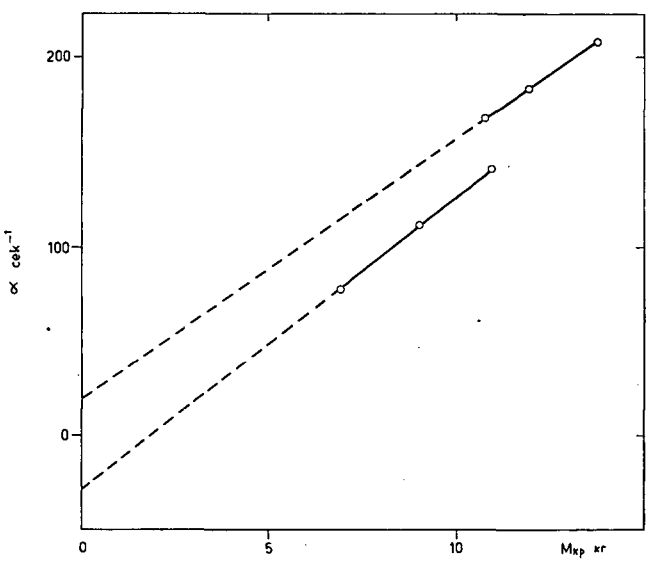
был смонтирован двухкаскадный предусилитель, соединявшийся 10-метровым коаксиальным кабелем с основным трехламповым усилителем. Сигнал с усилителя поступал на статистический анализатор через дискриминатор типа АД-1.

Для обеспечения приемлемой эффективности детектор помещался, как правило, в отражатель. Помещение детектора в активную зону не изменяло результата, хотя несколько увеличивало статистическую точность за счет некоторого роста ψ.

В физической сборке ПФ-4 [9] с гетерогенной центральной зоной ($\varrho_{Be}/\varrho_{U}=15{,}7$) последняя занимала 15% активной зоны, в остальной (гомогенной) части отношение ядерных концентраций бериллия и урана —235 составляло 31. Два борных регулятора с диаметром 8,4 мм, расположенные вблизи границ зон, были окружены бериллиевыми цилиндрами внешним диаметром 47 мм. В таком состоянии среднее время жизни мгновенных нейтронов оказалось равным $13{,}5\pm1{,}5$ мксек (принимая $\beta_{эфф}=0{,}007$).

Измерения показали, что перемещение бериллия толщиной 40 мм из торцевых отражателей гетерогенной части в середину центральной зоны увеличивает среднее время жизни до $20{,}6\pm0{,}3$ мксек. Последующее удаление бериллиевого чехла, окружающего один из регуляторов, снижает l до $16{,}9\pm0{,}2$ мксек. Эти данные подтверждаются измерениями методом частотного анализа шумов критического реактора.

Из приведенные результатов следует, что подавляющий вклад в среднее время жизни мгновенных нейтронов вносит время диффузии в бериллиевых блоках отражателя и активной зоны и что измерение величины $\alpha_0=\beta_{эфф}/l$ является чувствительным индикатором доли мягкой части спектра.



Фиг. 4

Зависимость α от критической загрузки M_{kp} для реактора ПФ-2а с водяным и бериллиевым отражателем.

Физический реактор ПФ-2а с водяным замедлителем собирался в двух вариантах: с боковым водяным отражателем толщиной 100 мм или такой же толщины бериллиевым отражателем. В реактор равномерно вводились различные поглотители, вследствие чего изменялась критическая масса M_{kp}. Зависимость α от M_{kp} при данном отражателе оказалась линейной. Этот результат не является неожиданным, учитывая, что утечка из активной зоны мало изменяется, так что можно ожидать, что α, характеризующая в данном случае скорость поглощения нейтронов, будет линейно зависеть от количества (концентрации) урана, (поглощение дополнительными поглотителями в первом приближении компенсируется пропорциональным возрастанием концентрации урана).

Две прямые (фиг. 4), на которые легли экспериментальные точки $\alpha = \alpha(M_{kp})$, имеют близкий наклон, по среднему сечению соответствующий тепловому спектру. Сдвиг прямых характеризует различие в диффузионных свойствах бериллиевого и водяного отражателей.

Простота экспериментальной техники и непродолжительное время эксперимента выгодно отличают настоящий метод, который может быть применен к решению широкого круга задач.

ЛИТЕРАТУРА

[1] ORNDOFF, J., *Nucl. Sci. and Engng* **2** (1957) 450.
[2] BENGSTON, J. *et al.*, Доклад № 1783 на 2-й женевской конференции по мирному использованию атомной энергии, 1958.
[3] COHN, C. E., *Nucl. Sci. and Engng* **5** (1959) 331.
[4] MOORE, M. N., *Nucl. Sci. and Engng* **6** (1959) 448.
[5] Научные и технические основы ядерной энергетики, т. 2. По ред. К. Гудмена М., изд. иностранной литературы, 1950, стр. 118.
[6] PETROFF, M. D., and DOGGETT, W., *Rev. sci. Instrum.* **27** (1956) 838.
[7] МОГИЛЬНЕР А. И. и ЗОЛОТУХИН В. Г., *Атомная энергия* **10** (1961) 377.
[8] ЗОЛОТУХИН В. Г. и МОГИЛЬНЕР А. И. *Атомная энергия* **10** (1961) 379.
[9] ЛЕЙПУНСКИЙ А. И., КУЗНЕЦОВ В. А. и др., Экспериментальные исследования некоторых физических особенностей промежуточных реакторов с бериллиевым замедлителем, доклад на Венском семинаре по физике быстрых и промежуточных реакторов, 1961.

IV. 2. STABILITY

SM-18/49

AN ANALYSIS OF THE STABILITY OF EBR-I, MARKS I TO III, AND CONCLUSIONS PERTINENT TO THE DESIGN OF FAST REACTORS*

R. R. SMITH, R. G. MATLOCK, F. D. MCGINNIS, M. NOVICK AND F. W. THALGOTT
ARGONNE NATIONAL LABORATORY, ARGONNE, ILL.,
UNITED STATES OF AMERICA

Abstract — Résumé — Аннотация — Resumen

An analysis of the stability of EBR-I, Marks I to III, and conclusions pertinent to the design of fast reactors. An intensive re-examination of the instabilities noted in EBR-I, Marks I and II, has resulted in a physico-mathematical feedback model which satisfactorily describes the frequency-dependence and the intensity of resonant effects. As the result of these studies it now seems firmly established that those features responsible for the instabilities may be directly attributed to two undesirable features of mechanical design. One concerns the existence of a perforated shield-plate system which controlled reactivity through the delayed expansion of ligaments contiguous with fuel-rod extensions. The effects of both were manifested, respectively, by two distinct power-coefficient components: one negative and strongly delayed; and the other, positive and prompt. The combination of the two resulted in a feedback which permitted constructive interference between input and feedback reactivities.

The results of stability tests conducted on Mark III strongly substantiate this conclusion since the elimination of the shield-plate system and the addition of stabilizing ribs to the fuel rods resulted in a system which displayed none of the undesirable features characterizing earlier loadings, and which was exceedingly stable under all credible operating conditions.

As a consequence of these studies it follows that the stability of fast systems may be controlled completely by proper mechanical design. Careful planning, with particular emphasis placed on matters of fuel-rod motion, should be sufficient to guarantee the stability of future fast reactors.

Analyse de la stabilité des réacteurs EBR-I, Mark I à III, et conclusions applicables aux études de réacteurs à neutrons rapides. Une nouvelle étude approfondie des instabilités constatées dans les réacteurs EBR-I, Mark I et II, a incité à concevoir un modèle physico-mathématique à rétroaction, qui indique de façon satisfaisante l'intensité des effets de résonance et la façon dont ils dépendent de la fréquence.

A la suite de cette étude, il semble maintenant bien établi que les causes d'instabilité peuvent être directement attribuées à deux aspects mécaniques du réacteur: d'une part, l'emploi d'un système de plaques de protection perforées qui agissaient sur la réactivité du fait de l'expansion retardée des ligaments voisins des prolongements des cartouches de combustible; d'autre part, l'absence de couplage radial entre les cartouches de combustible. Les effets des deux causes se manifestaient par l'apparition de deux composantes distinctes du coefficient de puissance: l'une négative et fortement retardée, l'autre positive et instantanée. Leur combinaison avait pour résultat une rétroaction permettant une interférence constructive entre la réactivité à l'entrée et celle due à la rétroaction.

Les résultats des essais de stabilité effectués sur le Mark III confirment la conclusion ci-dessus; la suppression du système de plaques de protection et l'adjonction de nervures de stabilisation aux cartouches de combustible ont donné un système extrê-

* Work performed under the auspices of the United States Atomic Energy Commission.

ment stable dans toutes les conditions de fonctionnement concevables et ne présentant aucun des défauts constatés précédemment.

Il s'ensuit que la stabilité des réacteurs à neutrons rapides peut être complètement contrôlée de manière purement mécanique. Une planification soigneuse, en s'attachant tout particulièrement au problème des mouvements des cartouches de combustible, devrait suffire à assurer la stabilité des futurs réacteurs à neutrons rapides.

Анализы устойчивости реактора EBR-I марки I-III и заключения, относящиеся к проектам реакторов на быстрых нейтронах. Интенсивное повторное исследование неустойчивости, замеченной в реакторе EBR-I марок I и II, имело своим результатом физико-математическую модель обратной связи, которая удовлетворительно описывает частотную зависимость и интенсивность резонансного эффекта, Как результат этих исследований, в настоящее время, по-видимому, твердо установлено, что те черты, от которых зависит неустойчивость, могут быть прямо отнесены к двум нежелательным особенностям механического проекта. Одна из них касается существования системы перфорированного экрана, которая контролировала реактивность через запаздывающее расширение конструктивных элементов связей в результате удлинений топливного стержня. Другая связана с отсутствием радиального сопряжения между топливными стержнями. Влияние обоих факторов соответственно сказывается на двух различных компонентах коэффициента зависимости реактивности от мощности: один отрицательный и сильно запаздывающий и другой положительный и быстрый. Сочетание этих двух явлений имело своим результатом обратную связь, которая допускала конструктированное вмешательство между вводимой реактивностью и реактивностью обратной связи.

Результаты испытаний на стабильность, проведенных на реакторе марки III, в значительной мере подтвердили этот вывод, после того как исключение системы экрана и дополнение стабилизирующих ребер к топливным стержням имели своим результатом систему, которая не обнаружила ни одного из нежелательных явлений, характеризующих более ранние нагрузки, и которая была чрезвычайно стабильной при всех вероятных рабочих условиях.

Как следствие этого изучения, вытекает, что стабильность быстрых систем может полностью контролироваться соответствующей механической конструкцией. Тщательное планирование с особым акцентом на вопросах движения топливного стержня должно быть достаточным, чтобы гарантировать устойчивость будущих реакторов на быстрых нейтронах.

Análisis de la estabilidad de los reactores EBR-I, Mark I, II y III, y conclusiones aplicables al diseño de reactores rápidos. Como resultado de un nuevo y meticuloso estudio de las inestabilidades observadas en los reactores EBR-I, Mark I y II, los autores han concebido un modelo físico-matemático de realimentación que describe satisfactoriamente la intensidad de los efectos de resonancia y su variación en función de la frecuencia. Gracias a dicho estudio, parece ahora firmemente establecido que la inestabilidad puede atribuirse directamente a dos inconvenientes del diseño mecánico; por una parte, el empleo de un sistema de placas de protección perforadas, que alteraba la reactividad debido a la dilatación retardada de los ligamentos contiguos a la prolongación de los elementos combustibles; por otra parte, la falta de acoplamiento radial entre los elementos combustibles. Los efectos de ambos se manifestaron por la aparición de dos componentes distintas del coeficiente de potencia: una negativa y muy retardada, y la otra positiva e instantánea. Su combinación se tradujo en una retroalimentación que permitió una interferencia constructiva entre la reactividad inicial y la debida a la realimentación.

Los resultados de las pruebas de estabilidad efectuadas con el reactor Mark III confirman esta conclusión; la supresión del sistema de placas de protección y la adición de nervaduras estabilizadoras a los elementos combustibles permite obtener un sistema que no presenta ninguno de los inconvenientes característicos de las cargas anteriores y que es extraordinariamente estable en todas las condiciones del funcionamiento concebibles.

Del estudio se deduce que es posible lograr la estabilidad de los reactores de neutrones rápidos mediante un diseño mecánico adecuado. Su concepción cuidadosa, particularmente en lo que se refiere a los movimientos de los elementos combustibles, debe bastar para asegurar la estabilidad de los futuros reactores de neutrones rápidos.

Introduction

One of the most significant conclusions resulting from the Mark III (EBR-I) stability studies consists in the fact that there is nothing intrinsic in the mechanical or neutronic features of a fast reactor which would cause it to be unstable [1, 2]. As the result of a very comprehensive series of experiments, it has been demonstrated conclusively that the Mark III loading of EBR-I is characterized by extreme stability under all credible operating conditions. As an immediate consequence of these experiments it follows that the resonant instabilities noted in Mark II (and in Mark I) were the result of certain mechanical features of design. A detailed reinvestigation of structural Mark II feedback-effects, coupled with information gained in the Mark III tests, has led to a credible and acceptable explanation of the Mark II instability.

The first attempt to study the dynamic behaviour of the reactor was initiated in May 1955 [3]. The results of these studies, in which the transfer function of the reactor was measured under various conditions of power and flow, demonstrated that the reactor could be brought into a resonant condition at certain frequencies. Unfortunately, the measurements were relatively crude and attempts to interpret the results in terms of physical feedback-processes were unsuccessful.

In November, 1955, a second, more comprehensive, series of tests was initiated. Again the same oscillatory phenomena were observed. A transient experiment carried out with the main coolant flow stopped demonstrated conclusively the existence of a prompt-positive power coefficient and led to an unintentional partial melt-down of the core [4, 5].

As the result of this incident a large amount of effort was devoted to an analysis of the instability and melt-down results. For the most part, earlier thinking was guided by the following well-established items of information. Under normal steady-state operating conditions the net power coefficient was negative. Reactivity had to be added at constant flow and constant inlet temperature to initiate and to sustain a power increase. However, it seemed clear then, as it does now, that the time behaviour of the overall power coefficient was dominated by two major components: one positive and prompt and the other negative but larger in magnitude and much more slowly acting. Direct evidence of their existence was provided by the results of flow-change tests. Immediately following a reduction in flow the power would increase, pass through a maximum and would eventually decrease to some lower-equilibrium valve. Following a flow increase the converse behaviour was noted. Evidently the sudden increase in fuel and coolant temperature following a flow reduction was sensed by the reactor through the prompt-positive component as an addition to reactivity later cancelled and over-ridden by the larger and more slowly acting negative component.

From the oscillator results it was known that the resonance peak shifted towards lower frequencies as the flow rate was reduced. This behaviour suggested that the process responsible for the negative component must in some way have been associated with the physical transport time of the coolant through

the reactor. The results of the excursion experiment substantiated the validity of this concept, since at greatly decreased flows the normally over-riding negative component essentially disappeared and caused the resultant power coefficient to be positive.

With these facts and concepts in mind a large effort was devoted to the interpretation of the dynamic behaviour of the reactor in terms of various feedback mechanisms which, in principle, should have accounted for the existence of the prompt-positive and delayed-negative power coefficient components.

To explain the resonant behaviour SIEGEL and HURWITZ [6] postulated a mathematical model based on the coupling of individual power coefficient components, each with its own characteristic time-dependence. As an extension of this concept THALGOTT [7] demonstrated that a feedback model based on prompt-positive and delayed-negative power coefficient components was qualitatively consistent with experimental facts.

In a similar direction KINCHIN [8] formulated a mathematical feedback model based on a positive component arising from a radial bowing of fuel rods and a delayed-negative component resulting from a delayed expansion of the tube sheet (the perforated shield plate located immediately above the Mark I and Mark II cores). By assigning credible values for time constants and power coefficients associated with the various feedbacks, Kinchin was able to reproduce in a qualitative manner the general structure of the EBR-I resonances.

Using an analytic rather than a synthetic approach BETHE [9], in his elaboration of the Kinchin concepts, arrived at a value for the effective transport lag associated with the delayed-negative component (10 s) and a partition of the net power coefficient into prompt and delayed components. While Bethe attributed the source of the positive term to an inward bowing of fuel rods, as did Kinchin, he was unable to identify the origin of the delayed-negative component with any specific structural member. It is interesting to note that Bethe commented on the substantial inconsistency existing between the analytically determined transport lag (10 s) and the actual physical transit time of the coolant through the reactor (about one second).

In view of the results of the Mark III tests, it now seems clear that rod bowing was indeed the source of the postulated positive component. The origin of the delayed-negative component was, however, a matter of some mystery. One of the first attempts to explain this effect was that of LICHTENBERGER [10] who postulated a mechanism based on the pre-heating of the core-inlet coolant by a transfer of heat across the flow divider from core outlet to blanket inlet. Such a mechanism was at one time considered credible since the physical transit time for coolant flowing from blanket to core was qualitatively consistent with the concept of a transport time lag. The results of intensive tests devoted to a study of such possibilities in Mark III have, however, reduced considerably the credibility of this and other pre-heating mechanisms.

A recent detailed consideration of structural feedback-effects peculiar to Mark II, coupled with information gained in the Mark III tests, points strongly to the conclusion that the mechanism responsible for the delayed feedback of Mark II involved thermally-induced motions in the lower shield plate, a perforated plate located immediately down stream from the reactor core. It is particularly significant that a mathematical treatment of the mechanical feedback arising from these motions predicts with a high degree of accuracy the natural resonance frequency of the reactor.

1. Previous feedback models

The efforts of both Kinchin and Bethe have been extremely useful in the identification of the origin of the delayed structural power coefficient component and, as a basis for later discussion, it is well to consider in some detail their mathematical concepts. In common terminology Kinchin [8] assumed that the fast or prompt contribution to the dynamic power coefficient was described by the following relations where X^0_f is

$$X_f(i\omega) = \frac{X^0_f}{(1 + i\omega\tau_f)} \tag{1}$$

the zero frequency (or steady-state) power coefficient, ω is the oscillation frequency, and τ_f is the time constant describing the fast response of the core to power changes. Kinchin further postulated that the prompt steady-state power coefficient is defined by the algebraic sum of the following components

$$X^0_f = X^0_b + X^0_c + X^0_u \tag{2}$$

where X^0_b, X^0_c and X^0_u are the steady-state contributions from rod bowing, coolant expansion and uranium expansion, respectively. To explain the resultant positive coefficient at zero coolant flow Kinchin assumed that the positive rod-bowing contribution more than compensated for the various negative effects or, in other words, that X^0_f is positive. To explain the overall negative nature of the power coefficient under normal flow conditions it was necessary to include a negative term for the effects of structural expansion described by the following relation

$$X_s(i\omega) = \frac{X^0_s}{(1 + i\omega\tau_f)(1 + i\omega\tau_s)} \tag{3}$$

where X^0_s is the zero frequency or steady-state value of the delayed structural power coefficient component. The magnitude of X^0_s is postulated such that the following inequality holds:

$$X^0_b + X^0_c + X^0_u + X^0_s < 0. \tag{4}$$

The sum of Eqs. (1) and (3) multiplied by the average power P defines the reactivity feedback given by

$$-H = P\left[\frac{X^0_f}{(1 + i\omega\tau_f)} + \frac{X^0_s}{(1 + i\omega\tau_f)(1 + i\omega\tau_s)}\right]. \tag{5}$$

Through the substitution of credible values for the various time constants and the power coefficient in Eq. (5), Kinchin was able to show that the feedback described by Eq. (5) does indeed lead to a resonant structure not unlike that experimentally observed in the Mark II oscillation studies. The physical origin of the feedback described by the delay term was attributed by Kinchin to the movement of fuel rods by thermally induced motions of the tube sheet (shield plate).

In his elaboration of the Kinchin concepts Bethe assumed that structural heating proceeds in two stages: first the convection of heat by the coolant from the fuel to the relevant portion of the structure, and then diffusion of heat into the particular component. Bethe preferred to describe the convection term by the cyclic operator $e^{-i\omega\tau}$ where τ is the physical transit time of the coolant from core to structure. Accordingly, his feedback is described by the following

$$-H = P\left[\frac{X^0_f}{(1 + i\omega\tau_f)} + \frac{X^0_s\, e^{-i\omega\tau}}{(1 + i\omega\tau_f)(1 + i\omega\tau_s)}\right] \tag{6}$$

where the various power coefficients and time constants have the same significance as those in Eq. (5). Bethe's expression for the feedback is superior to Kinchin's in that Eq. (6) accounts, qualitatively at least, for the displacement of the resonance towards higher frequencies at the higher flow rates.

From the results of Mark II transfer-function measurements and through a modification of Eq. (6), Bethe arrived at a value of 10 s for the transport lag τ, a value grossly inconsistent with the actual transit time between core and exit (about one second at one-third flow). This extremely significant discrepancy was later resolved by STORRER [11] who showed that at low frequencies a temperature signal transported by the coolant travels slower than the coolant by the factor $(\tau_f + \tau_c)/\tau_c$, where τ_f is the time constant for the fuel and τ_c is the time constant for the coolant. An evaluation of this ratio for Mark II results in a value of about 10. Hence, a physical transit time of one second is actually sensed by the structure as a 10-s lag. The importance of Storrer's contribution cannot be overemphasized since the solution of this perplexing discrepancy prepares the way for more rigorous mathematical approaches.

Unfortunately, the use of Eq. (6) with τ modified to include Storrer's refinement predicts the existence of significant harmonics in the amplitude of the transfer function, harmonics sufficiently large in magnitude that it is inconceivable they would have been "missed" in the measurements.

The failure to observe harmonics, whereas theory clearly indicates their existence, seriously challenges the validity of the transport-lag model. While certain items of information may be cited in support of this model other items of equal importance may be offered in contradiction. The explanation of why the transport-lag model fails to explain contradictory information is essentially this: the model is much too simple to include the effects of extremely complicated and important feedbacks which are now known to exist. To explain all experimentally observed phenomena (which a proper model must do) it is necessary to add a second transport-lag term to Eq. (6). As a further complication, a mathematical development reveals that both effective transport time constants and both power coefficients are frequency-dependent.

2. Application of Mark III results to the Mark II problem

To appreciate the striking differences in the dynamic performances of Mark II and Mark III it is necessary to understand the nature of the changes incorporated in the Mark III loading. Following the melt-down, all components enclosed within the inner tank were removed and were replaced with a core and support structure which embodied important mechanical changes [12]. For the most part, the nuclear and heat-transfer characteristics remained unchanged. Both loadings consisted of metallic slugs of highly enriched uranium and both were designed to operate at comparable power densities. In regard to the physical relationship of fuel with respect to structure, strong changes were effected. In brief, the Mark II fuel rods were loose, while those in Mark III were compacted into a rigid array through the action of longitudinal ribs and tightening rods. A further important difference was effected by the elimination of the perforated-shield-plate system used for the location and orientation of fuel rods in Mark II.

Other less important differences consisted of the elimination of the NaK annulus between fuel and cladding and the provisions made to permit parallel coolant flow.

Several important conclusions, each pertinent to an understanding of Mark II behaviour, have resulted from the Mark III oscillator studies. The delayed negative power coefficient component, so obviously present in Mark II, was eliminated, presumably through the elimination of the perforated-shield-plate system. The addition of stabilizing ribs to the fuel rods was beneficial in two respects: positive feedback from rod bowing was prohibited and the much higher degree of radial coupling between fuel rods promoted a much stronger prompt-negative component. It may also be argued that the inclusion of stabilizing ribs to the fuel rods reduced considerably the possibility of feedback arising from the delayed expansion of downstream structural members.

One of the most important conclusions resulting from the Mark III tests is concerned with the fact that inward rod bowing is a consequence of special conditions of rod restraint, conditions which are not easily realized in practice. In fact, the promotion of conditions conducive to a large positive rod-bowing effect proved to be one of the major difficulties encountered in the Mark III stripped-rib tests. Unless the fuel rods are rigidly fixed at upper and lower restraining points, reactivity effects associated with rod deformation may be positive, negative or even zero. Since the fuel rods in Mark II were essentially loose with respect to structural bearing points it is difficult to appreciate why rod-bowing effects were so pronounced. As described below, certain structural features effected conditions highly conducive to rod bowing whenever the reactor operated under high power-density conditions.

3. Description of Mark II

Since it will be necessary to refer frequently to various features of the Mark II core and structure, it is essential that some effort be devoted to a sufficiently detailed description of those features particularly pertinent to the various feedback processes. A cut-away view of the reactor as it existed during the Mark I and Mark II loadings is given in Fig. 1. Surrounding the reactor tank is a massive cup composed of stacks of keystone-shaped uranium bricks clad with stainless steel. Since there is no evidence that the cup entered into any of the feedback considerations subsequent discussion will be confined to those components included inside the reactor tank. A cross-sectional view through the reactor at core elevation is given in Fig. 2. Relevant radial and vertical dimensions are given by a sketch, Fig. 3. As an aid in later calculations pertinent features of the reactor are broken down into specific regions and are designated as such in Fig. 3.

The core region consists of a hexagonal assembly of cylindrical fuel rods each of which consists of a stainless-steel tube 0.448 in in diameter with a 0.020-in wall thickness containing concentrically spaced fuel and blanket slugs, both of which are 0.384 in in diameter. A 0.010-in annulus contains NaK to serve as a heat-transfer bond. The fuel rods (Fig. 4) are positioned at the bottom on 0.494-in centres by a plate perforated with triangular holes, with, in the case of Mark II, engaged conical-shaped rod tips. Dimensions and tolerances are such that a 0.005-in diametral clearance exists between rod tips and holes. Surrounding each positioning hole are six coolant passages $^3/_{16}$ in in diameter

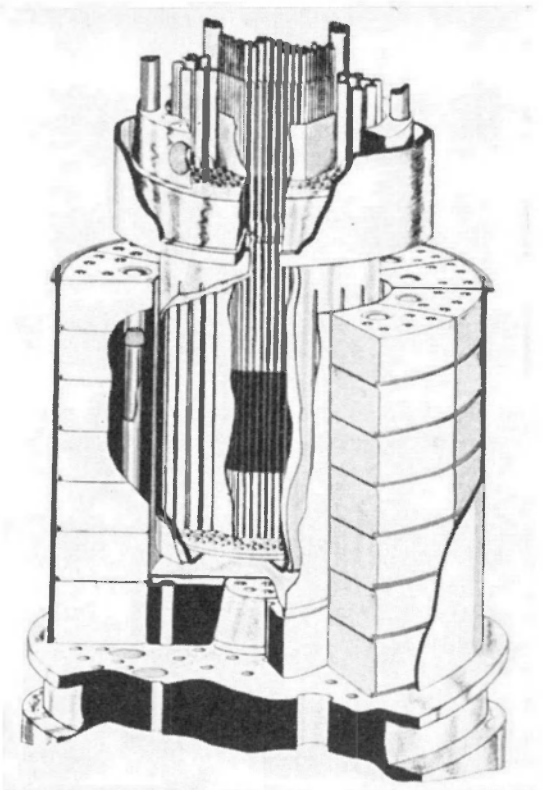


Fig. 1
Cross-sectional view through Mark I and Mark II cores.

and three $^{1}/_{16}$ in in diameter. Surrounding the hexagonal core (see Fig. 2) are 138 blanket rods nominally 0.963 in in diameter. These are located at the bottom by 0.498-in truncated-conical tips fitted into 0.502-in circular holes in the bottom plate.

Approximately 16 in above the centre of the core is a 4-in-thick shield plate illustrated photographically in Fig. 5. The plate consists of two regions: an inner hexagonal section containing 217 fuel rod holes nominally 0.460 in in diameter and an outer arrangement of 138 0.964-in holes for blanket rods. Separating the two regions is a hexagonal annulus approximately 0.20 in wide with grooves milled to a depth of 0.125 in in the upper and lower surfaces. A hexagonal flow divider of stainless steel 0.088 in in thickness fitted into grooves in the lower surface of the shield plate (and in the bottom plate) serves to separate inlet and outlet coolant. Immediately above the lower shield plate is a 3.812-in inlet spacer. This consists essentially of a massive (hollow) ring through which the main tie rods penetrate. Holes along the periphery admit inlet coolant. Immediately above the inlet spacer is the 2-in seal plate which is identical in cross-section with the lower shield plate. A flow divider approximately 4 in long fits into grooves milled in the upper surface of the lower shield plate and lower surface of the seal plate.

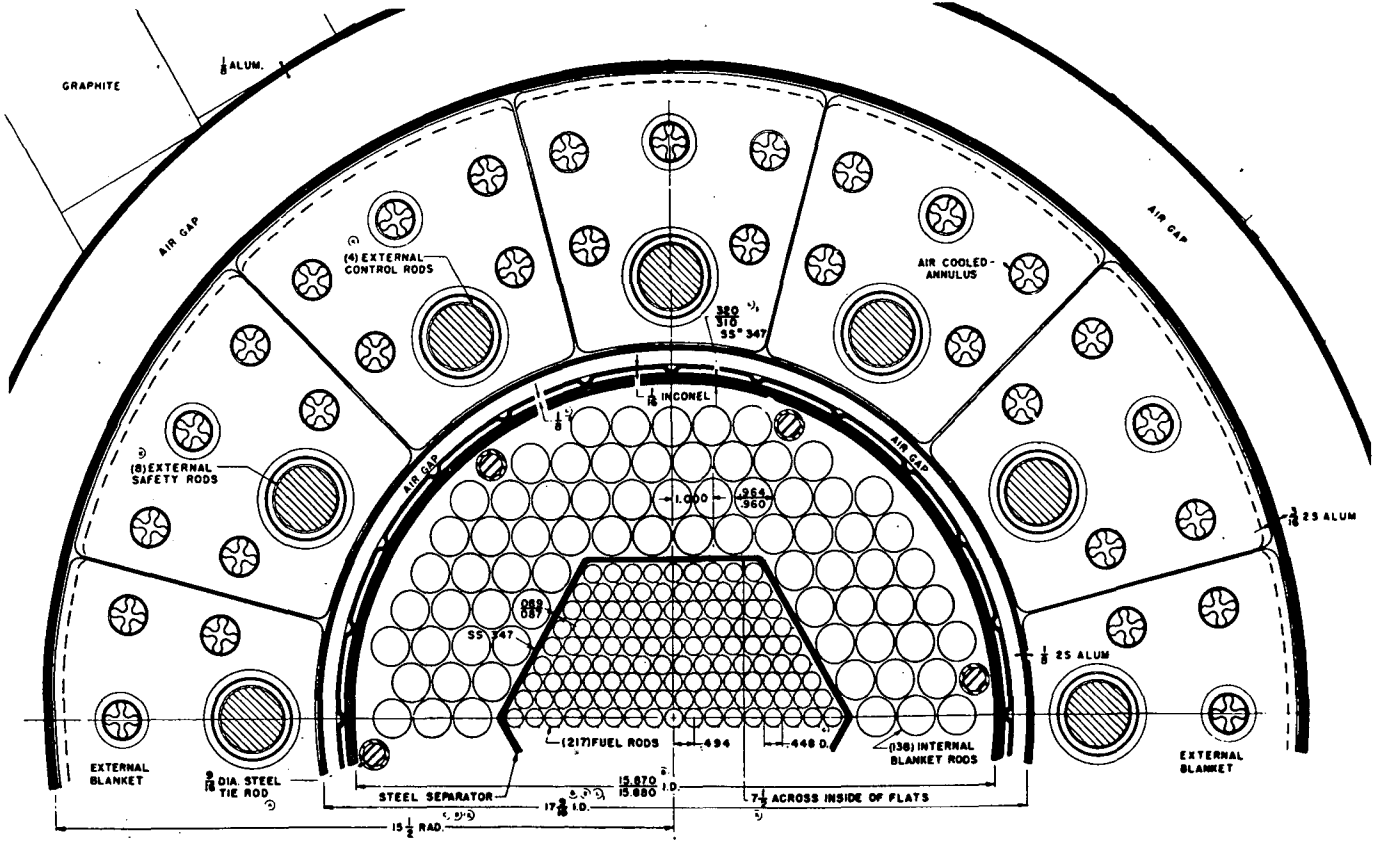


Fig. 2
Cutaway view of EBR-I, Mark I and Mark II loadings.

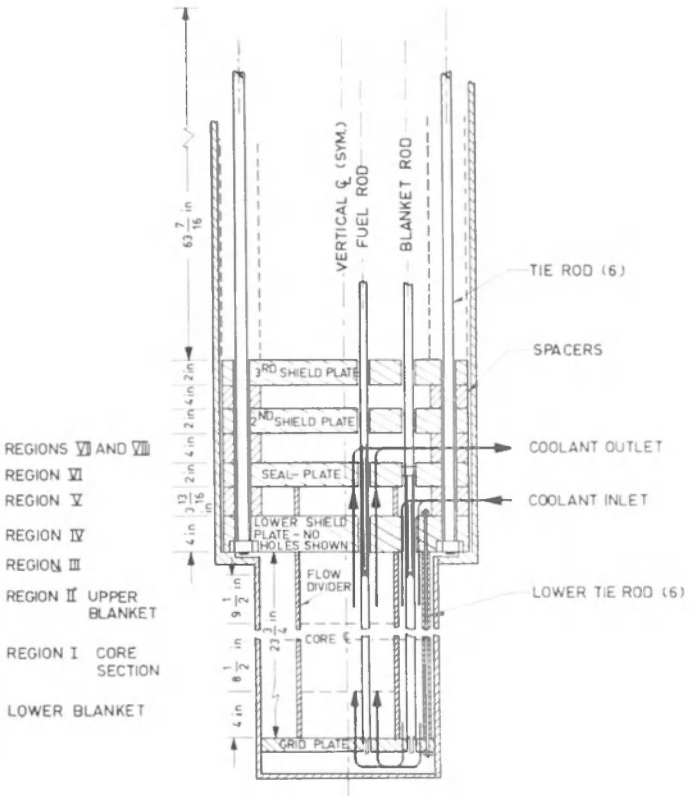


Fig. 3
Schematic view of EBR-I structure.

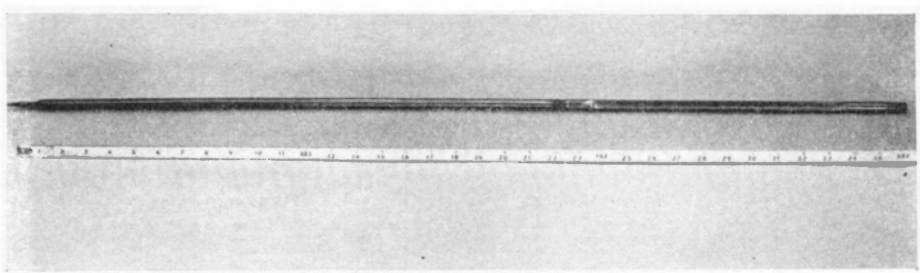


Fig. 4
EBR-I, Mark II fuel rod.

To permit the passage of coolant, blanket-rod extensions are fluted in the vicinity of the lower shield plate while fuel-rod extensions are fluted over the length extending from the bottom of the lower shield plate to the top of the seal plate. The nominal dimensions of fuel-rod extensions (at maximum diameter) and shield-plate perforations are 0.449 and 0.460 in respectively, giving a nominal diametral clearance of 0.011 in. A cross-section through a typical fuel-rod extension

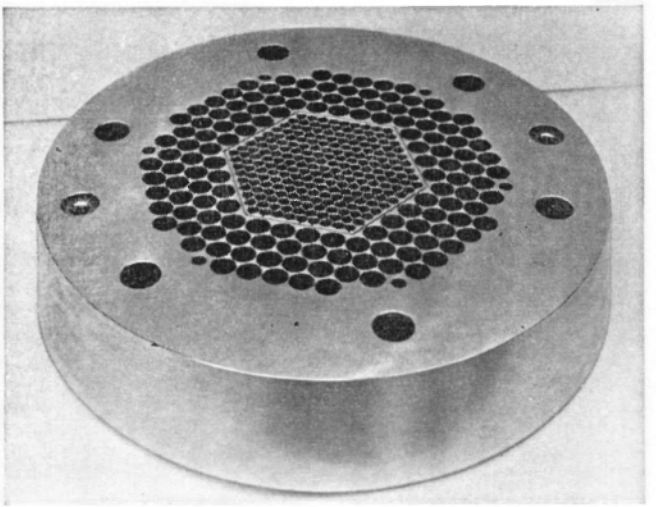


Fig. 5
EBR-I shield plate.

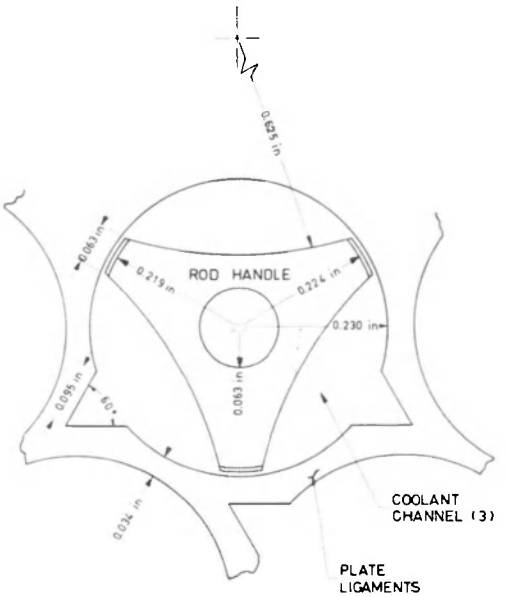


Fig. 6
Horizontal cross-section through lower shield plate and rod handle.

and shield-plate perforation at the lower surface of the shield plate is given in Fig. 6. The nominal dimensions of the fuel-rod extensions in the vicinity of the seal plate and at the lower surface of the lower shield plate are slightly different. The reduction in radius from 0.224 in at the lower surface of the lower shield

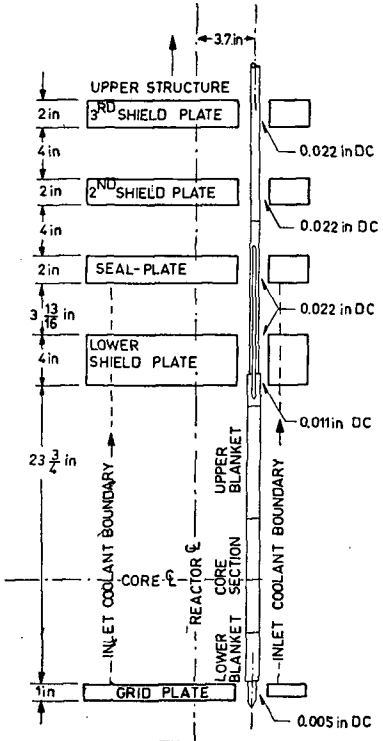


Fig. 7
Typical fuel-rod orientation at 25°C, zero power.

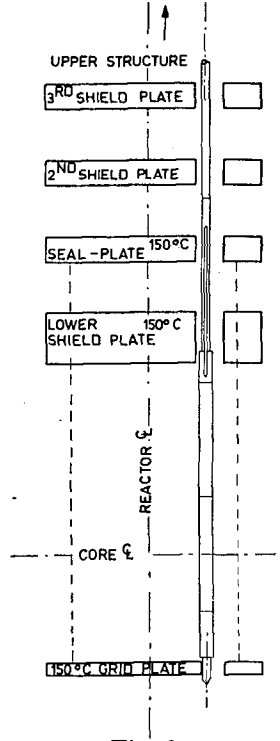


Fig. 8
Typical fuel-rod orientation at 150°C, zero power. Rods are preferentially gathered at the inner edge of grid and lower shield plates because of radial expansion.

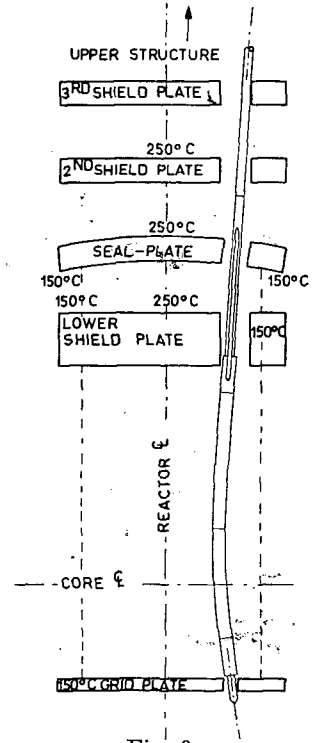


Fig. 9
Typical fuel-rod orientation at one-third full power and one-third flow, 150°C inlet temperature, 250°C outlet temperature. Rod tips are gathered at the outer edge of the grid-plate holes and the inner edge of the lower-shield-plate holes.

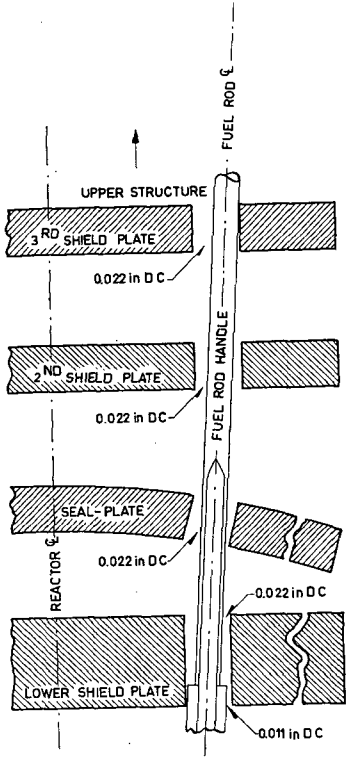


Fig. 10
Upper structure contact of a bowed fuel rod.

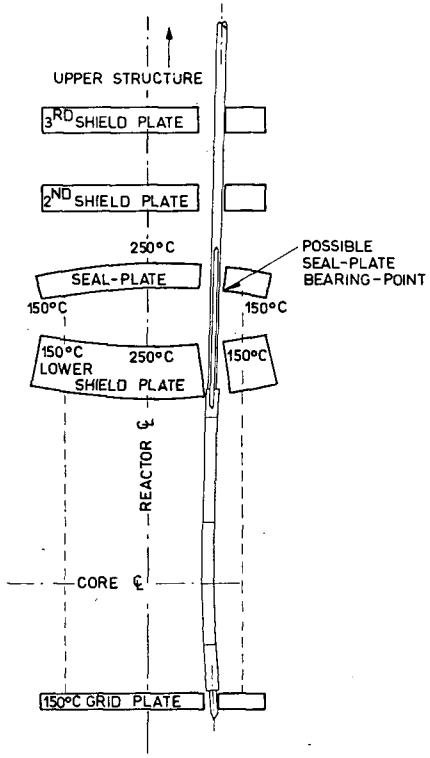


Fig. 11
Typical fuel-rod orientation during increasing portion of oscillating power cycle or immediately following a step-power increase.

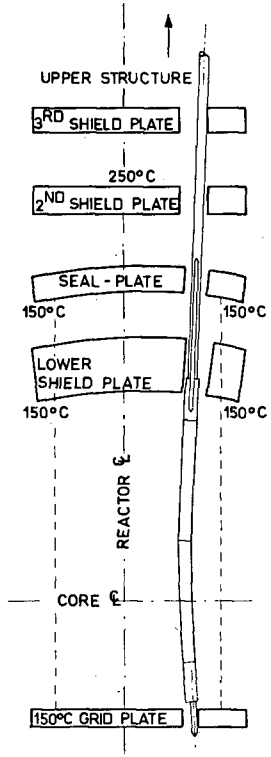


Fig. 12
Typical fuel-rod orientation during decreasing portion of oscillating power cycle or immediately following a step-decrease in power.

plate to 0.219 in in the vicinity of the seal plate is illustrated in exaggerated form in Fig. 6*.

In series flow, the only arrangement possible, the coolant flows downward around the fluting of the blanket rods into the lower plenum where the flow reverses and passes upward through the core, around the flutings of the fuel-rod extensions, and finally into the outlet spacer located immediately above the seal plate.

Above the seal plate is a series of 2- and 4-in shield plates alternating with 4-in structure rings running to an elevation of 86 in above the lower shield plate. Six 1.50-in tie rods penetrating the outer portion of the various shield plates and structure rings tighten the entire collection.

4. Mechanisms which affect fuel-rod movement

As discussed in Appendix A, the results of attempts to detect motions induced by temperature variations in the ligaments of a spare 2-in shield plate have demonstrated conclusively that such motions do exist. For the most part the motions are of two types: a rotational or tilting mode in which the vertical axes of the rod-perforations are tipped towards or away from the normal, and a translational mode in which the axes are moved radially inward or outward.

The magnitude and the time-dependence of the feedback from the plate depends on the manner in which the temperature of the plate is perturbed. Two cases are of particular interest. In one the temperature of the plate is affected by a step change in power; in the other the temperature is affected by a sinusoidal variation of power.

4.1. Fuel rod movement associated with a step-change in power

The manner in which shield-plate ligaments control reactivity is perhaps more readily understood in terms of elementary and greatly exaggerated illustrations. The relationship of fuel rods and their extensions with respect to pertinent structural features is illustrated in Fig. 7 for the conditions of zero power and a nominal inlet temperature of 25°C. For the most part the relationship between rod tips and locating holes and between rod extensions and shield-plate ligaments is one characterized by a complete lack of order. To illustrate this concept rod tips and rod extensions are shown at the centres of their respective positioning holes in Fig. 7. It is true, of course, that mechanical imperfections in the various components will permit a certain amount of physical contact but such physical contact will be a random phenomenon. The probability of a rod tip's bearing on the inner edge of a bottom plate hole is exactly the same as the probability for its bearing on an outer edge. The same argument follows for any physical contact between rod extensions and shield plate ligaments.

At elevated inlet temperatures and at zero power a reasonably high degree of order emerges. An illustration of this change is given in Fig. 8. As a consequence of the overall unconstrained radial expansion of the bottom plate, the rod tips are preferentially gathered at the inner edges of the locating holes. After the small radial clearance ($2^1/_2$ mil) is closed, the rod tips are moved farther outward as the inlet temperature increases. A similar situation exists for the rod extensions and shield plate ligaments but because rod tip clearances are

* See p. 53.

closed at a lower temperature, ligamental expansions never quite "catch up" with the rod movement effected by bottom plate expansion. A small clearance between rod extensions and ligaments will, in principle, always exist regardless of how high the temperature of the inlet coolant is raised. The actual amount of the clearance is dictated by the nominal radial clearance between the rod tips and locating holes. In practice, mechanical imperfections in the system will lead to physical contact between some rod extensions and ligaments. Such contact when it does occur will preferentially be along the inner edges of the shield-plate ligaments. This preferential orientation of rod tips and rod extensions, incidentally, explains why both Mark I and Mark II were characterized by a strong isothermal temperature coefficient of reactivity.

Near the centre of the shield plate there will, of course be less preferential orientation since the degree of radial expansion will be small. At greater distances from the centre the degree of preferential orientation increases and attains a maximum in the outer rows. The loss of coupling in the inner region is offset insofar as feedback is concerned by the much stronger coupling in the outer rows where flux bending is the greatest.

As the reactor is brought to power the fuel portion of the rod begins to bow (Fig. 9). At this point the rod tips move outward until they eventually come to bear against the outer edges of the locating holes. From this point on the tips can no longer move outward and the fuel portion of the rods bows inward. As a result of the bowing action, rod extensions not already gathered along the inner edges of the ligaments are forced inward until the system becomes highly ordered. With few exceptions (particularly in the very important outer rows) the rod tips and shield-plate bearing-point situations will be as illustrated in Fig. 9. A more detailed illustration of the contact between rod extensions and ligaments is given in Fig. 10.

An interesting consequence of full-power operation is a pronounced convex dishing of the seal plate, since the entire temperature differential across the core is manifested as a strong temperature differential across the outer edge of the seal plate.

The orientation of rod extensions with respect to the ligaments of the seal plate and the second and third shield plates is not clear. If, through mechanical imperfections and through a reverse bending of the extensions above the shield plate, contact occurs between the rod extensions and seal plate ligaments, the bearing points act as pivots and provide a mechanism for a limited amount of mechanical amplification. Ramifications of this possibility are considered in section 7. 4. For the present the development will be restricted to the simplest (non-amplifying) case and unless otherwise indicated the situation as illustrated in Fig. 9 will be assumed.

The situation existing immediately after a step increase in power is illustrated in Fig. 11. The bottom surface of the shield plate is heated relative to the upper surface. The temperature wave carried by the coolant is not "sharp" and for a limited period of time a temperature differential will exist across the plate. As a result of the preferential heating of the lower surface the plate will assume the form of a slight concave dish. (As described in Appendix A, such motions are not speculatory. They have actually been observed in simulations carried out with a spare shield plate.) Since the fuel-rod extensions bear at well-defined points along the inner edges of the shield plate ligaments, the action is manifested by an outward movement of the extensions at these points.

As the plate finally restores to thermal equilibrium (at a higher temperature), the temperature differential across the plate and consequently the concave dishing disappears. By this time, however, the effects of the much slower translational component of expansion are felt. Although moderately constrained, the ligaments undergo a slow overall radial expansion which effectively moves the rod extensions outward during and after a step-power increase.*

The situation existing immediately following a step-decrease in power is illustrated in Fig. 12. For a short period of time (s) the upper surface of the plate will be at a higher temperature than the lower surface. The plate assumes the form of a slightly convex dish and the rod extensions which are positioned with positive pressure along the inner edges of the ligaments move inward. Again such motions are known to exist from the simulations.

4.2. Fuel-rod movement associated with a sinusoidal variation in power

When the power is varied sinusoidally the various motions and associated feedbacks are somewhat different. During the rising portion of the cycle the lower surface of the shield plate will always be at a higher temperature than the upper surface. As a consequence the plate will assume the form of a concave dish. Since the action of the shield plate ligaments is essentially the same as that for a step increase in power, Fig. 11 will suffice as an illustration of this effect. During the falling portion of the cycle the upper surface will be at a higher temperature and the plate will deform to the shape of a convex dish. This situation is illustrated in Fig. 12. During the rising portion of the cycle, shield plate ligaments move outward and carry the rod extensions with them. During the falling portion of the cycle the bearing points move inward, and since the rod extensions are held with positive pressure against the bearing points, the rod extensions also move inward, tracing in reverse the path followed during the rising half of the cycle. Both effects, the outward movement of fuel during the rising portion of the cycle and the inward movement during the falling portion, are clearly negative.

Superficially, it is difficult to appreciate how a significantly large temperature gradient can exist across a 4-in plate immersed in a relatively fast-flowing stream of coolant, the temperature of which is being oscillated at low frequencies. However, several effects not immediately obvious combine to produce a situation conducive to strong temperature gradients. First of all the larger cross-sectional flow area of the shield plate effects a sharp reduction of flow velocity in this region. Secondly, the thermal capacitance of the rod extensions and ligaments causes the temperature wave to travel much more slowly than the physical velocity of the coolant. Finally, the amplitude of the temperature wave is decreased in passage through the plate. All three effects result in a situation in which a large fraction of the oscillating temperature amplitude of the coolant wave leaving the core can be manifested in the form of a temperature differential across the plate.

---

* The much slower radial expansion has been observed in the simulations. The fact that the radial mode is slower than the dishing action is attributed to the time required for the hexagonal annulus of the shield plate to expand and consequently to allow stresses built up in the ligaments to be partially relieved.

5. Oscillating temperature-differential across the plate

As Storrer [11] has shown, a temperature signal impressed at low frequencies may travel more slowly than the coolant itself. In the fuel region where heat is generated the delay is caused by an increase in the temperature of the coolant as it flows upward through the core. As a consequence a certain fraction of the heat produced in the fuel is expended in increasing the fuel temperature in order to maintain a required temperature gradient between fuel and coolant. The rate of temperature rise in the coolant is accordingly decreased and is manifested effectively as a lag in the velocity of the temperature wave.

As heated coolant flows past finely divided non-heat-producing materials, such as the rod handles, the coolant temperature is reduced through a loss of heat to the inert material. The effect of the loss is one of a reduction in the velocity of the temperature-wave front with respect to the physical velocity of the coolant. It is apparent, then, that the delay of the temperature wave (as well as the extent of amplitude reduction) will depend on the heat capacity of the inert material and the heat transfer rate between the coolant and material. For Mark II the effect is large. In general, throughout the core, upper blanket and rod extensions, the effective transport lag of a temperature signal is slowed by a factor of approximately ten.

For the most part the temperature differential across the plate arises from the fact that the temperature of the upper surface always lags behind that of the lower surface. For example, if the temperature of the upper surface lags behind that of the lower surface by 180° (which is certainly possible at high frequencies) a large fraction of the amplitude of the coolant temperature oscillation at the bottom of the plate will be manifested in the form of a temperature differential across the plate. The smearing of the amplitude of the temperature wave in its passage through the plate also contributes to the temperature differential although in a less important capacity. To calculate the temperature differential across the plate at any given time it is therefore necessary to establish not only the phase-lag across the plate but the amplitude of the temperature wave at both lower and upper surfaces.

The phase-lag across the plate is actually independent of coolant history. Its calculation relies solely on a knowledge of the flow velocity and the heat-transfer properties of the materials included in the shield plate region. The amplitude of the wave at both lower and upper surfaces does, however, rely on the history of the coolant since in its passage through the core and the upper axial blanket the amplitude of a temperature signal is smeared, i.e., what might have begun as a large oscillating signal leaving the core may arrive at the lower surface of the shield plate as a mere ripple. To establish the temperature differential across the plate at any given time and for any given frequency, it is therefore necessary to evaluate the attenuation of the oscillating temperature signal in the core, blanket and rod-handle regions. Since each of these regions differs considerably in matters of heat generation and heat-transfer properties it is necessary to evaluate the attenuation factors for each region separately. While the calculation of the temperature gradient across the plate does not require knowledge of the phase-lag across these downstream regions, an evaluation of the phase-lag between the core and the shield plate essentially defines the effective transport lag.

Since the seal plate and the second shield plate may under very special conditions also contribute to the feedback, it is necessary to carry out attenuation and phase calculations over the regions included between the lower shield plate

TABLE I

SUMMARY OF PHYSICAL PARAMETERS
470 kW, 108 gpm

Region	h BTU/h-ft °F	z in	v in/s	τ_c Coolant (s)	τ_f Rod (s)	τ_p Plate (s)	Material	$\frac{z}{v}$ (s)
I Upper half of core	4.21×10^4	4.25	28.6	0.093	0.84	—	347 stainless steel NaK U–2 wt.% Zr	0.298
II Upper axial blanket	4.21×10^4	7.5	28.6	0.093	0.84	—	347 stainless steel NaK U–2 wt.% Zr	0.260
III Rod handles	3.46×10^4	1.69	28.6	0.112	0.77	—	347 stainless steel NaK	0.059
IV Shield plate	20.8×10^4 5.94×10^4 14.8×10^4	4	20.2	0.021 — —	— $0.365(\tau_{f_1})$ —	— — $0.109(\tau_{f_2})$	NaK 347 stainless steel 347 stainless steel	0.198
V Inlet spacer	4.1×10^4	3.81	13.4	0.20	0.53	—	NaK 347 stainless steel	0.284
VI Seal plate	20.8×10^4 5.94×10^4 14.8×10^4	2	20.2	0.021 — —	— $0.365(\tau_{f_1})$ —	— — $0.109(\tau_{f_2})$	NaK 347 stainless steel 347 stainless steel	0.099
VII Inner section Outlet spacer	3.5×10^4	4	10.6 (minimum)	0.125	0.70	—	NaK 347 stainless steel	0.160*
VIII Outer section Outlet spacer	3.39×10^4	4	32.8 (minimum)	0.271	4.17	—	NaK 347 stainless steel	0.580*

Note. Regions IV and VI have three values for h, one each for coolant, rods and plate.
* Residence time.

and the second shield plate. In all, eight individual regions are involved. These are identified in Fig. 3.

With minor exceptions the treatment necessary to the evaluation of the various phase and attenuation factors is simply an application of the concepts developed by Storrer [11]. As Storrer has shown, the complex amplitude of the coolant temperature wave at an elevation z is given by

$$Y_z = Y_0 \, e^{-\lambda(z/v)} + \frac{e^{-\lambda(z/v)}}{v h \tau_c (1 + i\omega\tau_f)} \int_0^z a(z') \, e^{\lambda(z'/v)} \, dz' \tag{7}$$

where Y_0 is the amplitude (not complex) of the temperature oscillation at zero elevation. The parameter λ defined in Eq. (8) (below) is complex and is very strongly frequency-dependent. Since v is the coolant velocity, z/v defines the physical transit time of the coolant over the distance z. The time constants τ_c and τ_f are those associated with the coolant and fuel (or inert material) respectively. The evaluation of these constants for each of the eight regions is discussed in Appendix B. A summary of the various values for z/v, τ_c and τ_f, along with other pertinent information, is given in Table I. The heat-transfer coefficient h appearing in the denominator of the second term is the steady-state overall heat-transfer coefficient. Its evaluation for each of the various regions is also discussed in Appendix B. The increase in the temperature of the coolant in its passage through the core is considered through the inclusion of the specific heat generation term $a(z')$,which for this particular model (Storrer's axially continuous) is a function of elevation z' only.

The term λ appearing in the exponentials of Eq. (7) is defined by

$$\lambda = \frac{\omega}{1 + \omega^2 \tau_f^2} \left[\frac{\tau_f^2 \omega}{\tau_c} + i \left(1 + \frac{\tau_f}{\tau_c} + \tau_f^2 \omega^2 \right) \right] \tag{8}$$

where ω is the oscillation frequency (rad/s). In the absence of heat generation, for example in the upper axial blanket or in the rod-handle region, the heat generation term $a(z')$ vanishes and Eq. (7) reduces to

$$Y_z = Y_0 \, e^{-\lambda(z/v)}. \tag{9}$$

By breaking λ up into its real and imaginary parts Eq. (9) may be rewritten as

$$Y_z = Y_0 \, e^{-\alpha(z/v)} \, e^{-i\beta(z/v)} \tag{10}$$

where

$$\alpha = \frac{\omega^2 \tau_f^2}{\tau_c (1 + \omega^2 \tau_f^2)} \tag{11}$$

and

$$\beta = \frac{\omega}{1 + \omega^2 \tau_f^2} \left(1 + \frac{\tau_f}{\tau_c} + \tau_f^2 \omega^2 \right). \tag{12}$$

The term α determines the attenuation (or smearing) of the wave amplitude. From Eqs. (10) and (11) it is clear that as the frequency increases the amplitude of the temperature oscillation decreases. In a similar manner decreasing the flow velocity or increasing the flow path (at constant velocity) effects a decrease in the amplitude of the temperature oscillations. The term described by Eq. (12) defines the associated phase-lag over a path-length z and for a coolant velocity v. At low frequencies the phase lag across a region will be small: for increasingly higher frequencies the phase-lag increases rapidly.

Some qualitative insight into the concept of a temperature differential across the shield plate is provided through Eq. (10). Although the temperature wave suffers little reduction in amplitude at low frequencies, the temperature differential across the plate will be unimportant since the phase-lag across the plate is small. At high frequencies where the phase lag is large and conducive to strong gradients, the amplitude of the temperature oscillations is so reduced that the resulting differential is small. For intermediate frequencies the temperature differential will reach a maximum value. The frequency at which this occurs is extremely pertinent to an understanding of the delayed-negative feedback.

5.1 Attenuation factor and phase-lag across the core (Region I)

A rigorous solution of Eq. (7) over the length of the fuel region leads to an evaluation of the complex temperature amplitude at any given elevation in the core and for any given oscillation frequency. As a major simplification in the solution of Eq. (7) it has been assumed that power production is uniform throughout the core. This simplification is not seriously compromising since the attenuation and phase-lag of the temperature signal generated in the core are relatively minor compared with the corresponding cumulative effects of the other regions. The solution of Eq. (7) then becomes

$$Y_z = \frac{e^{-\lambda(z/v)}}{vh\tau_c(1+i\omega\tau_f)}\left[\frac{av}{\lambda}(e^{\lambda z/v}-1)\right]. \tag{13}$$

where a is the specific power generation under the conditions assumed above and where the limits of integration have been taken between $z'=0$ and $z'=z$.

A rearrangement of Eq. (13) leads to

$$Y_z = \frac{a(1-e^{-\lambda z/v})}{\lambda h\tau_c(1+i\omega\tau_f)}. \tag{14}$$

Thus, for specific operating conditions the complex temperature amplitude Y_z at the top of the core may be expressed in terms of the oscillation frequency. Since it becomes necessary eventually to compare the results of calculations with experimentally known facts, all subsequent calculations will be based on the conditions of 470 kW and 108 gpm flow, conditions for which experimental transfer-function information is available. Actually it is unnecessary to evaluate the amplitude (magnitude) of the temperature wave at the top of the core since these values may be obtained more reliably from the actual transfer-function results. It is still necessary, however, to establish from Eq. (14) the attenuation factors and phase-lag as a function of frequency. The results of a solution of Eq. (14) for the conditions specified above are summarized in Table II. The attenuation factors given consider the attenuating influence of the entire 8.5-in core. The phase-lag, however, considers only that lag occurring between the middle and the top of the core. In other words, insofar as phasing is concerned, the core midplane is considered as a reference.

5.2 Attenuation factor and phase-lag across the upper axial blanket and rod-handles (Regions II and III)

In calculating the complex temperature amplitude of the coolant wave leaving the upper axial blanket it has been assumed that the heat-generation term of Eq. (7) is zero or, in other words, that heat generation in the blanket may safely be neglected. Eq. (7) then becomes

$$Y_2 = Y_1 e^{-\lambda(v/z)} \tag{15}$$

TABLE II

PHASE LAGS AND ATTENUATION FACTORS, REGIONS I—VIII

f (c/s)	w (r/s)	Region I Core		Region II Blanket		Region III Rod handles		Region IV Shield plate	
		$\varphi_{1/2}$	μ_1	φ_2	μ_2	φ_3	μ_3	φ_4	μ_4
0.001	0.00628	0.15	1.000	0.935	1.000	0.167	1.000	1.56	1.000
0.005	0.0314	2.0	1.000	4.69	0.998	0.830	1.000	7.90	0.999
0.0089	0.0560	4.95	0.993	8.28	0.994	1.48	0.999	13.9	0.996
0.0185	0.1162	7.80	0.982	16.9	0.974	3.06	0.996	29.1	0.982
0.0233	0.1465	9.50	0.970	21.6	0.959	3.80	0.993	36.6	0.972
0.0284	0.178	11.8	0.940	26.0	0.940	4.65	0.990	44.5	0.958
0.0336	0.212	14.0	0.920	30.8	0.915	5.50	0.986	52.5	0.941
0.0376	0.236	16.0	0.880	34.0	0.896	6.10	0.983	59.2	0.927
0.0484	0.304	18.2	0.858	42.8	0.845	7.80	0.973	76.0	0.879
0.0584	0.368	23.7	0.771	50.6	0.787	9.15	0.961	92.8	0.843
0.078	0.490	28.0	0.698	64.0	0.673	11.5	0.935	122	0.741
0.098	0.615	37.6	0.595	74.3	0.560	13.6	0.904	152	0.620
0.150	0.942	40.0	0.394	92.5	0.344	17.5	0.835	233	0.353

f (c/s)	w (r/s)	Region V Inlet spacer		Region VI Seal plate		Region VII Core section, Outlet spacer		Region VIII Blanket section, Outlet spacer	
		φ_5	μ_5	φ_6	μ_6	φ_7	μ_7	φ_8	μ_8
0.001	0.00628	0.37	1.000	0.84	1.000	0.38	1.000	3.20	0.999
0.005	0.0314	1.90	1.000	4.25	1.000	1.89	1.000	15.9	0.983
0.0089	0.0560	3.30	0.999	7.48	0.998	3.38	0.998	27.2	0.945
0.0185	0.1162	6.90	0.995	15.5	0.991	6.96	0.991	48.8	0.790
0.0233	0.1465	8.70	0.992	19.6	0.985	8.73	0.987	55.7	0.560
0.0284	0.178	10.4	0.988	23.8	0.978	10.6	0.980	60.6	0.472
0.0336	0.212	12.5	0.982	28.4	0.969	12.5	0.972	64.0	0.392
0.0376	0.236	13.7	0.978	31.4	0.962	13.9	0.966	64.5	0.350
0.0484	0.304	17.5	0.964	40.3	0.938	17.7	0.945	65.7	0.267
0.0584	0.368	21.2	0.948	48.5	0.909	21.0	0.920	64.5	0.221
0.078	0.490	27.6	0.911	63.8	0.841	26.7	0.866	61.8	0.190
0.098	0.615	33.8	0.864	78.8	0.782	32.2	0.800	59.5	0.156
0.150	0.942	47.3	0.718	115	0.580	42.2	0.677	59.0	0.133

where Y_2 is the complex amplitude of the temperature wave at the top of the blanket and Y_1 is the amplitude (not complex) of the wave leaving the core. Substitution of values for z/v, τ_c and τ_f (from Table I) into Eq. (15) results in the values for the attenuation factors and phase-lags summarized in Table II. Thus, for 0.001 c/s the phase-lag between the top of the core and the top of the blanket is 0.935°. The attenuation factor for this frequency is 1.000, i.e., the amplitude of the temperature oscillation at the top of the blanket is the same as that at the top of the core. For the highest frequency considered, 0.15 c/s, the phase-lag has increased to 92.5° and the amplitude of the wave leaving the blanket has been reduced to 0.344 times the input value. Since the amplitude of the wave leaving the core (at 0.15 c/s) has already been attenuated by a factor of 0.394, the amplitude of the wave leaving the blanket is only 0.344×0.394 or 13.5% of that generated in the core.

Calculations of the attenuation and phase-lag of the temperature oscillations across the rod-handle region present no special problem since the heat generation term of Eq. (7) is clearly zero. Substitution of values for z/v, τ_c and τ_f (from Table I) into an expression similar to Eq. (15) results in the values summarized in Table II.

5.3 Attenuation factor and phase-lag across the shield plate (Region IV) and seal plate (Region VI)

In calculating the attenuation and phase-lag of the temperature wave across these regions it is necessary to extend the scope of Storrer's expressions, since two media involved, ligaments and rod extensions, each with widely differing heat-transfer properties, are present. Because the ligaments of the shield plate are thin, approximately 0.034 in, the time constant for response will be short. For this reason the phase-lag across the plate would be expected to be small. On the other hand, the large (by comparison) time constant for the rod-handles effectively slows the propagation of the temperature signal. The effects of both (ligaments and rod-handles) cannot be found from a simple lumping of the inert material. Instead, it is necessary to revise and to solve simultaneously three transient heat-balance expressions. A solution of these expressions, given in Appendix C, results in the following expression for λ:

$$\lambda = \frac{\omega^2(\tau^2_{f_1} + \tau^2_{f_2}) + 2\omega^4 \tau^2_{f_1}\tau^2_{f_2}}{\tau_c(1+\omega^2\tau^2_{f_1})(1+\omega^2\tau^2_{f_2})} + i\,\omega\left[1 + \frac{\tau_{f_2}}{\tau_c(1+\omega^2\tau^2_{f_2})} + \frac{\tau_{f_1}}{\tau_c(1+\omega^2\tau^2_{f_1})}\right] \quad (16)$$

where τ_{f_1} and τ_{f_2} are the time constants for the rod extensions and ligaments, respectively. A description of the method used to evaluate the time constants for these two special cases is given in Appendix B. Substitution of values for τ_{f_1}, τ_{f_2}, τ_c and z/v (from Table II) into Eqs. (15) and (16) results in values for the attenuation factors and phase-lags summarized in Table II.

5.4 Attenuation factor and phase-lag across the inlet spacer (Region V)

Since there are no ligaments in this region, capacitance effects are dictated solely by the fluted rod extensions and the coolant. An evaluation of the time constants for this special case is discussed in Appendix B. Substitution of values for τ_c, τ_f and z/v from Table I into Eqs. (8) and (15) results in the values for the attenuation factors and phase-lags summarized in Table II.

5.6 Attenuation factor and phase-lag across the outlet spacer (Regions VII and VIII)

In regions I through VI the transit time of the coolant is exactly defined by the length of flow path and the coolant velocity. In the outlet spacer (or outlet plenum) the coolant flows outward from the interstices between the fuel-rod extensions, around the blanket rod-handles and out through holes in the spacer ring into the outlet pipe. Here the concept of a physical transit time breaks down since the flow is continuously decelerating. To circumvent a detailed consideration of the velocity as a function of position, it has been assumed that the transit time may be approximated by the residence time of the coolant in this region where the residence time is defined as the time required to fill the region under consideration with coolant at the specified flow rate.

A further complication consists in the fact that the outlet spacer comprises two quite different regions: an inner, containing the fuel-rod extensions; and an outer, containing the much more massive blanket rod-handles. Since the heat-transfer

properties of each region are quite different, each region has been considered individually, i.e., residence times and time constants have been calculated for each region. A discussion of the various calculations for the inner region (VII) and outer (VIII) is given in Appendix B. Substitution of the appropriate values into Eqs. (8) and (15) results in the values for the attenuation factors and phase-lags summarized in Table II.

6. Temperature gradient across lower shield plate as a function of frequency

From Table II it is clear that sizable phase-lags can exist across the lower shield plate and that the phase-lag increases rapidly with frequency. It is also clear that the amplitude of the input temperature wave is smeared in its passage through the plate. Both effects act to produce across the plate a temperature gradient which is a strong function of the oscillation frequency. At low frequencies, for example at 0.001 c/s, the temperature gradient will be extremely small since the temperatures at the lower and upper surfaces of the plate at any given time will be essentially the same. This is, of course, a consequence of the small phase difference (actually 1.56°) at this frequency. At high frequencies, for example 0.098 c/s, the phase lag is much larger (152°) and the lower and upper surfaces may, in principle, be at considerably different temperatures. On the other hand, the amplitude of the temperature wave at this frequency is smeared considerably in its passage through the core, blanket, rod-handle and plate regions. Unless the amplitude of the wave at the bottom surface of the shield plate is significantly large, the temperature gradient across the plate may be of little consequence even though strong phase-lags may exist. Clearly, the maximum temperature gradient occurs at some frequency which effects a compromise between increasing phase and decreasing amplitude.

A summary of phase-lag and attenuation data pertinent to an evaluation of the frequency at which the maximum temperature gradient occurs is given in Table III. All phase data are quoted relative to the centre of the core. For example, the phase-lag (150°) quoted at the bottom of the plate at 0.150 c/s is simply the sum of the phase-lags occurring across regions I—III, i.e. over half of the core, the blanket, and the rod-handle region. The phase-lag quoted at the top of the plate is, accordingly, the phase-lags summed over regions I through IV. Similarly, the attenuation factors quoted are the products of attenuation factors over the downstream regions. For example, the attenuation factor at 0.150 c/s is simply the product of the attenuation factors over regions I through III, which, from Table II, amounts to $0.394 \times 0.344 \times 0.835\ Y_1 = 0.113\ Y_1$ where Y_1 is the amplitude (not complex) of the temperature wave leaving the core.

Since the attenuation factors and phase-lags are known (for any given frequency) at both lower and upper surfaces, it is possible to express the temperature of both upper and lower surfaces as a function of time. Hence, the temperature of the coolant at the bottom of the shield plate T_3 may be expressed as a function of time t by the following relation

$$T_3 = T_1 + \mu_3 Y_1 (\sin \omega t - \theta_3) \tag{17}$$

where T_1 is the average temperature of the coolant leaving the core, μ_3 is the cumulative attenuation factor over regions I through III, Y_1 is the non-complex amplitude of the temperature wave leaving the core, ω is the oscillation frequency

TABLE III

PHASE LAGS AND ATTENUATION FACTORS FOR SHIELD PLATE

f (c/s)	ω (r/s)	Bottom of plate		Top of plate		Phase lag across plate
		Phase lag φ	Amplitude $x\ Y_1$*	Phase lag φ	Amplitude $x\ Y_1$	
0.001	0.00628	1.24	1.000	2.80	1.000	1.56
0.005	0.0314	7.52	0.998	15.4	0.997	7.88
0.0089	0.0560	14.7	0.987	28.7	0.985	14.0
0.0185	0.1162	27.8	0.953	57.9	0.938	30.1
0.0233	0.1465	34.9	0.922	71.5	0.898	36.6
0.0284	0.178	42.5	0.875	87.0	0.837	44.5
0.0336	0.212	50.3	0.832	102.8	0.781	57.5
0.0376	0.236	56.1	0.772	115.3	0.716	59.2
0.0484	0.304	68.8	0.705	144.8	0.619	76.0
0.0584	0.368	83.5	0.582	176.3	0.492	92.8
0.078	0.490	103.5	0.440	225.5	0.332	122
0.098	0.615	124.5	0.300	278	0.181	154
0.150	0.942	150.0	0.113	383	0.040	233

* Y_1 = non-complex amplitude of temperature wave leaving the core.

and θ_3 is the cumulative phase-lag over regions I through III. The temperature of the coolant at the upper surface of the shield plate is given by

$$T_4 = T_1 + \mu_4 Y_1 (\sin \omega t - \theta_4) \tag{18}$$

where μ_4 and θ_4 are the cumulative attenuation factors and phase-lag, respectively, over regions I through IV.

To illustrate the concept of a temperature gradient across the plate, Eqs. (17) and (18) have been evaluated as a function of time for the fixed frequency 0.0376 c/s. The results are given in Fig. 13. To simplify the illustration the constant T_1 has arbitrarily been assumed to be zero, and the temperature of the coolant

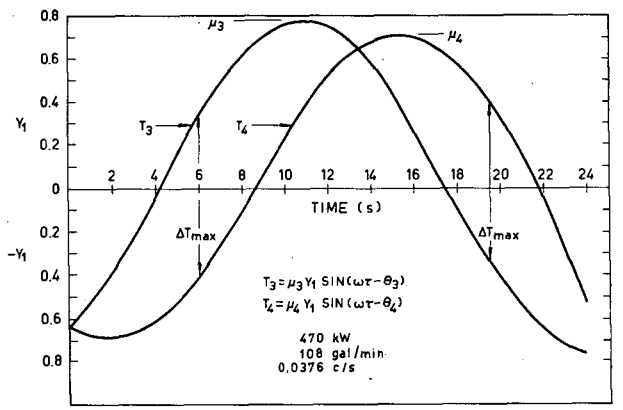


Fig. 13
Temperature, upper and lower surfaces of shield plate.

at lower and upper surfaces has been expressed in terms of Y_1, the amplitude of the temperature wave leaving the core. From Fig. 13 it is clear that the temperature differential across the plate in the course of one cycle goes from a maximum negative differential, to zero, to a maximum positive differential and finally back through zero to a maximum negative.

The temperature differential (or gradient) across the plate at a time t and for an oscillation frequency ω is therefore defined by the difference between Eqs. (17) and (18). Hence,

$$\Delta T = T_3 - T_4 = \mu_3 Y_1 (\sin \omega t - \theta_3) - \mu_4 Y_1 (\sin \omega t - \theta_4) \tag{19}$$

or

$$\Delta T = Y_1 [\mu_3 (\sin \omega t - \theta_3) - \mu_4 (\sin \omega t - \theta_4)]. \tag{20}$$

From Eq. (20) it is clear that the temperature differential across the plate is a time-dependent function of Y_1, the oscillation frequency and the attenuation factors and phase-lags accumulated over the various downstream regions.

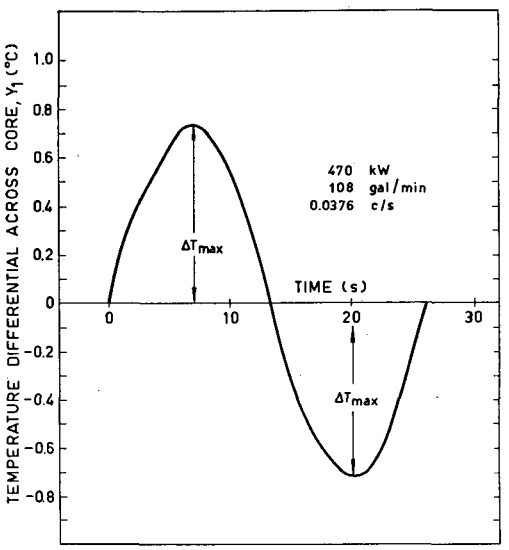


Fig. 14
Temperature differential across shield plate.

A plot of Eq. (20) for the fixed frequency 0.0376 c/s is given in Fig. 14. Note that the temperature gradient oscillates between maximum positive and negative values with a frequency defined by ω. Since the time constant for ligamental flexing is small (0.109 s, Table I), the shield plate during the course of one cycle (at low frequencies) will assume the form of a concave dish for one portion of the cycle, and after straightening will form a convex dish during the second portion. The significance of these actions will be discussed in section 7.

For a given frequency μ_3, μ_4, θ_3 and θ_4 are fixed. It is therefore possible to establish the time at which the temperature differential passes through a maximum (or minimum). This value may then be substituted back into Eq. (20) to establish

the value of the maximum temperature gradient. A partial differentiation of Eq. (20) results in the following

$$\left[\frac{\partial(\Delta T)}{\partial t}\right]_{\omega} = Y_1[\omega\mu_3\cos(\omega t - \theta_3) - \omega\mu_4\cos(\omega t - \theta_4)]. \tag{21}$$

Setting $\left[\frac{\partial(\Delta T)}{\partial t}\right]_{\omega} = 0$,

$$\mu_3\cos(\omega t - \theta_3) = \mu_4\cos(\omega t - \theta_4) \tag{22}$$

and since

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \tag{23}$$

$$\mu_3[\cos\omega t\cos\theta_3 + \sin\omega t\sin\theta_3] = \mu_4[\cos\omega t\cos\theta_4 + \sin\omega t\sin\theta_4]. \tag{24}$$

Dividing Eq. (24) by $\cos\omega t$

$$\mu_3[\cos\theta_3 + \tan\omega t\sin\theta_3] = \mu_4[\cos\theta_4 + \tan\omega t\sin\theta_4]. \tag{25}$$

A rearrangement of Eq. (25) leads to

$$\tan\omega t = \frac{\mu_4\cos\theta_4 - \mu_3\cos\theta_3}{\mu_3\sin\theta_3 - \mu_4\sin\theta_4}. \tag{26}$$

Since μ_3, μ_4, θ_3 and θ_4 are given (Table III), it is possible through Eqs. (26) and (20) to evaluate the maximum temperature gradient for each of the discrete frequencies listed in Table III. The results of this evaluation are given in the third column of Table IV. Note that the temperature gradient expressed in terms of Y_1 (the non-complex amplitude of the wave leaving the core) is small at the low frequencies, passes through a maximum at about 0.0584 c/s and falls off rapidly at higher frequencies.

A more realistic appraisal of the maximum temperature differential across the plate may be obtained from an evaluation of the non-complex amplitude of the

TABLE IV

TEMPERATURE GRADIENT ACROSS SHIELD PLATE
470 kW, 108 gpm, ΔT across core = 92°C

f (c/s)	ω (r/s)	Temperature gradient* across plate	$\Delta n/n$**	Attenuation across core	Core temperature amplitude Y_1	Temperature gradient across plate (°C)
0.0089	0.056	$0.242Y_1$	0.117	0.993	5.35	1.29
0.0185	0.116	0.438	0.106	0.982	4.80	2.10
0.0233	0.147	0.571	0.104	0.970	4.65	2.55
0.0284	0.178	0.648	0.114	0.940	4.93	3.19
0.0336	0.212	0.716	0.123	0.920	5.20	3.71
0.0376	0.236	0.735	0.137	0.880	5.55	4.07
0.0484	0.304	0.815	0.108	0.858	4.73	3.85
0.0584	0.368	0.780	0.0865	0.774	3.08	2.41
0.078	0.490	0.676	0.0660	0.698	2.12	1.44
0.098	0.615	0.302	0.0556	0.595	1.52	0.46
0.150	0.942	0.137	0.0475	0.086	1.17	0.01

* Y_1 = non-complex amplitude of temperature wave leaving the core.
** Δn measured as peak-to-peak value.

core temperature wave. Fortunately, this becomes possible through an analysis of existing Mark II data for 470 kW and 108 gpm flow. The fourth column of Table IV gives the actual measured fractional change in power resulting from the oscillator input. At 0.0089 c/s, for example, the power oscillates between the extremes of 5.85% above the average and 5.85% below the average. In the absence of attenuation effects, the amplitude of the temperature oscillation is simply the product of the power variation and the temperature differential across the core which for 470 kW and 108 gpm was measured as 92°C. Hence, the non-complex amplitude of the temperature wave leaving the core at 0.0089 c/s amounts to $0.0585 \times 92.0°C = 5.38°C$. However, from Table II it is clear that the temperature wave is moderately attenuated before it leaves the core. Since the attenuation factors in the core are known, the actual non-complex amplitude of the temperature wave at the top of the core Y_1 is simply the product of Y_0, the non-attenuated temperature amplitude, and the respective attenuation factor for that frequency. Knowledge of Y_1, then, permits an evaluation of the actual temperature gradient across the shield plate given in the last column of Table IV. To illustrate the dependence of maximum temperature differential on frequency the results of Table IV are illustrated graphically in Fig. 15. From this it is clear that the maximum temperature differential lies in the vicinity of 0.04 c/s.

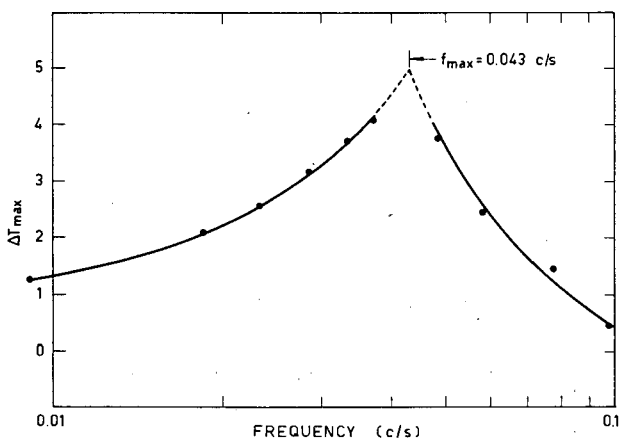


Fig. 15
Maximum temperature differential across shield plate.

Assuming that the amount of feedback reactivity controlled by shield-plate flexing is proportional to the temperature differential, it follows that the associated power coefficient is strongly frequency-dependent and reaches its maximum value at 0.043 c/s for 470 kW and 108 gpm. It is interesting to note that the resonance frequency of the reactor under these conditions of power and flow turns out to be 0.040 c/s.

7. Feedback model

Before attempting to incorporate the plate-flexing feedback into a mathematical model, it becomes necessary to scrutinize actual transfer-function results for the specific conditions under which the maximum temperature differential across

the plate was derived, i.e. 470 kW and 108 gpm. Unfortunately, all amplitude and phase data accumulated during the May and November (1955) tests are characterized by irregularities which are not understood. For this reason it is difficult to effect a rigorous correlation between experimental and calculated results. The extent of the irregularities in the existing transfer-function data is illustrated for the specific conditions of 470 kW and 108 gpm in Fig. 16 (amplitude) and Fig. 17 (phase).

For frequencies above 0.1 c/s the amplitude lies below the calculated curve to an extent which must be regarded as suspicious. Since all feedback processes are attenuated at the higher frequencies, the usual behaviour of the transfer-function amplitude is one characterized by an asymptotic approach to the calculated curve. An inspection of the phase-dependence, on the other hand, does not reveal any obvious irregularities.

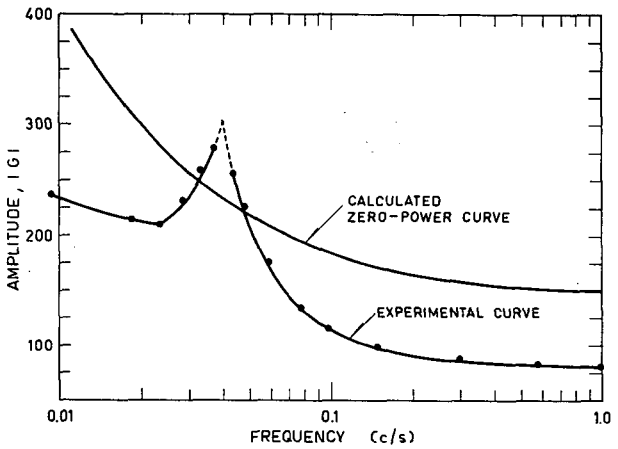


Fig. 16
Unnormalized transfer-function amplitude, 470 kW, 108 gal/min.

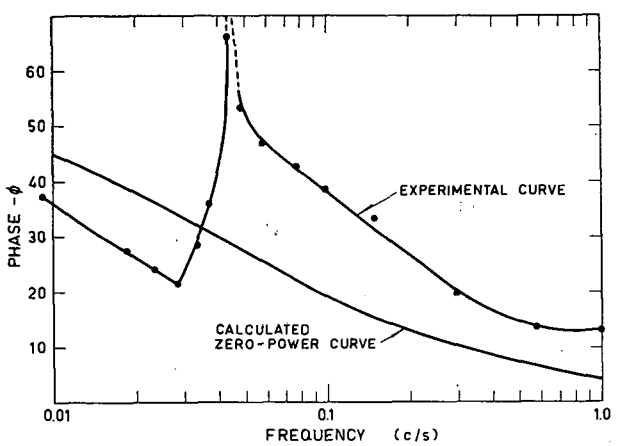


Fig. 17
Unnormalized transfer-function phase, 470 kW, 108 gal/min.

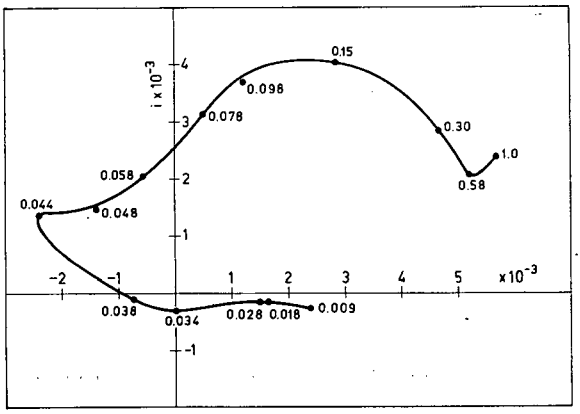


Fig. 18
Feedback for unnormalized data, 470 kW, 108 gal/min.

A more realistic appraisal of the untreated experimental data may be obtained through a study of the feedback, Fig. 18 separated from the information given in Figs. 16 and 17. A casual inspection of these results suggests a mechanism based on the existence of two power-coefficient components: one prompt and positive and the other strongly negative and varying with frequency according to a transport-lag dependence. Such an explanation would presumably account for the fact that the low-frequency feedback is smaller in amplitude than it is at higher frequencies. The fact that the high-frequency feedback is so strong in the I quadrant implies a feedback process which acts with a long transport lag. The fact that a significant transport lag is required, however, points up the fallibility of this simple explanation. If the region controlling the feedback is so far removed from the core that it must be described by a transport-lag dependence, it is certain from Storrer's developments that the amplitude of any temperature perturbation initiated in the core would be so small that dynamic feedback effects would be negligible. Even if strong temperature oscillations could be initiated at high frequencies the attenuation of the wave in its passage between the core and the pertinent structural region would reduce the oscillating amplitude to an ineffective ripple. From Table II it may be seen that a feedback effect originating in the vicinity of the lower shield plate for a frequency of 0.15 c/s would be attenuated by a factor between 0.113 and 0.04. Since the behaviour illustrated in Fig. 18 is characterized by little or no attenuation, it is difficult to accept the effect as real. Rather it is more logical to question the validity of the experimental results cited in Figs. 16—18.

The first clue regarding the origin of the inconsistencies is readily apparent from a scrutiny of transfer-function results obtained at 2 kW (essentially zero power), Fig. 19. At all frequencies the amplitude lies significantly and consistently below the calculated curve. Unquestionably, the origin of this discrepancy is associated with the use of an incorrect value for the absolute reactivity worth of the oscillator rod. Accordingly, the experimental amplitude results may be revised upward by a constant factor (1.25) which essentially brings experimental and calculated results into agreement. The same factor may then be applied with sufficient justification to all other amplitude results regardless of the actual power.

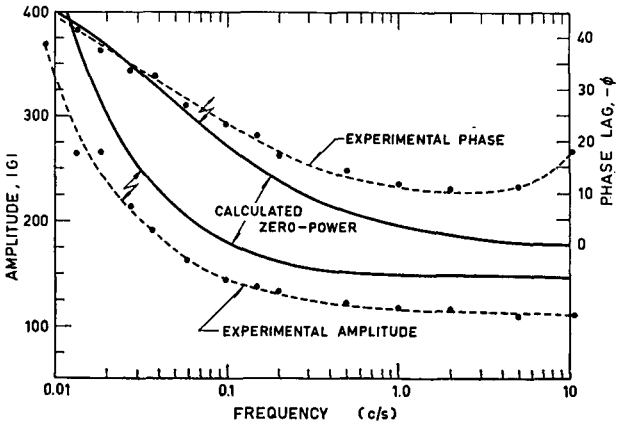


Fig. 19
Zero-power data, 2 kW, 108 gal/min.

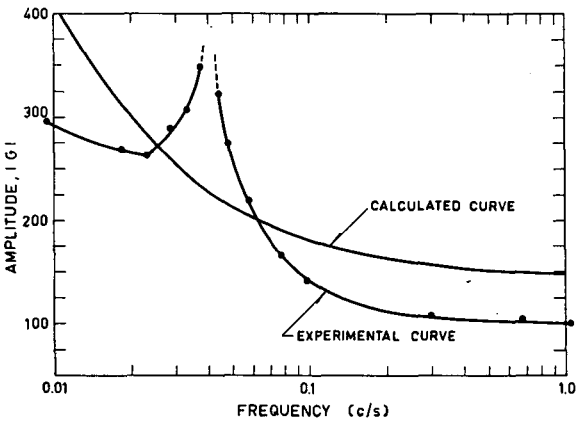


Fig. 20
Transfer-function amplitude, first normalization.

The experimental phase-lags deviate from the calculated phase curve for frequencies greater than 0.04 c/s, and as the frequency increases the discrepancy becomes larger. Essentially the same behaviour (although to a less important degree) has been noted in the Mark III oscillator studies [1]. The origin of this discrepancy has been associated with the time required for neutrons originating in the core to reach the detector. As the results of the Mark III tests have shown, the phase discrepancy for a given frequency is constant with respect to power. It is therefore permissible to apply the frequency—dependent discrepancies noted at zero power as corrections to the experimental phase values at other levels of power.

A normalization of the amplitude by a factor of 1.25 results in the values illustrated in Fig. 20. Again the experimental amplitude values lie significantly and consistently below the calculated curve. A separation of the feedback (with correc-

tions applied to the phase) still results in a feedback quite similar to that illustrated in Fig. 18 for the unnormalized data. No credible explanation for the amplitude discrepancies of Fig. 20 can be offered. It is difficult to resist the conclusion that the reactivity worth of the oscillator rod decreases strongly as the reactor power increases. On the other hand, temperature- and power-sensitive processes which are expected to affect the reactivity worth, i.e. neutron absorption and scattering, and thermal expansion are too small by a wide margin to account for the magnitude of the discrepancy.

To correlate the results of calculations with those obtained from experiment it becomes necessary to decide whether (*a*) to accept the partially normalized results of Fig. 20 as real or (*b*) to effect a further normalization. Since no credible explanation can be given for the partially normalized results at high frequencies above 0.06 c/s), the second alternative (*b*) is preferred. The results of a second normalization of the amplitude are given in Fig. 21. The additional normalization is based

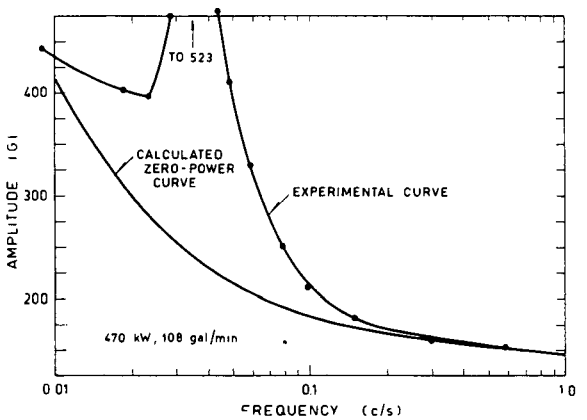


Fig. 21
Transfer-function amplitude, second normalization.

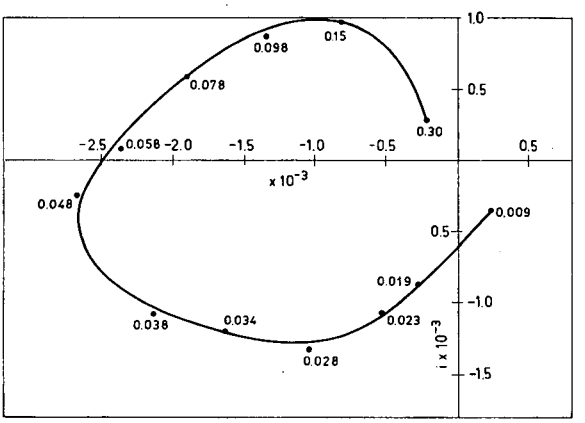


Fig. 22
Feedback, 470 kW, 108 gal/min, second normalization.

on the tacit assumption that feedback effects above 0.1 c/s are insignificantly small. As a consequence of the developments given below, such an assumption appears to be reasonable.

A separation of the feedback associated with the amplitude given in Fig. 21 and the phase in given Fig. 17 (corrected for zero-power discrepancies) results in the dependence given in Fig. 22. For the purpose of correlating calculated with experimental results it will be assumed that this particular feedback is peculiarly associated with the conditions, 470 kW and 108 gpm.

To explain the exceedingly complex feedback-dependence illustrated in Fig. 21, the following mathematical model is postulated.

$$-H = P\left[\underbrace{\frac{-A_0}{1+i\omega\tau_f}}_{\text{core feedback processes}} + \underbrace{\frac{\mu_b(\omega)\,B_0\,e^{-i\omega\tau(\omega)}}{(1+i\omega\tau_f)(1+i\omega\tau_p)}}_{\text{plate flexing}} + \underbrace{\frac{\mu_c(\omega)\,C_0\,e^{-i\omega\tau(\omega)}}{(1+i\omega\tau_f)(1+i\omega\tau_r)}}_{\text{radial-plate expansion}}\right]. \quad (27)$$

7.1 Prompt term

The first term describes the feedback originating in the core. As a simplification it has been assumed that all core feedback processes, i.e., rod bowing, axial and radial fuel expansion, and coolant expulsion act with the same time constant, τ_f. The algebraic sum of the individual components defines the net power coefficient A_0 which in the case of the Mark II is known to be positive in sign. Since the fuel rods in the Mark II were not radially coupled, the normally strong radial contribution to power coefficient was greatly reduced. As a consequence the net negative component resulting from the combined effects of fuel expansion and coolant expulsion was insufficiently large to compensate for the effects of rod bowing which for the Mark II were strongly positive.

7.2. Plate-flexing term

The second term describes the feedback associated with shield-plate flexing. The product $\mu_b(\omega)\,B_0$ defines the actual power coefficient for a given oscillation frequency ω where B_0 is the value for the power coefficient evaluated for the frequency at which the oscillating temperature differential across the plate reaches a maximum (Table V), and where $\mu_b(\omega)$ is the frequency-dependent attenuation factor summarized in Table VI. The time constant for flexing is given by τ_p and the effective transport lag between the core and the shield plate is defined by $\tau(\omega)$. As it turns out, $\tau(\omega)$ is also frequency-dependent and accordingly must be evaluated for each frequency from the accumulated phase-lag between the core centre and the shield plate (Table II).

7.3. Radial expansion term

The third and final term describes the delayed feedback arising from the overall radial expansion of the plate. Again the power coefficient $\mu_c(\omega)\,C_0$ is frequency-dependent. In this case the frequency-dependence enters because the amplitude of the temperature wave is strongly attenuated at the higher frequencies.

The attenuation coefficient $\mu_c(\omega)$ begins with a value of unity at low frequencies and decreases with increasing frequencies according to Eqs. (10) and (11). A summary of values for $\mu_c(\omega)$ as a function of frequency (evaluated from the information of Table II) is given in Table VI. The power coefficient C_0 is defined

TABLE V

POWER COEFFICIENT FROM SHIELD-PLATE FLEXING

B_0 = maximum value

f (c/s)	ω (r/s)	Temperature gradient across plate from Table IV (°C)	Power coefficient for shield-plate flexing
0.0089	0.056	1.29	0.280 B_0
0.0185	0.1162	2.10	0.455 B_0
0.0233	0.1465	2.55	0.555 B_0
0.0284	0.178	3.19	0.692 B_0
0.0336	0.212	3.71	0.808 B_0
0.0376	0.236	4.07	0.882 B_0
0.0484	0.304	3.85	0.838 B_0
0.0584	0.368	2.47	0.522 B_0
0.078	0.490	1.44	0.313 B_0
0.0978	0.615	0.46	0.100 B_0
0.150	0.942	0.01	0.002 B_0

TABLE VI

VALUES USED IN THE SOLUTION OF EQ. (27)

τ_p = 0.109 s (calculated value, App. B);
τ_r = 1.0 s (assumed value)

f	$\mu_b(\omega)$	$\mu_c(\omega)$	$\tau\omega$
0.0089	0.280	0.985	6.78 s
0.0185	0.455	0.940	6.35 s
0.0233	0.555	0.910	6.35 s
0.0284	0.692	0.875	6.35 s
0.0336	0.808	0.805	6.35 s
0.0376	0.882	0.742	6.35 s
0.0484	0.838	0.660	5.98 s
0.0584	0.522	0.535	6.15 s
0.078	0.313	0.381	6.17 s
0.098	0.100	0.243	5.91 s
0.150	0.002	0.079	4.95 s

as the steady-state (zero frequency) value for the effects of an overall radial expansion of the plate. The transport lag $\tau(\omega)$ is identical with that appearing in the second term since both describe the effective time required to transport a temperature signal from the core to the shield plate. The time constant τ_r describes the time-dependence of the overall radial expansion of the plate. Because the relatively massive hexagonal annulus (Fig. 5) is apparently involved, τ_r is considerably larger than τ_p, the time constant for simple ligamental flexing.

The frequency-dependence of each term is illustrated in Fig. 23. No attempt has been made to weigh the various terms according to their actual importance in the mathematical model, i.e., each term in Eq. (27) has been assigned an arbitrary weight of unity. Various values for time constants, attenuation factors and

transport lags are summarized in Table VI. With one exception, that of τ_r, values assigned to time constants and transport lags are directly the result of heat-transfer considerations (see Appendix B and Table I). In this case the arbitrary

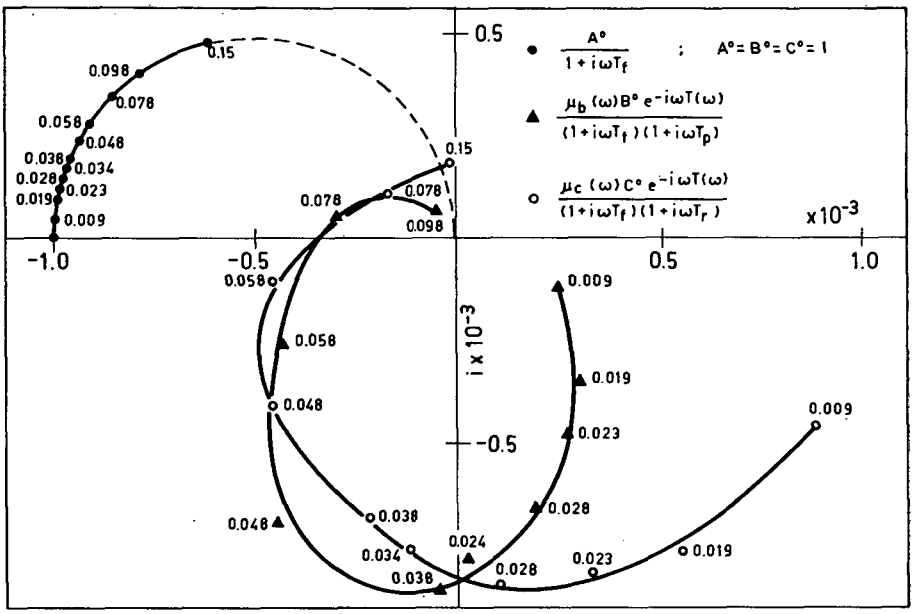


Fig. 23
Feedback components.

value of 1 s has been assigned since this value eventually proved to be more consistent with experimental results.

The attenuation factor $\mu_b(\omega)$ for the second term was evaluated from a normalization of the maximum temperature-differential frequency-dependence (see Fig. 15 and Table V) while the attenuation factor $\mu_c(\omega)$ for the third term is simply the cumulative attenuation factor established over the core, blanket and rod-handles, regions I through III.

The zero-frequency intercepts of the first and third term are respectively A_0 and C_0. The zero-frequency intercept for the second term is the origin since $\mu_b(\omega)$, in this case, approaches zero at low frequencies. All three terms eventually go to zero, but at different rates, as the frequency increases indefinitely. To combine the three terms into a simulation of the experimental feedback it is necessary to assign to the individual components weighting factors which provide a satisfactory compromise between phase and amplitude values over as wide a frequency spectrum as possible.

At 0.15 c/s nearly all feedback arises from the prompt-positive effect, and to a first approximation the measured feedback at this frequency may be equated to the first term. An evaluation carried out in this manner results in a value of $A_0 = 2.1 \times 10^{-6}\ \Delta k/k$ kW. A knowledge of A_0 considerably simplifies the evaluation of B_0 and C_0. Substitution of A_0, $\mu_b(\omega)$, and $\mu_c(\omega)$ into Eq. (27) for 0.0376 and 0.0484 c/s, where $-H$ is known, results in simultaneous expressions in terms of

B_0 and C_0. The solution of these equations results in values of $B_0 = 1.7 \times 10^{-6}$ and $C_0 = 2.5 \times 10^{-6}\ \Delta k/k$ kW.

A test of how well the various weighting factors fit the experimental data is given in Fig. 24. It is meaningless for two reasons to force a better agreement

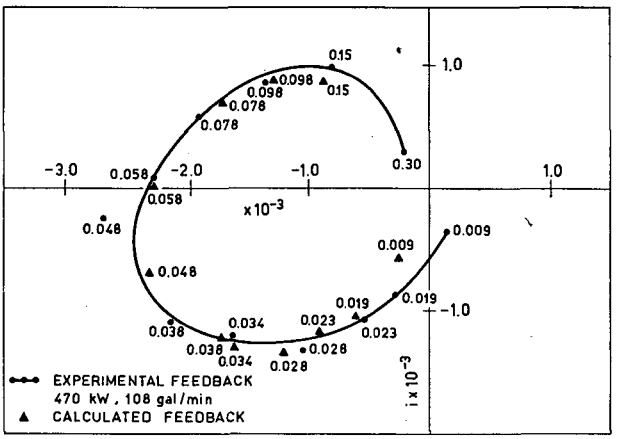


Fig. 24
Comparison of experimentally measured and calculated feedbacks.

between the calculated and experimental values by a "cut and try" shaping of the various parameters. In the first place, it has not been established unambiguously that the experimental feedback plotted in Fig. 24 is precisely defined. In the second place, the mathematical model given by Eq. (27) is somewhat oversimplified in that it fails to consider the effects of pivotal action at seal-plate bearing-points.

8. Credibility of power coefficient values

The most important criterion to be satisfied in the quasi-analytical fit of Fig. 24 is whether thermally-induced ligamental motions are sufficiently large to explain the magnitudes of the three power coefficients, A_0, B_0 and C_0, as indeed they must be if the postulated mechanism is to be regarded as valid.

8.1. Power coefficient, A_0

As defined by Eq. (27) the power coefficient A_0 is simply the algebraic sum of the steady-state power coefficients for all feedback processes limited in their action to the core. Hence

$$A_0 = A^0_{\mathrm{rb}} + A^0_{\mathrm{ax}} + A^0_{\mathrm{rad}} + A^0_{\mathrm{NaK}} \tag{28}$$

where A^0_{rb}, A^0_{ax}, $A^0_{\mathrm{rad}} + A^0_{\mathrm{NaK}}$ are the steady-state power coefficients for rod bowing, axial expansion, radial expansion and coolant expulsion, respectively.

From flow-reduction tests it is known that A_0 is positive. Since A^0_{rb} is also positive, it follows that the sum of the negative components is insufficiently large to compensate for the positive rod-bowing term. The simplest test of whether the quasi-analytically determined value of A_0 is reasonable is to estimate the magni-

tudes of the various negative components and to determine whether the magnitude of the remaining rod-bowing term is consistent with the amount of rod bowing expected at 470 kW and 108 gpm flow.

A reasonably satisfactory approximation of the axial power-coefficient component may be obtained from the following relation [13]:

$$\frac{\Delta k}{k} = -0.3\,\frac{\Delta h}{h} \tag{29}$$

where $\Delta k/k$ is the reactivity change associated with a fractional change in core height, $\Delta h/h$. The power coefficient for axial expansion is then

$$A^0_{\text{ax}} = \frac{\Delta k}{k\,\text{kW}} = \frac{-0.3\,\Delta h}{h\,\Delta P} \tag{30}$$

In terms of the linear expansion coefficient of the fuel, Eq. (30) may be rewritten as

$$A^0_{\text{ax}} = \frac{-0.3\,\Delta L}{L} \times \frac{\Delta T_{\text{eff}}}{\Delta P} \tag{31}$$

where $\Delta L/L$ for the uranium-zirconium fuel alloy at 200°C $= 13 \times 10^{-6}$/°C. To a first approximation the term $\Delta T_{\text{eff}}/\Delta P$ defines the effective temperature change of the coolant with respect to power. To account for the fact that the effects of a change in temperature differential across the core are fully felt at the top of the core and are not felt at the bottom, the effective temperature differential across the core is defined as one-half that of the total differential. In particular, at 470 kW and 108 gpm the actual measured temperature differential across the core was 92°C. The effective temperature differential across the core is then 46°C and the coefficient $\Delta T_{\text{eff}}/\Delta P$ is 46°C/470 kW or 0.10°C/kW. Power changes of 1 kW result in an effective coolant temperature change of 0.10°C. Substitution of values for $\Delta L/L$ and for $\Delta T_{\text{eff}}/\Delta P$ into Eq. (31) results in a value of $-0.39 \times 10^{-6}\,\Delta k/k$ kW.

In estimating the magnitude of A^0_{rad} it is necessary to consider the fact that the fuel rods in Mark II were not coupled radially. The effective change in core radius is therefore dictated by expansive effects occurring in the outer row of rods only. A measure of the power coefficient may be obtained from the relation [13].

$$\frac{\Delta k}{k} = -0.6\,\frac{\Delta R}{R} \tag{32}$$

where $\Delta k/k$ is the reactivity change associated with the fractional change in the core radius, $\Delta R/R$. Hence,

$$A^0_{\text{rad}} = \frac{\Delta k}{k\,\text{kW}} = \frac{-0.6}{R}\,\frac{\Delta r}{\Delta P} \tag{33}$$

where R is the effective core radius (3.7 in) and $\Delta r/\Delta P$ is the change in core radius effected by a power increase ΔP (r is the radius of a fuel rod). Since

$$\Delta r = r\,\frac{\Delta L}{L}\,\Delta T_{\text{eff}} \tag{34}$$

Eq. (33) becomes

$$\text{A}^0_{\text{rad}} = \frac{-0.6\,r}{R} \times \frac{\Delta L}{L} \times \frac{\Delta T_{\text{eff}}}{\Delta P} \tag{35}$$

where $\Delta T_{\text{eff}}/\Delta P$ is defined above. An evaluation of Eq. (35) results in a value of A^0_{rad} of $-0.048 \times 10^{-6}\,\Delta k/k$ kW.

The power coefficient for coolant expulsion may be estimated crudely from the relative NaK/fuel worth established from a comparison of wet and dry critical experiments in Mark I [10]. From these experiments it was found that 1700 g NaK reduced the critical fuel loading by approximately 2 kg. It follows, then, that one gram of NaK had an equivalent worth of 0.850 g of fuel. The change in the coolant mass with respect to effective temperature-differential change may be found from the following:

$$\frac{\Delta M_c}{\Delta T_{eff}} = \left(\frac{\partial M}{\partial d}\right)_M \frac{\Delta d}{\Delta T_{eff}} + \left(\frac{\partial M}{\partial v}\right)_d \frac{\Delta V_f}{\Delta T_{eff}} \tag{36}$$

where the subscripts c and f refer to coolant and fuel respectively, $\Delta d/\Delta T_{eff}$ is the temperature coefficient of NaK density and $\Delta V_f/\Delta T_{eff}$ is the volume expansion coefficient of the fuel. Since $M = dV$

$$\frac{\Delta M_c}{\Delta T_{eff}} = V_c \frac{\Delta d}{\Delta T_{eff}} + d \frac{\Delta V_f}{\Delta T_{eff}} . \tag{37}$$

In terms of fuel equivalent

$$\frac{\Delta M_f}{\Delta T_{eff}} = 0.850 \left(V_c \frac{\Delta d}{\Delta T_{eff}} + d \frac{\Delta V_f}{\Delta T_{eff}}\right) . \tag{38}$$

Multiplying Eq. (38) by the effective change in temperature across the core per kW, $\Delta T_{eff}/\Delta P$ results in

$$\frac{\Delta M_f}{\Delta P} = 0.850 \frac{\Delta T_{eff}}{\Delta P} \left[\frac{V_c \Delta d}{\Delta T_{eff}} + \frac{d \Delta V_f}{\Delta T_{eff}}\right] . \tag{39}$$

Using the approximation [13]

$$\frac{\Delta k}{k} = -0.27 \frac{\Delta M}{M} \tag{40}$$

the power coefficient A^0_{NaK} may be obtained from

$$A^0_{NaK} = \frac{\Delta k}{k \Delta P} = \frac{-0.27 \Delta M}{M \Delta P} =$$

$$= \frac{-0.27 \times 0.85}{M} \frac{\Delta T_{eff}}{\Delta P} \left[V_c \frac{\Delta d}{\Delta T_{eff}} + d \frac{\Delta V_f}{\Delta T_{eff}}\right] . \tag{41}$$

The power coefficient may then be evaluated from the following information: M (wet critical mass) = 51 kg, $\Delta T_{eff}/\Delta P = 0.10$°C/kW, $V_c = 2000$ cm³, $\Delta d/\Delta T_{eff} = 2.4 \times 10^{-4}$ g/cm³ °C, $d = 0.847$ g/cm³, $\Delta V_f/\Delta T_{eff} = 0.194$ cm³/°C. Hence, $A^0_{NaK} = -0.29 \times 10^{-6} \Delta k/k$ kW. Since expressions (29), (32) and (40) are based on dimensional changes of a core of constant composition, the values deduced for A^0_{ax}, A^0_{rad} and A^0_{NaK} are slightly in error. For the purpose of the following illustration, however, the effects of such errors are inconsequential.

A summation of the three negative power coefficients results in a value of $-0.73 \times 10^{-6} \Delta k/k$ kW. Since the net power coefficient is estimated to be $2.1 \times 10^{-6} \Delta k/k$ kW, the value of A^0_{rb} necessary to satisfy Eq. (28) is then $2.8 \times 10^{-6} \Delta k/k$ kW. The amount of radial fuel-displacement corresponding to a power coefficient of this magnitude may be found from Eq. (32). The change effected in the core radius through rod bowing in going from zero to 470 kW is

$$\Delta R = \frac{R}{0.6} \times A^0_{rb} \times 470 . \tag{42}$$

Substitution of the appropriate values into Eq. (42) results in a value of $\Delta R =$ 8.1 mil. Rod-bowing calculations based on a model in which the rod is fixed at the bottom by the grid plate and at the top by the shield plate result in the conclusion that the value of 8.1 mil is reasonable.

8.2. Power coefficient, B_0

If one assumes that the degree of unconstrained expansion of the shield plate is 0.75 (from Appendix A), it can be shown that the angle of incline for outer row ligaments is given by

$$\alpha = \frac{0.75\,\delta}{4} \tag{43}$$

where δ is the differential expansion between lower and upper surfaces for a given temperature differential across the plate. For an effective plate radius of 3.7 in and a value of $17 \times 10^{-6}\,\Delta L/L°\text{C}$ for the linear expansion coefficient of stainless steel $\alpha = 12 \times 10^{-6}$ rad/°C temperature differential across the plate. From Table IV it may be seen that a temperature wave with a half-amplitude of 6.30°C results in a maximum temperature differential across the plate of 4.07°C at 0.0376 c/s (close to the frequency at which the maximum temperature differential across the plate occurs). The angle of incline per degree centigrade change in core outlet temperature is then $12 \times 10^{-6} \times 4.07/6.30$ or 7.8×10^{-6} rad°C. Since the temperature differential across the core at 470 kW is 92°C, $\Delta T/\Delta P = 0.20°\text{C}/$ kW and $\Delta\alpha/\Delta P = 1.56 \times 10^{-6}$ rad/kW.From simple geometric considerations it can be shown that the physical movement of outer shield-plate bearing-points for a 1-kW power change is 0.006 mil. Assuming no amplification of motion and that a 1-mil ligamental displacement at plate level is sensed as a 0.5-mil displacement of fuel at core centre, it follows that the power coefficient B_0 is given by

$$B_0 = \frac{-0.3\,\Delta R}{R\,\text{kW}} = \frac{-0.3 \times 6 \times 10^{-6}}{3.7} = -0.50 \times 10^{-6}\,\Delta k/k\,\text{kW} \; . \tag{43}$$

A comparison of this value with the quasi-analytically determined value of $1.7 \times 10^{-6}\,\Delta k/k$ kW reveals an approximately threefold discrepancy. The treatment, however, fails to consider the existence of a strong amplifying mechanism which may account for the magnitude of the discrepancy. A discussion of this mechanism is given in section 8. 4.

8.3. The power coefficient, C_0

Assuming a value of 0.75 for the degree of unconstrained expansion (Appendix A) and a 1:0.5 ratio between displacements at plate and core levels, the power coefficient C_0 expected from thermal, mechanical, and neutronic considerations is given by

$$C_0 = \frac{-0.3 \times 0.75\,\Delta R}{R\Delta P} = -0.3 \times 0.75 \times \frac{\Delta L}{L} \times \frac{\Delta T}{\Delta P} =$$

$$= -0.77 \times 10^{-6}\,\Delta k/k\,\text{kW} \; . \tag{44}$$

Since the quasi-analytical-determined value of C_0 amounted to $2.5 \times 10^{-6}\,\Delta k/k$ kW, the calculated value of C_0 is too small by a factor of three to explain the experimental results. The reason for this discrepancy undoubtedly lies in the failure to consider the existence of amplifying mechanisms.

8.4. The effects of amplifying mechanisms

In the development given thus far, special emphasis has been placed on an attempt to explain the extremely complex frequency-dependence of the experimental feedback. Since the postulated model satisfactorily explains the general structure of the feedback (and transfer function), there seems little doubt that the structural member responsible for the delayed feedback is the shield plate. The results of heating experiments conducted on a perforated plate of nearly identical design strongly substantiate this conclusion. The failure of the model to explain the magnitude of the feedback is believed to be the result of assumptions which are overly conservative.

To explain the discrepancy it is necessary to reconsider the assumptions upon which calculations of B_0 and C_0 were based: (1) that the second normalization of experimental data is justifiable; (2) that a 1:0.5 ratio exists between displacements occurring in the shield plate and core, respectively; (3) that dishing of the shield plate may be described as a simple arc; (4) that ligamental expansion is somewhat constrained; and (5) that the effects of amplifying mechanisms may be neglected. For the most part the first four assumptions are reasonable. While these assumptions could be biased to some extent in one direction or another, it seems unlikely that any justifiable change in any of these would resolve the

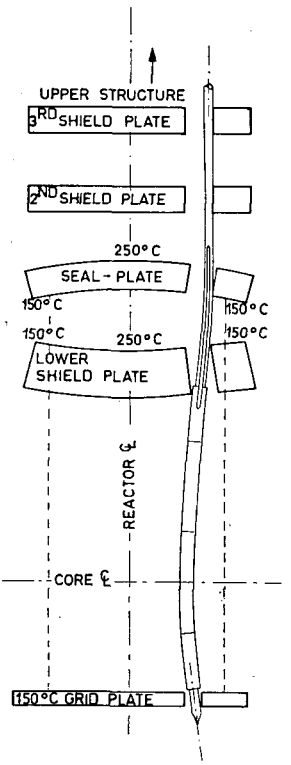


Fig. 25
Preferential orientation of rod extensions and seal-plate bearing-points.

discrepancy. The assumption that the effects of amplifying mechanisms may be neglected, however, may be seriously compromising.

The most likely mechanism for amplifying ligamental motions is one involving possible bearing-points along the lower outer edges of the seal-plate perforations. An illustration of this possibility is given in Fig. 25. The mechanism relies essentially on the possibility that the extremely strong temperature differential across the outer (blanket) portion of the seal causes the plate to assume the shape of an inverted dish. As a consequence of the dishing the lower outer edges tilt slightly upward and perhaps a little inward. In this position they effectively provide ideal pivotal bearing-points for rod extensions. The credibility of the mechanism relies on whether or not a substantial fraction of the rod extensions is in actual physical contact with these bearing-points.

The nominal diameters of the perforations and rod extensions in this region are 0.460 and 0.438 in respectively, giving a nominal diametral clearance of 22 mil. From purely statistical considerations it follows that a certain unspecified fraction of the total number of rods will accidentally bear at these points.

Another mechanism tending to orient the rod extensions against these bearing-points is concerned with expansive effects in the second and third shield plates. Since these and all other contiguous structural members (tank wall, tie rods, etc.) in the region above the outlet spacer are bathed in coolant at the outlet temperature their expansions will be unconstrained. Increases in reactor power will effectively move outer bearing-points outward. The seal-plate bearing-points are much less sensitive to power changes since the outer lower surface of the seal plate is always bathed in coolant at the inlet temperature. (Only 0.20 in of stainless steel separates the bearing points from the inlet coolant.) The effect of power increases on seal-plate bearing-points may actually be a slight inward movement caused by additional dishing as the temperature differential across the blanket portion of the plate increases.

Finally, rod bowing may be (and probably is) sufficient to form a slight reverse bow in the rod extensions above the shield-plate bearing-points (see Fig. 25). Contact between rod extensions and seal-plate bearing-points would require a bending of about 1.3 mil/in, an amount not at all implausible.

While the arguments outlined above apply more rigorously to rod extensions penetrating the outer row of holes, feedback effects arising from the movement of these rods are particularly important in view of the strong flux gradient at the edge of the core.

Assuming that rod extensions are oriented so that they are in physical contact with the seal-plate bearing-points, any displacement effected in the rod extensions by shield-plate flexing would be amplified at core level by a factor of roughly two. Displacements effected by the slower overall expansion of the shield-plate ligaments would also be amplified by the same factor.

If the seal-plate bearing-points tilt upward and inward as the result of a power increase they would force the rod extensions inward. Pivotal action at the shield-plate bearing-points would lead to an additional twofold amplification of motion. Unfortunately, the importance of this effect is practically impossible to assess without the benefit of extremely elaborate calculations of shield-plate deformation.

Because of the time required for an increase in coolant temperature to be sensed by the outer edge of the seal plate, the effects of seal-plate bearing-point motion would not influence the magnitude of the plate-flexing power coefficient, B_0. Feedback effects originating as the result of such motion should more properly

be considered through the inclusion of a third transport-lag term in Eq. (27). Any increase in the overall value of the steady-state power coefficient from this source would, of course, result in a reduction in the magnitudes of C_0 and B_0 required to explain the experimental feedback.

Admittedly, the inward motion of seal-plate bearing-points following a power change is highly speculatory and accordingly cannot be considered seriously as a means of explaining the discrepancy between quasi-analytic and calculated power-coefficient values. The amplifying mechanism based on the preferred orientation of rod extensions with respect to seal-plate bearing-points is, however, not only much more plausible but undoubtedly does exist to some extent. Whether the degree of orientation is complete is a question which cannot be answered with the information available. If the degree of orientation is high, and strong arguments can be cited to support such a postulate, the discrepancy between experimental and calculated results is reduced considerably. For a completely oriented system minimum values of the power coefficients B_0 and C_0 would be 1.00 and $1.60 \times 10^{-6}\, \Delta k/k$ kW, respectively, compared with the respective quasi-analytically determined values of 1.70 and $2.56 \times 10^{-6}\, \Delta k/k$ kW. Additional amplification arising from the tilting of seal-plate bearing-points would reduce the discrepancy even further. In view of the many uncertainties involved, particularly those involving the second normalization of the experimental transfer-function data and those associated with the assumptions upon which the calculated values were based, a disagreement of this magnitude is not surprising. Considerations of the fact that the postulated mechanism explains the frequency-dependence of the feedback and comes within a reasonable margin of explaining its amplitude lead to the conclusion that the lower shield plate was the structural member responsible for the Mark II delayed-negative coefficient.

9. Explanation of experimentally observed phenomena

To be considered valid the mechanism and its associated mathematical model must also account for all experimentally observed phenomena which have a bearing on the instability.

9.1. Isothermal temperature and power coefficients of reactivity

The isothermal temperature coefficient of Mark I was measured to be $31 \times 10^{-6}\, \Delta k/k$ °C, [10], a value which compares favourably with that of $28 \times 10^{-6}\, \Delta k/k$ °C measured for Mark III, [1]. Essentially no difference exists between these values.

The effect of an isothermal temperature change in Mark II was sensed as a reactivity effect through unconstrained expansive actions in the grid plate and in the lower shield plate. After closure of nominal clearances at these points all rod tips and rod extensions were preferentially gathered along the inner edges of their respective locating holes. Under these conditions the rods were radially coupled at top and bottom and, as a consequence of the coupling, expansive actions associated with the grid plate and shield plate were transmitted to the rods. In Mark III the system of stabilizing ribs provided a very strong radial coupling and the temperature coefficient, as expected, was also strong.

Superficially it may seem that the magnitudes of the steady-state power coefficients in both Mark II and Mark III would also be approximately the same.

While no direct measurements of the Mark II power coefficient were ever made, a reasonable estimate may be obtained from the results of the quasi-analytical treatment of section 7. From the values $A_0 = 2.13$ and $C_0 = -2.56 \times 10^{-6}\ \Delta k/k$ kW ($B_0 = 0$ at $w = 0$) it is clear that the steady-state power coefficient is $-0.43 \times 10^{-6}\ \Delta k/k$ kW. In Mark III on the other hand, the steady-state power coefficient under comparable conditions of power and flow rate amounted to $-4.05 \times 10^{-6}\ \Delta k/k$ kW [1] .Thus, while the isothermal temperature coefficients are approximately the same, the power coefficients differ by a factor of roughly ten. The explanation of this rather large difference is concerned with rod bowing which in the case of Mark II was particularly marked and in the case of Mark III was essentially non-existent. In Mark II a considerable freedom of radial motion permitted the rods to bow in the direction of high flux. In Mark III the system of stabilizing ribs prohibited radial motion effectively and greatly minimized bowing effects.

As a result of the Mark III tests it was established conclusively that inward rod bowing is a consequence of special conditions of end restraint. From these tests it was learned that rod bowing does not necessarily result in a strong positive effect unless the fuel rods are rigidly fixed at both top and bottom. Because of the clearances afforded the rod tips and rod extensions in Mark II, it is at first difficult to appreciate why rod bowing contributed so heavily to the power coefficient. As discussed in section 3, rod tips and rod extensions were preferentially oriented such that any deformation resulting from an increase in power would be manifested in the form of an inward bow.

9.2. The existence of prompt positive and delayed negative components

Immediately following a reduction in flow, the power would increase, pass through a maximum and would eventually decrease to some lower equilibrium level. Such behaviour can be explained in terms of the postulated model. The immediate result of a flow decrease would be a sudden increase in the coolant temperature throughout the core. Since the negative effects, i.e. axial and radial fuel expansion and coolant expulsion, were insufficiently large to cancel the positive effect from rod bowing, the sudden increase in coolant temperature was sensed by the core as an addition of reactivity. The power would, accordingly, increase. Approximately 6.5 s after the flow reduction the temperature signal would reach the shield plate. The expansive motions of the shield plate would then increase the core radius through the outward movement of rod extensions. The reactivity loss resulting from ligamental expansions would override the smaller positive effect and the power would eventually drop below the original level. The sequence of events occurring during and after a flow increase is exactly the converse of the above.

9.3. Excursion results

In the last experiment conducted on Mark II, the main coolant flow was reduced to zero and the reactor was placed on a positive period. As the coolant heated, additional reactivity was fed back to the reactor through the influence of the prompt-positive term of Eq. (27). Since the convection of heat to the shield plate was greatly reduced, the normally overriding negative effects associated with shield-plate expansion essentially vanished. The dynamic behaviour of the reactor under these conditions was governed almost entirely by a feedback given by the first term of Eq. (27).

It is interesting to note that at some flow rate between zero and 108 gpm the prompt-positive coefficient associated with the core and the delayed-negative coefficient associated with the structure would exactly cancel each other out. The dynamic behaviour of the reactor when placed on a positive period would then be governed by a zero-power coefficient. The power would increase indefinitely with a period which would neither increase nor decrease.

9.4. The absence of higher order resonance in the transfer-function amplitude

The shape of a feedback governed by a constant power coefficient and a long transport-lag dependence is one of a spiral which may complete several revolutions about the origin before decreasing to zero at high frequencies. For each entry into the III quadrant a partial phase and amplitude cancellation with the inverse zero-power gain $1/G_0$ always occurs. The effect would be manifested in the transfer function through the occurrence of higher order resonances lying on the high-frequency side of the fundamental. The fact that such peaks were not observed, whereas a simple transport lag model indicates their existence, has always been perplexing. The model described by Eq. (27) resolves this ambiguity. The second term which describes the feedback associated with plate flexing approaches zero rapidly at the higher frequencies. As the frequency increases the amplitude of the temperature perturbation arriving at the shield plate is considerably reduced. As a consequence the amplitude of the oscillating temperature differential across the plate decreases strongly with frequency. The magnitude of the feedback from shield-plate flexing is accordingly described by a power coefficient which is a strong function of the oscillating frequency.

The power coefficient in the third term of Eq. (27) is also affected by the amplitude attenuation of the coolant temperature wave in its passage between core and shield plate. At the higher frequencies the amplitude of the temperature oscillations in the vicinity of the shield plate are so strongly attenuated that little or no ligamental motion is effected.

Because of the strong non-linear behaviour of both B_0 and C_0, the feedback as a function of frequency approaches zero much mote rapidly than for the case of truly constant power coefficients.

9.5. The shift of resonant peaks to higher frequencies at higher flow rates

As the flow rate increases (at constant power) the physical transit time of coolant between core and shield plate decreases. The effects of the decrease are manifested through the influence of the cyclic operator $e^{-i\omega\tau(\omega)}$ [Eq. (27)] as a strong increase in the phase of the feedback for all frequencies. As a consequence constructive interference of the feedback with the input reactivity occurs at a higher frequency.

Conclusions

1. Because the model predicts the general shape of the experimentally measured feedback and comes within a reasonable margin of explaining the feedback magnitude, it may be concluded that the structural member responsible for the delayed-negative power coefficient was the lower shield plate. There seems little doubt that any credible interpretation of the feedback must involve the concept

of a delayed feedback which almost certainly must have originated at some structural member located downstream from the core. A careful scrutiny of all downstream structural members supplemented by Storrer's attenuation-concepts narrows the region of suspicion to that included between the lower shield plate and the seal plate. The facts that thermally-induced ligamental motions have actually been observed in such a plate and that the maximum temperature differential across the shield plate occurs close to the natural resonant frequency of the reactor strongly support this conclusion.

2. The instability of Mark II may be directly attributed to the combination of two undesirable features both of which are easily eliminated through elementary changes in mechanical design. The lack of radial coupling between fuel rods was detrimental in two respects: rod bowing was permitted and the normally strong prompt-negative feedback from radial fuel expansion was greatly weakened. The consequence was an overall prompt-positive power coefficient of reactivity. The other important feature was, of course, the shield-plate system which was responsible for the delayed-negative coefficient. The combination of the two effects resulted in a feedback which permitted constructive interference of periodic reactivity insertions. It is almost certain that the addition of stabilizing ribs to the fuel rods would have eliminated the instability since radial motion of fuel from rod bowing and from shield-plate ligamental motions would have been denied. For a rigidly coupled system of rods the dynamic behaviour of the reactor would have been dominated by an overall prompt and strongly negative power coefficient of reactivity.

APPENDIX A

The effect of temperature transients on shield-plate ligamental motions

The results of attemps to detect motions induced by temperature variations in the ligaments of a dummy 2-in shield plate have demonstrated conclusively that such motions do exist.

The first attempts to observe motions involved heating the ligaments with the exhaust from a hydrogen blast lamp. To prevent the heating of the outer portion of the shield plate (which remained at constant temperature in the reactor) a pyramidal shroud was installed beneath the plate in such a manner that the shroud effectively channelled the heated air through the central section and left unaffected areas outside the hexagonal annulus (see Fig. 5). To limit the extent of heating the blast was interrupted after approximately four seconds. Temperatures in the central portion of the plate rose rapidly to about 200°C. Measurements of expansive motions were carried out with a dial indicator butted against a quartz rod which in turn was sealed into a stainless-steel insert machined to provide a flush fit with the fuel-rod perforations. Asbestos shielding was installed around the measuring equipment to prevent extraneous heating effects. The responses of approximately 20 holes were tested. Although no two responded exactly alike, a general pattern of behaviour was clear. With the indicator butted against the outside of the quartz rod (approximately 4 in above the plate), heating the ligaments was followed immediately, i.e. within 2—3 s, by a movement of the rod inward. After approximately 5—6 s the inward motion ceased and the rod began to move outward at a much slower rate coming to rest eventually at a position approximately 4—5 mil outward from its initial position. With the indicator bearing on the top of the rod, i.e. in a position measuring vertical deflections, the initial movement was 3—4 mil downward over a period of approximately 2—3 s followed by a much slower return to a position approximately 4—5 mil above its initial elevation. The explanation for this behaviour is clear. Immediately following the 4-s heat blast the first motion consists of a warping or a dishing of the plate. A dial indicator butting against the outside of the rod senses the motion as an inward motion. An indicator mounted on top of a rod senses the motion as a decrease in elevation. The dishing is apparently the result of preferential

expansion in the lower portion of the ligaments. As the entire collection of ligaments reaches a more or less equilibrium temperature the dishing effect disappears, i.e., the axes of the perforations return to vertical. Such actions are consistent with the recoveries noted. The overshoot in elevation is the result of an overall vertical expansion which essentially raises the rod and bearing point of the indicator. The overshoot in radial displacement is a consequence of a much slower expansion of the hexagonal annulus which allows ligamental stresses to be at least partially relieved. Observations carried out on a $^1/_8$-in stainless-steel pin positioned in the annulus substantiated this conclusion. Indicator measurements carried out as a function of elevation along the outside of the rod verified the existence of the dishing effect, i.e., significantly larger inward displacements were measured at the upper portions of the rod. Other measurements carried out on the right- and left-hand sides of the rod demonstrated that lateral motion also occured.

In another series of experiments hot water was used as a heat-transfer medium. In these experiments water at approximately 50°C was stored in a tank contiguous with the lower broached portion of the shield plate. An overflow chamber fitting around the periphery of the hexagonal annulus was mounted on the upper side. By gasketing mating surfaces hot water could be forced upward through the rod perforations into the overflow chamber without sensibly affecting the temperature of the outer portion of the plate. Again dial indicators were used to measure displacements by expansive motions. The results of measurements conducted on approximately 25 rod locations confirmed the results obtained in the heated-air experiments. With an indicator butting against the outside of the quartz rod at an elevation approximately 4 in above the plate the initial motion was consistently inward. In general, the initial motion consisted of about 0.5 to 1.0 mil over a period of 2—3 s followed by a somewhat slower recovery to the initial position. In nearly all cases an overshoot, i.e. an outward motion, of approximately one mil taking place over a period of 5—10 s was observed. By reducing the flow of water through the plate the initial tilting could be exaggerated and the time required for tilting, recovery and overshooting could be increased. It was also observed consistently that the tilting effect was more pronounced in the outer two or three rows of rod perforations.

The results of experiments in which the bearing-point of the indicator was placed on top of the rod, i.e. in a position measuring vertical movement, were consistent with the results of similar experiments conducted with heated air. The initial motion consisted of a 0.2 to 0.3-mil dip over a period of 2 to 3 s followed by a slower recovery to initial elevation. In most cases a small overshoot of approximately 0.3 mil was observed.

The results of experiments conducted with two dial indicators both bearing against the inside of the rod but with one at 4-in elevation above the plate and the other at 8-in demonstrated conclusively that the initial action consisted of a tilt. In nearly all experiments of this nature the upper indicator registered significantly larger radial displacements than was the case for the lower indicator.

From these results it is clear that the immediate consequence of a sudden increase in the temperature of coolant flowing through the ligaments is a rapid dishing action which tilts the axes of the perforations towards the normal. Since the tilting is more pronounced along the periphery of the perforations, it seems likely that the thin (approximately 0.20-in) hexagonal annulus is also involved in the dishing action. Apparently the temperature gradient established across the plate effects a preferential expansion not only of the ligaments but of the annulus as well. The recovery and overshoot are consistent with the behaviour expected for a system which eventually comes to a higher (and uniform) equilibrium temperature.

Attemps to observe the reverse effect, i.e. a convex (upward) dishing, were successful. With the ligaments at an equilibrium temperature of 50°C the hot water was drained suddenly and was replaced with water at approximately 25°C. As the cold water reached the lower surface of the ligaments an indicator bearing against the outer edge of the quartz rod registered a sudden outward motion followed by a slow return to its initial position and a somewhat slower displacement towards the centre of the plate. Experiments conducted with two dial indicators, one bearing at 4 in above the plate and the other at 8 in, demonstrated that the initial outward motion was the result of a convex dishing, since the upper indicator registered displacements approximately twice those of the lower.

From the results of these experiments it is possible to estimate a crude value for the degree of unconstrained expansion. For a change of 25°C the average amount of overall radial displacement associated with holes in the outer two rods amounted to approximately 1.2 mil, in contrast with the 1.57 mil expected for a completely unconstrained system. The degree of unconstrained expansion is then approximately 1.2/1.57 or 0.75.

APPENDIX B

Evaluation of physical parameters

Time-constant calculations

In order to evaluate Eqs. (8) and (16) it is necessary to establish time-constant values for each material and each region where the time constant is defined by the following

$$\tau_x = \frac{C_p}{h} \tag{B-1}$$

where C_p is the total heat capacity per unit length of material x and h is the overall heat-transfer coefficient per unit length for the same material. Physically, τ_x is analogous to the RC time constant of an electrical circuit. More specifically τ_x is the time required, after a perturbation, for the temperature of a given component to reach 63.2% of its steady-state value.

Film heat-transfer coefficients

Regions I-VI. In these regions coolant flows in long vertical channels. The expressions used for determining the various film coefficients is [14]

$$h'_f = \frac{K}{D_e}[7 + 0.025\,(N_{Re} \times N_{Pr})^{0.8}]\ \text{BTU/h—ft}^2\,°\text{F} \tag{B-2}$$

$$h_f = h'_f \times S\ \text{BTU/h} — \text{ft}\ °\text{F} \tag{B-3}$$

where S is the flow perimeter of all coolant channels.

Regions VII and VIII. In these regions the coolant flow is essentially perpendicular to both fuel and blanket rod-handles, which in this special case, are of circular cross-section. The expression used for determining the film coefficients is [14]

$$h'_f = 1.17\,\frac{K}{D_e}\,(N_{Re} \times N_{Pr})^{0.8}\,N_{Pr}^{1/3}. \tag{B-4}$$

The value for coolant velocity used in determining N_{Re} is the minimum value for the particular region. This procedure results in a conservative estimate of the film coefficient.

Method of calculating the equivalent diameter (D_e) of flow channels in the inlet plenum (Region V)

In this region the fuel rod-handles are fluted. Since no provisions were made for preferential angular orientation of fuel-rod tips in the bottom plate, it is assumed that the orientation of one rod-handle with respect to its neighbours is random. The equivalent diameter in this region is assumed to be equal to four times the total coolant flow area divided by the total flow perimeter of all rod-handles. From these assumptions it may be shown that $D_e = 0.44$ in. A comparison of this value with that obtained for the shield plate channels ($D_e = 0.15$ in) shows that in the outlet plenum the equivalent diameter is larger by a factor of about three. This is to be expected in that the flow area per channel in the inlet plenum is increased by a factor slightly larger than three because of the absence of plate ligaments.

Heat-transfer-conduction coefficients

Regions I, II, III and VIII. In these regions the rods and rod-handles are solid and circular in cross-section. The expression used for determining the conduction

coefficient is [11]

$$h'_c = 8\pi K \qquad \text{BTU/h — ft°F} \tag{B-5}$$

$$h_c = nh'_c \tag{B-6}$$

where n is the total number of rods.

Regions IV, V, VI and VII. The coefficients of conduction for the rod-handles and plate ligaments in these regions may be calculated from

$$h_c = \frac{KS}{d} \tag{B-7}$$

where S is the flow perimeter and d is the mean distance for heat conduction. For the fluted portion of the rod-handles d is estimated to be 0.030 in (see Fig. 6). For the shield-plate ligaments d is assumed to be one-fourth of the ligamental thickness since coolant flows on both sides of the ligaments. It is also assumed that the coolant present in the 0.011-in annular segments between the fluted rod-handles ligaments is stagnant and that no heat is transferred from the coolant in these regions.

The fuel rod-handles in Region VII are circular, hollow and gas-filled. On the assumption that all heat conducted into the rod is carried by convection into the upper portion of the handles, the distance d for heat conduction is assumed to be the tube thickness.

Physical properties of NaK and stainless-steel-347 at 300°C

	C_p	ϱ	k	μ
	BTU / lb m °F	lb m / ft³	BTU / h ft °F	lb m / ft h
NaK	0.215	49.9	14.9	0.614
SS347	0.12	498	10.76	—

APPENDIX C

Calculations of the phase and amplitude reduction of a temperature wave in its passage through the shield plate (Region IV) and seal plate (Region VI) are complicated by the presence of an additional inert medium, the relatively thin 34-mil ligaments. Since Storrer's expressions [11] apply strictly to a single inert medium (rod handles in this case), it is necessary to extend the scope of his mathematical developments.

The following transient heat-balance expressions may be written for each component of the shield-plate region:

For the rod handles:

$$\frac{\tau_{f_1}[\partial T_{f_1}(z,t)]}{\partial t} = T_c(z,t) - T_{f_1}(z,t) \tag{C-1}$$

For the shield-plate ligaments:

$$\frac{\tau_{f_2}[\partial T_{f_2}(z,t)]}{\partial t} = T_c(z,t) - T_{f_2}(z,t) \tag{C-2}$$

For the collant:

$$\tau_c\left[\frac{\partial T_c(z,t)}{\partial t} + \frac{v\,\partial T_c(z,t)}{\partial z}\right] = T_{f_1}(z,t)\,T_{f_2}(z,t) - 2\,T_c(z,t) \tag{C-3}$$

where the above parameters are defined as

τ_{f_1} = the time constant for heating or cooling the rod handles,
τ_{f_2} = the time constant for heating or cooling the shield plate ligaments,
τ_c = the time constant for heating or cooling the NaK,
$T_c(z, t)$ = the bulk coolant temperature,
$T_{f_1}(z, t)$ = the average temperature of the rod handles,
$T_{f_2}(z, t)$ = the average temperature of the ligaments,
v = the coolant velocity.

Assuming that power changes in the core and temperature changes in the shield-plate region vary sinusoidally, the following expressions apply

$$T_{f_1}(z, t) = X_1(z)e^{i\omega t} + T_0 \tag{C-4}$$

$$T_{f_2}(z, t) = X_2(z)e^{i\omega t} + T_0 \tag{C-5}$$

$$T_c(z, t) = Y(z)e^{i\omega t} + T_0 \tag{C-6}$$

where $X_1(z)$, $X_2(z)$ and $Y(z)$ are non-complex amplitudes of the temperature oscillations and T_0 is the steady-state temperature. Differentiating and substituting into Eqs. (C-1), (C-2) and (C-3) yields

$$\tau_{f_1}(i\omega X_1 e^{i\omega t}) = T_c - T_{f_1} \tag{C-7}$$

$$\tau_{f_2}(i\omega X_2 e^{i\omega t}) = T_c - T_{f_2} \tag{C-8}$$

$$T_c\,(i\omega Y e^{i\omega t} + vY' e^{i\omega t}) = T_{11} + T_{f_2} - 2\,T_c. \tag{C-9}$$

Eliminating $e^{i\omega t}$ from these expressions

$$\tau_{f_1}\, i\omega X_1 = (T_c - T_{f_1})\, e^{-i\omega t} = Y - X_1 \tag{C-10}$$

$$\tau_{f_2}\, i\omega X_2 = (T_c - T_{f_2})\, e^{-i\omega t} = Y - X_2 \tag{C-11}$$

$$T_c\,(i\omega Y + vY') = (T_{f_1} + T_{f_2} - 2T_c)\, e^{-i\omega t} = X_1 + X_2 - 2Y. \tag{C-12}$$

Eliminating X_1 and X_2 from (C-12) gives

$$\tau_c\,(i\omega Y + vY') = \frac{Y(1 - i\omega\tau_{f_1})}{1 + \omega^2\tau_{f_1}^2} + \frac{Y(1 - i\omega\tau_{f_2})}{1 + \omega^2\tau_{f_2}^2} - 2Y\,. \tag{C-13}$$

Solving (C-13) for Y yields

$$Y_z = Y_0\, e^{-\lambda\frac{z}{v}} \tag{C-14}$$

$$\lambda = \frac{1}{\tau_c}\left[i\omega\tau_c - \left\{\frac{1 - i\omega\tau_{f_1}}{1 + \omega^2\tau_{f_1}^2} + \frac{1 - i\omega\tau_{f_2}}{1 + \omega^2\tau_{f_2}^2} - 2\right\}\right]. \tag{C-15}$$

Separating (C-15) into real and imaginary components

$$\lambda = \frac{\omega^2(\tau_{f_1}^2 + \tau_{f_2}^2) + 2\omega^4\tau_{f_1}^2\tau_{f_2}^2}{\tau_c(1 + \omega^2\tau_{f_1}^2)(1 + \omega^2\tau_{f_2}^2)} + i\omega\left[1 + \frac{\tau_{f_1}}{\tau_c(1 + \omega^2\tau_{f_1}^2)} + \frac{\tau_{f_2}}{\tau_c(1 + \omega^2\tau_{f_2}^2)}\right] \tag{C-16}$$

where

Y_z = complex temperature amplitude of the coolant in the shield-plate region,
Y_0 = non-complex temperature amplitude of the coolant at the inlet of the plate,
z = vertical distance at which the coolant temperature is evaluated,
v = coolant velocity,
ω = power oscillation frequency, rad/s.

REFERENCES

[1] SMITH, R. R. *et al.*, ANL-6266 (1960).
[2] SMITH, R. R. and MATLOCK, R. G., ANL-6354.
[3] JENS, W. H. and KLECKER, R. W., "EBR transfer function experiment", unpublished.
[4] BRITTAN, R. O., Proc. 2nd UN Int. Conf. PUAE, **12** (1958) 267.
[5] KITTEL, J. H., NOVICK, M. and BUCHANAN, R. F., *Nucl. Sci. Engng*, **4** 2 (1958).

[6] SIEGEL, R. and HURWITZ, H., Jr., KAPL-1138 (1956).
[7] THALGOTT, F. W., unpublished.
[8] KINCHIN, G. H., RP/M83, AERE, Harwell (1956).
[9] BETHE, H. A., APDA-117, (1956).
[10] LICHTENBERGER, H. V. *et al.*, ANL-5023 (1953).
[11] STORRER, F., APDA-132 (1959).
[12] RICE, R. E. *et al.*, ANL-5836 (1958).
[13] BAKER, A. R., Argonne National Laboratory, Idaho Division, 2 February 1961, unpublished.
[14] ETHERINGTON, H., Nuclear Engineering Handbook, McGraw-Hill Book Company, New York, (1958).

SM-18/28

ÉTUDE DES RÉGIMES TRANSITOIRES ET DE LA STABILITÉ DES PILES RAPIDES

R. ABDON, J. M. CHAUMONT, P. P. CLAUZON et G. GHESQUIÈRE
CENTRE D'ÉTUDES NUCLÉAIRES DE SACLAY,
FRANCE

Abstract — Résumé — Аннотация — Resumen

Studies on the transient operation and stability of fast reactors. These studies form part of the general programme of perfecting calculation methods for fast reactors.

The basic formulae are given for the layouts used, i.e. the classic kinetic and thermal exchange equations, etc. A description is then given of the digital computer methods employed for studying the stable functioning of the reactor and of the methods used for transient operation studies.

Finally, some examples of application are discussed and a comparison is made with parallel studies on the same subject.

Etude des régimes transitoires et de la stabilité des réacteurs à neutrons rapides. Ces études s'inscrivent dans le programme général de mise au point de méthodes de calculs concernant les réacteurs à neutrons rapides.

On présente la formulation de base, c'est-à-dire les équations classiques de la cinétique, des échanges thermiques, etc. pour les schémas utilisés, puis on décrit les méthodes de calcul sur machine numérique utilisées pour les études de stabilité de fonctionnement du réacteur et celles utilisées pour les études en régime transitoire.

Viennent ensuite la discussion de quelques exemples d'applications et la comparaison à des études analogiques menées parallèlement.

Изучение стабильности и переходных режимов реакторов на быстрых нейтронах. Эта работа входит в общую программу разработки вычислительных методов, касающихся реакторов на быстрых нейтронах.

Даются основные уравнения для применявшихся схем расчета, т.е. классические уравнения кинетики, теплообмена и т.д. Описываются методы вычисления на цифровых счетных устройствах, применявшиеся для изучения стабильности работы реактора, а также подобные методы, использовавшиеся для изучения работы реактора в переходных режимах.

Далее дается несколько примеров применения этих методов и сравнение с параллельно проводившимися аналогичными исследованиями.

Estudio de los regímenes transitorios y de la estabilidad de los reactores rápidos. Estos estudios forman parte del programa general de perfeccionamiento de los métodos de cálculo relativos a los reactores de neutrones rápidos.

Los autores presentan los planteos básicos, es decir, las ecuaciones clásicas de la cinética, de intercambios térmicos, etc., correspondientes a los sistemas utilizados, y describen los métodos mecánicos de cálculo numérico aplicados al estudio de la estabilidad de funcionamiento del reactor y a los estudios del régimen transitorio.

A continuación, los autores discuten algunos ejemplos de utilización de estos cálculos y comparan los resultados obtenidos con los de estudios analógicos realizados paralelamente.

Introduction

Certaines caractéristiques des piles rapides font que leur comportement dynamique est assez nettement différent de celui des piles thermiques. Rappelons les caractéristiques essentielles:

— Très fort couplage entre la puissance et la réactivité dû à la puissance spécifique élevée et au faible volume critique des piles rapides;

— Faible inertie thermique et fort gradient de températures, d'où possibilité de chocs thermiques importants;

— Temps de vie de neutrons prompts très bref, environ 10^{-7} s contre 10^{-3} à 10^{-5} pour les piles thermiques;

— Proportion effective de neutrons retardés (β_{eff}) très petite pour le plutonium-239, combustible indispensable pour les piles rapides;

— Possibilité d'excursion de puissance à allure explosive due à la présence de grandes quantités de combustible très enrichi, qui en cas de fusion peuvent théoriquement former plusieurs masses critiques.

On conçoit aisément combien est importante dans tout projet de pile rapide l'étude du comportement dynamique. C'est une étape essentielle, qui pourra influencer de façon importante les caractéristiques et le dessin de la pile.

Nous partagerons cette étude en trois parties en fonction du but recherché:

a) Etude de la stabilité de fonctionnement autour d'une puissance donnée quelconque dans la gamme des puissances prévues pour le fonctionnement de la pile; on peut tenter également d'étudier la stabilité globale.

b) Etude de distribution des températures et des contraintes dans les diverses parties de l'installation lors de régimes transitoires relativement probables: démarrage, arrêt, arrêt d'urgence de la pile, arrêt de diverses pompes, etc.

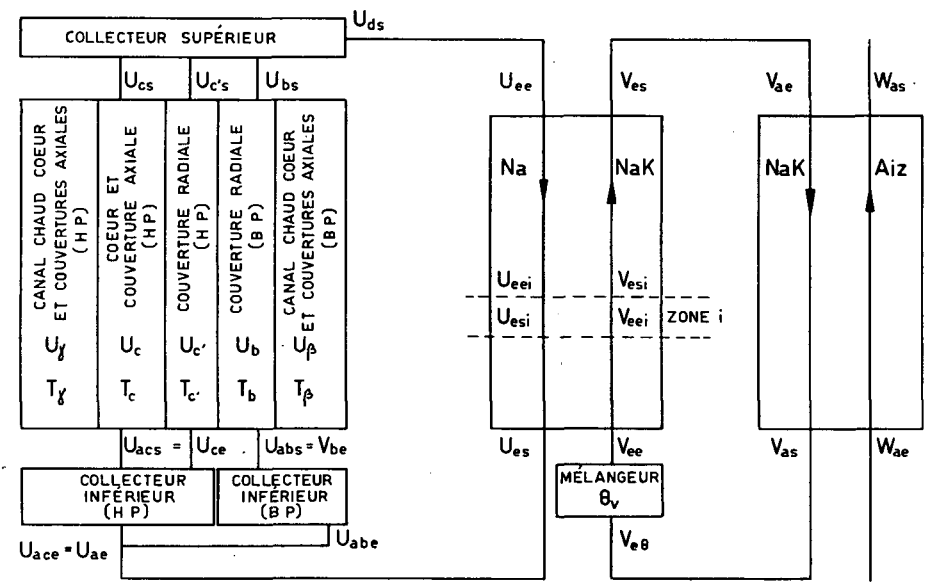


Figure 1
Programme dynamique P_2. Schéma de principe.

c) Etude du comportement de la pile lors d'incidents graves et relativement improbables (exemple: fusion et rassemblement rapide du combustible), avec pour but essentiel l'évaluation de l'énergie maximale libérée lors de tels incidents.

Ce partage se justifie non seulement par les buts très différents de chacune de ces trois parties, mais aussi par les approximations différentes que l'on est amené à faire dans chaque cas, et donc par les méthodes de calcul également différentes d'un cas à un autre. On présente dans ce papier les deux premiers aspects de l'étude (*a*, *b*) tels qu'ils ont été traités jusqu'à présent dans la Section d'études de piles rapides de Saclay.

Formulation de base

La formulation utilisée pour la cinétique neutronique et les échanges thermiques est classique. Nous avons choisi un modèle pour la pile et les échangeurs, où les températures sont des fonctions continues axialement et moyennées radialement 1 — 2 — 3. Le schéma en figure 1 explicite les notations utilisées par la suite. Nous avons pris, comme cas de figure, un échangeur terminal NaK-Air.

BLOC PILE

Echanges thermiques

La pile est découpée radialement en canaux j, lesquels sont découpés axialement en zones i. Les équations ci-dessous expriment un bilan thermique entre combustible et sodium plus structures:

$$A_{ij}\frac{\delta U}{\delta t} + B_{ij}\, m_u \frac{\delta U}{\delta z} = H_{ij}\,(T - U)$$

$$C_{ij}\frac{\delta T}{\delta t} + D_{ij}\frac{\delta U}{\delta t} = H_{ij}\,(U - T) + \lambda_{ij}(z)\, q\,(t)$$

A_{ij}, B_{ij}, H_{ij}, C_{ij}, D_{ij} sont des constantes thermiques [1],
m_u le débit sodium,
$q(t)$ la puissance neutronique,
$\lambda(z)$ la répartition axiale de la puissance,
T la température du combustible,
U la température du sodium.

Cinétique [4]

$$0 = \frac{\Delta k\,(1-\beta) - \beta}{l}\, q + \sum_i \lambda_i C_i$$

$$\frac{\mathrm{d}\, C_i}{\mathrm{d}\, t} = \frac{\beta_i\,(1 + \Delta k)}{l}\, q - \lambda_i C_i$$

Définition habituelle des notations utilisées où:
Δk la réactivité est égale à la somme de deux termes: Δk_c, réactivité externe fournie par le contrôle, et Δk_r, la réactivité de contre-réaction de la puissance:

$$\Delta k = \Delta k_c + \Delta k_r.$$

Δk_r est la somme de plusieurs effets (effet Doppler, déformation des assemblages, etc.). Nous nous bornons ici aux seuls effets de densité et de dilatation.

$$\Delta k_r = \sum_{ij}\left[\eta_i \cdot \overline{\Delta U_i} + \xi_i\, \overline{\Delta T_i}\right]_j + \overline{\mu_c} + \overline{\mu_b}$$

où
j représente l'indice de canal,
i l'indice de zone,
$\overline{\Delta U_i}$ et $\overline{\Delta T_i}$ sont les élévations moyennes de température du sodium et du combustible dans la zone considérée,
$\overline{\eta_i}$ et $\overline{\zeta_i}$ représentent les coefficients de réactivité isothermes dus au sodium plus structure et au combustible,
$\overline{\mu_c}$ et $\overline{\mu_b}$ sont par exemple ici des coefficients dus à la dilatation de la plaque de support inférieure (partie cœur $\overline{\mu_c}$ et partie couverture $\overline{\mu_b}$), qu'une étude simplifiée nous a donné comme solutions de:

$$\overline{\mu_c} + 2{,}5\,\frac{\delta\,\overline{\mu_c}}{\delta t} = \mu_{c_0}\,\Delta\,U_{ce}$$

$$\overline{\mu_b} + 2{,}5\,\frac{\delta\,\overline{\mu_b}}{\delta t} = \mu_{b_0}\,\Delta\,U_{be}$$

μ_{c_0} et μ_{b_0} étant des constantes,
et ΔU_{ce} et ΔU_{be} étant les écarts des températures d'entrée du sodium dans les parties cœur et couverture du sommier (voir fig. 1).

ECHANGEUR PRIMAIRE

On considère ici un échangeur à métaux liquides (liquide primaire Na cédant sa chaleur au liquide secondaire NaK). Les équations ci-dessous expriment le bilan thermique:

$$a\,\frac{\delta U}{\delta t} + b\,m_u\,\frac{\delta U}{\delta z} = h\,(V - U)$$

$$c\,\frac{\delta V}{\delta t} + d\,m_v\,\frac{\delta V}{\delta z} = h\,(U - V)$$

où
a, b, c, d, h sont des constantes thermiques,
m_u et m_v sont les débits de sodium et de sodium-potassium,
V représente la température du sodium-potassium.

ECHANGEUR SECONDAIRE (cas de RAPSODIE)

Dans le cas de RAPSODIE, l'échangeur secondaire est un échangeur NaK-air à courants croisés, donc de formulation délicate. Nous avons adopté la formulation simplifiée suivante (i représente l'indice de zone):

$$\chi\,(\overline{V_i} - \overline{W_i}) = m_v\,c_v\,(V_{asi} - V_{aei}) - M_i\,\frac{d}{dt}\,(\overline{V_i})$$

$$\chi\,(\overline{V_i} - \overline{W_i}) = m_w\,c_{pw}\,(W_{asi} - W_{aei})\,.$$

(On néglige la capacité calorifique de l'air dans l'échangeur.)

L'échange de chaleur se fait entre les températures moyennes $\overline{V_i}$ du NaK et $\overline{W_i}$ de l'air ($\overline{V_i} = (V_{asi} + V_{aei})/2$).
χ, c_v, c_{pw} sont des constantes thermiques,
m_v, m_w représentent les débits NaK et air,
M_i est la masse calorifique du NaK dans la zone i (on a négligé la masse critique de l'air dans la même zone).

COLLECTEURS

Nous désignons sous ce terme tout espace d'expansion où se mélange le réfrigérant et qui fait donc varier la température de ce dernier. Pour en tenir compte, nous avons adopté la relation suivante:

$$E\,k\frac{\mathrm{d}U_s}{\mathrm{d}t} = m_u \sum_j F_j\,(U_{e_j} - U_s)$$

U_s est la température de sortie du collecteur,
U_{ej} la température du Na à l'entrée j,
$F_j\,m_u$ le débit Na à l'entrée j ($\Sigma F_j = 1$),
j
m_u le débit total Na,
E la masse calorifique du collecteur,
k le coefficient de mélange.

TUYAUX

Nous désignons sous ce terme les tubes et tuyauteries reliant les diverses parties des circuits (bloc pile, pompes, échangeurs, etc.). Il s'agit alors de déterminer le retard τ entre le sodium à la sortie et à l'entrée. On part de la relation

$$\int_{t-\tau}^{t} m\,(\theta)\,\mathrm{d}\theta = N^*$$

m est le débit de réfrigérant,
N^* le volume du réfrigérant dans le tuyau corrigé pour tenir compte de la masse calorifique de l'acier.

La relation équivalente suivante a été utilisée dans la programmation:

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = \frac{m\,(t) - m\,(t-\tau)}{m\,(t-\tau)}\,.$$

Etude des régimes transitoires

PRÉSENTATION DES MOYENS DE CALCUL

On a vu que le principal objet de l'étude des régimes transitoires est l'évaluation des chocs thermiques en divers points sensibles des circuits, avec comme conséquence la modification possible de certaines parties des circuits, et la définition pour le contrôle de seuils de température et de gradients de température.

Ceci nous conduit, dans la plupart des cas, à étudier la pile dans son ensemble telle qu'elle est représentée sur la figure 1. Pour résoudre les diverses relations que nous avons définies dans le paragraphe précédent, de puissantes machines sont nécessaires si l'on veut une précision suffisante.

En calcul digital, les équations différentielles ont été traitées par la méthode classique des équations aux différences. Les limitations des programmes étudiés sont dues aux différents retards intervenant dans les circuits; en effet, ceux-ci nous obligent à conserver en mémoire les valeurs des températures aux entrées des différents tuyaux. De ce fait, nous ne disposions que de 150 pas en temps dans un premier programme sur machine Mercury Ferranti (P_1). Ce programme, basé sur un schéma un peu plus simple que celui donné en figure 1, nous a permis néanmoins de dégrossir l'étude des régimes transitoires.

Nous démarrons actuellement l'exploitation d'un second programme (P_2) sur machine IBM-7090 (langage FORTRAN), où nous disposons pour l'étude d'environ 1500 pas pour la variable temps. Le schéma de pile adopté est celui de la figure 1.

En période de test des programmes P_1 et P_2, nous avons rencontré quelques difficultés en ce qui concerne la formulation de l'échangeur NaK-air, le découpage en espace du bloc pile et des échangeurs, et le découpage en temps pour l'ensemble du calcul lui-même. Ceci nous a conduits a formuler deux programmes P_1' et P_2', où seul le bloc pile était étudié sur la base des équations utilisées pour les programmes P_1 et P_2. Le fait que les retards n'existaient plus nous a permis une étude très fine en temps et le volume moins grand du calcul une étude plus fine en espace.

Nous présentons dans le paragraphe suivant quelques résultats.

Indépendamment des calculs sur machines digitales, nous utilisons également des machines analogiques (PACE). Les résultats, évidemment moins précis que ceux fournis par les programmes P_1 et P_2, sont beaucoup plus aisés à obtenir, et la souplesse d'emploi des machines analogiques permet une étude plus facile des problèmes de contrôle, pilotage automatique, etc. La difficulté majeure rencontrée ici est la simulation des retards avec mise en mémoire des températures d'entrée aux différents tuyaux. Nous ne présenterons pas les résultats obtenus avec les machines analogiques. En effet, les discordances que nous avons notées peuvent être le fait des constantes et de la formulation quelque peu différente utilisées. Aussi est-il difficile de conclure, et de nouvelles études sont actuellement en cours pour résoudre ce problème et permettre en même temps la réalisation d'un simulateur de contrôle d'une pile rapide.

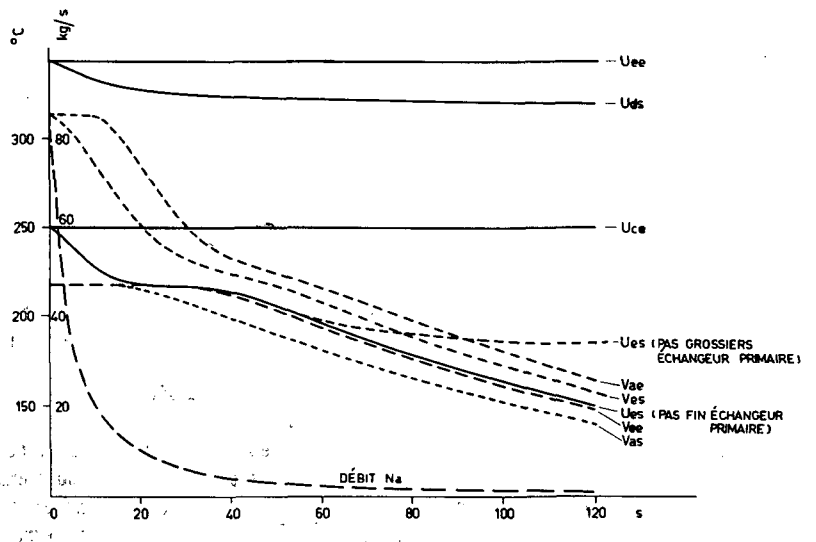


Figure 2

Décroissance des températures des réfrigérants en cas de chute d'urgence des barres de contrôle suivie après 0,25 s de l'arrêt des pompes primaires.

QUELQUES EXEMPLES D'APPLICATION DES PROGRAMMES P_1, P_1', P_2, P_2'

Ils concernent la pile RAPSODIE telle qu'elle était définie au début 1960:

Combustible (U-Pu-Mo) { 70% U enrichi à 20%
20% de Pu
10% de molybdène

Hauteur du cœur: 40 cm

Gradient de température axial: 90°C

(température Na sortie cœur = température Na couverture — 50° C)

La figure 2 donne la décroissance des températures lors d'une chute d'urgence des barres de sécurité avec arrêt des pompes Na, obtenue avec le programme P_1. On peut voir l'effet d'un découpage plus fin en espace dans l'échangeur primaire sur la régularisation des températures. Des résultats plus complets concernant la dynamique du réacteur rapide RAPSODIE sont donnés ailleurs [5].

La figure 3 présente la décroissance des températures lors d'une chute d'urgence des barres de sécurité avec arrêt des pompes Na, obtenue par les programmes P_1 et P_2. La légère discordance peut provenir de constantes quelque peu différentes.

La figure 4 présente l'évolution de la température du Na sortie cœur lors d'un arrêt debla pompe primaire Na, sans chute des barres. Le programme P_1 donne ici, semble-t-il, un résultat aberrant, certainement à cause de ses limitations. Par contre, les programmes P_1' et P_2' sont relativement concordants.

La figure 5 nous montre les résultats obtenus avec P_1 et P_1' lors d'une insertion de réactivité de 1 cent/s (incident de démarrage par exemple). On note ici un léger écart dû au pas en temps plus fin dans le programme P_1'.

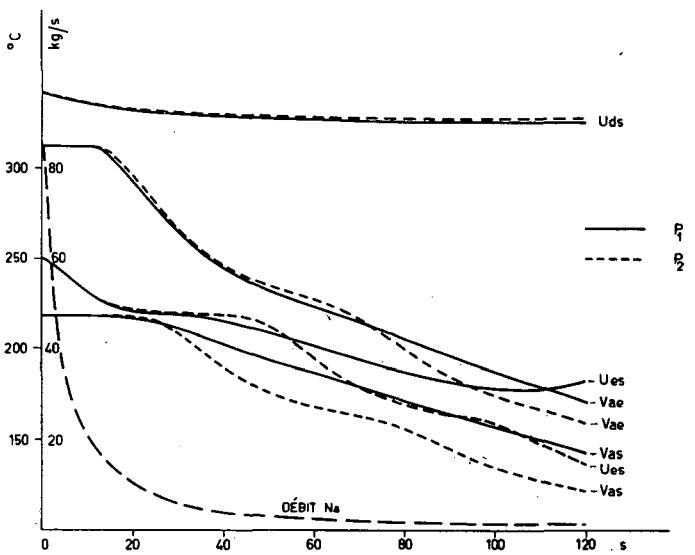


Figure 3
Chute d'urgence des barres de sécurité avec chute du débit Na.

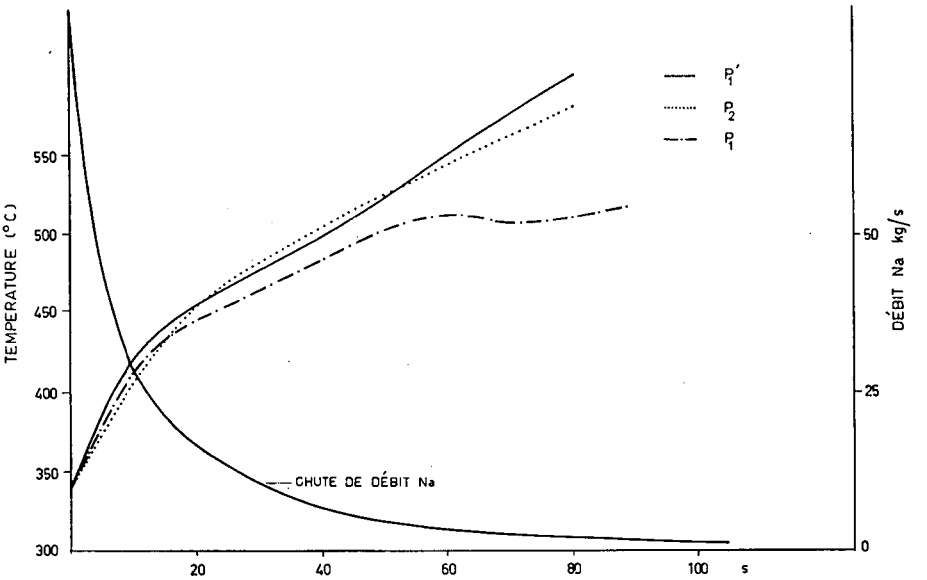


Figure 4
Arrêt de pompe Na. Température Na sortie cœur U_{ds}.
——— P_1'
..... P_2
—.—.—. P_1

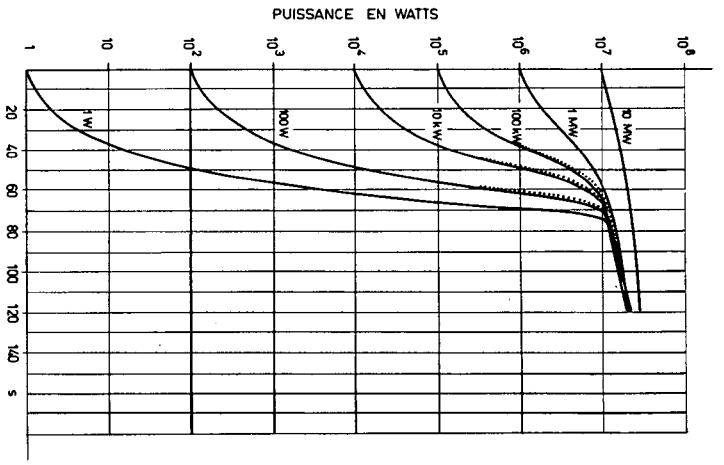


Figure 5
Accidents au démarrage. Introduction linéaire de 1 cent/s avec contre-réaction. A l'instant initial, la pile est à l'état critique. Débit sodium: valeur nominale à 10 MW.
——— P_1'
..... P_1

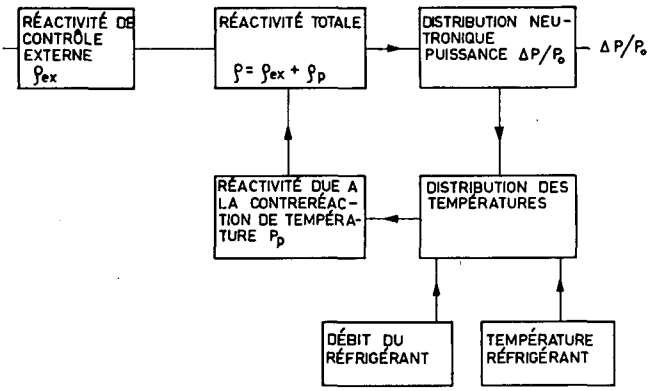


Figure 6
Schéma descriptif pour la stabilité du bloc pile.

Etude de la stabilité

Présentation du problème

Nous nous sommes bornés pour l'instant à l'étude de ce problème dans le cas linéaire. Nous espérons pouvoir utiliser dans un stade ultérieur une condition suffisante de stabilité globale dans le domaine non linéaire, telle qu'elle apparaît par exemple dans la référence [6].

Nous avons utilisé le critère de Nyquist comme outil mathématique. Dans le domaine des fréquences intéressant la stabilité de la pile, on peut raisonnablement penser que les circuits extérieurs au bloc pile n'interviennent pas (constantes de temps et retard très longs). Storrer en a donné une démonstration [7]. Nous avons donc dans une première étude considéré le bloc pile seul avec température d'entrée du sodium constante. L'analyse des phénomènes est donnée dans la figure 6.

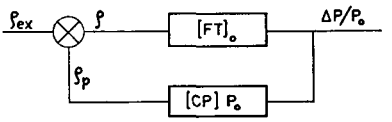


Figure 7
Boucle stabilité bloc pile.

La figure 6 nous montre qu'à une certaine excitation de la réactivité, la pile produit une puissance $\Delta P(t)/P$. Compte tenu du débit du réfrigérant et de sa température, on peut en déduire une certaine distribution de températures. La contre-réaction de réactivité due aux effets de température (effet Doppler, déformation des assemblages, dilatation et changements de densité, etc.), que nous appelerons ϱ_p, s'ajoute à la réactivité ϱ_{ex} du contrôle pour donner ϱ. La boucle est fermée et on la représente souvent selon la figure 7, où:

$$\varrho = \varrho_{ex} + \varrho_p,$$

P_0 est la puissance moyenne de la pile, $[F\ T]$ est la fonction de transfert faisant passer de la réactivité (ϱ) à la variation de puissance $\frac{\Delta P}{P_0}$.

C'est la fonction de transfert à puissance nulle que l'on obtient facilement à partir des équations de la cinétique linéarisées avant l'application de la transformation de Laplace:

soit:
$$\frac{1}{s\left[\Lambda+\sum_m \frac{\beta_m}{\lambda_m+s}\right]} \sim \frac{1}{s\sum_m \frac{\beta_m}{\lambda_m+s}}$$

$[CP]\ P_0$ nous fournit à partir de la variation relative de puissance $\Delta P/P_0$ le coefficient de réactivité de puissance ϱ_p. $[CP]$ est le coefficient de puissance par unité de puissance pour des conditions de fonctionnement bien déterminées de la pile.

On sait [4, 7] que l'étude de la stabilité revient à l'étude de la fonction de transfert de puissance $\Delta P/\varrho_{ex}\ P_0=[F\ T]\ P_0$. D'après la figure 7, $[FT]\ P_0$ s'écrit:

$$[FT]_{P_0}=\frac{[FT]_0}{1+(-[FT]_0\cdot[CP]P_0)}.$$

Le critère de Nyquist appliqué ici au dénominateur $1+(-[FT]_0\cdot[CP]_{P_0})$ ou plus simplement à $-[FT]_0\,[CP]_{P_0}$ nous permet de savoir s'il existe ou non des racines à parties réelles positives, sources d'instabilité pour la fonction $[FT]/_{P_0}$. Si l'on trace dans le plan complexe la courbe Γ représentant $-[FT]_0\cdot[CP]_{P_0}$, sa position par rapport au point -1 nous permet de résoudre ce problème.

APPLICATION DU CRITÈRE DE NYQUIST A LA FONCTION $-[FT]_0\,[CP]_{P_0}$

Nous avons vu dans le paragraphe précédent que:

$$[FT]_0=-\frac{1}{s\sum_m \frac{\beta_m}{\lambda_m+s}}.$$

Le problème est donc de déterminer $[CP]$.

Etude du coefficient de puissance $[CP]$

La puissance dégagée dans la pile conduit à une certaine distribution de températures réagissant sur la réactivité globale par les effets suivants.

Action sur les propriétés nucléaires des éléments. C'est l'effet Doppler, que l'on peut négliger dans les piles à combustible métallique lorsque l'énergie moyenne des neutrons est élevée. Pour RAPSODIE version combustible métallique, nous n'en avons pas tenu compte. Toutefois l'étude sera entreprise pour la version combustible oxydes.

Déformation du combustible sous l'effet du gradient de température. Dans le cas particulier de RAPSODIE, les assemblages sont soumis à deux effets d'origine thermique:

a) La différence de température entre la base du cœur et le niveau des plaquettes produit une dilatation différentielle de la section droite des assemblages. Il se produit une réduction des jeux entre plaquettes; ces jeux de 50 μ en isothermie s'annuleraient complètement pour une différence de température de 130°C environ entre base et sommet du cœur.

b) La différence de température entre faces intérieure et extérieure des assemblages de l'ordre de 20°C maximum pour la rangée périphérique du cœur produit

une déformation vers l'extérieur dans le cas de RAPSODIE. L'étude de la répartition des gradients montre que des arc-boutements ne se produisent pas en régime normal. Les déformations thermiques sont libres. L'accroissement équivalent du rayon est de l'ordre de 50μ entre l'isothermie et le régime nominal, correspondant en première approximation à 60 pcm.

Des calculs plus précis de réactivité introduite à partir de la variation de géométrie vont être poursuivis. La méthode a été exposée par STORRER [7]. La déformation des assemblages conduit à un coefficient négatif. Cependant, le jeu existant entre pied des assemblages et plaque supérieure du sommier et le jeu résiduel aux plaquettes en régime permanent peuvent permettre des variations non négligeables de la position des assemblages, surtout en l'absence de gradient axial, et peut-être occasionner certaines instabilités d'ordre mécanique (friction) ou hydraulique (vibration). L'effet de déformation des assemblages est certainement un effet non linéaire, et nous devrons faire une approximation pour l'étudier dans le cadre exposé ici. Dans l'exemple qui suit, nous n'avons pas tenu compte de cet effet — entre autres parce que la définition exacte du jeu entre assemblages et de leur tenue n'a pas été encore fixée. L'étude plus complète va être entreprise, mais on peut affirmer d'ores et déjà qu'une solution satisfaisante en ce qui concerne la stabilité n'est pas impossible à trouver. De toutes façons, nous pourrons tenir compte dans le programme P_3 décrit ci-après de l'effet de déformation des assemblages.

Action due à la dilatation des structures du sodium, du combustible et également à la variation de densité de ces matériaux. Des calculs neutroniques fournissent aisément les valeurs des coefficients isothermes de réactivité dus à la dilatation et au changement de densité. Nous avons supposé que les structures et le sodium étaient à la même température U, le combustible étant à la température T (voir les équations thermiques du chapitre précédent).

Si l'on veut obtenir une précision convenable, compte tenu du refroidissement envisagé pour RAPSODIE, ont est conduit à envisager sept canaux dans la pile, correspondant aux sept zones de refroidissement par le sodium. Si l'on veut suivre de près l'évolution des températures axialement, on voit que l'on est conduit à de très lourds calculs qui ne peuvent être assurés sans le concours de machines digitales.

Si l'on applique la transformation de Laplace aux équations thermiques du bloc pile, on obtient:

$$\begin{aligned} A_{ij}\,\Delta U\,s + A_{ij}\,v_j \frac{\partial U}{\partial z} &= H_{ij}\,(\Delta T - \Delta U) \\ C_{ij}\,\Delta T\,s + D_{ij}\,\Delta U\,s &= H_{ij}\,(\Delta U - \Delta T) + \lambda_{ij}\,(z)\,\Delta q\,(s) \end{aligned} \qquad (1)$$

$A_{ij}\,v_j$ étant le terme $B_{ij}\,m_u$ défini précédemment.

Si l'on pose:

$$\frac{A}{H} = \alpha;\ \frac{c}{H} = \gamma;\ \frac{D}{H} = \delta$$

et ensuite

$$B = \frac{s\,[\gamma\,(\alpha s + 1) + \delta + \alpha]}{\alpha v\,(1 + s\gamma)}$$

la résolution du système (1) conduit à:

$$\Delta U(Z,s)=\Delta q(s)\,e^{-Bz}\int_{o}^{z}\frac{\lambda(z')\,e^{Bz'}}{\alpha v H(1+s\gamma)}\,dz'$$

$$\Delta T(z,s)=\frac{\Delta q(s)}{1+s\gamma}\left[(1-s\delta)\left\{e^{-Bz}\int_{o}^{z}\frac{\lambda(z')\,e^{Bz'}}{\alpha v H(1+s\gamma)}\,dz'\right\}+\frac{\lambda(z)}{H}\right]. \tag{2}$$

Les fonctions de transfert $(\Delta U/\Delta q)(z,s)$ et $(\Delta T/\Delta q)(z,s)$, que l'on peut tirer du système (2), nous fournissent donc les variations de températures du sodium et du combustible par unité de puissance. Si l'on a déterminé les coefficients isothermes par unité de longueur dus au sodium et aux structures, soit $\varrho_u(z)$ et $\varrho_T(z)$, dans chaque canal, on pourra alors écrire:

$$[CP]=\sum_j\left[\int_o^{z_o}\left[\varrho_u(z)\frac{\Delta U(z,s)}{\Delta q}+\varrho_T\frac{\Delta T}{\Delta q}(z,s)\right]dz\right]_j$$

j étant l'indice de canal.

Cette dernière relation a été programmée sur IBM-7090 en langage FORTRAN (programme P_3). Pratiquement, ϱ_u et ϱ_T ont été calculés dans chaque canal pour un certain nombre de zones comme le montre la figure 8.

Nous rencontrons des difficultés en cours de test du programme P_3 en ce qui concerne la précision des résultats, le découpage axial pour l'intégration le long de l'axe 0_z étant en cause. Nous avons donc généralisé l'introduction des paramètres variables $\lambda(z)$ sous forme de fonctions analytiques, et nous programmons actuellement une nouvelle version du programme P_3 où les intégrations ont été faites. Ce programme P_3 nous fournira la phase et l'amplitude des fonctions suivantes:

$$[CP](i\omega)-[FT]_0(i\omega)-(-[FT]_0(i\omega)\,[CP)(i\omega)\,P_0)-[FT]_{P_0}(i\omega).$$

Nota bene

Compte tenu de la remarque faite au début de ce chapitre, il peut être utile, dans les faibles fréquences, de connaître la réaction de la pile dans son ensemble, en particulier lorsque la température d'entrée du réfrigérant varie.

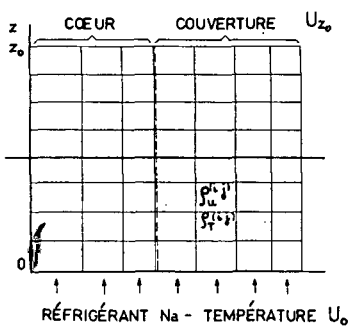


Figure 8
Bloc pile RAPSODIE pour étude de stabilité.

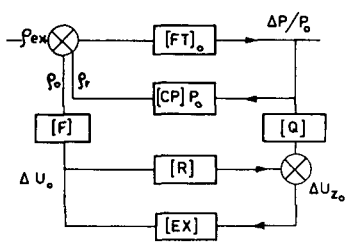


Figure 9
Boucle stabilité ensemble pile.

Lorsque l'on fait varier seule la température d'entrée du sodium, on est conduit à une expression du même type que celle étudiée précédemment. Les conclusions à tirer restent donc les mêmes [7].

Si l'on considère la pile dans son ensemble, on est conduit à un schéma du type de la figure 9, où Q est la fonction de transfert entre la puissance $(\Delta P/P_0)$ (s) et la température de sortie du sodium $\Delta U\ (Z_0, s)$. Or celle-ci par $[EX]$, fonction de transfert des circuits extérieurs, donne $\Delta U\ (O, s)$, qui réagit sur ΔU (Z_0, s) (fonction de contre-réaction $[R]$). $\Delta U\ (O, s)$, par l'intermédiaire de la fonction $[F]$, fournit alors le coefficient de réactivité k_0, dû au circuit extérieur.

On est donc conduit, tous calculs faits, à l'étude de la fonction:

$$[FT]_{P_0} = \frac{[FT]_0}{1 + \left\{ - \frac{[F]\,Q\,[EX]}{1-[R][EX]} - [CP]P_0 \right\} [FT]_0}$$

Nous espérons trouver une expression approchée de $[FT]_{P_0}$ pour cette étude ultérieure, tout en remarquant que les programmes dynamiques directs P_1 et P_2 peuvent être utilement employés ici.

REMERCIEMENTS

Nous tenons à remercier le Service de calcul arithmétique, qui a assuré la programmation des programmes P_1, P_1' P_2, P_2' et P_3.

RÉFÉRENCES

[1] BUMP, T. R. et SEIDENSTIKER, R. W., *Nucl. Sci. Engng* 4 (1958) 44-64.
[2] STORRER, F., APDA-132 (1959).
[3] SANDMEIER, H. A., ANL-6014.
[4] BETHE, H. A., APDA-117 (1956).
[5] CAMPAN, J. L. *et al.*, Vol. III des présents Comptes-rendus, p. 355.
[6] GILLES et WEGRZYN, *Automatisme* 6 (1961) 3.
[7] STORRER, F., Communication privée.
[8] Rapport CEA (*à paraître*).

STABILITY ANALYSIS OF EBR-II*

H. H. HUMMEL AND L. T. BRYANT
ARGONNE NATIONAL LABORATORY, ARGONNE, ILL.,
UNITED STATES OF AMERICA

Abstract — Résumé — Аннотация — Resumen

Stability analysis of EBR-II. Transfer function calculations have been carried out for the EBR-II by means of reactivity coefficients derived from distributed worths of fuel, sodium, and steel measured on the ZPR-III, taking into account temperature variation in the core. Temperature calculations are performed both on an IBM-704 using the method of Storrer and on an analogue computer. In the former case a more detailed description of the reactor is possible, but only linear problems can be considered.

Time constants based on the steady-state temperature distribution in a two-region pin with fuel and clad regions give, when used in a radially lumped pin model, axial temperature distributions which agree reasonably well with the exact calculations up to a frequency of 5 rad/s. For the EBR-II pin the effect of the clad on the time constants is significant.

At present the only feedback mechanism for which calculations have been made is a linear one corresponding to unrestrained thermal expansion. Because of the short time lags involved, no instability in the reactor seems possible. Bowing and irregular radial expansion of the core are some possible sources of non-linearity in the feedback which are being studied.

Analyse de la stabilité du réacteur EBR-II. La fonction de transfert pour le réacteur EBR-II a été calculée en utilisant des coefficients de réactivité dérivés des valeurs de combustible, de sodium et d'acier mesurées au moyen du ZPR-III, compte tenu des variations de la température dans le cœur. Les températures sont calculées avec un IBM-704, par la méthode de Storrer, et avec une calculatrice analogique. Dans le premier cas, on peut donner une description plus détaillée du réacteur, mais la calculatrice ne peut résoudre que les problèmes linéaires.

Les constantes de temps, basées sur la distribution stationnaire de la température dans une aiguille à deux zones, l'une étant le combustible et l'autre la gaine, lorsque ces constantes sont utilisées dans un modèle à aiguilles disposées radialement, donnent des distributions axiales de la température qui correspondent assez bien aux calculs exacts jusqu'à une fréquence de 5 rad/s. Pour l'aiguille du réacteur EBR-II, l'effet du gainage sur les constantes de temps est prononcé.

A l'heure actuelle, on n'a effectué des calculs que pour un mécanisme à rétroaction linéaire qui correspond à une dilatation thermique libre. Il semble que le réacteur ne soit pas capable d'instabilité en raison de la brièveté des retards. Il est possible que la non-linéarité de la rétroaction soit due à la déformation et à la dilatation radiale irrégulière du cœur; une étude sur ce point est en cours.

Анализ устойчивости реактора EBR-II. Были проведены расчеты передаточной функции для реактора EBR-II, для чего были взяты коэффициенты реактивности, полученные из распределенных реактивных способностей топлива, натрия и стали, измеренных на реакторе ZPR-III с учетом вариации температуры в активной зоне. Подсчеты температуры производятся как с помощью кода счетно-решающего устройства IBM-704 методом Сторрера, так и на моделирующем устройстве. В первом случае возможно более детальное описание реактора, но можно рассматривать только линейные проблемы.

* Work done under the auspices of the United States Atomic Energy Commission.

Когда постоянные времени, основанные на распределении температуры установившегося процесса в тонком двухзонном стержне, состоящем из зоны топлива и зоны оболочки, используются для стержня, находящегося в радиальном блоке, получаются распределения аксиальной температуры, которые вполне соответствуют точным расчетам вплоть до частоты 5 радиан/сек. Если взять тонкий стержень реактора EBR-II, то действие оболочки на постоянные времени является значительным.

В настоящее время единственным механизмом с обратной связью, для которого сделаны расчеты, является линейный механизм, соответствующий неограниченному тепловому расширению. Ввиду коротких запаздываний кажется невозможной неустойчивость в реакторе. „Изгибание" и неравномерное радиальное расширение активной зоны являются возможными источниками нелинейности в обратной связи, которые сейчас изучаются.

Análisis de la estabilidad del reactor EBR-II. Los autores han calculado la función de transporte para el reactor EBR-II, utilizando coeficientes de reactividad derivados de los valores del combustible, del sodio y del acero, medidos en el reactor ZPR-III, para lo que han tenido en cuenta la variación de la temperatura en el cuerpo del reactor. Las temperaturas se han calculado con una máquina IBM-704, por el método de Storrer, y con una calculadora analógica. En el primer caso es posible lograr una descripción más detallada del reactor, pero la calculadora sólo puede resolver problemas lineales.

Cuando las constantes de tiempo basadas en la distribución de temperaturas en régimen estacionario en una aguja de dos zonas (zona de combustible y zona de revestimiento) se aplican a un modelo de agujas dispuestas radialmente, se obtienen distribuciones axiales de la temperatura que concuerdan razonablemente con los valores exactos calculados hasta una frecuencia de 5 rad/s. En el caso de las agujas del EBR-II, el efecto del revestimiento en las constantes de tiempo es acusado.

Hasta ahora el único proceso de realimentación que se ha calculado es el proceso lineal correspondiente a una dilatación térmica libre. Dada la brevedad de los retardos que se producen, es improbable que el reactor sea inestable. Se están estudiando dos causas posibles de falta de proporcionalidad en la realimentación: la deformación del cuerpo y su dilatación radial irregular.

Introduction

Calculations have been made for predicting the behaviour of EBR-II during oscillator measurements to be made during start-up of the reactor. Because of assumptions made in the calculations, which are believed to be justified by the design of the reactor, no instability in its operation appears possible.

The design of EBR-II has been discussed in detail elsewhere [1]. Two vital factors affecting the safety of a fast reactor, which are both dependent on its specific design, are bowing of the fuel elements and the presence of a large delayed negative reactivity coefficient [2]. One would expect such a delayed coefficient to be caused by expansion of an upper supporting structure resulting in outward movement of the fuel. Because of the method of bottom support of the EBR-II sub-assemblies [1] it is assumed that no such effect takes place. The question of bowing in the EBR-II has been discussed in Ref. [3]. This matter is currently under reconsideration, but no bowing effects have yet been incorporated into the feedback model. It is not believed that these effects will prove to be significantly large. There is some question about how large the radial clearances between sub-assemblies will actually be. It is believed that under operating conditions probably no clearances will exist and that the core will expand radially according to the local sub-assembly wall temperature, assumed to be the same as the local coolant

temperature. The calculations have been performed with and without the radial expansion feed-backs to illustrate the magnitude of the effect involved.

There are adequate experimental and theoretical results to indicate that the Doppler effect will be insignificant in EBR-II, and it has therefore been ignored. The sodium void coefficient, which was found to be positive in certain large reactors, is strongly negative in EBR-II.

Because of the large sodium inventory in the primary coolant tank, the temperature of the sodium entering the reactor has been assumed constant.

Because of the above assumptions, no prompt-positive or delayed-negative reactivity coefficients are present in the feedback model, and the predicted behaviour is therefore quite stable. The assumed feedback is a prompt-negative one owing to the unrestrained thermal expansion of fuel and steel and coolant expansion. No non-linearities are present in the feedback model used so far. Limited bowing would be one possible source of non-linearity. Another possible source of non-linearities is phase transformation in the fuel. It is believed, however, that this will be too sluggish for fissium (uranium containing actual or simulated fission products) to affect the results of oscillator experiments.

1. Temperature calculations

Oscillating temperatures in the EBR-II fuel-pins are calculated both with an IBM-704 code prepared for LONG [4] using the exact integration technique of STORRER [5], and with an analogue computer. These calculations are for two-region pins, the inner region being the metallic fuel and the outer homogenized sodium bond and steel clad. Although the digital computer technique is more accurate, its application is limited to linear problems in which time-dependence can be separated out. The use of the analogue is desirable because it permits study of the effect of non-linear feedbacks (although none has so far been assumed in the EBR-II) and of non-linearity in the kinetics equation.

Because of the limited capacity of the analogue a simplified temperature calculation must be made when it is used. It has been found that in the EBR-II core acceptable accuracy in temperature calculation is obtained for frequencies up to 5 rad/s with the use of steady-state radial temperature distributions. That is, the oscillating component of the temperature in a fuel-pin at a given axial position is equal to the oscillating coolant temperature at that point plus a source-produced term which has the same ratio to the oscillating source as it would have in the steady state. The solution in this approximation for the oscillating temperature of the coolant $T_c(z)$ as a function of axial variable z for a heat source $a(z)$ per unit length of pin has been given by Storrer [5] as

$$T_c(z) = T_c(0)\, e^{-\frac{\lambda z}{v}} + \frac{e^{-\frac{\lambda z}{v}} \int_0^z a(z')\, e^{-\frac{\lambda z'}{v}}\, dz'}{\lambda h \tau_c (1 + i\omega\tau_f)} \qquad (1)$$

where:

h is an overall heat-transfer coefficient defined so that $h(\overline{T}_1 - T_c)$ is the total heat transferred per unit length of pin, $\overline{T}_1$ being the "lumped" fuel temperature,

v is the coolant velocity,

ω is the frequency, rad/s,

$\tau_c = C_c/h$, where C_c is the heat capacity of coolant per unit length,

τ_f is a fuel pin time constant, which represents the ratio of heat stored in the pin per unit temperature rise of the fuel to heat lost per unit temperature difference between fuel temperature and coolant temperature. This is related to the ratio of the average fuel temperature (relative to T_c) to the temperature at the outside of the pin (relative to T_c,) and this ratio is contained in the constant h, which can be defined as

$$h = 2\pi R h_F \frac{T(R) - T_c}{\bar{T}_1 - T_c} \tag{2}$$

where R is the outer radius of the pin and h_F is the coolant film heat-transfer coefficient.

Storrer's development was for an unclad fuel-pin, and for this case

$$\frac{1}{h} = \frac{1}{8\pi k} + \frac{1}{2\pi R h_F} \tag{3}$$

where k is the thermal conductivity of the pin. τ_f is given by C_1/h, where C_1 is the heat capacity of the pin per unit length. λ is then given by

$$\lambda = \frac{\omega}{1+\omega^2 \tau_f^2}\left[\frac{\tau_f^2}{\tau_c}\omega + i\left(1 + \frac{\tau_f}{\tau_c} + \tau_f^2 \omega^2\right)\right]. \tag{4}$$

It is desirable to treat the EBR-II pin as having two radial regions, a fuel region 1, and a homogenized bond and clad region 2. In this case Eq. (1) still applies, but it is necessary to modify the definition of λ to the following:

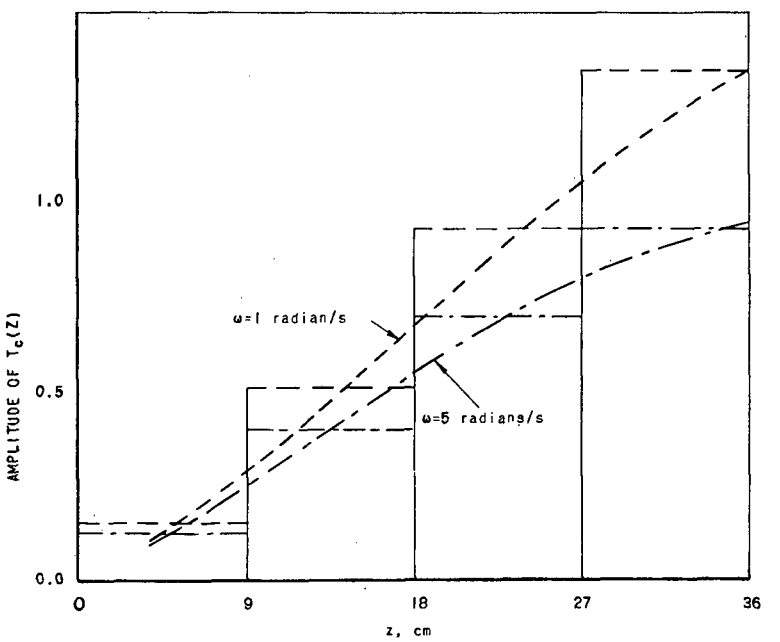


Fig. 1
Amplitude of T_c (z) in EBR-II core in degrees C for oscillations of 1% of full power multiplied by 270/537. Coolant velocity: 270 cm/s.

$$\lambda = \frac{\omega}{1+\omega^2 \tau_f^2}\left[\frac{\omega \tau_f \tau'_f}{\tau_c} + i\left(1 + \frac{\tau'_f}{\tau_c} + \omega^2 \tau_f^2\right)\right] \tag{5}$$

in which

$$\tau_f = \frac{C_1 + C_2[(\bar{T}_2 - T_c)(\bar{T}_1 - T_c)]}{h} \tag{6}$$

$$\tau'_f = \frac{C_1 + C_2}{h} \tag{7}$$

and h is found from Eq. (2) by using the two-region steady-state temperature solution with a constant heat source in region 1, rather than by using Eq. (3.)

The phase-shifts in EBR-II core temperatures relative to the power are very nearly linear in ω up to 2—3 rad/s. This is to be expected when typical values of constants for EBR-II are examined for a coolant velocity of 537 cm/s, the average full flow value. In this case

$$\tau_f = 0.104 \text{ s}$$
$$\tau'_f = 0.136 \text{ s}$$
$$\tau_c = 0.0410 \text{ s}$$
$$\tau'_f/\tau_c = 3.32$$

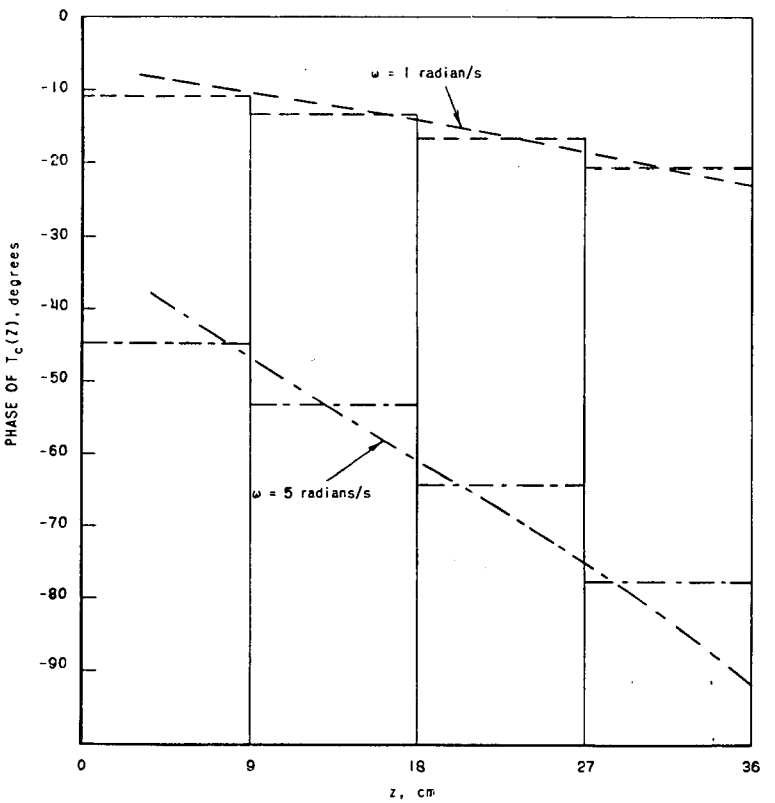


Fig. 2
Phase of $T_c(z)$ in EBR-II core relative to power. Coolant velocity: 270 cm/s.

$$\frac{z}{v} = \frac{36.12}{537} = 0.0672 \text{ s (for the full height of the core).}$$

For low frequency, λ is approximately

$$\lambda = i\omega\left[1 + \frac{\tau'_f}{\tau_c}\right] \tag{8}$$

the terms in ω^2 being small. The coolant transport lag in the core is then of the order of $[1+(\tau'_f/\tau_c)]\ z/v$ which is 0.29 s. A complete discussion of the validity of lumped parameter models and of low-frequency approximations for a bare fuel-pin is given in Refs. [5] and [6].

The application of the bare-pin approximation using (3) for h and a definition of τ according to (7) leads to a value of τ_f of 0.087 s. This would not make λ significantly different from the value given by (5) at low frequency since h divides out of the term linear in ω. An error of 20% would be produced in the part of the phase-shift independent of coolant velocity because of the $i\omega\tau/$ term in (1). It has been found that (1) with the use of (5), (6) and (7) gives phase-shifts very accurately in the low-frequency range.

Solutions for coolant temperature in the core have been obtained on the analogue computer, using four axial segments in the core, of the equation of which (1) is the continuous solution. This is compared with the rigorous solution from the IBM-704 in Figs. 1—4. While agreement is not perfect, the analogue computation is seen to be a reasonable approximation.

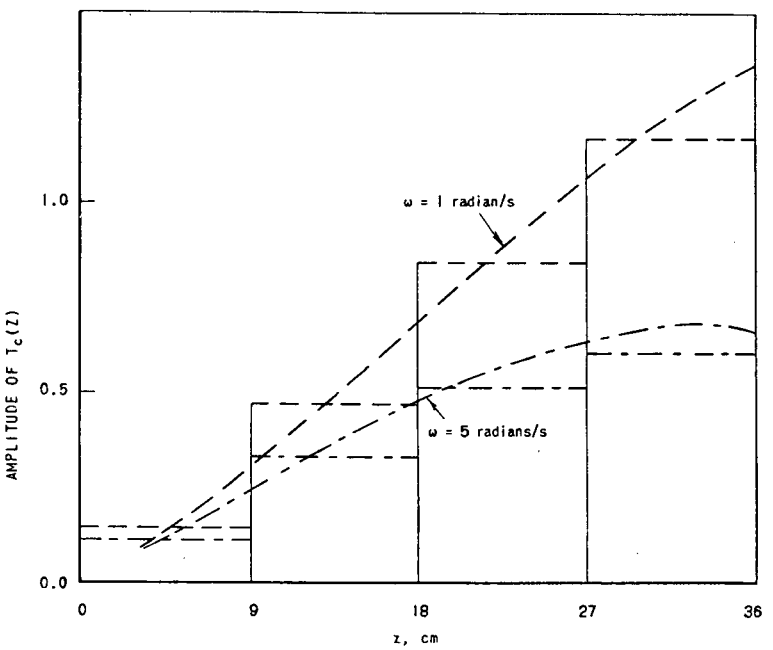


Fig. 3
Amplitude of T_c (z) in EBR-II core in degrees C for oscillations of 1% of full power. Coolant velocity: 537 cm/s.

The problem of calculating large temperature phase and amplitude changes which can occur between the reactor core and an upper supporting structure can be an important one for reactor stability. It was found to be so in studies of EBR-I, Mark II [7]. The radially lumped parameter model is probably of limited usefulness in attacking this problem. In this case, since there is no source, the steady-state temperature distribution is a constant, so that $\bar{T}_1 = \bar{T}_2 = T(R)$.

The term $1/8\,\pi k$ in Eq. (3) would not apply in the case of a one-region pin with no heat source. For fast reactors the frequency range of greatest interest for stability is below $\sim$1 rad/s. In this frequency range, if heat transfer is from the coolant to relatively thin pieces of metal, amplitude attenuation can probably be neglected and the phase calculated from

$$\lambda = \left[1 + \frac{C_M}{C_c}\right] i\omega \tag{9}$$

where C_M is the heat capacity of the metal per unit length. This is equivalent to assuming that metal and coolant at a given z are instantaneously at the same temperature. This assumption was made in this paper in dealing with the coolant-header gap regions immediately above and below the EBR-II core. If heat transfer is to thick pieces of metal or to material of low thermal conductivity, the

TABLE I

EBR-II REACTIVITY COEFFICIENTS FOR ISOTHERMAL EXPANSION*

$-(\delta k/\delta \tau) \times 10^5$/°C

Region	Axial expansion		Coolant		Radial structural expansion	
	Fuel	Steel	Expansion	Displacement ** by steel expansion	Fuel	Steel
Core—Δz, cm (from bottom)						
0 — 4.52	0.017	0.008	0.135	0.017	0.100	0.019
4.52— 9.03	0.060	0.006	0.115	0.015	0.142	0.013
9.03—13.55	0.086	0.004	0.108	0.014	0.157	0.009
13.55—18.06	0.100	0.003	0.101	0.013	0.174	0.006
18.06—22.58	0.098	0.003	0.101	0.013	0.168	0.006
22.58—27.09	0.080	0.004	0.108	0.014	0.141	0.009
27.09—31.61	0.059	0.006	0.115	0.015	0.125	0.013
31.61—36.12	0.027	0.008	0.135	0.017	0.087	0.019
Core total	0.527	0.042	0.918	0.118	1.094	0.094
Gap, unlagged	—	0.008	0.157	—	—	0.054
Gap, lagged***	—	0.005	0.115	—	—	—
Upper blanket	0.016	0.004	0.103	0.013	0.032	0.008
Radial blanket	0.104	0.051	0.128	0.066	0.216	0.102

* Assumed coefficients of expansion: sodium volume 2.9×10^{-5}/°C
steel linear 1.9×10^{-5}/°C
fuel linear 1.8×10^{-5}/°C
All expansions except axial fuel assumed proportional to local coolant temperature.

** This effect is cancelled if radial structure expansion occurs.

*** Lag is 0.24 s at full flow of 537 cm/s and is inversely proportional to coolant velocity.

lumped parameter model probably cannot be used at all, as temperature phase-shifts in the solid material and amplitude attenuation will be large even at low frequency. In this case, the exact solution of Storrer [5] must be employed.

2. Reactivity coefficients, open and closed loop transfer functions

Transfer-function calculations have been carried out so far only on the analogue. Core temperatures have been calculated for a single representative pin, while coolant gaps and blankets have been lumped both radially and axially. The reactivity coefficients of expansion of fuel, steel and sodium were obtained from ZPR-III material-replacement measurements on an EBR-II mock-up [8]. These reactivity coefficients are given in Table I for a core divided into eight axial slices, which were lumped into four slices for the feedback calculations. The lack of symmetry of fuel worth about the core mid-plane found in the measurements is presumably due to a similar dissymmetry in EBR-II geometry. The coefficients are divided into those which are based on no radial expansion and those which are associated with unrestrained radial expansion. Lumped coefficients are given for

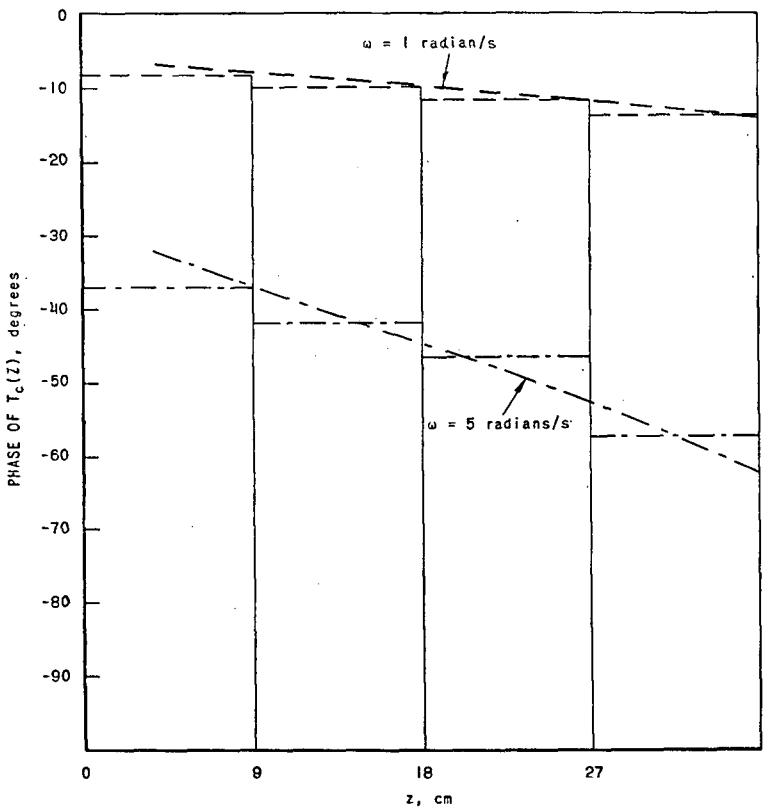


Fig. 4
Phase of T_c (z) in EBR-II core relative to power. Coolant velocity: 537 cm/s.

TABLE II

PHASE LAG IN DEGREES OF OPEN LOOP TRANSFER FUNCTIONS OF EBR-II RELATIVE TO POWER

Feedback, phase	$v = 270$ cm/s								$v = 537$ cm/s							
	Radial expansion				No radial expansion				Radial expansion				No radial expansion			
ω (rad/s)	0.1	1.0	2.5	5.0	0.1	1.0	2.5	5.0	0.1	1.0	5.0	2.5	0.1	1.0	2.5	5.0
Na in core	1.8	17.3	40.1	65.4	1.9	17.7	40.7	66.2	1.2	12.0	28.7	50.4	1.1	11.9	29.0	51.2
U in core	1.4	17.4	39.8	63.0	1.5	17.6	40.0	62.6	1.3	12.2	29.1	50.5	1.1	12.3	29.3	39.9
Total core	1.8	17.4	40.1	64.7	1.8	17.5	40.3	64.8	1.2	12.2	28.9	50.4	1.3	12.3	29.1	50.6
Gap (unlagged)	2.3	22.7	52.6	84.8	2.4	22.9	52.4	84.3	1.0	15.1	35.1	61.6	1.3	13.6	35.3	61.4
Gap (lagged) ..	5.5	48.9	120.7	213.5	3.4	49.3	121.6	224.1	3.8	27.9	69.6	135.0	1.8	30.4	69.8	—
Upper blanket.	3.8	58.7	131.5	230.4	6.1	60.8	133.3	235.1	3.8	36.7	82.2	141.2	4.3	37.9	79.1	139.1
Radial blanket	47.3	—	—	—	46.4	—	—	—	28.8	—	—	—	28.4	—	—	—
Total.....	4.1	22.9	48.9	69.9	3.4	24.4	51.5	70.1	2.4	13.9	34.9	58.5	2.5	15.8	36.3	60.5

TABLE III

AMPLITUDE OF OPEN LOOP TRANSFER FUNCTIONS OF EBR-II AT FULL POWER

Feedback $(\delta k/\beta)$ $(\delta n/n_0)$	$v = 270$ cm/s								$v = 537$ cm/s							
	Radial expansion				No radial expansion				Radial expansion				No radial expansion			
ω (rad/s)	0.1	1.0	2.5	5.0	0.1	1.0	2.5	5.0	0.1	1.0	2.5	5.0	0.1	1.0	2.5	5.0
Na in core	0.391	0.377	0.322	0.211	0.200	0.192	0.164	0.112	0.219	0.214	0.197	0.156	0.111	0.110	0.101	0.080
U in core	0.164	0.158	0.132	0.087	0.164	0.159	0.132	0.087	0.117	0.115	0.104	0.078	0.115	0.114	0.104	0.079
Total core	0.555	0.534	0.455	0.308	0.362	0.350	0.297	0.199	0.336	0.330	0.300	0.236	0.229	0.224	0.205	0.159
Upper gap (unlagged) ..	0.080	0.077	0.062	0.037	0.060	0.057	0.047	0.028	0.048	0.047	0.042	0.031	0.036	0.035	0.031	0.023
Upper gap (lagged)* ...	0.042	0.039	0.032	0.018	0.042	0.041	0.032	0.018	0.024	0.024	0.021	0.015	0.024	0.024	0.021	—
Upper blanket.	0.059	0.050	0.035	0.017	0.049	0.043	0.029	0.015	0.035	0.031	0.026	0.016	0.029	0.025	0.021	0.014
Radial blanket	0.033	—	—	—	0.013	—	—	—	0.021	—	—	—	0.009	—	—	—
Total.....	0.760	0.689	0.530	0.312	0.523	0.478	0.354	0.195	0.460	0.430	0.378	0.274	0.325	0.308	0.267	0.184

* Lag is 0.48 s at 270 cm/s, 0.24 s at 537 cm/s coolant velocity.

the upper coolant gap, upper blanket and radial blanket. The unlagged and lagged components of the gap feedback and the lag time were obtained from use of the temperature variation through the gap obtained as described above.

Open loop transfer functions relative to the power are given in Table II. Feedback for the radial blanket was found to be negligible above $\omega = 0.1$. Large axial phase shifts are expected to occur in this case because of the low coolant flow rate. A more refined treatment is clearly desirable here.

The 537-cm/s case corresponds to average full flow in the core. Amplitudes for both velocities are given as $(\delta k_{FB}/\beta)/(\delta n/n_0)$, where $\delta k_{FB}/\beta$ is the amplitude in dollars of the oscillating reactivity feedback for oscillation at full power, while $\delta n/n_0$ is the fractional change in power. In the calculation of the closed loop transfer function at reduced power, which would be necessary at reduced flow, the amplitude in Table II is to be multiplied by the fraction of full power at which the reactor operates (Table III).

In Figs. 5 and 6, the EBR-II closed loop transfer-function phase and amplitude are given for the various assumptions made in the feedback. The amplitude in this case is $(\delta n/n_0)/(\delta k_{APP}/\beta)$, where $\delta k_{APP}/\beta$ is the applied oscillating reactivity in dollars. The phase is that of the power relative to the applied reactivity. As expected, there is no indication of the formation of a resonance.

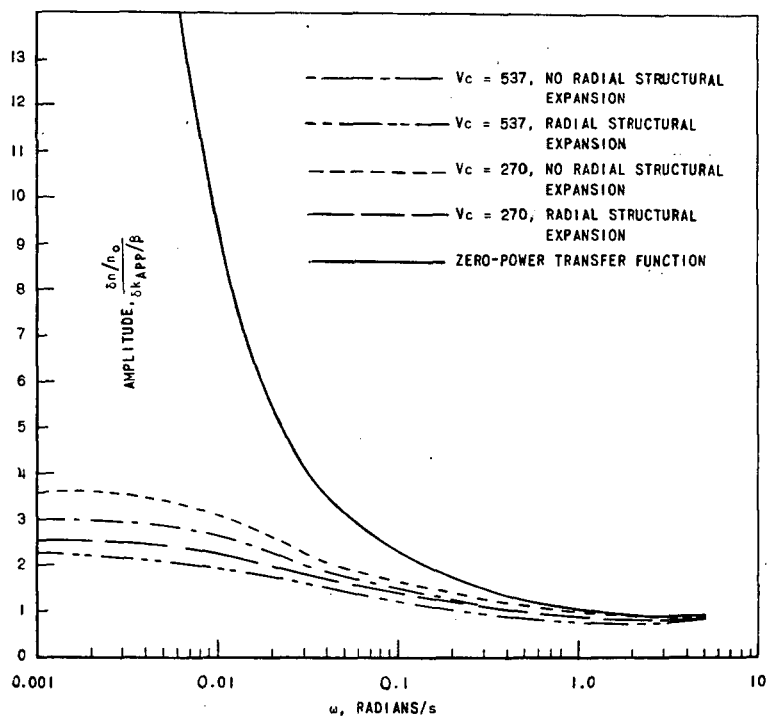


Fig. 5
Amplitude of closed-loop transfer-function of EBR-II. Curves for $v = 537$ cm/s for full power. Curves for $v = 270$ cm/s for full power multiplied by 270/537.

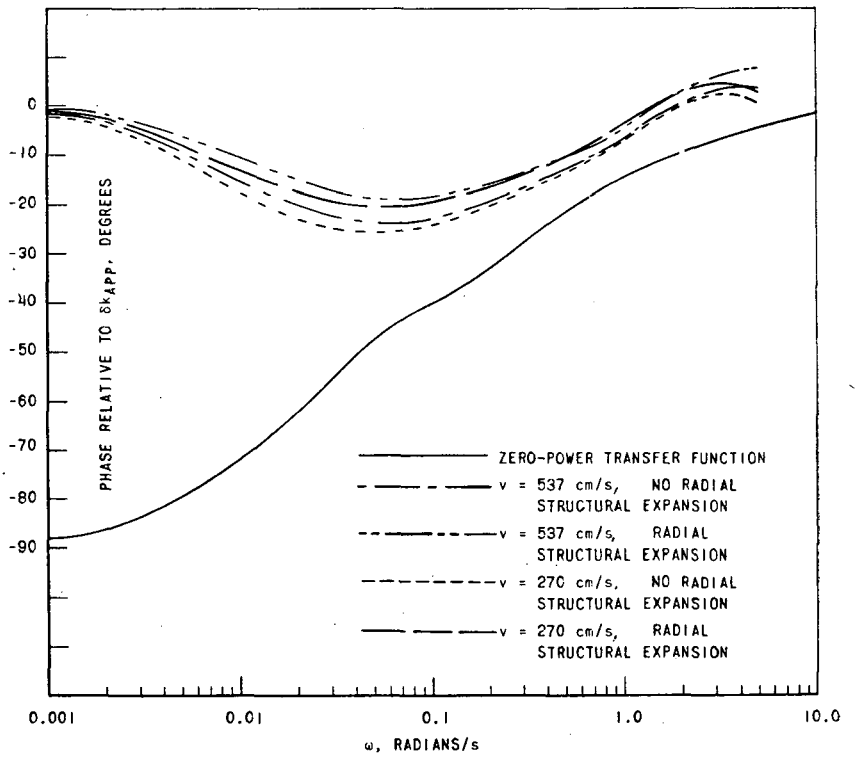


Fig. 6
Phase of closed-loop transfer-function of EBR-II.

Further work planned on EBR-II dynamics includes the performing of more detailed calculations for linear feedback on the IBM-704. A study of the possible effect of sub-assembly bowing is also planned.

ACKNOWLEDGEMENTS

Assistance with the computational work was given by Mildred Schlapkol, Stella Dean and K. E. Phillips. Helpful discussions were held with J. C. Carter and W. B. Loewenstein.

REFERENCES

[1] KOCH, L. J. *et al.*, Proc. 2nd UN Int. Conf. PUAE 9 (1958) 323.
[2] BETHE, H. A., APDA-117 (1956).
[3] KOCH, L. J. *et al.*, ANL-5719 (1957).
[4] LONG, J. K. and HAROLDSEN, R., "Analysis of EBR-I power temperature relationships", paper presented at Washington meeting of American Nuclear Society, November 1959.
[5] STORRER, F., APDA-132 (1959).
[6] STORRER, F., Proc. Conf. Transfer Function Measurement and Reactor Stability Analysis, ANL-6205 (1960) 251.
[7] SMITH, R. R. *et al.*, *These proceedings*, Vol. **III**, p. 43.
[8] KEENEY, W. P. and LONG, J. K., ANL-6301 (1960) 45.

IV. 3. DOPPLER EFFECT

THE DOPPLER EFFECT IN A LARGE FAST OXIDE REACTOR ITS CALCULATION AND SIGNIFICANCE FOR REACTOR SAFETY*

P. GREEBLER AND B. A. HUTCHINS
GENERAL ELECTRIC COMPANY, SAN JOSE, CALIF.,
UNITED STATES OF AMERICA

Abstract — Résumé — Аннотация — Resumen

The Doppler effect in a large fast oxide reactor—Its calculation and significance for reactor safety. The negative Doppler temperature coefficient of a large PuO_2—UO_2-fuelled fast reactor provides an important safety advantage. It is shown that, with appropriate selection of core size and composition, the prompt Doppler coefficient completely overrides delayed reactivity effects due to thermal expansion of sodium and other core materials. The significance of the Doppler coefficient for reactor safety is indicated by its effectiveness in limiting the fuel temperature rise during prompt-critical power transients initiated by accidental expulsion of control rods or of sodium coolant. Selection of design parameters and operating conditions (such as refuelling schedule) for reactor safety and stability is discussed on the basis of these calculated power transients.

The strong Doppler effect in a large fast oxide reactor results from the degradation of the neutron spectrum and the ability to use a low Pu^{239} to U^{238} ratio. Techniques used to calculate the negative contribution to the Doppler coefficient by U^{238} and Pu^{240} and the positive contribution by Pu^{239} are described, and multi-group cross-sections for these isotopes are given as functions of fuel temperature and of isotopic concentrations. The calculational techniques include the effects on the Doppler power coefficient of the spatial variation of neutron flux and importance, and of power and fuel temperature in an operating reactor.

L'effet Doppler dans un grand réacteur à oxyde à neutrons rapides — Calcul et importance pour la sécurité du réacteur. Dans un grand réacteur à neutrons rapides alimenté au PuO_2—UO_2, le coefficient thermique négatif de Doppler présente un grand intérêt du point de vue de la sécurité. On montre que si les dimensions et la composition du cœur sont choisies de manière appropriée, le coefficient d'effet Doppler instantané annule complètement la réactivité retardée due à la diltation thermique du sodium et d'autres matériaux du cœur. L'importance de l'effet Doppler pour la sécurité du réacteur est mise en évidence par l'efficacité avec laquelle il limite l'accroissement de la température du combustible lors de régimes transitoires de puissance, immédiatement critiques, provoqués par le retrait accidentel des barres de contrôle ou du fluide de refroidissement au sodium. Sur la base du calcul de ces régimes transitoires de puissance, les auteurs étudient le choix des paramètres de construction et des conditions d'exploitation (plan de rechargement, par exemple) pour assurer la sécurité et la stabilité du réacteur.

Dans un grand réacteur à oxyde, à neutrons rapides, l'importance de l'effet Doppler résulte de la dégradation du spectre de neutrons et de la possibilité d'utiliser un

* Work performed by the General Electric Co., San Jose, California, under the sponsorship of the United States Atomic Energy Commission.

faible rapport $^{239}Pu/^{238}U$. Les auteurs décrivent les méthodes utilisées pour calculer la contribution négative de ^{239}U et ^{240}Pu et la contribution positive de ^{239}Pu au coefficient Doppler, et donnent les sections efficaces de ces isotopes en fonction de la température du combustible et des concentrations isotopiques. Les calculs portent notamment sur les effets, dans un réacteur en fonctionnement, de la variation spatiale du flux de neutrons, de la puissance et de la température du combustible sur le coefficient de puissance de Doppler.

Эффект Допплера в большом оксидном реакторе на быстрых нейтронах — его расчет и значение для безопасности реактора. Отрицательный температурный коэффициент Допплера большого реактора на быстрых нейтронах, работающего на топливе PuO_2-UO_2, дает важные преимущества в отношении безопасности. Можно видеть, что при соответствующем подборе размера и состава активной зоны мгновенный коэффициент Допплера полностью превышает эффект запаздывающей реактивности благодаря температурному расширению натрия и других веществ активной зоны. Значение коэффициента Допплера для безопасности реактора подтверждается его эффективностью в ограничении повышения топливной температуры во время мгновенно-критических переходных изменений энергии, вызванных аварийным выбросом регулирующих стержней или натриевого охладителя. Обсуждается вопрос о выборе заданных параметров и эксплуатационных условий (таких, как график перегрузки) в целях безопасности и стабильности реактора на основе этих вычисленных изменений энергии в неустановившихся режимах.

Сильный эффект Допплера в большом оксидном реакторе на быстрых нейтронах является результатом смягчения нейтронного спектра и способности использовать низкое соотношение Pu^{239} к U^{238}. Описываются методы, применяемые при подсчете отрицательного действия на коэффициент Допплера U^{238} и Pu^{240} и положительного действия Pu^{239}, и многогрупповое поперечное сечение для этих изотопов дается как функция температуры топлива и изотопных концентраций. Методы расчета учитывают влияние на энергетический коэффициент Допплера пространственного изменения нейтронного потока, а также важность величины мощности и температуры топлива работающего реактора.

Efecto Doppler en un gran reactor rápido con combustible en forma de óxido — Su cálculo e importancia para la seguridad del reactor. En los reactores rápidos de grandes dimensiones alimentados con PuO_2—UO_2, el coeficiente térmico negativo de Doppler constituye una importante ventaja desde el punto de vista de la seguridad. Los autores demuestran que seleccionando adecuadamente el tamaño y la composición del cuerpo, el coeficiente inmediato de Doppler anula por completo los efectos de la reactividad retardada debido a la dilatación térmica del sodio y de otros materiales presentes en el cuerpo. La importancia del efecto Doppler para la seguridad de los reactores se pone de manifiesto por la eficacia con que limita el aumento de la temperatura del combustible durante los regímenes transitorios de potencia debidos a los neutrones inmediatos, provocadas por la expulsión accidental de las barras de control o del sodio refrigerante. Basándose en el cálculo de esos regímenes transitorios de potencia los autores estudian la selección de los parámetros constructivos y de las condiciones de funcionamiento (como, por ejemplo, el plan de carga del combustible), para garantizar la seguridad y estabilidad del reactor.

La intensidad del efecto de Doppler en los grandes reactores de neutrones rápidos con combustible en forma de óxido se debe a la degradación del espectro neutrónico y a la posibilidad de utilizar una baja razón $^{239}Pu/^{238}U$. Los autores describen los métodos aplicados para calcular la contribución negativa del ^{238}U y del ^{240}Pu al efecto de Doppler, y la contribución positiva del ^{239}Pu; expresan las secciones eficaces de estos isótopos para varios grupos de neutrones en función de la temperatura del combustible y de la concentración isotópica. Asimismo, calculan los efectos de la importancia y de la variación espacial del flujo neutrónico en el coeficiente de potencia de Doppler, así como los de potencia y temperatura del combustible, en el caso de un reactor en funcionamiento.

Introduction

A large fast oxide reactor (PuO_2—UO_2-fuelled) possesses several features which make it attractive for central power station applications. These include the ability to withstand high fuel burn-up and a high internal breeding ratio which facilitates attainment of high fuel burn-up and reduces reactor-control requirements. Major disadvantages to a fast reactor are the high concentration of fissile material and the short neutron lifetime, which result in a potential safety hazard in the event of a prompt critical excursion. An important safety advantage of a large oxide-fuelled core over a small metal-fuelled core is an overriding prompt negative power coefficient due to the Doppler effect. The large Doppler coefficient combined with the relatively long heat-transfer-time constant for oxide fuel promotes reactor stability and tends to terminate a power excursion resulting from accidental insertion of a fairly appreciable positive reactivity step. An essential safety-design criterion for such a reactor is that the Doppler effect be sufficiently large to terminate a power excursion resulting from credible reactivity insertions (such as control-rod ejection or loss of sodium coolant) before bulk fuel melt-down occurs.

The large Doppler temperature coefficient of reactivity results mainly from three interrelated factors. One is the degradation of the fast-neutron spectrum, with a significant neutron flux occurring in the energy region of strong U^{238}-resonances below 10 keV. A second factor is the low Pu^{239} to U^{238} isotopic ratio, limiting the positive Doppler contribution of the Pu^{239}. The third factor is the large scattering cross-section per fuel atom, which enhances the negative Doppler effect of the strongly self-shielded U^{238}-resonances and diminishes the positive Doppler effect of the weakly shielded Pu^{239}-resonances.

Method of calculation

Doppler reactivity effects were obtained by performing 18-energy group, one-dimensional diffusion calculations at several different fuel temperatures. Multigroup cross-sections for U^{238}, Pu^{239} and Pu^{240} were calculated as a function of fuel temperature and of isotopic concentrations. The resolved resonance region for U^{238} from 5 to 1000 eV was split into four groups. The region from 1000 to 9000 eV, containing unresolved but well-separated resonances, was divided into two groups. The fast region, above 9 keV with highly overlapping resonances, was divided into 10-energy groups utilizing an Argonne National Laboratory fast reactor cross-section set [1].

Effective resonance integrals were calculated for each energy group, using measured U^{238} resonance parameters below 1000 eV [2]. A constant radiative capture width $\Gamma_\gamma = 0.0246$ eV, a chi-squared distribution with one degree of freedom [3] of reduced neutron widths $\Gamma^0{}_n$ with $\langle \Gamma^0{}_n \rangle = 0.00176$ eV, and an average resonance spacing of 18.5 eV from 1000 to 9000 eV were assumed. For Pu^{240}, resolved resonance parameters were available only up to 120 eV [4], and the statistical methods for unresolved resonances were used from 120 to 9000 eV with $\Gamma_\gamma = 0.032$ eV, $\langle \Gamma^0{}_n \rangle = 0.0027$ eV, and an 11.0-eV mean spacing between resonances. The reactor was assumed to be homogeneous for these calculations, which is a very good approximation for an oxide-fuelled fast reactor. The RES Code, which was developed by NORDHEIM and ADLER [5] and utilizes the methods of DRESNER [6] for evaluation of Doppler broadened resonance integrals in homogeneous cores, was used for these calculations.

For Pu^{239} resolved resonance parameters were available only up to 60 eV. The following assumptions were made for unresolved resonance calculations:

1. The resonance fission width Γ_f follows a 3-degree-of-freedom chi-squared statistical distribution [7].
2. The ratio $\Gamma_\gamma/\langle\Gamma_f\rangle$ as a function of energy is the same as the energy dependence of the Pu^{239} α value (see Appendix A).
3. Γ_γ is constant at 0.039 eV, $\Gamma^0{}_n$ follows the usual 1-degree-of-freedom chi-squared distribution [3] with $\langle\Gamma^0{}_n\rangle = 0.000\,661$ eV, and the average resonance spacing is 2.5 eV.
4. Γ_f/Γ_γ is constant over each resonance (Breit-Wigner single level approximation).
5. A statistical weight factor of $^1/_2$ for each Pu^{239} spin state is adequate (instead of the actual $^1/_4$ and $^3/_4$ values which increase the computational requirements).

With these assumptions, the Pu^{239} resonance self-shielding could be treated the same as for U^{238}, simply by using the sum $\Gamma_\gamma+\Gamma_f$ in place of just Γ_γ and allowing for the statistical fluctuation of Γ_f. Appendix A describes in greater detail the calculational procedures used for evaluation of the Pu^{239} resonance integrals in the unresolved resonance region.

In the 10 high-energy groups, which do not contribute much to the Doppler coefficient, the method of FESHBACH *et al.* [8] was used to compute the change in the U^{238} capture cross-section with fuel temperature. In this high energy region, each Pu^{239}-atom was assumed to contribute a positive Doppler reactivity effect which has $1^1/_2$ the magnitude of that of each U^{238}-atom. This is based on BETHE's analysis [7] of the U^{235} Doppler effect for partially enriched uranium fast reactors and takes into account the higher $(\nu\sigma_f - \sigma_a)$ for Pu^{239}. The Doppler effect for Pu^{240} in the 10 high-energy groups was neglected, since it has almost zero reactivity worth over most of this energy region.

Spatial averaging of the Doppler reactivity change to yield an effective prompt negative power coefficient was carried out using the square of the power density as an approximate statistical weighting factor. Both the gross fuel-temperature distribution over the core and the radial temperature distribution over a single fuel rod were taken into account, as well as the non-uniform temperature rise in response to an increase of reactor power. The heat of fusion of the oxide fuel at its melting point was also taken into account in the computation of Doppler reactivity changes during a power excursion. Equations for these calculations and assumptions upon which they are based are given in Appendix B.

Results

Appendix C contains temperature-dependent fission and radiative capture cross-sections for U^{238}, Pu^{239} and Pu^{240} used for the multi-group calculations of Doppler reactivity changes. Fig. 1 shows the importance of the intermediate-energy resonance region to the Doppler coefficient of a large fast oxide reactor. Although 80% of the fission power is caused by neutrons above 9 keV, this fast-energy region contributes only 10% of the Doppler coefficient. About 75% of the Doppler reactivity change is contributed by the energy region between 300 and 4000 eV. The Doppler coefficient for the large fast oxide reactor is 5 to 10 times greater (depending on core size and composition) than that for a small relatively highly enriched fast reactor such as FERMI-I [9], which does not have the appreciable low-energy tail in its power spectrum. Recent calculations carried

out independently at APDA also yielded the appreciable Doppler coefficient ($-1 \times 10^{-5}\, \Delta k/k$–°C) for a large fast oxide reactor [10].

Fig. 1 also shows the contribution of each of the fuel isotopes to the Doppler coefficient. It is somewhat disturbing that the Pu^{239} contributes a positive Doppler

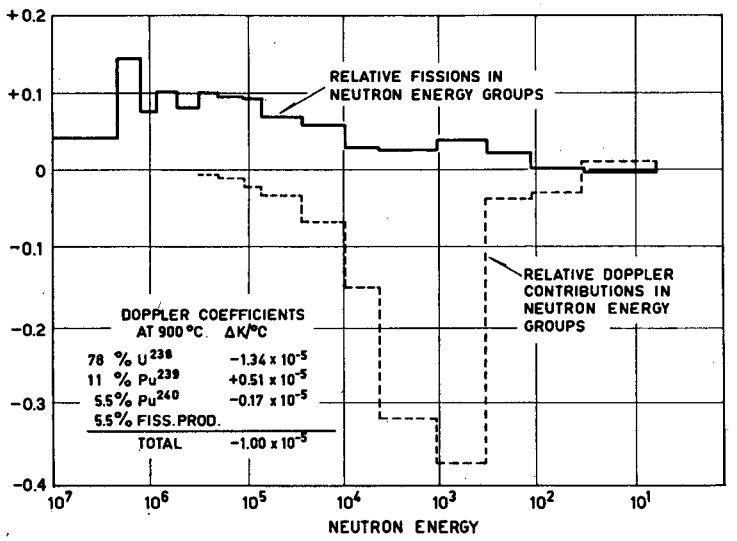


Fig. 1
Neutron power spectrum, energy distribution of Doppler coefficient and contributions by U^{238}, Pu^{239} and Pu^{240} to the total coefficient. The core is a 3-ft slab composed of 33% PuO_2—UO_2, 17% steel and 50% sodium.

effect about 40% the magnitude of the negative contribution by the U^{238}, in spite of the 1:7 isotopic ratio. This indicates that smaller fast reactors requiring higher Pu^{239}-content in the fuel may not have a significant negative Doppler coefficient and, in fact, very small reactors would have positive coefficients.

Fig. 2 shows the decrease in reactivity with increasing fuel temperature due to the Doppler effect. The four curves cover core-slab thicknesses of 2 and 3 ft, with sodium volume fractions of 30 and 50%. The Doppler coefficient at a given fuel temperature is the slope of these curves. For the 3-ft case with 50% sodium, the coefficient is -2.8×10^{-5}, -0.84×10^{-5} and $-0.37 \times 10^{-5}\, \Delta k/k$–°C respectively for 20, 900 and 2750 °C fuel temperatures. This is approximately a $1/T$ variation, where T is the absolute temperature. The curves of Fig. 2 are based on a uniform temperature distribution of the core fuel. Taking into account the non-uniform power and temperature distributions these coefficients are increased by a factor of 1.23.

Table I lists Doppler coefficients and the amount of useful shut-down reactivity available from the Doppler effect for several reactor core sizes and compositions. These values include the effects of the spatial variations of reactor power and fuel temperature. Sodium temperature coefficients, which tend to be positive for large fast oxide cores, are also listed for comparison with the Doppler

TABLE I

DOPPLER AND SODIUM TEMPERATURE COEFFICIENTS AND TOTAL AVAILABLE REACTIVITY DECREASE DUE TO DOPPLER EFFECT

Per cent sodium volume[1]	Core thick.[2] (ft)	Per cent Pu^{239} [3]		Temp. coef. ($\Delta k/°C \times 10^6$)		$(\Delta k)_{Dop}$ in dollars[4]	
		Core avg.	Blanket avg.	Dop. at 900° C avg. fuel	Sodium at450°C	Initial fuel temp. 20° C	900°C(avg.)
10	2	10.0	1.8	—8.6	+1.1	—4.4	—1.5
10	3	9.0	0.9	—8.7	+1.7	—4.5	—1.5
30	1.5	12.2	2.6	—8.5	+1.5	—4.6	—1.5
30	2	10.9	2.2	—9.3	+2.8	—4.8	—1.7
30	3	9.7	1.5	—9.7	+4.5	—5.0	—1.7
50	2	12.8	2.6	—9.3	+1.0	—5.4	—1.8
50	3	11.1	2.0	—10.3	+5.1	—5.6	—2.0
50	5	9.8	1.1	—10.9	+11.0	—6.0	—2.2
70	3	14.3	2.9	—10.5	—2.5	—6.1	—2.4
70	5	11.8	1.7	—13.9	+8.4	—7.4	—2.8

[1] Remainder of core is fuel and steel in a 2:1 volume ratio.

[2] All cores are slabs with 15-inch thick UO_2 axial blankets. See Fig. 3 for dimensions of cylindrical cores having nuclear properties equivalent to above slabs.

[3] The Pu^{240} content of the core fuel is one-half that of the Pu^{239}; the fission product pair concentration is 5.8%, corresponding to an average fuel burn-up of 50 000 MWd (t) per 2000 lb of (U + Pu). Blanket fuel isotopic compositions in each case correspond to the build-up in depleted uranium during 50 000 MWd (t)/T exposure of the core fuel.

[4] Available Doppler reactivity change for a fuel temperature rise from the initial temperature to a condition in which there is complete melting of the oxide at the reactor position of peak power (see Appendix B for details).

coefficients. Only slab cores are included in Table I; however, dimensions of cylindrical cores having the same neutron leakage and composition as the slab cases may be obtained from Fig. 3.

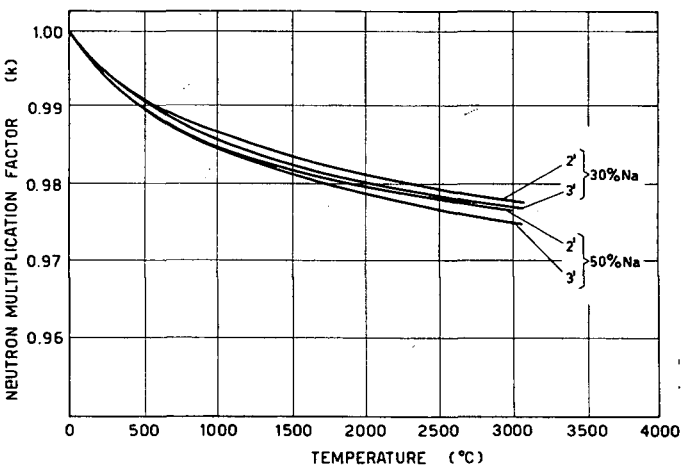


Fig. 2
Neutron multiplication factor versus average fuel temperature (Doppler effect).

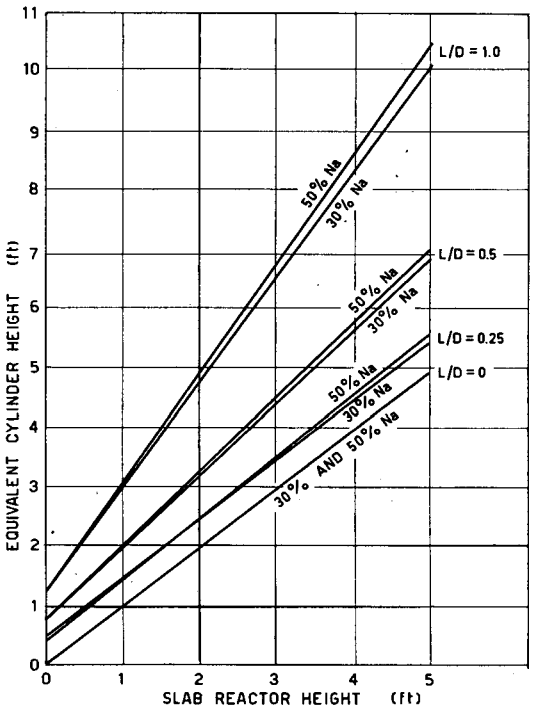


Fig. 3
Equivalent cylinder dimensions for blanketed slab reactors containing 30 and 50% sodium volume.

The values of $(\Delta k)_{\mathrm{Dop}}$ in Table I represent the negative reactivity provided by the Doppler effect in a power transient as the fuel temperature rises from its starting point to a condition in which the oxide is completely melted at the fuel rod position of peak reactor power. The peak energy density in such a transient is that required to raise the fuel to its melting temperature plus the heat of fusion. In such a transient, there is appreciable melting of fuel in the central regions of many fuel rods. This condition has been arbitrarily defined as the threshold of bulk fuel melting. Values of $(\Delta k)_{\mathrm{Dop}}$ are listed in Table I for (*a*) the fuel initially at room temperature, representing a power excursion at start-up, and (*b*) the fuel temperature distribution initially at operating conditions (900° C average), representing a power excursion from the normal full power operating conditions.

There are no experimental data available to verify the large calculated Doppler coefficient. Measurements by FROST [11] of the U^{238} Doppler effect in a U^{235}-fuelled critical assembly operating in an intermediate spectrum yielded a large negative coefficient. This result is in qualitative agreement with the present Doppler calculations, although the critical experimental conditions were too different from a fast oxide reactor to provide quantitative substantiation of the calculated results.

Significance for reactor safety

The effectiveness of the Doppler coefficient in providing reactor stability under normal power transients, such as load-following, is indicated by the magnitude of its negative prompt power coefficient relative to the power coefficients of other reactivity feedback mechanisms. Table II lists the principal contributions to the prompt and delayed temperature coefficients of a reference large fast oxide reactor (the 2-ft slab core with 50 % sodium of Table I), and also of two smaller metallic-fuelled fast reactors. The prompt negative temperature coefficient

TABLE II

TEMPERATURE AND POWER COEFFICIENTS OF REACTIVITY

	FERMI-I*	EBR-II**	Fast oxide reactor
Core volume (l)	330	66	3140
Temperature coefficient ($10^{-6}\Delta k/^0$ C)			
Fuel Doppler (prompt)	— 1.6	+ 0.4	— 9.3
Fuel expansion (prompt)	— 3.3	— 4.8	— 1.3
Coolant expansion (delayed)	— 6.1	— 8.7	+ 1.0
Radial core expansion (delayed)	— 3.0	— 9.7	—10.4
Power coefficient ($10^{-6}\Delta k/1\%$ *power*)			
Prompt	— 9.3	— 6.3	— 49***
Delayed	— 6.4	—10.1	— 7.0

* Temperature coefficients taken from APDA-124 [9].
** Temperature coefficients taken from ANL-5719 [14].
*** Based on Doppler coefficient only.

for the fast oxide reactor arises almost entirely from the Doppler coefficient, which is purely a nuclear process and, consequently, does not have some of the uncertainties associated with mechanical expansion of fuel, particularly of oxide and other ceramic fuels. For the smaller metallic-fuelled cores having harder neutron spectra, the prompt negative reactivity coefficient is due principally to mechanical expansion, and the Doppler effect is not very significant. Prompt and delayed power coefficients calculated from the temperature coefficients are also listed in Table II. The prompt negative Doppler power coefficient of the large fast oxide reactor substantially overrides the sum of all the delayed coefficients, thereby providing reactor stability.

The capability of the Doppler effect for terminating a prompt critical power excursion is an important factor in the selection of the size and composition of a fast oxide reactor core and in setting limits on reactor operating conditions. One example of this is the limitation on the size and composition imposed by the reactivity effect associated with total loss of sodium coolant.

The solid curves of Fig. 4 show the reactivity increment resulting from a total loss of sodium, plotted against the sodium volume fraction of the core. The three curves represent three different slab-core sizes from $1^1/_2$ to 3 ft thickness.

These curves have the same general dependence on core size and composition as the sodium temperature coefficient and show some of the characteristics of this coefficient in a large fast oxide reactor. As the core size increases, the neutron

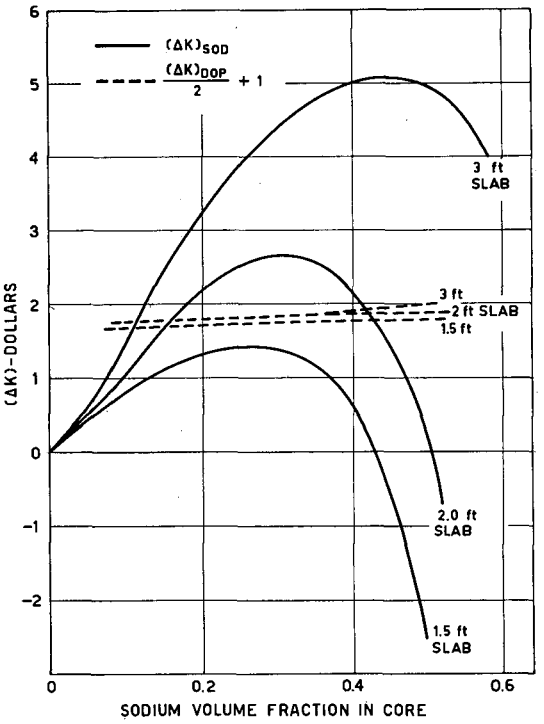


Fig. 4
Change in reactivity due to total sodium loss and total available Doppler effect.

leakage decreases and the sodium coefficient becomes larger and remains positive over most of the sodium composition range. This is because the change in neutron leakage represents the major negative contribution to the sodium temperature coefficient, which is competing with several positive contributions associated with a hardening of the neutron spectrum [12]. At very low sodium volume fractions, both the spectral hardening and leakage effects associated with sodium removal are extremely small; at large sodium volume fractions the negative leakage effect is dominant. The difference between the positive spectral hardening and the negative leakage effects, and hence the sodium coefficient, attains a peak value at some intermediate value of sodium volume fraction.

The quantity $[(|(\Delta k)_{\mathrm{Dop}}|/2)+1]$ plotted in Fig. 4 for the range of core sizes has the following significance: if a reactivity greater than 1 dollar is instantaneously inserted (a reactivity step), the resulting prompt-critical power excursion will be terminated by the Doppler effect before bulk fuel melting occurs only if the above quantity involving $|(\Delta k)_{\mathrm{Dop}}|$ is greater than the reactivity step which initiates the excursion (see earlier definition of $(\Delta k)_{\mathrm{Dop}}$ for the description of the threshold of bulk melting). For an instantaneous loss of the sodium

coolant (which perhaps is beyond the realm of credibility), the resulting power excursion will be terminated by the Doppler effect for those conditions of core size and composition where the solid curve falls below the corresponding dashed curve; for the 2-ft slab case, this is below 15 % or above 45 % sodium volume fraction.

In a fast reactor, it is difficult to conceive of a prompt-critical reactivity insertion of such speed that it is equivalent to a reactivity step. (For a typical large fast oxide reactor, the average neutron lifetime is about 5×10^{-7} s.) Fig. 5

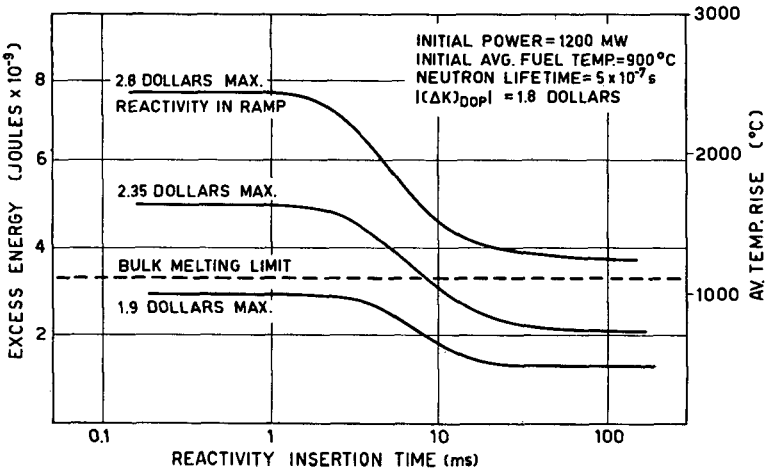


Fig. 5
Average fuel temperature rise and excess energy generation in prompt ramp excursions.

shows the calculated average fuel temperature rise and the excess energy generated in the fuel of a large fast oxide reactor during prompt power excursions initiated by the insertion of rapid reactivity ramps. The three curves are for three different values of maximum reactivity in the ramp, namely $[1 + {}^1/_2 \mid (\Delta k)_{\mathrm{Dop}} \mid]$, $[1 + {}^3/_4 \mid (\Delta k)_{\mathrm{Dop}} \mid]$, and $[1 + \mid (\Delta k)_{\mathrm{Dop}} \mid]$ expressed in dollar reactivity units, The Doppler effect is assumed to be the only reactivity feed-back mechanism available for terminating the prompt excursion. $\mid (\Delta k)_{\mathrm{Dop}} \mid$ is 1.8 dollars for the selected reactor which is initially at full power of 1200 MW (t) and possesses nuclear characteristics equivalent to the 2-ft slab core with 50 % sodium volume of Table I. The average fuel temperature rise listed on the ordinate scale of Fig. 5 is actually the temperature rise of the fuel which is at average temperature at the start of the ramp. This is slightly higher than the core-averaged fuel temperature rise due to the delay in temperature rise of the fuel undergoing a transition from the solid to the liquid phase.

It is seen in Fig. 5 that the reactivity ramp insertion time must be of the order of 1 ms or less to produce the effect of a reactivity step of equal magnitude. For slower ramps, the negative Doppler feed-back limits the core reactivity rise to a value below the maximum reactivity of the ramp. For a 100-ms ramp insertion time, the maximum inserted reactivity may be greater than 1 dollar by almost $(\Delta k)_{\mathrm{Dop}}$ without exceeding the temperature rise limit that yields bulk fuel melting (see dashed line of Fig. 5).

Excess operating reactivity for burn-up in the fast oxide reactor must be held to $[1+{}^1/_2\,|\,(\Delta k)_{\mathrm{Dop}}\,|\,]$ dollars if an instantaneous loss of all shim control-rods during full power operation is a credible accident. For the reference design with a $|\,(\Delta k)_{\mathrm{Dop}}\,|$ of 1.8 dollars, the permissible 1.9 dollars of excess operating reactivity allows about a 3-month operating time between shut-downs for partial core refuelling for a fuel specific power of 1000 W/g $Pu^{239+241}$. If the time for rod ejection is of the order of 100 ms in the worst credible accident involving loss of control-rods, the excess operating reactivity can be raised appreciably, but not more than $[1+|\,(\Delta k)_{\mathrm{Dop}}\,|\,]$, thereby allowing a corresponding increase of the refuelling interval.

The preceding examples illustrate the vital role that the Doppler effect plays in providing safety and stability for the large fast oxide reactor.

APPENDIX A

Pu^{239} resonance integral evaluation

The evaluation of the resonance integral for fissile material is more complex than for non-fissile material, since both capture and fission resonances must be evaluated. The absorption resonance integral for Pu^{239} was calculated using the relationship applicable to a homogeneous reactor,

$$I_{\mathrm{eff}}=\int_{-\infty}^{\infty}\frac{\sigma_{\mathrm{a}}\sigma_{\mathrm{p}}}{\sigma_{\mathrm{t}}}\,\frac{\mathrm{d}E}{E} \tag{1}$$

where σ_{a} is the absorption cross-section, σ_{p} is the potential scattering cross-section per Pu^{239} atom, σ_{t} the total cross-section, and E is energy. The RES Code used to evaluate the above integral assumes that the energy dependence of the cross-sections are given by the usual Doppler broadened, Breit-Wigner line shape.

In order to evaluate I_{eff} for Pu^{239}, Γ_γ was replaced by $\Gamma_{\mathrm{a}}=\Gamma_{\mathrm{f}}+\Gamma_\gamma$, where Γ_γ is the radiative capture width and Γ_{f} the fission width. This calculation did not present any problem in the resolved resonance region, which for Pu^{239} extends only up to 60 eV. For the unresolved resonance region, between 60 eV and 9000 eV, it was necessary to use a statistical distribution for both the neutron and fission widths based upon the analysis of PORTER and THOMAS [3]. The chi-squared distribution function was used, given by

$$p(x,\varrho)\,\mathrm{d}x=\Gamma(\varrho)^{-1}\,(\varrho x)^{\varrho-1}\,\mathrm{e}^{\varrho x}\,\mathrm{d}x. \tag{2}$$

For numerical computations, the neutron and fission width distributions were approximated by discrete widths, each representing a given fraction of the total distribution. Four widths were used to represent the distribution of neutron widths, given by

$$x_{\mathrm{n}}=\Gamma^0{}_{\mathrm{n}}/\langle\Gamma^0{}_{\mathrm{n}}\rangle$$

These were the values $\bar{x}^i{}_{\mathrm{n}}$, for $i=1,\ 2,\ 3\ \&\ 4$, corresponding to

$$\int_0^x P\,(x,\ 1/2)\,\mathrm{d}x=1/8,\ 3/8,\ 5/8\ \&\ 7/8 \tag{3}$$

but normalized to give

$$\sum_{i=1}^{4}\bar{x}^i{}_{\mathrm{n}}=4.$$

Only three representative values were used for the narrower fission width distribution ($\varrho=3/2$) as recommended by BETHE [7] of

$$x_{\mathrm{f}}=\Gamma_{\mathrm{f}}/\langle\Gamma_{\mathrm{f}}\rangle\,.$$

These were the values $\bar{x}$, for $k=1$, 2 & 3, corresponding to

$$\int_0^x P(x, 3/2)\,\mathrm{d}x = 1/6,\ 3/6\ \&\ 5/6 \tag{4}$$

but normalized to give

$$\sum_{k=1}^{3} \bar{x} = 3.$$

The average fission width was calculated as

$$\langle \Gamma_f \rangle = \Gamma_\gamma / \alpha(E) \tag{5}$$

where $\Gamma_\gamma = 0.039$ eV for the unresolved resonances and $\alpha(E)$, the ratio of capture-to-fission cross-sections, was obtained as a function of energy from the best estimate based upon the available experimental data. Table III shows $\alpha(E)$ values for each of the 18 energy groups used in these calculations.

TABLE III

EIGHTEEN GROUP VALUES OF $\alpha(E)$ FOR Pu^{239}

Group, j	$\bar{u}$	E_j (eV)	$\alpha(E)$*
1	0.75	4.8×10^6	0.02
2	1.75	1.77×10^6	0.04
3	2.25	1.05×10^6	0.05
4	2.75	0.65×10^6	0.07
5	3.25	0.40×10^6	0.10
6	3.75	0.28×10^6	0.16
7	4.25	1.45×10^5	0.23
8	4.75	0.88×10^5	0.30
9	5.50	0.41×10^5	0.37
10	6.50	0.15×10^5	0.47
11	7.4	6.2×10^3	0.52
12	8.5	2.0×10^3	0.56
13	9.75	550	0.62
14	10.9	185	0.65
15	11.8	76	0.98
16	13.6	12.6	0.60
17	15.8	1.4	0.41
18	—	~0.2	0.49

* The first 10 group $\alpha(E)$ values were taken from ANL-5800 [1]. Groups 11 through 14, the unresolved resonance groups, were estimated from experimental values given in KAPL 1793 [13]. These $\alpha(E)$ values are approximately the midpoint values over the experimental ranges of uncertainty. The $\alpha(E)$ values in groups 15 through 18 are according to resolved resonance parameters and thermal cross-sections in BNL-325 [4].

Table IV shows normalized values of x_n^i and $\bar{x}_f^k$, $\langle \Gamma^0_n \rangle$ and Γ_γ and the corresponding values of Γ_n, Γ_f and $\Gamma_a = \Gamma_f + \Gamma_\gamma$ used in the Breit-Wigner form for one energy value (550 eV). The combination of four $\bar{x}_n$'s and three $\bar{x}_f^k$'s yields a statistical distribution of twelve equally probable resonance absorption integrals for each resonance. The resonance absorption strength for each energy group was computed as the average of twelve single resonance integrals at the mean lethargy of the group. A resonance absorption strength was also computed for each group for the case of Pu^{239} at infinite dilution (σ_p large). The ratio of these resonance absorption strengths gives the Pu^{239} resonance self-shielding factor for each group. The infinite dilution fission cross-section for each group was obtained by lethargy averaging

TABLE IV

UNRESOLVED RESONANCE PARAMETERS FOR Pu^{239} (for $E_j = 550$ eV)

1. Γ^0_n calculation

i	$\bar{x}^i_n$	$\langle \Gamma^0_n \rangle$(eV)	Γ^0_n (eV)	Γ_n (eV)
1	0.032	0.000661	0.000021	0.000496
2	0.279	0.000661	0.000185	0.00432
3	0.947	0.000661	0.000625	0.0147
4	2.742	0.000661	0.00182	0.0425

2. Γ_f and Γ_a calculation

k	$\bar{x}^k_f$	Γ_γ (eV)	Γ_f (eV) (for $E_j = 550$ eV)	$\Gamma_a = \Gamma_f + \Gamma_\gamma$ (for $E_j = 550$ eV)
1	0.310	0.039	0.0195	0.0585
2	0.854	0.039	0.0536	0.0926
3	1.836	0.039	0.116	0.155

the fission cross-section curves in BNL-325 [4]. Multiplying this value by the group self-shielding factor yielded the effective Pu^{239} fission cross-section. The radiative capture cross-sections were then computed using the effective fission cross-section and the $\alpha(E)$ values listed in Table III.

APPENDIX B

Evaluation of Doppler effects with spatial temperature and power distributions

The Doppler coefficient for spatially uniform flux and fuel temperature is given by

$$(\mathrm{d}k/\mathrm{d}\bar{T}) = \Delta k/\Delta \bar{T} = \frac{k_2 - k_1}{\bar{T}_2 - \bar{T}_1} \tag{6}$$

where $\Delta \bar{T}$ is the change in the average fuel temperature.

The Doppler coefficient is of practical interest as a reactor power coefficient. Consequently, the effective Doppler coefficient must take into account the following space-dependent effects: *(a)* neutron flux and importance, *(b)* gross fuel temperature distribution over the reactor and radially across an individual fuel rod, and *(c)* fuel temperature response to a change of reactor power. Assuming a $1/T$ dependence of the Doppler coefficient on absolute temperature, T, the effective Doppler temperature coefficient was obtained from the uniform coefficient, $\mathrm{d}k/\mathrm{d}T$, using the relationship:

$$C_1 = \frac{(\mathrm{d}k/\mathrm{d}\bar{T})_{\mathrm{eff}}}{(\mathrm{d}k/\mathrm{d}\bar{T})} = \frac{\bar{C}_2 \bar{T} \int_0^{R_{\max}} \int_0^{Z_{\max}} [P(R,Z)]^2 [\mathrm{d}T(R,Z)/\mathrm{d}\bar{T}]\, R\,\mathrm{d}R\,\mathrm{d}Z/T(R,Z)}{\int_0^{R_{\max}} \int_0^{Z_{\max}} [P(R,Z)]^2 R\,\mathrm{d}R\,\mathrm{d}Z} \tag{7}$$

where $\bar{C}_2$ is a correction factor for the parabolic radial temperature distribution across an individual fuel rod, $\bar{T}$ is the average fuel temperature in °K, and $T(R,Z)$

and $P\,(R, Z)$ are the fuel temperature and relative power density at core coordinates R and Z. These two functions were approximated by

$$T\,(R, Z) = T_0 + P\,(R, Z)\,(\bar{T} - T_0) \tag{8}$$

with T_0 taken as the average coolant temperature, and

$$P\,(R, Z) = P_R P_Z\,[1 - 2\,(1 - [1/P_R])\,(R/R_{\max})^2]\,[1 - 3\,(1 - [1/P_Z])\,(Z/Z_{\max})^2] \tag{9}$$

where P_R and P_Z are respectively the peak-to-average radial and axial core power densities. Eq. (7) uses the square of the power distribution as an approximation of the product of the neutron flux and importance.

The correction factor $\bar{C}_2$ was obtained using the relationship

$$\bar{C}_2 = \frac{\int_0^{R_{\max}} \int_0^{Z_{\max}} [P(R,Z)]^2 C_2(R,Z)\, R\,dR\,dZ}{\int_0^{R_{\max}} \int_0^{Z_{\max}} [P(R,Z)]^2\, R\,dR\,dZ} \tag{10}$$

where

$$C_2\,(R, Z) = \frac{(T_c + T_0)}{r_{\max}^2} \int_0^{r_{\max}} r\,dr/T\,(r)$$

$$= \frac{1}{2}\left(\frac{T_c/T_0 + 1}{T_c/T_0 - 1}\right) \log_e\,(T_c/T_0)$$

and r is the radial coordinate for an individual fuel rod, T_c is the central fuel rod temperature at the core coordinates (R, Z), and $T\,(r)$ is the temperature across the fuel rod, given by

$$T(r) = T_c - (T_c - T_0)\,(r/r_{\max})^2.$$

In order to simplify evaluation of the integral in the numerator of Eq. (10), $C_2\,(R, Z)$ was fitted to the following linear expression in the ratio T_c/T_0:

$$C_2\,(R, Z) = 0.95 + 0.05\,T_c\,\,(R, Z)/T_0. \tag{11}$$

The linear fit is adequate, since C_2 does not depart appreciably from unity over the range of $T_c\,(R, Z)$ values in the core. The distribution of centre fuel rod temperatures over the core is given by $T_c\,(R, Z) = T_0 + P\,(R, Z)\,(\bar{T}_c - T_0)$, where $\bar{T}_c$ is the central fuel temperature at the point of average core power density.

For the scoping study covered in this document, the following values were assumed: $P_R = 1.6$; $P_Z = 1.25$; $T_0 = 725°$ K; $\bar{T} = 1170$ °K; and $\bar{T}_c = 1400$ °K. These yielded $\bar{C}_2 = 1.08$ from Eq. (10), and $C_1 = 1.23$ from Eq. (7) for the overall correction factor on the uniform-temperature Doppler coefficient.

Similar corrections for spatially non-uniform temperature distributions can be made for the total Doppler reactivity reduction $(\Delta k)_{\mathrm{Dop}}$ in a power excursion. Again assuming a $1/T$ fuel temperature dependence for the Doppler coefficient, this correction factor is given by

$$C_3 = \frac{[(\Delta k)_{\mathrm{Dop}}]_{\mathrm{eff}}}{[(\Delta k)_{\mathrm{Dop}}]_{\bar{T}}} = \frac{\bar{C}_4}{\log_e\,(\bar{T}_2/\bar{T}_1)} \frac{\int_0^{R_{\max}} \int_0^{Z_{\max}} [P(R,Z)]^2 \left[\log_e \frac{T_2(R,Z)}{T_1(R,Z)}\right] R\,dR\,dZ}{\int_0^{R_{\max}} \int_0^{Z_{\max}} [P(R,Z)]^2\, R\,dR\,dZ} \tag{12}$$

where $T_1(R, Z)$ and $T_2(R, Z)$ are the average fuel temperatures in the rod at core coordinate (R, Z) respectively before and after the excursion. These functions can be expressed in terms of average temperatures as follows:

$$T_1(R, Z) = T_0 + P(R, Z)\,(\overline{T} - T_0) \tag{13}$$

$$T_2(R, Z) = T_1(R, Z) + P(R, Z)\,(\overline{T}_2 - \overline{T}_1) \tag{14}$$

where $\overline{T}_1$ and $\overline{T}_2$ are core-averaged fuel temperatures respectively before and after the excursion.

The correction factor $\overline{C}_4$ takes into account the non-uniform temperature distribution across an individual fuel rod and is given by

$$\overline{C}_4 = \frac{\int_0^{R_{max}} \int_0^{Z_{max}} [P(R,Z)]^2 C_4(R,Z)\, R\, dR\, dZ}{\int_0^{R_{max}} \int_0^{Z_{max}} [P(R,Z)]^2\, R\, dR\, dZ} \tag{15}$$

where

$$C_4(R, Z) = \frac{2}{\log_e \dfrac{\overline{T_2(r)}}{\overline{T_1(r)}}} \int_0^{r_{max}} \log_e [T_2(r)/T_1(r)]\, r\, dr. \tag{16}$$

The radial temperature distribution $T_1(r)$, is given by a parabola

$$T_1(r) = T_c(R, Z) - [T_c(R, Z) - T_0]\,(r/r_{max})^2 \tag{17}$$

while

$$T_2(r) = T_1(r) + P(R, Z)\,(\overline{T}_2 - \overline{T}_1) \tag{18}$$

$$\overline{T_1(r)} = \frac{1}{2}[T_c(R, Z) + T_0] \tag{19}$$

$$\text{and } \overline{T_2(r)} = \overline{T_1(r)} + P(R, Z)\,(\overline{T}_2 - \overline{T}_1). \tag{20}$$

The correction factor, C_3, was evaluated for a power excursion occurring at full power operation using the following values:
$P_R = 1.6$, $P_Z = 1.25$; $T_0 = 725°K$, $\overline{T}_1 = 1170°K$, $\overline{T}_c = 1420°K$, and $\overline{T}_2 = 2260°K$. These values yielded $\overline{C}_4 = 1.037$, and $C_3 = 1.09$ for the overall correction factor to be applied to the uniform temperature $(\Delta k)_{Dop}$.

For a power excursion from a cold reactor, the values were taken to be: $P_R = 1.6$, $P_Z = 1.25$, $T_0 = \overline{T} = \overline{T}_1 = \overline{T}_c = 273°K$, $\overline{T}_2 = 2020°K$, which gave correction factors of $\overline{C}_4 = 1.0$ and $C_3 = 1.09$.

The values for $\overline{T}_2$ were computed by assuming that just enough fission energy could be generated in the fuel rod at the reactor centre (position of peak power density) to bring this fuel to its melting temperature, ~3023°K, and melt it. Since the heat of fusion is 750 times the specific heat of the fuel, the average fuel would have an additional increase in temperature of 375° K (for peak-to-average power density = 2) during the time that the peak fuel was undergoing the phase change at its melting temperature. In numerically evaluating the integrals in Eqs. (12) and (15), values of $\overline{T}_2(R, Z)$ which were above 3023°K were set at 3023°K before integrating.

APPENDIX C — CROSS-SECTIONS FOR Pu^{239} AND Pu^{240}

Group No.	Isotope	σ_p	300° K		750° K		1,500° K		2,500° K	
			σ_f	σ_c	σ_f	σ_c	σ_f	σ_c	σ_f	σ_c
1	Pu^{239}	370	2.00	0.040	2.00	0.040	2.00	0.040	2.00	0.040
2	Pu^{239}	370	1.95	0.078	1.95	0.078	1.95	0.078	1.95	0.078
3	Pu^{239}	370	1.86	0.093	1.86	0.093	1.86	0.093	1.86	0.093
4	Pu^{239}	370	1.75	0.123	1.75	0.123	1.75	0.123	1.75	0.123
5	Pu^{239}	370	1.70	0.170	1.70	0.170	1.70	0.170	1.70	0.170
6	Pu^{239}	370	1.68	0.269	1.68	0.269	1.68	0.269	1.68	0.269
7	Pu^{239}	370	1.73	0.398	1.73	0.398	1.73	0.398	1.73	0.398
8	Pu^{239}	370	1.80	0.540	1.80	0.540	1.80	0.540	1.80	0.540
9	Pu^{239}	370	2.00	0.740	2.00	0.740	2.00	0.740	2.00	0.740
10	Pu^{239}	370	2.25	1.06	2.25	1.06	2.25	1.06	2.25	1.06
11	Pu^{239}	370	2.57	1.33	2.58	1.34	2.58	1.34	2.59	1.34
12	Pu^{239}	370	3.83	2.14	3.89	2.17	3.91	2.19	3.93	2.20
13	Pu^{239}	370	7.24	4.50	7.60	4.73	7.81	4.86	7.92	4.93
14	Pu^{239}	370	15.2	9.86	16.5	10.7	17.6	11.4	18.2	11.9
15	Pu^{239}	370	13.8	17.1	15.4	19.1	16.9	20.9	18.1	22.4
16	Pu^{239}	370	26.5	15.9	28.2	16.9	30.1	18.1	31.8	19.1
17	Pu^{239}	370	56.0	23.0	56.0	23.0	56.0	23.0	56.0	23.0
18	Pu^{239}	370	986.0	480.0	986.0	480.0	986.0	480.0	986.0	480.0
1	Pu^{239}	247	2.00	0.040	2.00	0.040	2.00	0.040	2.00	0.040
2	Pu^{239}	247	1.95	0.078	1.95	0.078	1.95	0.078	1.95	0.078
3	Pu^{239}	247	1.86	0.093	1.86	0.093	1.86	0.093	1.86	0.093
4	Pu^{239}	247	1.75	0.123	1.75	0.123	1.75	0.123	1.75	0.123
5	Pu^{239}	247	1.70	0.170	1.70	0.170	1.70	0.170	1.70	0.170
6	Pu^{239}	247	1.68	0.269	1.68	0.269	1.68	0.269	1.68	0.269
7	Pu^{239}	247	1.73	0.398	1.73	0.398	1.73	0.398	1.73	0.398
8	Pu^{239}	247	1.80	0.540	1.80	0.540	1.80	0.540	1.80	0.540
9	Pu^{239}	247	2.00	0.740	2.00	0.740	2.00	0.740	2.00	0.740
10	Pu^{239}	247	2.25	1.06	2.25	1.06	2.25	1.06	2.25	1.06
11	Pu^{239}	247	2.55	1.32	2.56	1.33	2.57	1.33	2.58	1.34
12	Pu^{239}	247	3.73	2.09	3.82	2.13	3.86	2.16	3.88	2.17
13	Pu^{239}	247	6.76	4.21	7.21	4.48	7.49	4.66	7.65	4.76
14	Pu^{239}	247	13.4	8.73	14.9	9.68	16.0	10.4	16.9	11.0
15	Pu^{239}	247	11.7	14.5	13.1	16.3	14.5	18.0	15.7	19.5
16	Pu^{239}	247	22.5	13.5	23.9	14.4	25.6	15.3	27.1	16.3

17	Pu^{239}	247	56.0	23.0	56.0	23.0	56.0	23.0	56.0	23.0
18	Pu^{239}	247	986.0	480.0	986.0	480.0	986.0	480.0	986.0	480.0
1	Pu^{240}	740	1.60	0.020	1.60	0.020	1.60	0.020	1.60	0.020
2	Pu^{240}	740	1.50	0.080	1.50	0.080	1.50	0.080	1.50	0.080
3	Pu^{240}	740	1.35	0.180	1.35	0.180	1.35	0.180	1.35	0.180
4	Pu^{240}	740	0.70	0.200	0.70	0.200	0.70	0.200	0.70	0.200
5	Pu^{240}	740	0.18	0.200	0.18	0.200	0.18	0.200	0.18	0.200
6	Pu^{240}	740	0.03	0.200	0.03	0.200	0.03	0.200	0.03	0.200
7	Pu^{240}	740	0.00	0.300	0.00	0.300	0.00	0.300	0.00	0.300
8	Pu^{240}	740	0.00	0.450	0.00	0.450	0.00	0.450	0.00	0.450
9	Pu^{240}	740	0.00	0.700	0.00	0.700	0.00	0.700	0.00	0.700
10	Pu^{240}	740	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
11	Pu^{240}	740	0.00	1.15	0.00	1.17	0.00	1.18	0.00	1.19
12	Pu^{240}	740	0.00	2.98	0.00	3.10	0.00	3.17	0.00	3.22
13	Pu^{240}	740	0.00	6.88	0.00	7.50	0.00	7.96	0.00	8.28
14	Pu^{240}	740	0.00	15.9	0.00	18.3	0.00	20.5	0.00	22.1
15	Pu^{240}	740	0.00	16.6	0.00	19.4	0.00	22.2	0.00	24.3
16	Pu^{240}	740	0.00	4.62	0.00	5.43	0.00	6.17	0.00	6.76
17	Pu^{240}	740	0.00	193.0	0.00	193.0	0.00	193.0	0.00	193.0
18	Pu^{240}	740	0.00	208.0	0.00	208.0	0.00	208.0	0.00	208.0
1	Pu^{240}	494	1.60	0.020	1.60	0.020	1.60	0.020	1.60	0.020
2	Pu^{240}	494	1.50	0.080	1.50	0.080	1.50	0.080	1.50	0.080
3	Pu^{240}	494	1.35	0.180	1.35	0.180	1.35	0.180	1.35	0.180
4	Pu^{240}	494	0.70	0.200	0.70	0.200	0.70	0.200	0.70	0.200
5	Pu^{240}	494	0.18	0.200	0.18	0.200	0.18	0.200	0.18	0.200
6	Pu^{240}	494	0.03	0.200	0.03	0.200	0.03	0.200	0.03	0.200
7	Pu^{240}	494	0.00	0.300	0.00	0.300	0.00	0.300	0.00	0.300
8	Pu^{240}	494	0.00	0.450	0.00	0.450	0.00	0.450	0.00	0.450
9	Pu^{240}	494	0.00	0.700	0.00	0.700	0.00	0.700	0.00	0.700
10	Pu^{240}	494	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00
11	Pu^{240}	494	0.00	1.13	0.00	1.15	0.00	1.16	0.00	1.17
12	Pu^{240}	494	0.00	2.82	0.00	2.97	0.00	3.07	0.00	3.13
13	Pu^{240}	494	0.00	6.10	0.00	6.77	0.00	7.29	0.00	7.67
14	Pu^{240}	494	0.00	13.1	0.00	15.3	0.00	17.3	0.00	18.8
15	Pu^{240}	494	0.00	13.4	0.00	15.7	0.00	18.0	0.00	19.9
16	Pu^{240}	494	0.00	3.73	0.00	4.43	0.00	5.10	0.00	5.66
17	Pu^{240}	494	0.00	158.0	0.00	158.0	0.00	158.0	0.00	158.0
18	Pu^{240}	494	0.00	208.0	0.00	208.0	0.00	208.0	0.00	208.0

APPENDIX C (cont.)

CROSS-SECTIONS FOR U^{238}

Group No.	Δu	E_L (eV)	Isotope	σ_p*	300° K		750° K		1500 °K		2500 °K	
					σ_f	σ_c	σ_f	σ_c	σ_f	σ_c	σ_f	σ_c
1	1.5	2.25×10^6	U^{238}	40	0.590	0.015	0.590	0.015	0.590	0.015	0.590	0.015
2	0.5	1.35×10^6	U^{238}	40	0.450	0.062	0.450	0.062	0.450	0.062	0.450	0.062
3	0.5	8.25×10^5	U^{238}	40	0.003	0.130	0.003	0.130	0.003	0.130	0.003	0.130
4	0.5	5.0×10^5	U^{238}	40	0.000	0.143	0.000	0.143	0.000	0.143	0.000	0.143
5	0.5	3.0×10^5	U^{238}	40	0.000	0.130	0.000	0.130	0.000	0.130	0.000	0.130
6	0.5	1.8×10^5	U^{238}	40	0.000	0.150	0.000	0.150	0.000	0.150	0.000	0.150
7	0.5	1.1×10^5	U^{238}	40	0.000	0.200	0.000	0.200	0.000	0.200	0.000	0.200
8	0.5	6.7×10^4	U^{238}	40	0.000	0.299	0.000	0.300	0.000	0.301	0.000	0.301
9	1.0	2.5×10^4	U^{238}	40	0.000	0.397	0.000	0.400	0.000	0.402	0.000	0.403
10	1.0	9100	U^{238}	40	0.000	0.601	0.000	0.610	0.000	0.615	0.000	0.617
11	0.8	4000	U^{238}	40	0.000	0.399	0.000	0.440	0.000	0.467	0.000	0.487
12	1.4	1000	U^{238}	40	0.000	0.728	0.000	0.850	0.000	0.940	0.000	1.02
13	1.2	300	U^{238}	40	0.000	0.850	0.000	1.03	0.000	1.21	0.000	1.35
14	1.1	100	U^{238}	40	0.000	1.55	0.000	1.80	0.000	2.07	0.000	2.36
15	1.2	30	U^{238}	40	0.000	2.31	0.000	2.48	0.000	2.69	0.000	2.91
16	1.8	5	U^{238}	40	0.000	4.02	0.000	4.14	0.000	4.32	0.000	4.52
17	2.5	0.4	U^{238}	40	0.000	0.40	0.000	0.40	0.000	0.40	0.000	0.40
18	—	0	U^{238}	40	0.000	1.52	0.000	1.52	0.000	1.52	0.000	1.52

* σ_p is the potential scattering cross-section per isotopic fuel atom in barns. This was calculated for a reactor containing 40% sodium with a fuel-to-steel volume ratio of 2:1. The U^{238} atom fraction in fuel was taken to be 0.9. The higher values of σ_p for Pu^{239} and Pu^{240} were calculated for atom fractions of 0.10 and 0.05, respectively, while the lower values were for atom fractions of 0.15 and 0.075.

REFERENCES

[1] LOEWENSTEIN, W. B., ANL-5800 (1958) 418–419.
[2] ROSEN, J. L., CU-185 (1959).
[3] PORTER, C. E. and THOMAS, R. G., *Phys. Rev.* **104** (1956) 483–491.
[4] HUGHES, D. J. and SCHWARTZ, R. B., BNL-325 (1958).
[5] ADLER, F. T., HINMAN, G. W. and NORDHEIM, L. W., GA-350 (1958).
[6] DRESNER, L., *Nucl. Sci. Engng* **1** (1956) 68–79.
[7] BETHE, H. A., APDA-119 (1957).
[8] FESHBACH, H. GOERTZEL, G. and YAMAUCHI, E., *Nucl. Sci. Engng* **1** (1956) 4–20.
[9] Enrico Fermi Atomic Power Plant, APDA-124 (1959).
[10] NICHOLSON, R. B., *Trans. Amer. nucl. Soc.*, 12–15 (1960) 364.
[11] FROST, R. T., KATO, W. Y. and BUTLER, K., Proc. 2nd UN Int. Conf. PUAE **12** (1958) 79.
[12] NIMS, J. B. and ZWEIFEL, P. F., APDA-159 (1959).
[13] SIMPSON, J. B. and MOLINO, D. B., KAPL-1793 (1957).
[14] KOCH, L. J. *et al.*, ANL-5719 (1957).

EFFET DOPPLER DU PLUTONIUM-239 DANS LES RÉACTEURS RAPIDES*

E. A. FOSSOUL
BELGONUCLÉAIRE, BRUXELLES,
BELGIQUE

Abstract — Résumé — Аннотация — Resumen

The Doppler effect of plutonium-239 in fast reactors. The growing importance of plutonium as a fuel in fast power reactors, especially with nuclei loaded with oxide or uranium and plutonium carbide, prompted the author to study the Doppler effect of Pu^{239} in a fast spectrum.

Following the direction indicated by the studies of Goertzel, Lane and Bethe, and the formulation proposed by Nicholson, the Doppler effect of Pu^{239} in the energy range 1—200 keV was evaluated.

At these energies, the total macroscopic cross-section $\mu_t(E)$ scarcely deviates from its mean value $<\mu_t(E)>$, and the contribution to the Doppler effect by the group of neutrons centered on energy E can be evaluated by Nicholson's method "A". The function $Q_E^{(s)}$ is calculated, for various energies of E, on the basis of recent findings regarding the resonances of Pu^{239}.

Effet Doppler du plutonium-239 dans les réacteurs à neutrons rapides. L'intérêt grandissant du plutonium comme combustible dans les réacteurs de puissance à neutrons rapides, notamment pour les noyaux chargés d'oxyde ou de carbure d'uranium et de plutonium, a amené l'auteur à étudier l'effet Doppler du ^{239}Pu dans un spectre rapide.

Dans la ligne des travaux de Goertzel, Lane et Bethe et en suivant le formalisme développé par Nicholson, il a évalué l'effet Doppler du ^{239}Pu dans la bande d'énergie qui s'étend de 1 keV à 200 keV.

Pour ces énergies, la section macroscopique totale $\mu_t(E)$ s'écarte peu de sa valeur moyenne $<\mu_t(E)>$, et la contribution à l'effet Doppler pour le groupe de neutrons centré sur l'énergie E peut s'évaluer suivant la méthode « A » de Nicholson. La fonction $Q_E^{(s)}$ est calculée à partir des données récentes concernant les résonances du ^{239}Pu, et ce pour différentes énergies E.

Эффект Допплера плутония-239 в реакторах на быстрых нейтронах. Растущий интерес, проявляемый к плутонию как топливу в энергетических реакторах на быстрых нейтронах, в частности для сердечников из окиси или карбида урана и плутония, вынудил авторов изучить эффект Допплера плутония-239 в быстром спектре.

Основываясь на работах Герцеля, Лейна и Бете и следуя формальному методу, разработанному Никольсоном, оценивался эффект Допплера плутония-239 в диапазоне энергий от 1 кэв до 300 кэв.

Для этих энергий полное макроскопическое сечение $\mu_t(E)$ несколько отклоняется от своего среднего значения $<\mu_t(E)>$ и доля групп нейтронов, сосредоточенных вокруг энергии E, может определяться по методу „A" Никольсона.

$Q_E^{(s)}$ вычисляется, исходя из последних данных о резонансе плутония-239, для различных энергий E.

Efecto Doppler del plutonio-239 en los reactores rápidos. El interés creciente del plutonio como combustible para los reactores de potencia rápidos, principalmente en el caso de los que se alimentan con óxido o carburo de uranio y de plutonio, ha incitado al autor a estudiar el efecto Doppler del ^{239}Pu en un espectro rápido.

* Etude effectuée dans le cadre du contrat 003-61-4-RAPB avec la Communauté européenne de l'énergie atomique (Euratom).

Siguiendo la línea de los trabajos de Goertzel, Lane y Bethe y el formalismo desarrollado por Nicholson, el autor evaluó el efecto Doppler del ^{239}Pu en la banda de energía comprendida entre 1 keV y 200 keV.

Para estas energías, la sección macroscópica total $\mu_t(E)$ se aparta poco de su valor medio $<\mu_t(E)>$, y la contribución al efecto Doppler para el grupo de neutrones cuya energía se centra en torno a E puede calcularse por el método «A» de Nicholson. Para diferentes valores de E, el autor calculó la función $Q_E^{(s)}$ a partir de datos recientes relativos a las resonancias del ^{239}Pu.

Introduction

L'étude de l'effet Doppler dans les réacteurs rapides a déjà fait l'objet de nombreuses publications. Les principales contributions sont dues à Goertzel *et al.* [1], Feshbach *et al.* [2], Lane *et al.* [3], Bethe [4]. Récemment, Nicholson [5] a publié une synthèse du sujet tout en étendant la méthode de calcul pour tenir compte de la diffusion inélastique et pour traiter correctement le domaine d'énergie où les résonances peuvent être considérées comme isolées.

L'intérêt de développer des méthodes adéquates pour évaluer le coefficient de température Doppler semble assez évident. D'une part, cet effet joue un rôle important dans la sûreté de fonctionnement des réacteurs rapides actuellement envisagés (combustible à l'oxyde ou au carbure d'uranium et de plutonium), qui ont un spectre assez mou. D'autre part, la mesure expérimentale du coefficient de température Doppler est extrêmement difficile à réaliser.

Le but de la présente contribution est d'évaluer, à l'aide de données expérimentales existantes sur le plutonium-239, la contribution de cet élément au coefficient Doppler global d'un réacteur rapide.

Nous suivrons pour ce faire la méthode décrite par Nicholson (méthode A), valable au cas où la section macroscopique totale du milieu fluctue peu en fonction de l'énergie, c'est-à-dire pour des énergies supérieures à environ 10 keV dans le cas du plutonium-239.

Méthode « A » Nicholson

La contribution au coefficient de température Doppler d'un groupe de neutrons centré sur l'énergie E est donné par l'expression suivante:

$$T\frac{\delta k_E}{\delta T} = \frac{W(E)\,\Phi_E}{\sqrt{8\pi}\,\Delta\,\langle\mu_t\rangle}\sum_s \varrho^{(s)^2}\cos 2\delta_l\,\langle\sigma_c\rangle_s\,\langle\sigma_f+\sigma_\gamma\rangle_s\,\langle S\rangle_s\,P^{(s)}$$

avec

$$P^{(s)} = \frac{\left[\frac{\nu W_H}{W(E)}-1\right][E_f^{(s)}-e^{(s)}]-\alpha^{(s)}[E_\gamma^{(s)}-e^{(s)}]}{1+\alpha^{(s)}}$$

$$\alpha^{(s)} = \frac{\langle\sigma_\gamma\rangle_s}{\langle\sigma_f\rangle_s}.$$

L'indice s court sur les différents isotopes, sur leurs différents spins totaux J, ainsi que sur le moment orbital l du neutron incident. On remarque que l'expression

$$Q_E = \sum_s \cos 2\delta_l\,\langle\sigma_c\rangle_s\,\langle\sigma_f+\sigma_\gamma\rangle_s\,\langle S\rangle_s\,P^{(s)}$$

où s court uniquement sur J et sur l, ne dépend que de l'isotope envisagé et peut se calculer une fois pour toutes. Quel que soit le réacteur envisagé, la connaissance des fonctions Q_E permet d'évaluer aisément le coefficient de température Doppler en fonction des quelques données fondamentales sur le réacteur telles que

Φ_E: le spectre,
$W(E)$: l'importance,
$<\mu_t>$: la section totale,
$\Delta = \sqrt{\frac{4\,EKT}{A}}$ la largeur Doppler,
T: la température,
$\varrho^{(s)}$: les différentes concentrations des isotopes à considérer.

La détermination de Q_E exige la décomposition des sections efficaces en leurs composantes suivant le moment orbital du neutron incident l et le spin total J des séquences de résonances.

Il existe trop peu de données expérimentales pour reconstituer sans hypothèse ou sans ajustement de paramètres les sections efficaces nécessaires. Après avoir épuisé les hypothèses raisonnables, on est amené à ajuster certains paramètres. Suivant en cela une idée émise par Bethe [4], Nicholson ajuste deux paramètres, $<\Gamma_f>$, largeur moyenne de fission, et $<\Gamma_{n1}>_J$, l'une des largeurs de diffusion de $l=1$, de façon à reconstituer la valeur correcte de la section efficace de fission mesurée σ_f ainsi que le rapport capture sur fission globale α. Cette façon de procéder est assez commode, malgré les calculs par approximation successive, grâce à un découplage fortuit des paramètres. $<\Gamma_f>$ agit surtout sur α tandis que $<\Gamma_{n1}>_J$ influe surtout sur la grandeur de σ_f.

La procédure est en résumé la suivante. A l'aide du modèle statistique adopté pour le noyau et du paramètre libre $<\Gamma_f>$, on détermine les sections de formation du noyau composé $<\sigma_{c0}>_J$, de fission $<\sigma_{f0}>_J$, et de capture $<\sigma_{\gamma 0}>_J$ pour les neutrons de moment orbital $l=0$. On passe ensuite au calcul des sections pour les neutrons de moment orbital $l=1$, à l'aide notamment du deuxième paramètre libre $<\Gamma_{n1}>_J$ que l'on ajuste de façon à réaliser $<\sigma_{f1}> = \sigma_{f\,exp} - <\sigma_{f0}>$.

On calcule ensuite la valeur globale de α et on ajuste en conséquence le paramètre $<\Gamma_f>$.

Tous les paramètres étant ainsi fixés, on passe au calcul de Q_E proprement dit.

Dans le calcul de Q_E pour le plutonium-239, nous avons négligé la diffusion inélastique, l'effet étant faible comme l'a montré Nicholson dans le cas de l'uranium-235. Par contre, nous avons effectué le calcul de Q_E pour plusieurs cas correspondant à diverses données expérimentales, concernant notamment α et la statistique des résonances aux basses énergies.

Données expérimentales sur le plutonium-239

Section efficace de fission σ_f

Nous avons adopté pour σ_f la courbe donnée par Yiftah, Okrent et Moldauer [6]. Cette courbe est reproduite sur la figure 1 dans le domaine d'énergie qui nous intéresse, c'est-à-dire depuis 1 keV jusqu'à 300 keV.

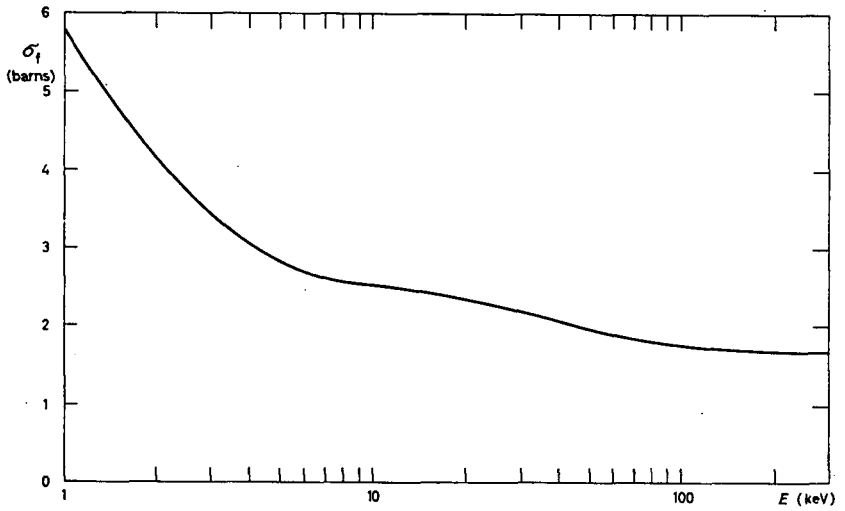


Figure 1
Section efficace de fission du plutonium-239.

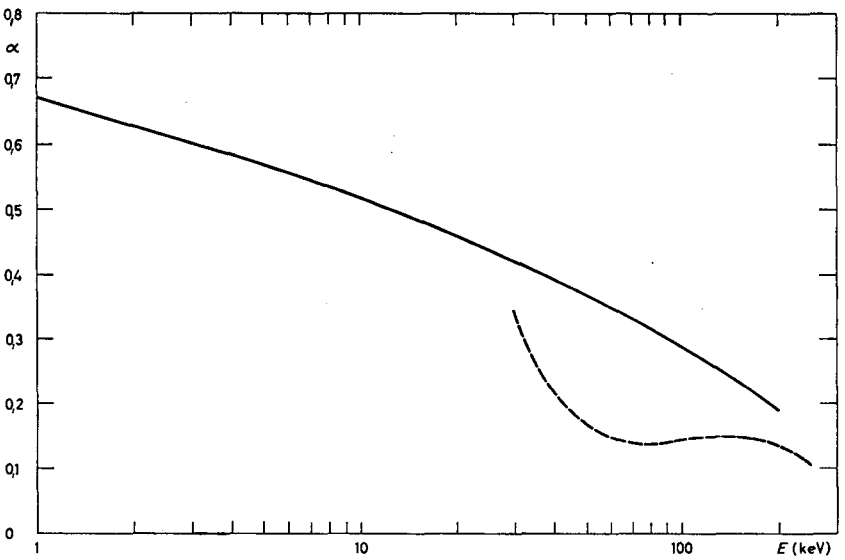


Figure 2
Rapport capture sur fission α du plutonium-239.
——— S-M
— — — — — D-H

RAPPORT DE CAPTURE SUR FISSION α

Pour α, deux références ont été utilisées. La première courbe est celle publiée par SAMPSON et MOLINO [7], reprise à la figure 2 sous la désignation S-M. La seconde courbe ne comprend en fait que les quatre points qui proviennent des mesures effectuées à Los Alamos par DIVEN et HOPKINS [8]. Cette courbe est

également tracée sur la figure 2 sous la désignation D-H. On peut remarquer une très grande différence pour α dans la bande d'énergie de 50 à 200 keV. Les nouvelles valeurs sont notablement plus basses.

DONNÉES STATISTIQUES AUX BASSES ÉNERGIES

Deux références ont été retenues: l'analyse des résonances du plutonium par EGELSTAFF, GAYTHER et NICHOLSON [9] et celle de BOLLINGER, COTE et THOMAS [10], références désignées respectivement par E et B dans la suite du texte.

Ces données sont résumées dans le tableau I.

TABLEAU I

DONNÉES STATISTIQUES AUX BASSES ÉNERGIES

	E	*B*
$\langle S \rangle$ (eV)	1,8	2,9
Γ_γ (eV)	0,038	0,039
$\langle \Gamma_f \rangle$ (eV)	0,046	0,099
$\frac{\langle \Gamma_n^0 \rangle}{\langle S \rangle}\left(\text{eV} - \frac{1}{2}\right)$	$1{,}15 \cdot 10^{-4}$	$1{,}0 \cdot 10^{-4}$

Définition du modèle de calcul

CONSTANTES DIVERSES

Le rayon du noyau ^{239}Pu a été pris égal à $R = 8{,}7 \cdot 10^{-13}$ cm. Cette valeur permet de déterminer en fonction de l'énergie les déphasages δ_l et les facteurs de pénétration v_l.

On ne considère que deux valeurs du moment orbital: $l = 0$ et $l = 1$

$$\delta_0 = \frac{R}{\lambda\!\!\!^{-}} \qquad \delta_1 = \frac{R}{\lambda\!\!\!^{-}} - \operatorname{artg} \frac{R}{\lambda\!\!\!^{-}}$$

$$v_0 = 1 \qquad v_1 = \frac{R^2/\lambda\!\!\!^{-2}}{1 + R^2/\lambda\!\!\!^{-2}}$$

où $\lambda\!\!\!^{-}$ est la longueur d'onde associée au neutron

$$\lambda\!\!\!^{-} = \frac{4{,}553 \cdot 10^{-13}}{\sqrt{E\,(\text{MeV})}}\,\text{cm}\,.$$

Le tableau II donne les différentes valeurs de ces constantes, qui interviennent dans le calcul de Q_E.

EXAMEN DES SPINS

Le spin du noyau ^{239}Pu étant $I = 1/2$, les valeurs de J possibles sont les suivantes:

$l = 0$	$j = \frac{1}{2}$	$J = 0$
		$J = 1$
$l = 1$	$j = \frac{1}{2}$	$J = 0$
		$J = 1$
	$j = \frac{3}{2}$	$J = 0$
		$J = 1$
		$J = 2$

TABLEAU II

CONSTANTES DIVERSES

E (keV)	$\lambda\!\!\!^{-}$ (10^{-13} cm)	δ_0	δ_1	cos $2\delta_0$	cos $2\delta_1$	v_1
1	144	0,060	0	0,993	1	0,004
5	64,4	0,135	0,001	0,964	1	0,018
10	45,5	0,191	0,002	0,928	1	0,035
30	26,3	0,331	0,011	0,789	1	0,099
50	20,4	0,427	0,024	0,657	0,999	0,154
60	18,6	0,468	0,030	0,593	0,998	0,180
100	14,4	0,604	0,061	0,355	0,993	0,267
175	10,9	0,799	0,125	—0,028	0,969	0,390
200	10,2	0,855	0,147	—0,138	0,957	0,422
250	9,11	0,955	0,193	—0,333	0,927	0,477

Pour un neutron de moment orbital $l=0$, seuls deux états de spin sont possibles: $J=0$ et $J=1$. Les résonances observées aux basses énergies résultent de la superposition de ces deux séquences. D'autre part, on remarque que, pour un neutron de moment orbital $l=1$, deux voies d'entrée (et de sortie) sont possibles pour les séquences de résonances $J=0$ et $J=1$ (états internes). Les facteurs statistiques $g_J=2J+1/(2s+1)$ $(2I+1)$ sont ici (avec $s=1/2$) $g_0=1/4$, $g_1=3/4$, $g_2=5/4$.

ESPACEMENT DES RÉSONANCES

Il s'agit de choisir et l'espacement moyen et la distribution de ces espacements. En ce qui concerne la distribution, l'analyse de Nicholson dans le cas de l'uranium-235 et 238 a montré qu'elle vérifiait la distribution en χ^2 pour un nombre élevé de degrés de liberté (de l'ordre de 10). Comme la seule quantité qui dépend de cette distribution, c'est-à-dire $e^{(s)}$, varie peu en fonction de v (de 0 à 1 quand v passe de 2 à l'infini), on s'est contenté d'adopter dans les calculs une valeur constante de $e^{(s)}$. On a pris $e^{(s)}=0{,}8$, identique à la valeur choisie par Nicholson dans le cas de l'uranium-235.

En ce qui concerne la valeur moyenne, on corrige en fonction de l'énergie la valeur moyenne observée aux basses énergies, sur la base d'une température du noyau composé θ égale à 0,6 MeV:

$$\langle S\rangle(E) = \langle S\rangle e^{-\frac{E\,(\mathrm{MeV})}{0{,}6}}.$$

L'espacement moyen d'une séquence se détermine en supposant les densités de niveaux respectives proportionnelles à $2J+1$ et en tirant parti de la relation valable aux faibles énergies:

$$\frac{1}{\langle S\rangle} = \frac{1}{\langle S\rangle_0} + \frac{1}{\langle S\rangle_1}$$

relation complémentaire qui permet de tirer les valeurs $\langle S\rangle_J$ à partir de $\langle S\rangle$

$$\langle S\rangle_0 = \frac{4}{1}\langle S\rangle$$

$$\langle S\rangle_1 = \frac{4}{3}\langle S\rangle$$

$$\langle S\rangle_2 = \frac{4}{5}\langle S\rangle.$$

Le tableau III donne les espacements en fonction de l'énergie.

TABLEAU III

ESPACEMENT MOYEN ENTRE RÉSONANCES
(électronvolts)

E (keV)	E			B			Facteur de correction
	$J=0$	$J=1$	$J=2$	$J=0$	$J=1$	$J=2$	
1	7,19	2,40	1,44	11,58	3,86	2,32	0,998
5	7,14	2,38	1,43	11,51	3,84	2,30	0,992
10	7,08	2,36	1,42	11,40	3,80	2,28	0,983
30	6,85	2,28	1,37	11,03	3,68	2,21	0,951
50	6,62	2,21	1,32	10,67	3,56	2,13	0,920
60	6,52	2,17	1,30	10,50	3,50	2,10	0,905
100	6,09	2,03	1,22	9,81	3,27	1,96	0,846
175	5,38	1,79	1,08	8,67	2,89	1,73	0,747
200	5,16	1,72	1,03	8,32	2,77	1,66	0,717
250	4,74	1,58	0,95	7,64	2,55	1,53	0,659

LARGEURS PARTIELLES DE RÉSONANCE

Comme pour les espacements des résonances deux choses sont à déterminer: la valeur moyenne et la distribution. Des considérations théoriques conduisent à admettre comme forme de distribution celle en χ^2 à ν degrés de liberté, ν correspondant cette fois au nombre de voies de sorties possibles. Dans le cas de la diffusion de neutrons de moment orbital $l=0$, une seule voie de sortie est ouverte et $\nu=1$, tandis que, pour la diffusion de neutrons de moment orbital $l=1$ et conduisant aux états internes de spin ($J=0$ ou $J=1$), deux voies de sortie sont ouvertes et $\nu=2$. Dans le cas extrême d'une infinité de voies ouvertes, comme par exemple pour la capture radiative, ν tend vers l'infini, c'est-à-dire que la distribution est tellement resserrée qu'une seule valeur est possible (Γ_γ est effectivement pratiquement constant).

Largeur de radiation Γ_γ

On suppose Γ_γ constant égal au Γ_γ observé aux basses énergies

$\Gamma_\gamma = 0{,}038$ eV [E]
$\Gamma_\gamma = 0{,}039$ eV [B].

Largeur de fission Γ_f

Si on suppose une distribution en χ^2 à ν degrés de liberté, la valeur quadratique moyenne s'exprime en fonction de ν et du carré de la valeur moyenne

$$\langle \Gamma_f^2\rangle = \langle \Gamma_f\rangle^2 \left(1 + \frac{2}{\nu}\right).$$

Suivant Bethe [4], on trouve pour les basses énergies

$$\frac{\langle \Gamma_f^2 \rangle}{\langle \Gamma_f \rangle^2} = 1{,}70$$

ce qui conduit à $\nu = 2{,}8$. On a adopté $\nu = 3$.

Quant à la valeur moyenne $[\Gamma_f]$, supposée indépendante du spin total J, elle reste comme paramètre libre pour l'ajustement des sections efficaces théoriques sur les données expérimentales σ_f et α.

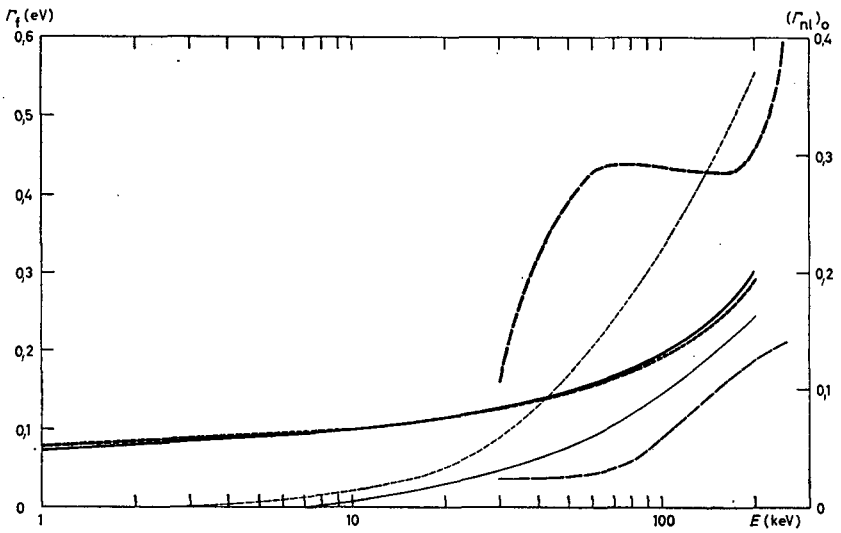


Figure 3
Largeur fission (Γ_f) et largeur diffusion $(\Gamma_{n1})_0$.
——— S-M, E (Γ_f)
– – – – – D-H, E (Γ_f)
····· S-M, B (Γ_f)
——— S-M, E $(\Gamma_{n1})_0$
– – – – – D-H, E $(\Gamma_{n1})_0$
····· S-M, B $(\Gamma_{n1})_0$

La figure 3 donne les valeurs trouvées pour $\langle \Gamma_f \rangle$ en fonction de l'énergie dans les diverses hypothèses de calcul. Le tableau IV reprend ces mêmes valeurs.

Largeur de diffusion Γ_n

Comme on l'a mentionné plus haut, la distribution sera celle en χ^2 avec $\nu = 1$, sauf pour les interactions caractérisées par $l = 1$ et $J = 0$ ou $J = 1$, où $\nu = 2$.

La valeur moyenne de la largeur de diffusion est donnée par

$$\langle \Gamma_n \rangle = \langle \Gamma_n^0 \rangle v_l \sqrt{E}$$

où $\langle \Gamma_n^0 \rangle$ est la largeur réduite de diffusion et v_l le facteur de pénétration tenant compte de la barrière centrifuge. Pour des énergies suffisamment faibles seules interviennent les interactions avec $l = 0$, c'est-à-dire avec $J = 0$ et $J = 1$.

TABLEAU IV

LARGEUR DE FISSION

E (keV)	$\langle \Gamma_f \rangle$ S-M et E (eV)	$\langle \Gamma_f \rangle$ H-D et E (eV)	$\langle \Gamma_f \rangle$ S-M et B (eV)
1	0,076	—	0,079
5	0,090	—	0,093
10	0,102	—	0,104
30	—	0,162	—
50	0,149	—	0,148
60	—	0,433	—
100	0,193	—	0,188
175	—	0,428	—
200	0,302	—	0,290
250	—	0,595	—

On a dans ce cas, pour la section de formation d'un noyau composé:

$$\langle \sigma_{co} \rangle = 2\pi^2 \lambda^2 \sum_J g_J \frac{\langle \Gamma_{n0} \rangle_J}{\langle S \rangle_J} = 2\pi^2 \lambda^2 \sqrt{E} \sum_J g_J \frac{\langle \Gamma^0_{n0} \rangle_J}{\langle S \rangle_J}.$$

La section est donc proportionnelle à $\sum_J g_J [\langle \Gamma^0_{n0} \rangle_J / \langle S \rangle_J]$. Si on suppose le rapport $\langle \Gamma^0_{n0} \rangle_J / \langle S \rangle_J$ indépendant de J, on a, tenant compte de $\sum_J g_J = g_0 + g_1 = 1$:

$$\frac{\langle \Gamma^0_{n0} \rangle_0}{\langle S \rangle_0} = \frac{\langle \Gamma^0_{n0} \rangle_1}{\langle S \rangle_1} = \frac{\langle \Gamma^0_{n0} \rangle}{\langle S \rangle} = \xi_0.$$

ξ_0 peut se déterminer à partir des résonances aux faibles énergies. Il est toujours de l'ordre de $1 \cdot 10^{-4}\,\mathrm{eV}^{-\frac{1}{2}}$. On a donc:

$$\langle \Gamma_{n0} \rangle_J = \langle \Gamma^0_{n0} \rangle_J v_0 \sqrt{E} = \xi_0 \langle S \rangle_J \sqrt{E}.$$

Ces relations conduisent en fait à supposer $\langle \Gamma^0_{n0} \rangle$ inversement proportionnel à $2J+1$ au même titre que $\langle S \rangle$.

Pour les largeurs neutrons ayant $l=1$, on conserve la même hypothèse de proportionnalité à $1/(2J+1)$.

Laissant $\langle \Gamma_{n1} \rangle_0$ comme paramètre libre pour l'ajustement des sections théoriques, on a pour $\langle \Gamma_{nl} \rangle_J$:

$$\langle \Gamma_{nl} \rangle_J = \frac{\langle \Gamma_{n1} \rangle_0}{2J+1}.$$

S'il se présente plusieurs voies d'entrées j, on suppose $\langle \Gamma_{nlj} \rangle_J$ indépendant de j, de sorte que dans le cas de deux voies d'entrées $\sum \langle \Gamma_{nlj} \rangle_J = 2 \langle \Gamma_{nl} \rangle_J$, ce qui revient à multiplier par 2 la section de formation d'un noyau composé dans ce cas particulier.

Le tableau V donne toutes les valeurs des $\langle \Gamma_{n0} \rangle_J$ pour les deux hypothèses de calcul.

TABLEAU V

LARGEUR NEUTRON (eV)

($l=0$)

E (keV)	$\sqrt{E}$ ($eV^{\frac{1}{2}}$)	E		B	
		$\langle \Gamma_{n0} \rangle_0$	$\langle \Gamma_{n0} \rangle_1$	$\langle \Gamma_{n0} \rangle_0$	$\langle \Gamma_{n0} \rangle_1$
1	31,62	0,026	0,009	0,037	0,012
5	70,71	0,058	0,019	0,081	0,027
10	100,	0,081	0,027	0,114	0,038
30	173,2	0,136	0,045	0,191	0,064
50	223,6	0,170	0,057	0,239	0,080
60	244,9	0,184	0,061	0,257	0,086
100	316,2	0,221	0,074	0,310	0,103
175	418,3	0,259	0,086	0,363	0,121
200	447,2	0,265	0,089	0,372	0,124
250	500	0,273	0,091	0,382	0,128

Pour $\langle \Gamma_{n1} \rangle_0$, on trouve, après ajustement, les valeurs données au tableau VI, valeurs reprises également à la figure 3.

TABLEAU VI

LARGEUR NEUTRON (eV)

($l=1$)

E (keV)	$\langle \Gamma_{n1} \rangle_0$ S-M et E	$\langle \Gamma_{n1} \rangle_0$ D-H et E	$\langle \Gamma_{n1} \rangle_0$ S-M et B
1	0	—	0
5	0	—	0,005
10	0,006	—	0,016
30	—	0,025	—
50	0,050	—	0,112
60	—	0,029	—
100	0,096	—	0,218
175	—	0,114	—
200	0,163	—	0,368
250	—	0,140	—

Calcul de la fonction Q_E

On a mené trois séries de calculs, que nous avons désignées par S-M, E (α Sampson-Molino, statistique Egelstaff), D-H, E (α Diven-Hopkins, statistique Egelstaff) et S-M, B (α Sampson-Molino, statistique Bollinger).

Tous les calculs d'ajustement de paramètres ($\langle \Gamma_f \rangle$ et $\langle \Gamma_{n1} \rangle_0$), ainsi que les calculs de détermination des sections efficaces et des moyennes nécessaires, ont été effectués sur la machine IBM-1620. Le programme a notamment comme sous-routine les intégrations numériques qui permettent d'opérer les moyennes directement sur les fonctions de distribution plutôt que d'effectuer les moyennes à l'aide d'un modèle discret. A titre d'exemple, les résultats du problème S-M, E

à 50 keV sont donnés au tableau VII. Les données expérimentales à reconstituer sont $\sigma_f = 1{,}95$ b et $\alpha = 0{,}364$. D'autre part, $\nu = 2{,}906$, $e^{(s)} = 0{,}8$, cos $2\delta_0 = 0{,}657$, cos $2\delta_1 = 0{,}999$.

TABLEAU VII

CALCUL DE Q_E A 50 keV

	$\langle \sigma_{cl} \rangle_J$ (*b*)	$\langle \sigma_{\gamma l} + \sigma_{fl} \rangle_J$ (*b*)	$\alpha^{(l,J)}$	$E_\gamma^{(l,J)}$	$E_f^{(l,J)}$	$P^{(l,J)}$	$\langle S \rangle_J$ (eV)	$Q_E^{(l,J)}$ (b^2xeV)
$l = 0$								
$J = 0$	0,525	0,178	0,318	1,840	1,983	1,471	6,62	0,598
$J = 1$	1,578	0,864	0,342	2,161	2,316	1,806	2,21	3,575
$l = 1$								
$J = 0$	0,311	0,199	0,354	1,635	1,718	1,074	6,62	0,440
$J = 1$	0,932	0,765	0,383	1,797	1,856	1,204	2,21	1,897
$J = 2$	0,780	0,653	0,386	2,638	2,746	1,886	1,32	1,268
							Total	7,778

On a consigné dans le tableau VIII les résultats obtenus pour la fonction Q_E dans les trois séries de calcul. A titre comparatif, on a également fait figurer la valeur Q_E pour l'uranium-235 telle que l'a calculée Nicholson dans le cas où l'on néglige la diffusion inélastique. Ces valeurs sont reprises à la figure 4.

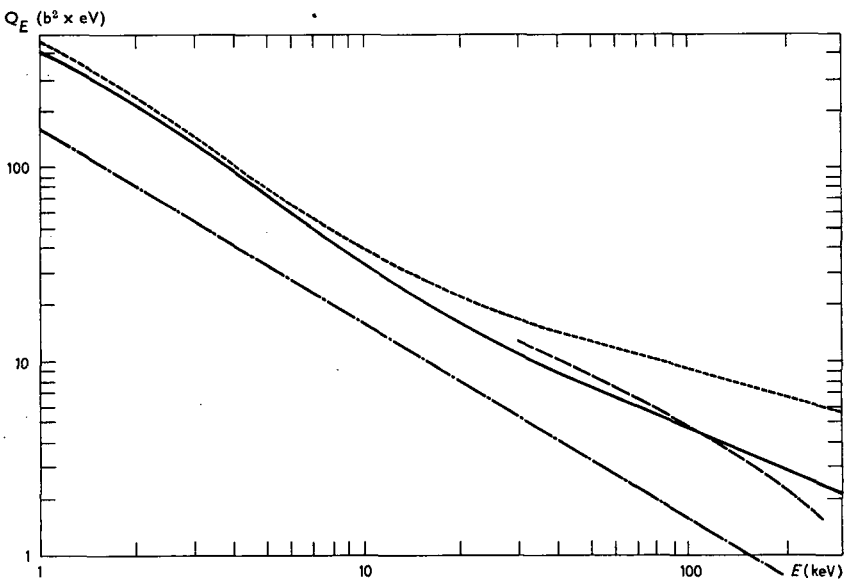


Figure 4
Fonction Q_E pour le plutonium-239.
——— S-M, E
– – – – – D-H, E
····· S-M, B
— · — · — · ^{235}U

TABLEAU VIII

VALEURS DE Q_E
(b^2xeV)

E (keV)	S-M, E	D-H, E	S-M, B	235-U
1	410	—	460	160
5	72	—	79	—
10	34	—	39	16
30	—	13	—	—
50	7,8	—	13	3,2
60	—	7,5	—	—
100	4,6	—	9,2	1,65
175	—	2,5	—	—
200	3,0	—	6,7	0,8
250	—	1,6	—	—

Conclusions

Avant toute autre chose, ces travaux nous ont permis de constater une carence profonde en matière de données expérimentales sur le plutonium-239 dans la bande d'énergie intéressant les réacteurs rapides là où l'effet Doppler se fait le plus sentir.

En ce qui concerne l'effet Doppler dans le plutonium-239, il apparait nettement plus élevé que dans l'uranium-235: un facteur 2 et plus. En outre, l'effet aux énergies élevées est relativement plus important, la fonction Q_E diminuant moins vite avec l'énergie dans le cas du plutonium.

Les différences obtenues dans les résultats correspondant aux diverses données expérimentales permettent d'émettre quelques commentaires d'ordre qualitatif.

En ce qui concerne α, malgré la grande différence apparaissant entre la courbe de Sampson-Molino et les mesures récentes de Diven-Hopkins, la fonction Q_E se modifie peu. Elle reste du même ordre de grandeur. Seule apparaît une légère augmentation dans la région où α est particulièrement faible.

Par contre, la méthode est fort sensible aux données statistiques adoptées pour le noyau ^{239}Pu aux basses énergies. Pratiquement, on constate un facteur 2 de différence entre les résultats [E] et [B]. Cette différence ne peut provenir que de la différence dans l'espacement moyen, qui est beaucoup plus grand chez Bollinger. Comme ξ_0 est pratiquement inchangé ($1 \cdot 10^{-4}$ au lieu de $1{,}15 \cdot 10^{-4}$), l'augmentation de $\langle S \rangle$ se traduit par une augmentation de la largeur réduite de diffusion $\langle \Gamma_n^0 \rangle$ sans incidence sur la section de formation d'un noyau composé $\langle \sigma_{c0} \rangle$ qui est proportionnelle à ξ_0. Cette augmentation de $\langle \Gamma_n \rangle$ entraîne finalement une réduction de $\langle \Gamma_f \rangle$ pour reconstituer correctement $\langle \sigma_{c0} \rangle$. La diminution de $\langle \sigma_{f0} \rangle$ doit être compensée par une augmentation de σ_{f1}. Ceci s'opère par le truchement d'une augmentation de $\langle \sigma_{c1} \rangle$ par réajustement de $\langle \Gamma_{n1} \rangle_0$ à une valeur plus élevée.

Cette contribution plus importance des interactions de $l=1$ augmente cons sidérablement l'effet Doppler, surtout aux énergies élevées où les contribution- de $l=1$ deviennent beaucoup plus importantes que celles de $l=0$.

Sur deux points au moins, les données de Bollinger semblent moins heureuses que celles d'Egelstaff. Elles conduisent tout d'abord à adopter des valeurs de Γ_{n1} trop élevées, qui s'écartent trop des valeurs que l'on pourrait raisonnablement estimer en multipliant les largeurs Γ_{n0} par le facteur de pénétration, Ainsi, à 50 keV,

$$\langle \Gamma_{n0} \rangle_0 = 0{,}239 \text{ eV}$$

$$\langle \Gamma_{n0} \rangle_0 v_1 = 0{,}239 \times 0{,}154 = 0{,}0368.$$

Or la valeur ajustée $\langle \Gamma_{n1} \rangle_0$ vaut 0,112 (environ 3 fois plus).

Un deuxième argument défavorable consiste en ce que la section de formation d'un noyau composé pour $l=1$ $\langle \sigma_{c1} \rangle$ est fort différente de celle que l'on obtient pour l'uranium-235. En effet, à 50 keV, $\sigma \langle_{c1} \rangle_0 = 2{,}781$ b, au lieu de 2,053 b pour l'uranium.

Les données d'Egelstaff conduisent quant à elles à

$$\langle \Gamma_{n0} \rangle_0 = 0{,}170 \text{ eV}$$

$$\langle \Gamma_{n0} \rangle_0 v_l = 0{,}0262 \text{ au lieu de } \langle \Gamma_{nl} \rangle_0 = 0{,}0503 \text{ (environ deux fois plus)}$$

$$\langle \sigma_{c1} \rangle = 2{,}023 \text{ (très proche de 2,053).}$$

RÉFÉRENCES

[1] GOERTZEL, G. *et al.*, Proc. UN Int. Conf. PUAE **5** (1956) 472.
[2] FESHBACH, H. *et al.*, *Nucl. Sci. Engng.* **1** (1956) 4–20.
[3] LANE, A. M. *et al.*, UKAEA Report AERE — T/M 137 (1956).
[4] BETHE, H. A., USAEC Report APDA — 119 (1957).
[5] NICHOLSON, R. B., USAEC Report APDA — 139 (1960).
[6] YIFTAH, S. *et al.*, Fast Reactor Cross-Sections, Pergamon Press (1960).
[7] SAMPSON, J. B. et MOLINO, D. F., USAEC Report KAPL-1793 (1957).
[8] DIVEN, B. C. & HOPKINS, J. C., Evaluation Group Newsletter No. 3 [BNL-653 (N-3)] (1961) p. 2.
[9] EGELSTAFF, P. A. *et al.*, *J. nucl. Energy* **6** (1958) 303—321.
[10] BOLLINGER, L. M. *et al.*, Proc. 2nd UN Int. Conf. PUAE **15** (1958) 127.
[11] LUK'YANOV, A. A. et ORLOV, V. V., *Atomn. Energiya* **10** (1961) 262—264.

IV. 4. SAFETY PROBLEMS

SM-18/78

A REVIEW OF THE NUCLEAR ASPECTS OF FAST-REACTOR SAFETY*

D. OKRENT
ARGONNE NATIONAL LABORATORY, ILL.,
UNITED STATES OF AMERICA

Abstract — Résumé — Аннотация — Resumen

A review of the nuclear aspects of fast-reactor safety. This paper is intended to provide a somewhat philosophical review of the progress, status and problems in fast-reactor safety. It begins with a discussion of the status of studies into the nuclear side of safety at the time of the 2nd Geneva Conference in 1958. Recent developments in the safety of moderate-sized, metal-fuelled reactors are then treated, including questions of stability, core melt-down, autocatalytic effects and explosion calculations. The philosophy applied in the choice of a "maximum accident" fur such reactors is examined.

The changes in safety characteristics which arise with different fuel element types and for rather large reactors are reviewed, including questions of zero fuel expansion, positive sodium reactivity coefficients and Doppler effect. The influence of these changed safety characteristics on design and on hazards evaluation is discussed. And finally, the outstanding safety problems presently facing fast reactors are listed.

Etudes sur la sécurité des réacteurs à neutrons rapides. L'objet du mémoire est de présenter quelques réflexions générales sur les progrès accomplis en matière de sécurité des réacteurs, sur l'état actuel de cette question et sur les problèmes qu'elle pose. Il débute par des remarques sur les résultats de sondages des aspects nucléaires de la sécurité à l'époque de la deuxième Conférence de Genève, en 1958. Les progrès récents accomplis dans le domaine de la sécurité des réacteurs de dimensions moyennes, à combustible métallique, font ensuite l'objet d'une étude, où l'auteur considère notamment le problème de la stabilité, de la fusion du cœur, des effets autocatalytiques et des calculs d'explosion. Il examine les considérations générales qui définissent un «accident maximum» de ces réacteurs.

L'auteur étudie les modifications qui interviennent dans les caractéristiques de sécurité lorsque changent les types d'éléments combustibles et que les dimensions des réacteurs deviennent assez grandes; il examine notamment le cas où la dilatation du combustible est nulle, la question des coefficients de réactivité du sodium positif et l'effet Doppler. Il étudie l'influence de la variation de ces caractéristique de sécurité dans les études de réacteurs et dans l'évaluation des risques. Enfin, il énumère les problèmes majeurs de sécurité qui restent encore à résoudre en ce qui concerne les réacteurs à neutrons rapides.

Рассмотрение ядерных вопросов безопасности реактора на быстрых нейтронах. Цель данного доклада — рассмотреть в несколько философском плане достигнутые результаты, положение дел и проблемы, связанные с безопасностью реактора на быстрых нейтронах. Вначале рассматривается положение дел с исследованиями ядерных аспектов безопасности во время второй Женевской конференции в 1958 году. Затем обсуждаются последние достижения в вопросе безопасности средних реакторов на металлическом топливе, включая вопросы устойчивости, расплавления активной зоны, автокаталические эффекты и вычисления взрыва. Рассматривается философское определение „максимального инцидента" для таких реакторов.

В докладе также обсуждаются изменения в характеристиках безопасности, которые возникают при применении различных видов топливных элементов и при увеличении размеров реакторов, включая вопросы нулевого расширения топлива, положительные коэффициенты

* Work performed under the auspices of the United States Atomic Energy Commission.

реактивности натрия и Допплер-эффект. Рассматривается влияние этих изменений характеристики безопасности на конструкцию и на оценку риска. И, наконец, перечислены нерешенные проблемы безопасности, существующие в настоящее время в отношении реакторов на быстрых нейтронах.

Examen general de los aspectos nucleares de la seguridad de los reactores rápidos. El propósito de la presente memoria es formular algunas reflexiones generales sobre los progresos realizados en materia de seguridad de los reactores rápidos, sobre el estado actual de la cuestión y sobre los problemas que ésta plantea. Comienza con un examen del estado de los estudios sobre los aspectos nucleares de la seguridad, en la época en que se celebró la Segunda Conferencia de Ginebra (1958). Luego, pasa a tratar los recientes progresos en materia de seguridad de reactores de medianas dimensiones con combustible metálico, incluyendo los problemas de estabilidad, fusión del cuerpo, efectos autocatalíticos y cálculos de explosiones. Examina el criterio que sirve de base a la definición del "accidente máximo" para tales reactores.

El autor estudia seguidamente las modificaciones que las características de seguridad experimentan al utilizarse diferentes tipos de elementos combustibles y al aumentar las dimensiones de los reactores; examina en particular el caso en que la dilatación del combustible es nula, así como la cuestión de los coeficientes positivos de reactividad debidos al sodio y la del efecto Doppler. Discute la influencia de la variación de esas características de seguridad sobre el estudio de los reactores y sobre la evaluación de los riesgos que su funcionamiento entraña. Por último, enumera los problemas de seguridad aún no resueltos que se plantean actualmente en las instalaciones de reactores de neutrones rápidos.

Status of metal-fuelled reactors in 1958

An extensive review of nuclear accidents in metal-fuelled, fast power reactors was presented at the 1958 Conference on Peaceful Uses of Atomic Energy by McCarthy *et al.* [1]. The major effort in the United States up to that time had been devoted to the fairly small (60 l) EBR-II [2, 3] and the medium-sized (330 l) FERMI reactor [4]. Both use single-piece, cylindrical pins of partly enriched uranium for their basic fuel element, at least in the first loading. Hence, the problems studied in detail and the conclusions drawn at the time reflected the needs of a limited type of fast reactor.

Early considerations of the safety of fast reactors had centered about the possibility of an explosive energy release occurring in the core, due either to large and sudden external reactivity insertions or to the presence of a positive power coefficient which was both strong and closely coupled time-wise to the power.

The study of possible causes of positive power coefficients which were strong enough to cause rapid internal reactivity insertions had been stimulated considerably by the demonstrated autocatalytic behaviour of the EBR-I [5, 6] under certain conditions. By 1958, however, theoretical and experimental work indicated strongly that the Doppler effect would be small in EBR-I, EBR-II and FERMI [7, 8, 9]. And bowing and other mechanical distortions which might add substantial reactivity were thought to be in hand, once the necessity of controlling or limiting fuel-element motion had been recognized.

Axial expansion of the one-piece metal fuel pin was calculated to give a substantial prompt negative reactivity coefficient in both EBR-II and FERMI. As designs for these reactors evolved, it appeared clearly possible to preclude beyond reasonable doubt the sudden insertion of considerable reactivity by the

control or fuel handling systems. Even if all instrumentation were inoperative, the small reactivity addition rates possible coupled with the prompt shut-down coefficient of the fuel led to relatively weak nuclear excursions. The first burst in the typical calculation was insufficient to cause melting [1, 3], and any secondary bursts began with so high a power level that little reactivity could get in beyond the prompt critical point.

The major nuclear safety problem facing these reactors was believed to be associated with core melt-down. Initial analyses of melt-down accidents usually assumed the rapid or instantaneous loss of coolant following a high power run. Melting followed, due either to reduced fission power or to fission-product decay heat. The fuel in the core was further assumed to reassemble rapidly into a more dense configuration under gravity [10]. The violence of the explosive disassembly then depended on assumptions regarding the equation of state and the conditions governing rate of reassembly.

The rapid loss of sodium accident soon lost favour, as the designers built extra safeguards into the coolant containment system. Such engineering care is a practical necessity. Even if the public safety is not threatened by a nuclear explosion, the economic loss incurred in a major core melt-down is formidable.

Undaunted, the theoreticians hypothesized temporary sodium loss from the core by boiling, and calculated large reactivity addition rates by adopting pessimistic assumptions on the effects of gravity or re-entering coolant [3, 11]. Armed with these large reactivity insertion rates, they calculated substantial energy yields and high pressures in the process of explosive disassembly. And while other studies had shown the reactor plants to be capable of containing fairly substantial explosions if designed appropriately, there was little or no margin of safety.

Thus it seemed vital to gain some understanding of the detailed course of core melt-down. To be able to state definitively the path in space and time of all the fuel under any conditions is beyond hope. But a better qualitative understanding of the important phenomena leading to dispersal or reassembly was an essential goal. And a better definition of the operating abnormalities or accidents which would lead to core melt-down was needed.

Reactor instability had been feared earlier as a prime source of overheating and melt-down. The unstable behaviour of the Mark-II core of EBR-I [6] had led to some suspicion that all fast reactors were subject to major unknowns in their dynamic behaviour. However, the predictably stable behaviour of the "rigid" Mark-III core and the partial theoretical understanding of the earlier instabilities had eased this concern. It was clear that bowing and fuel distortion had to be controlled, and that reactor kinetic behaviour needed careful analysis and experimental confirmation. But instability did not represent a universal danger of core melt-down accidents.

Recent developments in safety of moderate-sized metal-fuelled reactors

Reactor stability

The experimental kinetics programme on the Mark-III core of EBR-I during the intervening years has added to the belief that fast-reactor stability does not represent a basic safety problem. The dynamic characteristics of this reactor have been mapped out in great detail [12]. Different types of experiments correlate one with the other. And calculations are in rather good agreement with most

of the data. For example, LONG [13] has computed the amplitude and phase of the temperature at various positions in the fuel and coolant and obtained very good agreement with experiment. CARTER *et al* [14] have combined a theoretical treatment of the heat transfer with a semi-empirical formulation of some non-linear effects requiring only two parameters and explained a long series of experiments quite well. They later formulated a theoretical model for the semi-empirical assumptions.

When experiments were run in which controlled bowing of fuel elements was allowed, the reactor behaved as expected. Superimposed on the feedbacks originally seen was a small positive reactivity effect with a time constant typical of a fuel pin. And the results were in reasonably quantitative agreement with calculation [15].

It is also gratifying that good experimental and theoretical evidence of the physical phenomenon causing instability in the Mark-II core of EBR-I has been found. The unstable behaviour of this reactor core had long been attributed to a strong negative feedback with a long time constant [16, 17]. However, no acceptable physical cause of this mathematically required reactivity effect had been proven. Recent out-of-pile experiments by SMITH and MATLOCK [18] have shown that an appropriate type of dishing of the upper support plate does take place in the presence of temperature gradients. This motion can lead to fuel rod motions and reactivity effects of the type needed. Applying the theoretical techniques developed by STORRER [19], they have obtained very good agreement on phase of the required feedback, rough agreement on amplitude. They can also explain the absence experimentally of a second, smaller resonance at higher frequencies, a fault found with earlier analyses.

Theoretical analyses of the stability of the EBR-II reactor [20] and the FERMI reactor have found both these reactors to be very stable, within the assumptions of the calculation. Generally, the methods detailed by STORRER [19] have been applied to the calculation of radial and axial temperature amplitude and phase distributions. These temperatures, or approximations thereof in the form of solutions of a lumped thermal model, are converted into reactivity feedbacks for use in analogue computer solutions of the kinetic equations.

The major uncertainty in these studies seems to be in the specification of bowing, distortions or other non-linear motions which do not lend themselves to precise calculation. The design philosophy in both EBR-II and FERMI is to limit the amount of reactivity associated with bowing to an unimportant amount. Considerable analyses and out-of-pile testing leads the designers to expect actual kinetics experiments to demonstrate that this control has been achieved satisfactorily. It is worth noting that an early fuel sub-assembly design for one of these reactors proved to be rigid initially, but to develop appreciable clearances due to creep when tested out-of-pile under temperature and flow.

Clearly the problem of controlling fuel and blanket-element bowing and distortion is a continuing one in fast reactors, although very large systems should be less sensitive to such motions. The possibility of deliberately allowing bowing to occur in such a way as to provide a negative power coefficient remains. And the question of just what happens with completely unrestrained fuel pins needs to be answered. In one series of EBR-I Mark-III experiments, only very slight reactivity effects were observed upon such fuel-pin release [12].

One can anticipate the need of non-linear feedbacks for bowing effects [21] if theory is to match experiment. More important, it may be possible to monitor

the changing character of reactor behaviour with burn-up by analysis of kinetics experiments. Routine transfer function measurements throughout the reactor lifetime are probably well-justified.

Melt-down problem

The major effort in the core-melt-down problem has been directed toward the experimental programme in TREAT [22, 23, 24]. This pulsed, graphite-moderated, homogenous reactor is a unique tool designed and constructed by Argonne to meet the acute need for a facility permitting in-pile testing of fast-reactor fuel elements to failure. It can safely provide twice the flux needed to melt a natural uranium pin over a large test section. The available pulse times range from 100 ms to 30 s. Unusual access for photographic or other purposes is permitted.

The experimental programme in TREAT began in September, 1959. Since that time about 100 test samples have been heated by fission, mostly to destruction [25, 26, 27]. Most of these experiments have been performed on samples surrounded by an inert atmosphere, contained in an opaque capsule. Information on the effect of overheating was derived primarily by post-mortem examination. Recently, other experimental techniques, including melt-down in transparent containers permitting high-speed photography, and melt-down in the presence of sodium, have been developed and put into use.

Experiments to date have been run on the sodium-bonded, stainless steel jacketed EBR-II fuel pin [2], on the zirconium-clad, co-extruded FERMI pin [4], and on EBR-II pins jacketed with the refractory metals tantalum and niobium. The major area of the melt-down problem to which real knowledge has been added is the mechanism of fuel-element failure.

The EBR-II pins are observed to fail primarily due to dissolution of the steel by uranium over broad areas. The FERMI pins fail from dissolution of zirconium, sometimes locally, unless sudden expansion leads to fine cracks in the cladding. The tantalum and niobium pins fail from a build-up of internal pressure, arising from sodium bond vapour and compression of the inert gas atmosphere utilized during fabrication. Analysis of the different kinds of failure is in agreement with experiment [26, 27]. Of course, these observations were all made in the absence of a flowing sodium coolant. If temperature distributions are changed enough, some of these results may be altered.

In the absence of sodium, the uranium runs down under the influence of gravity upon escape from a FERMI element, with little lateral velocity. The same effect occurs in an unbonded EBR-II fuel pin. In the sodium-bonded pins, however, once the energy input passes a certain threshold, the uranium is pushed out laterally at high velocity by the vaporized bond. And experiments in which a buckled flux was impressed axially on the sample always produced failure at the hottest point, with all the uranium in the pin escaping the pin at this position.

The behaviour of the sodium-bonded pins upon failure immediately suggests a new hazard not previously considered in safety studies [1, 3]. Namely, the rapid expulsion of all the fuel in the pin by the sodium bond might occur coherently over a large number of fuel pins. This provides an obvious mechanism for obtaining a rapid rise in reactivity if failure cocurs near the horizontal centre line. Even if failure occurs near the top of the reactor, a sufficient number of pins failing simultaneously at this position can make for a new assembly which is supercritical.

Calculations for EBR-II [28] show that 3% $\delta k/k$ is available if the seven central sub-assemblies fail and send all their fuel to the core centre. If all 61 sub-assemblies did so, 14% $\delta k/k$ is available. Obviously the time scale and position of pin failure must be known, if one is to make a realistic appraisal of the hazard. And the mechanism of pin failure becomes vital in analysing the situation.

Actually, until one performs some detailed analysis of the course of pin failure in typical accidents, there is no basis for estimating the hazard. Large quantities of reactivity can be generated with any assumption which leads to the assembly of most of the fuel in a more dense configuration. Gravity has been calculated to perform the same role [3].

There may be other, unexamined phenomena which will play a major role in the bond-induced accident. For example, with relatively slow rates of overheating, fuel element swelling under the action of fission product gases may be sufficient to burst the jacket in advance of fuel melting. The melting fuel would no longer be subject to high sodium-vapour pressures and its escape from the jacket would take a different course.

However, reactors having sodium-bonded fuel clearly must be carefully examined for hazards from this mechanism.

A question not studied in the TREAT programme as yet is the effect of flowing sodium on the dispersal of the molten fuel. Some interesting model studies with a lead-water system by MORELLE [29] cast some light on the situation. Using system parameters scaled to resemble the FERMI reactor, he observed the transient behaviour and oscillations of the water when the heat input in steel pins exceeded the point of stable flow. The lead pins were then heated electrically to melting in the presence of such oscillations and the motion of the molten lead globules photographed. The limited series of experiments exhibits flow reversal, partial dispersal of the fuel (lead) and other effects which must be predictable theoretically in order to have a basic grasp of the melt-down problem.

Another experiment of interest was the rapid electrical melting of pre-irradiated FERMI fuel pins [30]. The foamy appearance of the escaping fuel, its frothy rather than dense nature, should have a considerable effect on the time and manner of its redistribution. Fuel density has a large influence both on available excess reactivity and the course of any explosion resulting from reassembly. Also, its failure to escape at high velocity, as did the fuel under action of bond sodium vapour in EBR-II pins, is of real interest.

A final region of out-of-pile experiments concerned with the melt-down problem dealt with the attack of various potential jacket materials by molten fuel, in this case uranium. MACINTOSH and BAGLEY [31] have reported one series of experiments of this type, but more data are needed. Unfortunately, further experimental data does not always simplify life for the theoretician. Some recent measurements on the penetration of stainless steel by molten uranium [32] have not shown dissolution rates increasing monotonically with temperature. Just above 1150 °C the rate of attack drops abruptly by a factor of $\sim$3, then starts increasing again slowly. Quantitative prediction of the time for pin failure by dissolution in an accident thus becomes very complicated.

AUTOCATALYTIC EFFECTS

The major effort in this area involved a study of melt-down configurations on the fast critical facility, ZPR-III. Two rearrangements of the FERMI reactor which simulated hypothetical reassembly of the molten fuel under the influence

of gravity were constructed [33]. Measurements of the power distribution and the worth of fuel as a function of position made it clear that these particular configurations would have no tendency to gain reactivity due to the fuel movement during the initial phases of disassembly in an explosion.

A further interesting aspect of this work was the reasonably good success encountered in calculating these experiments by two-group, two-dimensional diffusion theory [34]. This method shows sufficient promise to warrant its use for examining other potentially autocatalytic configurations on paper. Nicholson [35] has devised a simplified two-dimensional explosion code which examines the early phases of such explosions, using power and reactivity distribution curves as basic input information.

Nicholson [36] also has published an extension of earlier work on the Doppler effect. The effect has been calculated at lower neutron energies than before, and a new method for handling U^{238} at these lower energies was presented. In addition the effect of inelastic scattering was allowed for. Using these techniques and a new review of available resonance parameters, he once again calculates the effect to be small for the FERMI reactor, just as was predicted earlier [7].

Explosion calculations

By 1958, fairly good analytical methods of calculating the energy generated in a fast reactor were available [10, 1]. This method was subject to certain errors introduced by simplifying hypotheses; hence, accurate numerical solutions of the coupled hydrodynamics-neutronics equation had also been developed [37, 38]. Since then, effort has continued on improvements in the analytic technique and on an evaluation of its errors [39]. The influence of power distribution on energy yield has been examined [40], as have various other parameters [39]. Generally speaking, the calculation methods for spherical geometry are thought to be in fairly good shape, relative to our knowledge of initial accident conditions.

The equation of state for uranium, plutonium and other material of interest to fast reactors in the range 5000 to 50000 °C, at densities below normal and pressures up to hundreds of thousands of atmospheres, represents a big uncertainty in the calculations. The published efforts on uranium centre mainly about estimates of the critical temperature and pressure, and the vicinity thereof [41, 42]. Considerable doubt remains as to the prescription for the full region of interest. For example, the published work does not allow for ionization effects at higher temperatures, and some recent estimates indicate these to be important [43].

Philosophy toward maximum accident

As the design and construction of EBR-II and FERMI have progressed, safety considerations have been constantly in mind. Both reactors have double or triple walled containment of the primary sodium to guarantee against loss of coolant. Both have good shut-down cooling characteristics. Both have multiple instrumentation and low rates of reactivity insertion. Both have prompt and delayed negative reactivity coefficients. Both have sharply limited bowing. Both are expected to be very stable. And both have very good blast containment features.

Minor blockage of fuel elements due to swelling or foreign material in the coolant has been analyzed in each reactor and seems innocuous. And it is not anticipated that swelling could take place rapidly and simultaneously in all assemblies.

Thus, it is nigh impossible to postulate a "credible" chain of events which would lead directly to rapid, large-scale melt-down in these reactors. If it becomes possible to show that any local melting cannot spread rapidly, leading to large-scale melting, these reactors may have side-stepped the core melt-down problem, without proving that melt-down entails no dire consequences, once in motion.

The above approach suffers from one drawback, however. One needs to assume that all avenues have been explored and that all solutions and final designs are correct. But actual testing and proof is not always possible. In one early reactor design the secondary containment of the primary sodium coolant suffered from a basic design fault, such that one pipe failure automatically led to failure of the second, and escape of the coolant. This was noticed and corrected by the designers at an early stage. But the fact that it did occur makes one ask if the loss of coolant accident can be completely disregarded in evaluating the hazards of future reactors. Similarly, in careful testing of preliminary sub-assemblies, bad flow distributions which could have led to overheating have been uncovered. Already mentioned is the creep phenomenon, which would have permitted fuel element bowing some time after initial operation and testing of the particular reactor.

That these various design faults have been found and corrected is both reassuring and disturbing. It demonstrates the care with which these two reactors are being built. But it suggests that flaws in future reactors may get through undiscovered.

Hence, an appropriate philosophy in evaluating reactor safety might best be as follows.

Assume that the design does fail in the various supposedly incredible ways, i.e. loss of the sodium coolant, and study the hazards resulting therefrom. If the particular reactor design contains autocatalytic features or other built-in mechanisms which automatically make for a real threat to the public safety in this event, much greater assurance of the incredibility of the original assumption will be required.

In the case of EBR-II, the expulsion of molten fuel by sodium-bond vapour may be such a mechanism. Detailed study of the mode of failure for irradiated fuel and for fuel in the presence of flowing sodium is needed. Also needed are detailed calculations of the relative times of pin failure and fuel escape throughout the core in the event of an over-heating incident. In this way it may be learned that this mechanism really poses no threat to public safety. If the contrary opinion develops, however, proof that core melt-down on a major scale is truly impossible must be put to a very vigorous test indeed.

Changed safety characteristics of fast reactors having greater size and/or different fuel elements

Different types of fuel elements

Among the fuel elements being given serious consideration for fast power reactors at present are the restrained metallic fuel element, the oxide, the carbide and the cermet of enriched-uranium oxide (or plutonium oxide) particles in a matrix of steel or U^{238}. The primary fissionable materials include Pu^{239} and U^{233}, as well as U^{235}, and thorium is considered as a fertile diluent both for U^{233} and Pu^{239}. Various other material compositions have been and will continue to be suggested, and various geometric forms of the solid-fuel reactor may be considered. Attention herein will be given only to those materials just itemized, and the pin shape will generally be assumed if some mental geometric picture is needed. Work

continues on the molten-plutonium-fuelled reactor [44], but its safety characteristics will not be discussed here.

The restrained metallic fuel concept is an outgrowth of the swelling problem encountered with the "weakly jacketed" uranium or uranium-plutonium metallic fuel element. At fairly low burn-ups and at undesirably low metal temperatures, the EBR-II and FERMI fuel pins exhibit excessive swelling due to fission product gases. Plutonium-uranium metal acts worse in this respect than uranium, due to its lower melting point and its poorer strength at elevated temperatures. Primary emphasis in work on a metal-fuel element has thus turned to the concept of "weak-fuel, strong can." If this fuel element type proves to be successful in irradiation tests, it will make for very great interest in metal-fuelled fast reactors, since the pyrometallurgical methods offer considerable promise of inexpensive reprocessing and reconstitution [45, 46].

Such a reactor will have different dynamic nuclear characteristics from the FERMI or EBR-II reactors using their reference design fuel elements. The restrained fuel cannot be expected to grow axially or radially in a super-prompt critical excursion. And with little heat escaping into the clad or coolant, only the Doppler effect will be available for reactivity feedback prior to cladding failure. Its kinetic characteristics in regard to stability may also be influenced somewhat, since appreciable heat may have to flow before significant feedback from the jacket or coolant is available. The conditions of fuel-element failure and the dispersal or reassembly of fuel in a melt-down will remain difficult problems to solve, very sensitive to specific design.

The cermet-type fuel element will probably resemble the constrained metal-fuel element in some of its safety characteristics. In a prompt excursion most of the heat will remain in the oxide particles, and there may not be any prompt expansion effect. If the fuel is in the form of a highly enriched uranium or plutonium oxide, the Doppler effect will presumably be positive. In this case, a total Doppler effect between operating and uranium boiling temperatures of only 0.001—0.002 $\delta k/k$ can make any accident which just reaches the prompt critical region into a fairly violent reactor explosion.

Ceramic fuels, such as the carbide or oxide, pose a new set of unknowns regarding reactor safety. Since cracks and other types of structural deviations have been observed upon irradiation in both types, there is little guarantee that the fuel can be expected to expand axially in a prompt burst. Actually, one needs to examine closely whether some mass-transport phenomenon can relocate the fuel appreciably during irradiation. Should a low-density region develop near the middle of the fuel pin (in an axial sense), expansion conceivably could add reactivity during a prompt burst. A cracking and ratcheting of the fuel also might lead to the same effect.

A second problem, which has been mentioned particularly in connection with the oxide fuel, is that of fuel slumping [47]. If the ceramic is used at much below theoretical density, or if clearances and free space in the fuel element provide the equivalent effect, melting of the fuel may lead to a slumping within the individual jackets. The increase of fuel density in the foreshortened reactor can add considerable reactivity to the system, as much as 0.13 $\delta k/k$ having been calculated for a fuel of 65% original density [47].

The oxide and carbide fuels may also produce altered kinetic and stability characteristics. The low thermal conductivity of the oxide will increase some time

constants of the system; if this is coupled with a negligible prompt-reactivity effect from the fuel itself, the response to ramp reactivity insertions or oscillations may be appreciably more sluggish.

Sodium void coefficient

The major new observation in the past few years pertinent to fast reactor safety was that of Nims and Zweifel [48], concerning the possibilities of a positive sodium void coefficient in large fast reactors. The introduction of the sodium into the reactor adds reactivity by reducing neutron leakage. In most, if not all, large, plutonium-fuelled systems, it also tends to lose reactivity by its moderating effect. This results from the decrease in the number of neutrons emitted per absorption in all core material with a reduction in neutron energy. In smaller systems, the effect on neutron leakage predominates. In large systems, the situation may reverse. And it is particularly likely that in the inner portion of the core the local sodium void coefficient will be positive, even if the coefficient for the entire core is negative.

A considerable number of parameter studies and specific design calculations of the sodium void coefficient have followed this observation [47, 49—54]. Important new computation techniques for treating the elastic resonance scattering in sodium accurately have been developed [55, 56]. The coefficient is clearly a function of size and composition of the core [51]. It varies with plutonium isotopic composition [52]. And it is very poorly known.

Some rough calculations, assuming a reactor core which is 25% fuel, 25% steel and 50% sodium by volume, found the threshold core volume for a net positive sodium coefficient to be 1200 l for the metal fuel, 2000 for the carbide and 3000 for an oxide of low density. The introduction of niobium structure for the steel sharply reduced all thresholds by a factor of 2 or 3 [51]. Other calculations have shown the coefficient to become appreciably more positive with the build-up of large amounts of Pu^{240} [52] or with increases in the concentration of U^{238} present [47].

The sodium void coefficient will depend strongly on the neutron capture cross-sections of all the materials in the core, many of which are completely unmeasured. It will depend on the detailed neutron energy spectrum, with allowances for self-shielded resonances, as well as gross structure. And it will be a function of the specific engineering reactor design. Hence, the specification of the reactor size at which it changes sign or an accurate determination of its magnitude is impossible with any certainty.

If one could always stay below the reactor size at which this coefficient becomes positive, the problem would not be severe. However, design studies are currently being made for reactors well beyond this threshold [47].

The problem does not seem to be as severe for U^{233}-Th-fuelled reactors [51]. The lesser variation of $\eta(E)$ (the neutrons emitted per absorption) for U^{233} with energy makes for a smaller spectral effect. Also, reactors cooled with a Pb-Bi are less likely to suffer from a positive coefficient [51].

Doppler effect

Earlier calculations of the Doppler effect in fast reactors concerned themselves primarily with neutrons of $\sim$100 keV energy [7, 57, 58]. Neutrons much above this energy contributed little, while few neutrons much below this energy were to be found in the EBR-II or FERMI reactors.

Nicholson [36] and GREEBLER *et al.* [47] have extended the calculational technique to lower energies and applied the methods to larger reactors. Greebler has calculated the Doppler effect of a series of large, oxide-fuelled reactors, having a plutonium composition of 2/3 Pu^{239}, 1/3 Pu^{240}. Within the assumptions of the calculational method and data, the overall Doppler coefficient is found to be negative and large ($\sim 10^{-5}$/°C). The negative effect of the U^{238} overrides the positive contribution of the Pu^{239}, but the latter is appreciable. About 75% of the effect results from neutrons between 300 and 4000 eV.

Over a group of large, oxide-fuelled reactors Greebler found that the total Doppler coefficient from room temperature to infinity, calculated using the reactor spectrum with sodium present, generally exceeded the total reactivity to be gained by a complete loss of sodium. However, the reactivity held down by the Doppler effect between the fuel operating temperature of 900 °C and the melting point of 2750 °C was frequently less than the sodium effect.

The reduction in Doppler effect which would accompany loss of sodium was not calculated, but one can expect it to be appreciable.

OTHER ASPECTS

A major difference between the EBR-II and FERMI reactors on the one hand, and the plutonium- and U^{233}-fuelled reactors of the future, will be in the delayed neutron fraction. A typical effective delayed neutron fraction in these U^{235} fuelled systems was 0.007 or 0.008. Since Pu^{239} and U^{233} have delay fractions near to 0.002, a considerable shift in the margin of control is inevitable. In U^{233}-Th systems, the low fission cross-section of thorium means that the combined delay fraction will remain small, despite the very large contribution per fission from thorium. In Pu-U^{238} systems, the contribution of the U^{238} will be more significant, and combined delay fractions $\sim$0.30—0.45 can be expected.

In either system, as recycle increases the fraction of higher isotopes present, the delay fraction will increase, fortunately [52]. But not very much. The change with burn-up may prove to be an operational nuisance, in that the zero power transfer function will change. This may introduce calibration difficulties into routine measurements of the transfer function.

Of course, the changes in isotopic composition with burn-up and/or recycle will also make for changes in the Doppler coefficient [47] and the sodium reactivity coefficient [52].

Another way in which some future designs will differ from reactors under construction lies in the excess reactivity requirements. The reactivity in control rods required to maintain criticality with burn-up will depend sharply on the internal breeding ratio of the particular design, and the frequency of partial unloadings. U^{233}-Th systems as a class will have a considerable reactivity hold-up in the protactinium intermediary of the chain:

$$Th^{232}\,(n,\gamma)\,Th^{233} \xrightarrow[2.33\,m]{\beta^-} Pa^{233} \xrightarrow[27.4\,d]{\beta^-} U^{233}.$$

In one particular design, as much as 4% $\delta k/k$ could be held up in the protactinium [59]. With partial unloading every 15 days, there was still a decrease in reactivity of 0.6% due to protactinium build-up.

Similarly, a reactor having a large negative Doppler coefficient might easily require more than 1% $\delta k/k$ (or 3 dollars) [47] to get from sodium melting temperature to operating temperature.

Hence, the once traditional statement that fast reactors have low excess reactivity requirements may well not be universally applicable. The possibility of holding the excess reactivity in the hot, clean reactor to less than a dollar seems remote in any event.

Influence of changed characteristics on design and on hazards evaluation

Some general comments

The changed reactor characteristics having the greatest bearing on safety include:

1. The possible loss of the prompt negative reactivity coefficient.
2. The possible introduction of a prompt positive reactivity coefficient in certain designs.
3. The possible positive sodium void coefficient, locally or net over-all.
4. The reduced delayed-neutron fraction.
5. Probable increased excess reactivity requirements.
6. New modes of generating accidents, i.e. slumping, ratcheting, etc.
7. The potentially significant changes in reactivity feedbacks, modes of fuel-element failure, etc., with burn-up.
8. The altered time response of reactivity feedback in some designs.

The reaction to the new hazards problems posed by the changed nuclear characteristics has varied. Calculations indicate that a metal-fuel element jacketed with niobium or molybdenum can lead to a positive sodium void coefficient in reactors having a core volume ~500—600 l [51]. The exact volume depends on the details of geometry and composition. This is not a large reactor by present day conceptual standards. Yet, metallurgists are working on the development of such a fuel element, among others. Similarly, the possibility of large reactivity gains from slumping has not ended work on lower density oxide fuel elements.

In one design study on large oxide-fuelled reactors, the positive sodium coefficient was accepted as a fact of life, and a search was made for a reactor having a sufficiently large negative Doppler coefficient to override it [47]. Calculations therein and elsewhere [52] show that a negative sodium coefficient may be attainable for the same-size reactor if the volume fraction of sodium is increased to 0.6 or 0.7 at the expense of U^{238} and steel, but the study lays emphasis on a design with decreased sodium volume fractions. Of course, reducing the content of U^{238} in the core lowers the internal breeding, and hence makes for greater reactivity losses with burn-up.

Clearly, not all development work is safety-oriented. And clearly, there are frequently conflicting requirements imposed by safety considerations. But it may be vital to clarify certain general facets of fast-reactor safety and to adopt some guiding principles of design. For example, can or should one tolerate a fast reactor with a non-negligible prompt positive coefficient, even if the "maximum credible" accident appears to be containable? And should one deliberately design and build a fast reactor with an appreciable positive sodium void coefficient?

Neither the sodium coefficient nor the Doppler effect are well-known at present. Both are subject to considerable uncertainties and even a lack of basic cross-section data. Both will be sensitive to the detailed neutron-energy spectrum, the

variations in capture cross-section with energy of all isotopes present, and to the actual isotopic composition. If the critical mass were not accurately predicted, both could be changed appreciably by a change in fuel enrichment after the final critical experiment had been performed. If a delicate balance exists between the needs of safe design and the physically available Doppler and sodium reactivity coefficients, may not the flexibility of reactor design be restricted far more than is desirable? And is it not likely that in the absence of any initial guiding principles of hazards evaluation, the development of a design satisfactory from other considerations will lead to safety standards tailored to fit the reactor?

The latter question warrants considerable thought. It is likely that economic performance and safety considerations may conflict for many possible fast-reactor concepts. For example, to gain very high burn-ups in the oxide-fuel element, it may be necessary to use low-density fuel, subject to slumping. To obtain infrequent reloading, it may be necessary to achieve high internal breeding, with a consequent positive sodium coefficient. If a fuel element having satisfactory economic performance is found, its developers are certain to be swayed by arguments that its hazard characteristics are good enough. But what is "good enough"?

Some specific comments

1. The introduction of reduced delayed-neutron fractions, of zero or positive prompt-reactivity feedbacks, of longer time coefficients and of possibly positive sodium void coefficients requires a re-examination of the behaviour of various reactor designs in ramp reactivity insertion rate accidents. The conclusion previously drawn for metal-fuelled reactors with prompt negative feedback, that accidents involving the improper insertion of control-rods or fuel elements do not seem to represent a major hazard, may no longer hold. If the direct consequence of such an accident at its worst is not just melting but an explosion, new and greater attention in this area will be required.

2. The excess and total reactivity requirements of the control-rods needs to be evaluated as a function of reactor design and operation. The relation between available reactivity and the probability of a bad accident needs to be examined. Inevitably, some balance between economic factors and safety will be struck.

3. The stability of various new designs needs to be re-examined in the light of reduced delayed-neutron fractions and a different class of reactivity feedback amplitudes and phase relations. Since stability $\sim$delay fraction [17], other things being equal, one can anticipate less margin of safety, *a priori*.

4. A distinction between overall and local values of the sodium (and other) coefficient may become important. A positive sodium coefficient from the inner (high power) region of the core may introduce important perturbations in studies of stability or ramp-insertion type accidents.

5. The manner of fuel-element failure upon overheating and the characteristics of a melt-down accident will have to be studied. One will have to ask if there are new mechanisms, such as extremely rapid swelling of cladding or jacket materials under fission-gas pressure, which could lead to a large scale melt-down. And can a local melt-down rapidly spread?

6. For some designs, studies will be required of explosions which are somewhat autocatalytic in their early stages. That is, a prompt positive reactivity feedback in the solid or molten stage may exist.

Outstanding problems in fast reactor safety

1. The two major needs are precise determinations of the sodium reactivity coefficient and of the Doppler effect for each and any reactor.

2. The extent to which and the conditions under which positive components of the reactivity feedback can be tolerated combine to form another major need.

3. The question of what happens in a core melt-down becomes a greatly expanded need. As the number of fuel-element types multiplies, the overall problem increases in scope. Some of the major facets are: (*a*) What is the behaviour of the sodium coolant during a thermal transient? (*b*) What are the modes of failure of the various types of fuel elements? (*c*) What is the manner of fuel motion following jacket failure? (*d*) Can one design each specific reactor so that melt-down will not occur coherently and a rapid reassembly is impossible?

4. The true behaviour of the various fuel element types prior to melting during thermal transients, together with the resultant reactivity effects, is the basis of another problem.

5. The quantitative prediction of fuel bowing and distortion remains a problem. Can these phenomena be applied desirably in a controlled fashion?

6. The equation of state for the various reactor materials at temperatures, pressures, and densities generally associated with fast-reactor explosions remains an open question. What is the available work from such explosions and its damage potential?

7. Finally, the possible contribution of the coupled-reactor concept to fast-reactor safety needs careful study. The practicality of the design of large, coupled fast-thermal reactors should be resolved. The relative safety of coupled and all-fast reactors needs to be ascertained. The possibility that the coupled-reactor concept may render acceptable reactor fuel elements judged unsafe in the all-fast application should be evaluated.

REFERENCES

[1] McCARTHY, W. J. *et al.*, Proc. 2nd UN Int. Conf. PUAE **12** (1958) 207.
[2] KOCH, L. J. *et al.*, Proc. 2nd UN Int. Conf. PUAE **9** (1958) 323.
[3] KOCH, L. J. *et al.*, ANL-5719 (1957).
[4] AMOROSI, A. and YEVICK J. G., Proc. 2nd UN Int. Conf. PUAE **9** (1958) 358. See also APDA-115.
[5] LICHTENBERGER, H. V. *et al.*, Proc. UN Int. Conf. PUAE **3** (1955) 345.
[6] THALGOTT, F. W. *et al.*, Proc. 2nd UN Int. Conf. PUAE **12** (1958) 242. See also ANL-5577.
[7] BETHE, H. A., APDA-119 (1957).
[8] KATO, W. Y. and BUTLER, D. K., ANL-5809 (1958).
[9] BAKER, A. R. and JAQUES, T. A. J., AERE R/M 168 (1958).
[10] BETHE, H. A. and TAIT, J. H., UKAEA-RHM (56)/113 (1956). See also NDA 14—82.
[11] DAANE, R. A., MELA, R. and DE FELICE, J., NDA 14—107 (1955).
[12] SMITH, R. R. *et al.*, ANL-6266 (1960).
[13] LONG, J. K., *Trans. Amer. nucl. Soc.* **2** (1959) 214.
[14] CARTER, J. C., SPARKS, D. W. and TESSIER, J. H., *Nucl. Sci. Engng* **8** (1960) 326. See also ANL-6124.
[15] OKRENT, D., Proc. Conf. Transfer Functions and Reactor Stability, ANL-6205 (1960). Also P. CLAUZON, unpublished.
[16] KINCHIN, G. H., AERE RP/M 83 (1956).
[17] BETHE, H. A., APDA-117 (1956).
[18] SMITH, R. R. and MATLOCK, R. G., *Trans. Amer. nucl. Soc.* **4** (1961) 71 and 12.

[19] STORRER, F., APDA-132 (1959).
[20] HUMMEL, H. H. and BRYANT, L. T., *These proceedings*, Vol. **III**, p. 107.
[21] SANDMEIER, H. A., ANL-6014 (1959).
[22] FREUND, G. A., ISKENDERIAN, H. P. and OKRENT, D., Proc. 2nd UN Int. Conf. PUAE **10** (1958) 461.
[23] FREUND, G. A. *et al.*, ANL-6034 (1960).
[24] OKRENT, D. *et al.*, ANL-6174 (1960).
[25] DICKERMAN, C. E., SOWA, E. S. and OKRENT, D., *Nucleonics* (April 1961).
[26] DICKERMAN, C. E., OKRENT, D. and SOWA, E. S., *These proceedings*, Vol. **III**, p. 171.
[27] DICKERMAN, C. E. *et al.*, ANL-6334 (to be published).
[28] LOEWENSTEIN, W. B. and MAIN, G., Unpublished (ANL).
[29] MORELLE, J., Unpublished (APDA).
[30] FREAS, D. G. *et al.*, BMI-PRDC-656.
[31] McINTOSH, A. B. and BAGLEY, K. Q., *J. Brit. nucl. Conf.* (1958) 15.
[32] WALTERS, C. M. and KELMAN, L. R., Unpublished. See also ANL-6355, 43.
[33] GEMMELL, W., *Trans. Amer. nucl. Soc.* **3** (1960) 335.
[34] CHAUMONT, J. and O'SHEA, D. M., Unpublished (ANL).
[35] NICHOLSON, R. B., Unpublished (APDA).
[36] NICHOLSON, R. B., APDA-139 (1960).
[37] OKRENT, D. *et al.*, ANL-5977 (1959).
[38] STRATTON, W. R., COLVIN, T. H. and LAZARUS, R. B., Proc. 2nd UN Int. Conf. PUAE **12** (1958) 196.
[39] JANKUS, V. Z., *These proceedings*, Vol. **III**, p. 209.
[40] O'SHEA, D. M. and OKRENT, D., Unpublished (ANL).
[41] ELROD, H., NDA 14—121 and NDA 14—122.
[42] BROUT, R. H., APDA-118 (1957).
[43] COWAN, R. D., Unpublished (LASL).
[44] HALL, D. B., Proc. 2nd UN Int. Conf. PUAE **13** (1958) 300. See also LA-2112 and LA-2327.
[45] SCHRAIDT, J. H. and LEVENSON, M., Proc. 2nd UN Int. Conf. PUAE **17** (1958) 361.
[46] See Series of Papers on the "Melt Refining of Irradiated Uranium" in *Nucl. Sci. Engng* during 1959, 1960 and 1961.
[47] GREEBLER, P., HUTCHINS, B. A. and SUEOKA, J. R., GEAP-3646 (1961). See also GREEBLER and HUTCHINS, *These proceedings*, Vol. **III**, p. 121.
[48] NIMS, J. B. and ZWEIFEL, P. F., *Trans. Amer. nucl. Soc.* **2** (1959). See also APDA-135.
[49] OKRENT, D., Proc. Conf. Physics of Breeding, ANL-6122 (1959).
[50] GREEBLER, P. and ALINE, P., Proc. Conf. Physics of Breeding, ANL-6122 (1959).
[51] YIFTAH, S. and OKRENT, D., ANL-6212 (1960).
[52] OKRENT, D., *These proceedings*, Vol. **II**, p. 271.
[53] BHIDE, M. G. and HUMMEL, H. H., *These proceedings*, Vol. **II**, p. 177.
[54] APDA-129 (1959).
[55] HUMMEL, H. H. and RAGO, A. L., *These proceedings*, Vol. **I**, p. 231.
[56] NICHOLSON, R. B., ZWEIFEL, P. F. and FERZIGER, J. H., *Trans. Amer. nucl. Soc.* **4** (1961) 19.
[57] FESHBACH, H., GOERTZEL, G. and YAMAUCHI, H., *Nucl. Sci. Engng* **1** (1956) 4.
[58] LANE, A. M., LYNN, J. E. and STORY, J. S., AERE T/M 137 (1956).
[59] GOLDMAN, A. J., *Trans. Amer. nucl. Soc.* **4** (1961) 33. See also *Nucl. Sci. Engng* **10** (1961) 91.

SM-18/43

THE FAST-REACTOR SAFETY PROGRAMME IN TREAT*

C. E. Dickerman, D. Okrent and E. Sowa
Argonne National Laboratory, Argonne, Ill.,
United States of America

Abstract — Résumé — Аннотация — Resumen

The fast-reactor safety programme in TREAT. Because of the difficulties of specifying theoretically the many complex phenomena of a fast-reactor melt-down incident, experimental investigations are under way to study the mechanisms involved in fuel-element failure and subsequent movement of fuel. An in-pile programme is under way using TREAT, a special pulsed engineering test reactor built to provide a high integrated neutron flux over a large sample volume under transient conditions.

Special techniques are being developed to follow the course of melt-down experiments. Tests have been performed on EBR-II Mark I elements (clad with stainless steel, tantalum or niobium) and metallurgically-bonded zirconium clad Fermi-I samples.

Early results will be reviewed, more recent data presented, and their significance discussed. Failure of EBR-II elements has suggested a new type of hypothetical melt-down accident, not apparent from out-of-pile tests. Differences in EBR-II element failure produced by replacing steel by tantalum or niobium, and the characteristically different phenomena associated with Fermi-I samples are consistent with predictions of a simple model of element failure, incorporating a diffusion controlled mechanism of cladding dissolution by molten fuel and experimental data on cladding penetration.

Sécurité des réacteurs à neutrons rapides — Expériences à l'aide de TREAT. Etant donné les difficultés qu'on éprouve à déterminer théoriquement les nombreux phénomènes complexes liés à un accident de fusion dans un réacteur à neutrons rapides, on a entrepris des études expérimentales sur les mécanismes qui interviennent lors de la défaillance d'un élément combustible et sur les mouvements ultérieurs du combustible. Un programme d'expériences est exécuté à l'aide de TREAT, réacteur pulsé d'essai de matériaux, construit spécialement pour produire un flux neutronique intégré intense dans un échantillon de grand volume dans des conditions transitoires.

On met au point actuellement des techniques spéciales pour suivre les différentes étapes des expériences de fusion. On a fait des essais sur des éléments combustibles du réacteur EBR-II, Mark-I (gainés d'acier inoxydable, de tantale ou de niobium) et des échantillons de Fermi-I gaînés de zirconium en liaison métallique.

L'auteur passe en revue les résultats obtenus antérieurement et présente des données plus récentes, en analysant leur importance. Une défaillance des éléments du réacteur EBR-II a donné à penser à un nouveau type d'accident de fusion, qui n'avait pas été décelé dans les essais hors-réacteur. Les différences entre les défaillances des éléments du EBR-II qui se sont produites lorsqu'on a remplacé l'acier par du tantale ou du niobium et la nature nettement différente des phénomènes associés aux échantillons de Fermi-I confirment les prévisions établies avec un modèle simple de défaillance d'élément, comportant un processus contrôlé par la diffusion de dissolution de la gaîne par le combustible fondu, ainsi que des données expérimentales sur la pénétration dans la gaîne.

---

* Work performed under the anspices of the United States Atomic Energy Commission

Программа исследований на реакторе TREAT вопросов безопасной эксплуатации реактора на быстрых нейтронах. В связи с тем, что трудно дать точное теоретическое обоснование многих сложных явлений, в частности случая расплавления (тепловыделяющего элемента) в реакторе на быстрых нейтронах, сейчас проводятся экспериментальные исследования для изучения механизмов, вызывающих повреждение топливного элемента и последующее движение топлива. Осуществляется программа исследований внутри реактора с использованием реактора „TREAT" — специального импульсного опытного реактора, конструкция которого обеспечивает высокий интегральный нейтронный поток для большего объема образца при переходных условиях.

Разрабатываются специальные методы, позволяющие проследить экспериментальным путем случаи расплавления. Проведены испытания на реакторе EBR-II топливных элементов реактора Марк I (покрытых оболочкой из нержавеющей стали, тантала или ниобия) и образцов Ферми-I, покрытых оболочкой из металлургически связанного циркония.

В докладе будут рассмотрены ранее полученные результаты, будут приведены более поздние данные и будет обсуждено их значение. Повреждение топливных элементов реактора EBR-II натолкнуло на мысль о новом предполагаемом случае расплавления, который нельзя обнаружить при испытаниях вне реактора. Различия между повреждением топливных элементов реактора EBR-II, вызываемым заменой стальной оболочки оболочкой из тантала или ниобия, и совершенно разными явлениями, связанными с образцами Ферми-I, соответствуют предположениям простой модели повреждения топливного элемента, включающей контролируемый диффузионный механизм растворения оболочки расплавленным топливом и экспериментальные данные по проницаемости оболочки.

Seguridad de los reactores de neutrones rápidos — Experimentos con el reactor TREAT. Dada la dificultad de determinar teóricamente los múltiples y complejos fenómenos asociados a los incidentes de fusión en los reactores de neutrones rápidos, se ha emprendido una investigación experimental para estudiar los fenómenos que intervienen durante la avería de un elemento combustible y los movimientos subsiguientes del mismo. Se está ejecutando un programa de investigaciones con ayuda de TREAT, un reactor de ensayo de materiales pulsado, especialmente construido con el fin de obtener un intenso flujo neutrónico integrado, sobre un gran volumen de muestra en condiciones transitorias.

Se están elaborando técnicas especiales para observar el desarrollo de los experimentos de fusión. Se han efectuado ensayos con elementos combustibles del reactor EBR-II Mark I (revestidos de acero inoxidable, tántalo o niobio) y probetas de elementos combustibles del reactor Fermi-I revestidas de circonio en unión metálica.

Los autores examinan los resultados iniciales, exponen otros datos más recientes y discuten su significación. Las averías de los elementos EBR-II sugieren un nuevo tipo de accidente hipotético de fusión, que no se había observado en el curso de los ensayos realizados fuera del reactor. Las diferencias producidas en las averías de los elementos EBR-II al sustituir el acero por tántalo o niobio, y la índole netamente diferente de los fenómenos asociados a las probetas del Fermi-I confirman las predicciones basadas en un modelo simple de avería de los elementos combustibles, con intervención de un mecanismo de disolución del revestimiento por el combustible fundido, controlado por la difusión, así como los datos experimentales sobre la penetración en el revestimiento.

Introduction

In investigations of abnormal operating characteristics of typical fast-reactor designs, it was found that some conditions could exist in which the melting point of the fuel was exceeded. Since the concentration and amount of fissionable material in the fast power reactors of interest is sufficient to produce a number

of critical masses if arranged compactly, and since under pessimistic assumptions one can obtain a rate of assembly of this molten material sufficient to generate a nuclear explosion equivalent in destructive force to hundreds of pounds of TNT [1, 2], this hypothetical melt-down problem has become of increasing interest and is the focus of fast-reactor safety studies on metal-fuelled reactors today.

The early concerns about a possible explosive energy release due to either a large and sudden external reactivity insertion or the presence of a large positive power coefficient have become less serious, as development progressed on designs for actual metal-fuelled fast reactors. Such fast power reactors possess low excess reactivity requirements. In addition, there is close coupling between reactivity and temperature in metal-fuelled power reactors. As a consequence, it appears that it is possible to design such a reactor system so as to preclude beyond a reasonable doubt the insertion of enough reactivity to cause an explosive event [2].

Hence emphasis has been placed upon less severe incidents which lead to some degree of core melt-down.

It is informative to consider some melt-down conditions and the hypothesis which lead to reassembly into a super-prompt critical configuration. Three cases are discussed briefly below.

(*a*) A sudden, major break occurs in the primary sodium coolant system and all the coolant disappears from the reactor almost immediately. The reactor is shut down, but fission-product decay heating melts the fuel elements. These collapse as a unit under the force of gravity, and as a higher uranium-density region forms in the bottom of the core volume the assembly becomes super-critical [3].

(*b*) The power level somehow gets much too high and the sodium boils away from the centre of the reactor. The uranium from the middle of the core trickles down into the lower part of the core and freezes there upon coming in contact with the sodium remaining in the lower blanket, producing a region abnormally dense in enriched uranium at the core bottom, with a large gap at the core centre. This almost critical configuration is then made supercritical by the upper portion of the core falling as a single unit under gravity [1].

(*c*) The sodium is expelled from the core temporarily by boiling, then rushes back moments later at high speed. During this brief interval the fuel elements undergo some degree of failure. The onrushing sodium carries the melt-down fragments with it, collecting the fuel rapidly into a more compact configuration.

The rather extreme assumptions outlined above are characteristic: the major reason for them is the difficulty of specifying the course of a melt-down with accuracy. To obtain a completely definitive answer, one would need to determine the location in time and space of all the fissionable material throughout the course of the incident. However, the motion of the fuel is subject to the interplay of many complex forces and events.

The cause of the melt-down, the condition of the coolant system, the past history of reactor operation all play a role. The modes of failure of fuel cladding, the roles of sodium coolant and fission-product gases on fuel dispersal and the mechanical arrangement of core and blanket are equally important. And each of these variables, as well as some others not mentioned, has many facets. For example, when one considers the coolant alone, one must know if it is stagnant

or flowing, if it is boiling in a steady, two-phase flow or chugging violently in and out of the core, or indeed if it is leaking away and at what rate.

Since a satisfactory solution by solely theoretical means was impossible, an experimental programme, capable of providing significant information in this field, was sought.

This programme has taken form in the shape of several out-of-pile laboratory studies, plus a major effort centered about a continuing series of experiments delving into the failure behaviour of fuel samples in a pulsed, thermal, test reactor named TREAT [4], designed and built especially for this purpose.

The basic approach taken to the melt-down problem is to divide it into as many separate stages as feasible and to study each of these in the most appropriate fashion, experimentally. Thus, the rate of attack of various cladding materials by molten uranium can be studied in laboratory tests. Similarly, the dynamic behaviour of sodium under transient heating conditions might be studied in the laboratory, if an appropriate heat source and geometric arrangement could be devised.

Efforts have also been made to study the failure of individual fuel elements with electrical-resistance heating. While some interesting results have been obtained in this fashion [5], it is felt that localized heating could easily occur and produce hot spots and regions culminating in a non-realistic mode of failure, as observed in the use of the experiments performed. The application of electrical heating methods to fuel element melt-down can only be done with great difficulty except in the simplest configuration, and all the methods suffer from the cessation or severe perturbation of heat generation when the electrical circuit is broken upon movement of the fuel.

Electrical melt-down tests on EBR-II elements under flowing sodium have been attempted, but also encountered difficulties. It was found impossible to introduce sufficient power into the fuel element to produce adequate liquefaction prior to discontinuation of the circuit. Further, it was recognized that streaming, magneto-hydrodynamic forces, and short circuiting of the current in such tests could result in anomalous effects that would be difficult to explain and understand.

It should also be noted that inductive heating techniques suffer from difficulties of coupling under liquid metals and of non-representative distributions of power generation throughout the structure of the fuel element.

Hence, nuclear heating was needed to provide an experiment in which the energy is generated directly in the fuel pins, with a realistic geometry including sodium coolant, and where the energy would continue to be released properly despite partial pin failure.

The TREAT reactor is designed to accomplish this, to permit the introduction of sufficient heat to melt samples ranging from single pins up to full sub-assemblies in times between hundreds of milliseconds and tens of seconds without damage to the reactor proper, and to permit ready access for special instrumentation and test equipment.

The goals of the experimental programme in TREAT are threefold. First, it is intended to study the behaviour of metallic fast-reactor fuel elements in the simplest possible configurations under various conditions of overheating, up to and beyond melting, in the presence and absence of sodium and/or fission products and under various geometric conditions to provide basic data on their failure and insight into the phenomena involved. Second, it is planned to con-

duct "proof tests" in which various details of actual reactor operating conditions are mocked up, to provide direct experimental information on the courses of specific hypothetical accidents. The third and later goal is to extend the studies of overheating to other types of fuel element, such as oxides or carbides. By restricting experiments during the first stage to simple systems where possible, and by developing special measuring devices to describe the course of the experiment, it is hoped empirically to uncover the mannerisms of failure for each fuel-element type. As theoretical advances on individual aspects of the meltdown process susceptible to test in TREAT are made, special experiments will be devised to confirm or disprove the predictions. An example of such an experiment may be the time of failure of the cladding under attack by molten uranium in an actual pin when the energy input to the sample as a function of time is known.

In addition to providing inspiration for theoretical work, the simpler experiments provide a basis for predicting the course of more complicated experiments, involving large clusters of pins. They suggest mechanisms which can lead to dispersal or assembly in the actual reactor. For example, if fuel elements always failed at or near their centre, axially, and all the uranium tended to be expelled rapidly at this point, this would provide a mechanism for rapid reactivity addition and possible progression of a partial failure.

Finally, as one becomes quantitative in describing the modes of failure as a function of design and fabrication technique, of irradiation history and of local temperature, it may be possible to suggest core designs which render essentially impossible the coherent type of reassembly of large sections of the fuel alloy which is required to generate the large reactivity addition rates needed to generate substantial pressures before subsequent disassembly.

II. Experimental programme

A. General plan

The specific types of experiments performed and the pace at which the programme progresses are both intimately related to the technological progress in special engineering devices and new instruments. With this in mind, a sequence of experiments has been planned, starting with the experiment which is simplest, both phenomenologically and in its technological requirements. Single fuel elements, supported in an opaque container and surrounded by an inert gas, form this category. Instrumented primarily with thermocouples and relying on post-mortem examinations to give details on the behaviour of the sample, this experiment is the simplest to engineer. From examination of the type of cladding failure, the degree of solution of the cladding by the uranium and the orientation and form of the uranium after completion of the experiment, one can reconstruct a considerable portion of the course of the test.

The second type of experiment in a logical progression involves single fuel elements in an inert gas, but this time supported in a transparent container, enabling high-speed visual light photography of the course of events and thus possibly an exact description of the time of failure and the subsequent motion. Such an experiment requires two technological steps forward, a safe transparent capsule and the appropriate photographic techniques. The phenomena observed are the same as in the opaque capsule.

The third step involves the study of individual fuel elements surrounded by stagnant sodium. The role of the sodium as an agent for dispersal, solidification or acceleration of the uranium becomes the new variable in this test.

The fourth step involves the study of individual fuel elements in flowing sodium. This permits a generalization of the third step to the role of the sodium velocity and pressure drop on the melt-down.

The fifth step might logically go into more complicated experiments, such as clusters of pins in an inert atmosphere or in sodium. The interplay of one element on the other now enters. The effect of sub-assembly design can be examined and the motion of the molten alloy beyond the core into the upper or lower blanket is subject to test.

As one delves further into the phenomena of melt-down, other general types of experiments become of interest, because the acquisition of some knowledge generates ideas for more and more specific tests of the theory.

Since residual fission-product gases are expected to play a major role on the mode of cladding failure, the manner of fuel-alloy escape from the cladding and the physical form taken by the uranium after escape, a considerable number of experiments using pre-irradiated samples will be performed in the appropriate test categories.

B. Present status of programme

As of June 1, 1961, melt-down experiments have been performed on a total of 102 samples, divided for convenience into 21 experimental series.

Ninety-two tests were of the first type; that is, they were experiments on the behaviour of single fuel elements contained in an inert gas inside opaque capsules. The reactor power bursts used had durations in the approximate range of 200 ms to 25 s, including 19 flattened power transients. Most of the experiments utilized short power excursions, of $\lesssim 0.5$ s duration. Ten samples were run in capsules incorporating tantalum neutron absorbers to produce an axial power profile typical of that in fast-reactor cores. Sample cladding temperature and, where applicable, sodium bond release data were combined with the results of post-mortem inspections to determine types of sample failure, and their associated temperature thresholds, and to suggest the pertinent mechanisms of failure. Sixty-four of these TREAT experiments were performed on EBR-II Mark-I specimens, the type used for the initial studies because of their ready availability. Eighteen Fermi-I elements were also utilized, as were ten EBR-II fuel pins clad with the refractory metals tantalum and niobium rather than stainless steel. The in-pile tests were augmented by two series of furnace tests on EBR-II Mark-I elements, run up to and including the range of mild sample failure.

Recently, the first steps have been taken into the areas of melt-down in sodium and optical photography of dry melt-downs. Six preliminary experiments have been performed on melt-down of EBR-II Mark-I elements in stagnant sodium. Four tests have been run on such elements contained in a transparent facility—two check experiments with the windows covered by a back-up plate, and two tests with actual photography of the sample behaviour.

Preparations are under way to extend the scope of the programme to study melt-down in flowing sodium, the effects of fission-product gases in the fuel samples, and to survey phenomena concerned with oxide fuel.

III. Experimental equipment

A. TREAT

Since TREAT (Transient Reactor Test Facility) is the key tool in the experimental programme, its characteristics will be reviewed briefly here. TREAT is a graphite-moderated, pulsed test reactor constructed by Argonne National Laboratory at the National Reactor Testing Station in Idaho to generate safely high-instantaneous and high-integrated thermal neutron flux over a large sample volume. It was designed to enable the rapid heating to destruction of mock-ups of fast-reactor fuel elements under controlled conditions, without harm to the test reactor. The principle design goal was taken to be the near maximization of integrated neutron flux; the reference design core-averaged integrated neutron flux is $\sim 3 + 10^{15}$ nvt, thus providing experimenters with a high degree of flexibility in such matters as sample enrichment, containment materials, etc.

The fuel consists of a homogeneous dispersion of urania in a graphite matrix. This graphite matrix serves a double purpose: (*a*) it acts as a large heat sink in intimate contact with the fuel, absorbing the energy generated in the 16—18-μ urania particles with a time lag of the order of 1 ms, and (*b*) it is a moderator which, upon being heated, generates a sizeable prompt negative temperature coefficient of reactivity by raising the energy of the thermal neutrons and thus increasing their probability of leakage from the core.

TREAT design has been described by FREUND *et al.* [4]. Additional engineering details are contained in the TREAT Hazards Summary [6] and the TREAT Engineering Design Report [7]. Criticality calculations on the reactor design have also been reported [8], as well as modification of these predictions made upon the discovery of boron impurity in the core graphite [9]. A report on experimental reactor physics measurements on TREAT in its simplest form (a uniform cylindrical core without viewing slots, test hole or test sample) has been published [10], as has a report on the analysis of the kinetics of TREAT [11] with the simple, solid core loading. A study of the kinetics of TREAT, perturbed by loading with viewing slots, test hole, and test sample, is in preparation [12].

The actual core-averaged integrated thermal neutron flux for a transient initiated at 30 °C and producing a maximum local reactor temperature of 400 °C is approximately 3.8×10^{15}. This is more than twice the integrated flux necessary to raise an uncooled natural uranium EBR-II fuel pin from 30 °C to the melting point and to supply the uranium heat of fusion. A melt-down sample consisting of a cluster of pins would produce a greater flux depression than that due to a single sample element. However, calculations indicate that the integrated flux in the melt-down specimens due to a transient associated reactor temperature rise from 30 °C to maximum of 400 °C would be more than enough to melt a cluster of 19 uncooled natural uranium EBR-II pins originally at 30 °C.

Within the requirement that transients be performed in such a manner that the reactor be safely self-limiting by virtue of its negative temperature coefficient of reactivity, even if all scram rods were to fail, it is possible to obtain a wide range of burst durations from the order of 200 ms to 10 s or more. In addition, with the use of special control-rod drives, "flattened" transients can be performed in which the reactor output more or less approximates a constant power pulse over a time range of approximately 1.5 to 30 s.

A cut-away view of TREAT is shown in Fig. 1. The fuel elements consist of a 122 cm long zircaloy-clad core section attached to 61 cm long aluminium-clad upper and lower reflector sections. Each element has a 10.02-cm square cross-section with truncated corners which form the cooling channels. Hence,

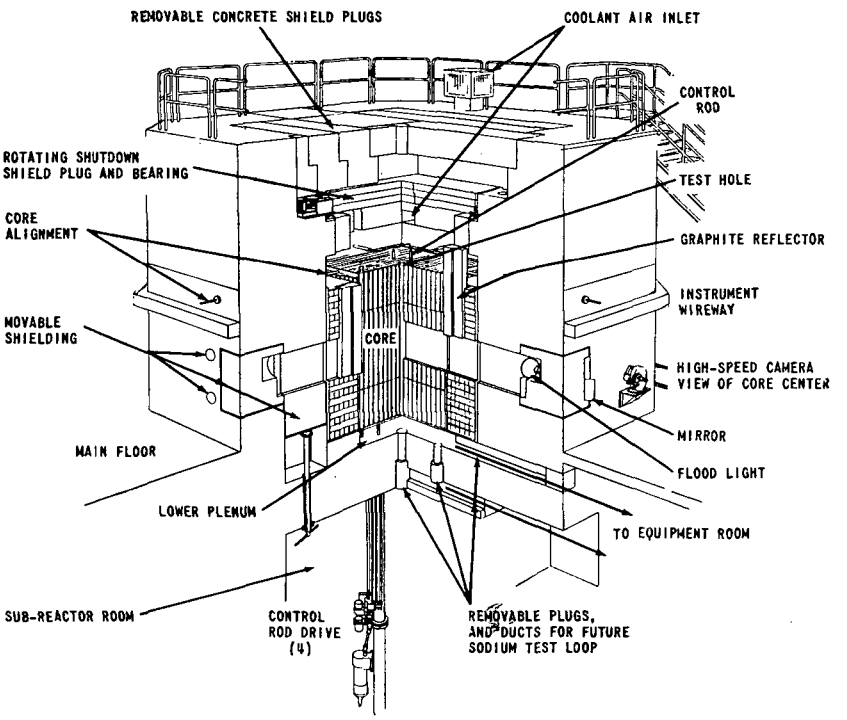


Fig. 1
TREAT perspective.

removal of a fuel element from the reactor leaves a test hole 10.16-cm square by 244 cm long. If desired, more than one element can be removed to form the test hole. Radial reflector elements are the same size as the fuel elements. Special elements are available which have a slot approximately 55.9 cm by 8.9 cm in the middle, thus making it possible to load one or more reactor slots for viewing the test hole. Fig. 1 shows one such slot set up with a camera.

Since TREAT is a thermal reactor, flux depression and sample self-shielding can be serious problems. The flux decrease between the surface and centre of a natural uranium EBR-II fuel pin is only about 3%. For 6% enrichment, the flux decrease rises to about 18%. Fig. 2 shows calculated thermal neutron flux plots along the horizontal midplane of TREAT loaded with an EBR-II fuel element in the standard dry opaque melt-down facility. Two curves are given: one for a natural enrichment sample element, and one for a 6% enrichment sample element. Experiments on clusters of fuel pins must be set up to minimize the effects of flux depression across the clusters. This requires in many types of experiments careful grading of enrichment from pin to pin in the clusters.

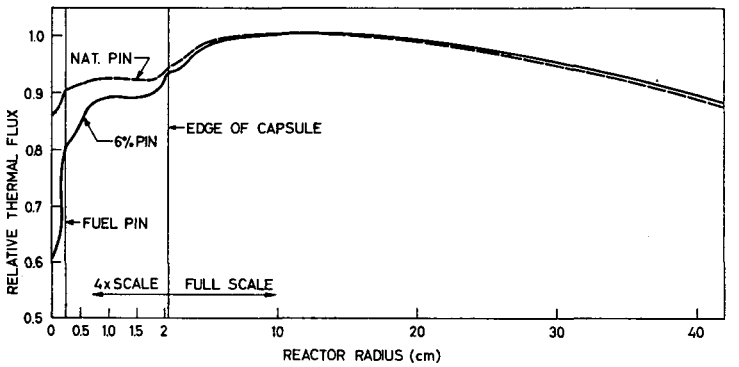


Fig. 2
Calculated thermal neutron flux along horizontal midplane of TREAT with sample.

Once melt-down products leave the pins, however, graded enrichments may cause uneven generation of power in the escaped material, an effect which must be considered in any special experiments for which detailed specification of post-failure power conditions is required.

B. In-pile facilities

1. *Dry opaque capsule*

Basic design for the dry, opaque melt-down capsule has been described previously [13, 14]. Each sample is held in a graphite crucible in a helium atmosphere, inside a stainless-steel can. The graphite crucible walls provide a convenient heat sink for "freezing" melt-down products, expelled from sample elements, *in situ*. A special TREAT dummy fuel-element holds the capsule. Since the experimental TREAT element is identical in outer dimensions with the regular TREAT elements, it can be handled using the TREAT fuel-element loading coffin and standard TREAT fuel-element procedures. Various modifications of this design that have been used include (*a*) extra graphite collars which served as shelves for catching melt-down products or preventing extensive post-experiment movement of sample fragments, (*b*) graphite holders which produced and maintained bowing of test samples, and (*c*) inclusion of tantalum neutron absorbers to produce a shaped axial power profile.

2. *Stagnant sodium capsule*

A modification of the dry opaque capsule was engineered to permit the performance of simple experiments on melt-down in the presence of stagnant sodium [15]. The sample is held, in a sodium bath, inside a stainless-steel inner capsule contained inside a graphite-lined steel capsule similar to the dry opaque type. Heaters are provided inside the capsule to raise the sample temperature to the desired pre-experiment level. Fig. 3 shows a simplified lay-out of the capsule. A refractory metal foil (zirconium, for example) is used as a liner inside the inner capsule to protect it from attack by molten melt-down products. Fast-response thermocouples are welded to the outside of the inner capsule. An expansion volume is provided above the sample and its sodium pool, which is sized so that the annulus of sodium around the sample is approximately the

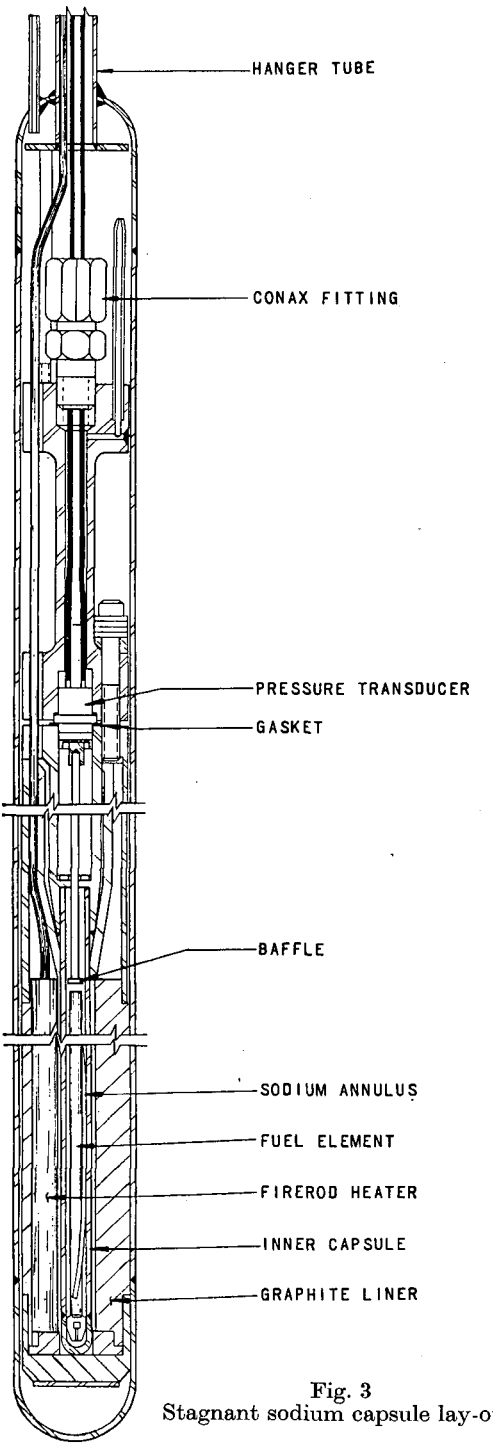


Fig. 3
Stagnant sodium capsule lay-out.

same as that around an EBR-II element in its core subassembly. The top flange of the inner capsule is designed so that it can contain either a miniature pressure transducer or a thermocouple which would extend into the sample to measure sample temperatures.

3. *Transparent melt-down facility*

The transparent melt-down facility [16] consists of two principal units, an inner capsule made of zircaloy which holds the melt-down sample and an outer sub-assembly made of steel which positions the capsule inside a reactor core slot and provides a leak-tight containment for a helium gas cover. Helium is introduced into the capsule from which it flows into the outer box, so as to purge the entire assembly. High-purity fused silica windows are used for both the capsule and the outer leak-tight sub-assembly. The outer unit is lined with graphite and additional zircaloy for protection from melt-down products in the event the capsule were breached during an experiment.

4. *Sodium loops*

Present development is being directed toward the construction of small package loops to permit the melt-down study to be performed upon single elements of the EBR-II Mark-I type. In addition, a large sodium loop is being designed to allow the melt-down of clusters.

C. Instrumentation

1. *Thermocouples*

For the determination of the temperature rise of the specimens exposed in TREAT, it is necessary to position rapid response thermocouples in strategic locations along the element. In the dry, single-element tests it was felt that the most significant temperature data would be those measured at the inner wall

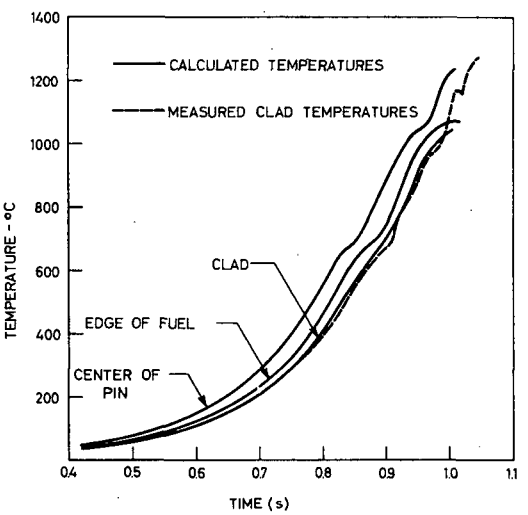


Fig. 4
Comparison of calculated and experimental sample temperatures for typical experiment.

of the cladding where the fuel-cladding interaction would take place upon contact between the two materials. Because of the difficulty of affixing thermocouples in this position, and because of the small temperature drop associated with the thin cladding, thermocouples were welded to the outer surface of the cladding. In the dry tests chromel-alumel or platinum-platinum 10% rhodium thermocouples were spot-welded near to the mid section of the element with the two thermocouple leads enclosing a 90° sector of the cladding perimeter [17]. This method provides an average temperature of the sector, with insignificant lag time due to conduction of heat into the thermocouple leads. Fig. 4 compares calculated and experimental cladding temperatures for a typical transient experiment.

Temperature measurements of the cladding in the presence of sodium presents some additional problems. A thermocouple technique has recently developed by the Advanced Technology Laboratory under Argonne subcontract, which should permit measurement of fuel central metal temperatures under sodium. This thermocouple consists of tantalum and molybdenum wires sheathed in a tantalum tube of about 0.114 cm diameter with magnesium oxide acting as the insulating medium. It is imbedded in 0.366 cm diameter EBR-II pin using zone melting techniques. Results of an in-pile test of this type of instrument are described elsewhere [14].

2. *Pressure measurement*

Miniature, fast response absolute pressure transducers were placed in the top of the graphite crucibles of the dry opaque capsules to note the release of sodium vapour from the sample pins. Release of this sodium, originally present in the EBR-II type samples as a thermal bond between the fuel and its cladding, is an indication of sample failure. Hence, pressure pulses superimposed upon the typical response [18] of the transducer to the high TREAT radiation levels, possibly co-ordinated with sudden marked irregularity in the sample temperature records, are indicators of sample failure.

3. *Optical cameras*

A dual optical camera system has been designed and built for taking both standard-speed and high-speed motion pictures of melt-down. A special xenon lamp provides a light source sufficiently bright to permit colour photography at the test section with camera speeds of the order of 1000 frames per second. Actual colour motion pictures have been taken of two melt-down experiments. No difficulties were encountered—either in those tests or in preliminary runs without fissionable samples—from darkening or glow of the fused silica windows. Times of appearance of the sodium cloud caused by release of bond sodium upon cladding failure were clearly marked.

IV. Experimental results

A. EBR-II Mark-I

1. *In-pile tests*

The EBR-II Mark-I fuel-element design has been described previously [2]. Briefly, it consists of a fuel-alloy cylinder 0.355 cm in diameter and 35.1 cm long, bonded to a 0.023-cm-thick stainless-steel jacket by a 0.015-cm sodium annulus. The steel jacket is welded to a steel bottom fitting and extends 7.85 cm

beyond the top end of the fuel cylinder, thus providing a reservoir and expansion space for the bond sodium. A helical spacer wire is wrapped about the outer periphery of the cladding to maintain separation of the elements in the sub-assembly. The "fissium" fuel alloy consists of 95 wt.% uranium, with the remaining small percentage consisting of a chemical mock-up of the fission products in the amounts expected to be attained at equilibrium during recycling of fuel using the EBR-II pyrometallurgical refining process. This alloy does not have a sharp melting point, but has been found to melt over the range 1002 °C—1081 °C [2].

Results of the TREAT experiments on EBR-II Mark-I elements subjected to short (~0.2—0.5 s duration) reactor power bursts may be summarized briefly in terms of the maximum cladding temperatures [14, 19]. Unless noted otherwise, all samples were given essentially uniform axial power profiles.

(*a*) Between approximately 900 °C to 950 °C, damage increased in severity in those elements with spacer wires, and voids in the sodium bond as well as patches of uranium-iron alloy on the fuel surface were seen. Below about 950 °C, the sample elements retained their integrity. It was observed that elements warped badly around the EBR-II element (helical) spacer wire.

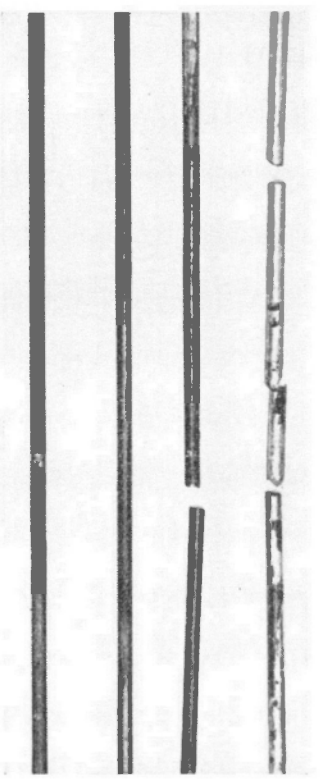


Fig. 5
Fuel alloy from EBR-II Mark-I samples. Maximum cladding temperatures were respectively, left to right, 530°C, 945°C, 965°C and 1000°C.

(*b*) Incipient failure occurred in the range 950 °C to 1015 °C. Failures were common, and the uranium alloy flowed from the cladding under the influence of the vapour pressure of the bond sodium. However, the sodium pressure was not high, and the escaping uranium left the cladding in a non-violent fashion.

(*c*) Above 1015 °C, the samples were found to fail rapidly and extensively. The fuel alloy was expelled violently from the cladding, tending to be sprayed out against the crucible walls in annuli of more-or-less uniform thickness and length under pressure of the sodium vapour inside the cladding. Essentially all the fuel alloy was found to be ejected.

Figs. 5 and 6 demonstrate the three ranges of effects described above. Fig. 5 is a close-up of the decanned fuel-alloy cylinders from samples whose cladding was not ruptured, and which had maximum recorded temperatures of 530 °C, 945 °C, 965 °C and 1000 °C, respectively. The bond sodium remaining on the fuel surface has been removed, but the rapidly increasing degree of eutectic formation as the maximum temperature approaches 1000 °C is quite apparent. Fig. 6 shows the opened crucibles of two specimens which experienced failure. The mild failure, in which the fuel-alloy-eutectic melt-down product flowed out into a puddle at the base of the sample, had a maximum temperature of 1015 °C. Its companion, which shows extensive spraying of fuel on the crucible walls, recorded a maximum temperature of 1030 °C.

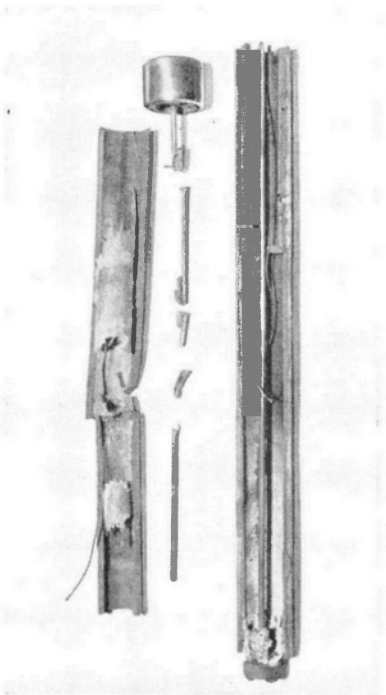


Fig. 6
Failed EBR-II Mark-I specimens. Maximum cladding temperatures recorded were 1030°C (left) and 1015°C (right).

These results, while not unexpected, were different from those obtained from earlier out-of-pile melt-down experiments, conducted using electrical heating during the EBR-II Mark-I fuel development programme. Both in-pile and out-of-pile experimentation demonstrated that unirradiated EBR-II fuel elements could be heated for limited times to temperatures considerably above 725 °C, the threshold for formation of the uranium-iron liquid eutectic, without failure. However, the types of failure observed earlier out-of-pile, when failure occurred, were all of a highly localized nature (presumably due to "hot spots"), and gave no indication of the more extensive failures and subsequent movement of liquid alloy under the influence of sodium pressure observed for the TREAT samples.

The range of energy inputs used for short duration transients extended up to a total sample energy about 250% greater than that necessary to produce failure. For these high energy inputs, the fuel alloy was found to be sprayed out laterally, relatively uniformly along the entire length of the cladding, as

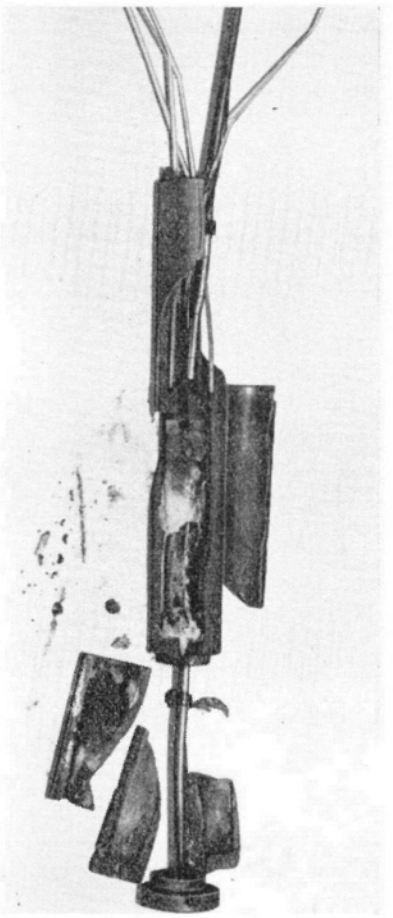


Fig. 7
Typical failed EBR-II Mark-I sample after exposure to shaped axial neutron flux.

though the molten fuel penetrated the cladding immediately adjacent rather than travel inside the cladding to a point of failure.

Examination of samples which were exposed to constant power transients showed that there is no significant change in the temperature threshold of failure for constant power bursts up to about 25 s duration. However, there appears to be evidence for somewhat less severe failure when failure does occur. No appreciable difference in post-experiment conditions was noted, for these specimens, between elements with a spacer wire and those without.

A series of elements were run with an axial "chopped cosine" power profile having a maximum-to-minimum ratio of about 1.6. Results were consistent with those on similar elements which were subjected to an essentially constant axial power profile: each element failed in the high-temperature central region. The fuel alloy was found to have been expelled upwards from the bottom portion of the element, and downwards from the top portion, with essentially all the fuel distributed within a length of crucible of about 18 cm (see Fig. 7). A small cylinder of fuel alloy about 1.5 cm long (presumably from one end of the element) was found among the spattered melt-down residue for a sample given approximately 20% greater energy input than that estimated to be sufficient to cause "extensive failure" in the central part of the sample. As the total energy input of the samples examined was increased, the amount of cladding remaining decreased and, for an input of about 70% greater than that necessary for extensive failure, only about one-half of the cladding remained.

Two samples run in stagnant sodium which received energy inputs estimated to lie in the approximate region of the temperature threshold of failure were found to be warped or to have a limited amount of fuel-cladding eutectic formation. But they did not fail. However, one element which did not fail displayed a localized area of severe fuel erosion when the cladding was removed. More severe energy inputs produced failure, and an input approximately 30% greater than that of the two lowest energies produced a failure pattern similar to that observed for dry elements which had been carried to the extensive failure region.

2. *Furnace tests*

Sample times-at-temperature were extended from the range of a few seconds encountered in the TREAT experiments to as long as a few thousand seconds using the electrical furnace. At temperatures of about 800 °C, the times to failure were ~2000 s. However, for temperatures of about 950 °C, the time-to-penetration had decreased to ~150 s.

Clad penetrations at temperatures of 900 °C and below were always very small, about the size of pinholes. However, when these elements were stripped of their cladding, it was found that the underlying uranium was covered with broad areas of eutectic. This suggests that the eutectic which forms at contact points melts and promotes further attack preferentially upon the fuel alloy. As the attack spreads, and if sufficient time is allowed, the alloy flows downward to fill the annulus and to contact the cladding again. After this, as attack on the cladding continues, a penetration is made in the cladding, and eutectic flows out. At about 1000 °C the cladding penetrations became more severe even for short times and large breaches were formed.

B. EBR-II FUEL PINS CLAD WITH REFRACTORY METAL CLADDING

EBR-II samples tested in TREAT include two series which were clad with either tantalum or niobium rather than stainless steel. Failure was found to

occur for temperatures of about 1400 °C. Essentially all the fuel alloy was expelled from the failed elements, which were found to contain localized penetrations rather than the broad areas of attack characteristic of the failure due to fuel-cladding interaction observed in the case of the steel-clad elements.

Elements which reached temperatures above the melting range of the fuel alloy but which did not fail showed results of movement of the fuel-alloy column to form a series of short lengths. These were easily detected in X-ray photographs taken before removal of the cladding. This movement of molten alloy was presumably caused by boiling of the sodium bond.

C. Fermi-I samples

The basic Fermi-I element is a simpler system than the EBR-II element. Its description [20] will be reviewed briefly here. The fuel is 10 wt.% molybdenium-uranium, in the form of 0.374-cm diam. cylinders metallurgically bonded

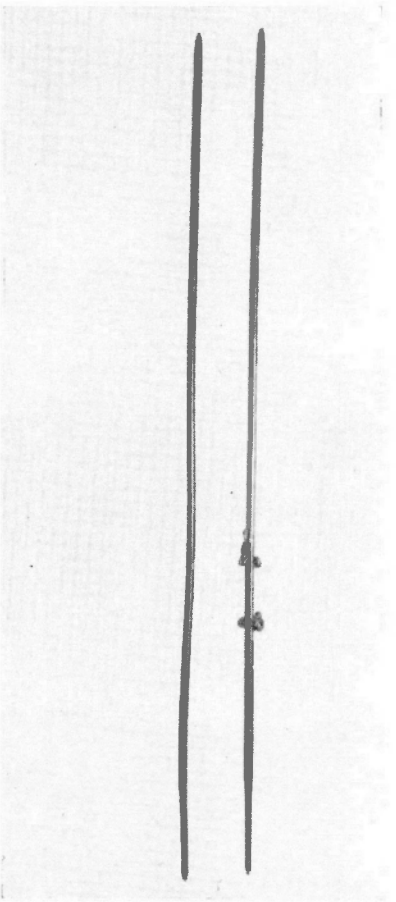


Fig. 8
Two Fermi-I TREAT specimens. *Left*: maximum temperature about 1150°C. *Right*: maximum temperature about 1250°C.

by coextrusion to 0.013-cm-thick zirconium cladding, with zirconium end caps. Overall length, including zirconium caps, is 81.5 cm.

For TREAT experiments, half-length fuel cylinders were used, capped with regular end fittings, resulting in an overall length comparable to that of EBR-II elements and permitting the use of standard dry opaque dry capsules.

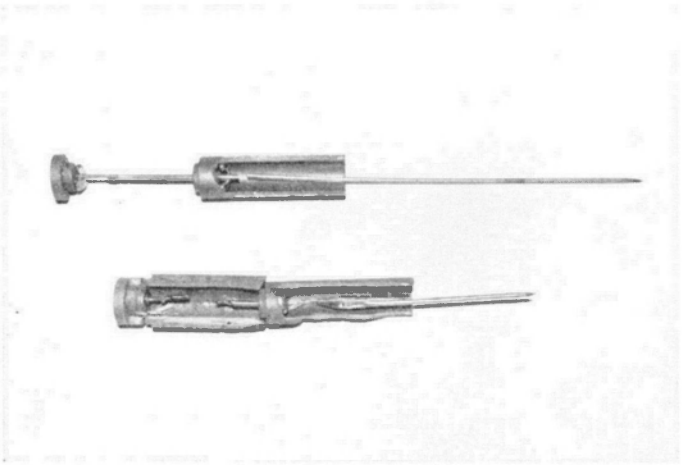


Fig. 9
Two Fermi-I TREAT specimens showing characteristics of high-temperature ($\geq$ 1400°C) exposure.

In the case of Fermi-I samples subjected to short power pulses, considerable fuel surface cracking of the cladding and sample warpage (but no failure) occurred when the maximum temperature was between the fuel melting point and ~1300 °C. In one such case only, a portion of the alloy "dribbled" from a crack in the cladding. Metallographic inspections of such samples, confirmed by positive identification of diffusion layer composition by "electron microsounding" inspection showed that no significant dissolution had taken place. Fig. 8 shows two specimens: one which was raised past the fuel melting point without failure and which shows considerable warping, and one which went to an estimated maximum temperature of ~1250 °C and shows a small failure with a small "dribble" of uranium alloy. Fig. 9 depicts the much more extensive failure and attack of cladding noted for peak temperatures ~1400 °C and higher. The cladding cracks were not found on samples heated during constant power TREAT bursts, and the threshold of failure for Fermi-I elements given such transients appears to be ~50 — 100 °C higher than that for the short ($<$0.5 s duration) excursions. All the Fermi-I samples show a fuel-alloy escape characteristic of flow under gravity, in contrast with the EBR-II type elements whose bond sodium can expel the fuel vigorously from the cladding.

V. Cladding failure calculations

A. EBR-II Mark-I

Failure of the cladding on an EBR-II Mark-I specimen exposed in these tests might be assumed to be due to one or a combination of the following:

bursting from the radial thermal expansion of the fuel, bursting from internal gas pressure, and dissolution of cladding by the fuel alloy.

Estimates of relative fuel and cladding thermal expansion for typical sample temperature distributions ruled out failure due to radial expansion. Bursting from internal gas pressure only can also be ruled out for most temperatures of interest. Fig. 10 is a graph of internal pressure and bursting pressure as a function

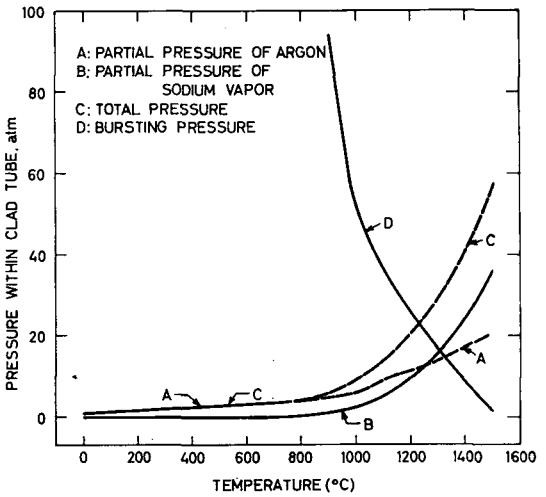


Fig. 10
Calculated internal and bursting pressures for EBR-II Mark-I element.

of temperature calculated for an isothermal EBR-II Mark-I element. At 1000 °C, the estimate of bursting pressure is about six times the total internal pressure. It should be noted that the sodium contributes to the pressure in two ways: by its vapour pressure, and by its thermal expansion which compresses the cover gas inside the cladding. However, the calculated bursting and internal pressures approach each other rapidly above 1000 °C, so that this could become significant in conjunction with cladding flows and/or appreciable cladding dissolution.

Table I summarizes the results of experiments by ANL Metallurgy Division [21] on time of penetration of steel and iron by molten uranium or molten uranium-5 wt. % fissium alloy. In the experiments, iron or steel sample tubes of the wall thicknesses indicated were plunged into a molten bath and the time for penetration of the tubing was measured. If one assumes that in the transient heating of a specimen, local defects occur in the oxide film coating the fuel pin, then local attack of the cladding results. No data are available on the effects of contact pressure upon attack of unprotected steel by uranium at the temperatures of interest here. However, measurements at 750 °C by Nuclear Metals Inc. [22] under ANL subcontract demonstrated rate increases $\sim$ a factor of 2 by measuring contact pressure from 0.58 to 1.05 atm. An increase from 1.05 to 2.07 atm increased the rates by an additional 70 %.

Dissolution of cladding unprotected in localized areas by the oxide film on the fuel pin, possibly assisted by internal pressure, is thus selected to be the most likely theoretical cause of EBR-II Mark-I sample cladding failure.

TABLE I

PENETRATION OF STAINLESS STEEL AND IRON BY MOLTEN URANIUM AND URANIUM-5 wt.% FISSIUM

Temp. (°C)	Type 304 SS				Type 430 SS		Armco Iron
	in U		in U-5 wt. % Fs		in U	in U-5 wt. % Fs	in U-5 wt. % Fs
	0.025 cm	0.102 cm	0.025 cm	0.102 cm	0.102 cm	0.102 cm	0.102 cm
1100	—	—	18.4	64.3	—	—	15.0
1125	—	—	0.61	3.8	—	—	—
1150	0.53 0.43 0.40	2.7 2.9 2.6	0.82 — —	3.4 3.76 —	1.93 — —	2.5 — —	1.9 — —
1187	2.8 1.92 1.2	11.6 — —	3.8 — —	16.75 — —	3.74 — —	8.8 — —	3.4 — —
1244	1.7	9.5	3.3	12.7	10.8	—	9.95
1300	—	9.0	—	10.1	—	—	9.6
1350	0.32	6.3	—	8.03	—	—	5.85

B. EBR-II MARK-I FUEL PINS CLAD WITH REFRACTORY METAL

The same three causes considered above in the case of steel-clad EBR-II pins are of interest in the case of refractory metal cladding.

Again, radial thermal expansion is not a possible explanation of cladding failure. However, because of the increased high-temperature resistance to attack by uranium, internal pressures become of greater importance. Figs. 11 and 12 present estimates of internal pressure and cladding bursting pressure as a function of temperature for isothermal samples with niobium and tantalum cladding respectively. The figures predict bursting in TREAT tests due to internal pressure at sample temperatures ~1500 °C if there is sufficient cladding resistance to attack by molten fuel to prevent significant attack during a short duration temperature excursion.

Fig. 13 shows experimental data [23] on the time for molten uranium to penetrate 0.054-cm-thick cans of Nb, Zr, and Ta as a function of temperature. If the penetration mechanism is diffusion controlled, the thickness of metal, t, dissolved by the uranium in a time, τ, may be described [24] by the equation $t(\tau)=k\sqrt{\tau}$, where k is a constant which depends upon materials, geometry, and temperature.

Using the equation to correct for the different cladding thickness and extrapolating the data [23] to shorter times, it is possible to estimate the time at maximum temperature required for complete dissolution. Times for complete penetration in the range 1300–1400 °C are typically in the hundreds of seconds. Hence, failures at temperatures ~1400 °C should be typical of bursting due

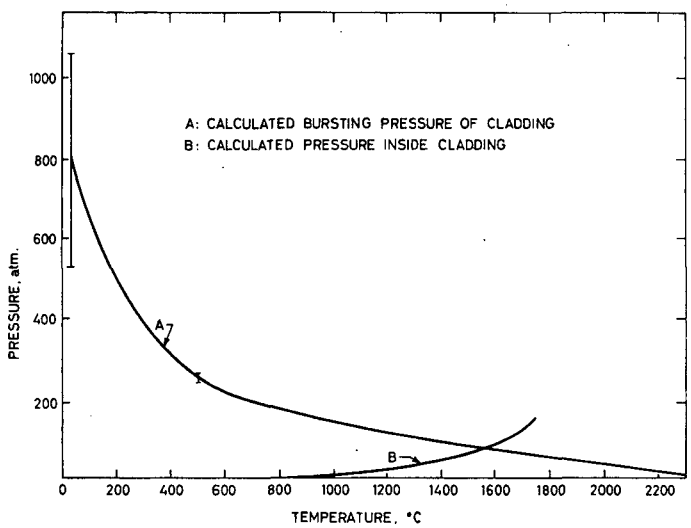


Fig. 11
Calculated internal and bursting pressures for niobium-clad EBR-II fuel pin.

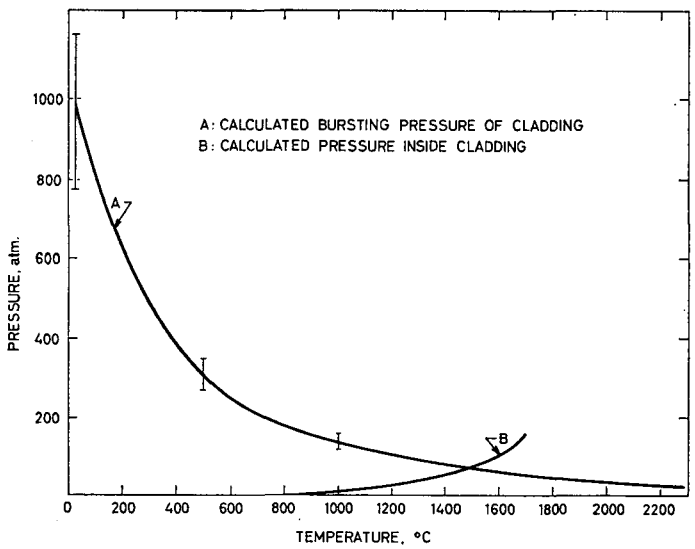


Fig. 12
Calculated internal and bursting pressures for tantalum-clad EBR-II fuel pin.

to internal pressure. At about 1500 °C penetration times are of the order of a few seconds, and cladding dissolution may be significant. The available data are not sufficient to tell its importance precisely since major extrapolations are required.

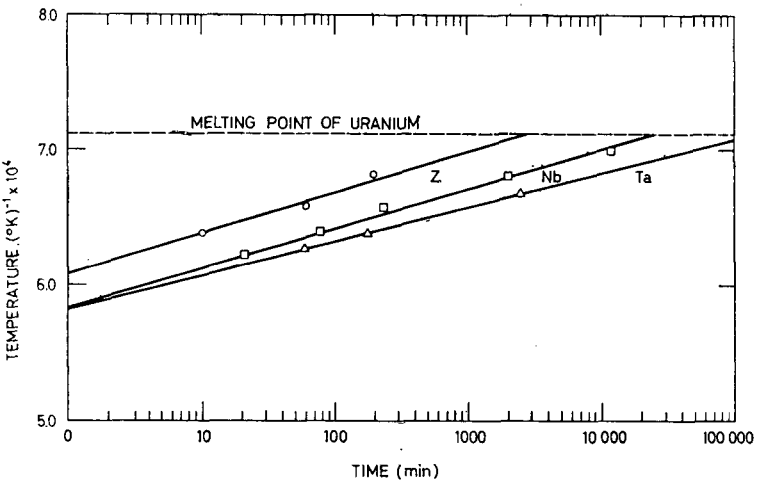


Fig. 13
Experimental data on time-to-penetration for molten uranium contained in Nb, Zr and Ta.

C. Fermi-I samples

For the metallurgically bonded zirconium-clad Fermi-I samples, cladding dissolution by the molten fuel core is the mechanism of interest. Using extrapolations of the zirconium penetration data [23] and the time-thickness dissolved relation, it can be estimated that cladding dissolution becomes significant in short temperature excursions like those of the TREAT experiments for sample temperatures ~1400 °C. It is estimated that 40 s is required to penetrate the cladding for a sample held at 1294 °C, but that this time is decreased to 4 s for a sample at 1370 °C.

VI. Discussion

The first year and one half of TREAT experiments has demonstrated several things.

1. Experiments and experimental results are reproducible to a highly satisfactory degree.
2. Much can be learned from experiments with simple instrumentation and careful post-mortem analysis.
3. The engineering development associated with each new type of experiment is considerable.
4. The TREAT reactor has much more capacity than has been needed for the experimental programme, to date.
5. In-pile tests turn up results different from those encountered in out-of-pile tests.
6. In addition to confirming ideas and calculations concerning fuel-element failure and fuel dispersal, the experimental programme can uncover new phenomena of basic importance in considering melt-down-induced accidents in fast reactors. In particular, the rapid expulsion of all the fuel at the hottest point

in the pin for sodium-bonded test samples suggests a mechanism for producing a supercritical assembly not previously considered in safeguard analyses. It has triggered extensive theoretical work on the details of pin failure as a function of time and position in the EBR-II under assumed overheating conditions to see whether dangerous reactivity insertion rates may result.

ACKNOWLEDGEMENTS

Assistance from many Argonne associates is gratefully acknowledged. In particular, the following should be mentioned:

R. Schiltz and L. E. Robinson of Reactor Engineering Division, for supervision of capsule construction and assembly, as well as many discussions of programme and techniques.

J. F. Boland, the TREAT manager, and other personnel at TREAT, including R. D. Cook, R. De Forest, F. Kirn and H. Lawroski. Their effective, efficient operation of the reactor and data collection was essential.

W. Newman, J. Burghart, and C. August of Reactor Engineering Division, who assembled test capsules and contributed to their design.

S. Skladzien and J. Monaweck of Reactor Engineering Division, who performed some of the sample post-mortem inspections.

REFERENCES

[1] McCARTHY, Jr. *et al.*, Proc. 2nd UN Int. Conf. PUAE **12** (1958) 2165.
[2] KOCH, L. J. *et al.*, ANL-5719 (1957).
[3] BETHE, H. A. and TAIT, J. H., UK AEA RMH (56)/113.
[4] FREUND, G. A., ISKENDERIAN, H. P. and OKRENT, D., Proc. 2nd UN Int. Conf. PUAE **10** (1958) 1848.
[5] FREAS, D. G. *et al.*, BMI-PRDC-656.
[6] MAC FARLANE, D. R., FREUND, G. A. and BOLAND, J. F., ANL-5923 (1958).
[7] FREUND, G. A. *et al.*, ANL-6034 (1960).
[8] ISKENDERIAN, H. P., ANL-6025 (1959).
[9] ISKENDERIAN, H. P., ANL-6115 (1960).
[10] KIRN, F. *et al.*, ANL-6173 (1960).
[11] OKRENT, D. *et al.*, ANL-6174 (1960).
[12] DICKERMAN, C. E., JOHNSON, R. D. and GASIDLO, J., ANL-6457 (1962).
[13] DICKERMAN, C. E. *et al.*, *Trans. Amer. nucl. Soc.* **2** (June 1959).
[14] DICKERMAN, C. E. *et al.*, ANL-6334 (to be published).
[15] SOWA, E. S., HEAP, J. C. and JANICEK, J. ANL Internal Memorandum (November 14, 1960).
[16] GOLDEN, G. H., DICKERMAN, C. E. and ROBINSON, L. E., Transparent Facility for TREAT Meltdown Experiments (in preparation).
[17] SCHILTZ, R. J., ANL Internal Memorandum (May 29, 1953). SCHILTZ, R. J. and CARTER, J. C., ANL Internal Memorandum (June 28, 1957).
[18] BOLAND, J. F. and KIRN, F., ANL-6301 (1960).
[19] DICKERMAN, C. E., SOWA, E. S. and OKRENT, D., *Nucleonics* (April, 1961).
[20] AMOROSI, A. and YEVICK, J. G., Proc. 2nd UN Int. Conf. PUAE **9** (1958) 2427.
[21] WALTER, M. and KELMAN, L. R., ANL-6355 (1961) 43.
[22] JENKINS, R. D., NMI-4815 and NMI-4816 (1960).
[23] McINTOSH, A. B. and BAGLEY, K. Q., *J. Brit. nucl. Energy Conf.* (January, 1958) 15ff.
[24] GOLDEN, G. H., ANL-6167 (1960).

SM17-18/46

MAJOR ACCIDENT ANALYSES FOR EXPERIMENTAL ZERO-POWER FAST REACTOR ASSEMBLIES*

G. FISCHER, E. W. BARTS, S. KAPIL AND K. TOMABECHI
ARGONNE NATIONAL LABORATORY, ARGONNE, ILL.,
UNITED STATES OF AMERICA

Abstract — Résumé — Аннотация — Resumen

Major accident analyses for experimental zero-power fast reactor assemblies. A study has been made of the possibility, mechanism, and consequence of melt-down and other major nuclear accidents for a ZPR-III type experimental zero-power fast reactor of the two-half type. This study has been supplemented by an evaluation of the importance of the Doppler effect for a wide range of nuclear reactor assemblies for such a reactor.

A melt-down event is highly improbable because of the restricted sequence of events which must be postulated. A discussion of the mechanism of the collapse is followed by the results of coupled neutronics-hydrodynamics calculations for two zero-power assemblies. A 1200-l core has been examined because it represents a relatively large reactor of common core composition. A smaller core with a high-void fraction has been examined as a potentially more dangerous system. Very different time-wise behaviour has been found for the two systems.

For sharp accidents in zero-power assemblies, the U^{235}-atoms, separated as plates of enriched uranium, will heat very rapidly while the remainder of the core remains essentially cold, so that a gas of U^{235}-vapour will provide the disassembly pressure. The adaption of the neutronics-hydrodynamics code AX-I to the use of a Van der Waals gas is described. Another important change in the equation of state used in the code is to employ a Mie-Grüneisen type equation derivable from solid state theory. This change provides a more satisfactory way to evaluate the pressure term for cores of variable composition.

Because the highly enriched U^{235} plates of a zero-power assembly will heat much more rapidly than the depleted uranium plates, the possibility of a net positive Doppler effect is much larger for an experimental assembly than for the equivalent power breeder reactor. This hazard has been examined for a range of possible assemblies. These calculations indicate that the Doppler coefficient for a zero-power assembly does not become important as a hazard until one approaches systems with the very soft neutron-energy spectra characteristic of large oxide power breeders.

Analyse des accidents graves pouvant survenir dans les réacteurs expérimentaux à neutrons rapides de puissance zéro. Les auteurs ont étudié la possibilité, le mécanisme et les conséquences de la fusion et autres accidents nucléaires graves dans les réacteurs expérimentaux à neutrons rapides de puissance zéro, du type ZPR-III, à cœur divisé. Cette étude a été complétée par une évaluation de l'importance de l'effet Doppler sur un grand nombre de réacteurs de ce type.

Les auteurs démontrent qu'il est fort peu probable qu'une fusion se produise, du fait que la conjonction des circonstances qui pourraient la provoquer est difficilement réalisable. L'exposé du mécanisme de fusion est suivi de l'analyse des résul-

* Work done under the auspices of the United States Atomic Energy Commission.

tats de calculs couplés neutronique-hydrodynamique relatifs à deux réacteurs de puissance zéro. On a choisi pour cette étude un cœur de 1200 l, qui correspond à un réacteur relativement grand à cœur normal. L'étude a également porté sur un cœur plus petit ayant un coefficient cavitaire plus important, qui pourrait présenter un plus grand danger. Chaque système a eu un comportement en fonction du temps tout à fait différent.

Si un accident grave survient dans un réacteur de puissance zéro, les atomes de ^{235}U, isolés dans les plaques d'uranium enrichi, s'échauffent très rapidement tandis que le reste du cœur demeure pratiquement froid; il y a ainsi formation d'un gaz du ^{235}U qui donne lieu à la pression de rupture. Les auteurs expliquent l'adaptation qu'ils ont faite du code AX-1 de neutronique-hydrodynamique pour l'appliquer à un gaz de Van der Waals. Une autre modification importante de l'équation d'état utilisée dans ce code consiste à employer une équation du type Mie-Grüneisen, dérivée de la théorie de l'état solide. Cette modification permet d'évaluer de façon plus satisfaisante le terme de pression pour les cœurs de composition variable.

Du fait que les plaques en uranium fortement enrichi d'un réacteur de puissance zéro s'échauffent plus rapidement que les plaques en uranium appauvri, la possibilité d'un effet Doppler positif net est beaucoup plus grande dans le cas d'un réacteur expérimental que dans celui du réacteur surgénérateur de puissance équivalent. Ce danger a été étudié pour un certain nombre de montages possibles. Les calculs montrent que, pour un réacteur de puissance zéro, le coefficient de Doppler n'atteint une valeur dangereuse que lorsqu'il s'agit de réacteurs ayant les spectres d'énergie neutronique très mous qui caractérisent les grands réacteurs surgénérateur de puissance à oxyde.

Анализ крупной аварии для экспериментальных реакторных установок нулевой мощности на быстрых нейтронах. Проведено исследование возможности, механизма и последствий расплавления, а также других крупных ядерных инцидентов для экспериментального реактора нулевой мощности на быстрых нейтронах типа ZPR-III двухполовинчатого типа. В дополнение к этому исследованию проведена оценка значения эффекта Допплера для многих ядерных реакторных установок такого реактора.

В докладе будет показано, что явление расплавления маловероятно ввиду ограниченного количества явлений, которые нужно постулировать. После рассмотрения механизма разрушения будут даны результаты расчетов, связанных с нейтронной физикой и гидро-динамикой, для двух реакторных установок нулевой мощности. Проведено исследование активной зоны емкостью 1200 литров, характерной для относительно большого реактора с активной зоной обычного состава. Была исследована меньшая активная зона с высоким пустотным коэффициентом, как потенциально более опасная система. У этих двух систем обнаружен очень различный временной режим. В случаях серьезных аварий на установках нулевой мощности атомы U^{235}, которые распределены в пластинах обогащенного урана, очень быстро нагреваются, тогда как остальная часть активной зоны по существу остается холодной, и таким образом газообразный U^{235} создает распределенное давление. В докладе будет дано описание применения к газу Ван дер Ваалса кода AX-I нейтронной физики и гидродинамики. Другим важным изменением уравнения состояния, использованного в коде, является применение уравнения Ми-Гринейзена, выведенное из теории твердого состояния. Это изменение дает возможность более удовлетворительно выразить член давления для активных зон различного состава.

Ввиду того, что пластины U^{235} с высоким обогащением в установке нулевой мощности нагреваются гораздо быстрее, чем обедненные урановые пластины, возможность получения результирующего положительного эффекта Допплера намного больше в экспериментальной установке, чем в реакторе-размножителе эквивалентной мощности. Этот риск был исследован в отношении ряда возможных установок. Эти расчеты указывают на то, что коэффициент Допплера установки нулевой мощности не приобретает опасного значения, пока не будут созданы системы крупных энергетических реакторов-размножителей на оксидном топливе с очень мягкими спектрами энергии нейтронов.

Análisis de los accidentes graves que pueden producirse en los reactores experimentales rápidos de potencia cero. Los autores han estudiado la posibilidad, el mecanismo y las consecuencias que produciría la fusión u otros accidentes nucleares graves en el caso de los reactores experimentales rápidos de potencia cero del tipo ZPR-III de cuerpo dividido. Este estudio se completó con una evaluación de la importancia del efecto Doppler en un gran número de reactores de ese tipo.

Los autores demuestran que es muy improbable que se produzca una fusión del reactor debido a que en la práctica difícilmente pueden concurrir las circunstancias necesarias. Después de discutir el mecanismo del fallo, presentan los resultados de cálculos neutrónicos e hidrodinámicos combinados relativos a dos conjuntos de potencia cero. Se ha escogido para este estudio un cuerpo de 1200 litros, porque corresponde a un reactor relativamente grande con un cuerpo de composición corriente. También, se estudió el caso de un cuerpo más pequeño con una elevada fracción de vacío, que puede presentar un peligro mayor. Se comprobó que el comportamiento en función del tiempo es muy distinto para cada sistema.

Al producirse un accidente súbito en un conjunto crítico de potencia cero, los átomos de ^{235}U, separados en forma de placas de uranio enriquecido, se calientan muy rápidamente, en tanto que el resto del cuerpo permanece relativamente frío, de manera que se formarán vapores de ^{235}U cuya presión provocará la ruptura del sistema. Los autores explican cómo han adaptado la clave AX-I para cálculos neutrónico-hidrodinámicos al caso de un gas que se ajusta a la ecuación de Van der Waals. Otra modificación importante introducida en la ecuación de estado utilizada en la clave, consiste en emplear una ecuación del tipo de Mie-Gruneisen, derivada de la teoría del estado sólido. Esta modificación permite evaluar de manera más satisfactoria del término de presión para el caso de cuerpos de composición variable.

Dado que en un conjunto de potencia cero las placas de uranio fuertemente enriquecido en el isótopo-235, se calentarán con más rapidez que las de uranio empobrecido, la posibilidad de que se produzca un efecto Doppler positivo neto es mucho mayor en un conjunto experimental que en el reactor de potencia reproductor equivalente. Se ha estudiado este peligro en el caso de diferentes conjuntos posibles. Los cálculos indican que en un conjunto de potencia cero el coeficiente Doppler sólo alcanza un valor peligroso en los sistemas que poseen un espectro de energías neutrónicas muy blando, característico de los grandes reactores de potencia reproductores, alimentados con óxido de uranio.

The melt-down accident

Since melt-down accidents are a problem of major concern when the safety aspects of fast power breeder reactors are considered, it seemed worth-while to make a study of the same type of accident for the zero power experimental system. This leads to a comparison of the differences between the two systems of the relative liability to this type accident, to its mechanism for the two systems and to a calculation of the resulting energy release. It will be seen that while this type of accident is highly improbable for the experimental assembly, the energy of the burst can be large.

Two types of experimental reactor systems will be considered. The first, the zero power mock-up of a fast power breeder reactor, will allow the most direct comparison to the power reactor melt-down. The second will be of a composition which might be explored for the purpose of a "pure" reactor physics type study. An assembly composed solely of enriched uranium and of low-density aluminium, and with a reflector of depleted uranium, is chosen because the high void fraction of this core, produced by the holes drilled in the aluminium plates, considerably

increases the range of melt-down accident which can occur. The reactor has a greater number of modes of collapse over which it can be brought to prompt critical. The power height reached before the voids are removed and disassembly begins can be considerably greater.

Comparison of melt-down accidents on fast power breeder and experimental assemblies

In order to improve the comparison between the two types of systems, the experimental zero power reactor will be assumed to have been loaded with a mock-up of a power breeder core at the time the accident on the experimental system occurs.

A rather important difference between a power and a zero power experimental reactor melt-down accident is that a melt-down accident on a zero power reactor can at most release only the fission products which are generated as a result of the burst which destroys the reactor. Essentially no fission products are formed during the normal operation of a zero power reactor, and new fission products produced by an accident will decay rapidly. The initial activity after the burst will fall by a factor of about 10,000 over a period of one half hour, as when any possible cloud would be dispersing. There would be very few long-lived isotopes and no built-up plutonium, unlike the case of a power reactor.

In the case of a fast power reactor, a melt-down has been hypothesized to follow loss of the sodium coolant from the core [1, 2]. The loss of sodium leaves a large void into which the molten core can collapse. Fission-product heating of the fuel of the power reactor, which was operating close to its melting point at the time of the Na loss, then causes a melting of the fuel: the fuel next freezes in some configuration that leaves the power reactor sub-critical by about 10 % to 15 %. The upper portion of the core, or core and reflector, then is assumed to fall freely for a considerable distance, so that at the time the system goes prompt critical there is a very high rate of reactivity addition.

For a zero power system there is no simple equivalent to the loss of sodium accident. The aluminium which frequently mocks up sodium would not easily leave the reactor after a serious accident burst which must first happen in order to raise the core to a high temperature. This lack of void space, fortunately, greatly limits the energy release of the melt-down accident. The core materials of the zero power reactor are initially cold and without the fission-product heating mechanism (or fission products). A rather particular accident must be hypothesized in order to produce the large amount of energy needed to achieve a general melt-down condition and yet not lead to a permanent shut-down by the expansion or motion of the metal or, for the more severe accident, by vaporization of the U^{235}.

Shut-down by vaporization of the U^{235} in a severe accident is particularly likely rather than a melt-down collapse. This is because the time required for the heat to diffuse from very hot U^{235} plates into the surrounding materials so that they are raised to their melting point is of the order of 20 s. Another important difficulty to production of a melt-down accident is that it is necessary to destroy the integrity of the stainless-steel matrix tubes which support the core. If there is sufficient impedance to the interaction of molten uranium with the steel matrix due to the presence of aluminium, carbon, or aluminium oxide, so that destruction does not occur by iron-uranium eutectic formation, then

the matrix will collapse only when the melting point of steel, about 1535 °C, is approached. A very unique set of conditions must be postulated in order to allow the enriched uranium to generate heat which can move into the neighbouring materials for a time very long by accident standards and yet exclude shut-down by any other mechanism, such as simple thermal expansion.

The void space which remains in a segment of a zero power core mock-up of the breeder reactor after a just molten state may be considered to be achieved is approximately 10 % of the total core volume. Because there may be as much as a 3-to-1 or 4-to-1 power ratio between the centre and the edge of the core of the reactor, there will be a void fraction considerably smaller than 10 % at the core centre when a molten state has been achieved at a distance from the centre sufficient for the collapse of the molten zone to give the desired reactivity gain. One should recognize as well that the melting aluminium may trap a fraction of the air enclosed in the holes drilled into this metal to give it its 63 % or 45 % density. After a degree of compression and heating this gas will be able to support the core weight, so that there must be a residual void space at the time the core collapses. A study of this void problem for a 1200-l equivalent power reactor case showed that it was necessary to postulate that the melt-down collapse must be assumed to start with the partially molten core very close to critical since, if the initiating accident left the reactor even a few per cent subcritical, the allowable collapse could not bring the core to prompt critical. If the incident were imagined to proceed by means of a slow heating of the core, the reactor operators or the several period and level trips would almost certainly stop the incident. The improbability of this situation is so very large that it might be considered negligible.

Scram of control- and safety-rods before the collapse of a fast power reactor tends to initiate the core collapse from a more subcritical state and yields a consequently higher rate of reactivity addition at the time prompt critical is reached. For a zero power mock-up of the power breeder core the collapse of the limited void space within the matrix tubes plus the collapse of the space formerly occupied by the control- and safety-rods would not give a reactivity gain equal to that lost by the removal of these rods. Therefore, rather than a severe accident, scramming the rods would lead essentially to no accident at all.

Because of the small void fraction which remains when the zero power burst begins, only a limited energy generation (or temperature rise) would be required to remove the remaining void space and begin disassembly pressures. This is again very much unlike the case in some power breeder analyses. Finally, the physical characteristics of the molten material of the zero power assembly are such that disassembly pressures are transmitted somewhat more rapidly than is the case for the power reactor melt-down.

In spite of the fortunate difficulties which tend to reduce greatly the importance of melt-down accidents for the experimental assembly, several cases of this type accident were examined. It will be seen that careful choice of relatively limited conditions can lead to severe energy releases.

Method of calculating severe accident energy releases

One of the most satisfactory tools presently available for the analysis of severe accidents is the AX-I code [3]. This code is capable of following a nuclear incident throughout the course of a severe burst, doing, in a space-dependent spherical

model, alternately neutronic calculations and then thermodynamic plus "hydrodynamic" calculations which, taken together, are called "hydrocycles" in the code description. The calculations can be started in such a way that one may specify the coefficient α' in the exponent $e^{\alpha' t}$ by which the power of the burst is to increase with time. The code will do a spherical SNG transport-theory calculation, changing all radii proportionally until the corresponding excess reactivity is reached and then turn to the "hydrocycle" portion of the code. The energy generated per unit time is calculated for each space interval of the reactor. By using this information, as well as the material specific heat parameters specified, the regional increase in temperature is found. The regional increase in temperature together with the corresponding change in density as a result of any initial, or for later time of calculated velocities, is the source of the calculation of the regional pressures. The equation used for pressure is:

Pressure $= (\alpha \times \text{density}) + (\beta \times \text{temperature}) + \tau$. Here α, β and τ are coefficients supplied by the physicist [3, 4]. If τ is made negative, the code will set the pressure which it calculates to zero whenever τ is larger in magnitude than the sum of the first two terms. This allows one to provide a threshold which must be reached before the disassembling pressure can begin to act.

The pressure for each region is used with Newton's law of motion to calculate the velocities induced for each radial segment of the reactor. Shock wave propagation, after the wave is built up, is treated by a check made by the code for the "Courant stability criterion", which watches the rate of propagation of the shock front, and by addition of a shock-smearing routine, which imitates the actual physical viscosity. The calculated velocities can in turn produce changes in density for the various regions. These changes, along with the further generation of fission energies, yield new internal energies. The number of hydrocycles of the above type which the code performs before returning to the SNG-calculation routine is controlled partly by the physicist and largely by the design of the code. The next SNG calculation determines the proper, probably changed, values for α' and spatial power generation rates. These revised values are then employed on another sequence of hydrocycles.

This process will continue until the calculation has followed the incident to the point where α' has become negative as a result of the displacements which have occurred and the burst is essentially over. If α' is negative and the rate of power generation is also small, the code continues only the hydrocycle calculations, making no more neutronic calculations. The print-out record of the code gives, as a function of time and position in the core, calculated values of pressure, temperature, and material velocities. It also gives, for various time steps α', the rate of power generation and the integrated power to that time, as well as data on the progress of the code. This code was used to analyse the energy releases discussed below.

Melt-down calculations

Power reactor mock-up

As a model for the equivalent power breeder mock-up, a system larger than those which probably will be explored initially has been chosen. This is a reactor having a 1200-l core and a reflector of 50 cm in both the radial and axial directions. The simplified core is chosen to be composed of uranium, iron, and aluminium (63 % density), with volume fractions of 0.40, 0.20 and 0.40 respectively, while

the reflector is composed of depleted uranium, iron, and aluminium with volume fractions of 0.6, 0.2 and 0.2.

Before the accident can occur, a partially molten condition must somehow be achieved while losing less reactivity than the small amount which can be gained upon collapse of the core. This might be imagined to happen if the reactor was being operated by an automatic level control device which became malfunctional. This malfunction must be of a very particular nature in order to achieve the large amount of preliminary heat which is needed. Other assumptions must be made as well. The operators must be inattentive to the instrumentation of their reactor for many seconds. Also, the control system must have sufficient excess reactivity available for the reactivity loss due to metal expansion to be overcome. This is more reactivity than is normally used for a fine control-rod. Finally, one must require that the level trips do not scram the reactor.

The difficulties of achieving a melt-down accident on an experimental assembly, which were discussed in the comparison to the equivalent power reactor, must also be surmounted. Nothing must occur during the relatively long time required for the zone of melting to spread through each matrix drawer which would lose enough reactivity for the system to be unable to go prompt critical.

A diffusion theory calculation was made to determine the temperature distribution across the radius of this core. From this distribution a model was chosen which gave a maximum void space at the core centre at the time prompt critical was reached. This choice allows the power to rise to a higher level before disassembly begins and leads to a larger energy yield for the accident. The choice

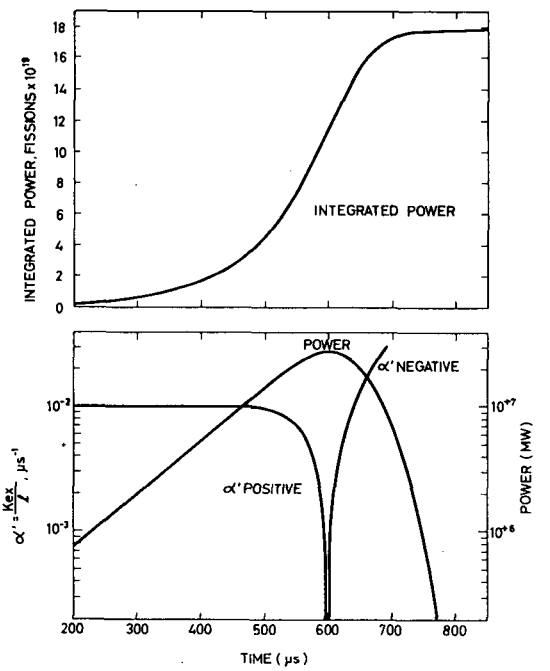


Fig. 1
Time behaviour of 1200-l experimental reactor following melt-down.

of the initial inverse period was made following a model outlined by JANKUS [5]. The value of $(d/dt)\,K_{ex}$ at prompt critical was determined by the pessimistic assumption that the entire barely molten matrix shall begin to collapse at the same time with an acceleration equal to 1/2 that of free fall.

The values calculated by AX-I, as a function of time, of $\alpha' = (K_{ex} - \beta)/l$, the rate of power generation, and the integrated power for this accident are shown in Fig. 1.

The choice of relatively high value of the inverse period, as well as the choice of a maximum void case, leads to an integrated energy release equal to almost 1.8×10^{20} fissions, indicating a severe accident. It should be emphasized that this is to some extent an upper limit of the energy that *could* be released if the more pessimistic assumptions are regularly chosen. As the earlier comparison to the power reactor case has shown, the much more likely event is that the very restricted set of circumstances by which prompt criticality can be achieved will not occur, so that a far smaller energy release would be found.

The influence of the size of the collapsing experimental core on the integrated energy release is an interesting question. A partial answer was obtained by repeating the pattern of assumptions chosen above when applied to a 400-l core of the same composition except for the degree of enrichment of the fuel. It was found that for this model the number of fissions in the integrated burst was reduced by a factor of 5.3.

There is an area of danger for zero-power study methods which has not been calculated as yet. An expedient manner of studying some of the characteristics of very large, dilute, low-enrichment cores is to place a portion of this core at the centre of the zero-power system and surround this segment with an enriched annulus of a "driver" region which would be followed by a usual blanket. Thus central danger coefficients, spectra, etc., can be studied for a range of dilute systems with a greatly reduced labour effort. The danger of this procedure is that during the course of a burst the outer driving annulus often will heat much faster than the dilute core so that resulting displacements may be partially inward toward the centre of the core. This motion, during the burst, will add reactivity so that an auto-catalytic nuclear accident has been set up.

This danger can be greatly reduced, if not removed, by careful choice of the composition of the central core and driving annulus segments.

HIGH-VOID EXPERIMENTAL CORES

An important use of a zero-power experimental reactor is to do "pure" reactor physics studies. The objective usually is to learn the characteristics of very simple systems. There have been many such studies on ZPR-III in Idaho Falls and at Los Alamos.

As an example of such a reactor system which offered the possibility of considerable danger in case of a melt-down accident, an experiment involving a core consisting of enriched U^{235} plates and aluminium of 63 % density was chosen. The particular danger of this system is that there is a large void fraction which must be filled by power generation before any shut-down disassembly can begin. In the time required for this power to develop, the rate of power generation of the super-prompt critical system can rise severely.

A relatively small core of 150 l was chosen for this case because this is a reasonable size for such an experiment. Based on previous ZPR-III work and calculations, a core of the following composition was chosen:

U^{235}	9.27 vol %
U^{238}	0.67 vol %
Al (full dens.)	42.87 vol %
Fe (matrix)	9.17 vol %
Void	38.08 vol %

The pattern of assumptions made for the power reactor mock-up studies was followed again for a melt-down of this core. In this case, however, it was felt that a more plausible accident would be calculated if some degree of shut-down had been achieved by the initial accident, due to the physical energy release, loss of control-rods, metal expansion, etc. It was postulated for the first study of this case that a gain of reactivity equal to 8 % was required to achieve prompt criticality. This assumption has the partially compensating consequences of increasing K_{ex} at prompt critical and of reducing some of the void to be filled by heating before disassembly begins.

A portion of the results of the AX-I calculation of this event are shown as curve A in Fig. 2 A. The total integrated number of fissions for this burst, shown by curve A in Fig. 2 B, is equal to 5.3×10^{19}. The relatively very rapid shut-down shown for this event probably is due to two conditions. The first helping condition is that the material to be displaced by the pressures generated is much lighter than usual, being largely aluminium with some iron and highly enriched uranium. The physical characteristics of aluminium are also such that pressures are transmitted well by this material. The second helping condition is that the core is relatively small. A small physical displacement therefore contributes a relatively large reactivity loss.

To the factor of almost 3 in reduced energy yield for the high-void accident, compared to that of the 1200-l core, one must unfortunately counterpose the recognition that the considerable difficulty of achieving a prompt critical melt-down collapse for the power reactor mock-up is no longer so prominent for this case.

In order to test the importance of the low density of the core to the integrated energy release, the accident was recalculated using the same assumptions and the same physical characteristics for the materials, with the single exception that the density of the core was changed from its initial value of 4.3 to that of the 1200-l case: 10.1. The effect of the increased inertia on the fall-off of reactivity, indicated by the behaviour of α' and the resulting increase in peak power and burst width, as well as the gain in integrated energy release, are clearly shown by curves B in Figs. 2 A and 2 B. The central core temperature has risen to 7500 °C at the end of this burst.

As a third calculation for this choice of core, the effect of changing the fraction of void remaining at prompt critical was examined. Instead of the previous requirement that the high-void core collapse should gain 8 % $\Delta k/k$ to reach prompt critical, it was postulated that only 2 % $\Delta k/k$ was needed, representing close to a lower limit of reasonable choice for $\Delta k/k$.

By following the previous pattern of assumptions, it was found that the period was considerably reduced, from 2.275×10^{-2} μs^{-1} to 1.404×10^{-2} μs^{-1}. This system went prompt critical, however, with a 34.5% void fraction at the core centre compared to a 24.9% void fraction for the 8% $\Delta k/k$ gain previously required. The behaviour of α' and power generation is shown in Fig. 3, while the integrated power is shown as curve C on Fig. 2B. The considerably heightened

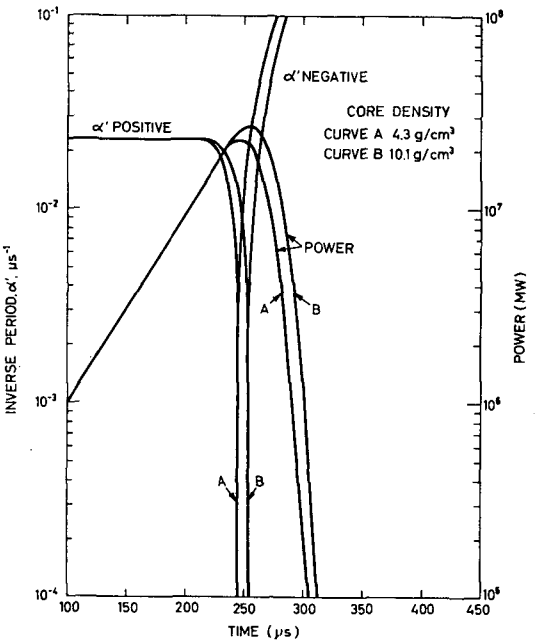


Fig. 2 A
High-void core melt-down. Influence of density change on burst behaviour.

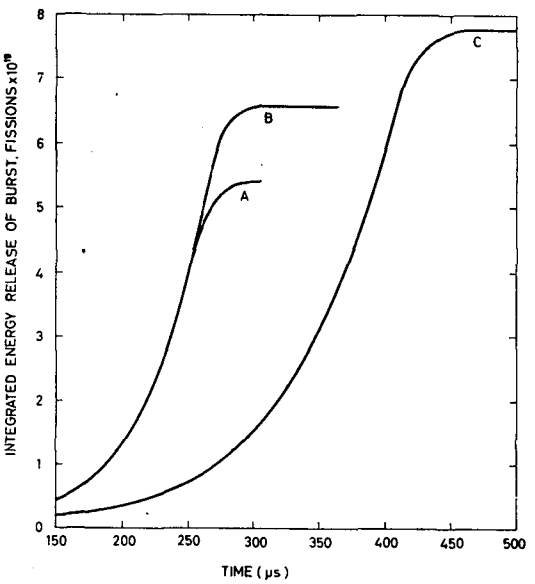


Fig. 2 B
High-void core melt-down. Integrated energy release patterns.
Curves A and B: high k, low-void core at prompt critical, and two core densities.
Curve C: low k, high-void core at prompt critical, density of core A.

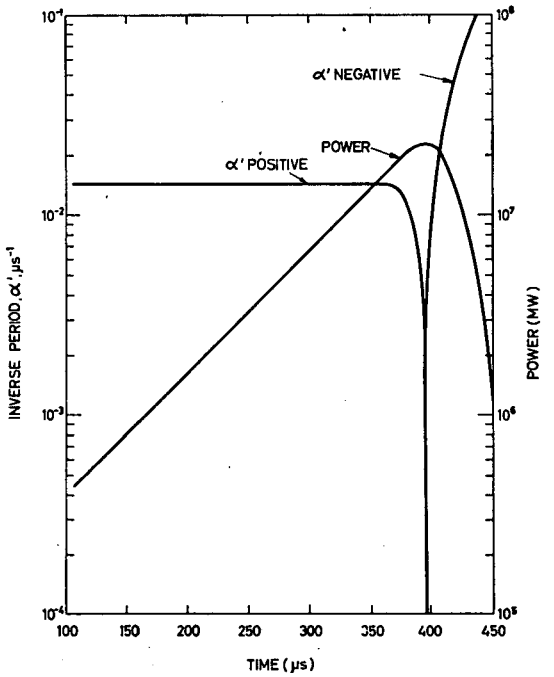


Fig. 3
High-void core melt-down. Influence of changed void fraction and k at prompt critical.

yield of curve C, compared to curve A, in spite of a marked reduction of the inverse period for that case, indicates the hazard of the presence of void spaces in a zero power assembly.

Non-melt-down severe accidents on zero-power experimental assemblies

The discussion preceding the calculations just provided showed the large degree of difficulty to postulating any means of bringing about such a burst, as well as that the most likely outcome of a melt-down incident would be a very small energy release.

Non-melt-down severe accidents may not require such a far-reaching range of implausable assumptions, and so it is important to examine this type of accident for zero-power experimental assemblies.

A characteristic of zero-power experimental assemblies, which makes them very different from power reactors for sharp severe accidents, is that the core fuel of the experimental reactors is composed of plates made of enriched uranium or plutonium and separate plates of depleted uranium. Other core components exist as still other plates. Before the accident occurs, all of these materials are at room temperature. Since a severe accident on a fast reactor will be essentially over in a time of about a few hundred microseconds, there will be no time for heat loss from the enriched uranium plates to any of the other materials of the core. Thus it is very possible that shut-down for a sharp burst on an experimental assembly will occur by expansion pressures produced as a result of vaporization of the enriched uranium plates.

There is sufficient void space in most configurations for the uranium vapour to be treated as a highly compressed gas. A variation of the AX-I code has recently been completed which treats the uranium vapour as a Van der Waals gas rather than a perfect gas because of the severe compressions. The two constants which are required for the equation of state can be estimated to some degree of certainty from works by BROUT [6] and DAANE *et. al.* [7].

The manner of analyses of this programme is to approximate the experimental reactor into a fairly large number of spherical zones. A proper portion of the core zones represent U^{235} and void space, and other core zones represent the remaining core materials. When the temperature of the enriched uranium has exceeded the critical temperature of the metal, the code, after two checks, applies the Van der Waals equation to this region, using the previous equation of state for other regions.

This new version of AX-I will be applied to a sequence of cases similar to those examined as melt-down systems.

Importance of Doppler effect for zero-power assemblies

The development of oxide and carbide fuels for large dilute power breeder reactors has brought the significance of the Doppler effect into prominence once again [8, 9]. For these systems the quite soft neutron-energy spectrum gives effective Doppler coefficients for the separated isotopes which are considerably larger than for the harder spectra of the EBR-I, EBR-II or FERMI metallic cores.

For a zero power experimental assembly in which the U^{235} is separated from the U^{238}, the enriched uranium plates, with positive Doppler coefficient of reactivity, will go through a much larger temperature rise than do the depleted uranium plates. Thus there is the possibility that the net Doppler coefficient might be positive for some experimental systems, and an auto-catalytic reaction could occur if the net reactivity coefficient due to the Doppler effect exceeded the negative reactivity coefficient due to metal expansion.

This problem, unfortunately, is very difficult to solve accurately. There have been no experimental measurements in this soft spectral area, and the problems of a satisfactory theoretical evaluation are considerable.

As a first attempt to answer this question for the Argonne assemblies, the basic terms of the Doppler reactivity coefficients as found in the recent work of NICHOLSON [8] were employed. These neutron-energy-dependent terms were matched with flux values and critical core compositions determined using the 16-group cross-section set of YIFTAH, OKRENT and MOLDAUER [10]. This procedure was applied to a range of possible assemblies with the results shown in Table I.

The first two examples merely reinforce the previous opinions that for relatively hard spectra the Doppler coefficient is a small number. The third example shows that even though $\sigma_f^{238}/\sigma_f^{235}$ is smaller than for the above cases, the 7:1 ratio of U^{238} to U^{235}, because the respective Doppler coefficients depend upon the square of atomic densities, yields a small Doppler coefficient even for this neutron-energy spectrum.

There is a considerable uncertainty in the coefficient for the 1500-l oxide case, because both the coefficients available from the work of Nicholson and the spectra of the cross-section set do not extend far enough down in neutron energy.

TABLE I

DOPPLER COEFFICIENT AT 20 °C FOR ZERO-POWER EXPERIMENTAL ASSEMBLIES

Type of Spectrum	Atomic Ratio U^{238}/U^{235}	Doppler coefficient ($\Delta k/k$ per °C, at 20°C)
EBR-I size metal core	~0	$+0.4 \times 10^{-6}$
56-l metal core (≈ EBR-II)	~1	$+0.3 \times 10^{-6}$
1500-l metal core	7	$+0.03 \times 10^{-6}$
1500-l oxide core	3	$+2.6 \times 10^{-6}$

Further study is being undertaken by others at Argonne which should reduce this doubt.

While this last coefficient is definitely competitive with the negative temperature coefficient of expansion, one should recall that the Doppler coefficient falls off approximately with a $T^{-3/2}$ power. Thus, for a burst which produces a large temperature rise in the enriched uranium, the effective Doppler coefficient will be considerably smaller than its room-temperature value.

ACKNOWLEDGEMENTS

The authors are very grateful to V. Z. Jankus for his discussions on major accident calculated methods. Don Johnson re-programmed the AX-I code to employ the Van der Waals equation of state and to allow a larger number of core zones.

REFERENCES

[1] BETHE, H. A. and TAIT, J. H., UKAEA-RHM (56)/113.
[2] KOCH, L. J. *et al.*, ANL-5719 (1957).
[3] OKRENT, D. *et al.*, ANL-5977 (1959).
[4] STRATTON, W. R., COLVIN, T. H. and LAZARUS, R. B., Proc. 2nd UN Int. Conf. PUAE **12** (1958) 196.
[5] McCARTHY, Jr. *et al.*, Proc. 2nd UN Int. Conf. PUAE **12** (1958).
[6] BROUT, R., APDA-118 (1957).
[7] DAANE, R., MELA. R. and De FELICE, J., NDA-14-107 (1955).
[8] NICHOLSON, R. B., APDA-139 (1960).
[9] GREEBLER, P., HUTCHINS, B. and SHEOKA, J. S., GEAP-3646 (1961).
[10] YIFTAH, S., OKRENT, D. and MOLDAUER, P., Fast Reactor Cross-Sections, Pergamon Press (1960).

SM—18/42

A THEORETICAL STUDY OF DESTRUCTIVE NUCLEAR BURSTS IN FAST POWER REACTORS*

V. Z. JANKUS
ARGONNE NATIONAL LABORATORY, ARGONNE, ILL.,
UNITED STATES OF AMERICA

Abstract — Résumé — Аннотация — Resumen

A theoretical study of destructive nuclear bursts in fast power reactors. An analytic method of calculating the energy release in destructive nuclear bursts in fast reactors was given by Bethe and Tait. This treatment made several major assumptions, the consequences of which needed investigation. Some changes in the formulation were made during maximum accident calculations for EBR-II. Three major assumptions in the original method involved the neglect of wave propagation, the use of a constant rate of exponential power rise during the positive period range, and the neglect of power generated thereafter. The latter two assumptions have compensating tendencies. An exact, numerical solution on the IBM-704 enabled a testing of the importance of these hypotheses, some results having been given in preliminary fashion.

Detailed comparisons of the exact solution with the original analytical method are presented herein. An improved quasi-analytical formulation, which still neglects wave propagation, is used to ascertain the importance of this assumption and define boundaries of its applicability.

Investigations of the sensitivity of energy yield to parameters in the equation of state have led to scaling laws, based on the proportionality of excess reactivity to displacements, which permit an expression of some uncertainties in the energy-pressure relationship in terms of the initial reactivity.

Etude théorique des bouffées nucléaires destructives dans les réacteurs de puissance à neutrons rapides. Bethe et Tait ont mis au point une méthode analytique de calcul de l'énergie libérée par les bouffées nucléaires destructives dans les réacteurs à neutrons rapides. Cette méthode repose sur plusieurs hypothèses majeures, dont les conséquences appelaient une étude. Certaines modifications ont été apportées à la formulation pour les calculs relatifs à l'accident le plus grave prévisible dans un réacteur du type EBR-II. Trois hypothèses majeures de la méthode initiale consistaient à négliger la propagation d'ondes, à utiliser un taux constant de l'augmentation de puissance exponentielle pendant la période positive et à négliger la puissance produite par la suite. Les deux dernières hypothèses tendent à se compenser. Une solution numérique exacte obtenue à l'aide d'une calculatrice IBM-704 a permis de vérifier l'importance de ces hypothèses, car quelques résultats préliminaires ont déjà été donnés.

L'auteur compare de manière détaillée la solution exacte et les résultats de la méthode analytique initiale. Une formulation améliorée, quasi analytique, dans laquelle on fait également abstraction de la propagation d'ondes, sera utilisée pour déterminer l'importance de cette hypothèse et définir les limites de son application.

Des études sur la sensibilité du rendement énergétique aux paramètres de l'équation d'état ont conduit aux lois de démultiplication, fondées sur le rapport entre l'excédent de réactivité et les déplacements qui permettent d'exprimer en fonction de la réactivité initiale certaines indéterminations dans la relation entre l'énergie et la pression.

* Work supported by the United States Atomic Energy Commission.

Теоретическое исследование разрушительных ядерных взрывов в энергетических реакторах на быстрых нейтронах. Бете и Тайт дали описание аналитического метода подсчета высвобождаемой энергии при разрушительных ядерных взрывах в реакторах на быстрых нейтронах. Этот подход привел к нескольким основным предположениям, выводы которых требуют исследования. Некоторые изменения в формулировке были сделаны при расчетах максимальных инцидентов для экспериментального реактора-размножителя EBR-II. Три основных предположения в первоначальном описании метода сводились к тому, что не учитывалось распространение волны, использовался постоянный темп экспоненциального повышения энергии в области положительного периода и не учитывалась получаемая после этого энергия. Два последних предположения имеют компенсирующие тенденции. Точное численное решение с помощью кода счетно-решающего устройства IBM-704 позволило проверить значение этих гипотез, и некоторые результаты даны в предварительном виде.

В докладе будет проведено подробное сравнение результатов точного решения с результатами первоначального аналитического метода. Улучшенная полуаналитическая формулировка, в которой еще не учитывается распространение волны, будет использована для установления значения этого предположения и для определения возможностей ее применения.

В результате исследований чувствительности выхода энергии к параметрам уравнения состояния были выведены пересчетные законы, основанные на пропорциональности избыточной реактивности к перемещениям, которые позволяют выразить некоторые неопределенности в отношении энергия-давление в членах первоначальной реактивности.

Estudio teórico de las explosiones nucleares destructivas en los reactores de potencia rápidos. Bethe y Tait expusieron un método analítico para calcular la energía liberada en las explosiones nucleares destructivas de los reactores de neutrones rápidos. Este método se basa en varias hipótesis principales cuyas consecuencias exigen una investigación. En la formulación del método se introdujeron algunas modificaciones con motivo de los cálculos referentes al accidente más grave concebible en el reactor EBR-II. Tres de las hipótesis principales del método inicial consistían en despreciar la propagación de las ondas, suponer un incremento exponencial constante de la potencia durante el período positivo y prescindir de la potencia generada después. Las dos últimas hipótesis tienen efectos compensadores. Una solución numérica exacta, obtenida con la calculadora IBM-704, permitió verificar la importancia de esas hipótesis; algunos resultados se han publicado a título preliminar.

Los autores comparan detalladamente la solución exacta con los resultados del método analítico inicial. Emplean una formulación mejorada cuasi analítica, en la que también se hace abstracción de la propagación de las ondas para determinar la importancia de esta hipótesis y definir los límites de su aplicación.

La investigación de la sensibilidad del rendimiento energético a los parámetros de la ecuación de estado, ha llevado a formular leyes de desmultiplicación basadas en la proporcionalidad entre el exceso de reactividad y los desplazamientos, que permiten expresar ciertas indeterminaciones de la relación energía-presión en función de la reactividad inicial.

Introduction

The yield of an excursion in a fast reactor depends upon a number of parameters: reactivity inserted k_0, prompt lifetime l, the flux shape, equations of state in the core and the blanket, etc. Many of these parameters cannot be determined very accurately for a given reactor. Thus it seemed useful to vary these parameters in order to ascertain which of them influences more the magnitude of the yield.

In a fast reactor, thermalization of the neutrons is negligible, and in excursions shut-down is accomplished by bodily motion of the reactor material. For small excursions, the reactor is shut down by thermal expansion of reactor core and blanket. For larger excursions, thermal conductivity lags considerably the heat production. And for still larger excursion even spreading of the pressure wave becomes negligible, and shut-down of large excursion in a reactor of a short lifetime is primarily affected by the inertia of the mass itself. Such a case has been treated analytically by BETHE and TAIT [1]. Later a somewhat generalized and improved version was presented at the Second Geneva Conference [2]. Both of these treatments have used one-neutron-group-perturbation theory to calculate the reactivity. In Section I of this report, the Bethe-Tait method will again be outlined, bypassing a multi-group treatment, still assuming that displacements of reactor material are small, and neglecting the propagation of elastic waves during the excursion. (Usually wave propagation is of less importance than one would expect for reasons considered later).

The following chapter will compare the results of a number of numerical AX-I [3] calculations against the Bethe-Tait theory. The salient outcome in all these cases is that the Bethe-Tait method always yields less energy and higher pressure than the more accurate AX-I calculation. Also, the Bethe-Tait results get closer to the results of AX-I as the magnitude of the excursion increases. Thus one can use the Bethe-Tait method to give a pessimistic estimate of pressure peak and an optimistic estimate of energy yield. The estimates are close to the actual situation, of course, only for very large excursions.

1. The Bethe-Tait method

1.1. MECHANISM OF REACTIVITY REDUCTION

In the previous paper [2], an expression for the reactivity reduction has been derived based on first order perturbation of the one-group diffusion equation. The validity of the one-group treatment is doubtful, and the afore-mentioned formulae could easily be generalized for multi-group treatment. Numerical calculation of the latter, however, is rather cumbersome. Thus, here we intend to derive equivalent expressions based on the (experimentally measurable) worth of reactor material. R. D. NICHOLSON [4] has applied this idea in the study of quasi-two-dimensional explosion calculations. Also we wish to extend the calculation (consistent with the Bethe-Tait assumptions) for the case in which the "microscopic" transport cross-section in the blanket is different from that in the core.

We will consider a reactor composed of a core and a blanket. We will assume that the core is composed of one homogenized material, the blanket of another, and that there may be variation of density of these materials. We will define the worth of reactor material $w(r)$ as the decrease in reactivity due to removal of unit mass of materials at the place r. The worth will be continuous in the reactor except for the interface of the core and the blanket. The change in the reactivity due to an increase in density, $\delta\varrho$, of the core material is

$$k - k_0 = \int \delta\varrho\, w_c(r)\, \mathrm{d}V + \ldots$$

If the change of density is caused by motion of the material

$$\delta\varrho = -\nabla \cdot \varrho u$$

where u is the displacement. Substituting this relation into the preceding one and integrating by parts over the whole core, we see that

$$k - k_0 = \int \varrho u \cdot \nabla w_c \, dV + \dots \tag{1}$$

Now taking the second derivative with time and substituting for $\varrho \ddot{u}$ its value from the equation of motion:

$$\varrho \ddot{u} = -\nabla p \tag{2}$$

we have

$$\ddot{k} = -\int \nabla p \cdot \nabla w_c \, dV + \dots$$

Integrating by parts we obtain

$$\ddot{k} = \int p \left(\nabla^2 w_c \right) dV - \int p \nabla w_c \cdot dS + \dots \tag{3}$$

Now we will consider the reduction of reactivity at the interface of the core and the blanket. We will assume that the velocity of wave propagation is very small. Then if the pressure inside the core is p, the pressure at the interface has dropped to an intermediate value

$$p_i = p \frac{\varrho_b c_b}{\varrho_b c_b + \varrho c}, \tag{4}$$

where ϱc and $\varrho_b c_b$ are wave impedances of the core and of the blanket. In the region of intermediate pressure the core and blanket particles are traveling at the speed

$$\dot{u} = \frac{p}{\varrho_b c_b + \varrho c}$$

and the region itself is increasing with appropriate velocity into the core and into the blanket. So that the motion of core part increases the derivative of reactivity at the rate

$$\int \varrho \dot{u} \cdot \nabla w_c \, c \, dS = \int p \frac{\varrho c}{\varrho_b c_b + \varrho c} \nabla w_c \cdot dS$$

and motion of the blanket

$$\int \varrho \dot{u} \cdot \nabla w_b \, c_b \, dS = \int p \frac{\varrho c}{\varrho_b c_b + \varrho c} \nabla w_b \cdot dS .$$

The sum of these two contributions is

$$\int p \nabla w_c \cdot dS - \int p \frac{\varrho_b c_b}{\varrho_b c_b + \varrho c} (\nabla w_c - \nabla w_b) \cdot dS .$$

Adding this to Eq. (3), we obtain that total reactivity reduction is

$$\ddot{k} = \int p \, (\nabla^2 w_c) \, dV - \int p \frac{\varrho_b c_b}{\varrho_b c_b + \varrho c} (\nabla w_c - \nabla w_b) \cdot dS . \tag{5}$$

If the gradients of worth are the same at the interface for core and blanket materials, the second term vanishes and we have quite a simple expression for the reactivity reduction.

Previously [2] we have derived an equivalent expression in terms of the flux in the diffusion theory formulation. Analytically the approach was even more simple and general, since we did not have to define separately worth coefficients for the core and blanket, and did not need to treat the interfaces differently from the rest of the reactor. In the one-group treatment we have expressed reactivity change by Eq. (18) of Ref. [2].

$$k - k_0 = \int \varrho u \left\{ \frac{2}{\varrho} (\nabla D)\, (\nabla \Phi)^2 + \frac{D}{\varrho} \nabla (\nabla \Phi)^2 - \frac{2}{\varrho} (\nabla \Phi)\, \nabla \cdot \right.$$

$$\left. \cdot (D \nabla \Phi) \right\} \mathrm{d}V \Big/ \int \nu \Sigma_\mathrm{f} \Phi^2 \, \mathrm{d}V.$$

This equation, which can be written in a form similar to Eq. (1)

$$k - k_0 = \int \varrho u \cdot f \, \mathrm{d}V \tag{6}$$

was valid over the whole reactor, even if it is composed of different materials. However, instead of ∇w_c of (1) we had

$$f = \frac{1}{(D\varrho)} \left\{ \nabla (D \nabla \Phi)^2 - 2 (D \nabla \Phi)\, \nabla \cdot (D \nabla \Phi) \right\} \Big/ \int \nu \Sigma_\mathrm{f} \Phi^2 \, \mathrm{d}V. \tag{7}$$

Now, applying to Eq. (6) the same procedure as we have used in deriving Eq. (3), but taking the volume of the whole reactor for our domain of integration, we obtain

$$\ddot{k} = \int p \, (\nabla \cdot f) \, \mathrm{d}V. \tag{8}$$

Since pressure is being generated only in the core, we subdivide our domain of integration into three regions: interior of the core, pill-box shaped volume enclosing the interface, and the rest of the blanket. The integral over the blanket vanishes if the velocity of wave propagation is small enough. The integral over the pill-box does not if f is discontinuous. Upon evaluating the latter using the divergence theorem, we obtain

$$\ddot{k} = \int p \, (\nabla \cdot f) \, \mathrm{d}V - \int p_\mathrm{i} \, (f_\mathrm{c} - f_\mathrm{b}) \cdot \mathrm{d}S \tag{9}$$

where p_i is the pressure at the interface, f_c and f_b are the values of function f on the core and the blanket side at this location. It is obvious that the surface integral may contribute a considerable fraction of the reactivity reduction if $(f_\mathrm{c} - f_\mathrm{b})$ is not small.

It is interesting to note that the expression for f, Eq. (7), simplifies considerably for a one-dimensional reactor. For a spherical reactor it becomes equal to

$$f = -\frac{4}{r} \frac{1}{(D\varrho)} \left(D \frac{\mathrm{d}\Phi}{\mathrm{d}r} \right)^2 \Big/ \int \nu \Sigma_\mathrm{f} \Phi^2 \, \mathrm{d}V; \tag{10}$$

for infinite cylindrical reactor

$$f = -\frac{2}{r} \frac{1}{(D\varrho)} \left(D \frac{\mathrm{d}\Phi}{\mathrm{d}r} \right)^2 \Big/ \int \nu \Sigma_\mathrm{f} \Phi^2 \, \mathrm{d}V;$$

and it vanishes for a plane reactor.

In the derivation of Eqs. (5) and (9), no mechanism of pressure generation has been mentioned and no assumption on its propagation has been made, except in the surface term. Bethe and Tait [1] have assumed that the pressure in this integral can be calculated adequately, neglecting local expansion and propagation of the pressure. This assumption, of course, usually will not give correct local pressures, but it is quite satisfactory for evaluation of the volume integral in Eq. (8) if $(\nabla \cdot f)$ can be considered roughly constant and if non-linear effects in the propagation of the pressure are not too prominent. In this case, the integral $\int p \, \mathrm{d}V$ does not change much during the time of power generation due to such effects. In actual situations, however, $(\nabla \cdot f)$ usually decreases going from the centre outwards, and $\int p \, \mathrm{d}V$ decreases as time flows by. Thus, a smaller reactivity reduction results and a higher energy yield is obtained. In the original paper, Bethe and Tait have assumed a mono-energetic parabolic flux

$$\Phi = 1 - q r^2/b^2 \tag{11}$$

for a spherical reactor, thus obtaining $(\nabla \cdot f)$ constant. Using this flux, and assuming that the macroscopic cross-sections are constant in core, from Eq. (10) we see that

$$(-\nabla \cdot f) = \frac{48 q^2 F}{4\pi \Sigma_{\mathrm{tr}} \nu \Sigma_{\mathrm{f}} b^7 [1 - (6q/5) + (3q^2/7)] \varrho_{\mathrm{c}}} \tag{12}$$

where

$$F = \int_0^b \nu \Sigma_{\mathrm{f}} \Phi^2 r^2 \, \mathrm{d}r \Big/ \int_0^\infty \nu \Sigma_{\mathrm{f}} \Phi^2 r^2 \, \mathrm{d}r$$

b is the radius of the core, Σ_{tr} and Σ_{f} in Eq. (12) are the transport and fission cross-sections in the core, ϱ_{c} is the density of core material. In the previous [2] derivation of this constant, we have committed an error in integration, as has been pointed out to us by A. P. Schmitt [5]. Thus, to be consistent with the initial one-group parabolic flux assumption, we should have $[1 - (6q/5) + (3q^2/7)]$ instead of $(1 - 0.6q)$ in Eqs. (21), (23), (28), (30), (32a) and (33a) of that paper. An average value of $(\nabla^2 w_{\mathrm{c}})$ can be estimated also using other experimental or multi-group calculated results. If, for a spherical reactor, we assume that $(\nabla^2 w_{\mathrm{c}})$ is constant, we see upon integrating that

$$\nabla w_{\mathrm{c}} = (1/3)(\nabla^2 w_{\mathrm{c}}) r \tag{13}$$

and on integrating again

$$w(b) - w(0) = (1/6)(\nabla^2 w_{\mathrm{c}}) b^2. \tag{14}$$

Thus $(\nabla^2 w_{\mathrm{c}})$ can be estimated from the difference of core material worth at the centre and at the interface. Similarly, another average value of $(\nabla^2 w_{\mathrm{c}})$ can be obtained if we know the reactivity change for a uniform expansion of the core alone $(-b \Delta k/\Delta b)$. Then, substituting Eq. (13) into Eq. (1), we obtain

$$(\nabla^2 w_{\mathrm{c}}) = -\left(-\frac{b \Delta k}{\Delta b}\right) \frac{15}{4\pi} \frac{1}{b^5 \varrho_{\mathrm{c}}}. \tag{15}$$

1. 2. Solution of kinetic equations

Assuming that the integral in Eq. (5) or Eq. (8) can be evaluated adequately, calculating the pressures according to the energy input and neglecting expansion, we need only the power shape and the equation of state for the core material

at the original density. Previously, for the sake of simplicity it was assumed that the power is of parabolic shape, given by Eq. (11), and that the pressure is vanishingly small till the treshold of energy density Q^* is reached and increases linearly thereafter:

$$\begin{aligned} p\,(e, \varrho_c) &= 0 && \text{for } e < Q^* \\ p\,(e, \varrho_c) &= (\gamma - 1)\,\varrho_c\,(e - Q^*) && \text{for } e > Q^*. \end{aligned} \tag{16}$$

Then, after evaluation of the volume integral in Eq. (5), we could express $\ddot{k}$ as a function of Q, the energy density at the centre of the core. Depending upon whether threshold energy density has been reached at all, in part, or in the whole core, we have obtained three analytic expressions for $\ddot{k}$ [2]:

$$\left.\begin{aligned} \ddot{k} &= 0, \text{ for } Q < Q^* \\ \ddot{k} &= -\left(\frac{1}{x}\frac{k_0^3}{l^2}\right)\frac{Q}{Q^*}\left(1 - \frac{Q^*}{Q}\right)^{5/2}, \text{ for } Q^* < Q < Q^*\,(1-q)^{-1} \\ \ddot{k} &= -\left(\frac{1}{x}\frac{k_0^3}{l^2}\right)\frac{5}{2}\,q^{3/2}\left[(1 - 0.6\,q)\,\frac{Q}{Q^*} - 1\right], \text{ for } Q > Q^*\,(1-q)^{-1} \end{aligned}\right\} \tag{17}$$

where $1/x$ is an abbreviation for

$$\frac{1}{x} = (-\nabla \cdot f)\left(\frac{4\pi b^3}{3}\right)\left(\frac{2}{5}\,q^{-3/2}\right)(\gamma - 1)\,\varrho_c\,Q^*\,\frac{l^2}{k_0^3}. \tag{18}$$

This is a fairly simple expression for $\ddot{k}$ as a function of the energy generated. It is quite clear, however, that starting with Eq. (5) we can evaluate $\ddot{k}$ for non-

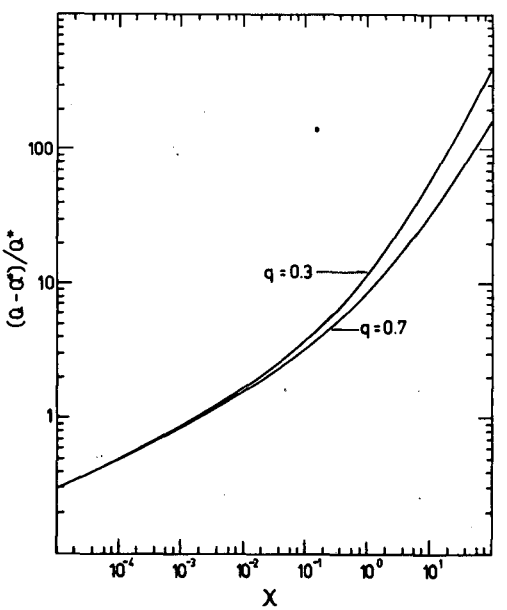


Fig. 1
Ratio of excess energy to threshold energy in simple Bethe-Tait calculation.

constant ($\nabla^2 w_c$) and any given power shape and equation of state in the core. Provided we calculate pressures neglecting local expansion and wave propagation, we would end up with an expression for $\ddot{k}$ as a function of energy input, though the expression may be more complex than Eq. (17). Thus one has to consider that the assumptions of constant ($\nabla \cdot f$), spherical geometry, parabolic power shape and simple equation of state for core material are only non-essential simplifications of the Bethe-Tait approach.

If only prompt neutrons are to be considered, the connection between power and reactivity is given by

$$k = l \frac{\mathrm{d}}{\mathrm{d}t} \ln \dot{Q}. \tag{19}$$

Eqs. (19) and (17) form two coupled second order differential equations. These equations were transformed into four first order differential equations using as dependent variables: k, $\dot{k}$, Q and $\ln \dot{Q}$. (The latter choice has been made to assure convergence at large values of time.) Then the equations were solved numerically on the LGP-30 using the Runge-Kutta routine. For a step input of reactivity, k_0, and an excursion starting at negligible power, results depend on k_0 and prompt lifetime l only through the combination $k_0^3 l^{-2}$, since $\ddot{k}$ is a function of the energy input only. For the present, simple Bethe-Tait case, Eq. (16), the energy yield compared to the threshold energy, depends only on x and the power shape q. The results have been displayed in Table I and Fig. 1.

TABLE I

RATIO OF EXCESS ENERGY TO THRESHOLD ENERGY $(Q-Q^*)/Q^*$ IN SIMPLE BETHE-TAIT CALCULATION

x \ q	1	0.7	0.5	0.3
10^{-5}	0.2981	0.2981	0.2981	0.2981
10^{-4}	0.4902	0.4902	0.4902	0.4902
10^{-3}	0.8431	0.8431	0.8431	0.8487
10^{-2}	1.561	1.561	1.562	1.633
10^{-1}	3.267	3.268	3.325	3.774
10^{0}	8.398	8.461	9.090	11.69
10^{1}	29.82	31.07	36.47	54.60
10^{2}	160.5	176.9	226.7	384.6

Calculations for larger and smaller values of x have also been performed. Determining asymptotes graphically we have seen that for large values of x the yield approaches slowly

$$\frac{Q-Q^*}{Q^*} \to \frac{17}{15}\,\frac{2}{5}\,\frac{q^{-3/2}}{1-0.6q}\,x.$$

For small values of x

$$\frac{Q-Q^*}{Q^*} \to \frac{9}{4}\,x^{5/27}.$$

In Fig. 2 we have also plotted as solid curves the "widths" of power bursts obtained in Bethe-Tait calculations. The "width" has been defined as the ratio

of the integrated power ($E_T = E^* + E_{ex}$) to the maximum power reached and measured in terms of the initial period: $k_0 E_T / l P_{max}$. In Bethe-Tait calculations, the "width" increases from one at very small values of x up to 2.90 for very large values of x. Open dots in Fig. 2 represent the widths of power bursts obtained in AX-I calculations described in Section 2.1 and Table VI.

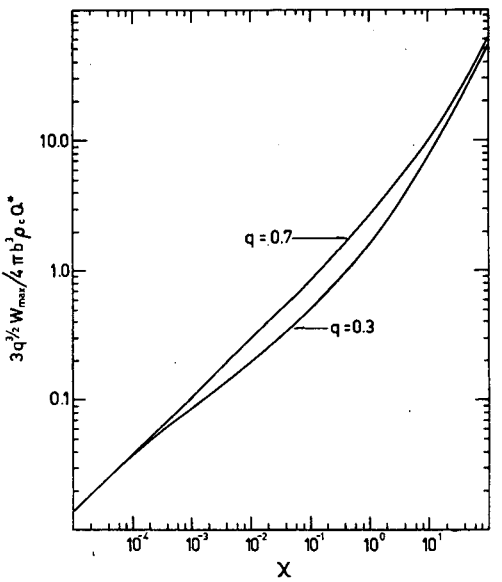


Fig. 3
An upper bound for maximum work as given by Eq. (21).

1. 3. Excess energy and an upper bound for mechanical work

In the derivation of formula (17), threshold energy density at the centre, Q, has been used as a parameter, and the ratio of this energy density over threshold energy density is displayed in Table I and Fig. 1. However, since power maintains spatially constant shape, this ratio is the same as the ratio of total energy in excess of threshold to the threshold energy. Where the latter, E^*, is understood as the amount of energy generated from the beginning of excursion, at negligible power, till the incipience of reactivity reduction mechanism,

$$E^* = (4\pi/3)\, b^3 \cdot \varrho_0 \cdot Q^* \,(1 - 0.6\, q)\,. \tag{20}$$

Only integrated quantities E_{ex} and E^*, of course, preserve their significance when we allow expansion during heat generation, as in Section 2. Energy in excess of threshold, E_{ex}, by itself, however, does not determine energy available for mechanical work.

An upper bound for it can be estimated easily assuming that energy density distribution by the end of heat generation still has parabolic shape and that threshold energy density does not increase with specific volume. Then energy density above threshold energy density:

$$Q\,[1 - (q r^2/b^2)] - Q^*$$

could be converted to work in an isentropic expansion. Simple spatial integration of the above expression gives an upper bound for mechanical work:

$$\left.\begin{aligned} W_{\max} &= \frac{4\pi}{3} b^3 \varrho_c Q^* \left(\frac{Q-Q^*}{Q^*}\right)^{5/2} \left(\frac{Q^*}{qQ}\right)^{3/2} \cdot \frac{2}{5} \\ &\text{if } Q^* < Q < Q^*(1-q)^{-1} \\ W_{\max} &= \frac{4\pi}{3} b^3 \varrho_c Q^* \left[\frac{Q}{Q^*}(1-0.6q)-1\right] \\ &\text{if } Q > Q^*(1-q)^{-1}. \end{aligned}\right\} \tag{21}$$

A dimensionless quantity $3q^{3/2} W_{\max}/4\pi b^3 \varrho_c Q^*$ has been evaluated using Eq. (21) and Table I, and the results have been presented in Table II and Fig. 3. One

TABLE II

AN UPPER BOUND FOR MAXIMUM WORK $3q^{3/2}w_{\max}/4\pi b^3 \varrho_c Q^*$ AS GIVEN BY (21)

x \ q	1	0.7	0.5	0.3
10^{-5}	0.0131	0.0131	0.0131	0.0131
10^{-4}	0.0370	0.0370	0.0370	0.0365
10^{-3}	0.104	0.104	0.104	0.0848
10^{-2}	0.297	0.297	0.281	0.190
10^{-1}	0.875	0.864	0.717	0.479
10^{0}	2.84	2.63	2.14	1.55
10^{1}	11.4	10.3	8.92	7.33
10^{2}	63.6	59.8	56.0	51.8

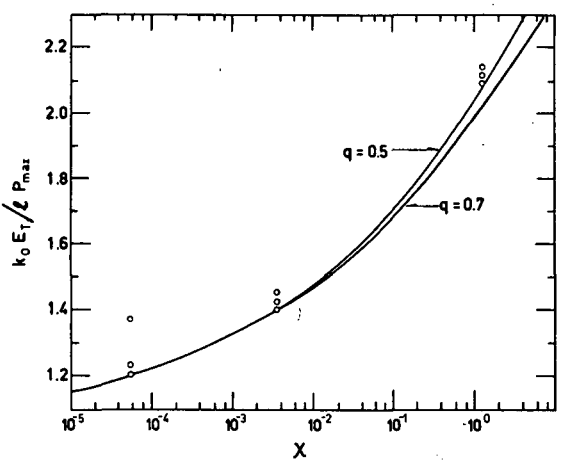


Fig. 2
Width of the power burst in terms of initial period.

notices that for small x the curves for various q's coincide. For large x, the curves have a common asymptote. Using previous expression for $(Q - Q^*)/Q^*$ we see that

$$\frac{3q^{3/2}W_{\max}}{4\pi b^3 \varrho c\, Q^*} \to 0.45\, x \text{ for } x \gg 1.$$

This simple estimate of upper bound for energy available for mechanical work, however, usually is too large, particularly for low energy yields. A better estimate can be obtained if equation of state for isentropic expansion is known. For example, if isentropic expansion is described by Eq. (16a) and $Q<Q^*\,(1-q)^{-1}$, one finds that $W_{\max}$ obtained from Fig. 3 or Table II should be multiplied by a function F of $\beta'\,(Q - Q^*)/(-\tau' A')$. This function is nearly equal to 1 if argument values are larger than 10, but

$$F = \frac{2}{7}\frac{\beta'\,(Q-Q^*)}{-\tau' A'}\left[1 - \frac{4}{9}\frac{\beta'\,(Q-Q^*)}{-\tau' A'} + \ldots\right], \text{ if } \frac{\beta'\,(Q-Q^*)}{-\tau' A'} \ll 1.$$

1. 4. Approximate solutions of kinetic equations

Previously [2, 6], some approximate solutions of coupled equations, Eqs. (17) and (18), have been made assuming that the heat input, Q, continues rising exponentially and calculating reactivity from Eq. (17) till it vanishes. The value of Q at this time was considered a fair approximation. It is nearly correct for very large values of the argument x, since the actual value is only 1.13 of the approximate value. However for smaller values of the argument this approximation procedure is a serious underestimate. This can be seen by examining Fig. 4, where the ratio of the actual value of excess energy to the approximate

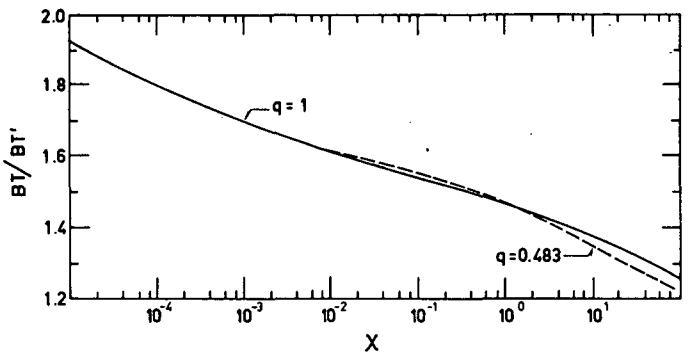


Fig. 4
Ratio of excess energy in BT to that in approximation BT'.

value, BT/BT', has been plotted for $q=1$ and $q=0.483$. Similar calculations using the IBM-704 have also been performed. In these calculations, AX-I has been modified so that the heat generation rate continues to rise exponentially. The programme, called AX-I′, terminated the calculation when the excess reactivity k_0 was reduced to zero. The ratio of the results for excess energy, AX-I/AX-I′, has been plotted in Fig. 5. One observes that in both figures a curious inflection region in the curve is obtained after the zone of pressure generation reaches the interface between the core and the blanket.

A better approximate solution of the coupled equations, Eqs. (17) and (19), could be obtained by using the long delay approximation for reactivity reduction. Then one determines reactivity from Eq. (17), assuming that the heat input continues to rise exponentially with time. Using this reactivity one finds numerically power and energy yield from Eq. (19).

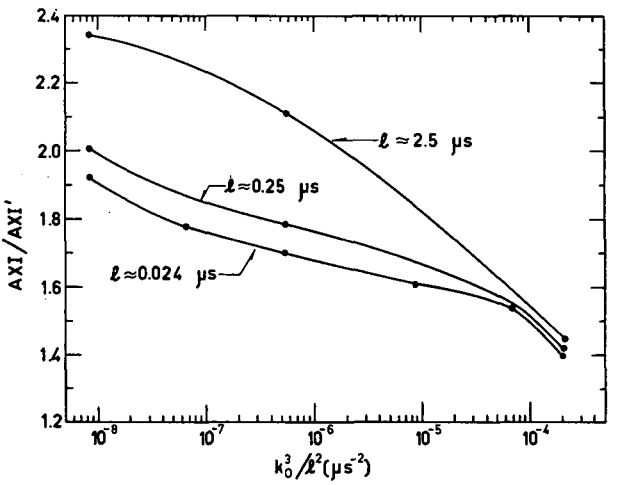


Fig. 5
Ratio of excess energy in AX-1 to that in AX-1'.
$x \approx 1.10 \times 10^4$ (k_0^3/l^2)

We have used this method to calculate the yield of an excursion when pressure is proportional to nth power of heat input. Assuming that heat input continues to rise exponentially, we see that reactivity reduction calculated from Eq. (8) increases also exponentially, at a different rate. Using reactivity found this way, we can perform integrations of Eq. (19) for power and total energy

TABLE III

THE PEAK OF THE (DIMENSIONLESS) AVERAGE CORE PRESSURE, EQS. (22) OR (8b), IF PRESSURE IS PROPORTIONAL TO nth POWER OF HEAT INPUT

n	$[\bar{p}]$	$n^3\left[\Gamma\left(1+\frac{1}{n}\right)\right]^n$
0.5	0.202	0.177
1.0	1.13	1.00
1.5	3.26	2.89
2.0	7.08	6.30
2.5	13.1	11.54
3.0	21.9	19.2
3.5	33.4	29.5
4.0	48.6	42.9
4.5	68.7	60.7
5.0	91.7	81.6

yield exactly. And we obtain that final value for "dimensionless" pressure averaged over the core is

$$\left[\bar{p}_{\max}\right] = (-\nabla \cdot f) \times \bar{p}_{\max}(4\pi/3)\, b^3 \times (l^2/k_0{}^3) \approx n^3 \left[\Gamma\left(1+\frac{1}{n}\right)\right]^n. \quad (22)$$

$\bar{p}_{\max}$ is the highest average pressure in the core. Exact solutions of kinetic equation were also obtained using numerical integration. They are compared with these approximate values in Table III. One notices that, for all n, the long delay approximation understimates pressure yield by about 13%. Thus, it is likely that long delay approximation will underestimate average peak pressure only by a little also for other connection between pressure and heat input.

2. AX-1 calculations

2. 1. General description of the programme

Given a spherically symmetric, superprompt critical system, the programme, computes the variation in time and space of the specific energy, temperature, pressure, density, and velocity. As a function of time it computes the reactivity (in the form of alpha, the inverse period), the power, the total energy, and the position of the boundaries of the various shells into which the system has been subdivided. All delayed-neutron effects are ignored, and no allowance is made for transfer of heat by conduction or radiation. The input information includes the initial reactivity or geometry, the initial velocities and temperatures of the mass points, the composition and disposition of materials, the appropriate equation of state constants, and the microscopic neutron cross-sections. For calculational purposes, the spherical assembly is divided into a number of hypothetical spherical shells or mass points. The neutronics of this system is calculated in conventional fashion, using the S_4 method, thereby providing a power distribution across the radial network, as well as the alpha of the system. From the neutronics calculation one goes to thermodynamics and hydrodynamics portion to calculate the variation of power, temperature, pressure, density, and velocity with time.

The calculation proceeds initially like the usual S_n calculation. After computing average cross-sections for each of the spherical shells, in the mixture code, the programme proceeds either to a calculation of alpha (k_{ex}/l) for the specified configuration, or to a scaling of the reactor radii to provide the alpha originally specified. Before proceeding to the hydrodynamics the code also computes k_{ex} ($= 1 + k_0$) for the initially converged configuration, if so requested.

Then, for one or more short time intervals, Δt, alpha is considered to remain constant while the power varies as exp ($\alpha \Delta t$). From the pressure gradients in the system the average accelerations of the mass points are computed and, hence, the new velocities at the end of a time interval. These, in turn, lead to the new radial positions of each shell boundary at the end of a time interval. The solution is performed in a Lagrangian co-ordinate system, i.e., the mesh is embedded in the material, and follows it along throughout its motion.

During the time interval energy is added to the system (the average power times Δt), and this is distributed among the shells in accord with the previously calculated fission distribution. By allowing for the work done by or on a shell in expansion or compression, the net change in internal energy is computed, and from the internal energy a new pressure and temperature are obtained.

The so-called viscous pressure, a mathematical procedure by VON NEUMANN and RICHTMYER [7, 3] is included to permit thermodynamic and hydrodynamic calculations in the presence of steep shock front. Hence, the total pressure used is the sum of the hydrodynamic pressure, given by the equation of state, and the synthetic viscous pressure, p_v.

$$p_v = C_{vp} \frac{1}{\varrho} \left(\Delta R \frac{\partial \varrho}{\partial t} \right)^2 \tag{23}$$

where ϱ is the density, ΔR is the mesh-width, and numerical constant C_{vp} is usually taken to be $1.5 \leq C_{vp} \leq 2$.

When calculation of thermodynamic and hydrodynamic changes during the time interval Δt is complete (a hydrocycle), a series of tests is run and the programme proceeds with another hydrocycle or goes back to the neutronics calculation. To control the pace of a problem, the code continually examines the magnitude or rate of change of certain crucial parameters, and varies Δt of a hydrocycle or the number of hydrocycles per neutron cycle accordingly. This latter number begins at unity and is allowed to build up gradually if the forces present are not changing alpha too rapidly or modifying the density of a mass point radically. When the power variation in a hydrocycle, or the change in alpha between neutron cycles, gets so large as to damage the accuracy of the solution, the pace of the calculation is slowed automatically — or stopped in extreme cases.

2. 2. COMMON PROCEDURE USED IN PRESENT STUDY

In the present calculations, it was assumed that the hydrostatic pressure, p, depends linearly on the temperature, θ, and the density, ϱ:

$$p = \tau + \beta\theta + \alpha\varrho$$

if the temperature and density are sufficiently large. In all the calculations, however, τ had a rather large negative value, and the hydrostatic pressure was assumed to vanish whenever the above expression yielded a negative value. All the calculations have been started at low temperature and density. Thus one needed to produce an appreciable amount of energy density to generate non-vanishing pressure in the core. The specific heat at constant density has been assumed to vary linearly with temperature.

$$C_v = \left(\frac{\partial e}{\partial \theta} \right)_v = A + B\theta.$$

A system of units: g, cm, μs, keV, has been used. Thus density has been measured in g/cm^3, temperature in keV, pressure in Mb (10^{12} dyn/cm^2), energy density e in 10^{12} erg/g. Since during the time heat is being generated in an excursion there is no appreciable thermal conduction, information about temperature is superfluous and only the relation connecting pressure with internal energy and density is needed. Eliminating θ we obtain (6a) that this relation is

$$e = \frac{A}{\beta}(p - \tau - \alpha\varrho) + \frac{B}{2\beta^2}(p - \tau - \alpha\varrho)^2 - \frac{\tau}{\varrho} + \alpha \ln p + \text{const.}$$

fairly complex, involving five free constants. By choosing them judiciously, we can hope to approximate even a fairly complicated equation of state governing the actual physical situation during an excursion.

In the initial calculations, reported in Section 2. 3. the equation of state constants, with the exception of τ, have been chosen the same as used in early calculations by STRATTON [8, 9]. Using his set of constants, Stratton was able to represent successfully the burst of Godiva. However, based on this success no claim can be made that the same numerical values are applicable to a reactor with voids, differing much in density from Godiva. Indeed, more sophisticated considerations of uranium properties point to the contrary [9]. So, in the present set of calculations, some of the parameters have been varied to obtain an indication of how they influence the energy yield in an excursion. After the initial calculations, some simplifications of the equations of state have been made. A few constants pertaining to the behaviour of the core material have been kept the same throughout the study (except as noted otherwise). In all the problems reported the core density has been the same

$$\varrho_c = 7.92 \text{ g/cm}^3. \tag{24}$$

Also the initial rate of pressure rise

$$\left(\frac{\partial p}{\partial e}\right)_\varrho = \frac{\beta}{A + B\theta_{sp}} = \frac{\beta^2}{A\beta - B(\tau + \alpha\varrho_c)} = 17.815 \text{ g/cm}^3 \tag{25}$$

has been kept the same throughout the study.

All the calculations have been performed using one-energy group neutrons. The lifetime of the reactor has been varied by varying the velocity of the neutrons. All calculations have been started with the reactor material at rest, with negligible power density, and with initial core temperature at

$$\theta_i = 10^{-4} \text{ keV}. \tag{26}$$

Actually, since in the initial stage the power and energy increase exponentially, some machine time has been saved by assuming an initial energy distribution of proper parabolic shape (and everywhere well below threshold) and starting the calculation with the power equal to the total energy rise at this time divided by the initial period, k_0/l. At criticality the reactor core was about 23.38 cm in radius and it had about a 20-cm blanket. The computing machine varies linear dimensions of the reactor keeping composition (density and cross-sections) constant to achieve the requested initial inverse period. Hence, in the calculations reported, the radius of the core was somewhat larger, roughly

$$b \approx 23.4 \text{ cm}. \tag{27}$$

The threshold energy (generated before non-vanishing pressure has been reached in the centre) was

$$E^* \approx 2100 \times 10^{12} \text{ erg} \tag{28}$$

with exception of Section 2. 4. Since b and E^* varied little among the problems, we have not given detailed account of their variation except in Figs. 8, 10 and 11.

We believe that keeping the above parameters ϱ_c, initial $(\partial p/\partial e)_\varrho$, b and E^*, the same and varying lifetime l and reactivity inserted, k_0, does not restrict much the generality of the study. The belief is based upon arguments advanced in Appendix B. There, relying on plausible assumptions (that during the excursion density variation is small and that reactivity is proportional to this variation), we are able to construct solutions that depend only on two of the dimensionless combinations of the above six constants.

2. 3. Variation of lifetime

In the initial set of calculations, the prompt lifetime has been varied by varying the neutron velocity. For neutron velocity $V_g = 1695$ cm/μs, a lifetime of approximately $l \approx 0.024$ μs has been obtained. Then the neutron velocity has been arbitrarily decreased by a factor of 10 and by a factor of 100, resulting in roughly proportional increases in the lifetime.

Here the core has composed of 36% enriched uranium. Microscopic cross-sections for U^{235} have been (in barns):

$$\nu\sigma_f = 3.75, \quad \sigma_{tr} = 7.0, \quad \sigma_{scatt} = 5.3. \tag{29}$$

Cross-sections for U^{238} have been chosen

$$\nu\sigma_f = 0.25, \quad \sigma_{tr} = 7.0, \quad \sigma_{scatt} = 6.7. \tag{30}$$

While in the blanket no heat generation has been allowed by choosing for the blanket material

$$\nu\sigma_f = 0, \quad \sigma_{tr} = 7.0, \quad \sigma_{scatt} = 6.8\,. \tag{31}$$

The constants for the equations of state of core material, of density 7.92 g/cm^3, have been:

$\alpha = 0.02873$ cm^2/μs^2, $\quad A = 12.163$ cm^2/μs^2 keV,
$\beta = 278.46$ g/cm μs^2 keV, $\quad B = 5780$ cm^2/μs^2 keV^2.
$\tau = -0.3946$ g/cm μs^2,

The density of the blanket has been chosen to be 15.83 g/cm^3. The constants for the equation of state in the blanket have been chosen the same as in the core with exception of τ. The latter has been chosen $\tau = -0.4687189$ in order to make blanket pressure exactly zero at initial density and the starting temperature of 5×10^{-5} keV. Starting temperature of the core was $\theta_i = 10^{-4}$ keV, thus requiring

$$Q^* = A\left[-\frac{\tau + \alpha\varrho_c}{\beta} - \theta_i\right] + \frac{1}{2}B\left[\left(-\frac{\tau + \alpha\varrho_c}{\beta}\right)^2 - \theta_i^2\right] = 0.007\,093 \text{ cm}^2/\mu\text{s}^2$$

of energy density to start generating pressure. With this set of constants the ratio of power at the interface to that at the centre was

$$1 - q = 0.517.$$

Thus if one computes x of the Bethe-Tait theory approximately, using Eqs. (18) and (12) while for $(\gamma - 1)\,\varrho_c$ taking the initial $(\partial p/\partial e)_\varrho$, one obtains

$$x \approx 1.10 \times 10^4 \ \mu\text{s}^2 \ (k_0^3/l^2).$$

The results of these computations are given in Table IV and Figs. 6 and 7. In these and other figures dots represent values obtained directly from an IBM-704 computation. Due to the presence of the threshold energy density Q^*, roughly the same amount of energy E^* had to be generated in all cases before pressures started to build up and the reactivity reduction became effective. Thus only the excess over actual threshold energy is given in this and the following tables and figures. The value, for pressure, p_{max}, given in this and the following tables, is the maximum pressure at the centre printed out by IBM-704. For small excursions the pressure varies considerably between successive print-

TABLE IV

VARIATION OF LIFETIME

Problem No.	k_0^3/l^2 (μs^{-2})	l (μs)	Excess energy yield (10^{12} erg)	Centre peak pressure (Mb)	Maximum kinetic energy (10^{12} erg)
1	4.609×10^{-10}	0.0206	621	0.0064	1.36
2	7.947×10^{-9}	0.0227	1000	0.0176	9.7
3	6.576×10^{-8}	0.0234	1600	0.044	34.5
4	5.387×10^{-7}	0.0240	2770	0.106	150*
5	8.602×10^{-6}	0.0246	7076	0.295	957*
6	6.950×10^{-5}	0.0248	19450	0.709	3540*
7	2.048×10^{-4}	0.0249	38400	1.18	8180*
14	8.834×10^{-3}	0.0252	1760000	11.3	376000*
15	7.143×10^{-2}	0.0255	55228000	65.7	2134000
8	8.69×10^{-9}	0.248	1178	0.0076	4.5
10	5.552×10^{-7}	0.248	3050	0.0594	224
12	2.081×10^{-4}	0.253	47000	1.056	15160
9	8.767×10^{-9}	2.51	1890	0.00254	1.05*
11	5.64×10^{-7}	2.52	4470	0.0228	40
13	2.180×10^{-4}	2.66	125000	0.755	60440

outs. Thus the value for pressure given in the tables may be not exactly the maximum pressure reached at the centre. One notices that while curves for excess energy yield have roughly the same shape as in the Bethe-Tait case, the pressure decreases faster with decreasing (k_0^3/l^2). Also there exists a considerable lowering of the pressure with increasing lifetime, while the concurrent raise of energy yield is usually of smaller size. In the tables we have given also the maximum kinetic energy obtained during the run of the problem. However, usually this is not the final value and further discussion of it is postponed until Section 2.7.

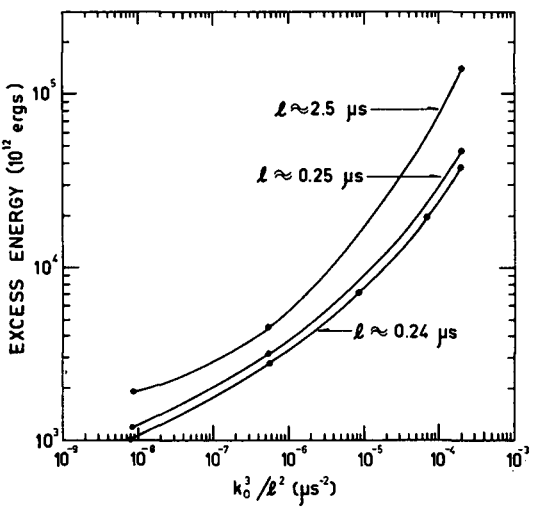


Fig. 6
Energy yield in excess of threshold for various lifetimes.
$E^* = 2100 \times 10^{12}$ erg

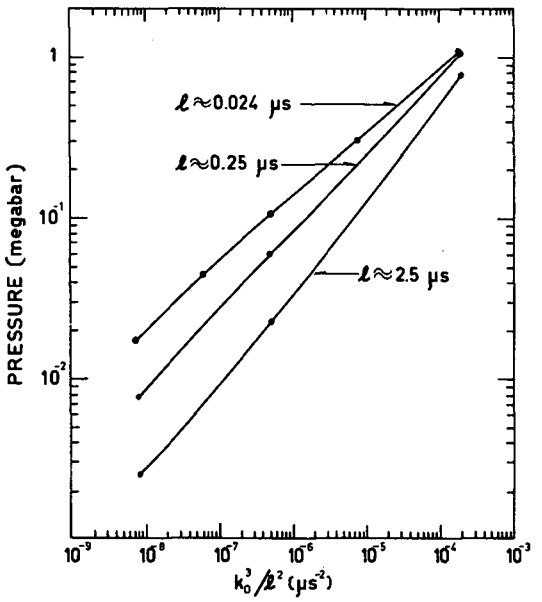


Fig. 7
Centre peak pressure for various lifetimes.
$\left[\left(\frac{\partial p}{\partial e}\right)_\varrho \times Q^* = 0.126\,\text{Mb}\right]$.

2.4. Variation of blanket density

Treating an excursion in the Bethe-Tait manner, one finds (provided the "microscopic" transport cross-sections, $D\varrho$, are the same in the core and in the blanket) that the energy yield depends only upon the functional relationship between pressure and energy in the core, and is entirely independent of the density and equation of state in the blanket. By rough physical considerations, however, one is inclined to expect a higher damage potential in a reactor with a heavy blanket, since this blanket provides a tamping tending to contain the reactor. The Bethe-Tait approximation asserts that displacements of the blanket reduce reactivity, just like displacements of the core, and the smaller displacement obtained in a heavier blanket is exactly compensated by a correspondingly larger transport cross-section of the blanket material. It was of interest to check this by a few numerical calculations.

In this set of problems the same microscopic cross-sections have been used as in the previous Section 2.3. Neutron velocity has been $V_g = 1695$ cm/μs. But three quite different blanket densities have been chosen, and appropriate enrichments of the core have been determined to keep the reactor critical. This procedure has caused different power levels at the core and blanket interface relative to the power level at the centre, $1 - q$, as well as some change in the lifetime. The equation of state here has been simplified somewhat by choosing $B = 0$, and changing A so that the initial rate of pressure rise has remained the same as in Eq. (25). Equation of state parameters have been

$\alpha = 0.02873$, $\beta = 278.46$, $\tau_c = -0.3946$
$A = 15.631$, $B = 0$.

TABLE V

VARIATION OF BLANKET DENSITY

Problem No.	k_0^3/l^2 (μs^{-2})	Blanket density (g/cm^3)	q	Excess energy yield (10^{12} erg)	Centre peak pressure (Mb)	Maximum kinetic energy (10^{12} erg)
27	8.642×10^{-9}	18.5077	0.476	1080	0.0197	11.55
28	5.475×10^{-7}	18.5077	0.476	2825	—	—
29	2.094×10^{-4}	18.5077	0.476	37510	1.87	12750
30	7.003×10^{-9}	7.92	0.586	870	0.0156	5.65
31	4.45×10^{-7}	7.92	0.586	2380	0.1008	132
32	1.665×10^{-4}	7.92	0.586	23840	1.376	8584
33	5.314×10^{-9}	3.96	0.691	700	0.0138	3.11
34	3.401×10^{-7}	3.96	0.691	1840	0.0875	68.25
35	1.257×10^{-4}	3.96	0.691	16370	1.094	5573

They have raised the threshold energy density to

$$Q^* = 0.007815 \text{ cm}^2/\mu s^2.$$

And, due to the variation in q, the threshold energy E^* has not been constant in this set of the problems.

The results of these calculations are given in Table V. To compare these results with the predictions of Bethe-Tait theory, we have plotted the results of calculations for $l \approx 0.024$ μs in the previous section as a solid curve in Fig. 8. The ordinate is ratio of energy in excess of threshold to the threshold energy, the abscissa is x defined by Eqs. (18) and (12):

$$x = \left[\frac{5}{32} \frac{\Sigma_{tr} \nu \Sigma_f [1 - (6q/5) + (3q^2/7)]}{\sqrt{q}\, F}\right] \times \frac{b^2}{(\gamma - 1) Q^*} \times \frac{k_0^3}{l^2}. \tag{32}$$

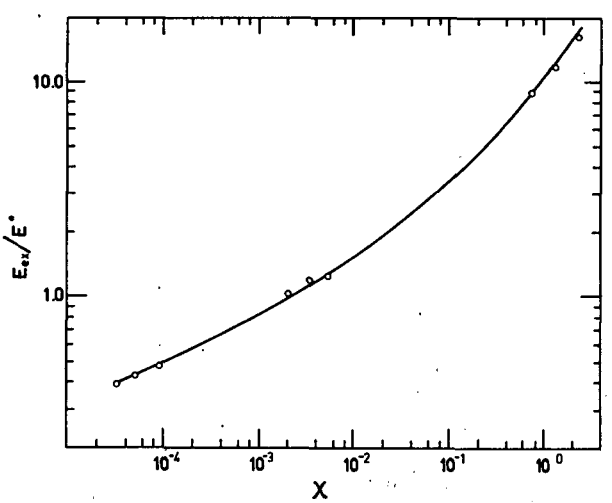


Fig. 8
Effect of blanket density.

The results of present calculations have been plotted as open dots. There is a slight deviation from the smooth curve at low yields. Since the pressure waves at low yields do not reach the blanket before shut-down, this certainly cannot be attributed to a difference in blanket density, but must be attributed to the inaccuracy in assigning q and a slight variation in l. There is a larger deviation for high energy yield, but a deviation of this magnitude is also predicted by the Bethe-Tait theory (see Fig. 1) as a consequence of a difference in q. Some decrease in energy yield relative to the solid curve for large x can be attributed to the change of equation of state at initial density by making B vanish. Thus we conclude that the density of the blanket has no significant direct influence for the short lifetimes considered in these calculations. Its influence may be larger for longer lifetimes.

2.5. Yield as a function of k_0 and k_0^3/l^2

In the reactor excursions in which we are now interested, excess reactivity is always a small number. Therefore, first-order-perturbation treatment can be expected to be valid, and reactivity reduction can be expected to be adequately described by equations of type (8). In a calculation of the Bethe-Tait type, Eq. (8) is used with an additional assumption that pressure and acceleration can be adequately calculated neglecting expansion entirely, and using the entire heat input to raise the internal energy. Thus inertial effects are considered of paramount importance. Then for a step of reactivity the computation gives energy yield as a function of k_0^3/l^2.

On the other hand in some cases it is permissible to neglect inertial effects entirely. Then expansion and corresponding reactivity reduction follow energy input immediately. In such cases the pressure during an excursion should also be considered negligible. This type of computation for step insertion of reactivity (neglecting delayed neutrons) gives energy yield as a function of k_0, and entirely independent of the lifetime l.

It is obvious that these two assumptions represent extreme cases. In actual situations, the pressure is not negligible and depends not only on heat input, but also on concurrent expansion. To demonstrate this, several calculations with different values of k_0^2/l^2 and k_0 have been made. In these calculations, enrichment of the core has been 44.15 % and the ratio of power at the interface to that in the centre has been $1-q=0.414$. The equation of state here has been simplified by choosing $\alpha=B=0$ and other constants have also been changed:

$$\beta=108.046, \qquad \tau=-0.15, \qquad A=6.065,$$

so that threshold energy density has been $Q^*=0.007815$ cm$^2/\mu$s^2 and common constants, Eqs. (25) and (26), of the present study have been preserved. The same equation of state has been used also for the blanket.

The results are displayed in Table VI. After reduction into dimensionless form, as explained in Appendix B, the results for energy yield and centre peak pressure are also displayed in Figs. 10 and 11. One notices that larger k_0 (for a given k_0^3/l^2) results in a somewhat larger energy yield and a considerably smaller peak pressure. From Fig. 10 one notices that, with decreasing k_0^3/l^2, curves of constant k_0 tend to become horizontal, thus indicating approach of the region where the energy yield becomes a function of k_0 only, and inertial effects tend to disappear.

TABLE VI

VARIATION OF REACTIVITY INPUT

Problem No.	k_0^3/l^2 $(\mu s)^{-2}$	k_0	Excess energy yield (10^{92} erg)	Centre peak pressure (Mb)	Maximum kinetic energy (10^{12} erg)
59	7.003×10^{-9}	0.000141	853	0.0208	10.2
170	4.45×10^{-7}	0.000141	2338	0.1423	199*
171	1.665×10^{-4}	0.000141	22892	1.570	1281*
76	7.003×10^{-9}	0.000560	921	0.00895	6.17
60	4.45×10^{-7}	0.000560	2378	0.113	58*
173	1.665×10^{-4}	0.000560	23040	1.544	4136*
77	7.003×10^{-9}	0.004088	1486	0.00479	1.68
175	4.45×10^{-7}	0.004088	2561	0.049	179
61	1.665×10^{-4}	0.004088	23752	1.391	8681*

The "widths" of the power bursts, $k_0 E_T/l P_{max}$, obtained in the calculations of Table VI, have been displayed as open dots in Fig. 3. They are always larger than the widths expected from Bethe-Tait calculations, and (for given k_0^3/l^2) the deviation increases with increasing value of k_0.

2.6 VARIATION OF WAVE VELOCITY

In Appendix A we have shown that for the simplified equation of state ($\alpha = B = 0$) the velocity of wave propagation is

$$c^2 = \left(\frac{\partial p}{\partial \varrho}\right)_s = (-\tau' + p)\frac{\beta'}{A'} \times v^2$$

if the pressure is positive and vanishes otherwise. A higher velocity of wave propagations results in a more rapid spreading of the pressure wave and should result in a higher energy yield. To check this, a number of calculations were performed keeping Q^* and β'/A' constant and varying τ'. In these calculations the lifetime has been kept short, $l \approx 0.02$ μs, and three cases of reactivity input have been considered.

TABLE VII

VARIATION OF WAVE VELOCITY

Problem No.	k_0^3/l^2 (μs^{-2})	$-\tau$ (Mb)	Excess energy yield (10^{12} erg)	Centre peak pressure (Mb)	Maximum kinetic energy (10^{12} erg)
59	7.003×10^{-9}	0.15	853	0.0208	10.20
56	7.003×10^{-9}	0.2682	856	0.0155	5.29
62	7.003×10^{-9}	0.5	885	0.0111	2.64
65	7.003×10^{-9}	1.0	956	0.0082	1.13
60	4.45×10^{-7}	0.15	2378	0.113	58*
57	4.45×10^{-7}	0.2682	2380	0.101	115*
63	4.45×10^{-7}	0.5	2399	0.0846	77.3
66	4.45×10^{-7}	1.0	2437	0.0646	38.8
61	1.665×10^{-4}	0.15	23752	1.391	8681*
58	1.665×10^{-4}	0.2682	23744	1.374	8621
64	1.665×10^{-4}	0.5	23919	1.357	5343*
67	1.665×10^{-4}	1.0	24183	1.313	4802*

The results are displayed in Table VII. In all the cases the energy yield increases faster than proportionally to the increase of $-\tau$. When $-\tau=1$, velocity of wave propagation is 0.53 cm/μs. Multiplying it with the length of time from the moment the pressure has started until most of the energy has been generated (as given by IBM-704), we obtain, for problems 65, 66 and 67, 29 cm, 17 cm and 8 cm respectively. A significant increase in energy yield however is realized only in problem 65, where "the distance the wave has travelled through" is larger than the radius of the core. We notice also that in all the cases pressure is a more sensitive function of τ than is the energy yield. This can be understood realizing that the propagation of the pressure wave at constant velocity does not change $\int p\mathrm{d}V$ and thus affects $\ddot{k}$, given by Eq. (8), only very little and causes only small increase in the energy yield. In Fig. 9 we have displayed the centre peak pressure as a function of initial wave velocity,

$$c_0=\tau'\beta' v_0^2/A'.$$

The decrease of the kinetic energy realized with increasing $-\tau$ looks even larger than the decrease in pressure. Thus it is likely that the destructive potential of the burst will decrease with increasing $-\tau$ in spite of increasing energy yield.

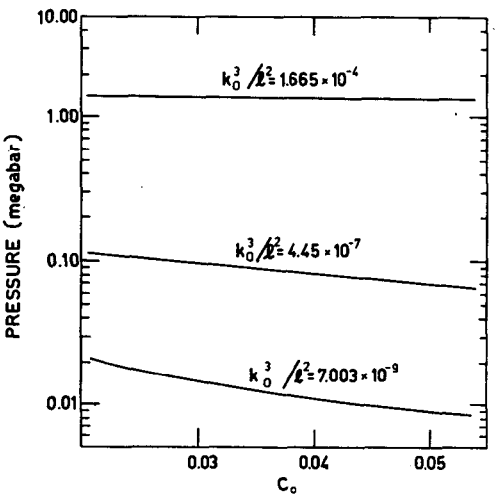


Fig. 9
Variation of centre peak pressure with wave velocity.

2.7. Kinetic energy

In all the tables of this report we have given also the maximum kinetic energy obtained during the running of the problem on the IBM-704. The kinetic energy at the end of the run has always been close to this value. The cases where kinetic energy has been rising at an appreciable rate at the end of the run have been indicated with an asterisk. During the run, kinetic energy keeps increasing when energy is being generated at an appreciable rate. It increases later, at a somewhat lower rate, when the pressure wave is reflected from a rarer medium and decreases somewhat when the pressure wave is reflected from a denser

medium. At other times the total kinetic energy keeps a roughly constant value. The value of kinetic energy given here is not always the maximum value obtainable from the excursion, since in most cases at the end of the run the pressure has not yet vanished from every place. Thus continuation of the run would very likely increase the kinetic energy.

In a few cases the maximum of kinetic energy that could be obtained by an isentropic expansion has been computed. We have taken Eq. (16a) as the maximum energy available per gram of uranium and summed over the core using our standard parabolic shape for power and energy distribution. The result of this calculation has always given a much larger number for kinetic energy than obtained at the end of IBM-704 run. Sometimes the kinetic energy has been only $^1/_{10}$ of what one expects to get by an isentropic expansion. Thus it seems probable that in all these cases a large amount of potential kinetic energy has been used up by heating the core and the blanket.

So, while our calculations indicate that the composition of the blanket has only a small affect upon the total energy yield, it may have a large influence on the amount of kinetic energy eventually realized. In like manner, only small changes of specific volume are realized during appreciable energy generation. But an equation of state valid for large changes of volume is needed to compute the kinetic energy. Since our simplified equation of state is very unlikely to be adequate in an actual situation, kinetic energies obtained here have only a qualitative theoretical significance.

APPENDIX A

On the equations of state

The AX-I programme uses both thermal and caloric equations of state. In the present calculations they were expressed in a simple linear form:

$$p = \tau + \beta\theta + \alpha p \tag{1 a}$$

$$c_V = \left(\frac{\partial e}{\partial \theta}\right)_\varrho = A + B\theta. \tag{2 a}$$

During a fast excursion, while heat is being generated, we can reasonably assume that there is no heat conduction. Thus information about temperature, θ, is superfluous. All we need for an adiabatic calculation is the relationship among pressure, p, density, ϱ, and internal energy, e. This gives us only one equation of state. Indeed it is quite thinkable to have two substances that have the same relationship among p, ϱ and e and differ in absolute temperature, θ. Those two substances would behave identically in our calculations. Thus it is of interest, starting from Equations (1a) and (2a) to obtain the relationship among p, ϱ and e.

Integrating Eq. (2a) we obtain

$$e = A\theta + \frac{1}{2} B\theta^2 + f(\varrho) \tag{3 a}$$

where $f(\varrho)$ is a yet undetermined function that has to be such that the second law of thermodynamics is satisfied. The second law says

$$\frac{1}{\theta}\left(\mathrm{d}e - p\,\frac{\mathrm{d}\varrho}{\varrho^2}\right) = \mathrm{d}s$$

or, after substitution of Eqs. (1a) and (3a):

$$\frac{1}{\theta}\left[(A + B\theta)\,\mathrm{d}\theta + \left(f'(\varrho) - \frac{\tau + \beta\theta + \alpha\varrho}{\varrho^2}\right)\mathrm{d}\varrho\right] = \mathrm{d}s. \tag{4 a}$$

This is a complete differential, thus

$$\frac{\partial}{\partial \varrho}\left(\frac{A+B\theta}{\theta}\right)=\frac{\partial}{\partial \theta}\frac{1}{\theta}\left[f'(\varrho)-\frac{\tau+\beta\theta+\alpha\varrho}{\varrho^2}\right]$$

or

$$0=-\frac{1}{\theta^2}\left[f'(\varrho)-\frac{\tau+\alpha\varrho}{\varrho^2}\right].$$

Integrating this expression we find that

$$f(\varrho)=-\frac{\tau}{\varrho}+\alpha\ln\varrho+\text{const.}$$

Substituting this into Eq. (4a) we obtain

$$s=A\ln\theta+B\theta+\frac{\beta}{\varrho}+\text{const.} \tag{5 a}$$

Similarly from Eq. (3a) we have

$$e=A\theta+\frac{1}{2}B\theta^2-\frac{\tau}{\varrho}+\alpha\ln\varrho+\text{const.}$$

Now solving for θ from Eq. (1a) and substituting this value into the preceding expression we obtain

$$e=\frac{A}{\beta}(p-\tau-\alpha\varrho)+\frac{B}{2\beta^2}(p-\tau-\alpha\varrho)^2-\frac{\tau}{\varrho}+\alpha\ln\varrho+\text{const.} \tag{6 a}$$

the desired relationship among e, p and ϱ.

This is a rather complex equation, even for a constant ϱ. But we know how to solve the problem of an excursion, neglecting the variation of density for any relationship between p and e, using the generalized Bethe-Tait procedure. So, in order to investigate the influence of pressure variation with density, in later calculations we have simplified the relationship between p and e taking $B=0$. Then the relationship between pressure and internal energy becomes linear like that used by Bethe and Tait [1]. Equation (6a) then simplifies into

$$e=\frac{A}{\beta}p-\frac{\tau}{\varrho}+\alpha\ln\varrho-\alpha\frac{A}{\beta}\varrho+\text{const.}$$

Here the variation of internal energy with density is still quite complex. However, since in the perturbation treatment only small (linear) variations of density are considered, we are interested only in

$$\left(\frac{\partial e}{\partial \varrho}\right)_p=\frac{\tau}{\varrho^2}+\frac{\alpha}{\varrho}-\alpha\frac{A}{\beta}$$

evaluated at the initial density. Thus, within first-order-perturbation theory, we should obtain identical results with any two sets of constants if A/β and $(\partial e/\partial p)_p$ are the same in the two sets. To have these conditions satisfied one constant is superfluous. Thus in our later calculations we have chosen

$$\alpha'=0 \tag{7 a}$$

(where the prime denotes a net set of constants).
And

$$\left(\frac{\partial e}{\partial \varrho}\right)_p=\frac{\tau'}{\varrho^2}=\frac{\tau}{\varrho^2}+\frac{\alpha}{\varrho}-\alpha\frac{A}{\beta}. \tag{8 a}$$

Our simplified equations of state then read

$$p=\tau'+\beta'\theta \tag{9 a}$$

$$c_v=A' \tag{10 a}$$

and the relationship among pressure, internal energy and specific volume, $v = 1/\varrho$, is then

$$p = \frac{\beta'}{A'}(e + \tau' v) + \text{const.}$$

The integration constant is eliminated by measuring energy density, e, from the initial conditions, when $\theta = \theta_i$ and $v = v_0$:

$$p = \frac{\beta'}{A'}\left[e - A'\left(-\frac{\tau'}{\beta'} - \theta_i\right)\right] + \frac{\beta'}{A'}\tau'(v - v_0). \tag{11 a}$$

To check our contention that only a linear variation of pressure with specific volume is of importance, we have chosen as a basis three excursions: problems 30, 31, and 32, with equation of state constants given in Section 2.4. Then we have determined a new set of constants so that Eqs. (6a) and (8a) are satisfied and the rate of change of pressure with energy density is the same

$$\beta'/A' = \beta/A \tag{12 a}$$

as well as the threshold energy density

$$\begin{aligned} Q^* &= A'(\theta'_{\text{sp}} - \theta_i) = A(\theta_{\text{sp}} - \theta_i) \\ &= A'\left(-\frac{\tau'}{\beta'} - \theta_i\right) = A\left(-\frac{\tau + \alpha\varrho}{\beta} - \theta_i\right). \end{aligned} \tag{13 a}$$

The calculations with the new set of constants:

$$\begin{array}{lll} \alpha' = 0, & \beta' = 1290.1, & -\tau' = 0.2682, \\ A' = 72.42, & B = 0, & \end{array}$$

are reported in Table VIII. Agreement is quite satisfactory, even for a sensitive quantity, centre peak pressure.

TABLE VIII

LINEARIZATION OF PRESSURE DEPENDENCE ON DENSITY

Problem No.	k_0^3/l^2 (μs^{-2})	Excess energy yield (10^{12} erg)	Centre peak pressure (Mb)	Maximum kinetic energy (10^{12} erg)
30	7.003×10^{-9}	870	0.0156	5.65
31	4.45×10^{-7}	2380	0.1008	132
32	1.665×10^{-4}	23840	1.376	8584
56	7.003×10^{-9}	856	0.0155	5.29
57	4.45×10^{-7}	2380	0.101	115 *
58	1.665×10^{-4}	23744	1.374	8621

For materials of nearly normal density, constants for the present equations of state can be simply estimated using experimental measurements. Usually measurements of specific heat are available, also the cubical coefficient of thermal expansion

$$-\frac{1}{\varrho}\frac{\mathrm{d}\varrho}{\mathrm{d}\theta} = \frac{\beta}{\alpha\varrho}$$

is known, as well as the velocity of propagation of sound, c. Using Eqs. (1a), (5a) and (2a), the sound velocity can easily be expressed in terms of our parameters

$$c^2 = \left(\frac{\partial p}{\partial \varrho}\right)_s = \alpha + \beta\left(\frac{\partial \theta}{\partial \varrho}\right)_s = \alpha + \frac{\beta^2\theta}{\varrho^2(A + B\theta)} = \alpha + \frac{\beta^2\theta}{\varrho^2 c_v}$$

where usually the second term is small and the square of the wave velocity is approximately equal to α.

Of course, for our simplified equation of state this is no longer true. When α and B vanish, the preceding equation and Eq. (9a) say

$$c^2 = \frac{\beta'^2 \theta}{\varrho^2 A'} = \frac{\beta'}{A'} v^2 (-\tau' + p). \tag{14 a}$$

Thus the velocity of wave propagation increases as p increases, and experiences a sudden jump when pressure goes from vanishing to small positive as voids are being eliminated.

After the heat generation is over, the core material expands, presumably doing some work and imparting some kinetic energy to the reactor material. The maximum work that a core element can produce is obtained assuming that the expansion is isentropic. For the simplified equation of state ($\alpha = B = 0$), taking Eq. (5a) and substituting θ from Eq. (9a), we see that during an isentropic expansion

$$A' \ln \frac{p - \tau'}{\beta'} + \beta' v = \text{const.}$$

Supposing that the expansion continues until the pressure vanishes, and substituting v from Eq. (11a) we see that the energy lost and the work done is

$$w = \frac{A' \tau'}{\beta'} \left(\ln \frac{p_1 - \tau'}{-\tau'} + \frac{p_1}{\tau'} \right) \tag{15 a}$$

where subscript 1 refers to the state preceding expansion. Now assuming that the initial volume is equal to v_0 and calling the energy density above threshold

$$e_{ex} = e - A' \left(\frac{-\tau'}{\beta'} - \theta_i \right)$$

we see from Eq. (11a) that the pressure is proportional to the excess energy density, e_{ex}. Thus Eq. (15a) can be rewritten

$$\frac{w}{e_{ex}} = 1 - \frac{-\tau' A'}{\beta' e_{ex}} \ln \left(1 + \frac{\beta' e_{ex}}{-\tau' A'} \right). \tag{16 a}$$

So the energy available to do work usually is considerably smaller than energy generated over threshold. If $\beta' e_{ex}/(-\tau' A') \ll 1$,

$$\frac{w}{e_{ex}} = \frac{1}{2} \left(\frac{\beta' e_{ex}}{-\tau' A'} \right) \left[1 - \frac{2}{3} \left(\frac{\beta' e_{ex}}{-\tau' A'} \right) + \dots \right].$$

APPENDIX B

Connection between nuclear parameters k_0, l, and the equation of state

First-order-perturbation treatment yields an energy reduction proportional to the displacement, Eq. (6):

$$k - k_0 = \int \varrho u \cdot f \, dV. \tag{1 b}$$

When the equation of motion, Eq. (2):

$$\ddot{u} = -\frac{1}{\varrho} \nabla p \approx -\frac{1}{\varrho_0} \nabla p \tag{2 b}$$

is used, the mechansim of reactivity reduction can be described by

$$\ddot{k} = \int p \nabla \cdot f \, dV. \tag{3 b}$$

If delayed neutrons are neglected, the reactivity is connected with the energy input by Eq. (19):

$$k = l \frac{d}{dt} \ln \dot{Q}. \tag{4 b}$$

The shape of the power is known and so are the equations of state. If one uses them for calculation of the pressure neglecting expansion and wave propagation, one can express p in Eq. (3b) in terms of energy input. Equations (3b) and (4b) become coupled ordinary differential equations describing the excursion in the Bethe-Tait approximation.

If a step input of reactivity is to be considered and the excursion is assumed to start with negligible power, initial conditions for these differential equations are

$$Q \approx 0, \qquad \dot{Q} = (k_0/l)\, Q, \qquad k = k_0, \qquad \dot{k} = 0.$$

We can eliminate these initial conditions formally by measuring time in terms of initial period, l/k_0, and the excess reactivity in terms of k_0. In terms of these dimensionless quantities, Eq. (4b) can be rewritten

$$\left(\frac{k}{k_0}\right) = \left(\frac{l}{k_0} \frac{\mathrm{d}}{\mathrm{d}t}\right) \ln \dot{Q}$$

free of any parameters. If in addition we decide to measure length in terms of bk_0, Eqs. (1b), (2b) and (3b) can be rewritten in dimensionless form

$$\left(\frac{k}{k_0}\right) - 1 = \int \rho \left(\frac{u}{bk_0}\right) \cdot f\, b\, \mathrm{d}V$$

$$\left(\frac{l}{k_0} \frac{\mathrm{d}}{\mathrm{d}t}\right)^2 \left(\frac{u}{bk_0}\right) \approx -\frac{b}{\varrho_0} \nabla \left(\frac{l^2}{b^2 k_0{}^3}\, p\right)$$

and

$$\left(\frac{l}{k_0} \frac{\mathrm{d}}{\mathrm{d}t}\right)^2 \left(\frac{k}{k_0}\right) = \int \left(\frac{l^2}{b^2 k_0{}^3}\, p\right) \nabla \cdot f\, b^2\, \mathrm{d}V.$$

Our simplified equation of state, Eq. (11a) can also be rewritten as

$$\left(\frac{p v_0 l^2}{b^2 k_0{}^3}\right) = \left[\frac{\beta'}{A'} v_0 k_0\right]\left(e \frac{l^2}{b^2 k_0{}^4}\right) - \left[\left(-\tau' - \beta' \theta_i\right) v_0 \frac{l^2}{b^2 k_0{}^3}\right] + \left[\frac{\beta'}{A'} v_0 k_0\right] + \left[\tau' \frac{l^2}{b^2 k_0{}^3} v_0\right]\left(\frac{v - v_0}{v_0 k_0}\right) \tag{5 b}$$

where the variables in parentheses and the constants in square brackets are all dimensionless. In addition we can also take a constant factor out of $\nabla \times f$ and include it into the definition of dimensionless u and p. Thus the progress of the excursion, described by these dimensionless variables, depends upon three dimensionless parameters in the equation of state. And the variation of the paramters k_0 and $k_0{}^3/l^2$ we have performed in Section 2 is equivalent to the variation of β'/A' and $(\beta'/A')Q^* = (-\tau' - \beta\theta_i)$, which have been kept constant. To check this contention, we have chosen problems 65, 66, 67, 72 and 75 as references. Then we have reduced k_0 by 2, kept l the same and simultaneously have changed the constants of equation of state,

TABLE IX

CONNECTION BETWEEN NUCLEAR AND EQUATION OF STATE PARAMETERS

Problem No.	Excess energy yield (10^{12} erg)	Centre peak pressure (Mb)	Problem No.	Excess energy yield (10^{12} erg)	Centre peak pressure (Mb)
65	956	0.0082	305	59	0.00104
66	2437	0.0646	266	151	0.0081
67	24183	1.313	267	1488	0.164
72	1169	0.008	272	76	0.0018
75	3014	0.06	275	195	0.0098

Eq. (11a), to keep the parameters in the square brackets in Eq. (5b) the same (θ_i has also been kept the same). Then, as seen from Eq. (5b), one expects to obtain only 1/16 of previous energy yield and 1/8 of previous pressure. The results of these calculations are displayed in Table IX. The discrepancy between expected and actually obtained values is noticeable in the last three problems. However, at least a part of it can be explained by failure of the synthetic viscous pressure, Eq. (23),

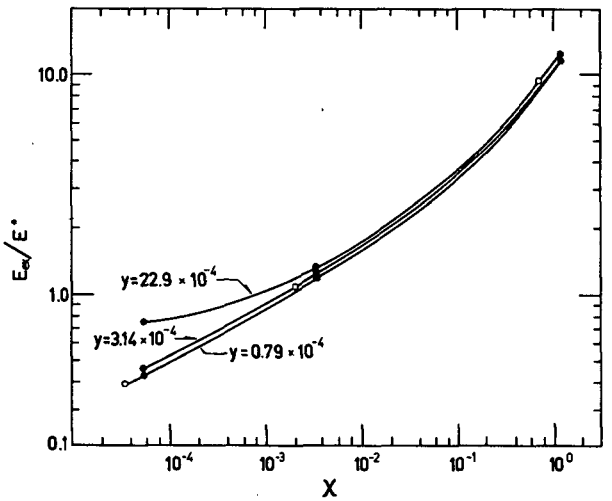


Fig. 10
Dimensionless excess energy yield, E_{ex}/E^*, as a function of dimensionless parameters x (32) and y (6b).
$\tau'/(\tau' + \beta'\theta_i) = 1.0776$

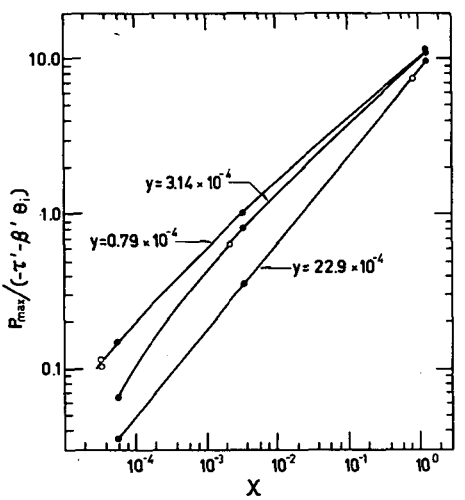


Fig. 11
Dimensionless centre peak pressure, $p_{max}/(-\tau' - \beta'\theta_i)$, as a function of dimensionless parameters x (32) and y (6b).
$\tau'/(\tau' + \beta'\theta_i) = 1.0776$

to conform with our scaling and other inaccuracies of the numerical procedure. Thus the agreement seems satisfactory. And we have plotted the yields of excursions in Table VI in dimensionless notation as solid dots. The dimensionless excess energy, E_{ex}/E^* has been plotted in Fig. 10, and dimensionless centre peak pressure, $p_{max}/(-\tau - \beta'\theta_i)$ in Fig. 11. As arguments here we have used x, of Eq. (32), and

$$y = \left[\frac{5}{32}\frac{\Sigma_{tr}\nu\Sigma_f[1-(6q/5)+(3q^2/7)]}{F\sqrt{q}}\right]\left(\frac{\beta' v_0}{A'}\right)k_0. \tag{6 b}$$

The third parameter, from the equation of state, Eq. (5b), $\tau'/(\tau'+\beta'\theta_i)$, was equal to 1.0776 for the whole set. In addition, we have performed three calculations with this third parameter equal to 1.2 and three calculations with parameter value equal to 1.0447. In these six calculations, threshold energy density has been chosen higher: $Q^* = 0.012546$, but β'/A' and y's have been the same as in the reference problems of Table VI. These calculations have been indicated by open dots and should lie on solid lines of Figs. 10 and 11. Two open dots indicating low pressure yield in Fig. 11 demonstrate that the yield is sensitive to the third parameter, $\tau'/(\tau'+\beta'\theta_i)$, if the excursion is small.

Though Figs. 10 and 11 have been plotted in dimensionless form, the calculations have been done for the same power shape and assuming the same material in the blanket as in the core. We already know that the power shape changes the yield even in the simple Bethe-Tait calculation (Fig. 1 and Table I). In addition, the change of material in the blanket may be expected to induce larger variation in the yield than obtained in Section 2.4. Calculations in that section have been performed for very short lifetimes and serve mainly to demonstrate the lack of influence of blanket material in the Bethe-Tait approximation.

As we have seen, when the Bethe-Tait method is applicable, expansion during an excursion is negligible as well as wave propagation. Then only the relationship between pressure and energy at the initial density of the core is needed to determine the yield. In addition, two relationships between pressure and energy that differ only in a constant scale factor for energy produce identical pressures and reactivities during the burst. Thus we have seen for simple Eq. (16) that progress of the excursion was described only by $(\gamma - 1)Q^*$, and did not involve $(\gamma - 1)$ by itself.

It is instructive to consider a simple equation of state without a threshold

$$p = AQ^n, \text{ for } \varrho = \varrho_c. \tag{7 b}$$

Then, since progress of the excursion does not depend on a scaling factor of power input, pressure and reactivity in the Bethe-Tait approximation do not depend on A. In particular, our dimensionless pressure average over the core

$$\frac{\bar{p}l^2}{\varrho\, b^2\, k_0^3}\left(-\nabla\cdot f\frac{4\pi}{3}b^3\varrho_c b^2\right) \approx \frac{\bar{p}l^2}{\varrho_c b^2 k_0^3}\left(\frac{16q^2F}{\Sigma_{tr}\nu\Sigma_f b^2[1-(6q/5)+(3q^2/7)]}\right) \tag{8 b}$$

is a function of dimensionless time and n only. We have computed the maximum of Eq. (8b) by the Runge-Kutta method on the LGP-30, and displayed the results in Table III. As demonstrated in the comparisons of Figs. 10 and 11, however, using the Bethe-Tait method, one expects to obtain more accurate values for the energy yield than the peak pressure. Thus the peak pressure obtained here should be regarded only as an intermediate step to estimate the energy yield from Eq. (7b).

ACKNOWLEDGEMENTS

The autor is grateful to Dr. D. Okrent for his interest in this work, his advice and encouragement. He is also pleased to acknowledge generous assistance of H. J. Wheeler, L. B. Miller, L. C. Kvitek, and D. M. O'Shea, who by preparing numerical programmes, running and processing problems and performing numerical calculations have made considerable contribution to this report.

REFERENCES

[1] BEHTE, H. A. and TAIT, J. H., UKAEA-RHM (56)/113.
[2] McCARTHY, W. J. *et al.*, Proc. 2nd UN Int. Conf. PUAE **12** (1958) 207.
[3] OKRENT, D. *et al.*, ANL-5977.
[4] NICHOLSON, R. D., APDA (Private Communication).
[5] SCHMITT, A. P., Saclay, France (Private Communication).
[6] KOCH, L. J. *et al.*, ANL-5719.
[7] VON NEUMANN, J. and RICHTMYER, R. D., *J. appl. Phys.* **21** (1950) 232.
[8] STRATTON, W. R. (Private Communication).
[9] STRATTON, W. R. *et al.*, Proc. 2nd UN Int. Conf. PUAE **12** (1958) 196.

V. PHYSICS OF SPECIFIC REACTORS

SM-18/33

PHYSICS OF THE DOUNREAY FAST REACTOR

D. C. G. SMITH
DOUNREAY EXPERIMENTAL REACTOR ESTABLISHMENT,
UNITED KINGDOM ATOMIC ENERGY AUTHORITY,
UNITED KINGDOM

Abstract — Résumé — Аннотация — Resumen

Physics of the Dounreay fast reactor. A brief description of the Dounreay fast reactor and its experimental facilities is given, followed by a survey of the experiments carried out during the low power commissioning of the reactor. Calculations in support of the experiments are described and comparison is made between predicted and observed results. In general the theoretical methods predict reasonably well the neutron energy spectrum and the distribution of reaction rates in the core but underestimate the total flux and the softness of the spectrum in regions outside the core. A few anomalies were found where the calculated and measured quantities disagreed by factors which lie outside the expected errors of calculation and measurement.

Description sommaire du réacteur à neutrons rapides de Dounreay. L'auteur décrit succinctement le réacteur à neutrons rapides de Dounreay et les installations expérimentales connexes; il passe également en revue les expériences qui ont eu lieu au cours des essais de fonctionnement du réacteur à basse puissance. Il expose les calculs qui ont servi de base à ces expériences et fait une comparaison entre les prévisions et les résultats enregistrés. De façon générale, les méthodes théoriques permettent de calculer de façon satisfaisante le spectre d'énergie des neutrons et la distribution des vitesses de réaction dans le cœur, mais sous-estiment le flux total et l'énergie des neutrons à l'extérieur du cœur. L'auteur a relevé quelques anomalies, en ce sens que les différences entre les quantités calculées et les quantités mesurées étaient nettement supérieures aux erreurs de calcul et de mesure auxquelles on pouvait s'attendre.

Физика реактора на быстрых нейтронах в Дунрей. Дается краткое описание реактора на быстрых нейтронах в Дунрей и его экспериментального оборудования; далее следует обзор экспериментов, проведенных во время пускового периода работы реактора на низкой мощности. Описываются расчеты, приведенные в подтверждение экспериментов, и дается сравнение между предполагаемыми и наблюдаемыми результатами. В основном теоретические методы дают возможность достаточно разумно предсказать спектр нейтронной энергии и распределение скоростей реакций в активной зоне, но недооценивают общий поток, а также мягкость спектра в районах, находящихся вне активной зоны. Было обнаружено несколько аномалий, когда подсчитанные и измеренные количества не согласовывались с факторами, которые лежат вне ожидаемых погрешностей расчета или измерения.

Breve descripción del reactor rápido de Dounreay. La memoria describe brevemente el reactor rápido de Dounreay y sus instalaciones experimentales, y resume los experimentos efectuados durante la puesta en marcha a baja potencia. Detalla los cálculos en que se basaron los experimentos y compara los resultados calculados con los obtenidos en la práctica. En general, los métodos teóricos permiten predecir con bastante precisión el espectro de energías neutrónicas y la distribución de las velocidades de reacción en el cuerpo, pero dan valores demasiado bajos para el flujo total y la blandura del espectro en el exterior del cuerpo. Se encontraron algunos casos anómalos en los que las diferencias entre los valores calculados y los medidos son mayores que los errores de cálculo y de medición previstos.

1. Description of the Dounreay fast reactor

Following the successful operation of low-power experiments at Harwell it was decided to build the Dounreay fast reactor (DFR). A full description of this reactor has been published [1] but a brief description of the reactor will be given, including a note of the available facilities for experiment.

The reactor is fuelled by enriched uranium and cooled by a 70–30 % NaK alloy. The core shape is approximately a right cylinder whose diameter and height are 21 in, and the core — volume 130 l — is constructed within a 2-ft-thick radial blanket of natural uranium. The core and blanket are contained in a stainless-steel vessel which is in turn surrounded by borated graphite and a concrete biological shield. Between these two shields are the 24 heat-exchangers and 24 primary coolant pumps which circulate the liquid metal through the reactor core and blanket. The liquid metal is covered by an atmosphere of nitrogen, and direct access to the core and natural-uranium blanket is not normally possible. The reactor vessel is sealed by two rotating shield plugs containing borated graphite.

The 367 core fuel elements are in the form of hollow cylinders (0.7 in outer diameter and 0.3 in inner diameter) sheathed on the outside by niobium and on the inside by vanadium, both these surfaces being cooled with liquid metal. The reactor is controlled by the movement of fuel. There are 12 control rods situated around the periphery of the core, each consisting of a cage containing six enriched and four natural U fuel elements. Six of the 12 are for shut-down purposes, and it is normal for a total of 10 control rods to be in the core, the remaining two being used for operational control.

The core fuel elements are arranged in a triangular lattice and are charged and discharged through the rotating shields. The minimum time between shut-down and the discharge of the first element is approximately 48 h. For the low-power experiments, access to the core and blanket was required and four

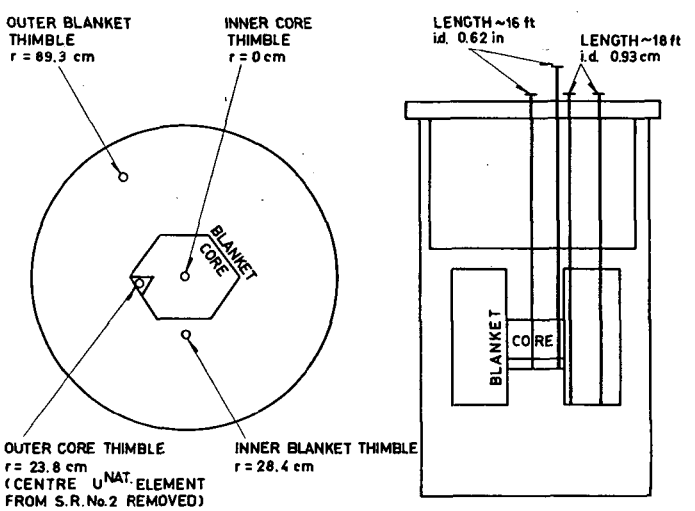


Fig. 1
Schematic diagram of reactor and thimble arrangement.

experimental thimbles were installed: at the core centre, the core edge, and at the inner and outer edges of the blanket. The thimbles consisted of stainless-steel tubes about 17 ft long sealed at the lower end. These were inserted in place of normal core or blanket elements.

The possibility of puncturing a thimble made it necessary to have a low blanket-gas pressure, and therefore coolant pumping power had to be limited in order to prevent cavitation at the pump inlet. The thimbles extended to the regions below the core and blanket, and 16-ft extension stems were required for the fission chambers and irradiation sample containers. A schematic diagram of the reactor and thimble arrangement is given in Fig. 1.

Additional irradiation facilities were provided by special fuel elements which were externally similar to core or blanket fuel elements. For the special core elements there was no inner tube and the outer tube (made of stainless steel) formed a sealed cylinder. Samples for irradiation were previously loaded into five core and seven special blanket elements. Being externally similar, they were loaded into the core and blanket in the same way as normal elements.

During the early design stages of the DFR the physics data were obtained mainly from the low-power experiment ZEUS which was constructed at Harwell [2]. As far as the physics is concerned, ZEUS was designed using simple, one-energy-group, cross-section data and very simple models although subsequently more sophisticated multi-group calculation methods were used and compared with experiment. ZEUS was intended as a close simulation of the DFR but, inevitably, the design of the latter was changed after ZEUS had been built. The core diameter, size, and pitch of the fuel elements, and the dimensions of the axial breeder were changed and measurements on ZEUS were not directly applicable to the DFR. Calculation was used to bridge the differences between the two reactors. Following a calculation and measurement on ZEUS, an adjustment factor was obtained which was applied to a similar calculation for the DFR.

The DFR has also been modified since it first became critical. After operation for a short time the reactor was shut down to permit the introduction of a modified section in the central 6-in diameter region of the core, which will be replaced later by experimental sub-assemblies. This arrangement, known as core B, is more complex from the calculation viewpoint; and most of the work reported in this paper will refer to core A.

The first calculations of critical enrichment for the DFR were made using one-group diffusion theory but the later calculations were more elaborate. A series of one-dimensional multi-group calculations was made using the Carlson S_4 method [3] for infinite slab and cylindrical models. The resultant flux and spectrum information along the principal horizontal and vertical axes was used to assess two-group constants for all parts of the reactor. These constants were used in the multi-region, cylindrical geometry, diffusion theory programme PDQ [4].

For the core B calculations the more recently developed multi-group, two-dimensional, transport-theory code TDC [5] was used. The advantages of this code were not fully realized as the complexity of the core B makes it difficult to design a sufficiently simple but realistic model. The 11 energy-group set of cross-sections published by LOEWENSTEIN and OKRENT [6] was used as the basis for the calculations.

Calculations fell broadly into one of two categories: those essential to the design and safe operation of the reactor and the more detailed calculations

relevant to the finer points in the design and performance. It is difficult and uneconomic to make extensive and detailed measurements in a large-scale reactor experiment such as the DFR. Comparison between calculation and experiment was therefore made at points accessible in four thimbles supplemented by activation samples in the special elements loaded into the core and breeder.

Fission chambers, activation and perturbation samples were used in the low-power experimental programme whose objectives were:

(*a*) To verify that predictions concerning the operation and safety of the reactor were soundly based, e.g. power-density distribution, control-rod worth, isothermal temperature coefficient of reactivity;

(*b*) To compare experimental with calculated values for a variety of reactions and reactor parameters and thereby check the validity of the calculation methods and constants used; and

(*c*) To measure a few reaction cross-sections of particular interest including those for which unexpected values had been found in ZEUS [2].

2. Experiments

Control-rod calibration

The first experiment on the critical reactor was to check the control-rod effectiveness. An estimate of this was obtained during the critical assembly but a more rigorous measurement was subsequently made (1) to determine the magnitude of the reactivity change caused by a 1-in movement of a control rod when it is half-way into the core and (2) to examine the variation of this reactivity change as the control-rod position was moved from fully out to fully in. These measurements will be described in reverse order.

Relative calibration

The relative effectiveness of a control rod varies with its position, being a broad maximum when half-way into the core. This was checked by adjusting the control-rod positions until the reactor was sub-critical (by about 3×10^{-3}) with the control rod under test in its "fully in" position. Under these conditions the neutron flux is inversely proportional to the amount of reactivity by which the reactor is sub-critical. The variation in the reciprocal of the flux (as measured on the installed counting channels) with control-rod position enabled the relative calibration to be made. A calibration curve was constructed, the abscissa being the control-rod position as shown on the control-room indicator and the ordinate being the reactivity change in units known as "effective inches", each unit being the reactivity change equal to a 1-in movement when the rod is at its most sensitive position.

Absolute calibration by static perturbation

The next part of the calibration was to find the reactivity change caused by a control-rod movement of one effective inch. From simplified perturbation theory the reactivity change caused by the insertion of a small sample of fissile material is known in terms of the size, composition and nuclear constants of the constituents of the sample, and a reactor "perturbation constant". The latter, which depends on the distribution of fission rates, had been calculated, and was checked later in the experimental programme.

Samples of Pu^{239} (50 g) and U^{235} (100 g) were lowered into the centre of the reactor and the change in the control-rod position required to maintain steady power was measured. Fissile material was used since the fission cross-section and number of neutrons per fission are believed to be more accurately known than capture cross-sections and are insensitive to spectrum characteristics, the latter being relatively uncertain in the DFR. The control-rod displacement was about 1 in and the reactivity change 4×10^{-4} per inch.

Absolute calibration by kinetic measurement

An independent assessment of the absolute sensitivity of a control rod was obtained by moving the rod by a known amount and measuring the change in the steady period of the converging or diverging flux. The period is related to the effective delayed-neutron fraction (known from ZEUS) and the reactivity change. In practice the reactor was balanced with a drift rate of $<1\,\%$ per minute, the control rod moved by an amount which varied from $^1/_4$ in to $1^1/_2$ in, and the flux monitored with a fission counter. The pulses from the counter were switched automatically between two counting channels, one being read and reset while the other was operating. The steady period was measured over a power range of 1–$1^1/_2$ decades starting from the time when the initial transients had decayed. Corrections were made for initial drift rate and, at low powers, for the effect of the source term.

Results

The results of the control rod calibration were as follows:

Static calibration	Core A	Core B
Pu^{239}	3.9	3.6
U^{235}	4.2	3.95
Kinetic calibration	4.2	3.95

Reactivity change per inch movement (units of 10^{-4}).

The difference in the magnitude between cores A and B is due to the different core loadings but the difference between the Pu and U^{235} calibration is anomalous and is repeated for core B. This difference is discussed later (Section 3, p. 261).

The total worth of a control rod, obtained by combining the relative and absolute calibrations, was found to be 0.67 % compared with 0.8 % and 0.75 % from one- and two-group perturbation calculations.

TEMPERATURE COEFFICIENT

The experimental determination of the temperature coefficient is a valuable partial check on the total power coefficient, which is derived from calculated temperature coefficients and temperature distribution.

The coolant temperature was raised from 140 °C to 170 °C by heat sources in the primary coolant circuit (immersion heaters and reactor vault heaters; total 200 kW) and in the secondary circuit (trace heating and electromagnetic pumps; total 500 kW, and by steam heating, 500 kW).

The reactor was maintained at a constant flux throughout two heating and two cooling runs, changes due to temperature being compensated by movement of control rod. When plotted against coolant temperature the compensating control-rod movement was linear and indicated a temperature coefficient of -3.4×10^{-5} per °C, which is reasonably close to the calculated value of -4×10^{-5} per °C.

Fission cross-section measurements

At the core centre

A measured reaction rate is proportional to reactor power, the reaction cross-section and the mass of the sample. In the case of the Pu^{239} and U^{235} fission chambers the calculated cross-sections and the measured effective masses were used to determine the true power in terms of the indicated power. This calibration was then used to determine those cross-sections known to a lower order of accuracy, e.g. U^{238} (n, f), Th^{232} (n, f) and also various (n, γ) and threshold reaction cross-sections.

The U^{235} and Pu^{239} counters were previously calibrated in the thermal flux of the DMTR and also in a 14-MeV neutron beam.

The values of the fission and also some capture cross-sections are given in Table I, and the agreement with calculations is within the errors of measurement and calculation except for the U^{238} (n, γ). This confirms the calculated neutron-energy spectrum for the core centre.

Table I

FISSION AND CAPTURE CROSS-SECTIONS AT THE CENTRE OF THE DOUNREAY FAST REACTOR

Reaction	Method	Threshold energy (MeV)	DFR cross-section (mb)	
			Measured	Predicted
Pu^{239} nf	fission chamber	—	1800*	1800*
U^{235} nf	fission chamber	—	1460	1420
U^{238} nf	fission chamber	1.35	80	80
Th^{232} nf	fission chamber	1.35	20	20
S^{32} np	activation	1	16	14
Al^{27} nα	activation	3.3	14	14
Mg^{24} np	activation	4.9	0.4	—
In^{115} nn′	activation	0.5	41	—
Au^{197} nγ	activation	—	210	220
Mn^{55} nγ	activation	—	9	9
In^{115} nγ	activation	—	220	227
I^{127} nγ	activation	—	210	190
U^{238} nγ	activation	—	133	155
B^{10} nα	perturbation	—	1100**	845

* Assumed value ± other cross-sections are relative to Pu.
** Includes cross-sections for (n, p) and (n, T2 α).

Spatial distribution of fission rates and the B^{10} (n,α) *reaction rate*

Vertical scans in the four thimbles are plotted in Figs. 2–5. The fission rate curves for U^{235}, Pu^{239} and B^{10} (n, α) show bumps in the two core thimbles at positions close to the top and bottom tube plate but the threshold detectors do not. Thus the spectrum must alter appreciably in these regions. Similar bumps occur in the inner blanket thimble at the level of the top tube plate of the breeder. A similar scan but extending further into the top shield of the reactor was taken in the inner blanket thimble of core B and is shown in Fig. 6. (The flux for cores A and B should be very similar in this region.) There is a pronounced

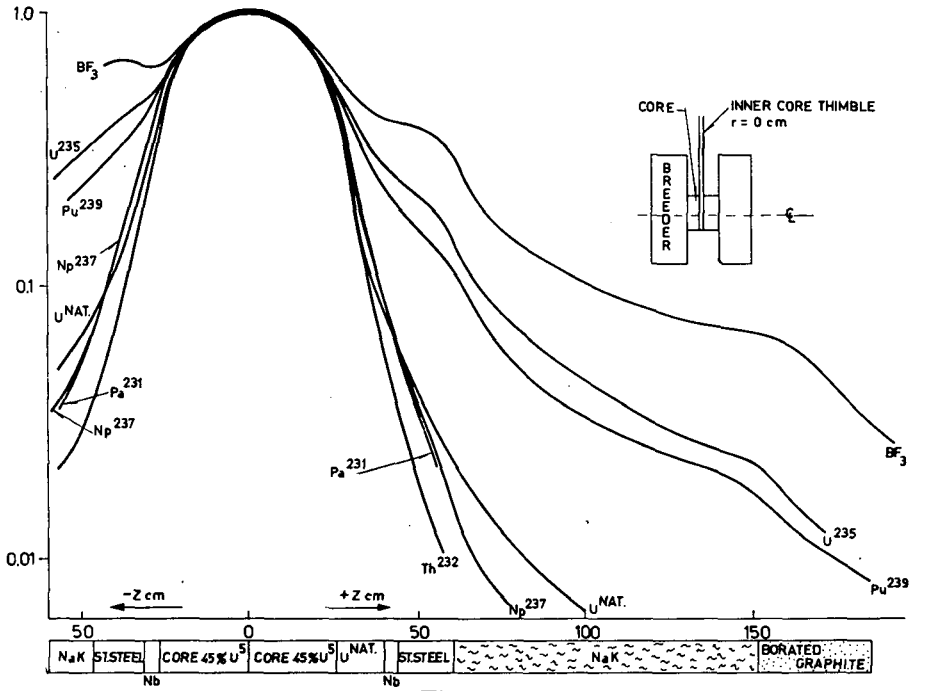


Fig. 2
Fission rates and B^{10} capture rate along the inner core thimble (normalized to unity at core centre).

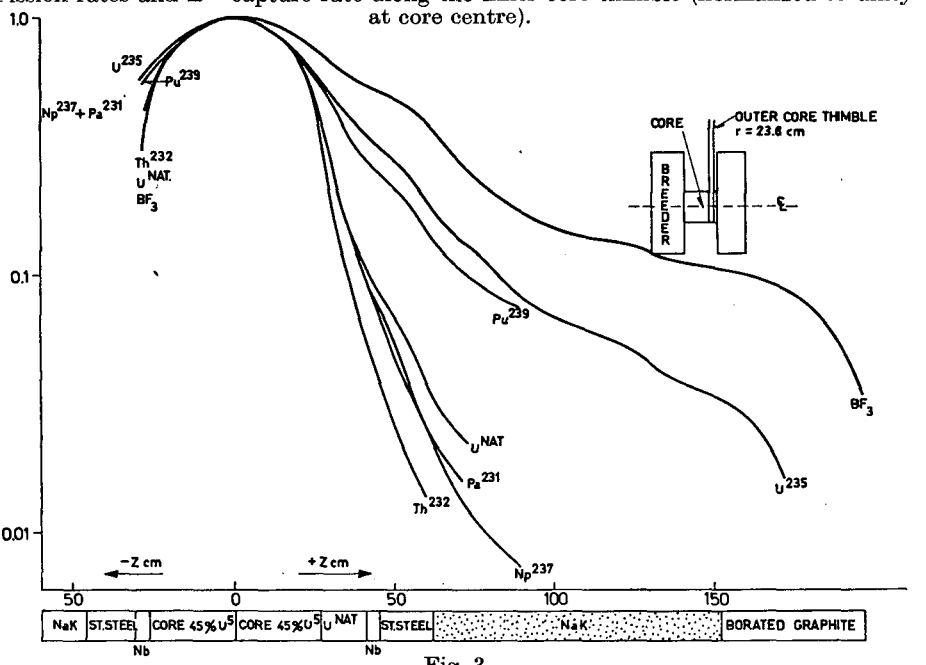


Fig. 3
Fission rates and B^{10} capture rate along the outer core thimble (normalized to unity at core centre).

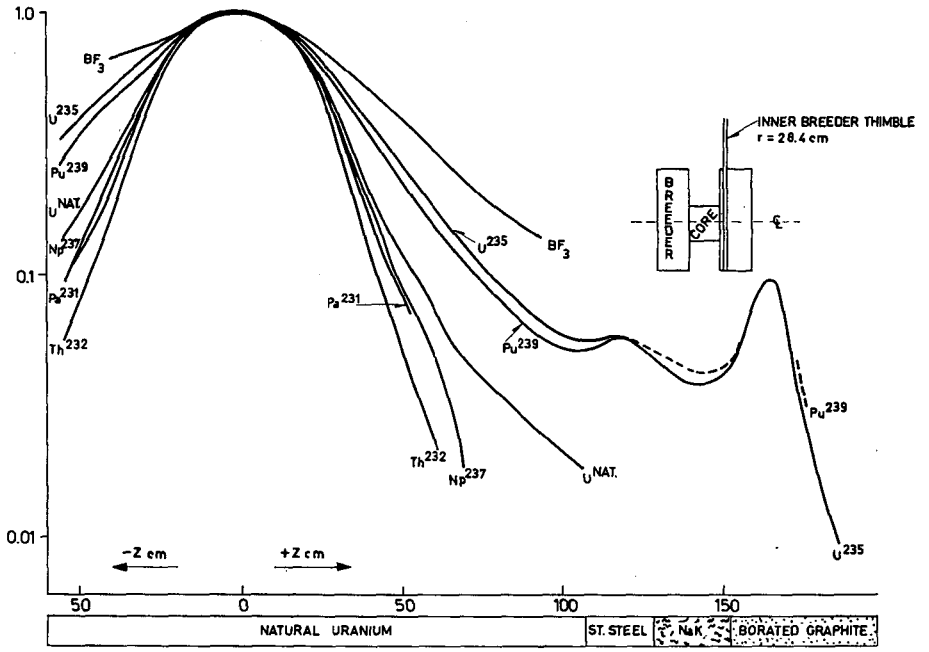


Fig. 4
Fission rates and B^{10} capture rate along the inner-blanket thimble (normalized to unity at core centre).

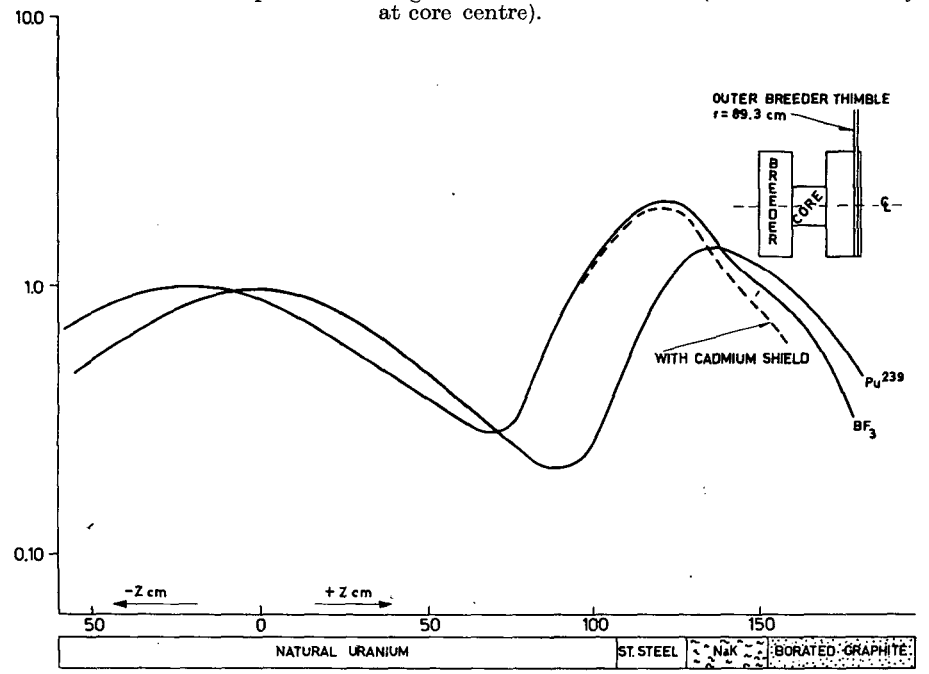


Fig. 5
Fission rates and B^{10} capture rate along the outer-blanket thimble (normalized to unity at core centre).

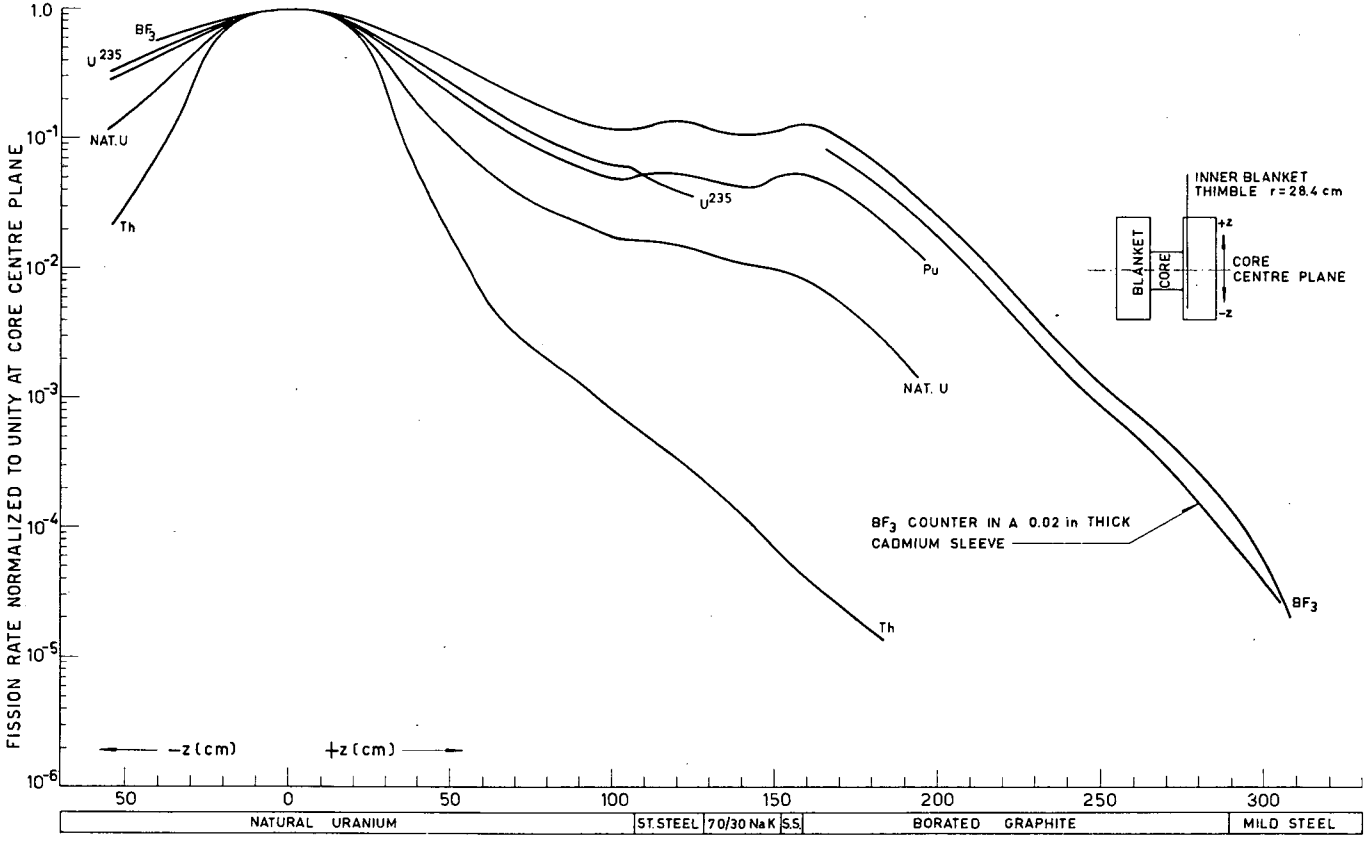


Fig. 6
Experimental fission and reaction rate scans for U^{nat} (n, f), U^{235} (n, f), Pu^{239} (n, f), Th^{232} (n, f) and B^{10} (n, α) along the inner-blanket thimble in the DFR (core B).

bump in the region of the steel at 120 cm and at the boundary of the borated graphite.

The observed bumps in the vertical scans were not predicted so that the cross-section data used are deficient, probably in the steel data or the sodium data.

TABLE II

COMPARISON BETWEEN CALCULATED AND MEASURED VALUES OF REACTION RATES IN A RADIAL SCAN THROUGH THE REACTOR CENTRE

(Values normalized to unity at the core centre)

Reaction	Outer core thimble			Inner blanket thimble		
	(a) Expt. value	(b) Calc. value	b/a	(a) Expt. value	(b) Calc. value	b/a
Pu^{239} (n, f)	0.52 ± 0.02	0.53	1.02	0.38 ± 0.02	0.40	1.05
U^{235} (n, f)	0.55 ± 0.01	0.56	1.02	0.43 ± 0.01	0.44	1.02
Nat. U (n, f)	0.44 ± 0.01	0.40	0.91	0.20 ± 0.01	0.23	1.15
Th^{232} (n, f)	0.42 ± 0.01	0.38	0.91	0.16 ± 0.01	0.20	1.25
Np^{237} (n, f)	0.47 ± 0.03	0.42	0.89	0.25 ± 0.02	0.26	1.04
Pa^{231} (n, f)	0.44 ± 0.02	0.39	0.89	0.22 ± 0.01	0.23	1.04
Au^{197} (n, γ)	0.66	0.64	0.97	0.60	0.53	0.88
B^{10} (n, α)	0.61 ± 0.02	0.68	1.11	0.50 ± 0.02	0.58	1.16
Mn^{55} (n, γ)	0.73	0.60	0.82	0.91	0.49	0.54
U^{238} (n, γ)	—	—	—	0.50	0.50	1
U^{238} but 86 cm above core centre plane in the inner blanket thimble				0.06	0.018	0.3

Radial measurements were possible only in the four thimbles and the results are given in Table II. The Pu^{239} and U^{235} fission rates were well predicted but the threshold reactions were underestimated (about 10 %) at the edge of the core and overestimated (about 10 %) in the inner blanket. The gradient of the fast flux was evidently severely underestimated at the core blanket boundary.

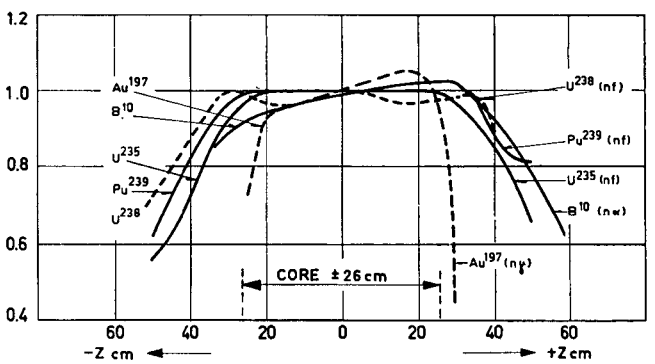


Fig. 7

Ratio of calculated/experimental reaction rates along core centre thimble.

For the core centre thimble a comparison between experimental results and calculation is given in Fig. 7. It is clear that there is good agreement within the core region but that calculated values underestimate the reaction rates outside the core. In particular this applies to the Pu^{239} fission rate. Since the Pu (n,f) cross-section does not vary appreciably with neutron energy until the spectrum is very degraded, the results indicate that the total flux has been underestimated. The discrepancy is generally worse for reactions which are more sensitive to low-energy neutrons, implying that there is an additional factor contributing to the disagreement, an underestimation of the softness of the spectrum.

It was generally concluded that the fission-rate distribution, and therefore power-density distribution, was adequately predicted by calculation and that this would not be a source of significant error in the expected temperature distribution.

Activation measurements

Measurements have been made of the reaction rates of various materials as a function of their position in the reactor.

Foils and non-hydrogeneous powders were irradiated in special stainless-steel capsules which could be lowered into, and removed from, the thimbles while the reactor was at power. This was done by means of long $^1/_4$-in diameter rods threaded to take the capsules, and of such length as to place the capsules in prescribed positions. Other samples were encapsulated in stainless steel and loaded into special fuel and blanket elements mentioned earlier.

Most of the activated samples were counted in an assembly which allowed geometrical reproducibility but was not designed for absolute counting, and so the results are quoted relative to the value at the core centre. In some cases additional measurements were made using 4π or coincidence measurements, or a calibrated gamma-spectrometer, to measure the absolute value of the reaction rate.

The measurements made include:

(*a*) Vertical scans with gold and manganese to supplement and complement the fission-chamber measurements;

(*b*) Irradiation of uranium at selected points to find the variation in the U^{238} (n, γ) U^{239} reaction rate, important in breeding considerations, throughout the reactor;

(*c*) Vertical scans with sulphur to provide an indication of the fast flux present, i.e. 2-MeV neutrons, producing the S^{32} (n,p) P^{32} reaction;

(*d*) Irradiation of iodine in selected positions, because of difficulty which had been found in work on ZEUS in finding a spectrum which was consistent with both the Au^{197} (n, γ) and I^{127} (n, γ) measurements; and

(*e*) Distribution of the radiative capture rate in the centre thimble for copper, sodium and U^{238}.

Absolute values at the core centre

Activated gold foil 10^{-4} in thick was measured in a coincidence and also a 4π counter. A 20 % self-absorption correction was applied to the latter and the two measurements were in agreement. Manganese, iodine, and U^{238} (following solvent extraction in ether) were deposited as a very thin film and measured in the 4π counter.

Sulphur was assayed in a liquid counter against a standard phosphorous solution. Manganese (n,p), indium (n,n′), sodium (n,γ) and (n,2n) and U^{238} (n,2n) were counted in a calibrated gamma-spectrometer.

The results are included in Table I and show general agreement with calculation with the exception that the measured capture cross-section for U^{238} was 20 % lower than calculation. The sodium n,2n reaction had not previously been measured in United Kingdom fast reactors because a substantial irradiation time is required. The U^{238} n,2n reaction is of importance since the U^{237} production is an important factor in the handling and processing of irradiated fuel elements. As mentioned on page 257, the activation results are consistent with the calculated spectrum at the core centre.

Relative scans

Measurements in the thimbles of core A had demonstrated the need for a more detailed determination of relative activation rates along the experimental thimbles. For example, the effective cross-sections for radiative capture in gold and manganese seemed to increase much more rapidly than had been expected, as the neutron spectrum became degraded away from the core. This was particularly evident with manganese, but the measurements were not detailed enough to indicate the mode of variation clearly. The more detailed measurement of these reactions was facilitated by irradiating the gold and manganese as continuous wires instead of foils.

In view of the unexpected peaks in the radiative capture of gold and manganese it was decided to supplement the U^{238} capture measurements with a more detailed scan of lower accuracy. A series of capsules containing uranium oxide was irradiated in the inner core thimble. The solvent extraction process was used to remove fission-product activity for the sample in the centre capsule but was omitted for other samples. A correction for fission-product activity was estimated for the core centre position by counting the activated uranium with and without the fission products. For other positions the correction was estimated by multiplying the centre value by the relative fission rate as determined by the U^{238} fission counter. The accuracy within the core was poor but outside the core the fission rate in the U^{238} had fallen sufficiently for a reasonable assessment to be made.

Radiative capture in copper was measured, to seek high-energy resonances suggested by self-screening effects observed in USSR measurements in BR1, [8]. The copper was irradiated as a wire suspended in the inner core thimble.

Sodium, in the form of sodium acetate, was melted and forced into a 150-cm tube of stainless steel. After irradiation the tube was cut into 3-cm lengths and the acetate transferred to aluminium capsules and counted under a scintillator. In addition, some heavily irradiated sodium carbonate was counted on a gamma-spectrometer and, by allowing the Na^{24} to decay, the production of Na^{22} was determined.

The (n,p) reaction on sulphur was measured in the inner core centre and the blanket thimbles, vertically into the inner rotating shield. Being a 2-MeV threshold reaction the scan was intended to check shielding calculations. The sulphur was irradiated partly as zinc sulphate and partly as elemental sulphur. The specific activity of the latter was increased by burning off the sulphur and counting the residual phosphorus. In this way it was possible to find the fast-neutron flux at positions high into the reactor shield.

Results of relative scans

The reaction rates for gold and manganese are given in Fig. 8 together with calculated values. The latter were determined from TDC fluxes but substantially similar results were obtained from the SNG/PDQ calculations. It is apparent that the manganese and to a lesser extent the gold capture rate has been seriously underestimated in regions just outside the core. The peaks are larger than for the experimental B^{10} capture rate (Fig. 2) so that either the Mn and Au cross-sections must depart from the $1/v$ characteristic in the 50 — 300 keV region or there is an unexpectedly large proportion of the neutron flux in these positions at about 1 keV where resonance capture is more effective.

The U^{238} scan (Fig. 9) has relatively large errors due to the difficulty of estimating the fission-product correction. There is, however, little doubt that the reaction rate distribution for U^{238} is similar to that for gold.

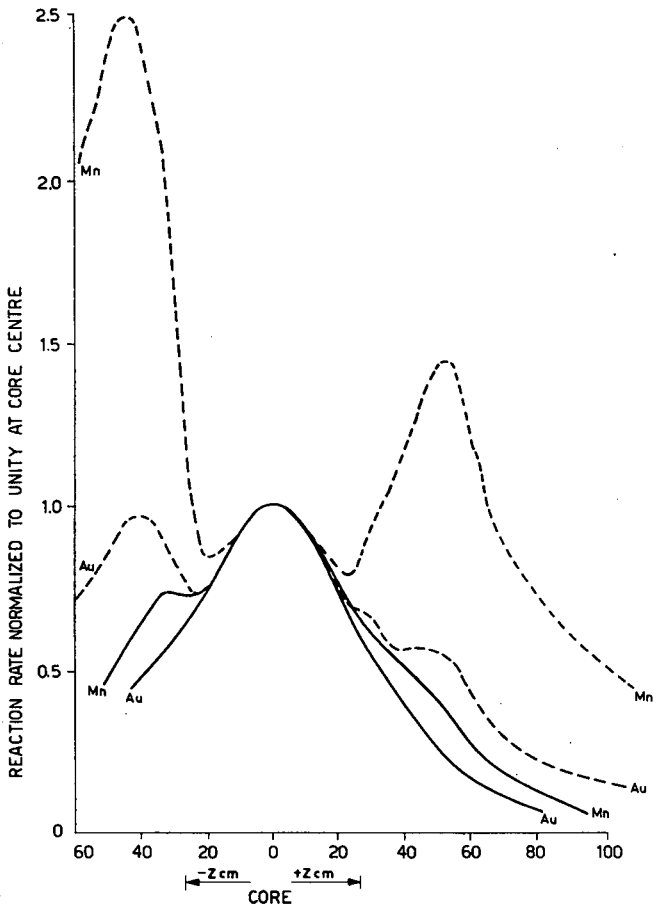


Fig. 8

Comparison between calculated and measured reaction rates for manganese (n, γ) and gold (n, γ) in a vertical scan along the centre core thimble of the DFR.

———— Calculated

– – – – – – Experimental

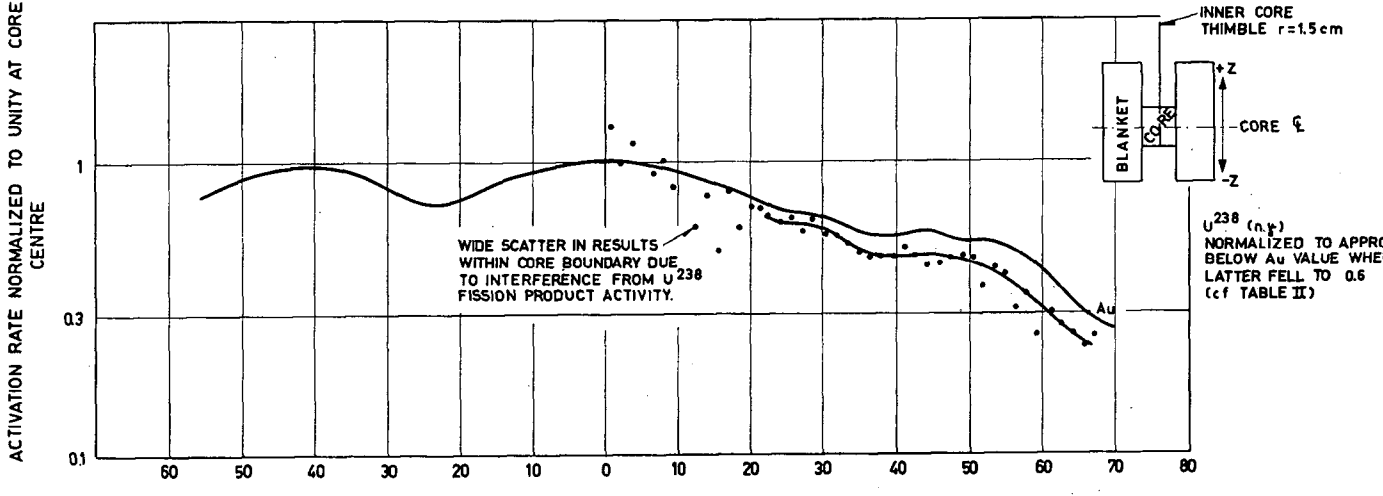


Fig. 9
Experimental activation rate scans for the Au^{197} (n, γ), I^{127} (n, γ) and U^{238} (n, γ) along the inner core thimble in the DFR (core B).

The gold capture rate was also measured in the inner blanket thimble and Fig. 10 shows the result together with a calculated scan. The calculated scan is normalized to unity at the centre plane and the experimental scan to a value of 1.2 at the centre plane corresponding to the difference between calculation

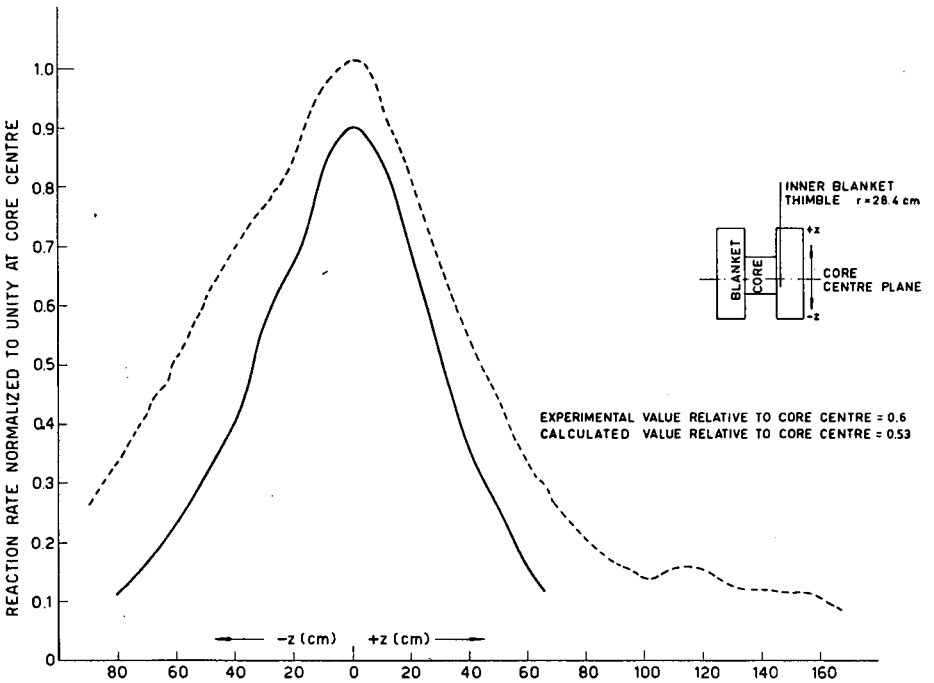


Fig. 10
Comparison between calculated and experimental values of the Au^{197} (n, γ) Au^{198} reaction-rate scan along the inner-blanket thimble in the DFR (core B).
_____ Calculated (ref.) (normalized to 0.9 at core centre plane)
_ _ _ _ _ Experimental

and experiment at this point as determined from a radial scan. There is a strong suggestion that, by analogy with gold, the capture rate for U^{238} is underestimated by an appreciable amount in the inner blanket. There is no evidence to show how much, if any, of the additional capture is due to extra neutrons being scattered from the coolant above the core. If this contribution is small then the underestimation of the breeding gain will extend radially into the blanket and the error in this region may be almost a factor of two. This may not mean a substantial increase in the overall breeding gain though some increase over the calculated value is likely. It is possible that the capture may be concentrated in the inner blanket at the expense of a reduction in the outer blanket.

The copper, sodium and sulphur scans for the centre thimble are given in Fig. 11. The copper results do not suggest the presence of large resonances and the shape of the curve is similar to the B^{10} and sodium curves. The sodium scan is also free from large peaks, which was surprising in view of the large cross-section resonance at 3 keV.

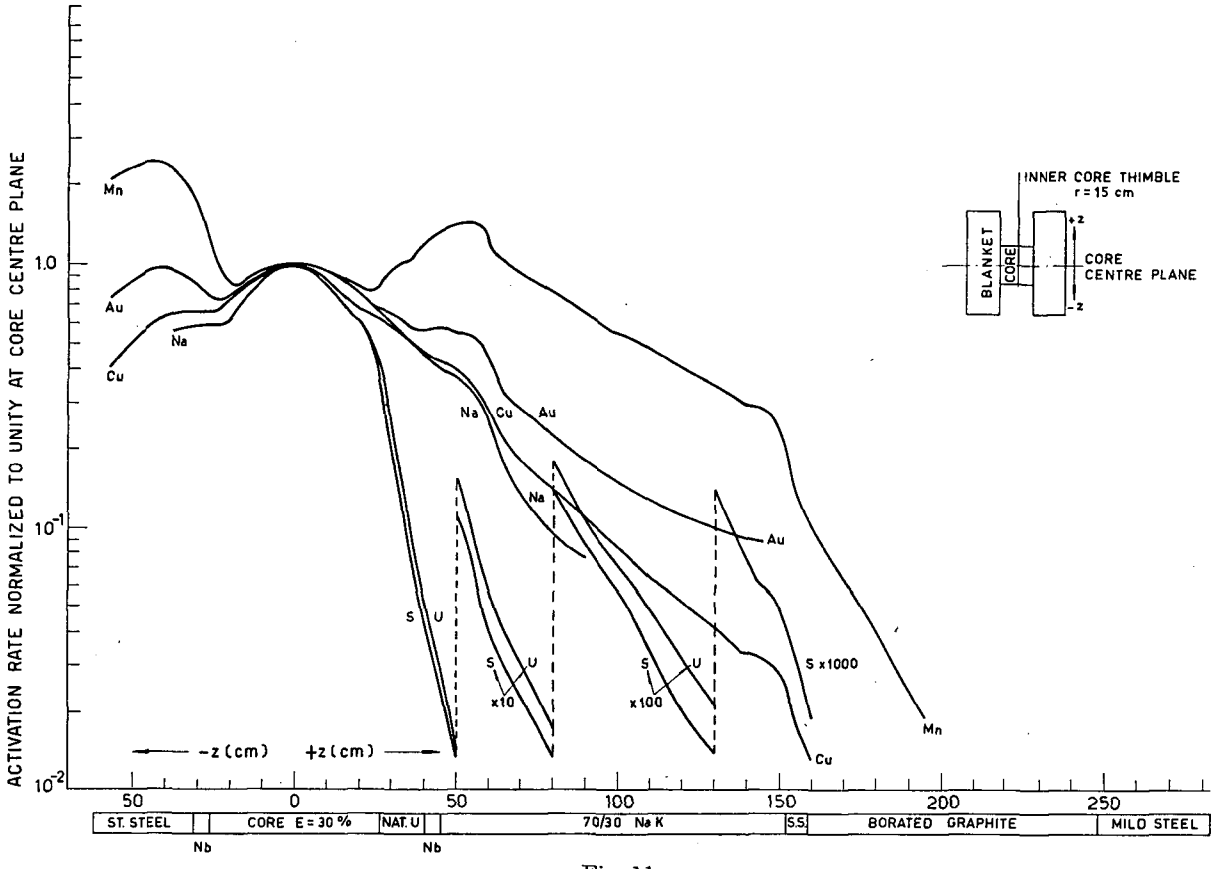


Fig. 11
Experimental activation-rate scans for the Cu (n, γ), Mn^{55} (n, γ), Au^{197} (n, γ), S (n p), U^{238} (n f) and Na^{23} (n, γ) along the inner core thimble in the DFR (core B).

The sulphur measurements follow closely the group-1 flux of the TDC calculation (>2.5 MeV). Above the core the gradient of the TDC group-1 flux is less than the measured sulphur flux, confirming the U^{238} fission-chamber results. At 100 cm above the core the disagreement was a factor of 3.

Radial scans

There is little information about the radial distribution since access was mainly restricted to the thimbles. The special fuel elements containing irradiation samples showed that the predicted activation rate for gold and sulphur was within errors of measurement at positions along the central radial plane and within the core boundary. Measurements in the thimbles are included in Table II. One measurement of the U^{238} capture rate at a point 86 cm above the centre plane and in the inner blanket thimble confirms the very large underestimation of this reaction rate at positions in the inner blanket.

NEUTRON SPECTRUM

Direct measurement of the neutron spectrum is difficult but some information may be obtained by comparing relative reaction rates. At the core centre the satisfactory agreement between calculated and measured cross-sections is indicative of a successful prediction of the neutron-energy spectrum. Only the U^{238} (n, γ) cross-section is discrepant and there is other evidence [8] to support the view that this cross-section should be reduced by about 20%. Above 10 keV the Pu^{239}, U^{235} fission, and the B^{10} (n, α) reactions have sufficiently dissimilar energy-dependence for the ratio of the relative reaction rates to be some guide to the spectrum. Below 10 keV the ratios become insensitive to energy. Some measured ratios are given in Table III. The mean neutron energy is defined, in this context, as the energy of a mono-kinetic neutron flux which would reproduce the measured cross-section ratio. In the inner core thimble the agreement between U^{235}/Pu^{239} and the B^{10}/Pu^{239} is reasonable from 0 — 50 cm above the core centre, corresponding to a mean neutron energy from 0.35 — 0.1 MeV, the outer core thimble suggesting a slightly softer spectrum from 0.3 to 0.095 MeV.

Above 120 cm in the centre thimble the U^{235}/Pu^{239} ratio falls to less than unity. This suggests the presence of neutrons having energies <1 keV since, apart from a few resonances, the fission cross-section for Pu^{239} is consistently lower than for U^{235} between 1 keV and 0.1 MeV. The B^{10}/Pu^{239} ratio in this region is rather low, indicating that the majority of the neutrons have energies greater than 100 eV.

In the inner blanket the U^{235}/Pu^{239} ratio of 0.8 suggests the presence of 1 keV neutrons at 140 cm above the core centre. When a cadmium sleeve was used round the B^{10} counter, the cadmium ratio was 1.05 at 140 cm rising to 1.25 at 170 cm. The low value (1.5 — 2.0) of the B^{10}/Pu^{239} ratio implies that most of the neutrons are in the energy range 100 — 1000 eV.

It appears that the spectrum of neutrons is very broad and that, while the 11 group cross-section set gives reasonable spectrum information for the core, a larger number of groups is required for estimating the spectrum outside the core.

EFFECTIVENESS OF 10-c Ra-Be SOURCE

Power, reactivity and source strength are related by the equation:

$$\mathrm{d}W/\mathrm{d}t = \varrho W + S$$

TABLE III

SUMMARY OF U^{235} σ_f/Pu^{239} σ_f AND B^{10} (n, α)/Pu^{239} σ_f and a rough estimate of the mean neutron energy through the core and blanket

Height above core (cm)	U^{235} (n, f) and Pu^{239} (n, f)								B^{10} (n, α) and Pu^{239} (n, f)							
	Inner core		Outer core		Inner blanket		Outer blanket		Inner core		Outer core		Inner blanket		Outer blanket	
	(a)*	(b)**	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
0	0.79	0.35	0.83	0.30	0.89	0.17	1.05	0.085	0.50	0.30	0.58	0.25	0.66	0.20	1.19	0.085
20	0.83	0.30	0.83	0.30	0.89	0.17	1.05	0.085	0.53	0.27	0.58	0.25	—	—	1.19	0.085
30	0.87	0.21	0.87	0.21	0.94	0.14	—	—	0.65	0.21	0.73	0.18	0.83	0.20	1.31	0.072
40	0.95	0.14	1.04	0.085	0.98	0.12	—	—	0.87	0.15	1.02	0.12	0.99	0.13	1.36	0.068
50	1.03	0.09	1.08	0.08	1.02	0.1	—	—	1.17	0.095	1.17	0.095	1.15	0.095	1.41	0.062
60	1.03	0.09	1.08	0.08	0.98	0.1	—	—	1.29	0.075	1.28	0.075	1.25	—	1.49	0.054
70	1.10	0.09	1.08	—	1.02	0.1	—	—	1.34	0.065	1.46	0.065	1.35	—	1.55	0.048
80	1.10	0.07	1.08	—	1.05	0.085	—	—	1.49	0.05	1.46	—	1.45	—	1.64	0.037
90	1.06	—	0.96	—	1.05	0.085	—	—	1.49	0.05	—	—	—	—	1.64	0.037
100	1.03	—	—	—	1.00	0.0001	—	—	1.54	0.048	—	—	—	—	1.79	0.02
110	0.99	0.0001	—	—	0.94	0.0001	—	—	1.54	0.048	—	—	—	—	1.79	—
120	0.95	0.0001	—	—	0.89	—	—	—	1.59	0.042	—	—	—	—	1.67	—
130	0.95	—	—	—	0.82	—	—	—	1.64	0.036	—	—	—	—	1.55	—
140	1.03	—	—	—	0.78	—	—	—	1.69	0.03	—	—	—	—	1.79	—
150	0.99	—	—	—	—	—	—	—	1.91	0.019	—	—	—	—	1.91	—
160	—	—	—	—	—	—	—	—	2.29	0.013	—	—	—	—	1.64	—
170	—	—	—	—	—	—	—	—	2.29	—	—	—	—	—	1.67	—

* (a) Measured cross-section ratio U^{235} (n, f)/Pu^{239} (n, f) or B^{10} (n, α)/Pu^{239} (n, f).

** (b) Neutron energy (MeV) at which the microscopic cross-section ratio = (a).

where W is the power, ϱ is the reactivity and S is the source term (in the same units as W). For steady state $\mathrm{d}W/\mathrm{d}t=0$ and $\varrho=S/W$. Thus the amount of reactivity which is equivalent to the source is inversely proportional to the reactor power. For the DFR $S=8\times10^{-4}$ so that at a power level of 1 W the source neutrons are equivalent to 2 in of control-rod movement. The experimental value of S is required for two reasons. If reactivity changes are being measured at low power and with the source in the reactor, then the effect of the source may influence the measurements and must be allowed for. Secondly, Fig. 12

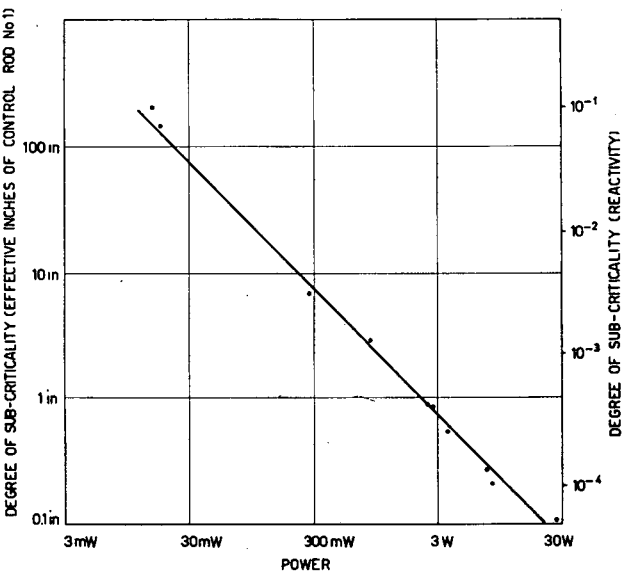


Fig. 12

Degree of sub-criticality of the DFR core A, as a function of reactor power (with 10-c Ra-α-Be source "in").

shows that by measurement of shut-down flux (or power) the degree of sub-criticality may be estimated to within 10% and this is a useful guide to reactivity changes produced during shut-down.

3. Principal inconsistencies between experimental and calculated results

The predicted critical mass, the neutron flux and power distribution for the core region and the neutron-energy spectrum at the core centre were reasonably well confirmed. The flux and the softness of the neutron-energy spectrum were underestimated away from the core centre.

The interpretation of results of integral experiments is seldom sufficiently clear to make definite pronouncements about nuclear data, but the following anomalies have arisen which cause doubt to be cast on the presently accepted values.

Radiative capture in U^{238}

The n, γ cross-section at the core centre was 20% below the calculated value (Table I). This was observed in cores A and B and for ZEUS and ZPR-III [7].

Radiative capture in manganese and gold

A comparison between the manganese and gold scans of Fig. 8 shows that within the region ± 40 cm of the core centre plane the ratio of the reaction rates is continually changing but outside this region the ratio is approximately constant. Published data for gold and manganese show that from 0.3 to 3 MeV the $1/v$ relationship holds for both reactions, implying that the ratio is insensitive to spectrum. The experimental results suggest that at least one of the reactions departs markedly from this law in the energy range encountered at the edge of the core. For lower energies, i.e. further from the core, the ratio is approximately a constant so that the anomalies are likely to be about 200 keV.

Radiative capture in copper

The absence of sharp peaks in the scan of the copper activation indicates that no sharp resonances were present in the energy range covered by the neutron spectrum or that if present, the change in the shape of the spectrum was insufficiently steep to resolve the peaks. The copper results were similar in character to the other radiative capture reactions. In contrast, the manganese scan exhibits large peaks.

Ratio of ν (U^{235})/ν Pu (Pu^{239})

The reactivity change induced by the insertion of a small sample of fissile material into the core centre is proportional to $[(\nu - 1) - \alpha]\sigma_f$ where ν, α and σ_f are the number of neutrons emitted per fission, the ratio of the capture to fission cross-section and the fission cross-section. The ratio of the value of $[(\nu - 1) - \alpha]\,\sigma_f$ for the two samples Pu and U^{235} may be experimentally determined to within a few per cent but the observed and calculated ratios differ

Table IV

MEASURED VALUES OF $(\nu-1-\alpha)_{Pu}$ / $(\nu-1-\alpha)_{U^{235}}$ FOR VARIOUS CORES

Core	$\frac{(\nu-1\alpha)\ Pu}{(\nu-1\alpha)\ U^{235}}$	Mean
ZEUS 1	1.434	
ZEUS 2	1.410	
ZEUS 3A	1.403	1.411
ZEUS 3B	1.435	
ZEUS 4A	1.407	
ZEPHYR	1.378	1.378
ZPR–III 5	1.47	
ZPR–III 11	1.46	1.493
ZPR–III 12	1.55	
DFR A	1.492	
DFR B	1.475	1.483
VERA	1.46	

by amounts which are larger than the believed experimental error. The ratio of the fission cross-sections is believed to be known within 3 — 5% and is in agreement with the experimental value; the discrepancy noted on page 245. may therefore be attributed to the ratio of the parameter $[(\nu - 1) - \alpha]$ for Pu^{239} to U^{235}.

The value of the ratio should be fairly independent of spectrum since α and $\bar{\nu}$ are not very energy-dependent. Values for this ratio are given in Table IV for a number of reactors. The values which are calculated from the 11-group set [6] and the 16 group set [9] both give 1.3 as the ratio. This differs from the experimental values by 5% for the comparatively hard spectrum of ZEPHYR to 14% for the DFR. The ratio is about 20 times more sensitive to changes in $\bar{\nu}$ than to changes in α so that there is a strong possibility that the ratio of $\bar{\nu}$ $(Pu^{239})/\bar{\nu}$ (U^{235}) is underestimated by the constants of refs. [6] and [9].

ACKNOWLEDGEMENTS

The work described in this paper was carried out by the whole of the Physics Group at DERE supported by their colleagues at Harwell and Risley.

REFERENCES

[1] Symposium on the Dounreay Fast Reactor, British Nuclear Energy Conference (1960).
[2] SMITH, R. D. and SANDERS, J. E., Proc. 2nd UN Int. Conf. PUAE **12** (1958) 89.
[3] CARLSON, G. G., LAMS–2201 (1958).
[4] BILODEAU, G. G. *et al.*, WAPD–TM–70 (1957).
[5] CARLSON, B. *et al.*, LAMS–2346 (1960).
[6] Reactor Physics Constants, ANL-5800.
[7] LOEWENSTEIN, W. B. and OKRENT, D., Proc. 2nd UN Int. Conf. PUAE **12** (1958) 16.
[8] LEIPUNSKY *et al.*, Proc. 2nd UN Int. Conf. PUAE **12** (1958) 3.
[9] YIFTAH, S. *et al.*, Fast Reactor Cross-sections, Pergamon Press (1960).

THE PHYSICS DESIGN OF EBR-II*

W. B. LOEWENSTEIN
ARGONNE NATIONAL LABORATORY, ARGONNE, ILL.,
UNITED STATES OF AMERICA

Abstract — Résumé — Аннотация — Resumen

The physics design of EBR-II. Calculations of the static, dynamic and long-term reactivity behaviour of EBR-II are reported together with results and analysis of EBR-II dry critical and ZPR-III mock-up experiments. Particular emphasis is given to reactor-physics design problems which arise after the conceptual design is established and before the reactor is built or placed into operation. Reactor-safety analyses and hazards-evaluation considerations are described with their influence on the reactor design.

The manner of utilizing the EBR-II mock-up on ZPR-III data and the EBR-II dry critical data is described. These experiments, their analysis and theoretical predictions are the basis for predetermining the physics behaviour of the reactor system. The limitations inherent in applying the experimental data to the performance of the power-reactor system are explored in some detail. This includes the specification of reactor core size and/or fuel-alloy enrichment, provisions for adequate operating and shut-down reactivity, determination of operative temperature and power coefficients of reactivity, and details of power- and flux-distribution as a function of position within the reactor structure. The overall problem of transferring information from simple idealized analytical or experimental geometry to actual hexagonal reactor geometry is described.

Nuclear performance, including breeding, of the actual reactor system is compared with that of the idealized conceptual system. The long-term reactivity and power behaviour of the reactor blanket is described within the framework of the proposed cycling of the fuel and blanket alloy.

Safety considerations, including normal and abnormal rates of reactivity-insertion, the implication of postulated reactivity effects based on the physical behaviour of the fuel alloy and reactor structure as well as extrapolation of TREAT experiments to the EBR-II system are analysed. The EBR-II core melt-down problem is reviewed.

Physique du réacteur EBR-II. L'auteur présente les calculs du comportement d'EBR-II statique, dynamique et sous évolution à long terme de la réactivité ainsi que les résultats et l'analyse des expériences critiques sèches faites sur EBR-II et en simulation sur ZPR-III. Il insiste particulièrement sur les problèmes de physique des réacteurs qui, dans l'élaboration du projet, suivent le choix du modèle théorique et précèdent la construction ou la mise en exploitation. L'auteur présente des analyses de la sécurité des réacteurs ainsi que diverses considérations sur l'évaluation des risques sous l'angle de leur influence sur le projet de réacteur.

Il décrit la simulation d'EBR-II, à partir des renseignements fournis par le ZPR-III ainsi que les mesures critiques sèches sur EBR-II. Ces expériences, leur analyse et les prévisions des calculs servent de bases pour prédire le comportement physique du réacteur. L'auteur approfondit quelque peu la validité intrinsèque de l'application des données expérimentales au fonctionnement du réacteur de puissance. Ceci comprend les données précises des dimensions du cœur et/ou de l'enrichissement de l'alliagne combustible, le choix convenable des valeurs de la réactivité prévues

* Work performed under the auspices of the United States Atomic Energy Commission.

en exploitation et pendant l'arrêt, la détermination des coefficients de réactivité à la température et à la puissance de fonctionnement, et la distribution précise de la puissance et du flux en fonction de la position dans l'ensemble du réacteur. L'auteur décrit le problème de l'application des renseignements obtenus à partir d'une géométrie simple, idéale, analytique ou expérimentale, à la géométrie réelle hexagonale du réacteur.

Il compare le rendement nucléaire, y compris la surgénération, du réacteur réel par rapport à celui du modèle théorique. Il décrit la réactivité à long terme et le comportement énergétique de la couche fertile du réacteur dans le cadre de l'étude du cyclage proposé du combustible et de l'alliage fertile.

L'auteur étudie les questions de sécurité considérant notamment l'introduction de taux de réactivité normaux et anormaux, les conséquences des effets supposés de réactivité, à partir du comportement physique de l'alliage combustible et de la structure du réacteur, ainsi que par extrapolation des expériences faites sur TREAT au système EBR-II. Il examine le problème de la fusion du cœur de EBR-II.

Физический расчет экспериментального реактора — размножителя EBP-II. Вычисления статистического, динамического и длительного режима реактивности экспериментального реактора-размножителя EBR-II даются совместно с результатами и анализом экспериментов на реакторе EBR-II по достижению критичности в сухом состоянии и отдельных экспериментов на реакторе ZPR-III. Особое внимание уделяется проблемам физического расчета реактора, которые возникают после определения принципиальной схемы и до сооружения или ввода реактора в эксплуатацию. Описываются анализ безопасности реактора и соображения по оценке опасностей, а также их влияние на расчет реактора.

В докладе описывается способ использования модели EBR-II на основании полученных на реакторе ZPR-III данных, а также данных сухой критичности реактора EBR-II. Эти эксперименты, их анализ и теоретические выкладки являются основой для определения физического поведения реакторной системы. Более подробно исследуются ограничения, присущие применению экспериментальных данных к рассматриваемой энергетической реакторной системе. Сюда относятся спецификация размеров активной зоны реактора и/или обогащение топлива, значения реактивности для нормального режима работы и остановки реактора, определение рабочей температуры и энергетических коэффициентов реактивности и распределение энергии и потока как функции положения в реакторе. Описывается общая проблема экстраполяции от простой идеальной аналитической или экспериментальной геометрии к фактической гексагональной геометрии реактора.

Ядерные характеристики, включая воспроизводство, фактической реакторной системы сравниваются с ядерными характеристиками идеальной принципиальной реакторной системы. Поведение реактивности в течение длительного времени работы и энергетический режим зоны воспроизводства реактора описываются в рамках предполагаемого топливного цикла и материала зоны воспроизводства.

В докладе анализируются соображения по безопасности, включая нормальные и анормальные скорости прироста реактивности, последствия ожидаемых эффектов реактивности, основанные на физическом поведении сплава топлива и структуры реактора, а также экстраполяция экспериментов, проведенных на опытном реакторе для изучения переходных процессов TREAT, на реакторе EBR-II. Изучается проблема расплавления в активной зоне реактора EBR-II.

Aspectos físicos del reactor EBR-II. La memoria informa sobre los cálculos del comportamiento estático, dinámico y a largo plazo de la reactividad del reactor reproductor experimental EBR-II, así como sobre los resultados y análisis de los experimentos críticos en seco del EBR-II y de los experimentos simulados en el reactor de potencia cero ZPR-III. Insiste particularmente en los problemas de física del reactor que, en la elaboración del proyecto, siguen a la elección del modelo, pero preceden a la construcción y puesta en marcha del reactor. Presenta diversos análisis del reactor desde el punto de vista de la seguridad y formula consideraciones sobre la evaluación de los riesgos y su influencia sobre el diseño del reactor.

El trabajo explica también la manera de emplear los datos obtenidos en los experimentos arriba citados. Estos experimentos, su análisis y sus predicciones teóricas constituyen la base para determinar el comportamiento físico del reactor. La memoria estudia detalladamente las limitaciones inherentes a la aplicación de los datos experimentales al funcionamiento del reactor de potencia. Ello incluye datos precisos sobre las dimensiones del cuerpo, el enriquecimiento de la aleación combustible, o de ambos factores; el establecimiento de una reactividad adecuada para el reactor funcionando o detenido, la determinación de la variación de los coeficientes de reactividad en función de la temperatura de funcionamiento y de la potencia generadora y detalles de la distribución de la potencia y del flujo en diversos puntos de la estructura del reactor. La memoria expone también el problema general que supone transferir a la verdadera geometría hexagonal del reactor los datos obtenidos en geometrías simples idealizadas, analíticas o experimentales.

Se compara el rendimiento nuclear, incluso el de reproducción, del reactor real con el del modelo teórico y se describen las variaciones a largo plazo de la reactividad y de la generación de energía en la envoltura fértil, refiriéndolas a los ciclos propuestos para el combustible y la envoltura fértil.

La memoria formula consideraciones sobre la seguridad estudiando en particular la introducción de índices de reactividad normales y anormales y la consecuencia de supuestos efectos de reactividad, que se basan en el comportamiento físico de la aleación combustible y de la estructura del reactor, así como en la extrapolación al sistema del EBR-II de los experimentos realizados con el conjunto TREAT. Por último, examina el problema de la fusión del cuerpo del reactor EBR-II.

Introduction

The Argonne Experimental Breeder Reactor II (EBR-II) is an unmoderated heterogeneous, sodium-cooled reactor designed to produce 62.5 MW of heat [1, 2, 3]. The reactor-system complex is primarily an engineering facility to determine the feasibility of this type of reactor for central-station power plant application.

The design emphasis on high thermal performance at high temperature with optimum burn-up in both reactor core and radial breeder reflector has placed severe requirements on detailed knowledge of the physics of the reactor. To this end the neutronics of EBR-II and similar systems have been extensively investigated [4, 5]. The correlation between the large amount of experimental data and theoretical prediction suggests that the static as well as many aspects of the kinetic behaviour are generally understood. It is recognized that certain detailed reactivity and spectrum effects cannot be properly predicted. It is not always obvious whether theory or experiment or both are in error when certain measured results cannot be properly predicted.

Reactor-physics calculations were originally initiated with few and multi-group diffusion theory and S_N analyses in one-dimensional geometry [6]. Few and multi-group two-dimensional diffusion theory analyses have been performed in both r, z and r, θ geometry. In addition, space-independent static and kinetic analyses have been freely used [3].

The physics design of EBR-II, or any other reactor, requires close liaison between and an understanding of many of the engineering aspects of the reactor system. For example, an error of 2% on the density of the fuel alloy appears as a 1% uncertainty in the reactivity. In the loading of the reactor to criticality,

a 2% uncertainty in fuel-alloy density alone suggests an uncertainty of at least 4% on the critical mass. The overall requirement on compositional precision cannot be overemphasized for good reactor design.

The critical experiment is performed principally to reduce uncertainties and thus reduce the amount of flexibility that must be built into the reactor. Flexibility in a critical assembly is an admirable feature. In a power-producing reactor, such flexibility is neither required nor desirable since it can lead to accident-prone situations which may not be readily recognized. Three distinct types of critical experiment have been performed on the ANL-ZPR-III facility which bear on the neutronics of EBR-II [5, 7]:

Fundamental experiments. These systems consist of single-region EBR-II-size cores surrounded by a uniform high- or low-density reflector.

Engineering mock-up. This system is designed to represent the best geometric and compositional mock-up of the actual reactor that may be assembled on the critical facility.

Clean mock-up. This system does not include detailed geometric and compositional asymmetries of the reactor design. This mock-up is of more general interest than the "engineering mock-up". It can be included in the class of "fundamental experiments" by lifting the restriction on reflector uniformity.

All three of these types of experiment have been used in specifying parameters for the design of EBR-II. Analysis of the experiments has shown that even with the detailed engineering mock-up and its peripheral reactivity measurements, analysis is required to effect a transition from this experiment to the actual reactor. The confidence in the analysis of transition is based upon analysis of the fundamental and clean mock-up experiments. These lend themselves to basic analytical investigations. Thus it might appear that all three types of experiment are necessary for a good understanding of the physics of a specific reactor design. This may become unnecessary when our understanding is broadened by time and experience.

The physics of an actual reactor, such as EBR-II, requires considerable attention to problems not faced during the general conceptual study. The precise determination of reactivity held by control mechanisms, the assurance that neutron-flux-monitoring equipment is adequate over all power ranges, and the assurance that neutron-source flux is adequate for reactor start-up are examples of problems peculiar to an actual reactor design. Another interesting problem requiring some precise understanding is the gradual shift of both power and reactivity from the reactor core to the radial breeder blanket. This effect comes about if fuel elements are more frequently cycled (replaced) than the inner portions of the radial blanket.

The EBR-II reactor programme is initiated with the performance of the dry critical experiment [8]. This experimental programme with no sodium in or near the reactor core (other than that contained for fuel-element bond purposes) will give insight into both the incremental and total reactivity worth of the sodium coolant in the reactor. This information will be obtained by comparing dry critical data with those from the wet critical experiments to be performed after coolant sodium has been introduced into the reactor. Subsequent to the wet critical experiments the reactor power will be gradually increased. The approach to power will include stability and kinetic investigations which will, in fact, determine the details of raising the reactor power level.

Description of the EBR-II reactor

The EBR-II power-plant complex is located at the United States Atomic Energy Commission National Reactor Testing Station (Fig. 1). The plant includes a complete remote fuel-processing and fuel-element-fabrication facility. EBR-II will be the first reactor in the United States Power Demonstration Program to operate on a closed fuel cycle. Partly spent or burned fuel can be pyrometallurgically reprocessed, re-enriched and returned to the reactor after being refabricated.

The reactor is submerged in the "primary tank" (Fig. 2) filled with about 90000 gal liquid sodium at 370°C. With a maximum coolant flow rate of about 9000 gpm, the large sodium reservoir insures that bulk coolant temperature transients are very slowly transmitted to the reactor. The large sodium reservoir also serves as a partial sink for an accidental energy release. The primary tank is suspended inside an air-tight containment vessel (Figs. 2 and 3) which serves to confine the accidental release of built-up fission products, plutonium and activated sodium from the primary system. The primary system confinement is designed to contain the energy release associated with a nuclear accident. The reactor building is designed to confine the effects of a maximum sodium-air interaction caused by a major sodium release. In a sense, the reactor is doubly contained.

The reactor is located in the reactor vessel near the bottom of the primary tank (Fig. 3). Coolant is taken from the bulk sodium in the primary tank, passed through two 5000-gpm mechanical pumps and introduced near the bottom of the reactor vessel. Flow is then upward through individual fuel and breeder reflector sub-assemblies. In order to achieve nearly uniform coolant outlet temperatures, the coolant flow is orificed consistent with axial and radial power-density gradients and discontinuities. The hot (482°C) coolant leaves the reactor near the top of the reactor vessel and then passes through the primary heat-exchanger submerged in the primary sodium. Sodium is the working fluid of the intermediate secondary cooling system.

Details of the reactor, the surrounding reactor vessel and the neutron shield are given in Figs. 4 and 5. The reactor (core and breeder reflector) is a hexagonal array of sub-assemblies (Fig. 5). The sub-assemblies are designed mechanically to prevent inadvertent interchange between enriched and depleted uranium-bearing sub-assemblies. Each sub-assembly contains the fissile or fertile material in the form of cylindrical fuel elements bonded with sodium and clad with stainless steel. The fertile material is unalloyed depleted uranium. The fissile material is contained in uranium-5 wt.% fissium fuel alloy. The uranium is 48.4% enriched in U^{235}. Each fuel sub-assembly contains 91 fuel elements. Each fuel element contains $\sim$67 g of the uranium – 5 wt.% fissium alloy.

Reactor control is effected by moving fuel into the core from below the core. There are twelve control and two safety sub-assemblies. Each of these contain 61 of the same fuel elements as in a normal fuel sub-assembly. Control- and safety-rod strokes are 14 in, which is approximately the length of the core. In the most reactive position, fuel elements in a control rod are at the same elevation as fuel elements in a fuel sub-assembly. In the least reactive position, the tops of the fuel elements in a control rod are at the same elevation as the bottoms of the fuel elements in a fuel sub-assembly. The 12 control sub-assemblies are activated from the top and must be disconnected, in their least reactive

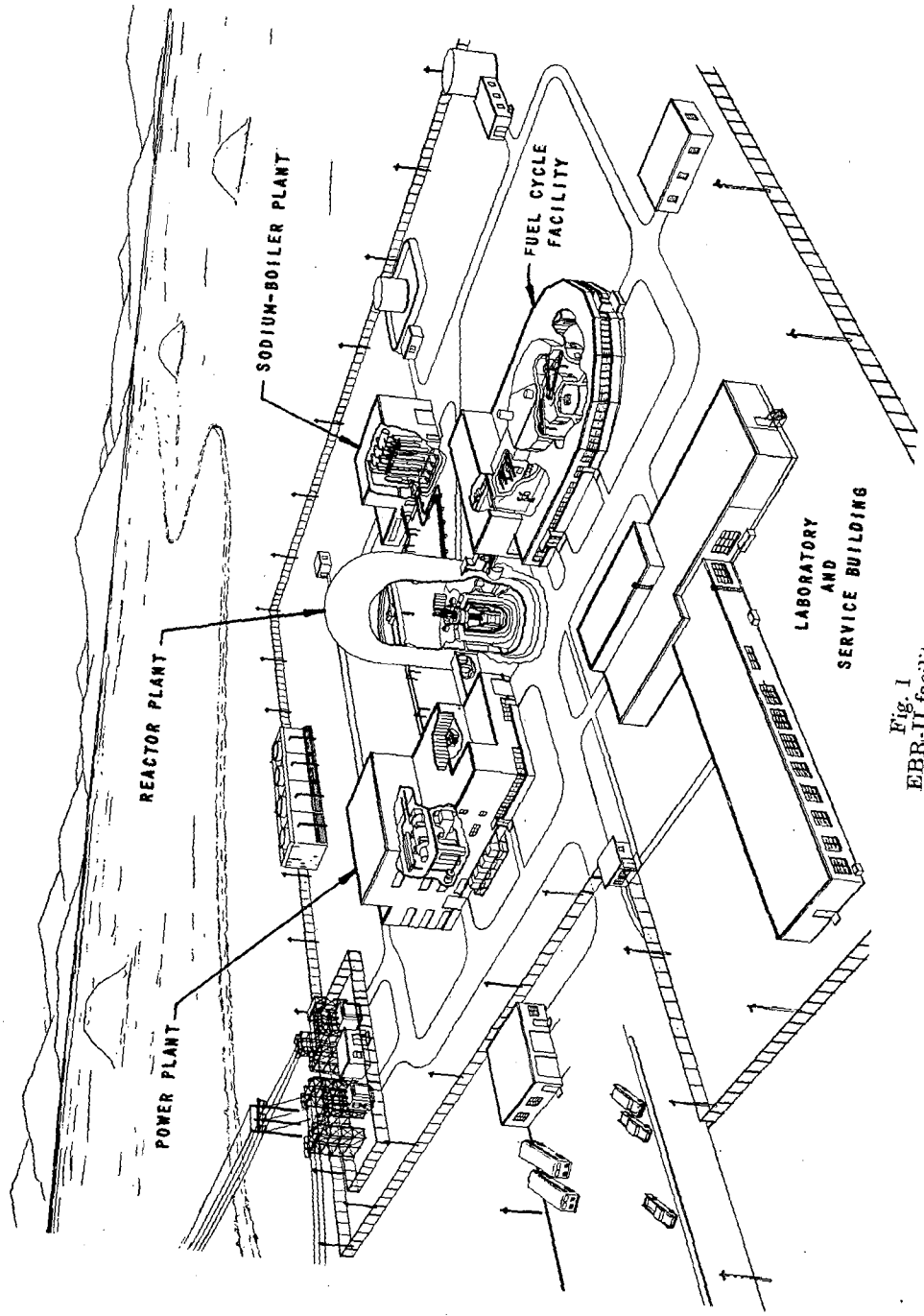


Fig. 1
EBR-II facility.

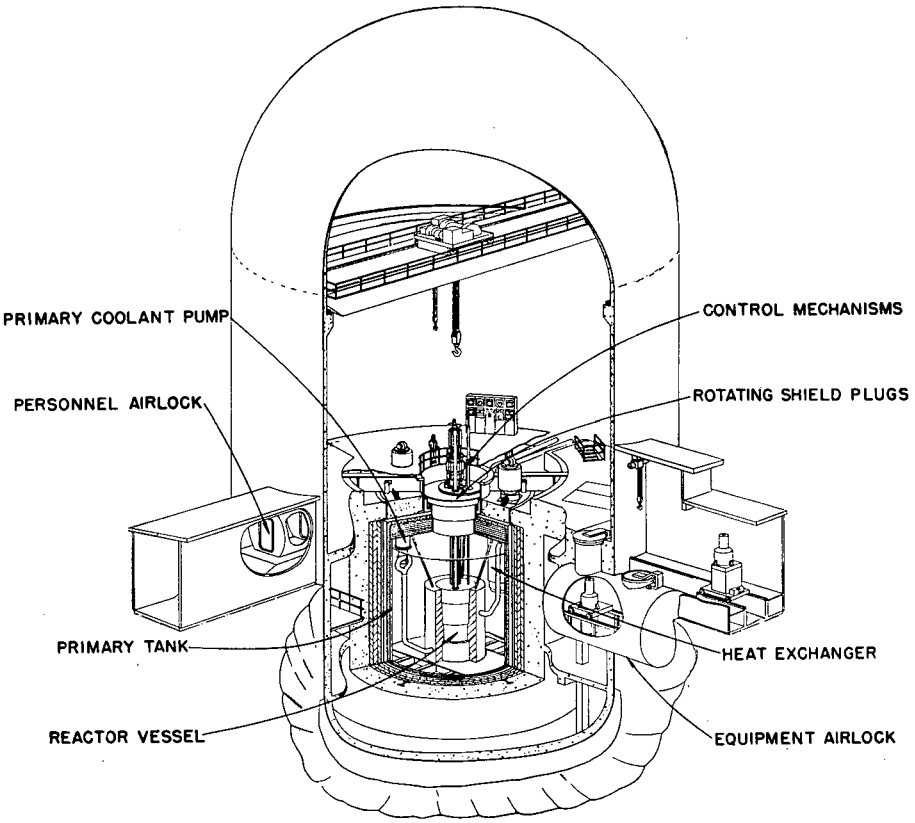


Fig. 2
EBR-II reactor plant.

positions, while fuel is being loaded (Fig. 6). The two safety rods are actuated from below the core (Fig. 4) and are operative while fuel is being loaded. Scram signals (emergency shut-down) actuate these rods depending on the "mode of operation". During "reactor operation", when the reactor is critical and producing power, an automatic scram signal will release the 12 control rods. With a pressure-assist they move to their least reactive positions. During "fuel handling", when the reactor is expected to be substantially sub-critical, an automatic scram signal releases the safety rods. The force of gravity causes them to move to their least reactive positions below the core. The safety rods may also be released manually during "reactor operation".

The neutron-flux-monitoring equipment (fission counters and ion chambers) are located in eight air-cooled ($<45°$ C) instrument thimbles. Four of these are embedded in the neutron shield surrounding the reactor vessel (Fig. 7). Four more are embedded in sodium just outside the neutron shield. The fission counters and ion chambers are located near the core central plane for maximum sensitivity but can be moved vertically inside the thimbles. The neutron shield consists mainly of graphite and borated graphite canned in stainless steel. Small

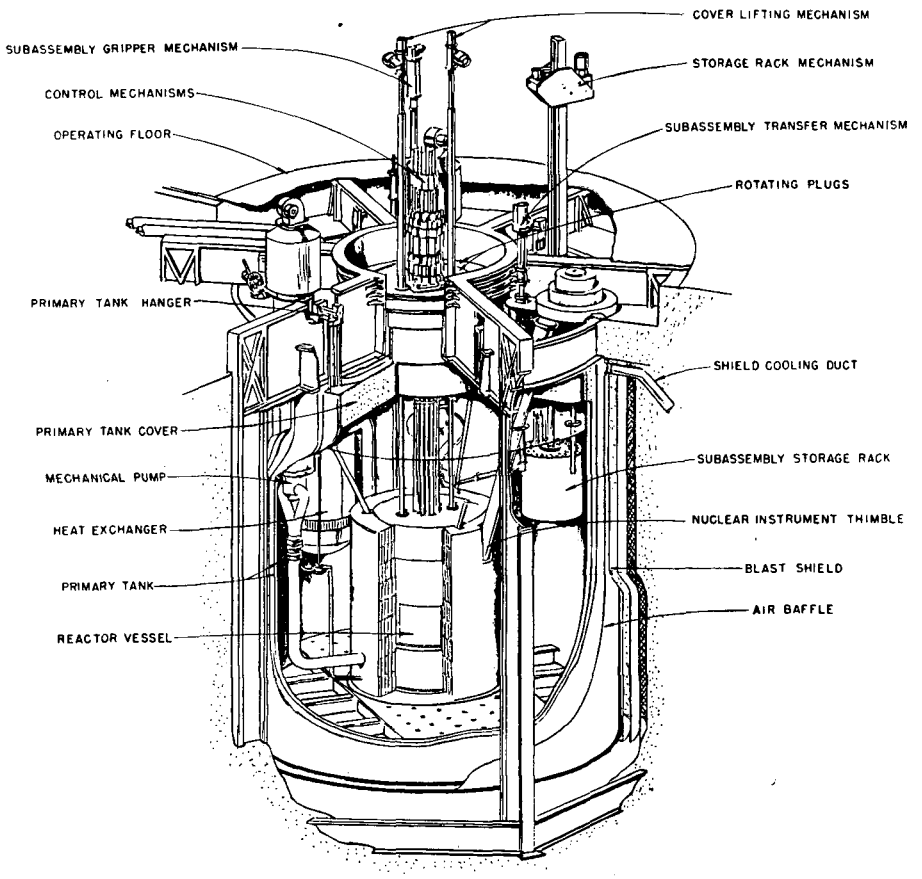


Fig. 3
EBR-II primary system.

amounts of sodium pass between the square shield cans. Borated graphite is used only in those shield regions where it does not interfere with instrument response. There is no borated graphite inside the reactor vessel. In the neighbourhood of the start-up channels (J_1 and J_2 thimbles of Figs. 5 and 7) there is no graphite inside the reactor vessel. There are 11 channels of nuclear instrumentation distributed throughout the eight instrument thimbles. Three log-count rate channels are operative from source power to about 2 kW. One linear-flux and three log-flux channels are operative from 100 W to full power. Three linear-flux channels are operative from 60 kW to full power. One channel is operative from 6 MW to full power and is to be used for automatic control of the reactor at power. Automatic control is not operative until the power level has been established manually.

An Sb-Be neutron-source sub-assembly will be located permanently in the inner blanket (Fig. 5b) of the reactor. A source activity of 100—200 c is required for reactor start-up. The Sb-Se source may be remotely disassembled by reactor-

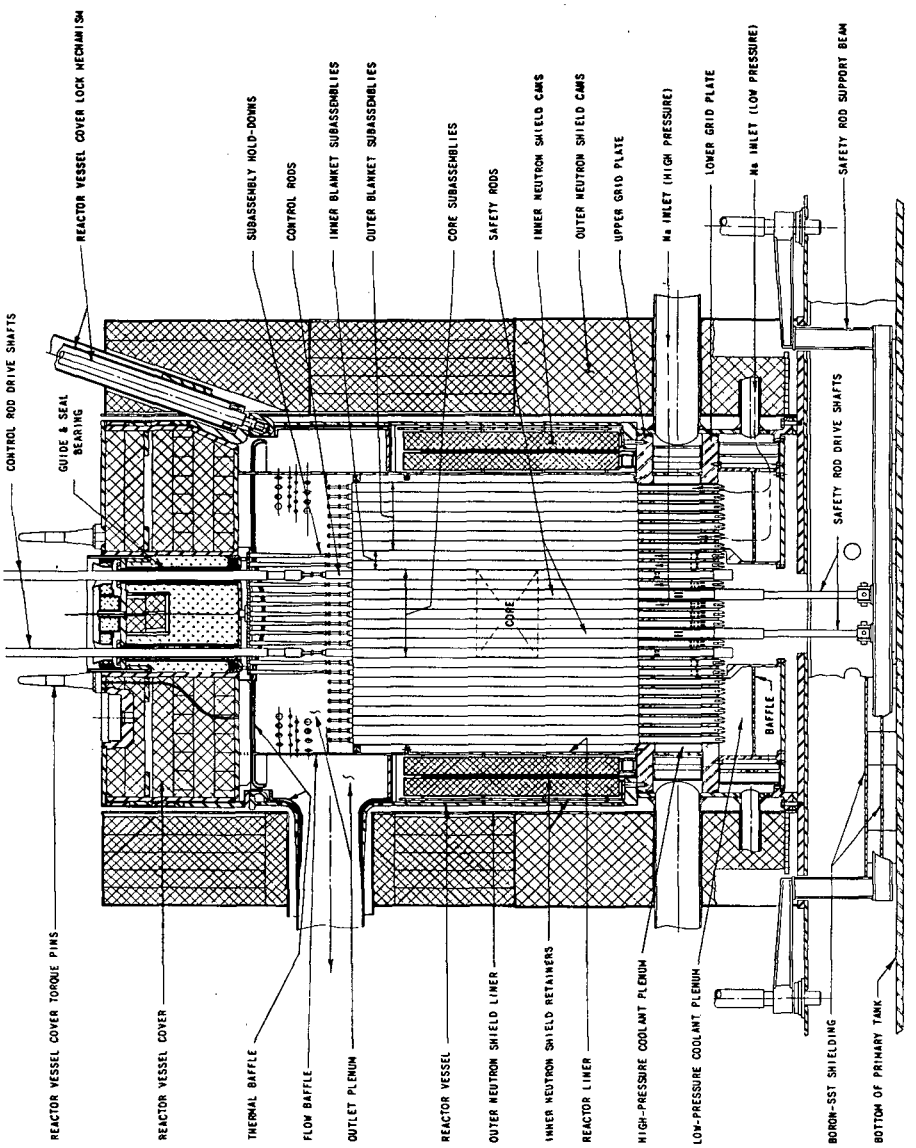


Fig. 4
EBR-II reactor (vertical section).

fuel handling equipment. The active Sb rod may be placed in a source shield sub-assembly located in the outer blanket (Fig. 5b).

In-core instrumentation (four fission counters) is provided for the dry critical experiment. If necessary, two such counters can be provided for the wet critical experiments. During the dry critical experiments, the in-core instrumentation

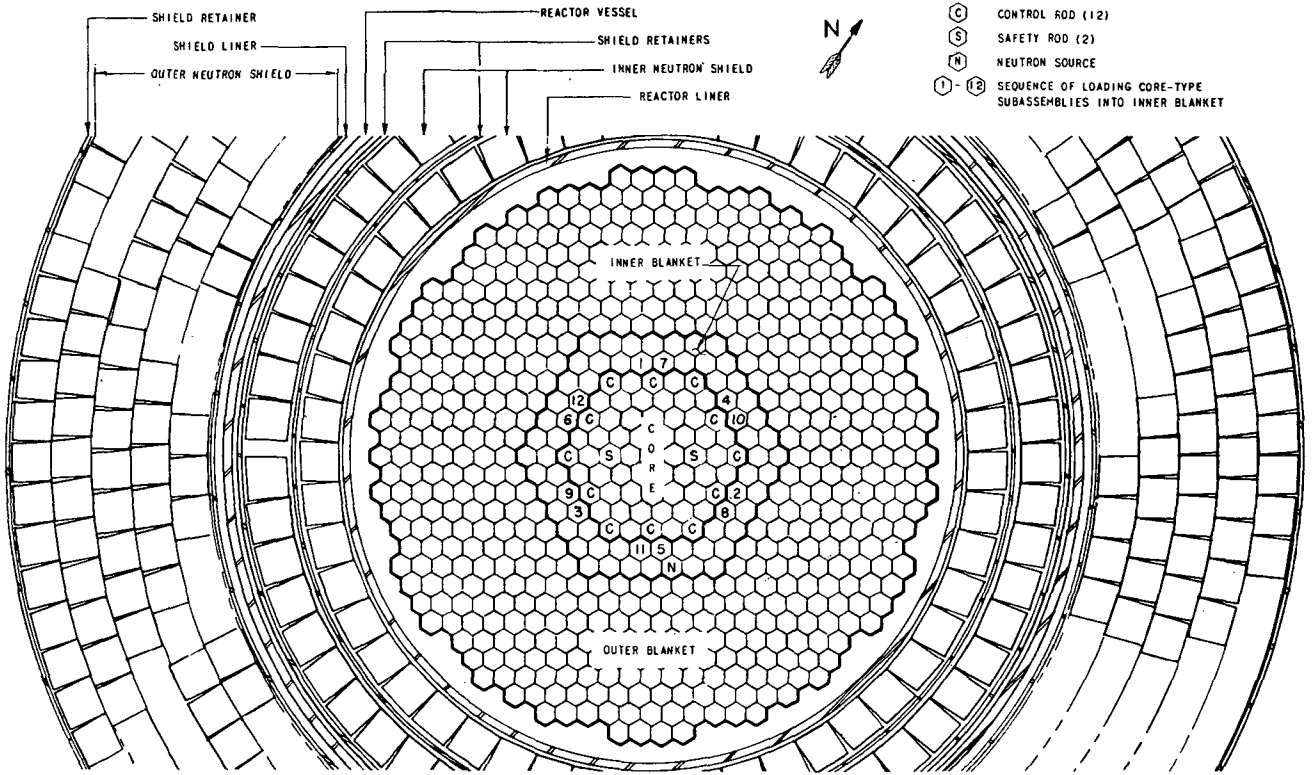


Fig. 5a
EBR-II reactor (horizontal section).

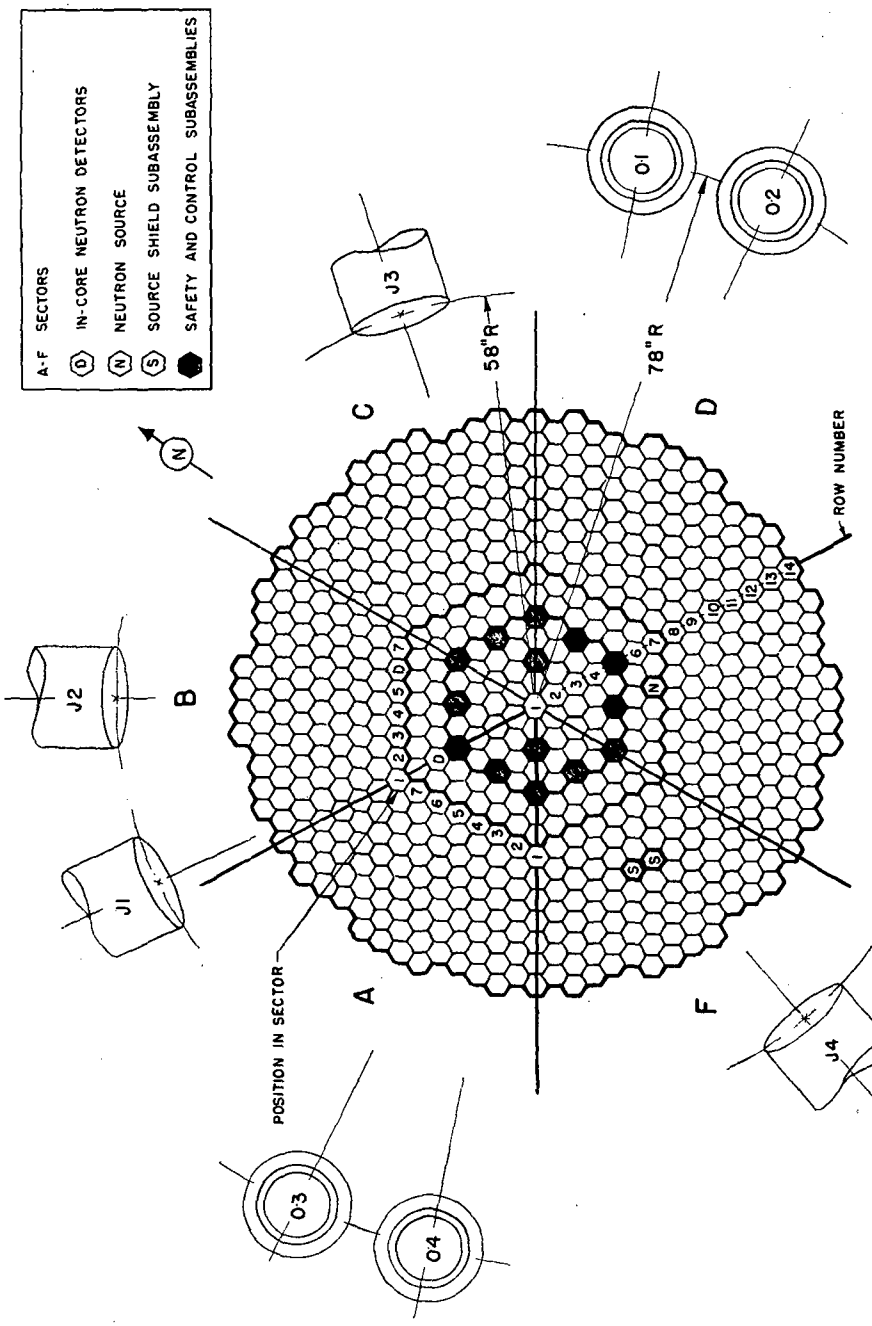


Fig. 5b
Sub-assembly locating diagram.

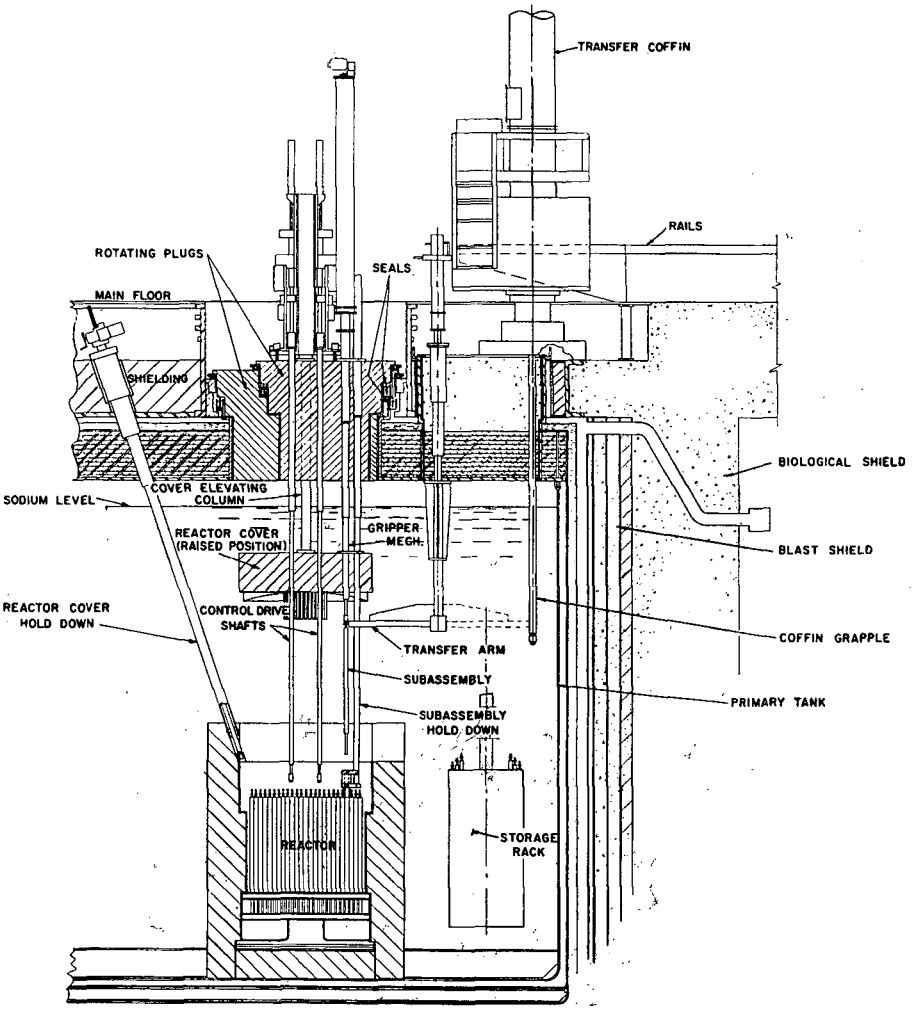


Fig. 6
EBR-II fuel-handling system.

will be permanently installed in the inner blanket (Fig. 5b). The wet critical in-core instrumentation may be located in a control-rod thimble.

EBR-II critical experiments

The EBR-II critical experiments on ZPR-III [5, 7] were designed to provide detailed information concerning the neutronics of the reactor. These were mock-ups of the EBR-II reactor configuration simulating the normal power loading (with sodium coolant). In addition, a large number of experimental investigations were made on systems similar to EBR-II [5]. The neutronics of EBR-II are

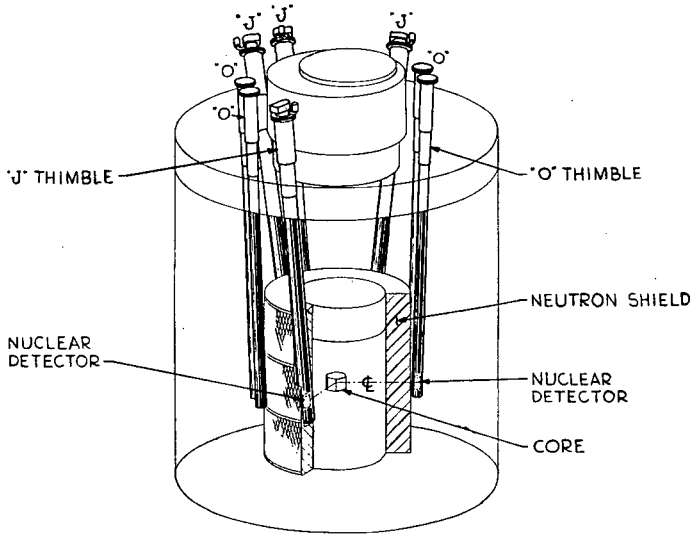


Fig. 7
Nuclear instrument thimbles.

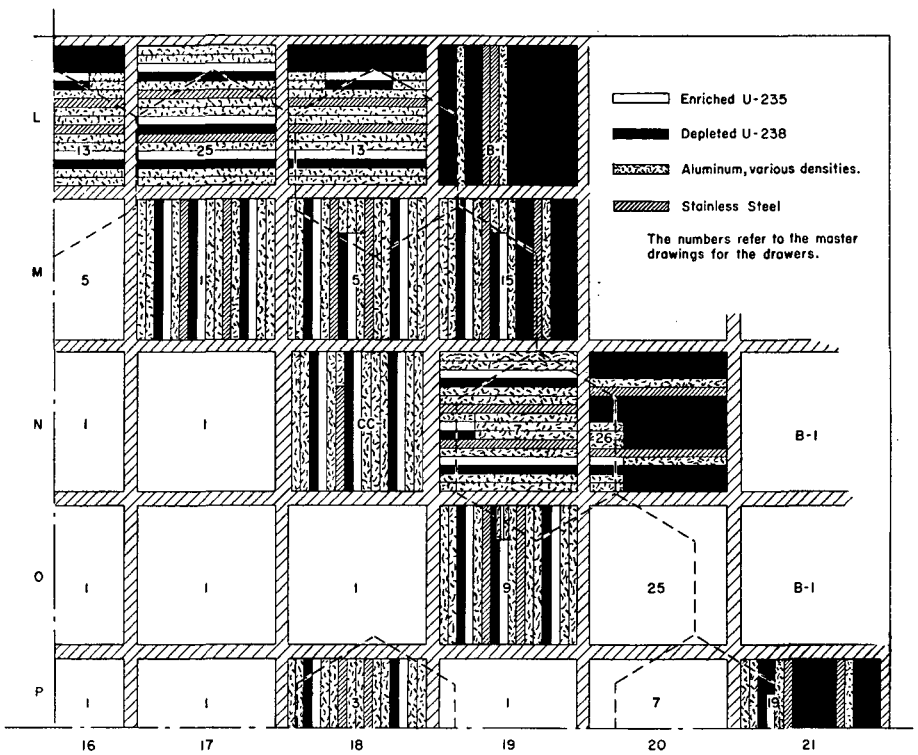


Fig. 8
EBR-II mock-up on ZPR-III (radial section at core mid-plane).

well understood on the basis of analysis which has been well correlated with a large amount of experimental data [4, 5].

The ZPR-III critical experiments utilized low-density aluminium for sodium in the mock-up of the coolant volume of the reactor system. Gross substitution experiments with both solid sodium and aluminium of different effective densities confirmed that removal of coolant material from the reactor core and reflector regions resulted in a loss of reactivity.

Fundamental experiments

Fifteen distinct critical experiments have been reported [5] for EBR-II-size cores. These experiments yielded information on reactivity effects caused by changes of core and reflector geometry and composition. Spectral measurements, neutron-flux traverses, material-replacement experiments and neutron-lifetime determinations are included. These experiments were instrumental in verifying the analytical techniques applicable to EBR-II physics.

All the experimental cores contained 14 vol.% U^{235}, 16 vol.% U^{238}, 12 vol.% stainless steel and 31 vol.% aluminium, a composition very similar to that of the EBR-II mock-up. These experiments included cylindrical, rectangular and annular cores with both high- and low-density reflectors.

Engineering mock-up

The detailed engineering mock-up experiments represent the best geometric and compositional version of EBR-II that can be assembled on ZPR-III. The radial properties of EBR-II were assembled as shown in Fig. 8. This is a cross-section of the core at the central plane of ZPR-III. The dotted hexagonal outline

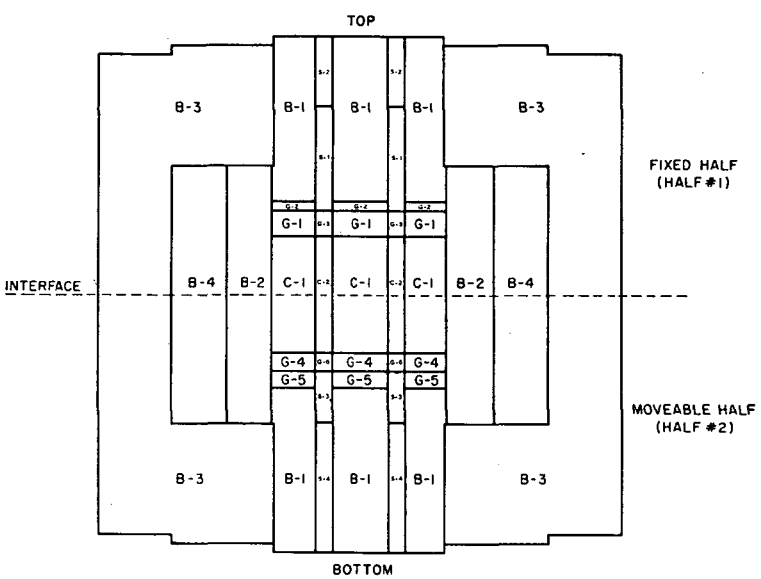


Fig. 9
EBR-II mock-up on ZPR-III (axial section through core centreline).

TABLE I

EBR-II AND EBR-II MOCK-UP ON ZPR-III
(Compositions and dimensions)

Region	Identification in Fig. 9	Mock-up on ZPR-III						Reactor (61-sub-assembly core)				
		Composition (vol. %)				Dimensions (in)		Composition (vol. %)			Dimensions (in)	
		U^{235}	U^{238}	Stainless steel	Al	Effective radius	Length	$U^{235}+U^{238}$	Stainless steel	Na	Effective radius	Length
Core	C-1	13.93	15.84	19.4	26.47	9.73	14.08	29.3	18.5	49.5	9.52	14.22
Control or safety rod	C-2	9.35	10.55	19.4	31.05	—	14.08	19.6	20.3	58.3	—	14.22
Structural gap regions	G-1	—	—	30.0	27.0	9.73	3.0	—	29.8	53.3	9.52	3.50
	G-2	—	—	56.3	9.0	9.73	1.0	—	52.2	47.8	9.52	0.87
	G-3	—	—	30.0	27.0	—	3.0	—	29.4	59.5	—	3.09
	G-4	—	—	40.5	22.3	9.73	2.0	—	39.5	60.5	9.52	2.06
	G-5	—	—	24.6	29.0	9.73	2.0	—	22.4	54.4	9.52	1.88
	G-6	—	—	35.1	24.6	—	2.0	—	34.8	65.2	—	1.63
Axial blanket	B-1	0.06	29.9	19.5	17.9	9.91	19	30.3	18.5	51.2	9.52	18
Radial blanket	B-2	0.13	61.26	19.6	6.43	15.5	30	60	20.9	19.1	30.75	55.0
	B-3	0.13	59.5	7.31	3.70	31.70	56—58					
	B-4	0.13	55.2	14.3	6.74	21.68	30					
Control- and safety-rod followers	S-1	—	—	12.3	35.4	—	12.0	—	12.2	87.8	—	12.66
	S-2	—	—	49.1	24.8	—	8.0	—	50.1	49.9	—	6.62
	S-3	—	—	72.1	8.94	—	6.0	—	73.1	26.9	—	20.31
	S-4	—	—	69.8	13.4	—	15.0	—	73.1	26.9	—	

shows the fifth row of the reactor core superimposed on the square ZPR-III matrix. The closed hexagons indicate the control-rod locations. The normal assembly matrix box (No. 1) contains three columns of enriched uranium, three columns of depleted uranium, two columns of stainless steel and eight columns of low-density aluminium. The axial properties of the mock-up core are shown in Fig. 9. This is a cross-section of the plane including the central axis of symmetry. This plane is symbolic in that it passes through the safety rods, but does not show the control rods. The composition of the various core and reflector regions is given in Table I.

The critical mass of the reactor configuration, which most closely resembles the composition and geometry of the EBR-II reactor, was measured to be 165 kg U^{235}. This critical configuration can be and was predicted to within a few kg U^{235} (calculated $k_{\text{eff}}=1.015$).

The average reactivity worth of a reactor control rod is measured to be 0.37% $\Delta k/k$.

The reactivity worth of two safety sub-assemblies is measured to be 1.36% $\Delta k/k$.

The reactivity worth of a core sub-assembly, relative to low-density aluminium (simulating the displacement of sodium) is a function of radial position in the core. These measurements are reported in Table II. The reactivity worth of inner blanket sub-assemblies relative to low density are also given in Table II.

TABLE II

REACTIVITY WORTH OF CORE AND BLANKET SUB-ASSEMBLIES AS A FUNCTION OF POSITION

(ZPR-III mock-up experiments)

Reactor row number (Fig. 56)	Type of uranium	Reactivity worth (% $\Delta k/k$)
1	enriched	1.53
2	enriched	1.48
3	enriched	1.31
4	enriched	1.05
5	enriched	0.77
6	depleted	0.18
7	depleted	0.04

A number of detailed experiments were performed to determine the reactivity worth of various materials uniformly distributed throughout the reactor core and reflector regions. The importance of these is to effect a transition from the "aluminium-cooled" ZPR-III to the sodium-cooled reactor. Results instrumental in effecting this transition are given in Table III.

A number of distributed material replacements in the reactor core are instrumental in accounting for the reactivity effect of the 5 wt.% noble metal fission products in the reprocessed fissium fuel alloy. These are given in Table IV. Measurements on synthesized fission-product mock-ups as well as other materials of interest to fast reactor design are also given.

TABLE III

REGIONAL MATERIAL-REPLACEMENT EXPERIMENTS
($^0/_0\ \Delta k/k$ per kilogram of material $\times 10^2$)

Material	Reactor region*			
	Core	Axial structural gaps**	Reflectors	
			Radial	Axial
Aluminium	7.4	5.8	3.7	1.6
Stainless steel	2.3	2.7	2.0	0.8
Sodium	11.6	6.4	5.8	3.7
U^{235}	33.5	—	—	—
U^{238}	1.5	—	—	—

* Core contains enriched uranium (C-1 in Fig. 9);
Reflector regions contain depleted uranium (B-1 in Fig. 9);
Structural gaps contain no uranium (G-1 in Fig. 9).
** The axial structural gaps are located between the core and the axial reflector.

TABLE IV

MATERIAL-REPLACEMENT EXPERIMENTS
(Distributed throughout EBR-II core)

Material	Reactivity ($\%\ \Delta k/k \times 10^2$/kg)	Material	Reactivity ($\%\ \Delta k/k \times 10^2$/kg)
Molybdenum	+2.4	Physicum II*	—2.5
Ruthenium	—0.2	V	+5.9
Rhodium	—5.6	Nb	+0.29
Palladium	—2.0	Al_2O_3	+13.1
Physicum I*	—3.3	—	—

* Synthesized fission product mock-ups, Ref. [5].

NEUTRON SOURCE AND INSTRUMENT THIMBLE EXPERIMENTS

Special experiments dealt with the attenuation and spectral shift of neutron flux from the centre of the core to the "J" and "O" thimbles (Fig. 7) where the normal fission counters and ion chambers are located. The "J" and "O" thimbles are located about 58 and 78 in respectively from the reactor centreline in the central core plane. The unperturbed neutron flux can be estimated within and outside the graphite neutron shield under the postulate of no thimble and uniform shield. The presence of the thimbles (up to 15 in outer diameter) appreciably complicates such an estimate and reduces the accuracy of the predicted result. The other aspect of these investigations dealt with determining the neutron yield of an Sb-Be neutron source of the type to be employed in EBR-II. Estimates of neutron yield are extremely uncertain. These experiments were designed to yield data relevant to: (*a*) the required Sb activity in the Sb-Be neutron source; and (*b*) the geometric and compositional arrangement outside the radial blanket and around the "J" thimbles to optimize low-power fission counter response.

The first experiment dealt with the early design of the neutron shield and reactor vessel [3]. The second series of experiments investigated several geometrical and compositional arrangements adjacent to the "J" thimbles in the final reactor configuration. The early shield configuration is shown in Fig. 17 of Ref. [3] while the final EBR-II radial neutron shield is shown in Fig. 5a. The early neutron shield was completely outside the reactor vessel which surrounds the radial outer blanket. The final reactor-vessel diameter is larger and thus the stainless-steel reactor vessel and its thermal baffling plates are almost in contact with the "J" thimble. Furthermore, some shielding material has been located between the reactor vessel and the outer boundary of the radial breeder reflector.

An absolute fission counter [9] was placed at the core centre and another at the radial core boundary during each investigation. Additional counters were placed throughout the radial blanket and neutron shield. However, the information of primary interest is that of low-power "J"-thimble fission-counter response. To enhance instrument response, moderating material was placed in the interior of the instrument thimbles immediately adjacent to the fission

TABLE V

SUMMARY OF EBR-II THIMBLE EXPERIMENTS

Shield configuration:	Final	Final	Final	Final	Final	Final	Final	Final	Early	Early
Thimble:	J	J	J	J	J	J	J	J	J*	O
Principal material in region (see Figs. 4 and 5)										
Inner shield:	Al	Al	Al	Al	Al	Al	C	C	Steel, Al, C	
Outer shield:	ZrH	ZrH	C	C	C	C	C	C	C	
Interior of thimble:	void	ZrH	void	ZrH	void	ZrH	void	ZrH	void	
"Bulk sodium":	Al	Al	Al	Al	C	C	Al	Al	Al	
Count-rate in thimble per watt of reactor power: (cps/W)	857	1345	731	2017	750	1841	395	826	1370	43
Cadmium ratio in thimble:	4.5	8.8	1.9	5.5	1.9	5.2	2.4	6.9	7.6	~5
Sub-critical count-rate with Sb-Be source* (~10 c)										
$k_{eff} \approx 0.97$:	12.6	—	10.7	—	11.4	—	5.8	—	~26	—
$k_{eff} \approx 0.94$:	6.3	—	5.8	—	5.6	—	2.9	—	~13	—

* 8-in outer diameter, final "J"-thimble diameter = 15-in outer diameter.

counters. This was accomplished by inserting 4.1 kg ZrH in the form of 10-in-long aluminium cans having a cross-sectional area of 9.8 in^2. Other experiments included the removal of sizeable sections of normal shield material (graphite) outside the "J" thimbles. The removed material was replaced by a better moderator (ZrH). Table V summarizes experimental fission-counter response data per watt of reactor power.

The neutron-shield configuration adjacent to the "J" thimble was the object of sub-critical investigations. With a 10-c Sb-Be neutron source located at the reactor core inner blanket interface, sub-critical count rates were obtained in the mock-up "J" thimbles, as well as at the core/inner blanket interface. These results are also given in Table V. The experiments demonstrated that the spatial distribution of neutron flux is essentially invariant between a multiplication of 20 and a critical reactor.

These neutron-flux-attenuation experiments served to specify the details of the final reactor neutron-shield configuration. Two "J" thimbles, containing the low-power fission counters, are immersed in a graphite outer shield with no graphite in the inner shield. Sodium is the major constituent of the inner shield region near these thimbles. All other portions of the inner shield contain graphite. These experiments showed that a 100—200-c Sb-Be source will supply 3—6 unmultiplied counts/s in the "J"-thimble start-up fission counters. More sensitive in-core instrumentation is provided for the dry critical experiments and is available for the wet critical experiments.

Power distribution

Uncertainties in the predicted power distribution may be estimated by comparing predicted with measured fission distributions from ZPR-III. Predicted fission distributions

$$F(r, z) \equiv \int \sigma_f(E)\,(E, r, z)\,\mathrm{d}E$$

are obtained from a multi-group two-dimensional (r, z) analysis. This calculation, using the 11-group constants from Ref. [4], will be described in a subsequent section. Figures 10 and 11 compare experimental with theoretical radial and axial fission distributions respectively. Since these experimental data are insufficient for an absolute power calibration, they are normalized to the predicted U^{235} fission density at the centre of the core.

The slight dip at $r \approx 11$ cm for the experimental U^{238} fission distribution in Fig. 10 is attributed to the presence of the safety rods. These were not properly described in the two-dimensional (r, z) analysis. Since mostly U^{238} fissions occur in the portions of the radial blanket shown in Fig. 10, it can be seen that dominant radial reflector fissions (U^{238}) are relatively well predicted. The data points in the outer portions of the radial reflector exhibit large uncertainties because of low power and thus low flux intensity in ZPR-III.

The rather striking deviations between theory and experiment in the gap regions in Fig. 11 must be attributed to an inadequate understanding of the neutron cross-sections of aluminium and stainless steel. However, since very few fissions ($<2\%$) occur in the axial reflectors these deviations between theory and experiment are not significant in determining the reactor-power distributions.

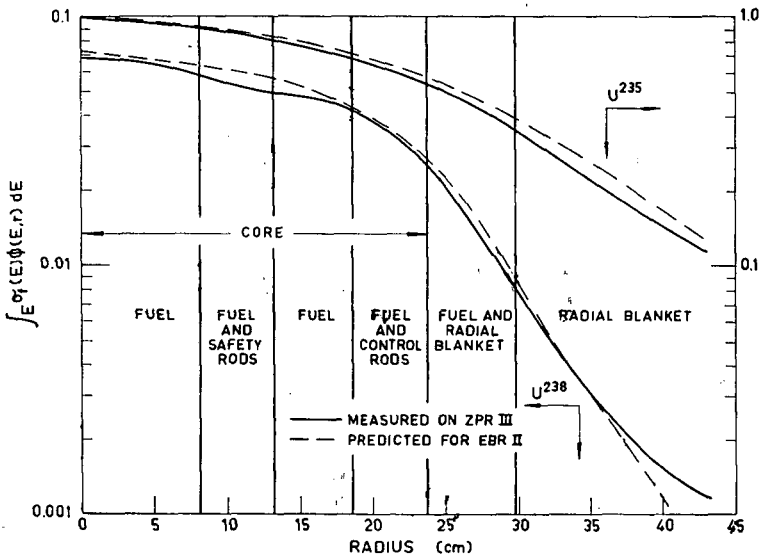


Fig. 10
Radial fission distributions at core mid-plane.

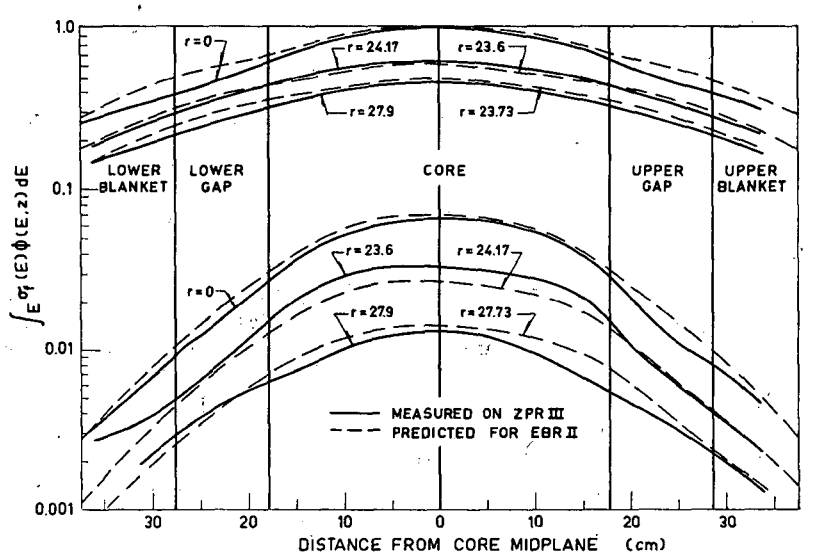


Fig. 11
Axial fission distributions at three radial positions.

Other experiments

General experiments dealing with inhomogeneity, neutron spectrum and lifetime, and material-replacement effects were included in the programme. These will not be described in the present paper. The EBR-II mock-up experiments cited in the previous sections are most significant for the reference en-

gineering design. Additional investigations will be described here. These were designed to give insight into details of providing experimental apparatus for reactor operation.

The control mechanisms were studied in detail. Six of the control rods are located on the corners and six are located on the "flats" of the hexagonal core (Fig. 5). The former are thus at a slightly larger distance from the reactor centre-line and hold about 81% of the reactivity of the latter. (The previously cited control-rod reactivity is an average.) Subsequent experiments showed that a control rod located on the corner of the hexagonal core requires the equivalent of 75 fuel elements (normal loading is 61 fuel elements) for constant reactivity worth of all 12 control rods.

The reactivity worth of a single control rod may be increased by inserting a highly absorbent material in the sodium void follower region of the control sub-assembly. This region is adjacent to the fuel in the fuel sub-assemblies when the control rod is in its least reactive (down) position. Distributing about 1.8 kg Ta in the void follower enhances the total worth of the control rod by less than 0.05% $\Delta k/k$. Distributing about 340 g $B_4^{10}C$ ($\sim$91 at.% B^{10} in B) in the void follower was found to double the total reactivity worth of the normal control rod. If adopted, the reactivity worth of a single control rod could exceed one dollar ($\Delta k/k \approx 0.73\%$). Special control rods utilizing about 250 g $B_4^{10}C$ are being designed as part of the EBR-II experimental programme.

A rotating oscillator rod was calibrated. The mechanism is designed to replace a control sub-assembly. Any rotary motion is confined within a hexagonal sub-assembly (2.3 in across flats). An eccentrically located $B_4^{10}C$ rod can rotate on a circle of about 2 in diameter. A 14-in-long rod containing 230 g $B_4^{10}C$ was found to have a reactivity amplitude of 0.034% $\Delta k/k$. This is about the maximum amplitude that can be obtained with such a rod within the framework of the EBR-II engineering design. The amplitude may be lowered by (*1*) reducing the $B_4^{10}C$ content or (*2*) fixing the oscillator in a less important position with respect to the core. Both of these conditions were experimentally investigated. Mechanical design considerations suggest that a vertically moving (up and down) oscillator mechanism may be preferable. Such a mechanism will be experimentally calibrated during the dry critical experiments in EBR-II.

Analysis of the engineering design

Fissionable and fertile materials

All of the fertile material outside of the reactor core is unalloyed depleted uranium ($\sim$0.2% U^{235}). A fabricated density of not less than 18.7 g/cm^3 was specified. The metallurgical and irradiation characteristics of this material have been studied extensively [10].

The fuel for the first loading is uranium-5 wt. % fissium alloy (Table VI) in the form of injection cast-pins weighing slightly more than 67 g each. About 100 small specimens and 20 full-size EBR-II fuel pins have been irradiated in the CP-5 and MTR reactors [11]. These irradiations, under a variety of temperatures to differing degrees of burn-up, have been encouraging. The fuel alloy may withstand from 1—2 at. % burn-up* at a maximum central metal temperature

* When 1% of the fuel alloy atoms are destroyed by fission, 1 at.% burn-up has been achieved. This represents the destruction of almost 2 at.% of the U^{235} contained in the fuel alloy.

of 650°C (1200°F). The dimensional stability of the alloy tends to improve as either fissium or zirconium content is increased (at the expense of uranium). It is known that zirconium content in the EBR-II fuel tends to increase with repeated fuel-alloy recycling. Hence the ultimate potential of this fuel-alloy is not exhibited by the first reactor-core loading.

TABLE VI

COMPOSITION OF URANIUM — 5 wt.% FISSIUM ALLOY

Element	Wt. %
Uranium	95.00
Zirconium	0.06
Niobium	0.01
Molybdenum	2.48
Ruthenium	1.97
Rhodium	0.29
Palladium	0.19

The phase diagram of uranium-fissium is very much like that of the U-Mo-Ru ternary system [12]. The phase transformation of interest under normal reactor operating conditions is that from the predominantly α phase to the predominantly γ phase. This transformation [13] occurs at about 550°C.

A steady-state heat-transfer analysis of the reactor at full power and full coolant flow develops the temperature distribution in each fuel element. At and near the core central plane, all of the fuel alloy is at temperatures higher than 550°C. Near the axial extremities of the fuel pins, the fuel-alloy temperature is generally less than or very near 550°C. There is an isothermal surface of revolution, tapered at each end, within the cylindrical fuel pin. Outside the surface all temperatures are less than 550°C; inside the surface fuel alloy temperatures exceed 550°C. Under steady-state conditions the unirradiated fuel will exhibit a mixed metallurgical phase structure. Whether this complex geometric phase structure of an unirradiated or low-burn-up pin causes geometric distortion remains to be observed. As the burn-up of the fuel proceeds, the low-temperature α-phase material may transform to the less dense γ-phase material even if irradiation temperatures are below 550°C.

The transformation kinetics of the 5 wt.% fission alloy are relatively slow. A complete transformation from α to γ phase would result in a fuel-alloy-density decrease of about 1.4%. The rate at which this transformation takes place depends on the rate at which the temperature rises and/or the stable temperature at which the transformation is carried to completion. Thermal-expansion measurements [12] suggest that with a temperature rise of 1°C/min, the total transformation, under various conditions, requires between 60 and 100 min. Hardness tests suggest that complete transformation can be effected in about 15 min at temperatures near the phase-transformation temperature. The transformation at bulk sodium temperature (~370°C) will require about 5 h.

The metallurgical phase phenomena may be the source of a reactivity drift. Whether such a drift occurs with unirradiated fuel only remains to be observed. If the expansion from α to γ phase is isotropic the 1.4% density change will cause a reactivity loss of 0.13% $\Delta k/k$.

U^{235} Enrichment of the EBR-II fuel alloy

The detailed engineering mock-up of EBR-II on ZPR-III was not precise enough to yield the required U^{235} enrichment of the fuel alloy directly. The critical-assembly results were interpreted on the basis of both perturbation measurements and reactor analysis.

The first critical core loading is designed to require 67 fuel sub-assemblies including control and safety rods. The firm requirement that the reactor exhibit criticality with a minimum size equal to the reference core (61 sub-assemblies) makes it necessary to provide six sub-assemblies ($<2\%\ \Delta k/k$) for "contingencies".

Significant differences between the critical experiment and the EBR-II include:

1. The mock-up core height is 14 in as opposed to 14.22 in for the reactor;
2. The mock-up exhibited the radial properties of a 64-fuel sub-assembly core.

A restricted set of conditions was the basis for fixing the fuel alloy atomic enrichment at 48.4%. These include:

1. The fuel alloy is the 5 wt.% fission alloy;
2. The as-cast density of the fuel alloy is 17.95 g/cm^3 (the density at which the fuel pins are cut to the proper length);
3. The density of the blanket uranium is not less than 18.7 g/cm^3;
4. The blanket uranium is depleted to about 0.22%;
5. The reference core will contain 67 sub-assemblies (53 fuel, 12 control and 2 safety);
6. The control rods are the 61-fuel-element control sub-assemblies;
7. With minor modifications, the configuration and compositions are as given in Table I and Ref. [3];
8. At full power (62.5 MW) and full coolant flow (8900 gpm) about \$ 1 of excess reactivity is available from control rod motion.

The enrichment (U^{235} in U) was necessarily specified before significant quantities of fuel elements were fabricated. Prototype studies indicated that fuel-alloy density would be ~17.95 g/cm^3. Production of the reactor fuel elements indicated slightly lower densities would be realized.

The enrichment of the fuel alloy was specified on the basis of two not entirely independent approaches. The first was to apply as much of the experimental mock-up data as possible with very little recourse to analytical results. The second was based principally on analytical results using experimental information only where analytical results were believed unreliable. The fuel-alloy enrichments obtained by the two methods are in substantial agreement and well within the uncertainty of the determination.

Table VII is a summary of the analytical results. The basic calculations are two-dimensional multi-group diffusion theory using the NICK-II [14] and CUREM [15] programmes with multi-group constants from Ref. [4]. Calculation No. 1 represents the reference 61-sub-assembly core while problems 2 and 3 are the 67-sub-assembly core. Control and safety sub-assemblies are homogenized with radially adjacent fuel sub-assemblies. The six core sub-assemblies in the inner blanket are homogenized with 24 depleted uranium-bearing sub-assemblies in problems 2 and 3. It is recognized that the latter calculations would tend to under-estimate the reactivity worth of the six peripheral fuel sub-assemblies. Corrections to the calculated reactivity are based on known diffusion-theory errors when compared with the more accurate S_N method [4, 6] as well as the known integral uncertainties in the multi-group constants [4]. The fissium correc-

TABLE VII

ENRICHMENT OF EBR-II FUEL ALLOY
(Several two-dimensional analyses)

Problem number:	1	2	3
Number of core, control and safety sub-assemblies:	61	67	67
Computing programme:	NICK II	NICK II	CUREM
Fuel-enrichment (U^{235} in U) for calculation, at. %:	49.5	48.4	48.4
Calculated K_{eff}:	0.9917	0.9992	0.9928
Corrections:			
Comparison between diffusion theory and S_4, δk:	+0.033	+0.033	+0.033
Basic errors in multi-group constants require, δk:	—0.015	—0.015	—0.015
Fissium correction, δk:	—0.0031	—0.0031	—0.0031
Corrected K_{eff}:	1.0066	1.0141	1.0077
Add six sub-assemblies to obtain 67-sub-assembly core, δk:	+0.0217	—	—
Provide:			
One dollar excess reactivity at full power, δk:	—0.0074	—0.0073	—0.0073
Power coefficient override (370°C→full power), δk:	—0.002	—0.002	—0.002
Temperature coefficient override (room temperature→370°C) δk:	—0.0067	—0.0067	—0.0067
Corrected K_{eff} (full power, full flow with one dollar excess reactivity):	1.0122	0.9981	0.9917
ΔK_{ex}	+0.0122	—0.0019	—0.0083
Correction* to initial fuel-alloy enrichment for just-critical reactor:	—2.68%	+0.0042%	+1.82%
Final fuel-alloy enrichment (U^{235} in U), at. %	48.2	48.6	49.2

* $\delta k/k \sim 0.455$ [(δ U^{235}/U)/(U^{235}/U)].

tion was obtained from the ZPR-III data for all constituents except molybdenum. Temperature- and power-coefficient calculations will be described in a subsequent section. The basic calculations predict different core alloy enrichments. Part of the cause may be ascertained by comparing problems 1 and 2. The difference is attributed to the homogenizing of six fuel and 24 reflector sub-assemblies around the 61-sub-assembly core boundary. This configuration is calculated to be less reactive than the more heterogeneous experimental mock-up. The

TABLE VIII

ENRICHMENT OF EBR-II FUEL ALLOY (EXPERIMENTAL DATA ANALYSIS)

Experimental core size: ~64 sub-assemblies
Experimental critical mass: 165 kg U^{235}
Experimental enrichment (U^{235} in U): 46.79%

Region	Corrections to composition and size of principal reactor regions		
	Designation (Fig. 9)	Most probable reactivity change (inhours)	Possible errors (inhours)*
Core:	C1, C2	—164	± 60
Upper gap:	G1, G2	+ 13	± 50
	G3	— 14	± 5
Lower gap:	G4, G5	— 58	± 20
	G6	— 28	± 20
Radial blanket	B2, B3, B4		
Total uranium		— 75	± 35
Structure and coolant:			
Inner		+ 88	± 40
Outer		— 32	
Axial blanket	B1		
Total uranium		+ 10	± 10
Structure and coolant:			
First 4 in		+105	± 10
Remaining 14 in		+ 4	—
Axial control and safety rods:	S1, S2, S3, S4	+ 28	± 10
Inhomogeneity effect in ZPR-III:		—332	±150
Add three sub-assemblies to obtain a 67-sub-assembly core:		+450	± 50
Temperature coefficient (room→370°C, does not include sodium**):		—278	± 50
Power coefficient (370°C→full power):		— 83	± 80
Provide one dollar excess reactivity at full power and full flow:		—307	± 10
TOTAL		—673	± ~500
Required reactivity:		+1.62% $\Delta k/k$	± ~1.2% $\Delta k/k$
Change in fuel-alloy enrichment (U^{235} in U) to provide 673 in-hours (1.62% $\Delta k/k$):		3.35%	± ~2.5%
Reactor core-alloy enrichment:		48.4 at.%	±0.012 at.%

* 415 in-hours = 0.01 $\Delta k/k$ = 1.0% $\Delta k/k$.
** High-temperature (370°C) sodium is included in the static configuration data analysis.

difference between calculations 2 and 3 is attributed to the convergence characteristics of the two computing programmes. It is not clear which of the two calculations is insufficiently converged. The predicted results are bracketed within 1.0% $\Delta k/k$. Fewer than three fuel sub-assemblies in the first row of the inner blanket represent this uncertainty.

The experimental analysis of the EBR-II mock-up on ZPR-III is summarized in Table VIII. The critical core alloy enrichment for a 67-sub-assembly core is 48.4 at.% U^{235} in U. The corrections are obtained by comparing the amounts of structural "coolant" and fuel material per unit volume in the mock-up with

those in the reactor having the dimensions of the mock-up (see Table I and Fig. 9). The distributed-material-replacement experiments (Tables III and IV) are then used to effect a transition from the aluminium-"cooled" ZPR-III to the sodium-cooled reactor.

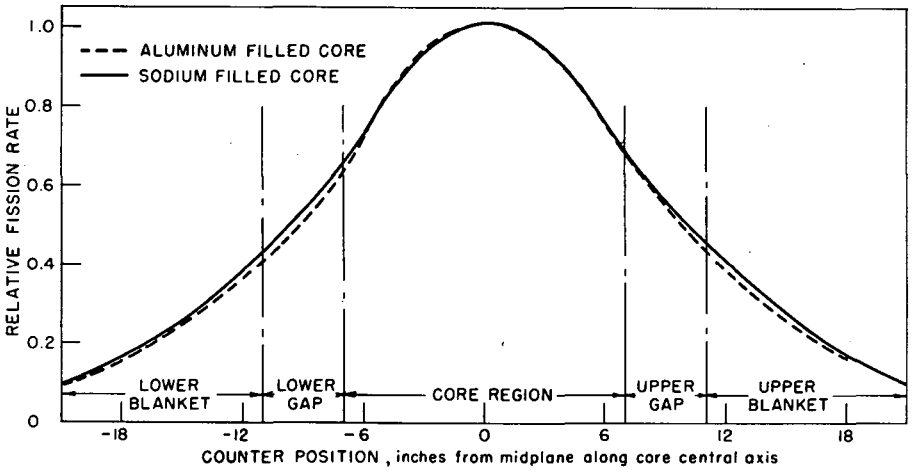


Fig. 12
Enriched-uranium fission distributions in aluminium- and sodium-"cooled" cores.

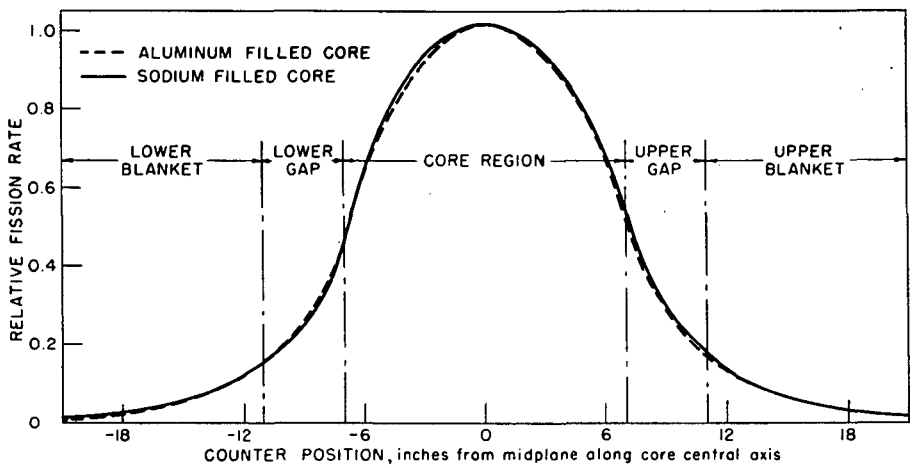


Fig. 13
Natural-uranium fission distributions in aluminium- and sodium-"cooled" cores.

It is recognized that the "linearity" principle used to correct the experimental fuel-alloy enrichment may not be completely valid. When the mock-ups are aluminium- and sodium-"cooled" the measured central worths of sodium are 0.06 and 0.096% $\Delta k/k$ per kilogram respectively. However, macroscopic material-replacement experiments, by substituting sodium for aluminium throughout the EBR-II mock-up core, are relatively insensitive to the amount

of material substituted. This same effect has been investigated analytically by removing different amounts of sodium from the reactor core. It is true that the first increment of sodium introduced into a "dry" reactor has a slightly greater reactivity effect than the average sodium worth. The difference between maximum and average sodium worth does not exceed 10% for the distributed replacements.

The neutron spectrum in an aluminium-"cooled" core is not the same as in the reference sodium-cooled reactor. Natural- and enriched-uranium fission distributions were measured throughout the core and reflector of both of these cores. The results, shown in Figs. 12 and 13, show that the spectral difference is small and probably within experimental error.

A small uncertainty is caused by the presence of U^{234} and U^{236} in the fuel alloy. Small amounts of these materials were also present in the ZPR-III fissionable materials. The EBR-II fuel alloy is "blended" by combining highly enriched (~93%) with slightly enriched (~1.5%) uranium. Table IX compares the U^{234} and U^{236} content of ZPR-III and EBR-II fuel.

TABLE IX

U^{234} AND U^{236} CONTENT OF FISSILE AND FERTILE MATERIAL

	Uranium			
	High enrichment		Low enrichment	
	EBR-II	ZPR-III	EBR-II	ZPR-III
U^{234} content (wt. %)	1.01—1.09	0.94—1.06	0.0076	0.00103
U^{236} content (wt. %)	0.109—0.163	0.402—0.511	0.021	0.0023

The effect of these "impurities" has been overestimated by assuming that an atom of U^{236} behaves like an atom of U^{234}. The removal of 1 at. % U^{238} gives a reactivity loss of 0.034% $\Delta k/k$. Addition of 1 at. % U^{234} is estimated to add 0.22% $\Delta k/k$ to the system. This perturbation calculation assumes that U^{234} and U^{238} differ only in the fission cross-section. The reactor multi-group calculations, which did not include U^{234} and U^{236}, are then subject to a maximum uncertainty of 0.19% $\Delta k/k$. This represents less than one fuel sub-assembly in the inner blanket (Row 6, Fig. 5b).

With the 48.4% fuel-alloy enrichment, 176 kg U^{235} and 190 kg U^{238} are required for the 67-element core. The available excess reactivity of \$1 suggests then that the hot clean critical mass at full power is 174 kg U^{235}.

The wet critical experiments will be conducted with a uniform sodium temperature of 260°C. If source strength effects are neglected, the clean system should have 1.32% $\Delta k/k$ excess reactivity with the 67-sub-assembly core.

The dry critical experiments will be conducted at room temperature with no coolant sodium in the reactor. About 50% of the core volume is filled with air. The reflecting regions contain similar or lesser air void volumes. The expected dry critical configuration will be compared with that actually observed. The degree of agreement between prediction and measurement will define, to some extent, the validity of the above predictions. The dry system is not expected to be critical on

less than an 81-sub-assembly core. The more likely requirement is between 85 and 90 fuel sub-assemblies, including control and safety rods*.

The major uncertainty in specifying the dry critical configuration is the total reactivity worth of the coolant sodium. The macroscopic sodium material-replacement experiments tend to suggest a lower total sodium reactivity worth than the purely analytical results. Measured sodium worth in the core is between 0.11 and 0.12% $\Delta k/k$ kg while calculations (with multi-group constants from Ref. [4]) tend to 0.14% $\Delta k/k$ kg. Similar discrepancies occur for the reflector regions.

Enrichment of EBR-II fuel alloy—Analysis after fabrication

The enrichment (U^{235} content in U) of the EBR-II fuel alloy is specified at 48.4 at. % (48.08 wt. %). This requirement is based on both experiment and analysis. The nuclear and thermal performance of the system depends on how well the fabricated properties of the fuel alloy meet this specification.

Fifty-seven process batches each containing about 10 kg of the 5 wt. % fissium alloy have been prepared and analysed. In each case calculated alloy enrichments, based on constituents to the 10-kg batch, are compared with sample analyses from the final process batch. The latter samples are mass-spectrometrically analysed to within 1%.

Of the 57 batches, seven were found to be less than 48.0 wt. % enriched while five were determined to be more than 48.2 wt. % enriched. Three of the five registering high enrichment are 48.3 wt. % enriched; the remaining two 10-kg batches are slightly less than 49 wt. % enriched. The seven registering less than 48.0 wt. % enrichment were all found to be 47.9 wt. % enriched.

If one were to assume the extreme case, that all of the fuel were 49-wt. % enriched, the reactor would exhibit 0.84% $\Delta k/k$ more reactivity than predicted. This represents between two and three fuel sub-assemblies at the core boundary.

The localized variation in enrichment will present itself principally as localized perturbations of the power. The magnitude of this localized power "peak" will be less (about 90%) than the percentage increase of enrichment. A 2.0% increase in enrichment throughout a single sub-assembly will give a 1.8% increase in the power generated by that sub-assembly. This sort of uncertainty, as suggested by the two fuel-alloy batches indicating close to 49 wt. % enrichment, is well within the "uncertainty factors" of the thermal analysis for EBR-II [3].

Reactor control

The physics of reactor control requires insight on the following questions:

1. What is the total reactivity worth of a single control or safety sub-assembly?
2. Are there any serious interaction (shadowing) effects?
3. What is the rate of inserting reactivity by control- or safety-rod motion?
4. What can be done to increase the reactivity worth of a single control rod?
5. What are the reactor conditions against which the control mechanisms can afford protection?

Both experiment and theory confirm that there is enough reactivity in the mechanisms for reactor control and shut-down. The conditions which cannot be protected against require simultaneous gross mechanical failure as well as loss of administrative control. These situations arise due to finite time requirements on

* The dry critical reactor initially diverged on 30 September 1961; 86 fuel sub-assemblies were loaded.

fast shut-down (scram). This problem cannot be remedied by enhancing the total reactivity held by the control rods.

The first control-rod-worth calculations [3] used one-dimensional cylindrical diffusion theory. The multi-group constants were from Table 13 of Ref. [5]. The two-group constants (Table 12, Ref. [5]) were also used. The analysis is straightforward. Fuel and void follower sections of the control sub-assemblies are homogenized with core sections of fuel sub-assemblies. Homogenization is accomplished by cylindricizing the hexagonal reactor. Equivalent radii of the sub-assembly row structure is obtained by equating cylindrical and hexagonal areas.

All of the calculations yielded between 0.5 and 0.6% $\Delta k/k$ per control rod. It was recognized that this calculation was not accurate because of three distinct approximations:

1. The cylindrical calculation tends to remove variation in control-rod worth. Six control rods are located on the "corners" and six control rods are located on the "flats" of the hexagonal core (Fig. 5).

2. What effect does the overall homogenization have on the calculated control-rod worth?

3. An axial buckling must be introduced into the cylindrical calculation. This provides for an equivalent core height of about 60 cm; actual core height is about 36 cm. When removing fuel for a control-rod-worth calculation, 60% more fuel is removed than in the actual reactor. This is believed to be the largest error introduced by these relatively simple control-rod evaluations.

The first uncertainty was resolved experimentally. It was found that the control rods on the corner of the hexagonal core hold about 81% of the reactivity of those on the more favourably placed "flats".

The effect of homogenization was analysed by performing two-dimensional, two-group calculations in cylindrical (r, θ) geometry. The radial asymmetries can be treated in detail. However, the axial-buckling problem is the same as in the simpler one-dimensional calculations. The r, θ calculation yielded about the same control-rod worth as the one-dimensional cylindrical analysis. The effect of homogenization is found to be small.

The problem of properly treating the axial motion of the actual control rod can be studied using two-dimensional calculations in r, z geometry. These calculations do give fair agreement with experiment. Table X summarizes a variety of control-rod calculations.

TABLE X

EBR-II CONTROL-ROD ANALYSIS*

Geometry		Multi-group constants		Computing programme		Calculated control-rod worth (% $\Delta k/k$)
No. of dimensions	Type	No. of groups	Reference	Name	Reference	
1	Cylindrical	2	[5]	RE-6	[16]	0.59
2	r, θ	2	[5]	MUG-II	[17]	0.58
2	r, z	2	[5]	MUG-II	[17]	0.46
1	Cylindrical	11	[4]	RE-6	[16]	0.52
1	Cylindrical	10	[5]**	RE-6	[16]	0.65
2	r, z	11	[4]	NICK-II	[14]	0.36

* Measured reactivity = 0.37 % $\Delta k/k$.

** Table 14 of Ref. [5].

Perturbation theory may be applied to the results of the one-dimensional cylindrical calculations. One may assume that the axial flux distribution can be given by $\cos k_z z$ for all groups; k_z is the axial buckling. By restricting the perturbation to the actual core height by means of a chopped cosine, this approach yields results similar to the r, z calculations. Table XI summarizes the correction factors, based on perturbation theory, which may be applied to one-dimensional cylindrical criticality calculations for EBR-II.

Table XI becomes very useful for semi-quantitative predictions without the time and expense of two-dimensional analyses. For instance, one-dimensional control-rod calculations yield $\sim 0.55\%\ \Delta k/k$ rod. Table XI suggests that a more realistic value would be (0.83) (0.55) $= 0.45\%\ \Delta k/k$ rod. While high, this is closer to the measured $0.37\%\ \Delta k/k$ rod. Considering the number of simplifying assumptions, including that of specifying a single k_z for all groups, the approach does improve the basic one-dimensional criticality calculation. A two-dimensional calculation and/or an experiment are required for precision reactivity determination.

TABLE XI

CORRECTIONS* TO ONE-DIMENSIONAL-SUB-ASSEMBLY REPLACEMENT CALCULATIONS (EBR-II)

Reactor location	Reactor row (Fig. 5b)	Composition**		Multiplication factor***
		Initial	Final	
Centre	1	Fuel	Na	0.72
Centre	1	Control rod	Na	0.76
Centre	1	Control rod	Blanket	1.24
Control rod	5	Fuel	Na	0.81
Control rod	5	Control rod	Na	0.83
Control rod	5	Control rod	Blanket	1.26
Control rod	5	Fuel	Blanket	1.07
Inner blanket	6	Fuel	Na	0.90
Inner blanket	6	Fuel	Blanket	1.06

* Based on perturbation theory using six energy groups.
** Fuel = composition of core section of fuel sub-assembly;
Control rod = fuel section of control rod;
Na = sodium or sodium-void-follower region of control rod;
Blanket = composition of blanket sub-assembly.
*** Results of one-dimensional cylindrical calculations are to be multiplied by this to obtain more realistic results.

Table XI may also be used to correct one-dimensional calculations of fuel sub-assembly removal. The one-dimensional calculated reactivity worth of the central core sub-assembly is $\sim 2.0\%\ \Delta k/k$. The correction factor (0.72) suggests that a more accurate value is $1.43\%\ \Delta k/k$. This replacement is measured to be $1.53\%\ \Delta k/k$.

The mechanical rates at which the control, safety and fuel sub-assemblies are inserted into the reactor are of considerable interest. These rates determine the rate of reactivity addition under both critical and sub-critical conditions. Before bringing the reactor critical, both control and safety sub-assemblies are in their least reactive positions. During fuel handling the safety rods are in the most

reactive position, but are "scrammed" before the critical approach. On the one hand, it is desirable that the time of approach to power does not exceed one hour. On the other hand, it is desirable that the reactor operator has at least one minute to act upon a malfunction.

Only the control rods can be actuated (other than for scram) at power. With a drive speed of about 5 in/min, all 12 rods can be raised individually in about 33 min. If the control rods are postulated to continue inserting reactivity once the reactor has reached delayed-critical it will require about 150 s to approach prompt-critical. If a single control rod holds about twice as much reactivity as the reference control rod, about 70 s is required to go from delayed- to prompt-critical.

The reactor system is instrumented with limit switches and power-level trips to prevent such inadvertent ways of adding reactivity. The system is limited mechanically to the motion of a single control rod at any one time.

TABLE XII

NORMAL RATES OF REACTIVITY INSERTION

	Control sub-assembly		Two safety sub-assemblies		Central core sub-assembly (sub-assembly loading mechanism)	
	Predicted *	Measured**	Predicted *	Measured**	Predicted *	Measured**
Total reactivity worth	0.5	0.37	~2.0	1.36	~2.0	1.5
Drive speed (in/min)	5	—	2	—	6	—
Effective stroke (in)	14	—	14	—	~14***	—
Rate of reactivity-addition % $(\Delta k/k)$/s						
Average	0.003	0.0022	0.0050	0.0032	0.015	0.011
Maximum	0.005	0.0038	0.0086	0.0055	0.025	0.018

* Reference [3], original one-dimensional calculations, before refinements. These were used to set mechanical drive speeds.

** On ZPR-III.

*** Important range.

Table XII summarizes the total reactivity worths and reactivity insertion rates for control, safety and fuel-handling systems. The latter is on the basis of maximum reactivity held by the central core sub-assembly. During fuel handling, the maximum reactivity insertion rate is about $1.8 \times 10^{-2}\%\ \Delta k/k$ s. The reactor is at least $3\%\ \Delta k/k$ sub-critical during fuel-handling procedures because the control rods are disengaged (Fig. 6) and in their least reactive positions. If administrative control should be lost during fuel handling and the reactor is near delayed-critical as the central fuel sub-assembly is inserted into the reactor core, about 40 s are required to go from delayed- to prompt-critical. The reactor is instrumented to scram the safety rods on both low power level and short period. If the fuel-handling safety circuits do not function, 40 s is available for either the reactor operator in the reactor control room or the operator of the fuel-handling system in the reactor plant to scram the reactor manually.

Certain experimental procedures such as stability studies will require that only 11 control rods are available for shut-down and power regulation. An oscillator rod will occupy the twelfth control-rod location. Furthermore, a future EBR-II loading may operate with a larger core. Investigations have proceeded to determine how the reactivity worth of existing control mechanisms may be enhanced. Three such approaches have been studied in detail:

1. Increase enrichment of the fuel alloy in the control sub-assembly fuel elements.
2. Increase the number of fuel elements in the control sub-assembly from 61 to 75. This provides for non-symmetric placement of fuel elements within the control sub-assembly.
3. Place poison material ($B_4^{10}C$) in the sodium void follower regions of the control sub-assembly.

Both experiment and analysis of the control sub-assembly show that the total reactivity worth is almost a linear function of the U^{235} content for a given core size. The reactivity worth is inversely proportional to core size. However, as core size increases the control sub-assemblies are located at positions of greater importance, which has some bearing on the total reactivity worth. Reactor physics can barely distinguish between 1. and 2. for a given U^{235} content in the control sub-assembly. The first approach requires the use of fuel elements having two different enrichments of U^{235} in the remote fabrication facility; the higher are for control sub-assemblies, while the larger number of lower enrichment are for fuel sub-assemblies. The possibility of inadvertent interchange could cause serious reactivity as well as heat-removal problems. A highly enriched central core sub-assembly would overheat and probably fail. In addition this sub-assembly could hold more than 2.5% $\Delta k/k$ total reactivity. The physics of this type of control are straightforward but the management and engineering problems dictated that it should be relegated to the future.

The provision of more fuel elements per control sub-assembly obviates the chance of inadvertent interchange cited above. However, non-symmetric internals of control sub-assembly fabrication as well as heat-removal problems made this approach not very desirable.

The final approach shows the most promise for the initial reactor loading. Experimentally it has been demonstrated that a single control-rod worth can be raised from 0.37% $\Delta k/k$ to ~0.7% k/k by inserting ~350 g $B_4^{10}C$ in the sodium void follower region of the control sub-assembly. (The reactivity worth may be slightly enhanced by using Ta.) The main problems of using $B_4^{10}C$ are also those of engineering design. Briefly they are high temperature or heat removal and carburization of clad material. The primary physics problem is long-term exposure of $B_4^{10}C$ in high neutron flux. It is also recognized that neutrons absorbed by $B_4^{10}C$ are not available for breeding. EBR-II will be operated with all control rods the equivalent of about 80% inserted. With a 14-in control-rod stroke, it was decided to place the poison material in a region about 7 in long. Thus the effective 28 in of "strong" control rod are fuel (14 in), sodium and steel void (7 in), and $B_4^{10}C$ (7 in). Poison material would not begin to enter the axial core extremity until half of the fuel section of the control rod has been moved below the core. Such control sub-assemblies are being provided for start-up experiments. They can be used to provide a relatively constant shut-down reactivity for all experiments.

Power distribution and calibration

The reactor design requires high thermal performance with fairly uniform coolant outlet temperature. This places severe requirements on knowing the neutron flux and power distribution throughout the reactor system. The comparison between theory and experiment has been demonstrated in Figs. 9 and 10 and in Ref. [5]. However, the power distribution for heat removal is slightly different from the fission distribution. A distinction must be made between the location where energy is created and where it is absorbed. A small fraction of the energy, principally gamma-rays, is absorbed some distance from the point of creation. This consideration poses some heat-generation problems in reflector regions where fission densities are low, located adjacent to core regions where fission densities are high.

The power distribution for reactor analysis is obtained from the 11 energy group CUREM [15] calculation (Figs. 9 and 10). Fig. 14 is a simplified sketch of the analysis in r, z geometry. Table XIII extends the description and gives the power produced in various regions of the reactor. This analysis assumes that all 12 control rods are fully inserted.

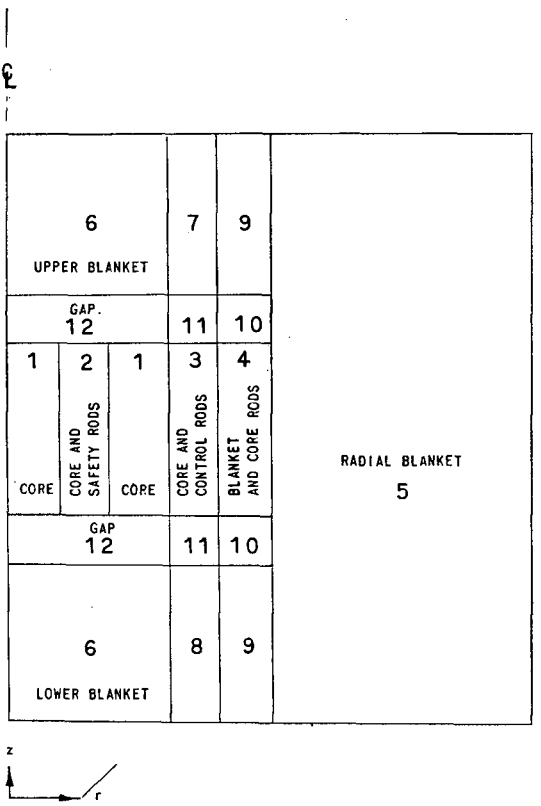


Fig. 14
Reactor model for two-dimensional multi-group analysis.

TABLE XIII

POWER PRODUCED IN VARIOUS REGIONS OF A 67-SUB-ASSEMBLY EBR-II CORE

(Total power = 62.5 MW)

Hexagonal sub-assembly Row No. (Fig. 5b)	Region of Fig. 14		Fraction of total power (%)		Total power (MW)		Upper and lower blanket bonus (%)*	Average total power per subassembly (MW)
1,2,4	1	Fuel sub-assemblies		39.11		24.44	1.65	0.9941
3	2	Fuel sub-assemblies	16.63		10.39		1.65	1.0561
		Safety sub-assemblies	2.23		1.39		—	0.695
				18.86		11.78		
5	3	Fuel sub-assemblies	14.68		9.18		1.22	0.774
		Control sub-assemblies	9.84		6.15		—	0.513
				24.52		15.33		
6	4	Fuel sub-assemblies	5.70		3.56		2.54	0.608
		Blanket sub-assemblies	3.21		2.01		36.25	0.114
				8.91		5.57		
7 ff	5	Radial blanket		6.32		3.95		
1,2,3,4	6	Upper and lower blanket		0.792				
5	7	Upper blanket		0.0887				
5	8	Lower blanket		0.0907				
6	9	Upper and lower blanket of 6 fuel sub-assemblies and upper and lower portions of blanket sub-assemblies		0.5071				
6	10	Portion of blanket sub-assemblies coinciding with structural gap of fuel sub-assembly		0.7996				

* From Regions 6, 7, 8, 9, 10.

Table XIV demonstrates the wide variation in power density throughout the reactor. The corrections for gamma-heating in the inner and first row of the outer blanket are given. The additional power produced due to 0.5 at.% plutonium production in reflector material is also given.

The cycling of blanket sub-assemblies cannot be specified until reactor experience has been obtained. Thus there is a certain arbitrariness in specifying the plutonium concentration and consequent contribution to reflector power density. For orientation, the first question to be answered is: When does this problem become important? Table XV gives the time, in years, required to achieve both 0.5 at.% burn-up of uranium and 1.0 at.% build-up of plutonium at the core mid-plane as a function of radius. Time required is based on full

TABLE XIV

POWER DENSITIES IN CORE CENTRAL PLANE ($\sim z = 0$)
67-sub-assembly core—Full power

Sub-assembly Row No.	Region	Maximum power density* (MW/l)					
		Fuel sub-assembly	Control or safety sub-assembly	Blanket sub-assembly			
				No γ-heat	With γ-heat	With 0.5 at. % Pu** (no γ)	With 0.5 at. % Pu** (with γ)
1	Core	1.23	—	—	—	—	—
2	Core	1.20	—	—	—	—	—
3	Core	1.11	0.740	—	—	—	—
4	Core	0.967	—	—	—	—	—
5	Core	0.803	0.538	—	—	—	—
6	Inner blanket	0.626	—	0.0832	0.120	0.0972	0.134
7	Inner blanket	—	—	0.0391	0.0637	0.0516	0.0762
8	Inner blanket	—	—	0.0157	0.0279	0.0234	0.0356
9	Outer blanket	—	—	0.0045	0.0050	—	—
10	Outer blanket	—	—	0.0034	—	—	—

* Averaged over radial area of sub-assembly.
** It is assumed that Pu power in blanket is greater than 62.5 MW (actual reactor power > 62.5 MW).

TABLE XV

PERTURBATION OF CLEAN REACTOR POWER IN REFLECTOR MATERIAL
by presence of bred plutonium

Region	r (cm)*	TPu** (yr)	TU*** (yr)	$\frac{P\,(1\%\,\mathrm{Pu})}{P\,(0)}$	P (0) (kW/l)
Inner blanket	22.4	0.85	0.93	1.22	222
Inner blanket	24.2	0.90	1.14	1.25	162
Inner blanket	26.0	0.98	1.56	1.31	111
Inner blanket	27.7	1.07	2.14	1.37	82
Inner blanket	29.5	1.18	2.8	1.43	60
Inner blanket	32.8	1.47	5.7	1.65	31
Outer blanket	36.0	1.87	11.0	1.91	18
Outer blanket	39.2	2.43	20.0	2.22	11
Outer blanket	49.0	5.76	94.0	3.30	2.7
Outer blanket	58.7	14.5	330.0	4.0	0.89
Outer blanket	68.4	39.0	—	4.35	0.29
Outer blanket	78.1	100.0	—	4.74	0.10

* Distance from centre of core in reactor mid-plane.
** Time required to achieve 1 at. % Pu build-up at core mid-plane.
*** Time required to achieve 0.5 at. % burn-up at core mid-plane.

power operation, 67 sub-assembly core and an assumed 100% plant factor. The ratio of the total fission power when 1 at.% plutonium is present P (1% Pu) to that when no plutonium is present (clean reactor, P (0)) is also given. Approximate local fission power densities are given for reference.

Some details of the power distribution are given in Refs. [2] and [3].

An absolute power calibration is required to (1) provide knowledge of power level before the heat-removal system becomes operative and (2) permit realistic power-level trip-settings before plant operation. This power calibration is performed relative to an absolute fission-counter [9] response. The counter containing 0.8 mg uranium (93% enriched) is located in the reactor mid-plane at the radial core boundary. This calibration has been obtained on the basis of integrating measured fission distributions from ZPR-III [7]. It has also been obtained from the multi-group analysis. There is a 15 to 20% difference between the experimental and analytical power calibration.

This agreement between theory and experiment is adequate for reactor start-up until the heat-removal system is operative. To a high degree of approximation, the 0.8 mg enriched uranium fission-counter will record:

$$\text{Count rate at core boundary (counts/s)} = 10^4 \frac{P}{M_c}$$

where

P = reactor power in W

M_c = critical mass in kg U^{235}.

These results are then related to the normal flux-monitoring instruments located in the J and O thimbles (Fig. 7). The data in Table V were normalized in this way. Experimentally this was accomplished by simultaneous count-rate measurements by two different counters. The absolute counter is in the reactor core while the normal reactor instrumentation is in a ZPR-III mock-up of the instrument thimble.

Neutron flux distribution and instrument response

The experiments designed to determine the attenuation of the neutron flux from the reactor core through the radial reflector and into the neutron shield are summarized in Table V. The experimental arrangement is shown in Fig. 15. On that figure, "W", "C", "J", "#5", "4" and "PC" refer to fission and pro-

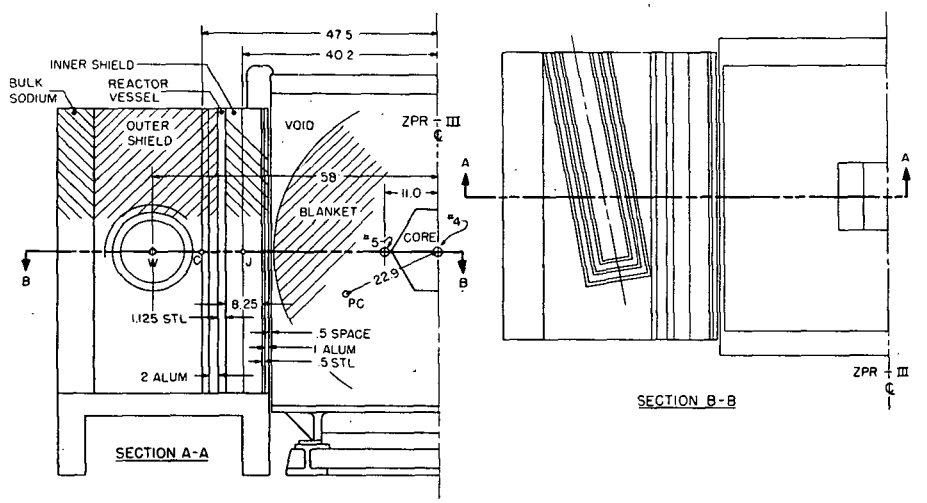


Fig. 15
Shield mock-up of EBR-II on ZPR-III (dimension in inches).

portional counters. The response of these instruments, when calibrated relative to each other [7], gives the spatial distribution of the neutron flux. (The early experiments with a different neutron shield configuration yielded similar information.)

It is recognized that the attenuation of the neutron flux through the neutron shield depends upon accurate treatment of the large instrument thimble. However, it is interesting to observe that relatively simple spherical multi-group calculations do predict the qualitative features of the neutron-flux attenuation in the outer reactor regions. Unpublished 20-group constants were used in the analysis. They are similar to the 19-group constants of Ref. [18].

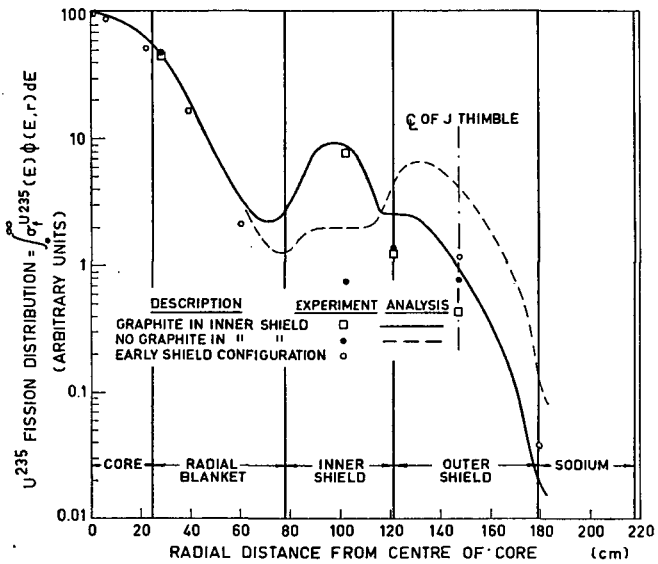


Fig. 16
Calculated and measured U^{235} fission distributions to the neutron shield (instrument response).

Figure 16 summarizes the measured and predicted U^{235} fission distributions for the graphite neutron shield. No moderating material is assumed in the interior of the instrument thimble. Two calculations are shown: one has graphite in the inner shield; the other inner shield is filled with coolant and structural material. The experimental data are normalized to the calculated results at the centre of the core. The qualitative features of the experimental fission distribution are predicted. However, it is quite apparent that the precision of such prediction is inadequate for precise reactor design. Predicted counter responses on the J thimble centreline are between two and five times greater than the measured values. There is some evidence in the comparison between theory and experiment that agreement could be improved by increasing structural (Fe) absorption cross-sections. However, it is also likely that the neutron spectrum is not accurately calculated. Multi-group calculations tend to predict harder spectra than are measured. Finally, the absorptions in the instrument thimble were not included in the analysis. A parametric study on the latter effect, by reducing

graphite and increasing Fe content in the outer shield, showed that the gross features of the fission distribution are as given in Fig. 16.

These analyses and experiments are basic to the construction of the nuclear detector range charts [8]. The most important considerations are:

1. Provision for adequate start-up flux. About 3—6 unmultiplied counts per second will assure reliable flux monitoring at all reactor multiplications.

2. Provision for "comfortable" overlap between start-up fission counters and intermediate- and high-power ion chambers.

Start-up fission counters are located in J thimbles (Fig. 7) with the "no graphite" inner shield. Intermediate-power-level ion chambers are located in J thimbles with the "graphite" inner shield. An overlap of about a factor of ten is provided for these instruments. High-power-level ion chambers located in the O thimbles are not operative below 10 kW.

Reactivity effects

The determination of reactivity effects due to motion or expansion of materials in the reactor system is not easily accomplished. Small sample substitutions, without appreciable spectral shifts due to the samples, can be treated by first-order perturbation theory. Macroscopic substitutions, with spectral shifts, are more readily handled with criticality analyses. The components of both the isothermal temperature coefficient and the power coefficient of reactivity are based on macroscopic substitution experiments or calculations. It would be desirable to use one-dimensional calculations as opposed to the more complicated two-dimensional analyses. The following procedure was adopted for calculating reactivity effects due to expansion and movement:

1. Devise both an "axial" and a "radial" spherical reactor calculation. The "axial" calculation has spherical reflector shells whose composition and dimensions are those along the reactor centreline (axis of symmetry). The "radial" calculation has spherical reflector shells whose composition and dimensions are those along the central core plane (almost a plane of symmetry). The core composition is uniform and the core radius is adjusted so that the reactor is critical with about 48% enrichment of the fuel alloy.

2. Reactivity effects are calculated with these models. Where applicable, both an "axial" and a "radial" calculation are performed. For example, a sodium-worth calculation in the axial structural gaps uses an "axial" sphere. A core sodium-worth calculation is performed with both an "axial" and a "radial" sphere.

3. Calculated results are then adjusted by combining the "axial" and "radial" calculations. Weighting is determined by the core surface area facing in the axial and radial direction.

Where feasible, the predicted results are compared with those measured at ZPR-III (Table III, etc.). Such comparison is not direct. To demonstrate, an important component in both power- and temperature-coefficient determination is the sodium density effect in the reactor core, predicted to be $\Delta k/k = -0.87 \times \times 10^{-5}/°C$. The experimental investigation at ZPR-III dealt with removing sizeable quantities of sodium from the reactor core to effect a gross density change of sodium. The measured worth is $\Delta k/k = -1.16 \times 10^{-3}/kg$ sodium. The mass of sodium in the core is about 29 kg and the coefficient of expansion is given as $2.9 \times 10^{-4}/°C$. Therefore, the sodium density effect inferred from the ZPR-III experiments is $\Delta k/k = -0.98 \times 10^{-5}/°C$. The experiment is an at-

TABLE XVI

COMPONENTS OF THE ISOTHERMAL TEMPERATURE and power coefficient of reactivity ($\Delta k/k°C$)

	Predicted*	Inferred from ZPR-III measurements
Core		
Axial growth of fuel	-0.39×10^{-5} $\Delta k/k°C$	$-0.34 \pm 0.2 \times 10^{-5}$ $\Delta k/k°C$
Radial growth of fuel (displacement of Na)	-0.09×10^{-5}	-0.057×10^{-5}
Axial growth of structure (density change)	-0.039×10^{-5}	-0.033×10^{-5}
Density change of coolant	-0.87×10^{-5}	-0.98×10^{-5}
Radial growth of supporting structure	-0.97×10^{-5}	-0.92×10^{-5}
Doppler effect	$+0.04 \times 10^{-5}$ average	
Bowing		
Gaps		
Density change of coolant	-0.38×10^{-5}	-0.33×10^{-5}
Density change of structure	-0.036×10^{-5}	-0.04×10^{-5}
Upper and lower blanket		
Density change of coolant	-0.21×10^{-5}	-0.21×10^{-5}
Radial growth of uranium and jacket	-0.016×10^{-5}	—
Axial growth of blanket uranium	-0.024×10^{-5}	—
Axial growth of jacket	-0.021×10^{-5}	-0.0064×10^{-5}
Inner blanket		
Density change of coolant**	-0.2×10^{-5}	-0.30×10^{-5}
Axial growth of blanket uranium	-0.066×10^{-5}	—
Axial growth of jacket**	-0.022×10^{-5}	-0.054×10^{-5}
Radial growth of uranium and jacket	-0.07×10^{-5}	—
Radial growth of supporting structure	-0.17×10^{-5}	—
Bowing	0	—
Outer blanket		
Density change of coolant**	-0.017×10^{-5}	-0.011×10^{-5}
Axial growth of blanket uranium	-0.014×10^{-5}	—
Axial growth of jacket**	-0.003×10^{-5}	-0.0012×10^{-5}
Radial growth of supporting structure	-0.034	—

* Reference [3]; multi-group constants from Tables 13 and 14 of Ref. [5].

** The experimental results are difficult to interpret because the radial boundaries, especially between core and blanket, are not well defined.

tempt to simulate power-reactor conditions with a zero-power, fixed-temperature machine. The calculations feature simple idealized geometric models. Comparisons between theory and experiment are encouraging. A number of such comparisons are given in Table XVI. If one allows fairly large uncertainties for those results not comparable with ZPR-III experiments, it is believed that the calculated isothermal temperature coefficient, given as $-3.6\times10^{-5}\,\Delta k/k$ per °C, is known to $\pm 10\%$.

One-dimensional reflected cylinders and slabs were used for some reactivity effect calculations. These calculations were "incorrect" in the same sense that the one-dimensional control-rod analyses (Table X) must be "corrected" (Table XI). For example, the incremental worth of core sodium, by one-dimensional cylindrical analysis, is calculated to be ~2.5 times as large as that obtained from the spherical analyses. Similarly, the incremental worth of sodium in the axial structural gaps, by one-dimensional slab analysis, is calculated to be ~33% less than predicted by spherical analysis.

Combinations of components of the isothermal temperature coefficient into the steady-state power coefficient of reactivity depends upon knowing the detailed temperature distribution throughout the reactor system. An attempt to combine basic reactivity effects with temperature distribution is given in Ref. [3]. The power coefficient, based on the reactivity difference between an isothermal system at 700°F and full-power, full-flow operation is predicted to be $-3.2\times10^{-5}\,\Delta k/k$ MW.

The influence of fuel element and sub-assembly bowing is not included in the above result. Uncertainties with respect to bowing are not completely resolved. If bowing were to take its most pessimistic course, a relatively prompt-positive component to the power coefficient of reactivity would appear. The magnitude may or may not be sufficient to yield a net positive power coefficient of reactivity at certain power levels below full power. However, it is significant to note that with pessimistic bowing analyses, there is a net reactivity loss between reactor start-up and full-power operation.

The most likely situation is that the power coefficient of reactivity is negative at all powers. A small prompt-positive component may be present at low powers but will not be present at or near full power. Principal evidence for this will be inferential. An increase in the magnitude of the steady-state power coefficient of reactivity with power level would be some evidence for this.

A long term reactivity drift is caused by the burn-out and subsequent irradiation growth of the fuel alloy. The reactivity loss due to uniform burn-out of fuel alloy is measured (Table III) and predicted to be:

$$\frac{\Delta k}{k} \approx 0.55\,\frac{\Delta M}{M}$$

where $\Delta M/M$ = fraction of U^{235} destroyed in the core.

The prediction of this value is relatively independent of both geometry and multi-group constants. The additional reactivity loss due to the irradiation growth of fuel alloy is difficult to estimate. There is some evidence [11] that the fuel elements experience a 2% volume increase per at.% fuel alloy burn-up. These irradiations were conducted near 600 °C. There is further evidence [11] which suggests that most of the irradiation growth is in the radial direction within a single fuel element. If this is true, the long-term reactivity effect is negligible. If it is assumed that the expansion is nearly isotropic, the 2% volume

change per at. % burn-up would appear as a 0.67% change in the length of the fuel pin. This change in length represents between 0.16 and 0.19% $\Delta k/k$. The first value is suggested by indirect ZPR-III experiment while the latter is predicted. Therefore, between 0.71 and 0.74% $\Delta k/k$ may be required for accommodating one at. % uniform core alloy burn-out and accompanying irradiation growth. This does not include the additional reactivity loss due to the presence of core fission products estimated at $0.1 \pm 0.05\%$ $\Delta k/k$. This effect requires about 60 days of full power operation at 80% plant factor.

Provision for transforming the fuel alloy from the more dense α-phase to the less dense γ-phase represents another 0.13% $\Delta k/k$.

Reactor operating experience will dictate the exact mode of cycling fuel sub-assemblies. However, the above estimates are indicative of the long-term core reactivity effects.

Nuclear performance

The breeding of nuclear fuel in EBR-II will initially be that of converting U^{235} to Pu^{239}. (True breeding, as demonstrated by simultaneous destruction and creation of Pu must await a future loading of the system.) As U^{235} is destroyed, principally in the reactor core, Pu^{239} is created throughout the system. Most of the bred Pu ($\sim$90 %) is distributed throughout the depleted-uranium reflectors. The bred plutonium is located at positions of low reactivity worth relative to the location of the fissioned enriched uranium. If no rearrangements of sub-assemblies take place during a reactor cycle, the reactivity of the system will not increase due to the bred plutonium. The overriding consideration in long-term reactivity behaviour is the loss associated with the destruction of U^{235} in the core.

Plutonium build-up in the radial blanket elements does have some influence on the critical size of the second and subsequent core loadings. Suppose the first reactor core is permitted to achieve one at. % average burn-up. A second core loading, identical to the first, will require one less core sub-assembly in the first row of the inner blanket. This analytical result assumes that the same amount of start-up excess reactivity is available in both cores. It further requires that the radial blanket is not recycled.

Reverting to the case of one at. % core burn-up, the smaller second core will dictate slightly higher power densities in the radial blanket for fixed total reactor power. Actually, inner-blanket sub-assemblies will initially require non-optimum recycling because of unknown optimum irradiation exposure for unalloyed depleted uranium.

The variation of reactor core size as a function of time has some effect on the nuclear performance of the system. Specific power (kW/kg), breeding gain and doubling time are slowly varying functions of the time.

With a total power output of 62.5 MW and a clean critical mass of 175 kg U^{235} the initial average specific power in the EBR-II converter is 360 kW/kg. A Pu-fuelled EBR-II with a 110-kg critical mass has an average specific power of 570 kW/kg.

An initial conversion ratio of 1.26 has been predicted for EBR-II. This prediction is uncertain on two counts:

1. More recent cross-section data [19] suggest that the neutron yield per fission of U^{238} is higher ($\nu\ U^{238} \approx 2.8$) than the analyses provided ($\nu\ U^{238} \approx 2.6$). This effect, while small, would tend to raise the predicted value.

2. It has long been known [4, 5, 19] that the predicted value of $\sigma_{n,\gamma}^{U^{238}} / \sigma_f^{U^{235}}$ exceeds the measured value by as much as 20 % at the centre of the reactor core. This could mean that measured breeding or conversion ratios will be less than predicted.

About 90 % of the predicted conversion is in the breeder reflector (external breeding ratio ≈ 1.16). The remaining 10 % of the predicted conversion is in the core (internal breeding ratio ≈ 0.10). This is where the nuclear performance of EBR-II differs from that of the larger (>400-l cores) more dilute conceptual fast-neutron systems. In the latter almost half of the breeding or conversion may take place in the reactor core. The predicted initial breeding ratio for the Pu-fuelled EBR-II is 1.7; the Pu-U-10 wt. % fissium is a possible fuel element for such a loading. For a ternary fuel element of Pu-U^{235}-U^{238}-10 wt. % fissium, the predicted combination of breeding and conversion ratio will be about 1.5. The upper limit of 20 % uncertainty on the parameter $\sigma_{n,\gamma}^{U^{238}} / \sigma_f^{U^{235}}$ does not seriously jeopardize the predicted breeding gain of a Pu-fuelled system.

The cited breeding constants are for the actual EBR-II engineering design. It is interesting to note that significant breeding parameters are less favourable than those obtained from the conceptual studies. The latter, performed in spherical geometry for an idealized reactor, do not include compromises to engineering design. Three such accommodations in EBR-II are:

1. The axial blankets contain only about 30 vol.% depleted uranium as opposed to 60 vol. % in the radial reflector.

2. None of the 14 control and safety sub-assemblies contain any fertile reflector material.

3. Critical-mass requirements are increased over those of the conceptual study. The presence of axial gaps for fuel-element expansion and structural reasons reduce the axial reflector saving.

TABLE XVII

INITIAL CONVERSION PARAMETERS FOR EBR-II

Description	No. of groups	Geometry	Conversion ratio				Estimated doubling time (years)
			Core	Axial reflector	Radial reflector	Total	
Reference EBR-II	11	r, z	0.125	0.109	1.03	1.26	35
Reference EBR-II	2	r, z	0.133	0.083	1.01	1.23	—
No uranium in axial reflector	2	r, z	0.122	—	1.06	1.18	—
Idealized EBR-II	11	sphere	0.11	1.23		1.34	20
Idealized Pu-fuelled EBR-II	11	sphere	0.21	1.63		1.84	6

These requirements suggest that the inhomogeneous axial reflector is quite "leaky". Furthermore, cycling of axial fertile material is dictated by core-alloy burn-up considerations. Unless a scheme is devised for reinserting the axial reflector elements, the use of the axial blanket is almost academic. When the core alloy reaches 1 at. % burn-up, maximum localized burn-up in the axial reflector does not exceed 0.04 at. %. The corresponding maximum localized Pu production does not exceed 0.1 at. % in the fertile material.

The above considerations prompted some investigations on the nuclear performance of EBR-II with no fertile material above and below the core. Replacing the axial reflector uranium by stainless steel tends to increase the reactivity of the system. As much as 3% $\Delta k/k$ can be obtained by such replacement. This would lower critical-mass requirements. The loss in conversion ratio accompanying such replacement is less than 10%. However, this may be as much as 30% of the breeding gain. Table XVII summarizes several predicted conversion parameters.

Safety considerations

A thorough review of the safety considerations pertaining to the design and operation of EBR-II requires the correlation of a number of diverse efforts and disciplines. Various aspects of this problem have been described elsewhere [3, 8, 20, 21, 22, 23]. This discussion is limited to a few topics dealing primarily with the causes of a nuclear accident. Some courses which may follow the initiation of such an event are also described.

The over-all safety of reactor operation depends on answering the following questions:

1. Are the mechanical components for inserting and removing reactivity reliable? If not, what may be the consequences?
2. Is the reactor system inherently stable during steady-state operation?
3. To what extent is administrative control required to avoid a nuclear event?

Mechanical malfunction

The reactor system, its mechanical components and flux sensing instrumentation provide a great degree of reliability [3, 8]. The design is such that a single mechanical failure or malfunction will not cause serious injury to the system or surroundings. For example, the remote possibility of fuel-handling-equipment failure to the point where a sub-assembly is actually dropped into the reactor core poses no serious hazards problem. Normal fuel-handling procedures require that the reactor is between 3 and 4% $\Delta k/k$ sub-critical while fuel is inserted into or removed from the reactor core. Should a core sub-assembly drop into the reactor core, the rapid rate of increasing source-multiplied flux will scram the safety rods (period trip). Even if this period trip is not operative, the reactor is still sub-critical after the sub-assembly has been dropped into the core. Such a malfunction (dropping of a fuel sub-assembly) becomes hazardous only if it accompanies or is accompanied by some other malfunction or loss of administrative control.

Dropping a fuel sub-assembly can have extremely serious consequences if one postulates the reactor at or near delayed-critical. The control rods are in their least reactive positions while fuel is being loaded; the safety rods are available for shut-down purposes. To postulate the reactor at delayed critical under these conditions requires a loss of administrative control and gross malfunction of

flux-monitoring equipment. (A malfunction of the latter will normally cause safety-rod scram and prevent further fuel handling.) Between 3 and 4% $\Delta k/k$ must have been unwittingly added to the system to realize a critical reactor.

A gross loss of administrative control or a gross system-failure is required to effect the addition of such large amounts of reactivity. The following four examples show the extent of failure or loss of control necessary to obtain a critical reactor during fuel-handling operations:

(*a*) About ten more than the six required fuel sub-assemblies must be loaded into the first row of the inner blanket (Fig. 5).

(*b*) There is an unknown fuel-alloy "blending" error with uranium enrichment about 6% uniformly higher than the specified 48.4%.

(*c*) Several sub-assemblies, between 5 and 10, contain highly enriched (~93%) uranium instead of the specified 48.4%.

(*d*) The gross failure of between 10 and 15 sub-assemblies with all contained fuel displaced towards the core mid-plane.

None of these situations should arise; if they do they should be noted by operating personnel. Sub-critical count-rates during fuel handling should not exceed 30 times unmultiplied-source count-rates. Rapid additions of reactivity will be noted by the automatic period trips. The power-level trip during fuel-handling procedures is set below 1 kW.

Six specific accidents caused by the malfunction of mechanical rod-drive mechanisms have been analysed in detail [3]:

1. The two safety sub-assemblies are driven into (upward) a critical reactor core at 2 in/min under zero-power conditions.
2. A fuel sub-assembly is driven (downward) into the centre of a critical reactor at a speed of 6 in/min under zero-power conditions.
3. A single normal control sub-assembly is driven (upward) into a critical reactor at a speed of 5 in/min under zero-power conditions.
4. A single normal control sub-assembly is driven (upward) into a critical reactor at a speed of 5 in/min under full-power and full-flow conditions.
5. A central fuel sub-assembly is driven (downward) into a critical reactor at 72 in/min under zero-power conditions.
6. A fuel sub-assembly is dropped into a critical reactor under zero-power conditions.

Of the six cases cited, only two (3 and 4) can occur without some gross reactor-loading error having taken place before the mechanical malfunction. Accidents 3 and 4 dealing with control sub-assembly motion, can be effectively prevented by power-level trip circuits, even though the reactor operator should note either unusual increase in power level or indication of rod motion before the trip circuits function.

The kinetic behaviour of the six EBR-II malfunctions has been studied [3] by applying the one-group space-independent model [23]. The objective is to obtain information concerning the time variation of the neutron flux and power. From these, the time variation of temperature is determined which in turn specifies limits on reactor operation. Several parameters are basic to all the kinetic analyses:

(*a*) Prompt-neutron lifetime for EBR-II is predicted to be 8.0×10^{-8} s. Measurements at ZPR-III (by Rossi-α) suggest from 7.5 to 8.5×10^{-8} s.

(*b*) The effective delayed-neutron fraction is $\beta_{\text{eff}} = 0.0073$ [4].

(*c*) Shut-down coefficients due to thermal expansion are based on the components given in Table XVI. Except for the full-power malfunction (Case 4), only core shut-down coefficients are operative. For rapid rates of reactivity insertion only fuel-alloy expansion is an initially effective shut-down mechanism. For extremely rapid rates of inserting reactivity, the fuel-alloy expansion is inertially inhibited [22] during the early stages of the excursion.

(*d*) Reactivity insertion rates are based on the "predicted" values in Table XII. These are higher than actually expected in EBR-II.

Even during a slow excursion, starting at zero power and flow, thermal-expansion effects in the reflectors are almost negligible. If it is assumed that most of the heat produced in a given reactor region remains in that region until the excursion is terminated, the temperature rise in the reflectors is small compared with that in the core. Table XVIII gives appropriate total temperature coefficients for the various reactor regions. The temperature rise of each region per degree (centigrade) rise in the core is also given. The latter is based on detailed power-distribution calculations whose integral results are given in Table XIII.

TABLE XVIII

REACTIVITY AND TEMPERATURE EFFECTS UNDER ZERO-FLOW CONDITIONS*

Reactor region (Figs. 5, 14)	$\Delta k/\Delta T$ region (per °C)	$\frac{\Delta T \text{ region}}{\Delta T \text{ core}}$	$\Delta k/\Delta T$ core (per °C)
Core	—1.28	1.0	—1.28
Inner blanket	—0.30	0.045	—0.014
Outer blanket	—0.03	0.010	—0.0003
Upper plus lower blankets	—0.26	0.02	—0.0052
Upper plus lower gaps	—0.42	~0	~0

* No radial expansion is included and no convection or conduction is assumed.

The consequences of the five cited zero-power malfunctions depend upon the initial temperature of the system. Except for the initial critical approach, the bulk system temperature will not be less than 315 °C. The melting-point of the fuel alloy is near 1130 °C. Another temperature of interest is the stainless steel/uranium eutectic formation temperature of 725 °C. It has, however, been experimentally demonstrated [24] that the latter temperature may be exceeded for short periods of time without fuel-element failure. The conclusions drawn from the six kinetic analyses are summarized in Table XIX.

The first four cases in Table XIX operate on a fairly long time-scale and can be terminated by the reactor operator as well as the automatic trip circuits. The last two cases (5) and (6) dealing with high-speed sub-assembly insertion into the centre of the core may not be terminated by automatic trip circuits and cannot be terminated by the reactor operator. However, it must be noted that the severe consequences are reduced, and more time is available for trip circuits to function under certain conditions. In particular, available time is increased if the reactor is somewhat sub-critical before the malfunction. However, the consequences may be more severe if the available time is still insufficient to shut the reactor down. In the gravity-induced reactivity insertion, the rate of adding

TABLE XIX

CONSEQUENCES OF MECHANICAL MALFUNCTION
coupled with loss of administrative control

Case No. (see text)	Initial power (W)	Malfunction	Will period-trip terminate excursion ?		Will power-level trip terminate excursion ?		If trips are inoperative, will melting be initiated ?	
				When (s) ?[1]		When (s) ?[1]		When (s) ?[1]
1	8	Safety rods	yes	<70	yes[2]	70	yes	120
2	8	Fuel sub-assembly loading mechanism	yes	<25	yes[2]	30	yes	55
3	8	Control rod	may-be	~100	yes[2]	~100	may-be	~230
4	6×10^7	Control rod	no	—	yes[3]	~17	yes	~160
5	8	Fuel sub-assembly loading mechanism (high-speed)	yes	<1	yes[2]	4	yes	~6
6	8	Fuel sub-assembly loading mechanism (gravity-induced)	prob-ably	imme-diately	no[2] maybe[4]	— imme-diately	yes	0.13

[1] After insertion of reactivity is initiated.
[2] ~ 10^3 W.
[3] ~ 78 MW.
[4] < 100 W.

reactivity when the reactor is near prompt-critical is greater if the reactor is initially below delayed-critical.

Further, it must be noted that the coincident malfunction of fuel-handling equipment along with the loss of administrative control must be accompanied by another coincidence. The latter refers to dropping the sub-assembly near the centre of the core. If the reactor is at or near delayed-critical and a fuel sub-assembly is dropped near the core boundary, the reactor will go on a period between 5 and 10 s. Power-level and period trips as well as operator action can shut the reactor down.

The reactor system is designed to cope with mechanical malfunction. However, these studies show that coincident malfunction and operational errors can lead to extremely severe consequences.

NUCLEAR ACCIDENTS

The TREAT (Transient Reactor Test) melt-down programme [24, 25] is an experimental investigation into the behaviour of fast-reactor fuel elements under transient power-excursion conditions. Mechanisms of fuel-element failure and the

distribution of resulting melt-down products are being investigated on an in-pile basis in an attempt to define accident conditions and determine the parameters of importance for describing or predicting the course of a fast-reactor melt-down.

Tests have been performed to study the failure of single, clad and bonded EBR-II-type fuel pins. These were contained in an inert atmosphere in a stainless-steel capsule. Similar experiments with clad and bonded EBR-II fuel pins suspended in stagnant or flowing sodium have been initiated. Postulated accidents are on the basis of these experiments. The results obtained, although not unexpected, were different from those of out-of-pile melt-down experiments using electrical heating. The latter investigations were conducted during the EBR-II Mark I fuel-element development programme. Very significantly, both in-pile and out-of-pile experiments showed that unirradiated EBR-II fuel (5 wt. % fissium) can be heated without failure to temperatures appreciably greater than 725 °C for limited times.

TREAT experiments [24] dealing with single EBR-II fuel pins contained in stainless steel clad and bonded with sodium suggest that rearrangements of the fuel alloy in the EBR-II reactor leading to a gain in reactivity may occur for very large temperature excursions. Pertinent experimental data from TREAT can be separated into roughly three broad categories. These are catalogued by the maximum clad temperature registered during an excursion while the fuel element is suspended in an inert atmosphere. The temperature ranges are:

(*a*) 950 to 970 °C, incipient failure,
(*b*) 970 to 1015 °C, mild failure,
(*c*) Above 1015 °C, energetic failure.

In order to reach such clad temperatures in the reactor, some gross malfunction must have taken place beforehand. (Maximum fuel-clad temperatures under normal conditions are expected to be between 480 and 560° C. Sodium boiling is initiated at 892 °C.)

The "energetic failure" results suggest that in an accident where the fuel-element clad temperature exceeds 1015 °C, molten fuel is violently ejected from the stainless-steel jacket at or near the point of initial jacket failure. The ejection is caused by very high pressure attributed to the vaporized sodium. In the usual TREAT experiment a nearly constant axial power distribution exists along the sample and hence failure is equally probable, anywhere along the pin and experiments tend to confirm this effect. Experiments were performed with a specially shaped axial power-distribution [24] closely resembling that in EBR-II. The results show that failure would occur at the highest localized temperature, possibly near the core mid-plane.

If operating conditions during an accident in the EBR-II reactor lead to maximum fuel-element temperatures at or near the horizontal mid-plane of the core, and if the "energetic" type of failure does occur only near the hottest point, it may be possible that the fuel alloy will be propelled from the axial extremities of the fuel elements toward the centre plane, leading to an increase in reactivity.

There are several unknowns in this problem. Does the hot spot always appear near the core centre in over-heating conditions? Does the position of maximum axial temperature at the time of failure vary considerably as a function of core radius? These two unanswered questions depend very much on reactor design and the course of the accident. Further, what happens if one or both ends of the fuel pin are not fully melted by the time the failure occurs at the hot spot? And what is the effect of pre-irradiation? Does swelling lead to a different mode of

cladding failure ? It is not clear that release of fission gases will serve to propel the fuel alloy from its container in the same way sodium does. Laboratory experiments [26] indicate a frothing action in the escape of uranium from pre-irradiated EFFBR pins [27].

The malfunction leading to major temperature rises may be caused by the reactor power-level being increased because of mechanical reactivity insertion at a rapid rate. A more likely cause of temperature increase might be obstruction or loss of coolant flow to a sub-assembly or region within a sub-assembly.

The estimated reactivity effects of concentrating fuel at the core central plane have been determined by using two-group diffusion theory calculations in r, z geometry. It is assumed in each case that fuel originally contained in a single sub-assembly remains within that sub-assembly. Thus, all fuel is dispersed throughout a central region about 11 cm thick, displacing all sodium and some stainless steel.

The first case assumes that such failure takes place in the seven central-core sub-assemblies (Fig. 5). The calculated reactivity effect is:

$\Delta k = 0.030$,

Δk/sub-assembly ≈ 0.0043,

Δk/fuel element ≈ 0.00044.

The second case assumes such failure takes place in the fifth row of the reactor core (Fig. 5). This is where the control sub-assemblies are located.

$\Delta k = 0.026$,

Δk/core sub-assembly ≈ 0.0013,

Δk/control sub-assembly ≈ 0.00086,

Δk/fuel element ≈ 0.000014.

The final investigation assumes complete failure within the 61-element core.

$\Delta k = 0.1443$,

Δk/fuel element (average) $= 0.000028$.

The calculations are pessimistic. It is assumed that no void formation due to sodium-boiling of the flowing reactor coolant has occurred.

The investigation dealing with complete core failure was repeated by assuming that sodium density in the core has been reduced by 50%.

$\Delta k = 0.1143$,

Δk/fuel element (average) $= 0.000022$.

This latter result suggests that the magnitude of the reactivity effect for a particular mode of failure is a function of, but relatively insensitive to, the exact disposition of the core sodium.

The "mild" and "incipient failure" experiments also suggest a fuel-alloy rearrangement after a stainless-steel-jacket failure due to eutectic formation. However, the rate and manner in which such rearrangement takes place is less pronounced than in the case of the "energetic failure" cited above. There is some suggestion from the "mild failure" experiments that clad penetration takes place at the bottom of the fuel-element jacket. However, the axial temperature gradients in EBR-II suggest that the experimental implication of failure at the bottom may not apply. In any event, it is the case of "energetic failure" which poses the major hazard on two counts:

1. Failure is caused by localized coolant passage obstruction within a single sub-assembly. Under these conditions, if all 91 fuel elements were to fail simultaneously the reactor would be on excursion, the magnitude depending on the location of the sub-assembly within the reactor core. For instance, in the control-rod ring (Row 5)

of the reactor a 35-s period is the result of fuel sub-assembly failure while a 65-s period is the result of control sub-assembly failure. The period trips will not terminate this malfunction. However, power-level trips are operative at all times. Near the centre of the core, complete sub-assembly failure (91 fuel elements) would exhibit a 3-s period which would activate both period and power-level trips before the excursion could proceed to other parts of the reactor.

2. Failure is caused by rapid over-heating such as primary pump failure coupled with the freezing of the control rods. Primary pump failure will normally scram the reactor. The likelihood of primary pump failure coupled with inability to scram the control rods is considered remote [3]. This remote set of circumstances is being used as a basis for generating a large-scale melt-down accident. The determination of realistic rates of reactivity-addition is one of the major objectives.

These speculations are based on rather recent and incomplete experimental results. The TREAT programme will continue to investigate failure mechanisms of single pins and clusters of EBR-II fuel elements in both sodium and an inert atmosphere. Additional experiments as well as analytical studies are required to confirm or disprove the assumption that the experimental results to date are characteristic of failure mechanisms in the actual reactor.

If further studies indicate the existence of failure modes capable of producing large reactivity gains, the basic question remains: What is the rate of reactivity addition? Preliminary estimates indicate that a small fraction of a second is needed for the fuel to travel from the end to the middle of the pin. In addition, there will likely be time delays of about a fraction of a second or more between the first pin failure and major pin failure in other sub-assemblies during a loss-of-coolant-flow accident. It will be possible to determine the consequences of such fuel-element failure only after all such effects are understood and considered in detail.

Core melt-down

The previous two sections describe situations which might conceivably give rise to a nuclear event. The discussion of malfunction demonstrates how temperature excursions may be initiated. The implication of the TREAT experiments demonstrates how the core may lose its mechanical rigidity under temperature excursions of various kinds. The consequences of such core failure are determined by the *rate* at which reactivity is added to the system. Some rather large reactivity effects are implied by the TREAT experiments. Without knowing the rate at which reactivity is inserted, an estimate of a nuclear burst can be obtained by considering the following two problems:

1. Following a loss of core integrity, what is an extremely rapid way of adding reactivity?
2. What are the mechanisms for limiting the extent and magnitude of the nuclear burst?

Detailed answers to these two questions were originally given by Jankus and Okrent [22] with more recent analyses by Jankus [28]. Without implying any credence to the accident, the following pessimistic situation was postulated:

(*a*) The sodium has boiled away from the centre of the core;

(*b*) The uranium from the middle of the core has trickled down into the lower part of the core and is retained there, producing a region abnormally dense in enriched uranium at the core bottom, with a large gap at the core centre;

(c) At the worst possible moment, the upper portion of the core falls as a single unit, producing a prompt-critical configuration at the highest possible reactivity-insertion rate.

TABLE XX

EXPLOSIVE-ENERGY YIELD
as a function of reactivity-insertion rate

Reactivity-insertion rate ($/s)	Explosive-energy yield (lb TNT)
200	370
600	850
1000	1240

After the excursion is initiated, the mechanism of reactivity reduction is caused by expansion of the reactor [22]. Table XX gives the predicted explosive energy yield for EBR-II as a function of the reactivity-insertion rate for the postulated excursion.

The two highest reactivity-insertion rates cited require many coincidences in timing and fuel disposition before and during the excursion. If this pessimistic accident were realized, the $ 200/s is a more probable reactivity-insertion rate. There is considerable evidence [21] that the primary containment system (see *Description*) is capable of withstanding at least 400 lb of TNT. In addition there is the comprehensive reactor-plant containment. The latter is designed to withstand a maximum sodium-air reaction.

REACTOR STABILITY AND KINETICS

Operating experience with the three EBR-I cores (Marks I, II and III) [29, 30, 31] had some influence on the design of EBR-II. The most important consideration dealt with the elimination of a prompt-positive power coefficient of reactivity. This is accomplished by reducing or eliminating bowing of the fuel sub-assembly as well as bowing of the fuel within the sub-assembly. If bowing does exist the reactor is designed such that such motion is principally outward, thus giving a negative component to the power coefficient.

Basically, the fuel and blanket sub-assembly support-structure makes the reactor a solid unit. However, clearances are necessary to effect insertion and removal of sub-assemblies. The sub-assemblies located in the grid plate are fixed, with respect to radial motion, at the bottom with essentially no restraining forces at the top. If there are thermal gradients to induce bowing of the sub-assembly walls, the ultimate net motion is outward. There may be increments of inward motion at some low powers. Such motion, if any, is further restricted by the presence of protrusions on the sub-assembly walls which tend to take up the clearances that may exist.

The sub-assembly bowing phenomenon may be qualitatively examined. The sub-assembly grid support-structure (Fig. 4) is at constant temperature ($\sim$350 °C). Radial sub-assembly motion pivots on the upper grid plate (Fig. 4). Initially, the sub-assemblies are free to move at the top, near the outlet plenum (Fig. 4). Bowing-induced motion will increase the core radius until the top end clearances are taken up. Further bowing will effectively reduce the core radius because subsequent motion is constrained at the top and the bottom of the sub-assembly.

Core-radius reduction will cease as soon as there is an effective contact between the sub-assembly and its inward neighbour. From this point on, all motion is outward, effectively increasing the core radius.

The fuel pins within the hexagonal sub-assembly are packed tightly through the use of helical spacer wires. Rigidity is further enhanced by indentations on the sub-assembly wall which restrict motion of the fuel pin where the spacer wires are not in contact with the wall.

It is not likely that a large delayed-negative temperature coefficient is present in EBR-II. Such a coefficient in EBR-I is attributed to the heating of structural components above the core. There are no such components in EBR-II.

Maximum bowing effects can occur in the control sub-assemblies. These effects are reduced by the incorporation of a flow "twister" just above the fuel elements. This tends to take the coolant from one side of the control sub-assembly and bring it to the other side. Thus, where the higher-temperature coolant is in contact with the sub-assembly wall in the core, the lower-temperature coolant is in contact with the same wall just above the fuel element. High and low temperature here refers to temperature distribution within a single control sub-assembly.

Detailed stability analyses for EBR-II have been reported [32, 33]. These studies find that the reactor exhibits no anomalous kinetic behaviour during steady-state operation.

ACKNOWLEDGEMENTS

The author is grateful for advice and encouragement from Mr. L. J. Koch, EBR-II Project Manager, and the EBR-II Project Staff.

The helpful comments, criticism and advice of Dr. D. Okrent are gratefully acknowledged.

Dr. H. H. Hummel originally suggested the use of ZrH to enhance instrument response.

The gamma-heating calculations were submitted by Mr. A. E. McArthy.

Mr. G. W. Main performed many of the detailed calculations in this report. His assistance and cooperation are hereby acknowledged.

The cited experimental data were obtained by the ANL-ZPR-III staff.

REFERENCES

[1] KOCH, L. J. *et al.*, Proc. 2nd UN Int. Conf. PUAE **9** (1958) 323.
[2] ZINN, W. H. and DIETRICH, J. R., Solid Fuel Reactors, Addison-Wesley Publishing Company (1958).
[3] KOCH, L. J. *et al.*, ANL-5719 (1957).
[4] LOEWENSTEIN, W. B. and OKRENT, D., Proc. 2nd UN Int. Conf. PUAE **12** (1958) 16.
[5] LONG, J. K. *et al.*, Proc. 2nd UN Int. Conf. PUAE **12** (1958) 119.
[6] OKRENT, D., AVERY, R. and HUMMEL, H. H., Proc. UN Int. Conf. PUAE **5** (1956) 347.
[7] ANL-6301 (1960).
[8] KOCH, L. J. *et al.*, EBR-II dry critical experiments—experimental program, experimental procedures and safety considerations, in press.
[9] KIRN, F. S., "An absolute fission counter", American Nuclear Society Second Winter Meeting (Oct. 1957) 61.
[10] Reactor Handbook, 2nd ed. **1** "Materials", Interscience Publishers, Inc. (1960).
[11] MONAWECK, J. H. and SOWA, E. S., ANL-6010 (1961); HORAK, J. A. and KITTEL, J. H., *Nucl. Metall.* VI (1959) 35.

[12] ZEGLER, S. T. and NEVITT, M. V., *Nucl. Sci. Engng* **6**, 3, 222.
[13] NEVITT, M. V. and ZEGLER, S. T., *J. Nucl. Materials* **1** (1959) 6.
[14] LEE, C. E., LONGMIRE, C. L. and STONE, P. M., Nick II, two-dimensional code for neutron diffusion in a finite cylindrical reactor, Los Alamos, unpublished.
[15] WACHSPRESS, E. L., CUREM, KAPL-1724, Modifications 1 and 2 (1957).
[16] BUTLER, M. K. and COOK, J. M., ANL-5437 (1956).
[17] STARK, R. H., KAPL-1068 (1954).
[18] DAANE, R., MELA, R. and DeFELICE, J., NDA 14—107 (1955). (The multigroup constants in this report were originally supplied by H. H. Hummel.)
[19] YIFTAH, S., OKRENT, D. and MOLDAUER, P., Fast Reactor Cross-Sections—A Study Leading to a Sixteen Group Set, Pergamon Press, New York (1960).
[20] BUMP, T. R. and MONSON, H. O., Proc. 2nd UN Int. Conf. PUAE **11** (1958) 404.
[21] MONSON, H. O. and SLUYTER, M. M., Proc. 2nd UN Int. Conf. PUAE **11** (1958) 124.
[22] McCARTHY, W. J. *et al.*, Proc. 2nd UN Int. Conf. PUAE **12** (1958) 207.
[23] BRITTAN, R. O., ANL-5577 (1956).
[24] DICKERMAN, C. E., OKRENT, D. and SOWA, E., *These proceedings* Vol. **III**, p. 171.
[25] DICKERMAN, C., SOWA, E. and OKRENT, D., *Nucleonics* **19** (1961) 4.
[26] FREAS, D. G. *et al.*, BMI-PRDC-656.
[27] AMAROSI, A. and YEVICK, G., Proc. 2nd UN Int. Conf. PUAE **9** (1958) 358.
[28] JANKUS, V. Z., *These proceedings* Vol. **III**, p. 209.
[29] LICHTENBERGER, H. V. *et al.*, Proc. UN Int. Conf. PUAE **3** (1956) 345.
[30] THALGOTT, F. W. *et al.*, Proc. 2nd UN Int. Conf. PUAE **12** (1958) 242.
[31] SMITH, R. R. *et al.*, *These proceedings*, Vol. **III**, p. 43.
[32] OKRENT, D. and HUMMEL, H. H., Proc. Conf. Transfer Function Measurement and Reactor Stability Analysis, ANL-6205 (1960).
[33] HUMMEL, H. H. and BRYANT, L. T., *These proceedings*, Vol. **III**, p. 107.

SM-18/82

ФИЗИЧЕСКИЕ ХАРАКТЕРИСТИКИ РЕАКТОРА БР-5

А. И. Лейпунский, О. Д. Казачковский, А. И. Абрамов, Ю. А. Александров, Н. Н. Аристархов, И. И. Бондаренко, Н. В. Краснояров, В. Н. Морозов, М. Н. Николаев, М. С. Пинхасик, О. А. Сальников, Г. Н. Смиренкин, Ю. Я. Стависский, Ф. И. Украинцев и Л. Н. Усачев
Академия Наук СССР, Москва
СССР

Abstract — Résumé — Аннотация — Resumen

Physical characteristics of the BR-5 reactor. The BR-5 reactor is an experimental fast reactor with a thermal capacity of 5000 kW. Its fuel is plutonium oxide and the coolant is sodium. The reactor's reflector consists of a thin layer of natural uranium surrounded by a thick layer of nickel.

This paper describes the physical features of the reactor as measured during start-up and operation. The neutron flux at the centre of the core is 8.2×10^{14} n/cm^2 s. The paper presents the results of studies of the reactor's kinetics, its temperature and power coefficients, and the distribution of the number of fissions and reaction rates of various reactions over the core volume. Neutron fluxes and spectra were likewise measured in the reactor's experimental assemblies.

Caractéristiques physiques du réacteur BR-5. Le réacteur BR-5 est un réacteur expérimental à neutrons rapides, d'une puissance thermique de 5 MW. Il utilise comme combustible de l'oxyde de plutonium. Ses réflecteurs se composent d'une mince couche d'uranium naturel entourée d'une couche épaisse de nickel.

Le mémoire indique les caractéristiques physiques du réacteur, mesurées lors de son démarrage et au cours de son exploitation. Le flux de neutrons au centre du cœur est de $8{,}2 \cdot 10^{14}$ n/cm^2 s. Les auteurs exposent les résultats des recherches sur la cinétique du réacteur; ils indiquent ses coefficients de température et de puissance ainsi que la distribution du nombre de fissions et des vitesses des différentes réactions dans le volume du cœur. Ils ont également mesuré les flux et spectres neutroniques dans les dispositifs expérimentaux du réacteur.

Физические характеристики реактора БР-5. Реактор БР-5 является опытным реактором на быстрых нейтронах с тепловой мощностью 5000 квт. Топливным материалом реактора служит окись плутония. Теплоноситель — натрий. Отражатель реактора состоит из тонкого слоя естественного урана, окруженного толстым слоем никеля.

В докладе приводятся физические характеристики реактора, измеренные в процессе пуска и эксплуатации. Поток нейтронов в центре активной зоны составляет $8{,}2 \cdot 10^{14}$ н/см^2 сек. Приводятся результаты исследования кинетики реактора, его температурный и мощностной коэффициенты, распределение числа делений и скоростей различных реакций по объему активной зоны. Измерены также потоки и спектры нейтронов в экспериментальных устройствах реактора.

Características físicas del reactor BR-5. El reactor BR-5 constituye un reactor experimental de neutrones rápidos con una potencia de 5000 kW (t). Emplea óxido de plutonio como combustible y sodio como medio de transmisión de calor. El reflector consiste en una delgada capa de uranio natural, rodeado por una gruesa capa de níquel.

En la memoria se exponen los parámetros del reactor, medidos durante la puesta en marcha y el funcionamiento normal. El flujo neutrónico en el centro del cuerpo es de

$8{,}2 \cdot 10^{14}$ n/cm^2 s. Se presentan los resultados del estudio de la cinética del reactor y de sus coeficientes de temperatura y de potencia, de la distribución del número de fisiones y la velocidad de las diversas reacciones en todo el volumen del cuerpo. También se midieron los flujos y se registraron los espectros neutrónicos en los dispositivos experimentales del reactor.

1. Введение

Реактор БР-5 представляет собой исследовательский реактор на быстрых нейтронах тепловой мощностью 5000 квт, предназначенный для накопления опыта и получения данных, необходимых для сооружения энергетических реакторов на быстрых нейтронах с расширенным воспроизводством ядерного горючего. Топливным материалом реактора служит окись плутония. В качестве теплоносителя используется натрий. Отражатель реактора состоит из двух слоев: тонкого слоя естественного урана и толстого слоя никеля.

Подробное описание конструкции этого реактора было дано в докладе на на 2-ой Международной конференции по применению атомной энергии в мирных целях [1]. В реакторе имеется большое количество экспериментальных устройств, предназначенных для проведения технологических, материаловедческих и физических экспериментов с быстрыми, медленными и тепловыми нейтронами. Большое количество мощных нейтронных пучков, экспериментальных каналов, тепловая колонка представляют широкие возможности для исследования ядерных констант, необходимых для расчета реакторов, и для проведения работ по физике ядра.

Продольный и поперечный разрезы реактора приведены на рис. 1 и 2. Вдоль вертикальной оси активной зоны проходит канал, предназначенный для проведения петлевых экспериментов с опытными образцами ТВЭЛ. На расстоянии 40 см от центра реактора через никелевый отражатель проходит вертикальный канал ⌀ 70 мм ОК-70. От поверхности активной зоны через отражатель и защиту реактора проходят два горизонтальных канала для вывода пучков быстрых нейтронов — каналы Б-1 и Б-3. Из никелевого отражателя выходит горизонтальный канал П-2, предназначенный для вывода пучка промежуточных нейтронов. Кроме того имеется широкий канал Б-2, предназначенный для экспериментов по радиационной защите. Реактор имеет тепловую графитовую колонну, имеющую ряд вертикальных каналов ⌀ 20 мм за исключением каналов ОК-3 и АК- имеющих ⌀ 70 и 100 мм, соответственно.

В настоящем докладе приводятся результаты измерений физических характеристик реактора БР-5.

2. Распределение нейтронов

Энергетические спектры нейтронов в различных частях реактора БР-5 различны. В активной зоне реактора спектр нейтронов довольно жесткий, значительная часть этого спектра лежит выше 100 кэв. Проходя через никелевый отражатель реактора нейтроны быстро теряют свою энергию за счет неупругих, а затем и упругих соударений, так что во внешних слоях экрана средняя энергия нейтронных спектров составляет всего лишь несколько десятков электроновольт.

Изменение нейтронных спектров, особенно в килоэлектроновой области энергий, представляет собой очень сложную экспериментальную задачу. Вследствие этого детальные измерения спектров проводились лишь в области сравнительно быстрых нейтронов ($E_n > 50$ кэв) для наиболее важных нейтронных пучков и каналов. В остальных же случаях в качестве характеристик спектра использовались сечения реакций с известным энергетическим ходом, усредненные по исследуемому энергетическому спектру. В качестве таких реакций использовались реакций деления U^{238} (n, f), Th^{232} (n, f) и Al^{27} (n, α), Na^{24}, обладающие эффективным порогом около 1,4 и 5 Мэв, соответственно и приблизительным постоянством сечения в области выше порога, реакция Pu^{239} (n, f), сечение которой почти постоянно в области от 50 кэв до 6 Мэв и реакции Na^{23} (n, γ), Gu^{63} (n, γ), Au^{197} (n, γ), являющиеся индикаторами главным образом медленных нейтронов.

Для того, чтобы иметь возможность получить качественную оценку смягчения спектра нейтронов в отражателе наряду с реакцией Pu^{239} (n, f) использовалась реакция деления U^{235}, сечение которой в области медленных нейтронов обладает более сильной зависимостью от энергии. Для получения информации о мягкой части спектра в отражателе реактора были также проведены исследования пространственных распределений нейтронов с $E_n \approx 5$ эв с помощью индикаторов золота Au^{197} — методом резонансной блокировки [2].

Облучение этих индикаторов производилось в экспериментальном, вертикальном канале, расположенном на расстоянии 5 см от оси реактора. Детекторами деления являлись малогабаритные камеры деления, содержащие тонкие слои делящихся веществ. Камеры были наполнены чистым аргоном до давления 10 ати. Режим наполнения обеспечивал независимость чувствительности камеры от времени и от температуры вплоть до 400° С с точностью до 2%, что являлось вполне достаточным, т.к. температура активной зоны при измерениях не превышала 250° С.

Активационные детекторы представляли собой фольги из Al, Cu, Au и таблетки из NaF. β-активность образцов измерялась с помощью торцевого счетчика Гейгера. Суммарный поток нейтронов всех энергий определялся только в тех местах реактора, где основная часть нейтронного спектра лежит в области постоянства сечения деления Pu^{239}. В этом случае камера деления с Pu^{239} являлась всеволновым детектором, счет которого $N_g = n_g \eta \sigma_f \varphi$ где n_g — число ядер Pu^{239} в камере деления;

η — эффективность регистрации актов деления;

φ — суммарный поток нейтронов;

σ_f — сечение деления Pu^{239}.

Величина n_g определялась, исходя из α-активности содержащегося в камере слоя Pu^{239}.

Поток быстрых нейтронов определялся аналогичным образом с помощью камер деления с естественным и обедненным ураном. Количества вещества в этих камерах определялись путем сравнения счета этих камер со счетом камеры деления с Pu^{239} при облучении их одинаковым потоком тепловых нейтронов. Непосредственные измерения спектров нейтронов проводились в вертикальном канале ОК-70, (см. рис. 1 и 2).

В канале ОК-70 спектр нейтронов измерялся с помощью ионизационной камеры, наполненной смесью He^3 и аргона. В канале Б-3 спектр нейтронов измерялся методом пропускания через водосодержащее вещество (n-гексан)

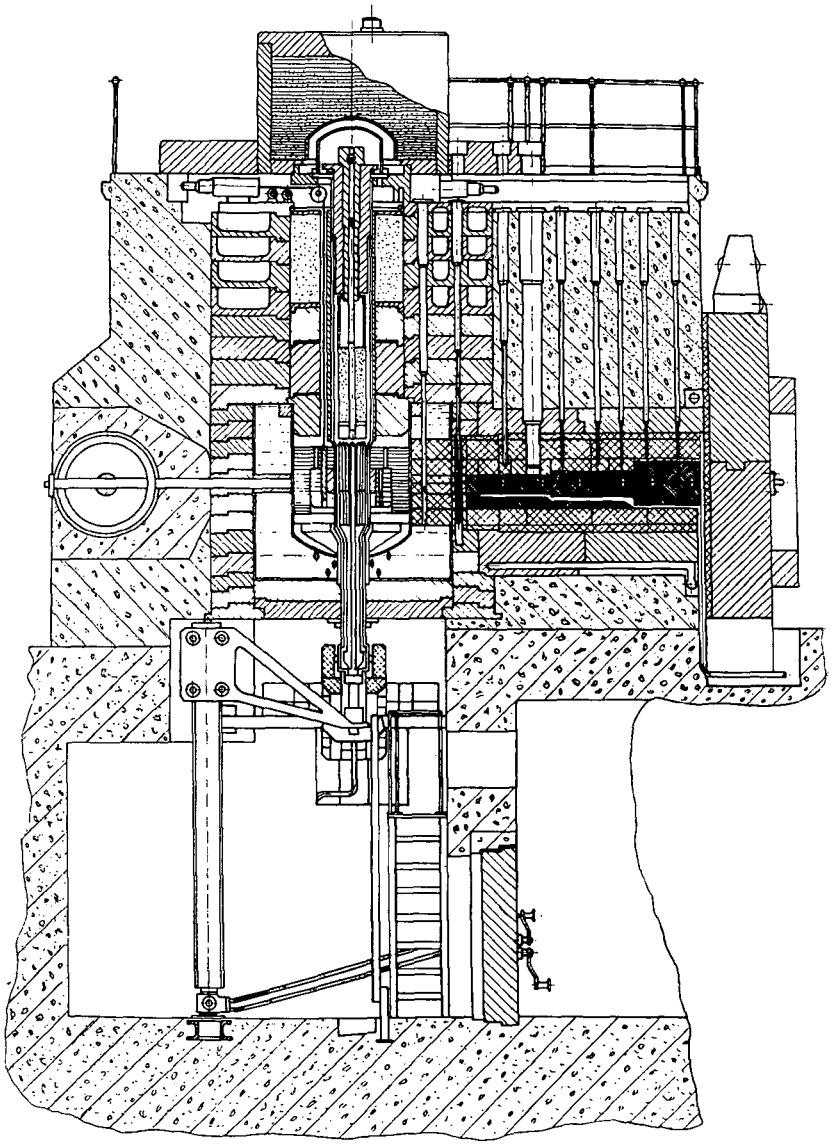


Рис. 1
Вертикальный разрез реактора БР-5.

в „хорошей" геометрии. В качестве детектора использовался борный счетчик. Кривая пропускания подвергалась обратному преобразованию Лапласа, полученный „оригинал" после учета энергетической чувствительности детектора и энергетического хода полных сечений водорода и углерода, содержащихся в рассеивателе, давал спектр нейтронов. Спектр нейтронов в канале Б-3 был измерен также методом фотоэмульсий. Ниже приводятся

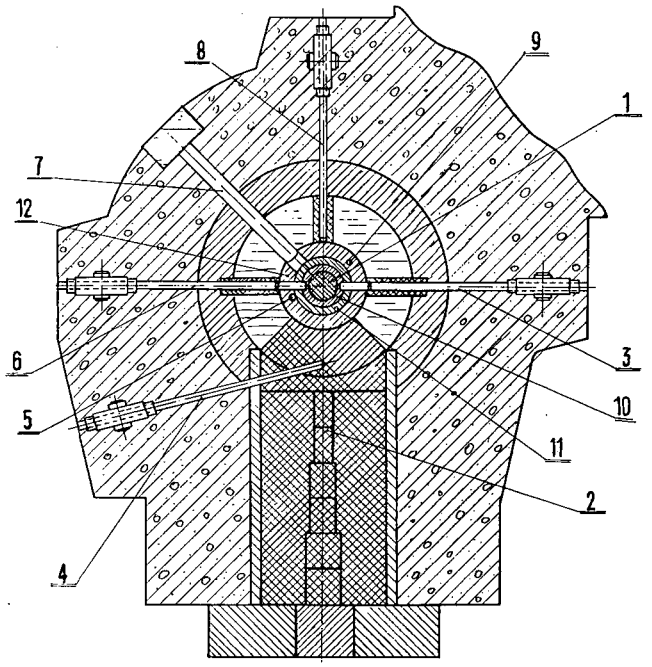


Рис. 2
Поперечный разрез реактора БР-5
1 — активная часть; 2 — выемная часть т. к.; 3 — канал Б-1; 4 — канал Т-4; 5 — канал ОК-70; 6 — канал Б-2; 7 — канал Б-2; 8 — канал П-2; 9 — канал ОК-50; 10 — компенсирующий цилиндр; 11 — экранный компенсатор; 12 — неподвижный никелевый экран.

результаты всех приведенных измерений. На рис. 3 приведены распределения скоростей счета различные детекторов вдоль вертикальной оси реактора. Граница активной зоны отмечена пунктиром. В центральной части активной зоны все распределения, в том числе и распределения делений Th^{232} и захватов в Au^{197}, имеющих совершенно различные энергетические зависимости сечений, подчиняются одному и тому же закону. Это указывает на то, что размеры активной зоны достаточно велики и в тех ее областях, которые далеки от границ, устанавливается постоянный энергетический спектр нейтронов. В этой области влияние отражателя не сказывается. В периферийных слоях активной зоны начинает сказываться влияние потока медленных нейтронов, идущего из отражателя. В результате этого распределения, снятые с помощью детекторов, обладающих различной энергетической чувствительностью, в этой области ведут себя по-разному. Наиболее сильно это различие проявляется в экране реактора, где нейтроны быстро теряют свою энергию в результате неупругого рассеяния, а затем продолжают упругое замедление. Благодаря сильной блокировке резонансного захвата резонансным рассеянием, вероятность избежать резонансного захвата при упругом замедлении в никеле достаточно велика, поэтому большая часть нейтронов проходит, замедляясь, через резонансную область в никеле не захватившись и дальше продолжает замедляться без заметного

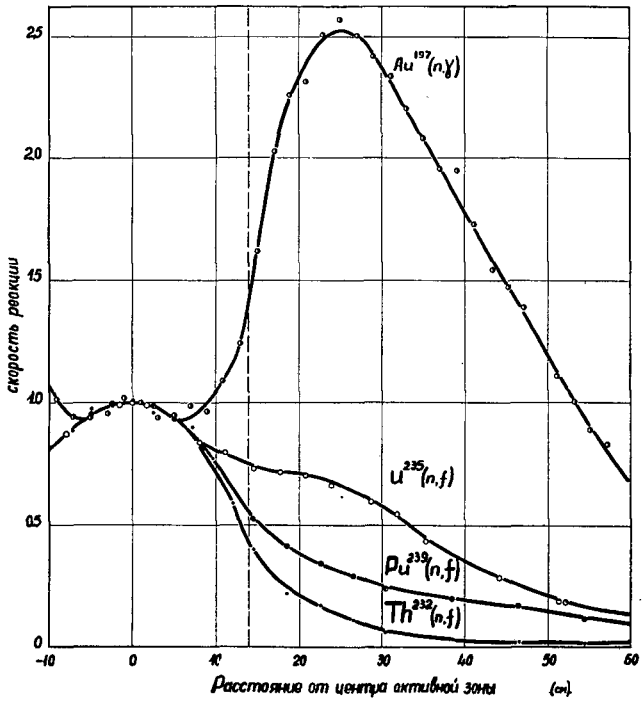


Рис. 3
Распределение скоростей различных реакций вдоль центрального канала реактора БР-5. Скорости всех реакций в центре активной зоны приняты за единицу.

поглощения до энергий порядка 10 эв, где начинает сказываться закон $1/V$. Описанная качественная картина объясняет тот факт, что плотность захватов нейтронов в золоте возрастает в отражателе почти в три раза по сравнению с центром активной зоны, несмотря на то, что нижние резонансы в использовавшихся золотых фольгах сильно блокированы. Резкое увеличение сечения при переходе от активной зоны к отражателю сказывается и на распределениях делений U^{235} и в меньшей степени Pu^{239}. Последнее обусловлено более слабой зависимостью сечения деления Pu^{239} от энергии.

На рис. 4 приведены распределения скоростей реакций Al^{27} (n, α) и захватов в Na^{23}, Cu^{63}, Au^{197} (тонкие индикаторы), измеренные в экспериментальном вертикальном канале, отстоящем от оси реактора на 5 см. Эти результаты подтверждают выводы, сделанные на основе рассмотрения данных, полученных в центральном канале реактора. Наиболее веско о сильном замедлении нейтронов в никелевом отражателе свидетельствуют распределения удельных активностей золотых фольг, закрытых с обеих сторон золотыми фильтрами различной толщины. Экстраполируя эти данные к „нулевой" толщине индикаторной фольги и „бесконечной" толщине фильтра, можно получить активность, обусловленную практически только нейтронами с энергиями в области резонанса $E_n = 4{,}9$ эв. На рис. 5 приведены распределения активности индикаторов без фильтра и с фильтрами раз-

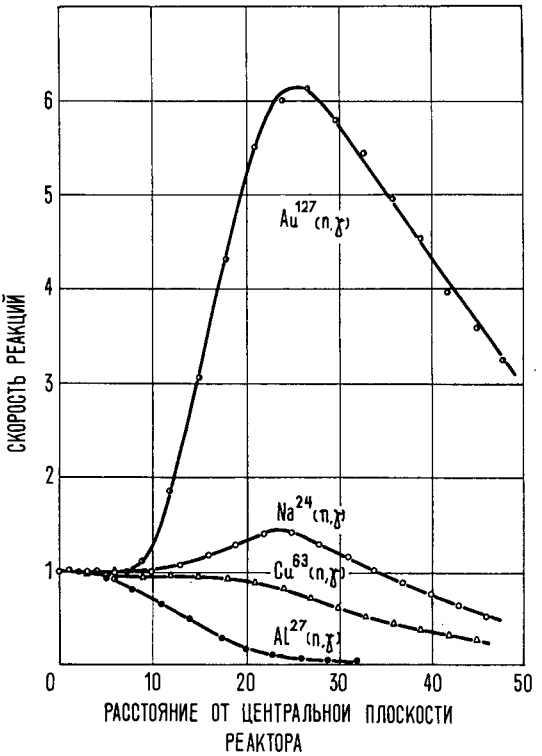


Рис. 4

Распределение числа реакций Al (n, d) и захватов в Na^{23}, Cu^{63}, Au^{197} (тонкий индикатор), измеренные в вертикальном канале, отстоящем от оси реактора на 5 см. Число реакций в центральной плоскости активной зоны принято за единицу.

личной толщины и распределения, полученные путем экстраполяции. На рис. 6 изображено пространственное распределение потока нейтронов с $E_n = 4{,}9$ эв по высоте вдоль экспериментального канала, вычисленное из экстраполированных активностей для номинальной мощности реактора.

В отличие от распределений, снятых с помощью детекторов, чувствительных к медленным нейтронам, распределение делений Th^{232} и реакций Al (n, α) обнаруживает резкий спад в отражателе реактора, что и следовало ожидать, т.к. быстрые нейтроны за счет упругого рассеяния в никеле быстро снижают свою энергию ниже порога этих реакций.

Распределения, приведенные на рис. 3, построены в относительных единицах. Для нормировки необходимо знать сечения соответствующих реакций в центре активной зоны реактора.

В центре активной зоны были измерены отношения сечений деления U^{235}, U^{238} и захвата в Au^{179} к сечению деления Pu^{239}. Полученные результаты приведены во второй колонке таблицы I.

Потоки нейтронов могли быть определены лишь в тех частях реактора, где основная часть нейтронов лежит в области постоянства сечения деления Pu^{239}, либо в области тепловых нейтронов. В экране реактора и выходящих из экрана пучках спектр нейтронов лежит в резонансной области, где се-

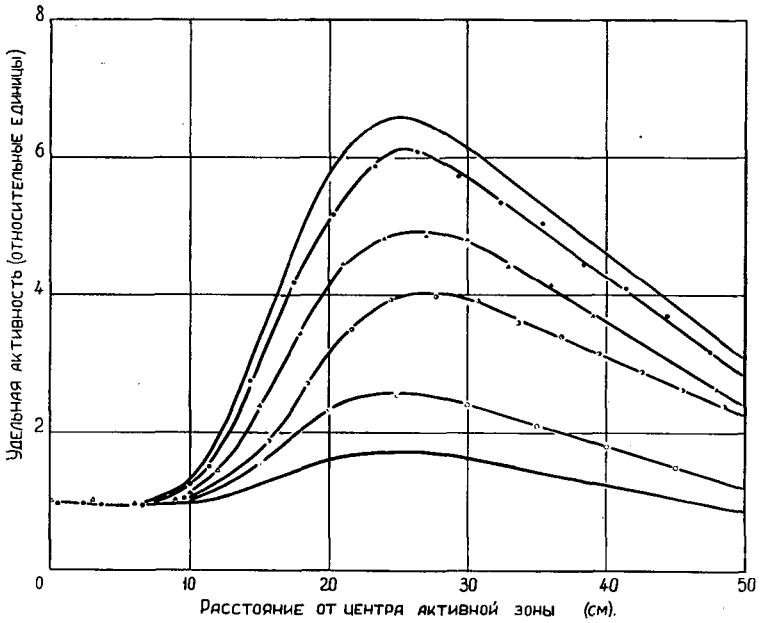


Рис. 5

Распределения удельной активности индикаторов без фильтров и с фильтрами различной толщины:

● — индикаторы толщиной 1,35 мг/см² без фильтра,

△, ◑ — те же индикаторы с фильтрами, толщиной 3,15 и 6,3 мг/см², соответственно,

○ — индикаторы толщиной 200 мг/см².

Верхняя кривая изображает удельную активность идеально тонкого индикатора. Нижняя получена путем экстраполяции к бесконечно толстому фильтру и представляет собой неблокируемую часть активности. Все применявшиеся в работе фильтры и индикаторные фольги были тонкими по отношению к энергетической области вне резонанса.

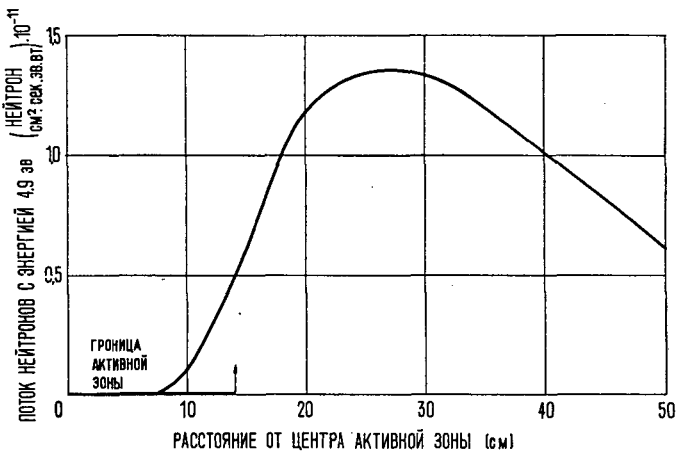


Рис. 6

Пространственное распределение потока нейтронов с $E_{п} = 4{,}9$ эв вдоль экспериментального канала.

Таблица I

СЕЧЕНИЯ РАЗЛИЧНЫХ РЕАКЦИЙ В ЦЕНТРЕ АКТИВНОЙ ЗОНЫ

Реакция	Эксперимент		Расчет
	σ/σ_f (Pu^{239})	σ (барны)	σ (барны)
Pu^{239} (n, f)	1	1,81	1,81
U^{235} (n, f)	0,82 ±0,03	1,48 ±0,06	1,46
U^{238} (n, f)	0,090±0,004	0,163±0,007	0,16
Au^{197} (n, γ)	0,121±0,005	0,22 ±0,01	0,21

Таблица II

ПОТОКИ НЕЙТРОНОВ В АКТИВНОЙ ЗОНЕ И ЭКСПЕРИМЕНТАЛЬНЫХ УСТРОЙСТВАХ РЕАКТОРА БР-5

нейтр./см^2 сек)

	Тепловые	Резонансные (на один log. интервал)	Быстрые (выше 1,4 Мэв)	Суммарный поток
Центр активной зоны	0	0	$(2,4\pm0,2)10^{14}$	$(8,2\pm0,3)10^{14}$
Пучок Б-1 и Б-3	0	0	$(2,2\pm0,2)10^{9}$	$(3,0\pm0,2)10^{10}$
Пучок П-2	0	0	$(1,2\pm0,4)10^{8}$	—
Пучок Т-4	$(8,5\pm0,5)10^{7}$	$(3,4\pm0,5)10^{6}$	$(1,0\pm0,3)10^{6}$	—
Максимальный поток нейтронов в тепловой колонке	$(5\pm2)10^{12}$	—	—	—

чение деление Pu^{239} сильно меняется, поэтому потоки нейтронов в этих случаях не могли быть определены с помощью использовавшихся методов.

Сводка данных о потоках дана в таблице II. Распределение потока тепловых нейтронов и кадмиевого отношения вдоль оси тепловой колонки измерялась с помощью радиоактивных индикаторов из Au^{179}, In^{115}, Cu^{63}. Усредненные данные приведены на рис. 7, кривая потока тепловых нейтронов нормирована к величине, полученной для канала АК. Кадмиевые отношения пересчитаны для детектора, обладающего чувствительность 1/V.

На рис. 8 и 9 приведен спектр нейтронов в пучке Б-3, идущем с поверхности активной зоны, определенный с помощью анализа кривой пропускания через водородосодержащую среду и методом фотоэмульсий. Второй максимум в этом спектре, лежащий при низкой энергии, обусловлен нейтронами, замедлившимися в отражателе. При энергиях выше 2,5 Мэв приведенный спектр совпадает со спектром нейтронов деления.

Спектр нейтронов в канале ОК-70, измеренный с помощью камеры, наполненной He^3, приведен на рис. 10.

Приведенные выше величины потоков нейтронов относятся к мощности, выделяемой в активной зоне (с учетом деления U^{238} и U^{235} в периферийных пакетах), равной 5000 квт. Определение мощности производилось, исходя из скорости счета камеры деления с Pu^{239} с известным количеством вещества и коэффициента неравномерности распределения делений по активной зоне. Кроме этого, на большой мощности реактора, порядка номинальной, была проведена градуировка мощности по тепловому балансу.

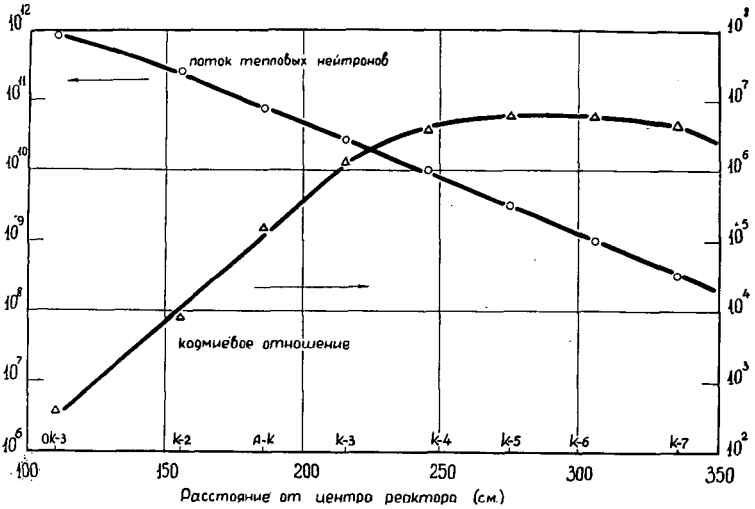


Рис. 7
Распределение потока тепловых нейтронов и R_{Cd} по тепловой колонке.

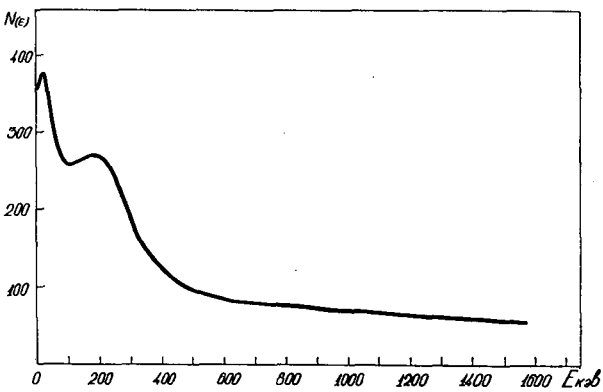


Рис. 8
Спектр нейтронов в пучке Б-3. Измерения выполнены методом пропускания нейтронов через водородосодержащую среду (*n* — гексан).

Количественный расчет пространственно-энергетических распределений нейтронов в реакторе БР-5 был выполнен для 9-ти и 18-ти групп. Распределение нейтронов, полученное для 9-ти групп путем решения кинетического уравнения в S_4-приближении методом Карлсона, удовлетворительно совпало с экспериментом Pu^{239}, U^{238} и активацией золота в активной зоне реактора (см. табл. 1). Это свидетельствует о правильности расчетного спектра в активной зоне реактора, приведенного на рис. 11. Спектр нейтронов, измеренный в интервале 0,1—1,2 Мэв в канале ОК-70, также дал удовлетворительное согласие с расчетным. Однако, экспериментальные кривые делений Pu^{239}, U^{235} и активации золота в отражателе, оказались к плохом согласии с теоретическими кривыми. Так, теоретическая кривая

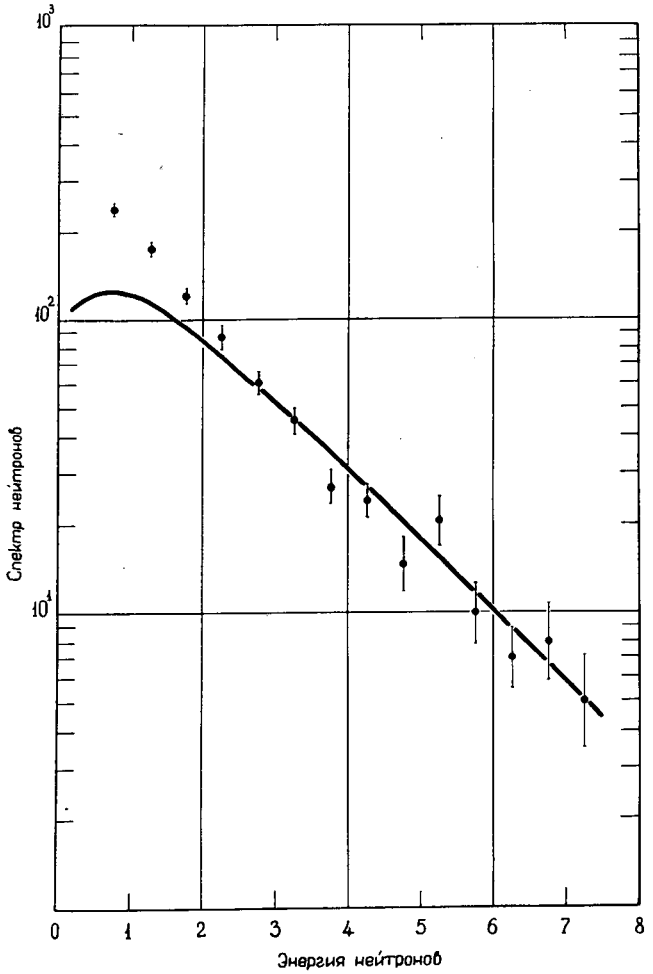


Рис. 9
Жесткая часть спектра нейтронов, выходящих из активной зоны, измеренная методом фотоэмульсий. Экспериментальные данные сравниваются с расчетной кривой $\sqrt{E_n}\, l^{-E_n/\theta}/\theta/\theta = 1{,}44$ Мэв [3] для спектра мгновенных нейтронов деления Pu^{-239}.

активации золота спадает в отражателе, тогда как экспериментальная кривая имеет резко выраженный максимум. В связи с этим были предприняты расчеты пространственно-энергетических распределений нейтронов в 18-ти групповом приближении для сферизованной модели реактора. Эти расчеты привели к лучшему согласию с экспериментом, что иллюстрируется рисунком 12, где сравнивается теоретическая и экспериментальная кривая активации золота. Несмотря на явное улучшение, полного согласия расчета с экспериментом нет. Повидимому, основной причиной этого расхождения является то, что исходный для расчета сферизованный и

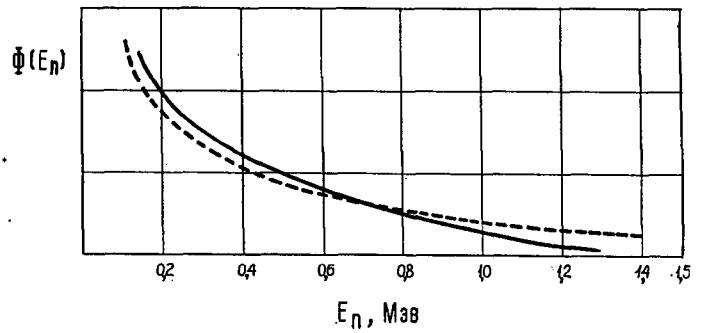


Рис. 10
Спектр нейтронов в канале ОК-70. Пунктирная кривая—расчетная.

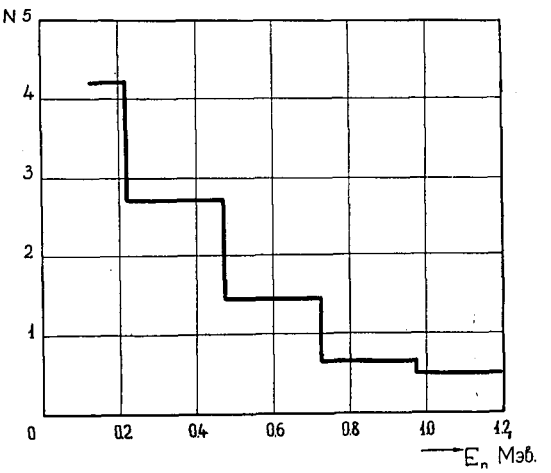


Рис. 11
Расчетный спектр нейтронов в центре активной зоны БР-5.

гомогенизированный вариант системы весьма далек от реальной конструкции реактора. Точный учет конструктивных особенностей (кольцевые зазоры, детали реактора за пределами отражателя и т.п.), представляет собой сложную задачу. Возможно, также, что при расчете была не совсем правильно подобрана система констант. Следует отметить, однако, что отдельно выполненное исследование пространственно-энергетического распределения нейтронов в никеле [4] дало удовлетворительно согласующиеся с расчетом результаты.

Длительное облучение урана в реакторе позволяет измерить $\alpha=\overline{\sigma_c}/\overline{\sigma_f}$, — отношение средних сечений радиационного захвата и деления — характеристику, в существенной степени определяющую воспроизводство ядерного горючего. Для этого один из урановых стержней кольцевого отражателя,

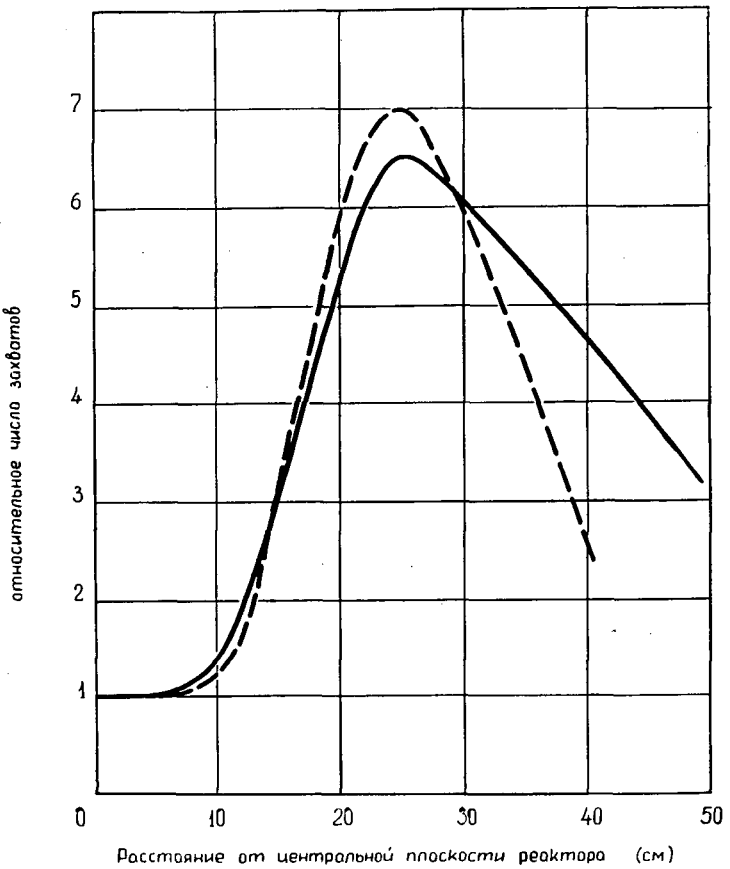


Рис. 12
Сравнение экспериментальной и расчетной кривых активации золота. *Сплошная кривая* — экспериментальная, пунктирная — расчетная.

облученный в потоке нейтронов $6{,}3 \cdot 10^{21}$ см^2, был подвергнут радиохимической переработке. Путем измерения концентрации Pu^{239} в уране (по удельной α-активности) и Pu^{240} в плутонии (по числу спонтанных делений) было определено отношение средних сечений радиационного захвата Pu^{239} и U^{238}. Значение этой величины, усредненное по длине стержня, оказалось равным $1{,}85 \pm 0{,}2$, что находится в удовлетворительном согласии с результатом расчета для 18-ти групп нейтронов — 1,95. Вычисленное из данных опыта и расчета отношение α составляет $0{,}19 \pm 0{,}01$.

3. Некоторые особенности кинетики и измерение возмущений

При исследовании реактора одной из важных проблем является проблема безопасной работы реактора. Существенную роль в этом вопросе играют мощностные эффекты, т.е. влияние изменений мощности на реактивность.

Происхождение мощностных эффектов в быстрых реакторах связано, главным образом, с разогревом и расширением элементов активной зоны

и отражателя. Неоднородность разогрева и различие в температурных коэффициентах расширения приводит к взаимному перемещению этих элементов и их прогибам. Расширение, связанное с однородным разогревом, и соответствующее изменение реактивности характеризуют обычно температурным коэффициентом. Условия однородного разогрева возникают при повышении температуры натрия на входе, и соответствующий температурный коэффициент равен на БР-5:

$$\alpha_T = -2{,}8 \cdot 10^{-5} \frac{1}{^\circ C}.$$

Изменение формы активной зоны, связанное с неоднородным разогревом от тепловыделения в реакторе (при постоянстве температуры натрия на входе), и соответствующее изменение реактивности характеризуют асимптотическим мощностным коэффициентом. На БР-5 этот коэффициент равен:

$$\alpha_M = -2 \cdot 10^{-4} \frac{1}{\text{мвт}}.$$

Изменение реактивности, обязанное изменению мощности, устанавливается не сразу. Характер такого переходного режима к асимптотическому мощностному и температурному эффекту играет очень большую роль для проблемы устойчивой работы реактора. В общем виде связь мощности и реактивности с учетом дополнительных внешних воздействий изображена на рис. 13.

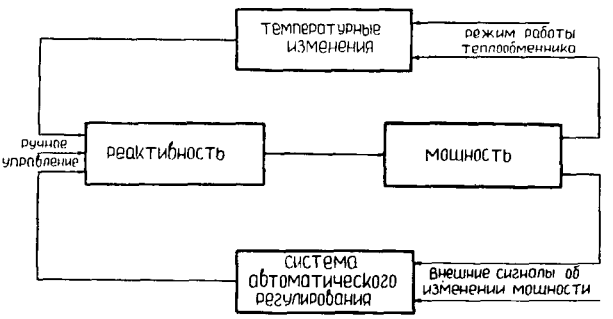


Рис. 13
Связь мощности и реактивности с учетом дополнительных внешних воздействий.

Исследование мощностных эффектов на БР-5 имело целью получение сведений о характере зависимости реактивности от мощности.

Если обозначить зависимость мощности от реактивности значком $m(\varrho)$, а зависимость реактивности от мощности $\varrho(M)$, то реактор можно представить (см. рис. 14) в виде четырехполюсника с обратной связью (при отключенной системе АР и постоянном режиме работы теплообменника), с зависимостью мощности от влияний реактивности $M(\varrho)$.

Аналогичное представление электрических и др. усилителей позволяет методы исследований, развитые в этой области, применить и к реактору. Для анализа свойств электрических усилителей широко применяют два способа:

1. Метод гармонического анализа, т.е. измерение частотнофазовых характеристик выходного параметра при синусоидальном изменении входного параметра на разных частотах.

2. Импульсный метод, т.е. наблюдение реакции выходного параметра при скачкообразном изменении входного.

При первом методе измеряется передаточная, функция $M_0(\gamma\omega)$, т.е. отношение комплексной амплитуды изменения мощности к комплексной амплитуде изменения реактивности. Когда измерения производят на малой мощности, то влияние мощности на реактивность мало (мощность на реактивность влияет пропорционально абсолютной величине изменения, а не относительной), и передаточная функция будет другой — $m_0(\gamma\omega)$.

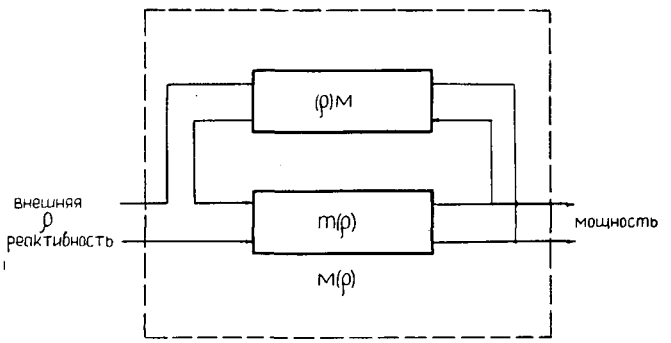


Рис. 14
Схематическое представление реактора в виде четырехполюсника с обратной связью.

Если обозначить через $M\varrho_0(\gamma\omega)$ передаточную функцию цепочки обратной связи мощности и реактивности (M — среднее значение мощности, $\varrho(\gamma\omega)$ — комплексная функция), то согласно теории систем с обратной связью

$$M_0(\gamma\omega) = \frac{M_0(\gamma\omega)}{1 - \bar{M}\,\varrho_0(\gamma\omega)\,M_0(\gamma\omega)}$$

отсюда

$$\bar{M}\varrho(\gamma\omega) = \frac{1}{M_0(\gamma\omega)} - \frac{1}{M_0(\gamma\omega)}.$$

Измерения на нулевой и некоторой отличной от нуля мощности позволяют в принципе рассчитать передаточную функцию для любой мощности.

Экспериментальное изучение $m_0(\gamma\omega)$ и $M_0(\gamma\omega)$ было широко использовано, например, на реакторе EBR-1 [5].

Второй метод, перенесенный на реактор, сводится к скачкообразному изменению реактивности и последующему наблюдению переходных процессов изменения мощности $M1(t)$ и $m1(t)$.

Принципиально оба способа эквивалентны, т.к. наблюдаемые функции взаимосвязаны [6]:

$$M_0(\gamma\omega) = \gamma\omega \int_0^\infty M_1(t)\,e^{-\gamma\omega t}\,dt$$

$$M_0(\gamma\omega) = \gamma\omega \int_0^\infty M_1(t)\, e^{-\gamma\omega t}\, dt$$

$$M_1(t) = \frac{1}{2\pi} \int_{\sigma-\gamma\omega}^{\sigma+\gamma\omega} \frac{M_0(\gamma\omega)}{\gamma\omega}\, e^{-\gamma\omega}\, d\omega$$

$$M_1(t) = \frac{1}{2\pi} \int_{\sigma-\gamma\omega}^{\sigma+\gamma\omega} \frac{M_0(\gamma\omega)}{\gamma\omega}\, e^{-\gamma\omega}\, d\omega.$$

На EBR-1 проводились эксперименты по определению $M_1(t)$.

Существует возможность использовать видоизменение этого способа, т.е. наблюдать изменение реактивности, соответствующее скачкообразному изменению мощности (входной и выходной параметры четырехполюсника меняются местами).

Пусть соответствующие функции будут

$R_0(\gamma\omega)$, $r_0(\gamma\omega)$ — передаточные функции, на мощности и достаточно малой мощности;

$R_1(t)$, $r_1(t)$ — переходные характеристики, т.е. изменения реактивности, соответствующие скачкообразному изменению мощности;

$\varrho_0(\gamma\omega)$, $\varrho_1(\gamma\omega)$ — передаточная функция и переходная характеристика цепочки обратной связи,

$$R_0(\gamma\omega) = \gamma\omega \int_0^\infty R_1(t)\, e^{-\gamma\omega t}\, dt$$

и аналогично для $r_0(\gamma\omega)$, $R_1(t)$ и $r_1(t)$.

При этом:

$$R_0(\gamma\omega) = \frac{1}{M_0(\gamma\omega)},\quad r_0(\gamma\omega) = \frac{1}{M_0(\gamma\omega)}.$$

Переходная характеристика зависимости реактивности от мощности получается здесь непосредственно, как разность двух измеренных функций:

Указанный метод и был использован при исследовании на БР-5.

Как это видно из приведенных соотношений, все методы принципиально эквивалентны, но практические особенности их осуществления оставляют каждый из методов хорошим для решения одних вопросов и ограниченным для решения других.

1. Частотно-фазовая характеристика может быть получена с большей точностью, чем переходные характеристики.

2. Частотно-фазовая характеристика непосредственно может дать ответ о частоте и граничной мощности, за которой возможных незатухающие и нарастающие колебания мощности.

3. Переходные характеристики более непосредственно могут быть использованы для расчета нестационарных режимов реактора (т.к. здесь непосредственно участвуют временные функции). Использование для этих целей передаточных функций включает трудности преобразований и соответствующее ухудшение точности при использовании приближенных способов.

4. Последний из поведенных методов при своем осуществлении использует систему автоматического регулирования („АР“), которая и обеспечивает

иско мое изменение реактивности, необходимое для скачкообразного изменения мощности. Использование системы „АР" создает преимущество, т. к. позволяет исследовать любые режимы, обеспечивая при этом безопасность проведения эксперимента. Пример расплавления активной зоны Марк II реактора EBR-1 показывает, что пренебрегать этим нельзя.

5. Инерционные свойства системы „АР" ограничивают возможность исследования этим способом процессов, протекающих достаточно быстро.

При проведении экспериментов скачкообразно изменялся потенциал задатчика мощности. Изменения составляли 5—20% от первоначального значения. Система АР обеспечивала изменение мощности за время —0,3 сек.

Положение сельсина — указателя фиксировалось самописцем, откуда было видно, что точность воспроизведения результатов по реактивности

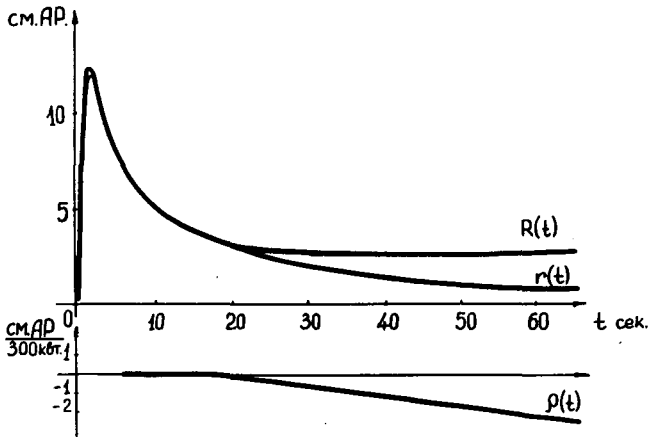


Рис. 15
Изменения реактивности, соответствующие скачкообразному изменению мощности при нормальном режиме работы.

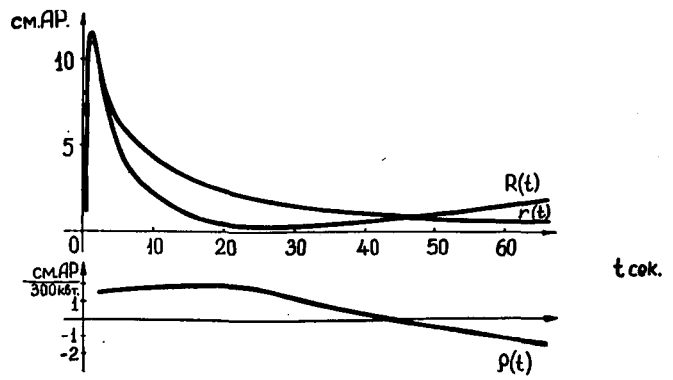


Рис. 16
Изменения реактивности, соответствующие скачкообразному изменению мощности при режиме работы, существующем в настоящее время.

была 5.10^{-5}%. Инерционность указателя и самописца не позволяет получить функции $R_1(t)$ и $r_1(t)$ в интервале времени от 0 до 3—5 сек после скачка мощности.

Соотношение $\overline{M}\ \varrho_1(t)=r_1(t)-R_1(t)$, получает здесь наглядное физическое объяснение.

Положение регулятора $r_1(t)$ обеспечивает такое изменение реактивности, которое необходимо для почти скачкообразного изменения мощности. Очевидно, что и на больших мощностях для того же относительного изменения мощности необходимо такое же изменение коэффициента размножения. Новое поведение регулятора $R_1(t)$ указывает на то, что часть изменения реактивности обязано влиянию мощности.

Наиболее интересные случаи кривых $R_1(t)$ и $\varrho_1(t)$ представлены на серии рисунков 15, 16, 17, где 1 см АР составляет 2.10^{-5} единиц реактивности.

Рис. 15 изображает нормальный режим работы, имевший место на первом этапе. Мощностной эффект отсутствует в течение 25 сек и затем становится отрицательным. Нарастание отрицательного эффекта обязано появлению на входе реактора более горячего натрия, совершившего полный круг в первом контуре.

Следует отметить, что перед скачком мощности имело место температурное равновесие.

Рис. 16 изображает характеристику режима работы, который существует в настоящее время. Отчетливо заметен положительный эффект, имеющий место в течение 30 сек.

Рис. 17 дает характеристику режима естественной циркуляции. Мощностной эффект положителен во все время наблюдения (до 3 мин.), хотя и уменьшается после максимума в 30—40 сек.

Таким образом, описанный выше метод позволил получить переходные характеристики влияния мощности на реактивность в режимах работы, где без системы „АР" имеет место неустойчивость, и где проведение исследований представляет немало трудностей другими путями.

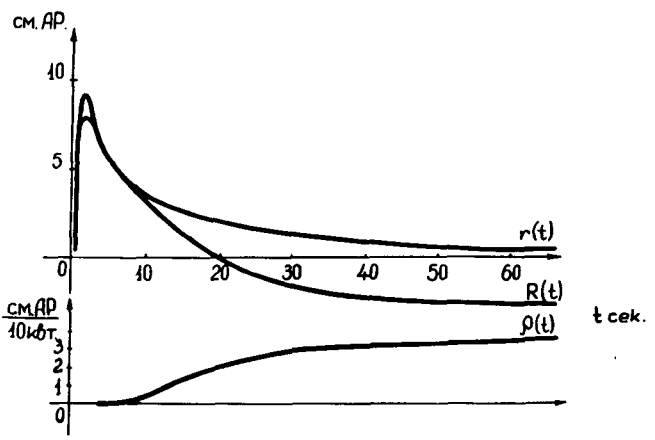


Рис. 17
Изменения реактивности, соответствующие скачкообразному изменению мощности при режиме естественной циркуляции.

В настоящее время работы по изучению мощностных эффектов и их природы продолжаются. Помимо изучения кинетики на реактора БР-5 в подкритическом и критическом режиме были измерены изменения реактивности при движении органов регулирования, заполнении активной зоны натрием,

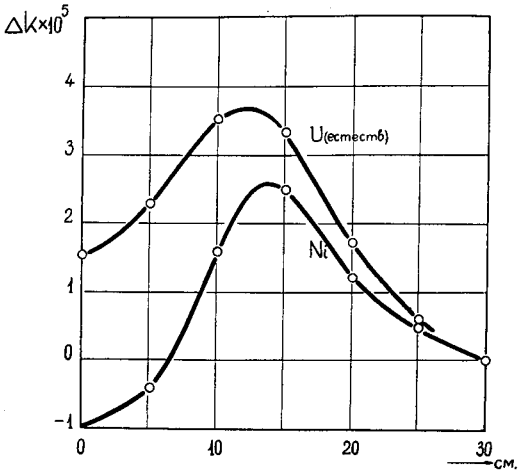


Рис. 18
Изменение реактивности при движении образцов по центральному каналу реактора.

извлечении отдельных топливных пакетов, а также при перемещении некоторых образцов по петлевому каналу. Некоторые результаты этих измерений приводятся на рис. 18 и табл. III. Данные, приведенные на рис. 18, относятся к 1 см^3 вещества.

ТАБЛИЦА III

ИЗМЕНЕНИЯ РЕАКТИВНОСТИ ПРИ ПЕРЕМЕЩЕННИЕ ОРГАНОВ РЕГУЛИРОВАНИЯ, ТОПЛИВНЫХ ПАКЕТОВ, ОБРАЗЦОВ И ЗАПОЛНЕНИЯ АКТИВНОЙ ЗОНЫ НАТРИЕМ.

Эффект	ΔK в %
1. Подъем компенсирующего цилиндра	3,6
2. Подъем экранного компенсатора	6,5
3. Подъем регулирующего стержня AP_1	0,057
AP_2	0,065
4. Извлечение центрального топливного пакета	1,1
5. Извлечение периферийного топливного пакета	0,6
6. Заполнение активной зоны натрием	4,0
7. Эффективная доля запаздывающих нейтронов	$0{,}247 \pm 0{,}00002$

ЛИТЕРАТУРА

[1] ЛЕЙПУНСКИЙ, А. И. *и др.* Экспериментальные быстрые реакторы в СССР. Доклад № 2129 на II Международной конференции по мирному использованию атомной энергии. Женева, 1958 г.
[2] ГОЛУБЕВ, В. И. *и др. Атомная энергия* (в печати).

[3] НИКОЛАЕВ, М. Н. *и др.* Распространение нейтронов в никелевом экране реактора на быстрых нейтронах. Доклад, представленный на Международный симпозиум по реакторам на быстрых и промежуточных нейтронах. Вена. 1961 г.
[4] ТЭТГОТТ, Ф. *и др.* Исследования устойчивости на реакторе EBR-1. Доклад № 1845 (США) на II Международной конференции по мирному использованию атомной энергии. Женева, 1958 г.
[5] РИЗКИН, А. А. Основы теории усилительных систем. 1954 г., стр. 55.

SM-18/61

ADVANCED EPITHERMAL THORIUM REACTOR (AETR) PHYSICS

A. V. CAMPISE
ATOMICS INTERNATIONAL, CANOGA PARK, CALIF.,*
UNITED STATES OF AMERICA

Abstract — Résumé — Аннотация — Resumen

Advanced epithermal thorium reactor (AETR) physics. The AETR concept is reviewed in reference to existing theory, nuclear parameters, and potential neutron economy.

The effect of thorium resonance capture in graphite-moderated systems with median absorption energies in the range from 0.10 to 100 keV have been studied. Narrow-resonance (NR) and wide-resonance (NRIA) formulations are used to obtain the temperature-dependent effective resonance integral of thorium rods which are expressed as equivalent multi-group cross-sections.

The need for nuclear data in the intermediate energy range led to design and construction of a critical assembly. Nuclear design of this assembly emphasizes the importance of cross-section data and the theoretical interpretation of these experimental results, both pertinent to the design of an AETR. The accuracy of the analytical techniques has been demonstrated in the analysis of ZPR-III experimental results.

Three heat-transfer configurations are compared using doubling time as an optimizing parameter. The effect of Pa^{233} and uranium-isotopes production on relative neutron economy, potential breeding ratios, and burn-up characteristics are evaluated in regard to the uncertainties in the nuclear cross-sections.

Physique d'un réacteur au thorium, à neutrons épithermiques, de type perfectionné (AETR). L'auteur analyse la conception de l'AETR du point de vue de la théorie actuelle, des paramètres nucléaires et du bilan potentiel de neutrons.

On a étudié l'effet de la capture par résonance dans le thorium, dans des systèmes ralentis au graphite pour des énergies d'absorption moyennes comprises entre 0,10 et 100 keV. On utilise des formules de résonance étroite et de résonance large pour obtenir l'intégrale de résonance effective en fonction de la température, relative aux barres de thorium dont on tient compte dans ces calculs en les exprimant sous forme des sections efficaces multigroupes équivalentes.

Pour obtenir les données nucléaires indispensables pour la gamme d'énergies intermédiaires, on a été amené à étudier et à construire un ensemble critique. L'étude de cet ensemble a mis en relief l'importance des données relatives aux sections efficaces et de l'interprétation théorique des résultats expérimentaux pour l'étude d'un réacteur au thorium de type perfectionné. La précision des méthodes analytiques employées a été démontrée lors de l'analyse des résultats expérimentaux obtenus avec le ZPR-III.

L'auteur compare trois configurations pour le transfert de chaleur, en utilisant le temps de doublement comme paramètre d'optimisation. Les effets de la production de ^{233}Pa et d'isotopes de l'uranium sur le bilan neutronique, les taux possibles de surgénération et les caractéristiques de la combustion sont évalués en tenant compte de l'imprécision des sections efficaces nucléaires.

---

* Under contract with Southwest Atomic Energy Associates.

Физика усовершенствованного надтеплового ториевого реактора. Рассматриваются принципы конструирования усовершенствованного надтеплового ториевого реактора с учетом существующей теории ядерных параметров и потенциального полезного использования нейтронов.

Был изучен эффект резонансного захвата тория в системах с графитовым замедлителем для нейтронов с энергиями от 0,10 до 100 кэв. Используются формулы узкого резонанса и широкого резонанса в целях получения зависимого от температуры эффективного резонансного интеграла ториевого стержня, который выражается в виде эквивалентных многогрупповых сечений.

Необходимость в получении ядерных данных в области промежуточных энергий привела к созданию проекта и конструкции критической сборки. Ядерный проект этой сборки подчеркивает важность данных поперечных сечений и теоретической интерпретации экспериментальных результатов, имеюших отношение к усовершенствованному надтепловому ториевому реактору. Точность аналитических методов была подтверждена при анализе экспериментальных результатов, полученных на реакторе нулевой мощности ZPR-III.

Проводятся сравнения трех конфигураций теплопередачи с использованием удвоенного времени в качестве оптимального параметра. Эффект производства изотопа протактиния-233 и урана при относительно полезном использовании нейтронов, возможные коэффициенты воспроизводства и характеристики выгорания оцениваются в связи с неточностями в ядерных поперечных сечениях.

Física del reactor epitérmico de tipo avanzado, alimentado con torio (AETR). El autor estudia la concepción del reactor AETR desde el punto de vista de la teoría actual de los parámetros nucleares y del balance neutrónico.

En los sistemas moderados por grafito examina el efecto de la captura por resonancia en el torio para energías medias de absorción del orden de 0,10 a 100 keV. Aplica fórmulas de resonancia angosta y de resonancia ancha para obtener la integral de resonancia efectiva en función de la temperatura, correspondiente a las barras de torio, y dicho parámetro se expresa como secciones eficaces equivalentes de varios grupos.

Se ha diseñado y construido un conjunto crítico para obtener datos nucleares indispensables en la gama de energías intermedias. En el diseño nuclear de dicho conjunto, se ha tenido particularmente en cuenta la importancia de los datos relativos a secciones eficaces y la interpretación teórica de estos resultados experimentales, cosas ambas relacionadas con el diseño del reactor AETR. La precisión de los métodos analíticos ha quedado demostrada por el estudio de los resultados experimentales obtenidos con el reactor ZPR-III.

Se comparan tres sistemas de transmisión de calor utilizando el tiempo de duplicación como parámetro óptimo. Se estudia el efecto de la formación del ^{233}Pa y de isótopos del uranio sobre el balance neutrónico relativo y se evalúa la probable razón de reproducción y las características de combustión teniendo en cuenta la imprecisión en el conocimiento de las secciones eficaces nucleares.

Introduction

The Advanced Epithermal Thorium Reactor (AETR)* is a sodium-cooled system designed to operate on the Th-U^{233} cycle with a neutron spectrum considerably above thermal, and as such represents a new concept in the field of power reactors. Although good data exist in the neutron energy range of from thermal to about 1 keV, and from 0.1 to several MeV, only limited data exist in the intermediate range of 1 to 100 keV, the range of interest in the AETR.

* Sponsored by Southwest Atomic Energy Associates.

The importance of integral nuclear data in this intermediate range has prompted the design of a flexible critical experiment.

In this paper, the diffusion theory utilized in the AETR nuclear analysis will be developed, existing nuclear data summarized, and new data computed. Nuclear design of the critical experiment will then be discussed, with special emphasis on the interpretation of results into information needed for a confident design of a full-scale power plant.

Theory

Multi-group diffusion theory was used in the analysis of all AETR nuclear problems*. The basic form

$$G(u, x)\,\varphi(u, x) = S(u, x) = S_f(u) + S_{i\text{-}j}(u, x) \tag{1}$$

indicates a source term (S), which contains fission (S_f) and scattering ($S_{i\text{-}j}$) contributions. An energy transfer matrix was used for the slowing-down of neutrons:

$$S_{i\text{-}j}(u, x) = \int_u \sigma_s(u \to u')\,\varphi(u, x)\,\mathrm{d}u\,. \tag{2}$$

The differential operator in Eq. (1) has the form

$$G(u, x) \cong \operatorname{div}(D \operatorname{grad} \varphi) \tag{3}$$

where D is the diffusion coefficient of the medium. Intermediate spectrum reactors like AETR are high-density-uranium systems due to the averaged neutron energy in the system, and hence an asymptotic solution of the transport equation, in an absorbing medium, was used to modify the diffusion coefficient; i.e.

$$D = \frac{\Sigma_a}{K^2}, \tag{4}$$

where

$$K^2 = 3\,\Sigma\Sigma_a\,(1 - \overline{\mu_0})\left(1 - \frac{4}{5}\frac{\Sigma_a}{\Sigma} + \frac{\Sigma_a}{\Sigma}\frac{\overline{\mu_0}}{1 - \overline{\mu_0}}\right).$$

One boundary condition permits the derivative of the neutron flux to be set equal to zero at the centre of the medium:

$$\left(\frac{\mathrm{d}\varphi}{\mathrm{d}x}\right)_{x=0} = 0. \tag{5}$$

An additional boundary condition [1] may be expressed by

$$\varphi + \omega\frac{\mathrm{d}\varphi}{\mathrm{d}r} = 0 \tag{6}$$

where ω is the extrapolation distance; i.e.

$$\omega = \frac{2.13D}{1 + 1.066\,PD/R} \tag{7}$$

where P=0, 1, or 2 for plane, cylinder or sphere,
R=radius of reactor.

These modifications to diffusion theory have increased the accuracy of the theory in studies of intermediate neutron energy spectrum reactors.

* A table of nomenclature is included at the end of this paper (see p. 353).

Nuclear data

The basic neutron cross-sections used in the analysis of the AETR concept were developed by C. B. MILLS [2]. Special attention was given to cross-sectional data for thorium and U^{233}.

THORIUM

Recent resonance parameters for thorium were obtained from Columbia University [3]. Effective resonance integrals were obtained for thorium-carbide (the fuel base for AETR) using these data. Narrow-resonance (NR) and narrow-resonance infinitely-heavy-absorber (NRIA) formulations were used in computing effective resonance integrals at different temperatures. Conditions for using NR or NRIA formulations are listed below.

a) NR/maximum energy loss per collision greater than practical width:

$$I_r = I_\infty \Big/ \left[1 + \beta \frac{\sigma_a}{\sigma_e}\left(1 - \frac{\Gamma_n}{\Gamma}\frac{\sigma_0}{\sigma_e}\right)\right]^{1/2} \tag{8}$$

b) NRIA/maximum energy loss per collision less than practical width:

$$I_r = I_\infty \Big/ \left(1 + \beta \frac{\Gamma_\gamma}{\Gamma}\frac{\sigma_0}{\sigma_m}\right)^{1/2} . \tag{9}$$

Only four resonances required the NRIA formulation. The unresolved region up to 30 keV was computed with the NR equation. The Porter-Thomas distribution of neutron widths was used in correcting the unresolved energy region. The corrections and results are listed in Table I.

TABLE I

INFINITELY DILUTE RESONANCE INTEGRAL

	(barns)
(a) Resolved	89.31
(b) Unresolved	
$L=0$; $j=1/2$	3.60
$L=1$; $j=1/2$	0.20
$L=1$; $j=3/2$	0.42
>30 keV	1.00
Total	94.53

TABLE II

RESONANCE INTEGRALS OF THORIUM CARBIDE RODS

(barns)

T (°F)	S/M (cm²/g)		
	0.498	0.435	0.348
200	16.76	16.02	14.95
650	17.96	17.20	16.05
950	18.66	17.84	16.63
1250	19.28	18.41	17.14

Resonance integrals for various surface to mass ratios of thorium-carbide rods were evaluated at AETR core temperature conditions. Results are listed in Table II.

Resonance integral data were averaged into multi-group cross-sections (Table III) used in the AETR diffusion programme.

TABLE III

AVERAGE THORIUM ABSORPTION CROSS-SECTIONS (from 0.1 MeV to 8.315 eV)

Lower energy of neutron group	200 °F	650 °F	950 °F	1250 °F
		$S/M = 0.498$		
17 keV	0.725	0.726	0.726	0.726
3 keV	0.810	0.827	0.836	0.844
0.454 keV	1.58	1.698	1.758	1.809
61.44 eV	3.038	3.336	3.500	3.651
22.6 eV	3.059	3.252	3.407	3.534
8.315 eV	1.865	2.010	2.093	2.168
		$S/M = 0.435$		
17 keV	0.725	0.725	0.726	0.726
3 keV	0.803	0.822	0.832	0.839
0.454 keV	1.536	1.655	1.715	1.766
61.44 eV	2.880	3.169	3.325	3.462
22.6 eV	2.864	3.069	3.189	3.299
8.315 eV	1.730	1.866	1.940	2.0077
		$S/M = 0.342$		
17 keV	0.725	0.725	0.726	0.726
3 keV	0.793	0.813	0.824	0.832
0.454 keV	1.471	1.590	1.650	1.702
61.44 eV	2.650	2.916	3.060	3.184
22.6 eV	2.585	2.771	2.866	2.959
8.315 eV	1.550	1.657	1.718	1.773

TABLE IV

ALPHA VALUES FOR U^{233}

E (keV)	$\overline{\alpha}$ (U^{233})
30	0.1085 ± 0.0220
60	0.1085 ± 0.0254
175	0.0983 ± 0.0202
250	0.0909 ± 0.0221
400	0.0780 ± 0.0103
600	0.0619 ± 0.0092
750	0.0472 ± 0.0086
900	0.0496 ± 0.0081
1000	0.0295 ± 0.0080

TABLE V

NEUTRON GROUP CROSS-SECTIONS FOR U^{233}

E_L (from 10 MeV)	η_g	$\nu\sigma f_g$	σ_a	σ_a	$\sigma_{g-1\to g}$	$\sigma_{g-2\to g}$	$\sigma_{g-3\to g}$	$\sigma_{g-4\to g}$	$\sigma_{g-5\to g}$
3.0 MeV	2.936	5.285	1.80	4.368	0	0	0	0	0
1.4 MeV	2.587	4.941	1.91	4.650	0.2	0	0	0	0
0.9 MeV	2.466	4.933	2.00	4.999	0.18	0.27	0	0	0
0.4 MeV	2.385	4.986	2.09	6.027	0.45	0.50	0.45	0	0
0.1 MeV	2.294	5.667	2.47	8.988	0.29	0.30	0.35	0 31	0
7 keV	2.283	8.107	3.55	12.58	0.05	0.05	0.06	0.05	0.04
3 keV	2.2817	15.00	6.574	17.62	0.05	0	0	0	0
0.454 keV	2.281	22.50	9.861	20.91	—	—	—	—	—
61.44 eV	1.33	60.0	45.0	56.0	—	—	—	—	—
22.60 eV	2.33	133.0	57.0	68.0	—	—	—	—	—
8.315 eV	2.14	255.0	119.0	129.0	—	—	—	—	—
3.059 eV	2.05	225.0	110.0	120.0	—	—	—	—	—
1.1256 eV	2.11	725.0	244.0	354.0	—	—	—	—	—
0.4141 eV	2.05	330.0	161.0	171.0	—	—	—	—	—
0.1523 eV	2.276	456.0	200.0	210.0	—	—	—	—	—
0.0561 eV	2.27	706.0	310.0	320.0	—	—	—	—	—
0.0253 eV	2.27	1104.0	485.0	495.0	—	—	—	—	—
Thermal	2.28	1187.0	520.0	530.0	0.05	—	—	—	—

URANIUM-233

Basic cross-section information published in the literature consists of fission and total cross-sectional data from thermal energies to 1 keV but having poor resolution at energies above 500 eV. Alpha values for U^{233} at energies from 30 keV to 1000 keV have been obtained at LASL by B. C. DIVEN [5]. These are given in Table IV.

These data have been weighed and averaged to produce multi-group cross sections (Table V). Fast cross-sectional data for U^{233} were obtained from LAMS--2255 [2] and used down to a neutron energy of 100 keV. BNL-325 data [4] were used from 100 keV. Previously mentioned MTR data were averaged from 1 keV to thermal. A 12.0-b scattering cross-section was used to obtain $\bar{\sigma}_c$ from fission and total cross-section measurements.

These data constitute the important nuclear parameters used in the nuclear analysis of the AETR concept.

Full-scale design studies

Nuclear performance of AETR cores having the characteristics shown in Table VI was studied.

TABLE VI

FULL-SCALE REACTOR CORE CHARACTERISTICS

Geometry	Spherical
Th/U^{233}	1.5 to 11.0
Graphite moderator (Volume %)	0 to 60
Averaged core temperature	950°F
Fuel/Na/SS	35/45/20
Fuel	UC-ThC

A standard blanket was used in all cases. The ratio of ThC/Na/SS in the blanket was 50/36/14. The blanket thickness was 50 cm.

The basic method used in all studies was to select a Th/U ratio for a given graphite volume fraction, and compute the critical radius of the reactor core. The results of these studies are shown in Fig. 1.

Breeding ratios for these cores are depicted in Fig. 2.

Of interest are those systems with high in-core breeding ratios; such systems would not be reactivity-limited since spent fuel is replaced by in-core breeding of new fuel where its reactivity worth is highest. However, high in-core breeding ratio systems necessarily have high critical masses, which is a disadvantage from a fuel inventory view-point (for short doubling time, the optimum breeder would have a high breeding ratio and low critical mass). As shown in Fig. 2, the change in critical mass in the unmoderated carbide core as the Th/U ratio is increased from 6.5 to 8 is 460 kg of U^{233}. This is an increase of 90% in mass, with a corresponding increase in in-core breeding ratio of only 20%. Similarily, increasing the Th/U ratio from 8 to 9.5 increases the mass of U^{233} by 1400 kg; the corresponding increase in the in-core breeding ratio is 17%. Thus a proper balance of breeding

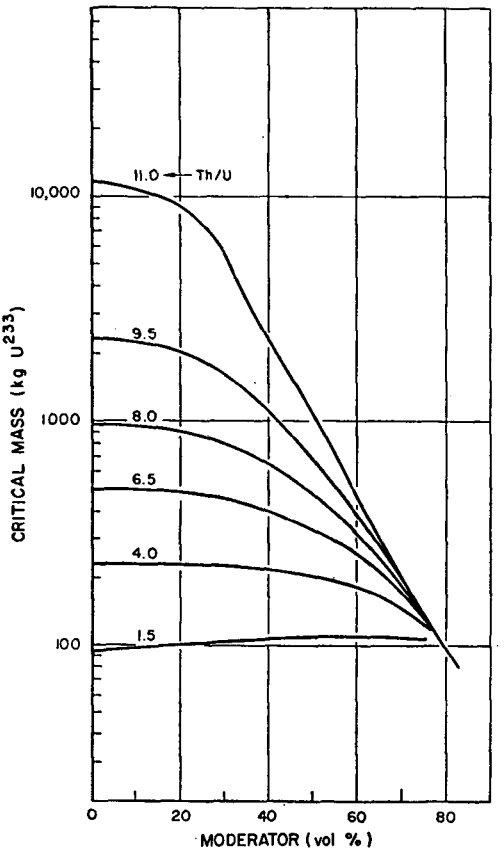


Fig. 1
Critical mass *vs* moderator fraction (Th/U).

ratio and critical mass is necessary in obtaining reasonable doubling time and attractive breeding systems.

Moderation of the neutron-energy spectra decreases both the critical mass and breeding ratio. The unmoderated systems shown in Fig. 2 have the highest critical masses and breeding ratios. Estimates of breeding ratio depend on the somewhat uncertain values of η, and there is a real incentive to obtain measurements of η in the energy range 0.1 MeV$\pm$1 keV. An additional factor complicating establishment of the breeding potential of a given AETR is the burn-up of the core. Doubling time calculations must include the effects of absorptions in isotopes formed during power generation in the core. Effects of burn-up are to reduce the effective breeding ratio and core reactivity, thus requiring new fuel to maintain criticality. Excess reactivity requirements affect the final choice of an optimum AETR power breeder core.

Burn-up studies assessed this problem of core poisoning and excess reactivity requirements. The basic thorium chain considered is shown in Fig. 3, and the results of the studies are given in Fig. 4.

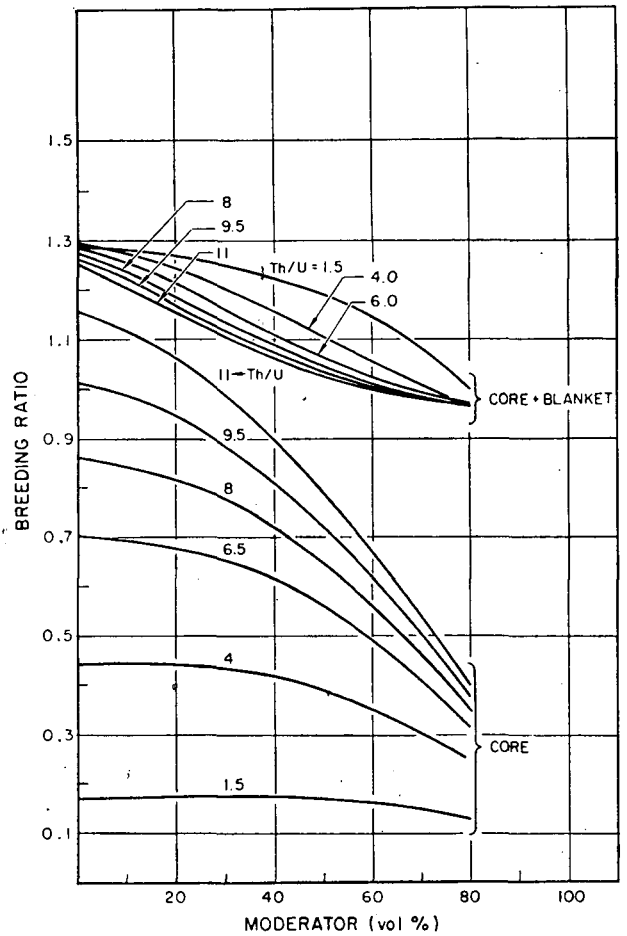


Fig. 2
Breeding ratio *vs* moderator fraction (Th/U).

As can be noted in Fig. 4, the excess reactivity requirements of an AETR core increase as the system is moderated. Increasing the Th/U ratio decreases the excess reactivity requirements. Problems of proper balance and moderator fraction will be solved from engineering considerations, where power generation is a design parameter predicting the required core size for heat transfer.

AETR engineering concept

The systems considered are listed in Table VII.
These systems are sized for the following heat transfer conditions:

Thermal power	1160 MW
Fuel	UC-ThC
Heat flux (maximum)	1.5×10^6 BTU/h
Coolant	Sodium

TABLE VII

NUCLEAR CHARACTERISTICS OF AETR ENGINEERING CORES

Mod. frac.*	Th/U	R (cm)	MFE (keV)	MAE (keV)
0.0	8	81.7	89.8	53.4
0.25	8	89.9	17.9	13.1
0.50	10	104.2	0.91	1.82

* Does not include carbon in dioxide.

TABE VIII

NEUTRONS ABSORBED U^{233} AND Th^{232}

Mod. frac.	Th/U	MAE (keV)	ABS U^{233}	Captures Th^{232}
0.0	8	53.4	53.52	43.84
0.25	8	17.92	55.15	42.86
0.50	10	0.91	57.33	40.53

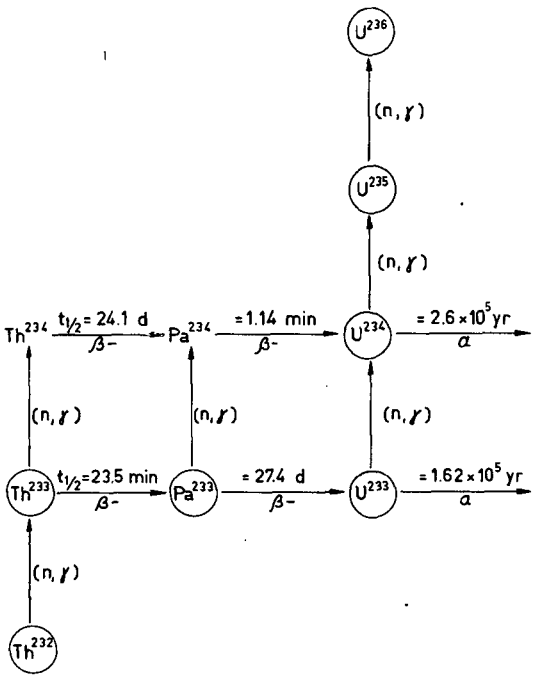


Fig. 3

Th-chain. (n, γ) calculation implies the absorption of a neutron and the emission of a γ-ray; β- indicates a β-decay with a half-life of $t_{1/2}$; α-indicates an α decay with a half-life of $t_{1/2}$.

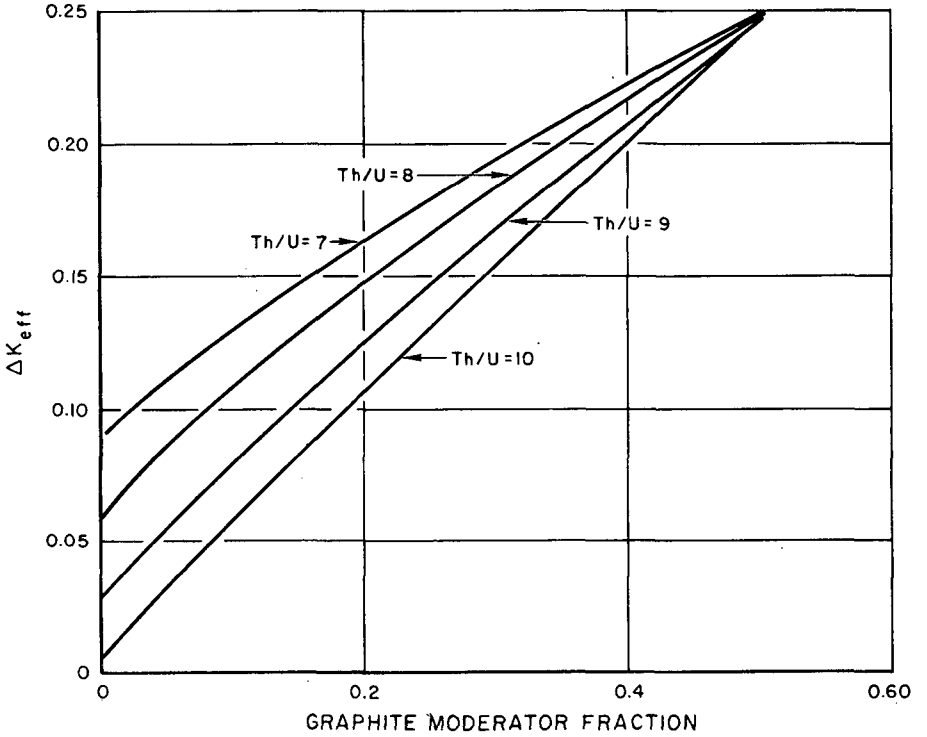


Fig. 4
Δk *vs* moderator fraction.

TABLE IX
NEUTRON BALANCE IN CORE
(based on 100 neutrons)

Mod. frac.	Th/U	$\overline{L}$	$\overline{A}$	$\overline{F}/k$
0.0	8	19.44	80.56	100
0.25	8	17.51	82.49	100
0.50	10	15.66	84.34	100

TABLE X
BREEDING RATIO — CORE (IBR) — BLANKET (BBR)

Mod. frac.	Th/U	MAE (keV)	IBR	BBR	TBR
0.0	8	53.4	0.8192	0.4146	1.2338
0.25	8	17.92	0.7772	0.3612	1.1384
0.50	10	0.91	0.7020	0.2985	1.0055

Neutron absorption balances, potential breeding ratios and neutron balances for these systems are listed in Tables VIII, IX and X.

Data presented in the above tables indicate nuclear characteristics of these systems at the time of initial criticality. Table XI lists the characteristics of the spent cores after an exposure of 50000 MWd/T.

TABLE XI

BURN-UP CHARACTERISTICS OF ENGINEERING DESIGNS

Mod. frac.	Critical mass (kg)	Δk	Decrease in U^{233} (kg)	Increase in U^{233} blanket (kg)	Total Pa^{233} in reactor (kg)	T (days)	Doubling time (years)
0.0	910	0.09	100.0	197.0	62.0	365	5.7
0.25	917	0.157	105.0	159.0	56.3	354	8.0
0.50	739	0.277	129.0	149.0	57.7	347	8.9

Doubling time of the unmoderated AETR core is the smallest of the three cores considered. The "50 %" core has the lowest critical mass and also the lowest breeding potential. Relative absorptions in all cores, listed in Table XII, give the reasons for the differences in neutron economy.

TABLE XII

RELATIVE ABSORPTION IN CORE MATERIALS
(based on 100 neutrons)

Material	"0 %" Moderated	"50 %" Moderated
U^{233}	49	45.48
Th^{232}	46	42.63
U^{234}	2.6	6.53
Pa^{233}	1.3	2.27
U^{235}	0.5	1.85
U^{236}	0.01	1.22

The averaged value of $\overline{(1+\alpha)}$ in the unmoderated system is 1.13, and in the 50 volume % graphite system the value is 1.24. In the unmoderated core, 5.7 neutrons are captured in U^{233} per 100 neutrons absorbed. In the 50 volume % graphite, 9.06 neutrons are captured in U^{233}. The total non-thorium captures in the unmoderated system are 10.1 neutrons. Correspondingly, there are 20.9 neutrons captured in the 50 % case. The number of neutrons lost in the 50 % case, in non-thorium captures, is approximately twice as large as in the unmoderated system. Neutron captures in U^{233} in the 50 % case are 1.59 times greater than in the unmoderated core. Therefore, the basic decrease in neutron economy of the 50 volume % graphite core is due to captures in U^{233} and heavier isotopes. A great need exists to measure the η- and α-values of U^{233}, other uranium isotopes, and other isotopes (including Pa^{233}) in the intermediate

neutron-energy range (0.1 MeV to 0.01 keV) in order to further evaluate moderated AETR cores having lower critical masses.

The AETR critical assembly was designed and constructed to explore this neutron-energy range and is discussed in the following section.

Critical experiment

Design

The primary function of the AETR critical assembly is to provide integral data needed for the design of a full-scale AETR power breeder core.

The small quantity of U^{233} (25 kg) available in the United States for this type of experiment necessitated the design of a multi-region critical assembly (see Fig. 5). The assembly design is of the fast-slow variety having a spherical

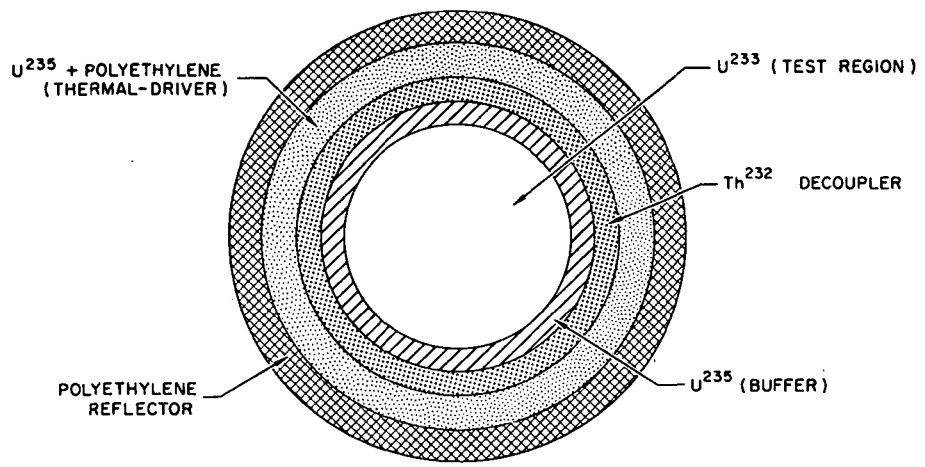


Fig. 5
AETR critical assembly.

central "test" section of the appropriate matrix of material that is proposed for a large AETR core. A butter region of U^{235} and thorium surrounds this "test" section. A decoupler region of thorium separates the buffer from the driver region. The largest reactivity worth of this system is held by the thermal driver portion of the core, which contains U^{235} and polyethylene. The entire spherical core is surrounded by a polyethylene reflector.

The following data were expected from the AETR critical assembly:

1. Integral data on the cross-section structure of all materials to be used in the full-scale core.
2. Reactivity worths of all materials in the full-scale concept.
3. Flux and adjoint functions that characterize the full-scale system.

To ensure that the experimental data obtained from this critical assembly could be readily transposed into data applicable to the full-scale system, the following criteria were followed:

1. The "statistical weight" of the test section where data will be obtained should be nearly identical to the full-scale system.

2. The neutron flux spectrum in the test core should be identical to that of the full-scale reactor.
3. The material composition of the test region should be nearly identical to that of the full-scale reactor core.

The final choice of the number of regions, the dimensions, and the material compositions were obtained by intensive analyses.

CRITERIA

The first criterion was derived by the following formulation.

The Boltzmann equation for the diffusion of neutrons may be written in the following normal form:

$$\begin{aligned} &\boldsymbol{\Omega} \cdot \nabla \varphi(u, \Omega, r) + \sigma_t(u, r)\, \varphi(u, \Omega, r) = \\ &\int\int \sigma_s(u', \Omega' \to u, \Omega; r)\, \Phi(u', \Omega', r)\, du'\, d\Omega' \\ &+ (1 - \varrho)\frac{\chi(u)}{4\pi} \int\int \nu \sigma_f(u', r)\, \varphi(u', \Omega', r)\, du'\, d\Omega' \end{aligned} \tag{10}$$

where

$\varphi(u, \Omega, r)$ = the neutron flux as a function of energy $[u = \ln(10^7/E)]$, angle of scattering Ω and position r,

$\sigma_t(u, r)$ = the total cross-section for collision as a function of energy and position,

$\sigma_s(u', \Omega' \to u, \Omega'; r)$ = the transfer scattering cross-section from $u'\ \Omega' \to u,\ \Omega$ at position r,

$\chi(u)$ = the relative fission spectrum,

ν = the average number of neutrons produced per fission,

$\sigma_f(u, r)$ = the fission cross-section as a function of energy and position,

ϱ = the reactivity of the reactor system.

The adjoint form of Eq. (10) is written as follows:

$$\begin{aligned} &-\boldsymbol{\Omega} \cdot \nabla \psi(u, \Omega, r) + \sigma_t(u, r)\, \psi(u, \Omega, r) = \int\int \sigma_s(u, \Omega \\ &\to u', \Omega'; r)\, \psi(u', \Omega', r)\, du'\, d\Omega' + \nu \sigma_f(u, r)/4\pi \int\int \chi(u')\, \psi(u', \Omega', r)\, du'\, d\Omega' \end{aligned} \tag{11}$$

where $\psi(u, \Omega, r)$ = the neutron importance.

For simplicity, we may consider the introduction of a pure absorbing material into a reactor core at position r_0. The perturbed flux $\varphi^*(u, \Omega, r)$ is obtained from the expression

$$\begin{aligned} &-\boldsymbol{\Omega} \cdot \nabla \varphi^*(u, \Omega, r) + \sigma_f(u, r)\, \varphi^*(u, \Omega, r) \\ &+ \sigma_a^*(u)\, \varphi^*(u, \Omega, r)\, \delta(r - r_0) = \int\int \sigma_s(u', \Omega' \to u, \Omega; r)\, \varphi^*(u', \Omega', r)\, du'\, d\Omega' \\ &+ (1 - \varrho^*)\frac{\chi(u)}{4\pi} \int\int \nu \sigma_f(u', r)\, \varphi^*(u', \Omega', r)\, du'\, d\Omega' . \end{aligned} \tag{12}$$

Multiplying Eq. (12) by $\varphi^*(u, \Omega, r)$ and Eq. (11) by $\psi(u, \Omega, r)$, adding, and integrating over the volume, all integrals vanish with the exception of the perturbation integral:

$$\varrho^*(r_0) = -\frac{\int\int \sigma_a^*(u)\,\psi(u,\Omega,r_0)\,\varphi^*(u,\Omega,r_0)\,\mathrm{d}u\,\mathrm{d}\Omega}{\int\int\int \frac{\chi(u)}{4\pi}\,\psi(u,\Omega,r)\,\nu\,\sigma_f(u,r)\,\varphi^*(u,\Omega,r)\,\mathrm{d}u\mathrm{d}\Omega\mathrm{d}r} \tag{13}$$

This integral (Eq. 13) is independent of reactor system. It is simply a statement that an absorption perturbation introduced at point r_0 will cause a reactivity change of $\varrho^*(r_0)$ in the total reactor.

We now set

$$\delta\sigma_a = -\int\int \sigma_a^*(u)\,\psi(u,\Omega,r_0)\,\varphi^*(u,\Omega,r_0)\,\mathrm{d}u\mathrm{d}\Omega \tag{14}$$

and

$$I = \int\int\int \frac{\chi(u)}{4\pi}\,\psi(u,\Omega,r)\,\nu\,\sigma_f(u,r)\,\varphi^*(u,\Omega,r)\,\mathrm{d}u\mathrm{d}r\mathrm{d}\Omega\,. \tag{15}$$

In the expression for $\delta\sigma_a$, the parameter $\sigma_a^*(u)$ describes the functional variation in absorption in the inserted material as it changes with lethargy. Therefore, $\sigma_a^*(u)$ is a property of the inserted material in the reactor and not of the reactor system.

The quantity $\varphi^*(u,\Omega,r_0)\,\psi(u,\Omega,r_0)$ is the statistical weight of the reactor system at point r_0 and is therefore a dependent variable of the reactor system. The quantity I is the total rate of creation of neutron importance [6]. Since all proposed critical experiments will be made in the central test section, the design criteria must produce the desired neutron environment in this region.

The associated reactivity change of a pure absorber in the critical experiment has the following form:

$$\varrho^*(r_0)\,|_{\text{Test Core}} = \delta\sigma_a^{TC}/I_{TC}\,. \tag{16}$$

The central reactivity change for this same absorber in a full-scale core would be

$$\varrho^*(r_0)\,|_{\text{Full-scale Core}} = \delta\sigma_a^{FC}/I_{FC}\,. \tag{17}$$

If the statistical weight function $\varphi^*(u,\Omega,r_0)\,\psi(u,\Omega,r_0)$ is identical in the two cores, the change in reactivity due to the sample in the test section can be interpreted as

$$\therefore \varrho^*(r_0)\Big|_{TC} \times \frac{I_{TC}}{I_{FC}} \approx \varrho^*(r_0)\Big|_{FC}\,. \tag{18}$$

Therefore, the first design criterion for the AETR critical assembly is that of nearly identical statistical weights. This first criterion is important in the analysis of reactivity coefficient measurements in the critical experiments.

The second criterion for design of the AETR assembly involves fission and capture foil measurements, where

$$F_{r_0}^x = \int \sigma_f(u)\,\varphi(u)\,\mathrm{d}u\,, \tag{19}$$

$F_{r_0}^x$ being the fission rate at r_0 of material x, and $\varphi(u)$ the flux spectra. Therefore, the flux spectra at r_0 in the two systems must be nearly identical for comparison.

The third criterion is concerned with the scattering properties of the media. To avoid additional problems of different materials, the third design criterion was met by having material compositions of the test section nearly identical to full-scale design.

After the required neutron environment was produced in this critical assembly, a detailed analysis of the experimental results was performed. A fundamental assumption in the analysis was the applicability of diffusion and perturbation theory.

Analysis

Results from the critical assembly will yield data on:

1. Reactivity coefficients of core materials.
2. Fission rate and ratio for different elements (i.e. U^{233}, U^{234}, U^{235}, U^{236}, etc.).
3. Neutron importance (obtained from neutron source measurements).

It should be noted at this time that the central test section of the critical assembly will be varied in a series of experiments. Initial experiments will contain a core whose composition is similar to that in a fast-reactor system. Assemblies will then be constructed with test cores that are partially moderated with graphite, i.e., 20 % to 50 % by volume, followed by a highly moderated system (>90 % graphite by volume) if time permits. Thus, test cores will cover a wide range of neutron-energy spectra. Measurements available for analysis will contain a definite energy dependence. Due to uncertainties in the cross-sectional data, disagreement between theory and experiment is expected. The basic analytical techniques used in studying the critical experiments account for this disagreement by empirical methods. Of the three classes of experiments mentioned, only the foil measurements and reactivity-coefficient determination are discussed. The source measurements will be used in checking different assumptions about scattering in the system. A block diagram of the analytical techniques and their relationship to the experimental results is shown in Fig. 6.

A fundamental assumption contained in the analysis is the applicability of perturbation theory. This implies small changes in the reactivity of the system

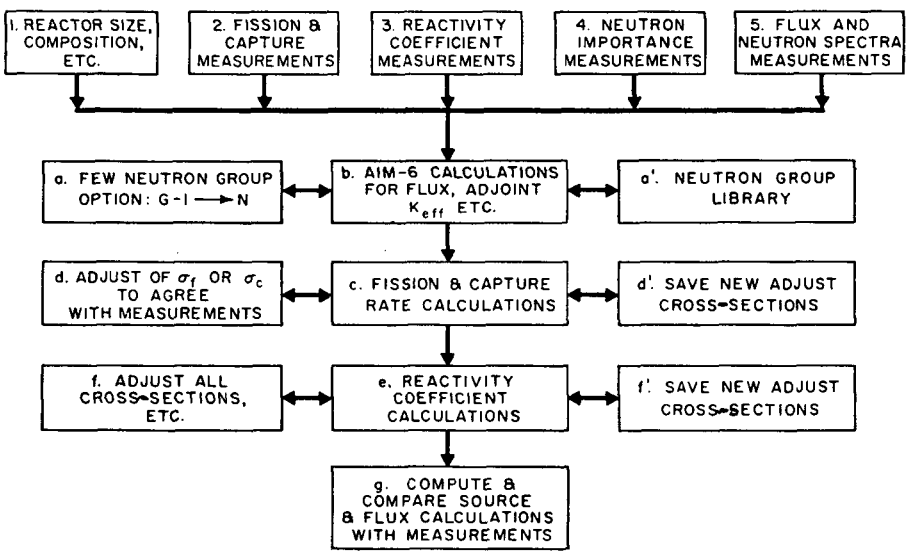


Fig. 6
Critical experiment flow diagram.

for a given perturbation. Thus the unperturbed flux and adjoint are used in these techniques. A second assumption is made that the values used for the input cross-sections represent a first-order estimate with the accuracy of the cross-section indicating second-order variations in the measured cross-section data.

Fission and capture rate measurement

The calculated fission rate has the following form (neutron group notation will be used in this discussion):

$$F_{\text{calc}} = \sum_g (\sigma_f)_g^\circ \varphi_g \tag{20}$$

where

$(\sigma_f)_g^\circ$ = the estimated fission cross-section in neutron group g,

φ_g = the neutron flux in group g.

Measured fission rate is expressed in the following form:

$$F_{\text{meas}} = \sum_g (\sigma_f)_g^\circ (1 + x_g) \varphi_g (1 + y)_g \tag{21}$$

where

$x_g (\sigma_f)_g^\circ$ is the second-order uncertainty in $(\sigma_f)_g^\circ$,

$y_g \varphi_g$ is the second-order uncertainty in φ_g.

Subtracting Eq. (20) from Eq. (21), we have

$$\Delta F = F_{\text{meas}} - F_{\text{calc}} = \sum_g (\sigma_f)_g^\circ x_g \varphi_g + \varphi_g y_g (\sigma_f)_g^\circ + (\sigma_f)_g^\circ x_g \varphi_g y_g . \tag{22}$$

Since $x_g (\sigma_f)_g^\circ \ll (\sigma_f)_g^\circ$ and $y_g \varphi_g \ll \varphi_g$, Eq. (22) may be rewritten excluding the product of the second order error terms:

$$\Delta F = \sum_g (\sigma_f)_g^\circ x_g \varphi_g + \varphi_g y_g (\sigma_f)_g^\circ \tag{23}$$

The basic technique in this proposed analysis sets $y_g = 0$ for all groups and accepts the flux calculation. Since the flux shape will change as further adjustments are made to the cross-sections, the system is iterated to obtain consistent fluxes.

In the adjustment routine,

$$x_g \approx \left(\frac{\partial F_{\text{calc}}}{\partial \sigma_f} \right)_g \tag{24}$$

where the terms $(\partial F_{\text{calc}}/\partial \sigma_f)$ are variable coefficients indicating the sensitivity of the calculation for fission rate to a given change in the neutron-group fission cross-section.

Since the only comparison between the measured and calculated values is the difference in the quantities, the variable coefficients $(\partial F_{\text{calc}}/\partial \sigma_f)_g$ are used to distribute the difference over the neutron-group structure.

The adjusted cross-sections would have the following form:

$$(\hat{\sigma_f})_g = (\sigma_f)_g^\circ + (\sigma_f)_g^\circ \Delta F \left(\frac{\partial F_{\text{calc}}}{\partial \sigma_f} \right)_g \tag{25}$$

Experiments planned for the critical assembly will be made in different intermediate energy spectra. The above technique will allow adjustment of the fission cross-sections of U^{233} over the intermediate energy range. This method will also be used for the fertile material, thorium, to obtain capture information from capture rate experiments.

Data on the other nuclear properties of the fission, fertile, structure, coolant, and moderator materials must be obtained from reactivity-coefficient measurements.

The reactivity change, associated with the replacement of an original reactor void in the centre of a core by a fissionable sample, is given by

$$\left(\frac{\Delta k}{k}\right)^*_{\text{calc}} = \psi_H \sum_g \nu_g (\sigma_f)_g^\circ \varphi_g - \sum_g (\sigma_f)_g^\circ \varphi_g \psi_g - \\ - \sum_g (\sigma_c)_g^\circ \varphi_g \psi_g - \sum_g \varphi_g \sum_g (\psi_g - \psi_g) (\sigma_f)^\circ_{g-g'} \tag{26}$$

where

ψ_H is the high-energy importance and is equal to

$$\psi_H = \sum_g \chi_g \psi_g . \tag{27}$$

Here

χ_g is the relative fission spectrum,

ψ_g is the neutron importance of group g.

Investigation of Eq. (27) shows six functions $[\nu_g, (\sigma_f)_g^\circ, (\sigma_c)_g^\circ, \sigma_{g-g'}, \psi_g, \varphi_g]$ of the neutron group. The assumption made early of accepting ψ_g and φ_g for the first iteration would reduce the number of unknown functions. Further reduction could be made by using values of $\hat{\sigma}_f$ for fissile material and $\hat{\sigma}_c$ for fertile material obtained from analysis of the rate experiments.

The measured reactivity change has the following assumed form:

$$\left(\frac{\Delta k}{k}\right)^*_{\text{meas}} = \psi_H \sum_g \nu_g (1 + \delta_g) (\sigma_f)_g^\circ \varphi_g - \sum_g (\sigma_f)_g^\circ \varphi_g \psi_g - \\ - \sum_g (\sigma_c)^\circ (1 + \gamma_g) \varphi_g \psi_g - \sum_g \varphi_g \sum_g (\psi_g - \psi_g') (\sigma)^\circ_{g-g'} (1 + \varepsilon_g^{g'}) . \tag{28}$$

The difference between the measured and computed value is assumed to have the following form:

$$\left[\left(\frac{\Delta k}{k}\right)_{\text{meas}} - \left(\frac{\Delta k}{k}\right)_{\text{calc}}\right] = \psi_H \sum_g \nu_g \delta_g (\sigma_f)^\circ \varphi_g - \\ - \sum_g \gamma_g (\sigma_c)_g^\circ \varphi_g \psi_g - \sum_g \varphi_g \sum_g (\psi_g - \psi_g') (\sigma)^\circ_{g-g'} \varepsilon_g^{g'} \tag{29}$$

or

$$\left[\left(\frac{\Delta k}{k}\right)_{\text{meas}} - \left(\frac{\Delta k}{k}\right)_{\text{calc}}\right] = \sum_g \left[\nu_g \delta_g \psi_H - \gamma_g (\sigma_c)_g^\circ \psi_g - \right. \\ \left. - \sum_g (\psi_g - \psi_g') (\sigma)^\circ_{g-g'} \varepsilon_g^{g'}\right] \varphi_g . \tag{30}$$

An approach similar to the above yields the following expressions for δ_g, γ_g and $\varepsilon_g^{g'}$:

$$\delta_g \approx \left| \frac{\partial \left(\frac{\Delta k}{k}\right)_{\text{calc}}}{\partial \nu} \right|_g \tag{31}$$

$$\gamma_g \approx \left| \frac{\partial \left(\frac{\Delta k}{k}\right)}{\sigma_c} \right|_g \tag{32}$$

$$\varepsilon_g^{g'} \approx \left| \frac{\partial \left(\frac{\Delta k}{k}\right)}{\partial \sigma_{g-g'}} \right|_g \sigma_{g-g'} \tag{33}$$

Distribution of the difference between theory and experiment is accomplished by three different coefficient sets. The merit of this technique is found in the set of experimental data. The data from all the critical experiments contain a definite energy dependence. The use of the above-mentioned coefficients presents an impress resolution of data according to theory. This imposed delineation of the data has a limited but definite utility. Incremental adjustments indicated by the proposed techniques will yield useful trends in the nuclear data and provide an adjusted set of cross-sections that will serve in the design of the full-scale AETR core.

Nomenclature

u = lethargy
x = position in reactor
$G(u, x)$ = differential operator
$\varphi(u, x)$ = neutron flux
$S(u, x)$ = source of neutrons
$\sigma_s(u—u')$ = transfer cross-section
Σ_a = macroscopic absorption cross-section
Σ = total macroscopic cross-section
$\bar{\mu}_0$ = cosine of the scattering angle
D = diffusion coefficient
I_r = resonance integral
I_∞ = infinitely dilute resonance integral
β = ratio of peak resonance cross-section
σ_0 = potential scattering cross-section of fuel
σ_ϱ = ordinary potential scattering cross-section per fuel atom
Γ_n = neutron width
Γ = total width
σ_m = potential scattering cross-section of moderator per fuel atom
$\overline{L}$ = leakage of neutrons
$A|$ = absorption of neutrons
$\overline{F}/k$ = fission neutrons
σ_c = capture cross-section
ν = neutron produced per fission
σ_f = fission cross-section
σ_a = absorption cross-section
σ_{tr} = transport cross-section
$\sigma_{g-g'}$ (etc.) = transfer cross-sections
σ_t = total cross-section
ϱ = density of material
$\chi(u)$ = relative fission density
ψ = neutron importance

$F_{r_0}^x$ = fission rate at point r_0 for material x
F = fission rate
$\hat{\sigma}$ = adjusted cross-section
δ = adjustment coefficients (ν)
γ = adjustment coefficients (σ_c)
ε = adjustment coefficients ($\sigma_{g-g'}$)
MFE = median fission energy
MAE = median absorption energy
ABS = absorptions
$\overline{(1+\alpha)}$ = averaged value of $1+\alpha$
E_L = lower energy of neutron group
α = ratio of captures to fissions
$\eta = \nu\sigma_f/\sigma_a$

REFERENCES

[1] FLATT, H. P. and BALLER, D. C., AIM-5—A Multigroup, One-Dimensional Diffusion Equation Code, Appendix IV, "Extrapolation Length at Reactor Boundaries," by E. R. Cohen, NAA-SR-4694 (1960).
[2] MILLS, C. B., LAMS-2255 (1959).
[3] Reports to the AEC Nuclear Cross Sections Advisory Group, ANL, WASH-1029 (1960).
[4] HUGHES, D. J. and SCHWARTZ, R. B., BNL-325 (1960).
[5] SHER, R. and MOORE, S., BNL-653 (N3) (1961).
[6] Reactor Handbook, AECD-3645, Vol. 1 (1955) 534.

SM-18-31

ÉTUDES PHYSIQUES DE RAPSODIE

J. L. Campan, J. M. Chaumont, P. P. Clauzon, G. Ghesquière, A. Kania, S. G. Kaufmann, P. Moinereau, A. P. Schmitt, M. Solanes et C. P. Zaleski
Centre d'études nucléaires de Saclay
France

Abstract — Résumé — Аннотация — Resumen

Physical research on RAPSODIE. The purpose of the paper is to give a review of the present state of research on the physics of the RAPSODIE fast reactor, to be constructed at the Cadarache Nuclear Research Centre.

The reasons for selecting and designing this type of reactor are exposed. An account is then given of various neutron studies, including critical masses, spectra, neutron economy, fuel development and plutonium production, reactivity co-efficients and control methods. Finally, some results of research on stability and dynamic behaviour (reactor shut-down, dropping of rods) and on explosion accidents are quoted.

Etudes physiques de RAPSODIE. L'objet de cette communication est de présenter l'état actuel des travaux intéressant la physique du réacteur à neutrons rapides RAPSODIE, dont la construction va être entreprise au Centre d'études nucléaires de Cadarache.

Les auteurs exposent d'abord brièvement les raisons qui ont conduit au choix et à la conception de ce type de réacteur. Ils font ensuite le point des études neutroniques: masses critiques, spectres, économie des neutrons, évolution du combustible et production de plutonium, coefficients de réactivité, moyens de contrôle. Puis ils donnent quelques résultats ayant trait aux études de stabilité et de comportement dynamique (arrêt du réacteur, chute des barres, etc.) ainsi qu'aux études d'accidents à caractère explosif.

Физика реактора РАПСОДИЯ. В данной работе описывается настоящее положение работ по физике реактора на быстрых нейтронах РАПСОДИЯ, строительство которого скоро начнется в Центре ядерных исследований в Кадараше.

Вначале авторы кратко излагают причины, приведшие к выбору этого типа реактора. Далее они описывают изучение нейтронных величин: критические массы, спектры, баланс нейтронов, изменение топлива и производство плутония, коэффициенты реактивности, средства контроля. Затом они дают некоторые результаты, касающиеся изучения стабильности и динамического поведения (остановка реактора, сброс стержней и т.д.), а также изучения инцидентов взрывного характера.

Estudios físicos del reactor RAPSODIE. La memoria tiene por objeto informar sobre el estado actual de los trabajos relativos a la física del reactor de neutrones rápidos RAPSODIE, cuya construcción se ha de iniciar en el Centre d'études nucléaires de Cadarache.

Los autores comienzan por exponer brevemente las razones en las que se ha basado la elección de este tipo de reactor y la concepción del mismo. A continuación, se refieren a los estudios neutrónicos: masas críticas, espectros, economía neutrónica, evolución del combustible y producción de plutonio, coeficientes de reactividad, medios de control. Luego presentan algunos resultados relativos a los estudios de estabilidad y de comportamiento dinámico (detención del reactor, introducción de las barras, etc.) así como a los estudios sobre accidentes de carácter explosivo.

Introduction

La pile RAPSODIE présentera sur le plan français une triple originalité: ce sera la première pile à neutrons rapides, la première à utiliser une matière fissile très concentrée (plutonium et uranium enrichi), la première à être refroidie par un métal liquide, en l'occurence le sodium. Elle fonctionnera à une puissance nominale de 10 MW avec extension possible à 20 MW. Sa construction, en ce qui concerne du moins les bâtiments et halls d'essais annexes, est commencée au Centre d'études nucléaires de Cadarache. La date de divergence prévue se situe dans le courant de 1964.

L'objet de cette communication est de présenter l'évolution et l'état actuel des travaux intéressant la physique de RAPSODIE. Dans une première partie, nous exposerons les objectifs de ce réacteur, nous signalerons ses caractéristiques principales et nous ferons l'historique du projet depuis sa conception jusqu'à la remise d'un avant-projet détaillé (fin 1958). Les deuxième et troisième parties seront consacrées à une revue des principales études de physique faites depuis lors, la deuxième partie traitant essentiellement des calculs de neutronique « statique » — masses critiques, spectres, production de plutonium, coefficients de réactivité —, la troisième des calculs se rapportant à la dynamique du réacteur.

En conclusion, nous indiquerons dans quel sens vont s'orienter les travaux futurs, en insistant plus particulièrement sur leur aspect expérimental.

1. Objectifs de RAPSODIE et historique du projet

La filière des piles à neutrons rapides apparaît comme complémentaire de la filière des piles thermiques à eau lourde et au graphite, puisque seules sans doute les piles rapides permettront d'utiliser le plutonium riche en ^{240}PU qui est produit en grandes quantités dans ces dernières. Il était donc naturel que le Commissariat à l'énergie atomique s'engageât dans la voie de l'étude et de la réalisation de telles piles. Au moment de la décision — dès 1957 — deux voies s'offraient à lui : soit réaliser une expérience critique rapide à puissance nulle, ce qui présentait les avantages d'être plus simple et d'exiger des délais plus courts, soit concevoir et construire une véritable pile développant une puissance d'au moins 1 MW. C'est la deuxième voie qui a été choisie parce que, du fait même des difficultés qu'elle présentait, elle permettait de se familiariser avec les problèmes qui se poseront dans la construction des futures piles rapides de puissance.

1.1. Objectifs de RAPSODIE et caractéristiques

Les caractéristiques de RAPSODIE ont été fixées en fonction de trois *objectifs* principaux :

a) Ce doit être un engin expérimental, donc doté d'une certaine souplesse dans sa conception. Les premières années de sa vie seront occupées par des études de comportement en régime statique et dynamique et à des puissances variées.

b) Il doit fournir une expérience valable pour les piles rapides futures de puissance; la puissance spécifique produite doit donc être assez importante pour que les problèmes technologiques des piles d'avenir apparaissent et soient étudiés.

c) Le flux de neutrons rapides doit être assez grand pour permettre l'irradiation d'éléments combustibles de piles de puissance dans des conditions intéressantes.

Parmi les *caractéristiques* principales de RAPSODIE, les points suivants nous paraissent à signaler:

Emploi du plutonium comme combustible. Son utilisation dès la première charge nous a paru s'imposer, à la fois parce qu'il sera produit en France en grandes quantités dans un avenir proche et parce qu'il offre les meilleures perspectives pour les piles rapides de l'avenir. Toutefois, étant donné le faible volume du cœur choisi — une quarantaine de litres —, l'emploi du seul plutonium comme matériau fissile aurait conduit à une teneur trop grande et entraîné des difficultés, de fabrication entre autres; d'autre part, il nous a semblé préférable d'employer le plutonium autant que possible dans les proportions qui seront celles des grosses piles futures. C'est pourquoi nous avons limité cette teneur à 15 à 20 % environ — dans le cas d'un combustible métallique U—Pu—Mo —, le complément de matière fissile étant réalisé avec de l'uranium enrichi en ^{235}U.

Flux de neutrons rapides élevé. La puissance maximale prévue pour RAPSODIE dans une première étape est 10 MW. Le flux maximal calculé, essentiellement constitué par des neutrons rapides, est alors de $1{,}1 \cdot 10^{15}$ n/cm^2 s au centre de la pile. La puissance moyenne dans le cœur est de l'ordre de 225 kW/l et la puissance au centre de 350 kW/l.

Emploi du sodium comme réfrigérant. Le sodium a été préféré aux autres métaux ou mélanges eutectiques liquides parce qu'il présente une meilleure conductibilité thermique, a de bonnes caractéristiques nucléaires, n'est pas toxique et est d'un prix de revient peu élevé. RAPSODIE étant la première pile française refroidie de la sorte, il y a là pour nos spécialistes un riche sujet d'études, et des travaux importants ont déjà été effectués à ce sujet.

1.2. Historique du projet

Les premières études ont débuté en 1957 et ont abouti à la remise d'un avant-projet en décembre 1958 [1]. Plusieurs notes internes du CEA, émanant du Service de neutronique expérimentale et du Service de physique mathématique, ont d'ailleurs précédé cet avant-projet ou lui ont été annexées [2—3].

Le combustible retenu finalement est un alliage ternaire U—Pu—Mo dont la composition pondérale est la suivante:

— 20 % de plutonium,
— 10 % de molybdène,
— 70 % d'uranium enrichi à 20 % en ^{235}U.

Des considérations métallurgiques et thermiques autant que neutroniques ont permis de fixer les grands traits du réacteur:

— Volume du cœur: 40 l environ.
— Répartition du combustible par assemblages hexagonaux contenant chacun 37 aiguilles gainées en acier inoxydable, le combustible occupant 40 % du volume du cœur.
— Sodium circulant occupant 33 % environ du volume du cœur.
— Choix de l'uranium naturel comme matériau fertile des couvertures, dans les proportions volumétriques suivantes: 60 % pour la couverture radiale et 50 % pour la couverture axiale.

A partir de ces données de base, les études neutroniques ont été faites sur l'ordinateur IBM 650, avec le code multigroupe PROD II et quelques petits codes annexes programmés à Saclay. C'est ainsi que l'on a pu préciser:

a) L'influence sur la réactivité des différentes couvertures et des matériaux qui suivent ces couvertures. La figure 1 montre que le rayon critique est peu

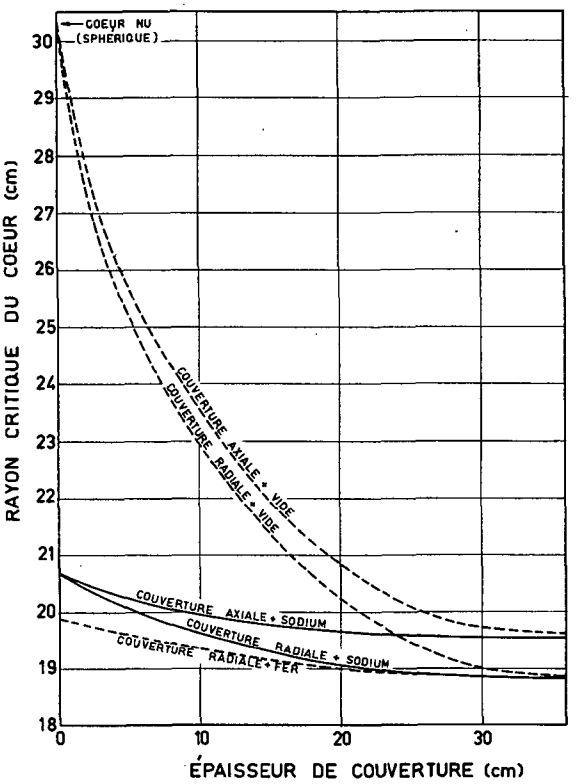


Figure 1
Etude des couvertures. Géométrie sphérique.

sensible à ce qui se trouve au-delà de 30 cm de couverture, et tout à fait insensible à ce qui se trouve au-delà de 40 cm.

b) L'influence des espaces de transition qui séparent le cœur des couvertures axiales.

c) La variation de la hauteur critique du cœur en fonction du rayon. On a tracé (fig. 2) les courbes donnant la hauteur critique en fonction du rayon successivement pour:

— le cœur nu,
— le cœur avec la couverture radiale, ce qui donne le gain dû à la couverture radiale,
— le cœur avec la couverture axiale, ce qui donne le gain dû à la couverture axiale.

En composant les gains dus aux deux couvertures, on a obtenu la hauteur critique en fonction du rayon pour la géométrie réelle.

d) La variation du rapport du volume critique en géométrie cylindrique au volume critique en géométrie sphérique en fonction de H/D, rapport de la hauteur du cœur à son diamètre. La figure 3 montre que ce rapport passe par un minimum, égal à 1,29 pour $H/D=1{,}1$, et varie peu dans un domaine assez étendu (de $H/D=0{,}95$ à 1,30).

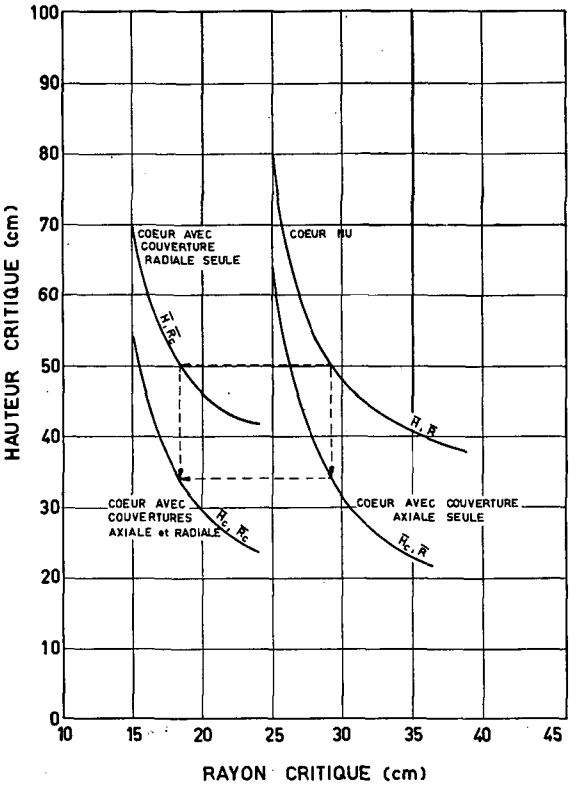


Figure 2
Courbes donnant la hauteur critique en fonction du rayon.

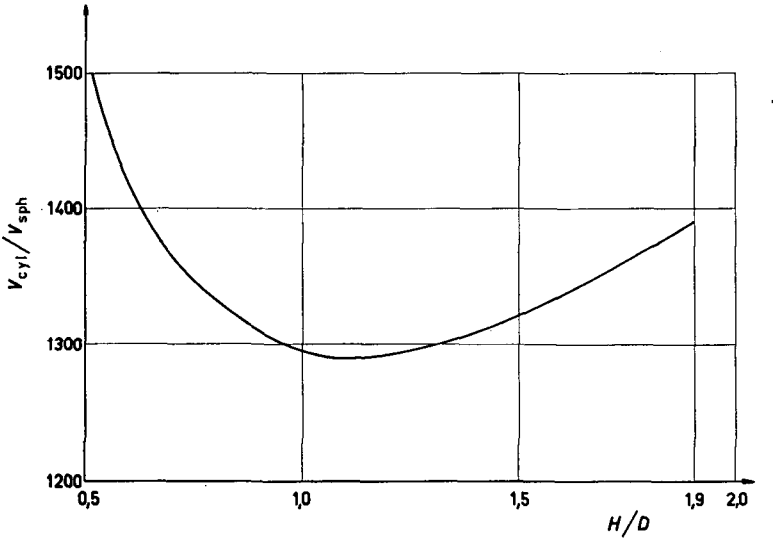


Figure 3
Variation de V_{cyl}/V_{sph} en fonction de H/D.

e) L'influence de l'épaisseur des couvertures sur l'économie des neutrons. Bien que le bilan des neutrons dans le cœur paraisse dépendre peu de l'épaisseur de la couverture radiale, on a constaté un accroissement important du taux de régénération total lorsque l'épaisseur de cette couverture passe de 20 à 40 cm, puis un accroissement beaucoup plus lent au-delà.

Simultanément, ont été abordées les premières études de protection: influence du sodium, des bétons normal et cellulaire sur la propagation des flux et dégagements de chaleur dans ces milieux. La mise au point à Saclay d'un programme de résolution des équations adjointes nous a permis également d'entreprendre les premiers calculs de perturbations, du temps de vie des neutrons — trouvé de l'ordre de 10^{-7} — de la fraction effective des neutrons retardés et de tracer la courbe période stable-réactivité.

Signalons également que dès cette époque des combustibles de remplacement ont été envisagés et ont fait l'objet d'études paramétriques, en particulier l'alliage Pu-Zr, les oxydes PuO_2—UO_2, et aussi l'alliage ternaire U—Pu—Mo avec d'autres compositions.

Finalement, c'est l'ensemble de ces études, associées à d'autres études de thermique, de mécanique et de mécanique des fluides (réglage des débits de

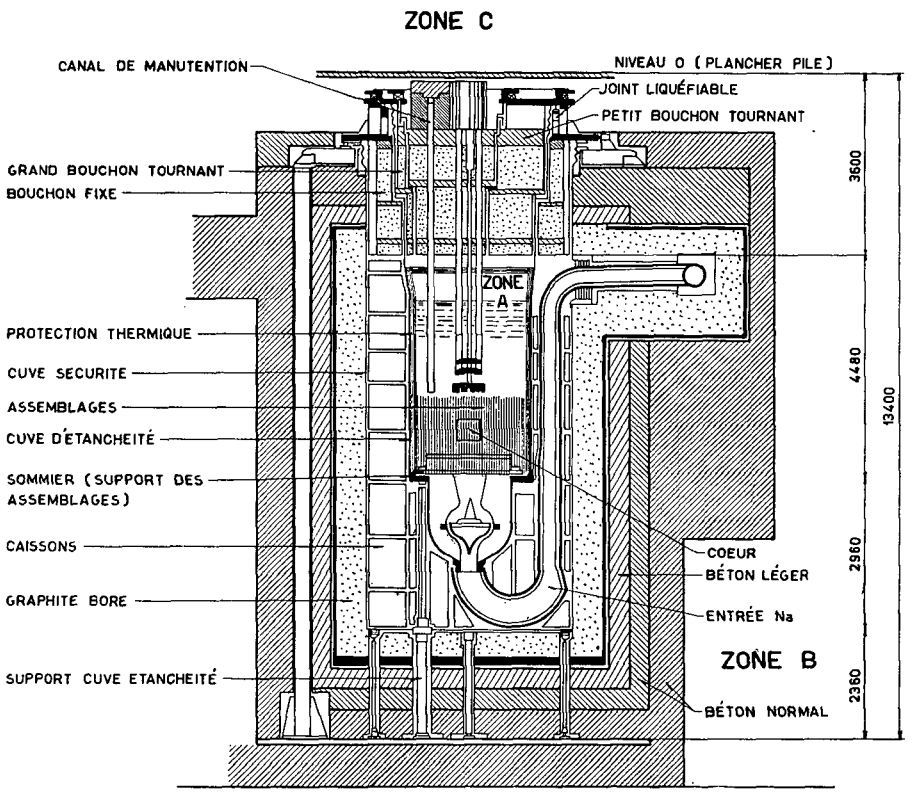


Figure 4
Vue en coupe dans le plan de l'entrée du sodium.

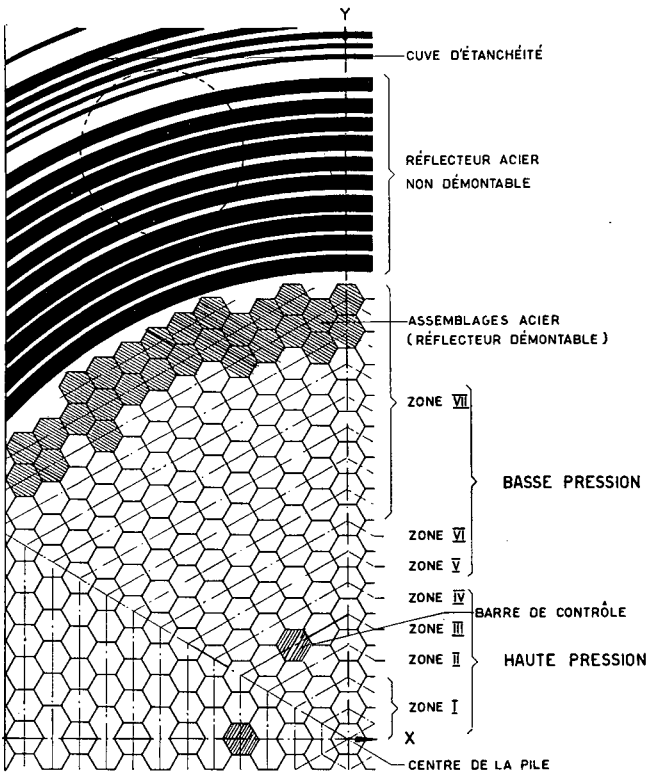


Figure 5
Section horizontale de la pile.

sodium), qui a constitué «l'avant-projet RAPSODIE» et ses annexes, et a conduit à la structure définitive de la pile, à quelques détails près peut-être.

Les figures 4 et 5 représentent respectivement une coupe axiale et une coupe radiale de la pile telle qu'elle se présente actuellement. On remarquera sur la figure 4 l'entrée du sodium, le système des bouchons tournants avec le passage du mécanisme des barres de contrôle, sur la figure 5 l'emplacement exact des barres et les zones de refroidissement numérotées de I à VII.

2. Etudes neutroniques statiques

Les études physiques du projet proprement dit, tant neutroniques que dynamiques, ont commencé en 1959, et leur développement a été permis par l'extension assez considérable des moyens de calcul dont nous avons pu disposer à Saclay.

2.1. Moyens de calcul

Après l'IBM 650 des débuts, des ordinateurs plus rapides et à plus grande capacité ont été mis en service au CEA: d'abord une machine Ferranti Mercury, puis une IBM 704, et enfin récemment une IBM 7090. Simultanément, des programmes nouveaux et aux possibilités plus grandes que PROD II ont pu être utilisés. Actuellement, nous utilisons les codes de criticalité suivants.

Codes basés sur l'approximation de la diffusion

1. A une dimension, code de LINDE sur Ferranti.
2. A deux dimensions:

— PDQ, CURE et CUREM sur IBM 7090,
— Code de Hassitt sur Ferranti.

Codes basés sur la méthode S_n

1. Un code, écrit à Saclay, sur Ferranti.
2. DSN (une dimension) et TDC (2 dimensions) sur IBM 7090.

Nous disposons aussi de programmes de calculs de perturbations à partir des flux directs et des flux adjoints sur Ferranti. Signalons également que deux codes utilisant la méthode de la diffusion prévus avec des sous-programmes annexes sont en cours de programmation à Saclay sur IBM 7090: l'un à une dimension à 50 groupes, l'autre à deux dimensions à 12 groupes.

Tous ces codes étant soit connus soit signalés par ailleurs [4], nous n'insisterons pas davantage.

D'une façon générale, les calculs de neutronique statique sont menés de la façon suivante.

a) Des calculs à une dimension avec un grand nombre de groupes permettent de déterminer le spectre des neutrons dans les différentes zones du réacteur et de calculer les flux dans les régions éloignées des régions actives.

b) Des calculs à deux dimensions, à 2 ou 3 groupes permettent de préciser, compte tenu de la géométrie réelle, les masses critiques, la carte des puissances, le bilan des neutrons, les caractéristiques de la pile avec les barres de contrôle insérées, les coefficients de température par zones. Du point de vue des constantes neutroniques de base, nous avons utilisé d'abord des sections efficaces à 10 groupes de Los Alamos, puis des sections efficaces à 10 groupes également, provenant d'Argonne. A partir de ces dernières, nous avons établi un jeu complet à 8 groupes qui a servi pour tous les calculs dont nous allons parler dans ce qui suit. Une étude comparée de ces sections efficaces et des erreurs que l'on peut en attendre fait l'objet d'une autre communication [4].

2.2. Caractéristiques neutroniques de RAPSODIE — Etude avec deux types de combustibles

Le combustible de RAPSODIE n'est pas encore fixé de façon définitive. Cependant, pour la première charge, le choix se fera entre un alliage métallique U—Pu—Mo et une céramique PuO_2—UO_2.

Nous allons ici passer en revue les caractéristiques principales de RAPSODIE avec ces deux combustibles pour lesquels les compositions suivantes ont été choisies:

— U—Pu—Mo: on a conservé le combustible de l'avant-projet: 10% en poids de Mo, 20% en poids de ^{239}Pu, uranium enrichi à 20%.
— PuO_2—UO_2: caractérisé par de l'uranium enrichi à 20% et un volume de cœur identique à celui obtenu avec le combustible précédent.

Dans la suite de l'exposé, nous désignerons RAPSODIE par R 59-1 lorsqu'il s'agira du premier type de combustible, par R 60-2 lorsqu'il s'agira du second type.

Remarque. Le critère du choix a été l'enrichissement de l'uranium que l'on a limité à 20%. Cependant nous nous orientons actuellement vers des compositions différentes: un combustible U—Pu—Mo à 15% en poids de Pu et 10% de Mo, et un combustible PuO_2—UO_2 à 25% en volume de PuO_2.

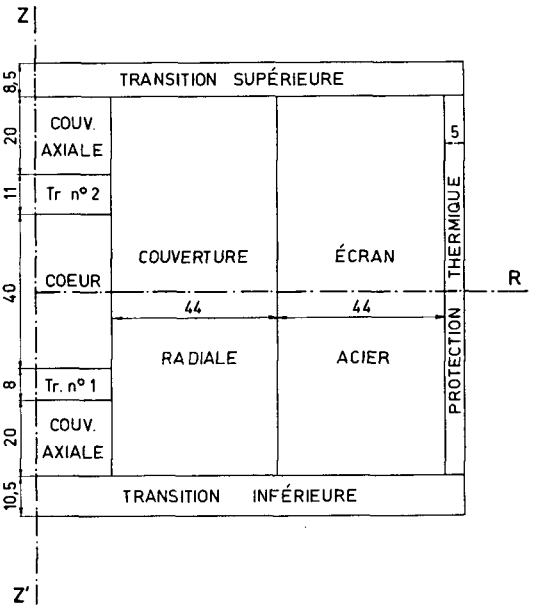


Figure 6
RAPSODIE 59–1. Coupe suivant l'axe. Cotes en centimètres.

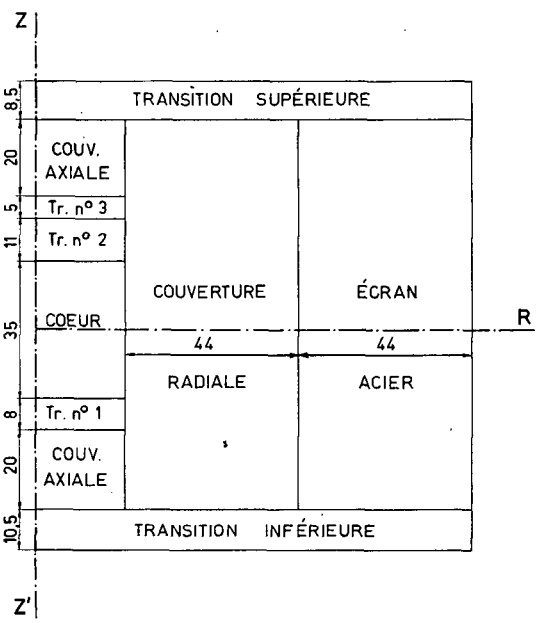


Figure 7
RAPSODIE 60–2. Coupe suivant l'axe. Cotes en centimètres.

Régions \ Constituants	Combustible	Fertile	Sodium de refroidissement	Sodium stagnant (joints)	Acier inox	Hélium
Cœur	42,2	—	33,05	6,17	18,58	—
Couverture radiale	—	56,05	25,35	3,03	15,57	—
Couvertures axiales	—	42,19	34,97	2,64	20,20	—
Ecran acier	—	—	10	—	90	—
Protection thermique	—	—	50	—	50	—
Transition inférieure	—	—	20	—	80	—
Transition n° 1	—	—	49,9	—	35,9	14,2
Transition n° 2	—	—	47,6	—	30	22,4
Transition supérieure	—	—	20	—	80	—

TABLEAU II

RAPSODIE 60-2 — COMPOSITIONS VOLUMÉTRIQUES A 250°C

Régions \ Constituants	Combustible	Fertile	Sodium de refroidissement	Hélium	Acier inox
Cœur	42,2	—	33,05	—	24,75
Couverture radiale	—	59,08	25,35	—	15,57
Couvertures axiales	—	44,83	34,97	—	20,20
Transition n° 2	—	—	33,05	42,2	24,75
Transition n° 3	—	—	47,50	—	52,50

Les compositions des régions qui ne sont pas indiquées sont identiques à celles des régions correspondantes de RAPSODIE 59-1.

Les figures 6 et 7 représentent les coupes schématiques des modèles calculés de RAPSODIE avec les deux types de combustible. On remarquera qu'ils diffèrent par la hauteur du cœur — donc son diamètre — et par la présence de deux zones de transition au-dessus du cœur avec R 60-2 : la hauteur de la chambre d'expansion des gaz de fission qui surmonte les aiguilles a été augmentée dans le cas des oxydes.

TABLEAU I

RAPSODIE 59-1 — COMPOSITIONS VOLUMÉTRIQUES A 250°C

Les compositions volumétriques des différents milieux sont indiquées dans les tableaux I et II. Dans les deux cas les assemblages cœur sont hexagonaux et possèdent les caractéristiques suivantes :

Largeur sur plat de l'enveloppe hexagonale	49,8 mm
Epaisseur de l'enveloppe hexagonale	1,0 mm
Largeur sur plat de l'hexagone de référence	50,8 mm
Nombre d'aiguilles de combustible par assemblage	37
Diamètre extérieur des aiguilles gainées	6,7 mm
Diamètre du combustible des aiguilles	5,7 mm

Pour R 59-1, la liaison combustible-gaîne d'acier inoxydable est assurée par un joint de sodium stagnant de 0,2 mm. Pour R 60-2, la gaîne d'inox est directement martelée sur le combustible donc plus épaisse.

Masses critiques

Le rayon critique de R 59-1, calculé sur le schéma de la figure 6 avec le code PDQ à 2 groupes d'énergie, est 16,44 cm. Compte tenu de la marge d'incertitude sur le résultat due à la méthode de calcul, à la différence entre la géométrie réelle du réacteur et la géométrie idéale de la figure 6, aux inhomogénéités, et aussi à ce qu'il est normal de prévoir une réserve de réactivité pour compenser l'effet négatif dû à l'insertion du système de contrôle dans le réacteur, nous avons fixé arbitrairement le rayon du cœur à 17 cm, ce qui conduit à un volume de cœur de 36,46 l et à une réserve de réactivité théorique de 1700 pcm environ. Pour le même volume, le rayon du cœur avec R 60-2 est 18,2 cm.

Les dimensions des deux cœurs et les masses critiques correspondantes sont indiquées dans le tableau III.

TABLEAU III

DIMENSIONS DES CŒURS ET MASSES CRITIQUES

	R 59-1	R 60-2
Dimensions du cœur		
Hauteur (cm)	40	35
Rayon (cm)	17	18,2
$\frac{\text{Nombre d'atomes d'uranium}}{\text{Nombre d'atomes de plutonium}}$	3,52	1,3
Masses critiques		
^{239}Pu	51,4	59,6
^{235}U	35,8	15,2
^{238}U	144	61,7

Flux et spectres des neutrons

Les flux ont été calculés à 8 groupes d'énergie, en géométrie sphérique, dans les directions radiale et axiale. Nous les avons représentés, dans le cas de R 59-1, en normalisant le flux total au centre à 1 n/cm^2 s (fig. 8 et 9). On remarquera la faible décroissance du flux dans le sens axial, due aux espaces de transition et au sodium, milieux dont les sections macroscopiques de capture sont très faibles.

Le flux maximal calculé à la puissance nominale de 10 MW est:

$1{,}07 \cdot 10^{15}$ n/cm^2 s avec R 59-1,

$1{,}17 \cdot 10^{15}$ n/cm^2 s avec R 60-2.

Dans le tableau IV sont comparés les spectres moyens, calculés également à 8 groupes, dans le cœur et la couverture radiale, pour les deux types de combustible. Les valeurs indiquées sont normalisées de façon que la somme soit égale à 1000.

Malgré la présence d'oxygène, le spectre n'est pas beaucoup moins dur avec le combustible oxyde. Il y a même davantage de neutrons dans le premier groupe au dessus du seuil de fission de l'uranium-238, sans doute à cause de la teneur beaucoup plus faible du cœur en cet isotope et de la moindre diffusion inélastique qui en résulte.

TABLEAU IV
SPECTRES MOYENS A 8 GROUPES
(théorie de la diffusion)

Groupe	Borne d'énergie inférieure (MeV)	Spectre de fission	Spectre dans le cœur		Spectre dans la couverture radiale	
			R 59-1	R 60-2	R 59-1	R 60-2
1	1,35	575	164	177	40	43
2	0,825	178	120	114	48	47
3	0,5	116	171	159	134	131
4	0,3	64	174	184	180	181
5	0,18	34	132	117	166	162
6	0,0674	25	158	136	259	250
7	0,0091	7,5	58	95	161	171
8	0	0,5	23	18	12	15

Economie des neutrons

Nous avons établi le bilan des neutrons, détaillé en distinguant les zones actives et normalisé à 1 neutron naissant dans le cœur, en géométrie (r, z) (PDQ) pour les deux combustibles R 59-1 (tableau V) et R 60-2 (tableau VI).

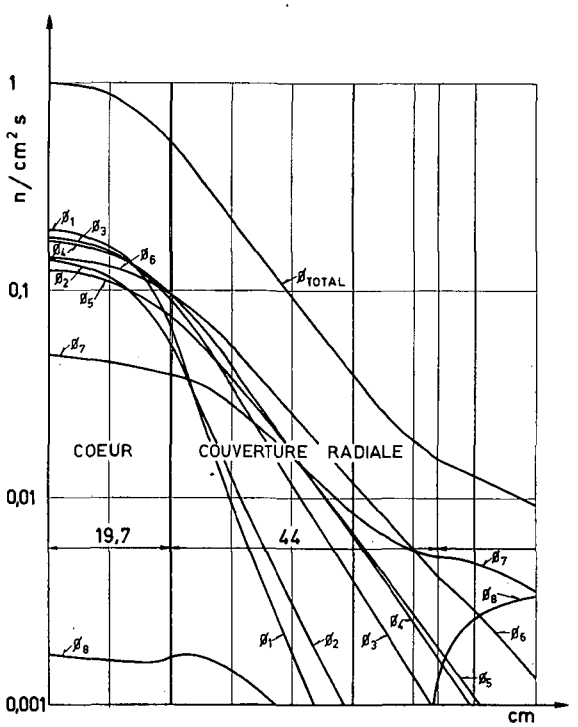


Figure 8
RAPSODIE 59-1. Distribution radiale de flux.

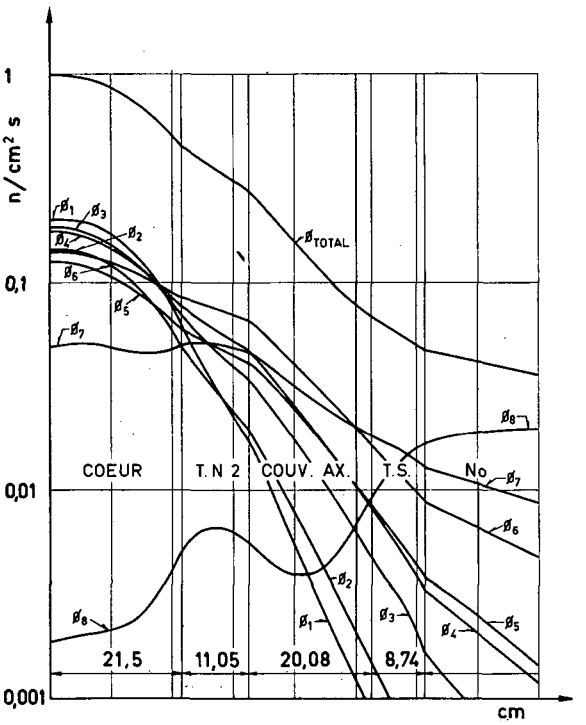


Figure 9
RAPSODIE 59–1. Distribution axiale de flux.

TABLEAU V

RAPSODIE 59-1—BILAN DES NEUTRONS EN GÉOMÉTRIE (r, z)

	Cœur 1 n. n.			Couverture radiale 0, 1925 n. n.		Couvertures axiales 0,0092 n. n.		Les deux couvertures 0,2017 n. n.	
	Fissions[1]	Captures	Fuites	Fissions[2]	Captures	Fissions[3]	Captures	Fissions[4]	Captures
^{239}Pu	0,2091	0,0341	—	—	—	—	—	—	—
^{235}U	0,1218	0,0191	—	0,0248	0,0049	0,0013	0,0003	0,0261	0,0052
^{238}U	0,0297	0,0545	—	0,0494	0,4616	0,0023	0,0256	0,0517	0,4872
Structures	—	0,0046	—	—	0,0521	—	0,0034	—	0,0555
Totaux	0,3606	0,1123	0,5271	0,0742	0,5186	0,0036	0,0293	0,0778	0,5479

[1] $\times \overline{\nu}$ (= 2,7731) → 1 n. n.
[2] $\times \overline{\nu}$ (= 2,5943) → 0,1925 n. n.
[3] $\times \overline{\nu}$ (= 2,5556) → 0,0092 n. n.
[4] $\times \overline{\nu}$ (= 2,5925) → 0,2017 n. n.

Il est à noter que le facteur « fissions rapides » (dues à l'uranium-238 dans le cœur) est beaucoup plus faible pour R 60-2 que pour R 59-1: 4,3% seulement des fissions contre 8,24% globalement; 18,6% des fissions se produisent dans l'uranium-238 avec R 59-1, 16,4% avec R 60-2.

TABLEAU VI

RAPSODIE 60-2—BILAN DES NEUTRONS EN GÉOMÉTRIE (r, z)

	Cœur 1 n. n.			Couverture radiale 0,2029 n. n.		Couverture axiale inférieure 0,0081 n. n.		Couverture axiale supérieure 0,0040 n. n.		Ensemble des couvertures 0,2150 n. n.		Fuites hors du cœur ou des couvertures
	Fissions[1]	Captures	Fuites	Fissions[2]	Captures	Fissions[3]	Captures	Fissions[4]	Captures	Fissions[5]	Captures	
^{239}Pu	0,2714	0,0465	—	—	—	—	—	—	—	—	—	—
^{235}U	0,0597	0,0102	—	0,0248	0,0050	0,0010	0,0002	0,0006	0,0001	0,0264	0,0053	—
^{238}U	0,0149	0,0278	—	0,0522	0,4639	0,0021	0,0184	0,0009	0,0117	0,0552	0,4940	—
Structures	—	0,0075	—	—	0,0738	—	0,0033	—	0,0020	—	0,0791	—
Totaux	0,3460	0,0920	0,5620	0,0770	0,5427	0,0031	0,0219	0,0015	0,0138	0,0816	0,5784	0,1170

[1] $\times\overline{\nu}$ (= 2,890) → 1 n. n.
[2] $\times\overline{\nu}$ (= 2,635) → 0,2029 n. n.
[3] $\times\overline{\nu}$ (= 2,61) → 0,0081 n. n.
[4] $\times\overline{\nu}$ (= 2,63) → 0,0040 n. n.
[5] $\times\overline{\nu}$ (= 2,635) → 0,2150 n. n.

Taux de régénération. Le taux de régénération, défini comme étant le rapport du nombre d'atomes de ^{239}Pu formés par capture de ^{238}U au nombre d'atomes de ^{239}Pu et ^{235}U du cœur détruits par capture ou fission prend les valeurs suivantes (calculs r, z à 2 groupes):

	Interne	Externe	Total
R 59-1	0,14	1,27	1,41
R 60-2	0,07	1,27	1,34

Remarquons que le taux de régénération total calculé à 8 groupes en géométrie sphérique est de l'ordre de 1,60 avec les deux combustibles. Une évaluation en géométrie sphérique apparaît donc comme très optimiste.

Taux de combustion. Après une marche continue du réacteur à la puissance constante de 10 MW pendant un an, les pourcentages de noyaux fissiles détruits sont:

	R 59-1	R 60-2
^{239}Pu	4,6	5,1
^{235}U	3,7	4,4
$^{239}Pu + ^{235}U + ^{238}U$ ...	1,94	3,03

Ils sont plus grands dans le cas des oxydes du fait surtout de la teneur moins grande en ^{238}U et du nombre plus faible des fissions qui lui sont dues.

Production de plutonium dans les couvertures

Une étude sommaire faite en supposant les phénomènes linéaires nous a permis d'évaluer les masses de plutonium et de ^{235}U dont on peut escompter la destruction ou la formation dans le cœur et les couvertures (tableau VII) après un an de fonctionnement à 10 MW.

TABLEAU VII

ÉVOLUTION DU RÉACTEUR EN UN AN A 10 MW
(masses exprimées en kilogrammes)

	R 59-1	R 60-2
^{239}Pu détruit dans le cœur ..	2,29	3,07
^{235}U détruit dans le cœur ...	1,31	0,67
^{239}Pu formé dans le cœur ...	0,51	0,27
^{240}Pu formé dans le cœur ...	0,32	0,45
^{239}Pu formé dans les couvertures	4,57	4,75

Une étude plus détaillée de la production de plutonium dans la couverture radiale a été faite en géométrie sphérique. Cette production peut se caractériser à un instant donné dans tout volume partiel V_i par deux grandeurs:

$$Y^i = \frac{\int_{V_i} N_9 \, dV}{\int_{V_i} N_8 \, dV} \qquad E^i = \frac{\int_{V_i} N_0 \, dV}{\int_{V_i} N_9 \, dV}$$

N_8, N_9, N_0 désignant les nombres d'atomes de ^{238}U, ^{239}Pu, ^{240}Pu contenus dans l'élément de volume dV. Y^i représente la teneur en ^{239}Pu, E^i l'empoisonnement en ^{240}Pu. A partir de E^i et Y^i, on peut définir ce que nous appellerons le «critère de pureté»:

$$G^i = \frac{E^i}{Y^i}$$

La figure 10 représente la variation de E^i, Y^i et G^i en fonction du rayon dans la couverture après une marche continue à 20 MW pendant trois mois.

Le critère de pureté est fonction du spectre: le plutonium formé est d'autant plus pur qu'il se forme plus près du cœur. On peut le modifier en modifiant les programmes de chargement-déchargement de la pile.

Supposons un programme de chargement-déchargement basé sur un partage de la couverture en trois charges égales:

— Charge I: le tiers intérieur — le plus proche du cœur — que l'on décharge chaque année.
— Charge II: le tiers central qui, au bout d'un an, est rechargé en position 1.
— Charge III: le tiers extérieur qui, au bout de deux ans, sera en position 1, après avoir occupé les positions III et II.

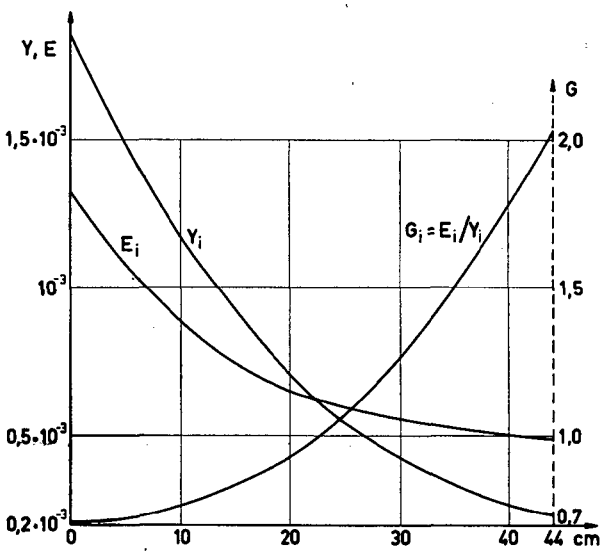


Figure 10
Production de plutonium dans la couverture radiale après 3 mois à 20 MW.

TABLEAU VIII
CRITÈRES DE PURETÉ COMPARÉS DE RAPSODIE-G1-G2

Pile	Critères de pureté
RAPSODIE	1
G 1	80
G 2	66,5

On a comparé (tableau VIII) le critère de pureté du plutonium obtenu après une irradiation complète de ce type dans RAPSODIE aux critères de pureté du plutonium produit dans les piles à graphite françaises G1 et G2, dans des conditions telles que leurs teneurs en ^{239}Pu soient identiques. Dans G1, il s'agit d'une irradiation globale (toute la charge restant dans la même position pendant l'irradiation), dans G2 d'une irradiation homogène (chaque élément, par une migration convenable, subissant le même degré d'irradiation).

On notera la très nette supériorité du point de vue pureté du plutonium produit dans une pile rapide du type RAPSODIE par rapport au plutonium venant de piles thermiques classiques.

Il est à noter aussi que le programme de chargement-déchargement que nous avons décrit pour RAPSODIE n'est pas le meilleur que l'on puisse imaginer pour le critère de pureté mais il a l'avantage d'être d'une grande simplicité.

Puissances développées

Des calculs détaillés des puissances spécifiques ont été faits pour permettre le réglage du débit de sodium par zones. Nous donnons dans le tableau IX la réparti-

TABLEAU IX

PUISSANCES ET PUISSANCES SPÉCIFIQUES

	R 59-1	R 60-2
Puissance totale (MW)		
Cœur	8,225	8,058
Couverture radiale	1,693	1,833
Couvertures axiales	0,082	0,109
Puissances spécifiques maximales (kilowatts par litre du milieu)		
Cœur	350	344
Couverture radiale	37,0	44
Couvertures axiales	8,25	13,6
Puissances spécifiques moyennes (kilowatts par litre du milieu)		
Cœur	225	221
Couverture radiale	1,60	1,66
Couvertures axiales	2,25	2,61

tion par milieu et quelques valeurs remarquables des puissances spécifiques pour une puissance totale de 10 MW. Ces valeurs résultent de calculs PDQ.

Le rapport de la puissance maximale à la puissance moyenne dans le cœur est dans les deux cas de 1,56. Environ 80% de la puissance totale apparaît dans le cœur.

Coefficients de réactivité

La plupart des études de réactivité dont nous parlons ici ont été faites en utilisant la théorie des perturbations, en géométrie sphérique à 8 groupes. Quelques résultats, cependant, comme les coefficients de température isothermes ou la valeur en réactivité des assemblages, résultent de calculs directs à deux dimensions (PDQ).

Flux adjoints — Importance des neutrons. La figure 11 réprésente les flux adjoints à 8 groupes calculés pour R 59-1 au moyen d'un code « adjoint » sur IBM 650.

Les spectres d'importance des neutrons ont été tracés également dans le cas de R 59-1 en différents endroits du plan médian de la pile, l'importance du groupe de plus grande énergie étant normalisée à l'unité (fig. 12). Dans le cœur, les groupes les moins rapides sont les plus importants: ceci résulte de la croissance des sections efficaces de fission du ^{235}U et du ^{239}Pu avec la léthargie. L'importance remonte cependant avec le groupe 1, qui correspond à des énergies supérieures au seuil de fission du ^{238}U. Dans la couverture et au-delà, l'importance décroît avec l'énergie: seuls les neutrons très rapides peuvent causer des fissions dans le ^{238}U de la couverture ou atteindre le cœur pour y maintenir la réaction en chaîne. Dans le tableau X sont notées les importances relatives des

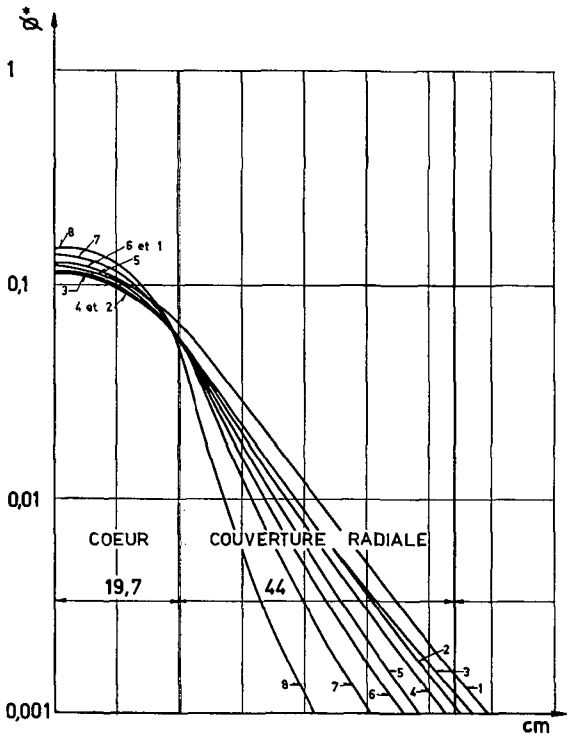


Figure 11
RAPSODIE 59–1. Flux adjoints GSR.

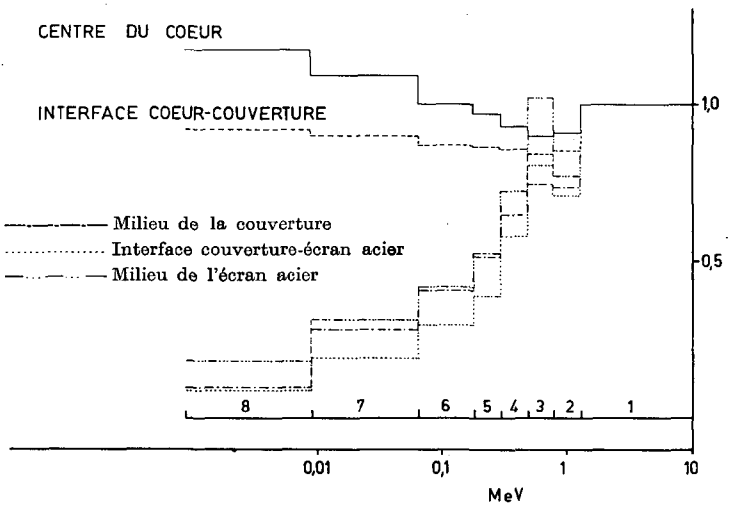


Figure 12
RAPSODIE 59–1. Importance des neutrons.

TABLEAU X

IMPORTANCE RELATIVE DES NEUTRONS DE FISSION DANS RAPSODIE AVEC COMBUSTIBLE MÉTALLIQUE

Elément fissile	^{239}Pu	^{235}U	^{238}U
Importance d'un neutron de fission prompte	1	0,85	0,61
Importance d'un neutron de fission retardée	0,95	0,78	0,56

neutrons de fission dans la pile chargée en combustible métallique. L'importance d'un neutron de fission prompte de ^{235}U et ^{238}U est inférieure à l'importance d'un neutron de fission prompte de ^{239}Pu, bien que les spectres de fission soient identiques parce qu'une partie des fissions de ces éléments se produit dans la couverture. Un neutron de fission retardée a moins d'importance qu'un neutron de fission prompte parce que les neutrons de fissions retardées naissent à des énergies pour lesquelles l'importance est minimale dans le cœur.

Fraction efficace des neutrons retardés (β_{eff}) — *Temps de vie des neutrons prompts* (l). A cause de la plus grande teneur relative en ^{239}Pu, élément dont la fraction

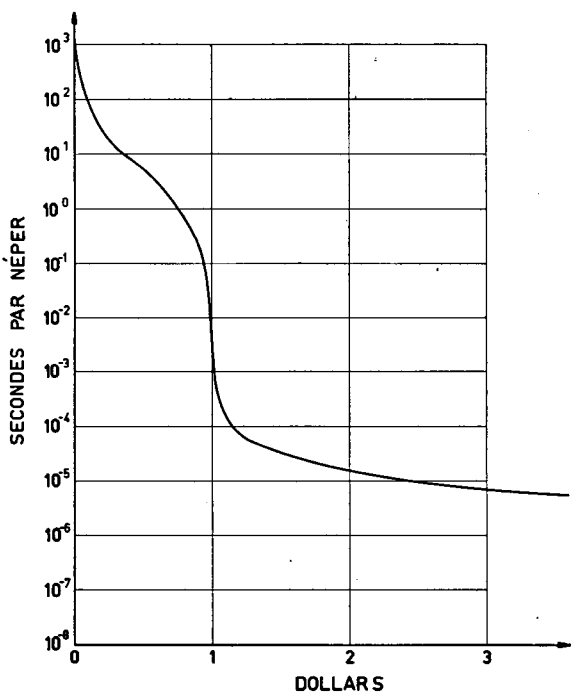


Figure 13
RAPSODIE 59-1. Période stable en fonction de la réactivité.

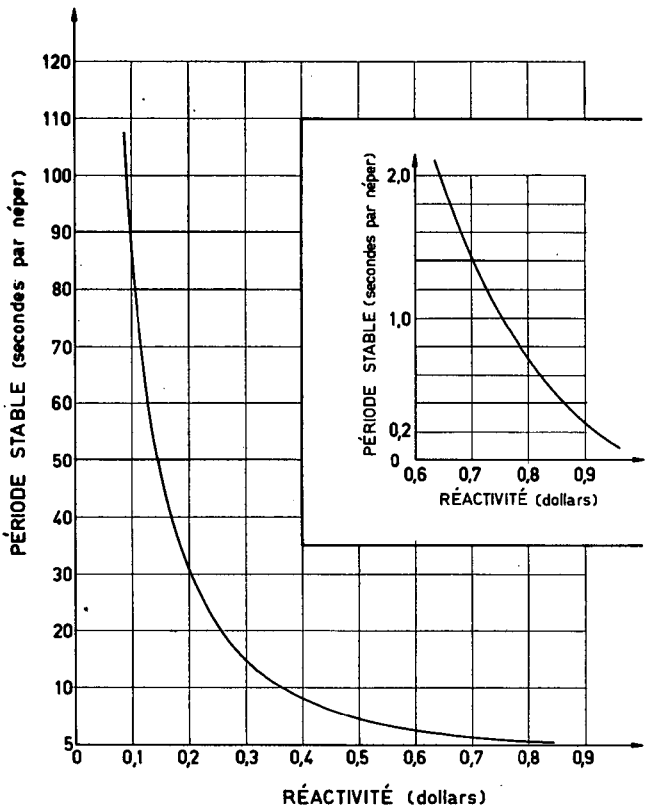


Figure 14
RAPSODIE 59–1. Période stable en fonction de la réactivité.

naturelle de neutrons retardés est faible, la fraction efficace des neutrons retardés est plus faible pour R 60-2 que pour R 59-1 (tableau XI). Par contre, le temps de vie des neutrons prompts est légèrement plus grand (spectre un peu moins dur).

TABLEAU XI

β_{eff} ET TEMPS DE VIE

	R 59-1	R 60-2
β_{eff}	0,00476	0,00386
l (s)	$7{,}29 \cdot 10^{-8}$	$8{,}92 \cdot 10^{-8}$

Les valeurs de β_{eff} ont été obtenues à partir des fractions naturelles de neutrons retardés de fissions rapides tirées de l'ANL 5800 [5].

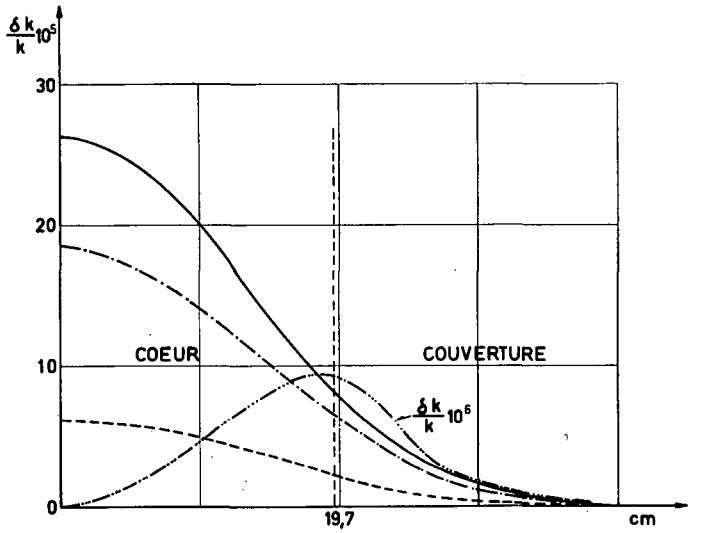


Figure 15
Combustible métal. Effet sur la réactivité.
Remplacement de 1 cm³ de sodium:
par 1 cm³ de ^{240}Pu – – – – –
par 1 cm³ de ^{239}Pu ————
par 1 cm³ de ^{238}U – · · · – · · ·
par 1 cm³ de ^{235}U – · – · – ·

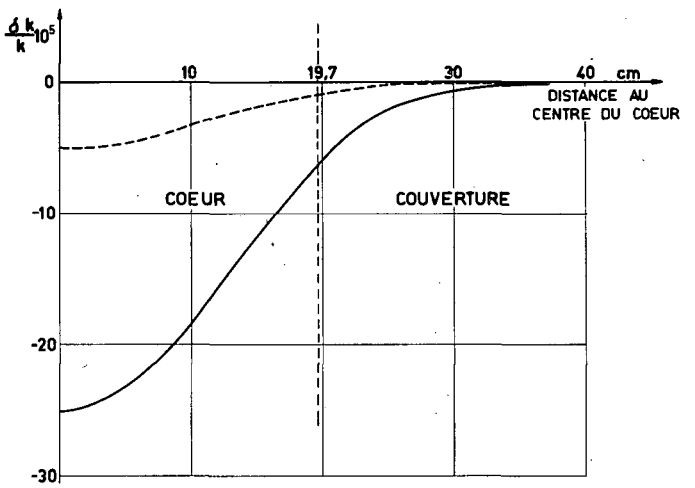


Figure 16
Combustible métal. Effet sur la réactivité.
Remplacement de 1 cm³ de sodium:
par 1 cm³ de B^{nat} – – – –
par 1 cm³ de ^{10}B ————

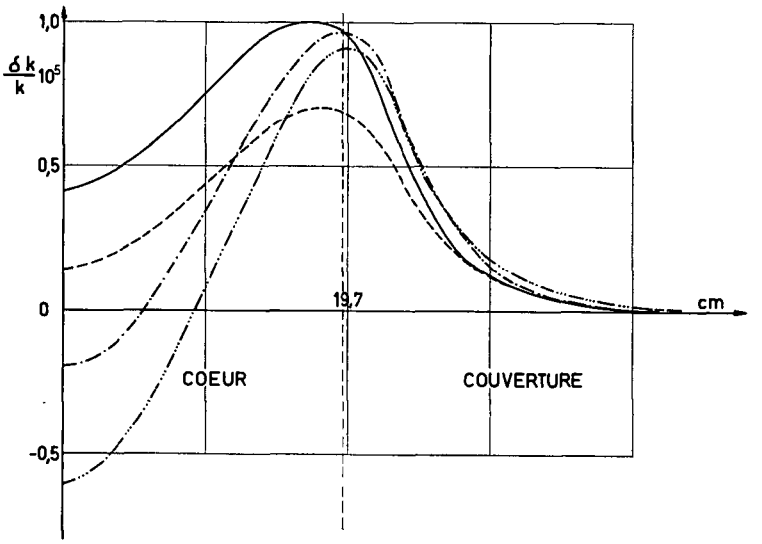


Figure 17
Combustible métal. Effet sur la réactivité.
Remplacement de 1 cm³ de sodium:
par 1 cm³ de Fe — · — · — · —
par 1 cm³ de Mo — · · · — · · · —
par 1 cm³ de O — — — — — —
par 1 cm³ de C ————————

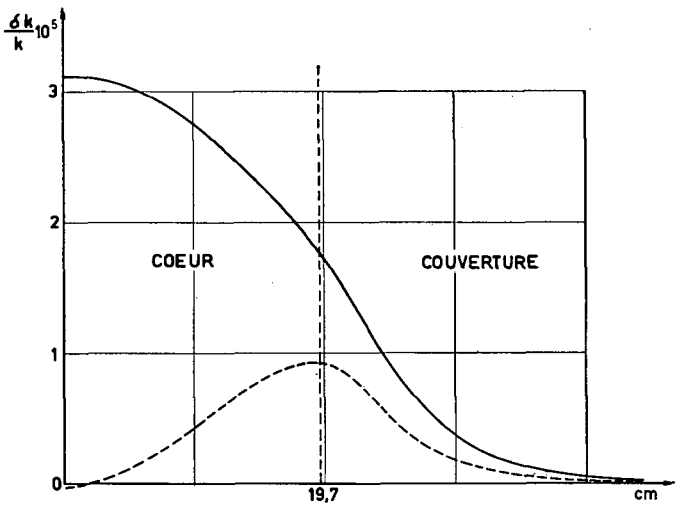


Figure 18
Combustible métal. Effet sur la réactivité.
Remplacement de 1 cm³ de sodium:
par 1 cm³ de milieu cœur————————
par 1 cm³ de milieu couverture — — — —

Courbe période stable-réactivité. La période stable est le temps nécessaire à la densité neutronique pour augmenter d'un facteur e. Elle est liée à la réactivité par la formule:

$$\frac{\varrho}{\beta_{\text{eff}}} = \frac{1}{\beta_{\text{eff}}\,(l + T)}\left[l + T\sum_{i=1}^{6}\frac{\beta_{i\,\text{eff}}}{1 + \lambda_i T}\right]$$

où

λ_i est la constante de désintégration du corps émetteur des neutrons retardés du groupe i,

T, la période stable (en secondes),

ϱ, la réactivité: $\varrho = k_{\text{ex}}/k_{\text{eff}} \simeq k_{\text{ex}}$.

La figure 13 représente la variation de T en fonction de la réactivité exprimée en dollars dans le cas de R 59-1. Nous donnons également des agrandissements de la courbe dans la zone intéressante pour le pilotage, au-dessous de β_{eff} (fig. 14).

Effets sur la réactivité de l'introduction de différents matériaux dans RAPSODIE

Les courbes des figures 15 à 18 représentent l'effet en fonction du rayon (géométrie sphérique) de la substitution à un centimètre cube de sodium d'un centimètre cube de matériau (^{239}Pu, ^{240}Pu, ^{235}U, ^{238}U, ^{10}B, B$^{\text{nat}}$, Fe, Mo, C ou O) ou d'un centimètre cube de milieu cœur ou couverture dans le cas du combustible métallique (R 59-1).

On remarque que:

a) Le plutonium-240 remplaçant le sodium, contrairement au ^{238}U, a toujours un effet positif sur la réactivité au centre du cœur;

b) Le remplacement du fer par le molybdène comme matériau de structure a un effet négatif sur la réactivité, surtout au centre du réacteur;

c) L'oxygène et surtout le carbone remplaçant le sodium ont un effet toujours positif;

d) Le remplacement d'un centimètre cube de couverture radiale par un centimètre cube de milieu cœur à l'interface conduit à un effet positif de 0.82 pcm.

Une étude similaire a été faite dans le cas du combustible oxydes, mais les résultats étant à peu de choses près les mêmes nous ne donnons pas ici les courbes correspondantes.

TABLEAU XII

RÉACTIVITÉ DES ASSEMBLAGES — EFFETS

(en pcm)

	Centre du cœur		Interface cœur-couverture radiale	
	R 59-1	R 60-2	R 59-1	R 60-2
Remplacement du sodium par un assemblage fertile	+ 400	+ 570	+ 840	+ 400
Remplacement d'un assemblage fertile par un assemblage combustible	+ 2.300	+ 1.940	+ 710* + 530**	+ 625

* Résulte d'un calcul (r, z).

** Résulte d'un calcul (x, y).

Réactivité des assemblages. On peut déduire de courbes telles que celles de la figure 18 une valeur approchée de l'effet sur la réactivité du remplacement d'un assemblage fertile par un assemblage combustible. Une méthode plus précise

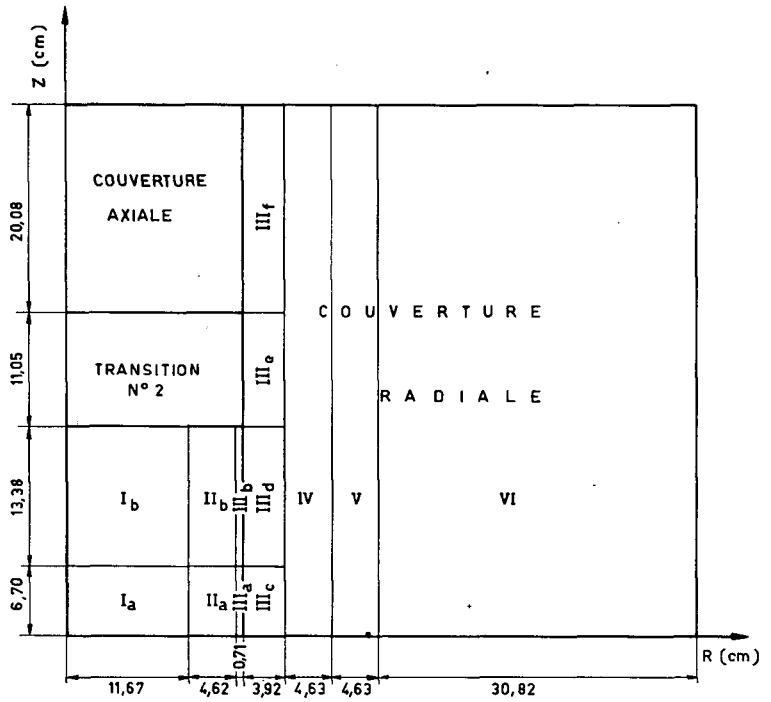


Figure 19
Découpage en zones de RAPSODIE pour le calcul des coefficients de réactivité isothermes.

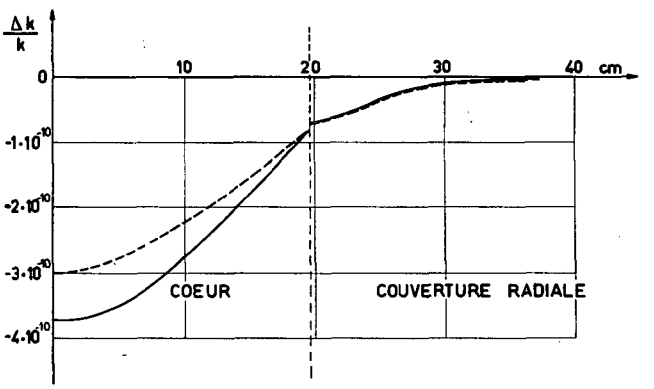


Figure 20
Effet sur la réactivité de la variation de température du combustible ou du fertile contenu dans 1 cm³ ($\Delta t = 1°$ C).
——— RAPSODIE 59-1
– – – – RAPSODIE 60-2

cependant est de le calculer directement en géométrie à deux dimensions (r, z) ou (x, y); c'est celle qui nous a conduits aux résultats du tableau XII. L'effet de remplacement d'un assemblage fertile par un assemblage fissile est plus faible avec R 60-2, qui comporte un plus grand nombre d'assemblages de type cœur.

Si tout le sodium de refroidissement quittait le cœur, il en résulterait un effet négatif sur la réactivité, de — 1900 pcm avec le combustible oxydes et de — 2150 pcm avec le combustible métallique.

Coefficients de température isothermes. Deux méthodes ont été employées: la théorie des perturbations à 8 groupes, en géométrie sphérique pour les deux combustibles et le calcul direct avec découpage en zones du réacteur (fig. 19), le découpage radial (I à VI) correspondant aux zones de réglage du débit de sodium (on a supposé la pile symétrique par rapport au plan médian: la zone

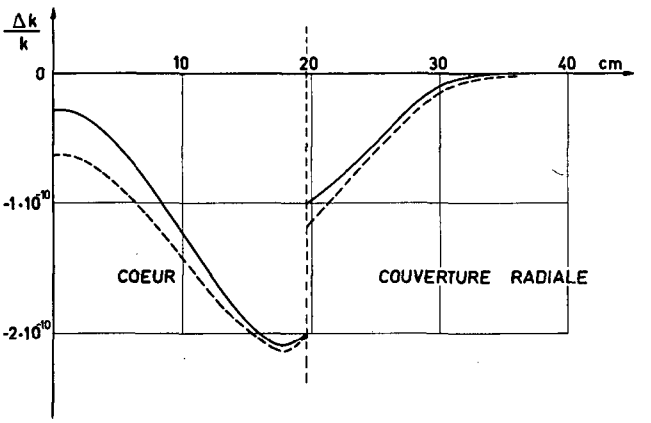


Figure 21
Effet sur la réactivité de la variation de température du sodium contenu dans 1 cm³ ($\Delta t = 1°$ C).
——— RAPSODIE 59-1
– – – – RAPSODIE 60-2

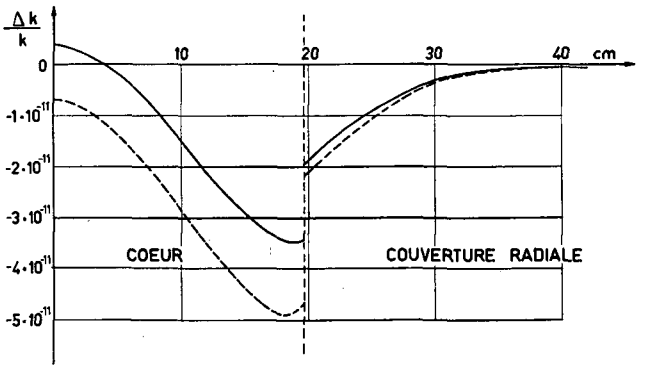


Figure 22
Effet sur la réactivité de la variation de température de l'acier contenu dans 1 cm³ ($\Delta t = 1°$ C).
——— RAPSODIE 59-1
– – – – RAPSODIE 60-2

III_d, par exemple, comprend la partie dessinée sur la figure 19 et sa symétrique). L'étude directe n'a été faite que pour R 59-1.

Les figures 20, 21, 22 donnent le coefficient local en fonction du rayon dans les trois cas où :

— Le combustible (ou le fertile) se dilate seul,
— Le sodium se dilate seul,
— L'acier des structures se dilate seul.

On remarquera que les effets de dilatation du sodium et de l'acier sont maximaux à l'interface cœur-couverture et que l'effet de la dilatation du sodium est négatif au centre du cœur. Le coefficient de dilatation linéaire des oxydes

TABLEAU XIII

COEFFICIENTS DE TEMPÉRATURE ISOTHERMES

$(\Delta k/k) \cdot 10^6$ par degré centésimal

		R 59-1	R 60-2
Cœur	Sodium	—7,48	—7,66
	Acier	—1,18	—1,76
	Combustible	—6,43	—4,35
Couverture radiale	Sodium	—4,72	—5,06
	Acier	—0,92	—1,02
	Fertile	—3,07	—3,30

TABLEAU XIV

COEFFICIENTS DE TEMPÉRATURE ISOTHERMES — DÉTAIL CŒUR ET COUVERTURE RADIALE *(R 59-1)*

$(\Delta k/k) \cdot 10^6$ par degré centésimal

Zone			Fertile ou combustible	Sodium	Acier
Cœur	I	a	—1,51	—0,29	—0,05
		b	—1,62	—1,56	—0,26
	II	a	—1,09	—0,84	—0,15
		b	—1,24	—1,66	—0,30
	III	a	—0,18	—0,37	—0,06
		b	—0,20	—0,34	—0,06
		Total	*—5,84*	*—5,06*	*—0,88*
Couverture radiale	III	c	—0,49	—0,67	—0,13
		d	—0,55	—0,98	—0,20
		e	—0,11	—0,23	—0,05
		f	—0,02	—0,12	—0,02
	IV		—0,80	—1,28	—0,26
	V		—0,35	—0,67	—0,13
	VI		—0,33	—0,61	—0,12
		Total	*—2,65*	*—4,56*	*—0,91*

étant plus faible que celui de l'alliage métallique, le coefficient négatif de température isotherme dû à la dilatation du combustible est moins important avec les oxydes.

Le tableau XIII indique les coefficients globaux par régions obtenus, en théorie des perturbations, avec les deux combustibles, le tableau XIV les coefficients détaillés correspondant aux zones de la figure 19 dans le cas de R 59-1.

En comparant les résultats se rapportant à R 59-1, on notera que la théorie des perturbations en géométrie sphérique conduit à des coefficients plus grands en valeur absolue que le calcul direct.

Autres effets sur la réactivité. *a*) Déformation des assemblages. La courbure des assemblages du cœur, due à l'existence d'un gradient de température radial, peut être une cause d'instabilité. Un calcul a montré que, dans le cas de RAPSODIE 59-1, une déformation libre des assemblages tenus simplement par leur base, s'écartant en gerbe, conduisait à un effet négatif sur la réactivité de — 114 pcm. Des calculs plus précis sont en cours à ce sujet dans le cadre plus général des études de stabilité. *b*) Effet Doppler. Il est certainement très faible pour RAPSODIE, quel que soit le combustible; étant données les teneurs en matériaux fissiles et fertiles, il est probablement négatif avec 59-1 mais peut être positif avec R 60-2. *c*) Influence de la combustion sur la réactivité. La marche du réacteur se traduit par une destruction des noyaux du cœur peu compensée par une conversion intérieure très faible. La baisse de réactivité que l'on peut en attendre est de l'ordre de 1000 pcm après 6 mois de marche continue du réacteur à la puissance constante de 10 MW.

Etude des barres de contrôle

Les barres de contrôle sont au nombre de six, elles occupent les emplacements d'assemblages normaux aux endroits indiqués sur la coupe de la figure 5. Ce sont des barres de «poison», l'absorbant étant du carbure de bore en poudre pressée à chaud, de densité 2,4. Pour l'instant sont prévues quatre barres de sécurité ou compensation à forte teneur en ^{10}B (bore enrichi à 90% en ^{10}B) et deux barres de pilotage dans lesquelles le bore serait naturel; les calculs qui suivent ont été faits dans cette hypothèse. Notre position n'est pas encore fixée, mais il se peut que nous adoptions finalement cinq barres de sécurité et une de pilotage s'il s'avérait que les calculs d'antiréactivité étaient par trop imprécis et optimistes quant à l'efficacité du bore-10.

Les barres sont cylindriques, de diamètre 45,8 mm, gainage acier compris. Le carbure de bore est réparti sur 450 mm de hauteur et les teneurs en bore-10 sont sensiblement 1125 g par barre de sécurité et 100 g par barre de pilotage (200 g si une seule barre est retenue)*.

Méthodes de calcul. Les calculs ont été faits avec le code PDQ, qui ne permet pas de représenter exactement la géométrie de la pile avec les barres.

Deux méthodes ont été employées:

a) En géométrie (r, z), on a remplacé les six barres par une couronne cylindrique dont la circonférence moyenne passe par l'axe des barres réelles (à 17,6 cm de l'axe de la pile) et dont la section droite est égale à six fois la section d'une barre. Un des inconvénients de ce procédé est qu'il escamote les effets d'ombre possibles entre barres.

* Pour le calcul de RAPSODIE 59-1, les valeurs adoptées ont été légèrement différentes: 1100 g de ^{10}B par barre de sécurité et 200 g par barre de pilotage, le carbure de bore étant réparti sur 400 mm seulement.

b) En géométrie (x, y), on a figuré les barres par des carrés à leur emplacement exact. La difficulté dans ce cas est inhérente à la géométrie (x, y): il est difficile de tenir compte des fuites axiales.

Valeurs en réactivité. Les deux méthodes ont conduit à des résultats tout à fait comparables. On trouvera dans le tableau XV la valeur en réactivité de chaque barre avec les deux combustibles, c'est-à-dire le gain de réactivité apporté par le passage de la barre de sa position basse à sa position haute.

TABLEAU XV

VALEURS EN RÉACTIVITÉ DES BARRES

(en pcm)

	R 59-1	R 60-2
Barre de sécurité ou compensation	2375	2400
Barre de pilotage	430	210
Total des 6 barres (4 + 2) ..	10360	10020

Le calcul montre qu'un gramme de bore-10 apporte en moyenne une antiréactivité de 2,2 à 2,3 pcm (on avait trouvé 3 pcm par la théorie des perturbations). Au total, avec les six barres, on dispose théoriquement d'une antiréactivité de 10%, ce qui est plus que suffisant.

Toutefois, il y a lieu d'être très prudent quant à ces valeurs. On sait en effet que l'approximation de la diffusion — dont procède PDQ — s'applique mal dans le cas des milieux très absorbants comme celui des barres au bore. C'est

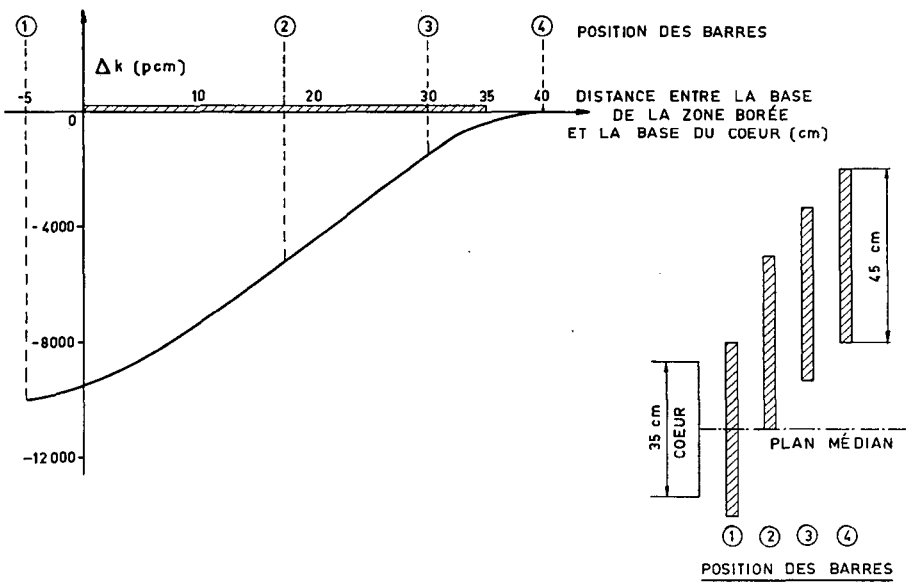


Figure 23
Variation de la réactivité avec la position des barres. RAPSODIE 60-2.

pourquoi nous envisageons de porter le nombre de barres de sécurité à cinq si les mesures expérimentales prévues sur ZPR III en montraient la nécessité.

La figure 23 représente pour RAPSODIE 60-2 la variation de la réactivité (en pcm) en fonction de la position des barres, supposées se déplacer toutes en même temps de la même quantité. La position 4 correspond à la remontée maximale des barres; la position 1 est la position basse; la position 3 est celle où toutes les barres sont légèrement enfoncées (au 1/8 du cœur environ).

On voit que la réactivité diminue lentement au début de la chute des barres, puis plus rapidement dès que le niveau inférieur de la partie borée arrive au sommet du cœur, la variation étant alors sensiblement linéaire. Ceci est un argument en faveur d'une insertion à peu près égale de toutes les barres pendant la marche du réacteur: leur action sera plus rapide en cas de chute d'urgence.

Effets sur les caractéristiques du réacteur. Les barres, du fait des captures parasites qui leur sont dues, ont un effet négatif sur l'économie des neutrons. On trouvera dans le tableau XVI quelques estimations des effets attendus sur les caractéristiques neutroniques du réacteur.

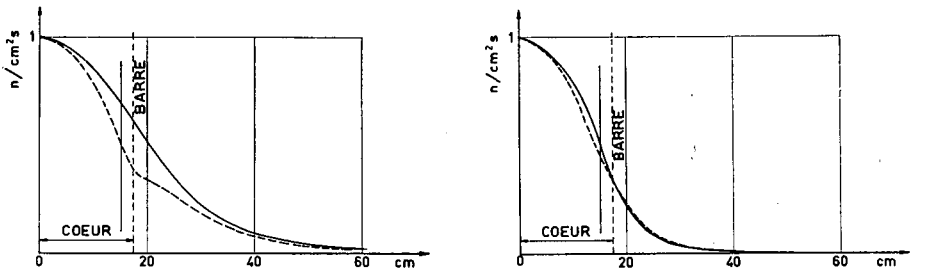


Figure 24
Effet d'une barre de sécurité sur les flux. *A gauche*: $E < 1{,}35$ MeV.
A droite: $E > 1{,}35$ MeV.
– – – – Barre en position haute
——— Barre en position basse

La figure 24 représente les effets d'une barre de sécurité sur les flux calculés (PDQ, géométrie x, y) dans le plan médian du réacteur, suivant un rayon passant par le centre de la barre, et normalisés à 1 n/cm² s au centre. Le groupe de haute énergie ($E > 1{,}35$ MeV) est peu affecté; par contre, le groupe correspon-

TABLEAU XVI

EFFETS DES BARRES DE CONTROLE SUR LES CARACTÉRISTIQUES NEUTRONIQUES DE RAPSODIE
(variations en %)

Flux maximal.	+5%
Puissance développée dans la couverture radiale	—11%
Taux de combustion maximal dans le cœur	+6%
Taux de combustion moyen dans le cœur	+2%
Taux de régénération total	—7%

dant à $E<1{,}35$ MeV subit une dépression importante mais qui ne semble pas se répercuter très loin. On peut donc penser que les effets d'ombre éventuels seront faibles.

Taux de destruction du bore-10. On a calculé que, pour une barre en position moyenne (position 2 de la figure 23) et au bout d'un an de fontionnement continu du réacteur à 10 MW, 1,2% des atomes de ^{10}B étaient détruits, le taux maximal étant de 1,7% dans la partie la plus proche du plan médian du cœur. Si la barre est en position haute, seulement 0,35% des atomes de ^{10}B sont détruits.

2.3. Etudes paramétriques de combustibles

Des calculs paramétriques avec d'autres combustibles métalliques et oxydes ont été faits pour RAPSODIE en vue de déterminer l'influence de la composition adoptée sur les masses critiques à volume de cœur constant (36,46 l), le cœur et tous les autres milieux ayant gardé les compositions volumétriques de RAPSODIE 59-1 ou 60-2.

Alliages métalliques U—Pu—Mo

Nous avons fait varier la teneur pondérale en plutonium, gardant celle du molybdène égale à 10%, et nous avons calculé l'enrichissement correspondant de l'uranium en ^{235}U et les masses critiques (tableau XVII). Les caractéristiques du cœur sont les mêmes que pour RAPSODIE 59-1: hauteur 40 cm, diamètre 34 cm.

Tableau XVII

COMBUSTIBLES U—Pu—Mo— — COMPOSITIONS ET MASSES CRITIQUES

% en poids de ^{239}Pu	Enrichissement de l'uranium (%)	Masses critiques (kg)		
		Total	Pu	U enrichi
20	20	257	51,4	179,8
15	26,9	257	38,5	192,6
0	49,15	264	0	238

Signalons que le combustible métallique qui a le plus de chances d'être retenu actuellement est celui à 15% en poids de plutonium.

Oxydes mixtes $PuO_2 — UO_2$

Nous avons pris comme paramètre de définition du combustible le rapport α du nombre d'atomes d'uranium enrichi au nombre d'atomes de plutonium, et pour densités 9,86 pour UO_2 et 10,31 pour PuO_2, soit 90% de la densité théorique. Les caractéristiques du cœur sont celles de RAPSODIE 60-2: hauteur 35 cm, diamètre 38,6 cm, avec deux zones de transition au-dessus du cœur. Le tableau XVIII indique comment varient l'enrichissement de l'uranium et les masses critiques en fonction de α. On remarquera que:

a) L'utilisation d'uranium enrichi à 60% au lieu de 20% réduit de près de moitié l'investissement en plutonium;

b) Avec un combustible UO_2 pur l'uranium doit être enrichi à 85% environ;

c) Avec un combustible $UO_2 — PuO_2$ où l'uranium est naturel le rapport U/Pu est sensiblement égal à 1.

TABLEAU XVIII

COMBUSTIBLES PuO_2 — UO_2 — COMPOSITIONS ET MASSES CRITIQUES

Combustible		% volumétrique de PuO_2 dans le combustible	Masse totale de combustible (kg)	Masse de PuO_2 (kg)	Masse de ^{239}Pu (kg)	Masse d'uranium enrichi (kg)
$\alpha = \frac{U}{Pu}$	Enrichissement de l'uranium (%)					
∞	84,2	0	151,9	0	0	133,8
20	80	4,62	152,1	7,3	6,46	127,4
3,5	60	21,6	153,3	34,3	30,3	104,8
1,9	40	33,7	154,1	53,4	47,1	88,7
1,3	20	42,6	154,8	67,5	59,6	76,9
1	0,7	49	155,2	77,7	68,6	68,3

Le combustible à oxydes que l'on envisage actuellement contiendrait environ 25% en volume de PuO_2 dans UO_2.

2.4. ETUDES SUR LA PROPAGATION DES FLUX A LONGUE DISTANCE

On a divisé le réacteur en trois zones, dans lesquelles les taux d'activité sont très différents (fig. 4).

La zone A comprend le réacteur lui-même; elle est inaccessible en tout temps. Dans la zone B se trouvent les pompes ainsi que les circuits de sodium et l'échangeur primaire; le flux neutronique qui y règne est suffisamment faible pour ne pas activer le sodium-potassium qui constitue le réfrigérant secondaire; par contre, l'activité du sodium primaire ne permet l'accès de cette zone qu'un certain temps (2 jours environ) après arrêt du réacteur. Sur la zone C, qui englobe le plancher de travail, ne doit régner qu'une intensité de rayonnement inférieure à la limite médicalement admise.

Pour se protéger latéralement du rayonnement intense provenant du cœur et des couvertures, on a disposé à l'intérieur de la cuve d'étanchéité du réacteur un réflecteur en acier qui atténue le flux gamma et donne à sa sortie un flux de neutrons partiellement ralentis par les chocs inélastiques qu'ils y ont subis. Une couche de graphite boré permet ensuite d'absorber la plus grande partie du flux neutronique, et un écran de béton sert de protection contre le rayonnement gamma.

On n'a pas voulu employer de béton dans les parties supérieures que constituent les bouchons fixes et mobiles, car on n'était pas sûr de sa tenue sous irradiation élevée et à haute température. Les bouchons sont donc constitués par une alternance de couches de graphite boré et d'acier.

Les épaisseurs des matériaux nécessaires ont été établies d'après des calculs préliminaires et quelques expériences de transmission de flux, effectuées en particulier derrière la colonne thermique du réacteur ZOÉ*. Par contre, on s'est attaché à déterminer de façon plus précise l'atténuation du rayonnement dans le réflecteur acier qui entoure les couvertures ainsi que dans le bain de sodium situé au-dessus du cœur. Il est en effet nécessaire de connaître avec un degré de précision assez élevé l'intensité des rayonnements neutronique et gamma qui pénètrent dans le graphite boré latéral et dans les bouchons, car sa valeur

* Ces travaux ont été effectués en collaboration avec le Service d'études de protection des piles.

détermine le dégagement de chaleur et conditionne par suite le système de refroidissement des premières couches de protection. Des études de transmission dans le sodium et dans l'acier ont été entreprises à l'aide de calculs à 20 groupes d'énergie de neutrons d'une part, et une approche expérimentale du problème a été lancée d'autre part. Dans ce but, on a construit dans la pile piscine TRITON des maquettes représentant certaines parties du réacteur, et on y a déterminé la transmission du flux neutronique par des calculs à 20 groupes. La confrontation des résultats de calculs avec les mesures effectuées à l'aide de différents types de détecteurs ou de chambres à fission est assez bonne, mais ces études ne sont pas achevées [6].

Il reste à résoudre les nombreux problèmes de protection locale posés par la présence des points faibles que sont les entrées et sorties des tuyaux de sodium, les divers circuits de refroidissement des bouchons et du graphite boré et du béton latéraux. Ces questions, n'étant pas susceptibles d'analyse précise, seront traitées à l'aide des règles habituelles, en particulier celles énoncées en [7].

2.5. Calculs préliminaires aux expériences sur RAPSODIE

Ces calculs n'ont porté jusqu'ici que sur les problèmes liés au démarrage de RAPSODIE.

Calcul d'une source Sb — Be

On a calculé le rendement d'une source de neutrons formée d'un cylindre d'antimoine dans un manchon de béryllium, en tenant compte, d'une part, de la dépression et de l'atténuation du flux thermique au cours de l'irradiation de l'antimoine, et, d'autre part, de l'auto-absorption dans l'antimoine des γ produits. On a trouvé un bon accord avec les calculs anglais (Crocker et Henry). Par contre, un écart sensible subsiste avec les quelques mesures faites à Saclay.

Le rapport des diamètres de l'antimoine et du béryllium a été déterminé pour un rendement maximum par curie.

Etude de l'approche critique

Calcul à un groupe. On a repris un calcul proposé par Hill et Syrett pour les réacteurs ZÉPHYR et ZEUS [8].

Il s'agit de suivre, en fonction du chargement en combustible, l'évolution des flux produits dans la couverture du réacteur par une source centrale.

Le modèle de réacteur utilisé remplace les couvertures axiales par un gain réflecteur, et considère comme infinie la couverture radiale.

Les flux, développés en série de fonctions de Bessel, sont calculés par un programme Fortran.

On constate (fig. 24 bis) comme pour les réacteurs déjà cités, que si l'on porte le rapport rayon-combustible/flux-couverture en fonction du rayon-combustible, la courbe obtenue devient rectiligne aux 2/3 du chargement.

Le rayon critique mis en évidence — point où cette courbe coupe l'axe des abscisses — ne diffère que de 7% de celui de RAPSODIE résultant des calculs multigroupes.

Calculs multigroupes de répartition des flux dus à la source, à l'aide des codes DSN et TDC. Une étude systématique de ces codes dans leur application à des problèmes de sources a été entreprise. Ces codes paraissent assez mal adaptés à ce genre de problèmes: on observe en effet une convergence très lente pour les flux de basses énergies. Cela serait dû, d'une part aux dimensions relativement

importantes du réacteur (1 m de rayon pour l'ensemble des couvertures), d'autre part au fait que la source débite dans les groupes lents.

Divers essais — accélération de convergence, facteurs d'échelle — sont en cours, en collaboration avec le Service de calcul électronique arithmétique.

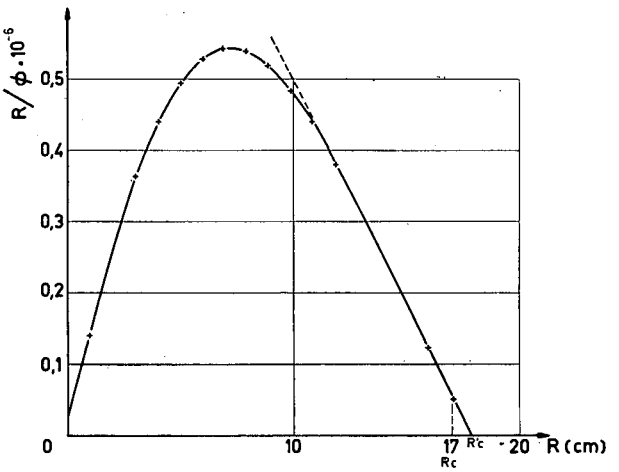


Figure 24 bis
Evolution des flux au cours de l'approche sous-critique.

3. Stabilité et comportement dynamique

3.1. Stabilité

Des études sont actuellement en cours, et un programme digital sur IBM 7090 (Fortran) est sur le point d'être achevé.

Parallèlement, des études tant expérimentales que théoriques ont été faites sur le problème de déformations des assemblages sous gradient de température. Dans le cas de RAPSODIE, les assemblages sont soumis à deux effets d'origine thermique :

a) Dilatation différentielle de la section droite des assemblages due au gradient axial;

b) Déformation vers l'extérieur due à la différence de température entre faces intérieure et extérieure des assemblages.

Pour un régime normal de fonctionnement de RAPSODIE, le phénomène peut être considéré comme linéaire, et nous espérons nous servir du programme digital P3 cité plus haut (traitement linéaire) pour résoudre le problème de stabilité. La seule difficulté que nous aurons à surmonter tient dans le fait que le réacteur n'est pas compact, et que certains déplacements peuvent être aléatoires dans la limite des jeux de construction.

De toutes façons, les études sur modèle simplifié montrent que des solutions satisfaisantes en ce qui concerne la stabilité de RAPSODIE existent.

Nous aurons simplement à choisir entre deux solutions : cœur compact, solution qui paraît plus sûre ou tout au moins plus facilement calculable du point de vue stabilité, et cœur libre, solution meilleure du point de vue mécanique et

des contraintes. Le choix pourra se faire après des essais sur maquette à l'échelle 1 (à Cadarache) qui, nous l'espérons, donneront des indications sur le problème des vibrations d'un cœur libre (avec jeu). Sinon, on pourra peut-être choisir après des premiers essais sur la pile elle-même.

3.2. Etude des régimes transitoires

La formulation utilisée et les méthodes de résolution pour l'étude du comportement dynamique des piles rapides sont exposées ailleurs [9].

Les résultats que nous donnons ici ont été obtenus à partir des codes P1 et P1′ en calcul digital sur ordinateur Mercury Ferranti. Les études se poursuivent actuellement avec un nouveau code P2 sur IBM 7090. Le schéma de principe des calculs est indiqué par la figure 25.

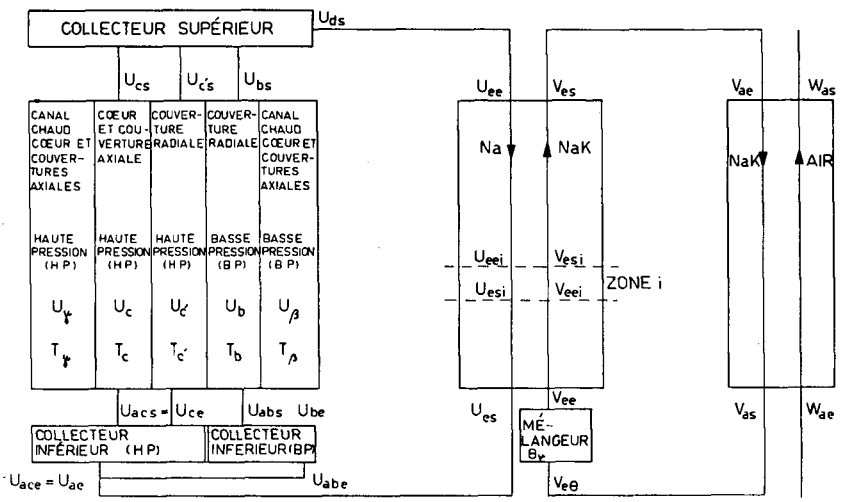


Figure 25
Programme dynamique P_2. Schéma de principe.

Les études ont porté sur la pile RAPSODIE 59-1, dont nous rappelons les caractéristiques essentielles:

— Combustible métallique (U — Pu — Mo),
— Hauteur du cœur: 40 cm,
— Gradient de température axial du sodium: 90°C,
— Gradient de température du sodium dans la couverture = gradient de température du sodium dans le cœur + 50°C.

Incidents concernant le contrôle

Chute d'urgence des barres de sécurité. Nous avons distingué deux cas suivant que cette chute s'accompagne (fig. 27) ou non (fig. 26) d'une chute du débit de sodium.

Le choc thermique qui en résulte est important dans les deux cas: 80°C/s et 70°C/s sur le combustible.

Une étude, assez détaillée dans le premier cas sur une possibilité de freinage du débit de sodium pour réduire le choc thermique nous a conduits à rejeter cette solution comme peu intéressante, et surtout comme dangereuse en cas de fonctionnement intempestif pendant une marche normale.

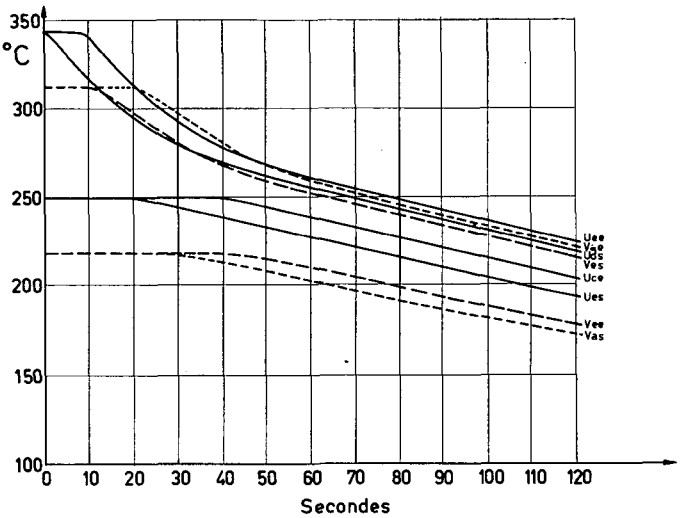


Figure 26
Décroissance des températures des réfrigérants en cas de chute d'urgence des barres de contrôle sans arrêt des pompes primaires. Coefficient de mélange: 0,3.

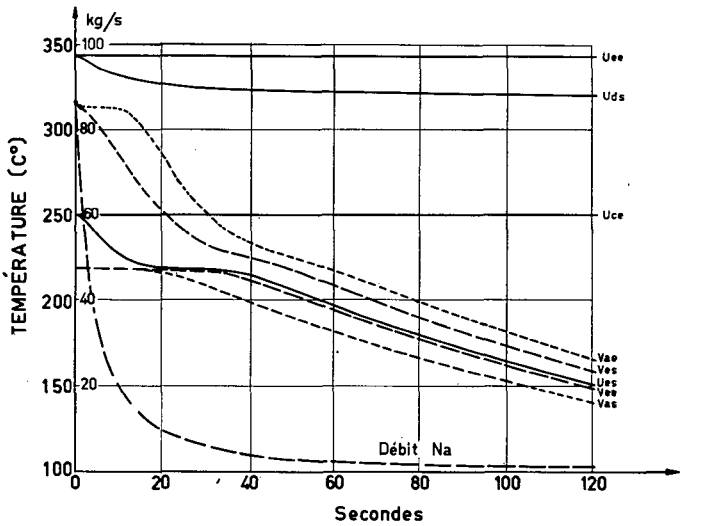


Figure 27
Décroissance des températures des réfrigérants en cas de chute d'urgence des barres de contrôle suivie après 0,25 s de l'arrêt des pompes primaires.

Par contre, nous avons envisagé la possibilité d'un arrêt rapide et non d'urgence pour réduire les chocs thermiques; on peut lire sur la figure 28 la valeur du choc thermique sur le combustible pour deux vitesses différentes d'insertion des barres.

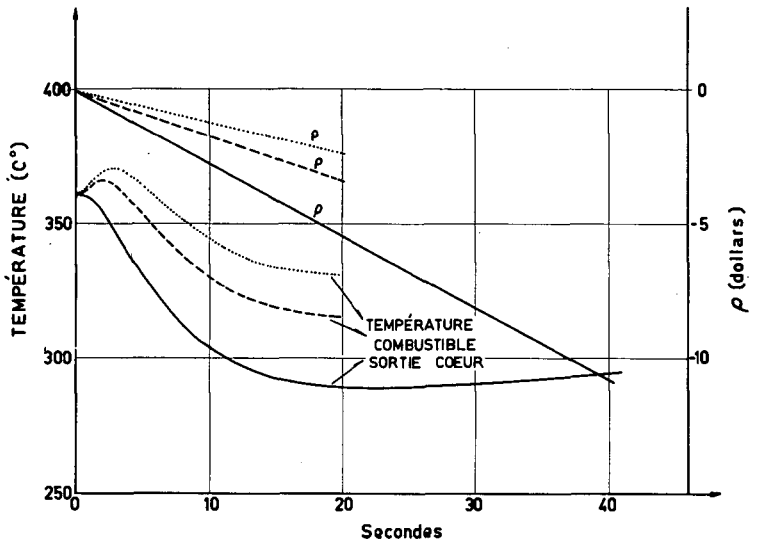


Figure 28
Introduction linéaire de 6500 pcm.
——— en 50 s
– – – – – en 80 s
..... en 100 s
Gradient maximum sur le combustible: en 50 s, 8°C/s—en 80 s, 5,5°C/s—en 100 s, 4°C/s.

La possibilité d'un arrêt rapide pour les incidents mineurs permettrait un usage moins fréquent de l'arrêt d'urgence et éviterait ainsi une fatigue importante dans le bloc pile.

Incidents au démarrage. Nous avons envisagé deux types d'incidents:

a) Introduction linéaire de réactivité de 1 cent/s pour le débit nominal et pour un débit réduit au millième de sa valeur (fig. 29 et 30). On peut noter l'heureux effet du coefficient de contre-réaction de la puissance.

b) Introduction linéaire de réactivité de 10 cent/s: l'incident est évidemment plus grave dans ce cas.

Ces divers cas peuvent correspondre à des incidents des types suivants: montée intempestive des barres de contrôle, incidents de chargement, etc. Ils donnent une idée de la rapidité de réponse du système, et dans une phase ultérieure ils permettront de prévoir des parades.

Incidents concernant le circuit primaire

Nous avons étudié les trois cas suivants.

a) Fluctuations du débit de sodium. Le danger de ces fluctuations croît avec leur amplitude et leur durée (fig. 31).

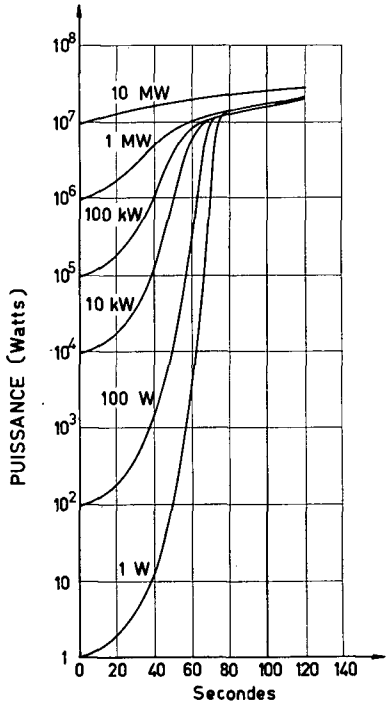


Figure 29
Accidents au démarrage. Introduction de 1 cent par seconde avec contre-réaction. A l'instant initial la pile est à l'état critique. Débit sodium: valeur nominale à 10 MW.

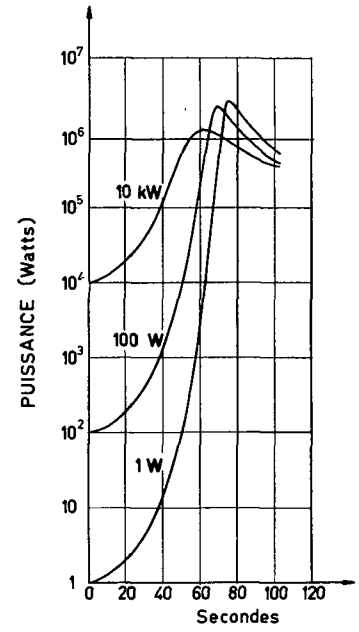


Figure 30
Accidents au démarrage. Introduction linéaire de 1 cent par seconde avec contreréaction. A l'instant initial la pile est à l'état critique. Débit sodium: 1/1000 de la valeur nominale.

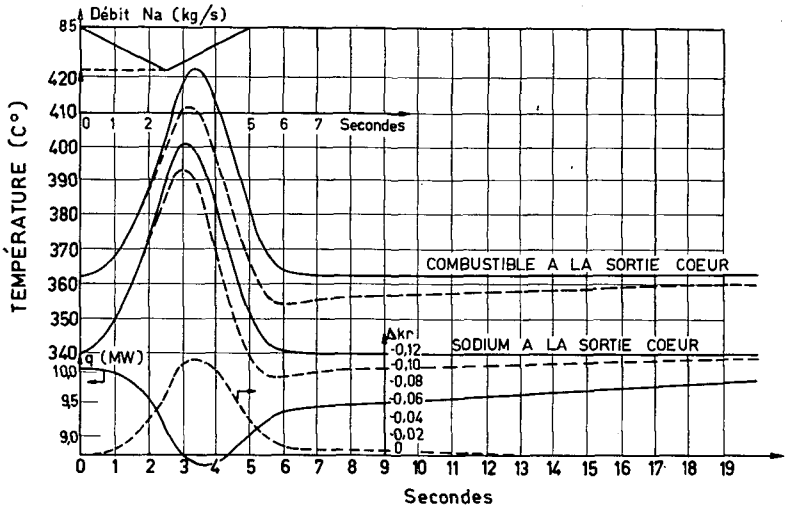


Figure 31
Transitoire de 5 s du débit sodium. Chute et reprise linéaire du débit. Variation de 50%.
..... avec contre-réaction
——— sans contre-réaction

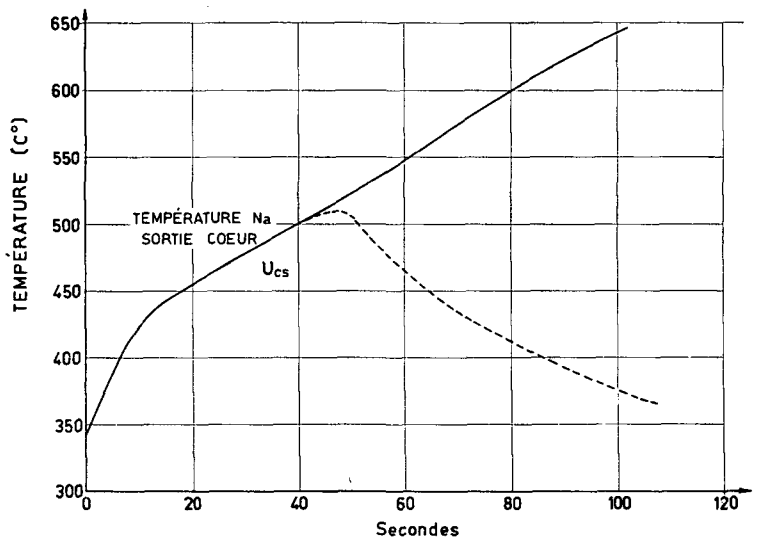


Figure 32
Arrêt de la pompe Na.
——— chute totale du débit
..... chute du débit jusqu'à 7% de la valeur nominale

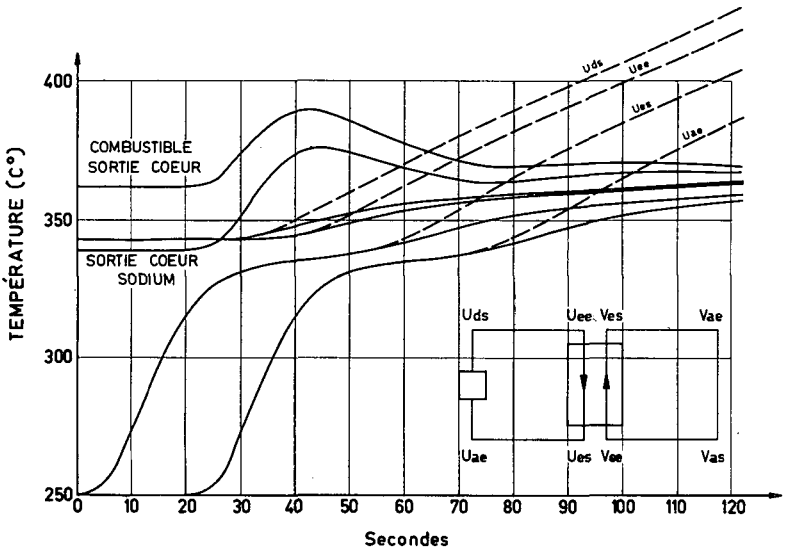


Figure 33
Arrêt de pompe du circuit secondaire. Variation de température dans le circuit primaire.
——— avec contre-réaction
..... sans contre-réaction

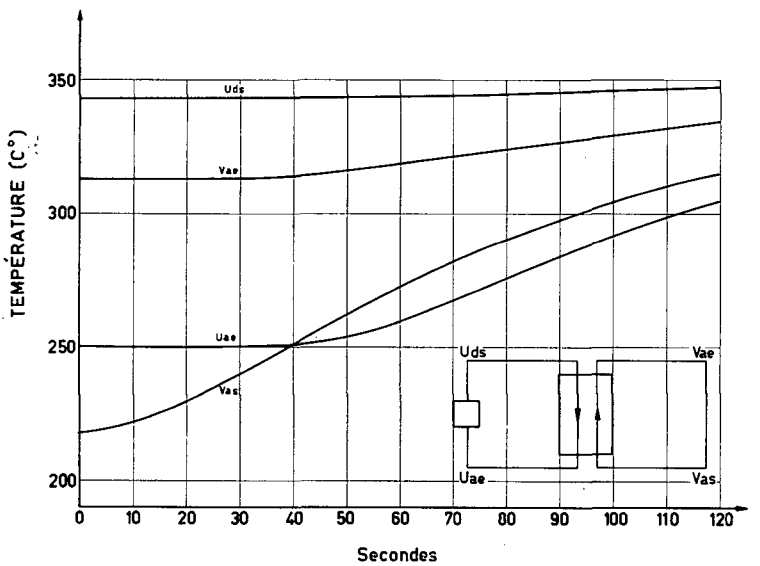


Figure 34
Arrêt de la pompe d'air. Variations de température avec contre-réaction.

b) Arrêt accidentel de la pompe à sodium. La contre-réaction de la température permet de stabiliser les températures dans le cœur. Il semble que l'opérateur puisse agir avant que les températures dépassent des seuils dangereux (fig. 32).

c) Augmentation linéaire du débit. Cela correspondrait au cas d'un emballement intempestif de la pompe à basse puissance.

Incidents concernant le circuit secondaire

Comme dans le paragraphe précédent, nous avons fait les hypothèses de fluctuations du débit (sans effet important) et d'arrêt accidentel de la pompe NaK. Les résultats sont présentés sur la figure 33.

La contre-réaction a pour effet de stopper la montée uniforme des températures suivant le premier choc thermique, mais ne supprime pas ce dernier qui reste important en particulier pour la température d'entrée du sodium dans la pile.

Incidents concernant la soufflante air

L'effet de l'arrêt accidentel de la soufflante air est donné par la figure 34. L'inertie des circuits est telle que le premier choc thermique ne se fait sentir à l'entrée de la pile qu'une minute après l'incident et qu'il est l'un des moins graves observés jusqu'ici: 0,4°C/s.

Tous ces résultats ont été obtenus au moyen de calculs digitaux et présentent un certain désaccord (20 à 40%) avec ceux des premiers calculs analogiques sur machine PACE. En fait, la formulation et les constantes adoptées étaient légèrement différentes. Nous nous garderons donc de conclure en attendant les résultats de nouvelles études qui sont actuellement en cours et permettront peut-être de lever le désaccord.

3.3. Régimes transitoires a caractère explosif

Cette étude a pour but d'évaluer l'énergie maximale qui pourrait être libérée dans RAPSODIE à la suite d'un accident.

Nous avons étudié le problème en deux temps:

a) Définition de la suite d'événements conduisant à la formation la plus rapide possible d'une pile la plus surcritique qui puisse être, compte tenu de la disposition générale de RAPSODIE, même si la réalité de cette suite d'événements nécessaires est très improbable;

b) Détermination de l'énergie maximale libérée et de l'allure du dégagement de cette énergie en fonction du temps. Ceci revient à déterminer le taux maximal d'introduction de réactivité correspondant aux accidents envisagés.

Hypothèses retenues pour l'accident maximal

Compression rapide du cœur. A la suite d'une excursion de puissance, le sodium bout et se trouve en grande partie expulsé du cœur de la pile; les éléments combustibles sont alors surchauffés et deviennent plastiques. La rupture de quelques aiguilles conduit à la formation d'une croûte mince infiniment rigide à la base du cœur. Les barres de contrôle sont dans leur position basse (position d'anti-réactivité maximale). A un moment donné, la pompe refoule du sodium frais, cette onde de sodium vient au contact de la croûte formée et exerce sur cette dernière une pression égale à la hauteur maximale de refoulement de la pompe. Cette force de pression a pour effet de comprimer le cœur. On suppose que cette compression est homogène, et à la limite on obtient une masse homogène de combustible et de matériau de structure au sommet du cœur.

TABLEAU XIX

CARACTÉRISTIQUES DE L'ACCIDENT MAXIMAL

Accident	Combustible	dk/dt (dollars/s)	E (kcal)	P (Mb)	t (μs)
Compression rapide du cœur	R.59-1	450	$190 \cdot 10^3$	0,2	60
Fusion du cœur	R.59-1	260	$157 \cdot 10^3$	0,145	80
	R.60-2	390	$115 \cdot 10^3$	0,145	80

Cette série d'événements conduisant à une telle compression est hautement improbable. Des résultats d'expérience sommaire récents (APDA) semblent montrer que l'écoulement du combustible fondu en présence de circulation de sodium en deux phases s'effectue de bas en haut. Si une telle situation se confirmait, les résultats obtenus avec cette série d'hypothèses seraient vraiment très pessimistes.

Fusion du cœur. Le sodium bouillant s'est échappé de la pile. Le combustible de la partie centrale du cœur fond et se rassemble à la base du cœur formant une région de grande densité en matière fissile, la partie centrale fondue étant pratiquement vide.

A un moment donné, la partie supérieure du cœur tombe par gravité sur la partie inférieure du cœur, créant ainsi un certain taux d'introduction de réactivité. Ce dernier sera maximal lorsque la configuration de l'ensemble, avant la chute de la partie supérieure, correspond à l'état le plus sous critique.

Calculs

Nous avons utilisé comme première approximation de la quantité d'énergie libérée la méthode analytique développée aux Etats-Unis par JANKUS et MCCARTHY [10]. Cette approche analytique est possible grâce à l'hypothèse simplificatrice suivante: on admet que le taux de réduction de réactivité pendant la destruction de la pile (par l'effet de pression dû à l'énergie libérée) est toujours plus important que le taux d'introduction de réactivité provoquant l'excursion de puissance.

Le taux de réactivité est défini par un certain nombre de calculs du coefficient de multiplication de la pile pour diverses configurations au cours du rassemblement, et par une estimation, compte tenu des hypothèses, de la variation de ces coefficients de multiplication en fonction du temps.

Les calculs ont été faits pour un cœur métallique dans les deux hypothèses; seule la seconde hypothèse a été utilisée dans le cas d'un cœur à oxydes.

Résultats

Les résultats des calculs pour RAPSODIE sont présentés dans le tableau XIX.

dk/dt (en dollars/seconde) représente le taux maximal d'introduction de réactivité correspondant aux hypothèses utilisées,

E (en kilocalories) l'énergie totale dégagée correspondante,

P (en mégabars) la pression maximale correspondante,

t (en microsecondes) le temps nécessaire pour la libération des 9/10 de l'énergie totale.

Remarques

a) Rappelons que par accident maximal nous entendons un accident survenant dans les conditions les plus pessimistes. De plus, les approximations mathématiques utilisées au cours des calculs sont toujours faites dans le sens le plus pessimiste. Il en résulte que les chiffres obtenus ne doivent être considérés que comme des chiffres extrêmes, et nous pensons que seul l'ordre de grandeur doit être pris en considération.

b) Sans prétendre réduire les chiffres « maximaux » indiqués dans le tableau XIX, nous pouvons essayer de dégager le caractère pessimiste des hypothèses utilisées. Si l'on suppose que l'accident est amorcé quand la pile est à sa puissance nominale la quantité d'énergie produite sera environ deux fois plus faible. En outre, l'effet Doppler a été négligé; or l'évolution de la température est très importante dans un accident de ce type. L'effet obtenu est compris entre 5 et 10 cents.

Conclusion

Aspect calcul

Ainsi que nous l'avons déjà signalé, les combustibles étudiés ici ne seront probablement pas exactement ceux retenus pour la première charge de RAPSODIE. Le choix se fera vraisemblablement entre les deux types suivants:

a) U — Pu — Mo avec
- 15% Pu en poids
- 10% Mo en poids
- 75% U enrichi à 27% environ

b) PuO_2 — UO_2 avec
- 25% PuO en volume
- 75% UO_2 en volume
- U enrichi à 57% environ

Des calculs détaillés sont en cours avec ces deux combustibles.

Aspect expérimental

Les expériences prévues sur la pile RAPSODIE ont essentiellement pour but de déterminer ou de préciser certains paramètres fondamentaux nécessaires à son fonctionnement. Compte tenu du but poursuivi dans son élaboration, cette machine n'est pas spécialement aménagée pour effectuer certaines expériences de physique des piles à neutrons rapides analogues à celles qu'il est possible de prévoir à priori dans des ensembles critiques dont la conception même doit tenir compte des impératifs liés à leur exploitation expérimentale.

Nous avons abordé le problème expérimental de RAPSODIE sous deux aspects:

1. *Etude et mise au point des méthodes de détection des neutrons rapides.* Cette partie du problème a en réalité été amorcée dès la mise en place à Saclay de l'expérience exponentielle rapide dans la colonne thermique d'EL2. Enumérons

brièvement les moyens de détection dont nous pouvons utiliser la technique correspondante au cours des études à froid et au cours de la montée en puissance de RAPSODIE.

a) Mise au point des chambres à fission : nous avons surtout étudié la miniaturisation de ce type de détecteur. Nous sommes actuellement en mesure de fabriquer mécaniquement des chambres de 4 mm de diamètre environ, d'effectuer de façon reproductible les dépôts de matière fissile sur une ou plusieurs électrodes et de faire fonctionner de telles chambres en valeur absolue ou relative. Des études systématiques de différents effets sur le fonctionnement des chambres — tels que la nature du gaz de remplissage, la pression de ce gaz, la tension de collection appliquée — ont permis de fixer une plage de fonctionnement pour chaque type. Nous avons également adapté les circuits électroniques de manière à éviter la contamination du comptage des produits de fission par les empilements α et les γ du milieu ambiant.

b) Mise au point de détecteurs par activation : toujours dans le cadre de l'expérience exponentielle rapide, nous avons mis au point un certain nombre de détecteurs par activation et les systèmes de comptage appropriés.

c) La technique des émulsions nucléaires nous est connue. Nous recherchons actuellement à systématiser l'identification et le comptage des traces pour permettre un dépouillement à la fois moins laborieux et plus précis. De nouvelles plaques Gevaert devraient conduire à différencier des neutrons d'énergie de 150 keV avec une résolution de 50 keV.

d) Nous suivons également avec beaucoup d'intérêt le développement des études et la mise au point de semi-conducteurs pour la détection des neutrons rapides.

2. *Etudes relatives au programme expérimental prévu sur RAPSODIE — Programme des expériences.* Nous ne parlerons ici que des expériences purement neutroniques axées sur l'étude du fonctionnement du réacteur. La plupart des expériences à faible puissance seront d'abord réalisées en l'absence de sodium dans la cuve dans le double but de tester les appareillages destinés à ces expériences et de vérifier les ordres de grandeur calculés des effets à mesurer.

a) Approche sous-critique. Voir à ce sujet le paragraphe 2.5. (p. 386.)

b) Mesures de neutronique statique. Ces mesures réalisées à l'aide des techniques décrites plus haut devraient permettre ,d'une part, d'obtenir la distribution des flux et puissances et de déduire de celles-ci l'étalonnage des chambres d'ionisation destinées au contrôle et à la sécurité du réacteur, d'autre part, d'obtenir le spectre des neutrons dans le cœur et dans les couvertures, et enfin d'obtenir le rapport de régénération global.

c) Etalonnage des barres de contrôle-perturbations.

d) Mesures de cinétique neutronique pour lesquelles on peut citer la mesure du coefficient de transfert à puissance nulle, l'étalonnage des barres de contrôle, l'étude de la réactivité d'un assemblage combustible en fonction de sa position, le coefficient isotherme de température.

A divers niveaux de puissance, en présence de sodium il est prévu de nombreuses mesures du coefficient de puissance par oscillation. L'ampleur à donner aux différentes mesures dépend des résultats de celles que nous pensons effectuer sur l'assemblage critique ZPR III, des possibilités expérimentales de RAPSODIE et du temps dont on disposera avant la montée en puissance.

RÉFÉRENCES

[1] Avant-projet RAPSODIE, Rapport interne CEA (29 décembre 1958).
[2] Notes bureau commun SNE—SPM n° 1 à 8 (notes internes CEA) (1957—58).
[3] LEDUC, J. *et al.*, Rapport DEP/SNE 8a/CD.
[4] SOLANES, M. et MOINEREAU, P., Vol. **II** des présents Comptes rendus, p. 27.
[5] «Reactor Physics Constants», ANL-5800 (1958).
[6] KANIA, A. *et al.*, Vol. **I** des présents Comptes rendus, p. 387.
[7] Reactor Shielding Design Manual, Rockwell éd.
[8] HILL et SYRETT, AERE-RPM-46.
[9] ABDON, R. *et al.*, Vol. **III** des présents Comptes rendus, p. 93.
[10] McCARTHY, W. J. *et al.*, Proc. 2nd UN Int. Conf. PUAE **12** (1958) 207.

SM-18/81

ИМПУЛЬСНЫЙ РЕАКТОР НА БЫСТРЫХ НЕЙТРОНАХ

Г. Е. БЛОХИН, Д. И. БЛОХИНЦЕВ, Ю. А. БЛЮМКИНА, И. И. БОНДАРЕНКО, Б. Н. ДЕРЯГИН, А. С. ЗАЙМОВСКИЙ, В. П. ЗИНОВЬЕВ, О. Д. КАЗАЧКОВСКИЙ, Н. В. КРАСНОЯРОВ, А. И. ЛЕЙПУНСКИЙ, В. А. МАЛЫХ, П. М. НАЗАРОВ, С. К. НИКОЛАЕВ, Ю. Я. СТАВИССКИЙ, Ф. И. УКРАИНЦЕВ, И. М. ФРАНК, Ф. Л. ШАПИРО и Ю. С. ЯЗВИЦКИЙ

АКАДЕМИЯ НАУК СССР, МОСКВА

СССР

Abstract — Résumé — Аннотация — Resumen

A pulsed fast reactor. A pulsed fast reactor (IBR) has been operating at rated capacity since December 1960 in the Joint Institute for Nuclear Research. This reactor is used as a pulsed neutron source for physical experiments carried out by the time-of-flight method. It is used for total cross-section and intermediate neutron capture cross-section measurements, for studying the interaction between slow neutrons and solids and liquids, and for measuring neutron spectra produced in various media.

The paper describes the basic structural features of the reactor and the results of the experiments for which it has been used. The reactor's operating system is based on recurrent pulses. Power pulses are produced when the mobile part of the reactor core moves swiftly through the stationary part of the core. The mobile part of the core is fastened to a rotating disc and travels at a speed of 230 m/s. The frequency of power pulses can be altered by means of an auxiliary mobile zone which has a range of 2.3—88 pulses per second. The mean power of the reactor is 1 kW, and the half-width of the power pulse in 36 μs.

The reactor is provided with a control and safety system which ensures automatic maintenance of mean power and swift shutdown in the event of any operational irregularity. It is fitted with a system of evacuated-neutron-flight tubes used in time-of-flight experiments. The main tube is 1000 m in length.

In the start-up process and during physical experiments carried out on the reactor, the influence on reactivity of displacing the controls and the mobile parts of the core was studied; the length of the pulse was measured under various operating conditions, and power pulse amplitude fluctuations were studied. Further measurements were made to establish the lifetime of prompt neutrons, the effective fraction of delayed neutrons, and coefficients of reactivity.

Un réacteur pulsé à neutrons rapides. L'Institut unifié de recherches nucléaires dispose d'un réacteur pulsé à neutrons rapides (IBR), qui fonctionne à la puissance nominale depuis le mois de décembre 1960.

Ce réacteur est utilisé comme source pulsée de neutrons pour les expériences de physique fondées sur la méthode du temps de vol. On l'emploie pour établir la section efficace totale et la section efficace de capture des neutrons intermédiaires, pour étudier l'interaction des neutrons lents et des corps solides ou liquides et pour mesurer les spectres neutroniques dans différents milieux.

Le mémoire décrit les caractéristiques essentielles de la construction du réacteur et les résultats d'expériences faites à l'aide de ce réacteur.

Le régime de fonctionnement normal est celui des impulsions périodiques. Les impulsions de puissance sont produites par un déplacement rapide de la partie mobile du cœur à travers sa partie immobile. La partie mobile se trouve fixée sur un disque tournant et se déplace à une vitesse d'environ 230 m/s.

Une zone mobile auxiliaire permet de modifier la fréquence des impulsions de puissance entre 2,3 et 88 ips. Le réacteur a une puissance moyenne de 1 kW. La demi-largeur d'une impulsion de puissance est de 36 μs.

Le réacteur est doté d'un système de commande et de sécurité qui assure le maintien automatique de la puissance moyenne et un arrêt rapide en cas de fonctionnement irrégulier.

Il est équipé d'un système de canalisations sous vide pour le passage des neutrons, qui permettent de mesurer le temps de vol. Le canal principal a 1000 m de long.

Lors du démarrage du réacteur et durant les expériences de physique dont il a fait l'objet, on a étudié l'effet que produit sur la réactivité le déplacement des organes de commande et des parties mobiles du cœur; on a mesuré la longueur des impulsions à des régimes de fonctionnement différents et étudié les fluctuations d'amplitude des impulsions de puissance.

En outre, les auteurs ont procédé à des mesures en vue de déterminer la durée de vie des neutrons instantanés, la fraction effective de neutrons retardés et les coefficients de température de la réactivité.

Импульсный реактор на быстрых нейтронах. Импульсный реактор на быстрых нейтронах (ИБР) работает на номинальной мощности в Объединенном институте ядерных исследований с декабря 1960 года. Реактор используется в качестве импульсного источника нейтронов для физических экспериментов, проводимых методом времени пролета. Проводятся измерения полного сечения, сечения захвата для промежуточных нейтронов, исследования взаимодействия медленных нейтронов с твердым телом и с жидкостью, измерения спектров нейтронов, устанавливающихся в различных средах.

В докладе описаны основы конструкции реактора и результаты его исследований. Основной режим работы реактора — режим периодических импульсов. Импульсы мощности возникают при быстром перемещении подвижной части активной зоны реактора через его неподвижную зону. Подвижная часть активной зоны закреплена во вращающемся диске и движется со скоростью ~230 м/сек. Частота импульсов мощности может изменяться с помощью вспомогательной подвижной зоны в диапазоне 2,3—88 им/сек. Средняя мощность реактора — 1 квт. Полуширина импульса мощности — 36 мксек.

Реактор снабжен системой управления и защиты, обеспечивающей автоматическое поддержание средней мощности реактора и его быструю остановку в случае нарушения режима. Реактор снабжен системой вакуумированных нейтроноводов, используемых в экспериментах по времени пролета. Главный нейтроновод имеет длину 1000 м.

В процессе пуска и физических исследований реактора изучалось влияние перемещения органов регулирования и подвижных частей активной зоны на реактивность, измерялась длительность импульса при различных режимах работы реактора, изучались флюктуации амплитуд импульсов мощности.

Проводились также измерения времени жизни мгновенных нейтронов, эффективной доли запаздывающих нейтронов, температурных коэффициентов реактивности.

Reactor rápido pulsado. Desde diciembre de 1960, el reactor de impulsos de neutrones rápidos IBR viene funcionando a su potencia nominal en el Instituto Central de Investigaciones Nucleares.

Dicho reactor se utiliza como fuente pulsante de neutrones para realizar experimentos de física por el método del tiempo de vuelo. Se llevan a cabo determinaciones de las secciones eficaces totales, de las secciones eficaces de captura de neutrones intermedios, estudios de las interacciones de los neutrones lentos con los sólidos y los líquidos y mediciones de los espectros neutrónicos en distintos medios.

Los autores describen las principales características constructivas del reactor y los resultados de los estudios realizados mediante el mismo.

Este reactor trabaja con arreglo a un régimen de impulsos periódicos. Los impulsos de potencia se originan cuando la parte móvil del cuerpo, fijada a un disco giratorio, atraviesa la parte estacionaria con una velocidad del orden de los 230 m/s.

Gracias a la presencia de una zona movible auxiliar, es posible variar la frecuencia de los impulsos de potencia entre 2,3 y 88 impulsos por segundo. La potencia media del reactor es de 1 kW y la duración media de los impulsos, de 36 μs.

El reactor está provisto de un sistema de mando y de seguridad que vela por el mantenimiento automático de la potencia del reactor en su valor medio, así como por su rápida detención en caso de perturbación del funcionamiento.

También posee el reactor conductores neutrónicos de vacío, que se utilizan en los experimentos de tiempo de vuelo. El conducto principal tiene 1000 m de longitud.

En el proceso de puesta en marcha y durante las investigaciones físicas realizadas con el reactor, se estudió el efecto del desplazamiento de los órganos de regulación y de las partes móviles del cuerpo sobre la reactividad, se determinó la duración de los impulsos a distintos regímenes de trabajo del reactor y se estudiaron las fluctuaciones de la amplitud de los impulsos de potencia.

Asimismo, se efectuaron mediciones del período de los neutrones instantáneos, de la fracción efectiva de neutrones retardados y de los coeficientes de variación de la reactividad en función de la temperatura.

Введение

Импульсный реактор на быстрых нейтронах (ИБР) является импульсным источником нейтронов, предназначенным для физических исследований и прежде всего для экспериментов по времени пролета. Он рассчитан в основном на работу в режиме периодических импульсов, однако может быть использован и для получения мощных одиночных импульсов.

Импульсы мощности в реакторе развиваются за счет быстрого изменения его реактивности, причем периодически на короткое время достигается мгновенная надкритичность реактора. В эти моменты и происходит основной рост мощности. В остальное время реактор является подкритическим.

Полное число делений в каждом импульсе (энергия импульса) определяется „подсветкой“ (интенсивностью постороннего источника нейтронов) до импульса и величиной мгновенной надкритичности. Подсветка при достаточно высоких средних мощностях обусловлена запаздывающими нейтронами, родившимися во всех предыдущих импульсах.

При некотором значении средней реактивности системы (и соответственно пиковой реактивности) имеет место равновесный режим работы, характеризующийся постоянной средней мощностью. При этом образование источников запаздывающих нейтронов во время каждого импульса компенсирует распад источников между импульсами и интенсивность подсветки остается в среднем постоянной.

Характер зависимости средней мощности реактора ИБР от времени в принципе такой же, как и для обычного реактора. При малом отклонении реактивности системы от равновесного значения средняя мощность изменяется медленно. С увеличением этого отклонения скорость роста или спада мощности увеличивается. Поведение реактора в этом отношении описывается обычными уравнениями кинетики для неимпульсного реактора. Особенностью поведения реактора ИБР является лишь то, что значение эффективной доли запаздывающих нейтронов очень мало. В номинальном режиме работы реактора оно составляет $\sim 10^{-4}$, т.е. почти в сто раз меньше соответствующего значения для обычного стационарного реактора с урановым горючим. Это обстоятельство, естественно, предъявляет особенно высокие требования к прецизионности системы регулирования. Теория реактора ИБР описана в работе [1].

Основные элементы конструкции реактора ИБР

Общий вид реактора ИБР и его активной зоны показаны на рис. 1, 2.

Активная зона реактора ИБР имеет неподвижную и подвижные части. Периодическое изменение реактивности системы происходит за счет перемещения подвижных частей, которые представляют собой два вкладыша из U^{235}, закрепленных в двух вращающихся дисках (см. рис. 2). Основной вкладыш запрессован в диск диаметром 1100 мм на расстоянии 440 мм от оси и может перемещаться с окружной скоростью до 276 м/сек. (при 6000 об/мин.), проходя через центр активной зоны. Вспомогательный вкладыш запрессован в малый диск и перемещается по краю активной зоны. Он служит для изменения частоты импульсов без изменения их формы и может вращаться с частотой различной кратности по отношению к частоте основного диска. Реактор становится надкритичным и импульсы мощности развиваются только в том случае, если основной и вспомогательный вкладыш одновременно совмещены с неподвижной частью активной зоны. Максимальное изменение реактивности при движении основного вкладыша достигает 7.4%, при движении вспомогательного вкладыша — 0.4%.

Неподвижная часть активной зоны состоит из плутониевых стержней в оболочке из нержавеющей стали. Каждый стержень с помощью конусообразного хвостовика фиксируется в верхней или нижней опорных решетках активной зоны. Одностороннее закрепление стержней обеспечивает отрицательный температурный коэффициент реактивности (за счет искривления стержней при нагреве в неоднородном нейтронном поле), и тем самым улучшает условия безопасной работы реактора.

Для обеспечения возможности обслуживания реактора после работы на номинальном уровне мощности предусмотрено приспособление для дистанционного надвигания на активную зону радиационной защиты из свинца толщиной 10 см (рис. 1). Это же приспособление используется для установки

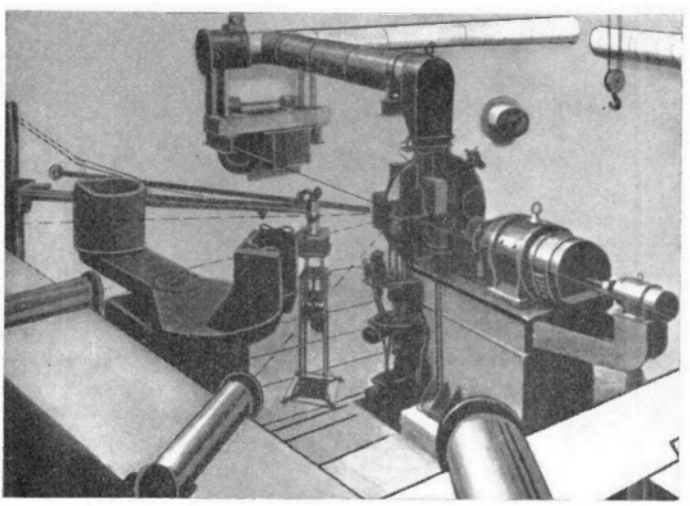


Рис. 1
Общий вид установки ИБР.

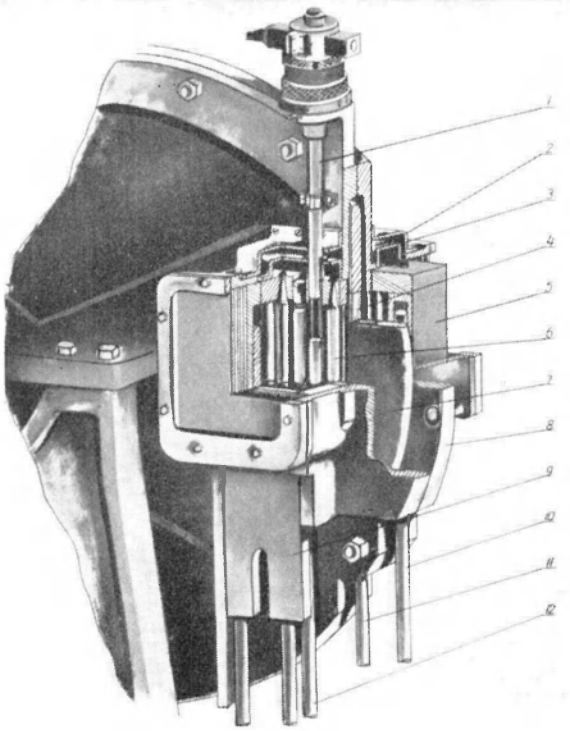


Рис. 2
Общий вид реактора ИБР.
1 — Бустер; 2 — Крышка; 3 — Коллектор; 4 — Основной вкладыш из урана-235; 5 — Отражатель; 6 — Плутониевый стержень; 7 — Основной диск; 8 — Корпус; 9 — Грубый регулятор; 10 — Ручной регулятор; 11 — Автоматический регулятор; 12 — Аварийный плутониевый стержень.

вблизи активной зоны парафиновых слоев, служащих для замедления быстрых нейтронов, испускаемых реактором.

Система управления и защиты реактора ИБР обеспечивает изменение реактивности системы при пуске, работе и остановке (в том числе аварийной) реактора, контроль уровня мощности во всех режимах работы и автоматическое поддержание уровня мощности при работе на мощности выше 1 ватта. Пуск реактора осуществляется с помощью грубого регулятора, являющегося подвижной частью отражателя. С помощью сервопривода грубый регулятор может перемещаться, обеспечивая изменение реактивности со скоростью $13 \cdot 10^{-5} - 1{,}3 \cdot 10^{-5}$ сек$^{-1}$. Ручной регулятор, представляющий собой стержень диаметром 20 мм, так же является частью отражателя. Автоматический регулятор (такой же стержень) соединен со своим сервоприводом. Для аварийной остановки используются два плутониевых стержня неподвижной зоны (АС-1 и АС-2), подвешенные на электромагнитах. При появлении аварийного сигнала стержни под действием пружин и собственного веса выбрасываются из активной зоны с ускорением около 20 g.

Отдельные органы регулирования вносят следующие изменения реактив-

ности: АС-1 и АС-2—1,1% (каждый); грубый регулятор — 2,4%; ручной регулятор — 0,1%; автоматический регулятор — 0,036%.

В системе управления и защиты используются две группы аппаратуры — пусковая и рабочая. Пусковыми датчиками служат пропорциональные BF_3-счетчики в парафине. При малых мощностях (глубокая подкритичность) усиленные импульсы от счетчиков длительностью ~0,5 мксек подаются на пересчетные схемы и измерители скорости счета. Используется схема временных ворот, синхронизированных с вращением основного диска при помощи фотодатчика. Эта схема обеспечивает регистрацию нейтронов лишь на время прохождения основного вкладыша через неподвижную зону. При больших мощностях импульсы от пропорциональных счетчиков интегрируются и результирующие импульсы длительностью ~600 мксек, амплитуда которых пропорциональна энергии импульса мощности, измеряются с помощью интегрирующих схем и наблюдаются на осциллографе. При мощностях выше 1 вт используется импульсная электронная аппаратура, датчиками для которой служат импульсно-токовые ионизационные камеры, заполненные метаном. Импульс ионизационного тока в камере, вызванный нейтронами и гамма-лучами, усиливается и подается на триггеры аварийной защиты (два независимых канала).

Система аварийной защиты прекращает реакцию, если амплитуда одного из импульсов мощности превосходит заданное значение.

Импульсы от метановых ионизационных камер идут и на систему автоматического регулирования реактора. Система автоматического регулирования включает в себя импульсный усилитель, преобразователь импульсов, схему сравнения с усилителем мощности, выход которого подается на электромашинный усилитель, управляющий двигателем сервопривода регулирующего стержня.

На выходе преобразователя импульсов поддерживается постоянное напряжение, пропорциональное амплитуде последнего пришедшего импульса. Каждый вновь пришедший импульс меняет это напряжение скачком до величины, соответствующей его амплитуде. Ступенчатое напряжение на выходе преобразователя сглаживается, сравнивается с опорным напряжением, определяющим уровень поддерживаемой мощности, а разбаланс обычным способом воздействует на движение регулирующего стержня. Использование преобразователя импульсов подобного типа позволяет получить значительный выигрыш в скорости срабатывания системы по сравнению со схемами, использующими интегрирование на RC-цепочках.

Для проверки системы автоматического регулирования и ее отработки в условиях разброса амплитуд импульсов (причиной такого разброса при малых мощностях является статистический характер размножения нейтронов в реакторе, при больших — вибрация отдельных элементов активной зоны, ведущая к колебаниям величины коэффициента размножения) была создана электронная модель ИБР. Модель, имитировавшая изменение во времени амплитуды импульсов мощности с учетом 6-ти групп запаздывающих нейтронов, включалась в цепь реальной системы автоматического регулирования. Изучались характеристики такой замкнутой системы при введении как одиночных скачкообразных возмущений, так и периодических возмущений различной частоты.

Реактор снабжен специальным устройством, предназначенным для создания мощных одиночных импульсов (бустер реактивности). Бустер сра-

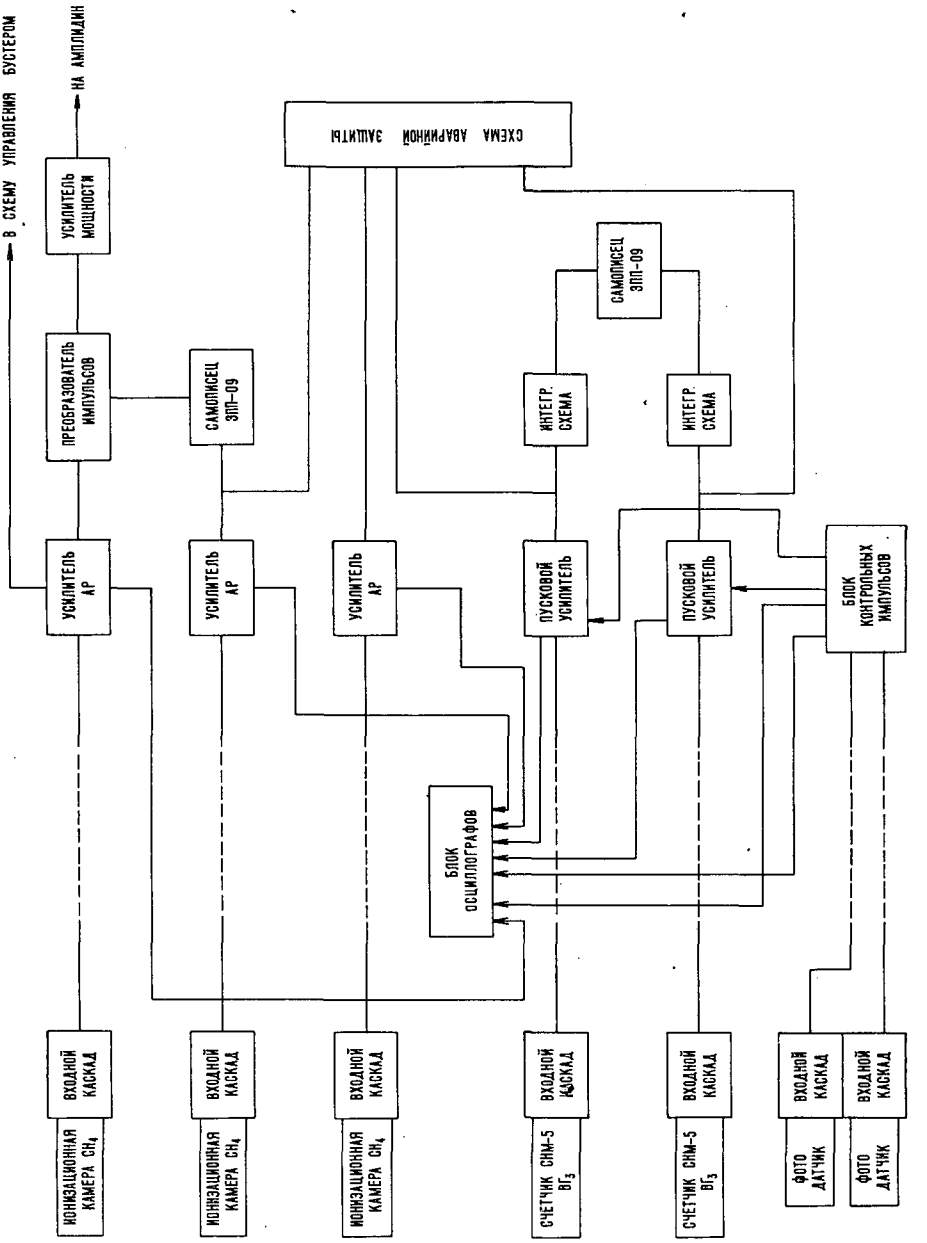


Рис. 3
Блок-схема электронной аппаратуры системы управления и защиты.

батывает от одного из импульсов мощности реактора и за время до появления следующего импульса забрасывает в активную зону плутониевый образец. Это приводит к резкому увеличению амплитуды следующего

импульса. От этого импульса срабатывает аварийная защита, реактор гасится и второй мощный импульс уже не развивается.

Блок-схема электронной аппаратуры системы управления и защиты приведена на рис. 3.

Вращение основного и вспомогательного дисков осуществляется с помощью силовой установки максимальной мощностью 100 квт. Установка включает в себя ускоритель с электроприводом, собранным по схеме Леонардо (трехмашинный агрегат). Предусмотрена система регулирования, обеспечивающая пуск, остановку машины и поддерживание постоянного числа оборотов с точностью ~2% в диапазоне 2000—6000 об/мин.

Для охлаждения воздуха, нагревающегося за счет трения в кожухе основного диска, предусмотрена циркуляция воздуха через систему водяных холодильников с выбросом в зал реактора. Неподвижная часть активной зоны охлаждается атмосферным воздухом, прокачиваемым с помощью компрессора. Расход воздуха ~60 м³/час.

Реактор ИБР расположен в центре зала размером $10 \times 10 \times 7$ м (рис. 1). Бетонные стены зала обеспечивают полную защиту от излучения. Реактор снабжен значительным числом специальных устройств, позволяющих проводить широкий круг нейтронных исследований. Основное экспериментальное устройство — главный нейтроновод длиной 1000 м.

Главный нейтроновод представляет собой металлическую трубу диаметром 400 мм на первом участке и 800 мм — на втором. В трубе поддерживается вакуум порядка 0,1 мм рт.ст. Нейтроновод снабжен цепью промежуточных павильонов, которые дают возможность устанавливать экспериментальную аппаратуру на расстоянии 70, 250, 500, 750 и 1000 м от реактора. Наряду с главным нейтроноводом сооружен вспомогательный нейтроновод длиной 100 м. Вплотную к залу реактора примыкает экспериментальный зал. В него могут быть выведены 4 нейтронных пучка диаметром до 800 мм. Можно выводить нейтронные пучки и в верхний экспериментальный зал, расположенный над залом реактора.

Пуск и физические исследования реактора ИБР

1. СТЕНДОВЫЕ СБОРКИ

Первый этап физических исследований реактора ИБР проводился на стендовых сборках с неподвижным и медленно перемещавшимся основным вкладышем. В остальном стендовые сборки полностью имитировали активную зону реактора.

Основная цель стендовых сборок — определение критической загрузки, измерение эффективности органов регулирования, изучение основных характеристик, определяющих длительность импульса — среднего времени жизни ценности нейтронов в реакторе (τ) и параметра скорости изменения реактивности при прохождении основного вкладыша через неподвижную зону (α). Действительно, полуширина импульса мощности реактора ИБР (θ) в равновесном режиме может быть определена (см. 1) по формуле

$$\theta = K\left(\frac{\tau}{\alpha}\right)^{1/3} v^{-2/3} \qquad (1)$$

Здесь v — скорость движения основного вкладыша; K — коэффициент, слабо зависящий от τ, α, v и энергии импульса; α — параметр параболы,

которой аппроксимируется зависимость реактивности ε от перемещения основного вкладыша в окрестности максимума коэффициента размножения;

$$\varepsilon = \varepsilon_m - \alpha x^2 \tag{2}$$

где ε_m — реактивность при максимальном коэффициенте размножения системы (для мгновенных нейтронов, $\varepsilon_m > 0$), x — смещение основного вкладыша.

При определении критзагрузки в центр активной зоны, заполненной стальными имитаторами плутониевых стержней, помещался нейтронный источник (Ra — α — Be). Нейтроны, испускаемые активной зоной, регистрировались пропорциональными счетчиками с BF_3, окруженными парафином. По мере замены стальных стержней на плутониевые строилась зависимость „обратного умножения" скорости счета детекторов от числа плутониевых стержней, помещенных в активную зону. Вносились поправки на регистрацию нейтронов от спонтанного деления.

Эффективность отдельных стержней и органов регулирования первоначально выражалась в единицах обратного умножения и в последующем пересчитывалась в единицы реактивности. Пересчет производился по формуле:

$$1 - k_{эф} = \frac{\nu_{эф} - k_{эф}}{\nu_ф} \cdot \frac{F_n}{F_\partial} \cdot \frac{1}{y} \tag{3}$$

где

$k_{эф}$ — эффективный коэффициент размножения;

$\nu_{эф}$ — среднее число вторичных нейтронов, отнесенное к акту захвата в горючем;

$\frac{F_n}{F_\partial}$ — отношение ценностей нейтронов точечного источника и нейтронов деления, распределенных по собственной функции.

y — умножение скорости счета детектора нейтронов;

При определении α изучалось изменение реактивности системы в процессе медленного перемещения основного вкладыша. Измерение среднего времени жизни мгновенных нейтронов осуществлялось методом Росси [2].

При исследовании стендовых сборок определялся также температурный коэффициент реактивности $\frac{\partial \varepsilon}{\partial \tau}$ для равномерного нагрева активной зоны.

Исследовался ряд вариантов компановки неподвижной части активной зоны. Для принятого оптимального варианта были получены следующие значения α, τ, $\frac{\partial \varepsilon}{\partial \tau}$:

$$\alpha = 0{,}7 \cdot 10^{-3} \text{ см}^{-2}$$
$$\tau = 1{,}2 \cdot 10^{-8} \text{ сек}$$
$$\frac{\partial \varepsilon}{\partial \tau} = -0{,}8 \cdot 10^{-5} {}^1/_{C^\circ}.$$

2. СБОРКА И ИССЛЕДОВАНИЯ РЕАКТОРА ПРИ НЕПОДВИЖНЫХ ВКЛАДЫШАХ

Реактор ИБР при неподвижных вкладышах не отличается от обычного быстрого реактора. Была проведена сборка варианта активной зоны, выбранного при стендовых исследованиях. Контрольные опыты позволили уточнить значения эффективностей органов регулирования. Был повторно измерен параметр α. Кроме того, измерялось изменение реактивности при

надвигании на активную зону парафинового замедлителя и свинцовой защиты. Соответствующие изменения реактивности составили:

$$\Delta\varepsilon_{\text{пар}} = 0{,}24\%$$
$$\Delta\varepsilon_{\text{св}} = 1{,}43\%$$

3. ПУСК И ИССЛЕДОВАНИЯ РЕАКТОРА ПРИ НЕПОДВИЖНОМ ВСПОМОГАТЕЛЬНОМ ВКЛАДЫШЕ (РЕЖИМ 83 ИМП./СЕК)

Этот режим работы реактора осуществлялся при номинальной скорости движения основного вкладыша (230 м/сек, 5000 об/мин).

При экспериментах использовался Ra-α-Be источник, помещенный в активную зону. По мере роста мощности все больше усиливалось влияние запаздывающих нейтронов, интенсивность которых сравнивалась с интенсивностью нейтронов источника при мощности $\sim 0{,}1$ вт.

Зависимость коэффициента размножения реактора ИБР от времени при выведенных органах регулирования приведена на рис. 4. По мере пуска

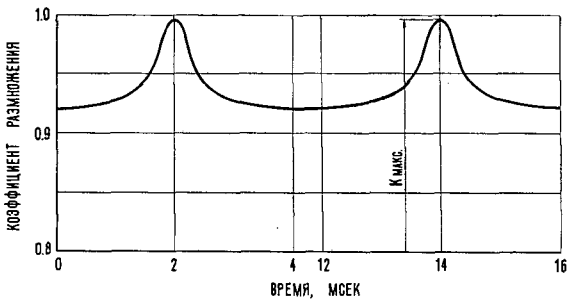


Рис. 4
Зависимость коэффициента размножения от времени (83 имп./сек).

реактора максимальный коэффициент размножения ($K_{\text{макс}}$) растет и кривая реактивности приближается к уровню $K = 1$. При этом система проходит следующие области:

а) Область квазистационарности (глубокая подкритичность на мгновенных нейтронах, $K_{\text{макс}} < 1$). В этой области поток нейтронов в реакторе успевает „следить" за изменением реактивности системы (исключая, конечно, запаздывающие нейтроны). Поэтому умножение интенсивности постороннего источника нейтронов для любого момента времени

$$y = \frac{1}{|\varepsilon|} \tag{4}$$

Зависимость полуширины импульса мощности (θ) от реактивности ($-\varepsilon_{\text{m}}$) в момент совмещения подвижной и неподвижной частей активной зоны можно выразить соотношением:

$$\theta = 2\sqrt{\frac{|\varepsilon_{\text{m}}|}{\alpha v^2}} \tag{5}$$

б) Область вблизи мгновенной критичности. Здесь уже максимальное умножение оказывается значительно меньшим определяемого формулой

(4). Наряду с этим и форма импульса отличается от квазистационарной; импульс начинает запаздывать относительно момента, соответствующего максимальному значению коэффициента размножения.

в) Область импульсной подкритичности. В этой области система на короткое время оказывается надкритичной на мгновенных нейтронах, однако эта надкритичность еще недостаточна для равновесной работы реактора (максимальное значение положительной надкритичности меньше равновесного). Для этой области энергия импульса может быть выражена так: (см. 1).

$$\left.\begin{aligned} E &= \frac{s}{v} K(\varepsilon_m) \\ K(\varepsilon_m) &= \frac{2{,}5}{v\sqrt{\alpha\varepsilon_m}} \cdot \exp \frac{1{,}33 \cdot \varepsilon_m^{3/2}}{v\tau a^{1/2}} \end{aligned}\right\} \quad (6)$$

где v — среднее число вторичных нейтронов при делении;

s — интенсивность постороннего источника нейтронов.

Поведение средней мощности реактора во времени в этой области аналогично поведению мощности обычного подкритического реактора, т.е. быстрое введение некоторой положительной или отрицательной реактивности ведет к скачку средней мощности и затем к медленному изменению ее до величины, соответствующей новому значению реактивности.

г) Область импульсной критичности. Реактор находится в равновесном состоянии, и поведение во времени его средней мощности не отличается от поведения средней мощности обычного критического реактора с уменьшенной эффективной долей запаздывающих нейтронов. Кинетика реактора ИБР в равновесном состоянии описывается уравнениями:

$$\left.\begin{aligned} W(t) &= \frac{\Sigma C_i(t)\lambda_i}{v} K(\varepsilon_m)\, n \\ \frac{dC_i(t)}{dt} &= -\lambda_i C_i(t) + W(t)\beta_i v \end{aligned}\right\} \quad (7)$$

Здесь: $W(t)$ — средняя мощность реактора;

$C_i(t)$ — концентрация источников запаздывающих нейтронов i-ой группы;

n — число импульсов в сек;

β_i — выход запаздывающих нейтронов i-ой группы;

Условие равновесной работы реактора записывается следующим образом:

$$K(\varepsilon_m)\, n\, \beta = 1 . \quad (8)$$

В процессе перевода реактора из области квазистационарности в область импульсной критичности изучалась форма импульсов мощности и регистрировалось нарастание средней мощности реактора.

Для наблюдения формы импульсов использовался сцинтилляционный счетчик с фосфором *ZnS*-плексиглас. Импульсы от этого детектора подавались на вход 1024-канального временного анализатора, пуск которого с помощью фотодатчика был синхронизирован с вращением основного диска.

Контроль средней мощности осуществлялся делительными камерами со слоями U^{235}. Две камеры, расположенные на различных расстояниях от

реактора, позволяли перекрывать диапазон средних мощностей от 10 мвт до 500 вт. Кроме того, для измерения средней мощности в диапазоне 50—200 вт использовалась делительная камера с известным количеством U^{235}.

Критическое состояние в процессе подъема мощности реактора определялось экстраполяцией, при которой использовалась зависимость реактивности от положения грубого регулятора (рис. 5). Экстраполяция сначала велась по „умножению в импульсе", определявшемуся с помощью временного анализатора как отношение максимальной скорости счета детектора нейтронов к скорости счета между импульсами (умножение подсветки между импульсами при выведенном из активной зоны основном вкладыше было измерено с достаточной точностью в экспериментах с неподвижным основным вкладышем). На кривой экстраполяции (рис. 6) виден прямой участок, направленный в область мгновенной критичности ($\varepsilon_m = 0$). Этот участок соответствует области квазистационарности (соотношение 4). Вблизи мгновенной критичности кривая начинает отклоняться от этой

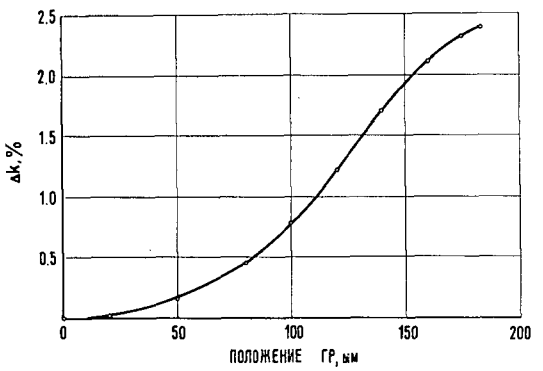


Рис. 5
Зависимость реактивности от положения грубого регулятора.

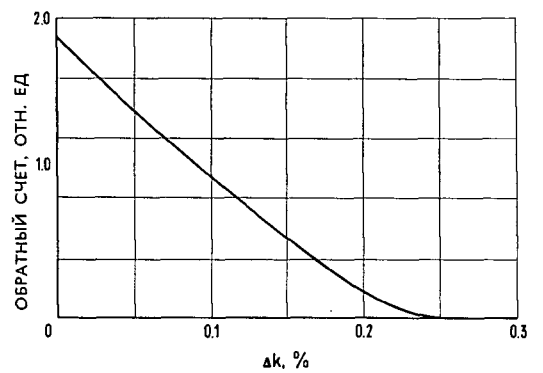


Рис. 6
Кривая экстраполяции на импульсную критичность.

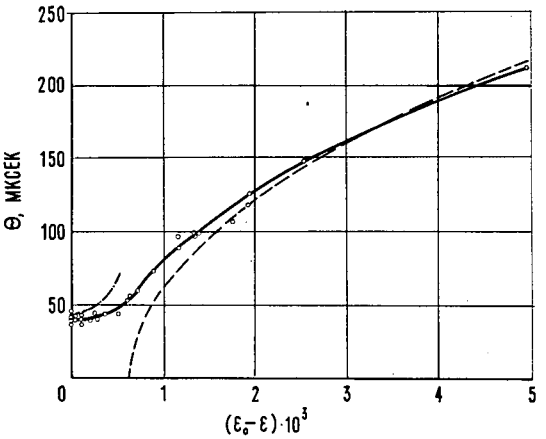


Рис. 7
Зависимость полуширины импульса θ от реактивности. Отсчет реактивности ведется от равновесной надкритичности.
— — — — — —квазистационарный случай, —.—.—.— соотношение (1).

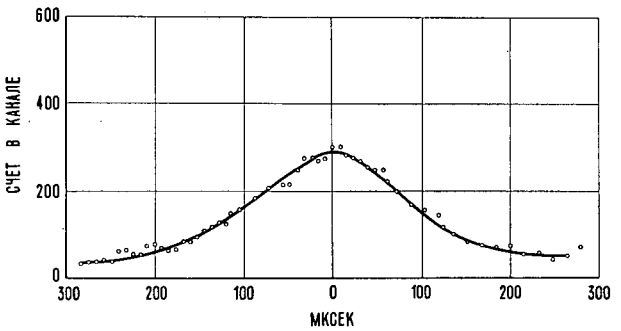


Рис. 8
Форма импульса мощности для $\varepsilon_m = -5\cdot 10^{-3}$.

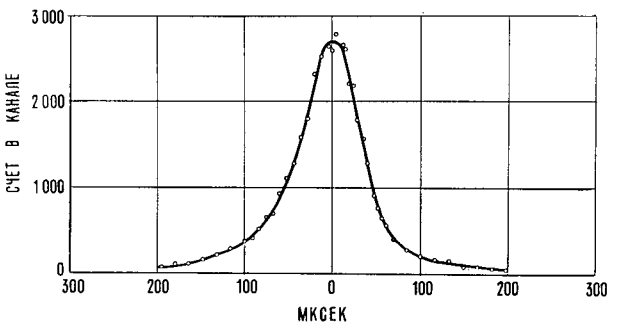


Рис. 9
Форма импульса мощности для $\varepsilon_m = -0{,}4\cdot 10^{-3}$.

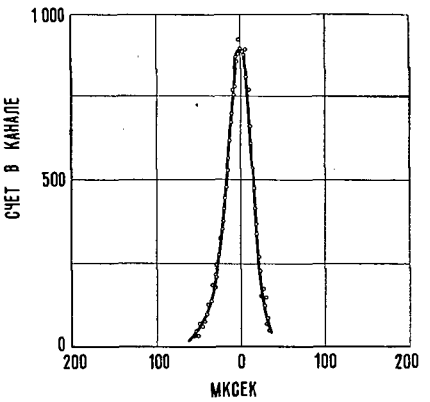


Рис. 10
Форма импульса мощности в равновесном режиме (импульсная критичность, $\varepsilon_0 = 0{,}6 \cdot 10^{-3}$).

прямой, и в дальнейшем экстраполяция ведется уже не по умножению в импульсе, а по скорости счета в пике, включающей и рост подсветки за счет запаздывающих нейтронов.

Область квазистационарности и переход к мгновенной и импульсной критичностям легко проследить и на кривой рис. 8, которая дает зависимость полуширины импульса от реактивности. Здесь пунктирные кривые — зависимости полуширины от реактивности для квазистационарного случая (соотношение 5) и для соотношения (1). Характерные формы импульса для различных значений реактивности приведены на рис. 8—10. Полуширина импульса в равновесном режиме составила 36 мксек.

Следует подчеркнуть, что статистический характер размножения нейтронов сильнее сказывается на работе импульсного реактора, чем на работе обычных стационарных реакторов. Такое положение связано с относительно небольшим, особенно при малых средних мощностях, числом первичных актов деления, определяющих развитие импульса. Это приводит к появлению значительных флуктуаций энергии (и соответственно амплитуды) импульсов.

Можно показать, что выражение для среднеквадратичного разброса амплитуд импульсов реактора ИБР имеет следующий вид:

$$\frac{\delta(J)}{\bar{J}} = \sqrt{\frac{\nu \cdot \Delta^2}{2S\tau}} \tag{9}$$

Здесь J — амплитуда импульса;
Δ — дисперсия коэффициента размножения для единичного акта деления.

Это выражение получено путем вычисления нарастания числа ветвей цепочек делений и дисперсии этого числа. При этом предполагалось, что все циклы размножения имеют одинаковую длительность, равную среднему времени жизни мгновенных нейтронов (можно показать, что в нашем случае, когда существенную роль играют лишь цепочки, возникшие в окрестности $\Delta k = 0$, при $\Delta k \ll 1$ разброс длительностей циклов несущественен).

Величина Δ^2 следующим образом связана с вероятностью $\varphi(n)$ того, что нейтроны возникшие в единичном акте деления вызовут n делений в следующем акте:

$$\Delta^2 = \sum_n n^2 \varphi(n) - \left[\sum_n n\varphi(n)\right]^2. \tag{10}$$

В частности, эффективный коэффициент размножения выражается через $\varphi(n)$ так:

$$k_{эф} = \sum_n n\varphi(n). \tag{11}$$

Строгий расчет Δ^2 достаточно сложен. При этом надо учитывать как статистический характер процессов испускания вторичных нейтронов при делении, так и статистический характер поглощения нейтронов, ведущих к делению. Для приближенной оценки можно воспользоваться тем обстоятельством, что вероятность испускания при делении определенного числа нейтронов сравнительно хорошо описывается распределением Пуассона. В этом случае и вероятности $\varphi(n)$ также будут подчиняться распределению Пуассона. Учитывая дополнительно, что $\bar{n}=k_{эф}=1$, можем принять $\Delta^2 \simeq 1$.

Тогда для равновесного режима реактора ИБР получим:

$$\frac{\delta(J)}{\bar{J}} = \frac{1}{\sqrt{2W\beta\tau_l}} \tag{12}$$

Здесь: W — средняя мощность (число делений в секунду);
β — эффективная доля запаздывающих нейтронов.

Флуктуации амплитуд импульсов мощности исследовались с помощью амплитудного анализатора, на вход которого подавались импульсы от метановой ионизационной камеры. Амплитудные распределения измерялись при различных средних мощностях. На рис. 11 приведена зависимость квадрата относительной полуширины распределения от средней мощности.

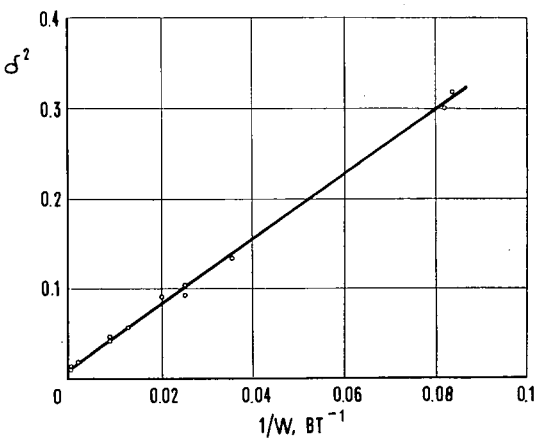


Рис. 11
Зависимость относительной полуширины распределения импульсов по амплитудам от мощности (83 имп./сек).

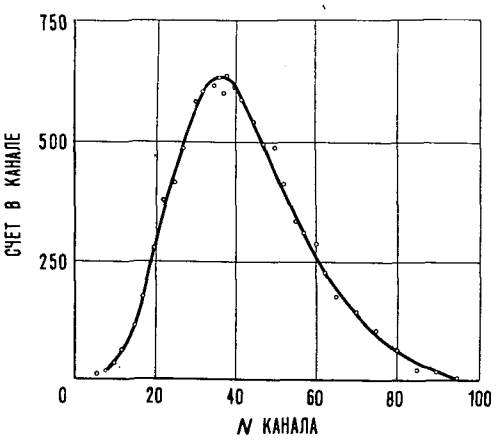


Рис. 12
Распределение импульсов по амплитуде при мощности 4 вт.

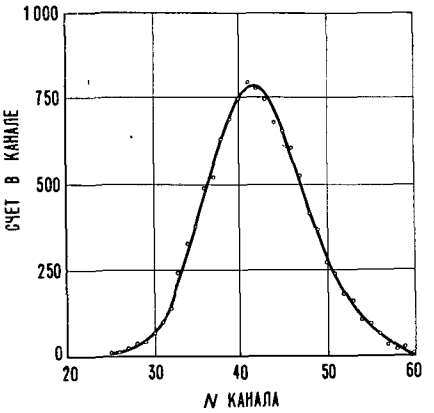


Рис. 13
Распределение импульсов по амплитуде при мощности 40 вт.

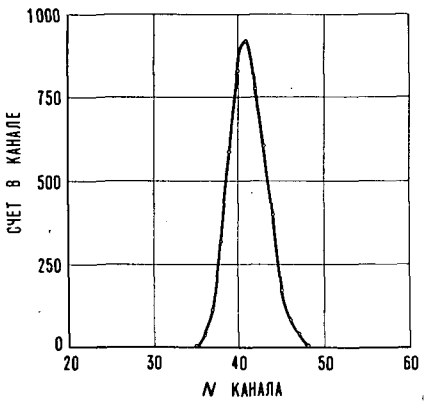


Рис. 14
Распределение импульсов по амплитуде при мощности 1200 вт.

На рис. 12—14 показаны распределения импульсов по амплитудам при средних мощностях 4, 40, 1200 вт соответственно. Интересно отметить, что значения средней мощности реактора, полученные из этих распределений с помощью соотношения (9), оказались в хорошем согласии с результатами измерения средней мощности двумя другими методами: калиброванным источником нейтронов и делительной камерой с известным количеством U^{235}.

Зависимость квадрата относительной полуширины амплитудного распределения от средней мощности (рис. 11). позволяет выделить разброс амплитуд, не зависящий от мощности. Этот разброс повидимому связан с вибрацией отдельных элементов активной зоны. Величина „вибрационной полуширины" при режиме 83 имп./сек составляет 10% и достаточно стабильна во времени.

Система автоматического регулирования работала вполне удовлетворительно даже при значительных флуктуациях. Оказалось возможным использование автоматического регулятора уже при мощности ~ 4 вт, когда относительная полуширина амплитудного распределения достигает 80%. При этом автоматический регулятор практически не влияет на картину флуктуаций (форма амплитудного распределения почти не менялась), средняя же мощность реактора поддерживалась постоянной с точностью $\sim 5\%$.

4. ОСНОВНОЙ РЕЖИМ РАБОТЫ РЕАКТОРА — 8,3 имп/сек

Частота импульсов 83 имп./сек слишком велика для ряда экспериментов и не позволяет работать с медленными нейтронами на базе 1000 м (появление рецикличных нейтронов). Поэтому основной режим работы реактора осуществляется при одновременном вращении основного и вспомогательного вкладышей, что позволяет снизить частоту импульсов при сохранении окружной скорости основного вкладыша, и, таким образом — длительности импульса.

При выводе реактора в основной режим значительное внимание было уделено фазировке движения главного и вспомогательного дисков. Необходимо было обеспечить одновременное вхождение обоих вкладышей в положение, соответствующее наибольшей эффективности. Так как механическая система, обеспечивающая синхронное вращение с кратными частотами главного и вспомогательного дисков имеет люфты и испытывает упругие деформации при работе с номинальным числом оборотов, такая фазировка не может быть осуществлена механически при остановленной системе.

Измерение временных сдвигов между прохождениями основного и вспомогательного вкладышей через активную зону осуществлялось с помощью импульсов от фотодатчика, связанного с главным диском и импульсов от магнитного датчика, связанного с вспомогательным. Степень фазировки контролировалась с помощью осциллографа и временного анализатора. Таким методом удалось осуществить фазировку с точностью до $0{,}25^0$ поворота вспомогательного диска.

Характер зависимости относительной полуширины амплитудного распределения от средней мощности при основном режиме не отличается от такового при 83 имп./сек. Однако „вибрационная полуширина" больше и составляет около 20% при условии точной фазировки (рис. 15). При нару-

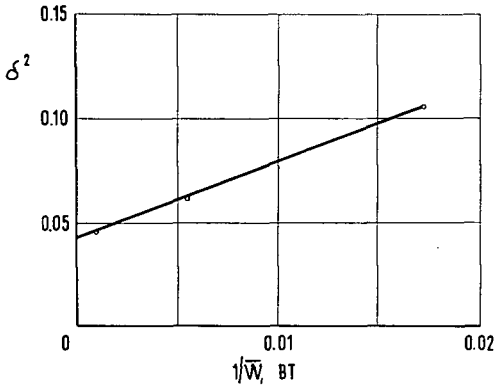


Рис. 15
Зависимость относительной полуширины распределения импульсов по амплитудам от мощности (8,3 имп./сек).

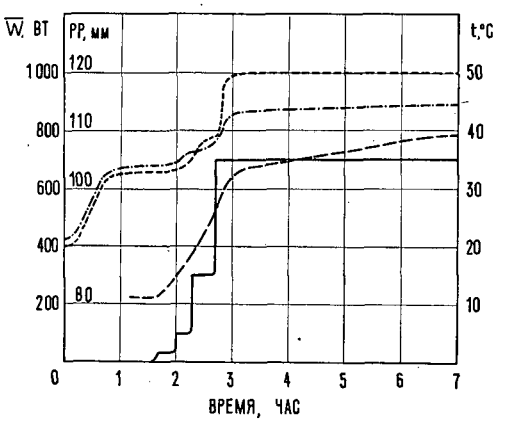


Рис. 16
Временной ход мощности реактора, температуры плутониевого стержня, воздуха, охлаждающего неподвижную часть активной зоны, и положения стержня ручного регулятора. ———— мощность реактора; — — — — температура плутониевого стержня; —.—.—.— температура охлаждающего воздуха; ———————— положение ручного регулятора.

шении фазировки вибрационная полуширина растет и в некоторых случаях достигает 40%. Можно считать, что этот рост вибрационной ширины связан с крутильными колебаниями вспомогательного диска. При нарушении фазировки зависимость реактивности от угла поворота вспомогательного диска становится более сильной, сильнее проявляются и крутильные колебания.

Необходимо отметить, что при основном режиме условия работы автоматического регулятора более тяжелы, чем при 83 имп./сек. Действительно, чем выше частота импульсов, тем в большей степени приближается ИБР по своим свойствам к обычному реактору. Однако, и в основном режиме имеет место вполне надежная работа автоматического регулятора при средней мощности выше 30 вт.

В процессе подъема средней мощности реактора начинается рост температуры плутониевых стержней и охлаждающего воздуха. Появляется еще одна возможность оценки средней мощности реактора — по балансу тепла. Проведенные оценки подтвердили результаты, полученные другими методами. На рис. 16 приведен временной ход мощности реактора, температуры плутониевого стержня и воздуха, охлаждающего неподвижную часть активной зоны. Здесь же приведена зависимость от времени положения стержня ручного регулятора, перемещаемого для компенсации температурного эффекта.

В настоящее время на реакторе проводятся эксперименты по измерению полных сечений, сечений захвата с использованием методов времени пролета. Начаты эксперименты по изучению спектров γ-лучей при захвате резонансных нейтронов, по измерению сечений резонансного рассеяния и деления. Проводятся работы по исследованию взаимодействия медленных нейтронов с твердым телом и жидкостью.

Продолжаются также физические исследования реактора, направленные на повышение средней мощности и сокращение длительности импульса.

ЛИТЕРАТУРА

[1] БОНДАРЕНКО, И. И. и СТАВИССКИЙ, Ю. Я., *Атомная энергия* **7** (1959) 417.
[2] ORNDOFF, J., *Nucl. Sci. and Engng.* **2** (1957) 450.

PROCEEDINGS

OPENING SESSION

INTRODUCTORY LECTURE

by

B. I. Spinrad

Argonne National Laboratory

FAST-REACTOR PHYSICS AND PROBLEMS *

At this meeting, we shall be considering the physics of reactors which are either very weakly, or not at all, moderated. Interest in these systems has three technical sources. First, at the high spectrum end fast reactors are good breeding systems. Second, many moderated reactors have high enough fuel enrichments so that there must be appreciable absorption at energies above thermal. Third, these reactors, which consist primarily of fuel and coolant, are attractive for compact power sources. I should like to comment on each of these advantages in turn.

Breeding reactors are important to the world's long-range energy future. Weinberg has pointed out that the energy content of the total uranium and thorium existing in granitic rocks is appreciably greater than the energy required to extract this uranium and thorium, but that this is not the case if only the U^{235} is used. Hence, our asymptotic power demand can be supplied by fission energy only through breeder reactors.

For this power demand, we have a comfortable development period, since our known energy reserves, including both fossil fuels and the U^{235} content of high-grade uranium ores, will supply the world's energy needs for at least a century. Nevertheless, this is a strong incentive for paying a good deal of attention to breeders and the fast reactors are proven breeders. Moreover, their breeding is not dependent on the use of extremely low cross-section materials in a reactor; typical breeding gains are 20 to 90% even when considerable parasitic material is included.

My second point is related, for even without considering breeding, fast or intermediate reactors can achieve high burn-ups without requiring the use of low cross-section materials. This is not, of course, to say that a high-absorption structural material is an unmixed blessing in any reactor; but such a material is much more tolerable in the concentrated systems which are typically fast or intermediate.

To illustrate this point, consider the direction being taken in many thermal-reactor designs. One way to decrease fuel-fabrication costs and insure integrity at high burn-up is to use a stout steel cladding; but this steel is a poison, and to compensate its effect the fuel must be appreciably enriched. This enrichment requirement is enhanced by burn-up allowances and control poisons. The result is a reactor with a significant fraction of epithermal fissions, and with a neutron spectrum whose Maxwellian component is not recognizable; or, in brief, an intermediate reactor.

My third point, that fast and intermediate reactors tend to be compact, suggests their use for interesting special applications. There obviously is a premium on compactness for all transport functions. There is also a current need for reactors in inaccessible places: these are either areas in which transport of fossil fuel is prohibitively expensive, or where construction is so difficult that pre-packaged plants are needed. Antarctica is an example of both requirements.

* Work performed under the auspices of the United States Atomic Energy Commission.

Thus, intermediate and fast reactors are of importance in the nuclear and total energy picture both now and in the future. There are other technical incentives for investigating these reactors which I need only mention:

1. Their compactness and high burn-up capabilities suggest their use as high-flux research reactors.
2. The rapid dynamic response of fast reactors makes them promising pulsed neutron sources.
3. High-temperature reactors tend to be hard-spectrum reactors, and fast reactors tend to use high-temperature coolants. Therefore this group of reactors is natural to consider for high-temperature application.

But this is sufficient introduction for the members of this Seminar. While fast reactors present many fascinating engineering problems which in turn react upon the physics of these systems, we are here to discuss quite specifically the physics problems themselves.

The state-of-the-art of reactor physics has recently been reported in TID-8210, "Current Outstanding Reactor Physics Problems", prepared by the United States Atomic Energy Commission's Advisory Committee on Reactor Physics. This report includes both general problem areas and, specifically, fast reactor physics problems. Rather than quote from this document, I shall refer you to it, and consider my remarks as commentary.

As with most other reactor types, the physics of fast and intermediate reactors has major subdivisions: reactor statics, commonly applied to design surveys and experimental analyses; long-term reactivity changes, or more generally, fuel-cycle problems; and short-term reactivity changes normally considered under the heading of reactor dynamics and safety.

The universal method used for reactor statics and reactor design is multi-group theory, generally with many groups and as detailed a set of group-transfer matrices as the cross-sections will justify and the computing device will admit. This procedure is proper and useful, but the use of complex computations for reactor surveys has a good many drawbacks. One of them is cost. Another is that we cannot make direct use of integral data obtained from critical and exponential experiments, threshold detector responses, or reactor substitutions. Such data can only be used as check points on our cross-section set. Perhaps the most important objection to the technique is that it tends to put a barrier between the design engineer and the reactor physicist who must collaborate on the reactor design. It is difficult for the physicist to explain his ideas when his only tool of communication is a multi-group calculation.

Therefore, although it would not improve the precision of fast- and intermediate-reactor theory, I think we need a heuristic description of the neutronic process in these reactors. The analogue in thermal reactor theory is the sort of description permitted by the four-factor formula, age-diffusion theory, and the perturbed Maxwellian spectrum. The practice of collapsing many-group into few-group calculations is not the proper answer, because it is primarily a computational technique which does not contribute to the reasoning process.

I have given some thought to the reasoning which might be used to construct a fast-reactor model of the type called for. The observation from which these ideas are derived is that fast-reactor spectra for systems as different as GODIVA and a natural uranium assembly can be reasonably well fitted to a linear combination of a fission spectrum and a "slow" spectrum centred between 1 and

200 kV. This "slow" spectrum is not particularly sensitive to changes in fuel concentration, but it becomes softer as elastic moderator is added.

I suggest that the fission spectrum is a legitimate canonical form for a fast-reactor spectral group; that neutrons tend to be depleted uniformly from the fission spectrum, by inelastic scattering at the upper energies and by absorption at lower; and that the inelastically scattered neutrons from a fission spectrum from a source for the "slow" group, whose spectrum is in turn governed by the balance between inelastic scattering and absorption.

The relative constancy of the slow-group spectrum is, I believe, the result of the similarity of inelastically scattered spectra from a fission source as we go from one element to another; it also derives from the uniformly small inelastic/absorption ratios in the 100-kV region. This is a balance which only appreciable moderation can disturb.

And this is as far as my reasoning has gone. If these ideas have merit, you are welcome to use them. You are also invited to criticize them, but not on the grounds that they do not allow a precise enough description of all processes; for a simple, qualitative, description is all that is sought, provided only that it applies to a usefully large set of cases.

Regardless of the foregoing, it is still true that we can predict reactor behaviour as precisely as our neutron cross-sections and fission parameters permit. Therefore, we must consider the status of our cross-section art for this type of reactor. My feeling is that we are getting quite satisfactory cross-section data for energies greater than about 100 kV. The energy region from 100 kV to the top of the useful fission spectrum is accessible to Van de Graaff machines, and the experimental techniques have improved very rapidly in the past several years. Where experimental data have not been accessible, we have been able to use the theory of nuclear structure and nuclear reactions to translate low-energy data, or data on similar elements, into useful reactor cross-sections.

Below 100 kV, Van de Graaff machines begin to exhibit poorer resolution, and below 25 kV they are no longer very precise accelerators. Yet many important phenomena of intermediate reactors and even of ceramic-fuelled fast reactors are due to reactions of few-kilovolt neutrons. As of now therefore, this spectral region is the one where data are most desirable. The problem is illustrated by the fact that, in ZPR-III, the best cross-sections available to us produced quite small errors in predicting criticality for completely unmoderated reactors. As moderators were added and the spectrum degraded to simulate ceramic-fuelled fast reactors, the error increased. We can decrease the error again by making changes in our cross-section sets which are, in fact, quite reasonable; but I do not call this a really satisfactory solution to the problem.

The next problem area pertains to fuel cycles, which I personally think present the most interesting problems in reactor physics today. Even excluding the complexities of fabrication and reprocessing, the fact of the matter is that we do not know how to define an optimum fuel cycle. For example, the steps we take to flatten power usually waste neutrons during the reactor cycle, or waste costly space in the reactor core; yet we cannot put a numerical value on the value of either the power flattening or these wastages. Similarly, we cannot state any rules to guide our attempts to utilize fuel loadings of spatially variable composition.

In the absence of strategic rules, our practice is to set up an initial core loading and an *ad hoc* technique of fuel management. We then calculate our material concentrations, reactivities and flux shapes as functions of reactor power and

time. We balance feed enrichments, control variations, and end-of-life exposures, which latter are generally externally imposed, against each other. We then repeat the problem under different initial conditions. Finally, we decide on our reference fuel cycle by making a qualitative judgement on the merits of each attempt with respect to cost and operational difficulty.

These remarks pertain to all categories of heterogeneous-reactor design; so it might be considered unfair to saddle a fast-reactor conference with the task of formulating fuel-management strategy. Nevertheless, I hope that in our fuel-cycle sessions, we come to grips with this problem.

Cumbersome as our procedures are, they are workable, provided that we know how to make the appropriate reactivity calculations. We can make these calculations if, in turn, we know the cross-sections of a host of materials not initially loaded into the reactor. These materials are the artificial heavy elements and the fission products.

For fast reactors, the artificial heavy elements—Pu^{240}, Pu^{241} and Pu^{242}, Pa^{233}, U^{233}, U^{234} and U^{236}—are the greater problem. They have large nuclear reaction cross-sections, with fission cross-sections that are rapidly varying with energy. Therefore, although even at the highest burn-ups they are minor reactor constituents, some precision in cross-section knowledge and reliable reactor-spectrum information is necessary if we are to gauge their effect.

The fission products, on the other hand, can probably be accounted for fairly accurately by statistical nuclear theory. For fast reactors, a small number of bulk-reactivity measurements will probably satisfy our needs.

For immediate reactors, other problems appear. The fission products are relatively more absorbing in intermediate spectra, and therefore their cross-sections must be known better. In addition, the statistics of overlapping resonances of the various materials will affect the reactivity contribution of each. This problem also appears in the physics of control-rod strength, and in the clean reactor as well, and is a very significant one to explore.

The final major category of physics effort is the study of reactor dynamics, which in turn bears on reactor safety. Since it is widely rumoured that fast and intermediate reactors have unique safety problems, I should like to address myself to that point first. Thus, let me say, for our record, that the presence of moderator, or the amount of moderator, in a reactor has absolutely no bearing on whether a reactor is safe.

The dynamic responses of different categories of reactors are different. Also, the mechanisms by which different classes of reactors shut themselves off are different. Therefore, a true statement is that the design techniques to be applied to fast and intermediate reactors in order to insure self limitation of power excursions are different from techniques used by thermal-reactor designers.

Our problem is to collect the information which will permit us to make good fast-reactor designs. This information consists of:

1. Mechanical distortion of the system under thermal and mechanical stresses, including such effects as fuel expansion, spreading, melting, and slumping, and coolant boiling and expulsion.
2. Changes in materials cross-sections due to atomic motions.
3. Changes in materials cross-sections due to mechanical distortions.

There are other data areas which are important: delayed-neutron yields, decay times and importances, and prompt-reactivity lifetimes. There is continuing

interest in experimental technique for these measurements, which indicates that we can expect very precise data.

This information will enable us to construct a reactor dynamic model, in which inputs of thermal, mechanical, and reactivity perturbations feed back into outputs of these quantities. We seek a design such that this feedback is always damped. To improve the probability of damping, reactivity feedback must be negative and prompt, and the system ought not to have transport-delay characteristics affecting its response.

To accommodate possible blunders in loading or operation, we must use designs which have also enough damping so that quite large perturbations do not have serious consequences. This means that perturbations beyond the expected operating range should have very large prompt negative feedbacks.

The collection of mechanical and thermal data is a significant problem to which, for example, the TREAT programme at Argonne is devoted. A major experimental effort is needed to procure these data, and it is up to the physicists. I think, in fact, that the time has arrived for physicists to devote their attention to problems of heat transfer, fluid flow, and mechanics of materials which we have previously ungraciously dumped in the laps of our mechanical engineering colleagues.

The collection of data on cross-section changes due to atomic motion, which we call Doppler effect, must be pursued; and so must the changes in cross-section following mechanical distortions.

These three data areas are exemplified by three safety problems which prophets of doom have learned to recite: melt-down reassembly, positive Doppler effect and positive coolant void coefficient. Good design renders them not only harmless, but uses them. Our problem is to design reactors which feature melt-down *dis*assembly, *negative* Doppler effect, and *negative* coolant void coefficient. When this is achieved, the short prompt-neutron response time becomes a virtue, and fast and intermediate reactors become the safest of all.

I hope that this Seminar will advance the acceptance of fast and intermediate reactors for those functions which I proposed at the beginning of this discussion, and of safe reactors according to the ideas I have just mentioned. I expect such an outcome, and therefore am eager to hear what you have to say. The best way to listen is to stop talking.

SESSION 1

Chairman: J. Horowitz

INTRODUCTION BY THE CHAIRMAN

The final objective of reactor theory is to be able to calculate the properties of a reactor solely from the basic data supplied by nuclear physicists. We are, however, at present very far from having attained this objective, either for fast and intermediate reactors which form the subject of this conference, or for thermal reactors. This is perhaps some consolation for those working in the less-developed field of fast and intermediate reactors. It does not seem, at least for the fast reactors, that the calculation methods are primarily to blame. It is the nuclear data which are either lacking or not known with sufficient accuracy. Considerable differences often exist between measurements made in different laboratories. These gaps and inaccuracies are particularly great in the case of large power reactors, whose very diluted cores contain great quantities of structural materials, and whose fissionable or breeder components are in the form of carbide or oxide.

When planning to calculate the critical mass, the breeding factor, the space-energy distribution of the neutrons and other quantities, the neutron specialist does not consider only what values he needs, but also the desired accuracy for each of them. This leads him to request the nuclear physicist to undertake a programme of measurements, and several authors will be speaking of this in their communications this-morning.

Among the principal requests made are those for measurements of the inelastic-scattering cross-sections, a precise figure for the average number of neutrons emitted by fission at different energy-levels, and the capture cross-sections, which are of particular importance in diluted cores where the spectrum is very degraded. Some of these values are very difficult to measure and theory has sometimes to supplement experiment. Finally, a particularly difficult problem is the calculation of the Doppler effect, which is involved in estimating the temperature coefficient; this requires a knowledge of the resonance parameters. The size and complexity of these programmes makes desirable a close collaboration between different laboratories and a certain division of labour. Such studies therefore lend themselves well to international collaboration.

However, in spite of all the efforts that have been made or are intended, it seems unlikely that it will be possible, for a long time to come, to undertake the neutronic part of a fast reactor project solely on the basis of the fundamental data. Integral measurements and critical experiments will continue to provide reactor physicists with their other indispensable avenue of approach. These form the subject of the second session of this conference.

In the last analysis, it is from a combination of these two methods of approach that reliable methods of calculation for fast reactors will emerge. It will be necessary, in the interests of efficiency and economy of resources, to make the best possible decision on what part each method should be asked to make.

Paper SM-18/4 was presented by J. J. Schmidt (see Vol. I, p. 3).
Paper SM-18/23 was presented by A. T. G. Ferguson (see Vol. I, p. 21).

Discussion

D. Okrent: I should like to thank Mr. Schmidt for pointing out the discrepancies in the low-energy molybdenum groups. This is the fifth or sixth error we have

found in the book. The one major printing error, which I should like to point out for the benefit of everyone present, is in the transport cross-section for oxygen, in Group I: this should be 1.14 and not 0.14. I cannot, offhand, give the history of the specific error mentioned by Dr. Schmidt; but Dr. Yiftah, who is now back in Israel, has volunteered to give a list of the errata of which we are aware to anyone who requests it.

B. Spinrad: I note that a number of the cross-section requests are cxpressed in terms of a certain percentage accuracy. I wonder whether the requester really means the point-by-point accuracy, or perhaps rather a certain precision regarding the integrals of the data; and whether the experimenter is not somewhat misled by this distinction.

A. T. G. Ferguson (United Kingdom): I think that in some cases—for example, the case of fission cross-sections at high energies, where the variation with energy is known to be smooth—the request could refer to the point-by-point accuracy. However, your suggestion is very relevant with regard to the resonance region; there, as an experimenter, I would interpret a request for 2% accuracy as referring to some local integral over the resonance and not, for example, to the cross-section at the position of maximum slope. In the latter case it would be utterly impossible to attain such precision.

Papers SM-18/76 and SM-18/36 were presented by A. B. Smith (see Vol. I, p. 29).

Discussion

J. L. Leroy: I should like to ask Dr. Smith whether he believes that the value of $\mathrm{d}\bar{\nu}/\mathrm{d}E$ is constant over the whole range from 2 MeV to 14 MeV.

A. B. Smith: No, I do not. I meant merely to imply a suspicion—and it is strictly a conjecture—that the lower value of $\mathrm{d}\bar{\nu}/\mathrm{d}E$ is vonstant to about 6 MeV, where the binding energy for the neutron in a heavy nucleus is reached. At that point I suspect it jumps or rises to the value of 15 MeV. The graph that we have shown really represents two linear slopes; it is not physically valid, in my opinion, and is strictly an empirical relation of this phenomenon.

Paper SM-18/24 was presented by J. L. Leroy (see Vol. I, p. 39).

Discussion

J. Chernick: I should like to ask Mr. Leroy whether any attempt has been made to combine the data on the fission cross-section of U^{233} with total cross-sections for U^{233} reported elsewhere, with a view to estimating the variation of η for U^{233} over the energy range between 7 and 60 eV.

J. L. Leroy: Not yet. Such measurements are planned, but I do not think that U^{233} will be dealt with at the beginning.

J.Chernick: I asked the question because I believe that such measurements will be quite significant for many breeder reactors, particularly thermal reactors.

GENERAL DISCUSSION

W. B. Loewenstein: I should like to ask a question of Mr. Schmidt. You showed a new alpha curve, and claim that this should improve agreement between theory and experiment in the outer blanket of the EBR-I. Have you investigated what happens in the core and inner blanket of the EBR-I with the alpha curve for both uranium-235 and plutonium?

J. J. Schmidt: No. In general I should say that the old alpha data are very much higher than the new. I discussed a little while ago the case of Pu^{239}; in my view, the integral alpha data for the EBR-I are lower than the theoretically calculated data (I refer for example to the work of Okrent *et al.*). The capture cross-sections are also lower by the new Diven curve, and that, I think, results in better agreement. However, one can lower the curve obtained by theoretical calculation to the experimental alpha curve. I hope that is an answer to your question.

W. B. Loewenstein: I should like to know whether you feel confident that you have calculated the right spectrum.

J. J. Schmidt: I think there are two possibilities in the case of the EBR-I. One possibility lies in measured spectra, which may not be altogether reliable at present. I believe such measurements were published in the *Journal of Nuclear Energy* in about 1957. The other possibility is to make theoretical calculations of the spectra; but then, of course, one must use the new values.

J. Chernick: I have a question for Dr. Schmidt. The fast reactor appears to have departed considerably from the original concept; it is becoming very large, very dilute and very safe as a result of the Doppler effect. I believe you mentioned that you have been calculating—for practical reactors of sufficient size to produce economic power—the uncertainty in the breeding ratio due to large discrepancies in the cross-sections of structural materials and the main components of the reactor—fissionable materials and coolant. Perhaps you could tell me what breeding ratio you believe is achievable and what the uncertainty is, based on present experimental data—not on data which may be obtained at some time in the future?

J. J. Schmidt: We have made calculations with the old molybdenum group cross-sections given by Okrent in his paper presented at the Geneva Conference in 1955, and we have recalculated the critical masses, for example, on the basis of the new molybdenum cross-sections. Our results differ by a factor of as much as 2 or 3, I believe, in the critical masses. I believe that breeding ratios such as 1.3 or 1.4 can be achieved, but I do not know exactly what differences will result depending on whether the old data or the new data are used.

W. Häfele: I think I can answer Mr. Chernick's question. If one uses the lower molybdenum values a great deal of structural material resistant to high temperatures is needed, but by adopting this concept one obtains breeding ratios in the range of 1.4. If the higher values—the so-called Russian values—are used, the ratio comes down to 1.2. Thus this discrepancy in the molybdenum data does have an important consequence: a difference of about 0.2 in the breeding ratio. The difference in the critical masses is higher, so that if the lower molybdenum values are applied, masses of between 200 and 250 kg can be expected (following the concept of external breeding); but if the higher values are applied, masses of 300 to 350 kg would definitely be used.

J. Chernick: Are you speaking of the initial values or the cycle values?

W. Häfele: The initial values, decidedly; but Mr. Ott will give a detailed talk on this question. One contention which will emerge from his talk is that over full cycles there is no definite decrease in breeding ratio.

D. Okrent: First, I should like to comment on Mr. Schmidt's reference to improved agreement between the new Diven data and the measurements in the EBR-I outer blanket. At Argonne, we feel that the measurements of α in the outer blanket of EBR-I cannot be reliably used for comparison with basic mono-

energetic data, as the reactor is not radially symmetric; in fact the spectrum varies as one moves out radially in various directions. We do not actually know what direction is applicable to the samples as the position of irradiation in the reactor was varied during their life-time. Measurements in this part of the reactor—direct spectrum measurements by cloud chamber, etc.—are therefore the least reliable. We do not question the Diven data; indeed we like them. My second point is that the new alpha data for Pu^{239} raise the breeding ratio of all designs, as the alpha is considerably lower in an important energy range. The actual value, of course, will depend on the specific design.

S. Yiftah: I should like to comment on the same point—the comparison of measured and calculated values of capture-to-fission ratios in EBR-I, using the new alpha data for Pu^{239} of Diven and Hopkins.

Calculations which I have made using the new alpha data indicate that the agreement of calculated and measured values is improved in the outer blanket—a result which agrees with Schmidt's calculations. In the core, on the other hand—and here I am replying to Mr. Loewenstein's question—my calculations show that by using alpha values one obtains lower calculated values, and the agreement is made somewhat worse.

J. J. Schmidt: What we mean, I think, is that the new Diven data are more reliable at lower energies than at high energies because of the rather hard spectrum in the core and the rather degraded spectrum in the blanket. Am I right?

S. Yiftah: No, I do not think you are.

C. P. Zaleski: I think that Dr. Chernick asked a question relating particularly to the breeding ratio in the reactor, and the inaccuracies which can at present be accepted for the breeding ratio in future reactors. We have, as a matter of fact, at Saclay, made some calculations of this kind for small reactors like RAPSODIE, and by making estimates—and reasonable estimates are all that can be made—with regard to the inaccuracies in the cross-sections themselves, we think that for reactors of the RAPSODIE type the inaccuracies in breeding ratio due to the cross-sections cannot be more than 15—20%. Some calculations which have recently been made at Saclay on large reactors, reactors of 1000 to 3000 or 4000 l, i.e. those which we think are likely to be optimum ones for the future, seem also to show that the figure of 20% should be about the maximum limit for the inaccuracies in breeding ratio, provided of course that one does not put into the reactor quantities of molybdenum greater than the fuel.

P. F. Zweifel: I should like to revert to a point made by Mr. Spinrad in his introductory lecture. He suggests the use of a simplified model for fast reactor calculations, similar to the "four-factor" formula applied to thermal reactors. However, such simplified models can apply only to bare (or very large) reactors. As fast reactors are essentially "multi-region" in that they contain core, blanket, etc., of vastly differing spectra, and as *fissions occur in both regions*, I doubt whether such a simplified approach would prove useful.

B. I. Spinrad: I think two points should be made clear. Firstly, I would deny the statement that we can calculate thermal reactors with the simple formulae only if they are bare or very large. It is true that our calculations are not altogether precise, but they are precise enough to make qualitative discussion possible. Secondly, I do not propose that the four-factor formula should be used for fast reactors of the sort we have to discuss; what I ask is that some model with similar precision and a similar direct connection to the reasoning process should be adopted for the purposes of our discussion. I think our discussion of the effect

of errors in the molybdenum cross-section and in certain alpha values on critical masses and breeding gains has shown how difficult it is really to isolate the source of these errors (without a total change in the cross-sections), and how much better it would be if we could apply an integral concept to the cross-sections. We would then be able to make qualitative statements without having to worry whether our cross-section set was in any way different from what we had assumed.

A. Campise: I should like to ask a question of the French group regarding the 15 or 20% uncertainty due to cross-sections. My question relates to the fact that the uncertainty in the inelastic scattering of all the materials used in the core is of the order of 50% when it takes the form of a slowing-down or transfer matrix. Do you still contend that 15 or 20% is the uncertainty which should be expected in calculating breeding ratios or critical masses of fast reactors?

C. P. Zaleski: I should point out that I was speaking of the breeding ratio and not of the critical mass. I cannot say exactly what the uncertainty in regard to the critical mass would be, and I think that errors of the order of magnitude you refer to in regard to the inelastic cross-sections will not lead to very large errors in breeding ratio.

A. Campise: My question concerned inelastic scattering. If we are to consider Pu^{239} for fast reactors, we must remember that the alpha value rises sharply, and with increased inelastic scattering this can lower the breeding ratio of a given reactor. Where there is a softer spectrum due to inelastic scattering, there will be a lower η value. What I really want to know is whether the uncertainty is not actually greater than 15 or 20%.

C. P. Zaleski: Of course the values which I have given are only in orders of magnitude and it is also difficult to know what uncertainties one should take for the inelastic cross-sections. I think that the values which Mr. Campise takes are extremes. I was speaking of a reasonable uncertainty for a given mean value. I do not think that this reasonable uncertainty would be as much as 50% of a reasonable mean value, particularly if the values one takes for inelastic cross-sections have been cross-checked by some integral experiments and fall into the category of "accepted" cross-sections. As Dr. Spinrad said just now, it is when one takes each cross-section separately that its influence can turn out to be greater that the overall uncertainties. In this way we arrive, as it were, at the idea of a mean uncertainty. That being so, the discussion would clearly be very long and very difficult because one would have to resort to multi-group calculations, but I think that in total the value of 20% is reasonable. I do not say it is the maximum which it would be possible to prove mathematically; I simply think that it is a reasonable value to take for the future.

D. Okrent: In ANL 6122*, there is a paper which gives the results of some calculations on the sensitivity of breeding ratio to cross-section variations; one of the cross-sections varied rather radically with inelastic scattering in U^{238}. It was shown that this did not have a very marked effect on a series of reactors of differing size and composition, so I think Mr. Zaleski's contention that the breeding ratio should not be very sensitive to inelastic scattering is sound.

A. Campise: As 50% of a power reactor is made up of sodium and 25% of steel, I feel the inelastic slowing-down in these two materials should also be studied and evaluated in relation to the breeding ratio.

* Proceedings of the Conference on the Physics of Breeding (19—21 October, 1959).

H. Soodak: Several speakers have mentioned the Doppler effect in connection with fast reactors. We all apparently agree that a fast reactor with a negative Doppler effect is better than one with no Doppler effect. I should like to ask whether anyone has actually studied the value of a negative Doppler effect to the safety of a fast reactor. How, for example, does one measure the importance of a coefficient of 10^{-6}/deg C or perhaps even 10^{-5}/deg C?

When a small amount of prompt-critical reactivity is inserted into fast pulse reactors such as GODIVA, the pulse is shut down, not primarily by the Doppler effect but by expansion.

B. I. Spinrad: I think that the remarks made by Mr. Zaleski in reply to Mr. Campise's question can be clarified by the sort of qualitative argument which I proposed in my introductory talk. Regardless of the amount of inelastic scattering which exists at high energies, the spectrum is not degraded below about 100 keV except by elastic moderation. Therefore, although the proportion of 100-keV neutrons to 1-MeV neutrons does rise with increased inelastic scattering, the alpha is quite flat in this entire region and the breeding gain is not greatly affected. That is the whole answer. Therefore, the important part of the spectrum which is affected by breeding gain is not the *inelastic* but the *elastic* scattering of the iron and sodium and other structural materials. This is not at all surprising.

S. Yiftah: My question relates to Mr. Ferguson's paper. In Table I you give values for the desired accuracies of the various cross-sections. How did you arrive at these figures?

A. T. G. Ferguson: I can give no more than a general answer, such as I gave during my paper. Certain arbitrary values were chosen for the acceptable error in both the breeding ratio and the critical mass. Perhaps Dr. Smith could comment in greater quantitative detail on the errors which we have chosen to accept.

R. D. Smith: As this is the subject of a paper which will be given in a later session*, we could perhaps defer detailed discussion until that time. Our principle was simply to choose a rather arbitrary level of accuracy for critical mass and breeding ratio. You can argue that the exact level which was chosen depends on economic assessments of reactors and so forth, and from such assessments we determined the errors which Mr. Ferguson discussed this-morning. I should prefer to leave any more detailed discussion until the later session.

The actual figures we chose were up to 1% for reactivity and between 2% and 5% for breeding ratios. The conversion to critical mass depends on how you calculate this—whether you alter the enrichment or the size of the reactor; but if you alter the size this could well mean 10% in critical mass.

J. Horowitz: When one looks at the list of papers to be read at the Conference, one might say that intermediate reactors are the "poor relations". In particular, at today's session, there are no papers which set out, as is done for the fast reactors, the requests made to the nuclear physicists by the reactor physicists who have to make the calculations for intermediate reactors. I would much appreciate it if, during this general discussion, someone could present—at least very briefly—his ideas on this subject.

B. I. Spinrad: As there are other people in the audience who are more expert, I shall limit myself to a few remarks. Firstly, it has been pointed out that our current understanding of intermediate reactors depends on quite precise know-

---

* SM-18/15 (see Vol. II, p. 111).

ledge of the resonance structure. Any improvement of our knowledge will require a great deal more work in the field of cross-section measurements—work of the sort which has been illustrated by Mr. Leroy's paper and which is becoming common in a number of accelerator centres throughout the world. Furthermore, one encounters a great deal of theoretical difficulty in using the cross-sections because techniques for very detailed analysis are the only techniques available to us. This means that a great deal of sample calculation study is inevitable—to determine the effects of overlapping resonances, for example—before we can make any sense at all out of our predictions. In fact the problem is similar to the one we encounter in the case of fast reactors, in that we have at present no way of interpreting our critical experiments. With regard to cross-sections, I think our problems are close to solution, in view of all the excellent new measurements which are being so commonly reported nowadays—and more of which will, I feel sure, be reported in the future. However, our ability to use these cross-sections presupposes the solution of a great many problems. At this point I should like to ask whether Dr. Hummel, Dr. Hansen or Dr. Zweifel would care to make further comments.

P. F. Zweifel: I would ask Dr. Spinrad to consider a bare thermal reactor in which all the fissions take place at thermal energy; assume that it is one-group, has a soft Maxwellian spectrum and is water-moderated; then make some minor change in the oxygen resonances. Now I should like to know how you would calculate the change in the criticality of that reactor without doing any detailed calculation.

B. I. Spinrad: If it is a well-moderated reactor I would predict no change.

P. F. Zweifel: Now let us be reasonable. You must calculate something—the age, or the slowing-down density, or the non-leakage probability, which is one of the six or seven factors in your four-factor formula; and that certainly requires a detailed calculation. It requires a solution of the space- and energy-dependent Boltzmann equation; and the fact that you separate the detailed calculations into discrete operations, as it were, and finally recombine them does not, I submit, mean that you will avoid detailed calculations.

B. I. Spinrad: I do not know in how much detail you want your answers, but I think it would be better to postpone this argument.

H. H. Hummel: I am not an expert on intermediate reactors, but I did have occasion last year to do some calculations on an intermediate reactor with a rather hard spectrum. I was shocked to discover how little information was available on resonance parameters in the energy range from 50 eV to 1 keV. An application of the statistical model in this energy range offers little promise because of the way the measured fission cross-sections behave. The cross-sections of G. Hansen and W. Roach of Los Alamos Scientific Laboratory were used in calculations at Argonne for the conceptual design of a fast-fuel test reactor. The problem of calculating the intermediate-energy neutron-flux distribution in a fast reactor subassembly placed in such a reactor appeared to be particularly difficult. The cross-sections of Hansen and Roach were based on the statistical model, using the narrow resonance approximation. It would be interesting to know how much confidence Hansen places in these cross-sections.

J. Chernick: I think we all realize that the calculation of intermediate-energy reactors is a relatively new and unexplored field. We have little information at our disposal, and the best we can do is to extrapolate the techniques that have been used for either fast or thermal reactors.

However, there are at least two other approaches which should be explored in addition to cross-section measurements, which are, in any case, difficult to interpret: one is the integral type of experiment for intermediate-energy reactors, such as has been developed by Dr. Horowitz; and the other, I believe, is the Monte Carlo method. We all have very large machines at our disposal now, and I think these can be tested, at least for simple thought-experiments, etc., to give us some information in the intermediate energy range.

SESSION 2

Chairman: C. H. Westcott

Paper SM-18/85 was presented by I. I. Bondarenko (see Vol. I, p. 65).

Discussion

H. H. Hummel: Have you considered experiments on mixtures of elements? Such experiments are instructive as a given element tends to perturb the neutron flux produced by other elements.

I. I. Bondarenko: Yes. We intend to make such experiments, and such experience as we have indicates that we can hope to find certain regular patterns, which may not be 100% reliable but nevertheless hold true in many cases. This will make it possible in other cases to find the values of all these quantities for mixtures of elements, on the basis of the measurements for the individual elements making up the mixture.

P. F. Zweifel: Could you explain the origin of the first formula you wrote:

$$D=\frac{1}{3}\frac{\langle 1/\Sigma\rangle}{\langle 1/\Sigma^2\rangle}.$$

I don't believe I have seen that before.

I. I. Bondarenko: The numerator of the formula indicates that the free path is averaged over the spectrum in the group in question. The spectrum of the group is by and large inversely proportional to the total cross-section, but this does not apply in every case. The denominator is the normalization.

A. Campise: Have you studied or do you plan to study the effect of heterogeneity on calculating the diffusion coefficient, as in the case of a fuel cell or a control cell? Does the diffusion coefficient, when calculated for a cell, differ from that of a homogeneous region, considering the resonance effect of a given geometry?

I. I. Bondarenko: We are considering solely the resonance effects in a homogeneous medium, and none of these measurements has a direct connection with any kind of heterogeneous effects. That is another question.

H. H. Hummel: May I also comment on Mr. Campise's question? Resonance scattering occurs in the high-energy regions, where heterogeneity is not really a source of concern.

I. I. Bondarenko: Yes, these measurements of ours are intended for calculating the resonance blocking in the scattering cross-section and have nothing to do with blocking in the capture cross-section, which is more important in the case of slower neutrons.

Paper SM-18/12 was presented by K. Tsukada (see Vol. I, p. 75).

Discussion

P. F. Zweifel: Could you tell me what penetration factor you used in doing these Hauser-Feshbach calculations?

K. Tsukada: We used the penetration factor of Beyster *et al.* We are now calculating the penetration factor with a nuclear potential of the surface absorption type.

Paper SM-18/17 was presented by A. Moat (see Vol. I, p. 87).

Discussion

A. T. G. Ferguson: I should be interested to learn how well the optical model parameters, which give such a good fit with the inelastic cross-sections, fit the elastic and total cross-sections.

K. Parker: The optical model parameters used in Hauser-Feshbach calculations on Na^{13} at 3.97 MeV give good agreement not only with Towle and Gilboy's measurements of inelastic scattering to various levels, but also with the total and elastic cross-sections (also measured by Towle and Gilboy) at this energy. The "compound-elastic" cross-section, deduced as the difference between the measured elastic cross-section and the shape-elastic cross-section as calculated on the basis of the optical model, agrees with that calculated by Hauser-Feshbach theory.

Paper SM-18/73 was presented by J. H. Neiler (see Vol. I, p. 95).

Discussion

J. J. Schmidt: I should like to ask whether Mr. Neiler has taken the following three things into account in the several 100 keV region: increased level-spacing, inelastic scattering competition, and **d**- and **f**-wave capture. Better agreement with the data shown on his slides should be obtained by taking these effects into account.

J. H. Neiler: We have restricted these fits to the range where it is hoped that the effects you mention will not be important.

J. Chernick: First, I have always been anxious to know whether any resonance self-shielding effects are operative, especially in the low-energy regions, and whether the experimentalists have eliminated these. My second question is more general: can you say anything now, at least in a statistical way, about the ratio of **p**-wave to **s**-wave strength functions for the nuclides which you examined?

J. H. Neiler: We have devoted considerable attention to self-shielding and resonance self-protection effects, and we hope they have been accounted for in these measurements. For the details I refer you to our paper which has recently been published in the *Physical Review*. However, I should prefer not to make any generalizations regarding strength function behaviour.

Paper SM-18/55 was presented by J. C. Hopkins (see Vol. I, p. 111).

Discussion

J. J. Schmidt: Have you any explanation for the sharp decrease in the alpha, and consequently in the capture cross-section of plutonium, between 30 and 60 keV?

J. C. Hopkins: No.

A. Campise: I have been studying the ZPR-III experiments as well as some of our latest experiments with reactivity coefficients of Pu^{239} and U^{233}; we have evidence that there is a break at about 100 keV, since U^{233} is fairly flat; but we have no explanation for it either.

GENERAL DISCUSSION

C. H. Westcott: If there are no further specific questions, I shall perhaps take advantage of the Chairman's prerogative for a few minutes. This session is rather

difficult to sum up in that some papers related quite specifically to a few important materials, whilst others covered a wider field. Moreover, we have not quite finished. Tomorrow, for example, there will be some treatment of the number of prompt neutrons per fission for some of the important materials. But if I may express a few thoughts of my own before opening the general discussion, the picture I see is one which confronts us time and again in physics: at one end of the scale the problems are simple—in nuclear theory for example, this is true of the theory of the deuterons; and at the other end of the scale—and the parallel in nuclear theory is infinite nuclear matter—I would hardly say the problems are simple, but theory is fairly well-advanced. In between are the actual conditions of great interest to most of us, where our theories are, I think I may say, inadequate. With regard to cross-sections for reactor design, I believe we are in very much the same position. I believe the situation at thermal energies, although not completely cleared up, is rather simple. Below 1, 2 or 3 eV we have only one, two or three resonances to deal with—and, in the worst cases, the tails of a few others. The measurements required to obtain the necessary data for developing reactors are therefore fairly simple in this energy range. As for the field that we are discussing and will continue to discuss tomorrow, information is rather harder to obtain. It is true that a very high degree of accuracy is required. It is also true that our measurements show discrepancies. Fundamentally, however, this field is not too difficult either, and from what has been said today I would gather that from 3 or 5 MeV upwards we are gradually reaching a stage where—although problems do remain—measurements can at least be made by known techniques within a certain range. There are rather few instruments that will operate in certain limited parts of this range, but our theories are beginning to give reasonable agreement with experimental results. Therefore, if we were dealing with fast reactors in which the neutron spectrum was an unperturbed fission spectrum, we might not be in a bad position; but in fact, even in the case of fast reactors (and more so in the case of intermediate reactors), any actual design has a large amount of diluent in the structure. We therefore take great interest in the energy range from 500 eV or 1 keV up to a few mega-electron-volts; this is precisely the range in which problems arise. Someone suggested this-morning—unless I have misinterpreted his statement—that we ought to know all the resonance parameters for all energy regions. I believe this is an impossible task for the experimenter; it may be reasonable to push resonance parameters from 50 eV to 250 eV, but the number of resonances involved is very large and the work is arduous. There we come to the region where resonances necessarily overlap and—worse still—experimental techniques are particularly difficult. In this region, measurements which were previously made with choppers are now being made with pulsed accelerators—a change-over from what, in classical terms, is the slow-neutron technique to the fast-neutron technique. I would merely emphasize that not only is theory insufficient to explain all we want to know about this region, but the measurements needed to obtain 2% accuracy in all the desired cross-sections are also very numerous. I am sure this is clear to all of you, but perhaps it is worth repeating. Now let us return to the topic which is of immediate concern to us in this session: how we can best obtain reasonably adequate data for reactor design. My words may have prompted discussion; I hope, at least, that they have not broken the train of thought of those who want to ask more general questions.

D. Okrent: I should have thought that one of the main purposes of the meetings devoted to neutron physics data was to reach some kind of agreement between

people who make measurements and people who use them, as to the degree of accuracy required. I believe I have heard only Dr. Smith of the United Kingdom give a specific figure for the accuracy which he regards as necessary in a particular case. This is a continuing problem, and one that is worthy of discussion. How precisely do we need to know the critical mass and other data for a reactor? These precisions can then be translated back into cross-section accuracy.

C. H. Westcott: I am not even certain that the problem is altogether clear. We have already heard an objection to requests such as for 2% accuracy in regard to a particular cross-section, and the suggestion that such a request made no sense on the side of a resonance. It was further suggested that one ought perhaps to request something like 2% accuracy with a resolution of 5 ns/m. This is a very important question. I do not know to whom it should be directed first, but I suspect we should begin with the reactor designers who make out the request lists.

A. Campise: In order to answer the question adequately, one must consider fuel-cycle costs, critical mass versus breeding ratio, reprocessing etc.; this would, I think, lead us well away from the topic which we are supposed to discuss at this meeting. Speaking from a strictly scientific standpoint, I think our data ought to be as accurate as possible. Perhaps Dr. Spinrad would care to comment on this subject?

B. I. Spinrad: There are actually two ways of specifying a reactor, particularly a fuel cycle; depending upon the flexibility which the designer allows himself, extreme precision with regard to reactivity can either be of very great or of almost negligible importance. The first way is to specify the reactor in terms of control margin, to be considered between beginning and end with due allowance for juggling of material in between; in this case it doesn't seem to make much difference whether one knows k precisely or not. The second way is to keep to an initial concentration and assume that the reactor is through when it has run out; in this case one must know k very precisely. I believe that the former way of specifying a reactor loading is more appropriate to the enriched types of reactors which we have today. The most obvious case in which there is a specified and set control limit which must be very accurately determined is the case of the natural-uranium system, where there is no concentration of flexibility. To summarize, I think we need only moderate precision of reactivity for fast and intermediate reactors, if our concern is to save considerable sums of money. I consider 1 or 2% a moderate error in k. If we do not have some check-point as a critical experiment, of course, we are not at all certain of even this moderate precision; and then we are led to demand that all factors entering into our prediction should be very much more accurate than to 1 or 2%. But, in general, I think that if we do not depart too much from systems which have been subject to criticality measurements, we do not need a great deal of extra precision in order to get a useful result.

W. Häfele: What is the situation with regard to neutron cross-sections and yields in the intermediate energy range? Is any work being done?

B. I. Spinrad: Limited data on resonance integrals of fission products are being published, largely derived from reactivity measurements in small assemblies. I think the largest number of such measurements was made recently at the Knolls Laboratory; but there are very few data (except for isotopes which also happen to be stable) on the detailed cross-section beyond 1 or 2 eV.

W. Häfele: Do you agree that the question of yield is as important as the cross-section itself? I feel that the yield might be even more important than the cross-section at our present stage of knowledge.

B. I. Spinrad: The chemists assure me—and I have no way of checking up on them—that this problem is almost completely solved; they are troubled by no more major uncertainties.

W. Häfele: I think that is true for thermal fission; but what about fissions induced by neutrons at higher energies?

B. I. Spinrad: I think we must simply rely on the chemists. They provide us with tables and we must use them.

R. Taschek: I should like to revert to the problem of accuracies, which was really only partially discussed. I have recently seen some requests—I think from Brookhaven—for accuracies in respect of certain particular quantities: for example, an accuracy of 2—5% in intermediate energy ranges for such obviously necessary things as η, ν, α, σ_f etc. primarily for the fissionable nuclides. The statement that, for physics purposes, the greatest possible accuracy should invariably be set as our goal is misleading, unless it refers to the situation existing at a particular moment. I should like to make a number of points in connection with the problem of absolute cross-section measurements with neutrons, or perhaps I could even refer back to the absolute measurement of any cross-section, even using charged particles. The best known cross-section—not in thermal regions but in the reasonably fast-neutron region with which part of our discussion has been concerned—is the hydrogen (n, p) scattering cross-section; here a great deal of work has been done on the total cross-section. Even the most optimistic, I believe, will not claim that that work has resulted in absolute accuracies better than 0.5%; 1% is more likely. If, eventually, you have to refer an absolute cross-section back to the (n, p) scattering cross-section—which, to the best of my knowledge, has to be done in fast-neutron cross-section work—then the various steps through which you must pass will limit you, in a very difficult experiment indeed, to something like 2 or 3% accuracy. I am not referring to a relatively simple measurement such as the fission cross-section of uranium-235 at 1 MeV. This one is a long, difficult experiment even if one aims at an accuracy of 2, 3, or 4%. If the reactor physicists could say that there were, for example, three cross-sections which they really must know to 1% accuracy (in my own experience even 0.25% has been requested for some cross-sections)—and could prove to us, the measurers, that the request was really well founded—we might eventually be able to provide the data; but that would take a long time, and you would not believe the data even when you had them. I should like to point out that absolute measurements to 20% accuracy are easy; to 10%, fairly hard; to 5%, at least three times as hard as to 10%; and, from there on, only back-breaking labour can result in better measurements. So please don't tell us that you have to know 50 cross-sections to 2% accuracy. As Spinrad has said, if there are three cross-sections that you really want to know to a high degree of accuracy—and by a high degree of accuracy I mean 1 or 2%—please tell us what they are soon, because the measurements will take a long time.

P. Greebler: With regard to the problem of precision in cross-section measurements I should like to ask the following corollary question. If we knew our cross-sections precisely, how accurately could we predict physics parameters such as initial critical mass and reactivity coefficients in the light of present uncertainties in computational methods? An answer to this question would provide a guide to the upper limit of precision required of the measurements.

P. F. Zweifel: I cannot answer that question with any certitude, but I doubt very much whether such knowledge would establish an upper limit of precision;

on the other hand, it might tell us whether or not we have to improve our methods of calculation.

P. Greebler: I think we can agree that we are limited in the fast-reactor field at the present time by cross-section measurements and not by our techniques. Would there be general agreement on that statement?

B. I. Spinrad: With regard to fast reactors I think there might be general agreement; but with regard to intermediate reactors I must disagree. The problem of treating overlapping resonances in space and energy is extraordinarily difficult.

C. P. Zaleski: I certainly think that, as Dr. Greebler and Dr. Spinrad have said, in the case of fast reactors accuracy depends mainly on the constants, but only if one is engaged in calculating critical masses and the breeding ratio. If one is interested in the propagation of flux at a certain distance, in other words a problem of protection, a problem of the heat source in protection, then I think that the calculation methods also can be improved.

Now I would like to make a small comment on what Dr. Häfele said. I wonder whether, as regards the evolution of the reactivity of a fast reactor, the most important factor is not the evolution of the heavy nuclei (I am thinking of Pu^{239}, Pu^{240}, Pu^{241} and U^{238}) and whether fission-products questions do not take second place. Furthermore, I wonder if these measurements are not of rather secondary importance by comparison with measurements in connection with the fissile, fertile nucleus.

P. L. Balligand: Could any of the participants give us some figures regarding the difference between calculated critical masses and actual critical masses as measured on reactors which have gone critical?

G. I. Marchuk: The problem of the accuracy of reactor calculations depends to a large extent on our knowledge of physical constants. Clearly it was first of all necessary to check our most important critical experiments against various systems of constants and also to compare the results of all calculations. Important work in this field has been done in the Soviet Union. Some of the systems of constants that have been analysed here are of interest to us, and we were especially interested in two systems of constants—the nine-group and the eleven-group. In this connection, I should point out that we have limited ourselves essentially to the investigation of those reactor systems which dispense, as far as possible, with the need for subsidiary moderators. Among the reactors examined were model reactors of the uranium-water, uranium-graphite and uranium-beryllium types, and those with admixtures of sodium.

From the comparison and analysis of these systems and the calculations which were carried out on the whole range of spectra, beginning with the fast and ending with the thermal, it emerges that the difference between the calculations based on the eleven-group system of constants on the one hand and on the nine-group system on the other is not large and may in general be taken as 15—20% in respect of the critical mass.

With regard to resonance effects, the following experiments were carried out. Firstly, an experimental calculation was made in the following manner: the uranium-water, uranium-graphite and uranium-beryllium systems were examined with 27 resonances for uranium-235 and then with four resonances. In the case of intermediate reactors the maximum difference resulting from this change in the resonances is 30 or 40% for reactors with different moderators. Finally we carried out the following calculating and computational experiment. The position of the resonances was varied within the limits of each group, beginning with the highest

group, where the total intensity of the resonances belonging to the group was recorded, and proceeding to the lowest group. With such a variation the difference in the critical masses in the 21-group calculations did not exceed about 15%.

J. Chernick: I wonder whether a general answer can be given to this very important question. Dr. Taschek has mentioned the problem of requests from users, and the fact that very difficult experiments are required to give the accuracy specified in these requests. Of course there are several agencies which have tried to filter these cross-section requests and evaluate their urgency, and further comments from members of the Seminar would be in order.

There is one other problem which concerns me. In discussing rather specific types of fast reactors at this stage, I wonder whether we are not forgetting the possibility that future fast reactors may be quite different. I think no one yet claims to have a competitive fast reactor; and only when such a reactor does in fact exist will it be necessary, in my opinion, to achieve the great precision which has been discussed here.

I was very much encouraged to hear that the molybdenum-steel systems which Dr. Häfele referred to had a breeding ratio of about 1.3; and the French delegation referred to breeding ratios of 1.4 in systems in which no molybdenum is used. In any case it is obvious that the type of structural material used is very important. If the economic reactor is eventually found to be based on tantalum, the number will change again; so I really think that some of these long-range problems need to be examined.

W. Häfele: I should like to comment on the question of fission products raised by Mr. Zaleski. He is absolutely right when he says that the cross-sections of the heavier isotopes, such as uranium or plutonium, are the most important problem; but as a lot of work is being done in that field I am not unduly concerned about that aspect of the problem. On the other hand, I should like to make the following shocking statement: any fast reactor which cannot achieve at least 10% burn-up —better yet, 35% burn-up—has no economic future; and if one really does achieve 10% (or 35%) burn-up, or if one uses an easy recycling system for the core itself, the question of the fission product becomes really important. But if one achieves only 1% burn-up, the fission-product question is reduced not merely to secondary but to tertiary importance.

C. P. Zaleski: I should just like to say that I do not quite agree with what Dr. Häfele said just now, on the necessity of obtaining a burn-up of 10% in order that a fast reactor may be economic. Secondly, I am not convinced that some form of chemical treatment for eliminating fission products cannot be worked out and will not end by being generally accepted... I just do not know; I think the question is still an open one. Finally, I agree that it is useful to know as precisely as possible the cross-section of the fission products; but I think that, at present, it would be more important to have greater accuracy in our knowledge of the cross-sections of the heavy isotopes and in particular Pu^{240}, and perhaps Pu^{241}, than to have a greater knowledge of fission products. I may be wrong; it depends mainly on the type of reactor chosen.

A. Campise: I should like to comment on a question asked earlier about the calculation of fast and intermediate reactors. About two years ago we decided, at Atomics International, to use the basic cross-section set of Gordon Hansen and others at Los Alamos, and to improve upon that. At the time they had demonstrated the usefulness of this cross-section set for calculations of very fast reactors at Los Alamos, water, graphite and beryllium systems at Oak Ridge, and some

other facilities. We have since used this set—modified at our discretion—for somewhat less than twenty ZPR-III experiments, and at present we are using it for our own experiments. Our errors have been in the neighbourhood of 5% for critical mass; from 2 to 10% for fission ratios; from about 10 to 15% for reactivity coefficients of fuels and fertile materials; and from 20 to 50% for reactivity coefficients of structural materials. These are the accuracies which we are working with at present.

D. Okrent: Firstly, I should like to say that there is not complete agreement regarding the need for 10—35% burn-up to achieve economic power in fast reactors. Secondly, if one does burn a reactor to 10 or 20%, the long-term effects will again depend on Pu^{240}, Pu^{241} and Pu^{242} more than on fission products, and in this connection I can quote a few calculations: 10% burn-up in a 1500-l oxide reactor showed that four or five times as much reactivity, in this particular design, was going into the disappearance of fuel and build-up of fuels which were, let us say, not so good. This is an internal breeding ratio of less than 1, and four or five times as much as our estimate of the fission-product effect. So while fission products are important, they still appear to be a less important factor than the other. Perhaps one could design a reactor in which there was a balance of reactivity effects, and if this were crucial to the operation all these factors would have to be known accurately. At the moment I know of no measurements being made on the capture of Pu^{240}, Pu^{241} or Pu^{242}, apart from an attempt to irradiate some large quantities of material in the EBR-II reactor and to measure these graphically by mass spectrum. But this is not yet a proper experiment, and the subject is in no sense well covered.

W. Häfele: To some extent the question is one of the basic philosophy of breeder design. However, I must re-emphasize that we still believe in the importance of fission products. If one re-examines all the re-cycling possibilities there might very well be one such possibility which is very cheap; and only one stage, with decontamination factors of 5 to 10, might lead to a sum-up of special fission products. In such a case the question would really acquire some importance.

R. D. Smith: In the first place I should like to support Dave Okrent in his analysis of the fission-product question. We have done similar calculations and come to similar conclusions. In other words, the reactivity changes due to fission products are usually about one-fifth of those due to the fuel, though, as Okrent says, this proportion does depend upon the particular reactor. Where there is a peculiar re-cycling process I think we shall probably find that we are interested in the cross-sections of one or two fission products only, and not in fission products as a whole.

Mr. Campise offered some guesses on accuracies. I can quote some accuracies very similar to his; about 5% in reactivity for a 200-kg reactor is our present estimate of the critical mass that we can be fairly certain of getting. Occasionally one does a lot better than this, as in the case of the Dounreay fast reactor, where, probably fortuitously, we got to within 0.5% of the actual critical mass; but for that we did use a zero-experiment, ZEUS, as well.

There is yet another point which I should like to make. Some breeding ratios have been quoted already, and I expect a lot more will be quoted in the course of the Seminar. However, I think it would be useful if our colleagues said what breeding ratios they meant, as there are a number of different ways of defining the breeding ratio which give quite different answers.

With regard to accuracies, I am well aware that it is difficult to measure anything to 1% or barely over 0.5%; but the figure of 1% for reactivity which I mentioned earlier—and which we chose, as I said, more or less arbitrarily—at least provides relative standards for cross-section accuracy. It remains a fact that if one wishes to know reactivity to 1% in the particular reactor which we considered, one must know fission cross-sections to 1% and to 0.5%. If it is too expensive to measure to 0.5%, one must simply reconcile oneself to knowing the reactivity to a less high degree of accuracy. But this does mean that if one cannot measure the fission cross-section to about 1%, one is wasting time in trying to measure the capture cross-section—of Pu^{240}, for example—to a higher precision than 20%. Thus, in some ways, we are letting the measurers off by saying that it is pointless to measure cross-sections to a higher accuracy.

A. T. G. Ferguson: I should just like to comment on one of Dr. Höfele's remarks regarding our knowledge of the yield of fission products from fast-neutron physics. There has been a recent study of this at Harwell by Cunningham and Rao, who have measured the yields of fission products as a function of neutron energy from 30 keV to several mega-electron volts. They found some variations which were extremely interesting and surprising from the nuclear physics point of view, but these variations consisted almost entirely of a somewhat anomalous and irregular filling of the minima, or "through" elements. These elements represent only one part in 500 as compared with the peak yields in the peak area, so that the average yields were very little changed over the whole energy range they covered—the range that is most important in the case of fast reactors.

SESSION 3

Chairman: E. Clementel

Paper SM-18/35 was presented by B. I. Spinrad (see vol. I, p. 171).

Discussion

J. J. Schmidt: I should have thought that a $1/E$ rather than a $1/V$ behaviour was to be expected for medium-weight structural nuclei (such as Ni, Co, etc.) below about 1 keV, because Γ_n is larger than Γ_γ by a factor of approximately 1000.

B. I. Spinrad: I disagree. The resonances are not encountered until one gets well above the 1-keV region, and one needs be concerned, therefore, only with the standard $1/V$ part of the extension.

P. F. Zweifel: I should like to know exactly what is meant by the "deformed nucleus", to which Dr. Spinrad referred several times. Does this mean that the imaginary part of the potential has been put on the surface, or that one has a rounded well rather than a square well?

B. I. Spinrad: No, it means that the potential cannot be described as a pure function of radius; it also includes an angular term—i.e. it is not spherically symmetrical.

A. B. Smith: The deformed nuclear well used in many of these calculations is a non-radial well due to collective deformation, particularly with the D-wave strength function. Both L—S coupling and nuclear deformation affect this strength function in the region of A often under consideration. A detailed discussion of this work can be found in the proceedings of the International Conference on the Optical Model held at Florida State College in 1959.

Paper SM-18/11 was presented by Mrs. Saruis (see Vol. I, p. 180).

Discussion

M. Levine: How high in energy can you go, in the case of these fission products, without having to use $l=2$ and still higher angular-momentum contributions?

A. M. Saruis: Our calculations fit the available experimental data quite well in the energy range which we considered; we think, therefore, that the $l=2$ contribution is not important in this range. It might become important at higher energies, however—for example, above 1 MeV.

J. J. Schmidt: Have you made any calculation of fission-product capture cross-sections at energies above 100 keV?

A. M. Saruis: No, for those cross-sections we took no account of any increase in level spacings as a function of energy; that is, we calculated only the experimental low-energy values.

B. I. Spinrad: I should like to ask Mrs. Saruis whether she has performed the statistical exercise of comparing the average values for the capture of fission products with theoretical values assumed *a priori*, as it were, without the benefit of any specific information for any given material.

A. M. Saruis: No, we have made no calculations of that kind.

L. N. Usachev: I would like to dwell for a moment on calculations of the ratio between the radiative capture cross-section and the fission cross-section for the three fissionable isotopes Pu^{239}, U^{233} and U^{235}. The importance of this value was

discussed yesterday, and therefore, although we have at present only preliminary results, I am going to speak about them now.

In order to calculate this ratio, it is necessary to know both the radiative capture cross-section and the fission cross-section, i.e. it is necessary to know the dependence of the radiation width, the fission width and the neutron width on energy, and also the neutron permeability. The values for neutron permeability were taken from a book by Nemirovsky published in the Soviet Union under the title *Sovremenny modely atomnykh yader* (Recent Models of Atomic Nuclei). An appendix to this book gives permeability values, calculated for a nucleus with a blurred edge, due account being taken of the spin-orbit coupling. We took these values for the appropriate radius of the atomic nucleus.

The radiation width was taken from data on low-lying resonances, which are known, and was extrapolated in accordance with a generally accepted model, such as, for instance, the one used by Lane and Lynn in their work on radiation capture. This give a 50% increase in radiation width up to 0.5 MeV.

The dependence of the density on energy and on the angular momentum was taken from the work of Lang and Lecouteur.

Although the fission width is a matter to which we are also devoting attention ourselves, the value we used was taken from the well-known formula of Bohr—$\Gamma_f = (D/2\pi)\, pN$—where Γ is the fission width, D is the distance between the levels in the compound (nucleus), p is the permeability—which was obtained in much the same way as was done in the work published by Hill and Wheeler in 1953, which also deals with a number of fission questions—and N is the number of channels, i.e. the number of levels, the number of possible states of the nucleus at the saddle-point.

This number has also been determined by us on the basis of a model which was first proposed by Aage Bohr, although it now has certain special features. In the first place, the nucleus was considered to be in rotation, but not in axial rotation; but, of course, this point of view is now widely accepted, in accordance with the work of A. S. Davidov. We assume that the nucleus is not axially symmetrical either at its saddle-point or in its basic state. We assume also that the nucleus at the saddle-point is not symmetrical with regard to the plane of reflection, i.e. the nucleus is pear-shaped. On such assumptions, we get a completely fixed sequence of rotation levels. We use this sequence of levels—actually we do not get a sequence, but only the number of levels: the position of the levels in terms of energy is not determined, since they are determined solely on the basis of the geometrical property of the nucleus. We selected the position of these levels in such a way as to obtain coincidence with the experimental data, and primarily with the experimental data on fission widths. You see in Figs. 1, 2 and 3 the experimental points and the theoretical curves for the fission cross-section of Pu^{239}, U^{235} and U^{233} respectively. The theoretical curve is in fairly good agreement. This gives us some hope that the fission width is correctly represented by the aggregate of the levels of the nucleus at the saddle-point, which is what we take.

On the basis of the fission width, which thus corresponds to the fission cross-section, and of the radiation width, of which I have already spoken, it is possible to calculate the ratio between the radiative capture cross-section and the fission cross-section, which is what we are after. In Fig. 4, you have the experimental points and the theoretical curves for Pu^{239}, U^{235} and U^{233}. I would like to draw attention to two points in the case of Pu^{239}, the one at 20 keV and the one at 200 keV. These points were obtained in 1957—58 in our Institute by Andreev. They are

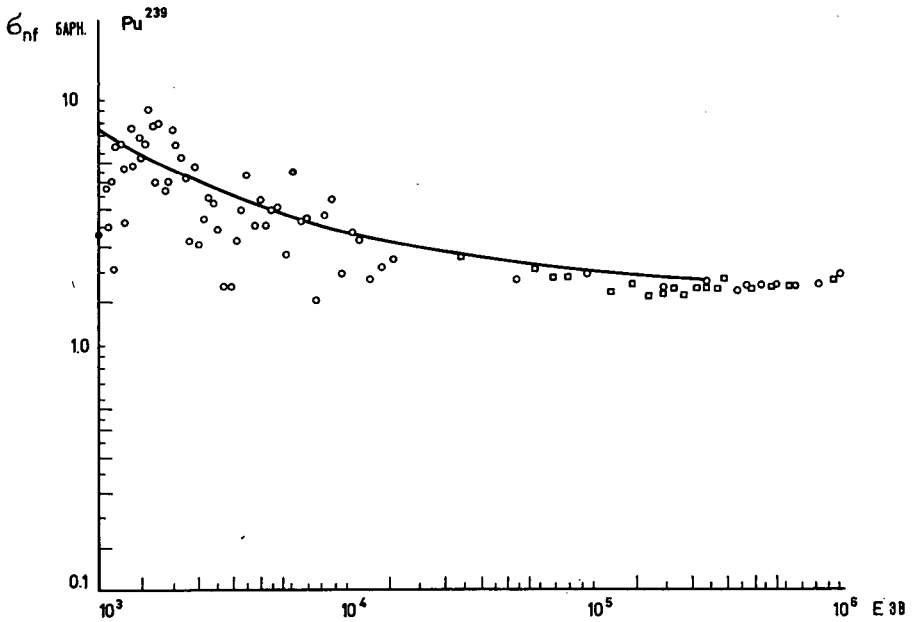


Fig. 1
Plutonium-239. Fission cross-section.
Points: experimental
Curve: theoretical

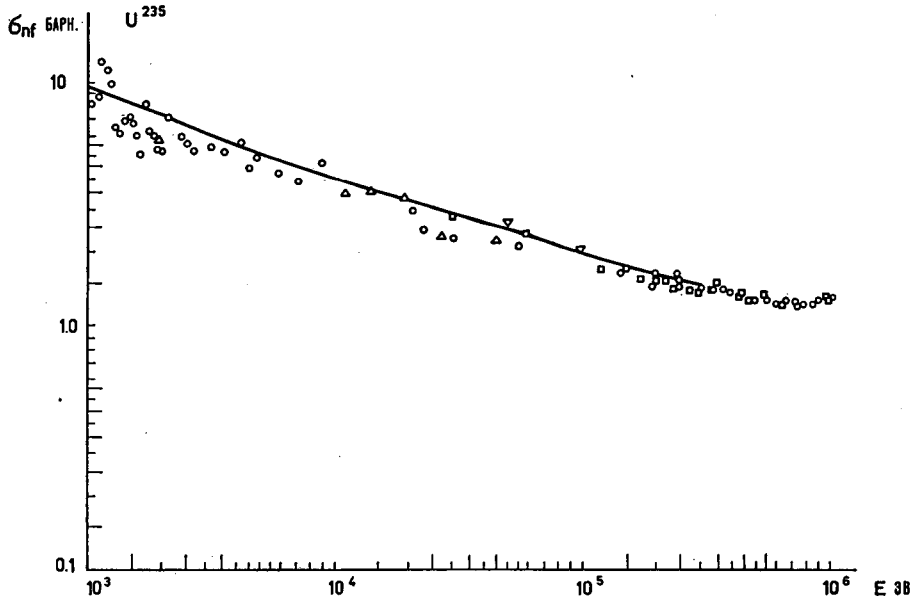


Fig. 2
Uranium-235. Fission cross-section.
Points: experimental
Curve: theoretical

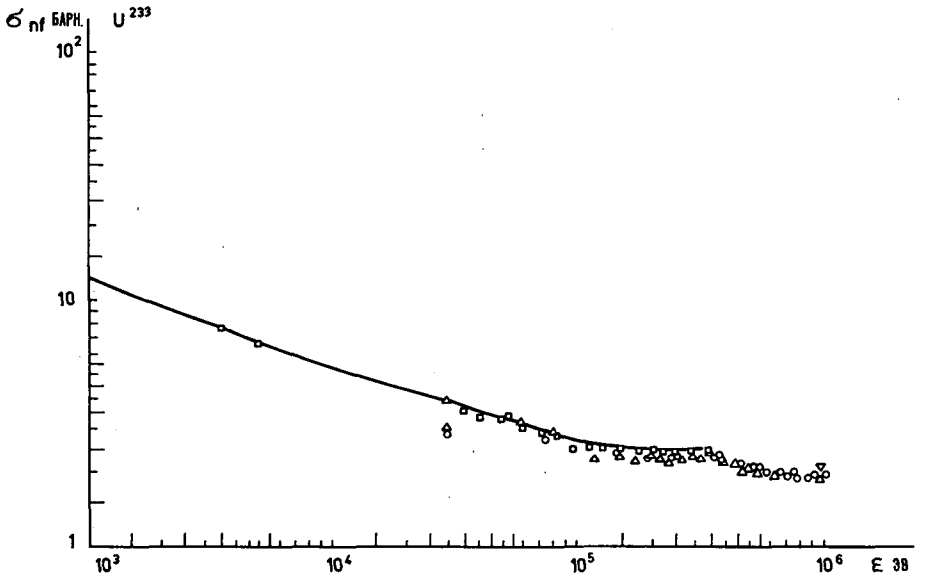


Fig. 3
Uranium-233. Fission cross-section.
Points: experimental
Curve: theoretical

in good agreement with those put forward yesterday by Hopkins and Diven. The point at 200 keV is lower than the Spivak point, presented at the first Geneva Conference by a group of Russian research workers. Unfortunately, the theoretical curve lies between them in the region of 200 keV. It would have been preferable if it had been a little lower in accordance with the more detailed data of Diven. As for U^{235} and U^{233}, there is good coincidence here also; perhaps better than in the case of Pu^{239}.

It should be said here that we also compared the number of channels at the saddle-point with the number of channels which it is possible to extract from the data on the distribution of fission widths at low energies. The number of channels for U^{235} and U^{233} is in this case two, from the s-waves, and for plutonium. In other words, the picture here of the number of channels corresponds to the data on resonances in the low-energy range.

It should be pointed out that these are preliminary results. This work was carried out by Gordeev, Rabotnov and Stavinsky; I also took part in some of it. It should also be said that it may be necessary to specify the neutron permeability, to compare it in greater detail with the inelastic scattering for the elements in question. In any case, the work will be continued.

A. Campise: I should be interested to know whether you have applied this theoretical treatment to a study of the capture cross-sections of these materials in the energy range in question.

L. N. Usachev: We used exactly the same methods for other elements, for example to calculate the radiative capture cross-section of tungsten. This is the work of Savitsky, Stavinsky and Tolstikov—measuring the radiative capture cross-section of tungsten and comparing the results with theory—but it is con-

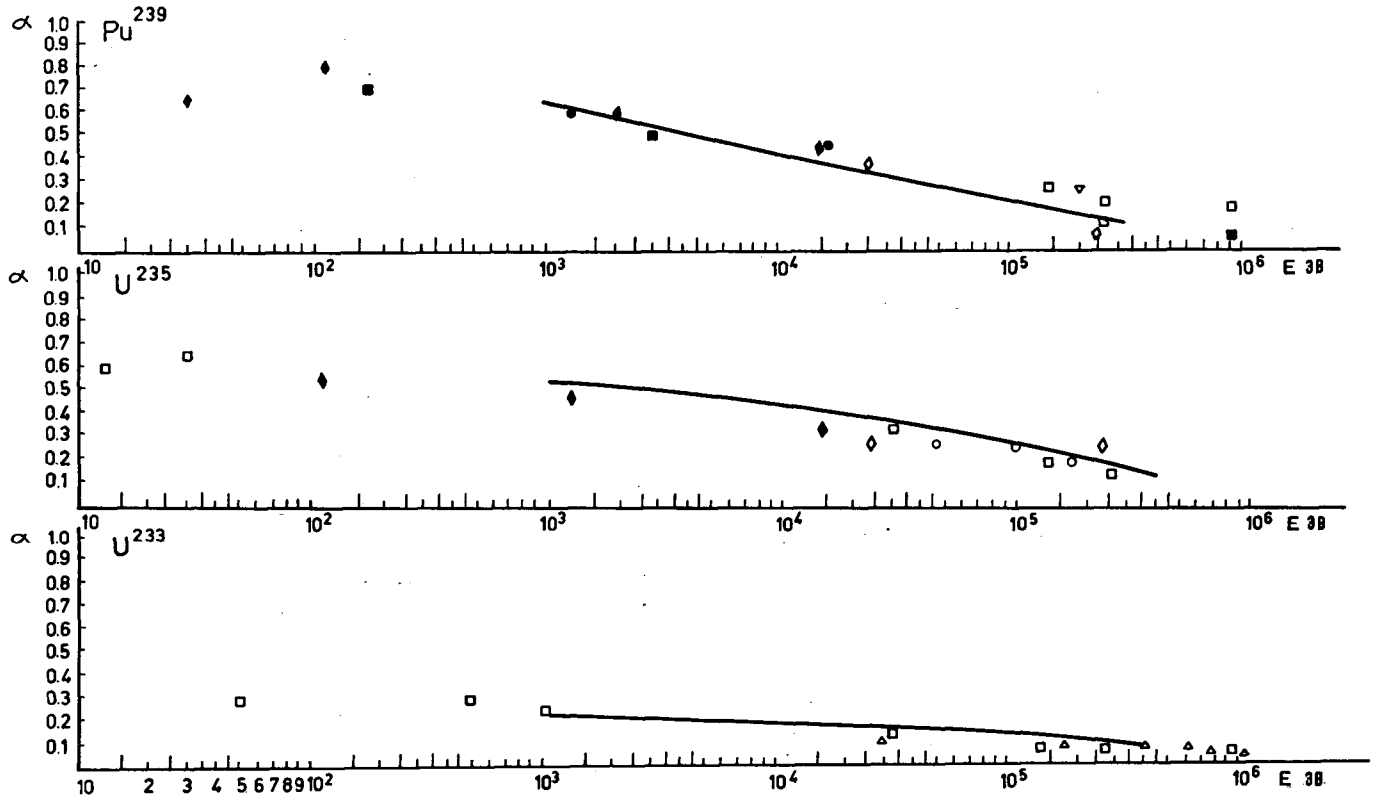


Fig. 4

Ratios of capture cross-section to fission cross-section for plutonium-239, uranium-235 and uranium-233 ($\alpha = \sigma_c/\sigma_f$).
Points: experimental Curves: theoretical

◆ Kann, Steward (1955)
◇ Andreev (1958)
□ Spivak *et al.* (1956)
○ Diven, *Phys. Rev.* **104** (1958) 144
△ Diven (most recent data, 1960)
● ■ KAPL

cerned only with methods of measuring the resonance capture cross-section by determining the dependence of level density on energy. As far as the fissionable isotopes of uranium are concerned, it is of course possible to obtain the radiative capture cross-section with reference to the fission cross-section and to establish the relation between these two cross-sections.

E. Clementel: I should like to make a brief comment of my own regarding the number of waves one should take into account in fitting the scattering cross-section data. There is no conclusive evidence that the fit between the theoretical and experimental curves can be improved by adding new values for the angular momentum of the incident neutron. Any aesthetical fit is misleading in this kind of analysis. The really valid argument is that the sum of the errors in a least-square fit should be minimum.

B. I. Spinrad: I should like to make another comment on the same subject. The parameters used to describe nuclear models are obtained by generalized fitting of a series of elements in a range of A, Z. They are fitted not for a particular isotope but for a range of the periodic table. They are subject to only minor adjustments; and, in the circumstances, the calculation of higher l terms is correct and profitable.

E. Clementel: I agree with Dr. Spinrad, but I was referring to the fitting of a specific cross-section, not to the relative cross-section. That is a different matter.

C. Erginsoy: I wonder whether any of the other participants has similar work or results to report on the calculation of fission cross-sections, especially as the last contribution of Dr. Usachev dealt with a very modern model of the nucleus at fission.

Paper SM-18/18 was presented by A. Moat. (see Vol. I, p. 139).

No discussion

Paper SM-18/56 was presented by J. C. Hopkins. (see Vol. I, p. 149).

No discussion

Paper SM-18/75 was presented by J. Kockum. (see Vol. I, p. 155).

No discussion

General discussion

R. Taschek: I think that those in charge of arrangements for the Seminar were very wise to schedule a discussion of neutron-physics data before the more general discussions on fast and intermediate reactors. It is important, in my opinion, that those of us who procure data should meet as frequently as possible with the people who use them, and also with those who do computation work. It seems to me that the papers already given have revealed four general areas of discussion, and these will perhaps become more sharply delineated as the Seminar goes on. In the first place there are the differential (sometimes called microscopic) data, of which one of the reports which we have just heard furnishes a very typical example. Then there are reactor-physics data, which we often call integral data, and which are more generally applicable to specific reactor configurations or types. Thirdly, there are the calculations and computations made in the general area of nuclear physics, corresponding, perhaps, to the differential nuclear data. In this field models are used in direct physical experiments to extrapolate and interpolate the

data necessary for reactor computations when such data are otherwise unavailable and perhaps likely to remain unavailable. Lastly, there are the rather straightforward uses of both extrapolated data and measured data in reactor-physics calculations.

I believe all four of these areas of work have been exemplified in the papers which we have heard thus far. Messrs. Schmidt and Ferguson told us what differential data the reactor physicists in their countries felt they needed. The required degree of accuracy is a question which necessarily arises in connection with these differential data; this is a problem which we have already discussed to some extent, and one to which I can perhaps give further consideration later. The two papers presented by Dr. Smith of the Argonne Laboratory were concerned in a more general way with the manner in which such information is obtained. They referred not merely to single items but to an overall programme. The paper presented by Dr. Leroy of Saclay than dealt with a specific problem about which we have heard a great deal in these two days: namely, the current situation with regard to ν, and particularly $\bar{\nu}$. This is perhaps of more interest to fast- and intermediate-reactor physicists than the multiplicity of ν. You may remember that only a few years ago there was no information on ν (apart from thermal ν) and some exceedingly crude values obtained by rather roundabout methods at the higher energies. It is now apparent that ν will become one of the well-determined quantities—and by this I mean that 1% accuracy, which I believe is nearly the same as that obtained for thermal energies, will be obtainable for most of the fast-neutron energies also. We have seen that there is fairly general agreement with regard to the energy dependence between the work of Saclay, Aldermaston, Los Alamos and the Research Institute of National Defence in Stockholm. You will recall that the theory for predicting the energy dependence of ν is really exceedingly crude, as indeed are present theories of the fission process in general if one considers how much we know about fission experimentally; so the linear dependence as predicted by the Bohr theory will, I feel sure, hold true only if there is an accidental composition of a number of factors such that they happen to coincide perfectly. On the other hand, I am not convinced, after hearing the last two papers, that there is any variation of the energy dependence other than linear below 1 MeV. It seemed to me—and I expect all who are engaged in measurements work would agree—that there is now an almost precise cancellation between the apparent energy variations as calculated at Aldermaston and at Los Alamos. Therefore, there still remains a value for $\mathrm{d}\bar{\nu}/\mathrm{d}E$ which we can take as being constant.

Let me consider now the papers on α. I wish I had had time to study the Soviet report on α and σ_f a little more closely. At this conference I have heard of no other calculational approach to a prediction of the actual fission cross-sections from basic considerations; and the fits for both α and σ_f appeared to be remarkably good. Unfortunately, I did not see—and I expect few of us did—what the energy abscissa was; my guess is that it started at either 1 or 10 keV. My recollection is that at least the plutonium fission cross-sections showed oscillations with the theoretical fit passing almost through the middle. It is unclear to me at the moment whether these are experimental oscillations or whether they are in the nature of resonance groups.

With regard to the general problem of neutron cross-section calculations, the paper by Moldauer, presented by Dr. Spinrad, and the paper of the CNEN Institute in Italy should be kept in mind because they indicate calculations which are strictly relevant to current reactor problems. These adopted the basic cross-

section approach, however, using a nuclear model along with available experimental data.

Dr. Neiler's paper was unique in dealing with the problem of capture (of which α is, of course, only a rather narrow example). This very important contribution provides an example of the marrying of two techniques which were unknown a few years ago: namely, the time-of-flight technique and the large-scintillation-tank detection technique. This now makes it possible to do certain things with fast neutrons which, in a sense, are not possible even with neutrons in the eV or a few hundred eV range (but which will, apparently, very soon become possible).

Thus we now have our capture cross-sections; but I should add that the discrepancies in the absolute values are quite appalling in that the values obtained by different laboratories for some of the basic cross-sections (the gold capture cross-section, for example) vary in certain energy regions by very large factors. I believe this gives rise to a basic difficulty when one is faced with the problem of how to measure absolute flux or how to normalize: namely, to what kind of experiment or to what general type of measurement should one normalize the data? In this connection quite a few approaches are available in fast-neutron physics—for dealing with capture in particular. There are methods which allow normalization to a measurement by radioactive decay, or to the sphere method, (which, I believe, is used quite extensively at Oak Ridge and Los Alamos); very often the sum of capture plus fission is taken as the basic reference cross-section. In any case, this is a problem which we must often bear with for a long time—the problem of discrepancies which cannot be readily resolved—and in the meantime the reactor physicist must either rely on his own guess-work or on the advice of someone else. It is regrettable that at least two other areas are not treated in our papers; the general problem of the measurement of inelastic scattering, and the measurement of gamma rays emitted in capture and inelastic scattering. The former was touched on to some extent in the paper of A. B. Smith, but the latter has not been dealt with in any of our papers. These are matters of considerable interest, however—especially the gamma rays produced in inelastic scattering—as we are particularly concerned with fast-neutron systems.

S. Yiftah: We have heard from Mr. Hopkins that the value of the slope $d\bar{\nu}/dE$ for U^{235} in the range from thermal to 4 MeV is deduced from his measurements as 0.122. Could he say anything about the corresponding slopes for other nuclides, such as U^{233}, Pu^{239}, Pu^{241}?

J. Hopkins: Yes. The slopes $d\bar{\nu}/dE_n$ are:

U^{233}: for E_n from thermal to 4.0 MeV, 0.121 ± 0.014
Pu^{239}: for E_n from thermal to 6.28 MeV, 0.098 ± 0.005
— for E_n from thermal to 4.0 MeV, 0.158 ± 0.012

I. I. Bondarenko: I should like to make a comment on the interesting paper of Moat, Mather and Fieldhouse on determining the relation between the number of secondary neutrons and the energy of the primary neutrons. In this connection I should like to mention the experiments carried out by Smirenkin and Kuznetsov in our Institute. They measured the relative change in the number of secondary neutrons as a function of energy in the energy region from 300 keV to 1.5 MeV. A Van-de-Graaff machine was used as the neutron source. A large sample of uranium-235 was placed in the neutron beam, and the neutron detector was a scintillation counter, which recorded neutrons of energy greater than 2 MeV, i.e. only part of the full neutron spectrum. A considerable amount of statistical information was obtained in these experiments and the error was less than 1%,

though they were not of course entirely satisfactory in that not all the neutrons were recorded. The results of these experiments, I think, confirm the results presented by Dr. Moat. These experiments showed that from 300 keV to 600 keV the average number of secondary neutrons of U^{235} hardly changes; it only starts to increase rapidly after 800 keV, and from about 1.2 MeV there is a normal linear dependence. Obviously, the peculiar nature of this dependence curve may be due to the fact that discrete fission channels appear here. If that is so, then clearly increase in the number of secondary neutrons can only be observed at the moment when a new fission channel actually, indubitably opens up.

In this connection I would like to mention some other experiments by Smirenkin and Nesterov, the purpose of which was also to find fission channels in the fission process for various nuclei. They measured the energy dependence of the angular anisotropy of fission for U^{235}, Pu^{239} and U^{233} in the energy region from 100 keV to 5 MeV at intervals of approximately 200 keV. It is interesting to note that for U^{235}, with smooth anisotropy, there are irregularities in the 1-MeV range, i.e. precisely in the range where there is a sharp increase in the number of secondary neutrons. It is also of interest to note that such irregularities were observed in all three isotopes mentioned, the spacing between them being almost the same for all three isotopes, i.e. approximately 0.6—0.8 MeV. It seems reasonable to suppose that despite the fact that in this neutron-energy region there may already be a considerable number of fission channels, only some of these channels play an important part. It seems to me that all these effects deserve more detailed study. In particular they may also be seen, for example, in the sudden jump in the curve showing the kinetic energy as a function of the energy of the fission-inducing neutrons.

R. Taschek: Might I ask Dr. Bondarenko how large the samples were—since he said they were large—that were put into the scintillation tank?

I. I. Bondarenko: A cylindrical sample of U^{235} with a diameter of about 3 cm and a thickness of about 3 mm was used. For counting the number of fissions in this sample, two fission chambers were used, one of which was placed before the sample and the other after the sample. The average of the readings from these two chambers was used for determining the number of fissions in the sample, and the same experimental method was used to arrive at the effects connected with the scattering and, in certain cases, with the secondary fissions in the sample.

R. Taschek: I asked that question because if the sample is thick (even though it may be thick only in one direction) one might be concerned by the possibility that self-absorption would reduce the neutrons from fission sufficiently to produce occasional energy variations below 1 MeV.

G. C. Tavernier: Probably an answer to the question I am going to put can only be given later; all the same, I think the question should be raised now.

From 1955 on, when the first publications by D. Okrent and other authors who have done work on the subject appeared, comparatively satisfactory agreement seemed to exist between experiments made on critical assemblies and calculations made by means of cross-sections, averaged according to one method or another. Recently, however, experiments carried out at Argonne have produced variations, or more precisely catastrophic differences, between the calculated values and the experimental values. To meet the most pressing needs, the cross-section values were adjusted in such a way that I think order has now been restored. If my memory is exact, what was done was to dry and make more detailed allowance for the scattering resonances of the aluminium and stainless steel, thus

altering the transfer cross-sections. In addition, the capture cross-sections for the stainless steel were increased and finally the transport cross-sections for the sodium —which seemed to be too reactive—were also reconsidered.

Contrary to the impression that might have been gained from certain discussions yesterday, from which it might appear that the values we have now should yield satisfactory results, this situation cannot be considered satisfactory. On the contrary, it seems that the more one turns to systems where the energy is degraded, the more necessary it is to make detailed calculations and to obtain extremely accurate experimental values if good agreement is to be had.

My question is as follows: if the adjustments in cross-section values that have just been mentioned were small, it would mean that very great accuracy is in fact required in all these experimental determinations. If, on the other hand, the differences involved were very large, I should like to know on what basis the changes were made and what the explanation for these differences is.

E. Clementel: I think we could perhaps postpone the answer to your question until the next session, when we shall hear a paper on the correlation of integral experiments and high-energy cross-sections.

D. Meneghetti: I shall present the results of detailed calculations at Session 8. But, briefly, I may say that the study of resonance-scattering effects in the calculation of two ZPR-III assemblies resulted in calculated reactivity reductions of about 1% k (assembly No. 23) and about 2% k (assembly No. 32).

D. Okrent: In view of the uncertainty of 3—4% in the fission cross-section of U^{235} and the obvious discrepancies in the value of ν (E) for U^{235}, we start the calculation of any U^{235}-fuelled assembly with a 3% uncertainty in k_{eff}. Any agreement between calculation and a long list of criticals must be fortuitous, and subject to change by more accurate measurements of microscopic data.

SESSION 4

Chairman: R. D. Smith

INTRODUCTION BY THE CHAIRMAN

We now turn to a discussion of integral data. The first paper of this afternoon will in fact summarize the general state of the art and show how far we have got with our present experiments. Later papers will describe particular machines, some of the results obtained recently and some of the experimental techniques used.

I shall not attempt to review the integral data experiments, as this will be done by Dr. Loewenstein in the first paper of this afternoon; but I should like, very briefly, to try and fit all these papers into a general pattern. I should perhaps begin by asking why we do integral experiments, and then consider briefly some of the reactor types used in the experiments. In the past such experiments have had four purposes: the determination and checking of basic data, the design of nuclear-weapon systems, the determination of criticality limits for storage and processing systems, and reactor design. The requirements of the reactor designer are our main concern today, and I think it is fair to say that his interest is concentrated in dilute plutonium-fuelled fast reactors rather than intermediate reactors because of the good nuclear performance of the fast reactor and the prospect of large surpluses of plutonium as a by-product of thermal reactors. On the other hand, Dr. Spinrad has already pointed out that the tendency is for the spectra of thermal reactors to become harder and the spectra of fast reactors to become softer; thus, in discussing fast reactors, we really cover very much the same ground as we would if we set out solely to cover intermediate reactors. It is, in fact, quite difficult to define a fast reactor and an intermediate reactor; this is something which, to the best of my knowledge, no member of the Seminar has yet attempted.

In trying to meet the requirements of the designer of a fast reactor, we must consider first of all whether calculations are not sufficient in themselves without integral experiments; and we have already heard some discussion of the use of multi-group calculations. I would like to point out some of the difficulties, however, and the first of these is that we have to use a large number of energy groups because cross-sections vary rapidly with energy. The number of groups is usually ten or more in the first instance, although it is frequently possible to reduce this number for later calculations when weighted cross-sections can be used, derived from a spectrum calculated with a large number of groups. Calculations are most readily carried out on spherical systems, but unfortunately the reactor engineers very rarely build spherical reactors. Secondly, one tends to calculate systems which are fairly uniform in composition, whereas practical systems are very far from being uniform. The reactor is divided up into a large number of zones of different compositions: axial blankets, radial blankets, different zones in the core, different zones in the blanket, different coolant fractions, etc.; and even within each region the reactor tends to be far from homogeneous because of the presence of control-rod guides and various other structural parts. Therefore the basically simple problem which can readily be attacked by calculation becomes in practice very complex; probably so complex that not even the largest computers are adequate to provide a solution. Also—and this is a point which we have dwelt upon a great deal in the last day or two—we need a lot of very accurate cross-section data covering a large number of materials over a large energy range. As we have heard in the last session, however, a broad

programme for the measurement of nuclear data is under way throughout the world and the data available to us are now improving rapidly. In fact I suspect that the designers' demands for data will prove to be insatiable: no sooner will the nuclear physicist measure something to 5% than we shall want to know it to 1%. Therefore, although in theory all the necessary information can be calculated, one concludes, because of the complexity of the calculations and the inadequacy of the data, that some form of integral experiments will become necessary. These data are necessary for two distinct reasons. In the first place, one must have the information needed for the design of a reactor well before construction is to begin. After the reactor has actually been built and loaded to criticality, further physics information is needed for its efficient operation. This information can be obtained partly from experiments on the reactor itself and partly from integral experiments; and the proportion in which these two methods are applied depends on a combination of technical feasibility and economics. On the one hand, access to a power reactor is very difficult, and it is hard to get measuring instruments or detectors into the desired parts of the core and blanket. The presence of the liquid-metal coolant produces further hazards and difficulties. Secondly a fast reactor is a very expensive device, and the cost of using it for long periods of time for physics experiments is very high. Such a cost is justified only if the experiments can be done in no other way (and the other way would, of course, be a zero-power assembly of some sort). On the other hand, one cannot heat the present zero-power assemblies; and since the temperature coefficients of fast reactors depend in any case very largely on structural movements, it would not improve the position even if one could heat them. Therefore, many temperature effects will have to be measured directly on the power reactor.

We may conclude, then, that integral experiments are needed in conjunction with theory for the design of reactors. Besides, one can gain the greatest possible experience from a zero-power reactor or from integral experiments, including quite probably a close mock-up of the actual power reactor. In this way one is left with a minimum of experiments to be carried out on the reactor itself.

I should like to consider some of the types of experimental machines which are used for these experiments. Firstly, one must refer to the sub-critical assemblies. There have been a good many fast exponential experiments done, notably in France and the United States. In these experiments a sub-critical assembly was supplied with neutrons from either a fast reactor or a thermal reactor. Although early experiments were somewhat misleading, later ones have shown that, while some useful results can be obtained from this type of experiment, it is, on the whole, very much less successful than experiments with actual critical systems.

Let us move on, therefore, to critical assemblies. Some of these, such as BR-I in the USSR or ZEPHYR in the United Kingdom were relatively inflexible in that the core size and composition could not readily be changed. In fact, however, there is still a use for this sort of assembly as a particular model of a particular reactor. At present, however, the tendency is to make the reactor core as flexible as possible, and in the majority of these machines the fuel and diluent material is in small and discrete pieces which are assembled in fuel tubes or drawers to form the fuel elements.

It is interesting to classify these machines, as there are now quite a number of them in existence or in the planning stage. They can be divided variously

into those with horizontal fuel elements and those with vertical fuel elements, or into those which can be separated into two halves to give shut-down and those which cannot be separated, and in which the required shut-down is obtained simply by moving a sufficiently large number of control rods. A good example of a zero-power fast reactor is the ZPR-III machine at Idaho Falls, which has now been in successful operation for many years. In this machine the core and reflector are assembled from a large number of 5-cm^2 plates. These are loaded into small drawers which slide horizontally into the two halves of a honey-comb structure. The two halves are placed on two tables which can be moved apart. We in the United Kingdom have devoted considerable thought to the relative advantages of this type of structure and a type such as our ZEUS, another zero-power reactor. As a result, our latest two reactors have two different arrangements. Of these two reactors (VERA and ZEBRA), VERA has vertical fuel tubes like ZEUS but is on movable tables like ZPR-III, and ZEBRA, which is now being built at Winfrith, will be more like ZEUS. The basic components of ZEBRA are also 5-cm^2 plates of various thicknesses, ranging from 1.5 mm to several centimetres. However, the plates are assembled into square steel tubes which are stacked vertically in the core, and the core cannot be split into two halves. Control is ensured by two independent sets of control mechanisms, one operating from the top of the reactor and one from the bottom. A major problem in designing a zero-power reactor of this type is the assembly of the very large number of core components into the fuel element. In the United Kingdom we are developing automatic machines to load and unload the components of ZEBRA.

Of course, the usefulness and accuracy of the integral experiments which can be carried out on these machines are limited. For experiments designed to check or extend our knowledge of nuclear data it is usually an advantage to have the core completely uniform. In fact, of course, this is not possible; even components of the smallest practical size may give appreciable flux variations within the dimensions of a plate. This is true even in a fast spectrum, and the position naturally becomes very much worse as one moves towards an intermediate-reactor mock-up. Of course, the power reactors themselves do have a very marked structure; unfortunately the exact geometry of the fuel elements is different from the zero-power structure and cannot normally be mocked-up with absolute precision in a zero-power reactor. Thus a two-stage correction has to be made: first one must allow for the inhomogeneity of the zero-power reactor and then for the effect of the different inhomogeneity of the power reactor itself. Fortunately, it seems that these effects are relatively small, at least in reactors where they have been investigated in detail. Another point is that although one may try to reduce the structure of a zero-power reactor to a minimum, there is in fact always some structural material left—usually 5 to 10% of the volume of the reactor—and this must form part of any assembly one makes. Thirdly, the variety of geometrical shapes which one can assemble is limited by the unit cell structure of the machine: thus it is difficult to assemble a sphere, for example, in ZPR-III or in ZEBRA, although an approximate sphere can be built. Another problem which I have already mentioned is that temperature effects are virtually impossible to simulate in a zero-power reactor, although it is possible to do some experiments on those effects which depend on nuclear properties. Here, of course, I am thinking of the Doppler effect. There is a further major difficulty, which arises from the almost universal use of sodium as a coolant

in fast reactors. It is very inconvenient to have liquid sodium present in the zero-power reactor, and in the past many measurements have been made by simulating the sodium coolant with aluminium plates. However, experiments on ZPR-III in which canned sodium was used and the effect with aluminium was compared, show that this procedure is not altogether satisfactory. This difficulty continues to concern the physicists who are working on these reactors.

Despite these difficulties zero-power reactors have in the past proved themselves to be extremely flexible machines. As for the future, it is possible that the change which will have the most far-reaching effect on the experimental programmes of these machines is the demand for ever-increasing accuracy in our results. Therefore, rather than simply measuring the critical size of a range of assemblies, we shall probably be more concerned with detailed measurements of the behaviour of these systems. Because of the present tendency to build larger and more dilute fast reactors, interest has shifted to the lower-energy end of the spectrum, so that the distinction I mentioned between fast and intermediate reactors has become less well-defined. This immediately increases the emphasis on Doppler effects, fine-structure effects, self-screening, resonance effects and various other complications about which we have already heard a good deal in this Seminar. In the past we have been able largely to ignore many of these effects; but in order to get the maximum information from our experiments it will be necessary to measure their properties in detail. The most useful measurement, in my opinion, would in fact be a direct measurement of the neutron spectrum. Unfortunately, we are only now developing semi-conductor devices which would have been very useful had they been developed five or six years ago. These look as though they will be useful only in the energy range above 200 keV or thereabouts, and the main region of interest is rapidly shifting to energies well below this level. However, I do think that this is a field in which a considerable effort should be made, and that a very great advance could perhaps be made if a spectrometer covering the range of interest were developed.

If one looks even further into the future, it is interesting to speculate on the extent to which measurements of the type I have been describing (and about which we shall hear more this afternoon) will eventually be supplanted by theoretical calculations with improved data. It seems to me that some characteristics of reactors will probably not be at all amenable to calculation in the foreseeable future, whilst others may involve a great deal of time and effort in the preparation of programmes and use up a very large amount of running time on the largest computing machines available. Therefore, even though zero-power reactors may be very expensive to operate—largely because of the cost of the fissile material fed up in them—it seems likely that, for some measurements at least, the integral experiment will always remain the cheaper and better method of obtaining the required information. Thus I think it is likely that we shall see at least some zero-power reactors running with full programmes for many years to come.

Paper SM-18/77 was presented by W. B. Loewenstein (see Vol. I, p. 415).

Discussion

D. Okrent: Firstly, I should like to ask, for the benefit of us all, when the second edition of Reactor Physics Constants will be available. Secondly, in your comparison of the difference in spectral indices between small and large

systems, I believe you omitted to mention the factor of geography. The small systems were measured primarily at Los Alamos whereas the large ones were done on ZPR-III. The Np^{237} detector was used at Los Alamos and the U^{234} detector on ZPR-III. This should be a significant factor.

W. B. Loewenstein: I implied that it was a significant factor when I mentioned that Gordon Hansen would report some slightly different results. As to when the constants compilation will appear, I should like to refer that question to Dr. Spinrad.

B. I. Spinrad: It will appear as soon as I manage to edit the last chapter; the rest of it is finished.

G. I. Marchuk: I would like to ask Mr. Loewenstein whether systematic calculations were carried out on the enriched material for critical assemblies which he referred to in his paper.

W. B. Loewenstein: The various systems that were shown have been extensively analysed by a number of people. I did not wish to imply that I had done such analyses myself; I merely wanted to report the data as they exist. There will be several papers—some of them this afternoon—pertaining to the analysis of these systems. For example, Dr. Hansen will present a paper on the analysis of the small systems, Dr. Battat on the analysis of the LAMPRE and Dr. Meneghetti on the analysis of the ZPR-III results. There is a wealth of published information on various aspects of these analyses, and by singling out a few names I had no intention of implying that these were the only people to have done such work.

Paper SM-18/51 was presented by J. Chernick (see Vol. **I**, p. 436).

Discussion

D. I. Spinrad: I am puzzled by the remark that the lack of equilibration of the spectrum in these Snell experiments would lead the experimenter to overestimate the relaxation length; it seems to me that he would underestimate it.

J. Chernick: We have done some calculations which involved putting a fast source under a Snell array. The results show an increase in relaxation length for about the first 10 cm, then a decrease which continues for some time. Similarly, the value of the ratio of U^{238} to U^{235} fissions approached equilibrium faster than did the relaxation length; however, this does not seem to be so serious. The percentage changes in the value of Φ^1/Φ are much greater than in this fission ratio.

B. I. Spinrad: Did I understand you to say that the apparent slope of the decay curve crosses the asymptotic value twice?

J. Chernick: Yes. These are the results of multi-group calculations, and crossing of this type apparently occurs in two groups; but we are giving the problem further study.

D. B. Hall: Could you estimate the effective neutron energy representative of the scattering-to-fission ratio as shown on your slide where the minimum values of η occurred—i.e. $\sim\Sigma_s/N_{233}\approx$ 100—200?

J. Chernick: It is in a fairly high intermediate-energy range. I think it would be serious only in very dilute fast reactors or very concentrated thermal reactors.

G. I. Marchuk: I should like to ask Professor Chernick his opinion on the following questions.

To what basic physical parameters relating to reactors with every conceivable type of spectrum did the system of constants which has just been described conform?

Secondly, in this system of constants, to what extent has account been taken of the resonance structure of the cross-sections? For example, could one use this system of constants for calculations on uranium oxide reactors by adding the constants for uranium and for oxygen?

J. Chernick: A detailed answer to your question can be found in our paper in *Nuclear Science and Engineering*, and it would take quite a while to give you that answer in full. In brief, however, we integrated over the actual resonances; there were no corrections for self-shielding; and the basic variable against which $\bar{\eta}$ was plotted was the slowing-down power of the moderator per uranium atom. This was the only parameter, and the curves would therefore require some modification for uranium oxide reactors.

P. Greebler: Your ability to predict the CA-18 experiment indicates good precision in the (n, 2n) cross-section of beryllium. As this was an experiment utilizing clean beryllium, a fairly appreciable error in the (n, α) cross-section would not have a large effect on the calculation. After the reactor has burned to some extent, however, the effect of the (n, α) cross-section becomes much more important because of the build-up of Li^6 and He^3. How good is the precision in the *ratio* of (n, 2n) to (n, α) cross-sections (important to reactivity in an irradiated system) relative to that of the *difference* between these cross-sections (important to reactivity in a clean system)?

J. Chernick: One must certainly know both the (n, 2n) cross-sections and the (n, α) cross-sections well. The (n, 2n) reactions give a plus effect of about 11% in pure beryllium and the (n, α) reactions about 4%; thus it is really the difference that gives 7%. I think the (n, α) cross-sections have been generally accepted for some years and are not at all in dispute. With regard to the (n, 2n) cross-sections, I think everyone is pleased that Cranberg has done these measurements again and that there is now such good agreement with Fisher's data. It is true that we got approximately the same results from both sets of data; however, they did cross, especially at low energies where this particular cross-section is very important. There are still fairly serious errors in the (n, 2n) cross-section near the threshold of the reaction. This is unfortunate; it is the reason why I think that the critical experiments which I mentioned would be extremely useful, as they do already seem to verify that the effect is approximately right. I admit there are some very difficult theoretical problems that we have not yet solved. I mentioned the resonance self-shielding factor which apparently—at least from the group viewpoint—alters reactivity by about 1%. Many other effects are mentioned in the paper, and I shall not enumerate them again. Thus there are some uncertainties, but I do think the position is much better since we included the (n, 2n) reaction in the analysis of this experiment.

W. Häfele: My question is closely related to Mr. Greebler's. How can one assess the difficulties associated with Li^6 poisoning following the (n, α) process in Be^9?

J. Chernick: Certainly it is possible that the helium and lithium will come off in some high-temperature systems. It is a fact that we must begin to consider specific processing systems. In any case, the 950-b cross-section of lithium is comparable to fission cross-sections at thermal energies; therefore, lithium poisoning would become a serious problem only if the fuel were taken to very high burn-up.

P. Greebler: Has any thought been given to a critical experiment on irradiated beryllium with a view to solving the problem of lithium build-up?

J. Chernick: It is really too early for that. We should obtain more information about unirradiated beryllium first; as I mentioned, we do not know the age or the absorption cross-sections very well. Our experiments have been rather rough thus far, and it is very unsatisfactory to do a calculation if one does not know the basic data for the assembly before putting the fuel into it.

D. Meneghetti: To what extent have you considered anisotropic scattering in your calculation of the (n, 2n) effects in beryllium at high energies?

J. Chernick: The relevant data can all be found in the *Physical Review* and there are compilations by Goldstein and by the Aldermaston group. The agreement is quite good, and I do not think we could have made any serious error. The data which we used may not be perfectly correct, but they are the best available. There is, for this particular assembly, a 4% difference in k as one goes from anisotropic to completely isotropic scattering. Surely we can make no very serious error in this range if we use the actual known angular distributions for beryllium.

Paper SM-18/55 was presented by G. E. Hansen (see Vol. I, p. 111).

Discussion

W. B. Loewenstein: By and large I agree with Dr. Hansen: the discrepancies to which I referred do not really apply to these systems. It has long been known that the agreement between theory and experiment is rather good for these small, high-density systems. For the larger, more dilute systems, however, the agreement between theory and experiment is still not altogether satisfactory. I am thinking, firstly, of the ratio of capture in U^{238} to fission in U^{235}. This is a very important constant, and our measurements still ten dto be smaller than our predictions. Secondly, I am thinking of the ratio of fission in U^{238} to fission in U^{235}. This is very well predicted in Dr. Hansen's system, but for the more dilute systems the best accuracy one can achieve is somewhere between 5 and 10%. Lastly, I am thinking of calculated vs. measured neutron lifetimes; for the more dilute systems the predicted lifetimes appear to be much shorter than the measured neutron lifetimes.

G. E. Hansen: I assume you are referring only to U^{235} systems.

W. B. Loewenstein: Yes. Some dilute plutonium-fuelled systems have been reported, but these are fairly small. To the best of my knowledge I have never seen a Rossi-α reported for dilute plutonium systems—not even in the 40- to 50-kg weight range.

G. E. Hansen: I should like to add a few comments regarding Pu^{240}, as Dr. Loewenstein promised I would do so. Some gross material-replacement measurements with alpha-phase plutonium balls of about 1.6-kg mass were made recently at Los Alamos. Three different alpha-phase plutonium balls used in critical-mass measurements were made critical by being surrounded with U^{235} metal. The three plutonium balls had different Pu^{240} concentrations, ranging from a few per cent to about 15% Pu^{240} content. We determined the variation in the amount of U^{235} which must be added to compensate the degraded worth of the plutonium fuel attributable to this Pu^{240} content; and it was then possible, experimentally, to assign an equivalent between Pu^{240} and Pu^{239} (e.g. approximately 0.6 g Pu^{239} would require a full gram of Pu^{240} to compensate the

reactivity loss). This is admittedly a very spectral-sensitive ratio. In order to get more lasting results we interpreted the critical-mass data in terms of an effective $\bar{\nu}$ for the number of neutrons per fission from Pu^{240}; that is, we assumed we knew what the fission cross-section was, essentially what the capture cross-section was (small) and obtained in this way a $\bar{\nu}$ of approximately 3.2 neutrons per fission.

J. J. Schmidt: Did you find any indication of appreciable sub-threshold fission in Pu^{240}?

G. E. Hansen: No. We did not look for sub-threshold fission.

S. Yiftah: Is the cross-section set called here "set x" the same as the one which you reported in your 1958 Geneva paper, or have changes been introduced since then?

G. E. Hansen: It is identical; that is why there was such a large discrepancy between observed and computed detector ratios.

Paper SM-18/19 was presented by J. W. Weale (see Vol. I, p. 253).

Discussion

R. D. Smith: Have you calculated the VERA system using the adjusted set of constants with a lower n, V and the different ν values, in order to see whether you do in fact get good agreement?

J. Weale: No, we have not yet calculated the VERA system with the data of O-28/60 adjusted as described by Pendlebury, but we shall do so shortly.

G. I. Marchuk: Allow me to speak again now although, basically, the question which interested me has been dealt with more fully in the papers that are to be discussed next. I would like, however, to dwell on a point closely connected with the question under discussion. What procedures, what methods of averaging physical constants can we use, in order to obtain a fundamental system of constants with the help of which further mathematical processing of nuclear-physics experiments on reactors can be carried out. It is clear that, at the present time, there are in existence a number of more or less completely worked-out systems of constants, based on various physical assemblies.

Manifestly, it will be pretty difficult to draw up a universal system of constants, seeing that each system is based on a particular reactor, with a particular spectrum and a particular set of nuclear physics characteristics. It is natural, therefore, that none of the proposed systems of constants can be satisfactorily used for all reactors; obviously they can be used only for those with nuclear physics characteristics very similar to those on which the system in question was based. The situation improves when we have to do, not with one set of characteristics nor with one critical assembly, but with a set of differing critical assemblies, i.e. assemblies which differ radically from one another in their spectra. It is clear that at a certain stage we encounter the necessity for a very wide programme of work on all these results; and to obtain an agreed system of constants for reactors of all possible spectra will be extremely difficult.

There is another way to calculate these group effects. It is based, as was shown at our previous sessions and as appears in the literature, on obtaining a multi-group system of constants, having a very large number of groups, which by their characteristics asymptotically approach the available differential cross-sections. Obtaining differential cross-sections is, at present, very difficult for many reasons, most of which have already been referred to at previous sessions. What

we are faced with, therefore, is a series of systems of physical constants derived, on the one hand, from macroscopic experiments and, on the other, from the generally contradictory data obtained from elementary experiments.

The question arises, how should we proceed in future in attempting to obtain a system of constants which could be used for a reactor with a wide range of spectra. Clearly (and in the Soviet Union the relevant processing work has now begun) we must start by constructing a system of constants with a significantly larger number of groups: for example, with a lethargy spacing of, say, 0.25, to begin with, or 0.125 later. It will clearly be very difficult to construct such a system of constants, and if we use elementary constants we shall not succeed, for many elements, in obtaining full agreement with the group constants which have already been obtained from various integral experiments with critical assemblies. In our view, however, it is absolutely necessary to develop the work along these lines in the future; because, if we can get agreement between the elementary constants (though in a certain sense they are also approximate constants — not simply elementary, but multi-group constants with a large number of groups) and experimental data, then we shall be able to develop appropriate theoretical methods of mathematical reactor calculations which can increase our accuracy in the basic integral and differential characteristics of reactor spectra. Here, naturally, a special role will be played by all conceivable kinds of resonance effects, which will clearly become (indeed are already) the object of numerous calculations. We have attempted the construction of multi-group systems of reactor equations, which take into account the main peculiarities of each particular reactor's neutron spectrum and adjoint function. These constants, this algorithm, will, we think, enable us in each specific case to obtain a system of constants capable in practice of yielding concrete results. The relevant mathematical processing work has been done and the preliminary calculations are being made.

W. Häfele: We feel that one has to do more than produce one, or even a finite number of multi-group sets using only a fixed flux-weighting. In producing group constants one is confronted by two types of cross-section variation: rapid variations, mostly connected with elastic-scattering processes, and rather slowly varying cross-sections, mostly connected with absorption and fission processes. We are concerned here only with the case of slowly varying cross-sections.

A group constant is defined as follows:

$$\sigma^{(i)} = \frac{\int_{(i)} \sigma \Phi \, \mathrm{d}u}{\int_{(i)} \Phi \, \mathrm{d}u} \quad \text{where } u = \text{lethargy.}$$

If one is faced with the task of investigating a large number of reactors which are not necessarily all of the same type, one must expect changes in the flux behaviour within one group as a consequence of the various dilutions and compositions of the reactors in question. One way of overcoming this difficulty is to enlarge the number of groups, but this method has natural limits. These limits are set, on the one hand, by the storage capacity of the available computer and, on the other hand, by the particular difficulties which arise in cases where more than one medium is present—for example, where the reactor configuration in question is a core and blanket. Of course, this difficulty appears primarily in cases where diffusion theory is applied.

We now proceed, putting for Φ in each group:

$$\Phi^{(i)} = a_0^{(i)} + a_1^{(i)} \cdot u + a_2^{(i)} \cdot u_2 + \ldots\ldots$$

(and still higher terms if desired). In each group a spectrum is assumed in the first step, and the relevant multi-group problem is solved. Before this is done, the following quantities are produced once and for all:

$$\sigma_{(n)}^{(i)} = \int_{(i)} \sigma \cdot u^n \, \mathrm{d}u.$$

These form a library.

The first run results in averages $\Phi^{(i)}$ in each group. One can now determine the values of $a_0^{(i)}, a_1^{(i)}, a_2^{(i)}$ — for example, in the following way:

(1) The average in the i^{th} group $\Phi^{(i)}$ shall be properly described by the $a^{(i)}$ set.

(2) The averages in the neighbouring groups $i-1$ and $i+1$, $\Phi^{(i-1)}$ and $\Phi^{(i+1)}$ shall also be described by the $a^{(i)}$ set.

The results are parabolae for the flux as a function of u within each group. However, the following should be noted: the parabolae $a^{(i)}$ describe the flux in the group i only, although the neighbouring groups were used for their determination.

One could use different conditions to determine the quantities $a^{(i)}$; for example, a connection at the boundary of each group could form the condition. However, it is our experience that this is an artificial condition. The best values for reaction rates are what one really wants — not smoothness of flux distribution. There are still other possibilities for determining the values (i), and these are described in a paper by Ott and Meets which will be published in *Nukleonik* by Springer Verlag in the near future.

The procedure continues as follows: the multi-group calculator is run a second time (the $\sigma^{(i)}$ values being taken from the $a^{(i)}$ values determined in the manner described above) and the $a^{(i)}$ values are determined once more. This iteration process is continued until convergence is achieved. It is our experience that, in spite of the fact that the steady connection at the group boundaries is not postulated, the flux shows an almost smooth overall behaviour specially at lower energies.

When one considers the procedure described here, one gets the impression that the calculation time would be longer than usual. This is true only to a certain extent, because every multi-group code has an inner and an outer cycle if criticality is to be achieved. One is able to correct the iterations of the outer cycle with the iteration of the $a^{(i)}$ set. It is our experience, therefore, that the calculation time is increased in most cases by a factor of about 1.5; for example, eight minutes are required on an IBM-704 computer to do an 11-group problem; but this code produces also the adjoint fluxes, which are used to accelerate the convergence.

We have used this method quite extensively in connection with a survey of possible reactor systems; about 300 11-group calculations were made. The influence of the proper weighting in each group described here gives differences of up to 20% in critical mass.

If one compares this procedure fairly with regular N-group or even $1.5\times$ N-group calculations, one must admit that this code produces the best group constants (in the sense described above) for each individual assembly.

The work described here was performed by Mr. Stittgen of the Kernforschungszentrum Karlsruhe.

G. E. Hansen: I should like to comment on the multi-group cross-section sets which it seems we shall be forced to adopt. The interest in intermediate systems with a corresponding resonance structure in the cross-sections has brought it about that we really cannot use linear mixing to obtain microscopic cross-sections with few-group treatments. Nevertheless, in the short run — or so it seems to me, at least — we are more or less bound to use mixing of cross-sections until we understand a little more about the details of resonance structure.

And now we must ask ourselves what would be an appropriate lethargy spacing for our eventual universal set; whether, for example, $^1/_8$ of a lethargy unit would be fine enough is very difficult to say at the present time. It can be said with certainty, however, that in order to obtain a universal set of multi-group cross-sections for the individual elements, the group structure must be very fine indeed—that is, if we are to be able to compute Doppler effects and resonance self-shielding automatically.

A. Kania: We should simply like to say that in our exponential experiment we have already used the weighting method just outlined by our German colleague—except that we reduced it to the first term, that is to say, represented the flux by a linear approximation. I should also like to say that in investigating the cross-sections we too had come to the conclusion that in calculating flux transmission a very large number of groups should be used to begin with, and we succeeded in establishing a group-structure with a lethargy unit of 0.25. So we fall back approximately on the value indicated by Dr. Marchuk. These 50-group cross-section studies are still in process of being worked out.

B. I. Spinrad: Multi-group theory means, after all, two different things. On the one hand it is used to describe the type of problem which one encounters in calculating leakage from small systems. This is not the general problem, however, except for some specialized systems such as Dr. Hansen has described. The other reason for using multi-group systems is to get spectral averages over rather large regions for which the leakage is small and generally constant. Under these circumstances the very fine group structure becomes simply a good method of presenting data. One actually uses the multi-group set to determine the slowing down spectrum or some equivalent as a function of energy. The problem is quite analogous to that which we encounter in thermal reactors, when we calculate the thermal neutron spectrum in the presence of heavy absorber; mathematically it is almost identical. I think we can see that there is a tendency to use a large number of groups in dealing with this restricted problem as well.

This does not mean, however, that there is not a great deal of regularity in the spectra which one derives from the large multi-group treatment. For example, it has been observed that it is the elastic moderater which renders the very fine group structure necessary at the low-energy end. And in the presence of rather heavy elastic moderation, one can make use of the continuous slowing-down model, which, I submit, is in general a good approximation and reduces a large number of algebraic manipulations to analytical formulation.

J. Chernick: I should like to comment briefly on Dr. Marchuk's suggestion. I certainly think that the fine multi-group approach has many uses, but I do not believe that it can be applied to all problems.

G. I. Marchuk: I completely agree with Professor Chernick. In raising this question for discussion, I intended to draw the attention of my colleagues to

the need to find new ways of developing fast reactor theory, which should be suited to the stage we have now reached. Obviously I do not hope that this method will prove a universal one: undoubtedly, for many other calculations it will be necessary to employ different means, using other algorithms.

A. Campise: In my opinion there are two aspects of the problem of choosing a universal group structure which we have not yet considered. One of these concerns the purpose of the universal set. Shall we use it to calculate criticality, to perform integral experiments, or to consider detailed flux distributions through a heterogeneous system? For what purpose is it needed in fact? Our principle at Atomics International is that, from the standpoint of design, there are certain integrals which one must be able to calculate in order to describe the reaction rates in a given heterogeneous lattice. We believe that the integrals which interest us can be obtained from integral experiments conducted in different energy spectra and interpreted into the integral cross-sections. In my opinion this process is more than sufficient for designing a power system. I can see the need for a detailed cross-section structure only if one is faced with the task of calculating the Doppler coefficient. In that case each resonance must be studied and its effect included in the calculation of the coefficient.

C. P. Zaleski: I think that in certain cases, for example the case of the exponential experiment with natural uranium, where the flux develops very fast and the spectrum changes considerably with it, it is useful to adopt a very fine group structure in multi-group calculations; otherwise the cross-sections would have to be re-weighted and altered again at each point. We made a number of trials with this in view and discovered that a very fine group structure gave distinctly better agreement with the experimental values than ten-group or five-group divisions.

J. J. Schmidt: Both today and yesterday we have discussed the problem of resonance structure in the medium-weight nuclei and even in the nucleus of U^{238}. I should merely like to emphasize that the method which Dr. Häfele has just described provides an easy way of obtaining these integrals by integrating over E times the Breit-Wigner formula; this is very easily done, once and for all. This method may be useful, for example, in spectral calculations with respect to Doppler coefficient weighting and so forth.

I. I. Bondarenko: I should also like to say a few words on the problems connected with choosing a particular number of groups for reactor calculations. Strictly speaking, we are dealing here with two different sets of circumstances. In the one case there are smooth changes in cross-sections, depending on the energy. The number of groups required in order to take these changes into account is not very large; larger perhaps than is used today, but at any rate of manageable size. For example, as Dr. Marchuk said, it is possible to divide each region into intervals of 0.25 by the appropriate lethargy transformations. That would probably be sufficient to solve all problems apart from those linked with specific resonance effects.

In their case it is clear that we cannot manage with a simple increase in the number of groups. Even if there is still some hope that such a simple increase would enable us to describe the resonance effects in the very light elements such as oxygen, such a method would clearly be inappropriate for elements of medium and heavy atomic weight; for these elements, quite different methods are necessary. Here there are many different effects. One method was briefly mentioned in my paper on the measurement of resonance parameters charac-

terizing the resonance structure*. A large number of effects may be linked with resonance effects, as, for example, the very interesting effects which occur on the boundary between two media. If, shall we say, you have an uranium reflector in a reactor, then, behind that, a copper reflector, then a steel reflector, at the boundaries the density of the reaction undergoes a peculiar effect which is linked with the presence of resonances. To describe these we must work out new methods; it is not enough merely to increase the number of groups.

Thus I wished to point out that we are here confronted with two problems and two sets of circumstances. Firstly, it is apparent that reactor calculation theory will soon reach a large number of groups — for example 50 groups — and that will be sufficient for a large variety of tasks; but in addition it will be necessary to work out methods for calculating the resonance structure, and this will not involve a further increase in the number of groups.

B. I. Spinrad: As Dr. Bondarenko said, a relatively small number of groups is sufficient for correct criticality production, provided suitable cross-section averages are available in the groups. I should also like to suggest that a statistical treatment of resonances (i.e. as randomly located cross-section peaks and valleys according to a known law) is likely to prove useful over a wider energy range than is now conceded. It seems that such a treatment is already required in the energy range above 10 keV for all but the light elements; I predict that it will work at lower energies and for lighter elements than we now expect.

* SM—18/85, see Vol. **I**, p. 65.

SESSION 5

Chairman: V. Raievski

INTRODUCTION BY THE CHAIRMAN

The previous meetings, at which we discussed neutron-physics data, have shown the number of experiments and their accuracy in the intermediate energy region to be insufficient. Again, though experimental results obtained with very concentrated assemblies seem to confirm the validity of the computing methods and sets of nuclear constants employed, the agreement for dilute systems does not appear to be satisfactory. In power reaction, where heat-exchange surfaces and the volume of coolants and structural materials are increased, dilution is inevitable, thus bringing about a degradation of the spectrum. The use of ceramic elements will produce an identical effect.

Whatever effort may be made with a view to avoiding spectrum degradation, it seems inevitable that a not inconsiderable part of the spectrum concerned will extend to the intermediate energy region, thus changing the reactor's static and kinetic behaviour.

Again, as Dr. Spinrad has pointed out on several occasions, there is a hardening of the spectrum in thermal-neutron power reactors, owing to the high temperatures obtained and the high rates of burn-up required; and that being so, the study of exponential critical assemblies operating in the intermediate energy region is clearly becoming of definite practical interest.

I should also like to express my opinion on the subject of the future of critical experiments and on the development of a method of calculation and a set of universal constants such as Dr. Marchuk outlined at yesterday's meeting, with, be it said, considerable caution and many reservations.

The experiments at present being carried out in the domain of fast or intermediate reactors are basic experiments. We represent the composition of the core by setting pieces of different materials side by side, without, however, reproducing the actual geometry of the reactor.

I think that when problems of economic competitiveness arise in the field of fast reactors we shall be led to build more and more accurate models and to elaborate more and more detailed theories, relating at the same time more specifically to the particular power reactor concerned. That is the direction being taken by thermal-reactor physics, though it is much more advanced. However, that stage is still a long way away, as we can see from the fact that when someone at our first meeting spoke about a 20% deviation in the breeding-factor, for instance, no one seemed shocked—quite the contrary.

Paper SM-18/52 was presented by M. E. Battat (see Vol. I, p. 263).

No discussion

Paper SM-18/48 was presented by J. K. Long (see Vol. I, p. 271).

Discussion

V. Raievski: We thank Mr. Long for the very interesting account he has just given. He has described for us what is in fact a considerable number of experiments with dilute systems. I think there is material there for trying out Dr. Spinrad's suggestion of adapting a very simple method of calculation with two-neutron groups. There are some extremely interesting points involved,

particularly the fact that the cross-sections for U^{235}, U^{238} and stainless steel probably should be changed, to allow verification of the experiments concerned. There is also the problem of heterogeneity which will certainly arise as soon as accurate forecasts on power reactors are wanted.

A. Campise: Could Mr. Long tell us what the units of his reactivity coefficients were ?

J. K. Long: They have been reduced to Σ_{eff} in millibarns; this was done by calculating the value for plutonium and comparing them with that. I believe that this is the best way to present data for a series of different assemblies, because one must cancel out the volume effects which would obscure the difference in the reactivity effects. This is a procedure outlined in the British reports on ZEUS and also in the reports from Los Alamos which appeared in ANS of last December. What was undertaken, I believe, was a reduction of the reactivity coefficients based on a comparison with a calculated value for plutonium.

G. C. Tavernier: I should simply like to mention that, according to some of the "Progress Reports" put out by the Argonne laboratory a few months ago, discrepancies in critical mass of the order of at least 100 kg had been encountered. The future user of the data might find such discrepancies rather considerable, even if the deviations are slight when translated into k_{eff}. I should be grateful if Mr. Long would comment on this point.

J. K. Long: Yes, this was true in some cases. Although k differes from 1 by only 5% in those two assemblies, this difference, when translated into an addition of mass around the edge of the reactor, amounts to between 20 and 25% of the mass of the reactor—in this case at least 50 kg; so the differences in mass are actually larger than the differences which appear here. In the case of the oxide reactors, the differences in mass sometimes appeared to be even larger than the ones listed here. It is true, therefore, that our preliminary estimates differed by about 100 kg. The picture is not as black as it might seem, however; these were only preliminary estimates, after all, and it may be possible to achieve a more satisfactory agreement now.

G. C. Tavernier: I have two questions. Firstly, we were told yesterday that the cross-sections of several elements had been re-adjusted as a result of these observations. I would mention again, as Mr. Long did just now, an increase in the capture cross-section for stainless-steel, a change in the transport cross-section for sodium, and a change in the elastic-transfer cross-section for aluminium and stainless steel. I should like to know whether these changes will be incorporated in the new tables promised us (in a re-issue of the report ANL-5800, I believe).

I should also like to know whether these changes will result in errors when they are applied to the older systems—errors which do not appear in the paper as now presented. In other words, will the changes in constants, which are now being introduced to correct the results of calculations on the latest assemblies, give the same good agreement we have had in the past when they are applied to the older assemblies ?

J. K. Long: I cannot answer your question in full because we do not know what results we will get when the various adjustments are made, and consequently we do not know what will be worthwhile to republish at that time. I intend to present the paper in its present form for the moment, and I have tried to indicate in each case what cross-section set was used. As cross-section sets are improved it will become apparent whether these values need to be recalculated. I think

there are others in a better position to tell us what recalculations have already been done.

J. Weale: The set of data to which I referred yesterday, which was published as AWRE 0-28/60, was adjusted in $\bar{\nu}$ to give good results for fast concentrated systems of U^{235} and Pu^{239}. It also gives fairly good agreement for the critical mass of the VERA assembly which I described yesterday, consisting of 20% U^{235} and 80% graphite by volume. This agreement is possible, I think, because there is no U^{238} present in that core; if the set of data in question were used to calculate a similar VERA assembly containing U^{238}, I believe the critical mass might be considerably in error.

D. Okrent: We must try to maintain some perspective. The original purpose of the Yiftah-Okrent-Moldauer study was to develop a set of cross-sections without making any attempt to adjust them to critical experiments. In the past, a number of people have demonstrated that they can fit a selected series of experiments, and Gordon Hansen has given us an excellent example of how one can fit the small concentrated criticals at Los Alamos. At the Geneva Conference in 1958, the Argonne laboratory presented a set of constants which fit all their medium-sized assemblies. But all of us who have done this are very conscious of the fact that we always adjusted our results to achieve the fit. Part of our purpose, in compiling the Yiftah-Okrent-Moldauer set, was to see how well the cross-sections would fit if they were *not* deliberately altered. Indeed, we know in advance that they would not fit extremely well, because we ourselves had had experience of adjusting data. But we thought our work would at least provide basic information for any given material; one could then either make adjustments or allow for new information. Thus if Mr. Long makes some calculations, using this cross-section set, before running a series of critical experiments on ZPR-III, we do not expect that he will get good agreement. Indeed, at the end of our study we show that we get bad agreement; we universally underpredicted the critical mass of all assemblies we calculated.

I should add that we showed one way of adjusting cross-sections which involved reducing $d\nu/dE$ for U^{235} from the value which we had assumed; happily, the new data indicate that this is perhaps more than an arbitrary adjustment, and that it will lead to good agreement. I feel that we should not be discouraged if our data result in a discrepancy of 100 kg in critical mass. If we wish to make the best possible predictions for any specific new assembly before constructing, it we can undoubtedly improve our predictions. In fact, the very first critical assembly built at ZPR-III agreed within 6 kg (less than 1% k). Therefore, if one adjusts the cross-sections they can be made to fit very well. Mr. Nims of Atomic Power Development Associates (APDA), who is not here, has a set of cross-sections which fit very well the same criticals with which the Yiftah-Okrent-Moldauer set disagrees by 100 kg. We are not surprised that he can achieve a good fit because, in fact, anyone can make these adjustments with a bit of careful study. I am convinced, therefore, that Mr. Tavernier should not be so concerned about these discrepancies.

Some of our critical experiments have been very interesting. We did some experiments with relatively little U^{238}, and these differed from earlier experiments because the scattering of the steel and the sodium played a more important role. The large discrepancies—larger than we encountered with the previous type of critical experiment run at ZPR-III, using large amounts of U^{233}—prompted many (like Dr. Hummel, Dr. Meneghetti, Dr. Nicholson at APDA and others)

to do more careful theoretical work on elastic scattering. We have always been concerned by the fact that elastic scattering was not being treated carefully; but it was not until we got large discrepancies which were clearly attributable to this phenomenon and no other that we were prompted to devote considerable effort to a theoretical study of it. Our experiments have shown that the analysis was decidedly worthwhile. In Paper SM-18/37* Dr. Meneghetti will discuss some analyses of critical experiments carried out on ZPR-III; and in Paper SM-18/45** Dr. Hummel will discuss a method which provides at least a partial solution of the problem of treating resonance scattering.

W. B. Loewenstein: I should like to point out that some of our difficulties may be due to cross-sections, and I think that the four cases presented by Dr. Long may serve as an indication of this. The first two assemblies feature stainless steel and sodium as diluent; the last two assemblies feature uranium and sodium as diluent. The ratio of core height to diameter in the first three assemblies is about 0.9, which is the optimum ratio. The last assembly has a ratio of core height to diameter of about 1.5, I believe. The gross difference between the first two systems and the last two systems, representing about 4%, is certainly a cross-section difference. However, the difference between assembly No. 101 and assembly No. 103 is clearly a difference attributable to core shape and not primarily cross-sections. This is simply because Dr. Long did neither a shape correction nor a heterogeneity correction, as he himself pointed out. The shape correction required for the last system would be larger than for the first three; some unpublished analytical work which we have done would seem to indicate that this is the case. The correction would lower the k_{eff} for all four systems and the k_{eff} for the last two would more nearly coincide.

G. C. Tavernier: Merely to sum up my ideas in the subject, I think I should be able to predict the critical mass of a given reactor to within, say, 10%. To do so I must have not only a vast knowledge of critical systems already constructed, but also a good team which can interpret the constants, and where necessary adjust them correctly for a particular case. I think that ties up with what Dr. Marchuk said yesterday and with what I have just heard from Dr. Okrent.

D. Meneghetti: I rather think Dr. Tavernier was really asking whether there would be a set of multi-group cross-sections which could be generally utilized. My own opinion is that we shall not succeed in producing such a set unless we adopt a great many more groups than we normally consider at present.

Paper SM-18/29 was presented by P. Clauzon (see Vol. I, p. 347).

Discussion

V. Raievski: Yesterday, Professor Chernick draw attention to the fact that asymptotic equilibrium had been achieved at a very large distance in a block of natural uranium. Here you have a block of considerable size then, of the order of 10 t I think, and it seems that the back boundary conditions are the greatest difficulty. If you ever wish to examine the relaxation length, you must obtain an exponential variation far from the front. In the curves which you have demonstrated, it appears that it is the reflection of the neutrons on the back which is causing the perturbation in precisely this asymptotic region. So it seems that

* See Vol. I, p. 457.
** See Vol. I, p. 231.

the back boundary conditions, for example, are particularly important. Do you think boundary conditions can really be arranged ?

P. Clauzon: We set up different thicknesses of steel and paraffin at the back of our block in order to alter precisely that boundary condition, and it was observed that in the sector between 400 and 500 mm the previous results were scarcely perturbed, despite fairly appreciable perturbations on the back.

V. Raievski: You also pointed out that the diffusion length was extremely sensitive to equilibrium. I think that is pointed out in your report.

P. Clauzon: In comparison with the results obtained in various other laboratories, yes.

V. Raievski: The spectrum index, on the hand, would perhaps be less sensitive. You seem to have obtained a very good spectrum index since it is exactly equivalent to that of the Chesholm experiments, and also to that of the calculations made with the Yiftah cross-sections.

Do you think the coaxial can change the spectrum, because you have either a vacuum—and then you have transport—or a hydrogenous substance ? Thus you can have a change in the spectrum as a result of slowing-down.

P. Clauzon: That is correct.

C. P. Zaleski: I should like to make a few comments on Mr. Raievski's question. Firstly, we have analysed spectrum indices as a function of the abscissa; we did not present our results in this paper, but they are to be found in an internal CEA report. In my opinion the analysis of diffusion length—the spatial analysis of a detector—is a more sensitive index, taking into account the accuracy of measurement obtainable, than the coarse or direct analysis of spectral indices. Be that as it may, we can say that we have a region in which there is a tendency towards equilibrium (in other words, the spectrum indices show plateaux), and that the region concerned cannot be longer than the region in which we have apparent agreement between the different diffusion lengths.

Mr. Clauzon did in fact emphasize that our first experiments resulted in quite considerable disagreement between the different diffusion lengths of the various detectors, since we took something like 200 or 300 mm as a base distance for studying the relaxation length. In the second part of the experiment, this length was reduced. Naturally the errors were a little larger, since we did not have such a great length at our disposal, but agreement seemed better. In my opinion the interesting point is that, in such studies as these, detailed analysis of spatial evolution can sometimes be better, more direct, more accurate, than purely spectral analysis, and can even furnish information on equilibrium.

As to Mr. Raievski's second remark, regarding the presence of hydrogenous substances, we have done some experiments with activation detectors which involved introducing polyethylene into the apparatus. For example we tried to analyse the results obtained by placing activation detectors alongside photographic plates, and found in fact relatively low sensitivity, provided the quantity of hydrogenous material did not exceed a certain given value.

L. N. Usachev (Union of Soviet Socialist Republics): My question concerns the processing of the experimental results, with reference to the diffusion approximation. Did I understand correctly that the way in which the propagation of neutrons was dealt with in the report involved a diffusion approximation, or was that not the case ?

P. Clauzon: Yes, that is right.

L. N. Usachev: Then I wish to make a brief remark. It is very simple here to

avoid using a diffusion approximation by taking an asymptotic approximation of the integral equation. To do so does not entail the use of any computors but, essentially, of the same formulae and the same spectra; though there will be a fairly appreciable difference, particularly as regards the number of neutrons in the higher group. The neutrons that induce fission in U^{238}, which are strongly and inelastically scattered, are described very badly by the diffusion approximation. The error in the spectrum can amount to 20%, as far as I recall the results of the analysis we carried out in similar experiments.

C. P. Zaleski: I think Dr. Usachev is speaking of the spectrum in equilibrium. Actually, the calculations we have done were not concerned with the spectrum in equilibrium but with real conditions, that is to say the propagation of flux starting from the graphite, graphite-uranium interface, uranium and uranium-paraffin interface. In that case one could certainly use better approximations, but I think it would require rather complicated machine calculations.

L. N. Usachev: Was the Monte Carlo method used in these calculations?

C. P. Zaleski: No, the calculations were made in diffusion approximation, with a uni-dimensional code which was certainly not perfect; but we simply wanted to compare these classical methods with the experimental result. I agree entirely with Dr. Usachev that it would be interesting to use other methods of calculating, but we have not done so; I think, however, that in our case it would have required very long machine calculations; of course the geometry was simple—one dimension—but there were numerous rather complex boundary conditions. It is not the spectrum in equilibrium that is being studied, but the evolution of the spectrum to see how it approaches equilibrium.

L. N. Usachev: You are quite right. When I mentioned such a simple method of refining the diffusion approximation I had in mind only the equilibrium spectrums.

P. F. Zweifel: I should like to address this question to Dr. Usachev. Is the asymptotic theory to which you refer not the same as diffusion theory, the diffusion rate being calculated from a transcendental equation—Placzek's equation, for example—so that the change would really be trivial?

L. N. Usachev: I did not understand your question very well, but the point is that the formulae obtained, as also exponentials for the asymptotic spectrum, are, properly speaking, almost identical; but instead of taking a specific value for the diffusion length, as in the diffusion approximation, we use the well-known transcendental equation with a hyperbolic tangent.

P. F. Zweifel: Yes, that is exactly what I meant. In fact, then, this more accurate calculation that you described entails no more work, no additional computing labour. It requires only the solution of a very trivial equation. I agree with you; the method should probably be used.

V. Raievski: It would perhaps be interesting to use other methods, not to calculate the diffusion length, but to ascertain whether equilibrium is reached. I am thinking, for example, of methods of the Monte Carlo type.

C. P. Zaleski: I agree that a detailed study of neutron propagation in the neighbourhood of the boundaries between, for example, the graphite and the uranium, or the uranium and the paraffin, could be done by methods which are more effective and more direct, but they would certainly be more costly in machine hours.

I should also like to add a comment regarding boundary perturbations. In our case the main perturbations were due, I think, to the transport of neutrons from the sides of the block, which gave strong contamination at the rear; that

is the reason why in practice we had the equilibrium region in the centre of the block. The contamination at the rear was almost as great as at the front. A system of the Los Alamos type is obviously very convenient, but we were not able to install such a system on the BL-2 at Saclay because of health considerations. However, the block was rather bigger than the Los Alamos block, so I think that in the last resort the equilibrium conditions were obtained in more or less the same way in both cases.

I. I. Bondarenko: I should like to make a few observations in connection with the last paper. Experiments of the type indicated were started a long time ago and have been carried out in many laboratories; that is a well-known fact. At the same time, I think that although they constitute one of the earliest types of experiment, they have retained their importance up to the present day and will remain important for some time to come. There are clearly two reasons for their importance. First of all, very simple conditions are present so that the experiments can be calculated easily and provide a system of constants for any single element straight away. I should like to emphasize that we can, or should, speak of two basic spectra in connection with these experiments. One of them is the asymptotic spectrum, which is established in a sub-critical medium of given composition at large distances from the neutron source. This is the spectrum investigated in the last paper. It is natural that such a spectrum cannot be established in all cases or in every medium, but only in a medium in which neutron breeding is observed; several additional conditions must also be met, involving the presence of an isotope fissionable at all energies for all neutrons.

In addition to the asymptotic spectrum, the equilibrium spectrum of the neutrons in a given medium is of interest; this corresponds to the spectrum which would exist in the medium if the neutron-fission sources were uniformly distributed throughout it. However, one can measure the properties of the equilibrium spectrum in other ways, i.e. not only by using uniform distribution of the neutron-fission sources, but also by using point or bounded sources and integrating the measurement results over the medium. Both these methods of measurement are therefore convenient because they can be easily interpreted. The second method clearly does not depend on transport cross-sections. The characteristics of equilibrium spectra are determined only by the energy-dependent cross-sections, the cross-sections of removal between groups of neutrons, the capture cross-sections and the production cross-sections. Thus the experiments in question allow us to check the energy-dependent cross-sections separately, leaving aside their dependence on the transport cross-sections. On the other hand, by the experiments with asymptotic spectrum which were discussed here, in addition to testing the energy-dependent cross-sections we can very simply test the transport cross-sections as well. In this connection I should like to underline what Mr. Usachev has already said, that theory in this field can be accurate without any need to use complicated machines. These are simple calculations which it is possible to do on a mass-scale and by means of which one can simply and methodically cross-check a large number of variations for systems of constants. It should be pointed out at the same time that for any particular type of assembly, a simple diffusion approximation does in fact still lead to considerable error in determining the ratio of the neutron flux in the higher group to the general neutron flux.

I should also like to say a few words on the experiments of this kind that have been carried out in the Soviet Union. The first experiments on neutron

equilibrium in natural uranium started in the Soviet Union in 1950 or a little earlier. Then there was a group of experiments using the BR-I reactor as neutron source, which we reported to the Second Geneva Conference in 1958. I think the results of these experiments were on the whole similar to those reported today. Experiments of this kind have been repeated recently as well, but with an attempt to obtain purer experimental conditions. The column of uranium was sited in such a way as to be at least 2.5 m distant from both walls and floor. In addition the thickness of the metallic-uranium layer was increased to 1.5 m. The measurements made with this large column showed that, all the same, effective equilibrium in the cross-section ratio is attained at an approximate distance of 1 m from the neutron-fission source. From, say, 70 cm up to 1 m, there is still, if I remember rightly, a divergence of about 3% in the cross-section ratio. At the same time it is precisely the cross-section ratio which appears to be a more sensitive quantity than the asymptotic length. However, I think that as regards this particular medium—natural uranium—the numerous experiments carried out in various laboratories have nevertheless led to satisfactory results as far as our experimental and theoretical knowledge are concerned. We carried out similar tests recently using a large prism of uranium oxide ($90 \times 90 \times 150$ cm). The BR-I reactor with a core of plutonium served as neutron source. We obtained the diffusion lengths and the cross-section ratio for various reactions in the asymptotic spectrum. I should add that we did not succeed in obtaining a satisfactory theoretical description of these experiments. Although many different types of calculation were tried, no agreement with the experiment was observed. The experiments testify that in this case the spectra become very soft. Apparently there are a substantial number of neutrons with an energy even below 1 eV. In other words, theoretical description of the asymptotic spectrum in uranium oxide is extremely complicated, and we have not so far succeeded in obtaining satisfactory agreement between theoretical calculations and experimental data. To put it another way, given as asymptotic diffusion length, the theory is described with an accuracy of about 10%, but the cross-section ratio—in particular, for example, the ratio of the capture cross-section for U^{238} to the fission cross-section for U^{235}—already contains an error of the order of a factor of 2. It is clear, therefore, that a great deal of work remains to be done in this field, and I should like to ask whether anyone has tried to carry out such experiments and compare them with theory.

H. Rief: It was mentioned earlier that one should apply more powerful mathematical methods to the calculations of these experiments. About a year ago I did a quite extended Monte Carlo study of the Snell experiment. In my study I used the Mandeville inelastic-scattering cross-sections, the Cranberg and BNL-3400 anisotropic cross-sections, and the BNL-325 fission cross-sections. If I remember rightly, I got a value of 0.43 for the ratio of primary neutrons to neutrons generated in U^{238}, and this is certainly below the value which was reported today. I should also be interested in carrying out a calculation reproducing exactly the experiment which was done at Saclay. I believe I could also introduce the graphite of the thermal column into the reflector, and then compare these results with your measurements.

C. P. Zaleski: I am pleased to learn that such interesting experiments are being carried out in the Soviet Union. I should have liked to acquaient myself with the experiments beforehand in order to discuss them at greater length. I hope that reports on the subject will be available.

Secondly, I believe Dr. Bondarenko said that spectral indices are more sensitive that relaxation lengths. I think one can hardly assert that categorically; perhaps one should consider the conditions in which measurements were made and the details of each measurement. In our case, we had excellent statistics on diffusion lengths: we had a vast number of detectors, very reproducible measurements, and we think that experimentally—I repeat experimentally—better results, or at least more sensitive results, are obtained by comparing the relaxation lengths of detectors with different cross-sections, than simply by local comparisons at a given point of the responses of two detectors. In that way, we were able to obtain better statistics on flux distribution, both in the spectrum and in space.

In reply to our colleague, I believe it would be very interesting indeed to take this matter up together again. I do think it is possible to do complete calculations using more effective methods, but I think that would mean considerable machine hours, and it seems to me Dr. Usachev agreed on that point.

L. N. Usachev: I quite agree that to obtain the full spatial energy distribution in a particular experiment, very lengthly calculations are required, and that to obtain the asymptotic spectrum and the volume-integrated spectrum very simple calculations are sufficient.

V. Raievski: I think Dr. Usachev is speaking of the asymptotic solution that can be obtained using simple methods, but what Dr. Zaleski was speaking of is the case where one takes account of the actual geometry, in other words the case where the asymptotic solution is not necessarily reached. It was precisely to check whether this asymptotic solution is reached that he proposes these calculations.

C. P. Zaleski: I think that on the whole we are all in agreement in France. The asymptotic spectrum is easy to calculate. The spectrum in more complex conditions is interesting to calculate; it must be done, but it requires a considerable amount of work.

H. Rief: The computer time is perhaps not as long as you think; I should estimate that one would need from 10 to 20 min on the IBM-704, which is quite reasonable.

C. P. Zaleski: I agree; it is not bad at all. We did our calculations with an IBM-150, a far less powerful machine.

L. N. Usachev: I should like to express my agreement with the last speaker; but I wish to stress, nevertheless, that for a comparison of theory and experiment it is, of course, better to use simple methods as there are then fewer unknown parameters.

Paper SM-18/6 was presented by O. W. Dietrich (see Vol. I, p. 377).

Discussion

V. Raievski: One can see that Mr. Dietrich is not now following the idea to develop the fast-neutron spectrum in a thermal reactor as a function of the cross-sections of the threshold detectors. This is, of course, a rather unsatisfactory proposal, since under those conditions the functions employed have no physical relation to the phenomena. It is interesting to see that it is in thermal reactors that concern is felt about the hardening of the spectrum in relation to the fission spectrum, whilst in fast reactors the opposite is the case; what causes concern is the softening of the spectrum in relation to the fission spectrum.

It is clear, then, that your proposal is valid only for the core of the reactor;

if, on the other hand, you ever wish to study the spectrum in the reflector, you are certainly obliged to use other functions.

O. W. Dietrich: I am fairly sure that this method could be used for other points in the reactor and in a fast reactor as well. The problem is one of finding a modification function for the fission spectrum. It is more or less a question of getting enough flux in the reactor, and for that reason these measurements have been used in reactor cores.

V. Raievski: Also, the hypothesis that a neutron which has undergone a collision with hydrogen can then be regarded as outside the fast-neutron spectrum is not strictly true; I am thinking, for example, of the case of the ε factor, which is an important factor calculated in thermal reactors. There is also an appreciable contribution from neutrons which have undergone a first collision.

R. D. Smith: I should like to make some general comments about the use of threshold detectors for finding the spectra of fast reactors. We have used this method very extensively in the past, and so, I think, has everybody else. We have tried a number of other methods of finding the spectrum and none of them, I think, is as good as the threshold detectors—or, more generally, a spectral index where one can include thermal cross-sections as well for measuring the low end of the spectrum.

Nevertheless, I think that this is fundamentally the wrong approach, and there are several reasons for this opinion. Firstly, to get good spectra one must measure the cross-sections in the reactor very accurately, and it is difficult to do this experimentally. Secondly, one must have precise knowledge of the cross-sections of the reactions as a function of energy. We do not know this at present, although this problem may be overcome in time. Thirdly, there is the difficulty of interpretation, and the paper we have just heard offered some ways of overcoming this difficulty. Another point is that there is a great scarcity of suitable detectors in the energy range which interests us for fast and intermediate reactors: some are available, but I fear there are not enough for this method to work very well. Thus I still feel that we need a good spectrometric method of determining spectra; in my opinion threshold detectors do not really offer a good method.

V. Raievski: Mr. Smith has just ruled out threshold detectors as a means of measuring spectra in fast and intermediate reactors. I understand that Mr. Balligand, would like to ask what other method Mr. Smith suggests.

R. D. Smith: Unfortunately I do not know of any good methods, but I am relying on my colleagues at this Seminar to produce one. I can only suggest the solid-state spectrometers which are being developed at the present time. These look as though they will give good spectrum at least down to 200 keV. Below that level, unfortunately, the problem becomes very difficult. However, I feel sure that it can be solved.

C. P. Zaleski: I wonder whether one could not say that threshold detectors, or more generally activation or fission detectors, can be used at least to supplement other methods, such as photographic plates. I think everybody is convinced of the difficulties of using such detectors, but I think that for lack of better methods they can nevertheless be of use in certain energy ranges. I should like to emphasize that not only threshold detectors, but also activation or fission detectors (for example, to compare Pu^{239} to U^{235} fission) can give results which in our opinion are of interest. The accuracy is not very good, but at least it is better than nothing. In other cases, of course, junction detectors or nuclear emulsions can also give very reliable information above a certain energy limit.

V. Raievski: It should also be noted that certain results may be directly utilizable, for example those which give the average value of the fission cross-section of U^{238}. That is an integral measurement of which direct use can be made in reactors.

C. Beets: From the point of view of detection techniques using nuclear emulsions, I think that considerable progress will be made in the years to come. On the Soviet side, use could be made of Perfilov-type emulsions, which are emulsions with a grain which is considerably finer than the Ilford emulsions; on the Western side, it appears that certain manufaturers of emulsions are going to make emulsions similar to the Demers type, and this would make it possible to bring the lower limit of validity of the emulsions to about 150 keV instead of the 300 keV normally permitted.

As regards the study of junctions, if one takes the lithium-6 reaction, i.e. with the production of tritons and alphas, the definition is not sufficient when using sandwich junctions, being at present only about 300 keV; the problem could probably be solved by confining ourselves to the spectrometry of only one of the charged particles, say the tritons, making certain hypotheses regarding the direction of the incident neutrons. In reactors, and particularly large reactors, the most simple hypothesis would be to say that the incident neutrons are isotropic and this would then make it possible, using only the spectrometry of tritons, to obtain a far lower limit of validity, at all events lower than 100 keV.

D. Okrent: I think it is worth pursuing measurements with high-energy threshold detectors. In the past it has been common to assume fission spectrum flux shape above a certain energy level—say 1.4 or 2.3 MeV. I think this assumption was made primarily because so little information was available on inelastic-scattering distribution; and the information which was in fact available was not considered reliable. However, an examination of calculations using the Yiftah-Okrent-Moldauer cross-section set shows that this assumption may be incorrect for large reactors and that significant changes in group cross-sections may result.

As far as threshold detectors are concerned, even if it is not possible to determine the spectrum with absolute prevision, one can certainly get relative measurements from one reactor to another; and these are, in themselves, of great interest to reactor physicists.

A. B. Smith: With regard to using fission thresholds for absolute spectral indices, I should like to bring to your attention a slide which was shown yesterday. The case in point is σ of Np^{237} at 2.5 MeV. Several recent measurements are in disagreement by approximately 30%, yet this cross-section has already been discussed as a spectral index. This may indeed be useful as a relative measurement, but as an absolute quantity it is open to very serious question.

Paper SM-18/60 was presented by K. Einfeld (see Vol. I, p. 287).

Discussion

D. Okrent: What do you mean when you say that the spectrum is identical with fission spectrum above 1 MeV? With what degree of accuracy is this statement meant to apply?

K. Einfeld: This spectrum has been compared with the theoretically calculated spectrum which we should expect to find in the centre region of the core. For the theoretical calculations I would refer you to paper SM-18/61*, which will be given next week.

* See Vol. II, p. 335.

D. Okrent: The fission spectrum for U^{235} is reasonably well defined; yet it is stated in the paper which you presented that the measurements made of the spectrum in this reactor were identical with measurements made elsewhere of the fission spectrum.

K. Einfeld: Yes, I believe so.

D. Okrent: Would Mr. Campise care to say whether, theoretically, he would expect this result? I would not expect it myself.

A. Campise: I agree with you; I was surprised at the results myself, and we are still examining the measurements to which Dr. Einfeld referred.

D. Okrent: May I ask another question? I believe your first assembly was fuelled with U^{235}. Was the composition of the central region exactly, or almost exactly, like ZPR-III?

K. Einfeld: Yes, it is comparable to ZPR-III. I have in mind the 2-A experiment on ZPR-III, which uses U^{235} and stainless steel.

D. Okrent: Have you estimated the critical mass from your AETR measurements?

A. Campise: No; our data are not sufficient for critical-mass determinations.

D. Okrent: Will you eventually be able to make reliable critical-mass estimates using this general method that you applied in your critical?

A. Campise: Only by testing the cross-sections that we use to analyse these integral experiments can we judge whether our calculated critical masses are correct or incorrect.

V. Raievski: These experiments are of considerable interest in that they make it possible to measure the ratios of cross-sections in small assemblies by using a breeding region. Clearly the important problem in such cases is to know whether you are really getting the spectrum in the central region of the critical assembly, because the central region is fairly weak. I believe you have made measurements with detectors, fission chambers, to see whether the spectrum was really such as one would expect to find in the critical assembly. Secondly, since these oscillation experiments are extremely accurate, I am wondering whether, for example, a large assembly using U^{233} would give the same results. Do you think it would be possible, in these experiments, to restart these oscillations in central regions having, for example, two different radii, in order to see if there is a difference in the results?

K. Einfeld: You are asking whether, if one takes two radii of U^{233} as a central test region, the same results would be expected? My answer would be that the fission count reverses have been taken in such a way that the neutron-energy spectrum is rather constant over a large range. Therefore, if you used a smaller central test region you need not expect large changes.

A. Campise: In the design of this multi-region critical assembly the outer regions were optimized to maintain the statistical weight and the flux spectrum over about 10 cm of the central test region. The results thus far indicate that the calculations were correct and that the reactivity is in fact maintained over about 5 or 10 cm of the test core. This would imply that the calculations are truly indicative of the flux spectrum with which we are working. Therefore, I would say that if you took a smaller radius you would have to reoptimize the dimensions to maintain a given spectrum. In other words, we are using the thorium as a filter to select the spectrum that we want, and we then optimize

the reactor accordingly. Every time we change the test region we reoptimize the outside.

V. Raievski: The question turns on the criteria which you apply in optimizing this particular region, because oscillation measurements are much more accurate than the tests which you carry out with the spectrum.

A. Campise: The criteria which we apply involve having the statistical weight flat over the region of the test core in which we plan to make our measurements. We also compare this statistical weight (by using the flux spectrum, which is another criterium) with a core which would extend to the outer boundaries of the physical reactor. In other words, the test core in the centre has the same characteristics, from a calculational standpoint, as a reactor which would extend all the way to the outer boundaries of the reflector of this particular core; although we have only a piece of this core, we contend that it has the right neutron environment.

J. Horowitz: I have two questions to ask, first of all a question of detail. In Table II, the value for the relative importance of the antimony-beryllium source in missing. Can you give us the figure please?

K. Einfeld: The neutron source length was calibrated after the paper had been prepared, by the manganese-sulphur method. The value for the neutron importance of the antimony-beryllium source is 3.88 for core No. 1. One has to consider that the neutron importance depends on the source strength, so one would have to have all these results together.

J. Horowitz: That leads me to a more general question. Mr. Raievski laid much stress on the fact of having a spectrum as near as possible to the one which would exist in an actual reactor. Personally, I have the impression that what would be much more difficult (especially if the spectrum is very degraded, which is the case for the lattices in which you are interested) is to ensure that the relative proportions of neutrons of different energies should be the same as in the reactor; and from this point of view the accuracy which you obtain by a purely experimental approach seens to be clearly superior to what one can interpret, to what one can deduce from the experiments. To take a particular case then, this first core is of no significance, but the value for antimony-beryllium was probably the most important one in Table II, since it is precisely the source whose energy is very different from the others.

Have you considered what exactly are the optimum geometries which make it possible to achieve a close replica of the spectrum and also a pattern of neutron distribution in terms of their different energies which resembles pretty faithfully the pattern that one would actually find in a large reactor? We have experience, as part of a much simpler problem, of oscillation measurements which are made for thermal reactors, where one has two or three parameters to consider rather than continuous neutron-energy values, and even in this case one has to take great precautions if one is to achieve any value from the experiments.

V. Raievski: The point raised by Mr. Horowitz links up with the question I asked, in the sense that there are two problems. It is first of all necessary to be certain of attaining, in the central region, the spectrum of the critical reactor. Secondly, the importance of a neutron in that central region should not depend on its energy, i.e. if you absorb on the one hand a neutron at 1 keV, and on the other hand a neutron at 100 keV, the effect on the reactivity should not be the same.

G. C. Tavernier: I think that the first two parts of my question have been

asked by other speakers and I will therefore not put them. I should like to ask what experimental evidence Mr. Campise can point to when he compares calculations for a small region and for a comparable large reactor — what experimental evidence is there that the flux, the spectral distribution of flux at the centre of the region, truly corresponds to the values that one has in large reactors?

Secondly, I seem to remember that in ZPR-IV or ZPR-V there was also a thermal breeding zone which automatically led to a fast sub-critical assembly, and that at one stage great difficulties were encountered by the Argonne experimenters in spite of the relatively large volume of the regions in question.

Perhaps comments from an Argonne expert would be useful at this point.

A. Campise: In the first place you ask whether there is actually experimental evidence that the small region indeed mocks up a full-scale system. The only experimental evidence we have is that, from a theoretical standpoint, the ratios or the reactivities that we calculate in the test region of this reactor are the same as these we would calculate for a large system; in other words, the calculated values are the same values one would get in the case of a full-scale system. We are now comparing these with our measurements; and this is, in fact, the only experimental evidence we have.

J. Weale: Considerable importance is being attached to the measured ratio of U^{235} to U^{238} fission (fission-chamber measurements). Our own errors in these measurements are of the order of 6%, and Clauson reported errors of 5%. I would like to ask Dr. Einfeld and Mr. Long what errors they have in their present measurements and how much they think these can be improved.

K. Einfeld: I am not sure what the errors are for the ratios given here. Other measurements on the same assembly produced errors of about 5%.

C. Beets: With regard to the problem of junctions, what is the permissible intergrated flux which does not destroy the properties of the junction — I mean in the case of sandwich junctions? Normally one permits an integrated flux of 10^{13} or 10^{14}; what value do you have?

K. Einfeld: I am sorry, I cannot quote a value.

B. I. Spinrad: I should like to return to Dr. Tavernier's point regarding the internal exponential experiments performed at Argonne which were reported some three years ago. In these experiments we did have a larger block and we encountered the same phenomenon of apparent equilibration of the spectrum. We are convinced now that this was something of an illusion, due to the fact that any radially symmetric spectrum will show a region of relatively flat response. The best indication of the equilibration was obtained by comparing a lattice in this experiment to a similar one measured in an exponential column. We were convinced not only that the spectrum was not equilibrated, but also that it was not close to being equilibrated in the rather large assembly. We would expect, therefore, that such things as danger coefficient and relative importance measurements would be in error in the second order rather than in the first order, and, therefore, might still be quite useful. However, the assembly reported by Mr. Einfeld is somewhat smaller; without a direct calibration of the spectral responses, one has some doubt as to the degree of equilibration.

V. Raievski: I think that the check of which Dr. Campise spoke — i.e. a comparison of the values calculated for a large reactor with those obtained experimentally — could not in fact be considered as an experimental check. I think that the only possible experimental check would consist in verifying that one has the same ratios in assemblies with different radii.

A. Campise: In connection with the AETR, we are interested in the U^{233} cycle; but to obtain this comparison would require an amount of fuel which is simply not available.

P. Schmid: I should like to draw your attention to the possibility of improving the statistical accuracy of such pile-oscillation methods. Our experience in pile-oscillation work, both in Norway and in Switzerland, was that the main statistical errors came from the low-frequency part of noise and from breaks. Such inaccuracies could be reduced to insignificance by the following method: the measurement cycle is subdivided into n measurements over half-periods, and those n measurements are given statistical weight according to binomial coefficients. It can be shown that this method eliminates aperiodic signals which are described by a power series up to the term T^{n-1}, where n is the number of such partial measurements.

V. Raievski: This is an observation on the accuracy of the measurements. I think, however, that it is the very principle on which the measurements are based that is being discussed. I think it might be useful if those whe are interested in the technique of these oscillation measurements could have a discussion with the proceding speaker on the possibility of reducing the background and other causes of fluctuation in these measurements.

D. Okrent: I find it hard to accept Mr. Campise's statement that he is unable to check his method because of the lack of U^{233}. U^{235} can be used to check the technique and this is available in larger amounts; indeed, one can compare the results with criticals. Before I am convinced I should like to see the measurements made on the AETR and then confirmed on the critical afterward. If one then got agreement, one could perhaps be sure of the results obtained with U^{233} thereafter.

A. Campise: We have only one comparison with a critical; that is the 2A core itself, which was put together on the ZPR-III machine. The fission ratios are in fact about the same.

J. K. Long: A question was asked about the accuracy of fission-ratio measurements. With regard to ZPR-III spectral indices, we have no difficulty in obtaining reproducibility of uranium-fission ratios to within $\pm 2\%$ (which may be attributed 1% to counter statistics, 1% to electronic drifts, minor geometrical effects, etc.). We are not confident that our measurements are entirely free from systematic errors such as isotopic analyses of the active material of the counters, and estimation of the effective mass of active material in the counter. Although it is difficult to estimate the extent of such errors, comparisons with calculations and such other cross-checks as we can make indicate that we might expect systematic errors as large as 10%. Our reported spectral indices are therefore more valuable for comparisons of assemblies than as absolute measurements. We hope to initiate intercomparisons of our counters with those used by other laboratories.

V. Raievski: I think that U^{233} has recently been re-evaluated. Could someone tell us what the $\bar{\nu}$ was for the U^{233} values calculated in Table I?

A. Campise: Do you mean the average over the spectrum?

V. Raievski: I should like to know the variation with energy as well.

A. Campise: The $\bar{\nu}$ values that we used in our analysis were taken from the work of Okrent and Yiftah.

D. Okrent: The value adopted for U^{233} was 2.5 at thermal; the value for $d\nu/dE$ was 0.123.

SESSION 6

Chairman: D. Okrent

INTRODUCTION BY THE CHAIRMAN

The topics which will be discussed this morning are essentially of a theoretical nature, dealing with methods of calculation and design. When I was asked to take the chair at this session I did not know what would be on the agenda, and I chose a few comments which, as it happens, bear no relation to the papers on the programme; nevertheless, I shall continue with my original choice.

At the stage which we have now reached in fast-reactor physics, it is to be expected that scientists in many areas of the world which lack large experimental facilities will be in a position to suggest useful experiments—experiments which will have to be carried out in the centres which do possess such facilities. This is certainly true in theoretical physics; a theoretician feels in no way obliged to suggest only such experiments as can be performed at his own installation; he is quite willing to suggest interesting experiments for any centre in the world, and I think it is entirely fair to ask that theoreticians in fast-reactor physics should do the same.

Mr. Long has said that he hopes to bring about a wider exchange of information with regard to measurements of spectral indices, threshold devices and so forth. I think this is something which should be strongly encouraged. By now our cross-section data are actually quite good, and we need really precise comparisons, at least for certain measurements. Indeed, it would be available for various installations to provide a local calibration, so that if one makes a series of different assemblies over a period of a few years, for example, one may know whether measurements made at the end of the period are on the same scale as those made at the beginning.

Instruments may change; new instruments may be developed. One way of accomplishing this would be to have a fixed neutron spectrum, aside from the general experimental facility, to which one could always refer for the purpose of normalizing measurements. In any case, I would strongly urge the various experimental groups to undertake this task before we have accumulated so many measurements that we cannot go back and recalibrate them.

More specifically, there are a number of particular fields in which theoreticians might suggest experiments for installations with large experimental equipment. One task would be to devise experiments for testing the microscopic data which we use, and to define the accuracy that is needed to provide a test. For example, we may soon have measurements of ν which, we shall be told, are accurate to 1% or 2%, and fission cross-sections accurate to 2% or 3%; similar accuracy may be quoted in regard to measurements of α for various materials. It should be possible to test the accuracy of these measurements, at least on a relative basis; and if we could test them on an absolute basis we should be that much further ahead. In the part, measurements on fast critical facilities have revealed cross-section errors in at least three cases which I know of: σ (n, p) for sulphur, for example, did not agree with the Los Alamos measurements in reactors; certainly σ (n, γ) for gold did not agree with Los Alamos measurements, nor, I believe, with certain measurements made in Britain; and we know that $d\nu/dE$ for U^{235} has been the object of suspicion on the part of reactor analysts for some time. At the moment we have a fairly consistent discrepancy in σ (n, γ) for U^{238} between the microscopic data and measurements in fast reactors, and there

may be a discrepancy in σ (n, γ) in iron analysis. These are cases in which the theoretician could suggest useful experiments, and there are other cases in which a good definition of a sophisticated experiment is needed. We need ways of defining the effective data for an assembly having three or four different fissionable materials, so that we do not have to rely completely on the measurements for the individual isotopes, which themselves are uncertain to perhaps 10%.

Another area directly related to experiments, in which I believe the theoretician could be of help, is what I will call the zoned critical problem. I think there are various installations which are planning or will inevitably plan zoned criticals, where the driving section is not a thermal but a fast section—driven, let us say, by U^{235}, which is generally available—and where the purpose is to make measurements on systems containing Pu^{239}, Pu^{239} plus Pu^{240} or even higher isotopes, which are harder to obtain. We must acquire some idea of how accurate this type of measurement can be, how much one can hope to learn from it, and by what methods. We need to know, moreover, whether this will be sufficient, or whether we shall be forced to build very large plutonium-fuelled critical assemblies (using, for example, 1000 kg) before we feel that we can give the engineers sufficiently accurate information on the physics design of the reactor. We may discover that it is a waste of time to try to make the same measurements with 50 kg. Attention should be devoted to these matters now before our experiments are too far advanced.

The last two problems I would like to mention are related to the sodium coefficient of large reactors. Later in the seminar we shall hear a number of papers dealing with the sodium reactivity coefficient in such systems, and we will see that a positive sodium coefficient is mentioned. I think it will help us to learn a few things about sodium coefficients in large reactors. In the first place it is important to know just what electrical power is wanted from a large reactor, because the word "large" is not sufficiently precise. I am aware that this is not a problem which normally concerns reactor physicists, but we do need some kind of guide. Once we have acquired a more precise idea of the size of a large reactor, it would be well to engineer and calculate a few different types of reactors of this size, in a rough way, to see whether they have a positive or negative sodium coefficient; how nearly negative or positive the coefficient is; whether one can readily redesign the reactors so that they are acceptable from an engineering point of view; whether the positive-sodium-coefficient problem can be eliminated; and whether it is a real problem or a somewhat artificial one, at least in certain types of reactor design. I feel we can be fairly certain regarding the importance of the positive sodium coefficient to practical reactor design.

I think that an evaluation of the importance of high internal breeding will be essential in connection with this problem, since we will find, I believe, that as the amount of U^{238} in the core is increased the difficulty of achieving a negative sodium coefficient simultaneously increases. We may then be faced with a choice, and we should realize that the effect of high internal breeding has a practical importance.

The last problem, which I think may prove to be of considerable importance, is connected with the design of the upper and lower blankets of practical reactors. If we lean towards reactors with a very low ratio of height to diameter, either because of the requirements of high power density or in an attempt to achieve a negative sodium coefficient, we will find that the upper and lower blanket sections, if they are attached to the fuel section, have a duration in the reactor which we cannot control. In many cases there is already a tendency to dispense with the

upper and lower blankets because they are uneconomic in the particular design in question. This problem may not be so serious where there is longer burn-up, but it will still be there. Some intelligently devised form of moderation in the upper and lower sections might be of help. At all events, we should devote some attention to this problem, with an eye to costs of fabrication, reprocessing and so forth; it will, beyond any doubt, turn out to be a fruitful area of investigation.

Paper SM-18/84 was presented by G. I. Marchuk (see Vol. II, p. 3).

Discussion

P. F. Zweifel: Could you compare the P_3 and S_4 methods with regard to accuracy and machine time? You said that they were roughly equivalent; but specifically, does the P_3 method take less time, and if so, how much less time? Have you found that it gives accuracy comparable to that obtained by the S_4 method?

G. I. Marchuk: I think that both methods undoubtedly are very efficient; in my paper I did not wish to criticize any one method, but merely tried to mention a few faults that we see. These faults are due, in particular, to the fact that for large systems we have to make a very large number of iterations. Consequently, even if this problem does not arise for small systems, so that these methods are widely applicable and effective, with large systems there are certain difficulties. We may therefore suggest not only P_3 or P_5 approximations; under this programme we also have others, which we think are highly efficient and in some cases—we have checked this—very greatly improve our representations and reduce the calculation time by comparison with the Vladimirov or Carlson method.

J. Chernick: I see that your methods of calculation are quite similar to those which are being used in the United States and elsewhere. On the other hand, Dr. Okrent has pointed out that experiments and calculations carried out at different places on identical systems appear to have produced varying results. Have you in the Soviet Union been at all aware of such "geographical effects"? In other words, are your results different from those which have been obtained elsewhere? If so, could you tell us of any specific cases?

G. I. Marchuk: I agree in the main with Mr. Chernick that there is no substantial difference in principle between our approaches and those used in the United States. There are only certain details, which are highly important not only with regard to the present state of the question, but also from the point of view of future progress. As regards your question—if I understood it correctly—whether we ever obtained different results by using one or other system of constants, the answer is that we did in fact try to do this and got different results. I cannot say now what the difference was, but there was one. I think this question should be studied carefully and its importance assessed, perhaps on a larger scale.

J. Chernick: So far the greatest emphasis has been laid, not only in your paper but in the Seminar as a whole, on the problem of getting group cross-sections. There is, however, another problem—that of calculating the neutron leakage accurately—and the methods for doing these calculations will undoubtedly become more complicated when we get the differential cross-sections which you have mentioned. To put it in simple terms, a defect of multi-group equations has long been the problem of the first-flight leakage. I wonder whether you could tell us how you deal with this problem. Do you take the fission neutrons at the point where the fissions occur or do you attempt to spread them out to take account of this effect?

G. I. Marchuk: We proceed in the following way: using P_1 or P_3 approximations to solve the kinetic equation, we obtain a certain system of differential equations which we then convert to a multi-group system in such a way as to avoid changing the value of k_{eff}. I think that this approach takes the problem of first-flight leakage into account.

H. Soodak: I should like to ask Dr. Marchuk to describe his work with Monte Carlo calculations in greater detail.

G. I. Marchuk: We have carried out a number of experiments using the Monte Carlo method, though we have relied on it only in isolated cases. This is due to the low efficiency of present computors and the fact that in each specific case the Monte Carlo method unfortunately gives too little information unless there are already a great many statistical data available; i.e. as I understand it, the Monte Carlo method is good for obtaining the critical mass, or some other critical integral parameter. And that was our initial aim. But that is not enough for our purposes; we are also vitally interested in the neutron spectrum. Our particular aim is to obtain a neutron spectrum at each point, and at the same time to obtain the critical mass.

E. D. Pendlebury: The general spherical harmonics method in P_3 and P_5 approximations has in the past been used quite successfully in the United Kingdom, but it has been abandoned—I think fairly generally—in favour of the S_n method. The P_1 approximation (diffusion theory) is of course still used. The difficulty with the spherical harmonics method is that the equations which one gets are essentially unstable—unstable in the sense that their numerical solution tends to lead to a large accumulation of numerical errors. I should like to ask whether Dr. Marchuk has had any experience of this.

G. I. Marchuk: The problem of a numerical solution of the transport equations in an approximation of the spherical harmonics method was solved three years ago in the Soviet Union. Very considerable mathematical difficulties were eventually overcome, and the result is that we now have two independent and effective methods of solving the problem. The first method involves the use of matrix factorization, a system of differential equations which was worked out by Keldish, Helfand, Lukutsiev and others in the Mathematical Institute of the Academy of Sciences. This method is stable in the sense that it is free of rounding-off errors. The second method, subsequently used in our work, was proposed by Godunov and is based on a solution of the Cauchy problem, with subsequent filtration of the rounding-off errors. This second method seems to be closer to your approach.

D. Okrent: We have experienced difficulties using the S_n method with thermal reactors or reactors that have a thermal region—difficulties, actually, regarding the convergence of the solution. Do I understand that with the spherical harmonics method you obtain accurate, swift solutions of such problems?

G. I. Marchuk: That is correct, but this applies only to one-dimensional geometries. We have not attempted to apply the spherical harmonics method in dealing with thermalization; or rather, to be more precise, one successful attempt was made. A general theory has been worked out in this connection, the results of which will be published within the next two months. The spherical harmonics method is used in this instance in combination with the multi-group method of calculating the slow-neutron spectrum, where both upward and downward transitions from a single group are taken into consideration. In our opinion this does constitute a certain degree of progress.

B. I. Spinrad: I should like to ask whether the various transport-theory methods

have been applied to the calculation of small and large heterogeneous effects in small cells of the reactor, as well as to the larger calculations of the total reactor.

G. I. Marchuk: Yes, I think you understand the question quite correctly. We average by the Wigner-Seitz method, and calculate the primitive cells of heterogeneous systems, mainly of course in the epithermal region where the influence of heterogeneity is very substantial. For this purpose we use as a rule the P_3 approximation and occasionally the P_5, which in fact gives you full information in 7 seconds. As regards resonance effects, we are now using the method outlined in the Spinrad-Chernick-Corngold paper. This same direction is being developed independently by Orlov, Lukyanov and others. We obtain the integral equations for the various layers and the approximate spectrum, but without the subsequent additional simplifying factors, which in general are very undesirable if one is to go on using this formula, as the spectrum must be obtained as accurately as possible within the limits of the group.

J. Chernick: We have also done quite a bit of work on neutron-thermalization problems, using some form of the integral transport equations. However, we have found that the Wigner-Seitz approximation is not adequate for tight uranium-water lattice cells and that it has to be modified. I wonder whether you have considered this problem yourself.

G. I. Marchuk: So far as resonance effects are concerned I fully agree with you; in this case we do not regard the Wigner-Seitz method as at all satisfactory, especially in the event of strong capture. For systems with large spacing, we adopt two methods. The first is that of numerical calculation in a specific cell of specific configuration, but only in the P_1 approximation. Secondly, to clear up possible errors, we always do an equivalent P_1 and P_3 comparison.

Paper SM-18/32 was presented by Miss Solanes (see Vol. II, p. 27).

Discussion

A. Campise: Did you employ, in your diffusion calculations, a diffusion coefficient corrected according to the asymptotic solution of the transport equation?

M. Solanes: Yes.

Paper SM-18/65 was presented by P. F. Zweifel (see Vol. I, p. 189).

Discussion

A. Campise: My question concerns the 10-group library referred to in Table IV of your paper. I should like to know the difference between the spectrum calculated for this carbon system and the spectrum used to average the cross-sections for the 10-group library.

P. F. Zweifel: The spectrum used for the carbon system appears in Fig. 1 of the paper. The dotted lines represent the first iteration, which we can assume to be nearly correct as the agreement is quite good. I expect the original averaging was done by someone who is present here, possibly Dr. Spinrad.

W. B. Loewenstein: Could you tell us more specifically which 10-group set you used?

P. F. Zweifel: It was an 11-group set, which appeared in ANL-5800 if I am not mistaken; this was then reduced to a 10-group set.

W. B. Loewenstein: If this was the 11-group set, the first two groups were averaged over a fission spectrum and the other groups over a flat spectrum. I believe the system that you analysed may have been the ZPR assembly No. 14.

P. F. Zweifel: The composition of the reactor is given in Table II, which shows the system with the largest amount of carbon in it.

W. B. Loewenstein: Yes, this is the ZPR assembly No. 14, and it is a well-known fact that the carbon constants involved do not properly predict this system. The fission spectrum weighting was used to make possible material replacement studies in systems which did not contain graphite. If the carbon constants were to be applied to the system which Mr. Zweifel studied, the first step would have been to alter the elastic-removal cross-section from above to below the U^{238} threshold. However, the choice of another set of constants, also published in ANL-5800, would have given a different result.

P. F. Zweifel: We chose this set of constants because of the high carbon number density; we knew that fact would be most important where there was a large amount of elastic scattering. We decided to use the 10-group set merely to see how it would work.

D. Okrent: In going from what you call the zero'th iteration to the first iteration you assumed a large difference in the flux shape within each group. Is that correct?

P. F. Zweifel: Yes.

D. Okrent: Therefore, as your zero-order approximation has a very bad flux shape at high energies, is not the difference between the zero- and first-order approximation misleadingly high, higher than is to be expected with a better first guess?

P. F. Zweifel: Quite right, and that is exactly why we used the 10-group library—to take what we thought was a reasonable estimate.

D. Okrent: Did you use the same microscopic cross-sections, in preparing your cross-section sets, as were used in the 10-group library?

P. F. Zweifel: No, the microscopic cross-sections that we used were taken from the MUFT library and from Henry's recent report, whereas the 10-group set was used unaltered.

D. Okrent: Then is not the reactivity comparison really meaningless? After all, one can generate an infinite number of 10-group cross-section sets, each of which will be different in calculating a given reactor.

P. F. Zweifel: I disagree entirely. Only carbon and uranium (and a small amount of U^{238} and iron) are present in this reactor, and we really assumed that the cross-sections of these materials had been well-known for a fairly long time—at least well enough known to give reasonable results. I doubt whether the microscopic data in the MUFT library are significantly different from the microscopic data used in preparing ANL-5800.

If the system had contained some unusual material, I might be more inclined to agree with you. I merely wanted to point out what can happen when one is careful enough to iterate the flux. We felt that the fairly rapid convergence shown by the dotted and solid lines in Fig. 1 was an indication that one iteration might be sufficient.

J. K. Long: The iterative procedure which Dr. Zweifel proposes for averaging cross-sections within a group omits the effect of spectral perturbations which are narrower than the width of the group, such as those caused by discrete resonances. Has Dr. Zweifel any further proposals for handling this averaging problem?

P. F. Zweifel: Yes, you are quite right, that is an important problem, and it just did not happen to be the subject of what we were looking at here and that is why I did not mention it. This afternoon, in the discussion period, I plan to present some remarks on exactly that problem and some work which we have done at

APDA, and if I could defer my answer until then I will show you exactly what we are trying to do.

G. I. Marchuk: What procedure does Dr. Zweifel suggest when this distribution of variables in the reactor is a rough approximation? This situation can arise particularly in the part of the core adjoining the reflector, and in the reflector itself.

P. F. Zweifel: We have considered this problem. I suggest that one should calculate the spectrum at each point by using a multi-group code and then iterate on the spatial dependence of the spectrum. I think the iteration scheme still has to be used.

G. I. Marchuk: Do you think that for the physics calculation of a reactor it is sufficient to apply the general transport approximation and to regard scattering as isotropic? Such an assumption does actually produce good results in critical-mass calculations; but in order to get more precise knowledge of the spectrum we in the Soviet Union have come round to the idea that we must reconcile ourselves to the necessity of taking into account the anisotropic nature of scattering.

P. F. Zweifel: This trend has been apparent in the United States as well, where such things as multi-group S_n codes have been written and are currently being used. I really think that there has not been sufficient study to determine how accurate the simple transport approximation can be. I think, furthermore, that many people have hastened to use transport codes, such as the S_n or P_3 methods, without studying carefully enough just how they should be used; I might mention, for example, that the group cross-section is a function of angle. I agree, however, that this approach will have to be explored more closely in the future, that we shall have to attempt more accurate calculations, although I hope they will be done with some discretion. I still think rather highly of the Monte Carlo method, and that may turn out to be the ultimate approach.

Paper SM-18/20 was presented by K. Parker (see Vol. I, p. 207).

Discussion

H. Rief: I should like to ask the speaker whether copies of this library are generally available. Is it free for distribution, or is it classified?

K. Parker: It is not classified and you are perfectly free to write for a copy. The only difficulty is that a limited number of copies was printed, and I do not know whether there are any more available.

J. J. Schmidt: What do you mean by the "new capture cross-section" for U^{238}? Do you mean the new Diven values?

K. Parker: No, this was meant simply as another (n, γ) cross-section.

Paper SM-18/7 was presented by R. Zelazny (see Vol. II, p. 55) and **Paper SM-18/21 by E. D. Pendlebury** (see Vol. II, p. 73).

Discussion

L. N. Usachev: Unlike all the other reports presented here, which dealt with numerical methods of calculation, Dr. Zelazny's report describes the development of analytical methods. I should like to point out their possible use in calculations, particularly as regards a full set of functions for multi-group theory. If we have such a set, we can develop a perturbation theory of the higher orders,

and it seems to me that the real interest of the paper lies just there. For the higher orders, in some cases—I do not say in all—the perturbation theory will surely have an effective field of application.

R. Zelazny: I quite agree with Dr. Usachev that the theory proposed offers great possibilities in the analytical solution of many problems. I do not think I have time to say any more about all these possibilities, but I can, in informal discussion, supply information on other problems which will soon be discussed in print.

General discussion

P. F. Zweifel: It seems to me that before the method outlined by Dr. Zelazny can become a really useful and practical method, one may to find approximations similar to those with which the Boltzmann equation is handled today. In other words, the method of Case does not really provide any solution to the Boltzmann equation that was not already available by the standard methods; it is just that the new methods are much more powerful and much simpler. I should like to ask Dr. Zelazny whether there might not be some simplification similar to that which could be done with this method, which would provide a high degree of accuracy and at the same time simplify the numerical calculations that might have to be done.

R. Zelazny: I agree completely with Professor Zweifel that this approach offers great possibilities for different approximations. Nevertheless, I must emphasize that in some problems, especially those I referred to in my paper, the last equations which must be solved are very amenable to numerical calculation. For example, in treating the critical problem for a slab one obtains a simple Fredholm-type integral equation with an exact critical condition of simple shape, which can be solved by easy, classical numerical methods. In such a case the simplification is quite considerable by comparison with other methods, such as the spherical harmonies or S_n methods.

A. Campise: Do you believe that this method will eventually prove more flexible than the present S_n method in treating 2-, 3- and 4-dimensional problems?

R. Zelazny: Thus far no really successful method has been developed for dealing with multi-dimensional problems, and a general theory of multi-dimensional singular integral equations is virtually non-existent. Plans and spherical problems can be treated very extensively, however, and this is sufficient in some important situations. There have been some attempts to come to grips with multi-dimensional problems, but it is really too early to say anything about these.

R. D. Smith: I should like, if I may, to comment on the introductory remarks which Mr. Okrent made this morning. He mentioned the necessity of establishing calibrations between various laboratories in the fast-reactor experimental field, and I think that some work which was initiated by Dr. Ferguson may be useful here. This is an arrangement somewhat similar to some experiments which have been reported from Aldermaston. An accelerator is used as a source of 14-MeV neutrons, and these are fed into the centre of a 4-ft cube of iron, which of course degrades the neutron spectrum; there are some holes into which one can put fission chambers or other apparatus that one wishes to calibrate. This gives a source strength of up to 5×10^{10} n/s into the 4π geometry at 14 MeV from the D-T reaction; and the importance of this is that one can establish the source

strength by counting the associated particles and get a reproducible and easily calibrated source strength. The cube is of solid iron and one would not, therefore, expect its characteristics to change with time; this is also something that could be reproduced in other labs. I suggest this as a partial answer to the need for calibration standards.

A. B. Smith: As we are discussing portable and handily calibrated sources, I should like to express a conviction that I have had for some years: the important $\bar{\nu}$ value is not U^{235} but Cf^{252}. This is an ideal source: you can count its fissions, you know its $\bar{\nu}$ value, you have a fission spectrum, and it is, in short, a very handy portable device for calibrating all sorts of fission ratios and detection devices.

L. N. Usachev: I should like to add something to Dr. Zelazny's reply to Dr. Campise on the subject of multi-dimensional problems. It is not necessary to look for a special set of functions for multi-dimensional problems. In order to apply perturbation theory, one must have a full set of functions, but not necessarily for a multi-dimensional system; a full set of functions for a single-dimensional system will suffice. The advantage of the theory of perturbation of the higher orders is precisely that it can be used to solve a multi-dimensional problem by applying a full set of functions derived for a single-dimensional case. This is what I had in mind when I spoke at the previous session.

A. Campise: My question to Dr. Zelazny was whether this method had more flexibility then the new Wick-Chandrasekhar application which Ben Carlson has made at Los Alamos in his new difference equation. He claims to have achieved a high degree of simplicity and to be able to apply the method easily to all multi-dimensional problems with very rapid conversions.

L. N. Usachev: Dr. Zelazny has mentioned the fact that the existence of a full set of eigenfunctions for multi-dimensional systems can be demonstrated. However, I have something else in mind: namely, that it is possible to use results already available for single-dimensional cases in order to solve multi-dimensional problems, by applying the theory of perturbation of the higher orders. I must confess, I do not know to what extent this procedure will prove useful, but I feel sure it will be effective in some cases.

J. Horowitz: When calculating biological shielding, one is interested in the propagation of radiation at large distances. Experience shows that in many cases one ends up with an exponential law governed by the propagation of very fast neutrons. This law can be described in first-order kinetic theory, but strong anisotropy in the diffusion must frequently be taken into account. In order to do this, Mr. Millot of the Atomic Energy Commission has developed a method similar to that described by Mr. Zelazny.

J. Chernick: I might add that at Brookhaven Dr. Takahashi has been applying the Case method to the solution of certain problems in slab geometry; these include the Milne problem of anisotropic scattering and also the slab-lattice problem.

R. Zelazny: May I repeat my point that no multi-dimensional set of eigenfunctions for the Boltzmann equation has been derived as yet, and there are bound to be rather considerable difficulties in this connection because we really have no general theory of multi-dimensional singular integral equations. There are other approaches. For example, there is an approach based on the possibility of constructing a general solution for cylindrical symmetry from one-dimensional solutions, or a possibility of constructing a solution from plane

waves. But if this approach is used certain singularities vanish which in some instances are of great help in explicit analytical treatment.

S. Yiftah: My first question is addressed to Dr. Marchuk. Your averaging of the microscopic cross-sections in order to get the group cross-sections was done according to the various regions of the reactor. While your equation certainly contains the correct factors in the averaging process, my question is practical. Suppose you have a specific reactor with a core, blanket and reflector; you would then have, if I understood correctly, several different cross-section sets for every region of the reactor—a few cross-section sets for the core, for example, a few for the blanket and possibly a few for the reflector; and the number of these different cross-section sets for calculating one specific reactor may tend to increase. My question is, then, how complicated can the process become for a practical calculation and how much does it add to the precision in calculating, say, critical masses, flux spectra and so on?

G. I. Marchuk: Yes, the actual core must, in my view, be broken down into regions, the number of which will depend on the specifications and spectral characteristics of the reactor. We must then obtain averaged constants, and solve the problems, for each region separately. I do not think this will be too difficult. In fact we have already started work in this direction, and I hope it will enable us to obtain more accurate results. If we take this process any further, however, we shall be averaging the constants for regions of ever decreasing size. To put it in a nutshell, we shall in that case already have reached a method of averaging in which there are no conjugate functions, but integrated functions for the corresponding energies of the cross-section. In this case, however, the cross-sections will be different at every point, which is inconvenient and does not really answer our purposes.

A. Kania: I arrived a few minutes ago and I am not sure whether I understood the discussion on a possible spectrum standard. There was a reference to californium-252 and to a block of steel, if I understood it correctly. I think, however, that one could also suggest a spectrum in equilibrium or in pseudo-equilibrium, in the manner of an exponential experiment. Such spectra exist; they are reproducible and can be used for calibrating instruments.

R. D. Smith: I think one difficulty with this method is that it would be more difficult to establish the absolute value of the flux than with either of the other methods suggested.

SESSION 7

Chairman: W. Häfele

Introduction by the Chairman

The paper to be presented by Mr. Häggblom deals with reactivity effects of the transverse gap in a reactor, a problem of special interest in fast reactors. The problem of air gaps is a very old one, in fact one of the oldest in reactor theory. The first report on the subject was written by Fermi himself in the summer of 1942, before the world's first reactor became critical.

But let us return to the main subject of the afternoon. It was pointed out earlier in the Seminar that today's fast reactors tend to become more intermediate. As long as the spectra of fast assemblies are really fast, the agreement between theory and experiment is fairly good. This has not always been so, but more precise measurements and improvements in the calculus have led to better agreement. Mr. Hansen's paper furnishes a good illustration of this. The shift of the spectrum to lower energies has one main reason: the use of plutonium and uranium oxide or carbides as fuels. It is more or less agreed that one can hope for higher burn-up if these are used instead of metallic materials. The presence of oxygen or carbide means that more elastic degradation of neutrons takes place, and therefore the spectrum is shifted to a region around 100 keV—sometimes, perhaps, to even lower values. The lower density of plutonium and uranium oxide or the corresponding carbides, and also the requirement for wide volume in the fuel elements in order to contain the fission-gas build-up, lead to large and dilute systems. The large oxide breeder studied by General Electric in San Jose about two years ago is a good example of such a dilute system. The new trend in design also suggests the use of new structural materials, although it is true that stainless steel is still the most important one; but the tendency to contain the high fission pressure build-up suggests greater amounts of structural materials. The concurrent tendency towards higher power densities leads to higher percentage volumes of the coolant, and above all to higher dilution. Thus we are again faced with the problem of deciding what set of group constants to use in order to fit our experimental results, because the earlier agreement between experiment and theory was established with more or less compact systems.

The situation is even more cumbersome where light elements are present; these have always been troublesome in fast reactors. The lighter elements tend to have a pronounced resonance structure in the intermediate range; the best examples of this are iron, aluminium and nickel. If the dilution and the amount of these materials are increased, one really has to meet the problem of scattering resonances in light elements. The scattering scheme of one single element alone causes great difficulties. If there are two scattering elements, the difficulty is that much greater. It is not possible directly to superimpose the results for each element because there is, concomitantly, a mutual interaction of flux depression as a function of energy. This problem may be of even greater importance in the future, when new alloys of scattering components such as inconel may be used. In the final analysis this raises once more the problem, formerly more or less solved, of the cross-sections of the fissile and fertile isotopes.

Paper SM-18/9 was presented by H. Häggblom (see Vol. II, p. 85).

Discussion

J. Chernick: This is a very old problem, and to the best of my knowledge it is still unsolved. It was first treated by Goldberger and Wilkins and Dr. Gregory Breit many years ago; later Freedman and Welton contributed to the subject, and particularly to the solution of the integral equation by perturbation methods, which you have shown. I would say that the expansion which you mention is capable of much greater precision than a simple diffusion theory, and it can certainly be carried out—provided, of course, the flux is slowly varying. Our approach is still largely physical; we try to find out why we get different results and where they come from. One of the problems that have always bothered me is that there are several terms which add to the leakage. These differ, of course, and in the end it is necessary to approximate the flux distribution by some zero-order approximation. One of the most disturbing things is that most of the physical leakage comes from near the boundary and is obviously very sensitive to transport effects.

One interesting question is how many gap widths one has to add to a pile for it to go critical. According to diffusion theory it should be 2 and according to the perturbation method 4, whereas our answer was actually 3. In general I do not think the correct answer, within the limits of very small gaps, is yet known. I am aware of several other contributions to the subject, at least one of which has been published in a British journal. This is by a Soviet author, I believe.

P. Benoist: I should like to make a few comments on Dr. Häggblom's paper. In 1955 we did, with Dr. Horowitz at Saclay, a variational calculation such as has just been described in order to determine the effect produced by the gap in the G-1 reactor. We observed at that time that even for a reactor of such large dimension (diameter approximately 6 m, gap approximately 8 cm), by neglecting the marginal effects, i.e. by admitting the adjoint-flux approximation on which Dr. Häggblom's calculation is based, one over-estimated the variation of reactivity by some 15 or 20%. It is possible to obtain a variational expression which is not based on this approximation if one begins with a more general equation than that of Friedman.

In the second place, I should like to suggest that one need not assume a reactor with a rectangular base. One can set up the expressions in a general form without that assumption, as the only parameter which needs to be considered is the transverse buckling.

H. Häggblom: Yes, that's right, the formula, is given for a square or rectangular reactor but it could easily be applied to others as well.

G. C. Tavernier: On a rather different subject, I would like to ask those who conducted the experiments at Argonne why, during the critical experiments made on ZPR-III, it was considered necessary to have extremely narrow gaps between the two halves of the core when they come together. In other words, there seemed to be a fear that gaps of a few mils might have very important effects on the reactivity or on the form of the flux. Was that predicted on the basis of an existing theory like the one just described, or some other one, or on experimental data?

J. K. Long: There were some corrections made for the gap effects in the reactors reported in our Geneva paper of 1958; to the best of my knowledge, these

were based on experiments rather than on theory. That is, I know we did experiments in which we varied the gap and estimated the effect, but I do not know whether Dr. Loewenstein has done theoretical analyses as well.

D. Okrent: Before the reactor was run we did, for safety purposes, a simple calculation on the effect of gap width. We used a method which, as I recall, was described by Tamor of Oak Ridge. It was, as I say, a relatively simple approach to the problem, but we needed only order-of-magnitude numbers for the purposes of safety. We ran one experiment, I recall, where we produced a gap of either 1/16 or 1/8 of an inch, and this was about as much reactivity as we cared to have available should the two halves run together further than planned. This experiment, fortuitously or otherwise, was in very good agreement with the simple calculation using Tamor's method, and it was on the basis of this combined approach, involving both theory and experiment, that we then made corrections for the small separation between the drawers loaded with fuel.

J. Chernick: I have a comment on the problem of trying to infer by experiment which of several differing theories is correct. I know of the paper by Tamor which Dr. Okrent mentioned: this is merely the diffusion-theory approximation as rediscovered at Oak Ridge by Tamor. As a matter of fact, Tamor found very good agreement with some data which were obtained at Oak Ridge. However, it is extremely difficult to know just what is meant by such agreement because there is a mean free path which varies, of course, with energy in the reactor. You have to assign some number for it—this is quite important—and you may have wall effects which perturb the perturbation even if there are no reflectors; if there are reflectors, of course, the situation is hopeless. I just wanted to add this word of caution with regard to the analysis of our experiments.

Paper SM-18/15 was presented by R. D. Smith (see Vol. II, p. 111).

Discussion

M. Levine: Your last remarks, about deducing the importance functions from the entries in this table, suggest that you could turn the problem round; that is, with the importance functions once calculated, you could deduce the entries in the table. Might that not have been a simpler way of doing this?

R. D. Smith: I think that is a very good question and I shall have to think about it. I think you would find that you could get some of the information that way but not all of it; I think it might be hard to get the magnitude of the effects. You would probably get the answers right qualitatively.

E. D. Pendlebury: We now have a machine code at Aldermaston which takes the output from S_n calculations—the ordinary solution and the adjoint solution—and calculates quantities required in the application of multi-group perturbation theory. Using these functions it is then a simple matter to calculate the effect on reactivity of changes in the cross-section data. All that is required, basically, is the two Carlson calculations followed by a bit of simple calculation on a desk machine.

R. D. Smith: In fact, we attempted to calculate the breeding ratios as well, and this took a rather disproportionate amount of time. It turns out that the cross-section limits you need in order to get a reasonable order of breeding ratios are, in fact, very similar to those you need for criticality calculations, and it may well be that it is necessary only to calculate k_{eff} for this type of survey.

B. I. Spinrad: I am interested to hear that the cross-sections for breeding ratios and for criticality are in quite good agreement. I think it has not yet been pointed out that the cross-sections for criticality must, in principle, be weighted with the adjoints, whereas the cross-sections for breeding ratio should not be.

L. N. Usachev: I should like to ask Dr. Pendlebury what variations he had in mind when he referred to perturbation theory, variations in the breeding ratio or variations in critical mass and k_{eff}.

E. D. Pendlebury: The results of the perturbation calculation can be used to calculate either the change in k_{eff} or the change in critical mass.

R. D. Smith: If I may just comment on Mr. Spinrad's observation, I think the fact that we are asking for the same accuracies is quite fortuitious, because we virtually guessed k_{eff} and the breeding ratio. There is no very strong mathematical relationship.

L. N. Usachev: I should like to point out that the work reported by Dr. Smith is extremely interesting. It describes the first systematic investigation of the effect of discrepancies in the constants on the breeding ratio. I wish to express our deep gratitude to our English colleagues for having passed on to us data which were reported a short time ago. This prompted us to consider how similar work could be carried out less laboriously, and perhaps more accurately, when the variations in constants are slight. Dr. Smith has just pointed out that care must be taken, since an error is possible when the constants vary slightly, in which case the effect will not be significant and there may be a loss of accuracy. We are, in fact, discussing the use of the perturbation theory in calculating the variation in the breeding ratio as a function of the variations in the constants. Dr. Pendlebury said that his laboratory has made various calculations of a similar nature using perturbation theory for a variation in k_{eff}. I should like to point out that the breeding ratio can be determined in precisely the same way, using a modified form of perturbation theory. For this purpose it is proposed to use the method evolved by B. B. Kademtsev, described in a work published in 1957. He generalized the perturbation theory, which had originally been evolved to determine k_{eff}—to be more precise, he did not generalize the perturbation theory, but extended the scope of the concept of the importance of neutrons to cover the general case, the general functional. Subsequently, in 1959, G. I. Marchuk and V. V. Orlov evolved the perturbation theory for the general functional. My aim is to apply this perturbation theory to a particular case of functional, which is capture in U^{238} and Pu^{239}. These two quantities give us the breeding ratio when the reactor remains critical. We have to ensure that reactivity remains constant, that k_{eff} is equal to unity. I have presented this theory here, but I should like to mention that it will be explained in detail in V. V. Orlov's work which will be published in the near future, and also in Marchuk's book. I shall give a brief account of this general theory.

An equation with some arbitrary general operator is examined. If we take, for example, the operator in a transport equation

$$\int \mathrm{d}x\, G^{+}(x', x);\ \int \mathrm{d}x\, G(x_0, x) \tag{1}$$

conjugated with an equation with delta-shaped sources

$$\begin{aligned} LG(x_0, x) &= \delta(x - x_0) \\ L^{+}G^{+}(x', x) &= \delta(x - x') \end{aligned} \tag{2}$$

the solution to this equation is, in fact, Green's function, in which x_0 is the source point and x the observation point. If we multiply this equation by

$$\int \delta (x - x_0)\, G^+ (x', x)\, \mathrm{d}x = \int \delta (x - x')\, G (x_0, x)\, \mathrm{d}x$$

and make a subtraction we obtain the reciprocity theorem

$$G^+(x', x_0) = G (x_0, x') . \tag{3}$$

These results are well-known. The equation is then set out with the same operator, but the delta-shaped source is replaced by source Q

$$LF = Q,\ F = \int Q (x_0)\, G (x_0, x)\, \mathrm{d}x_0 \tag{4), (5}$$

the conjugate equation is formulated with source

$$L^+ F^+ = \Sigma,\ F^+ = \int \Sigma (x_0)\, G^+ (x_0, x)\, \mathrm{d}x_0 \tag{6), (7}$$

will represent in one case the macroscopic capture cross-section, e.g. in U^{238}, and, in the other case, the macroscopic radiative capture cross-section plus the fission cross-section in Pu^{239}. Let us consider, for example, a reactor containing U^{238} and Pu^{239}. The solution to this equation is set out in the form of an integral from Green's function over the sources. This also applies both to F and F^+, i.e. to the conjugate function. We now form two functionals

$$I = \int F \Sigma \mathrm{d}x \tag{8}$$

and

$$I^+ = \int F^+ Q \mathrm{d}x \tag{9}$$

which prove to be equal. This can easily be demonstrated; by substituting expressions (5) and (7) for expression (8) and then for expression (9), we can see that $I = I^+$.

We can now vary the operator as follows: $L \to L + \delta L$. When we do this the function becomes perturbed ($F \to F'$) and the perturbed equation is written as follows:

$$\mathrm{F}^+ \cdot (L + \delta L)\, F' = Q . \tag{10}$$

The conjugate equation

$$F' \cdot L^+ F^+ = \Sigma \tag{11}$$

remains unperturbed; we repeat the same process of cross-multiplication and, by subtracting, we obtain expressions which show that this integral from the perturbation in the operator is exactly equal to the variation in the functional, i.e. to the value in which we are interested

$$\int F^+ \delta L F' \mathrm{d}x = \int Q F^+ \mathrm{d}x - \int \Sigma F' \mathrm{d}x = I^+ - I' = I - I' = \delta I. \tag{12}$$

If this represents the quantity of captured neutrons in the reactor, it would then exactly represent the change in that quantity due to the change in the integral-differential operator. Moreover, this can be any kind of change—the capture cross-section, transport cross-section, etc. Thus we obtained a formula for the change

in the number of captured neutrons of U^{238}, which is exactly the same formula for the change in Pu^{239} with fission and capture. These changes are reflected only in perturbations in the operator, i.e. changes in the operator. It is clear, therefore, that the use of such a formula solves the purely calculational problem to which Dr. Smith has referred. Furthermore, Dr. Zweifel said at our first meeting that there were too many parameters of all kinds which needed to be changed. And indeed the problem requires too complicated calculations for us to solve in our heads. This formula allows for every type of perturbation, having the integrals F^+/F'. If, however, the perturbations are slight, it may be assumed that $F' = F$; having, in that case, F^+ and F, i.e. two functions, we can study the effect of any perturbation in the operator on the quantity in which we are interested, that is, in the particular case in question, on the breeding ratio. I should like to refer, in addition, to the amount of work this involves. Furthermore, it can be taken that Q represents part of this operator and does not include all the terms in the integral-differential equation. Hence, the term representing fission neutrons can be excluded and inserted in the right-hand side as Q. This is in accordance with the procedure usually adopted in the case of successive approximations or source operations. This term, Σ, as I have already pointed out, is not included, nor are the conjugate terms. Then for F, we can use the neutron flux, which will already have been calculated when the critical size of the reactor was computed, i.e. F need not be taken into account again. This is exactly the same neutron flux as is used when calculating k_{eff} and in the perturbation theory for k_{eff}. The conjugate function must be calculated twice with different right-hand parts, once when Σ represents the macroscopic capture cross-section in U^{238} and again when it represents the macroscopic capture and fission cross-section in Pu^{239}. Thus, whereas it is usually assumed that the neutron flux, the importance of neutrons, has been determined, in this case it is necessary to do this twice. This, however, will no longer represent the importance of neutrons but will give information on the cross-section in question. Thus, this formula would appear to indicate that F^+ gives information regarding the cross-section with reference to a particular change in the functional.

Paper SM-18/45 was presented by H. H. Hummel (see Vol. I, p. 231).

Discussion

J. J. Schmidt: In the fine groups, have you taken into account the known fine structure of the inelastic-excitation cross-section of sodium and iron pertaining to the first few levels above threshold?

H. H. Hummel: No, for inelastic scattering I have used only what was available in the standard cross-section set. Otherwise, with this code, one would have to use a finer group structure.

G. I. Marchuk: Can you tell us, Dr. Hummel, whether you managed to make calculations for a uranium-oxide system? If so, what constants did you use?

H. H. Hummel: Mr. Bhide will discuss some results for uranium-oxide reactors in a paper to be given later.

P. Greebler: Your data show larger transport and elastic-removal cross-sections for sodium using Hibdon's results than with BNL-325. What is the physical explanation for this effect?

H. H. Hummel: This is true in the lower two groups, and I think the explanation is simple enough: the curve in BNL-325 follows the bottom of the resonances that were resolved by Hibdon.

P. F. Zweifel: I should like to discuss some preliminary results concerning flux depression in scattering resonances which were obtained at APDA by R. Nicholson, J. Ferziger, Mrs. N. Friedman and myself. An earlier report was given at the Pittsburgh meeting of the American Nuclear Society in June 1961. A final report, in which the techniques described here are applied to explain criticality and danger-coefficient measurements, with particular application to sodium-aluminium substitution experiments in ZPR-III, is now being prepared at APDA by J. Nims, C. Brunyum, R. Mueller, Nicholson, Mrs. Friedman and myself.

Basically, the problem facing us is that in performing the flux average of cross-sections over a group, the firm structure (i.e. resonance structure) of some elements causes a flux depression which makes the average cross-section lower than would be obtained with a more naive method of averaging. Now, such effects are extremely well known in the theory of resonance absorption and have been the subject of exhaustive theoretical and experimental investigations for the better part of two decades. However, the fact that the same effects (i.e. flux depression) may be of importance when the resonances present are primarily *scattering* resonances seems not to have been realized until very recently. High-resolution measurements of scattering cross-sections by Hibdon at Argonne have begun to provide the nuclear data necessary to calculate this scattering-resonance flux-depression effect, and as more data becomes available the calculations may certainly be expected to improve.

Before describing our approach to the theory of this effect, perhaps I should say that our analysis of various measurements in critical facilities, particularly the sodium-aluminium substitution experiments previously referred to, led us to suspect its importance; full details will appear in the final report.

In order to calculate explicitly the flux depressions and hence the group average cross-sections of resonance scatterers, it is necessary to obtain a detailed solution of the Boltzmann equation as a function of energy over the range in which the resonances occur. Since spatial effects are, to first order, separable from the fluctuations introduced into the energy spectrum by the scattering resonances, it is a reasonable approximation to study only the space-independent (infinite-medium) Boltzmann equation.

This equation may be written in the form

$$\Sigma_t(\mu)\,\varphi(\mu) = \sum_{i=1}^{N} \int_{\mu - \ln(1/\alpha^i)}^{\mu} \varphi(\mu')\,\Sigma_{si}(\mu' \to \mu)\,d\mu' + S(\mu). \tag{1}$$

Here Σ_t is the total cross-section of the reactor as a function of lethargy $\mu = \ln(10\ \mathrm{MeV}/E)$, φ is the neutron flux and $\Sigma_{si}(\mu' \to \mu)$ is the differential scattering cross-section for the i-th material in the reactor to scatter a neutron from lethargy μ' into lethargy $d\mu$ at μ. The summation, of course, is taken over all materials present in the system; for the i-th material $\ln(1/\alpha^i) = 2\ln[(A_i+1)/(A_i-1)]$ is the maximum lethargy change in an elastic collision where A_i is the mass of the i-th nucleus in units of the neutron mass. Because we are considering a small lethargy interval (i.e. one group), inelastic scattering may be lumped in with absorption. For simplicity we here neglect both inelastic scattering and absorption. $S(\mu)$ is the (fission) source which, for the purpose of the present calculations, will be assumed to be absent; furthermore, an asymptotic boundary condition will be placed upon φ, i.e. we take $\varphi(\mu) = \text{constant}$ for $\mu < \mu_0$, where μ_0 is some lethargy below the point at which the first resonance has become appreciably large.

In other words, $\mu_0 < \mu_1 - \Delta/2$ where μ_1 is the position of the first resonance and Δ is the "practical width" of the first resonance. The "practical width" may be defined as the lethargy at which the resonance cross-section has fallen below some multiple of the potential (non-resonance, assumed constant) cross-section. Wigner, in his original definition of practical width, took this multiple to be unity. Because the scattering resonances are much lower than the absorption resonances with which Wigner dealt, we generally take the constant to be 0.1. In any event, $\Delta \gg \Gamma$, where Γ is the "resonance width".

The application of this boundary condition assumes that there are no neutron sources (or sinks) or any scattering resonances within several collision intervals below μ_0. In practice, it is frequently possible to break the resonance region into a number of intervals for each of which this assumption is valid—in zeroth-order approximation, in fact, it may be assumed that this situation holds for each resonance, so that resonances are entirely non-interacting. This assumption is generally made in the case of resonance *absorption*, and the calculated resonance integrals compare favorably with experiment. In the case of scattering resonances, on the other hand, we shall see that this assumption is not justified, and we will be required to consider several resonances at a time.

We may divide Σ_{si} into the sum of a constant potential cross-section Σ_{pi} and a resonance contribution $\sum_r \Sigma_{0i}^{(r)} L^{(r)}(\mu)$; here the summation is taken over all resonances r associated with nuclear species i, $\Sigma_{0i}^{(r)}$ is the cross-section at resonance lethargy μ_r of the r-th resonance and $L^{(r)}$ is the line shape

$$L^{(r)} = \left(1 + \frac{4(E - E_r)^2}{\Gamma_r^2}\right)^{-1} \approx \left(1 + \frac{4(\mu - \mu_r)^2}{\Gamma'^2}\right)^{-1} . \tag{2}$$

Γ_r is the full width at half maximum while Γ'^2, the lethargy width, $= E_r \Gamma_r$. Equation (2) holds only if the resonance is sufficiently narrow that $(\mu - \mu_r)^4 \ll 1$ and that the variation of Γ with energy may be ignored; also, the interference between resonance and potential scattering has been neglected.

Under these assumptions, and setting $S(\mu) = 0$, Eq. (1) becomes

$$\Sigma_t(\mu)\,\varphi(\mu) = \sum_{i=1}^{N} \left\{ \Sigma_{pi} \int_{\mu - \ln(1/\alpha^i)}^{\mu} \varphi(\mu)\,d\mu' + \sum_r \Sigma_{\mu i}^{(r)} \int_{\mu - \ln(1/\alpha^i)}^{\mu} L^{(r)}(\mu')\,\varphi(\mu')\,d\mu' \right\} . \tag{3}$$

We may obtain some insight into the solution of this equation by considering the case of a non-interacting resonance. For a *single* element, independent of the variation of scattering cross-section with energy,

$$\varphi(\mu) = Q/\xi\Sigma_s(\mu) \tag{4}$$

where Q is a constant and ξ is the average logarithmic energy loss per elastic collision ($\xi = 1 - [\alpha/(1-\alpha)] \ln(1/\alpha)$. One possible approximation would be to take $\varphi = Q/[\overline{\xi\,\Sigma_s}(\mu)]$ for a mixture, where $\overline{\xi\,\Sigma_s} = \sum_i \xi_i \Sigma_{si}$. This, incidentally, is equivalent to assuming that the Fermi age equation holds, since that equation, under the assumptions made above, reduces simply to

$$\frac{\partial}{\partial \mu}\left(\overline{\xi\,\Sigma_s}\,\varphi\right) = 0. \tag{5}$$

Another possible approximation is the analogue of the NR (narrow resonance) approximation, familiar in the theory of resonance absorption. This consists essentially of setting $\varphi = Q/\Sigma_t = Q/\Sigma_s$ since, for our case, $\Sigma_s = \sum_i \Sigma_{si} = \Sigma_t$. In either the "age" or the NR approximation, an analytical expression may be obtained for the flux-averaged cross-section. We have, in the NR case, averaging over a width $\Delta\mu$ containing M independent resonances belonging to the i-th species,

$$\langle \Sigma_s \rangle = \int_{\Delta\mu} \Sigma_s(\mu)\, \varphi(\mu)\, d\mu \Big/ \int \varphi(\mu)\, d\mu \tag{6 a}$$

$$\langle \Sigma_s \rangle = \left[1 + \sum_{k,i} g_i^{(k)} \Big/ \left(\Delta\mu - \sum_{k,i} g_i^{(k)}\right)\right] \tag{6 b}$$

where

$$g_i^{(k)} = \frac{\pi}{2}\Gamma'_{(k)} = \frac{a_{ki}}{\sqrt{1 + a_{ki}}} \tag{6 c}$$

$$a_{ki} = \Sigma_{0i}^{(k)}/\Sigma_p. \tag{6 d}$$

The index k refers to the resonance, the index i to materials.

Sometimes it is desired to average mean free paths rather than cross-sections. We find still in the NR case

$$\langle \lambda_s \rangle^{-1} = \Sigma_p \left[1 + \sum_{k,i} h_i^{(k)} \Big/ \left(\Delta\mu - \sum_{k,i} (g_i^{(k)}) + h_i^{(k)})\right)\right] \tag{7 a}$$

where

$$h_i^{(k)} = \frac{1}{2}\left(\frac{2 + a_{ki}}{1 + a_{ki}}\right). \tag{7 b}$$

For calculation of danger coefficients, derivatives of cross-sections with respect to number densities N_i are needed. We find

$$N_i \frac{\partial \langle \Sigma_s \rangle}{\partial N_i} = \Sigma_{pi} \frac{\sum_{j,k} g_j^{(k)}}{\Delta\mu - \sum_{j,k} g_j^{(k)}} + (\Sigma_p - \Sigma_{pi}) \frac{\Delta\mu \sum_k h_i^{(k)}}{\left(\Delta\mu - \sum_{j,k} g_j^{(k)}\right)^2} \tag{8 a}$$

and

$$N_i \frac{\partial (\lambda)^{-1}}{\partial N_i} = \Sigma_{pi} \frac{\sum_{j,k} h_0^{(k)}}{\Delta\mu - \sum_{j,k} \left(g_j^{(k)} + h_j^{(k)}\right)} - \frac{(\Sigma_p - \Sigma_{pi}) \sum_k h_i^{(k)}}{\Delta\mu - \sum_{j,k} (g_j^{(k)} + h_j^{(k)})} + \tag{8 b}$$

$$+ \frac{(\Sigma_p - \Sigma_{pi}) \left(\Delta\mu - \sum_{j,k} g_j^{(k)}\right)}{\left[\Delta\mu - \sum_{j,k} (h_j^{(k)} - g_j^{(k)})\right]^2} \sum_k \left[\frac{\left(2 + \frac{3}{2} a_{ki}\right) h_i^{(k)}}{1 + a_{ki}} - \frac{\sqrt{2}\, g_i^{(k)} a_{ki}}{(1 + a_{ki})^2}\right].$$

The analagous expressions for the age-diffusion case $[\varphi \sim (\overline{\xi \Sigma_s})^{-1}]$ will not be

derived because that approximation is not useful in the case of rapidly varying cross-sections.

In Fig. 1 we have plotted $\varphi(\mu)$ throughout, below a single sodium resonance in a sodium-iron mixture as obtained from a numerical integration of Eq. (1) using the RESPECT IBM-7090 code. It was assumed that in this mixture 25% of the

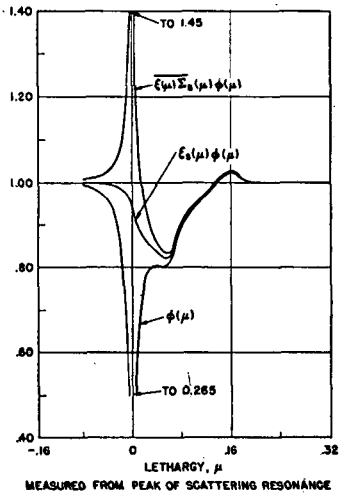


Fig. 1
Lethargy, μ, measured from peak of scattering resonance.

potential cross-section belonged to iron and 75% to sodium, and the Breit-Wigner sodium resonance cross-section was taken to have a peak 2.5 times the total potential cross-section; $\Gamma'=0.02$. Also included are curves of $\xi\,\Sigma_s\,\varphi$ and $\Sigma_s\,\varphi$, from which it is seen that the latter quantity is much closer to being constant than is the former, implying the superiority of the NR approximation over age theory. For this situation, we find the ordinary unweighted average of the resonance contribution to Σ_a to be 0.01. The flux-weighted average in the NR approximation is 0.0050; the correct flux-weighted average is 0.0045. These results indicate the possibly large effect of self-shielding or flux depression, and the fair accuracy obtainable from the NR approximation.

The large trough introduced into the neutron distribution below the resonance causes the interaction of adjacent resonances. In addition, the resonances will influence one another strongly if they overlap. In Table I we summarize the results of some calculations illustrating the effect of interaction between two resonances, considering the same mixture of iron and sodium. Each resonance has $\sigma_0=20$ and $\Gamma'=0.02$. The resonances were chopped off completely at a "practical width" of 0.1. Note that all the resonances are sodium resonances.

From Table I, we note that the NR approximation is fairly accurate for the isolated resonances; we note further that these resonances must be separated by the order five to ten times the width Γ' before they no longer interact strongly. The extremely strong interaction at small spacing is due to overlap of the resonances, but there is still an effect at a spacing of 0.1 which is due to the trough induced by the first resonance. This "sphere of influence" is of course dependent upon the

TABLE I

Spacing (lethargy)	$\overline{\Sigma}_{SR}$	$\langle\Sigma_{SR}\rangle$ (RESPECT)	$\langle\Sigma_{SR}\rangle$ (NR)	$\langle\lambda\rangle^{-1}-\Sigma_p$ (NR)
0.00	1	0.31	0.50	0.31
0.02	1	0.34	0.50	0.31
0.05	1	0.41	0.50	0.31
0.10	1	0.47	0.50	0.31
0.20	1	0.50	0.50	0.31

strength of the resonance. Also, for this particular case, the flux depression tends to reduce the average resonance cross-section by the order of 50 to 70%. Of course, the average scattering cross-section, which is the sum of the potential and resonance portion, will be reduced by a smaller amount. Defining the contribution of the resonances to the transport cross-section as $(\langle\lambda\rangle^{-1}-\Sigma_p)$, we note the even larger flux depression effect. (This calculation was done only in the NR approximation, however.)

For the same mixture of materials, using the resonance parameters published by Hibdon, and computing average cross-sections in the range 50—100 keV, we find (using bars to denote unweighted averages)

$$\overline{\Sigma}_s = 0.0477\ \text{cm}^{-1}$$

$$\langle\Sigma_s\rangle\,(\text{NR}) = 0.0387\ \text{cm}^{-1}, \quad \langle\Sigma_s\rangle = 0.0364\ \text{(machine result)}$$

$$(\langle\lambda\rangle^{-1}-\Sigma_p)\ (\text{NR}) = 0.0356\ \text{cm}^{-1}$$

$$N_{Na}\frac{\partial}{\partial N_{Na}}\langle\Sigma_{SR}\rangle\,(\text{NR}) = 0.037\,\text{cm}^{-1},\ N_{Na}\frac{\partial}{\partial N_{Na}}(\langle\lambda\rangle^{-1}-\Sigma_p)\,(\text{NR}) = 0.033\,\text{cm}^{-1}.$$

In the above results the potential scattering of Na contributed 0.019 cm^{-1}. The NR approximation looks particularly good here, and the unweighted average particularly bad.

TABLE II

Group	Energy limits (keV)	$\langle\overline{\Sigma}_R\rangle$	$\langle\Sigma_R\rangle$	$\langle\Sigma_R\rangle$ (NR)
4	240—300	0.0265	0.0242	—
5	190—240	0.0324	0.0283	0.0320
6	140—190	0.0161	0.0155	0.0163
7	110—140	0.0225	0.0212	0.0207
8	85—110	0.0192	0.0182	0.0195
9	70—85	0.0174	0.0156	0.0165
10	50—70	0.0495	0.0303	0.0422

The machine approach has been used to calculate flux-weighted average Na cross-sections for the Fermi core A; the results are given in Table II. (The NR approximation for this core gave extremely poor answers, a result which is somewhat surprising in view of its success in the calculations for iron-sodium

mixtures. The difficulty here may be due to the fact that the resonances are not narrow compared to the average decrement upon scattering by uranium. Neither are they so wide compared to this quantity that the analogue of the NRIA approximation could be used.) Again, Hibdon's data were used. Here $\bar{\Sigma}_R$ and $\langle\Sigma_r\rangle$ are the unweighted and flux-weighted resonance portions of the cross-section respectively.

We note that for this practical case the flux depression effect is somewhat smaller. When the non-resonant contribution to the cross-section (~ 0.02 cm^{-1}) is added in, the net effect is seen to be at most about 10% except for group 10. This leads one to believe that the effect may not be too important in the Fermi spectrum if the Hibdon data are correct. Theoretical studies of the Hibdon data are now under way in order to clarify this point.

A. Campise: Has there been any effort at Argonne to apply the Hibdon data to reactivity coefficient measurements on ZPR-III?

H. H. Hummel: We have not had time to do that yet. May I make a remark about the application of the narrow-resonance approximation? The error that you are likely to get in applying the narrow-resonance approximation depends, I should think, on the width of the energy region over which you apply it. If you apply it over an energy region which is small compared to the energy loss per collision in the lightest moderator, there would not be any important error. This factor has to be taken into account in assessing the accuracy of the narrow-resonance approximation. For example, I have applied this approximation over groups of about 0.01 in lethargy width.

P. F. Zweifel: The point is that you apply it over one complete resonance or more than one complete resonance, is that not right? As long as the interval over which you apply it encompasses more than one resonance, there is a chance, for example, of interference between the resonances.

H. H. Hummel: No, that is not true, if you apply it over a very narrow energy interval, because you are assuming isotropic scattering in the centre-of-mass system and the neutrons scattered are uniformly distributed in energy. Thus if you are talking about a very small energy interval I cannot see how the narrow-resonance approximation could fail.

P. F. Zweifel: The point is that these resonances are not so narrow.

H. H. Hummel: But I do not need to apply the approximation to a resonance solution; I apply it only to very narrow resonances—where several resonances fall within a lethargy width of 0.01.

J. Chernick: I have not studied this problem for some time. Some of the sodium resonances are quite wide, but I assume your widths are small enough for there to be no very strong change in the cross-section over the group you have chosen.

A. Campise: Could someone estimate how much of the reactivity coefficient of sodium is due to inelastic scattering and how much is due to elastic scattering?

H. H. Hummel: The work of Zweifel and Nims has already given some indication of this.

G. I. Marchuk: I should like to say a few words about the calculation of a neutron spectrum under the influence of a U^{238} resonance cross-section, which was systematically performed in the Soviet Union by Mikhailus. As the results will be published shortly, there will soon be an opportunity to study them more closely. For a long time we struggled with the problem of obtaining a sufficiently effective and accurate algorithm for solving the energy problem in an infinite homogeneous medium. This is the first step towards solving the problem I have

already referred to. As a result of our efforts we were able to make a large number of calculations for various compositions—U^{238}-oxygen, U^{238}-hydrogen, U^{238}-beryllium, and others—as over the full spectrum range from fast to thermal neutrons. Analysis of the results allowed us to draw the following conclusions: (1) In many cases the narrow-resonance approximation is insufficient, and in future we intend to avoid this approximation by attempting to solve equations with exact algorithm. (2) Interference between the resonance and the potential cross-sections is of major importance. (3) To our surprise, we found that interference in the flux, caused by the presence of a system of resonance levels, was very important. We intend to continue our investigations along these lines.

K. Einfeld: I have a question for Professor Zweifel. You said that one reason for performing your calculations was to see whether you could successfully replace sodium by aluminium, and I am wondering to what practical conclusions you came; whether it would be possible, for example, to replace sodium by some sort of aluminium alloy in critical experiments.

P. F. Zweifel: Our conclusion is that we can do it if we know how to do these calculations, so that in removing a certain amount of aluminium we know what the equivalent amount of sodium is. We think that, by means of this RESPECT code and by judicious use of the narrow-resonance approximation (although I must agree with Dr. Marchuk that there are certainly cases in which it does not apply), we will eventually succeed. If I may repeat the point I was trying to make to Dr. Hummel, the fact is that a lot of these resonances are simply not narrow, particularly compared to the energy loss in uranium, which is only 1%. The resonance which we chose there, for example, was 0.02.

P. Greebler: Dr. Hummel mentioned the importance of the resonance scattering of light elements in relation to the Doppler effect in a large, dilute fast reactor. This is certainly to be expected, because of the degradation of the neutron spectrum with the larger transport and elastic-removal cross-sections. However, the light elements do not exhibit significant resonance structure in the low-energy regions where most of the Doppler effect occurs for such a reactor; so the resonance structure of the light elements would not have a direct effect on the Doppler-broadened resonance integrals calculated at these lower energies.

H. Soodak: Both Mr. Okrent and Mr. Smith have spoken today about the need for some standardization of experiments. The point is well taken, and I should like to add that there is a similar problem with regard to calculations, using different data and different cross-section sets. It would be advantageous if there were such a thing as a basic set of calculations, a basic set of reactors or configurations of various kinds. This would enable us to compare different cross-section sets, just as standardized experiments would make possible a comparison of different experimental techniques.

E. D. Pendlebury: For our own convenience we have recently made a tabulation, at Aldermaston, of the critical sizes of various systems which we intend to use as standard systems in the future for the purpose of testing and adjusting group constants, or, more generally, for the purpose of testing and adjusting the microscopic scattering data.

SESSION 8

Chairman: L. Pál

Paper SM-18/37 was presented by D. Meneghetti (see Vol. I, p. 457).

Discussion

A. Campise: Have you used these modified cross-sections to calculate reactivity coefficients of either steel or aluminium measured in these assemblies?

D. Meneghetti: No, we have not. This work is still in progress and, furthermore, there is a real question as to how one should go about calculating the reactivity coefficients in this case. Even though these modified 16-group cross-sections may work for the critical mass, say, of the steel assembly, one may well wonder whether it is proper to use them for a reactivity calculation in the sense of a danger coefficient; it may be necessary to use a finer structure for this calculation, because the resonance effects influence reactivity calculations differently from critical-mass calculations.

W. Häfele: Did you use a one-dimensional or a two-dimensional code in studying heterogeneity effects?

D. Meneghetti: We used one-dimensional DSN slab-geometry cell calculations. The fine-structure geometries of ZPR-III cells are essentially slabs, if one disregards the top and bottom of the drawers and the so-called steel boxes. In this connection it should be mentioned that the use of SNG has, for such thin cells, led to unsymmetrical fine-structure flux plots, even though the cells are symmetrical. For this reason no definitive results could be obtained using the SNG code.

B. I. Spinrad: Have you made any estimates of the possible effects of the missed resonances which are likely to give "windows" in the transport cross-section? Corrections for resonance fine-structure scattering are likely to be underestimates, since the missed resonances (mostly p-waves) will create cross-section "windows", enhancing leakage.

D. Meneghetti: No, these are only preliminary calculations and we used the resonances listed in the library programming of Hummel and Rago.

H. H. Hummel: May I say in this connection that for most of the total cross-section data we simply took the curves in BNL-325, recognizing that the lack of resolution would probably introduce some error, and we have not attempted any theoretical corrections to account for missed resonances.

H. Soodak: Iron is supposed to have a resonance absorption integral at unknown energies. Have you considered the effect of this?

D. Meneghetti: Not as yet. The group cross-sections of steel are now being studied, to determine what aspect of the cross-sections is causing difficulty.

P. Greebler: Your comparison of experimental and calculated Rossi-α values, using uncorrected cross-sections, indicates that the predicted neutron spectrum is too hard. Have you calculated the neutron spectrum for a dilute system with the corrected cross-sections? If so, how much degradation of the spectrum took place (for example, how many more of the fissions were below 50 keV)?

D. Meneghetti: We have not yet reached the stage of plotting the curves. We feel that we need more shimming at the cross-sections, and at this stage we prefer not to make *ad hoc* adjustments. I do not know whether this opinion is generally shared, but I think we have now reached a point where we should

not arbitrarily adjust cross-sections simply to make ourselves feel happier in our calculations, even though this may be very useful in practice. For the most part, the analyses carried out on ZPR-III are not necessarily aimed at calculating a practical system. Much of our work is directed rather at the verification of cross-sections and computational procedures.

G. C. Tavernier: Have I correctly understood, from what Dr. Spinrad said, that in calculating multi-group constants you evaluate the structure of the sub-groups within each group without taking into account the fact that some of these scattering resonances have higher angular momenta?

D. Meneghetti: I think that is right; but perhaps Dr. Hummel will make a statement as he designed the programme with Mr. Rago.

H. H. Hummel: I think one would normally be more concerned with the behaviour of the total cross-section than with the angular distribution. We have to recognize that the angular distribution data have very poor resolution, and this certainly might introduce some error; but as far as the effective transport and elastic-removal cross-sections are concerned, one would be primarily concerned with the total cross-section. We have to recognize that these are not of the resolution we would like because some resonances may be lumped together and remain unresolved. This is bound to introduce some error, but we have not attempted to make any correction for it.

G. C. Tavernier: With the light elements, then, the resonances at angular momenta 1,2 etc. are particularly important. In connection, therefore, with what Mr. Spinrad was just saying about window-effect, could he tell us what experimental measurements have been made on this effect and what conclusions can be drawn from them?

B. I. Spinrad: I was referring to measurements that have not been available experimentally. Our theoretical predictions of cross-sections indicate that certain resonances with higher angular momenta will be completely missed with our present resolution: and since they show both rises and dips they will average out, in a poor resolution experiment, to no resonance at all. When these are taken into account, however, the cross-section dip creates a window.

H. H. Hummel: The windows are most likely to occur where you get interference between resonance and potential scattering.

E. D. Pendlebury: I would like to ask a question regarding the calculation of Rossi-α. I presume that in calculating these you make an exponential transformation of the form $e^{-\alpha t}$ and solve the resulting time-independent equations. When you do this the value of the alpha depends very much on the lowest energy group that you consider. Have you tried to find out whether you have not perhaps gone too low with this energy group? I might suggest that the only way to be sure that one is right is actually to do a time-dependent solution. Have you investigated this problem?

D. Meneghetti: No, we have not. Our method of calculating the Rossi-α is to calculate the effect of delayed-neutron fraction and the prompt-neutron lifetime which would give rise to the trouble you mentioned, and to take the ratio of these.

H. H. Hummel: If I may expand on your reply to Dr. Greebler's question, there is really not much advantage in computing the spectrum with the adjusted cross-sections, because you already have the detailed spectrum from the fine-group calculations, which give you much more information.

P. Greebler: If we already have the spectrum, could you give us a rough estimate of the degradation for the large dilute system that was shown here, as compared with the spectrum obtained using the uncorrected cross-sections?

H. H. Hummel: I am afraid I cannot give any figure.

D. Meneghetti: One really cannot be certain. If you try to draw a smooth curve through this adjusted fine-structure spectrum, you might come to one particular conclusion, but on the other hand the discrete wiggles in the curve might throw you off. It is a question of whether you are considering the mean value or the very abrupt troughs and peaks in the flux curve. It did seem to me, in one case which I examined superficially, that the effect was to make the flux more peaked in the central region, but I do not think that this is really meaningful.

Paper SM-11/26 was presented by A. Kania (see Vol. I, p. 387).

Discussion

W. B. Loewenstein: Were the curves abowing comparisons between theory and experiment absolutely normalized or simply shape-normalized?

A. Kania: We only compared the shapes of the curves; there was no attempt at absolute normalization. As was pointed out to me about ten days ago, these comparisons between shapes can be fairly dangerous. We already know of experiments, particularly with thermal reactors, in which comparisons were made between the relative shapes of calculated and measured responses, and the shapes of the responses were very similar; but, when an attempt was made to obtain absolute measurements, it was discovered that there were differences of the order of 2 or 3. Thus, in addition to the results which we have obtained on the relative values, it will be necessary to repeat these measurements in order to fix these curves in terms of absolute values.

A. Campise: What is the average energy of the neutron as a function of radius in this sodium pool? Have you estimated this, and how it changes?

A. Kania: We have not calculated it, although it would be very easy to do so; we have simply calculated the mean energy of the neutron spectrum which has been shown, i.e. 100 keV for the neutrons which enter the sodium. It seems, however, that we ought perhaps to check the value of this energy; I think, though, that in the sodium, where propagation is chiefly due to neutrons of intermediate energy, the mean energy varies relatively little.

M. Levine: It appears that your multi-group equations do not take the first-flight correction into account. Is that so, and is it not likely to have an important effect on the long-distance propagation of flux?

A. Kania: I do not quite know what you mean by "first-flight correction". We used a multi-group method of neutron transfer from one group to another. Are you speaking of the method of "un-collided neutrons"? If so, then in that sense, I can say that the correction has been made.

G. I. Marchuk: Protection against radiation is certainly one of the most important problems involved in reactor calculations. The simplest theoretical problems have already been solved. Thus, for example, an asymptotic calculation of large shielding can be done in some cases by applying the method of Wick; and calculations of shielding of the order of tons of mean free paths may be carried out by the momentum method. Quite recently, methods of calculating shielding in the immediate neighbourhood of a point source have been worked

out in the Soviet Union—this is the work of S. S. Nikolaishvili, L. N. Usachev, Petrov and others.

The calculation of finite shielding raises rather more serious problems. The basic mathematical method to be adopted in this case appears to be a multi-group approach to the solution of the Boltzmann equation, and such an approach is in fact being used in the Soviet Union. I should like to stress the fact that, in the shielding theory, the problem of averaging constants within the limits of the groups is even more important and more complex than elsewhere, because in this case we must obtain more sensitive characteristics than the critical mass of a reactor or other integral characteristics.

B. I. Spinrad: It appears that in this case, where there is no fission source within the medium, the multi-group equations represent, mathematically, an approximation to continuous slowing-down theory. I wonder, therefore, whether the actual use of the continuous slowing-down theory, appropriately adjusted for the multi-group constants that you have chosen, would not lead to a more convenient mathematical representation. It looks very much, from the spectra you have shown, as though a sort of Fermi age theory is indeed applicable, because at large distances the spectrum is appreciably softened toward the lower eV region.

A. Kania: I think that, in particular for sodium, one could try the Fermi age theory. I wonder, however, whether this method might not be more difficult to apply in the case of iron.

B. I. Spinrad: I think that the method would require more sophistication in the cross-sections, but I do not believe that there would be any difference from a mathematical point of view, and it does have the advantage of being fairly amenable to hand calculation. One can rather easily take the point-to-point kernels if one wants heating at any given spot; that is why I think it is useful.

A. Campise: With reference to Dr. Marchuk's comments on shielding, I think that in a practical reactor design one would not shield against the fast neutrons themselves, but thermalize them first and then shield the facility against thermal neutrons. Therefore, I think that our errors, while still serious from the standpoint of calculation, are not so serious from the standpoint of protection.

A. Kania: It seems to me that there are two problems. The first problem, which is one of biological protection, will be that of shielding against fast neutrons. I do not think that this problem is very real; at least it can be fairly easily resolved, because all that is required is to make sure that the personnel who work in and around the reactor are not submitted to too strong doses. Then there are all the problems at the industrial level connected with the reactor, such as problems of cooling and circulation; these have to be more clearly defined, and it is to resolve them that it is necessary to study this problem of transfer. All the same, I agree with you that it is not the fast neutrons which create the problem but the intermediate spectrum.

Donald Smith: The shielding materials considered in the paper were relatively non-absorbers. In practice the presence of absorbers would not soften the spectrum, as shown in Table III, but would harden it. When this occurs, the number of energy groups above 1.35 MeV may well need to be increased. A removal-theory multi-group programme (RASH) is in use in the United Kingdom and has been found to check with the preliminary measurements made in the shield of the Dounreay reactor.

In reply to Dr. Campise, approximate measurements of the mean neutron energy have been made in the sodium above the core of the Dounreay reactor by taking the variation, with distance, of the ratios of the U^{235} and Pu^{239} fission cross-sections and also the Pu fission to B^{10} (n, α) cross-sectoins. The results were in fair agreement and showed a slow reduction in the mean energy from about 90—70 keV.

C. P. Zaleski: I would like to make a comment on what Dr. Spinrad has just said on the subject of the age theory. I think that his remark is completely relevant; but I think that there may be certain difficulties in using that theory as one approaches the boundaries, i.e. near the fission source or near the source constituted by the blanket of the reactor or, on the other side, near the beginning of the shielding which may be in borated graphite or graphite-steel. At that point I am afraid that the age theory would not apply very well.

I have a second comment, with regard to what Dr. Campise said just now. I think that, as Mr. Kania has indicated, the essential problem which has been studied in this report is the question of the source of heat in the shielding and not that of biological shielding. With regard to this question of source of heat, we agree that a fairly high level of accuracy is necessary, at any rate in the construction of reactors which have certain special factors, i.e. when the shielding is not cooled by sodium but by gas.

Finally, with regard to the remarks of Mr. Smith of the United Kingdom, I think that in the case of the reactor at Dounreay (as indeed Mr. Smith has said himself) what was used was sodium with a thickness of about 40 cm. In our studies, we were interested in propagation at much greater distances, i.e. of the order of 2 m, and then the softening of the spectra is obviously greater than at 40 cm. In addition, we looked into what happens at the sodium-borated graphite or graphite-steel interface, where the conditions are still more delicate and more complex and where the spectrum becomes more and more soft.

I think that Dr. Campise has also mentioned, at one time or another, the softening of the spectrum in the plane perpendicular to the propagation of the neutrons, i.e. in the radial plane. I am not sure if I understood fully, but, if that is so, we have made some measurements radially in order to evaluate the radial leakage and the change of the spectrum. In our calculations and experiments we naturally took account of these factors.

J. J. Schmidt: With regard to Dr. Smith's remarks on the effect of absorbers, I think the rather high mean energy loss per elastic collision is sufficient to overcome the neutron absorption by radiative capture, which is relatively slow in sodium. One should therefore expect a weakening of the spectrum in any case but not a hardening.

Donald Smith: Yes, I think this is true; but once you get well into the sodium I think you may very well find that the spectrum hardens. Moreover, we are not so concerned with 2 m of sodium; in a practical reactor you do not want too much sodium above the core—you want to get really into the shielding. I was thinking more of the borated graphite. There, where inelastic scattering is no longer taking place, I think the spectrum may harden.

R. Zelazny: As shielding calculations can be performed in plane geometry, I might mention that some work is being done in Warsaw, on slowing-down and thermalization problems, using the approach which I described yesterday.

G. I. Marchuk: In reply to Dr. Campise's contention that, for large shielding, intermediate and thermal neutrons play a fundamental role in determining

the asymptotic spectrum of thermal neutrons, I should like to add a word of caution, for in my opinion the role of fast neutrons should not be underestimated. This is particularly important if we are to get a correct impression of the total spectrum of the neutron flux in the shielding; and in fact we are dependent on a knowledge of this for the solution of various problems.

Furthermore, I should like to point out that in the Soviet Union calculations of finite shielding are done in a P_n approximation of the spherical harmonics method. Our calculations show that the results obtained in P_3 and P_5 approximations differ substantially from the corresponding results obtained in the P_1 approximation.

In conclusion, I should like to make the brief, but possibly useful, comment that the P_2 approximation can be effectively applied in calculations of shielding. An important characteristic of calculations using the P_2 approximation is that at large distances from the source (or from the boundaries) the asymptotic flux in the P_2 approximation coincides with the flux in the P_3 approximation. The formal process of calculation is almost identical to that used in the P_1 approximation.

W. B. Loewenstein: I agree with Dr. Smith that one can get spectral hardening. We did an EBR-II mock-up on ZPR-III about a year ago, in which we mocked up the neutron shield and the sodium region outside the neutron shield with aluminium. We have experimental evidence, obtained from cadmium ratio measurements, that the spectrum is softer before one enters the aluminium than inside it. I believe the measured cadmium ratio shows a factor of 2 in this case.

A. Kania: Did the spectrum have a mean energy of 100 keV on entering the aluminium, or was it already much more degraded?

W. B. Loewenstein: I am not sure whether the spectrum is exactly 100 keV, but it is certainly in that neighbourhood. It is a leakage spectrum from the blanket.

D. Okrent: The problems of shielding are clearly diverse, and I should like to mention one which has not been brought up thus far. In the EBR-II reactor, we have the cooling equipment in a sodium pool alongside the reactor. Between the heat exchanger and the core we have a borated-graphite neutron shield which is intended to remove fast neutrons by thermalizing and trapping them. There is a considerable amount of sodium, measured in feet, between the borated shield and the heat exchanger. But one of the difficult problems in shielding is to determine the activation of the secondary sodium system, which is nominally non-radioactive since it does not pass through the core, but which does pass through this large sodium bath. Not only is there the borated graphite between this secondary sodium and the core, but one can actually put a layer of boron, or some similar material, around the heat exchanger during construction. Thus the only neutrons that get into the secondary sodium are those which are quite fast, having passed through the borated shield, a long distance of sodium and the secondary thermal absorber. This problem is rather important, as it decides whether one needs lead shielding around the supposedly non-radioactive secondary cooling; and in dealing with it one has to study the transmission of the faster neutrons over very long distances.

B. I. Spinrad: One is faced with rather different shielding problems depending on whether one uses a large light-metal bath or a hydrogenous medium. In the hydrogenous medium one would expect the very fastest neutrons to cause all the problems, because they are not subject to inelastic scattering except by incidental materials, and are not slowed down. In a bath of sodium or

aluminium, on the other hand, the energy degradation is greater for the higher energies rather than less, so the likelihood is that the higher-energy neutrons will be removed. In such a system one need not therefore worry so much about the high-energy neutrons as the intermediate-energy neutrons.

A. Kania: We quite agree with Dr. Spinrad and Dr. Okrent. Our sodium measurements revealed that the total flux was attenuated, not as a function of attenuation of the fast neutrons, but as a function of the attenuation of this intermediate flux. This was where the difficulty lay.

C. P. Zaleski: I should just like to comment on Dr. Okrent's remarks. In my opinion the problem of shielding, or rather of flux propagation at a great distance, occurs in a very different form in each type of reactor, and depends to a very marked degree on the design of the reactor. In one case the short- or medium-distance problem will be the most important as Dr. Marchuk has already stressed. In other cases it may in fact be long-distance propagation that is more important. Indeed, all these problems may one day be important for reactor designers. That is why in my opinion they deserve to be given rather detailed examination.

H. Soodak: I should like to mention two codes which are of value in shielding calculations. One of these is a Monte Carlo code called MIS. The other, called NIOBE, is a direct numerical integration of the Boltzmann equation, which takes into account all the various angular scatterings and uses exponentials along the neutron-flight paths.

Paper SM-18/71 was presented by H. Soodak (see Vol. II, p. 165).

Discussion

P. Greebler: At General Electric, we have done calculations on an integral superheating reactor consisting of a thermal BWR in the outer region and g fast steam-cooled central region. We were very much concerned with the floodina problem in the fast section. Our calculations showed that with the use of resonance absorbers the reactivity change could be held to less than 1% over the entire range of hydrogen concentration, from a void in the steam passages to complete flooding. We were further concerned about the spatially non-uniform flooding, such as might occur in an accident where water entered from the side or the top of the fast region. The calculations made on spatially non-uniform flooding gave the encouraging result that a solution obtained for the uniformly flooded case also satisfactorily resolved the flooding problem for the non-uniform situations studied.

H. Soodak: I remember being very surprised, when I read the abstract of the last Pittsburg meeting of the ANS, that the reactivity varied by only 1%. Theory shows that the various groups behave differently and that the resonance integrals are very complicated, so I am still surprised.

There is one interesting point which I ought to mention. If you study the reactivity of various reactors with different concentrations of uranium—for example if you take pure uranium-235, or a mixture of 7 to 1, and add hydrogen in a certain volume ratio such that there is 0.1 hydrogen atom per total atom of uranium—you find that the k value remains practically constant, for a given volume fraction of hydrogen, independent of the uranium concentration.

J. Chernick: The system described by Soodak is obviously a very interesting one, and one for which we have few experimental data to confirm the results of these calculations. Have you considered the reliability of your calculations

in the light of the uncertainty in the intermediate values of α? I doubt whether we can place such reliance on the α values for U^{235}, for example.

I should also like to ask how much excess reactivity you have to insert in the form of hafnium in order to be safe in the event of flooding.

H. Soodak: If I remember correctly, you can control the flooding—at least at some points—by adding about 10 at. %, and this does not cause a very great loss of reactivity in the unflooded state.

P. Benoist: I should like to ask Dr. Soodak whether, in the case of small additions of hydrogen, the decrease in leakage due to collisions with hydrogen is really of a negligible order of magnitude as compared with the spectrum attenuation effect?

H. Soodak: If you are speaking of the direct effect of the hydrogen scattering, I believe my calculations showed it to be about 20% smaller than the effect due to the softening of the spectrum.

P. Greebler: In answer to Dr. Chernick's question, the amount of resonance poison that is used depends very much on the system in question—its size, for example, and the leakage relative to other processes. The calculated reactivity changes are influenced by factors such as the uncertainty of the α value for Pu^{239}—in our case the α value for Pu^{239} is more uncertain than that for U^{235}. For a specific reactor design, we should have to know quite precisely how much resonance absorber should be used, and the amount would be influenced by these factors. I should add that the resonance absorber has little effect on the neutron economy in the unflooded condition. It is only when water is added and the neutron spectrum degraded that the capture by the resonance absorber becomes large.

J. J. Schmidt: I believe your α value for U^{235} in the region between 5 and 30 eV was 0.9. I think this is too high, by a factor of 2, as compared with the most recent measurements made at Saclay or Harwell.

H. Soodak: You are probably right. This was an old set of cross-sections, but a lower α value would not change the results I have presented. What the lower α value would change is the analysis of a flooding accident.

J. Chernick: I understand this work has so far been carried out only on U^{235} systems. Has anyone studied a true breeder of the same type of design?

H. H. Hummel: At Session 13 I shall present a paper on a coupled steam superheating reactor; it may well have some bearing on this point.

P. Benoist: I might mention a problem which is different from the one Dr. Soodak has just set forth and yet connected with it. It is a problem one comes up against in criticality studies, that of comparing a fast or intermediate infinite system with a finite system of the same composition, but one supplied with an infinite hydrogenous reflector. There is every reason to believe that the system with a hydrogenous reflector is less reactive than the infinite system, but we have not been able to find clear proof of this.

R. D. Smith: The effects of introducing small pieces of polythene into ZEUS and ZEPHYR have been reported and may be useful as check points for Dr. Soodak. It was shown that the geometrical distribution on the hydrogen added—i.e. whether it filled one channel or two—had a large effect on the reactivity change. The specimens used in ZEUS were of the order of a few centimeters. The effect per hydrogen atom decreased as the total number of hydrogen atoms decreased; reactivity was certainly not linear with the number of hydrogen atoms.

H. Soodak: The GODIVA results which I calculated were for small amounts, and there the effect was supposedly proportional to the hydrogen added.

Paper SM-18/72 was presented by M. G. Bhide (see Vol. II, p. 177).

No discussion

Paper SM-18/70 was presented by J. W. Webster (see Vol. II, p. 149).

Discussion

R. Zelazny: Could you tell us something about the boundary conditions which must be imposed on the functions, especially the adjoint functions?

J. W. Webster: The conditions we imposed there were the same as on the forward fluxes, namely continuity of the joint flux and d E times the derivative of the joint flux.

H. Soodak: I should like to demonstrate a known general method of obtaining the importance of a neutron for any desired effect. Thus the equation for the neutron flux φ may be written as $0=k\,P\varphi - \mathrm{L}\,\varphi$, where P is the fission-neutron production operator. The usually defined neutron importance m, is then obtained by demanding that the variation of k as expressed by $k=\int m\,L\,\varphi \Big/ \int m\,P\,\varphi$ be stationary with respect to variations in φ. There results the usual equation for the neutron importance m, along with the boundary conditions which arise out of integration by parts.

The same method may be used to obtain the importance, say a, for some absorption quantity A. Thus A might be $Q\,\varphi/P\,\varphi$, or the ratio of absorption in some element to total reactor power. In this case, the equation for importance a, along with boundary conditions, is obtained by demanding that the quantity $A=\int a\,Q\,\varphi \Big/ \int a\,P\,\varphi$ be stationary with respect to variations in φ. Then δA may be determined directly from δQ and δP (cross-section changes) without having to evaluate the effect of flux changes δQ,

$$\delta A/A = \int a\,\delta Q\varphi \Big/ \int aQ_Q$$

$$= \int a\delta P\varphi \Big/ \int aP\varphi.$$

P. F. Zweifel: I assume Dr. Soodak's point is that first-order perturbation theory is variationally stationary with respect to first-order changes in the functions. I should like to suggest another way of getting the same results, which in my opinion is far simpler. One simply uses the well-known adjoint equations. We can take the first equation in Dr. Webster's paper as an example; it is a diffusion equation, and he proves that it is self-adjoint. Actually, the one-velocity Boltzmann equation is self-adjoint by virtue of the well-known reciprocity theorem for one-velocity neutrons. I may be wrong, but I cannot see why it is necessary to use the complicated method shown by Dr. Soodak when the answers are already well known.

G. E. Hansen: I think Dr. Soodak means that there are really many adjoints, because there are many operators which one can form associated with the transjoint equation.

Paper SM-18/1 was presented by W. Häfele (see Vol. III, p. 19).

B. I. Spinrad: Is the reactivity step assumed partitioned according to the k_{11} in this particular example or is it taken in the fast system only?

W. Häfele: It is taken in the fast system only.

B. I. Spinrad: What are the numerical values of the strong and the weak coupling examples?

W. Häfele: The weak coupling is based on the set of Avery, which means assuming k_{11} equal to 0.94. In the first example $k_{11}=0.94$, $k_{22}=0.23$ and $k_{12}=0.462$, and the other quantity follows from the criticality condition. In the strong coupling $k_{11}=0.6$, $k_{22}=0.75$ and $k_{12}=0.5$.

J. Chernick: You mentioned two lifetimes, one of about 10^{-7} s in the core, and one of 5×10^{-4} s—a very large lifetime—in the reflector. I should think this might bring on a dangerous situation if you went above prompt critical; the lifetime in the reflector is now so long that I cannot see how these neutrons could be of much help if a runaway started.

W. Häfele: I am not sure that I understand what you mean, but this value of 5×10^{-4} refers to a real thermal component, not merely to the reflector. It is assumed that there really is a thermal component.

J. Chernick: I am referring to the time which the neutrons need to return from the reflector. A 1% increase in multiplication factor of the core would lead to a 10^{-5}-s exponential rise in power if the slow return of neutrons from the reflector were ineffective.

W. Häfele: The effect which you are mentioning is described by the so-called retardation effect $e^{\omega l_{22}}$, and this is precisely what we want here; otherwise one could not obtain a reactivity partition $l=0$.

B. I. Spinrad: I might try to put this into physical terms. We know that the fast reactors have relatively slow periods below prompt critical. The question is whether one really gains anything from the coupling. Could you give us an idea, then, of what excess k above prompt critical your so-called strong coupling case will handle?

W. Häfele: In my paper there is a sub-zero value approximated, which is between the stable and the unstable behaviour, and depends on the strength of the coupling. In the third example it is 3%; this still gives a stable behaviour, whereas 5% gives an unstable behaviour. However, this is simply a question of design, of the strength of the coupling.

D. Okrent: I would like to argue that it is not clearly necessary to design a coupled reactor so that the fast region has no possible way of reassembly into a supercritical reactor without a thermal component. If one considers a ramp addition reactivity rate rather than a step function, and assumes a zero or negative Doppler effect, the full reactivity potentially available will not get in before the reactor pushes or blows itself apart.

W. Häfele: Of course, if you operate around the stable condition you have this longer lifetime, and the melt-down might well be achieved before you have a shorter lifetime; I admit this explicitly for the case of a small reactivity step. But our idea was to find out how quickly the effective life-time changes if you have reactivity steps which are not necessarily small, i.e. how broad the region considered by Avery actually is, and whether his reactivity partition is valid.

P. Greebler: In the examples you gave, the Doppler coefficient of the fast region was positive. How much weaker could the coupling between the fast

and thermal regions be, and still have stability, if the Doppler coefficient of the fast region were negative? How sensitive is the required coupling to the magnitude of the Doppler coefficient?

W. Häfele: Much of the work which we are doing currently in Karlsruhe centres around this problem. You are quite right that a negative Doppler coefficient allows us to use a much weaker coupling; it may be enough merely to thermalize the blankets. On the other hand, one must face the following problem. I think Dr. Weinberg has always emphasized that one of the greatest disadvantages of fast reactors is their inventory; some thermal reactors, for example, have an inventory only one-tenth that of a fast reactor. If you want to approach a fast reactor with a low inventory, you must surely enlarge the enrichment; and then you really face this difficulty of a positive Doppler coefficient.

B. I. Spinrad: One point I should like to make is that the fast reactors used in coupled systems, even for quite large powers, tend to be of a rather smaller size than the all-fast reactors; this is because of the considerable amount of power in the thermal zone. For the same total power one may therefore expect that there will be less chance of getting a positive sodium coefficient, for example.

My second point is that, in terms of the strong coupling argument, it appears that the thermal contribution to the reactivity can be considered almost strictly as an extra group of delayed neutrons—a rather large number of short-time delayed neutrons—and it appears, moreover, that this time-delay does not change much. If the excursion continues to gain reactivity, it will certainly swamp this group along with any other delayed group, but it does seem that the gain would have to be very considerable. In other words, the non-linear effects of the excursion would have to persist; you would have to have a positive power coefficient well beyond where you might expect this to be reasonable.

W. Häfele: I might say that we have proven that these analytical equations can be written in a form which comes extremely close to the form which one obtains by assuming an additional group of delayed neutrons.

A. Campise: If we are discussing power systems in which the central fast region is part of a large dilute fast reactor, I wonder whether the critical mass would indeed be lower. The statistical weight drops off as you go out, and any thermal section you put on may require more fuel than you think.

W. Häfele: Our experience thus far has been that this does not happen, because the tendency of thermal reactors to have a very much lower inventory survives against the requirements in the intermediate range.

A. Campise: Nevertheless, you would have larger radius requiring more fuel.

W. Häfele: Yes, but our studies have shown that this effect is smaller than the other one I mentioned.

J. Chernick: While including the reflector neutrons as an additional delayed-neutron group may be a satisfactory approach for small perturbations, I think that many groups with various time constants would be needed for large perturbations.

W. Häfele: We have merely studied the possibility of using a delayed group; actually we use the difference equation in order to generalize the kinetic equations. Of course, if you wish to make a really careful study of a specific reactor, you must be more cautious and return to the full multi-group set. Our purpose was merely to get rough information as quickly as possible.

B. I. Spinrad: I would like to give one further example to show that the concept of the extra group of delayed neutrons can be useful. When the original coupled

reactor experiments were performed at Argonne on the ZPR-V system, there was difficulty in reconciling neutron lifetimes as measured with $1/\nu$ poisoning experiments with those measured by neutron statistics. This difficulty was resolved almost precisely by simply incorporating this one extra delayed group. For small perturbations it is indeed a very useful concept.

P. Schmid: My question concerns the excess temperature from Eq. (42). I conclude that you take the excess temperature as being proportional to the instantaneous fission rate. This seems to imply that your solutions for the excess temperature as a function of ω are valid only as long as the reactor period is longer than the thermal time constant.

I have a suggestion as well, and this concerns the representative value of λ. Your value is derived from an averaging of $1/\lambda$ values; but as the most interesting part of your diagrams is well above $\omega = 10$, I think it is advisable to take an average value for λ and not $1/\lambda$. This would mean a value of about 0.4, which is 5 times as high as the value you chose.

W. Häfele: I think Dr. Schmid is absolutely right, but we were really not interested in this at the time. We merely wanted an example with which to evaluate our equation, and were not concerned with the very specific values on which these assumptions were based. It is easy to adjust to a given situation, but that is not part of the problem considered here. Of course you are right that the equation $\theta_2/\theta_1 = S_2/S_1$ is not always valid; but if you consider a specific reactor system you can use your correct correlation, and that applies, of course, to the λ value as well.

SESSION 9

Chairman: B. I. Spinrad

Introduction by the Chairman

Perhaps I could begin this session most profitably by showing you a few simple equations relating to the cost of fuel cycles. I shall not explain their derivation in any detail, but I think it may be useful to see which elements of cost are due to the various physical and engineering factors comprised in a breeder or converter reactor.

I have divided these elements into four parts. The first are those processes which are dependent on the standard burn-up, i.e. burn-up in terms of fissions or megawatt-days per ton of total fuel:

$$\frac{F + P}{72\,B} \text{ mill/kWh.}$$

This represents the costs of fabrication and that part of the reprocessing which depends upon the handling of the total fuel. If the fabrication and processing costs are in dollars per kilogram (and we have here a conversion factor into U.S. currency) and the burn-up is in per cent, the result emerges in mills per kilowatt hour. To take an example, if we have costs of 500 dollars per kilogram for fabrication and 100 dollars per kilogram for processing (I have deliberately made these rather high), the result, in mills per kilowatt-hour, is 8/B, which indicates that, so far as this particular aspect of reactor economics is concerned, we stand to gain a great deal by increasing burn-up or decreasing the normalization within the actual charges.

The second element that enters into fuel cycle costs is that which is dependent on the burn-up per atom of fissile material loaded into the reactor. The equation we get here is essentially proportional to the enrichment over the burn-up. Actually, it has three parts which are, at the current state of the art, almost identical: one part is due to the processing of the fissile material—those charges which are incurred after the fissile material is separated; a second part is due to the investment charges on the critical system while the fuel is going into the process cycle; and a third part is due to the chemical losses of the process cycle. These are roughly equivalent; and if we take a 20% enrichment as a standard case, the figure we obtain is 1.87/B. This number can be reduced (which is what we want to do for all our fuel cycles) if we reduce the unit charges, the residence time of the processed materials, the chemical losses and the process charges. It can also be reduced by securing more fissions per atom of fissile material initially loaded, and one way of doing that is to reduce the enrichment.

The third element of cost is that which is dependent on the specific power, and this can be written as

$$0.47 \frac{\lambda}{P_s} V_p$$

where V_p is the value of the product material or the fuel material (they are taken as being equal) in dollars per gram; λ is the use charge on the critical investment in the reactor; and P_s is the specific power of the reactor in megawatts per kilogram of fissile material. If we use typical figures the result is $0.85/P_s$, which indicates that specific power levels of the order of 1 MW per kilogram of fuel are acceptable.

The fourth and final element is what we call burn-up costs. These can be expressed, very approximately but handily, as it turns out, in mills per kilowatt-hour, 2 (1 − CR) − CR being the conversion ratio. This represents a credit, therefore, where there is breeding, and a debit where there is not.

All of these elements indicate that there are certain parts of the fuel cycle which can be optimized only by increasing burn-up or reducing the out-of-pile cycle charges which are incurred in fabrication and processing. I think we can make our discussions clearer if we are careful to specify just what part of the fuel cycle we are referring to at any given moment.

Paper SM-18/13 was presented by S. Yiftah (see Vol. **II**, p. 257).

Discussion

P. Greebler: You stated that it appeared advantageous to use dirty plutonium since it yields a lower critical mass and a higher breeding ratio than clean Pu^{239}. Since the critical mass is based only upon Pu^{239} and Pu^{241}, should not the value of the Pu^{240} and Pu^{242} be taken into account in evaluating the relative merits of clean and dirty plutonium?

S. Yiftah: In my paper, I merely wanted to show what results one gets by taking only the critical masses of Pu^{239} and Pu^{241}, and when one speaks of critical masses one is usually referring to the thermally fissionable isotopes. I did not go into any calculation of the cost of Pu^{240} and Pu^{241} in order to find out what might be economically advantageous.

G. Vendryès: The comment I wish to make actually concerns Dr. Spinrad's introduction. I merely wished to say that at Saclay we have made a number of economic calculations on the cost of fast reactors to determine, in particular, the influence of the more important parameters, such as combustion rate and specific power; and using methods which do not go beyond simple arithmetic, we have arrived at expressions rather different from those mentioned by Dr. Spinrad just now. As will appear from the paper to be read later by Mr. Zaleski, the conclusions that can be drawn from them are quite substantially different. So I need not dwell on this point; I merely wished to mention that the economic calculations are not as obvious as they might appear.

R. D. Smith: Could Dr. Yiftah give us a physical as well as a mathematical description of the processes which result in a change of sodium temperature coefficients with plutonium isotopic composition?

S. Yiftah: The best I can do is to suggest that you study the spectra which are displayed in the appendix to my paper. I cannot discuss the problem in detail, but by examining the spectra in the core and blanket you can see the effect, the softening and hardening of the spectrum in the various cases.

G. C. Tavernier: My question relates to the composition of the plutonium B. I have not yet studied the original article mentioned in Ref. 3 of your paper, but the composition of the plutonium B seems to me rather unusual at first sight. Perhaps you could comment on it, because the Pu^{241} content seems to me to be very high as compared with the Pu^{240}. If I remember rightly, for an EBR-type reactor ranging from 7500 to 10000 MWd/t, the Pu^{241} content is still lower than the Pu^{240} content.

S. Yiftah: I am sorry, I can only refer you to the article itself where the details of the calculations are given. The next paper may well answer many questions concerning the various isotopic compositions that one can get—for instance when recycling plutonium several times until equilibrium composition is reached.

J. J. Schmidt: I understand it was with the plutonium C fuel that you obtained particularly high breeding ratios. I would like to ask you two brief questions. Firstly, what thermal value and what energy dependence of $\bar{\nu}$ did you assume for Pu^{241}? Secondly, what value did you assume for the energy dependence of α for Pu^{241}?

S. Yiftah: These values were all taken from the book, as I mentioned earlier. You can find them there along with the reasons why they were chosen.

H. H. Hummel: I believe the authors assumed that Pu^{241} has the same properties as U^{233}, and that may help to explain the effect of the isotopic composition on the sodium void coefficient.

S. Yiftah: If you compare the breeding ratios that we reported a year ago with those reported earlier, you will see that ours are higher in most cases. The main reason for this is that the ν of U^{238} was found to be higher than the value previously used, and this has been checked by various measurements carried out in the USSR, at Saclay and various other places. Another point is that the fission cross-section of Pu^{241} has been measured at Los Alamos, and we therefore have experimental results on the fission cross-section for the whole range of interest. The capture cross-section of Pu^{241} and some of its other characteristics were assumed to be similar to those of U^{233}, and not Pu^{239} as was formerly assumed. U^{233} and Pu^{241} have the same spin and the same lower resonance structure, and the new assumption was based on this analogy and a few supplementary calculations.

Paper SM-18/41 was presented by D. Okrent (see Vol. II, p. 271).

No discussion

Paper SM-18/2 was presented by K. Ott (see Vol. II, p. 213).

Discussion

G. C. Tavernier: There is just one point in the report of Mr. Ott and Mr. Jansen which I do not quite understand. It is this sentence: "The higher plutonium isotopes cannot be included in the definition of the breeding gain, because they have effects similar to those of catalysts unchanged by time." I do not quite understand the implications of the sentence and how these results are arrived at.

K. Ott: In the steady state, in this case, we have no net production of higher Pu-isotopes, therefore the breeding gain mentioned should be correct.

B. I. Spinrad: The seminar held at Argonne on the physics of breeding produced a number of alternative definitions of breeding. There is, I think, only one thing that must definitely be said about a definition of breeding gain, and that is that it must be applied to a system at steady state. It makes no sense at all to apply it to a system which is in a transitional phase.

Paper SM-18/50 was presented by M. M. Levine (see Vol. II, p. 247).

Discussion

G. C. Tavernier: For some types of fast reactors, fissium has been considered as an alloying element for the fuel elements. If I remember rightly, the volatile and gaseous fission products do not, in this case, form part of the technical fissium, and I should like to know the proportion of poisoning due to these

volatile and gaseous fission products as compared with that to the solids remaining in the technical fissium.

M. M. Levine: I think the gaseous fission products are made up of a very few species for the most part, but I can make no quantitative estimate at the moment. However, because there are so many fission products, and because their cross-sections are so nearly alike at high energy, I do not think that the omission of a few of them (or the evolution of a few of them into a gaseous state) would make a great deal of difference.

J. Chernick: If I may add a word to Mr. Levine's statement, although there are a great many gaseous fission products, most of them are not very serious as poisons. It is the rare earths and, in general, materials with very many resonances which constitute the greatest source of poisoning in all reactors.

Paper SM-18/3 was presented by P. Engelmann (see Vol. II, p. 229).

J. Chernick: Could you tell us what you include in your fuel inventory in estimating doubling times?

P. Engelmann: We include the internal and external inventory and assume a reprocessing schedule of 200 days. We have not yet considered whether the processing plant should be on site.

J. Chernick: Your central fuel temperatures of 2200 °C must be close to the vaporization point of the oxide, and this would leave no margin for the Doppler effect.

P. Engelmann: These figures are preliminary and should not necessarily be adopted for the purpose of design. We had assumed that an enrichment of 33% would just guarantee a zero Doppler effect, but we now find that we may have to use an enrichment of 25% or lower.

W. Häfele: May I clarify what I meant when I referred to 10% burn-up at Session 4? This should be defined as 10% burned fissile atoms per total fissile atoms at the beginning of the process. This refers only to Pu^{239}, not to all the heavy atoms.

Paper SM-18/30 was presented by C. P. Zaleski (see Vol. II, p. 301).

Discussion

P. Greebler: My question concerns your second conclusion regarding fuel cost *versus* burn-up. This is expected to be a parabolic relationship, yet the curve which you showed indicated a break at 30000 MWd/t, which appeared to me much sharper than a parabolic relationship. Could you explain that, and could you also say how sensitive that relationship is to your assumptions regarding the fabrication cost of the fuel?

C. P. Zaleski: I think Dr. Greebler's first comment may be partly due to confused presentation. I do not know whether you noticed that the zero was well below the diagram and that the asymptote was at 3 new centimes or thereabouts, which was due to the fixed cost. I think, therefore, that what you observed is more apparent than real, unless a mistake was made in the calculations; that is always possible as we have to do a good many calculations. We should be grateful if anyone who finds mistakes would point them out.

As regards the second point, we also varied the refabrication rate and the reprocessing rate, and of course we took different conditions for short cycles and long cycles. You may say that the short cycle for oxides is perhaps not very

realistic, but according to our pyro-metallurgy experts, it may not be impossible either, provided that the systems of fabrication are specially adapted for short cycles. For the short cycles we have costs, which I hope I can find for you as I do not know them by heart; in any case they are in an annex to the report. The cost is, I think, about 250 NF per kilogramme, or 600 NF per kilogram in the case of long aqueous treatments; but these figures should be checked as I may be mistaken. There is also of course, a further cost per pin, which is independent of the diameter—this is obviously an essential assumption in this study and one which is open to discussion. I apologize for this incomplete reply, but I think you will find the rest of the information in the paper.

J. Chernick: Please do not think this remark facetious. I do not see why you should worry about the cost of plutonium. If you really can get economical power from breeder reactors at the price you have accepted, I do not see why you cannot sell it at the same and thus have an internally consistent price scheme.

C. P. Zaleski: We took the cost of plutonium into consideration all the same, because we considered two possibilities: a completely closed cycle and a cycle involving thermal and fast reactors. In this case, it is important to have an equilibrium price in order to decide whether to go on building thermal reactors or whether to build only fast reactors. I would not go so far as to say that in 10 or 15 years only fast reactors should be built but, in fact, that is not far from our conclusions. And we should then have a completely closed cycle, for which the cost of plutonium is of no importance.

J. Chernick: Nevertheless, if the fast reactor is really the reactor of the future you need not concern yourself with thermal reactors; far from having plutonium to sell, operators of thermal reactors would have to buy it from you.

A. Kania: The curves show very clearly, in fact, that if the price of plutonium is varied, the cost does not change. This is a characteristic of the system in which the price paid for plutonium immobilized in the reactor is offset by the scale of plutonium obtained from breeding. That is why, for a given reactor design, the cost per kilowatt-hour is almost independent of the price of plutonium.

C. P. Zaleski: I should like to be a little more specific on this point. I think that on the whole what Mr. Kania has said is quite correct, but nevertheless the price of plutonium may be important in deciding whether the breeding rate should be increased or whether, if necessary, a lower breeding rate can be adopted. The shape of the curves might change slightly if the basic price of plutonium varied; because, after all, when investing new plutonium in new reactors (since there is still a surplus of plutonium—we have not yet reached a closed system, we are expanding), it is necessary to know what price to put on this surplus plutonium; and this provides a reference point for determining the importance of the breeding ratio.

J. Chernick: I assume what you mean is that you have to pay for some uranium besides ?

C. P. Zaleski: The essence of what I have said is that, of course, you have to pay for the uranium which goes into the reactor and that you must also know the new investment cost per kilowatt and the difference between this and any credit from the breeding gain, which will, of course, depend on the price of plutonium.

B. I. Spinrad: It seems to me that the reprocessing cost is the same regardless of specific power: it depends on a fixed burn-up and the necessity of processing a certain amount of fuel which has a very defininite quantity of fission products

representing energy that you have made; and this charge is independent of specific power. Therefore, the specific power affects only the interest charge in the critical mass of the core, and it should not show a minimum in the curve.

C. P. Zaleski: I think I mentioned, though perhaps I did not make it sufficiently clear, that the minimum comes not from the reprocessing cost, but from the refabrication cost; and we have taken the unit cost of refabricating pins as constant. It is obvious that if the pins have a greater diameter and if we have the same number of megawatt-days per ton, then fewer pins will have to be refabricated to obtain the same number of megawatt-days per ton. This is what gives you the minimum cost.

SESSION 10

Chairman: G. Vendryès

Introduction by the Chairman

The subject of kinetics and safety of fast reactors which we are dealing with today is one of particular importance. It is here that the most serious, and sometimes the most false, attacks have been launched against fast reactors, which have been suspected or accused of being fundamentally dangerous devices, or at least ones requiring very delicate operation.

A thorough examination of the matter is called for in order to dispel unfounded ideas and, in particular, to establish quite clearly the conditions to be fulfilled in constructing completely safe fast reactors.

The suspicion cast upon them no doubt arises from the parallel which may be drawn by some badly informed people between a fast reactor and an atomic bomb. It is hardly necessary to point out that a fast reactor, by its very structure, cannot under any circumstances behave remotely like an atomic bomb; they are as comparable as a horse and a fish might be on the ground that both have two eyes and a mouth.

It is nonetheless essential, in a fast reactor as in any other, to avoid reaching a state of prompt criticality by eliminating any possibility of a large and rapid insertion of reactivity and preventing the emergence of any positive power coefficient. The accident which might result in such a case could seriously damage or even destroy the reactor itself. The meeting to-morrow morning will be especially devoted to this question and to the precautions which must be taken to avoid such a situation, precautions which are becoming ever better known.

Below prompt criticality, the neutron kinetics of a fast core do not differ essentially from those of a thermal core. The fact that the percentage of delayed neutrons in plutonium is three times less than that in U^{235} is certainly a disadvantage, but this is largely compensated by the large quantity of delayed neutrons from the blanket which are normally present in the core, and by the fact that a fast reactor requires a lower reserve of reactivity than a thermal reactor.

Experience shows that, if care is not taken, fast reactors operating at a high specific power are liable to have an irregular kinetic behaviour and, in particular, to enter into oscillations which are unacceptable in normal operation. These phenomena are due to the conditions under which coupling between the power and the reactivity takes place. The reason why they are able to occur in the case of fast reactors lies ultimately in the small dimensions of such reactors and has absolutely nothing to do with the fact that the chain reaction is sustained by fast neutrons. This morning we shall hear several papers in which these problems are discussed at length, showing the extent to which our knowledge of the subject has developed in recent years and indicating that we now have available the necessary means for prediction and study of these instabilities.

In concluding this brief introduction, I should like to add two observations. The first is that studies on the dynamics of a fast reactor, to a far greater extent than purely static work, presuppose a very precise knowledge of its detailed mechanical structure; they provide the classic case of studies where neutron, thermal and hydraulic considerations—to confine myself to these three—must be closely associated.

The second observation is that an enormous amount of work remains to be done in studying kinetic problems and related matters. For example, there is reason to fear that the expulsion of sodium and other phenomena which prove effective in giving rise to negative prompt power coefficients in our present-day reactors will, in the larger reactors of the future, prove ineffective or even act in the wrong direction, and new methods will have to be found. The importance of this problem alone may be so great that the characteristics of future fast reactors will have to be determined primarily by the requirements of their kinetic behaviour.

Paper SM-18/5 was presented by F. P. Storrer (see Vol. **III**, p. 3).

Discussion

A. Campise: The problem of distortion to which you referred is not found only in fast reactors; in some cases it is more serious in intermediate and epithermal systems.

G. Vendryès: On this subject, it is well known, in the case of present-day fast reactors, that mechanical distortions of the core structure have a very great effect on reactivity. In the case of future reactors of larger dimensions, it is conceivable that these distortions will have less influence and that it will perhaps be difficult to derive an adequate negative reactivity coefficient from them. Have you had an opportunity as yet to apply the method which you described to large-scale fast reactors?

F. Storrer: No, I have not applied this method to any reactor other than the Fermi. However, it is my impression that the effect of structural distortion on reactivity is directly proportional to $(k_\infty - 1)/k_\infty$, and therefore it is the fraction of neutrons which leak from the core. When I say directly proportional, this is exactly the case for a bare reactor and approximately the case for a reflected reactor, as shown by Dr. Dietrich. As far as very large reactors are concerned, I think it possible that k_∞ is even higher: for example, it is my impression that dispersion reactors could have $k_\infty = 1.5$ and that the counterreaction from reactivity contributed by expansion might therefore be of the same order of magnitude as for small reactors. I do not know whether it is k_∞ which has been calculated for such reactors, but it is my impression that it might be rather high, and I think that this is the most important criterion.

It has, incidentally, been pointed out to me that my paper contains what might be called a gap: I present the formalism of the method and then I present Fermi's results, without showing the method of calculation. The only reason for this is that since the structure is a very complicated one, a report of some forty pages would have been necessary to formulate it. I could not consider including it in this paper, but there is an APDA memorandum setting out in detail our calculations for the Fermi reactor.

P. Greebler: In answer to Dr. Vendryès' question, we are now doing some stability calculations on the very large fast oxide reactor, and we find that the mechanical distortion effects are indeed very small for such a large reactor, as one might expect.

P. L. Balligand: What procedures—in particular, what modifications in the core structure—could be used in an attempt to avoid the drawbacks shown by your calculations?

F. Storrer: When the assemblies of fuel elements are attached to a downstream head plate, expansion or distortion of this plate must be prevented from giving rise to wide variations in reactivity. Such variations would be subject to a long delay and could therefore cause instability. In order to prevent substantial coupling between the plate and the core, it is desirable to make the core compact towards the centre. At this level the distortion will then be practically nil, and that which takes place above and below will be slight and in opposite directions. This compacting of the core is generally achieved by applying small plates to the assembly casings. Moreover, these small plates also eliminate the positive bowing coefficient which would normally be present. Another solution would consist in eliminating the head plate, in which case the assemblies would be held in place only at their upper end.

A. Campise: It is certainly true that by careful design—for example by removing the head plate and having the fuel elements in, say, hexagonal form—one can avoid bowing due to restraints at both the upper and lower extremities of the fuel elements.

F. Storrer: I did not quite understand your point about the hexagonal form. I think that another solution is obviously to do without this head plate and to have the fuel elements fastened only at the base. In this way, the bowing coefficient will always be negative and, as there is no structure to be considered, there will be no problem.

L. N. Usachev: In the reactors for which calculations were made, what numerical values did you obtain for the temperature coefficient, in degrees, and the power coefficient, in kilowatts ? It would be very useful if you could provide this data immediately.

F. Storrer: Later on I shall be giving a brief report on the Fermi reactor, and I shall take the opportunity to give these quantities.

Paper SM-18/49 was presented by R. R. Smith (see Vol. **III**, p. 43).

Discussion

G. Vendryès: Thank you, Dr. Smith, for your most interesting paper. We all know that the experiments in regard to stability or instability which have been carried out on EBR-I have had the great merit of bringing to light phenomena which are absolutely fundamental to the stability of fast reactors, and which also have an interest extending far beyond fast reactors themselves. The very thorough investigations to which these experiments have given rise have substantially increased our knowledge and their importance cannot, therefore, be over-emphasized, particularly in regard to the accident which occurred in EBR-I, and led to fusion of the core. I should not like to go as far as saying that we were very pleased about this accident, but I think that we must nevertheless regard it as an extremely useful experience, not only because it is an experiment which one would perhaps hesitate to repeat too often and which has itself been very instructive, but also because this accident, this experiment, has forcibly directed attention to the phenomenon of bowing. Turning to the results which you have presented, if I may be allowed to comment on them, it would seem that the prompt positive component of the power coefficient in the Mark II was fairly clearly explained by the phenomenon of rod bowing, which gives a quantitative explanation of the observed facts. As far as the delayed negative coefficient is concerned, I must say that the situation does not seem to be entirely satisfactory. I certainly agree

with you that not only are there very strong presumptions in favour of its being the lower shield plate which is responsible for the delayed negative component, but indeed it is difficult to see what else could be responsible. Again, it is quite clear that the mechanical and thermal phenomena in the areas above the core are extremely complex, and that their analysis undoubtedly presents great difficulties; it is also very difficult to obtain an exact picture of the distortion which takes place in all these upper areas of the reactor. However, as you yourself have told us, you had no experimental transfer function of which you could be absolutely certain, and your paper shows that the counter-reaction coefficients, adjusted to correspond to the measurements taken, are somewhat difficult to justify; I will not say that these facts give rise to doubt, but they appear to rule out any absolutely definite conclusion. I think that while the Mark II was undoubtedly too unstable, it may perhaps be a cause for regret that the Mark III and Mark IV are too stable, and do not enable us to get to the bottom of the problem. Perhaps an intermediate core in which, for instance, the positive coefficient due to bowing had been eliminated, but in which this retarded negative coefficient had been maintained more or less as it was in the Mark II, would have made it possible to settle the problem more conclusively; and I repeat that this problem is absolutely fundamental to our further studies on the kinetics of fast reactors.

C. P. Zaleski: I merely want to ask Dr. Smith, in regard to his suggestion for a completely compact and rigid core, whether he does not think that there are at least two difficulties: firstly, a simple question of machining tolerances, and secondly a question of the strain which might appear in completely compact cores, and which would increase the engineers' difficulties.

R. R. Smith: I was referring to the experience we have had with the Mark III core, which had a system of fuel rods with ribs and a central expandable tightening rod. It was not an expensive process to incorporate these ribs and to fabricate the tightening rods. There is some question, however, as to whether we ever really got a rigid core in the nominal Mark III loading. The reason why we did not observe rod bowing effects when the ribs were not actually tight was probably because, as the rods tended to bow, there were adequate clearances at the lower rib plate to give a very modest negative coefficient.

I really do not know what stresses in the mechanical design will have to be accepted in order to have rigid cores in future reactors. This will depend on the type of fuel and on the number of rods that have to be used. A rigid core may be more expensive, but I think that it will minimize rod-bowing effects and thereby almost certainly those movements in the upper structure which cause displacement of the fuel at the core centre.

Donald Smith: We have long felt that until the EBR-I Mark II behaviour could be quantitatively explained, the general stability of fast reactors would remain in question, so we are delighted to learn that Dr. Smith's work at Argonne has finally cleared up the situation fairly well. We have done some similar work—also on the EBR-I Mark II—and we came to much the same conclusions as the Argonne group. I think it is fairly clear from Kinchin's and Bethe's work that some form of transport lag has to be expected in the mechanism. This lag clearly points to the lower shield-plate, I think, but our explanation of what actually happens in the shield-plate was somewhat different from that suggested by the Argonne group. Essentially, we did not consider this dishing effect; we considered the effect of the various tube plates—the various components of the shield-plate bearing on each other thus causing the distortion. Some time ago we studied

your results to see how your model compared with ours. We had some difficulty in reconciling the amplitude of your terms with the physical dimensions and clearances in the reactor. I believe you mentioned that you had some 50% discrepancy in the amplitudes. It seems to us that our model gives better agreement with the amplitude, and like you we get very good agreement with the frequency. The essential point of our model is that we take into account the cosine distribution radially; in your model you ignored it. We think that this is probably the proper way of treating this tube plate region.

I should like to make a few other small points. You mention that Bethe's model had a spiral which will give a succession of resonances; but I think it is fairly clear that this would not occur because Bethe's model was lumped. If you took a distributed model at these higher frequencies, I think it is quite clear that there would be interference between them, and these higher modes would not be visible.

In the second place, you suggest that a blossoming effect would give a very large prompt negative temperature coefficient. I think this can be overdone in fast-reactor designs. We do not want too large a negative power coefficient, because it would oblige us to invest that much more reactivity. One big advantage of fast reactors over thermal reactors is that they enable us to work with a fairly restricted amount of available reactivity.

Lastly, I would like to mention a simple experiment which we did at Dounreay, one rather similar to your dishing experiment. We took a mesh rather like a car's radiator grid, cooled the outside and heated the inside in what we hoped was something like a cosine distribution. Our purpose was to find out what happens to the mesh at about half-radius. It is subject to forces from two directions, the expansion of the hotter inner zone and the pressures from the rather stronger hoop stresses which push it inwards. We wanted to know whether the points in this half-radius moved outwards or inwards. In fact our model collapsed, but before that happened we were fairly well satisfied that the half-radius points tended to move slightly inwards, as the model was heated up, and then outwards. The effect was very small, however, and I do not think that it would contribute to feedback.

R. R. Smith: When I mentioned a blossoming effect I only meant to suggest that this effect—which works to your advantage—is more desirable than a pure rod-bowing effect, which is positive and works to your disadvantage.

B. I. Spinrad: I fear that I must take issue with our Chairman in one respect at least, because I really think that further enquiry into the quantitative aspects of the EBR-I instability is scarcely justified. We have a qualitative explanation which is within the right order of magnitude, and we can intuitively see what corrections would be appropriate. We would probably not repeat the design in any case, precisely because of the difficulties associated with it.

A second point which I should like to discuss is that raised by Mr. Zaleski: namely, the engineering difficulties posed by a rigid core. The EBR-II core will be a fairly rigid one, and the problem is therefore of some concern to us. If the material of which the core is made weakens sufficiently and the core tends to grow, there will apparently be some slight permanent distortion. This will not affect the safety of the reactor, for it will still be possible to remove heat; but we do intend to study this matter when the fuel elements come out after irradiation.

Another point which interests me is that of providing for a large negative power coefficient in the actual design of a reactor. I think that the particular feedback

mechanisms we have discussed here, involving the clearances between successive plates, provide an excellent basis for a design principle which we can adopt. What we want is a small negative coefficient around the operating point and a large negative coefficient when there is any very strong deviation from it. One can get this by adjusting the clearances of the support points properly, and I think, therefore, that it is an extremely useful concept.

J. Chernick: Firstly, I should like to ask Dr. Spinrad what merit he sees in these temperature coefficients. If they are delayed by several seconds I think they may do more harm than good. Secondly, I wonder to what extent the transfer function, on which Dr. Smith's work and the British work is based, is itself based on the displacement model of Dr. Storrer, and how many of the physical parameters have been put in *a priori* and how many of them have been "fudged", i.e. adjusted in the light of subsequent knowledge, in order to get agreement with the frequency curve. Thirdly, I believe there were two power levels at which the EBR-I oscillations were observed, and I wonder whether the people who have done these analyses have explained both of them. Perhaps Dr. Spinrad could answer first, and the other parts of my question can be directed to either Dr. Smith.

B. I. Spinrad: The answer to the first question is very simple. The mechanism proposed by both Dr. Storrer and Dr. Smith to introduce a negative feedback depends primarily on making the rod bowing (which is very fast) push the rod outward, and therefore it is a prompt coefficient.

J. Chernick: I am satisfied with that answer.

R. R. Smith: May I answer the second part of Dr. Chernick's question? At the frequencies at which we find resonances Dr. Storrer's expression reduces to a very simple form, a factor $(1 + \tau_f)/\tau_c$, which is nothing more than the ratio between the total heat capacities of the components and the coolant divided by the total heat capacity of the core. Time constants are therefore not very important here. One can make great errors in one's heat transfer assumptions and still end up with a very small error in phase and amplitude calculations at the lower frequencies.

As to the "fudge" factor, we made no great attempt to force agreement between the model and experimental data because we did not feel that such agreement would prove anything. There is really no way of getting a unique solution of these components. What we wanted was to get some feeling for the magnitudes of the three power coefficients, and to see whether they were credible in terms of physical processes which we understand; and I think we have argued fairly cogently that they were. Now, it may well be—although I have no idea myself—that the transfer function data obtained in 1955 left quite a bit to be desired. There was some question about the amplitude for example, because at very high frequencies the amplitude of the transfer function was some 20 to 25% below the calculated curve. We attributed this to an inexact knowledge of the worth of the oscillator rod. The phase data were even worse. At the higher frequencies the phase values were unrealistically high, and the most we could do was to assume that at the higher frequencies our feedback effects were negligible and then make a normalization on that basis. If we are wrong about that, i.e., if there is a very large amount of feedback at the higher frequencies, I am sure that it would have a great effect on our results. For lack of something more definite we had to make this assumption, as otherwise there would be no way of comparing experimental data with calculated results. There were fudge factors in the transfer-function data, of course, but there was a normalization.

Donald Smith: With regard to this question of "fudge" factors, it has already been pointed out that the time scales involved are fairly well known. When you come to deal with the amplitude of these effects, however, you encounter difficulties with engineering clearances, as you must know whether these tolerances were met or not. We too made no attempt to force a fit in this direction. I should like to discuss sometime with the Argonne group whether the amplitude of the mechanism that they proposed is really large enough; to be out by a factor of 2 is really rather a lot.

In our calculations we started with the physical dimensions of the reactor as taken from the design, but, like you, we did not take fundamental heat-transfer coefficients, as these are not very well known. When possible we tried to use measured temperature distributions; in other words, wherever experimental data were available we used them, and in that sense we did not start from first principles.

R. R. Smith: After considering the transfer-function data—and some of us studied them in great detail—we concluded that the ion chamber used for the recording device seemed to be non-linear. This could result in a higher resonance peak than may actually have existed and would go a long way towards explaining the two-fold discrepancy between the calculated amplitudes and those obtained experimentally.

C. P. Zaleski: With regard to Dr. Spinrad's remark, I should like to make it clear that I personally classed the EBR-II reactor as among the less rigid of the fast reactors now being built and I thought there was no system which held the core together, at least in the cold state. Until the gradients are established, that is, there is no mechanical system of holding the core together to make it compact. Of course, what is or what is not reasonably compact is purely a question of definition.

J. K. Long: In further response to Dr. Chernick's remarks, I should like to point out that the later model—EBR-I Mark III—was analysed directly from the best possible assumptions with regard to the heat-transfer data, and no adjustment factors were used. It was possible to calculate the temperature patterns in the reactor with fair accuracy, using Storrer's equation and basic heat-transfer data. One has considerable latitude in choosing a model to translate these temperatures into a feedback, depending on how one assumes the reactor to expand. However, it is possible to find some models which will give the proper feedback for Mark III. Mark III is a simpler system than Mark II because the clearances which characterize the latter are absent.

D. Okrent: When Pierre Clauzon was at Argonne, he did a relatively simple, straight-forward calculation to estimate the change in the transfer function which one would expect if a limited amount of bowing were allowed to occur in the Mark III. This core is intended to be rigid, but by changing the configuration one can allow a certain amount of bowing to take place; and in this case the agreement between Clauzon's calculation and the experiment was really better than one might have expected, considering the errors in both calculation and experiment. It is generally felt then, I think, that while there is not perfect agreement between calculation and experiment, the agreement is nevertheless quite good. There are special problems, of course. For example, it is difficult to know just how much radial expansion occurs in a clamped core with some clearances and without a really rigid clamp.

The experience with Mark III shows that we can build stable fast reactors. It is regrettable, of course, that we do not have a physical model for the Mark II

instability which we can hold to be really trustworthy. A number of suggestions have been made, and some of them, I must admit, could be fudged into agreement with one experiment or another; but none was clearly acceptable. I believe that the work which Dr. R. R. Smith discussed today has largely dispelled the mystery surrounding the Mark II.

Whether the difficulty lay entirely in the lower shield-plate or whether three plates were involved is not, I think, a crucial matter, and I agree completely with Dr. Spinrad, that it is not worth a major investigation. In my opinion, future experiments should be designed to investigate important new problems; and one such problem, if I may offer an example, is whether an assembly can be designed with exactly the desired bowing coefficient. To sum up, it is most fruitful to look ahead, and we should avoid any excessive refinement of past problems.

R. R. Smith: With regard to Dr. Okrent's remarks, there is in fact a strong possibility that downstream perforated plates contributed to the difficulty. I mentioned, for example, that the seal-plates were also subject to motions, and at frequencies not far removed from the resonance. There is a kind of amplifying mechanism, a kind of harmony at the resonance frequency, and the action of the seal-plate could be positive. However, it would be 180° out of phase with the lower shield-plate and there would automatically be a two-fold increase in the feedback. If we take this factor into account, I think we can further close the gap between our calculated and experimental results.

Paper SM-18/28 was presented by P. P. Clauzon (see Vol. **III,** p. 93).

No discussion

Paper SM-18/44 was presented by H. H. Hummel (see Vol. **III**, p. 107).

Discussion

R. R. Smith: Have you considered the possibility of the control rods growing downward during a power increase, or the possibility of fuel-rod bowing in the control elements?

H. H. Hummel: No, but perhaps Dr. Loewenstein would like to comment on that question.

W. B. Loewenstein: The problem which Mr. Smith is referring to derives from the fact that the fuel sub-assemblies are supported at the bottom and the control sub-assemblies at the top; this means, in effect, that under heating the fuel rods tend to move upward and the control rods downward. So far we have investigated this effect for amplitude, but not for time-dependence. The magnitude of the effect for 12 control rods is not greater than 0.1% $\Delta k/k$; more likely 0.05% $\Delta k/k$.

R. R. Smith: Is there not a clearance between the rods and the shroud in the control elements? I will not suggest that this is important, but it might affect your results.

H. H. Hummel: You mean a clearance which will allow for bowing? Yes, I think that is true, but we have not done anything about it yet.

J. Chernick: Are your fuel rods capable of vibration or not?

W. B. Loewenstein: It is not at all obvious what "vibration" means here. The fuel pins are packed very tightly into the sub-assemblies, and of course there are spiral wires around the clad which fit into the sub-assemblies. There

is really very little room for vibration in the normal configuration. There might be if the initial structure were destroyed, but then, of course, there would be trouble on other counts as well.

General discussion

G. Vendryès: Before going on to a more general discussion, I shall ask Mr. Usachev, Mr. Donald Smith and Mr. Storrer to give us brief accounts of the BR-5 reactor, the Dounreay reactor and the Enrico-Fermi reactor, in that order.

L. N. Usachev: I shall very briefly indicate some of the results of measurements carried out on the BR-5 reactor. Let me begin by saying that there is some purpose in considering both the temperature coefficient and the power coefficient. The temperature coefficient of BR-5 is negative and is equal to $2.8 \times 10^{-5}\%(\triangle k/k)/°C$. It is brought about, obviously by the overall warming-up of the core, the expansion of the lower lattice and all rods, and the resultant decrease in activity. It is measured at constant power, with only the input-temperature changing; this can be done either by electric heating of the sodium or by accelerating the operation of the pump, which produces heat and warms the core.

I shall now pass on to the power coefficient. Even on the BR-2 reactor (a mercury-cooled reactor, much smaller than the BR-5, which stood on the site now occupied by the BR-5), we noticed the effects of a position power coefficient, which can, as we know, be very dangerous. We therefore took great care in designing the BR-5 to avoid a positive power coefficient, or at least to reduce it to a minimum. Potential bowing in the rods was analysed, the points where the rods could best be reinforced were studied, and a remote control system for packages of rods was installed. Perhaps I should point out that we did not manage to achieve everything that might have been done in this direction, since the constructors were unwilling to comply with all the physicists' requests. In any case, when the reactor was completed, the power coefficient was measured during the build-up process, before the reactor went onto full power. Measurements were made with an automatic regulator and a controller was used to effect a rapid change of power, the regulator reacting in approximately 2 s. In order to maintain a power level, say, approximately 10% above the previous level, the rod must first increase the reactivity and subsequently decrease it. Figure 16 of our paper SM-18/82* shows two curves, the low-reactivity curve and the high-reactivity curve. The low-reactivity curve corresponds to low power, i.e. to the low absolute power value. When the power effect is very small, then, as we can see, it can be disregarded. So, whenever possible, we specially select a power level at which this can be done.

However, we are also now carrying out measurements at high power. Then we get the curve shown in Fig. 16. This curve is a record of the behaviour of the control rod. We can see that in a given area the control rod has to be inserted lower than would be necessary if there were no power coefficient. This means that the power coefficient is positive. At about 20 s it begins to decrease under the influence of the negative temperature coefficient, caused by general heating. In Fig. 16, again, we can see the size of the power coefficient, measured in centimeters on the automatic regulator, during a 300-kW power change. The total length of the regulator can be taken as 20 cm, which corresponds

* See Vol. **III**, p. 315.

to approximately one quarter of the delayed neutrons or, to use the American notation, 25 cents for the entire regulator. We can see that 300 kW corresponds to 1 cm, which is a very low figure. For a power level of 5000 kW, a power coefficient of this size is not dangerous, since it does not go altogether beyond the limits set by the proportional number of delayed neutrons. Even if we increase power up to the 5000-kW mark, the positive change in reactivity will correspond to about one quarter of the proportional number of delayed neutrons. After obtaining these experimental results, the reactor was safely put into normal operation without untoward incidents.

I. I. Bondarenko: I should like to add something to Mr. Usachev's remarks. It should be emphasized that the behaviour of temperature and power coefficients in the actual reactor is extremely complex, since unforeseeable factors often take a hand. However, as Mr. Usachev has said, we took great care in designing the BR-5 to eliminate the positive temperature coefficients or, failing that, to reduce them to a minimum. All the measures outlined by Mr. Usachev were taken. What I wish to stress is that, although these efforts bore fruit, the actual results were more complicated than one could possibly have foreseen. In particular, it turned out that in the reactor there are both negative and positive components of the power coefficient. Here I am speaking of the prompt power coefficient, i.e. the change in reactivity which follows directly upon a power change, and not the delayed reaction which takes place asymptotically. These two components behave in a very complicated manner; in particular, they are not entirely in linear relation to the power level. The result is that when the reactor was operating at lower levels we did not observe a positive power coefficient, which only appeared at levels above approximately 60%, and even then not in all cases. However, measurements showed that it is not high, and can never lead to an uncontrolled excursion, since even in the worst of cases there is only a short delay before the heated sodium re-enters the reactor, i.e. before the long-term power coefficient becomes effective, and during that time nothing dangerous can happen.

My purpose in drawing attention to the complexity of these phenomena is to emphasize that we must come down to concrete details. For instance, the relation between the negative and positive components of the power coefficient depends on the placing of the control instruments, and principally on the position of the reactor's main control device—the corrective cylinder surrounding the core. These phenomena are now being studied, but although sensible hypotheses have been put forward to explain almost all our observations, the complexity of the processes involved is such that we cannot yet claim to understand fully the detailed behaviour of the reactor's temperature and power coefficients. Meanwhile we have proved to our satisfaction that the reactor's observed characteristics cannot give rise to an accident.

D. B. Hall: Is it your opinion that the observed small positive component of the power coefficient is due to structural variations of fuel or rods?

L. N. Usachev: Yes. We studied or rather assessed the influence of the Doppler effect, and found it had no appreciable influence whatever. The basic factors involved are expansion due to temperature, and bowing of various kinds, similar to those described in previous papers.

J. Chernick: Do you feel that you could go to much higher powers now? In other words, do you understand all the relevant phenomena well enough to extrapolate to, let us say, hundreds of megawatts?

I. I. Bondarenko: The BR-5 reactor has reached its planned power level, and we shall not increase it with the present design. I doubt whether all the results can be applied to more powerful reactors. In my opinion, the most important lesson we have learned is that the phenomena are more complex than any simplified diagram, so that it is essential to take great pains over the design. When we come to design other reactors, this experience will be useful. All piping, joints between metal parts, cables and rods used to operate the control mechanisms—all these may give rise to unexpected circumstances and must be carefully studied, as they have a considerable influence on the working of the reactor. In other words, there are two aspects to the problem. On the one hand, there are all those questions that can be regarded from the point of view of pure physics. But these do not cover all aspects of the problem, which is also, to a considerable extent, one of detailed design.

Donald Smith: We have seen in the earlier session of this Seminar that important differences exist between the comparatively simple but calculable reactors favoured by reactor physicists, and the more complex designs which are inevitably evolved by reactor-design engineers. A similar state of affairs exists, and will exist, for kinetic studies. Since these cannot readily be made on the versatile low-energy assemblies, the interpretation of the results of experimental kinetic studies will be difficult, and a large number of such studies will be required. At the present time there are many theories chasing few results. At present, the Dounreay Fast Reactor (DFR) cannot contribute much, but it is hoped that a programme of kinetic studies will commence shortly. A few remarks about our present knowledge may be of interest.

Calculation of temperature coefficient. The power coefficient is derived from the calculated space-dependent temperature coefficient and the temperature distribution. The latter is difficult to measure, so the former is a valuable partial check on the power coefficient. Originally, one-group perturbation theory was used and the important contributors were, in about equal proportion, longitudinal fuel expansion, radial structural expansion and coolant density reduction. Recently, more sophisticated treatment has been found to give about the same result.

Measurement of temperature coefficient. In measuring the uniform temperature coefficient, we raised the coolant temperature from 140° to 170° in 12 hours using non-nuclear heating. The reactor power was at 3 W. Experimental details included the effect of relative movements of core structure, control-rod-positioning mechanisms and control-rod-position indicators. The effect of a very small amount of entrained gas was also determined (it required a 1% correction for the temperature coefficient). The result was 33×10^{-6} per deg C, which is to be compared with the calculated value of 40×10^{-6} per deg C.

Calculation of power coefficients. A study of the power coefficients and their time characteristics are required for two purposes: safety assessment, and to estimate the confidence which can be placed in theoretical predictions of reactor behaviour. The accuracy required for the safety assessment is less, and simple models are adequate. The kinetic behaviour of the DFR has been studied using seven temperatures—coolant inlet, core coolant, blanket coolant, core fuel and blanket fuel, core can and blanket can. The basic equations were very simple $P - a\,(\theta_F - \theta_C) = b\dot{\theta}_F$. It is possible that for slowly varying temperature changes these equations are fairly sound, but to satisfy ourselves that we understand all the complexities of reactor behaviour it will be necessary to reconcile

calculated and experimental results over time intervals or for oscillation frequencies at which distributed effects preclude the use of simple lumped models. Calculations have been made giving the reactor response in which discrete radial and axial regions have been used for a unit cell comprising a characteristic fuel channel. The results are broadly similar to the very simple calculations though they differ especially at the high frequencies.

In a practical reactor system, there are a number of design features which tend to confuse the experimental data. For example, an important contributor to the DFR power coefficient comes from the radial movement of the control-rod carriages. This depends partly on the radial movement of the stainless-steel skirt which surrounds the stationary part of the core, but also on the heat flow from the core radially outwards and from the blanket radially inwards since the control rods are over-cooled compared with the core.

Thus the control-rod-carriage movement is a function of both core *and* blanket coolant. At high core-coolant velocities, the control-rod carriage is well cooled, but at lower core-coolant velocities the distributed thermal capacity of the carriage has to be considered.

This is an example of the complications that a practical reactor entails. The removal of the fast-acting temperature coefficient due to fuel-element bowing has made the calculated behaviour of DFR appear very stable indeed.

At Dounreay we propose to measure the steady-state power coefficient for a range of core- and blanket-coolant flows and thereby endeavour to separate the various components. The reactivity oscillator will also be used firstly as an overall safety check and then to examine more closely the precision with which the kinetic behaviour of a reactor can be predicted.

A few preliminary results have already been obtained. The temperature coefficient has been mentioned. By reversing the blanket-coolant flow we endeavoured to separate the core and blanket contributions. It was not a clear-cut experiment, but the result left us in no doubt that we had considerably overestimated the core and underestimated the blanket coefficients though the sum was reasonable.

Similarly, when the reactor operated at 1.5 MW with reduced coolant flow, we were able to obtain a measurement of the steady-state power coefficient. This unfortunately agreed with our prediction. In between the prediction and the experiment we had reconsidered the control-rod-carriage effect mentioned earlier, and we have difficulty in convincing some people that our apparent agreement is fortuitous.

The oscillator has been used at low power (100 kW), and a number of improvements have been incorporated. It has not yet been used at high power.

We are anxious to obtain experimental data with which to test our ideas and which will enable us to judge the degree of complexity which is required in the calculations.

F. Storrer: Firstly I shall give the information requested by Dr. Usachev regarding the temperature and power coefficient in the Enrico Fermi reactor. These are all calculated values, of course, as the reactor will not go into operation before the end of the year. The calculated isothermal temperature coefficient is $34.7 \times 10^{-6}\%$ $(\Delta k/k)/°C$, just a bit larger than the figure Dr. Usachev quoted for the BR-5 reactor. The power coefficient is —0.2458 cents per megawatt, and β_{eff} is 0.0066. The override, that is the reactivity to compensate for the isothermal increase of temperature from 517°C to 550°C (the operating inlet-coolant temperature), is 9.58 cents; and the override at constant inlet temperature

going from 0 to 200 MW is just about half a dollar—49.16 cents. The total override from the shut-down condition to full power, including the isothermal override and the power override, is 58.74 cents. The main components of the power coefficient are the radial expansion of the core due to expansion of the stainless-steel cans, which touch each other, and the fuel-pin axial expansion, which is the second largest component of the total coefficient. The figures for override from 0 to 200 MW, in cents, are 18.84 in the first case and 12.96 in the second.

These coefficients were calculated from basic data, and we used the danger-coefficient measurements made at the ZPR-III facility when they were available. For some materials we measured only the average worth over the whole core, and we used a calculated distribution of the worth to obtain the coefficients. For other materials we also measured the worth, as in the case of U^{235}, for example, where we measured the shape of the worth.

Let me say a few words about how we solved the problem of bowing. As you know, the reactor has a top hold-down plate, so that if nothing were done there would be a positive bowing coefficient. We decided to use pads a little above the middle of the sub-assemblies, and in this way we eliminated the positive coefficient. In fact, we obtained a very definite negative coefficient, because the sub-assemblies touch each other and expand against the pads. We had considered other possibilities, one of which was to have a flexible structure so that the hold-down plate would prevent the sub-assemblies from rising but would not prevent the top from moving radially. Another possibility which we investigated was a double-walled can in which the sodium would flow spirally between the two walls, thereby eliminating a large fraction of the temperature gradient. But we eventually abandoned those two solutions in favour of the pads. We think that with this system we have also eliminated the delayed negative coefficient due to expansion or distortion of the hold-down plate, since the core is essentially compact at the pad lavel. When the hold-down plate is lowered onto the top of the sub-assemblies, the tops are gathered together so that there is contact at all times between the sub-assemblies—or at least we hope there will be. Therefore, we do not expect any discontinuity or any appreciable non-linearity.

We have done some dynamic studies of the Enrico Fermi reactor, using both analogue and digital computers. Of course we will verify the stability of the reactor with oscillator tests, both during operation of the automatic control system and without it. But I should like to talk briefly about the work that we have done with analogue computers. We have used two of them, one for high frequencies and the other for problems at lower frequencies. At high frequencies, where there is no appreciable feedback around the coolant, we needed a more detailed representation of the core than a lump model. In our analogue study, therefore, we divided the core and blanket into twelve sections. On the other hand, the computer which we used for the lower frequencies simulated the whole system—the reactor, the primary coolant loop, the primary heat exchanger, the secondary loop and the steam generator. In this latter case we were interested primarily in control problems rather than stability problems, and as the whole thing was on a larger time-scale we did not make a detailed representation of the reactor; we used only one section for the core and one for the blanket.

Our calculations indicate that the reactor will be stable. Of course we have calculated detailed feedback transient functions and total reactor transient

functions. While we do not presume to say that they are correct, we feel sure that they would have to be wrong by many orders of magnitude before the reactor would be unstable. When I say many orders of magnitude, I mean, making the most pessimistic assumptions, something like a factor of 50 on the power level. But even if you neglect the amplitude attenuation of the feedback with frequency, and assume that with increased frequency you can introduce phase lags but no amplitude attenuation, it can be shown that this reactor will be stable by a large factor, primarily because the total override in going from zero to full power at constant inlet-coolant temperature is half a dollar. If it had been 2 or 3 dollars, a detailed examination of the feedback attenuation would be required. In fact, we have made such a study, but as we are not too sure of it we think it is well to have a simple order-of-magnitude estimate.

We have also studied the possibility of instabilities at low frequency, where the whole system would oscillate, and we found that this, too, was impossible by a substantial factor. I might also mention that in our analogue computer representation we automatically included the heat produced by the decay of fission products; but that is just a small point. Thus we have eliminated, as far as we know, all positive power coefficient. Furthermore, we feel sure that the Doppler effect is negative, considering that the enrichment is 25%. In our simulator runs we have simulated start-of-accident conditions with a one-cent-per-second insertion of reactivity up to an arbitrary total insertion, and it was found that even with this relatively high insertion rate there was no power over-shoot. This means that if, for example, you insert one dollar in this fashion, the power will never go above the equilibrium power level at one dollar, and the feedback will catch up soon enough to prevent any over-shoot. Thus if we limit the insertion rate, there is absolutely no danger of reaching prompt criticality.

A. Campise: I should like to know whether a stability analysis was performed on the EBR-I before it was constructed. If so, was the analysis similar to the one Mr. Storrer has described? What improvements does Mr. Storrer's description reveal, or rather what features were not included in the analysis of EBR-I?

B. I. Spinrad: I can only reply that the EBR-I was constructed before stability analysis became a common art. There was a kind of preliminary analysis in the case of Mark III, but neither Mark I nor Mark II was analysed in advance.

J. Chernick: I should like to ask Dr. Storrer what steps will be taken during the start-up of the Enrico Fermi reactor to avoid a problem such as the one Dr. Bondarenko mentioned, namely that the character of the transfer-function curves may change with power?

F. Storrer: Since I have left APDA, I can only tell you how matters stood eight months ago. We are going to carry out measurements of temperature coefficients at different temperatures, and measurements of the power coefficient at successive stationary power levels. We are going to carry out more accurate measurements also, in particular to determine the sodium danger coefficient or that of special fuel-element assemblies where there will be no sodium; and this will be done by putting the assemblies into particular areas of the reactor. At each level where we increase the power (I do not remember exactly what power levels were decided on, ranging up to 200 MW), we shall carry out all the tests I have mentioned and also oscillation tests. By isolating the feedback function from the results obtained and seeing how it develops

as a function of the power round which the oscillation tests are taking place, we shall be able to see whether any considerable non-linearity exists, or whether there is any non-linearity at all. We assume that since our reactor is compact and there is no discontinuity (we do not think there will be any discontinuity) the feedback will be relatively linear, even where it differs quite considerably from what we have predicted.

G. C. Tavernier: The question I am going to raise has no direct bearing on the reports which have been presented this morning. I purposely kept it back for that reason and I want to put it this way:

The analyses we have just seen relate in fact to reactors, and therefore presuppose certain well-known kinetic equations. I wonder if it would not be advisable in some cases to bring in a new factor which may well have remained hidden until now for the simple reason that, except possibly for the Russian reactor BR-5, the spectra of existing reactors are not yet sufficiently degraded. This factor is the following, that in order to undergo a certain number of elastic collisions and establish a sufficiently degraded spectrum from a fission-neutron pulse, for instance, a time comparable to the lifetime of the neutrons in the reactor is required. I wonder whether low kinetic equations or other processes should not be considered, owing to this fact.

H. H. Hummel: As we are concerned with a prompt lifetime of about 0.1 μs, I cannot see that this is really a problem in fast reactors. But perhaps I did not understand the question.

G. C. Tavernier: The fact is that in very fast transient operating conditions, the reactor spectrum may differ considerably from the cross-section spectrum (spatial distribution, etc.), which is clearly found under stable conditions but which one often assumes to be valid when adopting equations to allow for transient effects. In other words, seeing that the time required to establish a degraded spectrum corresponds to the lifetime of the neutrons in the reactor, we seem to have a term here which ought to be introduced into the equations.

H. B. Smets: The problem raised by Mr. Tavernier seems to occur only in the case of transient operating conditions which are so fast that, in my opinion, we do not have to worry about them in the case of power reactors. If my memory is accurate, the problem was examined at Los Alamos for transient operating conditions with, I think, microsecond periods, and even then it was not of outstanding importance. I have carried out a few computations in that direction myself, and for all practical purposes (leaving aside explosions, of course) the factor which Mr. Tavernier has mentioned seems to play a negligible note.

G. Vendryès: Would anyone from Los Alamos, Dr. Hansen perhaps, like to make a comment on this subject?

G. E. Hansen: Yes, I agree with Mr. Smets.

SESSION 11

Chairman: G. C. Tavernier

Paper SM-18/59 was presented by P. Greebler (see Vol. III, p. 121).

Discussion

A. Campise: I have two questions. Firstly, considering sodium as a delayed rather than a prompt effect, what time constants did you take for the sodium versus the fuel? Secondly, in the case of a rapid ramp, did you assume any boiling to take place in the channels such that a void coefficient would have to be considered?

P. Greebler: The time constant for the fuel (and for what we might call our reference reactor) is of the order of 2 s. This is an oxide fuel with rather low thermal conductivity, and the time constants are appreciably lower than they would be in the case of metallic fuels, for example. The coolant-time constant is therefore appreciably less than that of the fuel. We did not have a coolant-time constant in these calculations because we did not consider the coolant effects in the excursions.

The second part of your question concerned, I believe, an accident initiated by removal of the coolant. Could you repeat it please?

A. Campise: If you got boiling in the channels and had to consider a void coefficient, what would the sign of that coefficient be and how would it affect your results on shut-down with reference to the Doppler coefficient?

P. Greebler: Depending on the composition and size of the reactor, the sign of the coefficient can change and its magnitude can cover a very considerable range. For our reference design reactor we have selected one with a zero sodium temperature coefficient—that is, we have something like 50% sodium, which gives a zero coefficient. I might add that some of the data presented at this meeting, which showed greater elastic-removal cross-sections for some of the moderating materials, would perhaps force us to have a larger sodium volume fraction or a smaller core if we were to have a zero or negative sodium coefficient.

S. Yiftah: Have you done comparative calculations for a carbide fuel?

P. Greebler: Yes, we have done some calculations for carbide fuel—of reactivity coefficient only—and compared the results with those obtained from an oxide fuel. Our conclusion was that, with a somewhat smaller core, carbide fuel shows the same general characteristics as an oxide fuel.

B. I. Spinrad: My first question concerns the fact that the averaging of the temperatures apparently increased the Doppler effect with respect to the individual rods. I wonder whether this method of averaging is really appropriate, because the centre, being hot in any case, is a well-broadened absorber; it cannot broaden much as it gets hotter, and it therefore catches the neutrons which leak through the outer part of the fuel rod. Have you done any calculations, considering the actual temperature distribution, and using a method such as Monte Carlo, to ascertain whether your method of averaging is proper?

P. Greebler: No, we have not done any calculations of that kind. Our calculations were basically of the perturbation type, and are valid to the extent that the changes of cross-section in all cases are extremely small relative to the unperturbed system.

B. I. Spinrad: My second question concerns the space averaging over the whole reactor. What you showed was an augmentation of the effect, whereas it would

appear that the centre of the reactor, which is hotter, has a higher statistical weight and therefore should carry more weight rather than less.

P. Greebler: Your observation is correct. However, this must be balanced against the fact that, because we have a higher power in the centre, the response of the temperature to an increase of power is greater in the centre than it is on the outside. This effect is more important than the fact that the coefficient is greater at low temperature than at high temperature.

B. I. Spinrad: If I may make one more comment in connection with the first question, I can see that the original assumptions regarding the temperature distribution and the reactivity could lead to an increase over the average point due to the spatial effect within a rod; but I wonder then whether the base calculation was properly done.

P. Greebler: Could you be more specific as to what you mean by the base calculation?

B. I. Spinrad: The calculation of the effective absorption cross-section of the material in the rod. This is still a resonance calculation, but for a heterogeneous temperature distribution, and it could be appreciably different from a calculation for an average temperature distribution.

P. Greebler: Yes, I see; in fact, you are referring to a very difficult problem: how to do a resonance integral calculation when there is a temperature distribution across the rod. This is a very difficult question, and one which we did not attempt to solve. I think the effect would be small, but I should be very happy to hear whether others have done this.

G. C. Tavernier: Before we finish discussing Dr. Spinrad's question, I should like to know whether you have already done such calculations, taking into account the effect of the temperature variation in a given medium on resonance absorption.

P. Greebler: The problem is perhaps not quite so difficult as may be thought by those who are accustomed to working with thermal reactors, where there is a very pronounced *spatial* shielding problem added to the energy shielding. In the present case, however, the flux is relatively flat, even at the resonances, because the system is homogeneous; consequently, our method of averaging is far closer to being correct that it would be if applied to a thermal system.

B. I. Spinrad: I think that is correct, and it does weaken the objection somewhat because the slowing-down by oxygen is at least a very high fraction of the total slowing-down. However, those neutrons which enter the rod from the scatterings in sodium might be affected by the considerations I mentioned.

P. Engelmann: In calculating the Doppler coefficient in power excursions, you assumed a parabolic shape for the temperature distribution in the fuel rods. Do you assume that it will remain unchanged in fast excursions? It seems to me that it would become flatter in shape.

P. Greebler: We are considering only very steep excursions, for which we disregard conduction effects. In this situation, the parabolic temperature distribution exists only if it was present at the beginning of the excursion, where we started from full-power operating conditions. When we started from the cold condition there was no temperature distribution across the individual rod: I may not have pointed that out, but you will notice that the correction factor was zero for that case.

P. Engelmann: Have you done any calculations of the influence of a negative Doppler effect in a melt-down accident? If you have a ramp insertion of reactivity

much higher than you considered in your cases of sodium loss, the Doppler effect could lead to a lower excursion energy.

P. Greebler: I assume you are referring to a situation in which all of the sodium is lost, whether or not there is an excursion, and melt-down occurs simply through loss of the coolant, and that you are asking whether the Doppler effect limits the excursion energy in such cases. Yes, it does. Again, we have no good quantitative evaluations of this effect; that is something we are working on right now. But the Doppler effect certainly does act as a mitigating factor, and you can readily see that it would.

J. K. Long: I presume you have found that the spectral differences across the core introduce negligible changes, as you seem not to have corrected for them in your spatial averaging.

P. Greebler: As the basic, isothermal calculation of the Doppler effect was done with multi-group calculations, these spectral changes would have been taken into account at that stage. However, in going from isothermal to spatial averaging, we did not superimpose a correction for these changes: we used the square of the power denstity directly as a statistical weighting factor. We are now repeating all these calculations with perturbation, using flux and flux adjoint as spatial weighting factors, and there these spectral effects will be taken into account.

W. Häfele: First, I have a comment on Mr. Long's question. We have studied the spectral changes over the core, and it develops that they are extremely small except in the neighbourhood of the interface between core and blanket; but there one has a low statistical weight, and I think it is proper to disregard these changes.

My own question is this: you mentioned the χ-squared weighting, but don't you also have to take the lower temperatures of the outer core region into account?

P. Greebler: Yes, and that was included in the averaging.

W. Häfele: So after all it is somewhere between χ-squared and χ-cubed?

P. Greebler: Yes.

W. Häfele: Have you any idea for what ratio of U^{238} to Pu^{239}—I mean enrichment or inverse enrichment—the Doppler effect will be about zero in the oxide breeder?

P. Greebler: We have never gone to high enough enrichments with Pu^{239} actually to erase the Doppler effect, but I should guess that it would be in the neighbourhood of 30% enrichment. It is not easy to extrapolate this because many effects take place. The spectrum changes as one changes the enrichment—it becomes harder as one increases the enrichment, so that many compound effects occur.

D. Okrent: I might point out that when Dr. Clair was at NDA he wrote a paper (which may have been just an NDA report) on the problem of making a measurement with a hot sample in a cold core. This involves the interplay of atoms of one temperature with atoms of another temperature, and while the solution was of course specific the technique may apply to a fuel pin which has a varying temperature.

My question is somewhat related to the last point. Allow me to read a few lines from your paper: "It is somewhat disturbing that the Pu^{239} contributes a positive Doppler effect of about 40% the magnitude of the negative contribution by the U^{238}, in spite of the 1:7 isotopic ratio. This indicates that smaller fast reactors requiring higher Pu^{239} content in the fuel may not have a significant negative Doppler coefficient and, in fact, very small reactors would have positive coefficients." The meaning of "very small" is a bit vague, but if I look at the paper

by Mr. Greebler on which these calculations are based, I find, for example, that above the energy of, say, 1000 keV, the U^{238} contribution to the Doppler effect is between 5 and 10 times as great as the plutonium contribution, and it is only the relative equality at very low energies that brings these two contributions closer together. The implication is that if you design a reactor which has no neutrons, or a negligible number, below 100 eV, you are out of the region in which Pu^{239} is a strong competitor, and I would then expect the ratio to be closer to 1:1—perhaps not exactly, but less than 2:1.

I cannot agree, therefore, that small reactors will present the problem of a positive Doppler coefficient unless they use pure Pu^{239} or have a ratio of less than, say, 1:1.

P. Greebler: I agree that the wording is nebulous. In fact, we have not calculated how small the reactor must be before the coefficient would become either positive or insignificant, but let us say 35%. For U^{235} it is probably a 1:1 ratio, i.e. 50% enrichment; for Pu^{239} one would expect it to be more nearly 2:1.

D. Okrent: If one uses your own figures, the ratio would not be as high as 2:1, although that is just a rough estimate.

P. Greebler: That may be so, but you must admit that a small reactor will have something like a 2:1 ratio, and the Doppler effect will be insignificant if not positive.

J. J. Schmidt: What is the worth of the dollar in the oxide breeder which you considered?

P. Greebler: The delayed-neutron fraction is approximately 0.4%.

A. Campise: I have made calculations of U^{233}-thorium and U^{238}-plutonium systems to see at what point the Doppler coefficient becomes negative, and I agree with Dr. Okrent that it is about 1.8 for both U^{233} and plutonium.

W. Häfele: I might mention that when Dr. Dresner was in Karslruhe he did some calculations for us concerning the resonance integrals for a parabolic temperature shape within the rod. He concluded that in this particular case you have to take the average temperature in order to get the proper resonance integral.

P. Greebler: That is precisely what we did.

Paper SM-18/10 was presented by E. A. Fossoul (see Vol. III, p. 139).

Discussion

G. C. Tavernier: We thank Mr. Fossoul for his paper. Before opening the meeting for questions I should like to ask Dr. Greebler a question myself. Mr. Fossoul has just mentioned that the Q_E function is virtually independent of the α values adopted for plutonium-239. I should like to know whether Dr. Greebler's calculations led to the same conclusion.

P. Greebler: Without attempting to analyse the physical significance of the Q_E function, I see that it depends very much upon using more than just the $l=0$ state. Our detailed calculations for the fissile material Pu^{239} were done only in the energy region below 10 keV, where one is not justified in using the $l=1$ state; consequently, we did not get this particular function. However, the Doppler coefficient is certainly sensitive to the α value of Pu^{239}, particularly in the low-energy region. It is also sensitive to the α value in the high-energy region, but for quite a different reason. If the α value in the high-energy region is low, we can have a low enrichment, and the Doppler coefficient is therefore less positive. The ideal thing is to have a low α at high energies and a high α at low energies, in order to get the least positive Doppler effect from the Pu^{239}.

C. P. Zaleski: My question is addressed to both Mr. Greebler and Mr. Fossoul.

I merely want to ask them what precision they think can be attained in calculations of the Doppler effect, and more particularly, what precision we can hope to attain in determining the influence of the Doppler effect on the reactivity of a fast reactor.

P. Greebler: I can only guess at a figure such as $\pm 50\%$ for precision, because we really do not have very good experimental information. However, I should add that the question of precision has not been carefully analysed. There are two important elements in this calculation, the most important of which relates to the U^{238} because that is where we get our major negative Doppler coefficient. In this case I think the precision is reasonably good, because the statistical method which we use for the non-fissile material is quite well established. We have a great deal of information on resolved resonances at low energies which we can use to obtain the average statistical parameters. There is far less precision in the case of the fissile material, but this is, fortunately, relatively unimportant.

We are now evaluating the sensitivity of our calculated Doppler effect—particularly that of the fissile material—to possible errors in the parameters. There is one particular problem here on which Mr. Fossoul may be able to shed some light: at low energies, where only the $l=0$ wave can justifiably be used, one does not have enough free parameters to specify simultaneously the measured fission cross-section (at room temperature, let us say) and the α value. If one uses a Breit-Wigner single-level approximation, one must then revert to a multi-level treatment which can become very tedious indeed from the standpoint of numerical calculation. There is a method of getting average parameters from the resolved resonance data, but this is very limited because we do not know the spin state of many resonances at low energies.

E. A. Fossoul: Up to now we really have no precise measurements of the Doppler effect, and the best we can do is to compare the results of various calculations. I think the figure of 50% mentioned by Mr. Greebler is in fact quite reasonable. The only experiment I know of (I believe it was carried out in the United Kingdom) consisted in measuring a coefficient of the order of 10^{-6}, which turned out to be in agreement with the figure calculated by Dr. Bethe.

S. Yiftah: I should like to ask Dr. Fossoul or Dr. Greebler whether any attempt has been made to calculate the same effect for the higher plutonium isotopes.

P. Greebler: We have done such calculations for Pu^{240}. The problem is essentially the same as for U^{238}, except at very high energies where the Pu^{240} is fissile, and there we did no detailed calculations. At the lower energies we did the same sort of calculation as for U^{238}, although they were not so accurate because we did not have so many resolved resonances.

E. A. Fossoul: The reply to Dr. Yiftah's question is no, but we are now studying the low-energy end of the spectrum for Pu^{239} and will soon go on to calculations of Pu^{240}.

L. N. Usachev: I should like to ask Mr. Greebler whether, in the work reported, account was taken of the statistical distribution of fission widths, which are generally described by the Porter-Thomas formulae. If so, what was the distribution of fission widths for the s- and p-waves assumed to be?

P. Greebler: Yes, we used the Porter-Thomas distribution with $\nu=1$ for the reduced neutron width and with $\nu=3$ for the fission width; this was for calculations at low energies. At high energies we applied the Feshbach-Goertsel formulae directly to calculations of the Doppler effect, so that we used whatever assumptions their method of calculation included.

E. A. Fossoul: If your question refers to the degrees of freedom for the χ-squared function which was adopted for the different distributions, those are given in my paper. Briefly, we took $\nu=1$ for the diffusion of neutrons with orbital momentum $l=0$ and $\nu=3$ for the fission width; $\nu=1$ for neutrons of orbital momentum $l=1$ which do not lead to internal states of spin; and $\nu=2$ for neutrons of orbital momentum $l=1$ which do lead to internal states of spin $J=0$ or $J=1$).

L. N. Usachev: I did not fully understand Mr. Greebler's reply regarding the number of fission channels. Three fission channels were calculated for s-waves; how many were calculated for p-waves?

P. Greebler: For calculations below 10 keV, which was the important energy range for us—the range in which we did our own detailed calculations—we used only the s-wave because the p-wave is not significant. For the energy region above 10 keV, we used three channels for the fission width, applying directly formulae which were developed by Feshbach and which appeared in a 1956 paper in *Nuclear Science and Engineering*. I believe they did not apply the statistical methods to the fissile material in their treatment.

L. N. Usachev: I should like to comment further on this question of the number of fission channels. I had gained the impression that we should assume one fission channel for s-neutrons, as indicated by the most recent experimental data. At any rate, that is the hypothesis we adopted when we were considering, at Session 3, reports on the work done in calculating capture to fission ratios. We used one channel for plutonium, for s-waves, and found there was fairly good agreement with the experiment. We have, in addition, direct experimental data regarding the distribution of fission cross-sections and the distribution of fission widths.

C. Erginsoy: We see from a formula in Mr. Fossoul's paper that the Doppler coefficient is expressed as a long series of products and it looks as though it is really proportional to this function Q_E. In Table VII we see that an $l=1$ contribution to Q_E is fairly substantial, in fact almost one-half of the $l=0$ contribution. This would make the Doppler coefficient itself proportionally higher by about 50% in this case, whatever the spectrum and the importance of neutrons in the different energies might be. Two questions come to mind: whether the $l=2$ contribution should not also be taken into account and, secondly, whether the importance of the $l=1$ contribution would not perhaps make Dr. Greebler's results incomplete in a certain sense.

P. Greebler: I think your question is actually more relevant to Dr. Fossoul's presentation, but in any case the $l=1$ state contributes less than 4% at 10 keV.

A. E. Fossoul: Clearly, I think it would be an improvement if one succeeded in calculating the cross-section with $l=2$, but I seriously doubt whether that is practical. Even with the $l=1$ contribution one needs one machine-hour per point in order to do the fitting. So one must not have too many parameters to adjust, because even if one is satisfied from a theoretical point of view, that does not mean that the result is any the more valid.

J. J. Schmidt: I think the $l=2$ contribution begins somewherea round 100 or 200 keV, so that the Doppler effect is very small there, even in metallic breeders; and I think the inelastic competition might be considerably more important, because the first inelastic excitation lies, for example, at 8 keV in the case of Pu^{239}. In fact, there must be some rotational levels between this first level in Pu^{239} and the second at 57 keV. I think this is a more important effect.

G. C. Tavernier: I do not think, at least I do not suppose, our Soviet colleagues started up the BR-5 reactor without themselves estimating the significance of the Doppler effect in the plutonium oxide. I should be interested to know if they have any comment to make either on their method of calculation, or on the results obtained as far as that effect is concerned.

A second small point with reference to a remark made yesterday. I noticed from the slide you showed, Dr. Greebler, that when passing from room temperature to operating temperature, you took up in effect almost two-thirds of the reactivity by the Doppler effect, the remaining third being available before reaching a possible melt-down. How do you reconcile this policy of, as it were, investing a few dollars of reactivity in the Doppler effect in passing from cold to hot with what one speaker mentioned yesterday, to the effect that for fast reactors it was desirable to have only a fairly small total negative coefficient, just so as not to have too wide a margin of reactivity invested in the controls in the cold state?

P. Greebler: What we really want, I think, is to have a low negative delayed temperature coefficient, and I must admit that the distinction between delayed and prompt is sometimes very arbitrary. In the case of a fuel like oxide, which has a low conductivity, the situation is a little more clear-cut. I see no objection whatsoever to having as large a prompt negative coefficient as possible. It is true that one must have more control rods for the cold situation, but as these rods are removed and the power rises, the fuel temperature rises instantaneously and overrides any reactivity increment resulting therefrom.

J. Chernick: At about how many temperature points was this Doppler coefficient calculated, and have any calculations been done in the neighbourhood of the operating temperatures?

P. Greebler: Yes. We covered a range of temperatures from 300 to 2500°K, and we calculated four different temperatures in that range'.

J. Chernick: In view of the fact that two contradictory factors enter into the design—firstly, the temptation to push fuel temperatures rather close to the danger point, and secondly, considerations of safety — could you say what reactivities you hope to overcome, when you are actually operating the reactor? What Δk do you think the Doppler effect will handle?

P. Greebler: If you are referring to a rapid ramp, it was of the order of two dollars for the specific example which I showed in connection with our excursion calculations. The two-dollar reactivity insertion would cause an energy release in the reactor such as to produce complete melting of the oxide across the rod at the hottest point in the core.

J. Chernick: Would your company want to invest in such a reactor?

P. Greebler: The company would not invest in such a reactor until studies on experimental transient facilities had thoroughly demonstrated the possibilities of Doppler shut-down.

Papers SM-18/78, SM-18/43, SM-18/46 and SM-18/42 were presented by Dr. Okrent (see Vol. **III**, pp. 155, 171, 195 and 209).

Discussion

D. Okrent: May I be the first to ask two questions? I would like to ask two questions of the Soviet delegation. At the Geneva Conference in 1958 Dr. Alexandrov was asked his opinion of fast-reactor safety, and his answer was that there were

lots of wide open spaces in the Soviet Union. I should like to know whether that is the prevailing attitude, or whether you think that large, fast power reactors can safely be built near large cities. I should also like to repeat Mr. Tavernier's question: in view of your actual experience with plutonium oxide reactors, do you feel that such reactors still pose any major safety problems?

I. I. Bondarenko: I should like to make the following comments regarding the safety of fast reactors. We are optimistic enough to hope that physicists and designers, applying all their knowledge and skill, will be able to ensure the safe operation of fast power reactors. We also believe, of course, that no reactor should be constructed, no matter how extensive the available space may be, until it has been clearly demonstrated by a careful analysis of all the possible accidents and potential damage which might occur, that the damage could not be severe. In Mr. Okrent's very interesting report the general aspects of the safety problem were considered and the hazards of a general nature were indicated. I believe, however, that we should rather proceed on a case-by-case basis, i.e. that we should consider separately the problems involved in designing each reactor and that, by applying physical principles which are related to the Doppler effect and making use of various refinements of designs connected with rod-bowing effects, we should be able to achieve our aim. This depends on the ingenuity shown in designing the reactor. We believe and hope that, in the long run, physicists and designers will be able to ensure the safe operation of fast power reactors.

G. C. Tavernier: Before going on, I should like to ask Dr. Bondarenko whether the pulsed reactor—a description of which we all read with interest about a month or two ago in Russian periodicals and which will be described again tomorrow—is intended to play a role like that of the United States reactor TREAT, i.e. whether it can be adapted for tests on the dynamic distribution of fuel elements.

I. I. Bondarenko: If I understood the question correctly, you asked if our pulsed fast reactor is suitable for dynamic tests of fuel element break-down. I think it is not. On the contrary, we tried to adapt it for the opposite purpose, namely to ensure that no dynamic or static break-down occurred. We took a number of precautions with this end in view, and we shall, of course, describe them in the report on this reactor (SM-18/81).

D. Okrent: As Dr. Häfele has expressed considerable interest in a helium-cooled fast reactor, I should like to ask him whether he thinks that emergency shut-down cooling can be guaranteed in the event of major difficulties with the helium cooling system. Is the helium-cooled reactor not more prone to core melt-down accidents than the sodium-cooled reactor, where shut-down cooling does seem to be achievable? Also, do you feel that the coupled reactor concept will play a truly important role solving the safety problems of fast reactors?

W. Häfele: We admit that the use of helium as coolant increases the possibility of a melt-down, but behind the use helium there is a special philosophy of design. Everyone knows that the efficiency of atomic weapons is greater than that of chemical weapons by several orders of magnitude. It is not so generally realized, however, that in reactor accidents the chemical dangers are greater than the nuclear ones. Let me just mention the situation which one finds in thermal reactors using, for example, heavy water and natural uranium in a metallic form. A possible runaway may result in a power excursion of between 100 and 200 MW/s, equivalent to between 10 and 30 kg of TNT.

But if the excursion triggers a chemical reaction between metal and water—I am still speaking of the thermal situation—one may have to face the consequences of an energy release equivalent to between 3 and 5 t of TNT. This is in fact the basis of our reasoning. If a melt-down occurs in a sodium-cooled fast reactor, one must face a power runaway with an energy equivalent of about 50 kg of TNT. That may be enough to trigger a chemical reaction between the sodium and the air or water surrounding the reactor building. The energy inherent in that chemical reaction is of course far higher, especially if there are large pools of sodium. The possible explosive energy then approaches some 50 t of TNT. Please do not rely on these figures, which are merely intended to show that the energy which one would have to cope with from the chemical reaction is far greater.

The situation can then become very serious. The activity in a reactor's sodium pool can be calculated as some 10^7 c—possibly even a bit more. Sodium has a short half-life, to be sure, but the situation is still serious if the sodium from within the core comes to a reaction with the air outside the reactor building. Such an event would mean the end of reactor progress in Germany and I expect the same would be true in other countries as well.

Our idea, then, is to reduce the consequences of a major accident by avoiding the use of sodium, and we pay for this, quite frankly, by ncreasing the probability of a melt-down. Assuming that the energy released in the event of an accident would then be equivalent to no more than 50 kg of TNT, we feel that it could be contained in a special building.

That is the general background, but I should add that there are some possibilities for emergency cooling. One scheme, which is admittedly not very attractive, but perhaps ought not to be dismissed entirely, is to have a chimney some 10 or 20 m high over the reactor so that there is energency cooling by natural convection after a shut-down. Amother possibility—and this brings me to Mr. Okrent's second question—is that offered by coupling. The problem of melt-down really has two aspects. In the first place there is melt-down in the core, but it is afterwards that the main difficulty appears. If some molten material runs out of the core and forms a ball of molten material, it may very well lead to a large increase of reactivity. With no empty channels one may eventually have a very large super-critical ball, a situation which can be avoided if the fast-reactor component is so small that it is virtually impossible for it to form a critical configuration after a melt-down. This, rather than increased neutron lifetime, is the main advantage sought in coupled reactors. We feel that a lifetime of 10^{-5} s is still too short to prevent a runaway by mechanical devices; for this reason, it is essential to have a large negative Doppler coefficient.

C. P. Zaleski: I should like to make a remark on what Dr. Häfele has just said. It seems to me that as far as the sodium effect is concerned it is difficult to compare it with an explosion. The rapidity of the effect is not at all comparable to an explosion. Clearly then, even if the total energy liberated is very considerable, comparison with so many kilograms of chemical explosive is not perhaps very helpful.

I should like to add that any energy stored by helium under pressure in a gas circuit is also an energy which can result—in fact, has already resulted—in break-downs, and can therefore contaminate the environment.

W. Häfele: Dr. Zaleski is absolutely right to emphasize that the problem arises not from an explosion but from a chemical reaction, and I meant the

figure of 50 t of TNT merely as an energy equivalent. However, this energy cannot be contained, and even a slow build-up of pressure might be very serious. One must envisage not just the likely contingency but the seemingly unlikely accident. It may be difficult to argue that it can happen, but it is still more difficult to demonstrate that it cannot. Of course, there is some energy in helium as well, but I think we have a way of handling it. There are two projects under way at the moment which involve the use of helium as a coolant. One of these is in Germany—Dr. Schulten's so-called "potato reactor", which has high helium pressure—and the other is the DRAGON project. We should learn a great deal from them.

C. P. Zaleski: I think that we understand one another reasonably well, that basically we are in fairly good agreement with Dr. Häfele. Perhaps it is a question of how we evaluate the difficulties in the two cases. I think that the studies which have been carried out by the APDA group and by Argonne, and certain calculations which we have made in France, show that the sodium effect can very probably be contained within reasonable bounds. There are evidently difficulties, and it is, if I may put it that way, a question of how we evaluate the respective difficulties of the two systems. Personally, I do not think that the sodium system is more dangerous than the gas system, but it is a question which deserves further study.

D. Okrent: Our French colleagues have been giving serious consideration to both metallic and oxide fuels for RAPSODIE. I should like to know whether there appear to be any major differences in their safety characteristics, and whether those provide a basis for choice. Secondly, since you have considered large reactors with fuel contained in an inert matrix, and therefore using, let us say, an enriched plutonium oxide, do you feel that the problem of the Doppler coefficient is serious in such a reactor?

G. Vendryès: In reply to Dr. Okrent's two questions, let me first of all deal with the relative merits of oxide and metal in RAPSODIE from the safety angle. I should point out that much more research has been done on the metal variant so far, particularly from the standpoint of kinetics and safety. The reliable results so far obtained—at least on paper—suggest that stability with this variant would be extremely good. We consider that the Doppler coefficient would be practically negligible, but there is a high negative coefficient, mainly due to the expulsion of the sodium and the deformation of the elements.

The Doppler coefficient has not yet been worked out for the oxide. We think, however, that it might possibly be slightly positive, but we are confident that it would in any case be more than offset by a negative coefficient of approximately the same order of magnitude as has been worked out for the metal—again because of the elements and, particularly, the expulsion of the sodium.

On the question of major accidents, the calculations for both possibilities have been carried out by one and the same person, so that, although the problems are not absolutely identical, they have been approached, as it were, with the same degree of pessimism. In both cases, values of exactly the same order of magnitude were obtained for the release of energy and for the duration of energy releases. Thus, as things stand at present, there is no reason on safety grounds to adopt one variant rather than another for RAPSODIE. I should add that at the moment we do have a slight preference in favour of metal for the first RAPSODIE core. This is not because we think that this will provide a better basis for extrapolation—that would be quite a wrong interpretation of

our decision; it is merely that for the first core, which will in any case be subjected to a fairly low level of irradiation, we consider that it will be better to adopt the method that has been more thoroughly investigated, not only from the neutron-physics point of view but also from the standpoint of metallurgy and manufacture. I should stress, however, that nothing definite has been decided. With regard to the second core, the situation is even more undecided. We may continue with metal, but we are more likely to try something else. This would not necessarily be oxide, we might use, say, carbide.

As for Dr. Okrent's second question, i.e. the risks arising if plutonium oxide is dispersed in an inert stainless-steel matrix—a solution mentioned by Mr. Zaleski the other day when he compared a whole series of different possibilities for future reactor types—I should merely like to say the following. At the present time, these investigations are very much of a preliminary nature; they are parameter studies designed to provide information on possible future lines of development. As such, they have not gone very deeply into the problem involved. On the point you mention, however, we do consider this to be a very considerable problem, and the presence of a positive Doppler coefficient will probably make this method quite unsuitable. This is precisely the sort of possibility that can be ruled out *a priori* by safety considerations.

G. P. Zaleski: After hearing Mr. Vendryès' very clear reply, I should merely like to emphasize that the problem of the Doppler coefficient in large reactors which have the fissile material dispersed in an inert matrix has not escaped us. In fact, it was this problem which had led us to suggest a reactor which would have the fissile material dispersed in an inert matrix and mixed almost homogeneously with the breeding material (one element out of every two).

D. Okrent: I should like to ask Mr. R. D. Smith of the United Kingdom whether the new ZEBRA critical assembly has a fast fuel expansion coefficient. Secondly, can you have a positive Doppler coefficient of appreciable size for certain assemblies in this new critical, and what effect does such a coefficient have on safety, assuming that somehow the super-prompt-critical configuration is reached?

R. D. Smith: I should like to thank Dr. Okrent, although I would have thanked him even more enthusiastically if he had provided some of the answers. Before ZEBRA is operated, all considerations relevent to safety will have to be set out in an official report. The report will cover all the points raised by Dr. Okrent, but, unfortunately, it has not yet been completed. I can only offer some personal suggestions at this stage.

In answer to the first question, ZEBRA will have a prompt fuel expansion coefficient. The next question was whether we could build assemblies with positive Doppler coefficients. Since this is a very flexible device, I feel sure that we could, but our first assemblies will in fact have negative coefficients. I must say that I would not be altogether happy about building an assembly with a prompt positive coefficient, but as far as I know we have taken no definite decision in this respect.

As a matter of fact, our major effort is devoted to avoiding any situation in which a power excursion could become serious. In particular, we are paying very close attention to the design of the control-rod systems in order to eliminate any possibility of their not operating. We do this by being extremely careful in the construction of the electronic system and by making sure that no failure of any part of these systems can do other than increase the safety of the whole assembly. We have two different types of control-rod mechanisms, with double

bearings throughout, so that no seizure of a single part can stop the operation of the mechansism.

This leads me to a general point. Zero-power assemblies present somewhat different safety problems from those one encounters in power reactors. In the latter case, it is possible to make a specific safety assessment of a given configuration. In the case of a zero-power assembly, on the other hand, one proposes to build a whole series of very different assemblies. It would seem very important, therefore, to make a rather more general assessment of safety in so far as it is possible to do so—one which does not depend too much on a particular type of assembly. Otherwise one must, practically speaking, prepare a separate safety report for each experiment, and that involves a lot of time and money.

D. Okrent: The point of my question to Dr. Greebler has been somewhat lessened by his announcement that the General Electric group are now working on a reactor with zero sodium coefficient. Nevertheless, he has shown us a design with a positive sodium coefficient, and in any case, even when the uniform coefficient is zero, there may well be a positive component in the hottest region. I should like to know why you are devoting more attention to large fast reactors with a positive sodium coefficient—which would appear to be the case—than to similar designs with a negative sodium coefficient.

In the second place, I should like to pose a specific accident for the oxide reactor with a positive or nearly positive uniform sodium coefficient. Imagine an operating situation at high power which leads to sodium boiling. This is an assumption, and I shall not attempt to prove that it can happen, but I do not think it altogether unlikely. Analyses and experiments carried out at APDA by Morell and Nicholson indicate that the core will rapidly empty of liquid coolant, at least momentarily, and the coolant flow will then probably oscillate between half-full and almost empty. But in any case this is a mechanism for emptying the core of coolant; the coolant need not be leaked out of the whole system. If the sodium coefficient is positive or about zero, there will probably be a gain in reactivity from the loss of sodium. Furthermore, assuming that the Doppler effect is in general negative, it too will produce a gain in reactivity during the loss of sodium, because the spectrum gets harder and there is less negative Doppler effect. Thus the Doppler effect is actually harmful at the initial stage of this hypothetical accident because it adds reactivity. We can assume that the oxide was very hot at the start because we were running at power. There is very little additional negative Doppler effect between that point and melting and vaporization to assist in shut-down. This is, I think, a reasonably interesting hypothetical accident, and I should like to hear comments on it.

P. Greebler: Dr. Okrent has asked two very provocative questions. The answer to the first—why we want to design a reactor with a positive sodium coefficient—is of course that we do not. I believe I mentioned earlier that the design which we now recommend is one with a zero sodium coefficient; we would prefer a negative coefficient were there not other considerations. However, we should remember one pertinent comment made by Dr. Okrent: that even in a reactor with a zero or slightly negative sodium coefficient, the coefficient can still be positive in the central region of the core. That is certainly true. It may be worthwhile, therefore, to review our reasons for adopting a design with a sodium coefficient which at least is not substantially negative.

In the first place, economic considerations can lead one to a large dilute reactor such as we recommend. It is economically very attractive to have a reactor

with a large *internal* breeding ratio, one which produces most of the plutonium right inside the core, so that it can be burned over and over again without reprocessing. Blankets are very questionable from an economic point of view.

In the second place there are important safety considerations, one of which concerns the Doppler effect. Only in large dilute systems can we get this very substantial prompt negative Doppler coefficient, and if we had to choose between a substantial negative Doppler coefficient combined with a zero or slightly positive sodium coefficient on the one hand and a zero Doppler coefficient with a substantially negative sodium coefficient on the other, I believe we should choose the former alternative, although it is only fair to add that we have not analysed this situation in all its detail. Another safety problem is raised by the excess operating reactivity. This reactor does not have continous refuelling; it has to be shut down periodically while some of the fuel is removed, and there is a certain amount of excess operating reactivity which must be accommodated between refuelling. Of course, we want to avoid the kind of problem which the Fermi-I reactor presents, where, in order to keep the excess operating reactivity within safe limits, the management must shut down about every two weeks. This is very uneconomical. Therefore, in order to get a reasonable refuelling interval and still maintain excess operating reactivity at what we consider a safe level, we adopted a fairly large core size with a large internal breeding ratio. We may have been very severe in setting a limit on excess operating reactivity—this is part of what I believe Dr. Spinrad would call fuel management. We have said that even if the reactor were operating at full power and all the control rods were blown out, we would still expect the Doppler effect to terminate such an accident. This may be too severe; perhaps we should assume that if two control rods were blown out the Doppler effect would terminate the resulting excursion. In that case, we could certainly adopt a somewhat smaller core with a more negative sodium coefficient.

Dr. Okrent's second question involved a complex accident. While I am not sure that I fully understood it I should say that the hardening of the spectrum with the loss of the sodium coolant gives a decreased Doppler effect. Your point is, then, that because the Doppler effect is smaller it cannot be counted on so heavily for shut-down. Is that correct?

D. Okrent: Assume that the Doppler effect reduces k by 1% as the reactor goes up to operating temperature. If the spectrum suddenly became very hard, so that there was a zero Doppler effect, this 1% k would be recovered.

J. Chernick: May I interrupt this argument? I think the question is poorly defined. We have not been told under what conditions this accident might occur, whether the control rods work, what the time constants are, etc., and I therefore see no point in continuing the discussion.

P. Greebler: I agree, and I think Dr. Chernick has raised two very pertinent points: namely, could this accident happen so quickly that the control system would fail to take hold, and would the Doppler effect be lost instantaneously? I do not believe that it would; in any case, both these questions have to be seriously considered. In the third place I think the effect would be severe only if there were a very substantial positive sodium coefficient. Altogether it seems this is a well-nigh impossible accident we are being asked to circumvent.

D. Okrent: At any rate, no one has suggested means of circumventing it.

P. F. Zweifel: I must agree with Mr. Okrent that the problem may, after all, be a real one, and I hope that we shall have a chance to discuss it in greater detail.

D. Okrent: This question is addressed to Dr. Campise. How much reactivity hold-up will there be in the protactinium in a typical design of a fast or nearly fast U^{233}-thorium reactor? What effect does this have on the shim and shut-down requirements of the reactor? Will such a reactor have to have large excess reactivity in portions of its operating life, and is this hard to achieve? If it does have to have a large excess reactivity, do not start-up and mal-operation accidents become a serious threat, particularly in view of the very low delayed-neutron fraction to be expected in the U^{233}-thorium system, where these is a very small increase in β owing to the thorium fissions?

A. Campise: In the first place, our studies have indicated that there is virtually no difference, from the standpoint of the fuel cycle, between fuelling with plutonium and fuelling with U^{233} in large dilute fast reactors. I might say that the reactor types studied by our group invariably have negative Doppler and sodium coefficients. We are interested in the complete energy range from fast to epithermal, as U^{233} is a superior fuel over the entire range. This may be a factor of some significance in the competitive nuclear market of the future.

The problem of reactivity hold-up in protactinium is in fact one of the few problems that one has to face in a U^{233}-thorium system. In a fast carbide system, such as we are considering at present, the problem exists in a range between 20 and 50 kg at saturation. The safety shut-down is about 10%. Admittedly, if one is to overcome the protactinium problem, proper care must be exercised during shut-down and restart, when re-calibrating control rods, to account for the amount of excess reactivity that builds up depending on how long the reactor is shut down. In my opinion the problem could be handled simply by issuing detailed instructions for start-up, as this would be the most serious problem in a U^{233} system after it had once been irradiated.

W. Häfele: I should like to make a general statement. It is always supposed that the Karlsruhe group are interested only in a helium-cooled breeder, and that is not true. We are studying a sodium-cooled breeder at the same time and comparing the two systems with a view to solving precisely those problems which have been raised here. We feel that there is no straight-forward way of deciding the issue. Detailed studies will be needed in order to decide which type of accident is the more likely and the more serious.

J. Chernick: I wonder whether Dr. Häfele has compared the likelihood of fuel-element failure in the two systems. Most utilities would want their reactors to run at least 20 or 30 years in order to recover their investment. This is a question of financial risk rather than safety.

W. Häfele: I must confess that we are not yet in a position to answer that question. In order to do so, one would have to have completed a full study of the two reactor types.

Now, if I may change the subject slightly, I should like to ask a question of Dr. Okrent. Will there not be a slightly positive Doppler coefficient in EBR-II, although possibly a negative fuel-expansion coefficient?

D. Okrent: The Doppler coefficient will probably be zero, or else slightly negative or positive. However, it will apparently be very small compared to the equally prompt axial expansion of the fuel pin.

W. Häfele: Do you have any idea how small the Doppler coefficient must be in order that one may neglect it?

D. Okrent: I can answer that question only with reference to a specific reactor and a specific accident. If we are considering the EBR-II in a solid condition,

I should say that 1/10 of the axial expansion, for example, is small enough. However, if we consider the EBR-II in a molten condition and assume that there are no longer any reactivity effects due to expansion until thorough vaporization has taken place, an addition of 0.001 or 0.002 $\Delta k/k$ after the reactor has become super-prompt-critical will appreciably change the course of the explosion.

I should like to ask Dr. Smith how the negative Doppler coefficient will be obtained in ZEBRA. Will plutonium or uranium infinitely mixed with the U^{238} be used?

R. D. Smith: If you mean a negative Doppler coefficient, I misunderstood you before; I thought you were referring to the thermal expansion of the fuel. Part of the fuel is dilute uranium and part is enriched uranium. The plutonium is concentrated, of course, and that part may therefore have a small positive Doppler coefficient.

F. Storrer: I would like to say that I quite agree with Dr. Okrent that, during the normal running of a reactor and for relatively moderate excursions, the axial expansion of the fuel elements is quite as rapid as the Doppler effect. However, for extremely rapid transitions, i.e. for the very short periods such as one might meet with during a very serious accident or during the fusion of the core, the inertia would cause a certain delay between the—more rapid—Doppler effect and the axial expansion of the fuel elements. In that case it would perhaps be possible to calculate the minimum period during which the net Doppler effect is always less than the net effect of axial expansion, taking into account their attenuation as a function of the period.

G. C. Tavernier: I have a question, although I am not sure to whom it should be addressed. I suppose that the occurrence of the Doppler effect in a rapidly heated fuel is something which takes place in 10^{-15} or 10^{-14} s—or is my estimate altogether wrong? After all, a peak of very high temperature is formed locally in each case, but over a very small number of atoms; and this is then diffused and reaches equilibrium in a macroscopic region.

B. I. Spinrad: I should like to emphasize that both the Doppler effect and fuel expansion can be insignificant in many types of accidents involving both thermal and fast reactors. No matter how one approaches the problem, I think one must finally assume a certain limitation on the degree of blunder which is made in reactor operating or loading. For example, Atomics International operate a sodium-graphite thermal reactor and rely heavily on the Doppler effect for their safeguards mechanism. However, if one assumes that the management allowed a blunder exceeding the reactivity to be lost in Doppler effect, this reactor could have just as serious an excursion and explosion as a fast reactor, because it has a good deal of melt-down potential. I have calculated potential accidents involving very large reactivity insertions in water-cooled and moderated reactors. It was clearly shown that if the excursion exceeded the intrinsic shut-down mechanism of the system one could get a very large energy release indeed. This is not to say that the Doppler effect, the expansion effect and so on are unimportant, but merely to emphasize that one must assume reasonably good management. We might distinguish three ranges of mishap, as it were, the first relating to what I shall call normal errors around the operating range, the second to a very limited range between the normal operating range and some assumed maximum blunder which still allows the shut-down mechanism to take hold, and the third to accidents or blunders which exceed the postulated ability of the shut-down mechanism to take hold. I think one must depend upon managements to prevent

blunders of the third category, and one might expect reasonably good management to confine most irregularities to the first category.

R. D. Smith: I would like to support what Dr. Spinrad has just said. In fact, one has to assume good management for a zero-power assembly, and there is no point in carrying to an extreme the misfortune which you assume might occur in the reactor itself.

P. Greebler: I prefer to disagree with the opinion that good management is adequate. Experience has shown, I think, that if it is possible for an accident to occur, it will occur—the SL-1 accident might serve as an example. All designs should be inherently safe, in my opinion, and although inherent safety may be difficult to define we could perhaps say that the built-in shut-down mechanisms, such as the Doppler effect, must be adequate to terminate any accident which might conceivably occur within the reactor's lifetime—control-rod blow-out, loss of coolant and the like. We must ensure that the damage which does occur is merely local, and that we are not faced with complete destruction of the facility or, worse still, a community problem.

B. I. Spinrad: I think Dr. Greebler has in fact agreed with me, because the sort of accident he postulates as being the natural consequence of a reactor mishap can be prevented if one makes the most of certain features of design within the operating range of the reactor. No single design can incorporate every possibility, and the particular requirements of every individual reactor must therefore be submitted to thorough analysis. The main point, of course, is to ensure that no major increment of reactivity can be inserted rapidly in any reasonable operating mishap. Since the SL-1 incident was mentioned, I might point out that it was very instructive from the standpoint of design. The reactor structure allowed the system to be auto-catalytic, as subsequent analysis showed, and this is clearly something to be avoided in any case because an auto-catalytic system will inevitably be unstable under certain conditions. This is a particular case where a single error, even under less peculiar circumstances, might also have caused a severe accident.

P. Greebler: This question is addressed to Dr. Okrent. Your presentation seemed to indicate that, even in your severe-accident calculations, there was a larger energy release as the fuel lifetime became longer. Is that correct? If so, at what point does the long fuel lifetime such as one gets from coupled reactors diminish the energy released in an excursion? To phrase the question in more general terms, what relationship have you found between neutron lifetime and the energy release to be expected in an excursion?

D. Okrent: I think your first impression was wrong. In other words, other things being equal, the shorter lifetime will give the more powerful explosion. But this applies only to a step function of k and not to the ramp type of accident, where each case must be studied individually.

P. Greebler: Perhaps someone from Argonne could tell us under what conditions a long neutron lifetime is advantageous, and just how advantageous it is. Is there any definite answer to that question?

B. I. Spinrad: I can only repeat the general arguments which were advanced when the coupled reactor concept was first formulated. If there is a shut-down mechanism, whether it be a natural one or one depending on a special safety circuit with a delay time of the order of 1 ms, a coupled reactor would allow that mechanism to come into play, whereas a fast reactor would by-pass it completely.

A. Campise: My question is directed to Dr. Okrent and to the Argonne group in general. You referred to the necessity of building large dilute critical assemblies. It seems to me that with a little ingenuity you might be able to prove that multi-region criticals of the ZPR-III type, with test-sections in the centre to mock up dilute assemblies, would provide the data normally obtained from the large assemblies (with the possible exception of critical mass). At a recent meeting in Washington Dr. Spinrad seemed to support that view, and perhaps he would care to comment on it now.

B. I. Spinrad: I can only repeat what was said at the meeting in Washington. If it could be shown that the internal-zone type of critical assembly really does give the proper answers, we too would devote most of our time to that type of experiment, even though we have a large assembly. But, firstly, we do not consider that the value of such experiments has yet been clearly demonstrated (though we were among the first to try them), and secondly, even if it were demonstrated, occasional checks on complete large criticals would be essential.

SESSION 12

Chairman: M. Čopič

Paper SM-18/33 was presented by Donald Smith (see Vol. **III**, p. 241).

Discussion

A. Campise: Have you investigated what change there would be in your calculated critical mass if you arbitrarily increased the (n, γ) cross-section for U^{238}?

Donald Smith: The effect of changes in the (n, γ) cross-section for U^{238} on critical mass and breeding ratio has been studied as part of a general survey by Moorhead. A 20% reduction of σ (n, γ) will have less than 20% effect on breeding ratio and much less effect on critical mass.

I. I. Bondarenko: I did not quite understand your reference to our measurements in copper and would ask you to specify where you see the discrepancy or in what respect what you discovered was unexpected.

Donald Smith: At the second Geneva Conference you showed that there was a substantial difference in the capture cross-section depending on whether your measurements of copper were made in a copper environment or in a nickel environment. Our idea was to measure the capture cross-section in copper as a function of position in our reactor—in a position where the spectrum changes from a comparatively hard to a soft spectrum. If there were any capture resonances we would have expected some curious behaviour in the scan for the copper; but in fact it looked very much like the scan for gold and the other (n, γ) processes.

I. I. Bondarenko: Did I understand rightly that these measurements were made in the uranium screen of the Dounreay reactor?

Donald Smith: No, our measurements were not the same as yours, in which there was a full copper environment. We merely had a copper wire stretched from the core centre up through the core, through the top blanket, and a long way into the sodium, where the spectrum was changing. In fact we were not looking for the same effect as you; we were merely wondering whether we would see any sharp discontinuities or changes in the capture rate in copper as we went rapidly from one spectrum into another.

I. I. Bondarenko: I should like to point out that you should not really have expected to observe the effects which we found, as they are possible only in a large copper environment, where the copper itself is activated. Naturally, if, in the case in point, you measure with copper or with gold indicators in a uranium environment, the effects due to copper or gold should not appear. I am therefore not in the least surprised that nothing of the sort was observed.

Donald Smith: Without pressing the point, I merely thought that if capture resonances were present to the extent which seemed possible from your measurements, we should, in a scan through the centre, have seen a change in the capture rate for copper as opposed to the gold capture rate, for example. We do not doubt your measurements, but we were rather surprised, and in fact a bit disappointed that we were unable to shed some light on the matter.

Paper SM-18/39 was presented by W. B. Loewenstein (see Vol. **III**, p. 263).

Discussion

A. Campise: Could you tell us something more about the perturbation programme which you have at Argonne?

W. B. Loewenstein: It has been applied essentially to the CUREM programme, which is a multi-group CURE code. We have a set of fluxes and adjoints, and all effects can be calculated with it—leakage, absorption, scattering, etc. Actually, I believe the programme was done at the instigation of Dr. Okrent.

Donald Smith: May I ask how much confidence you place in your control-rod calculations? We have done both one-group and two-group perturbation calculations at Dounreay, and found that the former gave rather better results than the latter. Of course we have been able to measure the control-rod worth.

I was surprised to hear that you find leakage to be the most substantial effect in EBR-II. In our case the U^{235} component is by far the largest effect in our control-rod calculations.

W. B. Loewenstein: There is no doubt that the leakage effect is small. If you examine Table X of our paper, you will find that we have measured a control-rod worth of 0.37% $\Delta k/k$ on ZPR-III. Perhaps the most sophisticated control-rod calculation gives a value between 0.36 and 0.46. We have done another calculation which gives 0.65, and that gives the criticality exactly. The calculations which give 0.46 have not yet been corrected by the perturbation correction factors given in Table XI. By and large, however, I think our results are quite good.

Donald Smith: Would you agree that control-rod calculations are still so doubtful that it is almost worthwhile having a low-power experiment to check them for any reactor design? We have not yet reached the point where we can accurately predict control-rod worth.

W. B. Loewenstein: In my opinion, control-rod-worth experiments are almost more important than criticality experiments.

Donald Smith: That is exactly what I mean; but the worth cannot be calculated very well as yet. One further point: are you satisfied that the results which you obtain on ZPR-III are so typical that for EBR-II you can afford to do away with detailed experiments such as we have done at Dounreay? I am thinking particularly of the fact that you will then have your reactor immersed in sodium. We have come to the conclusion, in fact, that it would be possible to eliminate both the top and the bottom axial blanket in the core, because we found that in sodium the scattering is high enough and the capture low enough, so that neutrons which escape from the core are scattered until they return either to the core or to the radial blanket. In other words we would not lose our breeding ratio by eliminating those blankets. This was not evident from the results of measurements carried out on ZEUS.

W. B. Loewenstein: I think I can bear out your contention to some extent. I would not worry about the absolute numbers too much. There is a reference calculation for EBR-II, a two-group calculation which gives a total breeding ratio of 1.23; another calculation, which includes no uranium axial reflector, gives, essentially, an initial conversion ratio of 1.18. These numbers may not be entirely accurate, but I agree that the loss is small—of the order of 5%.

Donald Smith: My real point, however, is that when you come to measure this you may get a different answer.

W. B. Loewenstein: That is certainly possible.

Donald Smith: And you have no facilities for making the measurement?

W. B. Loewenstein: The breeding ratio will eventually be determined physically; we shall actually measure what goes into and what emerges from the reprocessing plant.

P. Moinereau: With regard to perturbation calculations and direct calculation

of control-rod worths by a homogeneous approximation, I should like to mention that at Saclay we have done both types of calculation for RAPSODIE and got fairly good agreement. In eight-group perturbation theory we got a value of 3×10^{-5}, and in (r, z) geometry a value of 2.3×10^{-5}.

D. B. Hall: Would Mr. Loewenstein care to guess whether control-rod calculations would be improved by cross-sections or by calculational techniques? What is lacking in fact?

W. B. Loewenstein: I doubt that the real problem lies with the cross-sections in a fuel-controlled reactor, although that might be the case in a poison-controlled reactor. In a fuel-controlled reactor, I think the real problem is that it is very difficult to mock up the control rods precisely. And no matter how precisely you determine the control rods, the engineers will want still more precise information.

A. Campise: You just mentioned that you had calculated a breeding ratio of 1.23. Could you characterize the median fission energy of the core?

W. B. Loewenstein: The figure which I mentioned for the breeding ratio was probably not, as I pointed out, absolutely correct. I feel that the calculated breeding ratio of EBR-II—or the conversion ratio of a U^{235}-fuelled reactor—is perhaps closer to 1.3. The medium fission energy of the core is perhaps in the neighbourhood of 300 to 400 keV.

Donald Smith: I should like to underline what Dr. Loewenstein said just now about the difficulty of modelling a control rod. In the Dounreay Reactor we took out the middle region, which is 6 in in diameter, and replaced it with another section—simply a block as I said before—into which we can put these experimental sub-assemblies. Using TDC calculations we found that the reactivity, which we then calculated, was rather sensitive to the way in which we modelled the reactor. Essentially we used a 1-in thick annulus of steel, and in a calculation where this steel was smeared into the inner zone or into the next radial zone we got an error of the order of 1 or 2%, which is a lot in terms of fuel elements. These calculations were done with a 16×16 mesh size, and that is simply not enough. I would not care to guess how far one would have to go to get an accurate control-rod calculation, but it would certainly have to be very detailed.

W. B. Loewenstein: It is fortunate, at least, that the shadowing effects are small.

J. Chernick: Am I correct in assuming that the phase transition will happen quite frequently?

W. B. Loewenstein: We are still rather uncertain about that. The phase transformation has been measured, and it is very similar to the uranium-molybdenum phase diagram, except that the temperature is at 550°C. Experts on irradiation seem to think that as burn-up increases in the fuel pin it may not be possible to revert to the high-density phase. But the difficulty is that this material, initially at least, exhibits this phase structure.

J. Chernick: I gather, then, that this phenomenon has been thoroughly investigated, at least for the fissium fuel element, that you are satisfied with its integrity and that you also know the kinetics of the phase transition.

W. B. Loewenstein: Yes. As a matter of fact the kinetics and the expansion have been rather extensively studied not only for the 5-wt.% fissium alloy, but for 3-wt.%, 7.5-wt.% and 10-wt.% fissium alloys as well.

W. Häfele: In EBR-II you have ordinary water refining at every tenth step, as it were. I should like to know how the figure of 10% was arrived at—whether

the consideration of certain nuclear aspects of the system or the requirements of fuel handling led you to adopt it.

B. I. Spinrad: It should be remembered that this 10% process—the so-called drag-out—is also a pyrometallurgical process. The determination is based on a balance of two factors. One of these is the degree of fissium which appears to be beneficial, for after a certain amount has been added no further improvement appears in the alloy unless quite a bit more is added. The second factor is that we want to keep as much material in as possible and make the drag-out process do as little work as possible. These two effects are therefore counterbalanced. Naturally we want to have a small enough quantity of the fissium so that the neutron poison is not too significant. In my opinion the most important single factor is the fissium content, because about 10% appears to be the ideal amount for this particular alloy; in this respect it compares with the 10% molybdenum alloy, which is also good.

Paper SM-18/82 was presented by L. N. Usachev (see Vol. III, p. 315).

Discussion

A. Campise: I should like to ask Dr. Usachev to characterize the flux spectrum in this reactor by a median fission or a median absorption energy, and I should also like to know how the spectrum in this reactor compares with the three other fast systems which the Soviet Union.

L. N. Usachev: With regard to the spectrum in the core, the calculated spectrum is given in our paper (Fig. 11) and it can be compared; I have not compared it. I cannot at present answer the second part of the question.

W. B. Loewenstein: You indicated that there was a 4% burn-up in the core. Firstly, how do you define burn-up, and secondly, how much burn-up do you have in the reflector?

L. N. Usachev: The figure of 4% refers to maximum burn-up. The average burn-up is lower; in other words the figure of 4% refers to burn-up in the central rods. These figures apply only when all the rods are working, including the central rods in which the burn-up takes place and which sustain the burn-up. In the peripheral rods the burn-up is less. Since the figure of 4% was maximum burn-up, it seems to me that I do not need to reply to the second part of the question.

W. B. Loewenstein: I think that answers my question. My next question concerns your Table III, where you give the sodium worth in the reactor as 4%. Is this a measured or a calculated value? Furthermore, I believe you said the core height was 28 cm. Is the core diameter about the same? What is the volume fraction of the coolant?

L. N. Usachev: All the data given in Table III were experimentally obtained; there are unfortunately no calculated values in the table. I was speaking of those cases in which the calculated values coincide closely with the experimental data—that is in practically all cases. The plutonium control rods are satisfactorily described in theory, as are the automatic regulating rods. But there is poor agreement where the core is filled with sodium, and also much worse agreement when we calculate the large nickel elements which are used as reflectors. For them too there were errors of about 30%.

W. B. Loewenstein: I had a special reason for asking about your sodium worth. Both dry and wet critical experiments were performed for the EBR-I, fuelled with uranium-235. As the core size is about the same the spectrum should

not be very different, but the worth of the sodium was considerably less than 4% there. This result rather surprised me.

J. Chernick: In your calculations, did you try to take into account the higher resonances in gold?

L. N. Usachev: We accounted for the higher resonances in gold experimentally.

J. Chernick: I notice that you used water as a partial shield. Are you satisfied with this arrangement or have you had trouble with it? Water can, after all, cause a great deal of trouble in such a reactor, precisely because it is a fluid shield. Have you experienced any such difficulties?

I. I. Bondarenko: I would like to say a few words regarding the danger of using water shielding in a sodium-cooled reactor. In our case, a considerable number of measures has been taken to avoid this danger. In particular, between the space filled with water and the sodium there is not one partition but several, including partitions in which water can flow. Thus, even if more than one partition were perforated, there would be no mixing of the water with the sodium.

In addition, I would like to say a few words on the large effect due to sodium. This effect is not surprising because in the sodium system sodium accounts for a fairly large part of the volume of the core (about 30%, if I remember rightly). This reactor is too small for sodium capture to occur in it, and naturally this factor plays an important role. The sodium effect in this reactor is much greater than in all the other reactors which have been described so far at this Seminar, and that is not surprising. In large reactors the sodium effect will obviously be less and may even have a different sign. In small reactors the effect is always greater and the sodium invariably improves the reactivity. I can even say that we under-estimated the sodium effect in BR-5 to some extent, i.e. it turned out to be greater than in the calculation, although not by much. If I remember rightly it was about 25% greater.

J. Chernick: I wonder whether you could give us the maximum burn-up of total atoms, as that is what really counts in an alloy (I assume this is an alloy). Also, could you tell us whether there have been any leaky or failed fuel elements thus far in the operation of the reactor?

I. I. Bondarenko: I would like to point out that in general the sole purpose of this reactor is to test fuel elements. Because of that, we wished to construct this reactor at little cost, and we had recourse to several special design features. In particular, this reactor, for simplicity's sake, has no large uranium reflector, but only a thin internal layer of uranium reflector, the remainder of the reflector being made of nickel. The reactor is air-cooled, and by means of a built-in blower it was possible to achieve a cooling effect such as could not be achieved with an equivalent layer of uranium. There is, however, also the thin layer of uranium, which allows us to study the behaviour of the fuel elements, the reflector and the shielding. This layer is a few centimeters thick, and in it we can study the behaviour not only of the active elements but also of the reflector elements. The last time we ran it, about two weeks before we came here, we took stock of the reactor's operation and determined the maximum burn-up of the fuel elements. By burn-up we mean the ratio between the number of plutonium atoms which have undergone fission and the total number of plutonium atoms in the fuel element. I would remind you that the core consists of pure plutonium oxide and does not contain any other substances. I think the burn-up must now be approaching 5%, because a short while ago it was more than 4%. All fuel elements are working normally. The presence of plutonium activity in the sodium

circuit would show that one of the elements was no longer hermetically sealed. This point has been carefully checked throughout the operation of the reactor; there is a system for disclosing plutonium activity in the sodium circuit and up to now no such activity has been disclosed. But at the same time there is some increase in activity in the sodium, which could be partly attributed to the escape of gaseous and volatile fragments. In particular, in the gas above the sodium, we have found fragments of radioactive inert gases. By activation analysis we succeeded in establishing the presence of, for example, caesium. But this escape of volatile fragments is considerable and, apparently, the main part of the fragments is retained in the fuel elements. In any event, I repeat, no escape of plutonium has been observed.

J. Chernick: Thank you for your very complete answer. Do you think that these fission products escape by diffusion?

I. I. Bondarenko: We have directly observed the presence of a certain amount of fission-fragment activity in the sodium circuit. This means that inert gases may escape from the fuel elements to some extent, but in very small quantities. It is difficult to say just how they do escape but I do not think it is simply a question of diffusion through the jacket. Most likely there is some sort of diffusion through the less tightly scaled parts of the welds.

A. Campise: Do you use stainless-steel cladding with a sodium bond between the plutonium oxide and the stainless steel?

I. I. Bondarenko: No.

Donald Smith: You mentioned just now that you do watch closely for the presence of fission products to see whether any of your fuel elements have failed. Could you briefly tell us what your detection method is, and what is the minimum unclad area of plutonium that you would expect to observe?

I. I. Bondarenko: I shall not be able to answer the second part of the question, as to the sensitivity of the methods employed. Several monitoring methods are employed. In the first place, we simply measure the radioactivity in the circuit. Secondly there is monitoring by delayed neutrons. If there are any fission fragments, they will be revealed by the presence of delayed neutrons after the coolant has left the core. This does not occur in the reactor but just outside it. These are the two main methods. There is also simple radiochemical analysis of the sodium and of the gas-blower which is located over the sodium.

J. J. Schmidt: Why have you chosen nickel for your reflector? Is it by reason of the specific energy dependence of the mean cosine of the elastic scattering?

L. N. Usachev: Nickel gives very good scattering, a very large scattering cross-section and is therefore a good reflector. Obliged as we were to work with the apparatus which already existed, we had to make as good a reflector as possible.

J. J. Schmidt: Don't you think, Mr. Usachev, that there will be a more considerable leakage of neutrons through the dips in the nickel resonances than there would be if you used alternating layers of different materials such as nickel and iron?

L. N. Usachev: Obviously a mixture would be better. But in the case of iron, for instance, we already know that there is a very pronounced dip; as far as nickel is concerned, I cannot remember at the moment. In a 24-keV dip a pure iron reflector would allow all the neutrons to leak away, but with nickel I do not think there are any such pronounced dips.

I. I. Bondarenko: Nickel has two isotopes. Consequently the resonances of one isotope, roughly speaking, plug the holes in the other. In fact, therefore,

by using pure nickel we are already doing as you suggest. For where you have a single element which has two isotopes, each with different nuclear properties, then of course you already have a mixture. If you look carefully at the periodic table, therefore, you will see that nickel is a good reflecting medium, not to mention the fact that its technical properties are also favorable. It stands up well in air and is good from the corrosion point of view.

D. B. Hall: We have examined the scheme that Mr. Schmidt suggests, and we find no particular advantage in mixing reflectors in a fast small plutonium reactor. I might add, by the way, that we have found very little absorption of plutonium in the sodium coolant. Have you made any observation of the fuel plutonium oxide in sodium?

I. I. Bondarenko: I cannot answer the last question, because after all I am a physicist and not a metallographer. Some experiments were of course carried out before the reactor was built, but I cannot give you details of the experiments on the plutonium oxide—sodium interaction.

B. I. Spinrad: What particular oxide of plutonium do you use in these fuel elements?

I. I. Bondarenko: PuO_2.

R. E. Mueller: Although there is no formal paper submitted to this Seminar on the Enrico Fermi Atomic Power Plant, I should like to take a few minutes to give you some of the important reactor physics data connected with this reactor.

The reactor is a fast neutron breeder. The fuel for the first loading consists of zirconium-clad pins containing U—10 wt. % Mo alloy with the uranium enriched to 25.6 % U^{235}. The blanket alloy is depleted U—3 wt. % Mo. There is no moderator. The average fission energy is approximately 0.2 MeV and is maintained at this high value by design to take advantage of the fast-fission effect in U^{238} and of the low absorption cross-section of structural materials at high energy. By maintaining a high-energy spectrum, more Pu^{239} is bred than U^{235} destroyed.

Major design and performance data are summarized in Table I.

TABLE I

PLANT DESIGN AND PERFORMANCE DATA WITH INITIAL ALLOY CORE

Plant performance

Reactor power (MW)	200
Gross electric power (MW)	65.9
Net electric power output (MW)	60.9
Net thermal efficiency	30.5

Design characteristics of core and blanket regions

	Core region	Axial blanket region	Radial blanket region
Number of subassemblies or sections	105	105*	531
Elements per subassembly or section	140	16	25
Element shape	round pin	round rod	round rod
Active volume (ft^3)	13.4	7.5*	156

Dimensions (in)

Uranium alloy outside diameter	0.148	0.395	0.395
Uranium alloy length	30.5	14.0*	61.75
Clad inside diameter	0.148	0.423	0.423
Clad outside diameter	0.158	0.443	0.443
Clad thickness	0.005	0.010	0.010
Inside diameter of region	—	—	32.7
Outside diameter of region	32.7	32.7	79.9
Length of region	32.78	17*	65.0

Composition

Type of uranium alloy (wt.%Mo)	10	3	3
U^{235} enrichment (wt.%)	25.6	0.35	0.35
Percentage of U^{235} (vol.%)	7.03	0.09	0.14
Percentage of U^{238} (vol.%)	20.27	25.51	39.86
Percentage of Zr (vol.%)	4.6	—	—
Percentage of Mo and SS (vol.%)	22.9	17.9	20.9
Percentage of Na (vol.%)	45.2	56.5	39.1
Power (MW)	174	1.2*	23.6
Power density (MW/ft^3)	13	0.16	0.151
Specific power (MW/kg U^{235})	0.35	—	—
Maximum heat flux (10^3 BTU/h ft^3)	640	72.2	217
Maximum to average heat flux	1.69	4.1	21.7
Sodium flow through region (10^6 lb/h)	7.10**	7.10	1.47***
Sodium velocity through region (ft/s)	15.7	14.1	—
Maximum nominal coolant temperature (°F)	1014	959	899
Maximum nominal outer clad temperature (°F)	1051	960	900
Maximum nominal uranium temperature (°F)	1115	991	950
Maximum local removal burn-up (at.%)	0.4	0.25	0.25

General nuclear characteristics (static)

U^{235} enrichment in fuel (wt.%)	25.6
Critical mass (105 subassemblies) (kg U^{235})	496
Core conversion ratio	0.29
Blanket conversion ratio	0.87
Total conversion ratio	1.16
Radial axial blanket Pu production	8.9
Median energy of core flux (MeV)	0.32
Mean fission energy (MeV)	0.25
Average core flux (n/cm^2 s)	3×10^{15}
Maximum to average core power density	1.69
Radial maximum to radial average power ratio	1.38
Axial maximum to axial average power ratio	1.23
Blanket fissions total fissions	0.076
Average neutron-generation time (s)	0.084

Reactor kinetics

Effective delayed-neutron fraction	0.006 6
Prompt-neutron lifetime (s)	1.4×10^{-7}

Delayed neutron groups

Group	1	2	3	4	5	6
Effective fraction ($\times 10^2$)	0.021	0.127	0.123	0.261	0.104	0.026
Decay constant (s^{-1})	0.0127	0.032	0.120	0.323	1.40	3.94

Isothermal temperature— reactivity coefficients (10^6 $\delta k/k$ °C)	Core region	Axial blanket region	Radial blanket region
Sodium expansion	—4.4	—5.2	—1.7
Uranium alloy expansion	—6.2	—0.1	—0.2
Hold-down plate expansion	0.0	—0.4	—0.3
Doppler (at 550 F)	—2.6	—	—
Subassembly can radial expansion	—11.3	—	—0.7
Lower support expansion	—1.5	—	—
Power coefficients (10^{-2} c/MW)			
Sodium expansion	—2.1	—2.7	—0.9
Uranium alloy expansion	—6.8	—0.1	—0.2
Hold-down plate expansion	0.0	—0.2	—0.1
Bowing	+0.2	—	—
Doppler (at 200 MW)	—1.9	—	—
Subassembly can radial expansion	—9.4	—	—0.3

* Each axial blanket section (upper or lower).

** Does not include 0.15×10^6-lb/h flow through control rods, neutron source and surveillance channels.

*** Does not include 0.14×10^{-6}-lb/h flow through vessel shielding.

Reactor control

Reactivity budgeting

Temperature override to 200 MW (c)	60.0
Total reactivity loss, weekly unloading (c)	15.4
Control margin (c)	16.6
Total reactivity in operating control rods (c)	92.0
Net reactivity loss (per day of operation)	2.2

Operating control rods

Boron-10 contained per rod (g)	87
Weight per rod (lb)	16
Poison section length (in)	10
Operating stroke of each rod (cm)	40
Reactivity in each operating control rod (c)	46
Maximum reactivity per rod-inch (c/in)	4
Maximum to average reactivity per rod-inch	1.37
Regulating rod design speed range (in/min)	1 to 10
Regulating rod runaway (synchronous) speed (in/min)	15
Regulating rod maximum reactivity insertion rate, at runaway speed (c/s)	1
Regulating rod drive type	Velocity servo
Shim rod speed (in/min)	0.4
Shim rod average reactivity insertion rate (c/min)	1.17

Safety rods

Boron-10 contained per rod (g)	535
Weight per rod (lb)	23.5
Poison section length (in)	36
Total operating stroke (in)	54
Active reactivity stroke (in)	38
Retraction speed (in/min)	1.6
Follow down speed (ft/min)	10
Total reactivity in 8 rod bank ($)	9.20
Minimum reactivity in individual rods ($)	1
Maximum negative reactivity withdrawal rate (c/s)	1
Reactivity withdrawal rate at $1 subcritical (c/s)	0.6
Delay of reactivity inserton after scram signal (s)	0.25
Total scram duration after scram signal (s)	0.8
Average rate of reactivity insertion on scram ($/s)	16.7

Sodium-system and steam-system performance

Sodium-system operating characteristics

Temperature of sodium leaving reactor (°F)	800
Temperature of sodium entering reactor (°F)	550
Temperature of sodium entering steam generator (°F)	767
Temperature of sodium leaving steam generator (°F)	517
Primary of secondary Na 200-MW reference flow (lb/h)	8.86×10^6

Steam parameters

Feedwater temperature (°F)	340
Steam temperature, at steam generator outlet (°F)	764
Steam pressure, at steam generator outlet, (psia)	600
Steam temperature, at turbine (°F)	760
Steam pressure, at turbine (psia)	576
Steam flow (lb/h)	6.4×10^5

Sodium-system pressure drops (at 200 MW reference flow)

Primary sodium pump head limit (ft)	260
Total primary sodium system pressure drop (ft)	126
Core and axial blanket pressure drop (ft)	106
Secondary sodium pump head limit (ft)	100
Total secondary sodium system pressure drop (ft)	36

Reactor and containment vessels

Reactor vessel

Height (ft)	36.3
Maximum diameter (ft)	14.5
Maximum wall thickness (in)	2
High pressure plenum design pressure (psig)	110
High pressure plenum design temperature (°F)	750
Radial blanket plenium design pressure (psig)	50
Radial blanket plenum design temperature (°F)	1000
Upper reactor vessel design pressure (psig)	50
Upper reactor vessel design temperature (°F)	1000

Primary shield tank

Overall height to top of dome (ft)	62
Height of primary shield tank (ft)	40.2
Maximum diameter (ft)	24
Wall thickness (in)	5/8
Design pressure (psig)	7.38
Design temperature (°F)	250

Reactor building

Overall height (ft)	120
Inside diameter (ft)	72
Average thickness (in)	1.03
Design pressure (psig)	32
Design temperature (°F)	650

The operating characteristics of the core and blanket are given in Table II. The detailed compositions of the various regions are given in Table III.

Methods of analysis. The nuclear characteristics have been determined by standard methods of analysis and from measurements made in the critical experiments for this reactor. Reactivity effects, critical-mass determinations and neutron-flux distributions have been determined from solutions to the Boltzmann equation by approximate methods in common use. One-dimensional multi-group diffusion theory has been used to obtain most of the necessary

TABLE II

OPERATING CHARACTERISTICS OF CORE AND BLANKET

General characteristics	Core section	Axial blanket section	Inner and outer radial blanket section
Maximum average power density	1.69	4.1	21.7
Coolant inlet temperature (°F)	552	550 (lower)	550
Average coolant temperature rise (°F)	272	4	175
Average coolant outlet temperature (°F)	824	826	725
Removal burn-up (max. at. %)	0.40	—	0.25
Power (MW) (at 0.25 at. % average core burn-up)	174	2.4	23.6
Coolant flow rate (lb/h)	7.10×10^6	7.10×10^6	1.47×10^6
Maximum heat flux (BTU/h ft^3)	640 000	72 200	217 000
Coolant velocity (ft/s)	15.7	14.1	—
Δp through subassembly (psi)	40.0		
Maximum nominal coolant temperature (7) (°F)	1014	959	899
Maximum nominal clad temperature (°F)	1051	960	900
Maximum nominal uranium temperature (°F)	1115	991	950
Maximum coolant temperature (9) (°F)	1099	1008	985
Maximum outer clad temperature (8) (°F)	1139	1009	989
Maximum uranium temperature (9) (°F)	1209	1043	1063

nuclear design data. The PROD-II IBM-650 code, written by KAPL, has been used extensively, along with programmes which have been written by APDA to be used in conjunction with PROD-II. These auxiliary programmes compute input needed for PROD-II, and use PROD-II output to calculate a variety of data such as neutron-absorption breakdowns, isotopic-reaction rates, and plu-

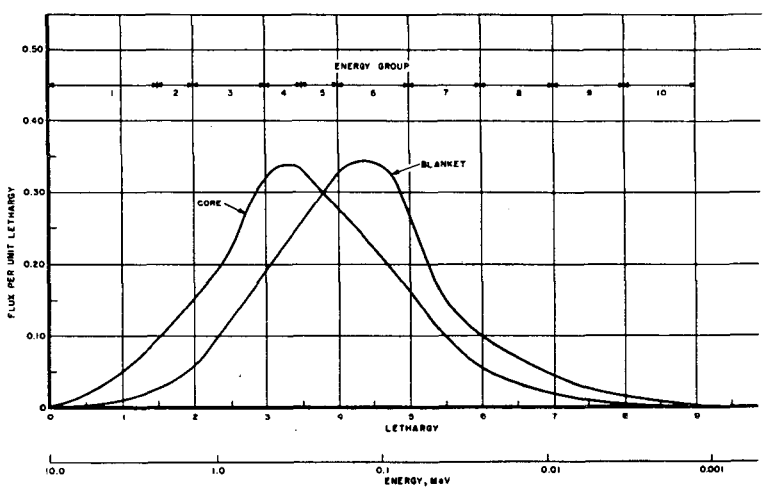


Fig. 1
Neutron-flux spectra in core and blanket.

TABLE III

REGION COMPOSITION

Material	Clean core — Reactor model composition by region*											Dirty core** — Core burn-up = 0.25 at. % Blanket burn-up = 0.14 at. %***		
	Material densities (g/cm³)	Empty control-rod channel	Control rod poison channel	Empty safety rod channel	Safety rod poison section	Active core	End cap	Radial blanket	Active axial blanket	Axial reflector	Radial reflector	Active core	Radial blanket	All other regions
U^{235}	18.5	—	—	—	—	0.0705	—	0.0014	0.0009	—	—	0.0696	0.0013	Same as in the clean core (axial blanket burn-up too low to be considered)
U^{238}	18.7	—	—	—	—	0.2027	—	0.4006	0.2552	—	—	0.2024	0.3980	
Mo	10.2	—	—	—	—	0.0556	—	0.0210	0.0136	—	—	0.0556	0.0210	
Zr	6.5	—	—	—	—	0.0466	0.04	—	—	—	—	0.0466	—	
Stainless steel	7.9	0.145	0.180	0.145	0.160	0.1640	0.20	0.1860	0.1670	0.25	1.00	0.1640	0.1860	
Na	0.86	0.855	0.560	0.855	0.630	0.4606	0.76	0.3910	0.5633	0.75	—	0.4606	0.3910	
Pu^{239}	19.6	—	—	—	—	—	—	—	—	—	—	0.0002	0.0020	
U^{236}	18.7	—	—	—	—	—	—	—	—	—	—	0.0002	—	
Fission products .	18.7	—	—	—	—	—	—	—	—	—	—	0.0008	0.0006	
B^{10}	1.9	—	0.0395	—	0.0649	—	—	—	—	—	—	—	—	
B^{11}	1.9	—	0.1830	—	0.0319	—	—	—	—	—	—	—	—	
C	1.9	—	0.0664	—	0.0282	—	—	—	—	—	—	—	—	

* Composition of each region by volume fraction of that region.
** Composition of equilibrium core for removal at 0.5 at. % burn-up.
*** Equivalent to 75 MW of reactor operation.

TABLE IV

SUMMARY OF NUCLEAR CHARACTERISTICS

U^{235} enrichment in fuel	wt. %	25.6
Critical mass U^{235} (105 subassemblies)	kg	496
Core conversion ratio		0.29
Blanket conversion ratio		0.87
Total conversion ratio		1.16
Radial axial blanket Pu production ratio		8.9
Median energy of core flux	MeV	0.32
Average core flux at 200 MW	$n/cm^2\ s$	3×10^{15}
Prompt-neutron lifetime	s	1.4×10^{-7}
Average generation time	s	0.09
Delayed-neutron fraction		0.0076
Effective delayed-neutron fraction		0.0066
Maximum to average core-power density		1.69
Blanket fission total fissions		0.0826

tonium concentrations, plutonium quality, and heat-generation rates as functions of burn-up.

Ten-neutron energy groups were normally used in the diffusion-theory calculations. These groups cover an energy range that extends down to approximately 1 keV. This range covers the core and blanket spectrum adequately as can be seen from the core and blanket spectra given in Fig. 1. For problems concerned with regions outside the blanket, it was necessary to use additional groups to represent the more degraded spectrum.

In determining power distribution, control-rod worth at various positions, and flux distributions in the vicinity of the actual irregular core periphery, one-dimensional calculations are inadequate. To obtain these data, two-dimensional diffusion theory was used. Two-neutron energy groups were used in the two-dimensional calculations. The spatial variation of spectrum was taken into account by weighting with the 10-group spectra in reducing the 10-group constants to 2 groups, the 2-group constants thus differing for each region.

Diffusion theory is adequate in nearly all of the nuclear analysis of the reactor except in determining the subcritical neutron fluxes at the neutron detectors which are located in the graphite shielding. Solutions to these problems were obtained by means of transport theory based on the S_n method.

The cross-section data used in the diffusion theory calculations are primarily those given in ANL-5800. The data taken from this source were altered to 10-neutron groups.

In many cases nuclear-design data can be obtained with greater accuracy and in more detail by use of perturbation theory. APDA has modified PROD-II to enable it to solve the flux adjoint equations for 10 groups with inelastic scattering. An auxiliary programme, using the PROD-II real and adjoint fluxes, was used to compute material danger coefficients at each space point. The individual components of the danger coefficient (viz., fission, capture, spectrum, and leakage) were obtained as output to permit a better understanding of the effects of the material on reactivity. In addition to calculations of material danger coefficients, perturbation theory was used to estimate neutron lifetime and the effective delayed-neutron fraction.

Statics. The nuclear characteristics of the reactor are summarized in Table IV. For the estimated clean critical loading of 105 core subassemblies, the critical mass is 496 kg of U^{235}. This loading provides an excess reactivity of 92 cents at the isothermal reactor start-up temperature of 517°F.

The in-pile conversion ratio is 1.16. The conversion ratio as it is given in Table IV is the ratio of U^{238} captures to the fissions and captures of enriched U^{235}. The destruction of U^{235} in the depleted uranium is ignored, since it has low economic worth. Although poison control is used, neutron losses to the poison are small since only 40 cents at operating temperatures is compensated by B^{10} absorption. The blanket is designed for a leakage of less than 0.5% of the neutrons. Absorptions in the molybdenum alloy material and the steel structure amount to about 4% of all neutrons. Most of the parasitic losses are due to U^{235} captures.

The prompt-neutron lifetime is estimated to be 1.4×10^{-7} s. The average generation time close to delayed critical is 0.09 s.

The delayed-neutron fraction in the clean core is 0.0076. It is considerably larger than that of U^{235} because approximately 15% of the fissions occur in U^{238}. The effective delayed neutron fraction, however, is 0.0066. The delayed neutrons are relatively less important than the prompt-fission neutrons because their energy is below that of the U^{238} fission threshold, whereas approximately 68% of the prompt neutrons are capable of causing U^{238} fission. This can be seen from the iterated fission probability curve as shown in Fig. 2. For the Fermi core, neutron importance increases with energy above the U^{238} threshold, and decreases with energy below it.

Core-power distributions obtained from two-dimensional calculations for the x, y and r, z planes are given in Figs. 3 and 4. The r, z power distribution shown corresponds to the shim and regulating rods having been withdrawn. Although this condition gives the highest maximum to average power density ratio, in this case 1.65, this ratio does not change significantly with rod position as can

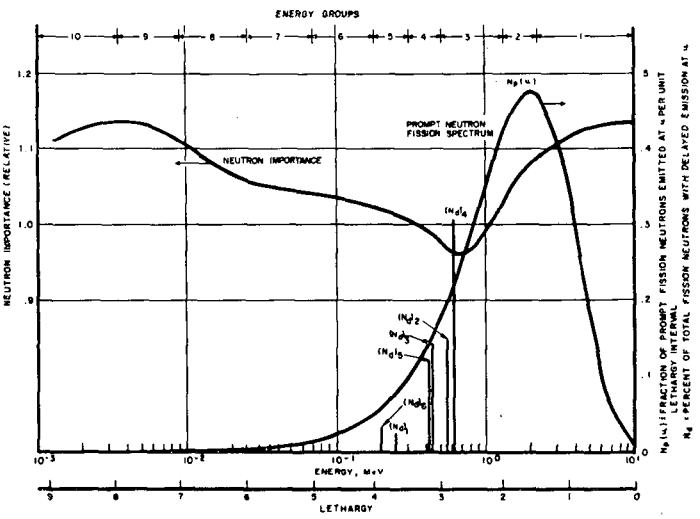


Fig. 2
Iterated fission probability in core and fission spectrum.

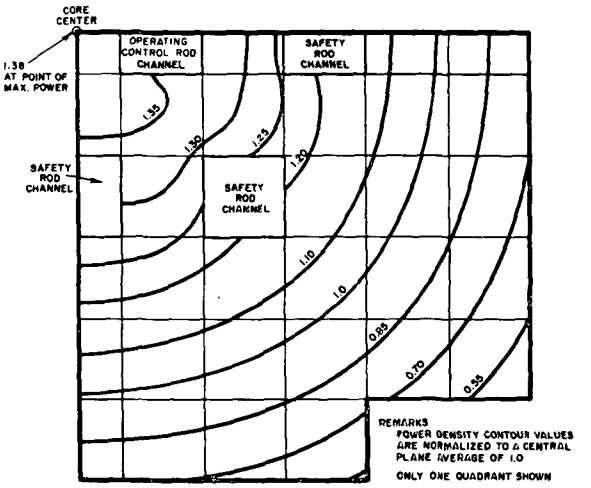


Fig. 3
Central plane radial power distribution in the core.

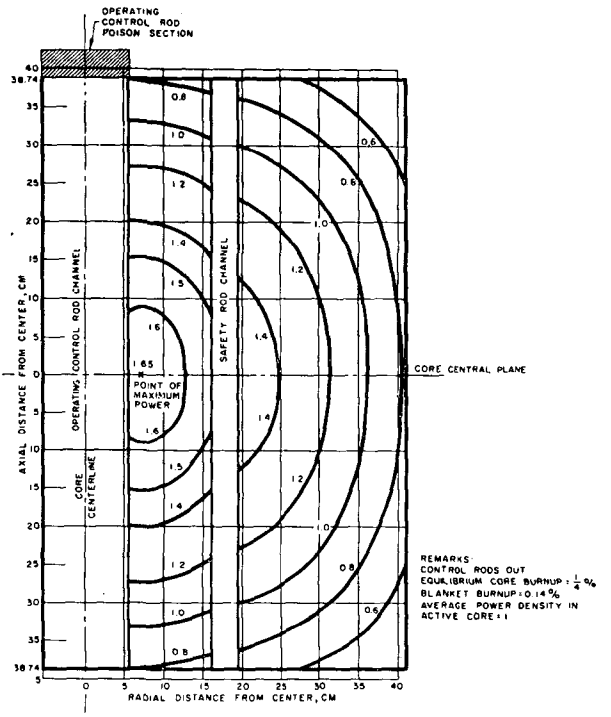


Fig. 4
Core power distribution with control rods out.

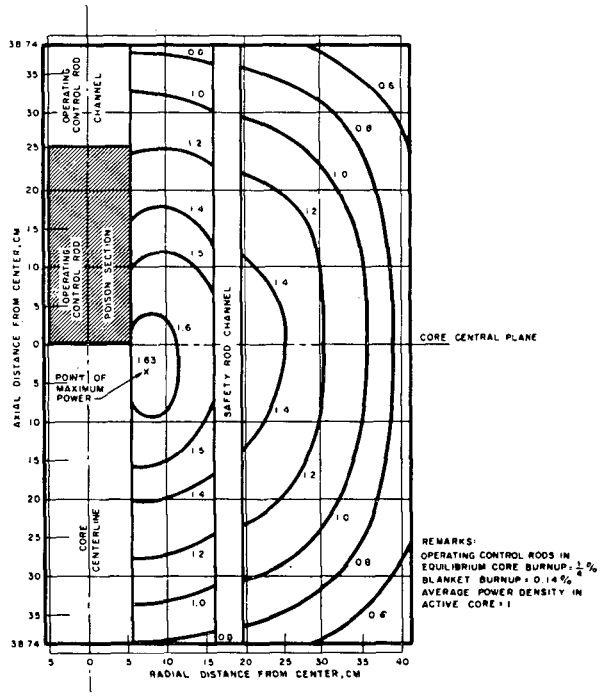


Fig. 5
Core power distribution with operating control rods in.

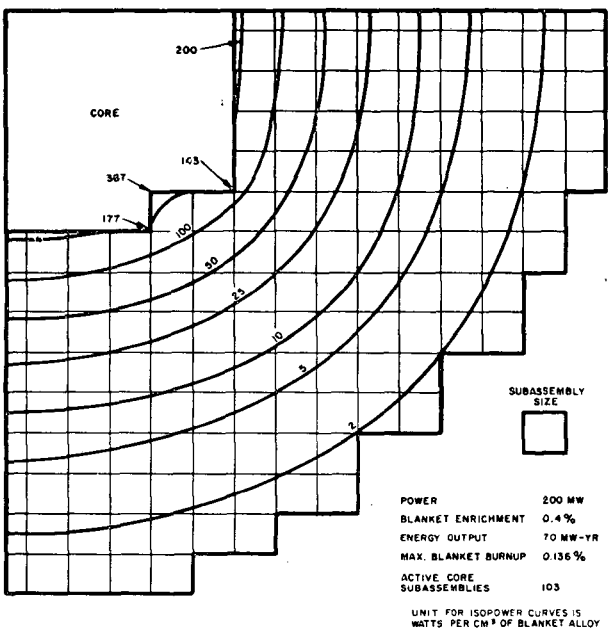


Fig. 6
Central plane radial blanket heating values.

TABLE V

NEUTRON ABSORPTION BREAKDOWN

	Neutron production			Neutron Absorption		
	Core	Blanket	Total	Core	Blanket	Total
U^{235} fission	0.824	0.021	0.845	0.340	0.009	0.349
U^{238} fission	0.092	0.063	0.155	0.037	0.025	0.062
U^{235} capture	—	—	—	0.066	0.002	0.068
U^{238} capture	—	—	—	0.119	0.353	0.472
Mo capture	—	—	—	0.012	0.007	0.019
Zr capture	—	—	—	0.002	—	0.002
SS capture	—	—	—	0.008	0.013	0.021
Na capture	—	—	—	0.001	0.002	0.003
B_4C capture	—	—	—	0.002	—	0.002
Total	0.916	0.084	1.000	0.587	0.411	0.998
						0.002
	Neutron leakage to shield					1.000

be seen by comparing Fig. 5 with Fig. 4. Note that the model used in obtaining these results does not contain a central core subassembly. The actual highest maximum to average ratio is 1.69, rather than the 1.65 obtained here.

The blanket-power distribution is complicated because a large part of the heating in this region is due to absorption of U^{238} capture gammas. In a clean blanket, about one third of the power is due to this heating component. In addition, fission of plutonium formed in the blanket results in heat generation which changes significantly with irradiation. The blanket heating contours shown in Fig. 6 were calculated using the Pu concentration which would exist after 200 core subassemblies have been cycled to 0.5% burn-up, which is greater than the burn-up actually planned.

A neutron-absorption breakdown is given in Table V for the clean reactor. The effective values of γ are 2.51 and 2.60 for U^{235} and U^{238} respectively, giving an overall effective value of 2.52. As stated in the table, 8.3% of the fissions occur in the blanket. The ability to breed is due to the low parasitic-neutron losses and the large fast-fission effect. U^{238} contributes about 15% of the total fissions. Parasitic-neutron absorptions account for 12% of the neutrons with more than one half of these being captured in U^{235}. Molybdenum is a relatively strong absorber and absorbs about 2% of the neutrons. Steel contained in the cans and supports is present in larger quantities and, although a weaker absorber than molybdenum, accounts for about the same percentage of neutrons. Because of the small amount of excess reactivity, the shim and regulating rod absorptions are negligibly small. Also, the loss of neutrons by leakage is small because of the density and thickness of the blanket.

The neutron-flux spectra in the core and blanket regions as well as the group structure used in the calculations are shown in Fig. 1. The flux peaks are at approximate energies of 400 keV in the core and 120 keV in the blanket. The flux in the lowest energy group in both core and blanket is small indicating that the spectrum is adequately covered by the 10 groups used.

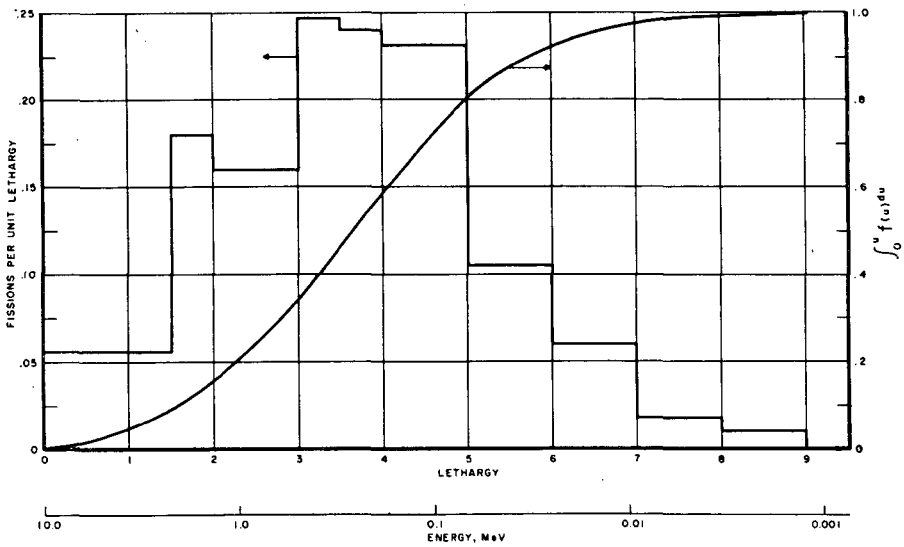


Fig. 7
Distribution of core fissions as a function of energy.

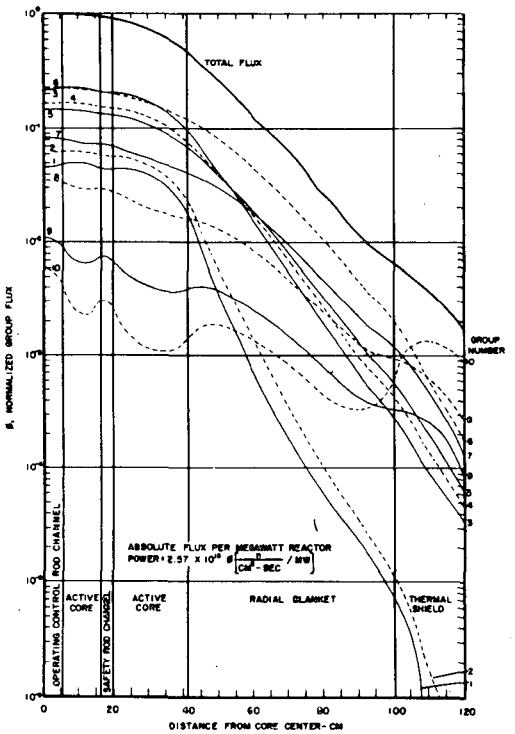


Fig. 8
Radial flux distribution at central plane.

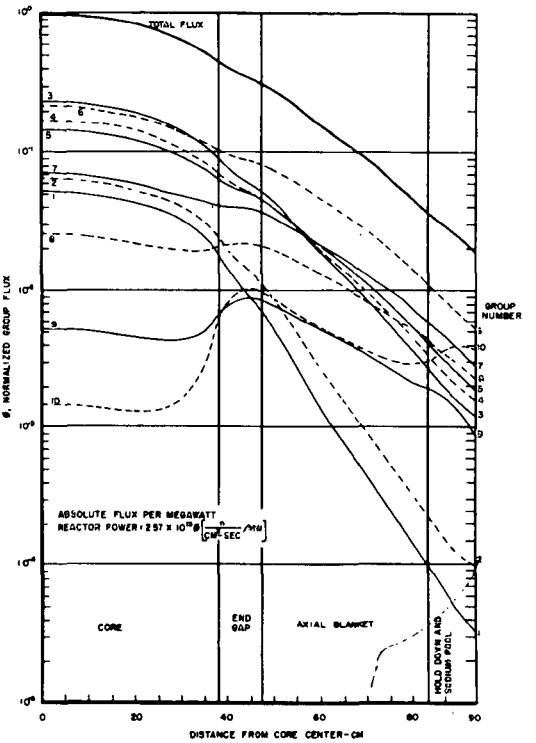


Fig. 9
Axial flux distribution along core centre-line.

The fissions per unit lethargy for the core are shown in Fig. 7. The median fission energy is about 250 keV. The dip in the fission distribution which occurs between lethargies 2 and 3 is due to the U^{238} fission threshold.

The radial and axial neutron flux distributions are shown in Figs. 8 and 9 respectively. The total flux is attenuated by a factor of 70 in the radial blanket.

Burn-up effects. The burn-up of the alloy fuel will be limited to a maximum value of 0.4 at. % pending a fuel surveillance programme demonstrating that this burn-up can be safely exceeded. This maximum burn-up corresponds to an average subassembly burn-up of about 0.3 % of the alloy atoms. Based on this burn-up, one core loading will produce 20 MWa of heat. An average core subassembly will therefore have a life of 38 d of continuous reactor operation at 200 MW. Core-subassembly lifetimes vary from 30 d for central subassemblies to 55 d for subassemblies located adjacent to the radial blanket.

For this relatively low burn-up, the core composition and hence the nuclear characteristics of the core do not change significantly. The composition changes of the fuel are shown in Table VI. These changes were calculated on the basis of 0.5 at. % average burn-up instead of the 0.3 at. % average burn-up planned. For an average removal burn-up of 0.5 at. %, it is seen that 2.7% of the U^{235} is destroyed in the core. In a subassembly ready for removal, the U^{235} to Pu^{239}

TABLE VI

EFFECT OF BURN-UP ON CORE AND BLANKET COMPOSITION

	Core composition (per 100 fuel atoms)	
	Clean	At 0.5 at. % burn-up*
U^{235}	25.8	25.1
U^{238}	74.2	74.0
U^{236}	—	0.1
Pu^{239}	—	0.2
Pu^{240}	—	0
Fission products**	—	0.6
	100.0	100.0

MIDPLANE RADIAL BLANKET COMPOSITION AT CORE-BLANKET INTERFACE
(per 100 uranium atoms)

	Clean	After 70 MWa	At 0.5 at. % blanket burn-up
U^{235}	0.4	0.3	0.3
U^{238}	99.6	99.0	97.6
U^{236}	—	0	0
Pu^{239}	—	0.5	1.6
Pu^{240}	—	2.4×10^{-3}	2.3×10^{-2}
Fission products**	—	0.2	0.5
	100.0	100.0	100.0

* Based on removal burn-up of 0.5 at. % rather than 0.3 at. % to be attained without inspection.
** Based on one fission-product atom formed per fission.

atom ratio is then 125. Looking at the core as a whole, the ratio is actually larger since it contains subassemblies at all stages of burn-up.

The build-up of plutonium in the blankets will be greater than in the core, but will not be large enough to significantly effect nuclear characteristics. The blanket material is expected to be capable of withstanding a maximum local burn-up of 0.5 at. %. However, the burn-up will be restricted to one half this amount until a blanket inspection programme indicates that the higher burn-up can be tolerated. At 0.5 at. % burn-up, a Pu^{239}-to-U^{238} ratio of 0.0164 will occur in the central plane of the innermost radial blanket subassemblies. The plutonium concentrations become increasingly higher with distance into the blanket for the same burn-up limit. The explanation for this effect is that the blanket spectrum becomes softer with distance from the core, and the U^{238} fission rate becomes smaller in comparison to other reaction rates. However, during the period in which 200 core subassemblies are used, none of the blanket subassemblies will reach its burn-up limit with the inner radial blankets only reaching a burn-up of 0.138 at. %. The plutonium concentration is also considerably less than will

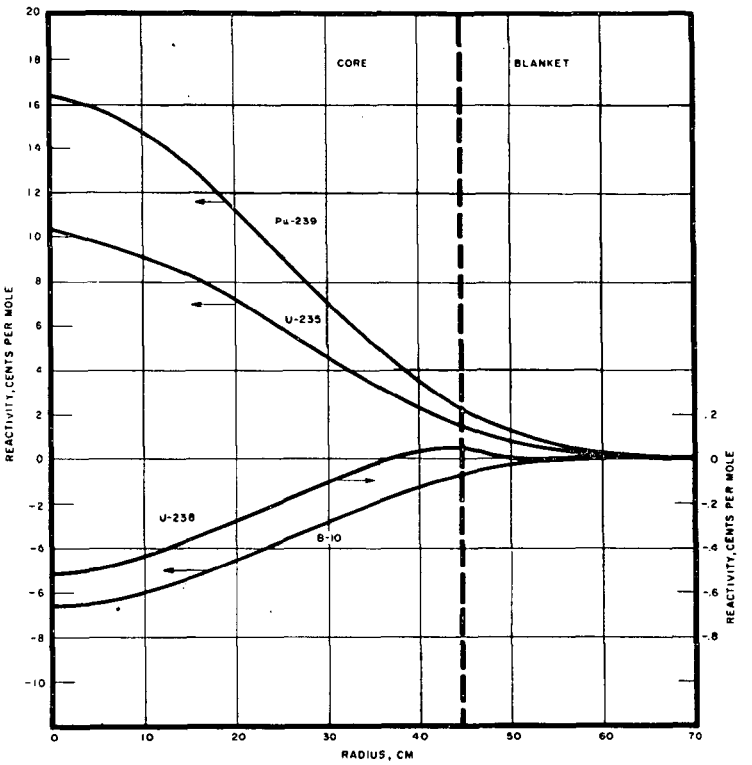


Fig. 10
Danger coefficients of U^{235}, U^{238}, Pu^{239} and B^{10}.

be present after 0.5 at. % blanket burn-up. It is estimated that after 200 core subassemblies have been used the plutonium in both core and blanket will contribute only about 1.6% of the fissions. For this composition the breeding ratio is reduced by 0.01. Similarly, the effect on the delayed-neutron fraction and other kinetic parameters is very small, amounting to less than 1%.

Material danger coefficients. Material danger coefficients have been calculated by means of perturbation theory for most of the reactor materials. These quantities are useful for estimating the reactivity effect of small changes in composition and to calculate temperature coefficients of reactivity. The computed danger coefficients for several important reactor materials are shown in Figs. 10 and 11.

A comparison of the calculated danger coefficients with those measured in the critical experiment is given in Table VII. Central and average core values are listed. Two sets of calculated values are given. One applies to the critical experiment and the other to the design as it has been modified since the critical experiments were performed.

The Doppler coefficient is the only truly nuclear reactivity coefficient in the reactor. All other reactivity coefficients result from changes either in composition or in geometry. These changes are sufficiently small for their reactivity effects

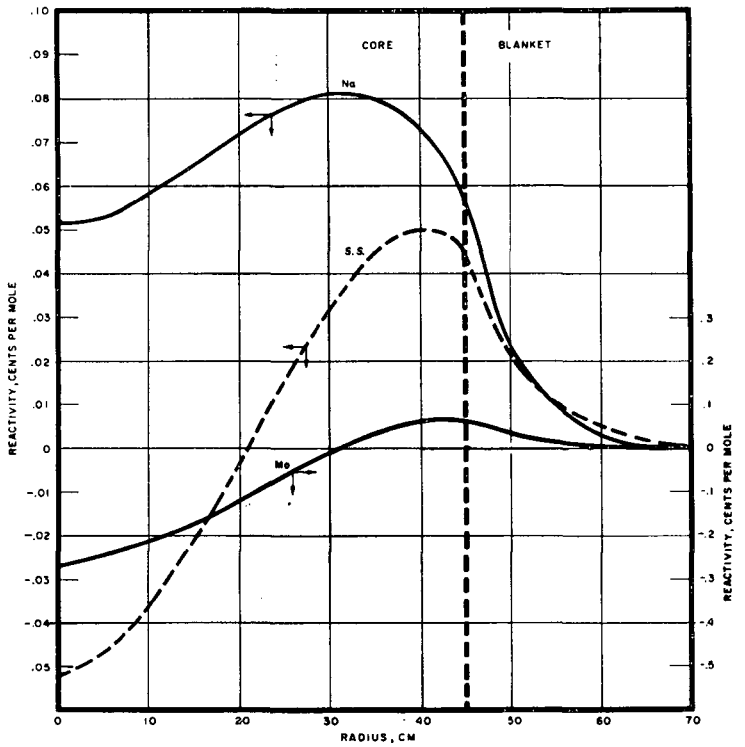


Fig. 11
Danger coefficients of sodium, stainless steel and molybdenum.

to be calculated by perturbation theory which benefits from the principle of superposition. Furthermore, these changes themselves can be calculated as a sum or integral of the changes induced by local temperatures or temperature

TABLE VII

MEASURED AND CALCULATED MATERIAL COEFFICIENTS (cents/kg)

Material	Central			Core average		
	Experimental from critical assembly	Calculated for critical assembly	Calculated for Fermi core	Experimental from critical assembly	Calculated for critical assembly	Calculated for Fermi core
U^{235}	44	43	43.3	17	16	16.1
Pu^{239}	73	67	68.1	26	25	24.7
U^{238}	—1.8	—2.1	—2.1	—	—0.329	—0.291
Mo	—3.5	—2.6	—2.72	—0.41	0.11	0.079
SS	—1.3	—0.85	—0.914	0.30	0.61	0.610
Na	—	2.8	2.25	2.4	3.4	3.2
B^{10}	—590	—630	—645	—	—220	—224

TABLE VIII

TEMPERATURE AND POWER COEFFICIENTS OF REACTIVITY

	Isothermal range		Power range				Total override (cents)
	Temperature coefficient (10^{-6} $\Delta k/k°C$)	Override (cents)	Temperature coefficient (10^{-6} $\Delta k/k°C$)	Power coefficient (cents/MW)	Override (cents)	Base temperature	
Na expansion	—4.44	1.22	—4.16	—0.0209	4.18	Average Na	5.40
Fuel pin radial expansion	—0.36	0.10	0.33	—0.0030	0.61	Average fuel	0.71
Fuel pin axial expansion	—4.80	1.61	6.96	—0.0650	12.96	Average fuel	14.57
Doppler	—2.59 to —2.47	0.70	—2.47 to —1.68	—	3.78	Fuel	4.48
Bowing	—	—	0.44	0.0023	—0.46	Average Na	—0.46
Lower support expansion	—1.47	0.43	—	—	—	Inlet Na	0.43
Core subassembly wrapper tube radial expansion	—11.33	3.14	—17.96	—0.0942	18.84	Average Na	21.98
Hold-down plate expansion	0.02	0.01	0.02	0.0002	—0.04	Exit Na	—0.05
Na expansion in IRB (inner radial blanket)	—1.70	0.47	—1.70	—0.0089	1.78	RB Na, average	2.25
IRB pin expansion	—0.24	0.07	—0.24	—0.0022	0.45	IRB alloy, average	0.52
IRB displacement due to core expansion	—0.67	0.17	—0.63	—0.0033	0.66	Average Na	0.83
IRB due to hold-down expansion	—0.28	0.08	—0.14	—0.0015	0.29	Exit Na	0.37
Na expansion in UAB (upper axial blanket)	—5.22	1.44	—2.61	—0.0274	5.48	Exit Na	6.92
AB axial expansion	—0.09	0.02	—0.04	—0.0007	0.13	UAB alloy, average	0.15
AB radial expansion	—0.06	0.02	—0.06	—0.0002	0.06	Average Na	0.08
AB due to hold-down expansion	—0.42	0.12	—0.21	—0.0020	0.44	Exit Na	0.56
		9.58		—0.2458	49.16		58.74

gradients throughout the reactor. This second superposition property arises from the fact that the reactor structure is elastic, and that such material properties as the thermal expansion coefficient and the modulus of elasticity are nearly independent of temperature. Hence, one can define local coefficients of reactivity which give the reactivity effect of a unit change of local temperature gradient. The knowledge of these local coefficients is required in the analysis of transients, since the temperature response during a transient is dependent on position. These local temperature coefficients can be obtained from structural deflection calculations, and from local material coefficients of reactivity.

In special cases when the temperature pattern is known, overall or integral coefficients of reactivity can be defined. They are determined by integrating the local coefficients of reactivity, weighted by the temperature pattern, over the volume of the reactor. Two such overall coefficients are the isothermal coefficient of reactivity, and the power coefficient of reactivity.

Local temperature coefficients. The calculation of the local effects requires the use of sodium and U^{235} material coefficients of reactivity. The U^{235} worth distribution measured in the critical experiment was done in sufficient detail to be used in the temperature coefficient calculations. However, not enough sodium was available to get a sodium worth distribution with sufficient detail for use in the temperature coefficient calculations. The calculated distribution of sodium worth was used in this instance, normalized so that the average sodium worth agrees with the experimental value.

Isothermal temperature coefficients. The total isothermal temperature coefficients are obtained by integrating the local temperature coefficients over the entire core volume. In addition, the Doppler coefficient must be corrected for temperature. The integrations were done numerically and the results are given in Table VIII.

The operation programme for the reactor contemplates that the coolant inlet temperature will be increased linearly with power from 517 to 550°F over the power range of 0 to 200 MW. The total reactivity override can be considered to be separable into an isothermal override from 517 to 550°F and a power override in which the inlet coolant temperature is maintained at 550°F while the reactor is raised from 0 to 200 MW. Such a separation makes the calculation of the override much simpler.

The isothermal override is the integral of the isothermal temperature coefficient over the temperature range from 517 to 550°F. The results, in cents, are given in Table VIII. The conversion from $\Delta k/k$ to cents was made using a total delayed neutron fraction of 0.0066.

Power coefficients. The power coefficients for the point effects are obtained by integrating, over the entire core volume, the product of the local temperature coefficients and the derivative of the local temperature with respect to power. The override is obtained by integrating the power coefficient over the power range of interest.

In reactor simulation work it is necessary to have temperature coefficients at powers which are based on an average temperature, such as average fuel temperature or average coolant temperature. Such temperature coefficients can be obtained by multiplying the power coefficients by the derivative of power with respect to the selected average temperature. A summary of power and temperature coefficient data is given in Table VIII.

Paper SM-18/40 was presented by J. K. Long (see Vol. I, p. 295).

Discussion

A. Campise: Could you comment on any of the special experiments which you plan to perform with this facility?

J. K. Long: A general outline of the programme is given in the first two pages of the paper, and I am not really in a position to describe it in greater detail. I notice that an investigation of the Doppler temperature coefficient is included as a possible experiment, but to the best of my knowledge no detailed experimental programme has yet been prepared.

A. Campise: Is there any thought of using an oscillator mechanism for the reactivity coefficient measurements, rather than material replacement experiments?

J. K. Long: Not at the present time, as far as I know. We know that oscillator measurements have some advantages and we never rule out the possibility of using them, but no definite plans have been made.

P. Clauzon: Could you give us the percentage of fixed structure, i.e. stainless steel, in ZPR-VI?

J. K. Long: ZPR-VI will have a stainless-steel, and ZPR-IX an aluminium, fixed structure. ZPR-VI, which is to be interchangeable with ZPR-III, will have the same percentage of fixed structure as the latter, i.e. 9.3% or 9.5% for the matrix and the drawers.

I. I. Bondarenko: I would like to ask a question which concerns the actual structural features of this machine. I do not quite follow the considerations which led to the construction of a two-half type of facility. We also deliberated at length as to what form the spherical stand should take, and for various reasons we decided against the split-table construction. Actually, when you design a facility, one of the first and fundamental problems is to ensure that every possible precaution is observed in all operations involving an increase of reactivity. Yet here we would seem to have the most dangerous type of case—you are taking two halves of an apparatus and bringing them close together. Why is it not after all better to assemble the core bit by bit so that no single operation increases the reactivity of the system to any great extent? Does it not seem to you that certain unforeseen circumstances are within the realm of possibility, and that the two halves might accidentally come together? That is clearly always possible. There might be an earthquake, for example, at the very moment when the two halves were approaching each other. It is a crude example, perhaps, but it seems to me that the point is not entirely clear and I would like you to explain to me what considerations led you to choose that design.

J. K. Long: Your arguments are interesting, and I know very well that many people prefer the single, fixed machine. As to the addition of reactivity by small increments, our contention is that we use both systems. In building up to a critical mass we add small increments of fuel, run the halves together, measure the multiplication, separate the halves, add another small increment and so forth. As long as we follow this procedure I think we have some of the advantages of both methods, and we also have the great advantage of being able to get right into the reactor and make internal changes—a relatively simple process in a two-half machine.

On the other hand, I realize that if we are to gain anything from this advantage we must be able to manipulate the fuel without remote handling equipment, and that in turn means that we are confined to operating at low powers. The

advantage of a fixed machine, I think, is that you may be willing to go to a somewhat higher power because you do not expect to handle the fuel quite so often, and that opens up the possibility of using different instruments and obtaining greater precision. However, we feel that we have not exhausted the possibilities of the two-half machine. Even at very low powers—most of our measurements are made at a fraction of a watt of power—we feel that there is still much information to be gained; and we are convinced that the flexibility afforded by the two-half machine is highly desirable. We have found that it is possible to tear apart a complete assembly and assemble a new one in about one week, which means that we can change our compositions fairly frequently. I know that in at least one instance our British colleagues have attacked this problem in a different way. As they want both the flexibility of the two-half system and the advantage of going to higher powers, they have taken the trouble to build automatic devices for loading their drawers. It will be very interesting to see how this works out, because it may permit higher powers and consequently greater accuracy in measurements while still allowing easy and flexible changes of the reactor.

B. I. Spinrad: Perhaps I will answer a number of these questions together if I explain some of the basic considerations underlying the design and operation of ZPR-VI. We thought it important that the materials used in ZPR-III and ZPR-VI should be largely interchangeable, so that if any unusual materials had to be obtained the orders would not have to be duplicated to supply the two sites. That is one reason why we preferred a two-half machine. Clearly, loading by increments is still essential, and the split table can be regarded as a safety device much the same as the outer blanket of the BR-5, for example, or a similar device in EBR-I.

The aluminium structure of ZPR-IX, which will come into service later, was chosen because it will allow coupled reactor experiments—fast-thermal experiments. In this case the aluminium is not so important in the part occupied by the core of a coupled system; we would not want a strong thermal absorber in the reflector of such a system. However, the current programme calls for doing most of our large-scale experiments on ZPR-VI, which will be completed earlier. At the moment we are thinking of using ZPR-IX as a facility for special experiments such as Doppler measurements, oscillation experiments, spectrum measurements and so on. In such cases we should prefer to construct assemblies with special, well-defined characteristics, which might have no particular bearing on a proposed practical reactor design, but which would help us to understand certain specific phenomena—the material replacement coefficient, the temperature coefficient or what have you. We are rather vague about it because these experiments are all extremely difficult, and we do not want to promise that we will perform then unless we are sure they will be workable. We are developing our technique now to that end.

K. Einfeld: Can you describe the test section which you will use in ZPR-VI for measuring the Doppler temperature coefficient?

J. K. Long: I have no details regarding the plans for future Doppler experiments. You probably know that a Doppler experiment was run on ZPR-III some years ago with a helical sample which was alternately heated and cooled.

K. Einfeld: You said that you would use this facility for experiments with uranium carbide. Could you say in what physical form the uranium carbide will be used?

J. K. Long: I feel quite sure that we shall use uranium metal and introduce carbon into the assembly in the form of graphite plates; it will be heterogeneous to that extent.

A. Campise: May I reply to Dr. Bondarenko's question concerning the safety of split-table critical assemblies? In the case of the AETR—and I assume in the case of ZPR-III as well—three different drives are used to bring the halves together. On the AETR we have definite stopping points where we turn over to slower drives, and by the time the halves are only 6 in apart they are being driven together very slowly. Even when they close the assembly is not critical because the rods are withdrawn; and only after the halves have come together are the rods driven very slowly to criticality. It would therefore seem very difficult to get accidental criticality. Of course one can advance the hypothesis, but every precaution is taken to prevent the halves from coming together quickly.

J. K. Long: I am reminded that we did have an earthquake in Idaho, and although the epicentre was not far from the testing site the facility was not affected. The halves are, after all, held by a drive screw; they are not free to move on the ways, which merely support their weight.

R. D. Smith: I think there is little point in arguing the relative merits of two-half machines, horizontal machines, vertical machines, etc.; we have thought about this for a long time, and I really think there is very little to choose between them. In the final analysis one's choice will probably depend on the particular experiments one wents to perform. I fail to understand, however, why your desire to have parts which will be interchangeable between ZPR-III and ZPR-VI should force you to adopt a two-half machine. After all, the pieces we propose to use in ZEBRA are virtually identical with the ZPR-III components. Another short question: how do you propose to introduce oxide into ZPR-VI?

J. K. Long: I think your first point is well taken; there are other designs which would probably allow for interchangeability of pieces. As for the introduction of oxide, we have two materials available at ZPR-III—aluminium-oxide and iron-oxide—either of which can be introduced into the assembly. The iron-oxide is in ceramic form. There are two possibilities; I do not know whether there are any other plans for ZPR-VI.

J. Chernick: Naturally, everyone has considered the possible hazards of these assemblies, but our experience is very limited. Completely unforeseen accidents can occur, and I think, therefore, that remote assembly of the critical facility should be given serious consideration. If the efforts of our British colleagues in this direction are successful they might profitably be imitated elsewhere. There is always some hazard, and where there is more than one way of assembling a super-critical fast reactor that hazard is multiplied.

R. D. Smith: The automatic assembly for ZEBRA is in fact designed only to load the fuel coupons into the drawers or tubes, and not to load the tubes into the assembly; so it would not meet the point which has just been made. Its purpose is to save time and ensure greater accuracy.

B. I. Spinrad: We did consider a fuel-loading machine, but we decided that it would be too awkward a device to adopt at the present time—although in principle I agree that its adoption would be a step in the direction of maximum safety. In a sense, however, we have struck a compromise, for we have a technique whereby the fuel elements—and these are rather small quanta in this case—are brought into the reactor cell one at a time, actually through a small window between the cell and the place where they are assembled. In these circumstances it would be very difficult to do anything other than load one piece at a time into the reactor, unless one were consciously trying to make a mistake.

SESSION 13

Chairman: D. B. Hall

Papers SM-18/25 and SM-18/27 were presented by C. Clair (see Vol. I, pp. 313 and 321).

Discussion

D. B. Hall: I did not fully understand the geometry of your assembly. How did you arrange the surface increments of mass to evaluate the effect of gap widths and of the delayed-neutron fraction? Were these buttons or plates which you applied to the plutonium?

C. Clair: I shall revert to some details of the structure of the system. As regards its initial geometry, Fig. 2 of the report describing the assembly itself (SM-18/25) shows a section of the reflector assembly. The upper part consists of a central spindle on which are placed a number of caps. The object is to be able to vary the thickness of certain reflectors as required, or to make perturbations by replacing one of the caps by a material other than natural uranium. Lower down you will see the uranium rings which are stacked one on top of the other. Three are shown in the drawing, without the corresponding plutonium. No plutonium is shown in this figure. Then, still lower, is the lower reflector ring which is fixed, except for the safety plug, which is shown in the raised position; two holes for rods can also be seen. So much for the reflector. The plutonium is placed entirely in the lower cavity, first a hemisphere, then three plates of the same thickness as the uranium rings, and on top of that the hemisphere corresponding to the first one.

H. Häggblom: Was your perturbation theory carried out in the diffusion theory approximation or in transport theory?

B. Lemaire: We used Carlson S_n theory.

Paper SM-18/61 was presented by A. Campise (see Vol. III, p. 335).

Discussion

W. B. Loewenstein: How will you provide the 10 to 20% excess reactivity needed to take care of the burn-up, and what effect will this have on the nuclear-performance constants which you quoted?

A. Campise: In our opinion the excess reactivity should be included as fuel control; it should be kept outside the core until needed. If you evaluate the the amount of excess reactivity which you need to burn to 25000 MWd/t and introduce that excess at the beginning of the process, the conversion ratio will be lowered considerably; that is particularly true of thermal systems. We hope that fuel control will help to solve this problem. Of course we can also adopt higher thorium-uranium ratios in order to provide an adequate breeding ratio.

W. B. Loewenstein: Does this mean that your system will operate at a lower plant factor than a normal all-fast system, i.e. that you will have to shut the plant down periodically in order to put fresh fuel into it?

A. Campise: No, the control rods will be fuel elements.

W. B. Loewenstein: I see; but that will require a very large number of control rods.

A. Campise: Yes.

D. Meneghetti: Would you care to comment on experiments or calculations you may have done in order to study the transmission of the thermal and near-

thermal neutrons from the thermal driver region through the internal blanket into the fast central region?

A. Campise: We are studying this at the present time and hope to have some data ready for the next ANS meeting in the United States.

C. P. Zaleski: Have you made measurements similar to those mentioned for Th^{232}, but with U^{233} or an isotope with a cross-section sensitive to the lower end of the spectrum? If so, did you obtain equally good agreement between theory and experiment?

A. Campise: Yes, we have used U^{235}; our U^{238} foils were not ready for the first core. The reactor has many regions, viz. the test region, the buffer, the coupler, the driver and the reflector. In fact we were able to reproduce the shape of a U^{235} traverse, and we plan to do the same thing with Pu^{239} and U^{233}. The U^{234} and U^{236} traverses were in very good agreement, but we did not carry them out past the decoupler because of the contaminant and the corrections which have to be introduced. I might point out for general interest that if you plot median absorption energy *versus* radius, the curve is fairly flat in the test region, drops very sharply in the driver and then goes over to thermal. Thus the data which we have accumulated thus far seem to indicate that we have an extremely hard spectrum in the centre being driven by an extremely soft reactor.

D. Okrent: Do you think that the useful data which you can obtain from this experiment would be appreciably changed if you used U^{235} as the fissile material in each case (plus thorium and carbon and whatever else you may wish to have in the central region), and then made very accurate measurements with U^{233}, U^{235} and other materials? After all, the spectrum is not determined to any great extent by the properties of the U^{233} itself.

A. Campise: They would not, and I think you have raised a very important point. Theory predicts that U^{233} should be replaced by U^{235} at a ratio of 2:1. We made the replacement experiment, using Hopkins' latest data, and as we observed no change in criticality apart from some 5 cents in the control rods it would seem that the 2:1 replacement ratio is nearly correct. We intent to prove experimentally that U^{235} can be used; we have already proved the point in theory. This is particularly important because irradiated U^{233} is a difficult material to handle in a zero-power critical experiment. If it is possible to avoid using it except in certain special cases, we would prefer to use U^{235}.

P. Greebler: You mentioned the reactivity penalties associated with the build-up of Pu^{233} and the higher uranium isotopes. Could you comment further on this problem, particularly the build-up of the higher isotopes, as U^{234} capture leads to a poorer fissile material (U^{235}) and U^{236} is a purely parasitic absorber in the neutron spectrum?

A. Campise: If you take your neutron balance as being 100, in an unmoderated carbide system you will lose about 10 neutrons and in a moderated system you will lose about 40 or 50. That is roughly the variation in absorption.

B. I. Spinrad: A simple estimate of the effect of U^{234} build-up can be made by doubling the effective α of U^{233} in the spectrum, since at steady state there will be one U^{234} capture for each U^{233} capture, and the 235—236 pair will not maintain itself. Therefore, the steady-state breeding gain will be controlled by $\eta_{233}/(1+2\alpha_{233})$. Will this system breed?

A. Campise: The α value which we now use in the intermediate range is about 0.15. If it were 0.3, I think we should have to harden our spectrum in, say, the 15-vol. % graphite system, which is just about the point of marginal breeding

with graphite. We should have to decrease the moderator fraction in order to compensate for the increase in α.

L. N. Usachev: Did you take into account the difference between the importance of the neutrons captured in the material under investigations and that of the fission neutrons emitted as a result of this capture?

A. Campise: As I pointed out earlier, we are trying to make off-centre measurements in our critical assembly, where we have moderated spectra. Successful interpretation of these data would speed up our research in moderated systems. We were recently asked to compare U^{233} to Pu^{239} at a meeting in Washington, D.C. We therefore made some measurements off centre in order to compare U^{233} and Pu^{239} with respect to reactivity worth and breeding potential. For the curve which we presented to the AEC we plotted our data as a function of median flux energy, and we made a quick survey of the ZPR-III experiments as they had extremely hard spectra—harder than we had planned to study in fact. We plotted the reactivity worth per gramme of U^{233} and Pu^{239} samples at high energy—roughly 10^7—10^5 eV, although I am not sure of the exact scale. When we compared our results with the ratio of α for U^{233} to α for Pu^{239} which has been presented by Diven and Hopkins we found that the curve did in fact follow the same trend. The break-point in the integral data is about 100 keV, while the break-point in Hopkins' data is about 60 keV. We must remember that these results were obtained over average spectra, where there was no differential beam of neutrons; we were considering a reactor spectrum. Diven had a highly defined beam of neutrons. It seemed interesting to us that these trends could be revealed, and we are hoping to make the same type of measurement for U^{233} to lower values, in order to see whether the Diven data or the extrapolations which our Soviet colleagues have made are correct.

L. N. Usachev: I must insist that Dr. Campise's formula can be used only where the neutron importances are identical. Normally they are not identical and one has to make appropriate allowance for that fact; and if they happend to be identical in this particular case, that is a point which ought to be established beyond any doubt by a suitable investigation.

D. Okrent: I must confess that I am a bit sceptical about the possibility of setting α values for, say, U^{233} from measurements such as you have described. Even if we discount Dr. Usachev's point and assume that you can account for the difference in the importance of neutrons with reasonable accuracy, I think you are still bound to face uncertainties of the order of 1 or 2% in the value of ν for each material, if you are taking a ratio. The α value of the other material will be substantially uncertain, Diven and Hopkins notwithstanding, and you will face more uncertainty in the ratio of the fissions in the two materials—here I think accuracy to within 2% is probably the best you can hope to attain. Your accumulated errors, I think, would add up to something like the magnitude of α, so it would hardly be surprising if you failed to get agreement.

A. Campise: I must say that I agree with Dr. Okrent. After making this study about a year ago I concluded that we were really on very uncertain ground. By making a few arbitrary changes in ν, either plus or minus, you can get, in some cases, absurd values for α. I brought this point up primarily because the agreement surprised me; when we got even more moderated systems we may get the disagreement which one would normally have to expect. In the meantime, I am presenting these results, not as an exact measurement of α, but merely to show you our data.

J. J. Schmidt: If I may add something to Mr. Okrent's statement, the only measurement of real value would be an integral α or η measurement for U^{233} in the resonance region, say between 10 eV and 1 keV, in a well-moderated system.

A. Campise: I must say that if we can calculate the reactivity coefficient to within 5 or 10%, our data are, in my opinion, good enough for reactor design.

J. J. Schmidt: So long as your measurements are not conclusive regarding the possibility of breeding, I should think that you must apply an integral method.

A. Campise: We have not obtained these data so far, and we are not sure what they will indicate.

Paper SM-18/31 was presented by P. Moinereau (see Vol. III, p. 355).

No discussion

Paper SM-18/38 was presented by H. H. Hummel (see Vol. II, p. 373).

Discussion

D. B. Hall: Before asking for questions I should like to make a few comments regarding the influence of neutron lifetime on melt-down. We have done no calculations for large power reactors, but we have studied the problem in LAMPRE, a very compact molten-plutonium system. In LAMPRE the expansion of the fuel provides the shut-down mechanism; it has a very large coefficient of expansion, something like 100×10^{-6} volumetric expansion per degree centigrade. We varied the neutron lifetime to see what the effect of a ramp input—not a step input—would be and found that the formation of a shock condition occurred somewhere between 1000 and 5000 dollars per second ramp input—we do not know exactly because the simple theory broke down at this point. We also observed that the peak power was greater with a longer neutron lifetime—10^{-3} or 10^{-4} s as opposed to the plutonium lifetime in this reactor of 5×10^{-9} s. An explanation of this, I believe, lies in the fact that the short lifetime is very closely bound up with the physical properties of the plutonium, and using the single shut-down mechanism comprised in the fuel expansion one has little or no overshoot, whereas with the longer lifetime one has essentially an inertial effect, and the overshoot is considerably greater. I do not think that this conflicts in any way with the work of Jankus, however.

D. Okrent: I think that the original Bethe-Tait paper, by applying some rough scaling laws, arrived at the same conclusions. If you consider a ramp type of accident, in which you allow as much reactivity to be added as time will permit—since there is no limit on the total available amount of reactivity—the yield of what I shall call the first burst is very insensitive to the neutron lifetime because you can insert more reactivity before the burst terminates the addition. That is why it is important to distinguish between a ramp and a step input.

J. Chernick: In the first place I might mention that we have also been doing some exploratory work on thermal-thermal systems. These are systems with rather large clusters of plutonium-oxide alloyed fuel elements in which one can get a very large fast effect; in fact it turns out that such systems yield fairly high conversion ratios even in thermal reactors.

May I ask you what the effect of power peaking is on your very fine power density—0.5 MW/l? Have you studied any possible buffer systems to reduce this peak?

H. H. Hummel: I should have thought that the power peaking was accounted for in the value of 0.5 MW/l.

J. Chernick: Even in our intermediate research reactor we got very considerable power peaking at the interface. I think this is a problem which commonly arises in what one might call externally moderated reactors with a very strong absorbing core.

H. H. Hummel: One can adjust the thickness of the isolation blanket to take care of this problem, and in practice I think that is what one would do. In other words, it is a phenomenon over which we do have some control.

B. I. Spinrad: Dr. Hummel mentioned the shut-down mechanism of ZPR-V, which was actually rather similar in its characteristics to the system described as thermal-fast rather than fast-thermal. As I recall, the course of the maximum accident for this system was roughly the following: there was little enough reactivity in the fast system so that, taken by itself, it could not go fast-critical. However, a situation could arise where reactivity might unintentionally be added to the system as a whole, bringing the whole reactor to a very high power. As the case in point is an uncooled critical assembly, any reasonable power at all would be regarded as high. Under these circumstances, the reactivity would ultimately limit itself by the feedback of boiling in the thermal zone, but this would be at a higher power than that for which the thermal-fast system was designed. As a consequence one might expect a melt-down of the fast core. The interesting feature of this particular case is that the melt-down of the fast core, occasioned by a power generation rise with thermal lifetime characteristics, would be pseudo-static as far as fast reactor excursions are concerned. As the fast core gained reactivity, and even as it began to approach a more nearly fast-critical state, it would remain for all practical purposes in power equilibrium with the thermal core. As it melted and expanded, the ultimate catastrophe—which of course never occurred—would be a rather gentle boiling-off of the uranium of the fast core.

W. Häfele: May I ask Dr. Chernick to describe the fast-fission factors in the thermal-thermal components?

J. Chernick: The large clusters of fuel elements will give a factor of 1.10, I think, in arrangements such as CANDU or arrangements using pressurized water. Of course one needs low neutron temperatures in such a reactor.

W. Häfele: I might say that, judging by our experience, 0.5 MW/l is probably an average rather than a peak value, because we feel it would not be very difficult to obtain it with steam cooling.

Paper SM-18/81 was presented by I. I. Bondarenko (see Vol. III, p. 399).

Discussion

A. Campise: How large was the U^{235} sample which you used to pulse this reactor?

I. I. Bondarenko: 10 cm in diameter and about 2.5 to 3 cm thick.

B. I. Spinrad: Do you think that the power of this device can be increased to the megawatt range, which would be required to make it competitive with the most modern of the point accelerators?

I. I. Bondarenko: I believe it would be comparatively easy to increase its power to the order of, say, 10—15 kW. It would be difficult—though feasible—to achieve a further increase because of the operational difficulties which would

arise if it should be found desirable to experiment with the reactor, e.g. by inserting samples or incorporating moderators to produce slow neutrons. There would also be operational and engineering difficulties to overcome in view of the radiation hazards, so that this is another limiting factor which would probably make it uneconomic to increase the power beyond 10 or 10 kW.

B. I. Spinrad: I asked that question because the physicists in the United States have placed a requirement of this sort; in other words they tend to prefer accelerators because of the very high intensities which they yield.

J. Chernick: As you had a choice between plutonium and uranium, why did you choose the former? There must be good reasons for using either of the two fuels.

I. I. Bondarenko: Plutonium gives a shorter neutron lifetime, and that is precisely what enables us to obtain a short pulse. With U^{235} the pulse duration would be twice as long, which would be undesirable.

J. Chernick: Is it not true, however, that it increases your dispersion and also makes the reactor rather small? In other words, would you change your mind if you decided to go to higher powers?

I. I. Bondarenko: Possibly.

J. Chernick: Have you done any experiments with this instrument the results of which will be published?

I. I. Bondarenko: As far as I know, many experiments have already been made for measuring capture cross-sections of various substances by gamma rays. They will be published, of course.

G. C. Tavernier: I noticed in your paper that you have a negative intrinsic coefficient due to the fact that your plutonium rods are only attached at one end, so that you can expand the core radially by heating. Does this mechanism not interfere with the pulse formation, or are the temperatures which you reach during the pulse not sufficient to cause a fall in reactivity because of this effect?

I. I. Bondarenko: The effect you speak of is small. That is to say the drop in reactivity due to the rise in temperature within the limits of the pulse itself, at any rate with a periodic pulsed system, is negligible compared to the effect produced by the movement of the sample. With large single pulses—which, incidentally, we have not yet tried—the effect you speak of would become noticeable, though even here it would be bound up with the movement of the sample and not with expansion. The temperature coefficient does not take the rod bowing into account since the reactor is air-cooled and the coefficient of heat transfer from wall to air is small, and thus, as leakage inside the rods is considerably higher than heat transfer to the air, the temperature of the rods is uniform. We have not measured the rod-bowing effect nor do we intend to do so as this would involve a dangerous experiment. Our calculations indicate that the coefficients should be several times greater because of rod bowing, but we have not measured this value and do not intend to do so.

V. Raievski: According to your paper it seems that in trying to get very short pulses one also gets increased pulse-amplitude fluctuations. Have you studied the fluctuations of the time interval which separates the passage of the U^{235} in the reactor from the moment when the pulse begins, and do you think that this time could have an influence on the accuracy, in that it defines the zero instant of the time of flight?

I. I. Bondarenko: I am not certain that I grasped all the nuances of the problem, so I will first answer the question as I understood it.

Power fluctuations were studied in the reactor and they are described in our

paper. They behaved as predicted theoretically. But apart from such statistical pulse fluctuations, it is the intensity of the pulse which fluctuates, not the period. This is understandable because the pulse-height fluctuations originate at the start of the pulse when there are still few fission chains. The pulse size is determined at that point. However, as the pulse develops, there are many more cross-sections and many more fission chains, so that the pulse is perfectly smooth and everything averages out. There are thousands or millions more chains than at the beginning, and the form and position of the flux do not therefore fluctuate. It is only the pulse-height that fluctuates, and this is associated with the fact that sometimes there are only a few neutrons. Apart from such statistical fluctuations, there are also fluctuations due to mechanical vibrations; these make up approximately 10% of the total in the 80—100-pulses-per-second range and approximately 20% in the 10-pulses-per-second range.

P. L. Balligand: I have a minor question which has already been touched on by Mr. Tavernier. I wished to ask what temperatures were reached by the fuel elements and why you chose air as the coolant. For it seems that if, as Dr. Spinrad suggested, it was desired to raise the power to 1 MW, it would probably be necessary to change the cooling system.

I. I. Bondarenko: At approximately 1.5 kW—the power used so far—the temperature was only about 100°C or even lower, so that air cooling was perfectly adequate. In fact it would even be possible to increase the power a little. When I was considering some such increase, I had in mind, among other things, going over to a gas with greater heat conductivity, such as helium. As I have said already, however, it is difficult to envisage really high powers, in the 1-MW range. In any case, this is another question altogether and not simply a matter of perfecting the reactor we are considering here.

P. L. Balligand: I think that if you were to increase the power to 1 MW it would probably be necessary to adopt another cooling system—which may be an answer to Dr. Spinrad.

I. I. Bondarenko: Yes, of course.

F. Storrer: I should like to ask whether, for calculating the kinetics of this reactor, a point model is sufficient, or whether you have to use a model which takes account of variation in space; in the same connection, I should like to ask whether you measured any variations in the shape of the flux as a function of time during a pulse.

I. I. Bondarenko: I am not sure whether I have understood the question correctly but isn't it one we have already discussed ? I believe Dr. Okrent said that in any case the reactor could be considered as being in equilibrium and in this sense the kinetics could be simplified. I think that this point is relevant but perhaps I have not seized the full purport of the question.

F. Storrer: I thought that the point model might perhaps not be sufficient, as the pulse lengths are relatively short and the pulse is produced by a sample of uranium which passes through the reactor at high speed.

I. I. Bondarenko: I still do not quite follow your question. The prompt-neutron lifetime in the reactor is far shorter than the times involved even in the rapid movement of the disk. Therefore, at any given moment the reactor is in the steady state from the point of view of spectra and all the other factors.

G. C. Tavernier: With regard to the value of this instrument as a pulsed-neutron source for time-of-flight experiments, I should like to ask what was the contamination in neutrons leaving the channel between each pulse, in order

to know whether it is an acceptable level; and I should also like to know where the neutrons are obtained. Is it from the outer surface or the centre, by means of a small channel?

I. I. Bondarenko: Neutrons are obtained from the whole of the reactor, and from its reflector. To obtain intermediate or slow neutrons, a layer of paraffin is placed behind the reflector to serve as moderator. For practical purposes, a layer of paraffin near a reactor has practically no effect on its pulse operation, though it does have an effect during steady-state operation, so that slow neutrons may enter the core, though they are not able to do so during the pulse.

H. B. Smets: Could you specify the maximum, minimum and mean reactivity values during one periodic oscillation?

I. I. Bondarenko: During the pulse, the excess above prompt criticality is about 0.001 in absolute units; between pulses, the neutron multiplication coefficient in the reactor varies from unity by approximately 8%.

H. B. Smets: I assume that the figure of 0.001 corresponds to a value above the prompt-criticality value, and that the mean reactivity value is therefore slightly positive with respect to the critical state.

I. I. Bondarenko: If your apparatus is working and the wheel is turning, the resulting changes in reactivity just sustain the chain reaction. The reactor is critical in the sense that a sustained chain reaction takes place, maintained by prompt neutrons during the pulse and by delayed neutrons in between pulses. But as a whole there is a continuous fission chain sustained *via* delayed and prompt neutrons. As for the normal neutron multiplication coefficient as a whole—i.e. the coefficient referring to all the neutrons and not just the prompt neutrons—its average value is equal to unity, i.e. the reactor is critical.

I have already said that the kinetics of this reactor are very similar to the kinetics of an ordinary reactor, i.e. the same processes take place here as in an ordinary reactor. When we say that a steady-state reactor which is just critical does not undergo any power changes in time, this is of course not strictly accurate, since in actual fact cosmic rays and other neutron sources are always present, so that if the power of a reactor does not change, this means it is very slightly sub-critical. If there were no extraneous neutron sources, then the power of a reactor which was just critical would not vary: the power of a supercritical reactor would increase exponentially, and that of a sub-critical reactor would fall. The position here is precisely the same. If you take the power averaged out over many pulses, the position is precisely the same as for a steady-state reactor.

P. L. Balligand: I should like to ask again, as I may not have understood quite correctly, what is the instantaneous flux during a pulse occurring in this channel which, after all, is similar to an ordinary experimental channel fitted with a chopper for time-of-flight measurements? I assume that the flux value must vary according to the pulse rate which one chooses. What are the practical values you are considering? This is in order to compare the flux produced by this reactor in an experimental channel with the flux of other reactors which are much more expensive to build.

I. I. Bondarenko: In the reactor there are large-diameter neutron tubes through which the whole of the reactor is visible, so I cannot tell you offhand what the neutron flux is through 1 cm^2/s because we were not interested in that particular value. If necessary, it can be computed, of course, but we were interested only in the total quantity of neutrons. I did indicate the total number of neutrons to you, but I find it difficult at the moment to give the flux per square centi-

meter. There is a difference between having a pulsed source and obtaining a pulse by interrupting the flow of neutrons. In the latter case you are interested in what the neutron flux is, because you want to known how many neutrons you will get, and in the former case you know the number of neutrons right away, so you are not very interested in the flux.

P. L. Balligand: I think I understand what you are saying. Perhaps I might put the question to a physicist in the audience. This would be equivalent to asking what flux in a standard reactor channel is needed to produce the same results on a time-of-flight selector? Is it possible to make a comparison (the figure given being 10^{14} n/s, if I remember rightly)?

I. I. Bondarenko: Yes; that was the figure for the total flux. I hardly think we shall be able to work out the details here and now, but it is clear that to obtain the same values in ordinary reactors very high powers and flux would be necessary. This particular reactor, in spite of its small size, is able to reach several thousands or even tens of thousands of kilowatts during a pulse, and besides you do not need collimator-choppers, the geometry of which diminish the flux even more. In my view, competition can be expected not from other reactors but from the electronic accelerators, which are now rapidly being developed with very promising results.

J. L. Leroy: I should like first to answer Mr. Balligand's question. If this pulsed reactor is compared with the linear accelerator at Saclay, it appears that the intensity of the neutrons delivered by the pulsed reactor of is the order of 10 to 20 times greater than that of the linear accelerator. Moreover, the intensity produced by the reactor is about 200 times that produced by the chopper fitted to the EL-3 reactor.

I should also like to comment briefly on the comparison between the pulsed reactor and the electron accelerators. I think that Dr. Bondarenko analysed the respective advantages and disadvantages of each of these arrangements very well. I think, however, that these two pieces of equipment are in fact complementary, since one, the electron accelerator, provides very high resolution, while the pulsed reactor provides very high intensity. Moreover, as we have been given an opportunity to hear about the development of pulsed reactors, I should also like to recall briefly the development possibilities of linear accelerators, as they are envisaged at the present time. Time resolutions for accelerators are now of the order of 10^{-7} s, which corresponds to flight paths of the order of 50—100 m at resolutions of the order of 1 ns/m. These are about the present limits, but it would seem fairly easy to increase the time resolution to 10^{-8} s. However, the accelerator builders now know, theoretically, how to reduce the pulse lenghts of linear accelerators to values as short as 10^{-10} to 10^{-11} s, with currents which are nevertheless quite strong. In order to make a fair comparison between such a machine and the pulsed reactor, it is necessary to consider not only the neutron source, but also the whole arrangement for neutron detection, and, of course, we do not yet possess slow-neutron detectors having a resolution of the order of 10^{-10} or 10^{-11} s, or a time-of-flight selector system capable of measuring times of flight of several microseconds with such precision. All the same, I believe that in some, perhaps different, time-of-flight experiments these accelerators can produce interesting results.

K. Einfeld: Between the pulses you have a background of delayed neutrons; my questions is whether you have some sort of mechanical device which prevents the delayed neutrons from entering the flight tube between the pulses.

I. I. Bondarenko: There are no special devices for keeping the beam clear of slow neutrons. If what you are thinking of is the possibility of slow neutrons being recycled, i.e. of their reaching the detector from previous cycles because of their slow speeds, that danger can be avoided by going over to a considerably lower pulse frequency. In fact, this is precisely where the advantage of a pulsed reactor lies in comparison, say, with an electronic accelerator, where a decrease in the number of pulses is always associated with a proportionate decrease in the neutron flux: the electron current constitutes a limiting factor. With the reactor, however, if you decrease the number of pulses, then all the neutrons you previously obtained for many pulses you now obtain for fewer; the number of neutrons remains the same.

J. Chernick: You have already indicated the average temperature in the reactor. Can you say something about the temperature variation that has been observed as an indication of how well it is controlled?

I. I. Bondarenko: During each pulse the reactor heats up by only a very small fraction of a degree, and the increase is therefore not measurable. For all practical purposes, the reactor's temperature remains constant, and its inertia is sufficiently high for power fluctuations not to cause temperature fluctuations. In other words, the reactor's thermal inertia is very high, and its temperature does not fluctuate with the power but remains constant with a variation of considerably less then 1°C.

J. Chernick: Do you have a BR number for this reactor which would help us to classify it?

I. I. Bondarenko: No. The numbering system to which you refer is used in the Physics and Power Institute. This reactor was designed in the Physics and Power Institute but has been built in the Joint Institute.

J. H. Neiler: What are the dimensions of the reactor and reflector?

I. I. Bondarenko: The entire reactor with its reflector has a diameter of approximately 30 cm.

If I may comment on Mr. Leroy's statement, I should like to say that I agree that a pulsed reactor and a linear accelerator can, in some cases, be mutually complementary although, of course, the very short pulse time which can be obtained in linear accelerators cannot always be used. In this connection I have already mentioned various factors which are involved. In addition to the limitations bound up with the detectors, there is another considerable limiting factor in connection with the slowing-down of the neutrons. If one concentrates on the very slow neutrons, the very short pulse times of electron accelerators cannot be turned to advantage. The same considerations apply to experimental studies of spectra—a subject of some interest, since we have not so far evolved a method for measuring neutron spectra in the 10- to 100-keV range. The fact is that no one has yet produced a good enough detector. In conclusion, I should like to say that only time and further study will enable us to judge the relative value of the two methods and say how useful the pulsed reactor will be.

D. Okrent: Clearly, in building and running such a reactor you wish to be sure that the disk does not stop at the maximum point of reactivity. Could you tell us briefly what features of design you have incorporated into the reactor to ensure that this does not happen?

I. I. Bondarenko: The disk has such a high kinetic energy that none of the other mechanisms involved can stop it. The possibility you refer to is therefore out of the question. Moreover, if the rotational speed of the disk begins to decrease, the emergency shut-down system comes into operation.

PANEL SESSION

Chairman: C. Sánchez del Río (IAEA)

National programmes in fast power development

C. Sánchez del Río: Gentlemen, the scientific sessions of the Seminar are over. This afternoon we want first to review something of the background against which all the physics work which you have been discussing during the last eight days is taking place. This background is, of course, the development of power reactors which will generate electricity economically. To provide a lead in the discussion, we have on the rostrum representatives of the four countries which now have large experimental programme in nuclear power development. There is really no need to introduce them to you since you have been working together during the past week and you have already heard them speak on several occasions. However, I should like to express our thanks to them in advance for agreeing to lead the discussion this afternoon. They are Dr. G. Vendryès, of the French Commissariat á l'énergie atomique, who is in charge of the fast reactor development programme in France; Dr. I. I. Bondarenko of the Physical Energy Institute of the USSR State Committee on the Use of Atomic Energy, a very well-known experimenter who has contributed greatly to the understanding of the physics of fast-reactor systems; Dr. Derek Smith of Winfrith Establishment of the United Kingdom Atomic Energy Authority, whose association with fast-reactor physics is probably one of the oldest; and Dr. B. I. Spinrad, director of the reactor engineering department of the Argonne National Laboratory of the United States of America, who was our key-note lecturer on the first day of the Seminar. A very important part of the United States' fast reactor programme is carried out in his department.

G. Vendryès: It is now about four years since the "Commissariat à l'énergie atomique" included fast reactors in its programme. It did so, of course, not only because of the long-term advantages of fast-breeder reactors, but also because it hoped, in particular, that the cost of generating power with that type of reactor might become competitive at a moderately early date, i.e. during the next decade, 1970—1980. The conditions in France are particularly suitable for this development, since the start-up of a new type of graphite reactor by Electricité de France will provide us with large quantities of plutonium for civil use during the next few years. Although it is expected to be at least ten to fifteen years before it is possible to use fast reactors for industrial purposes, we have recognized the need to start up this new type without delay. The techniques involved are advanced, and we think the problems that arise can best be solved if we tackle them in a progressive and prudent manner and, when designing each reactor, apply the lessons learned from the operation of the preceding reactor.

The first objective the "Commissariat à l'énergie atomique" set itself was the RAPSODIE experimental reactor. Since the principal characteristics of this reactor were described this morning, I shall not mention them again now. The selection of those characteristics was based on a compromise between our anxiety to avoid embarking on an excessively ambitious undertaking for this initial project and a desire to ensure that it would subsequently be possible to make direct use of the experience gained from the construction and operation of this first reactor. That was why we selected 10^{-15} n/cm^2 s as the central neutron flux (this is in fact the primary characteristic of the RAPSODIE reactor), since we considered that value was the minimum for carrying out, in practical con-

ditions, significant experiments on the irradiation of fuel elements. Such experiments will form one of RAPSODIE's main functions once it has been brought up to its power rating in operating conditions. Similarly, even when designing the reactor, we sought those solutions which we thought could most easily be extrapolated to more powerful facilities. It is with that in mind that we have decided to use plutonium even for the first loading of the reactor, probably in the form of a metal alloy consisting of plutonium, uranium and molybdenum, although a variant consisting of an oxide mixture is also being carefully studied.

Since 1958 we have worked very hard on pre-constructional studies, since we wanted to solve as many of the technological problems as possible, one by one, before finalizing the intricate details of the reactor design. A large number of tests of all kinds have been carried out in every field reactor: physics, neutron economy, control, safety, fuel cycle, mechanical, hydraulic and thermal. A group of laboratories and workshops have been built specially for the purpose in our new Cadarache centre, and in these we are at present erecting actual-size models of the reactor vessel, the liquid-state cooling circuits and the principal bits of machinery. These various pieces of apparatus will be tested in conditions approximating as closely as possible to those of actual operation, and sometimes in even tougher conditions.

Whilst these various studies are being carried out, the construction of the reactor is being started, at this very moment, at the Nuclear Research Centre of Cadarache in South-Eastern France. We hope that it will be finished towards mid-1964, and that it will become critical before the end of the same year. No firm decisions have yet been made about our subsequent programme. But we are already considering the possibility of constructing a reactor of the order of 250 MW as a second stage; this is merely a logical sequel. The reactor would be constructed in the last years of the present decade and would prepare the way for a true prototype of a fast-neutron plant on which work would not be started until ten to fifteen years from now; but you will realize that at that distance of time our plans must necessarily be very vague. We shall continue to concentrate on plutonium fuels, but we shall extend our field of research to materials other than the metal or oxide: the carbide seems to us specially promising. In regard to recovery and reprocessing, we are planning to study processes giving a short cycle, such as pyrometallurgy; nevertheless we think that if it is really to give us the savings we expect from it a short cycle should be specially adapted to the type of fuel used. This, incidentally, is the reason why we have not planned to use a short cycle for RAPSODIE, in which various types of fuel will be tried in turn, and in which we shall simply make use of water recovery.

As from next year the studies we have hitherto confined almost exclusively to RAPSODIE will be extended in preparation for the following stage. The necessary facilities will for the most part be concentrated at Cadarache where new laboratories and test workshops will next year be added to the existing installations. We are also planning to transfer to Cadarache next year a team to measure the main nuclear constants affecting fast reactors and also to install there a critical mock-up for the study of plutonium cores of the largest size.

In concluding this brief summary of our programme, I wish to point out that Euratom, wishing to undertake large-scale studies on fast reactors, has, among other things, suggested to the "Commissariat à l'énergie atomique" that the two organizations collaborate in that field, and that the "Commissariat" has warmly welcomed the proposal. I cannot tell you more at the present stage,

but it is highly probable that a great part of the work on fast reactors undertaken by France during the coming years will be carried out in association with Euratom.

I. I. Bondarenko: The Soviet Union possesses considerable reserves of fossil fuels—oil, coal and natural gas—sufficient to meet requirements for many years ahead. The change-over to nuclear fuel is, therefore, not such an acute problem for our power economy. Nevertheless, with electricity requirements constantly on the increase, we do regard the task of harnessing the new sources of energy as one of particular importance. Obviously fast reactors occupy a special place in this context. With their capacity for utilizing almost all uranium and thorium reserves for the generation of electricity, they open up practically unlimited prospects for the development of power production, their significance in this respect being comparable to that of thermonuclear reactors.

Our interest in fast reactors, however, is not confined to long-term prospects. It is our hope that we shall be able to obtain cheap electricity from them in the relatively near future. There are two major groups of problems to be solved, however, before this is possible. The first group relates to the reactor itself. What we need is a reactor with high conversion ratio and high burn-up values. Once we can get both these properties in combination, it will be possible to reduce the fuel-doubling time, and this in turn will provide the foundation for ensuring a steady increase in the capacity of nuclear power plants. There are many possible approaches to this problem: use can be made of various types of fuel—oxide, metal, carbide—or of non-fissionable nuclear fuel diluents; consideration should be given to the problem of using U^{233} in the thorium cycle in fast reactors; and it might also be useful to consider the possibility of coupling fast with thermal reactors, for example by combining both regions in one unit and using the plutonium directly produced by the unit, without further processing, for nuclear power production.

The second group of very important problems concerns the chemical reprocessing of the material used. Obviously, the successful development of fast reactors hinges on the solution to this problem. If fast reactors are built with high burn-ups, the volume of material needing to be chemically reprocessed will be small, which is one of the main justifications for hoping that fast reactors will be able to provide a source of cheap electricity in the near future.

I should like to say a few words on how this problem has been approached in the past in the Soviet Union and how we propose to tackle it in the future.

The first real work in this field dates back to 1949, when Leipunsky drew attention to the major characteristics of fast reactors and to their prospects. An initial period of investigation was devoted to the physics of these reactors. These investigations, which were carried out in many countries, confirmed our original suppositions and justified our original optimism. The opportunity for building up technological experience came with the construction and operation of the 5000-kW plutonium-oxide BR-5 reactor. Plutonium-oxide was adopted as fuel because, at the time, a large number of experiments had indicated that the oxide would produce high burn-ups, and this was of course a factor of fundamental importance. This assumption has been confirmed by the experience gained with BR-5, and the high burn-ups obtained give grounds for optimism about future developments.

Parallel with the operation of BR-5, work was proceeding on the development of the BN-50 project, a power plant using a fast reactor with a capacity of 50000 kW(e) or 250000 kW(t). This design, which is now completed, has taught

us a great deal. During the design stage, a number of technological and other problems came to light and possible ways of solving them were outlined. At the present time, however, we feel that it would be unwise to go ahead with the actual construction of this reactor. Operation of BR-5 already provides us with the experience we need for building large-scale industrial reactors, and, we hope, will continue to do so in the future. Furthermore, a 250000-kW(t) reactor is not the right size for making optimum use of the characteristics of fast reactors. For this reason, and in view of the successful operational experience gained with BR-5, we are now thinking of making a study of reactors of considerably higher power than BN-50, i.e. in the power range which can be considered optimum for fast reactors. This would seem to be, say, 800000 kW(t), or even considerably higher. We are still planning on the basis of oxides, i.e. a mixture of plutonium and uranium oxide, since results based on large-scale experiments are only available for this type of fuel. We are extremely interested, however, in the possibility of using carbides, and if we find from experience that we can obtain the same burn-ups with a carbide as we can with uranium oxide, then we shall probably turn to it. Further planning in the field of fast reactors will depend on the results obtained with the present programme.

R. D. Smith: Interest in fast reactors in the United Kingdom is based very much on the belief that they will produce power at both low capital cost and low fuel cost. The interest in their possibilities is heightened, as it is in France and other countries, by the prospect of abundant supplies of plutonium from the natural uranium graphite of central power stations which are now being built in the United Kingdom. It is expected that nuclear power stations of this type with an output of some 5000 MW will be operating by 1968. These stations will be producing plutonium by the ton in the 1970's, so that by then there will be no shortage of plutonium for the first fast reactors. This plutonium will, of course, be degraded and probably will contain about 20% Pu^{240}; but as we have seen during this Seminar, this is no disadvantage in fast reactor systems. We are also developing the advanced gas-cooled (AGR) type of reactor which may come into large-scale commercial operation toward the end of this decade, and these reactors will produce more plutonium. Over an even longer period we are looking toward the development of the high-temperature graphite reactor, using impermeable graphite for fuel element cladding. As a start, the DRAGON reactor, which is a small experimental reactor of this kind, is being built at Winfrith in co-operation with other European countries. This type of reactor is very promising so far as power generation costs are concerned and could, I think, prove a competitor to the fast reactor. Alternatively, since this reactor has a better neutron economy with U^{233} fuel, it might perhaps be developed on a family basis with fast reactors, using fast reactors with thorium blankets to supply the U^{233} for reactors of the HTGC type.

You will notice that I have not mentioned the question of breeding when discussing the present interest in fast reactors in the United Kingdom. This is indeed a very powerful long-term consideration; though breeding is marginally possible in some thermal reactors, it is probable that the much larger breeding ratios which can be achieved in fast reactors will turn out to be the practical way of utilizing the world's resources of U^{238} and thorium. Many years ago, when I first became associated with the fast-reactor programme, the prospect of high breeding ratio was considered a valuable short-term advantage, because the world's known resources of easily obtained uranium were then very much smaller than

they are now. At present, however, with the prospect of a surplus of plutonium, one is more inclined to find the best way of using this plutonium as a good economical fuel than to concentrate on methods of making more. At a succeeding stage of the programme we would then develop designs of reactors with high breeding ratios, so that we could increase the number of power stations to keep pace with the probable increase in demand for electricity. Thus we are now studying the design of fast-reactor power stations for use in the 1970's. It is likely that at that time the electricity generating boards will want power stations with an output of the order of 1000 to 2000 MW(e), probably with two or more reactors per station. Fast reactors tend to get more economical as they increase in size, at least on paper; but a reasonable extrapolation of present knowledge suggests that we should perhaps evolve a reactor with output of the order of 1000 MW(t). We should have plenty of plutonium, as I have already said, so that by then we should have no great incentive to adopt methods of reducing the critical size. For economic reasons, however, we do need a high fuel rating, somewhere in the region of 1/3 to 1 MW/kg, according to the details of the reactor design. Accordingly, we are studying reactors of the 1000-MW size, and we have initiated development programmes to study specific engineering problems relating to pumps, heat exchanges and, not least of course—though it may be only a small part of the actual structure—, the reactor itself.

The central problem is undoubtedly, I think, the design of the fuel element, and we are considering a number of possible fuels. These include uranium-molybdenum alloy elements canned in a refractory metal, and also pure uranium in a very strong refractory can, which would have a small gap and would be strong enough to allow fission gases to accumulate without bursting the can itself. Other possibilities are oxide canned in stainless steel, carbide canned in stainless steel, and the steel UO_2 cermet. We have studied the influence of the choice of the fuel element on the fuel cost of the reactor by means of a parametric survey, and we have used this to indicate the regions of greatest interest. Unfortunately, we do not yet possess sufficient knowledge of the irradiation behaviour of these various fuels to choose the fuel for a large power reactor. The situation is something like a horse race at this stage, I think, and considering the odds on the various starters we do not consider the prospects of the uranium-molybdenum alloy to be very good because of its apparent limitations in burn-up. Uranium oxide is not looked upon with particular favour at the present time, either chiefly because at acceptable temperatures below thermal conductivity it makes the pins come very small; this presents difficulties with regard to fabrication costs and possible distortions which may block the coolant channels. I think that probably cermet fuels and carbide fuels are the favourites. Cermet fuels will of course have the low breeding ratio, but as I have indicated this is perhaps not too important in the first reactors. Carbide is undoubtedly the best fuel—on paper, but a lot of it remains at present on paper—and until we have the irradiation data we cannot decide to use this fuel. We have, accordingly, a fairly large programme for the irradiation of fuel-element materials in our three material-testing reactors, DIDO and PLUTO at Harwell and DMTR at Dounreay, and we shall soon begin the irradiation programme in the fast reactor at Dounreay. As has been mentioned previously, this reactor has been modified to allow whole sub-assemblies to be placed in the centre of the core, with a view to testing fuel elements as distinct from fuel materials. When sufficient information becomes available, therefore, we shall choose our fuel material from among these possibilities.

Turning from the future to the present, the central part of our programme is of course the Dounreay fast reactor. You have already heard a good deal about this reactor during the Seminar, and I should merely like to repeat that it will help the British programme in many ways; first of all, by providing experience in the operation of a big sodium-cooled reactor, and secondly, in the way I have already mentioned, by allowing irradiation testing of fuel elements and materials. Thus we hope very shortly to have experience of the dynamic behaviour of, if not a large fast reactor, at least a larger fast reactor than any for which we now have data. Moreover, we shall get at least some cross-section information from such measurements as mass spectrographic determinations of α. Lastly, we shall gain experience in both the fabrication and processing of fast-reactor fuels.

With regard to physics studies, we have a very large nuclear data programme, concentrated mainly at Aldermaston and Harwell, using various facilities which Dr. Ferguson reviewed briefly in his paper. As to reactor physics, VERA has already been described by Dr. Weale, and this assembly is being used at Aldermaston in a programme of measurements on small dilute assemblies. ZEBRA is now being built at Winfrith; the buildings are nearly complete and we expect the reactor to be operating in about a year's time. In the first instance we shall have about 500 kg of U^{235} and 100 kg of plutonium. Our programme with this reactor is roughly the following. First of all we will investigate dilute U^{235}-fuelled assemblies in order to extend our knowledge of these, using up to 500 kg of fuel. We then hope to develop the two-zone technique which has already been the subject of considerable discussion, and use this method to investigate more dilute U^{235} assemblies than we can assemble with the 500 kg of fuel initially available, and also plutonium assemblies which will resemble the 1000-MW of reactor which I mentioned earlier. We do not yet know for certain whether this two-zone approach will be successful, but we are loath to use more plutonium in this sort of assembly than absolutely necessary because we have found from the ZEPHYR experience that the inconvenience of handling it—the considerable health hazard, the high spontaneous-fission rate from the Pu^{240}, and the high cost of canning the plutonium pieces—adds very considerably to the cost of the experiment. Whatever success we have with this approach will no doubt be reported in due course at another conference. When the design of the next big power reactor is itself further advanced, we shall almost certainly do physics measurements on mock-ups of this particular reactor—to determine such things as control-rod worth—using techniques which are now well established. ZEBRA will in fact have space for the assembly of systems up to nearly 3 m^3, though in the first instance we shall not use such large assemblies. It has already been mentioned, I think, that one of the main problems presented by this sort of machine is the time taken to assemble and disassemble the cores, while keeping track of all the pieces so that you do not accidently assemble a reactor somewhere else. Unfortunately, the pieces must be small, so that the total number of pieces in the last dilute core is of the order of 100 000. In order to alleviate this problem, we are developing automatic leading machines to load and unload the core pieces from the fuel elements.

In summary, therefore, we in the United Kingdom are anticipating the construction of 1000-MW commercial fast reactors in the 1970's, and will probably construct a prototype reactor in the latter half of this decade. The size of this prototype is still under discussion—sometimes quite heated discussion—but it might be 300 MW(t) or possibly some other value. The programme is supported

by the operation of the Dounreay fast reactor and by the nuclear-data programme, the zero-power-reactor programme, and extensive irradiation programme and development programmes in all other fields which promise to be important.

B. I. Spinrad: It is extremely difficult for one person to discuss the fast-reactor programme of the United States, because it is extremely diverse, it is looking into almost everything that is attractive, it is carried on in a great many separate institutions, and it has only an informal programmatic guidance.

Perhaps the best way of understanding this state of affairs is to take a look at what the aims of our total power programme are and what our putative schedule for fast-reactor development is. The aims of the United States power programme, notwithstanding the fact that there is no need of nuclear energy in the sense that we are running out of other forms of fuel, are: first, to achieve economic nuclear power in high-cost areas of the United States by the end of this decade; second, to assist friendly countries who for various reasons might prefer other types of reactors than we consider attractive internally in carrying out their aims; and third, to achieve breeding, which we consider useful enough in the long range to be worth a small (admittedly quite small) economic sacrifice in order to achieve it.

To these ends, the current programme features two major reactor constructions, the Enrico Fermi reactor being built by the Power Reactor Development Corporation, and the EBR-II being built by Argonne. These reactors are both quite complete in their major construction. The entire primary system of EB-II is complete, and the secondary system is in its final stages of construction; in fact, we had hoped to have started loading to criticality by this meeting, but I am afraid that this month has slipped by too fast.

It is expected that these reactors will provide enough experience so that by 1964 or 1965 it will be considered advantageous to build a new, more advanced, power reactor of undetermined size to feature the best of the construction features and operating features determined by the operation of these two systems. And it is hoped that on such a basis, starting in about 1968, one could begin construction of a fast reactor which might be truly economic.

Now this is a very vague programme indeed, because I keep qualifying it with "it is hoped" and "it is expected", but that is about all that can be said. For one thing we cannot be sure that everything we do will be economically successful; for another, we have a host of other parallel developments, including the LAMPRE programme which has been described, the fast-oxide-breeder programme, the carbide-breeder programme, the development of other types of fuels including pastes and vented fuels, and the development of other methods besides raising steam for generating the electricity. And we would like to leave the door open in our plans for what might be called the prematurely successful conclusion of one or another of these concepts.

The history of the first reactor programme in the United States is, of course, quite old. The first fast reactor was built in the United States; this was the Clementine reactor at Los Alamos, which was fuelled by plutonium metal and cooled by mercury. The second fast reactor was the EBR-I, whose concept was developed before 1946 and which was announced as a project in 1946. Conditions being as they were in those days, knowing as little as we did about sodium systems, it took quite a while to bring this reactor into operation. Its main purpose was to demonstrate that the physics predictions were qualitatively correct as to the fact of breeding, and this it did to our satisfaction.

We have had other reasons for being interested in sodium systems. Our thermal-reactor programme, for example, includes sodium-graphite reactors which have made a major contribution to the technology of sodium throughout the world. One cannot mention the fast-reactor programme without mentioning this parallel programme as well.

As far as our basic research is concerned, you have heard a great deal about it and I do not want to repeat too much of it. The United States programme of cross-section measurements is very widely dispersed throughout many universities and other institutions; it comprises the measurement of reactor cross-sections and cross-sections of secondary reactor interest such as might be required for experimental detectors, shields and so on.

I see no other way to describe the programme more fully except to go through the list of organizations and what they are contributing. I hope I am excused for starting with my own. At Argonne, we have been developing EBR-II, we have been building it, and we expect to be running it (giving the lie, I might say, to some people who have insisted that this reactor was a career rather than a project). I have mentioned its status. The development and operating programme is based on the use of metal fuel and pyrometallurgical recycling. The reason for this is that we decided quite early that, at the very least, a fast reactor must have a doubling time short enough so that, should it turn out that the fuel was expensive and the charges on the fuel high, the breeding could at least compensate these high charges. For this, the breeding itself is useful, and, as we have seen, the metal fuel allows very comfortable margins for the insertion of engineering materials. The programme features ultimate conversion to plutonium although the reactor will start with U^{235}. The reprocessing and the fuel fabrication are an integral part of this project, and we feel that we have achieved a quite satisfactory development of the technology of melt refining of the radiated fuel and of injection casting of the purified reprocessed material. All of this must be done remotely. We have particularly emphasized development of remote fabrication techniques, because we started off right at the beginning in the feeling that we would have plutonium to handle, that this plutonium would contain a good proportion of the higher isotopes, and that it would be not only alpha-active, requiring glove-box operation, but neutron-active from the Pu^{240} and gamma-active from the Pu^{241}. Thus, as far as we are concerned, there is not too much to be gained by undertaking very large decontamination; one needs a remote process anyway, and we have crossed this bridge right at the beginning. We intend to continue the development of this type of fuel, although there have been some reports of discouraging behaviour of metal fuels under irradiation. We have, however, discovered that most of the deleterious effects of irradiation of metal fuels can be counteracted by either lowering the temperature of the reactor, which is rather high and can afford to be lowered without great penalty, or by introducing a very stout cladding, a very strong one which will not react with the plutonium-containing metal. Therefore, we are developing alloys of some of the refractory materials for this purpose. We have demonstrated the ability of the metal when it is so restrained to undergo burn-ups of at least 5% in experimental samples.

Besides this programme at Argonne we have heard a good deal about the reactor-physics programme and about the programme of experimental reactor operations on the EBR-I, which has been so fruitful with regard to stability problems. We have heard something about the large number of reactor concepts under study. I will also mention that, in addition to the metallurgical processes for

EBR-II, we have also been concerned with the general problems of sodium bonding and the problems of fabrication of ceramics containing plutonium. Moreover, we, as well as most of the other organizations in the business, have quite heavy engineering programmes on the various components of a fast-reactor system. The critical components appear to be the sodium-to-steam generators, the units which have incompatible fluids on the two sides.

Now it is our intention to follow the metal programme for as long as it appears to be hopeful. And it is also our intention, should other fuels turn out to be the preferred ones, to work with the same philosophy as for EBR-II, namely that of developing the remote fabrication and the remote reprocessing steps, so that we can be assured that the reactor can be fuelled over and over again. This seems to us to be the major defect of many current concepts.

I will turn to the Los Alamos programme, which is the oldest in the United States, and has been quite famous for its very basic measurements on the properties of pieces of matter containing or forming fast-reactor spectra. I refer to the experiments with Godiva, Topsy and similar devices which determined for us the basic properties of reactors with very fast neutron spectra. The Los Alamos experiments on exponential columns of mixtures of U^{235} and U^{238} gave us most of our best early information to guide the development of fast-reactor theories, and the basic multi-group theoretical structure is a Los Alamos product. In addition, there has been going on at Los Alamos a good deal of work on the chemistry of plutonium alloys in particular, and it is this research which has led to the LAMPRE programme.

LAMPRE is admittedly a futuristic reactor, but it features several very elegant solutions to some of the problems which plague the designers of fast reactors; and if these solutions can be actually obtained, then the LAMPRE machine is a very promising type of device indeed.

For one example we have had a good deal of discussion about safety of fast reactors and a good deal of discussion of the type of response we want. The molten-plutonium type of fast reactor, in particular, has a large volumetric fuel expansion on heating, which is a fast-response negative power effect, and therefore this can be expected to be a safe type of machine. It has other advantages in that, as you know, very high power densities can be achieved in fluid-fuel reactors. Although we have pushed the performance of solid-fuel reactors very high in terms of power density, probably the fluid-fuel systems of the LAMPRE type can go a bit higher. The other attractive feature is that one does not have to worry about fuel integrity, except for keeping the fuel where one put it in the first place. The major problem, therefore, at the moment is the question of handling this alloy which has so many promising features, but which is a highly corrosive material to virtually every metal; it appears to be containable in tantalum, but only with difficulty.

In addition, at Los Alamos there is also the full complement of engineering research, physics research, metallurgy and chemical research.

Finally, there is a core-test facility under construction which will do, for the molten fast reactors, some of the concept developments that otherwise might be of academic interest only. It is the intention at Los Alamos to develop advanced concepts of the molten-plutonium type and to test them here, and I believe that there is already a programme of several years' work for this facility.

I will now turn to the Atomic Power Development Associates (APDA), who are the designers of the Enrico Fermi plant. In addition to developing the most

advanced heavy sodium-to-steam power equipment in the United States—for this type of reactor—the APDA group have been particularly strong on the question of economic analysis of fast-reactor concepts. This group has also been promoting the use of cermet fuels containing uranium and/or plutonium-oxide dispersed, in the first instance, in stainless steel, but preferably later in uranium. They have also worked on paste fuels, which consist of compacts of uranium-oxide or uranium-carbide or uranium metal, in which the interstitial spaces are filled with sodium and which, when warm, have about the same consistency as a paste. They have likewise advanced the concept of vented fuels, which are, as the name implies, fuel elements which afford a moderate degree of restraint to the particular fuel loaded, but which are vented so that the fission gases can escape through a porous plate and prevent unusual build-up of pressure inside the fuel. They have pointed out that the sodium of the primary system becomes quite active, and unless it is very pure it will probably stay permanently active, so that there is no particular virtue in preventing fission products from getting into the sodium stream.

I have to back-track just a little because I left out one facet of the Argonne and Los Alamos programmes in which APDA have also participated. That is the very extensive fast-reactor safety programme. The major part of this programme has been at Argonne under the very able leadership of Dr. Okrent, and I think we have heard at this Seminar a good deal of information concerning it. It is perhaps less well known that a good bit of the work, particularly in the development of the mathematical techniques originally used for dealing with safety problems, was performed at Los Alamos, and that some of the experimental studies on which we rely for knowledge of behaviour of fuel bundles comes from APDA.

I will turn now to the General Electric Company which has been working on a fast-oxide-breeder programme, which in its turn is historically linked to earlier intermediate-reactor studies at the Knolls laboratory. This earlier experience had shown that a slight amount of moderation could be tolerated without too great a loss in the potential breeding of some fast systems; and it had produced an experimental fuel element, the so-called "pin type", which is a restrained fuel element consisting of uranium-oxide packed into a strong steel pin. This was put into a reactor, and it just keeps going. In experimental irradiations, pin-type and other oxide fuel elements have achieved burn-ups of 10%. Up to about 3% burn-ups, one does not need the stout cladding, as the lattice holds most of the fission gases, but beyond that point the problem is the same as with the metal. The oxide has, however, the significant advantage that it does not alloy or react chemically with steel, which can therefore be used even at high temperature as a cladding.

The irradiation programme has been the major portion of the General Electric Company's activities, but they have also carried out safety studies and done conceptual work on the vented fuel system, which is particularly appropriate for the oxide type of fuel. They and Atomics International, which is the next organization I will mention, have both shown interest in carbide fuels for reasons already stated this afternoon.

The Atomics International programme you have also heard about. This is one of the few programmes that are not supported by the Government, but it is, nevertheless, integrated in the National Programme. In this case the aim is, of course, to achieve a high burn-up, primarily by diluting the fuel. It is proposed to do this by allowing some degradation of the neutron spectrum in order to achieve

a more stable matrix for the fuel material. The programme is tied closely to the use of U^{233} as the fuel and it depends on the fact that U^{233} now appears to have a significantly higher η than the competitive fissionable materials in intermediate spectra. It also appears to prevent, to some degree, the positive sodium coefficient disease.

This programme benefits particularly from the sodium graphite work which has been also performed at Atomics International.

Other organizations in the United States engaged in fast-reactor studies include the United Nuclear Corporation through its NDA Division, who have been participating primarily in safety studies. They did the basic work on the Doppler effect; I would call it the first modern work on this particular problem. They have continued their interest in this problem as well as in other aspects of reactor safety.

There is also a good deal of work which I think I should mention more or less as an appendix. We have not dealt as much with intermediate reactors as was suggested by the title of this Seminar, but I do feel it is significant to mention particularly the work of Brookhaven National Laboratory in this field. There is now being built the Brookhaven Beam Research Reactor, which will be a true soft-spectrum intermediate reactor. It is basically a large bath of heavy water containing a very high-density core of plate-type fuel. The concentration of fuel is high enough for the medium fission energy to be in the resonance region. It is expected that this type of reactor will have particular advantages for beam experiments because of the fact that the most useful energy range for the experimental physicist at Brookhaven occurs around the core spectrum peak. Thus, the epithermal spectrum can be extracted with beams which go straight to the core of the reactor. In addition to this, Brookhaven is doing a good deal of basic work on similar soft-spectrum intermediate systems containing both U^{235} and U^{233} with thorium-dioxide in water, and on graphite-uranium-aluminium systems.

From this very quick survey of the United States programme, I think you can get the idea, which is probably correct, that we are trying to do everything at once. We have keen competition for the honour of developing the most successful fast reactor, and a diversity of talent in the country which makes this feasible.

Discussion

W. Häfele: I shall speak about a special project of our nuclear centre in Karlsruhe which has its own place in the German reactor programme, but I shall not attempt to cover that programme as a whole. The Kernforschungszentrum Karlsruhe has not long been in the fast-breeder field. As you probably know, our work began in 1955 when we built the Karlsruhe research and testing reactor FR-2, a large heavy-water natural-uranium reactor, and the facilities around it. The FR-2 is now almost complete and is running already at zero power. We then decided to focus our attention on the fast power breeder type, and we intend to re-examine every decision formerly made in order to determine whether it suits our present plans or not.

A great many problems must be considered if one is to develop a satisfactory approach to reactor design, and in the course of time we have evolved certain specifications for our own programme. In the first place we want a true breeder, i.e. a reactor with a breeding ratio clearly larger than one. We should be pleased if the reactor had a short doubling time, but we feel that that is not the most important point these days. Secondly, the reactor must be really safe, and this

applies especially to the area surrounding the reactor building. We are even prepared to sacrifice safety within the building to a certain degree if, but only if, this leads to increased safety for the surrounding community. I explained this point to some extent yesterday. In the third place we want to achieve the highest possible degree of economy. As long as the reactor is a true breeder, it is our impression that good economy is more important than a short doubling time; but of course this is a complex problem, and many factors must be weighed.

We believe that the vital problems in the fast-reactor field today are of a technological rather than a physical kind. To reach the goal of good-economy we lay strong emphasis on burn-up studies, because we consider this a problem of first importance. Immediately after its completion, the Karlsruhe FR-2 reactor will be used for large-scale burn-up experiments.

Another problem of vital importance is that of cooling. One has to achieve power densities between at least 0.5 and 1 MW/l, and we are convinced that helium cooling is a good way to meet this requirement. However, it has never been finally demonstrated that 1 MW/l can be obtained, so we are planning large-scale out-of-pile experiments in this field. At the same time I want to emphasize that we consider sodium cooling as an already established alternative. Experiments on a reasonably large scale should familiarize us with the special technology of sodium.

At present we tend to consider plutonium-oxide and uranium-oxide most seriously, but later we shall also give serious thought to the carbide fuels. The fast reactor under study in Karlsruhe will have the specific power and power density of a full-power station, and it will have the minimum power level at which the desired specific powers, power densities and other characteristics can be achieved. Thus the reactor in question is the prototype of a larger station towards which all our economic studies are aimed.

Another point is that we want to demonstrate a closed fuel cycle. We must therefore devote some attention to problems of reprocessing of fuel and refabrication of fuel elements, and we are re-examining every possibility of cheap recycling with a low decontamination factor. Technological experiments are planned for somewhat later.

We are starting Van de Graaff experimental work to obtain specific differential cross-sections, and we are also examining the possibility of other experimental work. A fairly large amount of theoretical work lies behind all these activities; our efforts in theory have been devoted to parameter studies, nuclear-data work and detailed investigations of special problems such as fuel cycling and explosions. The Karlsruhe fast-reactor project has two stages. As a first stage, we want to sort out, during the next years, the main problems of technology and design. The second stage will be that of deciding upon the construction of a particular fast power reactor, and if we can reach a positive decision in favour of one particular design, the reactor will then be built.

E. Hellstrand: As you all know, Sweden is a very small country, one which cannot hope to cover all branches of fast-reactor physics and engineering. In fact, the first stage of our programme is quite limited, and any increase in our effort will depend on how successfully fast power reactors are developed in other countries. By and large, we are doing two types of work in the fast-reactor field. Firstly, we are making measurements of fast-neutron cross-sections and ν values for substances of interest in fast reactors. The most important tool used in these investigations is a newly installed 5.5-MeV pulsed Van de Graaff machine. Secondly, a small fast critical assembly will be built in the near future. The design

of the reactor is nearly complete, and it is scheduled to go critical at the beginning of 1963. The reactor will be quite similar to the VERA reactor at Aldermaston: it is to be a two-half machine with the fuel elements placed vertically. In parentheses, I might use this occasion to thank our colleagues in England and at Argonne for all the information they have given us in connection with the design of the reactor.

At first, the reactor will be used for criticality studies on small systems, as the available amount of U^{235} is quite limited; the enrichment will in fact be limited to 10% or less. Most of what we will be able to do with the reactor during this first stage has already been done elsewhere. We hope, however, that besides getting a good training in operation a fast system, we will obtain some results of international interest. Special emphasis will be laid on the correlation of results from experiments and calculations.

Later on, of course, we hope to obtain more enriched material—both U^{235} and plutonium—so that investigations can continue on geometries of more interest in power reactors. At the same time we shall start work on heat-transfer problems specifically connected with fast reactors, fuel recycling, sodium technology, etc. However, as I mentioned earlier, the rate at which we increase our effort will depend very much on how rapidly competitive fast power reactors are developed abroad.

J. van der Spek: In Belgium, the study of fast-neutron reactors and, as a logical corollary, that of the plutonium cycle has been undertaken systematically by private industry, with subsequent support from official sources. We have taken the question of fast reactors and the associated question of plutonium as forming a whole, and endeavoured to study the various links of the cycle simultaneously. As regards fast reactors, we contracted in 1956 to work together with Atomic Power Development Associates, Detroit, as Dr. Spinrad mentioned just now, and I wish to pay tribute to the excellent results we have obtained from this association and to the genuinely co-operative spirit Detroit has shown towards us. We sent a team of five technicians over there, and they acquired some very valuable experience—several of them are here today. In Belgium we are also carrying out theoretical and industrial studies on fast-neutron reactors.

As regards the reprocessing of irradiated fuels, the Eurochemic reprocessing plat at Mol, outside Bussels, is obviously a most valuable source of experience and information.

After reprocessing, the next problem for consideration is the manufacture of nuclear fuel and fuel elements with the use of plutonium. We in Belgium began to study these questions in 1957 and 1960, and we have obtained a major contract from Euratom for studying the fabrication of uranium and plutonium mixtures. Like most other countries, we have directed our attention to the oxides, the carbides and ceramets. We, for our part, believe that there is a great future for fast reactors, and I myself consider that, in an expanding economy, the problem is not entirely (at least from our point of view) as stated by Dr. Smith just now; it is not so much a question of finding a use for the plutonium produced as, at least in my opinion, of producing sufficient plutonium to meet the demand that will arise in the future.

D. B. Hall: I have a question to put to Dr. Bondarenko. At the 1958 Geneva Conference we heard a description of some work being done in the Soviet Union on a type of reactor which interests us at Los Alamos. This is what we call an equilibrium reactor, a one-region reactor in which the uranium starts as a blanket

material in which plutonium is bred. The fuel is then programmed towards the centre in some undefined way until it becomes a fuel instead of a blanket. We have done some calculations on this design, but it would be interesting to hear whether you have done any work beyond what was reported in 1958.

I. I. Bondarenko: This concept is extremely interesting and would appear in some sense to be an ideal towards which we might strive. In fact the prospect of doing without chemistry and of getting an analytical reactor which uses only U^{238}, produces power and does not require any chemical reprocessing of the materials used, is very attractive. I think that the difficulties in the way of its realization, however, are clear to all. Such a reactor is possible only where a very high burn-up is attainable—and also, only where there is not much non-fissionable material introduced into the reactor which might diminish the breeding ratio. At the moment it is not clear whether or how these two requirements can be combined. In striving to obtain a high burn-up, you are compelled to introduce much extraneous material into the reactor and diminish the breeding ratio to values where breeding itself takes place only with difficulty or not at all. I think therefore that the question requires further study, together with other possibilities for the longer-term development of fast-neutron reactors.

D. Okrent: This question is also addressed to Dr. Bondarenko. In the United States there has been some discussion of the need for a special engineering test reactor, designed specifically to test the fuel elements of fast reactors. Do you feel, in the Soviet Union, that such a test reactor is needed, or do you think that thermal reactors are adequate for testing the fuel elements of fast reactors or sub-assemblies ?

I. I. Bondarenko: We have in mind two possible ways of testing fuel elements; first, in fuel reactors, in particular the SM-2, which was described at the Second Geneva Conference; and secondly, on the BR-5, which is, to a large extent, ideally suited for fuel element testing. We have not yet thought of any other fast-neutron reactor for material-testing purposes.

G. Vendryès: I have one comment to make on the question just raised by Dr. Okrent. It seems to me that in a large-scale programme for the development of fast reactors, such a "special test reactor" is of particular importance, and that possible fuel element tests, though numerous, can be divided into a number of different types. There are those which are primarily long-term irradiation tests, that can be carried out in a reactor which might itself be the prototype of some future industrial fast reactor, such as in varying degrees ZPR-V, EBR-II, Dounreay and possible RAPSODIE. Then, there are other tests, which I should rather call destruction tests, where the fuel element is systematically subjected to conditions more severe than would occur in actual practice, both with a view to safety considerations and in order to narrow down the choice of fuel elements suitable for subsequent long-term irradiation tests. The project evolved by Argonne seemed to us (and still seems) extremely interesting from that point of view, and I would like to know what the prospects are of actually setting up such a reactor in the United States.

B. I. Spinrad: At the moment it appears that the development of a fast fuel test reactor would not be justified for the purposes of the United States programme alone. In other words, our programme, large as it is, is not large enough to support such a facility. If the project were to receive strong international support, however, it would become more practicable.

Dr. Hall's question to Dr. Bondarenko suggests one other comment which

I have wanted to make for some time. One of the possible virtues of fast reactors is that a considerable quantity of fissionable material can be built up in the blanket elements without having undergone a large amount of fission. It appears that with relatively modest fission exposures one can build up several per cent of new isotopes. It has occurred to us that this is a very great virtue of a high external breeding system, because it means that we can consider an interaction between a fast- and thermal-reactor complex in which the fuel for the thermal reactor is fabricated and subsequently enriched, instead of the other way round. This ought to be useful for both systems, because it would enhance the value of the product bred by the fast system and decrease the cost of the thermal fuel element.

A. Campise: We have given some thought to this question, and the point of most interest to us is that you can fabricate thorium-carbide rather inexpensively, expose it, and build up enough enrichment to use it as fuel for a thermal reactor, which would then have a high neutron economy.

D. B. Hall: To conclude our work, I have been asked to try to summarize the results of the Seminar, in as objective and balanced a manner as I can. In the first place, I should point out that this eight-day seminar on the physics of fast and intermediate reactors represents the first international meeting devoted solely to this important subject. The only previous conference on the physics design of fast power reactors was held in 1959 at the Argonne National Laboratory in Chicago. This present meeting, with much more widespread participation, has covered more topics and has revealed the wealth of information of increased precision which has been developed in the years since the 1958 Geneva Conference. These meetings, and the stimulation which inevitably results from the close association of workers in a common field, have been immensely valuable.

The Scientific Secretary, Dr. Erginsoy, provided a very real service by establishing late deadlines for the submission of contributions, thus permitting current work to be presented even though it made his own work considerably more difficult.

Reactor calculators, as users of data, and neutron physicists, as producers of data, were both represented. Both groups have their own points of view regarding what work is useful, and each individual has his own shade of interest. In general, the theoretical physicists want to have more detailed numbers and, indeed, they have been able to point out inconsistencies revealed by the calculations. They have, in several instances, tried to evaluate—by variational techniques—the effects of inaccuracies or errors in reactor calculations which would be produced by errors in cross-section measurements. This approach is helpful to the experimentalist, because it gives him a rational criterion by which to judge what precision is worth striving for. It is evident, however, that the liberties the theoretician takes with the datum points in constructing a particular multi-group constant set frequently distort the original numbers. It is admitted that most sets of group constants have been tailored to give satisfactory results in a particular design area, and the creator of the set is sometimes horrified to find that someone else has been using them for another design. That differences do exist in group sets used in different laboratories is hardly surprising in view of the fact that appropriate computing facilities, programme and techniques are not everywhere available. It would seem appropriate if the limitations of a set could be clearly identified to avoid its misuse.

The theoretical physicists have been explicit in presenting requests for detailed neutron measurements together with what they felt to be necessary accuracy. In some cases, the precision requested was high—in fact it bordered on the

unreasonable. The analysis on which these requests were based concerned for the most part the determination of the core mass or k_{eff}. This is important, but not as crucial to the practical reactor design as calculations concerning control effectiveness, or temperature and power coefficients. One might get the impression that the theoretical physicist considered his work done when he had solved the static equations predicting the core mass and the initial clean breeding ratio. It is highly probable that these calculated breeding ratios will not be measured in any actual power reactor. For most designs, one would have to wait for many years for the outer regions of the blanket to be evaluated. In that time, the core of any reactor must surely be changed to improve fuel elements with somewhat altered characteristics.

It is impressive to consider the many hours of reactor computations which are represented in the several parametric survey papers which were presented. One laboratory (Karlsruhe) alone mentioned some 300 problems, although not all were computed to the same degree. In taking advantage of new, powerful computing facilities, the number of groups for which constants must be determined is increasing from a few to twenty and even more. New intra-group averaging methods have been developed and programmes have been expanded. This field of experimental mathematics must have check points for cross-comparison. Several laboratories have selected, for their own use, a listing of experimentally measured assemblies which they consider to be reliable. These systems cover at least the range of reactor designs which are of interest to the particular laboratory. It would be useful if there were an interchange of these listings together with the significant parameters appropriate to each and, more important, a general agreement as to the relative accuracy. This procedure might eliminate unnecessary duplication and help to evaluate the merits of the several computing programmes.

As Dr. Okrent pointed out, the critical assemblies that are taken as standards are also in need of inter- and intra-laboratory standardization. Techniques and instrumentation change in subtle ways over a period of years, and it is possible that results reported several years ago are not directly comparable with measurements taken now by the same laboratory. Hidden systematic differences may also exist between laboratories. Thus, a simple reproducible reference standard adopted by the various laboratories may eliminate confusing conflicts of data. Standards suggested here were the use of Cf^{252} as a convenient fission spectrum source of known strength or an iron cube with predetermined detector locations which is irradiated by an accelerator-generated neutron source. More leisurely thought on the subject will surely result in a practical and generally acceptable proposal.

The French group from Saclay reported an extremely well-done experiment designed to measure the asymptotic neutron spectrum in a natural-uranium medium. This experiment is one that may be called a classic in that the objective is easily defined and, in principle, is calculable if the appropriate constants are known. It is tempting, therefore, to use it as a calculational check point. Since A. H. Snell made the first attempt in 1942 with limited facilities, experimenters in several laboratories have attempted to obtain definitive measurements which were free of edge effects and extraneous perturbations. To be accurate, it was stressed that the Saclay measurements were primarily concerned with observations on the manner with which the neutron flux in the uranium approached an equilibrium, and no claim to a positive equilibrium observation was made. However, it will be difficult to improve upon the reported experiment.

A session devoted to breeding and fuel-cycle problems made it clear—and all who are working on fast-power-reactor development must sooner or later become convinced of this—that fast reactors are a potentially economic way of obtaining useful energy. No one can claim now with conviction that he has the answer, and many searching investigations are in progress, looking for the most feasible approach to take. All the implications of the many fuel types mentioned could not be considered in this specialized meeting, but sufficient data will be accumulated to permit choices to be made in the near future. It is unlikely that fast-reactor programmes will ever, in any country, receive the same generous support that the thermal-reactor programmes have enjoyed. The next generation of expensive reactor experiments had therefore better live up to the claims made for them.

The two sessions devoted to the kinetics and physics of safety in fast reactors did little more than introduce that complex subject. It is rendered unnecessarily complex, perhaps, by the mistaken notion that the existence of a very short neutron lifetime creates a unique hazard, distinct from other reactor types. Clearly, all power reactors are inherently dangerous if improperly designed or carelessly operated; but so are automobiles and aeroplanes. The difference lies in what might be called acceptable risks. The aeroplane and automobile offer unique services which, according to some undefined process of reasoning, justify the hazard they bring with them. The central power station reactor, on the other hand, does not offer equivalent advantages, and so it cannot assume any privileges. As Dr. Vendryès pointed out at the opening of these sessions in question, safety must be considered in the conceptual reactor as a major factor of design.

The great attention focused on attempts to predict the Doppler contribution to the temperature coefficient is indicative of the concern for safety. In the absence of experimental measurements, the uncertainty regarding the magnitude—and even the sign—of the reactivity changes is large enough to be of considerable concern. This uncertainty is not acute for the existing fast reactors because their neutron spectrum is so high as to render the resonance area unimportant. In the large power reactors which are planned for the future, however, the dilution in the larger core will result in a softened spectrum; and this will increase the importance of the Doppler effect.

We had an active discussion on reactor safety, and the various designs under consideration were discussed. Divergent opinions were expressed regarding the degree of security which could be obtained from design features as opposed to operational, or management, control. These differences of opinion are understandable, as those who are oriented primarily towards critical assemblies favour management control. The flexibility required in their work makes other forms of control difficult, if not impossible. Power station reactors, on the other hand, must contemplate a very long life under the supervision of operators who generally will have less technical training than the experimenters who work with critical assemblies.

To close, I believe that the proponents of fast reactors as practical, economic power-producing machines of the future are coming into prominence. There is still a long way to go before we can hope to attain the same facility in calculation and in design that exists with thermal reactors; but, in my opinion, the reward which awaits us when we produce a successful fast breeder reactor is immeasurably greater, for we shall then be able, in the words of Dr. Alvin Weinberg, to "burn the rocks" of which there are well-nigh unlimited supplies.

CHAIRMEN OF SESSIONS

Session 1	J. Horowitz (France)
Session 2	C. H. Westcott (Canada)
Session 3	E. Clementel (Italy)
Session 4	R. D. Smith (United Kingdom)
Session 5	V. Raieski (Euratom)
Session 6	D. Okrent (United States of America)
Session 7	W. Häfele (Federal Republic of Germany)
Session 8	L. Pál (Hungary)
Session 9	B. I. Spinrad (United States of America)
Session 10	G. Vendryès (France)
Session 11	G. Tavernier (Belgium)
Session 12	M. Čopič (Yugoslavia)
Session 13	D. B. Hall (United States of America)
Panel Session	C. Sánchez del Río (IAEA)

SECRETARIAT

Scientific Secretary: C. Erginsoy (IAEA)
Executive Secretary: R. de Mazancourt (IAEA)
Executive Editor: R. Bourdariat (IAEA)
Records Officer: S. A. Edminster (IAEA)

LIST OF PARTICIPANTS

A. Ancarani	Euratom, Brussels	Euratom
A. Ascari	Centro Ricerche Nucleari SORIN Saluggia (Vercelli)	Italy
T. Auerbach	Eidg. Institut für Reaktorforschung, Würenlingen (AG)	Switzerland
M. E. Battat	Los Alamos Scientific Laboratory, Los Alamos, New Mexico	United States of America
G. H. Bauchard	Los Alamos Scientific Laboratory, Los Alamos, New Mexico	United States of America
C. Beets	Centre d'étude de l'énergie nucléaire, Mol	Belgium
P. Benoist	Centre d'études nucléaires de Saclay	France
V. Benzi	Centro di Calcolo del CNEN, Bologna	Italy
J. Berg	Institutt for Atomenergie, Lillestrøm	Norway
M. G. Bhide	Atomic Energy Establishment, Trombay (Bombay)	India
A. Birkhofer	Technischer Überwachungs-Verein, Munich	Federal Republic of Germany
M. V. Bodnarescu	Centro Comune di Ricerche, Euratom, Ispra (Varese)	Euratom
V. Boffi	Centro di Studi Nucleari della Casaccia, Rome	Italy
M. Boggardt	Reactor Centrum Nederland, The Hague	Netherlands
I. I. Bondarenko	Physical Energy Institute of the State Committee on the Use of Atomic Energy, Moscow	USSR
E. L. Brady	United States Mission to the IAEA, Vienna	United States of America
A. Campise	Atomics International, Canoga Park, California	United States of America
J. Chernick	Brookhaven National Laboratory, Upton, Long Island, N. Y.	United States of America
C. Clair	Commissariat à l'énergie atomique, Montrouge	France
P. Clauzon	Centre d'études nucléaires de Saclay	France
E. Clementel	Centro di Calcolo del CNEN, Bologna	Italy
J. Codd	Atomic Energy Establishment, Winfrith, Dorset	United Kingdom
M. Console	FIAT, Sezione Energia Nucleare, Turin	Italy
M. Čopič	Nuclear Institute "Josef Stefan", Ljubljana	Yugoslavia
B. Coppi	Gruppo Calcolo Reattori, University of Milan	Italy
W. Dabek	Institute of Nuclear Research, Polish Academy of Sciences, Warsaw	Poland
B. Defreyn	Euratom, Brussels	Euratom
O. Dietrich	Danish Atomic Energy Commission, Department of Physics, Risö	Denmark
K. Effat	Atomic Energy Establishment, Inchass	United Arab Republic
K. Einfeld	Interatom, Bensberg/Köln	Federal Republic of Germany

P. Engelmann	Kernforschungszentrum, Karlsruhe	Federal Republic of Germany
A. T. G. Ferguson	Atomic Energy Research Establishment, Harwell, Berks.	United Kingdom
P. Fornaciari	AGIP Nucleare, Milan	Italy
E. A. Fossoul	BelgoNucléaire, Brussels	Belgium
P. Greebler	General Electric Co., Atomic Equipment Department, San Jose, California	United States of America
W. Häfele	Kernforschungszentrum, Karlsruhe	Federal Republic of Germany
H. Häggblom	AB Atomenergi, Department for Reactor Physics, Stockholm	Sweden
D. B. Hall	Los Alamos Scientific Laboratory, Los Alamos, New Mexico	United States of America
I. Hamouda	Atomic Energy Establishment, Inchass	United Arab Republic
G. E. Hansen	Los Alamos Scientific Laboratory, Los Alamos, New Mexico	United States of America
H. Harder	Deutsche Bakcock und Wilcox-Dampfkesselwerke AG, Oberhausen-Rhld.	Federal Republic of Germany
J. Hejtmanek	Atominstitut, Wien	Austria
E. Hellstrand	AB Atomenergi, Stockholm	Sweden
P. Hemmig	U. S. Atomic Energy Commission, Washington, D. C.	United States of America
J. Hopkins	Los Alamos Scientific Laboratory, Los Alamos, New Mexico	United States of America
J. Horowitz	Centre d'études nucléaires de Saclay	France
M. Hoyaux	Groupement Professionnel de l'Industrie Nucléaire, Ateliers de Constructions Electriques de Charleroi	Belgium
H. H. Hummel	Argonne National Laboratory, Argonne, Illinois	United States of America
J. Jacquesson	Commissariat à l'énergie atomique, Montrouge	France
J. Jägersberger	Böhler Bros., Edelstahlwerke, Kapfenberg	Austria
A. Kania	Centre d'études nucléaires de Saclay	France
W. Keeney	Argonne National Laboratory, Argonne, Illinois	United States of America
A. Kind	Centro Comune di Ricerche, Euratom, Ispra (Varese)	Euratom
J. Kockum	Research Institute of National Defence, Stockholm	Sweden
M. Laffaire	Centre d'études nucléaires de Saclay	France
B. Lawrence	Australian Atomic Energy Commission Research Establishment, Sydney	Australia
D. O. Leeser	Atomic Power Development Association, Detroit, Michigan	United States of America
G. Leeth	General Electric Company, Hamilton, Ohio	United States of America
B. Lemaire	Commissariat à l'énergie atomique, Montrouge	France
P. Lénárd	Central Research Institute for Physics, Hungarian Academy of Sciences, Budapest	Hungary

J. L. Leroy	Centre d'études nucléaires de Saclay	France
M. Levine	Brookhaven National Laboratory, Upton, Long Island, N. Y.	United States of America
W. B. Loewenstein	Argonne National Laboratory, Argonne, Illinois	United States of America
J. K. Long	Argonne National Laboratory, Idaho Falls, Idaho	United States of America
C. Lo Surdo	SORIN (Società Ricerche Impianti Nucleari), Milan	Italy
J. E. Mann	UKAEA Reactor Group, Risley near Warrington, Lancs.	United Kingdom
G. I. Marchuk	Physical Energy Institute of the State Committee on the Use of Atomic Energy, Moscow	USSR
M. Marseguerra	Centro di Studi Nucleari della Casaccia, Rome	Italy
K. Matsuoka	Central Research Laboratory, Hitachi Ltd., Kokubunji, Tokyo	Japan
J. McCullen	Division of Reactors, IAEA, Vienna	IAEA
D. Meneghetti	Argonne National Laboratory, Argonne, Illinois	United States of America
A. Moat	Atomic Weapons Research Establishment, Aldermaston, Berks.	United Kingdom
P. Moinereau	Centre d'études nucléaires de Saclay	France
V. Molinari	Centro di Studi Nucleari della Casaccia, Rome	Italy
J. Moteff	General Electric Company, Cincinnati, Ohio	United States of America
F. Motte	Centre d'étude de l'énergie nucléaire, Mol	Belgium
B. Mueller	Atomic Power Development Associates, Inc., Detroit, Michigan	United States of America
P. D. de Myttenaere	BelgoNucléaire, Brussels	Belgium
P. Nathschläger	Österreichische Studiengesellschaft für Atomenergie GmbH, Vienna	Austria
J. H. Neiler	Oak Ridge National Laboratory, Oak Ridge, Tennessee	United States of America
W. W. Nijs	Reactor Centrum Nederland, The Hague	Netherlands
D. Okrent	Argonne National Laboratory, Argonne, Illinois	United States of America
P. L. Olgaard	Danish Atomic Energy Commission, Research Establishment, Risö	Denmark
K. Ott	Kernforschungszentrum, Karlsruhe	Federal Republic of Germany
K. Parker	Atomic Weapons Research Establishment, Aldermaston, Berks.	United Kingdom
R. Pasquer	Electricité de France, Paris	France
E. D. Pendlebury	Atomic Weapons Research Establishment, Aldermaston, Berks.	United Kingdom
F. Pierantoni	Centro di Calcolo del CNEN, Bologna	Italy
V. Raievski	Centro Comune di Ricerche Euratom, Ispra (Varese)	Euratom
H. Rief	Centro Comune di Ricerche, Euratom, Ispra (Varese)	Euratom
O. von Roos	Jet Propulsion Laboratory, California Institute of Technology, Altadena, California	United States of America

T. Rzeszot	Institute of Nuclear Research, Polish Academy of Sciences, Warsaw	Poland
N. Sack	Department of Theoretical Physics, Hebrew University, Jerusalem	Israel
W. Sahl	Euratom, Brussels	Euratom
A. M. Saruis (Mme)	Centro di Calcolo del CNEN, Bologna	Italy
W. Schikarski	Technischer Überwachungs-Verein, Munich	Federal Republic of Germany
P. Schmid	Eidg. Institut für Reaktorforschung, Würenlingen (AG)	Switzerland
J. J. Schmidt	Kernforschungszentrum, Karlsruhe	Federal Republic of Germany
R. T. Shanstrom	General Atomic Europe, Zürich	United States of America
D. Sigmar	Reaktor-Interessengemeinschaft, Vienna	Austria
H. B. Smets	OEEC, European Nuclear Energy Agency, Paris	ENEA (OEEC)
D. Smidt	Kernforschungszentrum, Karlsruhe	Federal Republic of Germany
A. B. Smith	Argonne National Laboratory, Argonne, Illinois	United States of America
D. C. G. Smith	Dounreay Experimental Reactor Establishment, Caithness, Scotland	United Kingdom
R. D. Smith	Atomic Energy Establishment, Winfrith, Dorset	United Kingdom
R. R. Smith	Argonne National Laboratory, Idaho Falls, Idaho	United States of America
M. Solanes (Mlle)	Centre d'études nucléaires de Saclay	France
G. Solinas	Centro di Studi Nucleari della Casaccia, Rome	Italy
H. Soodak	United Nuclear Corporation, N.D.A., White Plains, N.Y.	United States of America
B. I. Spinrad	Argonne National Laboratory, Argonne, Illinois	United States of America
A. de Stordeur	Euratom, Brussels	Euratom
F. Storrer	Euratom, Brussels	Euratom
F. Szabó	Central Research Institute for Physics, Hungarian Academy of Sciences, Budapest	Hungary
R. Taschek	Los Alamos Scientific Laboratory, Los Alamos, New Mexico	United States of America
G. C. Tavernier	BelgoNucléaire, Brussels	Belgium
A. Trani	Cantieri Riuniti dell'Adriatico, Trieste	Italy
K. Tsukada	Japan Atomic Energy Research Institute, Nuclear Laboratory, Tokai-mura, Ibaraki-ken	Japan
L. N. Usachev	Physical Energy Institute of the State Committee on the Use of Atomic Energy, Moscow	USSR
J. van der Spek	BelgoNucléaire, Brussels	Belgium
J. B. van Erp	Euratom, Brussels	Euratom
G. Vendryès	Centre d'études nucléaires de Fontenay-aux-Roses	France
J. Weale	Atomic Weapons Research Establishment, Aldermaston, Berks.	United Kingdom

J. W. Webster	Division of Reactors, IAEA, Vienna	IAEA
C. H. Westcott	Atomic Energy of Canada Ltd., Chalk River, Ontario	Canada
H. Weyand	Bundesministerium für Atomenergie und Wasserwirtschaft, Bad Godesberg	Federal Republic of Germany
M. Wladarz	Brown Boveri-Krupp Reaktorbau, GmbH, Heidelberg	Federal Republic of Germany
F. Woloch	Reaktorzentrum, Seibersdorf	Austria
Y. Yeivin	Department of Theoretical Physics, Hebrew University, Jerusalem	Israel
S. Yiftah	Israel Atomic Energy Commission, Rehovot	Israel
C. P. Zaleski	Centre d'études nucléaires de Saclay	France
J. J. Zamparo	ANSALDO, Genoa	Italy
R. Zelazny	Institute of Nuclear Research, Polish Academy of Sciences, Warsaw	Poland
G. Zorzoli	CISE (Centro Informazioni Studi Esperienze), Milan	Italy
P. F. Zweifel	University of Michigan, and Atomic Power Development Associates, Inc., Detroit, Michigan	United States of America

AUTHOR INDEX

The volume numbers are indicated by boldface Roman numerals. The page numbers of papers are printed upright, and those of contributions to discussions in italics. The page numbers of introductory talks by Chairmen of Sessions are underlined.

OTHER IAEA PUBLICATIONS IN THE PROCEEDINGS SERIES

Nuclear Electronics — L'électronique nucléaire (2 Vols.)	STI/PUB/2
Medical Radioisotope Scanning	STI/PUB/3
Large Radiation Sources in Industry (2 Vols.)	STI/PUB/12
Metrology of Radionuclides	STI/PUB/6
Disposal of Radioactive Wastes (2 Vols.)	STI/PUB/18
Codes for Reactor Computations	STI/PUB/24
Selected Topics in Radiation Dosimetry	STI/PUB/25
Small and Medium Power Reactors (2 Vols.)	STI/PUB/30
Effects of Ionizing Radiations on Seeds	STI/PUB/13
Radioisotopes in the Physical Sciences and Industry (3 Vols.)	STI/PUB/20
Inelastic Scattering of Neutrons in Solids and Liquids	STI/PUB/35
Chemical Effects of Nuclear Transformations (2 Vols.)	STI/PUB/34
Nuclear Ship Propulsion	STI/PUB/37
Pile Neutron Research in Physics	STI/PUB/36
Radioisotopes in Tropical Medicine	STI/PUB/31
Radioisotopes and Radiation in Entomology	STI/PUB/38
Power Reactor Experiments (2 Vols.)	STI/PUB/51
Effects of Ionizing Radiation on the Nervous System	STI/PUB/46
Whole-body Counting	STI/PUB/47
Tritium in the Physical and Biological Sciences (2 Vols.)	STI/PUB/39
Nuclear Electronics (3 Vols.)	STI/PUB/42
Plasma Physics and Controlled Nuclear Fusion Research (3 Vols.)	STI/PUB/50

IN PRESS

Radioisotopes in Soil-plant Nutrition Studies	STI/PUB/55

All the above publications are obtainable from the Sales Agents listed overleaf.

A complete Catalogue of all Agency publications will be gladly supplied by any of the Sales Agents or directly by the Editorial and Publications Sections, International Atomic Energy Agency, Kärntner Ring, Vienna I, Austria.

IAEA SALES AGENTS

ARGENTINA
Editorial Sudamericana, S. A.
Alsina 500
Buenos Aires

AUSTRALIA
Melbourne University Press
369, Lonsdale Street
Melbourne, C. 1

AUSTRIA
Georg Fromme & Co.
Spengergasse 39
Vienna V

BELGIUM
Office International de librairie
30, avenue Marnix
Brussels 5

BRAZIL
Livraria Kosmos Editora
Rua do Rosario, 135—137
Rio de Janeiro

Agencia Expoente Oscar M. Silva
Rua Xavier de Toledo, 140—1° Andar
(Caixa Postal No 5.614)
São Paulo

BURMA
See under India

BYELORUSSIAN SOVIET SOCIALIST REPUBLIC
See under USSR

CANADA
The Queen's Printer
Ottawa

CEYLON
See under India

CHINA (Taiwan)
Books and Scientific Supplies
Service, Ltd.,
P.O. Box 83
Taipei

DENMARK
Ejnar Munksgaard Ltd.
6 Nörregade
Copenhagen K

ETHIOPIA
G. P. Giannoppoulous
International Press Agency
P.O. Box 120
Addis Ababa

FRANCE and FRENCH UNION
Masson et Cie, Editeurs
120 bd Saint-Germain
Paris VIe

GERMANY, Federal Republic of
R. Oldenbourg
Rosenheimer Strasse 145
Münich 8

GREECE
C. Eleftheroudakis and Son
Constitution Square
Athens

ICELAND
Halldór Jónsson
Mjóstraeti 2
Reykjavík

INDIA
Orient Longmans Ltd.
17, Chittaranjan Ave.
Calcutta 13

ISRAEL
Heiliger and Co.
3 Nathan Strauss Street
Jerusalem

ITALY
Agenzia Editoriale Internazionale
Organizzazioni Universali
(A.E.I.O.U.)
Via Meravigli 16
Milan

JAPAN
Maruzen Company Ltd.
6, Tori Nichome
Nihonbashi
P. O. Box 605
Tokyo Central

KOREA, Republic of
The Eul-Yoo Publishing Co.
5,2-ka Chong-ro
Seoul

MEXICO
Librería Internacional
Av. Sonora 206
Mexico 11, D. F.

MONACO
The British Library
30, bd des Moulins
Monte Carlo

MOROCCO
Centre de diffusion documentaire
du B.E.P.I.
8, rue Michaux-Bellaire
(B.P. N° 211)
Rabat

NEPAL
See under India

NETHERLANDS
N. V. Martinus Nijhoff
Lange Voorhout 9
The Hague

NEW ZEALAND
Whitcombe & Tombs, Ltd.
G.P.O. Box 1894
Wellington, C. 1

NORWAY
Johan Grundt Tanum
Tidl. Aschehougs Boghandel
Karl Johans Gate 43
Oslo

PAKISTAN
Karachi Education Society
Haroon Chambers
South Napier Road
P.O. Box No. 4866
Karachi, 2

PARAGUAY
Agencia de Librerías
de Salvador Nizza
Calle Pte. Franco No. 39—43
Asunción

PERU
Librería International del Perú S.A.
Boza 879
(Casilla 1417)
Lima

PHILIPPINES
The Modern Book Company
508 Rizal Avenue
Manila

POLAND
Distribution Centre for
Scientific Publications
Polish Academy of Sciences
Palac Kultury i Nauki
Warsaw

PORTUGAL
Livraria Rodrigues
186, Rua do Ouro, 188
Lisbon 2

SOUTH AFRICA
Van Schaik's Bookstore (Pty) Ltd.
Libri Building
Church St.
(P.O. Box 724)
Pretoria

SPAIN
Librería Bosch
1, Ronda Universidad
Barcelona

SWEDEN
C. E. Fritzes Kungl. Hovbokhandel
Fredsgatan 2
Stockholm 16

SWITZERLAND
Librairie Payot
40, rue du Marché
Geneva

SYRIA
Georges N. Coussa
Imm. Chanan
rue Khan el-Harir
(B.P. 779)
Aleppo

TURKEY
Librairie Hachette
469, Istiklâl Caddesi
Beyoglu, Istanbul

UKRAINIAN SOVIET SOCIALIST REPUBLIC
See under USSR

UNION OF SOVIET SOCIALIST REPUBLICS
Mezhdunarodnaya Kniga
Kuznetsky Most, 18
Moscow G-200

UNITED KINGDOM OF GREAT BRITAIN
AND NORTHERN IRELAND
Her Majesty's Stationery Office
P.O. Box 569
London S.E. 1

UNITED STATES OF AMERICA
National Agency for
International Publications, Inc.
801 Third Avenue
New York 22, N.Y.

YUGOSLAVIA
Jugoslovenska Knjiga
Terazije 27
Belgrade

IAEA publications can also be purchased retail at the United Nations Bookshop at United Nations Headquarters, New York, at the newsstand at the Agency's Headquarters, Vienna, and at most conferences, symposia and seminars organized by the Agency.

Orders and inquiries from countries where sales agents have not yet been appointed may be sent to: Distribution and Sales Unit, International Atomic Energy Agency, Kärntner Ring, Vienna I, Austria

INTERNATIONAL
ATOMIC ENERGY AGENCY,
VIENNA 1962

PRICE (A): North America: US $12.00
Elsewhere: Sch 252,-
(72s.stg; NF 48; DM 42,-)