

TECHNICAL REPORTS SERIES No.12

Light Water Lattices

REPORT OF A PANEL
HELD IN VIENNA,
28 MAY - 1 JUNE 1962



INTERNATIONAL ATOMIC ENERGY AGENCY - VIENNA 1962

LIGHT WATER LATTICES

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FOREWORD

The progress of nuclear power and its application to the needs of man depend to a large extent on progress in many fields of science and technology. Among these fields, that of reactor physics is assuming particular importance and a deeper understanding of the processes taking place within a reactor, the possibility of calculating its critical mass and time-behaviour, the control of the reactor and the burn-up of its fuel and other problems are at present the subject of intensive study.

The investigation of the physics of various reactor lattices plays an important part in the solution of these main reactor problems. This topic has been included in the programme of panels on reactor physics initiated by the International Atomic Energy Agency, and the first of the series was a panel on heavy-water lattices held in Vienna in 1959.

A similar panel on light-water lattices, convened in Vienna from 28 May to 1 June 1962, was attended by prominent physicists from most of the well-known laboratories in this field, who exchanged the latest information on the status of work in their countries and discussed both the theoretical and the experimental aspects of the subjects. The supporting papers covered most problems, including criticality, resonance absorption, thermal utilization, spectrum calculation and the physics of plutonium bearing systems.

The present publication represents the report of the panel; it was compiled by the Chairman, Dr. H. Kouts of Brookhaven National Laboratory. It is hoped that this book will be of value to all scientists working in this field of reactor physics.

SIGVARD EKLUND

Director General

November 1962

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GENERAL SUMMARY

H. KOUTS - CHAIRMAN

The panel consisted of participants from France, the United Kingdom and the United States of America, as well as observers from the United States. The latter group joined fully in presenting information and in general discussion.

The members of the Panel were:

France:

M. J. Bailly
M. R. Delayre
M. Y. Girard
M. C. Guionnet
M. J. Larrivé

United Kingdom:

Dr. D. Hicks
Mr. I. Johnstone

United States of America:

Dr. H. Kouts (Chairman)
Mr. P. Gast
Dr. R. L. Hellens
Dr. D. Klein
Mr. G. Minton
Dr. D. Wehmeyer

In addition to the participants, three observers from the United States of America attended the meeting. They were:

Mr. H. Honeck
Mr. A. Radkowsky
Mr. T. M. Snyder

The meeting was very informal. The detailed papers, which are part of the record, were not read; instead, they served as bases for discussion. This procedure permitted extracting from each pertinent paper in turn the information bearing on a specific topic. It also allowed the introduction of other relevant matter which had not been formally prepared. The neutron physics of light water lattices was in this way reviewed in a logical manner. This arrangement is thought to have worked very well.

The initial discussions centred on the ability of present criticality calculations to reproduce the available experiments. Particular emphasis was given to eigenvalue calculations. It was seen that reasonably uniform practices are used at present. These are based on Fourier transform multi-group calculations of the neutron spectrum, including various methods of estimating cell effects on fast fission and resonance capture. Following this, few-group methods are used to find the eigenvalues. The spectral calculations are based

on the P_1 approximation (MUFT codes) or the B_1 approximation. There was some tentative indication that the latter gives closer agreement with experiment, but at this stage no conclusions on this matter can be justified. Attention has been given both in the United Kingdom and the United States to simplifying the spectral calculation to make it suitable for smaller computing machines. These methods depend on assumptions about the general shape of the spectrum, and they can be expected to be reasonably reliable for well moderated systems where these assumptions are nearly met. It appeared that in almost all analyses now done by these methods, the calculated eigenvalues are low. The reason for this situation is not at present clear; the problem is discussed in more detail in the summary on criticality calculations.

Calculated eigenvalues for plutonium fuelled systems studied in the United Kingdom are even lower. It is believed that additional effects are responsible here; in particular there is doubt about the correctness of present best values of the thermal and near thermal cross-sections of Pu^{239} .

Most criticality calculations were seen to use models which assume space and energy separability of the thermal neutron flux. The most common of these is the SOFOCATE code, which calculates a Wigner-Wilkins spectrum. The most successful of these models has been that proposed by Amouyal and Benoist. Recent efforts to place the calculation of the thermal neutron flux distribution on a sound theoretical footing have had a large measure of success, but there still remains a discrepancy whose origin is not clear. This discrepancy has little effect on purely uranium fuelled systems. When substantial amounts of plutonium or other strongly non- $1/v$ absorbers are present, the effect can be large. The problem is dealt with more fully in the summary on thermal flux distribution.

Resonance absorption calculations are done by many methods. The most common are either the NR-NRLA analytical methods supplemented by a statistical analysis of the unresolved resonance region, and the use of experimentally measured resonance integrals. The mutual shielding effects of neighbouring rods are estimated by the original analytical methods of Dancoff and Ginsberg, or they are found by Monte-Carlo methods. In either case, they are applied according to equivalence relations due to Bell or Bakshi. Some complete Monte-Carlo calculations have been run to test these methods; the results are described in the summary on resonance absorption and in the paper by Hellens and Honeck.

The fast fission effect was also treated in this paper. Calculation of the $\text{U}^{238}/\text{U}^{235}$ fission ratio through use of an equivalence relation suggested by Hellens agrees well with the available measurements, except for a systematic deviation between the two in wider spaced lattices. It also agrees well with single rod measurements up to a rod diameter of about 9 cm.

Following the detailed discussion of these clean lattice questions, there were presentations by the French participants on a measurement of the effective hafnium resonance integral by a critical assembly poisoning method, and of various experimental techniques which have been developed at Saclay. The hafnium measurement yielded a result in agreement with the 1900 b value now commonly accepted. The experimental techniques were directed at two problems which commonly trouble light water lattice critical experi-

ments. The first is the prevention of excess boron deposit on fuel elements in poisoned experiments. The second is the achieving of a constant, uniform water height in partially submerged lattices. Considerable success was met in treating both problems by methods reported in the paper by Bailly and Girard. Also reported were some measurements of control effectiveness by plate geometry absorbers. The agreement with calculations was good.

The attention of the panel then turned to the usefulness of the analytical methods used for clean, simple lattices when they are applied to more involved systems such as multi-region assemblies and complex reactor cores.

The degree of success in reproducing eigenvalues seems to be carried through into the multi-region systems. The cell reaction rates are also predicted reasonably well as a function of position, although there is a systematic deviation, particularly in the distribution of power throughout the lattices. Considerable improvement is obtained by the use of mixed number density methods.

The analysis of complex reactors such as the PWR and boiling water systems requires adjoining to the more basic theoretical methods a set of prescriptions. These account for geometric and structural complexities whose treatment would require too much computing machine time and capacity. Specific examples of these are the effects of water channels on power distribution and reaction rates in nearby rods, the poisoning effect of fuel element boxes, and the calculation of control rod worth and control rod interaction. These prescriptions add uncertainties of unknown size to those already present when they are extended to systems for which they have not been tested. At this stage, however, it is difficult to see how relatively intuitive methods of this kind can be avoided in practical cases, and the economics of nuclear power plant design should continue their attractiveness for some time.

In the discussion of PWR performance, Radkowsky pointed out that the trend in burn-up calculations is away from the use of single effective fission-product poison cross-sections. The presence of large resonance integrals among the fission products has been recognized, and these are now included in the analysis. The most complex burn-up calculations have been done by three dimensional few-group methods. These require considerable machine time, and efforts have been made to shorten this to manageable size. Considerable preliminary success has been achieved.

At the end, the panel tried to estimate the individual contributions to the uncertainty in the eigenvalue calculations. These uncertainties depend on the water to fuel ratio, the rod size and the fuel enrichment, as well as on other things such as clad thickness and composition. Furthermore, in most cases the estimates of uncertainty were the result of intuition. Therefore not much credence is to be placed in the Table which was arrived at. For what it is worth, the Table is as follows:

Source of uncertainty	Effect on eigenvalue (%)
Resonance absorption	0.5
Thermal flux spacial distribution	0.3
Thermal η (U^{235})	0.7

Fast fission factor calculation	0. 3
Leakage calculation	0. 5
Epithermal η (U^{235})	0. 7
Cross-sections used in thermal utilization	0. 1

If these uncertainties were really present and if they all acted in the same direction, they would amount to 3.1% in the eigenvalue. If they were combined statistically (being nearly independent), their net contribution would be 1.3%. The actual difference which is seen between calculation and measurement is somewhat smaller than this. Clearly there is a cancelling of errors, or the values are in general better known than the table indicates.

This question is touched on in the summary on criticality calculations.

The recommendations of the panel were as follows (some are repeated in the detailed summaries):

1) There should be accurate measurements of those activation resonance integrals which would be useful for spectrum measurements.

2) There should be accurate measurements of the low energy activation cross-sections of non-1/v absorbers, as a function of energy, to aid in measurements which test calculations of the thermal neutron spectrum. The use of these non-1/v detectors to provide spectral indices should be extended.

3) Intracell thermal flux distribution should be measured in more slab lattices, particularly those with large disadvantage factors and large spectrum shifts.

4) Monte-Carlo calculations should be done to further investigate the errors introduced by cell boundary conditions in the cylindrical rod lattices.

5) The size of the experimental error introduced by using all metal foils or all plastic foils in all regions of the cells should be measured.

6) There should be good danger coefficient measurements of boron and non-1/v absorbers dissolved in the water, to test the ability to calculate the low energy neutron spectrum.

7) Critical experiments should be performed on clean water lattices which have spectra more epithermal than those so far studied.

8) Slowing down distribution measurements in pure water and in metal-water mixtures should be resumed. The goal should be accurate measurement of the second and higher moments of thermalized neutrons in these media. These should include assemblies containing no U^{238} .

9) There is a 3% discrepancy between the values of η (Pu^{239}) (2200 m/s) measured directly and the value inferred from individual measurement of $\bar{\nu}$, δ_f , and δ_a . This discrepancy should be resolved and values of η (Pu^{239}) established to better than $\pm 1\%$ over the thermal and epithermal energy ranges.

10) Further nuclear physics measurements are required to improve the accuracy of the published cross-sections for Pu^{240} and Pu^{241} .

11) Homogeneous critical experiments with uranium-water and plutonium-water solutions should be compared to give information on effective η values for Pu^{239} in typical reactor spectra.

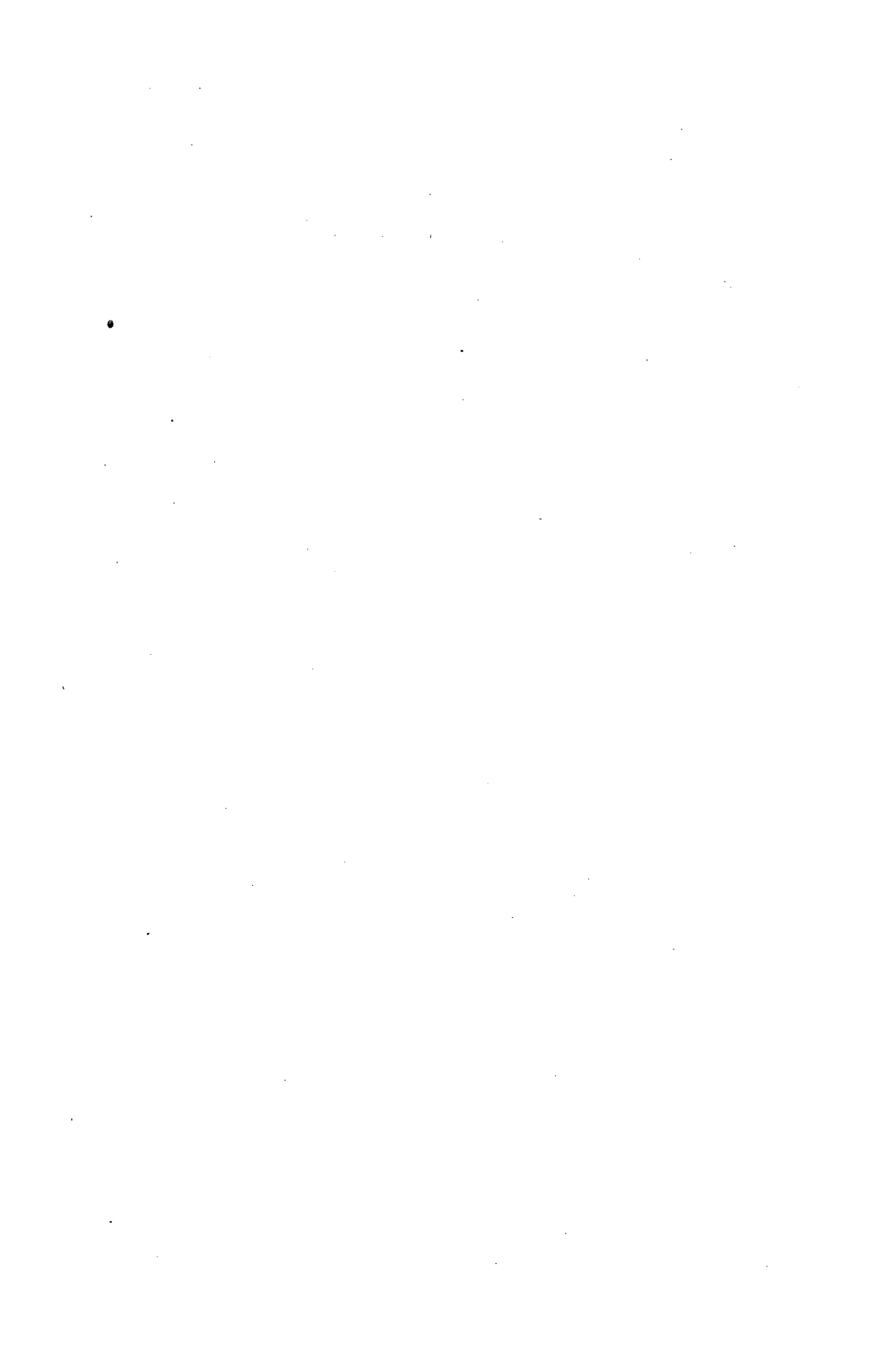
12) Experiments carried out to date with plutonium-aluminium fuel in overmoderated systems should be extended into the undermoderated region, under conditions where sufficient plutonium is present to distort the shape of the neutron spectrum near the 0.3 eV resonance.

13) Measurements of material buckling and neutron spectrum should be made in lattices containing sufficient quantities of Pu^{240} , and later Pu^{241} . It may be possible to obtain a preliminary check on the methods of calculation for these systems by the substitution of (say) indium for the Pu^{240} , but the success of this method is dependent on the knowledge of the nuclear data for the substitute material. Due to the local nature of the Pu^{240} resonance effect, these experiments will require the Pu^{240} or substitute material to be homogeneously dispersed in typical reactor fuel elements. Experiments will also be required with Pu^{240} in the presence of significant quantities of other resonance materials (e. g. U^{238}) so that the effect of resonance interaction can be gauged.

14) Isotopic analysis of power reactor fuel at various stages during burn-up will provide a critical test of methods of calculation of reactivity lifetime, power distribution changes, etc. Due to the large number and complex nature of the physical processes involved, comparison with calculations may be difficult in many cases. Power reactors without control rods or burnable poisons would seem to give the greatest possibility of a valid comparison.

15) This panel meeting should be followed by others:

- (a) A meeting on practical reactor core physics design methods in about a year.
- (b) A successor meeting on light water lattice methods in two to three years, when much new data should exist, especially on plutonium fuelled systems and long term behaviour.
- (c) A meeting to review in detail the advances in analysis of resonance absorption.
- (d) A successor to the IAEA sponsored meeting on reactor codes.
- (e) A meeting on reactor kinetics.



STATUS REPORTS

THE FRENCH PROGRAMME

Y. GIRARD

The French light water lattice research programme has always been limited to a few objectives.

The preliminary stage (1959) covered the study of control rods in a lattice using the only fuel available at the time (lightly enriched oxide) in the Alizé I critical facility.

We now have two critical facilities, the Alizé II (in operation 1951) and the Azur (in operation 1962), the cores studied being 90% uranium-zirconium-aluminium assemblies.

The main work on the experimental side has been on the development of techniques (boron solutions, pulsed neutrons, analysis of constituents). The studies relate to control rods, uniformly distributed or localized poisons (hafnium, boron), neutron flux and power density distributions, and temperature and vacuum coefficients.

On the theory side, our work has been very largely limited to the adapting of already published methods, with the stress on problems that are emerging, and techniques involving different combinations and permutations.

Experimental work on plutonium is to begin (under the heavy water and graphite research programme) on uranium-plutonium metal fuel. Water-plutonium research has been considered but no decision has yet been taken.

LIGHT WATER LATTICE RESEARCH IN THE UNITED KINGDOM

I. JOHNSTONE

Present United Kingdom work in the light water field forms part of a more general programme covering both light and heavy water moderation. This programme includes the use of critical and exponential assemblies, the development of design analysis methods and also more fundamental theoretical investigations.

The most recent experiments have extended the range of lattices studied to include highly enriched U^{235} and plutonium fuelled cores. Two series of light water lattice experiments have been carried out. In the first series highly enriched cores containing appreciable fractions of stainless steel were investigated. Material buckling and detailed thermal flux distributions have been measured in both critical and exponential facilities and the feasibility of small sized light water exponentials demonstrated. Particular at-

tention was paid to the form of the thermal and near epithermal spectra, using both integral reaction rate and time of flight techniques.

The second series of experiments in the HELEN I exponential assembly employed Pu-Al fuel. After initial comparison experiments with U^{235} fuel, three dilute lattices were studied as a check on the behaviour of plutonium in an essentially Maxwellian neutron spectrum. Later measurements will extend the data to harder spectra.

Techniques have been developed for measuring conversion ratio in low enrichment uranium dioxide lattices by measuring the ratio of neptunium production to the fission rate. This procedure does not involve cadmium but a separate normalization experiment in a thermal column is required. Experience has shown that the use of metal foils in oxide lattices can lead to unacceptable errors.

Present design methods are based on the MUFT-SOFOCATE computer codes written by the Westinghouse organization in the U. S. A. Computer programmes have been developed embodying tabulations of standard SOFOCATE results and three group fast neutron cross-sections fitted to the MUFT calculations. These programmes enable a complete lattice calculation to be made in a single computer run. Point reactor burn-up programmes have also been developed.

The resonance escape model in use is based on Hellstrand's integral measurements for single rods. The effective value in the lattice is computed using the Wigner rational and Bell approximations.

Reactivity predictions have achieved a root mean square deviation of approximately 0.7% and the mean values lie generally close to unity. The few-group design methods give rather low eigenvalues for the Helen plutonium cores and a more detailed analysis is now in progress.

Further experiments both with low enrichment U^{235} and plutonium fuels are planned. The design methods are being extended to space dependent burn-up and are being backed by further Monte-Carlo and multi-group transport calculations.

LIGHT WATER LATTICE RESEARCH IN THE UNITED STATES

H. KOUTS

The pertinent theoretical research is not limited to light water as a moderator, because many of the theoretical methods being developed are more general. The experimental work mentioned is specifically that done on light water moderated systems.

The experimental survey programme on lattices with uranium metal rods at Brookhaven is formally ended. There is still a continuing study directed toward improving the techniques in measurement of resonance capture and thermal flux distribution, so that the results of these will be more consistent and reliable. A similar study of slightly enriched slab lat-

tices is nearly finished. A survey programme on lattices of 3% enriched, steel clad UO_2 fuel elements will end in 1962. Again, measurements of microscopic parameters will continue. An experimental survey programme with U^{233} oxide, thorium oxide fuel elements will begin in about January 1963. The fuel elements for this set of experiments are now being fabricated.

The theoretical studies under way at Brookhaven encompass all areas of water moderated lattice systems. Analytical and computing machine studies are being done to improve the theory of the fast effect. Resonance capture is being studied both by analytical methods such as NR-NRIA and by machine methods such as Monte Carlo. Further work is being done to improve the THERMOS analyses of thermal flux distribution. Other work pertinent to this matter is devoted to better understanding and exploring the inelastic neutron moderation by water. A study of reactor leakage by other than Fourier transform methods is under way.

The experimental work being performed at Westinghouse Bettis Atomic Power Laboratory now stresses the enlarging of the area over which resonance capture probability is measured. The theoretical studies are considerably more broad, being concerned with most of the areas mentioned above, as well as with the long term reactivity behaviour of seed core reactors. Additional experimental work is done at the Shippingport PWR, consisting of studies of stability and of long term behaviour.

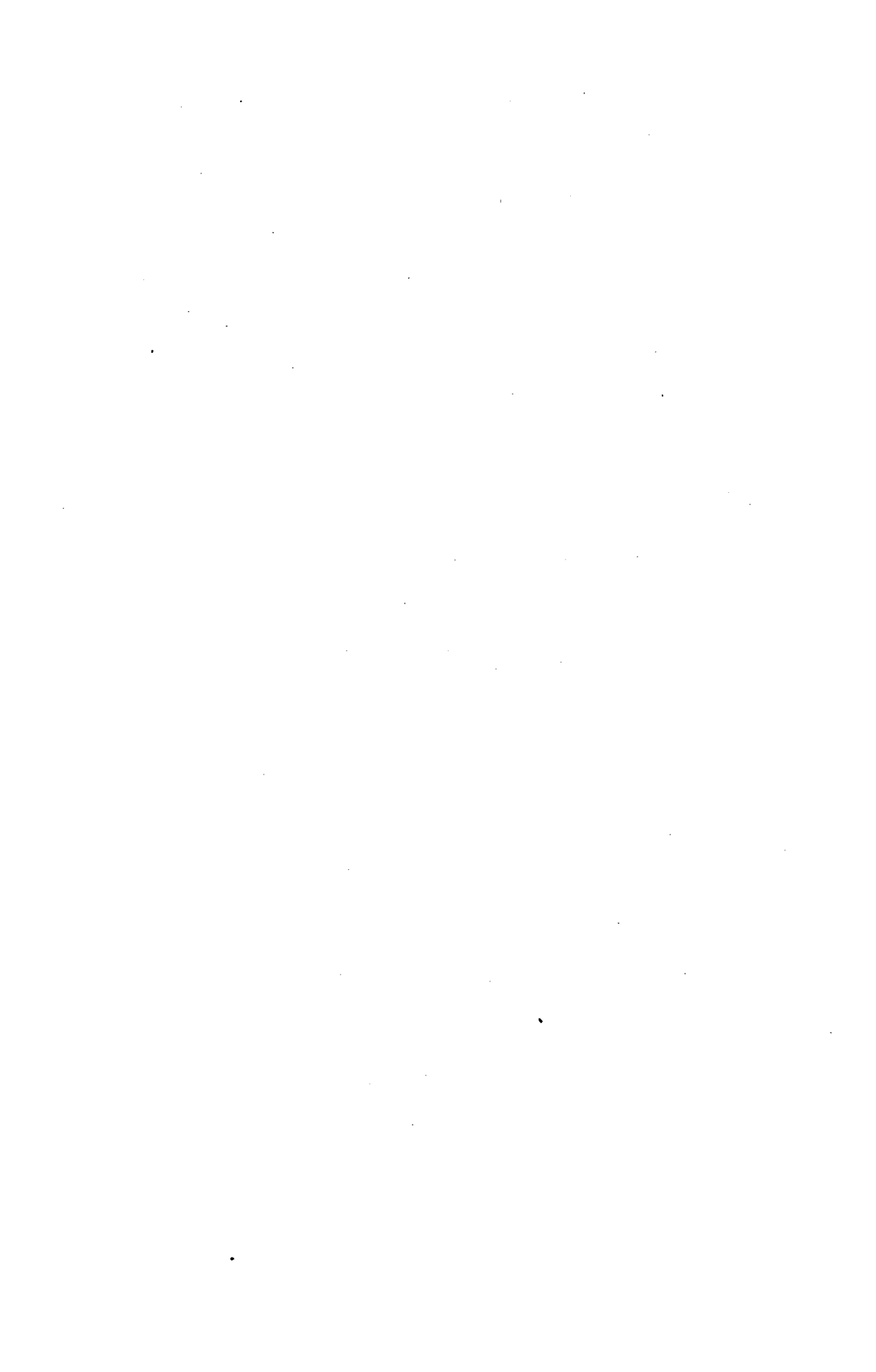
The experimental and theoretical programmes at Hanford are associated with the Plutonium Recycle Test Reactor. The experimental work is now done with plutonium fuel in heavy water; later work will use light water as a moderator. The theoretical work consists mostly of the long term behaviour surveys of various kinds of reactors fuelled with plutonium.

An experimental programme has just begun at Argonne to measure criticality and relative reaction rates in close packed lattices of uranium rods in light water. These relatively dry assemblies will have substantial epithermal fission fractions.

The principal experimental and theoretical work at Babcock & Wilcox is in support of the spectral shift reactor. Analysis of the experiments is done by the methods reported in the paper by Wehmeyer, and to a great extent this work can be considered as enlarging the region over which the light water lattice programme is carried out. The experimental programme contains measurements of critical mass, buckling, reflector savings, thermal flux distribution in cells and relative reaction rates. Additional work is being done at Babcock & Wilcox on the effective resonance integral of uranium oxide and thorium oxide rods as a function of rod diameter.

The programme at General Electric Vallecitos is more applied, being closely related to the development and design of specific boiling water reactors. A number of analytical and machine code models have been developed to these ends. The dynamic response of boiling water reactor systems of several kinds is under continuous active study.

The programme at Westinghouse Atomic Power Division has centred on multi-region lattice studies, both experimental and theoretical. Criticality surveys have been supplemented by detailed lattice parameter measurements. Similar specific studies have been done in connection with the design of various reactors.



SUMMARY OF DISCUSSIONS

CRITICALITY PREDICTION

R. L. HELLENS

The compounding of uncertainties which occurs in the computation of criticality makes the value of specific statements on the present agreement between theory and experiment somewhat dubious. The contribution from errors in the theoretical description of lattice constants (e. g. fast fission ratio, resonance capture ratio) are at present difficult to judge. These specific effects can be dealt with individually, and more adequate conclusions on the accuracy of their calculation must await better measurements of them.

An analysis of criticality prediction should distinguish between the present "best" theory and the simpler design models which are frequently called theory. The former consists of methods generally derivable from the transport equation. The latter uses methods which result from shrewd speculation about the physical nature of the effect described, and which derive their value from the fact that they seem to work. There is no clear boundary between the two classes, because simplifying assumptions are often needed to make the best methods useable.

The theory of individual lattice constants in water lattices has now become rather conventional, particularly for simple clean systems. Unfortunately, computing difficulties often reduce the usefulness of these methods for complex reactor systems. As a consequence, analytical models are needed for simplicity in analysis of the complex reactors. These must be justified by their success in predicting experimental results in the complicated cases, or by their ability to reproduce the results of more exact theory when comparison can be made. Speculative models frequently have more apparent accuracy than the best theory, especially when that best theory must be simplified to make the mathematics tractable. The speculative approach is more suspect when it is used to extrapolate from the region where it has been tested.

The estimates of uncertainties in our knowledge of the principal events important to criticality show that we should not expect theory and experiment to agree generally to closer than a few percent in the eigenvalue. A priori, better agreement than this would seem to imply an unusual amount of cancelling of errors. The errors present depend on the atom ratios of the lattices and similar physical parameters, so their degree of cancellation should vary with these quantities. On the other hand, the analysis of the broad range of Brookhaven slightly enriched uranium metal lattices shows little significant trend as these parameters are varied. The closeness of agreement between the experiments and the analysis by best theory is nearly constant throughout. In general, the calculated eigenvalues are about 1% low. This observation sheds doubt on the existence of large compensating errors.

The theory in use has been examined both from the point of view of mathematical precision and agreement with experiments. The impression gained from the panel discussion is that the primary difficulty in achieving better agreement with experiments lies in cross-section uncertainties, particularly in the epithermal neutron range.

The panel recommended that certain experiments be performed to test the theory further and to aid in resolving questions about it. These were:

- 1) Critical experiments should be performed on clean water lattices which have spectra more epithermal than in those so far studied.
- 2) Slowing down distribution measurements in pure water and in metal-water mixtures should be resumed. The goal should be accurate measurements of the second and higher moments of thermalized neutrons in these media.

THERMAL NEUTRON CELL DISTRIBUTION

H. C. HONECK

A variety of techniques is used to calculate the thermal neutron flux distribution in water lattices. The most detailed methods use integral transport theory and a Nelkin scattering kernel (THERMOS codes). Some methods use several thermal neutron energy groups with various scattering kernels, and treat the space and angle dependence in the diffusion approximation. Quite often calculations are done assuming the spacial and energy dependence to be separable. The energy variation is usually then taken from an infinite medium spectral calculation (such as that of the SOFOCATE code, which uses a hydrogen gas kernel). The spacial variation is found by a one-group calculation which uses average cross-sections from the spectral calculation, and which treats the angular distribution in the P_3 or S_4 approximation. The variability of methods is quite large, local practice and application causing much of the difference.

Thermal neutron activation experiments in cells have been directed toward measuring the disadvantage factor. Two kinds of measurements have been done. The activation of very small foils placed throughout the fuel and the water has given the detailed spacial variation. Activation of single foils of selected size and shape in the fuel and the moderator has given the disadvantage factor directly. In these measurements, the material activated (e. g., dysprosium, U^{235}) has a nearly $1/v$ cross-section, and the disadvantage factor found is essentially that of the neutron density. The neutron sensitive material is usually embedded in aluminium or a plastic.

Some measurements in the water have been made with non $1/v$ detectors such as lutetium and plutonium. These allow comparisons with the calculated values based on the non-Maxwellian character of the spectrum.

Comparisons have been made of computed and measured distributions of foil activation. Computations made by the most exact methods in use yield disadvantage factors which are uniformly higher than the experiments by

about 5% in cylindrical rod geometry, and about 3% in slab geometry. The difference between these two discrepancies is at present believed to be caused by the treatment of the boundary condition at the edge of a hexagonal cell in which a cylindrical rod is placed. This question needs more attention.

The use of metal foils in both fuel and water could lead to a systematic experimental underestimate of the disadvantage factor. The use of plastic base foils in both media could have a similar effect. The question of how accurate the best calculations are must await an experiment designed to test this possible source of experimental error.

The agreement between experiments and the less exact methods has been mixed. Some members of the panel reported considerable success by these methods. Others stated disagreement of several per cent in the disadvantage factor. The methods used are described in the papers. It would be very useful to make a systematic study of the precise relation between these and the more exact methods, to show what simplification in the latter can be made with little penalty in accuracy, and to what degree the former profit from cancellation of error.

The following recommendations were made:

- 1) Intracell thermal neutron distribution should be measured in more slab lattices, particularly those with large disadvantage factors and large spectrum shifts.
- 2) The use of non $1/v$ detectors to provide spectral indices should be extended.
- 3) Monte-Carlo calculations should be made to further investigate the errors introduced by cell boundary conditions in the cylindrical rod lattices.
- 4) The size of the experimental error introduced by using all metal foils or all plastic foils in all regions of the cells should be measured.
- 5) There should be good danger coefficient measurements of boron and non $1/v$ absorbers dissolved in the water, to test the ability to calculate the low energy neutron spectrum.

RESONANCE ABSORPTION

D. KLEIN

Klein presented the results of his isolated rod resonance integral measurements using metal and UO_2 . He also gave a correlation of these results with lattice measurements using the equivalence relations suggested by Bell and Bakshi. The correlation was satisfactory using either formulation for the effective surface to mass ratio. Hellens presented Monte-Carlo calculations for a large number of unclad lattices, and plotted the results as a function of the effective surface-to-mass ratio given by the Bell and Bakshi formulations. Two distinct straight lines were generated. He also discussed analytical calculations using the NRA-NRIA theory, and inquired as to the equivalent single rod diameter necessary to give agreement with

the Monte-Carlo results. The Bakshi equivalent rod size agreed well with the Monte-Carlo results. Its success, however, depends on the accuracy with which the Dancoff correction, c , is computed. The shadowing effect of outer rods and the effect of the cladding must be calculated accurately. Further Monte-Carlo calculations are now under way for clad fuel lattices. Measurements of ρ_{28} made at Brookhaven and Bettis were compared to the recent calculations done at Brookhaven. Except for 0.250 in rod lattices, the general agreement is good.

Wehmeyer presented results of the measurement of isolated rod resonance integrals using UO_2 , and results of the measurement of ρ_{28} . The results for the isolated rod resonance integrals for UO_2 obtained by Hellstrand, Klein and Pettus are given below ($1/v$ not included).

$$\text{Hellstrand:} \quad I = 4.15 + 26.6 \sqrt{S/M_{\text{UO}_2}}$$

$$\text{Klein:} \quad I = 5.0 + 24.8 \sqrt{S/M_{\text{UO}_2}}$$

$$\text{Pettus:} \quad I = 0.8 + 28.2 \sqrt{S/M_{\text{UO}_2}}$$

The results of Hellstrand and Klein agree very well over the range considered. The Pettus results are 2.5 to 3 b lower. More measurements of UO_2 resonance integrals are necessary to resolve this discrepancy.

As for the measurement of ρ_{28} , new experiments as well as new reduction techniques are being used. Coincidence counting techniques, β -counting techniques, chemical separation techniques and the standard γ -counting technique are variously used in the different laboratories. It is suggested that all techniques be used by a single group to measure ρ_{28} , so as to eliminate any possible systematic errors.

Since it is very difficult to measure ρ_{28} when the ratio exceeds 5, other techniques and experiments are recommended. Babcock & Wilcox as well as Westinghouse Atomic Power Group use an intermediate foil irradiation. Foils such as manganese, U^{235} , or dysprosium can be used to subtract out the thermal part of the activation. Measurements of the conversion ratio can also be used to provide information similar to that obtained from the ρ measurement. These latter experiments are providing much better agreement with theory for ρ_{28} values exceeding 5. Experiments such as these which provide less perturbing effects than the original ρ_{28} measurements are recommended for high ρ_{28} values. It is also recommended that samples of the fuel rod be examined for activation instead of using perturbing foils. As for the counting of the sample, the British use a guard ring of uranium which then results in all parts of the sample being counted with the same efficiency.

There was some discussion of measuring epicadmium spectra by use of activation. This would be very useful information. Before this can be done, the necessary infinite dilution resonance integrals should be measured. It was suggested that many more activation resonance integrals be measured with an accuracy which would permit interpretation of epicadmium spectral measurements.

Two recent sets of measurements and re-evaluations of the ThO_2 reso-

nance integral were noted. Pettus has re-evaluated his older data and now obtained:

$$I = 0.2 + 16.4 \sqrt{S/M_{\text{ThO}_2}}$$

Preliminary results of Hellstrand and Weitman give:

$$I = 1.5 + 18.1 \sqrt{S/M_{\text{ThO}_2}}$$

To aid in the analysis of resonance absorption, more resonance activation measurements of fertile materials are needed in lattices. These will help in unravelling the relations between resonance parameters, single lump resonance integrals, and lattice resonance integrals.

PLUTONIUM FUELLED SYSTEMS

I. JOHNSTONE

By comparison with the situation in uranium-water systems, experimental information on plutonium systems is very incomplete. Some critical size determinations of bare and reflected spheres of plutonium-water solutions have been made over a considerable range of atomic ratios. A small number of sub-critical lattice experiments have been carried out in the U. S. and in the U. K. using plutonium-aluminium alloy with plutonium contents in the range 1.8% to 20% by weight. In all cases for which published data are available, the Pu^{240} content was small (<3%).

In the U. S. experiments, material bucklings of rod lattices were measured and compared with the corresponding plutonium solution experiments to highlight the effect of self-shielding in the plutonium-aluminium fuel rods. The U. K. experiments used thin strips of the alloy in widely spaced slab geometry, and were compared directly with experiments in identical lattices in which the plutonium-aluminium fuel strips were replaced by uranium-aluminium strips. Additional activation measurements were made with a number of detectors to give information on the form of the neutron spectrum, in the centre of a water gap and near the fuel strips. So far, all these lattice experiments have been analysed with simple spectral models only; more sophisticated calculations are in progress.

The growth of plutonium during the life of a power reactor is discussed in several of the papers presented at this meeting. Operating experience with the seed-blanket PWR at Shippingport is reviewed in one of these papers and is compared with the results of reactor physics calculations. Many more comparisons of this kind should become available during the next few years, and these will provide an important test of the adequacy of present methods of calculation.

RECOMMENDATIONS

1. There is a 3% discrepancy between the value of η (Pu^{239}) (2200 m/s) measured directly, and the value inferred from individual measurements of $\bar{\nu}$, σ_f and σ_a . This discrepancy should be resolved and values of η (Pu^{239}) established to better than $\pm 1\%$ over the thermal and epithermal energy ranges.
2. Further nuclear physics measurements are required to improve the accuracy of the published cross-sections for Pu^{240} and Pu^{241} .
3. Homogeneous critical experiments with uranium-water and plutonium water solutions should be compared to give information on effective η values for Pu^{239} in typical reactor spectra.
4. Experiments carried out to date with plutonium-aluminium fuel in over-moderated systems should be extended into the undermoderated region, under conditions where sufficient plutonium is present to distort the shape of the neutron spectrum near the 0.3 eV resonance.
5. Measurements of material buckling and neutron spectrum should be made in lattices containing significant quantities of Pu^{240} - and later Pu^{241} . It may be possible to obtain a preliminary check on the methods of calculation for these systems by the substitution of (say) indium for the Pu^{240} , but the success of this method is dependent on the knowledge of the nuclear data for the substitute material. Due to the local nature of the Pu^{240} resonance effect, these experiments will require the Pu^{240} or substitute material to be homogeneously dispersed in typical reactor fuel elements. Experiments will also be required with Pu^{240} in the presence of significant quantities of other resonance absorbers (e. g. U^{238}), so that the effect of resonance interaction can be gauged.
6. Isotopic analysis of power reactor fuel at various stages during burn-up will provide a critical test of methods of calculation of reactivity life-time, power distribution changes, etc. Due to the large number and complex nature of the physical process involved, comparison with calculation may be difficult in many cases. Power reactors without control rods or burnable poisons would seem to give the greatest possibility of a valid comparison.

SUPPORTING PAPERS

A SUMMARY AND PRELIMINARY ANALYSIS OF THE BNL SLIGHTLY ENRICHED URANIUM, WATER MODERATED LATTICE MEASUREMENTS

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Abstract — Résumé — Аннотация — Resumen

A SUMMARY AND PRELIMINARY ANALYSIS OF THE BNL SLIGHTLY ENRICHED URANIUM, WATER MODERATED LATTICE MEASUREMENTS. The measurements carried out at BNL in some 96 light water moderated, uranium metal fuelled reactor lattices are presented in summary form. These results are used to evaluate the capability of a variety of theoretical methods to predict the neutron behaviour in the lattices as indicated by the material buckling and lattice parameters. Particular attention is given to the fast effect, resonance capture and thermal neutron distribution problems.

RÉSUMÉ ET ANALYSE PRÉLIMINAIRE DES MESURES FAITES AU LNB SUR DES RÉSEAUX URANIUM FAIBLEMENT ENRICHI ET RALENTIS À L'EAU. Le mémoire donne une récapitulation des résultats de mesures effectuées au Laboratoire national de Brookhaven sur quelque 96 réseaux de réacteurs à uranium métallique et ralenti à l'eau légère. Les résultats obtenus sont utilisés pour évaluer jusqu'à quel point différentes méthodes théoriques permettent de prévoir le comportement des neutrons dans les réseaux en tenant compte du foisonnement de la matière et des paramètres des réseaux. On traite tout particulièrement les problèmes suivants: effet des neutrons rapides, capture par résonance et distribution des neutrons thermiques.

КРАТКОЕ ИЗЛОЖЕНИЕ И ПРЕДВАРИТЕЛЬНЫЙ АНАЛИЗ ИЗМЕРЕНИЙ, ПРОВЕДЕННЫХ В БРУКХЕЙВЕНСКОЙ НАЦИОНАЛЬНОЙ ЛАБОРАТОРИИ НА РЕАКТОРНЫХ РЕШЕТКАХ СО СЛЕГКА ОБОГАЩЕННЫМ УРАНОМ И С ВОДНЫМ ЗАМЕДЛИТЕЛЕМ. Кратко излагаются результаты измерений, проведенных в Брукхейвенской национальной лаборатории на 96 реакторных решетках с топливом из металлического урана и замедлителем из легкой воды. Эти результаты используются для оценки того, насколько различные теоретические методы позволяют предсказать поведение нейтронов в решетках на основе параметров материального лапласиана и решеток. Особое внимание уделяется проблемам эффекта деления на быстрых нейтронах, резонансного захвата и распределения тепловых нейтронов.

RESUMEN Y ANÁLISIS PRELIMINAR DE LAS MEDICIONES EFECTUADAS EN EL LNB, EN RETICULADOS DE URANIO LIGERAMENTE ENRIQUECIDO-AGUA. Se presentan en forma resumida los resultados de las mediciones efectuadas en el Brookhaven National Laboratory, con 96 reticulados de elementos combustibles de uranio metálico, moderados con agua ligera. Se utilizan dichos resultados para evaluar la eficacia de una serie de métodos teóricos en lo que respecta a la predicción del comportamiento de los neutrones en los reticulados, en función del laplaciano del material y de los parámetros del reticulado. Se ha prestado particular atención a los problemas del efecto de fisión rápida, de la captura por resonancia y de la distribución de los neutrones térmicos.

INTRODUCTION

During the past nine years an extensive experimental programme has been underway at BNL to measure the behaviour of neutrons in slightly enriched uranium metal fuelled reactor lattices moderated by light water. The

intent of this work has been to provide a fund of precise data which can be used to judge the capabilities and limitations of the theoretical methods used in reactor design. With this object in mind the assemblies studied have been limited to simple cylindrical arrangements of metal fuel rods in a triangular lattice without the presence of control rods or other unnecessary perturbations. Earlier papers [1, 2, 3] have described the progress and some of the results of these measurements, giving at least in outline the techniques employed. In several papers [4, 3] the theoretical problems encountered in the description of such lattices have been discussed and in one [3] an early theoretical analysis of some of the lattices was given. The present paper presents a relatively complete summary of the experimental data accumulated for some 96 clean and poisoned lattices. The principal purpose of this paper, however, is to describe a theoretical study of these lattices which should help to identify the limitations of the theory as it now stands.

For the sake of completeness, a brief description of the fuel and lattices investigated will be given, but for detail the reader should refer to the earlier experimental reports cited above. Three batches of fuel were employed in the measurements having the enrichments of 1.027%, 1.143%, and 1.299% (by weight) in U^{235} . The fuel rods were encased in 0.028 in 2S aluminium tubing with a nominal 0.005 in air gap between the fuel and the inside of the tube. In the earliest series of measurements reported here the uranium metal was of 0.600 in diam; in subsequent series the fuel was reduced first to 0.387 in diam and finally to 0.250 in diam, except for the 1.3% fuel which was not reduced to 0.250 in. For each rod diam and enrichment, measurements were made at five water-to-uranium volume ratios, namely 1, 1.5, 2, 3, 4, except for the 0.250 in series for which the closest lattice packing was not attempted. In addition to the 38 clean lattices resulting from permutation of the rod diameter, enrichment and water-to-uranium volume ratio, measurements were made in 58 lattices with various concentrations of boric acid in the moderator water.

In general, two types of measurements have been attempted in these lattices. The material buckling of the lattice was obtained from the measurement of flux distributions in lattices of 4 ft long rods in an exponential experiment facility on the top of the BNL Research Reactor. In addition to these buckling measurements for each lattice, detailed measurements were made for a selected group of clean lattices of the spatial fine structure for fast, resonance and thermal neutrons in a miniature lattice facility. The latter contained 18 in long fuel rods in a 12 in diam tank reflected by paraffin. There is evidence that the size of this assembly is adequate to generate an asymptotic lattice neutron spectrum near the centre. These miniature lattices were irradiated in a tunnel under the BNL reactor which provided a thermal neutron flux about 100 times greater than that available in the exponential experiment facility.

In the comparison of our theoretical analysis with experimental results we have also made use of cell measurements [5, 3] carried out at the Bettis Atomic Power Laboratory in critical assemblies which employed the same fuel rods as the BNL exponential experiments. Since the cell measurements are frequently difficult to carry out with high precision and are subject to systematic errors peculiar to the particular method of measurement em-

ployed, these independent measurements at BAPL have been of considerable value.

I. THEORETICAL DESCRIPTION OF THE FINITE REACTOR LATTICE

In the 1958 Geneva Conference paper by KOUTS et al. [3], which described the U.S. work on the physics of slightly enriched water moderated lattices, a theoretical analysis was given by Hellens and Stein of 30 clean, uranium metal rod lattices. The methods employed in the 1958 analysis to compute the lattice flux spectrum have not been greatly changed in the present analysis; the principal improvements made in the intervening four years lie in the theory of the lattice fine structure effects in the fast, resonance and thermal neutron energy ranges. Of course, the continual improvement in cross-section information during this interval, particularly for U^{238} , has also had a significant influence on the precision which the theory can be expected to attain. Since by now much of the theory can be regarded as conventional, only a brief outline of the lattice calculation is given in this section to show how the present analysis of the BNL lattices was carried out; in section II, a somewhat detailed description is given of those parts of the more recently developed fine structure or lattice cell theory which are of particular importance in the present analysis.

Since measurements have been made of the material buckling for each lattice, we have computed the lattice eigenvalue for an asymptotic reactor having the measured mode shape. This of course avoids the computation of the worth of the reflector and permits a more detailed treatment of the neutron spectrum than would be convenient if the detailed boundaries of the lattice were considered. The behaviour of both the thermal and slowing-down neutrons has been computed by multi-group methods and the lattice eigenvalue is given in terms of the integral behaviour of the neutrons in these two energy regions. The resulting critical equation has a simple two-group form which facilitates the computation of the theoretical material buckling.

The neutron slowing-down probability in the asymptotic lattice has been computed for the measured buckling with the MUFT-IV [6] code. The principal purpose of this code is to provide the neutron leakage and reaction rates in the various isotopes, including the interaction between these effects, in the energy range of $10 \text{ MeV} \geq E \geq 0.625 \text{ eV}$. The equations [7] on which the code is based relate the flux, current, slowing-down density and current in each of 54 groups by the spherical harmonic P-1 approximation for a given spatial variation of the flux at all energies, $\cos Bz$. The representation of the scattering kernels is exact only for hydrogen; for energy loss by elastic collisions with elements of mass greater than unity the GREULING-GOERTZEL [8] approximation is used. Inelastic scattering is represented by an isotropic re-emission matrix. The neutron spectra resulting from fission in U^{235} and U^{238} are assumed to be the same although the average numbers of neutrons per fission are different and are taken to be functions of the energy of the neutron inducing fission. Resonance capture and fission is computed from resonance parameters in the code library by the NR-NRIA homogeneous mixture approximation. Heterogeneous effects and Doppler broadening are

incorporated as input to the code by replacing the homogeneous resonance integral of each individual resonance by the corresponding heterogeneous lattice value.

It is difficult to assess the accuracy of the P-1 approximation in predicting the slowing-down probability except for the fact that, if the slowing-down probability is expanded in powers of the buckling B^2 , the P-1 approximation gives the coefficients of the first two terms of the expansion exactly. In addition, the P-1 solution provides an estimate of the higher coefficients [9] in the series, but unfortunately little is known from either measurements or theory about the influence of these higher terms in uranium-water mixtures.

From the multi-group MUFT spectra of flux and current an effective diffusion constant (D_1) is defined as the ratio of the integrated current to the integrated flux gradient. By introducing, as well, the analogues of one-group diffusion theory defined in terms of the multi-group results we can write the neutron balance for the fast neutron spectrum in a simple form. The removal cross-section (Σ_{R1}) is taken as the rate of slowing down out of the fast group per unit fast flux; the absorption (Σ_{a1}) and fission (Σ_{f1}) cross-sections are simple averages over the fast group.

In the thermal neutron group, (which is discussed briefly in section II-C and more extensively in the accompanying paper by Honeck), the energy dependence is again treated by multi-group methods but the principal spatial problem is the neutron distribution in a lattice cell rather than the leakage out of the lattice. Because of the highly anisotropic flux in the cell and the nearly isotropic scattering in water, a direct numerical solution of the transport equation with isotropic scattering (the THERMOS code [10, 11]) is used to give the thermal neutron disadvantage factor and spectrum averaged cross-sections. If the latter are used in the method of AMOUYAL, BENOIST and HOROWITZ [12], rather good agreement with the results of lengthy two dimensional THERMOS cell calculations can be obtained. For the lattice calculations given in this paper, we have used this combination of methods to give the conventionally defined thermal group averaged η_2 and f_2 . The thermal neutron diffusion length, L , used to compute the lattice leakage, has been obtained by averaging the inverse of RADKOWSKI's [13] transport cross-section model for water over the lattice spectrum with the SOFOCATE [14] code.

The eigenvalue of the lattice at the measured material buckling is then composed of a fast (subscript 1) and thermal (subscript 2) term,

$$\lambda = \frac{(\eta f)_1 (1 - p_1)}{(1 + \tau B^2)} + \frac{(\eta f)_2 p_1}{(1 + \tau B^2) (1 + L^2 B^2)}, \quad (I-1)$$

with

$$p_1 = \frac{\Sigma_{R1}}{\Sigma_{R1} + \Sigma_{a1}}, \quad (\eta f)_1 = \frac{\nu \Sigma_{f1}}{\Sigma_{a1}}, \quad \tau = \frac{D_1}{\Sigma_{R1} + \Sigma_{a1}},$$

where each term in (I-1) represents the number of neutrons born by fast or thermal fission per neutron entering the group. The fast group non-absorp-

tion probability includes capture and fission in both U^{238} and U^{235} as well as the small capture in hydrogen, oxygen and boron.

In Fig. 1 the spectra generated by the multi-group calculations for two lattices are given as a function of lethargy. The high-energy dips in the

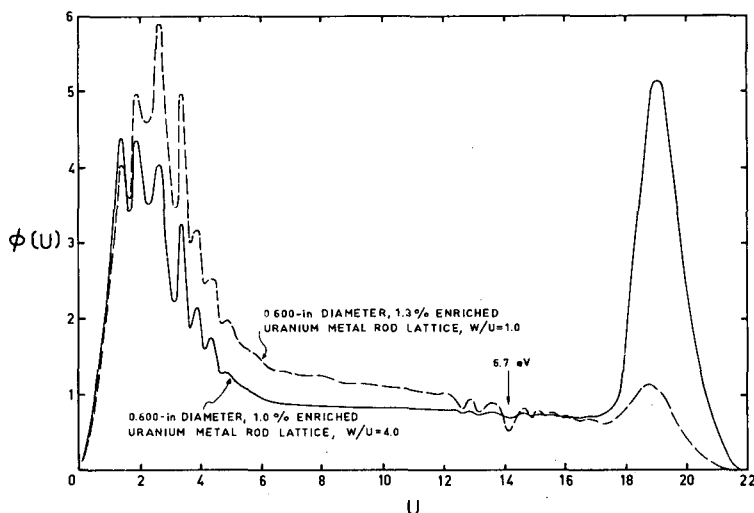


Fig. 1

Typical multi-group neutron spectra for the BNL lattices

flux are caused by the oxygen resonances; however, the three small dips on the low side of the fission peak are spurious and arise from the use of difference equations to represent the heavy element slowing down. Similar spurious detail is seen below the 6.7 eV resonance of U^{238} . These effects are not observed in calculations with hydrogen moderation alone.

The cross-sections used in the MUFT calculations have been tabulated by HENRY [15]; the particular element numbers used in his list are U^{238} (88), U^{235} (85), H (1), O (72) and Al (9). The thermal group cross-sections were taken from BNL-325, except for the U^{235} cross-section which was taken from the tabulation of SAFFORD and HAVENS [16].

II. THEORY AND MEASUREMENTS OF LATTICE FINE STRUCTURE

Although the theoretical description of the reactor lattices considered here is based on multi-group spectrum calculations, we shall discuss the lattice fine structure effects in the traditional four factor fashion, considering in turn the resonance capture, fast fission effect and thermal utilization. These results are used, as described in section I, in the multi-group analysis.

A. Resonance capture in U^{238}

The problem of resonance capture in a close packed reactor lattice seems at first sight so complex that only Monte-Carlo computations, such as those developed by RICHTMYER *et al.* [17], could be expected to provide an adequate description for various rod sizes and water-to-uranium volume ratios. However, in 1955, CHERNICK [4] proposed a semi-analytical approach which, with its subsequent development [18, 19], now provides an accurate method of computing the resonance integral for a lattice. In addition, because of its analytical features, the theory has led to a number of useful, though approximate, equivalence principles relating the resonance integrals of homogeneous mixtures, single absorber lumps, and lattices of absorber lumps in moderator. In this section, we shall not present the theory in any detail, since a number of excellent review papers [19, 20] exist, but will limit our discussion to those points which are of immediate interest to the BNL lattice calculations. We shall also present some preliminary Monte-Carlo results for the BNL lattices which give an indication of the errors to be expected from the use of Chernick's theory.

The central features of the semi-analytical theory as used here can be stated quite briefly. The average flux in the absorber and moderator at a given energy are related by a pair of integral equations involving escape probabilities for the two regions and the scattering down integrals. The latter can be simplified by the assumption that every scattering leads to either a large energy loss (NR approximation) or very small energy loss (NR1A approximation) compared to the width of a given resonance. This important step reduces the integral equations to simple algebraic equations from which the resonance capture probability can be computed. NORDHEIM [21] has obtained direct numerical solutions of the integral equations which show that the error introduced by the NR-NR1A approximation is probably no more than 1% of the resonance integral for the BNL lattices. Only for very small rods does the error become appreciable.

For a single fuel rod in an infinite sea of moderator the escape probability for a flat source distribution has been given by CASE, de HOFFMANN and PLACZEK [22] for a variety of geometrical shapes. By introducing the Wigner rational approximation for the escape probability, $P = 1/[1 + \Sigma(4V/S)]$, where V and S are the volume and surface of the rod, CHERNICK and VERNON [18] found an approximate equivalence relation between single rods and homogeneous mixtures. This approximate result for a Breit-Wigner resonance without Doppler broadening can be integrated directly and shows that a linear dependence of the resonance integral for single rods on $\sqrt{S}/M$ is to be expected for a large range of rod sizes. More accurate computations with the exact escape probability and Doppler broadening also show this type of dependence on rod size.

In reactor lattices, such as those used for the BNL measurements, the calculation of the escape probabilities for fuel and moderator is much more complex because of the possibility of a neutron traversing several alternate layers of material before making a collision. ROTHENSTEIN [23] has shown that the reciprocity principle

$$\Sigma_0 V_0 P_0 = \Sigma_1 V_1 P_1 \quad (\text{II-1})$$

links the probability P_0 , that a neutron which is scattered at any point in the fuel volume V_0 makes its next collision somewhere in the moderator volume V_1 , with the corresponding escape probability for the moderator. BELL [24] obtained an accurate, though not exact, expression for the escape probability of a neutron born uniformly in the fuel in terms of the escape probability for a single fuel element $P_0 (\Sigma_0 \ell_0)$ and the transmission probability $\Gamma_1 (\Sigma_1 \ell_1)$ of the moderator layer between fuel elements

$$P_0^* = P_0 (\Sigma_0 \ell_0) \frac{(1 - \Gamma_1)}{1 - \Gamma_1 [1 - \Sigma_0 \ell_0 P_0 (\Sigma_0 \ell_0)]} \quad (\text{II-2})$$

Here $\ell_0 = 4V_0/S_0$ is the average chord length of a single fuel element, Σ_0 is the total cross-section of the fuel element, and Σ_1, ℓ_1 are the corresponding quantities for the moderator. The term in the square brackets in the denominator is the transmission probability of the fuel rod. For a lattice of circular cylinders $\Gamma_1 (\Sigma_1 \ell_1)$ will be recognized as the Dancoff correction, C . It is worth noting that if the fuel rods are black the lattice escape probability has the same functional form as that of a single rod but is always smaller by the factor $(1-C)$; if the fuel rod is grey there can be no simple equivalence between the lattice and single rod, since the functional forms of the escape probabilities are different.

Further simplification of the lattice escape probability (II-2) can be made by representing the single rod escape probability by the rational approximation to give

$$P_0^* \approx 1/[1 + \Sigma_0 \ell_0 / (1-C)] \quad (\text{II-3})$$

This is similar to BAKSHI's [25] suggestion of using the exact single rod function with an average fuel element chord length augmented by the factor $1/(1-C)$. As the comparison with the Monte-Carlo calculations indicates, this approximation is very good but still requires the detailed calculation of the Dancoff correction. BELL [24] used the rational approximation for both fuel and moderator to get

$$P_0^* \approx 1/[1 + \Sigma_0 \ell_0^*], \quad (\text{II-4})$$

where $\ell_0^* = \ell_0 + \lambda_1 (V_0/V_1)$, and λ_1 is the mean free path in the moderator. This equivalence principle relating the augmented chord length for the fuel ℓ_0^* to the geometrical value for a single fuel element ℓ_0 , while convenient in that it avoids the detailed Dancoff correction calculation, always under-

estimates the resonance integral. The reason for this consistent error is that the rational approximation always over-estimates the transmission probability of the moderator so that the Dancoff correction is too large.

NORDHEIM in his review paper [19] compares these various escape probability approximations for lattices as a function of fuel cross-section; in Table I we give a comparison of the resonance integrals obtained using

TABLE I

RESONANCE INTEGRALS FOR 18 RESOLVED RESONANCES

$$\lambda_W = 0.6671 \text{ cm} \quad \rho_U = 18.7 \text{ gm/cm}^3$$

0.600-in metal rods					
W/U	Bell $\sqrt{S/M}$	Bell- σ_a	Bakshi $\sqrt{S/M}$	Bakshi- σ_a	Monte Carlo σ_a
1	0.312	7.40	0.334	7.87	7.85 $\pm$ 0.03
1.5	0.330	7.77	0.351	8.27	8.34 $\pm$ 0.04
2	0.339	8.00	0.359	8.47	8.48 $\pm$ 0.05
3	0.350	8.25	0.369	8.68	8.78 $\pm$ 0.06
4	0.356	8.38	0.372	8.76	8.85 $\pm$ 0.07
∞	0.375	8.83	0.375	8.83	-
0.387-in metal rods					
W/U	Bell $\sqrt{S/M}$	Bell- σ_a	Bakshi $\sqrt{S/M}$	Bakshi- σ_a	Monte Carlo σ_a
1	0.360	8.49	0.379	8.94	8.88 $\pm$ 0.04
1.5	0.387	9.13	0.411	9.68	9.68 $\pm$ 0.05
2	0.403	9.50	0.426	10.04	10.07 $\pm$ 0.05
3	0.421	9.92	0.446	10.52	10.47 $\pm$ 0.07
4	0.431	10.17	0.455	10.73	10.74 $\pm$ 0.08
∞	0.466	10.99	0.466	10.99	-
0.250-in metal rods					
W/U	Bell $\sqrt{S/M}$	Bell- σ_a	Bakshi $\sqrt{S/M}$	Bakshi- σ_a	Monte Carlo σ_a
1	0.405	9.56	0.425	10.02	9.74 $\pm$ 0.04
1.5	0.445	10.51	0.471	11.11	10.96 $\pm$ 0.05
2	0.470	11.09	0.493	11.64	11.56 $\pm$ 0.06
3	0.500	11.79	0.527	12.45	12.48 $\pm$ 0.07
4	0.517	12.20	0.545	12.87	12.94 $\pm$ 0.09
∞	0.580	13.75	0.580	13.75	-

Bakshi's lattice average chord length, $\ell_0/(1-C)$, as well as Bell's from (II-4) to find the single rod surface-to-mass ratio equivalent to the lattice. Also shown in Table I are resonance integrals obtained by M. Levine at BNL from 70 000 history runs on the IBM-7090 Monte-Carlo code REPETITIOUS provided by G. Minton of Westinghouse. This code is the outgrowth of an earlier IBM-704 code called REP by RICHTMYER and van NORTON [26].

The Monte-Carlo calculations include the effects of finite uranium and oxygen mass, Doppler broadening and the overlap of nearest neighbour resonances. In all of these calculations the 18 resolved resonances given in BNL-325, Supplement 1, have been used for illustrative purposes; the calculations for Chernick's theory have been taken from the curves of the resonance integral versus $\sqrt{S/M}$ given by DRAWBAUGH [20] for the same set of resonances.

In Table I, it is seen that Bakshi's equivalent rod size used in the NR-NRIA theory gives quite good agreement with the Monte-Carlo computations. The estimate of the Dancoff correction for open lattices may be slightly too large while for close packed lattices it appears to be too small for the 0.250 in rods. Bell's augmented chord length used in the correct single rod escape probability function appears to yield an error which arises from a large over-estimate of the Dancoff correction of the open lattices; the error apparently decreases for the close packed lattices giving a trend of resonance integral with decreasing water-to-uranium volume ratio much like Bakshi's formula. Nordheim's more exact treatment of the scattering integrals would tend to make the disagreement between the Bakshi and Monte-Carlo resonance integrals worse since it would decrease by $\sim 1\%$ the open 0.250 in lattice values. Fortunately the difference between the NR-NRIA values and those of Nordheim appear to be so small as to be negligible. In Fig. 2 the relation between the NR-NRIA theory and the Monte Carlo results is shown on the basis of Bakshi's and Bell's identification of the effective $\sqrt{S/M}$ of the lattice.

Although Bakshi's formula for the single rod, which is equivalent in resonance integral to the rod lattice, gives fairly good agreement with the Monte Carlo results, its success depends on the accuracy with which the

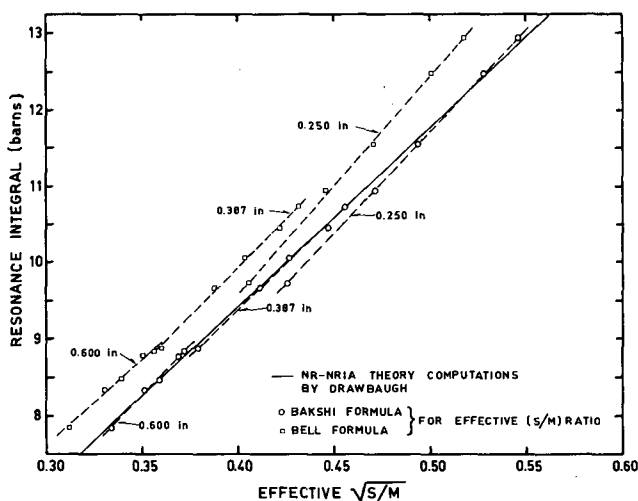


Fig. 2

Correlation of Monte-Carlo resonance integrals by several effective chord length models

Dancoff correction, C , is computed, particularly for the close packed lattices of small rods. Accurate computations of C are seldom done because of the geometrical complexity of excluding shadowed neighbouring rods. A review of the various methods in common use has recently been made by FUKAI [27] along with comparisons of these methods with Monte-Carlo one-velocity calculations for a few lattices. His computations of the Dancoff correction are in general slightly lower than the presumably correct value of C obtained from Monte Carlo. He also finds, as expected, that Bell's simple formula $C = [1 + (4V_1/S_1) \Sigma_1]^{-1}$ provides a considerable over-estimate. Of course, it must be emphasized that the error in the Dancoff correction is not related in any simple way to the error in the resonance integral because of the manner we have adopted of using the Bakshi and Bell equivalent chord lengths in the exact single rod escape probability. The direct comparison of resonance integrals given in Table I is probably the best measure of the uncertainties involved in the NR-NRLA lattice approximations.

Our lattice reactivity calculations have been based entirely on Bakshi's effective surface-to-mass ratio computed using the Dancoff corrections obtained by WANG [28] at BNL some years ago. These values of C have been summarized by ROTHENSTEIN [23] for the six nearest and six next nearest neighbours. We have followed Rothenstein in arbitrarily taking one half of the second ring contribution to the Dancoff correction for lattices with water-to-uranium ratio less than two to account roughly for the shadowing of the outer ring of rods by the nearest neighbours. This procedure yields values of $(1-C)$ which disagree with Fukai's Monte-Carlo calculations by less than 2% in the four lattices for which comparisons can be made.

Having related the effective chord length or surface-to-mass ratio for a lattice to that of an equivalent single rod, we can make use of a number of tabulations for single rods of the contributions to the resonance integral from the resolved resonances in the energy range up to 1000 eV, the unresolved $\ell = 0$ resonances from 1 keV to 30 keV and the small additional contribution from higher ℓ -wave resonances. Because of the extensive range of calculations given for both metallic uranium and uranium dioxide by DRAWBAUGH [20], we have based our lattice resonance integrals shown in Table II on his results. The cross-sections in the MUFT code are not arranged in such a way that the unresolved $\ell = 0$ resonance contribution for U^{238} can be conveniently varied to account for self-shielding; in order to do this we have simply removed and entered as a separate isotope that part of the capture cross-section lying between 454 eV and 40 keV so that in a $1/E$ spectrum, the resonance integral would coincide with the values given in Table II. Of course, in the neutron spectrum generated by MUFT for the BNL lattices, the resonance integral will be increased from that obtained from a $1/E$ flux spectrum by the decrease of the hydrogen cross-section above 10 keV and by the uncollided fission neutrons. This increase in the resonance integral which has also been computed explicitly by KLEIN [29] and LEVINE [30], is at most 1.2 b for the cross-sections used in the present lattice calculations.

In the last two columns of Table II the results of calculations by NORDHEIM [21] and ROTHENSTEIN [23] are listed for comparison. As mentioned before, Nordheim has used Bell's grey rod escape probability

TABLE II

RESONANCE INTEGRALS FOR BNL LATTICES FROM DRAWBAUGH [20]

$$\rho_U = 18.9 \text{ gm/cm}^3 \quad \sigma_a = 1.63 \text{ for } l \geq 1 \text{ in all lattices}$$

W/U	$\sqrt{S/M}$	Resolved $l=0^*$	Unresolved $l=0$	Total $l=0$	Nordheim [21]	Rothenstein [23]
0.600 in						
1	0.332	8.48	1.19	9.67	9.80	9.57
1.5	0.349	8.88	1.22	10.10	-	-
2	0.357	9.09	1.24	10.33	10.51	10.29
3	0.367	9.32	1.25	10.57	10.77	10.50
4	0.370	9.39	1.26	10.65	10.85	10.58
0.387 in						
1	0.363	9.21	1.25	10.46	-	-
1.5	0.409	10.34	1.32	11.66	-	-
2	0.424	10.73	1.35	12.08	-	-
3	0.443	11.21	1.38	12.59	-	-
4	0.452	11.43	1.40	12.83	-	-
0.250 in						
1	0.423	10.69	1.35	12.04	12.00	11.41
1.5	0.468	11.83	1.42	13.25	-	-
2	0.490	12.38	1.45	13.83	13.83	13.66
3	0.525	13.25	1.50	14.75	14.80	14.67
4	0.542	13.67	1.53	15.20	15.27	15.11

* The resonance integrals used in the multi-group calculations have this component reduced by 4%.

(II-2), which is probably the most accurate of the several formulae in use, and has avoided the NR-NRIA reduction of the scattering integrals. The latter improvement is apparently of little consequence in the BNL lattice range. Rothenstein's values of the resonance integral were obtained by using a modified form of the rational approximation for the lattice escape probability without Doppler broadening in the NR-NRIA theory. He then computed a host of corrections for Doppler broadening, the interference between resonance and potential scattering, iteration of the NR and NRIA approximations, and corrections for the use of the Wigner rational approximation of the escape probability. The rather close agreement shown in Table II between a variety of calculations of the lattice resonance integral, which are all based on Chernick's integral equations, is certainly gratifying when coupled to the indication from the Monte-Carlo calculations of the accuracy of Bakshi's method of relating lattices to single rods.

Measurements have been made both at BNL and BAPL of ρ^{28} , the ratio of epicadmium-to-subcadmium captures in U^{238} foils placed in a typical cross-section of the lattice fuel rod. The values of ρ^{28} are enrichment dependent so that the BNL measurements, done in 1.027% fuel, cannot be compared with the BAPL data which used both of the higher fuel enrichments.

It is found from our lattice calculations, however, that the ratio of thermal absorptions in the lattice cell-to-epicadmium captures in U^{238} is not only rather insensitive to enrichment but is essentially linear with respect to water-to-uranium volume ratio. This permits the comparison for a given

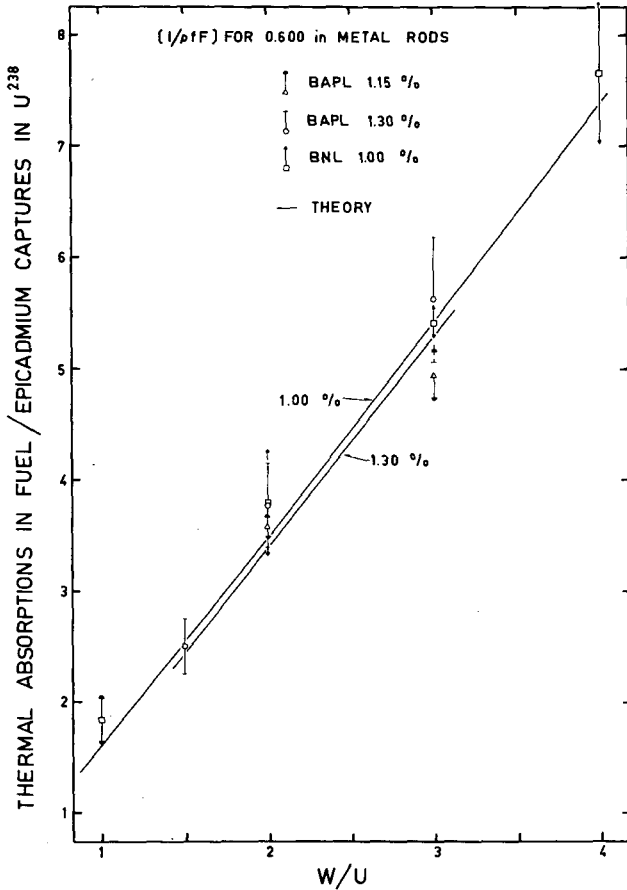


Fig. 3

1/ρff for the 0.600-in metal rod lattices

rod size of the measurements for all enrichments with the theoretical results, as shown in Fig. 3 and Fig. 4. Except for the 0.250 in lattices, where the disagreement between theory and measurement is very large, the general agreement is fairly good as is shown in Table III. The precision attained in these lattice measurements unfortunately falls far short of that indicated by the various measurements made with single rods [31]. The comparison of the measured and computed values of ρ^{28} in various lattices is given in Tables XII-XIV.

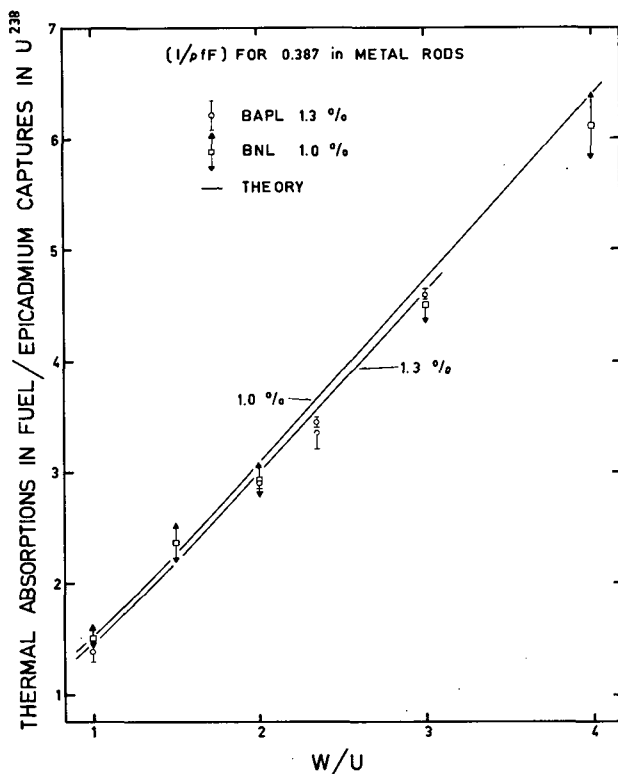


Fig. 4

$1/\rho fF$ for the 0.387-in metal rod lattices

B. Fast fission in U^{238}

In the theoretical analysis described in section I, the fast fission in U^{238} is obtained from multi-group calculations of the fast neutron spectrum of the finite homogenized lattice. Because of the long mean free path (~ 10 cm) of the fast neutrons contributing to δ^{28} , the ratio of fissions in U^{238} to those in U^{235} , the influence of lumping the fuel into rods is small but it cannot be entirely neglected if high precision is desired in the predicted values of the material buckling. In this section we will develop a simple, though hardly novel, method for the calculation of the rod size contribution to δ^{28} and to the lattice reactivity which can then be applied to the homogeneous multi-group results.

The simplest way of obtaining a description of the heterogeneous fast fission effect is to write the equations for neutrons capable of producing fast fission in terms of one group. The principal justification for this procedure lies in the observation that infinite medium multi-group calculations yield values of $(\delta^{28})^{-1}$ which are proportional to the water-to-uranium volume ratio. In addition, from homogeneous multi-group P-1 calculations with

TABLE III
MEASURED AND COMPUTED VALUES OF $1/\rho^{28}$ ff FOR METAL
ROD LATTICE

	1.03%		1.15%		1.3%	
W/U	Theory	Experiment (a)	Theory	Experiment (b)	Theory	Experiment (b)
0.600-in						
1	1.615	1.85 ± 0.19	-	-	-	-
1.5	2.536	-	-	-	2.463	2.51 ± 0.25
2	3.476	3.80 ± 0.46	3.429	3.58 ± 0.09	3.386	3.78 ± 0.38
3	5.426	5.42 ± 0.13	5.356	4.95 ± 0.21	5.305	5.63 ± 0.56
4	7.396	7.66 ± 0.61	-	-	-	-
0.387-in						
1	1.518	1.52 ± 0.08			1.465	1.39 ± 0.09
1.5	2.261	2.37 ± 0.15				
2	3.076	2.93 ± 0.12			2.995	2.91 ± 0.06
3	4.728	4.50 ± 0.13			4.623	4.59 ± 0.05
4	6.378	6.10 ± 0.27				

(a) BNL measurements

(b) BAPL measurements - private communication from D. Klein

$$\frac{1}{\rho^{28} \text{ ff}} = \frac{\text{total subcadmium absorptions}}{\text{epicadmium captures in } U^{238}}$$

leakage, an effective one-group transport cross-section can be fitted over an adequate range of water-to-uranium ratio so that the homogeneous system with leakage can be represented by a diffusion equation for that part of the neutron spectrum lying above the fast fission threshold of U^{238} :

$$\left[D B^2 + \Sigma_a + \Sigma_R \right] \phi = \chi \nu^{25} F^{25} + \chi \nu^{28} \Sigma_f^{28} \phi, \quad (\text{II-5})$$

where

- χ = fraction of fission neutrons born above the fast fission threshold,
 F^{25} = U^{235} fission rate,
 $\Sigma_a, \Sigma_R, \Sigma_f$ = absorption, removal and fission cross-sections,
 D = ratio of integrated current to integrated flux gradient.

Thus the ratio of fissions in U^{238} to fissions in U^{235} is given by

$$\delta^{28}(B^2) = \delta^{28}(0) / \left[1 + L_f^2 B^2 \right] \quad (\text{II-6})$$

$$\delta^{28}(0) = \frac{\chi \nu^{25} \Sigma_f^{28}}{\Sigma_a + \Sigma_R - \chi \nu^{28} \Sigma_f^{28}}$$

$$L_f^2 = D / \left[\Sigma_a + \Sigma_R - \chi \nu^{28} \Sigma_f^{28} \right].$$

The effective one-group cross-sections derived from the multi-group calculations are given in Table IV for the isotopes needed in the lattices considered in this paper.

TABLE IV

EFFECTIVE ONE-GROUP CROSS-SECTIONS FOR FAST FISSION

	σ_{tr}	$\sigma_a + \sigma_R - \chi \nu \sigma_f$	σ_f
U^{238}	4.5	1.61	0.38
Hydrogen	1.56	1.45	-
Oxygen	1.72	0.36	-
Aluminium	2.03	0.34	-
$\chi = 0.752$	$\nu^{28} = 2.69$	$\nu^{25} = 2.43$	$\alpha^{28} = 0.13$

The leakage from the lattice corresponding to a certain spatial mode is appreciably enhanced by the fast fission itself so that, as shown in Table V, the leakage is not much affected by the volume ratio in a simple water-uranium mixture. Comparisons with multi-group calculations show that the representation of the homogeneous δ^{28} by one group is usually in error by less than 5%. Similar comparisons for L_f^2 show that the error in this leakage coefficient is usually less than 10%.

TABLE V

HOMOGENEOUS FAST EFFECT IN URANIUM METAL-WATER LATTICE

	One-group	MUFT	One-group	MUFT
$W/U = 1$	$\delta^{28}(0) = 0.177$	0.170	$L_f^2 = 22.0$	20.8
2	0.112	0.111	20.8	21.0
4	0.064	0.066	20.0	21.6
Aluminium content of mixture corresponds to 0.600 in metal rods.				

Since more precise values of both δ^{28} and L_f^2 can be obtained from the multi-group calculations, the principal value of the one-group cross-sections is in the calculation of rod size effects.

The influence of lattice fine structure on the value of $\delta^{28}(0)$ can be computed by the simplification of a method proposed by CHERNICK [4]. Consider a lattice for which P_0 is the probability that a neutron born at any point inside the fuel by fission or elastic collisions makes its next collision in the moderator. An analogous probability exists for the moderator but need not be explicitly included since we shall represent the escape probability from the moderator by the Wigner rational approximation and in this approximation the moderator scattering has no effect on the flux in the fuel. Thus for an infinite lattice the flux in the fuel is determined by

$$\Sigma_0 V_0 \phi_0 = (1-P_0) \left[\Sigma_{s0} + \chi \nu^{28} \Sigma_{f0}^{28} \right] V_0 \phi_0 + (1-P_0) V_0 \chi \nu^{25} F^{25}, \quad (\text{II-7})$$

in which F^{25} is the fission rate in U^{235} . The fission ratio is then given in terms of the cross-sections and escape probability of the fuel; in a lattice the latter is of course dependent on the transmission properties of the moderator.

$$\delta^{28}(0) = \frac{(1-P_0) \chi \nu^{25} \Sigma_{f0}^{28}}{\Sigma_0 - (1-P_0) (\Sigma_{s0} + \chi \nu^{28} \Sigma_{f0}^{28})} \quad (\text{II-8})$$

The calculation of P_0 for a lattice is in general quite complex; a brief review of various approximations which have been studied has been given in the previous section of this paper on resonance capture. In the case of transparent lattices a convenient approximation is that suggested by BELL [24] based on the Wigner rational approximation:

$$P_0(\Sigma_0 \bar{\ell}) = \left[1 + \Sigma_0 \bar{\ell} \right]^{-1} \quad (\text{II-9})$$

$$\bar{\ell} = \beta \ell_R + \lambda_M V_0 / V_1,$$

where λ_M is the mean free path for removal in the moderator, ℓ_R is the average chord length of the fuel element and β is a constant factor used to improve the accuracy of the rational approximation for rather transparent rods. With the aid of this set of approximations the fission ratio can be written in a form which clearly gives the correct homogeneous mixture limit,

$$\delta^{28}(0) = \delta_{\infty}^{28} \bar{\ell} / (\gamma + \bar{\ell}). \quad (\text{II-10})$$

Here δ_{∞}^{28} is the fission ratio for an infinite block of fuel with a flat flux and γ is the mean free path for neutron removal in the fuel,

$$\gamma = \left[\Sigma_0 - \Sigma_{s0} - \chi \nu \Sigma_{f0}^{28} \right]^{-1}.$$

The effective one-group cross-sections from Table IV give the mean free path for water as $\lambda_M = 9.16$ cm and $\gamma = 13.1$ cm for uranium metal. It will be seen, however, from single rod fast effect measurements, that the mean free path in uranium metal should be of the order of 20 cm. This difference arises largely from the tendency of the rational approximation (II-9) to under-estimate the escape of neutrons from a single fuel rod. Comparison with the exact escape probability for cylinders given by PLACZEK [22] shows that a reduction of the effective single rod radius to 65% of the geometrical value results in a marked improvement in the accuracy of the escape probability for rods as large as 4 cm radius.

By this combination of approximations, a simple method is obtained for relating single rod and lattice measurements of the fast effect. It should be pointed out that the inclusion of moderator scattering in (II-7) will not describe the back-scattering into single fuel rods, which becomes important for large rods, because the use of escape probabilities computed for flat neutron sources makes the probability of return to any single rod vanish. As mentioned before, inclusion of moderator scattering in lattices does not alter the result since the rational approximation does not distinguish between back-scatter from and transmission through the moderator.

In the analysis of the lattice experiments described in section III, the homogeneous contribution to the fast effect has been taken from the MUFT multi-group calculations with heterogeneous corrections to δ^{28} and to the lattice reactivity taken from (II-10). For small rod sizes the expression (II-10) for the lattice fission ratio (δ_L^{28}) can be expanded into explicit homogeneous (δ_H^{28}) and heterogeneous contributions:

$$\delta_L^{28} = \delta_H^{28} + \frac{2\delta_\infty^{28}}{(1+a\xi)} \frac{\beta}{\gamma} R \quad (\text{II-11})$$

where

$$\delta_H^{28} = \delta_\infty^{28} \left[a\xi / (1+a\xi) \right]$$

ξ = the volume ratio of fuel to moderator;

a = the ratio of fuel to moderator removal cross-section;

β = 0.655;

R = actual rod radius.

The corresponding increase in the lattice eigenvalue is given roughly by the multiplicative factor:

$$K = 1 + \left(\frac{\nu^{28} - 1 - a^{28}}{\nu^{25}} \right) \frac{2\delta_\infty^{28}\beta}{\gamma(1+a\xi)^2} R. \quad (\text{II-12})$$

Although for the close packed lattices the reactivity contribution from the rod size effect is small, it does amount to as much as 1.4% in the 0.600 in metal rod lattice at $W/U=4$.

Some confidence in the simple expression (II-10) for the lattice fission ratio can be gained by relating measurements of δ^{28} in single rods and lattices through the parameter $[\bar{i}/(\gamma + \bar{i})]$. Two independent sets of measurements of δ^{28} have been completed in all the unpoisoned metal rod lattices, one by ERDIK [32] and one accumulated by a number of BNL experimentalists. Some parts of the latter data have been published before but the results given here are slightly different due to changes in the fission product β -decay calibration curve. In addition to the lattice measurements, PRICE *et al.*, [33] have published an extensive series of measurements of δ^{28} in single metal rods, some of which were so large that they overlap many of the lattice values. Two types of corrections must be first applied to these measurements:

(1) The lattice measurements made in the appropriate asymptotic flux spectra must be adjusted to infinite lattice values which should in principle remove the differences between measurements at the three enrichments;

(2) The single rod values made at diameters greater than 0.5 in must be corrected for moderator back-scattering and thermal flux depression. The former corrections can be obtained from either multi-group or one-group estimates of L_f^2 and the measured buckling of the appropriate lattice. The latter corrections for single rods have been estimated by Honeck using the MOCCA Monte-Carlo fast fission code developed by RIEF [34].

In Table VI the leakage corrected lattice values are given with the values of $[\bar{i}/(\gamma + \bar{i})]$ obtained from the cross-sections of Table IV. For comparison, the MUFT multi-group results, corrected for rod size effect are given along with the one-group values from Eq. (II-10). The latter appear to be in fair agreement with the measurements, the greatest disagreement being a tendency to over-estimate δ^{28} for the larger W/U volume ratio lattices.

A better basis for judging the ability of the one-group theory to account for rod size effects is provided by the measurements by PRICE *et al.*, [33] in single uranium rods. For large rods the comparison is made uncertain by the large effect of back-scattering from the surrounding moderator. However, for rods of less than one inch diameter the correction is small and the agreement with Eq. (II-10) appears to be satisfactory. Except for the smallest rod in which δ^{28} itself is very small, the difference between measurement and calculation is less than 10% as is seen in Table VII. A comparison of the simple theory, the rod size corrected multi-group calculations and the measured values, is shown in Fig. 5.

C. Thermal neutron distributions in lattices

The conventional parameters describing the thermal neutron multiplication properties of a lattice are f , the thermal utilization defined as the number of absorptions in the fuel per neutron slowing down into the thermal region, and η , the number of fission neutrons produced per absorption in the fuel. Since most absorption cross-sections are proportional to $1/v$, the reaction rate $\int v \Sigma_a(v) N(v) dv$ is proportional to the integrated neutron

TABLE VI
COMPARISON OF INFINITE LATTICE VALUES OF $\delta_L^{28}(0)$

		Erdik			BNL							
W/U	$\overline{E}/(\gamma+\overline{E})$	1.0	1.15	1.3	1.0	1.15	1.3	Erdik Av.	BNL Av.	MUFT Heter.	Eq. (II-10)	Ref M. C.
0.600 in												
1	0.427	0.183	0.187	0.188	0.196	0.187	0.194	0.186	0.192	0.181	0.184	0.165
1.5	0.346	0.153	0.153	0.139	0.155	0.150	0.152	0.146	0.152	0.149	0.149	0.136
2	0.295	0.125	0.124	0.119	0.129	0.123	0.128	0.123	0.127	0.129	0.127	0.117
3	0.235	0.086	0.094	0.094	0.100	0.090	0.101	0.091	0.097	0.104	0.101	0.091
4	0.200	0.075	0.075	0.078	0.076	0.080	0.083	0.076	0.080	0.089	0.086	0.075
0.387 in												
1	0.415	0.188	0.182	0.183	0.185	0.186	0.183	0.184	0.185	0.174	0.179	0.161
1.5	0.330	0.141	0.147	0.137	0.141	0.138	0.147	0.142	0.142	0.142	0.142	0.137
2	0.278	0.115	0.115	0.120	0.123	0.120	0.122	0.117	0.122	0.121	0.120	0.116
3	0.217	0.086	0.088	0.086	0.092	0.087	0.098	0.087	0.092	0.096	0.093	0.092
4	0.181	0.072	0.069	0.073	0.080	0.074	0.082	0.071	0.079	0.081	0.078	0.073
0.250 in												
1.5	0.320	0.132	0.131		0.133	0.147		0.132	0.140	0.136	0.138	0.129
2	0.268	0.106	0.109		0.110	0.118		0.108	0.114	0.115	0.115	0.109
3	0.205	0.080	0.083		0.087	0.090		0.082	0.089	0.089	0.088	0.083
4	0.169	0.069	0.065		0.066	0.071		0.067	0.069	0.074	0.073	0.067

TABLE VII

COMPARISON OF δ^{28} IN SINGLE URANIUM METAL RODS

Rod dia. (in)	$\bar{\ell}/(\gamma+\bar{\ell})$	δ^{28} meas. (a)	Correction (b)	δ^{28} meas. -corr.	Eq. (II-10)
0.250	0.0310	0.012	0	0.012	0.0134
0.387	0.0470	0.021	-0.001	0.020	0.0203
0.600	0.0715	0.035	-0.002	0.033	0.0308
0.750	0.0877	0.044	-0.003	0.041	0.0378
1.10	0.129	0.062	-0.006	0.056	0.0556
1.345	0.147	0.077	-0.008	0.069	0.0634
1.938	0.199	0.095	-0.013	0.082	0.0858
2.898	0.270	0.128	-0.014	0.114	0.116
3.636	0.318	0.145	-0.009	0.136	0.137

(a) Average of values given in BNL-616.

(b) Monte-Carlo correction for backscattering and thermal flux dip.

density $\int N(v) dv = N$. The parameters f and η are then determined by the average neutron density, $\bar{N}$, in each region and the non $1/v$ correction factors which depend on the neutron spectrum. A comparison paper [35] describes in some detail the techniques used at BNL for computing the thermal neutron density distribution in a lattice. We give only a brief summary here.

The neutron density $N(r, v)$ is computed from the integral form of the transport equation

$$vN(\underline{r}, v) = \int d\underline{r}' T(\underline{r}, \underline{r}'; v) \left[S(\underline{r}', v) + \int_0^{v*} dv' P(\underline{r}; v, v') N(\underline{r}', v') \right], \quad (\text{II-13})$$

where T is a transport kernel relating the flux at $\underline{r}$ to a unit isotropic source at $\underline{r}'$; P is the probability per cm that a neutron scatters from velocity v' to velocity v ; S is the slowing down source of neutrons, and v^* is the upper limit of the thermal group (taken at 0.785 eV). We have assumed isotropic scattering in the above equation. Calculations of $N(r, v)$ in slab lattices which include linear (P_1) anisotropic scattering have shown that, for the lattices considered here, the effect of anisotropic scattering on f is small and can be accurately approximated by using a transport correction in the scattering kernel P .

Eq. (II-13) is solved numerically by a code called THERMOS [11]. Thirty velocity groups and twenty spatial regions are used. Two cylindrical geometry options are available.

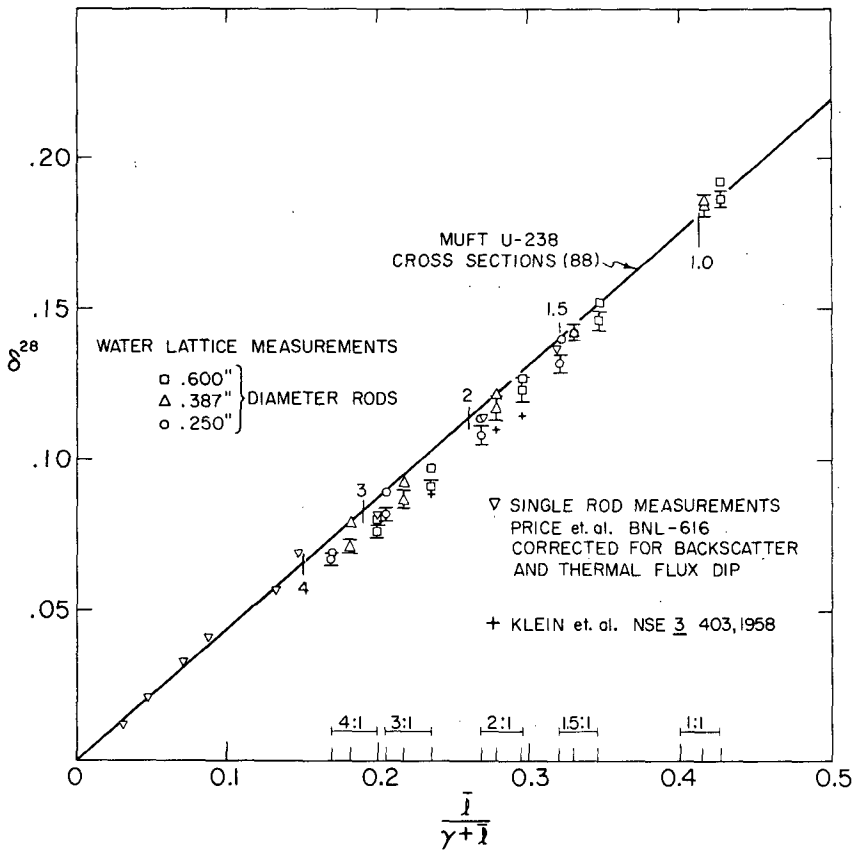


Fig. 5

Measurements of δ^{28} in single metal rods and metal rod lattices

- (1) Cylindrical cell geometry - the neutron density varies only in the radial direction. The outer cylindrical boundary of the cell is a reflecting boundary.
- (2) 2D geometry - the neutron density varies in two dimensions, x and y .

The exact reflecting boundaries for the triangular cell are used. The scattering kernel used for the calculations reported here is the NELKIN [36] kernel for water. This kernel accurately predicts the total cross-section, the diffusion coefficient, and the diffusion cooling coefficient of water and should be more than adequate for computing intracell neutron density distributions.

The results of the THERMOS multi-group calculations were used to obtain region-wise average cross-sections for use in one-group theories. We will see later that the disadvantage factor obtained from these one-group calculations are in fairly good agreement with the multi-group results. A simple but accurate approximation to the one-group transport equation is the method of AMOUYAL, BENOIST, and HOROWITZ [12] (called ABH).

This method is much the same as the collision probability approach used for computing resonance capture. Two additional simplifications are appropriate for thermal neutrons. First, since the optical thickness of the moderator region is usually large for thermal neutrons, single rod collision probabilities are used in computing the absorption probabilities in the fuel and Dancoff corrections are ignored. Secondly, most moderators are weakly absorbing so that the absorption probability in the moderator can be computed from diffusion theory. With these simplifications, the flux disadvantage factor δ_Φ can be expressed by a simple equation involving a few tabulated quantities.

$$\delta_\Phi = \frac{1}{P_0} + \frac{a^2 \Sigma_a}{c} \left[3c \Sigma_{tr} \frac{b^2 C}{b^2 - c^2} + \frac{3}{2} \lambda - 2 \right], \quad (11-14)$$

where P_0 , C , and λ are given by tables and graphs in [12]. The dimensions a , c , and b are the outer radii of the fuel, cladding, and cell, Σ_a is the average absorption cross-section of the fuel, and Σ_{tr} is the average transport cross-section of the moderator. Absorptions in the moderator have been ignored and the cladding is treated as a void. The neutron flux and density disadvantage factors, δ_Φ and δ_N , are related by

$$\delta_N = \delta_\Phi (\bar{v}_f / \bar{v}_m), \quad (11-15)$$

where $\bar{v}$ is the average neutron velocity in the region. The quantity $1/P_0$ is the ratio of the flux at the rod surface to the average flux in the rod and is used to estimate the average flux in the cladding.

Neutron density disadvantage factors for the 0.250 in, 1% enriched rods are shown in Fig. 6. The multi-group THERMOS calculations for a cylindrical cell and for the triangular cell show that the approximation of a cylindrical reflecting boundary is poor for small W/U ratios (thin water regions). The neutron spectra were found to be insensitive to the description of the outer boundary so that reliable spectrum averaged cross-sections can be obtained. The one-group cross-sections were then used in an S_4 cylindrical cell code and a P_3 code (CEPTR [37]) which has an option of reflecting boundary conditions (RBC) or cell boundary conditions (CBC). In the former case (RBC), all odd components vanish at the boundary, and in the other case (CBC), the normal derivative of the flux and the remaining odd components vanish at the boundary. These results for δ_N are also shown in Fig. 6. The cylindrical cell error is shown by all transport theory approximations which describe the outer boundary as a reflecting cylinder. This error in the cylindrical cell approximation occurs when the water channel thickness is small. Under these conditions the angular distribution of neutrons entering the fuel from isotropic sources far from the fuel is strongly peaked forward leading to a rather flat absorption distribution in the rod. The angular distribution of neutrons entering the fuel rod from isotropic sources near the surface of the rod is nearly isotropic leading to a large dip in the absorption

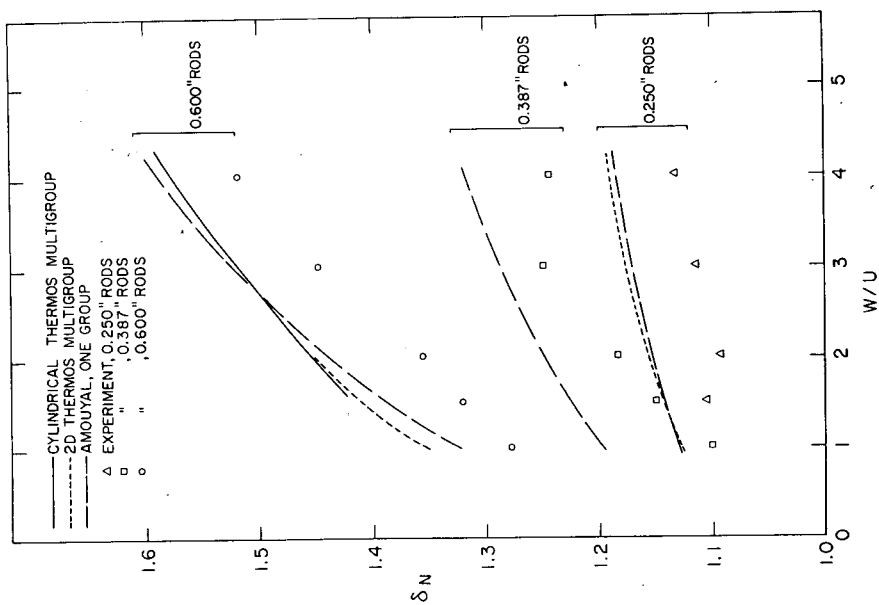


Fig. 6
Neutron density disadvantage factors for the 0.250-in,
1.0% enriched uranium metal rod lattices

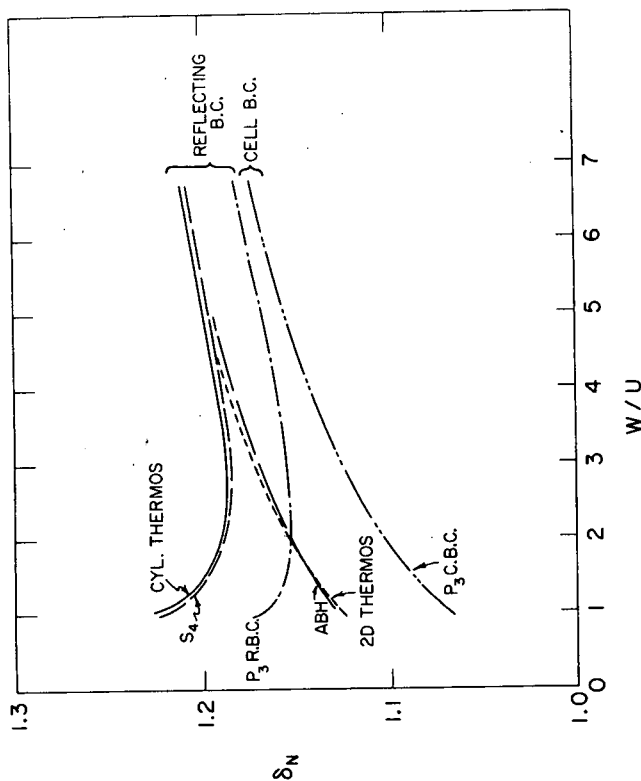


Fig. 7

Neutron density disadvantage factors for the 1.0% enriched uranium metal rod lattices

distribution in the rod. The cylindrical cell approximation places these scattering sources, on the average, too close to the rod; the dip in the fuel and the disadvantage factor are consequently over-estimated. Examination of the surface to average flux in the rod shows that this is indeed the case. As the diameter of the rod increases at constant W/U ratio, the thickness of the water channel increases and the cylindrical cell error decreases as is apparent in Fig. 7.

The ABH method with spectrum averaged cross-sections gives values of δ_N which agree well with our best two dimensional multi-group calculations (Fig. 7). Only a few of these two dimensional calculations have been done at present so that we use the ABH results to give a complete set of δ_N and f for all the lattices. In no case is the error expected to be more than 1.5% in δ_N .

The good agreement between the one-group and multi-group calculations encourages one to give prescriptions for average cross-sections. We find that in most cases it is possible to correlate the average velocity of the spectrum in each region with some simple parameters. For example, the spectral parameter τ_m defined by

$$\tau_m = (\bar{v}_m / 1.128)^2 \quad (II-16)$$

can be correlated with the parameter θ :

$$\theta = \left[\Sigma_{am}^0 + \Sigma_{ac}^0 \left(\frac{V_c}{V_m} \right) \left(\frac{\bar{N}_c}{\bar{N}_m} \right) + \Sigma_{af}^0 \left(\frac{V_f}{V_m} \right) \left(\frac{\bar{N}_f}{\bar{N}_m} \right) g_a \right] / M_2 \quad (II-17)$$

where Σ_a^0 is the 2200 m/s absorption cross-section, V is the volume, g_a is the non $1/v$ absorption factor for the fuel, and subscripts m , c , and f denote moderator, cladding, and fuel. The quantity M_2 is the second energy transfer moment of the scattering kernel. This θ is similar to the parameter $\Sigma_a / \xi \Sigma_s$ and Δ . It is unfortunately necessary in (II-17) to use the unknown quantities $\bar{N}$ and g_a to obtain a good correlation. These quantities can be estimated and an iteration procedure used. The τ_m as a function of θ is shown in Fig. 8. The correlation is good for all geometries and fuels for a given scattering kernel. There is some spread, however, in the correlations obtained from the various scattering models.

The experimental values shown in Fig. 8 were computed from spectra in homogeneous boric acid solutions measured by BEYSTER [38]. The value of $M_2 = 3.34 \text{ cm}^{-1}$ for the Nelkin kernel was used to compute θ . A value of M_2 which is 8% lower would place these experimental points on the curve. The average velocity in the fuel and cladding can be correlated in a similar way with the blackness of the rod and the average velocity in the moderator. From these prescriptions it is possible to obtain a complete set of one-group cross-sections. All quantities were averaged in the region $0 \leq E \leq 0.785 \text{ eV}$. Studies are now in progress to determine how these averages change with the assumed cut-off energy.

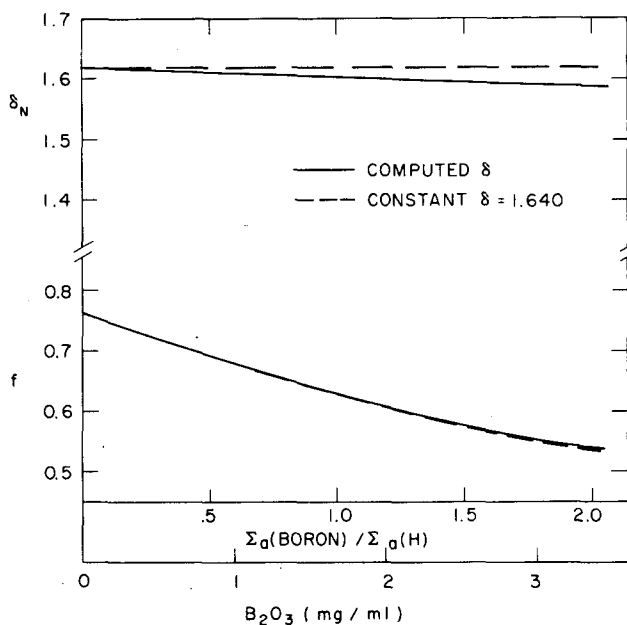


Fig. 8

Neutron density disadvantage factors and thermal utilization for an 0.600-in, 1.15% enriched uranium metal rod in a 4/1 lattice as a function of the boron concentration in the water

A measure of the non-Maxwellian character of the spectrum in the moderator is the ratio of the reactivity worths of boron and cadmium dissolved in the water. To a good approximation, the reactivity ratio is equal to the ratio of the spectrum averaged thermal absorption cross-sections of boron and cadmium. Experimental and calculated values of this ratio are given in Table VIII. Neutron temperature shifts derived by assuming that the spectra are purely Maxwellian are only half as large as deduced from the shift of the peak of the multi-group spectrum. This can be explained by noting that the $1/E$ tail increases the boron to the cadmium cross-section ratio which results in an apparently smaller temperature shift.

In the study of lattices with boron poisoning in the water it is customary to assume that the effect of the poison on the disadvantage factor is negligible. In addition, the hydrogen absorption is ignored in the ABH calculation of δ_N . To test these assumptions we have computed δ_N and f for a lattice as a function of the amount of boron in the water. The results are shown in Fig. 8. Ignoring hydrogen absorption in the ABH calculation of δ_N over-estimates δ_N by at most 1% and under-estimates f by less than 0.2%. In the largest poison concentrations encountered in the present analysis the error is less than 2% in δ_N and 0.4% in f .

III. COMPARISON OF THEORY WITH MEASUREMENTS

A summary of the calculations of the lattice eigenvalue, the components of Eq. (I-1), the measured material buckling and the difference between the

TABLE VIII
CADMIUM-BORON CROSS-SECTION RATIOS

Rod dia. (in)	Enrichment %	W/U	T _m (°K)	$\left(\frac{\sigma_{cd}}{\sigma_B}\right)^{(1)}$ exp.	$\left(\frac{\sigma_{cd}}{\sigma_B}\right)^{(2)}$ theory	$\Delta T_{exp.}^{(3)}$	$\Delta T_{theory}^{(4)}$
0.387	1.3	2	290	4.98 ± .30	5.00	35 ± 17	63
		3	290	5.26 ± .35	4.82	57 ± 21	43
		4	290	4.74 ± .30	4.72	14 ± 17	28
		3	290	4.63 ± .50	4.80	4 ± 30	38
	1.5	4	290	4.62 ± .50	4.70	3 ± 30	29
0.600	1.15	3	297	4.73 ± .50	4.81	5 ± 30	22

(1) An uncertainty of ± 10% was assigned to each measurement

(2) Based on 2200 m/s cross-sections of $\sigma_B = 755$ b and $\sigma_{cd} = 2450$ b

(3) Computed from Maxwellian averaged cross-sections

(4) Computed from the most probable speed

theoretical and measured bucklings is given in Tables IX-XI for the 96 clean and poisoned lattices. The column headed λ_1/λ gives the ratio of fast fission (U^{235} and U^{238}) to total fission contributions to the eigenvalue λ . As shown in section II-B, the values of K give the approximate heterogeneous fast fission correction to λ . A grouping of the eigenvalues with respect to rod size, enrichment and W/U ratio, shown in Fig. 16, shows no marked trend which is clearly beyond the probable error.

Perhaps the most instructive curves are those showing the computed and measured bucklings as a function of boron poison content of the moderator water, for if there were substantial systematic errors in either the leakage coefficients or the thermal flux disadvantage factor, the computations would not display the same slope as the measurements. In the $W/U = 1$ series for both 0.600 in and 0.387 in lattices and in the 1.0%, $W/U = 1.5$ for the 0.250 in rods (Figs. 13-15) there is an indication of this lack of parallel behaviour, but it is not observed consistently for the other W/U ratios. In general there appears to be no clear indication of better or worse agreement in the neighbourhood of $B^2 = 0$ than elsewhere so that large errors in the leakage calculation seem doubtful. The poison measurements apparently show that the offset between theory and measurement, which is appreciable, lies in the infinite multiplication factor. The poisoned lattice curves, of course, are also helpful in appraising the consistency of the buckling measurements themselves.

The eigenvalues computed for the clean lattices are substantially different from those given in the 1958 analysis [3]. The latter showed a marked

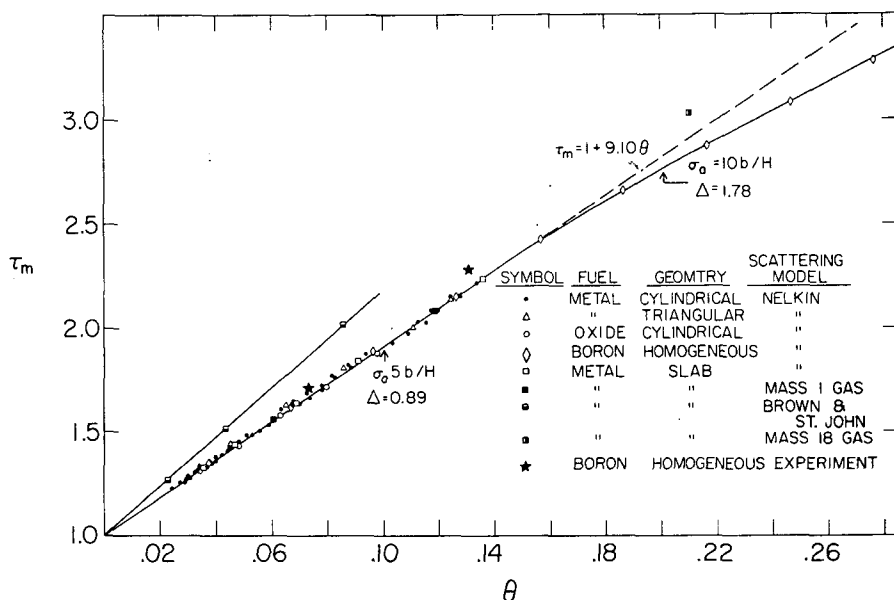


Fig. 9

Correlation of the spectral hardness parameter τ_m with the thermalization parameter θ

TABLE IX
SUMMARY OF BUCKLINGS AND COMPONENTS OF THE MULTI-GROUP
ANALYSIS OF THE 0.600-in METAL ROD LATTICES

% U ²³⁵	W/U	B ₂ O ₃ (mg/ml)	η_{th}	f_{th}	η_f	f_f	P	L ² (cm ²)	τ (cm ²)	λ_1/λ	K	λ	B ² × 10 ⁴ (cm ⁻²)	$\Delta B^2 \times 10^4$ (cm ⁻²)
1.00	1.0	0	1.468	0.9325	0.6628	0.9809	0.5285	2.61	33.52	0.2381	1.0068	1.0013	9.90 ± 0.54	0.40
	1.5	0	1.474	0.9015	0.6849	0.9719	0.6324	2.56	31.05	0.2268	1.0093	0.9989	29.63 ± 0.42	-0.39
	2.0	0	1.477	0.8712	0.6983	0.9628	0.6984	2.63	29.64	0.1855	1.0110	0.9974	36.07 ± 0.39	-0.97
	3.0	0	1.480	0.8131	0.7165	0.9445	0.7777	2.89	27.94	0.1397	1.0134	0.9979	33.15 ± 0.31	-0.82
	4.0	0	1.482	0.7594	0.7251	0.9267	0.8226	3.20	26.99	0.1158	1.0148	0.9968	20.96 ± 0.26	-1.25
1.15	1.0	0	1.512	0.9362	0.6875	0.9812	0.5193	2.50	33.84	0.3072	1.0068	0.9914	21.33 ± 0.41	-2.72
		2.980	1.512	0.8494	0.6797	0.9700	0.5182	2.11	32.98	0.3230	1.0068	0.9986	-2.54 ± 0.58	-0.42
		3.327	1.512	0.8403	0.6786	0.9685	0.5183	2.07	32.85	0.3244	1.0068	1.0038	-6.44 ± 0.46	1.13
	1.5	0	1.518	0.8966	0.7079	0.9724	0.6251	2.46	31.29	0.2326	1.0093	0.9948	40.23 ± 0.30	-1.87
		2.022	1.518	0.8202	0.7013	0.9605	0.6241	2.13	30.52	0.2463	1.0093	0.9974	13.17 ± 0.43	-0.89
		3.926	1.518	0.7527	0.6966	0.9484	0.6226	1.89	29.93	0.2592	1.0093	0.9950	-8.18 ± 0.40	-1.63
	2.0	0	1.521	0.8774	0.7206	0.9635	0.6921	2.53	29.88	0.1897	1.0110	0.9956	48.22 ± 0.31	-1.68
		1.911	1.521	0.7744	0.7137	0.9480	0.6910	2.16	29.02	0.2048	1.0110	0.9962	12.36 ± 0.62	-1.36
		3.000	1.521	0.7259	0.7105	0.9389	0.6901	1.99	28.59	0.2132	1.0110	0.9971	-5.96 ± 1.00	-1.00
	3.0	0	1.524	0.8210	0.7378	0.9456	0.7729	2.80	28.18	0.1424	1.0134	0.9955	47.12 ± 0.33	-1.81
		1.308	1.524	0.7232	0.7324	0.9296	0.7719	2.43	27.42	0.1547	1.0134	0.9967	7.70 ± 0.58	-1.23

TABLE IX (cont'd)

1.30	4.0	0	1.526	0.7685	0.7461	0.9282	0.8187	3.10	27.23	0.1168	1.0148	0.9937	36.03 ± 0.16	-2.54
		0.344	1.526	0.7344	0.7443	0.9223	0.8184	2.95	26.99	0.1204	1.0148	0.9919	22.16 ± 0.36	-3.18
		0.746	1.526	0.6983	0.7428	0.9162	0.8180	2.80	26.71	0.1246	1.0148	0.9939	5.58 ± 0.48	-2.32
		1.300	1.526	0.6539	0.7411	0.9072	0.8172	2.61	26.40	0.1306	1.0148	0.9906	-13.57 ± 0.46	-3.43
1.5	1.0	0	1.560	0.9404	0.7172	0.9816	0.5087	2.37	34.17	0.3184	1.0068	0.9848	32.11 ± 0.54	-4.94
		2.587	1.560	0.8682	0.7103	0.9728	0.5080	2.06	33.43	0.3311	1.0068	0.9977	10.67 ± 1.48	-0.71
		0	1.566	0.9122	0.7360	0.9730	0.6162	2.35	31.58	0.2401	1.0093	0.9915	51.87 ± 0.50	-3.13
		1.039	1.566	0.8680	0.7324	0.9676	0.6157	2.19	31.20	0.2471	1.0093	0.9927	38.63 ± 1.13	-2.42
2.0		3.452	1.566	0.7803	0.7266	0.9528	0.6140	1.89	30.42	0.2630	1.0093	0.9926	10.84 ± 1.22	-2.51
	2.0	0	1.570	0.8843	0.7481	0.9643	0.6845	2.43	30.17	0.1956	1.0110	0.9919	61.08 ± 0.32	-3.18
		2.587	1.570	0.7558	0.7398	0.9440	0.6827	1.99	29.09	0.2154	1.0110	0.9929	16.38 ± 0.34	-2.56
		0	1.572	0.8299	0.7642	0.9469	0.7671	2.69	28.44	0.1461	1.0134	0.9944	60.99 ± 0.26	-2.31
4.0		1.724	1.572	0.7097	0.7579	0.9265	0.7656	2.26	27.50	0.1620	1.0134	0.9944	12.87 ± 0.83	-2.11
	4.0	0	1.574	0.7789	0.7717	0.9299	0.8141	2.99	27.46	0.1195	1.0148	0.9941	50.38 ± 0.27	-2.45
		0.500	1.574	0.7317	0.7697	0.9223	0.8134	2.80	27.17	0.1249	1.0148	0.9872	33.21 ± 0.81	-5.14
		0.855	1.574	0.7016	0.7682	0.9167	0.8132	2.68	26.89	0.1283	1.0148	0.9953	16.93 ± 1.00	-1.83
		1.059	1.574	0.6854	0.7677	0.9134	0.8129	2.16	26.79	0.1305	1.0148	0.9913	10.97 ± 1.30	-3.34

TABLE X
SUMMARY OF BUCKLINGS AND COMPONENTS OF THE MULTI-GROUP
ANALYSIS OF THE 0.387-in METAL ROD LATTICES

% U ²³⁵	w/U	B ₂ O ₃ (mg/ml)	η_{th}	f_{th}	η_f	f_f	P	L ² (cm ²)	τ (cm ²)	λ_f/λ	K	λ	B ² × 10 ⁴ (cm ⁻⁴)	$\Delta B^2 \times 10^4$ (cm ⁻²)
1.00	1.0	0	1.468	0.9348	0.6352	0.9800	0.5170	2.77	36.17	0.2979	1.0046	1.0020	3.23 ± .80	0.56
	1.5	0	1.474	0.9091	0.6367	0.9723	0.6125	2.62	32.95	0.2271	1.0061	0.9982	19.70 ± .34	-0.58
	2.0	0	1.477	0.8829	0.6458	0.9641	0.6792	2.62	31.18	0.1851	1.0072	0.9983	29.02 ± .34	-1.29
	3.0	0	1.480	0.8343	0.6573	0.9476	0.7608	2.78	29.15	0.1379	1.0087	0.9984	31.39 ± .19	-0.60
	4.0	0	1.482	0.7895	0.6618	0.9316	0.8076	3.01	28.04	0.1123	1.0097	0.9949	25.68 ± .24	-1.95
1.15	1.0	0	1.512	0.9394	0.6591	0.9804	0.5083	2.64	36.44	0.3063	1.0046	0.9982	12.03 ± .91	-0.52
		0.627	1.512	0.9215	0.6561	0.9792	0.5031	2.55	36.22	0.3089	1.0046	1.0080	5.40 ± .89	2.25
		1.119	1.512	0.9079	0.6556	0.9772	0.5076	2.48	36.15	0.3119	1.0046	1.0031	3.64 ± 1.57	0.87
	1.5	0	1.518	0.9138	0.6597	0.9728	0.6051	2.51	33.25	0.2333	1.0061	0.9901	31.21 ± .35	-3.29
		0.793	1.518	0.8816	0.6573	0.9691	0.6044	2.37	33.02	0.2386	1.0061	0.9842	24.15 ± .90	-5.17
2.0		1.493	1.518	0.8551	0.6547	0.9650	0.6045	2.27	32.66	0.2421	1.0061	0.9967	12.87 ± 1.02	-1.05
		2.930	1.518	0.8053	0.6516	0.9566	0.6033	2.07	32.23	0.2511	1.0061	0.9924	-0.50 ± .54	-2.35
	2.0	0	1.521	0.8893	0.6680	0.9647	0.6728	2.51	31.48	0.1897	1.0072	0.9882	42.26 ± .51	-4.25
		0.793	1.521	0.8485	0.6648	0.9593	0.6725	2.35	31.10	0.1950	1.0072	0.9918	28.17 ± 1.00	-2.87
		1.493	1.521	0.8155	0.6626	0.9539	0.6720	2.23	30.79	0.1998	1.0072	0.9940	16.74 ± .57	-2.05
		3.660	1.521	0.7278	0.6568	0.9375	0.6701	1.92	29.98	0.2146	1.0072	0.9951	-13.85 ± .71	-1.57

TABLE X (cont'd)

1.30	3.0	0	1.524	0.8427	0.6785	0.9485	0.7559	2.67	29.44	0.1408	1.0087	0.9908	46.18 ± .37	-3.55
		0.793	1.524	0.7873	0.6755	0.9400	0.7551	2.46	29.01	0.1472	1.0087	0.9890	26.43 ± 1.01	-4.08
		1.499	1.524	0.7441	0.6724	0.9325	0.7546	2.30	28.61	0.1526	1.0087	0.9961	7.31 ± .54	-1.39
		2.247	1.524	0.7027	0.6710	0.9236	0.7535	2.15	28.31	0.1590	1.0087	0.9888	-6.97 ± .67	-3.88
1.30	4.0	0	1.526	0.7994	0.6822	0.9328	0.8037	2.91	28.28	0.1142	1.0097	0.9921	40.14 ± .17	-3.11
		0.358	1.526	0.7679	0.6808	0.9279	0.8033	2.78	28.08	0.1175	1.0097	0.9865	29.48 ± .52	-5.21
		0.793	1.526	0.7327	0.6794	0.9214	0.8027	2.63	27.83	0.1214	1.0097	0.9840	15.76 ± .85	-6.00
		1.501	1.526	0.6820	0.6770	0.9109	0.8019	2.43	27.38	0.1274	1.0097	0.9938	-9.50 ± .46	-2.21
1.30	1.0	0	1.560	0.9431	0.6883	0.9808	0.4981	2.50	36.75	0.3173	1.0046	0.9962	20.90 ± .46	-1.11
	1.5	0	1.566	0.9197	0.6870	0.9733	0.5967	2.38	33.51	0.2406	1.0061	0.9928	40.51 ± .30	-2.44
	2.0	0	1.570	0.8966	0.6946	0.9654	0.6656	2.39	31.73	0.1950	1.0072	0.9936	52.19 ± .36	-2.35
	3.0	0	1.572	0.8522	0.7043	0.9496	0.7502	2.55	29.70	0.1444	1.0087	0.9927	59.25 ± .33	-2.89
1.30	4.0	0	1.574	0.8105	0.7075	0.9343	0.7991	2.78	28.54	0.1168	1.0097	0.9932	54.69 ± .36	-2.75

TABLE XI
SUMMARY OF BUCKLINGS AND COMPONENTS OF THE MULTI-GROUP
ANALYSIS OF THE 0.250-in METAL ROD LATTICES

%U ₂₃₅	W/U	B ₂ O ₃ (mg/ml)	η_{th}	f_{th}	η_f	f_f	p	L ² (cm ²)	τ (cm ²)	λ_f/λ	K	λ	B ² × 10 ⁴ (cm ⁻²)	$\Delta B^2 \times 10^4$ (cm ⁻²)
1.00	1.5	0	1.474	0.9085	0.5920	0.9729	0.5927	2.88	36.38	0.2288	1.0040	0.9860	12.14 ± 1.03	-4.02
		0.249	1.474	0.8984	0.5908	0.9715	0.5929	2.82	36.19	0.2297	1.0040	0.9949	7.54 ± 0.97	-1.70
		0.480	1.474	0.8892	0.5902	0.9703	0.5928	2.77	36.10	0.2311	1.0040	0.9942	5.23 ± 0.95	-1.64
		0.823	1.474	0.8758	0.5886	0.9683	0.5930	2.70	35.87	0.2325	1.0040	1.0039	-0.63 ± 1.36	1.08
2.0	2.0	0	1.477	0.8856	0.5981	0.9652	0.6608	2.79	33.87	0.1845	1.0047	0.9932	19.95 ± 0.47	-2.15
		0.480	1.477	0.8611	0.5962	0.9617	0.6608	2.67	33.58	0.1884	1.0047	0.9997	11.15 ± 0.45	-0.09
		0.733	1.477	0.8488	0.5956	0.9598	0.6605	2.61	33.50	0.1903	1.0047	0.9959	8.69 ± 0.92	-1.26
		1.057	1.477	0.8334	0.5940	0.9574	0.6606	2.54	33.25	0.1918	1.0047	1.0084	0.87 ± 0.99	2.84
3.0	3.0	0	1.480	0.8424	0.6016	0.9502	0.7436	2.81	31.05	0.1372	1.0056	0.9963	25.15 ± 0.27	-0.94
		0.376	1.480	0.8162	0.6006	0.9462	0.7431	2.69	30.89	0.1404	1.0056	0.9962	16.04 ± 0.32	-1.29
		0.678	1.480	0.7962	0.5996	0.9429	0.7428	2.60	30.73	0.1428	1.0056	0.9959	9.31 ± 0.49	-1.37
		1.028	1.480	0.7744	0.5986	0.9392	0.7425	2.51	30.54	0.1454	1.0056	0.9962	1.57 ± 0.68	-1.25
4.0	4.0	0	1.482	0.8025	0.6026	0.9357	0.7920	2.93	29.59	0.1114	1.0062	0.9945	22.07 ± 0.21	-1.98
		0.251	1.482	0.7811	0.6017	0.9322	0.7918	2.83	29.43	0.1134	1.0062	0.9944	14.26 ± 0.16	-1.98
		0.518	1.482	0.7594	0.6008	0.9285	0.7915	2.74	29.27	0.1157	1.0062	0.9930	6.44 ± 0.35	-2.44
		0.680	1.482	0.7468	0.6002	0.9263	0.7913	2.68	29.17	0.1170	1.0062	0.9937	1.35 ± 0.23	-2.17

TABLE XI (cont'd)

1.15	1.5	0	1.518	0.9142	0.6136	0.9735	0.5859	2.74	36.62	0.2342	1.0040	0.9882	19.93 ± 0.94	-3.28
		0.635	1.518	0.8900	0.6122	0.9700	0.5854	2.61	36.40	0.2380	1.0040	0.9861	14.49 ± 1.08	-4.02
		1.077	1.518	0.8740	0.6111	0.9676	0.5852	2.53	36.23	0.2405	1.0040	0.9863	10.43 ± 0.67	-3.92
		2.007	1.518	0.8421	0.6098	0.9625	0.5841	2.37	36.03	0.2467	1.0040	0.9748	5.34 ± 1.72	-7.13
2.0		0	1.521	0.8924	0.6194	0.9658	0.6547	2.65	34.17	0.1900	1.0047	0.9887	31.07 ± 0.37	-3.66
		0.453	1.521	0.8704	0.6180	0.9625	0.6544	2.55	33.96	0.1927	1.0047	0.9894	24.44 ± 0.90	-3.38
		0.919	1.521	0.8489	0.6168	0.9592	0.6541	2.45	33.76	0.1958	1.0047	0.9889	18.35 ± 0.59	-3.49
		2.028	1.521	0.8017	0.6139	0.9513	0.6532	2.25	33.30	0.2029	1.0047	0.9890	4.22 ± 0.67	-3.35
3.0		1.969	1.521	0.8040	0.6136	0.9517	0.6535	2.26	33.27	0.2022	1.0047	0.9952	3.12 ± 1.14	-1.46
		0	1.524	0.8512	0.6226	0.9510	0.7385	2.68	31.41	0.1404	1.0056	0.9897	38.41 ± 0.18	-3.67
		0.358	1.524	0.8273	0.6213	0.9474	0.7383	2.58	31.20	0.1430	1.0056	0.9909	30.05 ± 0.24	-3.19
		0.793	1.524	0.8035	0.6202	0.9434	0.7379	2.48	31.01	0.1458	1.0056	0.9895	22.28 ± 0.58	-3.62
4.0		1.085	1.524	0.7826	0.6190	0.9398	0.7375	2.40	30.83	0.1484	1.0056	0.9891	15.04 ± 0.32	-3.70
		1.583	1.524	0.7547	0.6175	0.9346	0.7371	2.29	30.57	0.1520	1.0056	0.9911	4.32 ± 0.59	-2.95
		0	1.526	0.8129	0.6224	0.9369	0.7882	2.79	29.86	0.1132	1.0062	0.9904	36.31 ± 0.18	-3.56
		0.298	1.526	0.7884	0.6214	0.9328	0.7878	2.69	29.67	0.1156	1.0062	0.9904	27.27 ± 0.17	-3.50
		0.565	1.526	0.7677	0.6205	0.9291	0.7875	2.60	29.52	0.1178	1.0062	0.9886	19.87 ± 0.26	-4.09
		0.914	1.526	0.7424	0.6195	0.9244	0.7871	2.50	29.31	0.1205	1.0062	0.9890	9.91 ± 0.30	-3.86

TABLE XI A
GEOMETRY OF THE BNL LATTICES

Rod dia. (in)	R_o (cm)	t_g (cm)	t_{al} (cm)	$V_{al}/V + V_{al}$
0.250	0.3175	0.0127	0.0711	0.8634
0.387	0.4915	0.0127	0.0711	0.8585
0.600	0.7620	0.0127	0.0711	0.8551

Rod dia. (in)	V_w/V_u	a (cm)	R_c (cm)	V_u/V	V_g/V	V_{al}/V	V_w/V
0.250	1	0.9746	0.5117	0.3850	0.0314	0.1986	0.3850
	1.5	1.0643	0.5588	0.3228	0.0263	0.1665	0.4844
	2	1.1471	0.6023	0.2779	0.0227	0.1434	0.5567
	3	1.2967	0.6808	0.2175	0.0177	0.1122	0.6526
0.387	4	1.4308	0.7512	0.1786	0.0146	0.0921	0.7147
	1	1.4412	0.7567	0.4219	0.0221	0.1340	0.4220
	1.5	1.5860	0.8327	0.3484	0.0182	0.1107	0.5227
	2	1.7188	0.9024	0.2967	0.0155	0.0942	0.5936
0.600	3	1.9573	1.0276	0.2288	0.0120	0.0727	0.6865
	4	2.1697	1.1392	0.1862	0.0097	0.0591	0.7450
	1	2.1684	1.1384	0.4480	0.0151	0.0889	0.4480
	1.5	2.3988	1.2596	0.3660	0.0123	0.0726	0.5491
	2	2.6093	1.3703	0.3092	0.0104	0.0614	0.6190
	3	2.9855	1.5674	0.2363	0.0079	0.0469	0.7089
	4	3.3198	1.7429	0.1911	0.0064	0.0379	0.7646

 R_o = fuel radius a = rod pitch t_g = thickness of air gap R_c = radius of cylindrical cell t_{al} = thickness of aluminium tube wall

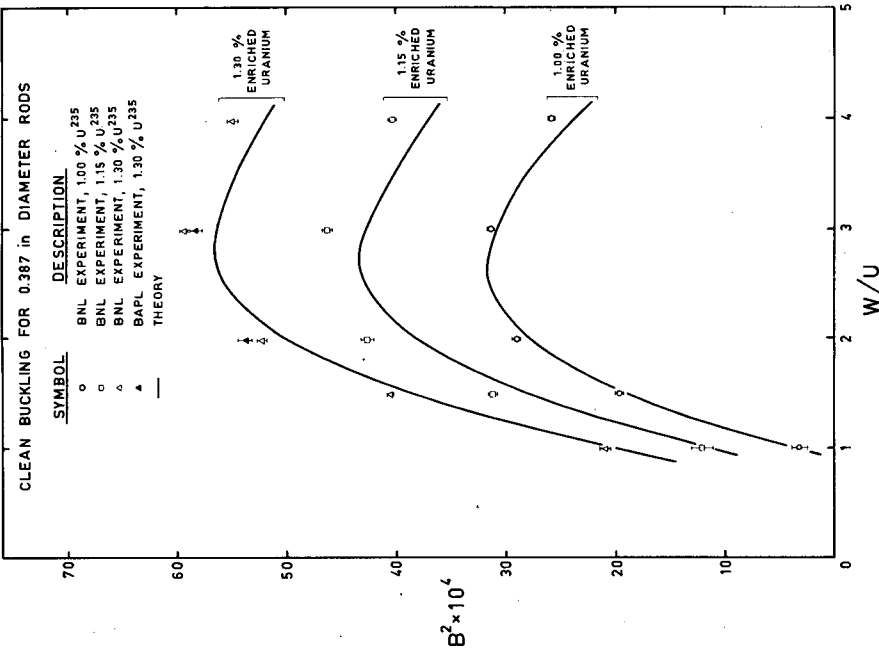


Fig. 11

Measured and computed bucklings for the clean 0.387-in rod lattices

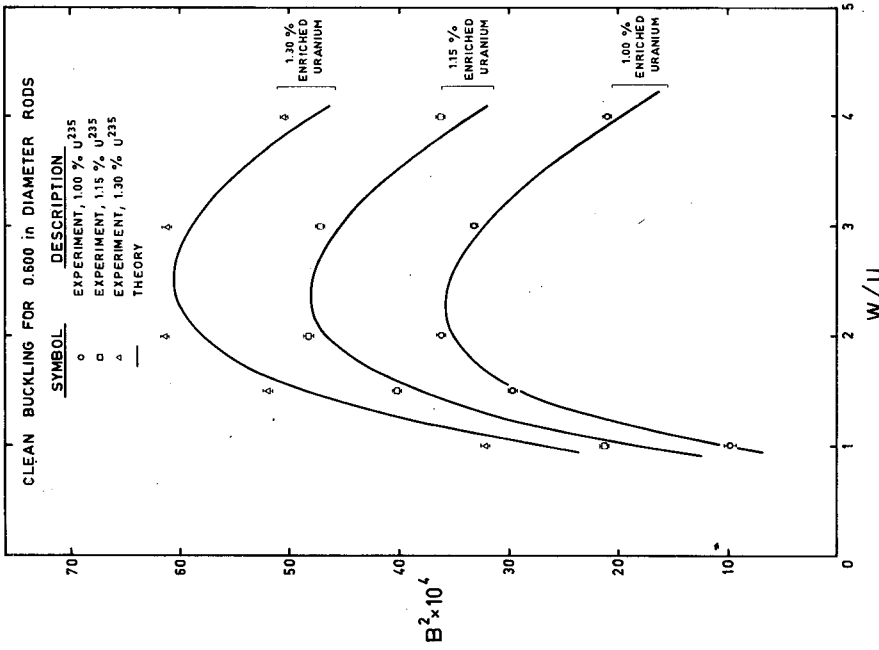
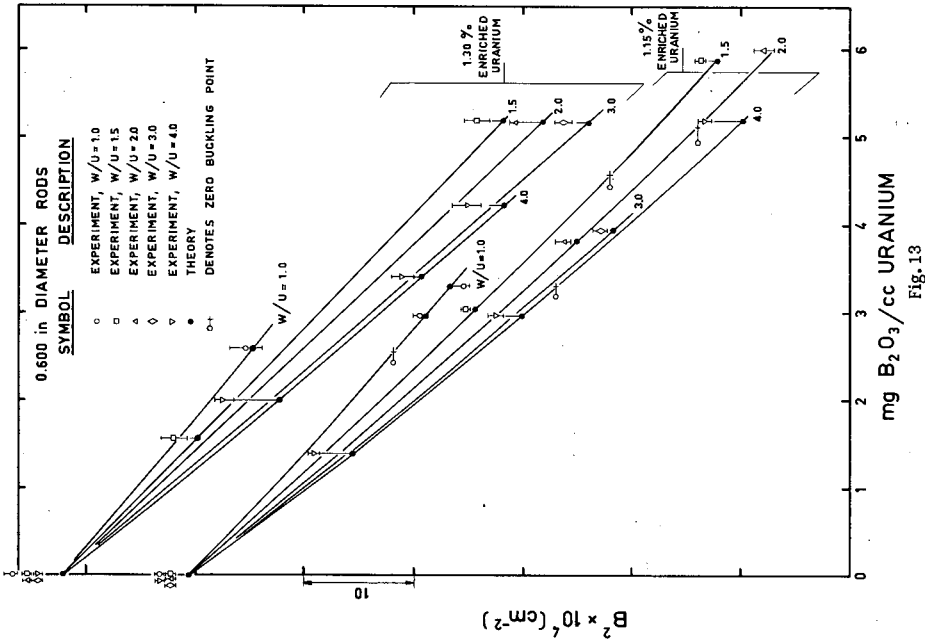


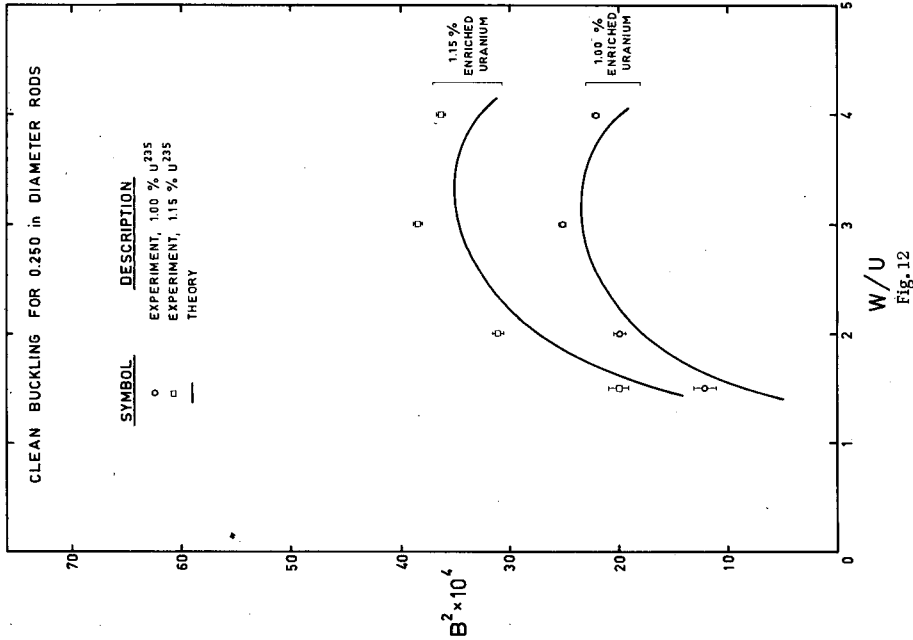
Fig. 10

Measured and computed bucklings for the clean 0.600-in rod lattices



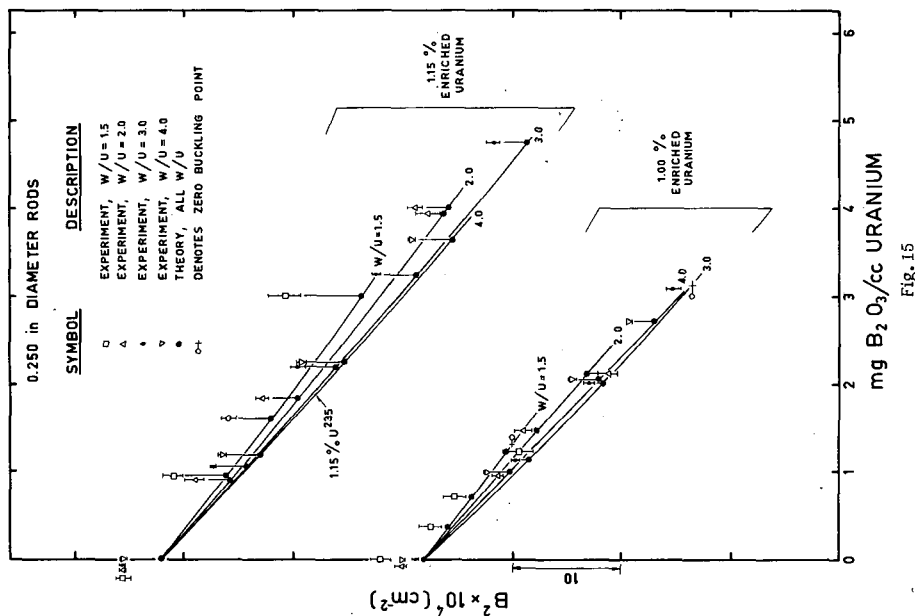
Measured and computed bucklings for the poisoned 0.600-in rod lattices

Fig. 13

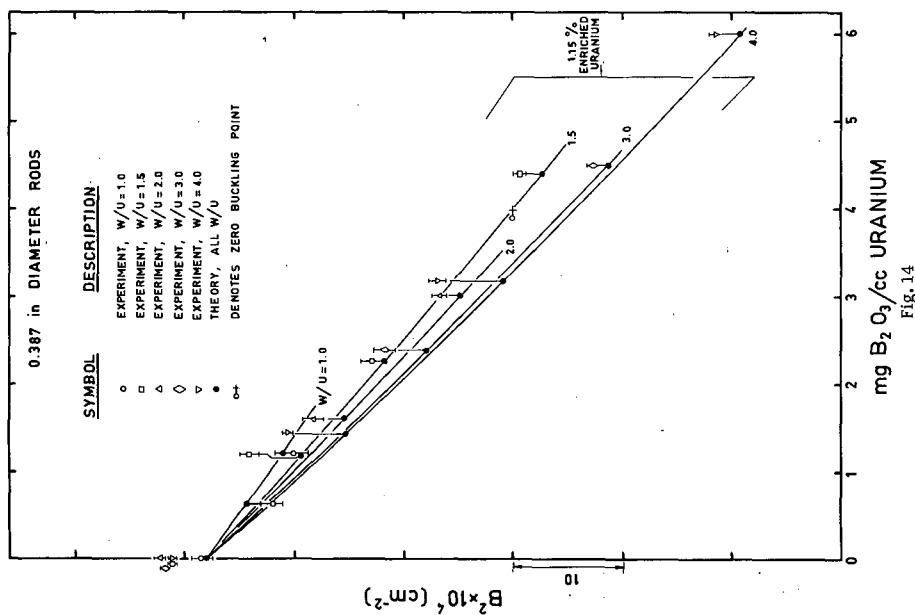


Measured and computed bucklings for the clean 0.250-in rod lattices

Fig. 12



Measured and computed bucklings for the poisoned 0.250-in rod lattices



Measured and computed bucklings for the poisoned 0.387-in rod lattices

TABLE XII
SUMMARY OF MEASURED AND COMPUTED CELL PARAMETERS
FOR THE 0.600-in METAL ROD LATTICES

% U ²³⁵	W/U	ρ			δ_{28}			δ_N		
		Theory	BNL exp.	BAPL exp.	Theory	BNL exp.	ERDIK exp.	Theory	BNL exp.	BAPL exp.
1.00	1.0	2.294	2.0 $\pm$ 0.2		0.1768	0.193 $\pm$ 0.002	0.180	1.324	1.278	
	1.5	1.520			0.1398	0.146 $\pm$ 0.008	0.144	1.388	1.321	
	2.0	1.151	1.05 $\pm$ 0.14		0.1194	0.121 $\pm$ 0.003	0.116	1.439	1.356	
	3.0	0.793	0.79 $\pm$ 0.02		0.0972	0.098 $\pm$ 0.003	0.081	1.523	1.448	
	4.0	0.624	0.60 $\pm$ 0.05		0.0861	0.073 $\pm$ 0.002	0.072	1.590	1.519	
1.15	1.0				0.1740	0.179 $\pm$ 0.004	0.179	1.346	1.270	
	1.5				0.1388	0.138 $\pm$ 0.005	0.142	1.415	1.289	
	2.0	1.252		1.20 $\pm$ 0.03	0.1185	0.113 $\pm$ 0.004	0.113	1.471	1.383	1.45
	3.0	0.860		0.93 $\pm$ 0.04	0.0965	0.082 $\pm$ 0.001	0.086	1.562	1.449	1.45

TABLE XIII
SUMMARY OF MEASURED AND COMPUTED CELL PARAMETERS FOR THE 0.387-in METAL ROD LATTICES

% U ²³⁵	W/U	ρ			g^a			δ_N		
		Theory	BNL exp.	BAPL exp.	Theory	BNL exp.	ERDIK exp.	Theory	BNL exp.	BAPL exp.
1.00	1.0	2.437	2.44 ± 0.10		0.1731	0.184 ± 0.006	0.187	1.197	1.101	
	1.5	1.693	1.61 ± 0.10		0.1366	0.136 ± 0.006	0.136	1.235	1.149	
	2.0	1.286	1.35 ± 0.06		0.1147	0.115 ± 0.002	0.109	1.249	1.133	
	3.0	0.889	0.93 ± 0.03		0.0904	0.087 ± 0.002	0.081	1.287	1.249	
	4.0	0.698	0.73 ± 0.03		0.0771	0.076 ± 0.002	0.068	1.318	1.243	
1.15	1.0				0.1698	0.181 ± 0.005	0.177	1.210	1.142	
	1.5				0.1335	0.130 ± 0.004	0.138	1.241	1.182	
	2.0				0.1119	0.110 ± 0.002	0.106	1.267	1.193	

TABLE XIII (cont'd)

	3.0				0.0882	0.079 ± 0.001	0.081	1.308	1.246	
	4.0				0.0743	0.0680 ± 0.0005	0.064	1.341	1.302	
1.30	1.0	2.969		3.13 ± 0.20	0.1664	0.176 ± 0.003	0.176	1.227	1.161	
	1.5				0.1309	0.1360 ± 0.0005	0.127	1.261	1.167	
	2.0	1.546		1.59 ± 0.03	0.1097	0.1106 ± 0.0002	0.108	1.290	1.210	1.21
	3.0	1.058		1.066 ± 0.012	0.0861	0.0874 ± 0.0006	0.077	1.335	1.272	1.31
	4.0				0.0734	0.0735 ± 0.0004	0.066	1.372	1.307	

TABLE XIV
SUMMARY OF MEASURED AND COMPUTED CELL PARAMETERS
FOR THE 0.250-in METAL ROD LATTICES

% U ²³⁵	W/U	ρ			δ^{28}			δN		
		Theory	BNL exp.	BAPL exp.	Theory	BNL exp.	ERDIK exp.	Theory	BNL exp.	BAPL exp.
1.00	1.0							1.129		
	1.5	1.892	2.35 ± 0.06		0.1321	0.129 ± 0.004	0.129	1.141	1.105	
	2.0	1.439	2.00 ± 0.04		0.1102	0.106 ± 0.005	0.102	1.152	1.091	
	3.0	1.002	1.27 ± 0.02		0.0857	0.084 ± 0.005	0.077	1.169	1.114	
	4.0	0.790	1.02 ± 0.01		0.0713	0.062 ± 0.002	0.066	1.183	1.132	
1.15	1.0							1.138		
	1.5				0.1297	0.141 ± 0.003	0.126	1.151	1.081	
	2.0				0.1077	0.111 ± 0.002	0.103	1.162	1.087	
	3.0				0.0830	0.084 ± 0.007	0.077	1.181	1.128	
	4.0				0.0594	0.066 ± 0.002	0.061	1.196	1.146	

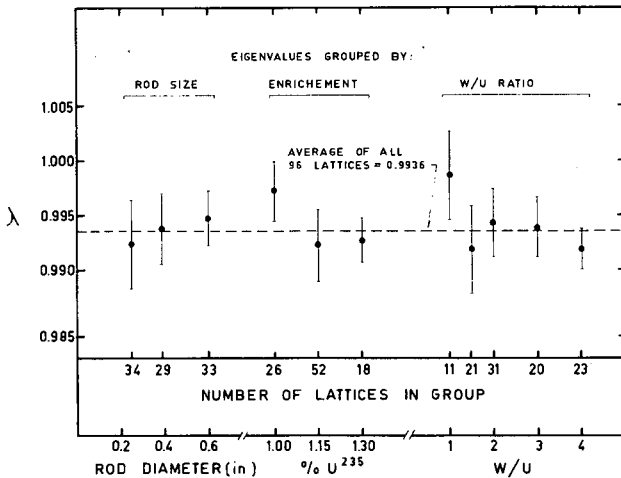


Fig. 16

Distribution of eigenvalues for all uranium rod lattices

variation in eigenvalue of 1% to 1.5% with the W/U ratio. In addition, the average eigenvalue was 2% higher than that of the present analysis. There are a number of reasons for this difference which can be summarized as follows:

(1) In the present computations the thermal neutron value of $\nu^{235} = 2.43$ is 1.6% smaller than the value 2.47 used before. This change alone accounts for the principal difference in thermal neutron η_2 ; the differences between the SOFOCATE homogeneous cell spectrum averaged cross-sections and the corresponding THERMOS heterogeneous spectrum averages are small - of the order of 0.2% in η_2 .

(2) The THERMOS computed values of f_2 are lower by 0.3% to 1.7% than the values based on measured disadvantage factors which were used in 1958.

(3) The neutron ages given by the new U^{238} and O^{16} cross-sections are essentially unchanged in the W/U = 4 lattices but are about 6% higher in the close packed lattices.

(4) The new U^{238} fast neutron cross-sections yield an appreciably lower homogeneous δ^{28} in the open lattices, but the heterogeneous correction discussed in section II-B more than compensates for this decrease. This correction results in the present results showing less variation of eigenvalue with W/U ratio than appeared in the 1958 calculations.

(5) The new fast cross-sections for U^{235} , with explicit fission and capture resonances, yield a higher fast neutron average of $\alpha_1^{25} = 0.50$ than the earlier cross-sections which gave $\alpha_1^{25} = 0.38$. This, of course, results in a decreased eigenvalue, particularly in the close packed lattices. Because of the difficulty of computing with precision the cadmium ratio of U^{235} fissions we have not yet attempted to compare the computed and measured values of δ^{25} , the epicadmium-to-subcadmium fission ratio in U^{235} .

It is clear that in any analysis of this sort there is a host of small sources of error which, in their net effect, can appreciably change the theoretical estimate of the neutron reproduction in the lattice. The present analysis is no more than an attempt to combine readily available cross-section information and computing techniques for the analysis of a long series of reactors. Later work will be directed towards improving the certainty of such features as the heterogeneous fast effect, the hexagonal cell boundary effect on the thermal disadvantage factor, and the proper calculation of the fast fission and capture in U^{235} .

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The authors wish to acknowledge the assistance of a number of people in both the experimental and theoretical reactor physics groups at BNL. Both M. Levine and J. Chernick have contributed substantially in discussions of the resonance capture and fast fission problems. H. Kouts, under whose direction the measurements were made, has been most helpful in making much of the previously unpublished data available. The assistance of W. Bornstein and Mrs. J. Weisenbloom with the numerical work is gratefully acknowledged.

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SOME LIGHT WATER LATTICE EXPERIMENTS IN THE UNITED KINGDOM

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Abstract — Résumé — Аннотация — Resumen

SOME LIGHT WATER LATTICE EXPERIMENTS IN THE UNITED KINGDOM. Critical and exponential measurements have been made in a number of light water moderated lattices, fuelled with highly enriched uranium. The results are compared with theoretical predictions based on the MUFT-SOFOCATE few-group model, and shown to be in good agreement. Further work is reported on an experimental comparison between uranium-water and plutonium-water lattices which may point to a breakdown in the methods of calculation or to a discrepancy in the nuclear data in current use for plutonium.

QUELQUES EXPÉRIENCES AU ROYAUME-UNI SUR DES RÉSEAUX A EAU LÉGÈRE. On a procédé à des mesures critiques et exponentielles sur plusieurs réseaux de réacteurs ralentis à l'eau légère et utilisant comme combustible de l'uranium fortement enrichi. Les résultats obtenus sont comparés aux prévisions théoriques fondées sur le modèle MUFT-SOFOCATE à petit nombre de groupes; la concordance s'est révélée satisfaisante. Le mémoire fait également état d'autres travaux qui ont été exécutés en vue de comparer les réseaux uranium-eau et plutonium-eau sur une base expérimentale; cette comparaison peut faire apparaître une incohérence dans les méthodes de calcul ou une contradiction dans les données nucléaires qui sont actuellement utilisées dans le cas du plutonium.

НЕКОТОРЫЕ ОПЫТЫ С ЛЕГКОВОДНЫМИ РЕШЕТКАМИ В СОЕДИНЕННОМ КОРОЛЕВСТВЕ. Были проведены критические и экспериментальные измерения на ряде решеток реакторов с замедлителем из легкой воды и топливом из высокообогащенного урана. Результаты сравниваются с теоретическими предсказаниями, основанными на нескольких групповых моделях MUFT-SOFOCATE и находятся с ними в полном соответствии. Сообщается о дальнейшей работе по экспериментальному сравнению урановодяных и плутониеводяных решеток, которое может указать на неточность в методах расчета или на расхождение ядерных данных, используемых в настоящее время для плутония.

ALGUNOS EXPERIMENTOS CON RETICULADOS DE AGUA LIGERA REALIZADOS EN EL REINO UNIDO. Se determinaron datos críticos y exponenciales en una serie de reticulados de combustible de uranio muy enriquecido moderados por agua ligera. Se compararon los resultados de estas mediciones con los cálculos teóricos basados en el modelo MUFT-SOFOCATE de pocos grupos, comprobándose que concuerdan satisfactoriamente. Se dan a conocer los resultados de otros trabajos acerca de una comparación experimental entre reticulados uranio-agua y plutonio-agua que podrían indicar una falla en los métodos de cálculo o bien una discrepancia en los datos nucleares corrientemente utilizados para el plutonio.

INTRODUCTION

In the United Kingdom, experimental physics effort in the field of liquid moderated reactors is mainly concerned with pressure-tube systems containing both light and heavy water; however, a number of basic experiments have been carried out in lattices moderated by light water only, and these

experiments are described in this paper and the results compared with theoretical predictions. Details of the theoretical methods in current use for predicting light water lattice parameters in the United Kingdom are discussed in a separate paper [1].

A considerable number of light water moderated, low enrichment UO_2 lattices have been investigated, particularly in the United States of America, and methods of calculation based on the MUFT-SOFOCATE few-group model have been developed to predict the effective reproduction factor of such lattices in the cold clean condition to within about $\pm 1\%$ which is adequate for most purposes. One of the primary aims of the early work undertaken in the U. K. was to test the use of these methods of calculation in water moderated lattices fuelled with highly enriched uranium and containing a significant quantity of additional absorber in the form of stainless steel. Both critical and small exponential assemblies were used and the results are shown to be in good agreement in Part A of this paper, thus demonstrating the feasibility of making exponential measurements with small quantities of fuel. The highly heterogeneous nature of the assemblies necessitated detailed measurements of the thermal neutron density distribution, and these were compared with the results of calculations following the methods described in the companion paper [1]. In general it was found possible to account satisfactorily for the observed lattice reactivities although diffusion theory calculations underestimated the flux peaking in water gaps.

The use of highly enriched uranium and stainless steel in these lattices gave a high ratio of absorption to scattering and for this reason, particular attention was paid to the study of the neutron spectrum by both integral [3] and differential [5] techniques. The results of these measurements are also summarized in Part A of this paper.

On the completion of the above experiments, a further experimental investigation was undertaken to test the validity of the same methods of calculation for lattices fuelled with plutonium. To date, three uranium-water lattices have been compared with the three corresponding plutonium-water lattices with the results given in Part B of this paper. There is a significant discrepancy between the results for the two types of fuel, which could result from the use of an inadequate reactor model for plutonium fuel or from an error in the basic nuclear data used for plutonium in the present calculations.

Further experimental work on plutonium fuelled, light water moderated lattices is planned in the U. K. and this work is outlined in Part C of this paper.

CRITICAL AND EXPONENTIAL MEASUREMENTS IN LATTICES FUELLED WITH HIGHLY ENRICHED URANIUM

1. DESCRIPTION OF LATTICES INVESTIGATED

1.1. Critical experiments

Two core compositions (designated type 1 and type 2) were investigated, each containing significant quantities of absorber (stainless steel) in addition to the fuel and light water moderator. The fuel and stainless steel were in-

troduced in the form of thin rectangular plates approximately 0.018 in and 0.030 in thick respectively with an area of 3 in by 12 in. The fuel plates were uranium aluminium alloy containing about 11g of 93% enriched uranium in each plate. Fuel packs, 3 in $\times$ 3 in $\times$ 12 in were formed by clamping together steel and fuel plates using four fixing bolts and separating the plates by polythene spacing washers 0.05 in to 0.01 in thick. Two or three of these packs were loaded into a stainless steel box of open framework construction to form a fuel element. The boxes were positioned in the reactor by a lattice plate at the bottom and by drilled lattice beams at the top. In order to ease core construction and to allow for the insertion of control plates, a lattice pitch of 3.35 in was chosen, thus leaving water gaps approximately 0.35 in wide between adjoining fuel boxes. A typical unit cell in the reactor lattice is shown in Fig. 1 and the relative volumes of the constituent materials are given in Table I.

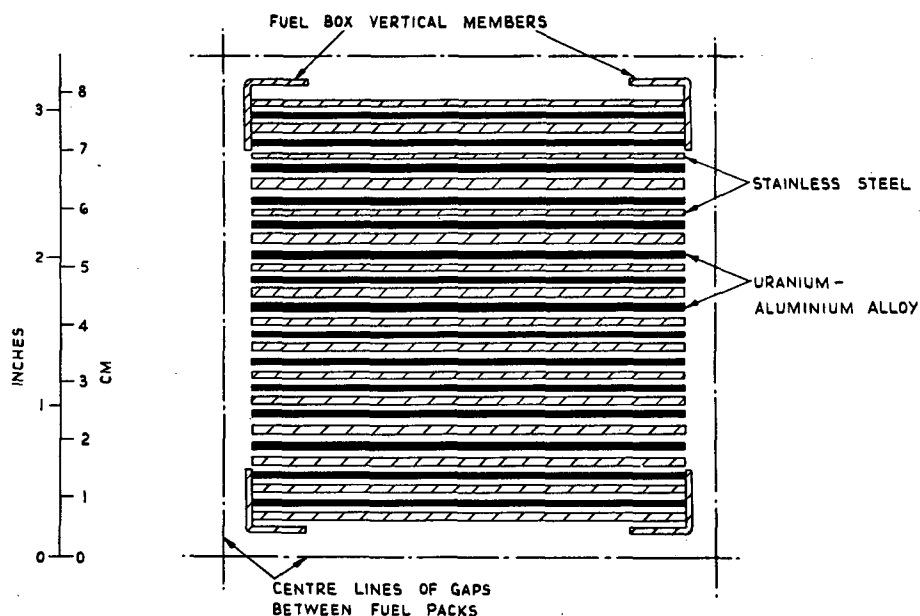


Fig. 1

Plan of a unit cell of type I fuel

The fuel packs used for the critical compositions were arranged in a series of rectangular cores ranging from 6 $\times$ 6 elements to 10 $\times$ 11 elements. In general the cores were critical in the "partially flooded" condition with the water level below the top of the fuel packs, and the reactor was controlled by the adjustment of the water height. However, for some experiments it was more convenient to work with a "fully flooded" core and in these cases control was exercised by the use of one or two fine control rods, each worth less than 0.5% reactivity.

1.2. Exponential experiments

The exponential facility consists of a shielded water tank above antimony-beryllium source of neutrons (strength approximately 1000 c giving approximately 10^{10} neutrons/s). Two methods of core construction were employed.

(a) Heterogeneous arrangement

Complete fuel elements, each containing two fuel packs were arranged to simulate a section of the critical cores described in 1.1. The fuel elements were held in position by a lattice plate at the bottom of the tank and by lattice beams at the top, as in the reactor. Buckling measurements were repeated in the lattice types 1 and 2 (See Table I) as well as in an additional lattice of similar construction, type 3.

(b) Near homogeneous arrangement

The gross water gaps between packs were eliminated by stacking fuel packs together to form a close packed rectangular array 2 ft high, using a light aluminium framework to prevent movement of the packs. Buckling measurements were made on 4 of these stacks, types 4 to 7 inclusive.

For both the critical and the exponential measurements, a range of core sizes was examined as a check on the validity of the measurements. Details of the lattices studied in each case are summarized in Table I.

2. SURVEY OF EXPERIMENTAL MEASUREMENTS

The spatial variation of the neutron flux in these systems may be approximately divided into three main components, namely:

(a) Macroscopic flux distribution

This is the overall flux distribution which would exist in an equivalent homogeneous reactor of identical shape and dimensions; the form is controlled by the leakage from the core boundaries and hence determines the buckling.

(b) Fine structure

This effect arises from the presence of water channels crossways 0.35 in wide between the 3 in squared fuel elements. The resulting local thermal flux variation decreases k_{eff} by about 3%.

(c) Hyperfine structure

This small thermal flux variation occurs because of the construction of the fuel packs from discrete plates separated by small water gaps. The effect of this small ripple in the thermal flux is important for these systems

TABLE I

DETAILS OF LATTICE COMPOSITIONS

Lattice Type	1	2	3	4	5	6	7
1. Specific volumes.							
Uranium 235	0.00415	0.00470	0.00331	0.00319	0.00389	0.00490	0.00590
Uranium 238	0.00031	0.00035	0.00025	0.00024	0.00029	0.00036	0.00044
Water	0.70430	0.69483	0.71851	0.65661	0.64653	0.62428	0.63158
Steel	0.22473	0.22473	0.22473	0.28961	0.28770	0.29277	0.26744
Aluminium	0.06651	0.07539	0.05320	0.05035	0.06159	0.07769	0.09464
2. Type of experiment							
Critical (C)	C	C					
or							
Exponential (E)	E	E	E	E	E	E	E
3. Physical arrangements							
Size of unit cell (in ²)	3.35	3.35	3.35	3.00	3.02	3.02	3.00
	× 3.35	× 3.35	× 3.35	× 2.95	× 3.03	× 3.03	× 2.98
Size of fuel pack (in ²)	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	× 2.84	× 2.88	× 2.88	× 2.88	× 2.88	× 2.88	× 2.88
4. Fuel pack construction							
Average number of:							
Fuel plates	15	17	12	9	11.5	14.5	17
Steel plates	16	18	24	18	11.5	14.5	18
Thickness in inches:							
Fuel plates (all)	0.018	0.018	0.018	0.018	0.018	0.018	0.018
Steel plates							
average	0.053	0.047	0.035	0.048	0.076	0.062	0.047
Water gaps							
average	0.057	0.051	0.052	0.071	0.082	0.061	0.051

NOTE: These specific volumes apply to a horizontal plane not including any pack bolts or fuel box webs but including the vertical fuel box numbers where applicable.

since the fuel and steel plates are separated and it reduces k_{eff} by about 2%.

Buckling measurements were made in each of the lattices of Table I and, in addition, the detailed local flux variations were measured in lattice types 1 and 2.

2. 1. Macroscopic flux distribution and buckling measurements

Because of the use of highly enriched uranium in these experiments, some of the cores investigated were unusually small, particularly in the case of the exponential experiments. It was important to establish that, under these conditions, the macroscopic flux distribution is spatially separable, i. e. $\phi(xyz) = X(x) Y(y) Z(z)$. In addition, both the critical and exponential systems were surrounded by effectively infinite water reflectors,

and the analysis of the experimental results carried out with an equivalent bare core model, assuming a horizontal extrapolation distance independent of core dimensions. The validity of this assumption was checked experimentally as described below.

2.1.1. Critical buckling measurements

The vertical flux distribution was measured by traversing small fission chambers up the core either in the centre of a fuel pack or in the inter-pack water gaps. The extrapolated height, and hence the extrapolation distance, was determined by fitting a cosine function to the flux values measured in the central core region using a least squares fitting programme. Points within 6 cm of the core boundary were omitted because of the perturbing effect of the reflector in this region.

Since the reactor was brought to the critical condition by raising the water level, the top reflector consisted, in general, of a short length of unimmersed fuel pack which reflected fast neutrons. The top extrapolation distance was found to be a function of the height of the unimmersed fuel, rising from about 3 cm with the water surface at the top of the fuel to about 7 cm for more than 30 cm of fuel projecting. There is also a small dependence on the transverse buckling of the core, probably caused by its effect on the fast neutron leakage from the sides of the dry packed region. A typical vertical scan made with a bare U^{235} fission chamber is shown in Fig. 2 together with the calculated U^{235} reaction rate. In view of the uncertainty associated with the extrapolation distance into the dry fuel pack region, the agreement is quite satisfactory.

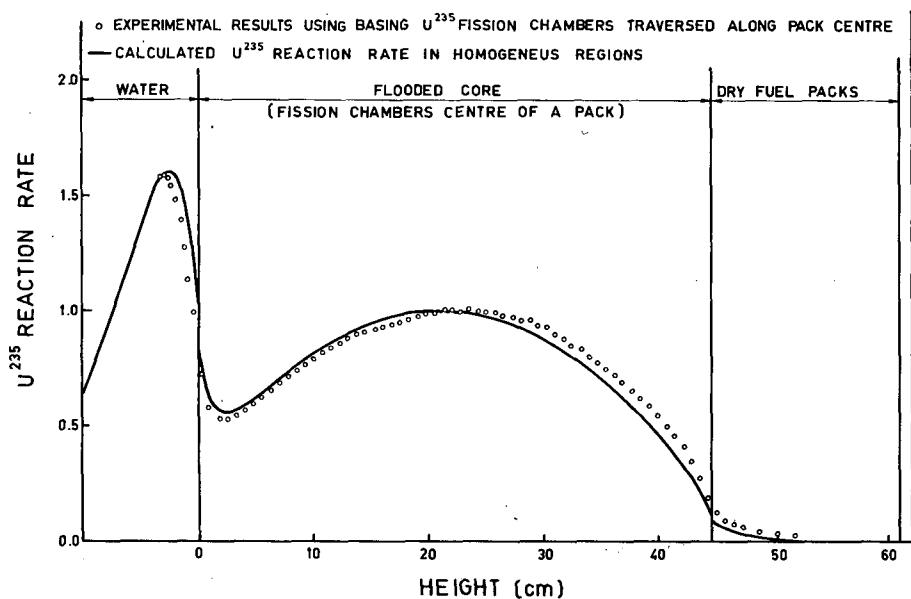


Fig. 2

Vertical distribution of U^{235} fission rate in critical core

The separability of the spatial variables was checked by measuring the vertical flux distribution at various distances from the vertical axis in two of the cores investigated. The results showed no evidence of a trend in the vertical extrapolation distance with horizontal position to within 4 cm of the core boundary.

The horizontal flux measurements with detectors sensitive to thermal neutrons were complicated by the fine structure caused by the transverse water gaps as shown in Fig. 3. It was found that measurements made at

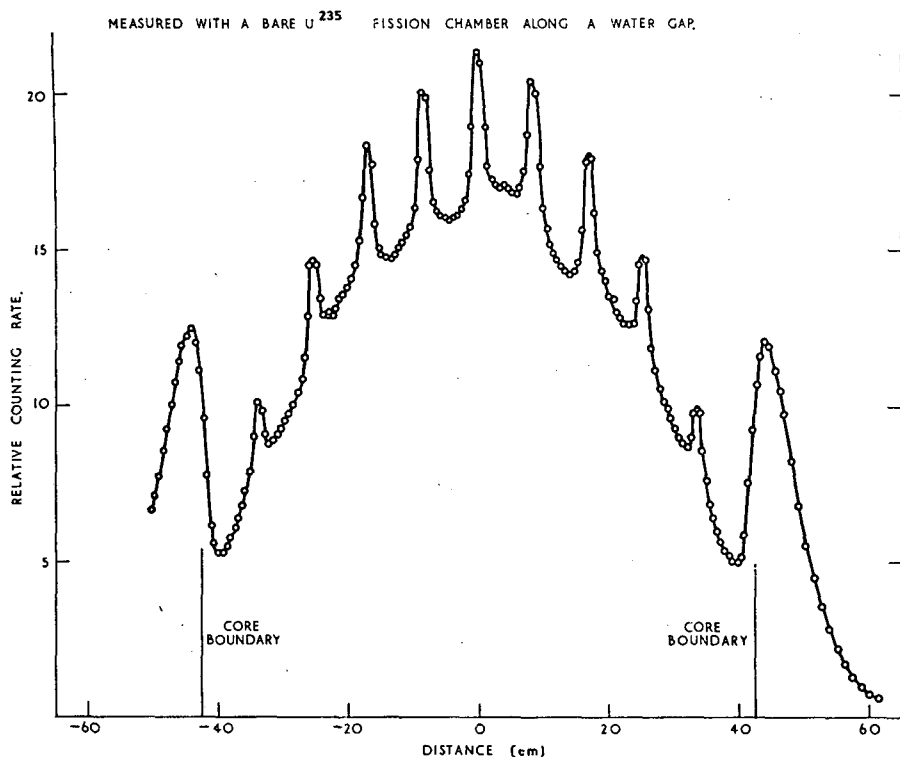


Fig. 3

Horizontal distribution of U^{235} fission rate in critical core

corresponding points in different cells were well fitted by a cosine function, and that when the original distribution was divided by the cosine function, the fine structure obtained was constant across the core to within the experimental limits. The extrapolated widths were obtained by fitting a cosine function to corresponding points in each lattice cell. Within the experimental limits (± 1.5 cm) the total horizontal extrapolation distance was found to be insensitive to changes in core size within the range considered, i. e. from 51 cm to 85 cm.

As an overall check on the critical buckling data, measurements were made by the above methods on five cores of the same composition but different dimensions. The cores were not uniform in the vertical direction because

TABLE II

MATERIAL BUCKLING DEDUCED FROM CORES OF
DIFFERENT DIMENSIONS

Core dimensions length $\times$ breadth $\times$ critical height (in cm.)	Measured buckling $\times 10^4 \text{ cm}^{-2}$	Perturbation $\Delta k_{\text{eff}}/k_{\text{eff}}$	Corrected buckling (for a homogeneous core) $\times 10^4 \text{ cm}^{-2}$
85.1 $\times$ 85.1 $\times$ 33.8	63.5 $\pm$ 0.5	1.16%	67.6 $\pm$ 0.5
68.1 $\times$ 68.1 $\times$ 41.5	63.0 $\pm$ 0.8	1.16%	67.4 $\pm$ 0.9
59.6 $\times$ 68.1 $\times$ 45.4	63.5 $\pm$ 0.9	1.30%	68.0 $\pm$ 1.0
59.6 $\times$ 59.6 $\times$ 47.9	63.6 $\pm$ 0.8	1.31%	68.2 $\pm$ 0.9
51.1 $\times$ 51.1 $\times$ 68.0	63.4 $\pm$ 1.2	1.33%	68.0 $\pm$ 1.3

of the fuel pack fixing bolts and structural webs on the fuel boxes, and the statistical weights of these items varied with the shape of the core. These effects were calculated using two-group perturbation theory and used to correct the measured material bucklings as shown in Table II. Within the experimental errors, results from the various cores are in good agreement.

2. 1. 2. Exponential buckling measurements

As in the critical experiments, macroscopic flux distributions in the exponential assemblies were measured with small fission chambers, $\frac{1}{8}$ in or $\frac{1}{4}$ in diam, but check experiments were carried out with small manganese foils to ensure that :

- (a) The cadmium ratio was constant in the region of measurement, (true up to 6 cm from the core reflector interface);
- (b) The contribution of higher harmonics to the flux distribution was small, (less than 1% above 25 cm from the base of the assembly);
- (c) The vertical stem of the fission chamber did not represent an unacceptable perturbation. (Relaxation length was the same whether measured by foils or fission chambers.)

The data from vertical fission chamber measurements were fitted to the function $\Phi = A \sinh [(c-z)/b_{11}]$, where b_{11} is the relaxation length, and c is the extrapolated core height.

Horizontal fission chamber measurements were extended into the reflector region on either side of the core, and the results fitted to the function $\Phi = B \cos (\pi x/a) + C \cosh vx$ by an iteration procedure (See Fig. 4). It was found possible to determine the extrapolated widths to within ± 1.0 cm by these means in the lattices (types 4 to 7) which were nearly homogeneous. In the bunched lattices, however, this method was impracticable and a different method was used as outlined below.

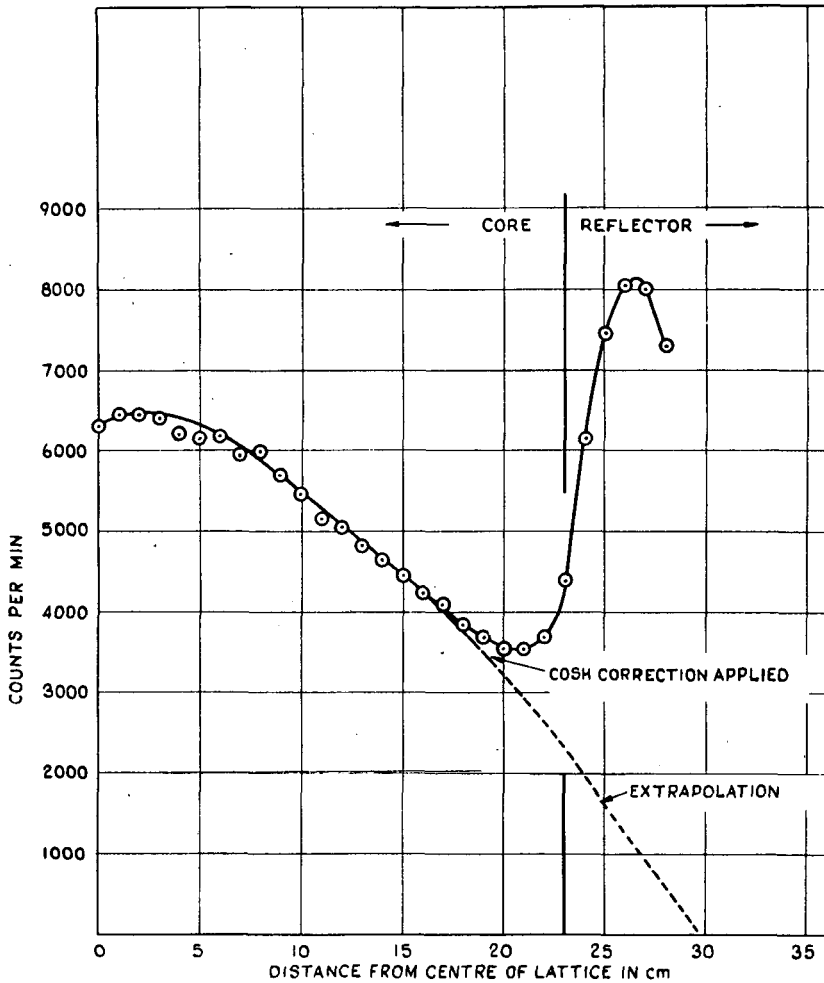


Fig.4

Horizontal flux scan in exponential assembly

Since the horizontal size of the systems was limited to an area less than 18 in square by the exponential criterion, it was clearly important to establish the validity in this case of the methods of analysis normally applied to exponential measurements. This was done in two ways:

- (a) Vertical relaxation measurements were made in a number of cores of the same composition but different size. The material buckling of the lattice was calculated as a function of assumed extrapolation distance and plotted for each core size as shown in Fig. 5. Provided that the extrapolation distance is truly independent of the core size, one would expect the lines representing the various assemblies to meet near a

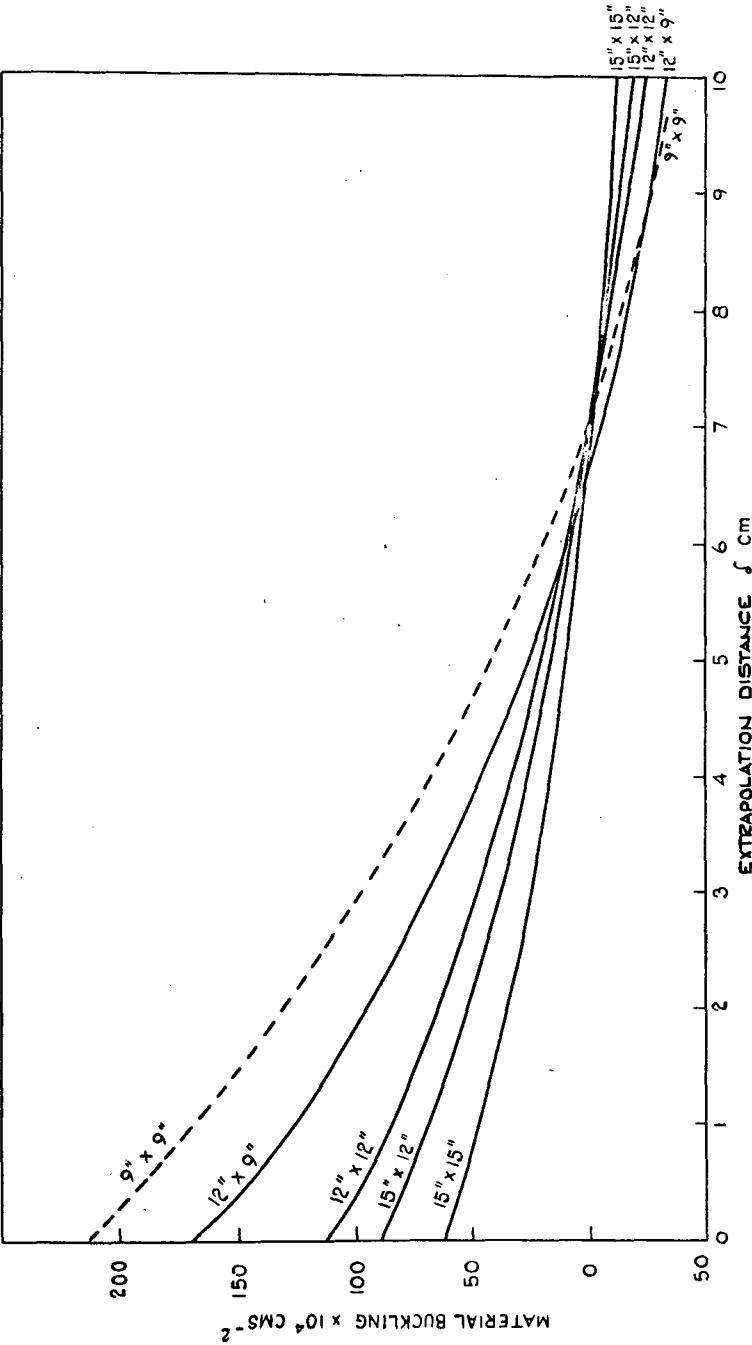


Fig. 5
Determination of extrapolation distance

common point giving the "best" values of B_m^2 and horizontal extrapolation distance. It may be seen that a common area of intersection may be ascribed to the lines representing the four larger assemblies, but the line representing the smallest assembly, (9 in $\times$ 9 in), does not pass through this area. On these grounds, a minimum size of 12 in $\times$ 12 in was defined for assemblies of this kind. In cases where the horizontal flux distribution was perturbed by the presence of water gaps, this measurement provided an alternative method of determining the extrapolated width of these systems to within an accuracy of ± 1.5 cm.

- (b) Exponential measurements of the material buckling of lattice types 1 and 2 were compared with critical buckling measurements in considerably larger lattices of the same composition. The results are included in Table III and it is clear that within the limits of accuracy of the experimental determinations, results from the two different methods are in agreement.

TABLE III

MATERIAL BUCKLING MEASUREMENTS IN CRITICAL AND
EXPONENTIAL ASSEMBLIES ($\times 10^4$ cms^2)

Lattice type	1	2	3	4	5	6	7
(a) Critical	46.7 $\pm$ 0.3	63.5 $\pm$ 0.5	-	-	-	-	-
(b) Exponential	46.6 $\pm$ 3.5	62.0 $\pm$ 3.0	19.0 $\pm$ 5.0	8.8 $\pm$ 5.4	38.4 $\pm$ 5.1	62.9 $\pm$ 4.9	80.8 $\pm$ 3.0

2.2. Fine and hyperfine structure measurements

Detailed flux distribution measurements were made in lattice types 1 and 2 to check the methods of calculation used to estimate the fine and hyperfine structure in the thermal neutron distribution. No significant fine structure was observed in the epicalcium flux distribution (i.e. $E > 0.45$ eV.)

The fine structure in the thermal neutron density was measured by irradiating small metal foils at about 120 different points in a typical cell, on the surfaces of fuel and steel plates, in the small intervening water gaps, and in the larger water gaps between fuel packs. In cases where foil holders were required, perspex was used since it has been shown [2] to minimize the perturbation introduced into a light water moderated system. Several irradiations were required so that adjacent foils were never closer than 1 cm in any one run. The observed activities were corrected for the effect of the macroscopic flux curvature and the epicalcium activation. A typical result is shown in Fig. 6 where the thermal activity of the manganese foils is compared with theoretical predictions using SOFOCATE constants in a diffusion theory calculation as discussed in [1]. As in all the cases investigated, the peaking in the neutron density distribution is underestimated by the diffusion theory calculation.

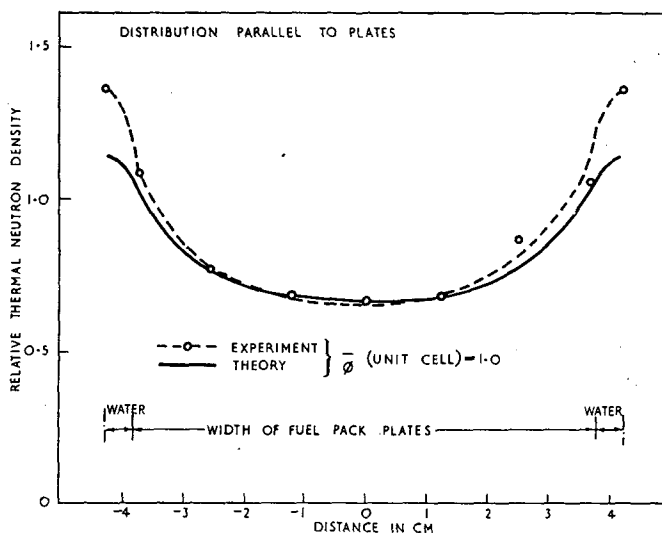


Fig. 6

Fine structure in the thermal neutron density

The above measurements provided average thermal neutron densities in the water gaps between plates and between pellets, and at the surface of the absorbing plates. However, the neutron density within an absorbing plate is depressed owing to the self shielding effect. This was measured in a single fuel plate *in situ* in the centre of a fuel pack, using a multiple foil technique with foils of the fuel plate alloy.

2.3. Measurements of neutron spectrum

2.3.1. Integral spectrum measurements [3]

A wide variety of activation detectors were used to calibrate the thermal and epithermal neutron spectra in the critical lattices 1 and 2, by means of cadmium ratio and fission rate ratio measurements. A cadmium box thickness of 0.025 in was used for the measurements in the centre of the fuel pack region, and the results were analysed with the assumption of an equivalent cadmium cut-off energy of 0.46 eV [4] and a Wigner-Wilkins form of the low energy neutron spectrum as used in SOFOCATE. Each reaction rate ratio was used to deduce a value of the spectrum index $\beta\gamma = \Sigma_a(KT)/\xi \Sigma_s$, where $\Sigma_a(KT)$ is the macroscopic absorption cross-section of the lattice at the moderator temperature, and Σ_s is the macroscopic scattering cross-section of the moderator, and the results are shown in Table IV and compared with calculated values of $\beta\gamma$. These errors quoted are statistical errors only and do not take into account any errors in cross-section data.

The assumption of a Wigner-Wilkins spectrum in the analysis of the experiments is shown to give consistent results over a wide range of

detectors of varying energy sensitivity and to give good agreement with theoretical prediction.

TABLE IV

VALUES OF $\beta\gamma$ FROM INTEGRAL SPECTRUM MEASUREMENTS

Material		Type 1 fuel pack	Type 2 fuel pack
Vanadium	C	0.276 ± 0.008	0.327 ± 0.010
Manganese	C	0.285 ± 0.014	0.306 ± 0.020
Gold	C	0.260 ± 0.018	-
U ²³⁵ foil	C	0.290 ± 0.008	0.32 ± 0.010
U ²³⁵ chamber	C	0.286 ± 0.008	0.316 ± 0.010
U ²³⁵ chamber	C	0.287 ± 0.009	0.320 ± 0.008
Pu ²³⁹ chamber	C	0.285 ± 0.010	0.311 ± 0.008
U ²³⁵ /U ²³⁸	F	0.270 ± 0.040	0.29
Pu ²³⁹ /U ²³⁵	F	0.270 ± 0.030	
Mean experimental value		0.282	0.317
Predicted value		0.282	0.309

2.3.2. Differential spectrum measurements [5]

As an additional check on the validity of the Wigner-Wilkins spectrum model in these systems, differential spectrum measurements were made in subcritical assemblies of lattice types 1 and 2 using the time-of-flight technique. Each assembly was approximately 17 in $\times$ 17 in $\times$ 12 in high and was surrounded by a water reflector. Using pulses of electrons from a linear accelerator, incident on a uranium target, normal time of flight measurements were made over the energy range 0.005 eV to 15.8 eV in the centre of a fuel pack and in an interpack water gap. Additional foil activation measurements were made near the end of the flight path tube, (0.75 in O.D.) to provide data for deriving suitable correction factors to the observed spectra. The results of the measurements in fuel packs of type 1 are shown in Fig. 7, and are compared with the predicted Wigner-Wilkins spectrum for an infinite medium of type 1 composition. Generally the spectra are in good agreement, the maximum discrepancy, approximately 8%, occurring at epithermal energies.

3. Comparison of theory and experiment

The MUFT-SOFOGATE computer codes can be used to predict average group spectra for homogeneous reactor systems. The lattices investigated in this work could not be considered as homogeneous, and hence the atomic number densities of the core constituents were weighted by the average neutron densities appropriate to each material. Since the discrepancies

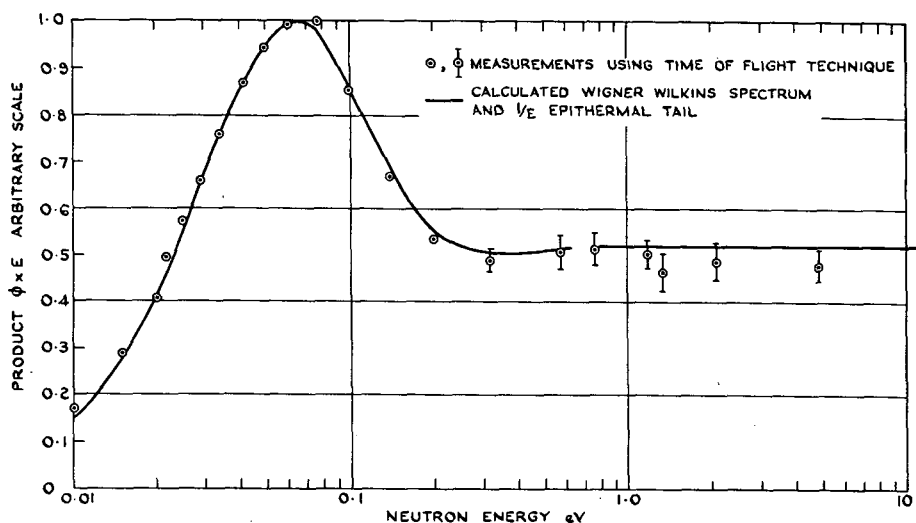


Fig. 7

Time-of-flight measurements compared with Wigner-Wilkins spectrum

between measurement and prediction of the thermal neutron distribution are significant in the water gaps only, difference in the effect on reactivity is small, and it was found that measured or predicted weighting factors could be used without appreciably affecting the calculated result.

It is convenient to compare the measurement of material buckling with theoretical prediction by using it to estimate the k_{eff} of the critical system as given in Table V. The departure of k_{eff} from unity then gives a measure of the consistency of the calculations with the experimental results. Values of k_{eff} within 1% of unity were obtained in all cases.

TABLE V

k_{eff} FOR SYSTEMS MEASURED IN CRITICAL AND EXPONENTIAL EXPERIMENTS

Lattice	1	2	3	4	5	6	7
k_{eff}	1.003 ± 0.001	0.999 ± 0.001	1.007 ± 0.012	0.990 ± 0.017	0.993 ± 0.015	0.990 ± 0.014	1.005 ± 0.008

4. Conclusions

- It has been shown that methods of calculation based on the use of a MUFT-SQFOCATE few-group model are adequate to determine the reactivity of highly enriched uranium-water systems, containing significant quantities of additional absorber, to within $\pm 1\%$.

- (b) It has been shown that material buckling measurements can be made in reflected light water moderated lattices as small as 12 in $\times$ 12 in in area if the lattices are nearly homogeneous, but the accuracy is limited by the width measurements to about $\pm 3 \times 10^{-4}$ cm².
- (c) Differential and integral spectrum measurements have shown that the Wigner-Wilkins model gives a good representation of the neutron spectrum in these undermoderated lattices.

EXPERIMENTAL COMPARISON OF URANIUM-WATER AND PLUTONIUM-WATER LATTICES

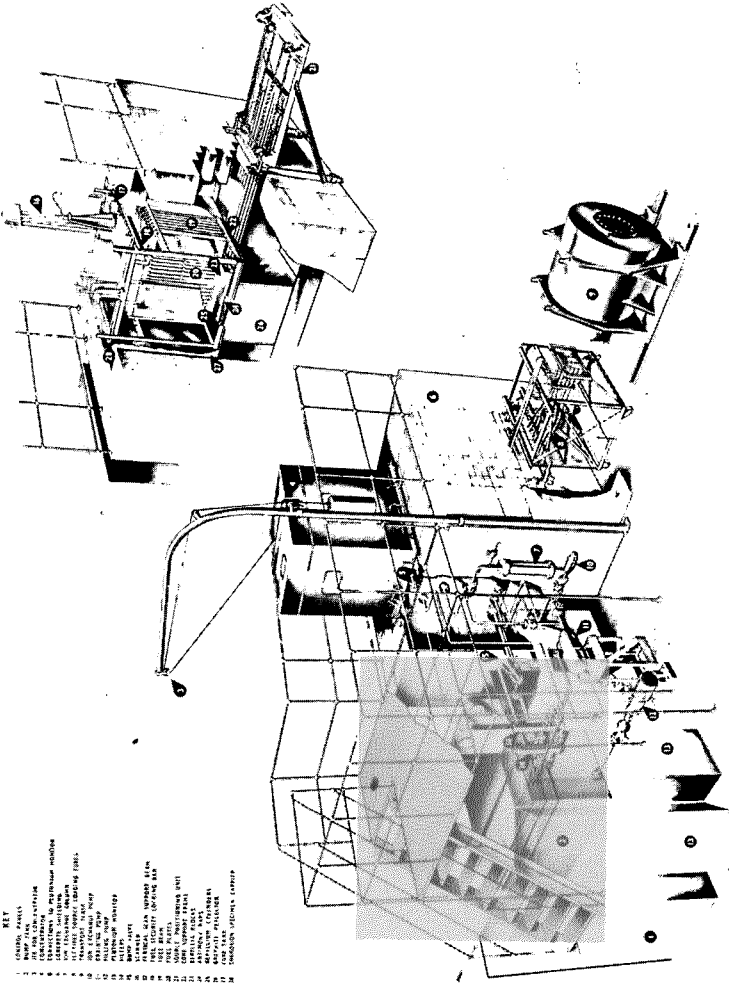
1. AIMS OF EXPERIMENTS

To provide a rigorous test of methods of calculation for water moderated systems containing plutonium, it is essential to make measurements in undermoderated systems where the hardening of the thermal neutron spectrum will significantly affect the number of neutrons captured in the plutonium resonance at 0.3 eV. However, as a preliminary to this work, it was decided to carry out some check experiments in well moderated uranium-water and plutonium-water systems to test the methods of calculation used for such systems in the United Kingdom [1] and to confirm the basic nuclear data in current use for plutonium. These preliminary experiments have now been completed, although some of the supporting theoretical work is still to be done, and the results are reported briefly in the following pages. The corresponding measurements in under-moderated lattices are planned at AEE Winfrith as discussed in Part C of this paper.

2. DESCRIPTION OF EXPERIMENTAL PLANT HELEN I (Hydrogeneous Exponential Liquid-Moderated Experiment.)

HELEN I is an exponential assembly (Fig. 8) consisting essentially of a core tank 4 ft $\times$ 4 ft $\times$ 4 ft high, situated over a plane antimony-beryllium source of neutrons, and surrounded on all sides, but not on top, by biological shielding. The neutron source strength is approx. 10^{10} n/s., giving a maximum thermal flux in the core tank of 5×10^5 n/cm²/s. A dump tank, dump valve, iron exchange column and pumping circuit are provided for the liquid moderator, and two neutron detectors are positioned close to opposite faces of the core and are interlocked with the pump and dump valve to safeguard the plant under all operating conditions.

Automatic scanning equipment is provided to measure macroscopic flux distributions along horizontal or vertical paths anywhere in the core. Once the equipment has been set in motion, it moves a $\frac{1}{4}$ in diam fission chamber along the scanning line to a pre-set starting point, measures the time to obtain a given number of counts and prints out this time and position, moves a pre-selected interval, counts again, etc. In this way good statistical accuracy can be obtained overnight while the plant is unattended, and a noticeable improvement has been observed in the reliability of the results, compared with those obtained previously when moving the fission chamber by hand.



- KEY
- 1 COVER, MAIN
 - 2 BOLT, 1/2 IN.
 - 3 GASKET, 1/2 IN. X 1/2 IN.
 - 4 CRANKSHAFT, 1/2 IN. X 1/2 IN.
 - 5 CRANK PIVOT, 1/2 IN. X 1/2 IN.
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Fig. 8
HELEN I

3. DESCRIPTION OF LATTICES INVESTIGATED

Since the aim of these experiments was the direct comparison of theory and experiment, it was important to maintain simple core geometry. The fuel was in the form of uncanned uranium/aluminium alloy strips, 0.040 in thick, 1.2 in wide, 28 in long, and plutonium/aluminium alloy 0.010 × 1.2 in wide × 28 in high canned in 0.010 in thick aluminium. A small quantity of air was trapped inside the aluminium cans of the plutonium fuel strips: the volume of this air was added to the volume of the aluminium can for the purpose of comparison with theory. These fuel strips were fitted side by side into slotted perspex sub-assemblies to form quasi continuous plates of fissile material extending right across the core (Fig. 9).

In this way, fine structure in the thermal neutron density distribution was limited to the direction perpendicular to the fuel plates only. The perpendicular distance between plates could be varied by using different

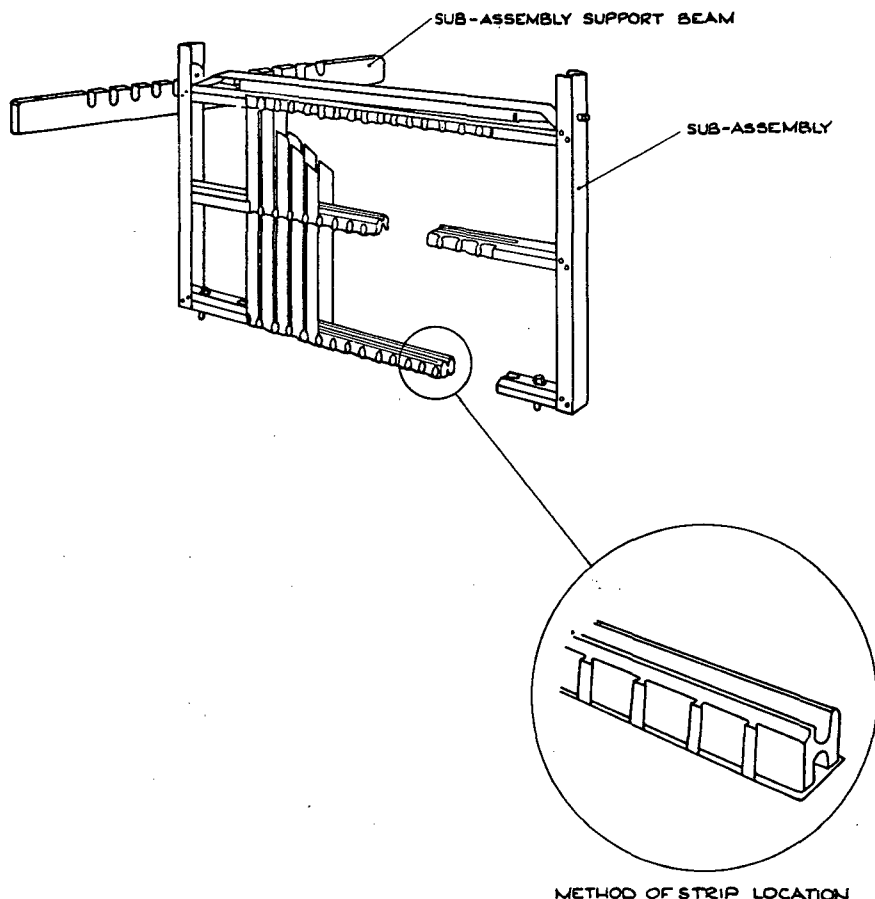


Fig. 9

Diagram of sub-assembly with fuel strips

sets of sub-assemblies and changing the slotted support bars in the beam support system. The cores were typically about 20 in square and 28 in high so that when built in the 4 ft square core tank they had a radial reflector which was effectively infinite. Three uranium-fuelled cores were compared with the three corresponding plutonium fuelled cores. Details of the lattices are given in Table VI.

TABLE VI
DETAILS OF LATTICES STUDIED IN HELEN I.

Lattice number	Fuel	Pitch in	Specific volumes			$N_H/N_{U^{235}}$	N_H/N_{Pu}
			Fuel	Water	Aluminium		
1	U	1.32 in	0.001000	0.971749	0.027252	1.37×10^3	-
2	U	1.07 in	0.001233	0.965149	0.336180	1.10×10^3	-
3	U	0.82 in	0.001609	0.954523	0.043869	0.84×10^3	-
4	Pu	3.32 in	0.000393	0.973800	0.027807	-	2.98×10^3
5	Pu	1.07 in	0.000485	0.965210	0.034304	-	2.40×10^3
6	Pu	0.82 in	0.000633	0.954600	0.044762	-	1.81×10^3

4. SURVEY OF EXPERIMENTAL MEASUREMENTS

The following measurements were made;

- Material buckling.
- Fine structure in the thermal neutron density distribution across the water gap between fuel strips, and inside fuel strips. (Uranium cores only.)
- Relative reaction rates of various activation detectors, to give information on thermal and epithermal neutron spectra.

4.1. Measurement of material buckling

The vertical flux distribution was measured with the automatic scanning equipment described above. The fuel strips are of uranium aluminium alloy and bend easily, and in some cases this changed the vertical flux distribution slightly by its differential effect on the thermal fine structure at various heights. However, due to the random nature of this effect in different parts of the core, and the repeatability of the relaxation length measurements in any given core position, (± 1 mm), it was considered adequate to average relaxation length over a range of measuring positions. Vertical flux scans were made in a minimum of seven different horizontal positions in each core.

In the horizontal direction the extrapolated width of each core parallel to the plates of fissile material was obtained by scanning along the centres of the water channels and fitting the measurements to the form

$$\phi(x) = A \cos (\pi x/a),$$

where a is the extrapolated width. The results are shown in graphical form in Fig. 10, where the extrapolated width a is plotted against the physical width a_0 for each core. The points representing both uranium and plutonium

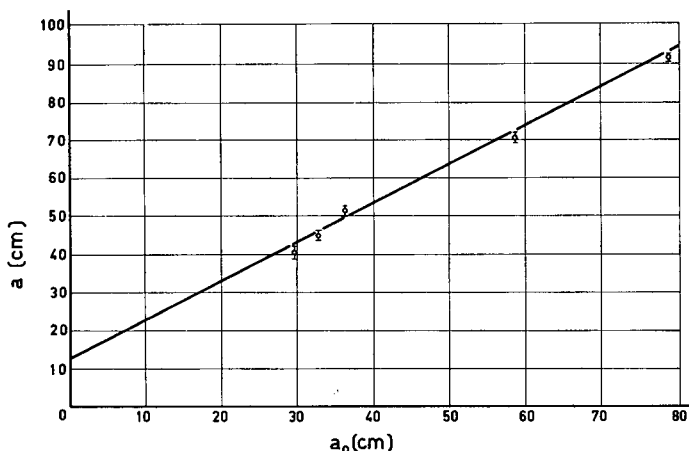


Fig. 10

Plot of extrapolated width (a) as a function of physical width (a_0)

fuelled cores are well fitted by a straight line inclined at 45° to the horizontal axis. Least square fits to the data yield the following values for the intercepts on the vertical axis:

Uranium fuelled cores, intercept = 12.97 ± 0.69

Plutonium fuelled cores, intercept = 13.00 ± 0.36

While the close agreement between these two values is certainly fortuitous it is clear that:

- (i) The extrapolated width of each core can be found by adding a fixed extrapolation distance to the physical width;
- (ii) The extrapolation distance does not vary with type of fuel or core pitch in the range of cores investigated.

This is probably explained by the fact that all the cores are well over-moderated ($NH/FUEL$ in the range 9×10^2 to 3×10^3), and contain very little fuel so that the slowing down and diffusion lengths in the core are determined almost entirely by the properties of the moderator. The result of a least squares fit to a straight line of slope 45° , gives the extrapolation distance = 6.50 ± 0.18 cm, and this value was used to compute all extrapolated widths in the direction parallel to the fuel strips.

The measurement of extrapolated widths in the direction perpendicular to the fuel strips was complicated by the very significant fine structure in the thermal neutron density distribution (Fig. 11). Values of the extrapolation distance were inferred from measurements of the vertical relaxation

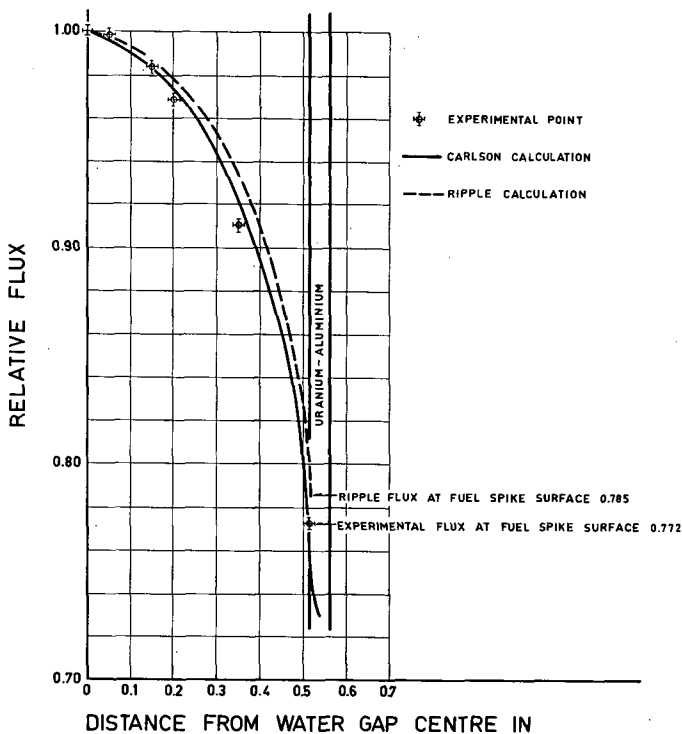


Fig. 11

Fine structure in the thermal neutron distribution in Core 3

distance in a range of cores of varying physical size by the method described in Part A. (2.1.2) of this paper. As with the measurements in the direction parallel to the fuel plates, a common extrapolation distance was used for equivalent uranium and plutonium cores. Results of these measurements are combined with the determinations of average vertical relaxation distance in each core to give the material bucklings in Table VII.

4.2. Measurement of thermal fine structure and flux depression

The thermal neutron density distribution perpendicular to the fuel strips in HELEN I varies rapidly with the width of the intervening water gaps, and it was therefore necessary to make special arrangements for the accurate location of the rather flexible fuel strips in the locality of the measurements. This was accomplished by the use of light perspex holders which combined the functions of:

- (i) Holding adjacent fuel strips at the correct pitch at the position of measurement; and
- (ii) Locating bare manganese foils ($\frac{1}{4}$ in O. D., 0.005 in thick) at known positions in the water gap between the two fuel strips.

TABLE VII
MATERIAL BUCKLING MEASUREMENTS IN HELEN I

Lattice number	Fuel	Extrapolated widths (cm)		Vertical relaxation distance (cm)	Material buckling $\times 10^4 \text{ cm}^2$
		$\frac{a}{x}$	$\frac{a}{y}$		
1	U	71.7 ± 0.4	73.4 ± 2.5	18.58 ± 0.10	$+8.6 \pm 1.3$
2	U	54.9 ± 0.4	55.5 ± 2.0	17.31 ± 0.08	$+46.3 \pm 2.4$
3	U	42.3 ± 0.4	44.8 ± 1.7	27.12 ± 0.22	$+90.9 \pm 3.9$
4	Pu	91.0 ± 0.4	80.5 ± 2.7	15.39 ± 0.05	-15.1 ± 1.1
5	Pu	91.0 ± 0.4	66.7 ± 2.3	28.06 ± 0.19	$+21.4 \pm 1.5$
6	Pu	65.0 ± 0.4	44.8 ± 2.0	36.77 ± 0.39	$+64.7 \pm 4.4$

During an irradiation, the vertical side members were attached to adjoining fuel strips using "Lassothene" tape and monitor foils were similarly secured in shallow depressions provided in the top and bottom members. After the irradiations, the foil activities (β and γ) were measured with scintillation counters using 1 mm plastic phosphors.

The measured fine structure in each core were corrected for the overall cosine distribution using data from Table VII, and the results, normalized to unity at the foil position in the centre of the water gap, were compared with theoretically predicted density distributions. Typical results for core 3 are shown in Fig. 11. The error limits ascribed to the experimental points are standard deviations and take into account both positioning errors and counting statistics.

In comparing theory with experiment infinite slab geometry was assumed and two methods of calculation were used. In the first, a collision theory programme RIPPLE [6] was used to calculate the neutron density distribution assuming an average core spectrum throughout the cell. It can be seen from Fig. 11 that this calculation underestimates the moderator peaking. The second calculation using the CARLSON SNG transport theory code, allowed variations in the neutron spectrum across the cell and gave a much better fit to the experimental results.

The flux depression in the uranium fuel strips was measured by the multiple foil method described in Part A 2.1.1 of this paper.

For the initial comparison with theory described in section 5 below, RIPPLE calculations were used to weight the atomic number densities to those for the equivalent homogeneous core. Further calculations using the CARLSON SNG code have not yet been completed.

4.3. Measurement of neutron spectrum

In the very undermoderated cores investigated in these experiments it was anticipated that the thermal neutron spectrum in the centre of the

water gap would be a Maxwellian with a peak near $E_0 = kT$, (where T is the physical temperature of the moderator), and that the proportion of epithermal neutrons would be small. To verify these assumptions and to provide a basis for comparison with future spectrum measurements in under-moderated systems, a series of integral spectrum measurements was made using a variety of activation detectors in the centre of the water gaps and at positions adjacent to the fuel strips.

The analysis of these experimental results is in progress and is based on a comparison between the measured reaction rates and the theoretical predictions using tabulated cross-sections integrated over neutron spectra calculated by the CARLSON SNG code. However, in the case of these well moderated systems, it is possible to obtain some preliminary information by assuming a much simplified form of the neutron spectrum and using published tables of effective cross-sections [7, 8]. Typical results for core 3, based on the Westcott spectrum model [7] are shown in Fig. 12. Reaction rate ratios for Pu/U, Lu/Mn, and Bare U/(Cd. covered U) are plotted as a function of r , the epithermal index and T , the "Effective neutron temperature". The results are in good agreement and show that the assumption of a Maxwellian peaking at ambient temperature is reasonable in this simple case, and that the number of neutrons per logarithmic energy interval in the $1/E$ tail is about 4% of the thermal neutrons. Later experiments planned in HELEN I should provide quantitative information on the extent to which such simple models breakdown in under-moderated systems.

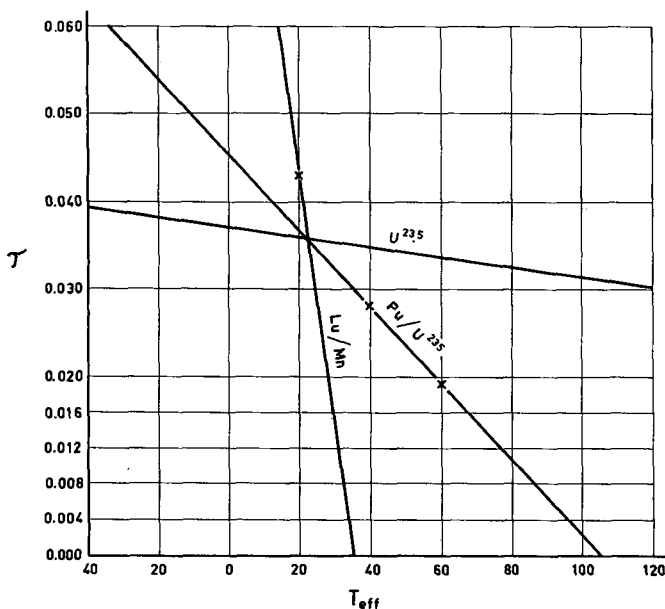


Fig. 12

Plot of r as a function of T_{eff} in centre of water gas (Core 3)

5. COMPARISON OF THEORY AND EXPERIMENT

The comparison of theory and experiment for these uranium and plutonium fuelled cores was carried out as for the uranium-steel-water systems described in Part A of this paper, i.e. the measured material buckling was introduced into the theoretical calculation to predict the k_{eff}

TABLE VIII

k_{eff} FOR URANIUM AND PLUTONIUM FUELLED LATTICES IN HELEN I

Core	Fuel	Pitch (in.)	Measured Buckling $\times 10^4 \text{ cm}^2$	k_{eff}	
				RIPPLE	CARLSON
1	U ²³⁵ /Al	1.32	8.6 ± 1.3	1.004 ± 0.004	0.984 ± 0.004
2	U ²³⁵ /Al	1.07	46.3 ± 2.4	1.016 ± 0.007	0.996 ± 0.007
3	U ²³⁵ /Al	0.82	90.0 ± 3.9	1.007 ± 0.011	0.989 ± 0.001
4	Pu ²³⁹ /Al	1.32	-15.1 ± 1.0	0.980 ± 0.003	-
5	Pu ²³⁹ /Al	1.07	21.4 ± 1.5	0.980 ± 0.004	-
6	Pu ²³⁹ /Al	0.82	64.7 ± 4.4	0.974 ± 0.013	-

of the equivalent critical systems with the results given in Table VIII and illustrated in Fig. 13. For the uranium cores, the fine structure was calculated by the one thermal group RIPPLE programme and by the CARLSON code. For the plutonium cores, only the RIPPLE results are at present available.

In each case, the k_{eff} for the uranium fuelled lattice is nearer unity, and if the calculations based on RIPPLE fine structure are compared, there is a discrepancy of about 3% between the values of k_{eff} for corresponding uranium and plutonium cores. Some of this discrepancy may result from the different effects of spectrum variation in the two cases, which is not taken into account in the RIPPLE calculations. This discrepancy could also point to an error in the value of η_0 (Pu²³⁹) in current use (i.e. $\eta = 2.079$ at 2200 m/s); the relative importance of these effects will be shown by the comparison of CARLSON type calculations for the uranium and plutonium cores when these are available.

6. CONCLUSIONS

Three uranium fuelled cores have been compared with three similar plutonium cores, using identical measuring techniques and methods of lattice calculation. Values of k_{eff} for the equivalent uranium fuelled cores are nearer unity and are 2 - 3% higher than corresponding values for the plutonium fuelled cores when the calculations of thermal fine structure are based on a one group collision theory calculation. This discrepancy may well

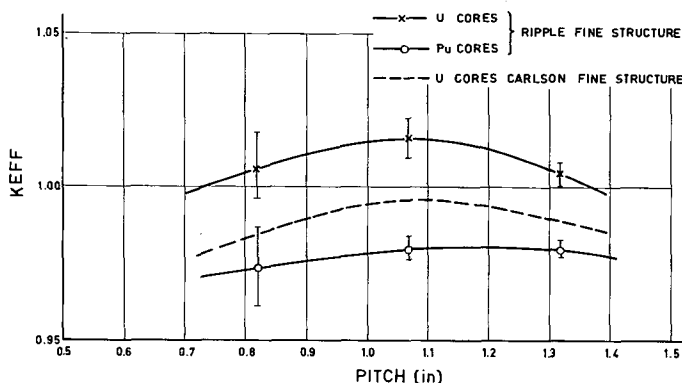


Fig. 13

Values of k_{eff} for U and Pu cores

be reduced when CARLSON type calculations of fine structure are available for both plutonium and uranium cores, but may still point to an error in the value of η_0 (Pu^{239}) = 2.079, used in the world consistent set. The present fit between theory and experiment would be improved by an increase in the assumed value of $\eta(\text{Pu}^{239})$.

PART C

PLANS FOR FUTURE EXPERIMENTAL WORK IN LIGHT WATER MODERATED LATTICES AT AEE WINFRITH.

The work in over-moderated plutonium and uranium systems described in Part B of this paper has confirmed that the methods of lattice calculation in current use in the United Kingdom for highly enriched water moderated lattices are sufficiently accurate for most purposes, but that some further work may be required for systems containing plutonium. Any inadequacy in these methods of calculation will be highlighted by the second series of experiments planned at Winfrith, in which identical under-moderated uranium and plutonium cores will be intercompared (See Fig. 14). It is anticipated that in some of the cores there will be a significant depression in the neutron spectrum at 0.3 eV due to absorption in the plutonium resonance. Since the material bucklings of these systems will be very sensitive to the shape of the thermal and epithermal spectra, these measurements should provide a very critical test of the methods of calculation for plutonium water systems.

Some of the other experimental and theoretical work being undertaken at Winfrith, as part of the general United Kingdom programme for liquid moderated systems, will be of considerable interest in the light water field. In particular, as part of the investigation of steam generating heavy water systems, measurements of neutron spectrum, initial conversion ratio and fast fission factor will be carried out in pressure tube systems, fuelled with uranium oxide rods containing varying proportions of plutonium oxide.

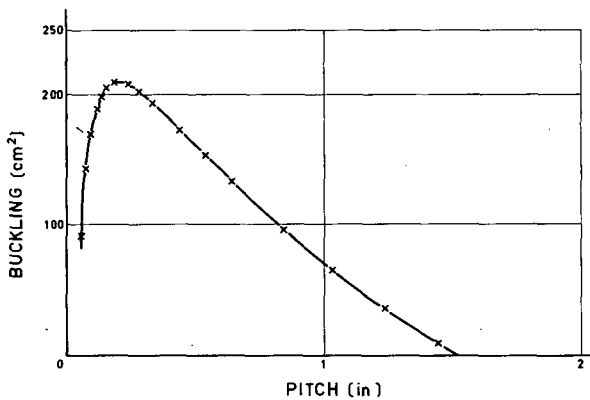


Fig. 14

Buckling of plutonium-aluminium spikes in water

The spaces between the pressure tubes will be filled with heavy water, while the fuel rods will be immersed in a mixture of light water and reduced density polystyrene to simulate the boiling coolant in an SGHW power reactor. Other work, planned in uranium oxide lattices moderated with mixtures of light and heavy water, will also be relevant to the better understanding of light water physics.

ACKNOWLEDGEMENTS

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LIGHT WATER LATTICE CALCULATIONS IN THE UNITED KINGDOM

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Abstract — Résumé — Аннотация — Resumen

LIGHT WATER LATTICE CALCULATIONS IN THE UNITED KINGDOM. United Kingdom calculations for light water lattices have been based on the MUFT-SOFOCATE few-group model. A complex analytical scheme based on the use of a number of separate computer programmes has been built up and has been shown to account satisfactorily for the behaviour of a number of United Kingdom and United States lattices. This scheme has been superseded for many purposes by the comprehensive SANDPIPER I and II computer codes developed at Winfrith. These have been incorporated in the REDSHANK-GREENSHANK burn-up codes which have given the typical results quoted in this paper. The work described forms part of a wider programme covering the whole field of light and heavy water moderation.

CALCULS FAITS AU ROYAUME-UNI SUR DES RÉSEAUX A EAU LÉGÈRE. Les calculs faits au Royaume-Uni au sujet des réseaux uranium-eau légère étaient fondés sur le modèle MUFT-SOFOCATE à petit nombre de groupes. On a élaboré un système analytique complexe basé sur divers programmes de calculatrices; il semble fournir une explication satisfaisante du comportement des réseaux de plusieurs réacteurs construits au Royaume-Uni et aux Etats-Unis. Pour de nombreux travaux, ce système a été remplacé par les codes très complets SANDPIPER I et II, mis au point à Winfrith. On les a incorporés aux codes REDSHANK-GREENSHANK qui sont destinés au calcul des taux de combustion et à l'aide desquels on a obtenu les résultats caractéristiques mentionnés dans le mémoire. Les travaux décrits ont été exécutés dans le cadre d'un vaste programme d'études qui couvre tous les aspects du ralentissement à l'eau légère et à l'eau lourde.

РАСЧЕТЫ ЛЕГКОВОДНЫХ РЕШЕТОК В СОЕДИНЕННОМ КОРОЛЕВСТВЕ. Расчеты в Великобритании для легководных решеток были основаны на нескольких групповых моделях MUFT-SOFOCATE. Была построена сложная аналитическая схема, основанная на использовании ряда отдельных счетно-решающих программ, и она оказалась удовлетворительной для определения поведения ряда английских и американских решеток. Эта схема заменялась для многих целей исчерпывающими счетно-решающими кодами SANDPIPER I и II, разработанными в Уинфрите. Эти коды были включены в коды выгорания REDSHANK-GREENSHANK, которые дали типичные результаты, приведенные в данном докладе. Описанная работа составляет часть более широкой программы, охватывающей всю область легководного и тяжеловодного замедления.

CÁLCULOS RELATIVOS A RETICULADOS DE AGUA LIGERA EFECTUADOS EN EL REINO UNIDO. El método de cálculo que se aplica en el Reino Unido a los reticulados de agua ligera se basa en el modelo MUFT-SOFOCATE de pocos grupos. Se ha preparado un sistema analítico complejo basado en la utilización de una serie de programas separados de calculadoras y se ha comprobado que explica satisfactoriamente el comportamiento de muchos de los reticulados existentes en el Reino Unido y en los Estados Unidos. En numerosas aplicaciones, se ha reemplazado ahora dicho sistema por las amplias claves de cómputo SANDPIPER I y II, ideadas en Winfrith. Se han incorporado estas últimas a las claves de combustión REDSHANK-GREENSHANK, que han conducido a los resultados característicos mencionados en esta memoria. Los trabajos descritos forman parte de un programa más extenso que abarca todo el campo de la moderación con agua ligera y agua pesada.

1. INTRODUCTION

In the UKAEA interest in water reactors has ranged over a wider field than is covered by the scope of this meeting, since attention is being given to reactors moderated by both light and heavy water. However, a number of investigations in the strictly light water field have been undertaken and are likely to be of general interest. The theory will be dealt with in this paper while the experimental aspects are considered separately [1].

At an early stage it was recognized that the relatively simple forms of two-group theory which can be used with considerable success in D_2O or graphite moderated reactors are not suitable for use with light water lattices. This arises from the nature of the neutron spectrum in such lattices (see Fig. 2), there being a well marked peak at the high energy end of the distribution. Since the relative intensity of the peak and the approximately dE/E epithermal region change from lattice to lattice it is not possible to establish a set of single fast-group cross-sections for the elements of interest which are valid in a wide range of lattices. This led to the formulation of a 3-group model similar to that proposed by DEUTSCH [2, 3]. By separating the fast region into two groups, one corresponding to the peak and the other to the approximately dE/E tail, it was clearly possible to obtain group cross-sections valid over a wider range. In the thermal region it also became apparent that the spectrum in the lattices then of interest was likely to be so hard that models based on a moderator temperature Maxwellian distribution and a dE/E tail were unlikely to define reaction rates with sufficient precision.

The adoption of a 3-group model (2 fast; 1 thermal) brought with it new problems since there were now more parameters to fix than in a two-group model: Yet the necessity to use enriched fuel precluded, on grounds of expense, the very wide range of correlation experiments which is feasible with natural uranium, D_2O or graphite moderated, lattices. It was therefore necessary to establish the parameters by calculation from fundamental nuclear data, and to extract from each experiment the maximum of information possible in the form of a wide range of reaction rate measurements. At about this time (1957) a stream of important publications and IBM 704 computer codes began to be published by the Westinghouse Atomic Power Department. In particular the MUFT (5) and SOFOCATE (6) computer programmes provided a ready means of calculating few-group neutron constants. Since an IBM 704 was available it seemed futile to proceed with an independent development and work in the United Kingdom has subsequently made considerable use of the Westinghouse methods. Independent theoretical investigations have been made to fill gaps in the coverage provided by the codes and comparisons made with experimental data from both the United States and the United Kingdom.

As will be shown, the available Westinghouse methods require the use of a number of separate computer programmes in order to obtain a complete picture of lattice behaviour. This is inconvenient in survey, initial design, or burn-up calculations. A single computer programme SANDPIPER I [7] has been developed at the cost of some simplifications and has been shown to give good agreement with the more elaborate procedure. The methods

employed here may be of particular interest to groups which do not have access to a computer suitable for the Westinghouse codes. Later work which will be reviewed includes studies of plutonium effects, burn-up, etc., and forms part of a wider programme covering both light and heavy water moderation.

2. FEW-GROUP LATTICE CALCULATIONS BASED ON THE MUFT AND SOFOCATE COMPUTER CODES

2.1. Principles

The MUFT [5] and SOFOCATE [6] computer codes were written at Westinghouse with the object of providing fast and thermal neutron spectra or group averaged constants respectively. Both deal with a flux mode of specified buckling in an essentially homogeneous medium and must therefore be supplemented by independent assessments of the effects of the heterogeneities present in an actual lattice. These heterogeneities are usually represented by assigning appropriate flux weighting factors to the concentrations of the nuclear species present and using the resulting flux weighted quantities as input to computer programmes written for homogeneous media. Since typical power reactor lattices are doubly periodic with the regular array of fuel pins broken at intervals by spaces for control rod insertions, it is useful to distinguish two classes of flux weighting factors. In the first place there are the hyperfine factors relating to flux variations in individual fuel pin cellules; secondly there are the gross fine structure effects associated with control rod water gaps. In these circumstances somewhat elaborate procedures are needed to synthesize the group parameters for a homogeneous lattice.

The homogenization procedures used at Winfrith have been described fully in a recent report [4]. For the sake of completeness a brief outline will be included in this paper; the various features of the procedures will then be related both to published U.S. lattice data and to previously unpublished results obtained with critical assemblies in the U.K. This presentation will also serve as an introduction to the functions of the comprehensive SANDPIPER I computer programme (see section 5), which for many purposes has now superseded direct use of the Westinghouse programmes at Winfrith.

Conceptual diagrams showing the main features of typical power reactor lattice cells are shown in Fig. 1 and a flow diagram for the homogenization procedure in Fig. 3. Details are summarized briefly as follows:-

2.2. Fast neutron processes

In the fast neutron region the neutron mean free paths are long compared with the heterogeneities and it is a good first approximation to regard the structure as effectively homogeneous except in the resonance energy region. Here the self and mutual shielding effects give rise to effective cross-sections which are strongly dependent on the details of the lattice; this effect is particularly important for U^{238} in a low enrichment system. The MUFT [5] programme has been written to include the so called "L factors" which

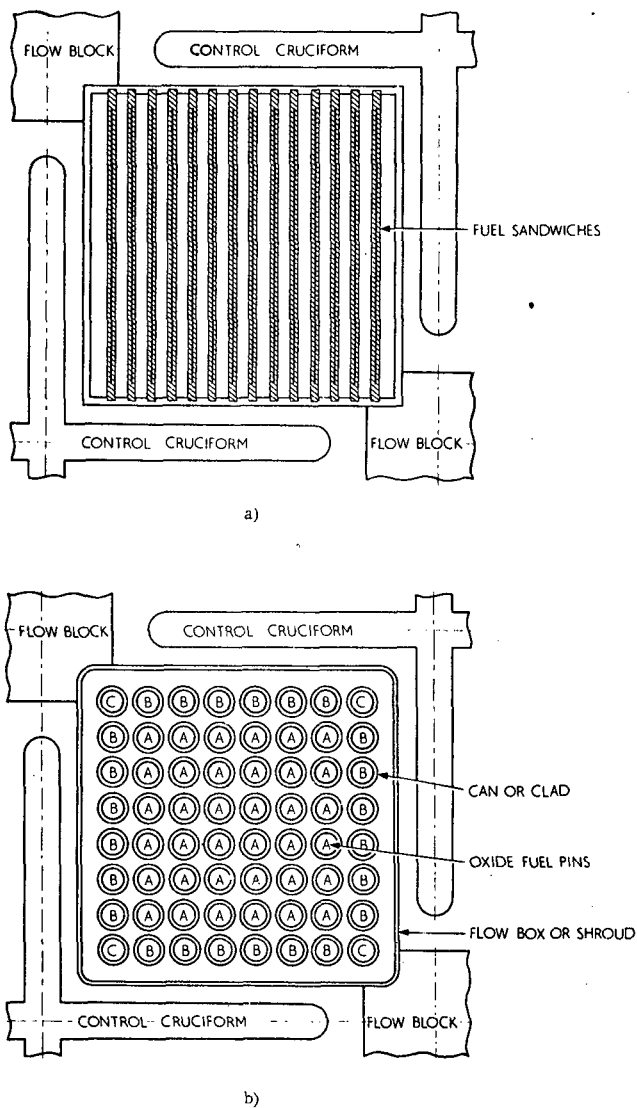


Fig. 1

- a) High enrichment plate type fuel assembly
- b) Low enrichment pin type fuel assembly

permit the effective resonance integrals to be adjusted to agree with independent calculations. The first step in the Winfrith procedure is to obtain a set of 3 MUFT calculations with 3 different overall L values. Each calculation corresponds to a different value of the U^{238} resonance integral which may be inferred from the "few group edit". The appropriate value for the

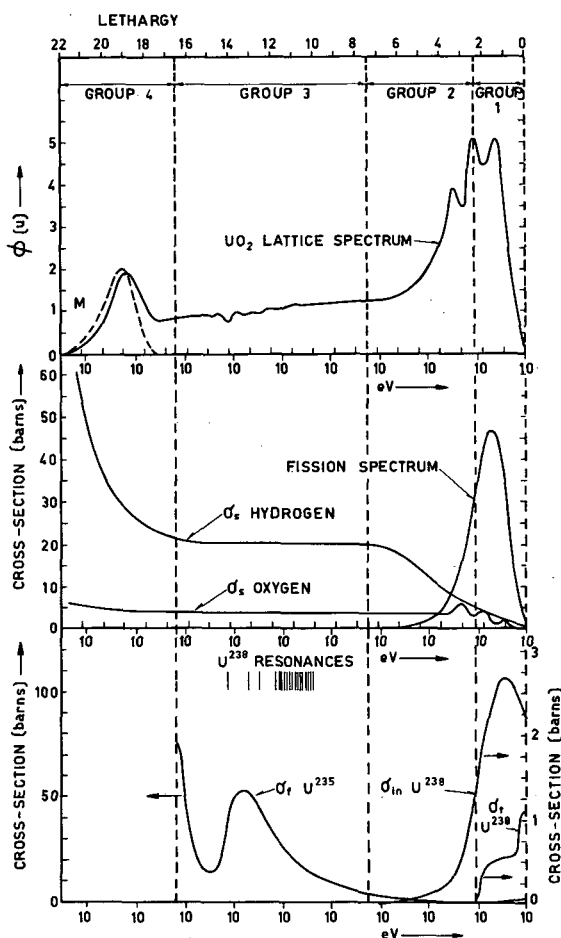


Fig. 2

Neutron spectra and cross-sections

lattice in question is then inferred from HELLSTRAND'S [8] experiments by the following procedure:-

(i) The experimental results are recorrelated with a formula of the form suggested by LEVINE [9] i. e. ;

$$R.I. = A + B (\sigma_p^c + \gamma_A S/4NV)^{\frac{1}{2}} \quad (2.1)$$

where A, B are constants

σ_p^c = potential scattering cross-section per resonance absorbing atom

S = surface of the fuel lumps

V = volume of the fuel lumps

N = number density of absorbing atoms within the fuel lump

γ_A = a mutual screening factor

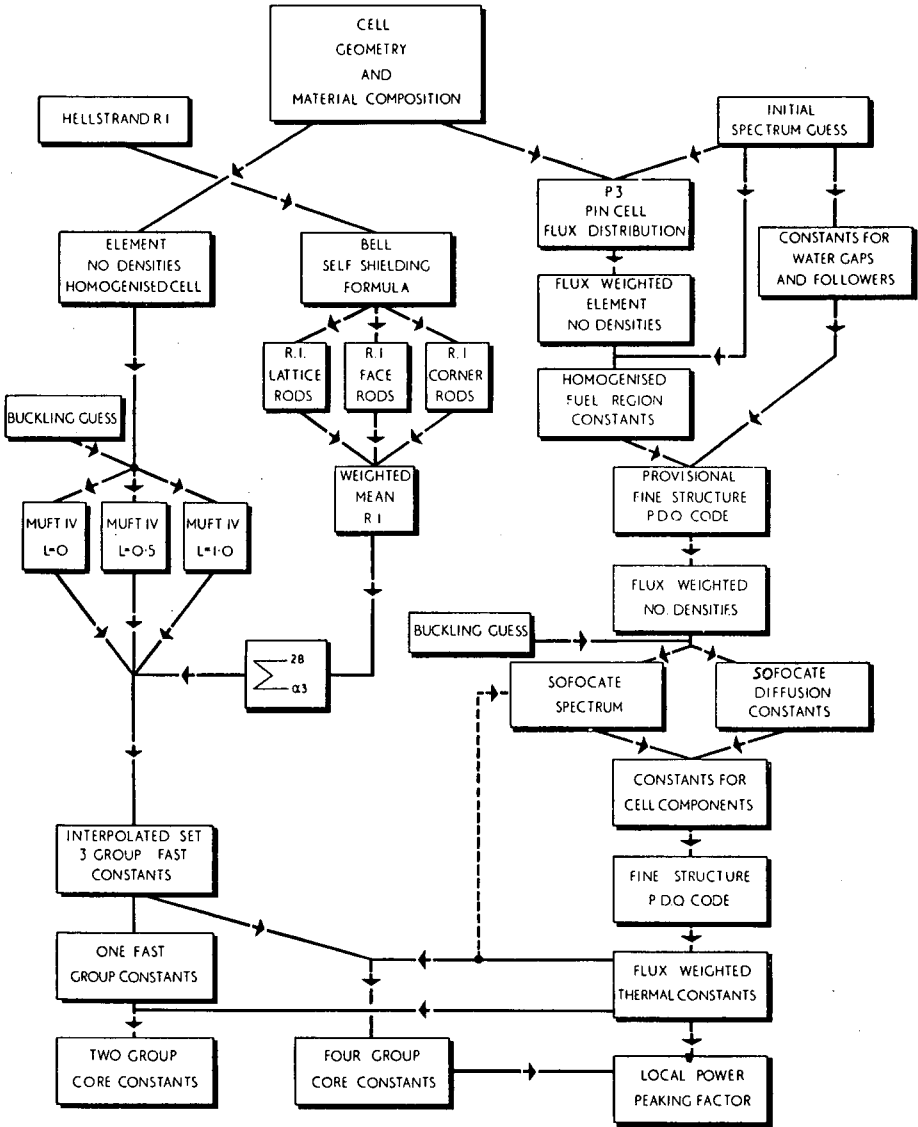


Fig.3

Flow diagram for lattice calculations

(ii) The self shielding factor γ is calculated by the BELL [10] approximation;

$$\gamma_A = [1 + S/4(\Sigma_m V_m + \Sigma_c V_c)]^{-1} \quad (2.2)$$

where

Σ_m = Macroscopic total cross-section of moderator for resonance neutrons

Σ_c = Ditto for cladding material

V_m = Volume of moderator associated with one fuel lump

V_c = Volume of cladding material associated with one fuel lump

(iii) In lattices containing control rod water gaps it has been assumed that these gaps are wide compared with the mean free paths. Thus shielding effects are reduced on the outer row of pins in each cell. This is dealt with approximately by assuming that the effective value of $(1 - \gamma)$ is reduced by a ratio dependent on the number of regular lattice neighbours missing (see [1] for details). For example for type B pins (see Fig. 1)

$$(1 - \gamma_b) = \frac{3}{8} (1 - \gamma_A) \quad (2.3)$$

and for type C

$$(1 - \gamma_c) = \frac{5}{8} (1 - \gamma_A) \quad (2.4)$$

(iv) A weighted mean resonance integral for all the fuel pins in the cell is then formed. (Generally this is very close to the value for type A pins so great refinement in the treatment of surface pins is not required).

Finally the appropriate set of MUFT constants is obtained by interpolating to give a set corresponding to the U^{238} resonance integral derived in (iv) above. These constants refer to the three fast-groups:-

(1) 10 MeV - 821 keV

(2) 821 keV - 5.5 keV

(3) 5.5 keV - 0.625 eV

which have been extensively used by Westinghouse.

2.3. Thermal neutron processes

A full solution of the thermal neutron problem in a power reactor lattice would recognize that the distributions in space and energy are not separable. Since the problem is essentially two dimensional, rather elaborate calculations near the limit of present computer capacities would be required, and it is not feasible to consider this approach if a wide range of cases are to be investigated. The method evolved at Winfrith rests on the following simplifications:-

- (i) A one group thermal model is used to represent the spatial variation
- (ii) The parameters for the single group are calculated for a Wigner-Wilkins spectrum appropriate to a homogeneous medium. The composition of this medium is arrived at by considering the "fine" and "hyperfine" flux weightings appropriate to the various constituents of the cell.

Referring to the flow diagram (Fig. 3) the calculation proceeds as follows. An initial guess (usually a moderator temperature Maxwellian) is made for the spectrum and the appropriate microscopic cross-sections evaluated. The hyperfine flux distribution within a fuel pin or plate cellule is then calculated by some form of transport theory. At Winfrith a P3 method [11] has

been used in cylindrical geometries, and the Carlson S [12] or collision theory methods [13] in plane geometry. Hyperfine flux weighting factors for the cell constituents are then used to homogenize the fuel bearing region. One-group two dimensional diffusion theory calculations with sources proportional to the moderating powers are then made with the PDQ code [14, 15] to determine the "fine structure" i.e., the partitioning of the flux between the homogenized fuel bearing region and the other constituents of the cell. Since the spectrum guess was too soft the fine structure is overestimated at this stage of the calculations. This is not very important from the point of view of fixing the reactivity of the system, since most of the absorption occurs within the fuel bearing region and the thermal utilization is therefore only weakly dependent on the fine structure. On the other hand, the power distribution within the fuel bearing region is of great importance from the thermal performance point of view. It is therefore worthwhile to re-evaluate the fine structure using a better estimate of the average spectrum based on flux weighting factors derived in the first round. This (see flow diagram Fig. 3) therefore requires a spectrum calculation with the SOFOCATE [6] code and a further estimate of the fine structure using PDQ. Finally, a set of thermal group constants

$$D, \Sigma_A, \Sigma_R \text{ and } \nu\Sigma_F$$

are calculated using microscopic cross-sections from SOFOCATE and elementwise number densities which take account of the fine and hyperfine flux weighting factors.

2.4. Overall reactor calculations and reactivity estimates

The procedure described in 2.2 and 2.3 above provide sets of four-group parameters

$$D, \Sigma_A, \Sigma_R \text{ and } \nu\Sigma_F$$

which may be used as input to space diffusion codes to solve problems of overall lattice or reactor behaviour.

A useful comparison with experiment may be made in the following manner. The measured buckling B^2 is input to the four-group equations written in the following form:-

$$-(B^2 D_1 + \Sigma_{1A} + \Sigma_{1R}) \phi_1 + (\chi_1/\lambda) \sum_i \nu_i \Sigma_{iF} \phi_i = 0 \quad (2.5)$$

$$-(B^2 D_2 + \Sigma_{2A} + \Sigma_{2R}) \phi_2 + \Sigma_{1R} \phi_1 + (\chi_2/\lambda) \sum_i \nu_i \Sigma_{iF} \phi_i = 0 \quad (2.6)$$

$$-(B^2 D_3 + \Sigma_{3A} + \Sigma_{3R}) \phi_3 + \Sigma_{2R} \phi_2 = 0 \quad (2.7)$$

$$-(B^2 D_4 + \Sigma_{4A}) \phi_4 + \Sigma_{3R} \phi_3 = 0 \quad (2.8)$$

where ϕ_i is the flux in group i

subscripts A, R and F refer to absorption, removal and fission respectively.

χ_i is the fraction of fission neutrons born in group i.

TABLE I
U. K. HIGHLY ENRICHED $U^{235}/Al/S.S.$ CORES

Ref. No.	Measured buckling m^{-2}	Calculated k_{eff}
H.1	46.7 ± 0.3	1.003 ± 0.001
H.2	63.5 ± 0.5	0.999 ± 0.001
H.3	19.0 ± 5.0	1.007 ± 0.012
H.4	8.8 ± 5.4	0.990 ± 0.017
H.5	38.4 ± 5.1	0.993 ± 0.015
H.6	62.9 ± 4.9	0.990 ± 0.014
H.7	80.8 ± 3.0	1.005 ± 0.008

Equations (2.5)–(2.8) are then solved for the eigenvalue λ . If the measured buckling refers to a just critical reactor or a steady state exponential measurement the λ (or the estimated k_{eff}) should be unity. The departure of λ from unity gives an indication of the effectiveness of the calculation scheme in reactivity calculations.

2.5. Comparison with experimental results

2.5.1. Overall reactivity

Reactivity comparisons made in the manner described in section 2.4 have been made for a number of highly enriched U.K. lattices and low enrichment U.S. lattices. Table I summarizes the results for the highly

TABLE II
SPECIFIC VOLUMES OF CORE MATERIALS

U. K. highly enriched cores

Ref. No.	U^{235}	U^{238}	Water	Steel	Aluminium
H.1	0.00415	0.00031	0.70430	0.22473	0.06651
H.2	0.00470	0.00035	0.69483	0.22473	0.07539
H.3	0.00331	0.00025	0.71851	0.22473	0.05320
H.4	0.00319	0.00024	0.65661	0.28961	0.05035
H.5	0.00389	0.00029	0.64653	0.28770	0.06159
H.6	0.00490	0.00036	0.62428	0.29277	0.07769
H.7	0.00590	0.00044	0.63158	0.26744	0.09464

TABLE III
COMPARISON OF CALCULATED AND MEASURED LATTICE DATA

Case	W/U		ρ_{28}	δ_{28}	δ_{25}	f	$\bar{\phi}_m/\bar{\phi}_u$	λ
L. 1	3.0	Calculated Measured	1.276 1.19 $\pm$ 0.04	0.073 0.071 $\pm$ 0.010	0.087 0.078 $\pm$ 0.006	0.864 0.873 $\pm$ 0.004	1.14 1.09 $\pm$ 0.03	0.994
L. 2	4.0	Calculated Measured	1.009 0.994 $\pm$ 0.013	0.060 0.059 $\pm$ 0.003	0.068 0.073 $\pm$ 0.005	0.832 0.837 $\pm$ 0.005	1.15 1.14 $\pm$ 0.03	0.999
L. 3	5.0	Calculated Measured	0.827 0.807 $\pm$ 0.014	0.051 0.051 $\pm$ 0.004	0.056 0.068 $\pm$ 0.006	0.800 0.805 $\pm$ 0.005	1.15 1.16 $\pm$ 0.03	0.995
L. 4	4.0	Calculated Measured	1.147 1.04 $\pm$ 0.05	0.061 0.063 $\pm$ 0.003	0.069 0.074 $\pm$ 0.006	0.834 0.836 $\pm$ 0.004	1.09 1.09 $\pm$ 0.01	0.991
L. 5	5.0	Calculated Measured	0.958 0.901 $\pm$ 0.070	0.052 0.054 $\pm$ 0.003	0.057 0.059 $\pm$ 0.006	0.805 0.807 $\pm$ 0.004	1.09 1.10 $\pm$ 0.01	0.992
L. 6	2.9	Calculated Measured	1.347 1.43 $\pm$ 0.01	0.076 0.004 $\pm$ 0.004	0.090 0.089 $\pm$ 0.004	0.873 0.874 $\pm$ 0.004	1.09 1.10 $\pm$ 0.01	1.000
L. 7	3.6	Calculated Measured	1.115 1.15 $\pm$ 0.01	0.064 0.070 $\pm$ 0.004	0.073 0.072 $\pm$ 0.001	0.849 0.848 $\pm$ 0.004	1.09 1.13 $\pm$ 0.01	1.004
L. 8	4.9	Calculated Measured	0.867 0.934 $\pm$ 0.01	0.053 0.057 $\pm$ 0.003	0.057 0.055 $\pm$ 0.001	0.810 0.803 $\pm$ 0.004	1.09 1.13 $\pm$ 0.01	0.998
L. 9	2.2	Calculated Measured	3.754 2.77 $\pm$ 0.08	0.083	0.236	0.834	1.22	1.003
L. 10	2.9	Calculated	2.801	0.068	0.171	0.822	1.21	1.007
L. 11	3.9	Calculated Measured	2.170 1.83 $\pm$ 0.12	0.056	0.129	0.806	1.21	1.006

TABLE III (cont'd)

COMPARISON OF CALCULATED AND MEASURED LATTICE DATA

Case	W/U	ρ_{28}	δ_{28}	δ_{25}	f	$\bar{\phi}_m/\bar{\phi}_u$	λ
L.12	2.6	3.264	0.073	0.206	0.823	1.315	0.996
L.13	3.5	2.401	0.058	0.148	0.807	1.318	1.006
L.14	8.1	1.149	0.032	0.067	0.731	1.368	0.987
L.15	2.6	4.176	0.069	0.265	0.851	1.409	0.999
L.16	3.5	3.053	0.055	0.188	0.837	1.412	0.994
L.17	8.1	1.428	0.029	0.083	0.768	1.473	1.001
L.18	10.0	1.115	0.023	0.064	0.730	1.485	0.975

TABLE IV

DETAILS OF EXPERIMENTS
(U. S. low enrichment lattices)

Case	W/U	UO ₂ Density g/cm ³	Enrichment %U ²³⁵	Canning material	UO ₂ pellet radius in.	Buckling m ⁻²	Ref.
L.1	3.0	7.53	1.3	Aluminium	0.600	28.37	16
L.2	4.0	7.53	1.3	"	"	30.17	"
L.3	5.0	7.53	1.3	"	"	29.06	"
L.4	4.0	7.52	1.3	"	0.388	25.28	"
L.5	5.0	7.52	1.3	"	"	25.21	"
L.6	2.9	10.53	1.3	"	"	32.59	"
L.7	3.6	10.53	1.3	"	"	35.47	"
L.8	4.9	10.53	1.3	"	"	34.22	"
L.9	2.2	10.2	2.7	Stainless Steel	0.300	40.70	17
L.10	2.9	10.2	2.7	"	"	53.20	"
L.11	3.9	10.2	2.7	"	"	63.30	"
L.12	2.6	9.45	3.0	"	0.564	50.75	18
L.13	3.5	9.45	3.0	"	"	60.00	"
L.14	8.0	9.45	3.0	"	"	68.81	"
L.15	2.6	9.45	4.0	"	"	69.25	"
L.16	3.5	9.45	4.0	"	"	85.52	"
L.17	8.0	9.45	4.0	"	"	92.84	"
L.18	10.1	9.45	4.0	"	"	91.79	"

enriched lattices with the volumetric compositions listed in Table II. The bucklings for lattices H. 1 and H. 2 were measured in a critical assembly; the remainder are exponential measurements. The errors quoted in the calculated k_{eff} correspond to the errors in the measured bucklings.

Table III presents similar calculations for the U.S. low enrichment lattices listed in Table IV.

For each series of experiments the mean eigenvalues and standard deviations of the distributions have been evaluated. The results are as follows:-

$$k_{\text{eff}} = 0.998 \pm 0.007 \text{ (U.K. High Enrichment)}$$

$$k_{\text{eff}} = 0.997 \pm 0.007 \text{ (U.S. Low Enrichment)}$$

The degree of agreement obtained by a single method over such a wide field gives confidence in its use as a design tool.

2.5.2. Overall reaction rates

Table III also includes comparison between the experimental and calculated thermal disadvantage factors for a $1/v$ absorber, i. e.,

$$\frac{\bar{\phi}_m}{\bar{\phi}_f} = \frac{\text{Mean flux in moderator}}{\text{Mean flux in fuel}}$$

and traces the influence on the thermal utilization factor f . The three Westinghouse reaction rate parameters ρ_{28} , δ_{28} and δ_{25} are also included. These are defined as follows:-

$$\rho_{28} = \frac{\text{epi-cadmium capture in U}^{238}}{\text{sub-cadmium capture in U}^{238}}$$

$$\delta_{28} = \frac{\text{fission rate in U}^{238}}{\text{fission rate in U}^{235}}$$

$$\delta_{25} = \frac{\text{epi-cadmium fission rate in U}^{235}}{\text{sub-cadmium fission rate in U}^{235}}$$

In general the agreement is good though a large disagreement in the ρ_{28} values occurs for lattices 9 and 11 (see Table III). If the U^{238} is forced into agreement with the experiments, reactivity discrepancies would appear amounting to 5% on lattice 9. If the experimental value is correct, it is difficult to see how large compensating errors could have occurred elsewhere in the calculations to give the satisfactory value obtained for k_{eff} . The lattices in question do, however, have harder neutron spectra than the remainder and under these conditions recent work suggests that cadmium ratio measurements are particularly difficult and some of the earlier measurements may be in error.

2.5.3. Spatial distribution of reaction rates

The measure of agreement achieved in fine structure reaction rate calculations carried out by the technique described in section 2.3 is illustrated

by the typical result shown in Fig. 4 which refers to lattice H. 2. Reaction rate contours derived from the output of the second PDQ calculation (see flow diagram Fig. 3) are compared with spot measurements made with small

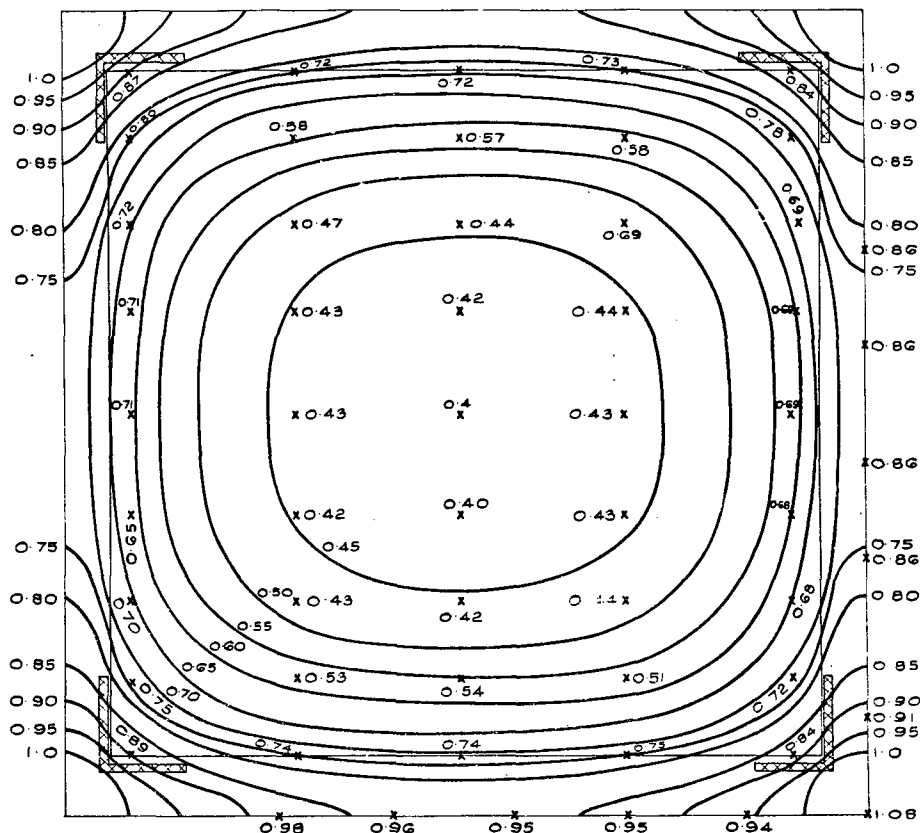


Fig. 4

Fine structure flux distribution in a normal lattice cell

manganese foils. The quantity plotted is the bare foil activation minus the cadmium covered activation and a correction has been applied to remove the overall reactor distribution. It can be seen that within the fuel bearing area (defined by the four shaded brackets clearly visible in Fig. 4) the agreement is good. This is a surprising result since the calculation is based on diffusion theory. However, it should be noted that the measured reaction rates in the water gaps are significantly higher than the calculated values.

The results of calculations which throw more light on this discrepancy are shown in Fig. 5. Here a one-dimensional one-group diffusion theory calculation of the flux distribution in an infinite slab system of similar composition and dimensions to the H-2 cell is compared with a one-group Carlson S_8 calculation. The agreement is very good within the fuel bearing region but the Carlson calculation shows an appreciably higher flux in the water

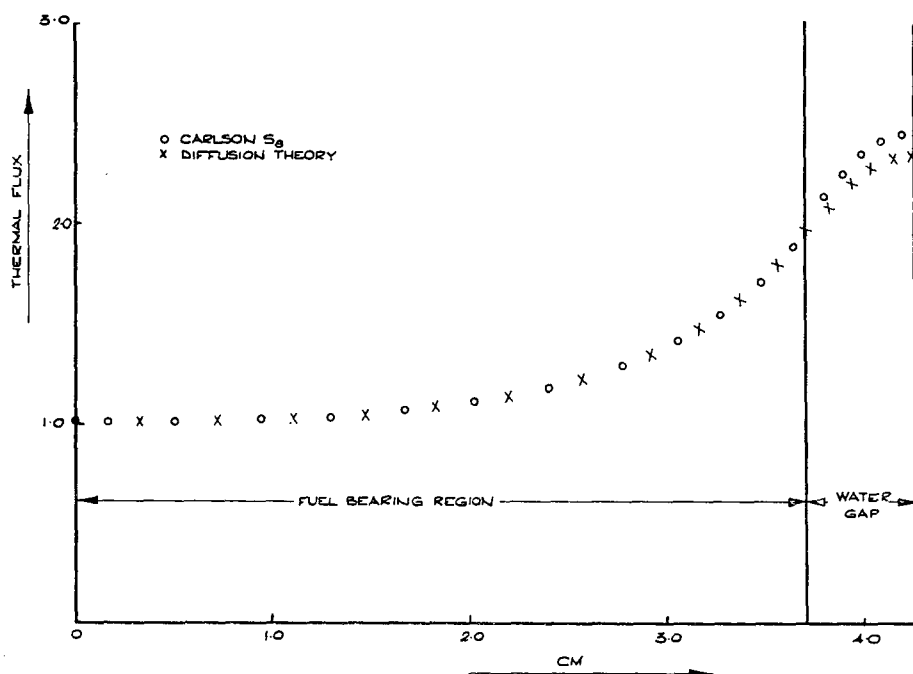


Fig.5

Comparison between diffusion and Carlson S_g calculations

gap. It appears likely that the observed discrepancy between the PDQ calculation and the measurements is due to transport effects. Nevertheless the technique based on diffusion theory appears to be adequate to predict the local power peaking factor required in thermal design calculations; the error in the water gap flux distribution leads to inappreciable reactivity errors. It is sometimes claimed that the use of Maxwellian rather than Wigner-Wilkins cross-sections in power peaking calculations accentuates the fine structure and so approximately compensates for the shortcomings of diffusion theory. This is not the case in the H-2 lattice, where the use of Maxwellian cross-sections destroys the agreement between experimental and calculated reaction rates within the fuel bearing region, as the following figures illustrate:-

$$\text{ppf} = \text{Power Peaking Factor} = \frac{\text{Peak Power}}{\text{Average Power}}$$

Maxwellian PDQ	ppf = 2.04
Wigner-Wilkins PDQ	ppf = 1.73

and it has already been shown that the Wigner-Wilkins PDQ is in good agreement with experiment (see Fig. 4).

2. 5. 4. *Validity of the Wigner-Wilkins spectrum model*

Integral reaction rate measurements have been undertaken to test the validity of the Wigner-Wilkins model in the lattices of Table II. In general good agreement was obtained. These results are discussed in more detail in the companion paper [1].

2. 6. General conclusions concerning MUFT-SOFOCATE lattice calculations

The general conclusion which may be drawn from the foregoing paragraphs is that MUFT-SOFOCATE lattice calculations provide a powerful design tool over a wide range of situations. However, the method of application proposed is somewhat tedious, as the flow diagram shows (see Fig. 3) and these methods are not convenient in design surveys or burn-up calculations. The resort to a correlation of Hellstrand's resonance integral data is also inelegant though this seems to be the only alternative to somewhat elaborate Monte-Carlo calculations [19, 20].

Recent work at Winfrith has resulted in the development of methods resting on a common conceptual basis to those described above but requiring only a single computer run for a complete lattice calculation or burn-up life history. However, before this work is described (sections 4 and 5) brief reference will be made to the special problems arising in lattices containing control absorbers.

3. SPECIAL PROBLEMS IN LATTICES CONTAINING CONTROL ABSORBERS

The methods used for treating control absorbers at Winfrith follow the lines described by HENRY [21] though there are some differences in detail which have been described elsewhere [4]. In principle, boundary conditions are fixed by high order spherical harmonic calculations in slab geometry. Special PDQ lattice calculations are then made to determine the properties of cells containing the control distribution.

Surprisingly good agreement has been obtained between calculated and measured thermal reaction rates in cells containing the absorbers. Figure 6 shows an H-2 cell containing a cadmium cruciform, while Fig. 7 shows the same cell containing a boral cruciform. These diagrams should be compared with Fig. 4 which shows the corresponding distribution for a normal cell. Figures 8 and 9 show the correspondingly good agreement obtained for the axial power distribution in reactors with partially inserted banks of cruciforms.

The controlled region of these two region reactors was treated as an exponential assembly and the appropriate buckling deduced from the flux distribution. Eigenvalues for the controlled regions were then deduced from the calculated cell constants and the measured bucklings with the following results:-

Cd Cruciforms	1.017 ± 0.004
Boral Cruciforms	0.992 ± 0.006

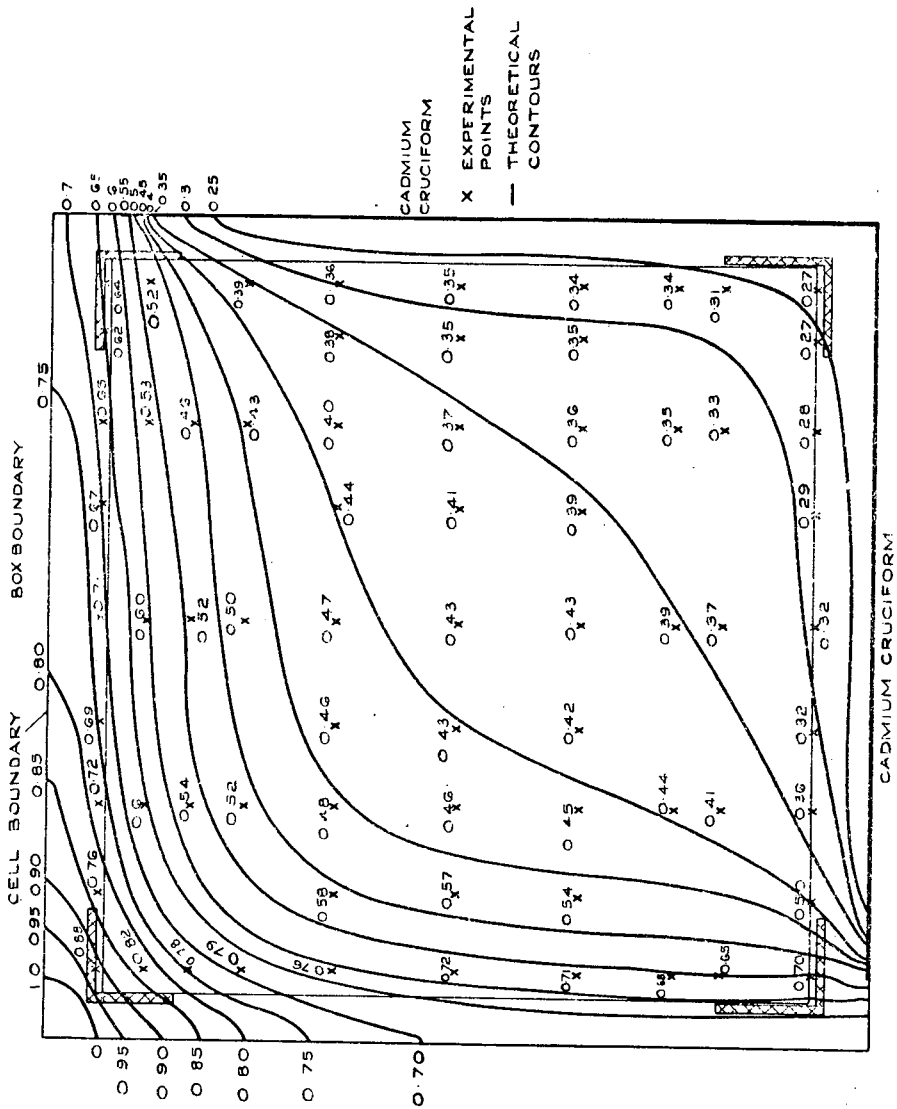


Fig. 6

Flux distribution in a lattice cell containing a cadmium cruciform

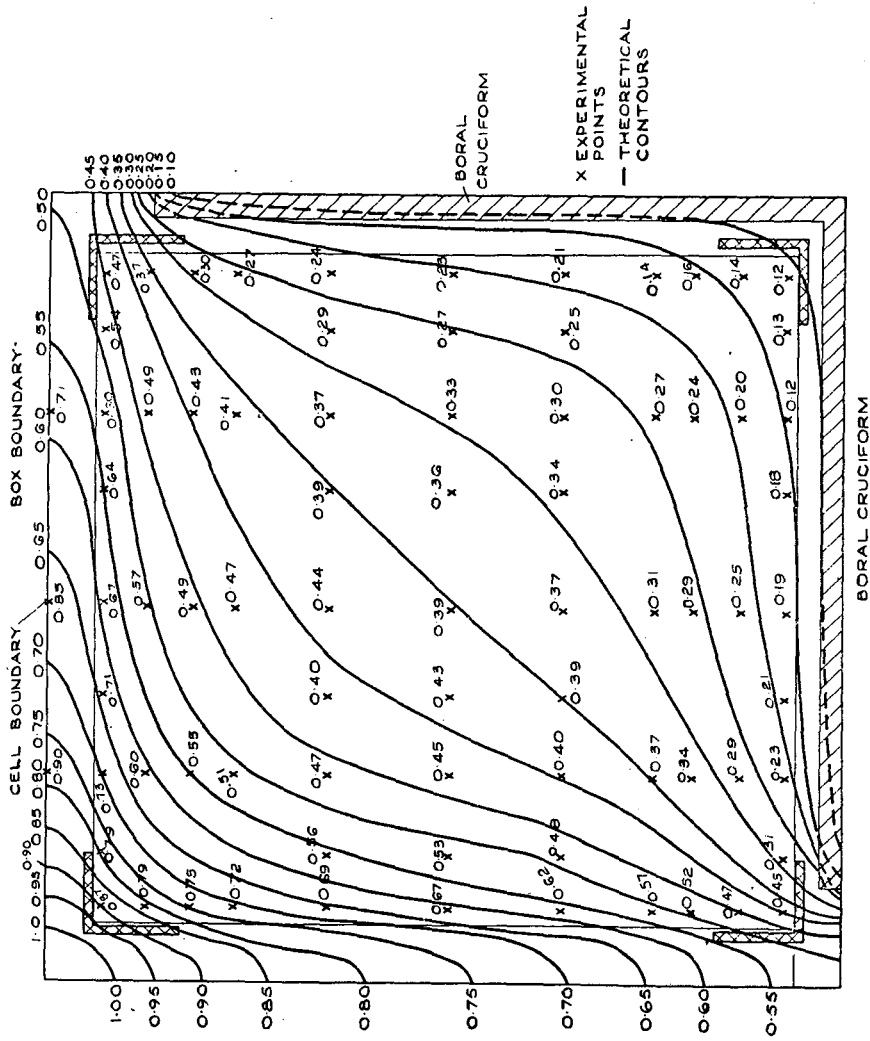


Fig. 7
Flux distribution in a lattice cell containing a boral cruciform

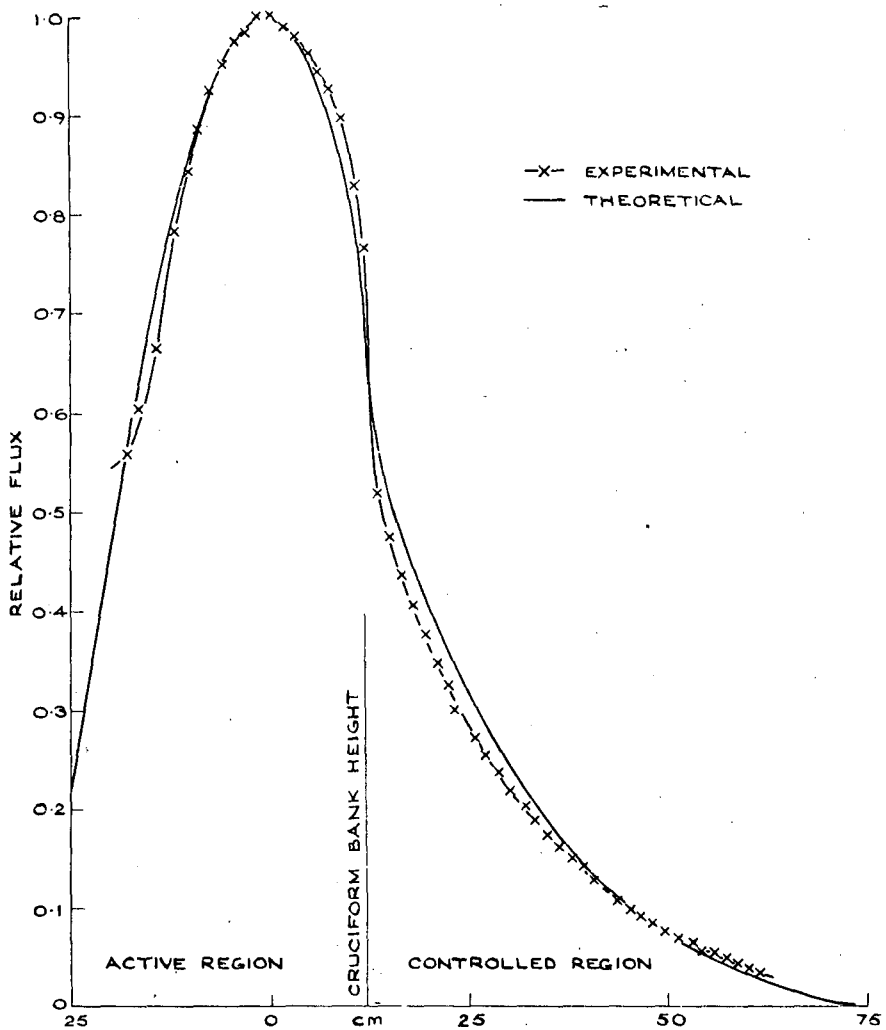


Fig. 8

Axial power distribution in a reactor with partially inserted cadmium absorbers

the corresponding figure for the clean core was listed in Table II as 0.999 ± 0.001 . Since the Cd cruciforms were estimated to control 21% and the boral cruciforms 35% in reactivity the above figures imply that these large reactivity changes are being estimated with errors of approximately 2% for the Cd and 0.7% for the boral cruciforms respectively.

These results provide additional confirmatory experimental evidence to that already quoted by HENRY [21], but it is important to reiterate that author's conclusion, that the success of the methods must be to some extent fortuitous in view of the simplifying assumptions made in the theory.

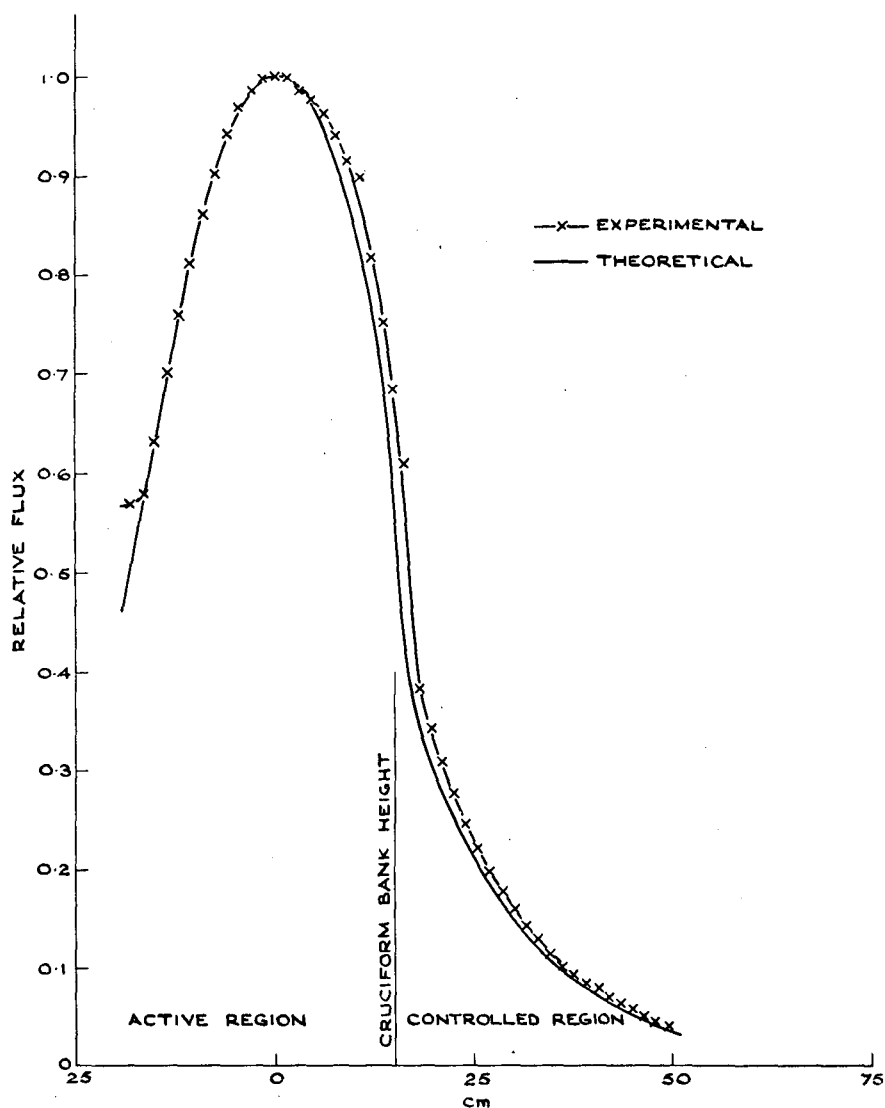


Fig. 9

Axial power distribution in a reactor with partially inserted boron absorbers

4. A SIMPLIFIED BASIS FOR FEW-GROUP CALCULATIONS

4.1. Fitted few-group cross-section

The first step in the simplification of the calculation scheme is to replace MUFT itself by an equivalent library of fitted cross-sections. This procedure was first suggested by OMBRELLARO [22], who showed that if the

standard energy groups referred to in section 2.2 were employed then the MUFT code yielded approximately constant elementwise few-group microscopic cross-sections over wide ranges of water metal mixtures. This is a consequence of the behaviour of the cross-sections and the resulting spectrum form illustrated in Fig. 2. The spectrum in group 1 is dominated by the fission energy distribution itself, group 3 is approximately dE/E , and group 2 forms a buffer region in which precise cross-section values are relatively unimportant. This implies that the range of fast neutron spectra encountered in water lattices can be represented rather well by changes in the relative intensities of groups 1, 2 and 3 without drastic changes of the energy distribution within each group. In turn this is the condition required to give substantially constant few-group microscopic constants appropriate to a wide range of lattice calculations. A fuller discussion has been presented elsewhere [4].

The Ombrellaro fitted cross-section scheme is an example of a correlation of a simple calculation scheme to a more detailed theory rather than to experimental results. It has been widely used as a data source for burn-up calculations, but has the further significance that laboratories with only limited computing facilities, or facilities incompatible with the MUFT code, can use calculation methods ultimately based on the comprehensive collection of multi-group cross-sections embodied in the MUFT library. In the work described in this paper the fitted cross-sections have made it possible to organize the flow diagram (Fig. 3) into a single computer programme of modest proportions. This simplification has proved particularly important in the more complex schemes required for the pressure tube reactor geometries where two different moderating regions can occur.

4.2. Special problems in group 3

In the original version of the fitted cross-section scheme due to OMBRELLARO [22] it was necessary to obtain an independently determined value of p_3 , the resonance escape probability for group 3, in order to fix the value of the U^{238} absorption cross-section and the effective value of the removal cross-section. The necessity for such a variable removal cross-section stems from the fact that the spectrum within group 3 is distorted from a dE/E form by an amount which depends on the strength of the U^{238} resonance absorption. Since most of the removal is associated with neutrons making their last collision with energies below the lowest U^{238} resonance at 6.68 eV the removal processes are associated with a depressed flux. This is formally equivalent to a reduction in the effective removal cross-section in the group to below the value appropriate in the absence of U^{238} .

A reformulation of Ombrellaro's procedure was suggested by the present author [4]. It was proposed that the U^{238} effective cross-section be derived from HELLSTRAND'S experiments [8] in the manner described in section 2 of this paper. At the same time the removal cross-section is calculated from the formula:-

$$\Sigma_{3R} = \sum_i (N^i \delta_{3R}^i) - (a/a + 1) \Sigma_{3A} \quad (4.1)$$

where

$$\Sigma_{3R} = \text{Total macroscopic removal cross-section}$$

- N^i = Number density of nuclear species i
 δ_{3R}^i = Microscopic removal cross-section for the i th nuclear species appropriate to a dE/E flux
 Σ_{3A} = Total macroscopic removal cross-section
 a = A spectrum distortion parameter.

It was also shown that a good fit to the full MUFT based calculation procedure, described in section 2 of this paper, could be obtained by taking

$$a = 0.862$$

in a range of UO_2/H_2O lattices. However, more recent work has shown that a modified treatment is required in highly undermoderated lattices.

4.3. Extensions required in undermoderated lattices

In highly undermoderated or "dry" lattices the original Ombrellaro scheme begins to break-down [23]. Since such situations are of potential interest, considerable attention has been devoted at Winfrith to the production of special devices to extend the validity of the scheme. These are all concerned with improved treatments of the group 3 processes.

Comparisons with MUFT calculations for undermoderated lattices, particularly those containing H_2O/D_2O mixtures, showed that a formula similar to (4.1) could not be made to fit the behaviour of the removal cross-section over the whole range of interest. Instead it has been assumed that:-

$$\Sigma_{3R} = f(\rho_3) \sum_i (N^i \delta_{3R}^i) \quad (4.2)$$

Figure 10 shows that the fitting function $f(\rho_3)$ is well defined by MUFT calculations covering a wide range of lattices. The most highly under-moderated system considered here has a coolant to uranium dioxide ratio of 0.97 and is moderated by pure D_2O , the most dilute has a coolant to UO_2 ratio of 15 and is moderated by pure H_2O .

In an under-moderated system an appreciable proportion of the fissions occur in the epithermal group 3 (0.625 eV - 5.5 keV). It is, therefore, important to note that the fission yield cross-section of U^{235} is also dependent on the spectrum distortions produced by resonance absorption. Figure 11 shows that the variations in the U^{235} fission yield cross-section can be correlated quite well with a simple polynomial function of the resonance escape probability ρ_3 i.e.,

$$\text{where} \quad \nu_3 \delta_{3F}^{235}(\rho_3) = g(\rho_3) \nu_3 \delta_{3F0}^{235} \quad (4.3)$$

$\nu_3 \delta_{3F}^{235}(\rho_3)$ = Effective U^{235} group 3 fission yield in a lattice where the group 3 resonance escape probability is ρ_3 .

$g(\rho_3)$ = A polynomial function of ρ_3

$\nu \delta_{3F0}^{235}$ = The original constant OMBRELLARO[[22] fission yield cross-section.

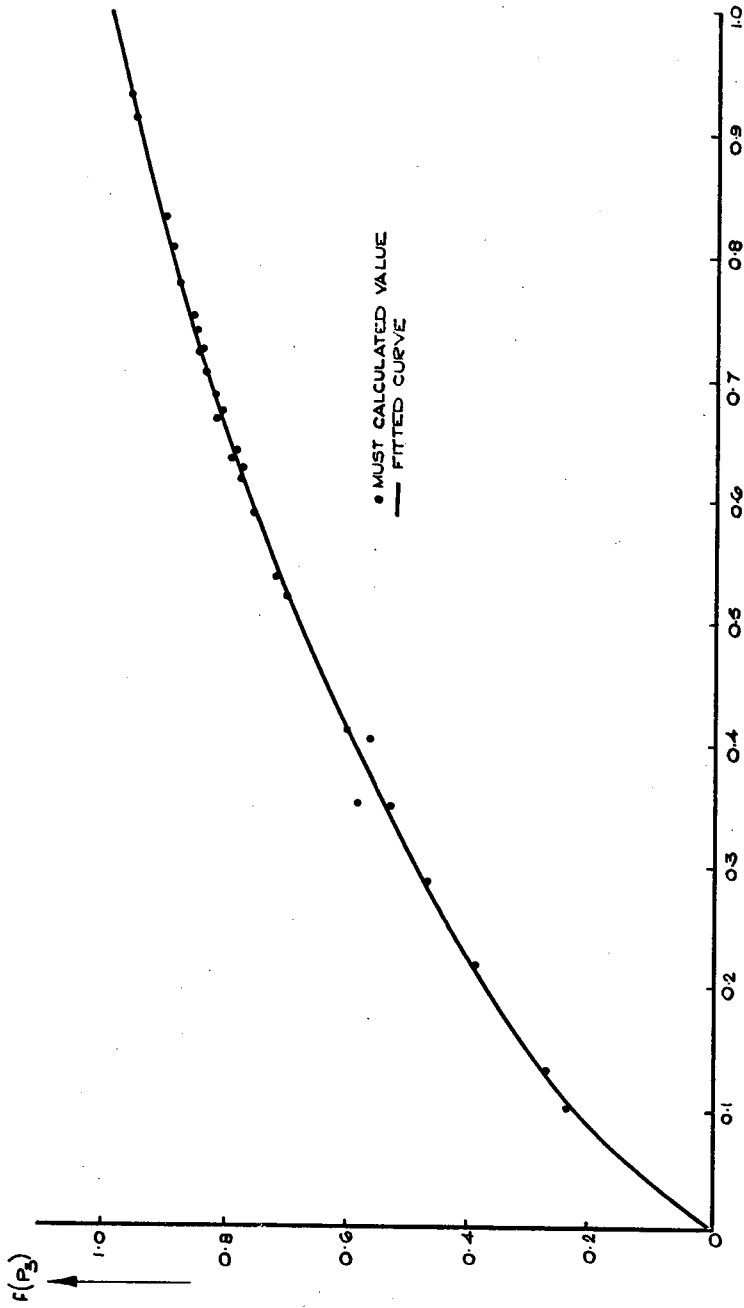


Fig. 10
Correction factor to group 3 removal cross-section

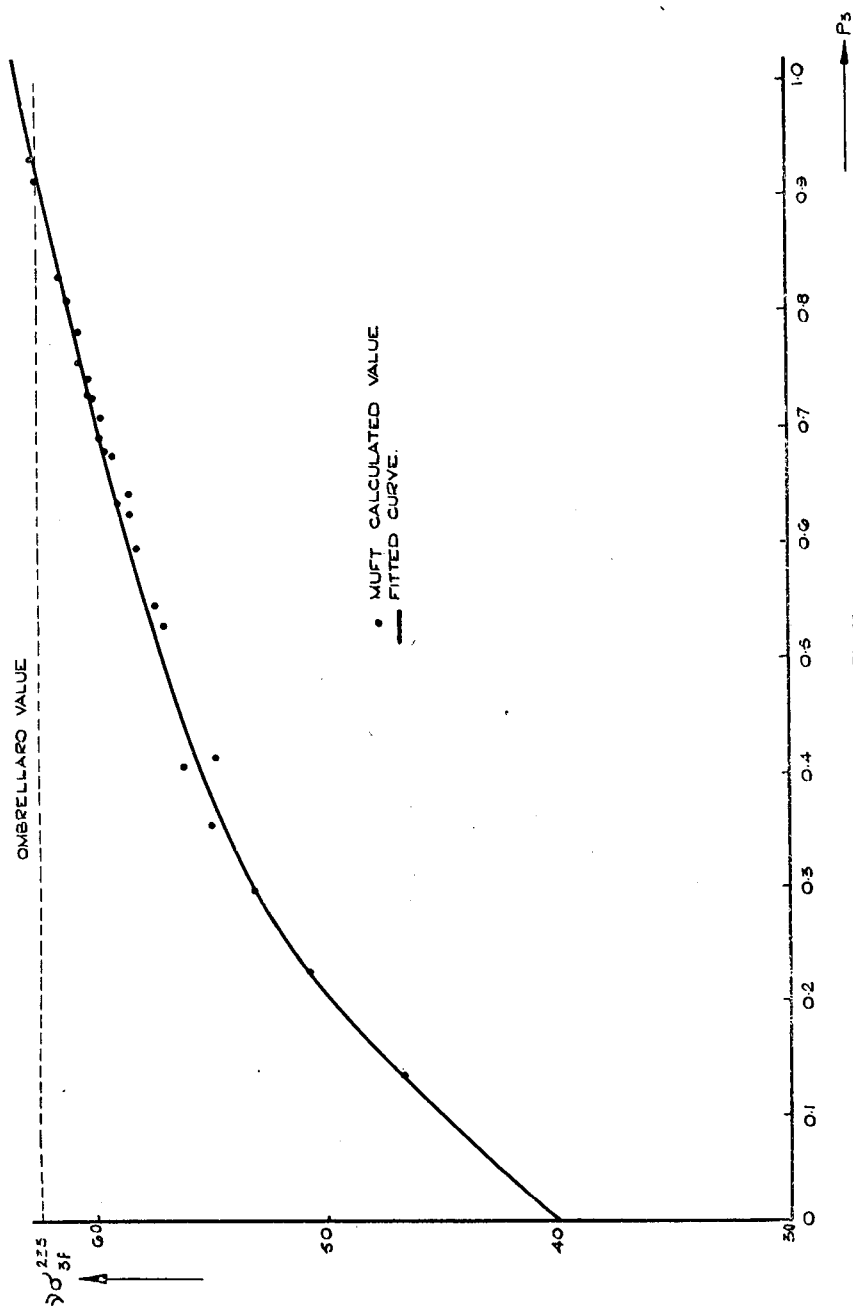


Fig. 11
Correction factor to U^{235} fission yield

Since the group 3 resonance escape probability is defined by

$$\rho_3 = \Sigma_{3R} / (\Sigma_{3A} + \Sigma_{3R}) \quad (4.4)$$

a dependence of cross-sections on ρ_3 which in turn depends on the cross-sections, implies that the determination of effective cross-sections in group 3 becomes an iterative procedure. This is, however, simple in nature and rapidly convergent.

The special devices represented by equations (4.2) and 4.4) were used in the first comprehensive Winfrith lattice programme, SANDPIPER I, which was written early in 1961 (see section 5 below). More recently the special group 3 devices have been placed on a systematic basis which will be described in the next section.

4.4. Cross section libraries for the SANDPIPER programmes

The first step in the creation of the Winfrith series of comprehensive lattice programmes was the replacement of MUFT by a suitable library of fitted cross-sections.

In his publication OMBRELLARO [22] listed cross-sections for the following elements,

$$\text{H, O, Zr, C, Fe, Ni, Nb, Cr, U}^{235}, \text{U}^{238} \text{ and B}^{10}$$

these have been supplemented at Winfrith by additional MUFT calculations which gave constants for

$$\text{Al, Pu}^{239}, \text{Pu}^{240} \text{ and Pu}^{241}.$$

The resulting tabulations of fitted cross-sections, together with the special devices, equations (4.2) and (4.3), forms the basis of the SANDPIPER I [7] programme library of fast neutron data. Since most of the data stems from the MUFT IV production tape released by Westinghouse in 1957 this is referred to as the "1957 Data Set" at Winfrith.

In 1960 HENRY [24] published an updated set of cross-sections for use with MUFT and related codes. This has been converted [25] into a compilation of fitted cross-sections referred to as the "1960 Set". The 1960 MUFT library differs considerably in detail from the 1957 version particularly in the fact that many elements other than U^{238} have resonances explicitly represented by Breit-Wigner parameters. This has led to a systemization of the group 3 special devices in use at Winfrith. In general cross-sections in group 3 are represented by a parametric display of the form:-

$$\delta_3 = g(\rho_3) \left\{ A + \frac{C\delta\rho}{\sqrt{\delta\rho(\delta\rho + D)}} \right\} \quad (4.5)$$

where

$g(\rho_3) = A$ polynomial function of ρ_3

A, C, D are constants

$$\delta \rho' = (\delta \rho^c + \gamma_s/4NV) \quad \text{in the notation of equation 2.1.}$$

The contents of the curly bracket approximate to equation (2.1) for concentrated absorbers such as U^{238} or Th^{232} . However, the form used in (4.5) has the advantage that it tends to an infinite dilution limit equal to (A+C) and is therefore useful for elements likely to be present in small quantities, for example Pu^{240} .

Thermal cross-sections in the SANDPIPER lattice programmes are derived from the AMSTER [26] tabulations of standard SOFOCATE [6] results which are stored in the programme libraries. Small changes in the generally accepted cross-section values which have taken place since the publication of the "SOFOCATE Second Deck" [27] are dealt with by the inclusion of appropriate normalizing factors. Deuterium moderator is regarded as being equivalent to an amount of hydrogen with the same slowing down power.

5. THE SANDPIPER I AND II LATTICE CALCULATION PROGRAMMES

5.1. Sandpiper I

SANDPIPER I [7] is a computer programme which carries out a modified form of the lattice calculation scheme discussed in section 2. It is written in Autocode for the 4 drum version of the Ferranti MERCURY computer. The simplifications which result in the replacement of the MUFT and SOFOCATE computer codes by library tabulations have already been discussed. Additional changes are summarized below; a fuller discussion will be presented elsewhere [28].

- (i) It was not feasible to include a two dimensional routine to calculate the fine structure. A one-dimensional calculation is performed for a system in which the volume ratios and the surface to volume ratio of the fuel bearing region are preserved. It has been shown that this leads to negligible reactivity errors but the ability to calculate local power peaking accurately has been lost. The latter still requires an independent PDQ [4] calculation.
- (ii) Hyperfine structure in the pin cellules is calculated by the collision theory technique described by NEWMARCH [13]. Here again the cellule geometry is transformed to an equivalent slab system with preservation of volume ratios and surface to volume ratio of the fuel.

The result is a convenient means of generating lattice constants requiring approximately 2 min of MERCURY time per case. It is thus a useful tool in survey type calculations.

5.2. Sandpiper II

The SANDPIPER I programme was applicable to UO_2 lattices only, accepted only pin type fuel geometries and used the "1957 set" of cross-section data. Since the "1960 set" with its more general representation of group 3 processes necessitated programme alterations, the opportunity was taken to increase the coverage of the programme. SANDPIPER II will therefore accept a general fuel composition, pin or plate fuel geometries, and uses the "1960 set" of cross-sections. In all other respects it is identical to SANDPIPER I.

5.3. Comparison with experimental results

The 18 low enrichment U.S. lattices analysed by the MUFT-SOFOCATE method (see section 2.5 and Tables III and IV) have also been analysed with the SANDPIPER codes. Table V shows a comparison of the values of k_{eff} calculated for the measured bucklings.

The MUFT-SOFOCATE and SANDPIPER I results are both based on the "1957" nuclear data. The use of SANDPIPER I results in a small loss of accuracy in reactivity predictions since the standard deviation rises from 0.7 to 0.9%. It is interesting to note that the use of the more refined data in SANDPIPER II restores the accuracy to $\pm 0.7\%$. Shifts of the mean eigenvalues between 0.997, 1.000 and 1.001 are scarcely significant in assessing the values of the various schemes of calculation since the neutron yield per fission is not known to this order of accuracy.

These results, and additional analysis to be presented elsewhere [28], have led to considerable confidence being placed in the SANDPIPER codes as survey and experiment design tools. It also provided an incentive to develop burn-up programmes which use SANDPIPER to recalculate lattice constants at intervals during life.

6. PROBLEMS ASSOCIATED WITH PLUTONIUM ISOTOPES

6.1. Review of theoretical problems

The presence of Pu isotopes in the lattice, either by deliberate introduction in the initial charge, or as the result of burn-up of low enrichment uranium, introduces a number of new physical problems.

In the first place, the presence of the 0.3 eV resonance in Pu^{239} implies that the fuel, fertile, moderator and structural materials no longer have a common essentially $1/v$ cross-section dependence. This means that a single thermal group model extending from 0-0.625 eV may no longer be adequate in assigning fine and hyperfine structure weighting factors to the cell constituents. Secondly, the presence of the resonance modifies the neutron spectrum and the very simple, mass unity, Wigner-Wilkins scattering model may not be good enough to define the relative reaction rates adequately. The Pu^{240} resonance at 1eV requires special consideration since it lies in the intermediate energy range between the slowing down and thermalization energy regions.

TABLE V

ANALYSIS OF U.S. LOW ENRICHMENT LATTICES

Lattice No.	k_{eff}		
	MUFT-SOFOCATE	SANDPIPER I	SANDPIPER II
	1957 Data	1957 Data	1960 Data
L.1	0.004	0.992	0.993
L.2	0.999	0.996	0.997
L.3	0.995	0.996	0.998
L.4	0.991	0.989	0.991
L.5	0.992	0.990	0.992
L.6	1.000	0.996	0.997
L.7	1.004	0.996	0.998
L.8	0.998	0.996	0.998
L.9	1.003	1.010	1.005
L.10	1.007	1.012	1.010
L.11	1.006	1.009	1.010
L.12	0.996	1.007	1.003
L.13	1.006	1.014	1.013
L.14	0.987	0.993	0.998
L.15	0.999	1.007	1.004
L.16	0.994	1.001	1.001
L.17	1.001	1.008	1.014
L.18	0.975	0.983	0.990
MEAN	0.997 ± 0.007	1.000 ± 0.009	1.001 ± 0.007

At Winfrith the problems of plutonium in water lattices are being tackled at two levels. The comprehensive lattice programmes are being written on the assumption that single thermal group disadvantage factors and the Wigner-Wilkins spectrum representation are still valid. A crude prescription for the Pu^{240} effective group 3 cross-section of the form shown in equation (4.5) has been evolved on the assumption that the resonance is narrow with respect to hydrogen or deuterium collision energy exchanges, but broad with respect to collisions with other elements. The simple models used in the comprehensive lattice programmes are being backed by more extensive studies using multi-group methods, and particular emphasis is being placed on detailed reaction rate comparisons between Carlson S_n and the few group diffusion methods. In this way it is hoped to establish the range of validity

of the simpler methods and to improve the range by, for example, replacing the crude Pu^{240} effective cross-section prescription by a correlation to more detailed multi-group calculations.

6.2. The Helen I experimental programme

The importance of the problems associated with the plutonium isotopes has led to the launching of an experimental programme in the HELEN I assembly at Winfrith. This work is described in more detail in the companion report [1]. The aims are:

- (i) To calibrate the assembly with U^{235}/Al fuel;
- (ii) To undertake comparison experiments with $\text{Pu}^{239}/\text{Al}$ relatively soft neutron spectra;
- (iii) To extend the plutonium measurements to harder spectra and the effects of the higher isotopes.

At present it is possible to report on items (i) and (ii). Three lattices fuelled with U^{235}/Al and three lattices fuelled with $\text{Pu}^{239}/\text{Al}$ plates have so far been studied. Values of k_{eff} corresponding to the measured bucklings have been calculated by the few group methods described in section 2. More elaborate calculations using the Carlson S_4 method and 40 thermal groups are also in progress. Table VI summarizes the results of the calculations to date.

The MUFT-SOFOCATE eigenvalues for the U^{235} lattices are significantly higher than those obtained with the more detailed Carlson method. In both cases the departure from unity is scarcely significant within the present accuracy of nuclear data. The lower MUFT-SOFOCATE values possibly arise because the one group collision theory treatment of local flux variations ignores changes in thermal spectra which may be important in the rather widely spaced lattices considered. The single group gives too soft an estimate of the spectrum in the fuel and hence overestimate both η and f .

Systematically lower eigenvalues than for the U^{235} experiments have been obtained for the plutonium lattices with the MUFT-SOFOCATE scheme. It is not yet clear if this is due to a failure of the one thermal group treatment,

TABLE VI

ANALYSIS OF HELEN I EXPERIMENTS

Core	Fuel	Pitch (in)	Measured buckling m^{-2}	Eigenvalue	
				MUFT-SOFOCATE	CARLSON
1	U^{235}/Al	1.32	8.6 ± 1.3	1.004 ± 0.004	0.984 ± 0.004
2	U^{235}/Al	1.07	46.3 ± 2.4	1.016 ± 0.007	0.996 ± 0.007
3	U^{235}/Al	0.82	90.0 ± 3.9	1.007 ± 0.011	0.989 ± 0.011
4	$\text{Pu}^{239}/\text{Al}$	1.32	-15.1 ± 1.0	0.980 ± 0.003	-
5	$\text{Pu}^{239}/\text{Al}$	1.07	21.4 ± 1.5	0.980 ± 0.004	-
6	$\text{Pu}^{239}/\text{Al}$	0.82	64.7 ± 4.4	0.974 ± 0.013	-

the Wigner-Wilkins spectrum model, or anomalies in the nuclear data. The 2200 m/s values of the cross-sections used are normalized to the "1960 World Consistent Set" [29]. It is hoped that the Carlson calculations and a careful study of the relative reaction rate measurements will throw more light on this discrepancy.

7. BURN-UP CALCULATIONS

7.1. The REDSHANK I programme

REDSHANK I is a point reactivity four group burn-up programme based on the SANDPIPER I lattice constant generator. It covers burn-up in low enrichment UO_2 fuelled lattices controlled by conventional poison devices.

Operation begins with an evaluation of the four group constants and k_{eff} for a system of specified lattice geometry, fuel loading and geometric buckling. Thermal and epithermal poisons appropriate to full insertion of a specified control absorber system are then estimated by the amplified one-dimensional methods described by HENRY [21]. The percentage control insertion required to achieve a target initial reactivity is then estimated and the appropriate poisons added in groups 3 and 4. Few-group fluxes are then calculated and burn-up commences. At intervals during life the SANDPIPER I routine is called in to recalculate the lattice constants and the control absorber percentage insertion adjusted to maintain a specified mean value of k_{eff} . Burn-up ceases when the control absorbers are fully withdrawn.

7.2. The GREENSHANK I programme

The GREENSHANK I programme is very similar to REDSHANK I except that control is exerted by varying the $\text{H}_2\text{O}/\text{D}_2\text{O}$ ratio in the coolant. In this case the programme is provided with a specified lattice geometry and fuel enrichment and the computation begins with an evaluation of the reactivity with pure H_2O moderator. This gives an indication of the reactivity excess potentially available for burn-up. The programme then hunts for the $\text{H}_2\text{O}/\text{D}_2\text{O}$ concentration appropriate to a specified target initial reactivity and burn-up commences. The SANDPIPER I routine is called in and the lattice constants recalculated at intervals during life, simultaneous adjustments of $\text{H}_2\text{O}/\text{D}_2\text{O}$ concentration being made to preserve a specified mean value of k_{eff} . Burn-up ceases either when the moderator has been diluted to pure H_2O or a state of optimum moderation corresponding to control reversal is reached.

7.3. Presentation of typical results

As an example of the type of calculation which can be undertaken with REDSHANK and GREENSHANK reference should be made to Fig. 12. This shows a comparison of the reactivity versus irradiation life curves for conventional absorber and spectrum shift reactivity control. The case illustrated is for a large power reactor fuelled with 3.5% enriched UO_2 ; the fuel pins were 0.3 in diam, the cladding material Zircaloy and the coolant to UO_2 volume ratio 1.5.

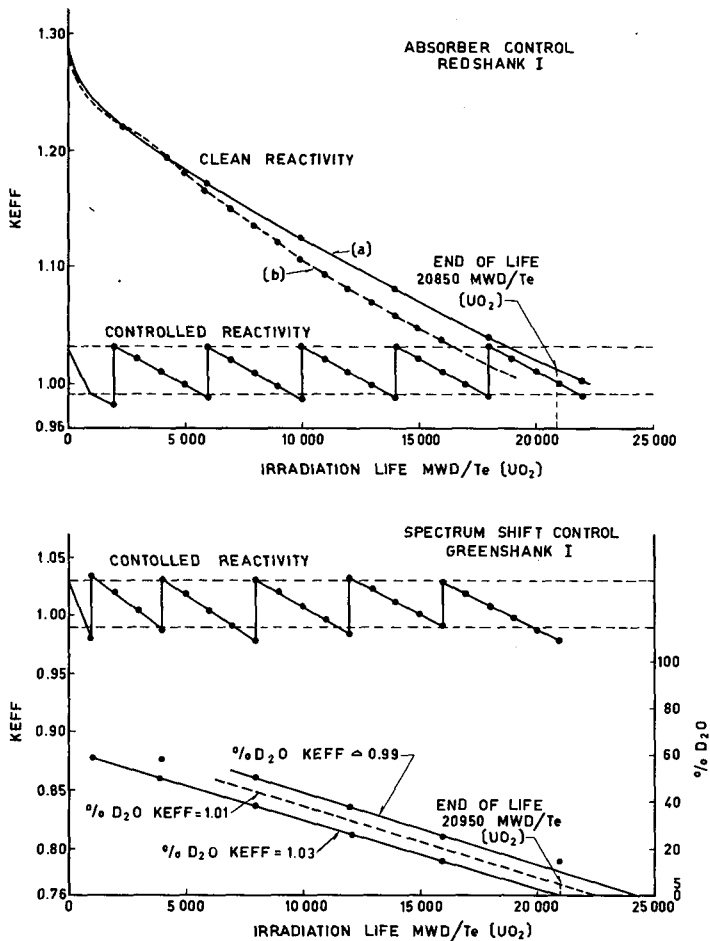


Fig. 12

Comparison of absorber control with spectrum shift

The upper set of curves for absorber control shows the initial estimate of the clean k_{eff} to be 1.30. Control rod insertion reduced this to 1.02 and burn-up commenced. The reactivity was allowed to wander over the range $k_{eff} = 1.01 \pm 0.02$; the mean value of 1.01 including a nominal 1% allowance for Doppler effect. Each time the reactivity fell below $k_{eff} = 0.99$ the SANDPIPER routine came into operation, the control rods were withdrawn and a fresh point on the clean reactivity curve (a) was obtained. The control rods were then re-inserted until $k_{eff} = 1.03$ and burn-up recommenced. In this particular case core life terminated at an irradiation of 20 850 MWD/Te UO_2 where the clean reactivity curve intersects the line $k_{eff} = 1.01$.

This calculation does not demonstrate that a high burn-up of this magnitude would necessarily be achievable in practice since the specified control rod array required a formal insertion of greater than 200% to hold down

the initial reactivity. These absorbers would therefore need to be supplemented by burnable or soluble poisons.

The broken clean reactivity curve (b) shows the effect of eliminating the intermediate SANDPIPER recalculations of the lattice constants. This reduces the lifetime estimate by approximately 2000 MWd/T(UO₂) which seems sufficiently large to justify the extra complication.

The corresponding curves for Spectrum Shift control are illustrated in the lower part of Fig. 12. As before the reactivity was allowed to hunt over the range $k_{\text{eff}} = 1.01 \pm 0.02$. After the initial Xe transient the reactor continued burn-up with a $k_{\text{eff}} = 1.02$ and a D₂O volume fraction of nearly 60%. The two lower full curves show the D₂O percentages immediately before and after each SANDPIPER reset and therefore correspond to reactivities of $k_{\text{eff}} = 0.99$ and 1.02 respectively. For practical purposes burn-up is regarded as ceasing when the dotted line corresponding to $k_{\text{eff}} = 1.01$ intersects a D₂O purity of 5% which occurs at 20 950 MWd/T(UO₂). This figure is remarkably close to that achievable with conventional control. However, the comparison is slightly misleading since the spectrum shift burn-up could be continued to nearly 22 500 MWd/T(UO₂), if dilution of the coolant to pure H₂O were economically justifiable.

Further comparisons between the burn-up characteristics obtainable with conventional and spectral shift control which have been obtained from the output of the REDSHANK and GREENSHANK codes are presented in Fig. 13. The upper diagram shows the net consumption of fissile material as a function of water to UO₂ volume ratio. The net consumption is defined in the manner suggested by BENGSTON *et al.* [23], i.e., as:-

$$(x - y - z) \text{ g/MWd}$$

where

$$\begin{aligned} x &= \text{U}^{235} \text{ burnt } \text{ g/MWd} \\ y &= \text{Pu}^{239} \text{ formed } \text{ g/MWd} \\ z &= \text{Pu}^{241} \text{ formed } \text{ g/MWd} \end{aligned}$$

These curves clearly show the superior neutron economy of the SSCR over the conventional PWR. Moreover, the increased conversion factors attainable in the drier lattices are clearly indicated. The trends for the PWR are very similar to those obtained by BENGSTON *et al.* [23].

The second diagram of Fig. 13 shows that it should not be assumed that the drier lattices are necessarily the most economically attractive, for here the consumption of U²³⁵ is greatest and since a greater value is generally assigned to U²³⁵ than to Pu²³⁹ minimum fuel costs do not necessarily occur at minimum net consumption. This point is further reinforced by the lower-most figure which shows that in the drier lattices lower burn-ups result for a given enrichment. During the earlier stages of irradiation the conversion factor is high but insufficient plutonium has formed to compete effectively with the U²³⁵. The short irradiation cores therefore are predominantly U²³⁵ burners.

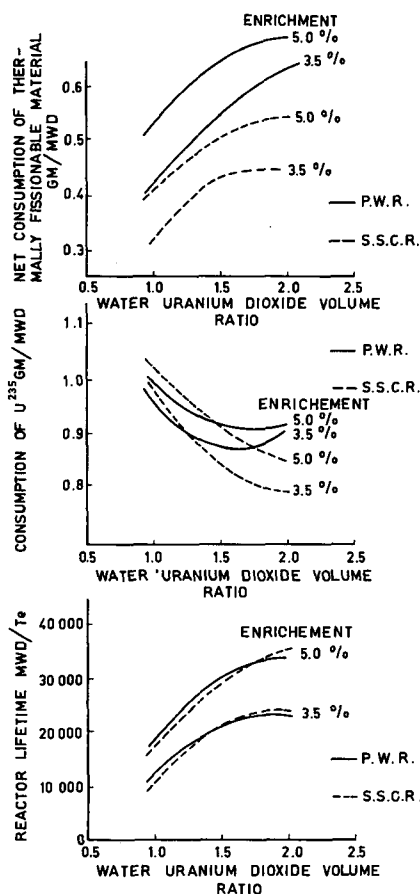


Fig. 13

Dependence of burn-up characteristics on coolant to uranium dioxide volume ratio

8. CONCLUSIONS AND REVIEW OF WORK IN PROGRESS

This report has shown that comprehensive few-group methods have been developed at Winfrith on the foundations laid by Westinghouse. These methods are capable of predicting the behaviour of a wide range of lattices with an accuracy acceptable in survey or initial design calculations. There are undoubtedly many ways in which the scheme could be improved. For example, it is desirable to replace the U^{238} resonance integral correlation with a treatment resting on a sounder theoretical basis, including an allowance for Doppler effects, and applicable with a greater degree of confidence in more undermoderated lattices. In general, the few-group calculations require more extensive backing by detailed multi-group calculations especially when plutonium isotopes are present. The work described here forms only a part of a wider programme covering both light and heavy water moderation with some emphasis on the problems associated with pres-

sure tube reactors. Few group design methods are under development in a form suitable to provide coverage over the whole field. A comprehensive lattice and burn-up programme METHUSELAH has been coded in Fortran II for the IBM 7090 and is now under development. METHUSELAH will perform any of the functions of the SANDPIPER-REDSHANK-GREENSHANK series as well as comparable types of calculations for the pressure tube systems.

More fundamental theoretical investigations are directed towards evaluations of the few group design scheme, studies of Pu^{240} effects, Monte-Carlo investigations of resonance absorption, etc. The further development of METHUSELAH into a space dependent burn-up programme is also in hand. It is apparent that there will be no lack of theoretical problems to be tackled in this field for some time to come.

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ANALYSE DES EXPÉRIENCES EFFECTUÉES DANS ALIZÉ I

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Abstract — Résumé — Аннотация — Resumen

ANALYSIS OF EXPERIMENTS CARRIED OUT IN ALIZÉ I. Alizé I experiments with a light water uranium oxide 1,5 wt.% lattice were mainly devoted to the study of absorbing plates (silver - indium - cadmium, boron). We tried to get some information on epithermal absorption, taking displaced volume and surrounding water gaps carefully into account. Exploitation of experimental results leads to empirical parameters, but some other experiments are needed to extend the range of validity as far as spectrum and geometry are concerned.

ANALYSE DES EXPÉRIENCES EFFECTUÉES DANS ALIZÉ I. Les expériences effectuées dans Alizé I sur un réseau d'oxyde d'uranium enrichi à 1,5% et modéré à l'eau légère avaient pour but l'étude de propriétés absorbantes de matériaux de barres de contrôle (argent - indium - cadmium, et bore sous forme de plaques). On s'est efforcé d'atteindre des paramètres caractéristiques des matériaux dans le domaine épithermique, en cherchant à éliminer les effets parasites. L'exploitation des résultats expérimentaux a permis de définir des paramètres utilisables pour des projets; il conviendrait toutefois de vérifier leur validité pour des spectres neutroniques et des géométries d'absorbants différents.

АНАЛИЗ ОПЫТОВ НА УСТАНОВКЕ АЛИЗЕ I. Эксперименты Ализе I с легководной решеткой с 1,5 вес % окиси урана были посвящены главным образом изучению поглощающих пластин (серебро-индий-кадмий, бор). Мы пытались получить некоторые данные относительно поглощения надтепловых нейтронов, тщательно учитывая при этом смещенный объем и окружающие водяные зазоры.

Использование экспериментальных результатов дает эмпирические параметры; однако необходимы некоторые другие эксперименты для расширения предела обоснованности в отношении спектра и геометрии.

ANÁLISIS DE LAS EXPERIENCIAS EFECTUADAS EN ALIZÉ I. Los experimentos efectuados en la instalación Alizé I, con un reticulado de óxido de uranio enriquecido al 1,5% en peso y moderado con agua ligera, tuvieron por principal propósito el estudio de las propiedades absorbentes de los materiales de las barras de control (plata-indio-cadmio y boro). Los autores procuraron reunir datos sobre la absorción epitérmica teniendo debidamente en cuenta el volumen desplazado y los intersticios del agua circundante.

Los resultados experimentales obtenidos han permitido determinar los parámetros empíricos, pero es preciso efectuar nuevos experimentos para extender los límites de validez a otros espectros neutrónicos y otras geometrías.

1. DESCRIPTION D'ALIZÉ I

L'installation ALIZÉ I a été construite au CEN, Saclay, en vue de l'étude des réseaux à eau légère et uranium légèrement enrichi. En fait, une modification du programme nécessitant la transformation de l'installation n'a permis pratiquement qu'une étude assez systématique d'éléments absorbants dans un seul cœur.

a) Caractéristiques générales

L'ensemble a été monté dans une fosse de section $4 \times 3,30 \text{ m}^2$ et d'environ 9 m de profondeur. Ce puits contenait :

- la cuve ($\varnothing = 2 \text{ m}$; $H = 2,5 \text{ m}$; $e = 6 \text{ mm}$) en aluminium prolongée par une virole en acier inoxydable supportant la charge du réseau, le tout fermé par un couvercle et calorifugé par un matelas de laine de verre,
- à l'intérieur de la cuve, le panier support de cœur, prenant assise sur une portée horizontale circulaire de la virole, contenant le cœur et ses accessoires,
- un réservoir de vidange,
- l'ensemble des circuits d'eau comprenant un réchauffeur et les divers mécanismes.

La fosse était fermée par deux dalles de protection se déplaçant sur rails, constituée par 7 cm de tôles d'acier, 60 cm de paraffine et 2 cm d'acide borique. Des sorties en chicanes permettaient le passage des canalisations et des nappes de câbles.

b) Installations

Outre les circuits de montée et vidange d'eau, l'installation comprenait un circuit de brassage avec réchauffeur et un ballon calibré permettant de faire varier le volume d'eau de manière reproductible.

Le niveau d'eau était repéré par un ensemble de pointes électriques calées en hauteur absolue ainsi que par un palpeur électrique permettant les interpolations.

Les organes de détection logés dans des chaussettes étanches placées dans le réflecteur comprenaient : deux compteurs BF3, deux chambres d'ionisation avec amplificateur linéaire, une chambre à amplificateur logarithmique de doublement (1%) muni d'un différentiateur.

c) Réseau

Pour le seul réseau étudié les éléments combustibles étaient disposés suivant un pas carré de 11,1 mm grâce à trois grilles de polyéthylène PBP ($d \text{ PBP} = 0,96$, $(MA/PBP)/(MH/H_2O) = 1,23$, dilatation linéaire = celle de l'eau, $e = 0,5 \text{ mm}$, cote par rapport au point bas du combustible $+ 0$, $+ 500 \text{ mm}$, $+ 1000 \text{ mm}$). Ces grilles assuraient un positionnement moyen correct et reproductible, l'équidistance entre barreaux étant particulièrement bien respectée dans les zones prévues pour des mesures.

Six irrégularités dans les distributions des éléments combustibles permettaient le passage des plaques de sécurité et de contrôle (cadmium : $e = 1 \text{ mm}$, largeur 120 mm).

Les modifications importantes du réseau étaient effectuées à l'extérieur de la cuve, le panier support du cœur étant transporté sur un bâti.

d) Eléments combustibles

L'élément unitaire était constitué par des billettes d' UO_2 .

Densité 10,16; $\phi = 6,935$ mm; $L = 12$ mm

E (% poids) $1,51 \pm 0,01$ (USA) ou $1,495 \pm 0,015$ (France)

Nous avons cherché à utiliser ces billettes de manière à permettre facilement et sans provoquer des bris d'oxyde

- 1° de modifier la longueur utile des éléments,
- 2° de permettre des mesures à l'intérieur du combustible,
- 3° de modifier éventuellement la nature du gainage.

La solution du pré-gainage indiquée ci-dessous a donné toute satisfaction et a été retenue ultérieurement pour le gainage d'oxyde à 3,5% ou d'oxyde naturel de divers diamètres.

Les billettes ont été pré-gainées par séries de quinze dans des tubes d'aluminium mince ($\phi : 7,1/7,3$ mm) puis placées dans les véritables gaines d'aluminium ($\phi : 7,5/9$ mm) étanches. Une tige de PBP de 15 cm avait été placée à chaque extrémité pour servir de réflecteur - la hauteur totale d' UO_2 dans les expériences d'ALIZÉ I était de 108 cm.

2. BUT DES EXPÉRIENCES

Le but recherché était essentiellement la vérification des méthodes de calcul des barres de contrôle et la détermination de paramètres caractérisant l'efficacité épithermique de deux matériaux de barre de contrôle d'usage courant, savoir : l'argent-indium-cadmium et le carbure de bore à l'état de dispersion dans une matrice support. Les expériences n'ont porté que sur un seul réseau propre ou avec un élément absorbant. Tous les absorbants considérés successivement (sous la forme d'une plaque) ne différaient que par leur nature. Quelques configurations géométriques de pile dépourvue d'élément absorbant mais utilisant le même réseau ont été également étudiées.

3. MÉTHODE DE CALCUL

a) Calcul des cœurs sans barre de contrôle

Les calculs de réactivité sont effectués en théorie de diffusion à deux ou quatre groupes en deux dimensions (xy) avec le calcul PDQ [1]; la troisième dimension est prise en considération à l'aide d'un laplacien axial B_z^2 déterminé expérimentalement.

Chaque barreau de combustible et la portion de modérateur qui lui est associé dans le réseau carré sont remplacés pour le calcul des constantes thermiques par la cellule cylindrique équivalente selon la technique classique. Ces constantes sont alors obtenues en utilisant des facteurs de désavantage calculés selon la méthode d'AMOUYAL et BENOIST [2]. Les sections efficaces microscopiques nécessaires sont pondérées sur un spectre de Wigner Wilkins à l'aide du code SOFOCATE [3] et avec la librairie de constantes du

« 2ème deck » [4] ; on a tenu compte de la dépression du flux dans l'uranium par introduction dans le code SOFOGATE des densités atomiques modifiées proportionnellement aux facteurs de désavantage.

Les constantes rapides du réseau infini sont calculées en considérant celui-ci comme étant homogène; elles sont pondérées sur le spectre de ralentissement de la pile nue équivalente. Les calculs sont faits par le code MUFT IV [5] avec l'approximation B1 et en utilisant les données les plus récentes pour les sections efficaces [6]. La capture de l'uranium-238 donnée par le code MUFT a été remplacée dans le domaine d'énergie allant jusqu'à 180 keV par celle déduite de l'intégrale effective ajustée sur les mesures expérimentales d'Hellstrand (y compris la capture résonante et la capture en $1/v$).

$$I_{\text{eff}} = 5,23 + 26,5 \left(X \frac{S}{M} \right)^{\frac{1}{2}}$$

où X est une correction d'effet d'ombre équivalente à la correction de Dancoff. L'effet de la capture réelle de ^{238}U sur le spectre en ralentissement est pris en considération grâce à un facteur de « self shielding » constant L, égal au rapport de l'intégrale effective réelle à l'intégrale effective en milieu homogène.

Le découpage en groupes est fait aux énergies habituelles, soient : 0,821 MeV, 5,53 keV et 0,625 eV.

Pour la configuration avec lame d'eau, on considère séparément la région du combustible qui est traitée comme le réseau infini et l'eau qui est traitée comme le réflecteur.

Les constantes multigroupes utilisées sont rassemblées dans l'annexe I.

b) Calcul des cœurs avec plaque absorbante

La plaque est considérée comme un corps noir dans le domaine thermique dans tous les cas considérés, ce qui est représenté dans le calcul par une condition de dérivée logarithmique sur sa surface $C_2 = D_2 \phi'_2 / \phi_2 = 0,47$. Dans le domaine épithermique nous avons essayé deux possibilités : soit adopter une condition de dérivée logarithmique sur la surface avec $C_1 = D_1 \phi'_1 / \phi_1$, soit considérer la barre comme un milieu doté d'un coefficient de diffusion $D = 1$ et d'une section efficace de capture Σ_1 convenable. En 4 groupes, on néglige la capture épithermique de la barre dans les deux groupes d'énergies les plus élevées où celle-ci est considérée comme un milieu de capture nulle et de coefficient de diffusion $D = 1$.

L'eau adjacente à la barre et n'appartenant pas au réseau est traitée comme une région indépendante dont les constantes sont calculées en milieu infini.

Le calcul est fait avec le code PDQ en deux dimensions (x y), la troisième dimension étant prise en considération grâce à un laplacien axial. Une étude initiale des courbes de flux axial calculées avec le code PDQ pour une pile avec une barre centrale annulaire équivalente en efficacité à l'un des absorbants essayés avait montré que l'économie de réflecteur inférieur paraissait sensiblement la même qu'en l'absence d'absorbant; faute d'un procédé plus satisfaisant, nous avons donc fait l'hypothèse que les écono-

mies de réflecteurs axiaux étaient, dans toutes les configurations avec barre, celle de la pile de référence.

Les paramètres C_1 ou E_1 en un groupe (C_3 ou E_3 en quatre groupes) seront tirés des expériences. Deux méthodes se présentent: dans la première on détermine le paramètre épithermique par la condition que la réactivité λ calculée avec le code PDQ pour la configuration critique avec absorbant soit égale à la réactivité λ calculée pour la configuration critique de la pile sans absorbant (pile de référence), dans la deuxième on impose à la réactivité λ d'être égale à la réactivité calculée pour la configuration critique avec du cadmium de même géométrie et disposé identiquement. Les deux méthodes ont l'avantage d'éliminer les erreurs pouvant résulter de l'incertitude sur la composition et dans une certaine mesure des incertitudes du calcul; cependant, dans la seconde méthode, les distributions des flux dans les deux configurations sont plus voisines et les erreurs qui résultent d'une mauvaise estimation des faits sont mieux compensées, d'autre part l'hypothèse de l'économie de réflecteur est plus justifiée, enfin les incertitudes résultant du calcul de l'efficacité de l'absorbant dans le domaine thermique sont réduites au minimum.

Les valeurs de E_1 et C_1 (ou E_3 et C_3) sont en principe liées par la relation [8]:

$$C_1 = \sqrt{E_1} D \operatorname{th} b \sqrt{\frac{E_1}{D}} \neq E_1 b$$

où $2b$ est l'épaisseur de la plaque absorbante.

L'influence du nombre de points dans la barre dans le calcul PDQ a été étudiée dans un cas: elle est négligeable.

D'autre part dans l'hypothèse où toute la capture épithermique de la plaque est concentrée dans le troisième groupe, une autre relation devrait tenir en principe entre C_3 et C_1

$$C_3 = C_1 \frac{\int_{0,625 \text{ eV}}^{10 \text{ meV}} \varphi(E) dE}{\int_{0,625 \text{ eV}}^{5,5 \text{ keV}} \varphi(E) dE}$$

Le rapport des intégrales de flux calculées par le code MUFT est de 3,148. Une même relation existe entre E_3 et E_1 .

Pour l'interprétation de certains résultats nous ferons également appel à la méthode de l'aire d'absorption en se basant sur le fait que l'efficacité relative de deux absorbants d'efficacité voisine, de même géométrie et disposés identiquement dans le réseau, est égale aux rapports de leur aire d'absorption. Les formes et les constantes utilisées sont reportées en annexe II.

4. EXPÉRIENCES EN RÉSEAU PROPRE

Un seul réseau de base est utilisé: les barres sont disposées uniformément selon un pas carré de 11,1 mm, l'uranium est enrichi à 1,5% les autres caractéristiques ont été données précédemment. Avec ce réseau,

quatre configurations de cœur ont été réalisées. Les résultats relatifs à chacune de ces formes de cœur sont reportés dans le tableau I; ils concernent :

- N : le nombre des barres de combustible,
- M : la masse critique d' UO_2 ,
- H : la hauteur d' UO_2 immergé critique,
- δ_l : l'économie de réflecteur latéral,
- δ_{zi} : l'économie de réflecteur axiale inférieure,
- δ_{zs} : l'économie de réflecteur axiale supérieure,
- B^2 : le laplacien matière.

Les limites d'erreur, compte tenu des incertitudes dues à la capillarité, sont les mêmes dans tous les cas :

$$-0,3 \text{ et } +1,5 \text{ mm.}$$

Ces résultats sont corrigés pour toutes les anomalies (boîtiers de détection, lame d'eau pour le passage des éléments de contrôle) et ramenés à la température de 18°C : l'ensemble des corrections est inférieur à 10 mm.

Le laplacien matière du réseau a été mesuré en dressant des cartes de flux à l'aide de détecteurs au manganèse. Ces détecteurs ($e = 0,15 \text{ mm}$, $\varnothing = 6 \text{ mm}$) étaient logés à l'intérieur des barres de combustible et protégés des produits de fission par une feuille d'aluminium. Un certain nombre de mesures ont été nécessaires pour éliminer les perturbations dues aux anomalies. Les ajustements des cartes de flux sur des distributions théoriques en $J_0 \times \sin$ ou $\sin \times \sin$ ont été faits grâce à une méthode de moindres carrés par la calculatrice IBM 650.

Les valeurs calculées pour la réactivité λ en deux et à quatre groupes sont également reportées dans le tableau : dans ces calculs, faute de résultats expérimentaux suffisants, les économies de réflecteurs ont été supposées les mêmes pour les cœurs à base carrée et polygonale, et avec quadrillage de barre d'eau, ce qui peut expliquer les écarts constatés.

Equivalent en réactivité d'un millimètre d'eau

1° Cœurs homogènes

Grâce aux mesures d'absorbants, l'équivalent en réactivité d'un millimètre d'eau a pu être mesuré pour des valeurs de H, hauteur d'uranium immergé allant de 640 à 790 mm. La relation cinétique a été corrigée pour tenir compte de l'effet des variations de laplacien géométrique. L'erreur systématique à craindre sur la valeur absolue de la réactivité est de l'ordre de 5%.

Dans le domaine de mesures, $\delta\rho/\delta h$ est représenté à $\pm 2\%$ par :

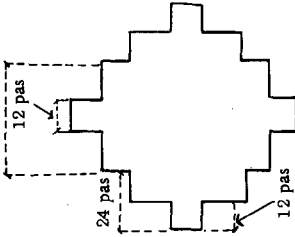
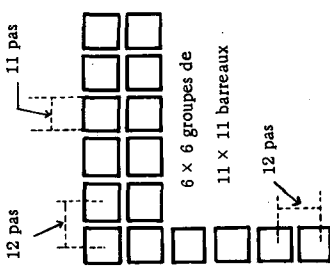
$$\frac{\delta\rho}{\delta h} = \frac{K}{(H + \Delta H)^3} = \frac{93\,500}{(H + 165)^3} \cdot 10^{-5} / \text{mm}$$

(ajustement par moindres carrés).

Si l'on admet $k_{\infty} = e^{M^2 B^2}$, d'une part la valeur de l'aire de migration

TABLEAU I

RÉSULTATS RELATIFS A CHACUNE DES FORMES DE CŒUR

	Cœur cylindrique	Cœur à base carrée	Cœur à base polygonale	Cœur avec quadrillage de lame d'eau
N barres	5185	5329	5184	4356
M kg	1280,4	1348,6	1331,5	1153,7
H mm	639,9	655,8	665,6	686,3
δq mm	$74,1 \pm 2,4$	$78,5 \pm 4,2$		
δz_1 mm	$84,3 \pm 3,8$	$79,3 \pm 3,4$		
δz_2 mm	$73,3 \pm 3,8$	$77,5 \pm 3,4$		
$B^{-2} m^{-2}$	$36,5 \pm 0,3$	$36,1 \pm 0,5$		
K_{eff} calculé (2 groupes)	0,99968	0,99942	1,00014	1,00325
K_{eff} calculé (4 groupes)		0,99984		1,00467
Rayon équivalent 45,1 cm		côté: 81,04 cm		
Géométrie				

M^2 est 47,4 cm², d'autre part la valeur des économies de réflecteur axiales est de 165 mm à comparer à $156,8 \pm 2,9$ mm pour le cœur carré.

2° Cœur quadrillé

En admettant encore $k_{\infty} = eM^2B^2$ et la valeur ajustée précédemment pour les économies de réflecteur, on obtient :

$$M^2 = 44,2 \text{ cm}^2.$$

Remarque : Dans l'ajustement utilisé, l'imprécision sur ΔH est relativement grande. Une erreur sur la relation cinétique se reporte directement sur K . Les variations des paramètres K et ΔH sont liées par $\delta\Delta H = \delta k/307$, c'est-à-dire une incertitude de 2% sur la relation cinétique, incertitude de 6 mm sur ΔH .

5. EXPÉRIENCES AVEC DES ÉLÉMENTS ABSORBANTS

5.1. Technique expérimentale

Toutes les mesures indiquées ici ont eu pour référence la pile carrée. Les absorbants testés se présentaient sous forme de sandwiches réalisés par assemblage de plaques de diverses natures. Ces plaques planes avaient en commun la hauteur de 1 000 mm et la largeur de 120 mm. Les sandwiches étaient placés au centre d'une lame d'eau créée par enlèvement de douze barres de combustible dans un plan diamétral (barres n°7 à 18 enlevées, barre centrale n°1). Le point du sandwich coïncidait avec le point bas du combustible : la taille critique était déterminée par approche cinétique.

Des expériences préliminaires avaient montré l'importance très faible de la présence de l'absorbant dans les réflecteurs : l'essai avait été fait avec l'absorbant A-12 (voir plus loin) successivement, enfoncé dans le seul réflecteur supérieur (position I), jusqu'à la partie inférieure du combustible (position II), enfin enfoncé totalement, réflecteur inférieur compris (position III) (fig. 1).

En position II l'absorbant vaut environ $1850 \cdot 10^{-5}$, la mesure donne $10 \cdot 10^{-3} \pm 4$ pour la position I et $8 \pm 6 \cdot 10^3$ pour l'écart entre les positions II et III.

5.2. Nature des absorbants

Trois types d'absorbants ont été essayés : cadmium, argent-indium-cadmium, aluminium boré. La nature exacte des sandwiches est précisée par un code dont la signification est indiquée ci-dessous.

a) Cadmium : Cd e = 0,51 mm.

Le cadmium est utilisé sous forme de plaques 1 000 X 120 mm ou de boîtes (e = 0,51 mm) enveloppant de tous les côtés un ensemble de plaques. La largeur des boîtes est 122 ± 1 mm.

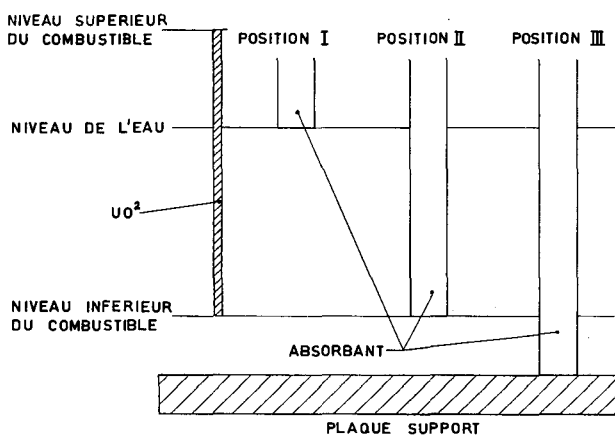


Figure 1

Dispositif d'expérience avec les éléments absorbants.

b) Argent - indium - cadmium : AIC

Ag 80%, In 15%, Cd 5% en poids.

e = 1 mm.

Type	e total mm	teneur en bore
B ₁	3,85	120 mg/cm ²
B ₂	5,75	240 mg/cm ²
B ₃	7,55	360 mg/cm ²

Granulométrie du bore

Taille max

B4C en μ	16	$6\sqrt{2}$	6	$3\sqrt{2}$	3	$3/\sqrt{2}$	1,5	$1,5/\sqrt{2}$	0,75
%	100	63	53,5	42	23,6	15,2	9,4	4,9	3,2

c) Plaques aluminium : Al et acier inoxydable : Ac

Pour chaque essai nous indiquons la disposition relative des plaques. Le chiffre précédant la nature des plaques donne le nombre de ces plaques, le chiffre suivant, l'épaisseur de la plaque en mm. En général, les sandwiches ont une épaisseur théorique de 10 mm.

5.3. Corrections - Erreurs

Les résultats ont été corrigés pour la température, les anomalies du réseau et les écarts connus du sandwich par rapport à sa position de référence.

L'évaluation des erreurs tient compte :

- du niveau (l'erreur absolue due à la capillarité disparaît),
- de l'incertitude sur le positionnement,
- de l'effet de foisonnement du sandwich.

La valeur de chaque sandwich a été mesurée une fois au moins et cinq fois au plus (nombre moyen de mesure : 2).

TABLEAU II
RÉSULTATS CARACTÉRISANT LES ABSORBANTS

Désignation de l'absorbant	Nature	H (mm)	$\Delta\lambda_{ex}$
A-1	Boîte Cd (Al9)	102,5 ± 1,1	1370 ± 10
A-2	Boîte Cd (Cd + Al 8 + Cd)	106,2 ± 1,9	1410 ± 20
A-3	Boîte Cd (3cd + Al 6 + 3cd)	109,2 ± 2	1447 ± 21
A-4	Al 1,5 + Boîte Cd (Al 6,5) + Al1	100,9 ± 3,1	1354 ± 32
A-5	Al 2 + Boîte Cd (Al 5) + Al 2	95,4 ± 1,7	1296 ± 18
A-6	Al 3,5 + Boîte Cd (Al 2) + Al 3,5	95,6 ± 2	1298 ± 21
A-7	Al 4,5 + 2 Cd + Al 4,5	92,2 ± 1,4	1263 ± 15
A-8	Al 1 + 2 Cd + Al 1	91,0 ± 1,4	1250 ± 15
A-9	Boîte Cd (3 AIC)	150,4 ± 2,1	1865 ± 22
A-10	Boîte Cd (AIC + Al 3 + AIC)	142,9 ± 2	1791 ± 23
A-11	Boîte Cd (Al 3 + AIC + Al 3)	131,8 ± 1,9	1679 ± 20
A-12	AIC + Al 0,5 + AIC Al 0,5 + AIC	143,1 ± 1,4	1797 ± 15
A-13	AIC + Al 4 + AIC	138,1 ± 2	1745 ± 21
A-14	Al 3,5 + AIC + Al 3,5	116,0 ± 1,8	1513 ± 33
A-15	Al 2 + 2 AIC + Al 2	135,4 ± 2,1	1716 ± 22
A-16	2 AIC	131,0 ± 2,1	1670 ± 22
A-17	AIC	114,4 ± 1,3	1496 ± 14
A-18	Boîte Cd (Al 0,5 + B 3 + Al 0,5)	153,2 ± 2,5	1903 ± 26
A-19	Boîte Cd (Al 1,5 + B 2 + Al 1,5)	139,5 ± 2,2	1757 ± 23
A-20	Boîte Cd (Al 2,5 + B 1 + Al 2,5)	128,9 ± 2,1	1648 ± 22
A-21	Al 1 + B 3 + Al 1	143,6 ± 2,2	1803 ± 23
A-22	Al 2 + B 2 + Al 2	131,7 ± 2,2	1675 ± 23
A-23	Al 3 + B 1 + Al 3	117,1 ± 2,3	1524 ± 24
A-24	AC 2 + B 2 + AC 2	126,0 ± 2,1	1618 ± 22
A-25	B 2	129,4 ± 2,1	1654 ± 22
A-26	B 2	116,7 ± 2	1520 ± 21
A-27	Al 5 + Cd + Al 4,5	88,4 ± 1,7	1223 ± 18
A-28	Al 10	8,6 ± 0,6	
A-29	Al 3	0,5 ± 0,4	
A-30	AC 8	39,0 ± 1,2	
A-31	Lame d'eau	- 2,3 ± 0,5	

5.4. Résultats expérimentaux

Les valeurs des absorbants sont finalement caractérisées par l'accroissement de la hauteur critique ΔH par rapport à la pile de référence, résultant de leur introduction. Nous avons traduit cette efficacité en réactivité en évaluant avec le code PDQ l'excédent de réactivité par rapport à la pile de référence, $\Delta\lambda_{ex}$, de la pile sans absorbant mais avec une hauteur accrue de ΔH .

Les principaux résultats sont condensés dans le tableau II.

a) Expériences avec le cadmium

1° Epaisseur de cadmium variable

Les absorbants A-1, A-2 et A-3 sont constitués par la boîte de cadmium remplie successivement avec 0, 2 et 6 plaques de cadmium et des plaques d'aluminium : dans tous les cas le volume d'eau déplacé reste pratiquement constant (122×10 mm) (fig. 2).

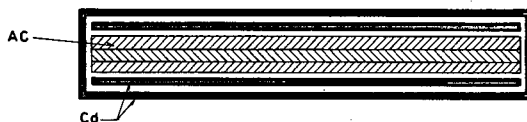


Figure 2

Exemple de la constitution du sandwich A-2 (tous les matériaux sont évidemment aussi jointifs que possible).

L'épaisseur de cadmium par cm^2 de surface exposée aux neutrons varie de $i = 1$ mm à $i = 4$ mm et l'augmentation totale d'efficacité atteint presque 5,6% ($\pm 2\%$ près). Cet effet avait déjà été signalé par BECKER et RUSSEL [7]. Les résultats ont été analysés par la méthode de l'aire d'absorption et sont représentés sur la figure 3. Le rapport de l'aire d'absorption relatif à une épaisseur i de cadmium à l'aire d'absorption relative à l'épaisseur 1 mm a été évalué, les conditions de dérivées logarithmiques respectivement pour les groupes thermiques et épithermiques ayant été calculées comme il est spécifié en annexe III. Les valeurs du rapport des efficacités expérimentales sont jointes par la courbe I. La courbe II représente le rapport des aires d'absorption calculé en négligeant l'absorption des neutrons dans le cadmium au-dessus de 0,625 eV et la courbe III inclut la totalité de la capture. De l'examen des courbes il ressort que l'accroissement d'efficacité du cadmium ne peut être imputable qu'à sa capture épithermique, le cadmium pouvant être considéré comme constamment noir en ce qui concerne le groupe thermique.

Ces résultats sont complétés par deux autres expériences qui concernent seulement des plaques : un absorbant A-27 de structure similaire à A-7 a été constitué en remplaçant deux épaisseurs de cadmium par une seule (0,51 mm) et en faisant l'appoint avec des plaques d'aluminium. L'écart entre A-27 et A-7, environ 3%, est prévu théoriquement (voir figure 3); il est dû à la diminution de l'efficacité thermique et surtout épithermique;

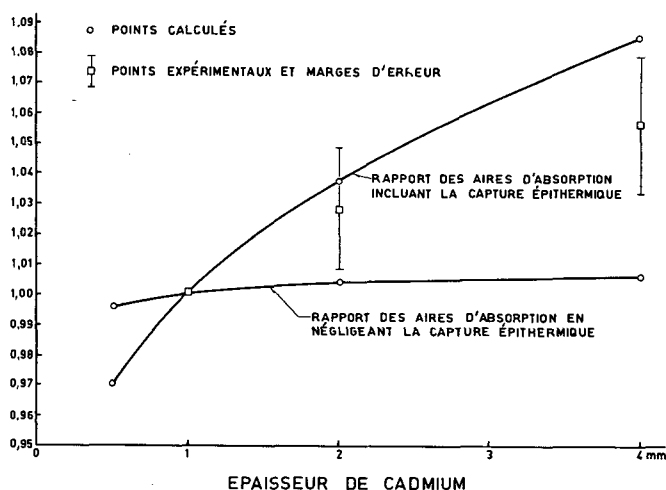


Figure 3

Efficacité relative du cadmium en fonction de l'épaisseur à périmètre extérieur constant

vu l'erreur expérimentale l'accord doit être surtout considéré comme qualitatif.

La réactivité calculée avec le code PDQ en négligeant cette capture épithermique pour les configurations critiques avec A-1, A-2 et A-3 est supérieure à celle calculée pour la pile de référence et les écarts sont respectivement de $140 \cdot 10^{-5}$, $170 \cdot 10^{-5}$ et $220 \cdot 10^{-5}$. La capture épithermique peut être prise en considération à l'aide de conditions de dérivée logarithmique déduite des valeurs calculées en annexe III et résumées dans le tableau ci-dessous.

	A-1	A-2	A-3
Théorie à 1 groupe C_1 rapide	$2,9 \cdot 10^{-3}$	$4,7 \cdot 10^{-3}$	$7,2 \cdot 10^{-3}$
Théorie à 3 groupes C_3 rapides	$9,1 \cdot 10^{-3}$	$15 \cdot 10^{-3}$	$23 \cdot 10^{-3}$

Les écarts se réduisent alors respectivement à $90 \cdot 10^{-5}$. Il n'en reste pas moins que le calcul sous-estime encore l'efficacité d'environ 7% : ceci peut être imputable à l'hypothèse de l'identité des réflecteurs axiaux entre pile propre et pile avec barre, à des effets de coins qui seraient mal interprétés par une longueur d'extrapolation constante sur tout le périmètre ou à d'autres causes : il doit être noté à ce propos que l'usage de la longueur d'extrapolation déduite de la théorie de diffusion ($\frac{2}{3} \lambda \mu$) au lieu de celle déduite du problème de Milne ($0,71 \lambda \mu$) réduirait l'écart de $20 \cdot 10^{-5}$. A titre d'indication,

TABLEAU III

PARAMÈTRES ÉPITHERMIQUES

Référence de l'absorbant	Epaisseur du « sandwich » = 2 B cm	Estimés par comparaison avec la pile de référence					Estimés par comparaison avec la pile avec cadmiun (A-1)				
		2 groupes		4 groupes			2 groupes		4 groupes		
		$\Sigma_1 b$	C_1	$E_3 b$	C_3	$C_3 (C_1)$	$\Sigma_1 b$	C_1	$\Sigma_3 b$	C_3	$C_3 (C_1)$
A-1	1	0,0088	0,0089		0,019	0,028		0,0029 (théorique)		0,0091 (théorique)	
A-2		0,013									
A-3		0,0157									
A-9		0,0482	0,0451	0,194	0,172	0,142	0,0390	0,0373	0,168	0,149	0,117
A-10		0,0414	0,0392	0,156	0,140	0,123	0,0332	0,0314	0,134	0,120	0,0988
A-11		0,0319	0,0305		0,099	0,096	0,0242	0,0212		0,083	0,0667
A-12		0,0442			0,154		0,0357			0,135	
A-13		0,0394			0,133		0,0303			0,116	
A-16		0,0396			0,134		0,0306			0,116	
A-17		0,0258			0,078		0,0192			0,062	
A-18		0,0506	0,0474	0,207	0,184	0,149	0,0412	0,0395	0,181	0,160	0,124
A-19		0,0385	0,0364	0,142	0,126	0,115	0,0302	0,0283	0,122	0,108	0,089
A-20		0,0293	0,0282		0,091	0,089	0,0220	0,0199		0,075	0,063
A-25	0,575	0,0382		0,129		0,0295			0,111		
A-26	0,385	0,0277		0,086		0,0208			0,070		
Marge d'incertitude due aux erreurs expérimentales		± 0,0018	± 0,0018	± 0,006	± 0,006		± 0,0018	± 0,0018	± 0,006	± 0,006	

les valeurs des paramètres épithermiques qui permettraient d'annuler cet écart sont données dans le tableau III. Mais l'écart avec les valeurs théoriques, qui semblaient même un peu élevées à la suite de la comparaison des rapports des aires d'absorption, laisse bien à penser que ces valeurs incluent surtout les erreurs dues à l'interprétation.

Cette discussion nous conduit à conclure qu'il est sans doute préférable de déduire la capture épithermique par comparaison avec la pile avec plaque de cadmium; l'incertitude résultant de la capture épithermique de ce dernier est bien inférieure à celle qui résulte de l'insuffisance de la méthode de calcul dans le domaine thermique.

2° Périmètre absorbant variable

Les absorbants A-4, A-5, A-6 et A-7 ont été surtout essayés pour servir de base de comparaison pour des absorbants épithermiques de même contour. Les épaisseurs apparentes des absorbants sont respectivement de 7,5 mm, 6 mm, 3 mm, 1 mm et 10 mm pour l'absorbant A-1, mais ils sont en fait constitués d'une boîte de cadmium dont la paroi, qui a toujours 0,51 mm d'épaisseur, est garnie de plaques d'aluminium; d'autres plaques d'aluminium sont disposées à l'extérieur pour que le volume d'eau déplacé reste constant.

Le rapport de l'efficacité de chaque absorbant à l'efficacité de A-1 est reporté à la figure 4. Les rapports d'aires d'absorption correspondants sont également donnés; ils ont été calculés en supposant que nous avions affaire à un absorbant de section rectangulaire ce qui n'est guère justifié pour des

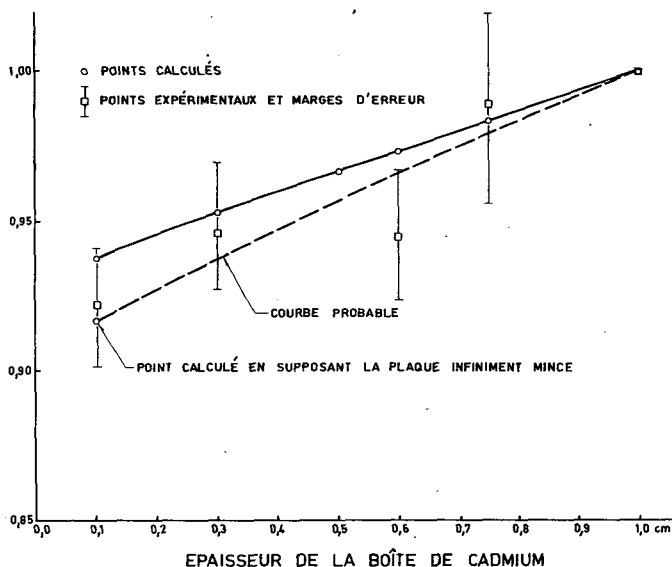


Figure 4

Efficacité relative du cadmium en fonction de l'épaisseur de la boîte de cadmium.

épaisseurs inférieures à la longueur de diffusion. La courbe ainsi déterminée est une droite; en fait si on calcule A-7 en la supposant infiniment mince, ce qui est plus réaliste, on peut concevoir que la courbe réelle serait incurvée vers le bas comme il est suggéré sur la figure et s'accorderait également bien avec les points expérimentaux, compte tenu des marges d'erreur. L'erreur expérimentale est cependant trop élevée pour justifier tout raffinement de ce type. Il y aurait lieu aussi de tenir compte de l'accroissement possible de l'efficacité par suite des fuites de neutrons thermiques dans les plaques adjacentes d'aluminium comme nous le verrons au paragraphe suivant.

3° Effet des lames d'eau adjacentes à un absorbant

L'absorbant A-8 est constitué de deux plaques de cadmium maintenues au centre de la lame d'eau par deux plaques d'aluminium d'épaisseurs telles que la section d'eau déplacée soit de 120×3 mm. Les efficacités de A-7 et de A-8 sont pratiquement les mêmes. Mais cela semble bien dû au fait que les couches d'aluminium adjacentes, par suite des fuites de neutrons, ont les mêmes conséquences que les lames d'eau qu'elles remplacent. En effet on peut évaluer l'efficacité du cadmium sans lame d'eau par différence entre A-7 et A-28 (plaque d'aluminium de 10×120 mm) et celle du cadmium avec lame d'eau par différence entre A-8 et A-29 (plaque d'aluminium de 3×120 mm centrée au milieu de la lame d'eau); la comparaison de ces deux efficacités montre que, compte tenu des marges d'erreur, la présence de lame d'eau de 3 mm de part et d'autre de la plaque entraîne un accroissement d'efficacité de celle-ci compris entre 4% et 10%.

b) Expériences avec l'argent - indium - cadmium (AIC)

1° Absorbants sous cadmium

Les absorbants A-9, A-10 et A-11 sont constitués en garnissant successivement la boîte de cadmium de 122×10 mm de 9,6 et 3 mm d'AIC. L'examen du tableau II montre que dans le réseau considéré le gain en efficacité dû à la capture épithermique est respectivement de 36%, 30% et 22%.

Les valeurs qui en sont déduites pour les paramètres épithermiques $E_1 b$ et C_1 , à la fois par rapport à la pile de référence et par rapport à la pile avec cadmium, sont reportées dans le tableau III.

Des écarts constants sont enregistrés entre $E_1 b$ et C_1 ($E_3 b$ et C_3): ils sont inexplicables. D'autre part l'évaluation de C_3 , faite à partir de C_1 , est très écartée de sa valeur réelle; ceci conduit à penser que la relation proposée entre ces deux quantités est trop simpliste.

La présence de la boîte de cadmium et l'existence d'une certaine capture épithermique de celle-ci fausse sans doute quelque peu les résultats; mais il semble difficile de traduire cet effet quantitativement et il sera négligé dans l'élaboration de $E_1 b$ et C_1 ($E_3 b$ et C_3).

Les valeurs brutes de C_1 et C_3 sont reportées sur la figure 5 avec en addition des valeurs calculées extraites des références [9] et [11]. Il est intéressant de noter que dans le domaine d'épaisseur étudiée, C_1 et C_3 semblent

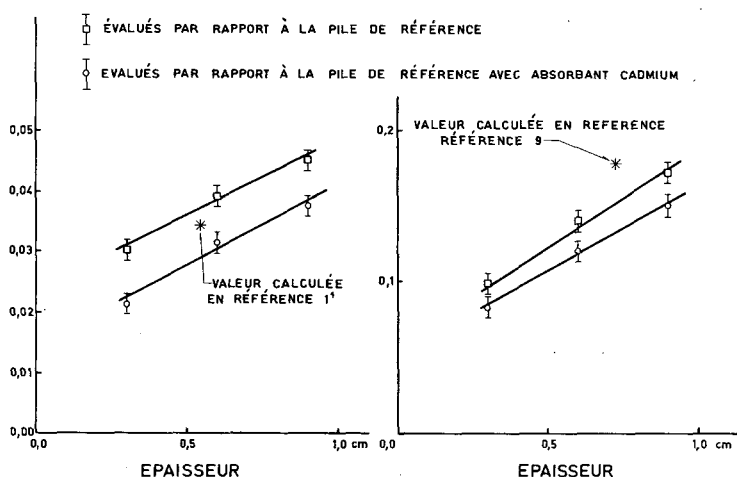


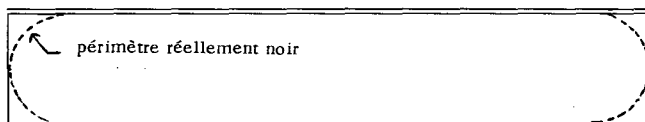
Figure 5

Paramètres épithermiques déduits des expériences sous cadmium.

assez approchés par une fonction linéaire de celle-ci. La structure de résonance de l'argent, du cadmium et de l'indium étant bien connue, il est probable que les valeurs calculées aux références [9] et [11] sont obtenues avec les mêmes sections efficaces; dans ces conditions, la comparaison des valeurs calculées et expérimentales confirme bien la nécessité de pondérer par le spectre épithermique $[\alpha(E)]/[1 + \sqrt{3}\alpha(E)]$ selon la méthode proposée en référence [11] (voir annexe III) au lieu de $\alpha(E)$ [9].

2° Absorbant nu sans lame d'eau

Les absorbants A-12 et A-13 sont constitués comme A-9 et A-10 mais sans la boîte de cadmium et avec des plaques complémentaires d'aluminium pour maintenir une épaisseur de 100 mm. La réduction en efficacité peut s'expliquer par une réduction de la longueur (120 contre 122 pour le sandwich) et peut-être par un effet de coin: le périmètre réellement noir pouvant être réduit par rapport au cadmium par suite de la section d'absorption



finie de AIC dans le domaine thermique. Ce dernier effet ne peut être pris en considération aisément dans la détermination des paramètres épithermiques et leurs écarts avec ceux calculés au paragraphe 2° peuvent peut-être s'expliquer ainsi.

L'absorbant A-15 constitué de deux plaques d'AIC avec deux plaques d'aluminium adjacentes peut se comparer à A-13, mais la réduction du périmètre absorbant est noyée dans l'erreur expérimentale.

L'absorbant A-14 constitué de 3 mm d'AIC contre deux plaques d'aluminium se compare avec l'absorbant de cadmium (A-6) dans la même géométrie; si on tient compte des écarts de longueur (120 contre 122 mm) on voit que 3 mm d'AIC nu apporte encore un gain d'efficacité d'environ 20% par rapport au cadmium. Le léger écart avec le gain d'efficacité des 3 mm d'AIC sous cadmium, 22%, s'explique par une moindre noirceur de 3 mm d'AIC nu dans le domaine thermique (voir annexe III) et peut-être par des effets de coins.

3° Absorbant nu avec lame d'eau

6 mm d'AIC avec lame d'eau adjacente de 1,5 mm (A-17) ou avec aluminium (A-15) ne présentent pas d'écart vraiment significatif, vu les incertitudes expérimentales; il en est de même pour 3 mm d'AIC avec lame d'eau de 3,5 mm (A-16) et avec aluminium (A-16). Comme pour le cadmium, il s'agit sans doute d'une compensation entre l'effet des fuites dans l'aluminium et l'augmentation d'efficacité due à la présence d'eau adjacente.

Les valeurs de L_b et de C_3 calculées dans ce cas pour 3 mm d'AIC apparaissent significativement plus faibles que les valeurs calculées précédemment. Cela peut s'expliquer en partie par une noirceur plus faible dans le domaine thermique, par les effets de coins signalés au paragraphe 2° et éventuellement par un traitement non satisfaisant de la lame d'eau, considérée comme un milieu ordinaire dans PDQ.

Les valeurs obtenues pour les paramètres épithermiques pour 6 mm d'AIC se recoupent avec les précédentes à l'intérieur des marges d'erreurs.

c) Expériences avec le bore

Une série analogue de mesures furent pratiquées avec des absorbants chargés respectivement à 360, 240 et 120 mg/cm² de bore naturel. L'examen du tableau I permet de constater que les efficacités du bore sous cadmium sont sensiblement égales aux efficacités du bore nu. Comme ci-dessus cette efficacité est déterminée par rapport à la boîte de cadmium sans bore A-1 pour les absorbants sous cadmium (A-18, A-19 et A-20) et par rapport à l'épaisseur apparente de cadmium convenable (A-4 à A-7) pour les absorbants nus (A-21, A-22 et A-23) et corrigée pour les écarts en périmètres absorbants. Le tableau ci-dessous résume les conclusions:

	nu	sous cadmium
Cadmium seul	1	1
Bore {	120 mg/cm ²	1, 20
	240 mg/cm ²	1, 30
	360 mg/cm ²	1, 37
		1, 21
		1, 29
		1, 39

A la suite de l'analyse de DAYTON [12] du gainage des matériaux de contrôle, l'effet du remplacement des plaques d'aluminium de 2 mm d'épaisseur adjacentes à l'absorbant A-22 par des plaques d'acier identiques (A-24) a été mesuré; la diminution de l'efficacité due à l'acier est juste significative, mais les marges d'erreur ne permettent pas d'en préciser l'ordre de grandeur.

Les valeurs des paramètres épithermiques correspondants sont reportées dans le tableau III.

d) Essais divers

D'autres matériaux ont été encore disposés à la place du réseau dans l'emplacement prévu pour la plaque absorbante; on se reportera au tableau II.

6. CONCLUSION

Dans les expériences décrites ci-dessus on s'est efforcé d'atteindre les paramètres épithermiques de l'argent-indium-cadmium et du bore en cherchant à séparer les effets parasites (déplacement du réseau, épaisseur des lames d'eau, économie de réflecteur).

En fait l'obtention de la criticalité par variation du niveau d'eau limite la précision des résultats: les variations d'économie de réflecteur sont difficiles à évaluer, et l'efficacité de la plaque est limitée; il semble préférable d'opérer en pile noyée et d'ajuster la criticalité en faisant varier la concentration en poison mis en solution dans l'eau.

L'étude expérimentale d'autres géométries d'absorbant (croix) et l'extension des expériences à des réseaux à spectre différent paraissent souhaitables. En dépit de ces réserves nos résultats peuvent déjà être utilisés avec une certaine confiance.

ANNEXE I

Constantes

1. Constantes du groupe thermique

	D	Σ_a	$\nu \Sigma_f$
Réseau infini	0,31582	0,076771	0,10932
Lame d'eau et réflecteur	0,1589	0,01928	

2. Constantes rapides (trois groupes rapides)

	Groupe	D	Σ_a	Σ_r	$\nu \Sigma_f$
Réseau infini	1	2,0246	0,004302	0,073953	0,007350
	2	1,0569	0,000750	0,072183	0,000432
	3	0,83283	0,020716	0,064645	0,006472
Lame d'eau et réflecteur	1	1,9975	0,001406	0,10836	
	22	1,0535	0,000013	0,14844	
	3	0,59685	0,000949	0,15102	

REMARQUES : Ces constantes ont été calculées par MUFT avec un laplacien de $31,76 \cdot 10^{-4} \text{ cm}^{-2}$ correspondant à une pile avec plaque absorbante. Pour la pile de référence on a utilisé des constantes calculées pour un laplacien de $36,01 \cdot 10^{-4} \text{ cm}^{-2}$.

3. Constantes rapides (un groupe rapide)

	D	Σ_a	Σ_r	$\nu \Sigma_f$
Réseau infini	1,2712	0,008140	0,020532	0,004392
Lame d'eau et réflecteur	1,2323	0,000794	0,048573	

Même remarque que pour les constantes 3 groupes.

ANNEXE II

Nous utilisons, pour les calculs d'aire d'absorption, la formule donnée par DEUTSCH [8] qui néglige l'épaisseur de la plaque.

$$A = 4a \left[\frac{1}{1 + \Sigma_a^1 / \Sigma_r^2} \cdot \frac{L_3}{1 + d_3 / L_3} \left(1 - \frac{1}{(1 + L_3 / L_2) (1 + d_2 / L_2)} \right) \left(1 + \frac{1}{1 + d_4 / L_3} \cdot \frac{L_3}{2a} \right) + \frac{L}{1 + d_2 / L_2} \left(1 + \frac{1}{1 + d_2 / L_2} \cdot \frac{L_2}{2a} \right) \right]$$

où : $2a$ désigne l'envergure de la plaque,

d_i est la longueur d'extrapolation dans le $i^{\text{ème}}$ groupe,

L_i est la longueur de diffusion dans le $i^{\text{ème}}$ groupe,

Σ_a^i est la section efficace de capture de la $i^{\text{ème}}$ groupe,

Σ_r^2 est la section efficace de transfert du deuxième groupe dans le troisième.

La coupure du groupe thermique est toujours à 0,625 eV, celle du deuxième groupe à 180 keV. Les constantes adoptées sont cohérentes avec celles données en annexe I.

Groupe	D	Σ_a^i	Σ_r^2	L_i	d_i
2	0,85897	0,014168	0,040544	3,963	
3		0,076771		2,028	0,6727

En réalité l'épaisseur de la plaque absorbante (1 cm) n'est pas négligeable, on peut se rendre compte de l'erreur en adoptant la formule pour une plaque rectangulaire. Celle-ci est obtenue en remplaçant l'envergure $2a$ par le demi-périmètre $P/2$ et le terme de correction de coin.

$$\left(1 + \frac{1}{1 + d_i/L_i} \cdot \frac{L_i}{2a}\right) \text{ par } \left(1 + \frac{2,41}{1 + d_i/L_i} \cdot \frac{L_i}{P}\right)$$

Cependant cette façon de procéder n'est pas très justifiée, la largeur de la plaque étant de l'ordre de grandeur de la longueur de diffusion.

Les paramètres C_1 et C_3 précédemment définis seront reliés à la longueur d'extrapolation d_2 par les relations :

$$C_1 = \frac{D_2}{d_2} \frac{\int_{0,625 \text{ eV}}^{180 \text{ keV}} \varphi(E) dE}{10 \text{ MeV}}$$

$$C_3 = \frac{D_2}{d_2} \frac{\int_{0,625 \text{ eV}}^{180 \text{ keV}} \varphi(E) dE}{5,5 \text{ keV}}$$

Les deux rapports d'intégrale de flux évalué comme précédemment par MUFT, valent respectivement : 0,5054 et 1,596.

ANNEXE III

Calcul de la condition de dérivée logarithmique

a) Domaine thermique

Elle est définie par l'expression classique.

$$C = \frac{\int_0^{0,625 \text{ eV}} C(E) \varphi(E) dE}{\int_0^{0,625 \text{ eV}} \varphi(E) dE}$$

$\varphi(E)$ est la distribution du flux thermique telle qu'elle est calculée par le spectre SOFOCATE.

$C(E)$ est calculé par la méthode d'AMOUYAL et BENOIST [2] étendue à la géométrie plane par THEYS [9].

$$\frac{1}{3C(E)} = \frac{4}{3} \left(\frac{1}{\Gamma(E)} - 1 \right) + 0,7104$$

avec

$$\Gamma(E) = 1 - 2 E_3(z)$$

$$E_n(z) = \int_1^\infty e^{-zu} u^{-n} du \quad z = 2 \Sigma_a(E) t$$

$2t$ est l'épaisseur de la plaque et Σ_a la section efficace macroscopique d'absorption.

Cette expression de $C(E)$ diffère de l'expression classique [10], en particulier elle conduit à une longueur d'extrapolation pour le corps noir de 0,7104 au lieu de $\frac{2}{3}$.

Les valeurs de C calculées pour différentes épaisseurs de cadmium et d'argent-indium-cadmium et de bore sont données ci-dessous :

Cadmium $2t$ (mm)	0,05	0,1	0,2	0,4
C	0,447	0,456	0,462	0,467
Argent-indium-cadmium $2t$ (mm)	0,3	0,6	0,9	
C	0,439	0,459	0,464	
mg/cm ² de bore naturel	120	240	360	
C	120 0,443	240 0,465	360 0,468	

Ces valeurs sont à comparer à la valeur obtenue pour le corps noir : 0,47. À l'aide de la forme d'aire d'absorption on peut montrer que l'assimilation de 3 mm d'AIC à un corps noir diminue d'environ 1,5% son efficacité thermique.

b) Domaine rapide

Le calcul a été fait seulement pour le cadmium selon la méthode de la référence [10].

$$C = \frac{\int_{0,625 \text{ eV}}^{190 \text{ keV}} \frac{\alpha(E)}{1 + \sqrt{3} \alpha(E)} \frac{dE}{E}}{\int_{0,625 \text{ eV}}^{190 \text{ keV}} \frac{1}{1 + \sqrt{3} \alpha(E)} \frac{dE}{E}}$$

avec :

$$\alpha(E) = \frac{1 - 2 E_3(z)}{2[1 + 3 E_4(z)]}$$

$$z = 2 \Sigma_a(E)t$$

La contribution des résonances est prise en considération par la formule de Stein.

Les valeurs de C calculé pour le cadmium pour le groupe d'énergie 0,625 eV - 180 keV sont tabulées ci-dessous :

$2t$ mm	0,05	0,1	0,2	0,4
sans les résonances	$2,5 \cdot 10^{-3}$	$4,6 \cdot 10^{-3}$	$7,6 \cdot 10^{-3}$	$12 \cdot 10^{-3}$
avec les résonances	$3,3 \cdot 10^{-3}$	$5,7 \cdot 10^{-3}$	$9,2 \cdot 10^{-3}$	$14 \cdot 10^{-3}$

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PROBLÈMES TECHNIQUES LIÉS A L'UTILISATION DES INSTALLATIONS A EAU

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Abstract — Résumé — Аннотация — Resumen

TECHNICAL PROBLEMS IN USING WATER SYSTEMS. This report concerns two very different but common technical problems encountered in light-water critical experiments. The first is the problem of the exact level of water in very tight lattices. The second problem is the study of the best conditions for using borated solutions in order to obtain very high accuracy.

PROBLÈMES TECHNIQUES LIÉS A L'UTILISATION DES INSTALLATIONS A EAU. Le présent rapport traite de deux problèmes techniques très différents mais fréquemment rencontrés dans les expériences critiques à eau. Le premier problème est celui de l'établissement du niveau réel de l'eau dans des assemblages serrés de tubes cylindriques. Le second problème est celui des conditions d'utilisation de solutions borées pour obtenir un maximum de précision.

ТЕХНИЧЕСКИЕ ПРОБЛЕМЫ, СВЯЗАННЫЕ С ИСПОЛЬЗОВАНИЕМ ВОДНЫХ СИСТЕМ. Данный доклад касается двух обычных технических проблем, встречающихся в критических экспериментальных реакторах с водным замедлителем.

Первой из этих проблем является проблема поддержания точного уровня воды в очень плотных решетках.

Второй проблемой является проблема изучения лучших условий использования борированных растворов в целях получения очень большой точности.

PROBLEMAS TÉCNICOS QUE PLANTEA EL EMPLEO DE INSTALACIONES DE AGUA. La memoria examina dos problemas técnicos muy diferentes que se presentan frecuentemente en los experimentos críticos con reticulados de agua ligera. El primero de esos problemas consiste en determinar el nivel exacto del agua en reticulados muy compactos. El segundo es el estudio de las condiciones óptimas de utilización de las soluciones boradas con miras a obtener un máximo de precisión.

I. PROBLÈMES DE CAPILLARITÉ DANS L'INSTALLATION A EAU ALIZÉ I

Toutes les expériences d'ALIZÉ I sont faites en cœur non noyé, c'est-à-dire tous les éléments de contrôle et de sécurité étant extraits du cœur. La réactivité est contrôlée essentiellement par le niveau d'eau. La hauteur d'uranium non immergé (plus de 30 cm dans tous les cas) fournit un réflecteur pratiquement infini.

Il est apparu rapidement que, pour le réseau étudié (pas carré: 11,1 mm, diamètre extérieur des gaines en aluminium: 9,0 mm) il n'y a pas identité entre le niveau d'eau effectif dans le cœur et le niveau, mesuré à 0,1 mm près, dans le réflecteur.

Les constatations suivantes ont été faites:

- a) Pour un niveau donné par montée d'eau, la réactivité mesurée à inter-

valles de temps courts augmente pendant environ une demi-heure, l'écart total correspondant à 0,5 mm environ.

b) Pour un niveau donné obtenu par vidange d'eau, les ordres de grandeur sont les mêmes mais la réactivité décroît.

c) Enfin pour un même niveau dans le réflecteur, la différence entre les réactivités limites mesurées par la vidange et par la montée correspond à 0,5 et à 1,5 mm.

Pour préciser ces phénomènes, des essais sur maquette ont été entrepris. La maquette comprenait essentiellement une cuve en verre (diamètre : 25 cm), une portion de réseau reconstitué avec les mêmes matériaux, un dispositif permettant de faire monter et de vidanger l'eau à la même vitesse que dans le cœur ainsi que divers dispositifs d'éclairage. Deux gaines placées l'une au centre, l'autre à la périphérie, étaient utilisées pour la mesure de niveau par l'intermédiaire d'un tube en U et d'un cathétomètre (précision : 0,03 mm).

Il a été vérifié qu'à la température ordinaire les bulles d'air sont négligeables tandis que les phénomènes de capillarité sont importants. Deux grandeurs caractéristiques ont été mesurées : h (mm), différence entre le point haut du film d'eau sur la gaine et le point bas du ménisque d'une part et δ (mm), différence entre le point bas du ménisque au centre du réseau et le niveau dans le réflecteur d'autre part.

Les résultats suivants ont été obtenus (les fourchettes correspondent non à des erreurs de mesure individuelle mais à des défauts de reproductibilité) :

a) Cœur dont le rayon vaut 10 cm :

Montée d'eau : h : 0 à 0,5 mm	stationnaire,
δ : + 0,8 à 0,6 mm	baisse légère en fonction du temps.
Vidange d'eau : h : 2 à 4 mm	immédiatement,
2 à 3 mm	1 heure plus tard,
1 à 3 mm	5 heures plus tard,
δ : -0,2 à 0,3 mm	immédiatement,
négligeable	3 heures plus tard.

b) Cœur dont le rayon vaut 5 cm :

h est identique mais δ est limité par la montée de + 0,2 à 0,4 mm et est négligeable par la vidange.

Nous avons cherché à supprimer h et δ en utilisant soit des agents mouillants dans l'eau, soit un traitement de surface des gaines, soit une combinaison des deux. Les produits utilisés doivent :

- avoir une section efficace faible,
 - être non moussant,
 - être chimiquement neutres et stables à 90°C,
 - ne pas trop réduire la résistivité (mesures électriques de niveau).
- Les conclusions des divers essais sont les suivantes :
- les agents mouillants annulent δ mais conservent h ,
 - le traitement de surface des gaines (siliconage) annule h mais accroît notablement δ .

La meilleure solution ($h = \delta = 0$) correspond à l'addition à 1000 cc d'eau de 1 cc de UCEPAL PO (Paix) plus 5 gouttes d'antimousse 426 (Rhodersil) associée à un siliconge Si4S (sans bore) (SIS) des gaines par trempage suivi d'une polymérisation de deux heures à la température de 200°C. Faute de temps (à cause de l'opération de siliconge) cette solution n'a pas été appliquée. Pratiquement, pour obtenir des mesures reproductibles de réactivité, nous avons adopté une procédure stricte, opérant toujours par montée d'eau et tenant compte des variations de niveau que peut introduire l'insertion des éléments de contrôle et de réactivité.

II. EXPÉRIENCES DANS L'INSTALLATION A EAU ALIZÉ II PAR EMPOISONNEMENT DU MODÉRATEUR A L'ACIDE BORIQUE

Après une brève description de l'installation, nous ferons un tour d'horizon rapide des problèmes technologiques que posent ces expériences, des solutions qui se sont dégagées des études effectuées et enfin, nous évaluerons les précisions que l'on peut espérer des mesures faites selon cette technique.

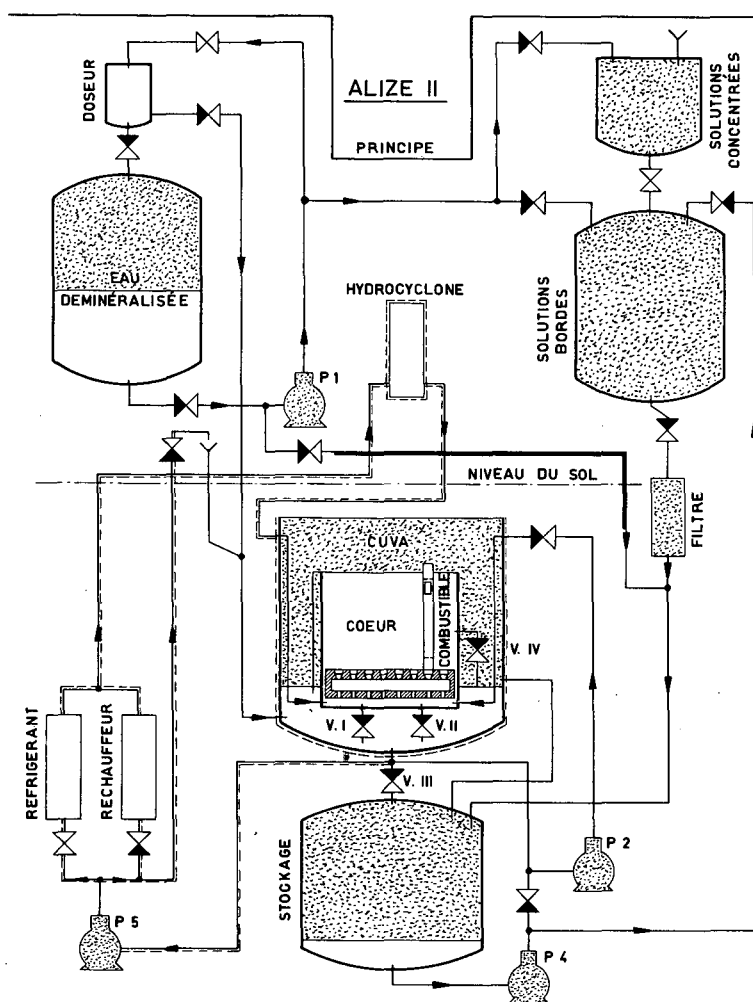
1. Description de l'installation

1.1. Généralités

ALIZÉ II est situé à Saclay dans le Hall des expériences critiques. La cuve expérimentale et certains circuits hydrauliques sont logés dans un puits de $3,2 \cdot 3,4 \text{ m}^2$ et de 9 m de profondeur qui contenait précédemment l'installation ALIZÉ I. Les circuits de préparation des solutions borées et d'autres circuits auxiliaires sont placés sur une charpente métallique construite au-dessus de l'aire de débattement des dalles mobiles de protection qui ferment le puits pendant le fonctionnement. Le tableau de contrôle et la cabine de stockage du combustible sont situés de part et d'autre de l'installation.

L'installation comprend essentiellement (fig. 1) :

- Le cœur, constitué d'un assemblage de boîtes contenant des faisceaux de plaques (de nature diverse, combustible U-Al 90%; aluminium, aluminium boré, zirconium naturel, zirconium déshafnié) qui définissent le motif du réseau. Les boîtes reposent sur une grille et sont bloquées à leur partie supérieure par des lattes de positionnement. Cet assemblage est construit dans une cuve d'aluminium de 1 250 mm de diamètre, laquelle est contenue dans une cuve calorifugée de 2 m de diamètre qui joue le rôle de réservoir de rechange du cœur et de bac tampon pour la charge de la pompe de circulation.
- Un réservoir de stockage de 8 m^3 situé sous la grande cuve.
- Deux réservoirs de $0,2 \text{ m}^3$ et 4 m^3 nécessaires à la préparation des solutions borées.
- Un réservoir de 6 m^3 qui constitue la réserve d'eau déminéralisée.
- Des mécanismes de commande des rideaux de sécurité, des plaques de contrôle, des sources et des dispositifs de lecture du niveau de la solution dans le cœur.
- Des circuits hydrauliques et des appareillages qui réalisent les différentes opérations désirées.
- Un ensemble de chaînes de détection liées à un tableau de contrôle.



1.2. Cœur et auxiliaires

Les plaques sont montées suivant un certain pas dans des crémaillères constituant ainsi des faisceaux placés ensuite dans les boîtes. La grille dans laquelle s'introduisent les pieds des boîtes peut être changée aisément pour modifier l'épaisseur des lames du modérateur entre les boîtes. Les différents cœurs présentent une taille sensiblement constante. Un casing (tôle légère d'aluminium) sépare le cœur des réflecteurs latéraux. Dans ces espaces réflecteurs sont logées des chaussettes étanches contenant les organes de détection dont la position par rapport au cœur est modifiable.

Deux groupes de rideaux de sécurité (cadmium gainé d'acier inoxydable) se déplacent dans l'espace libre entre des rangées de boîtes. Ils sont commandés par 2 mécanismes à embrayage liés à la chaîne de sécurité de l'installation.

La plaque de contrôle (cadmium gainé de plastique) dont l'efficacité est de $400 \cdot 10^{-5}$ au maximum n'est utilisée pratiquement que pour assurer la divergence du réseau.

Une source de Ra-Be de 75 mc est placée dans le fond d'une boîte spéciale. Un mécanisme l'amène dans le plan inférieur du combustible ou la retire au delà du réflecteur inférieur.

Un palpeur électrique effectue les mesures de niveau en approche sous critique. Des pointes électriques fixes, outre les verrouillages qu'elles commandent, contrôlent les indications du palpeur.

1.3. Circuits hydrauliques (schéma de principe)

Ces circuits permettent les opérations suivantes :

- la préparation des solutions borées,
- le remplissage de la cuve expérimentale et l'immersion du réseau (temps nécessaire : 70 minutes),
- les variations de concentration des solutions borées,
- la circulation des solutions entre le cœur et la grande cuve pour réaliser différentes opérations (renouvellement de la solution en contact avec le cœur : 10 minutes),
- le chauffage (depuis la température ambiante jusqu'à 95°C; durée : 36 heures),
- la réfrigération (retour de 95°C à la température ambiante; durée : 36 heures également),
- l'homogénéisation en température ($\pm 0,2^\circ\text{C}$ entre deux points quelconques du cœur entre la température ambiante et 95°C),
- l'homogénéisation en concentration après des modifications du titre,
- le prélèvement de solution pour le dosage.

1.4. Contrôle

Cinq organes de détection assurent le contrôle et la sécurité.

Un compteur à BF_3 attaque un intégrateur linéaire. Cette chaîne donne un contact de sécurité lorsque le signal de sortie est compris entre un seuil minimal. Ces niveaux de sécurité d'un facteur égal à celui de la sensibilité évoluent par bonds à chaque commutation de sensibilité de l'appareil de lecture.

Deux chambres d'ionisation au bore compensée sont accouplées à des amplificateurs linéaires à courant continu équipés d'un dispositif de sécurité analogue au précédent. L'ensemble, compteur à BF_3 - chambre à courant, ainsi constitué est tel que l'opérateur est obligé de suivre le niveau de flux du réseau pour maintenir la ligne de sécurité fermée, et donc les rideaux de sécurité hors du cœur. De plus, les sécurités maximales ne se trouvent jamais à plus de $\frac{2}{3}$ de décade.

Deux chambres d'ionisation, analogues aux précédentes, sont associées à des amplificateurs logarithmiques à courant continu qui délivrent un contact de sécurité fixe. Un circuit de différentiation, alimenté par ces amplificateurs, assure une ouverture de la ligne de sécurité pour une période stable inférieure à 5 s. Deux chaînes sont en outre spécifiquement affectées aux mesures.

Un compteur à BF_3 muni d'une échelle de comptage permet de suivre l'approche de la criticalité.

Une chaîne à courant, équipée d'une chambre d'ionisation identique aux chambres de sécurité, mesure le temps de doublement du flux sur $5 \frac{1}{2}$ octaves (l'écart relatif à craindre est de l'ordre de 1%).*

2. Problèmes technologiques

L'acide borique employé se présente sous forme cristallisée ayant la qualité requise pour l'analyse (teneur des cristaux en $\text{B}(\text{OH}_3)$ supérieure à 99,5%).

La limite de solubilité dans l'eau est de 55 g/l à 20°C.

Les excédents de réactivité à contrôler nécessitent des concentrations en bore naturel comprises entre 50 et 500 mg B/l. Quelques réseaux ont été contrôlés avec des solutions contenant moins de 50 mg de B/l, mais la perte de précision inhérente à ces faibles concentrations devient importante.

2.1. Préparation des solutions

Les solutions sont préparées à partir d'eau déminéralisée à 500 000 $\Omega \text{ cm}^2/\text{cm}$ au moins, sans difficulté particulière.

Un concentré de 2 001 est d'abord réalisé dans de l'eau à 50 - 60°C. Après un brassage durant une heure environ, la mise au titre désiré s'effectue dans le réservoir de dilution dans lequel la solution est agitée entre 1 heure et 3 heures.

2.2. Dosages

L'acide borique, trop faible pour pouvoir être dosé par acidimétrie avec une précision suffisante, donne avec les oses ou les polyols des complexes à fonction acide nettement plus forte.

Le lévulose a été choisi pour sa grande solubilité et la possibilité de l'obtenir très pur (4 g de lévulose pour 2 mg de bore).

Après la formation du complexe (10 minutes environ) le dosage s'effectue avec de la potasse; la courbe de titrage est suivie au pHmètre.

On détermine préalablement les courbes de titrage de deux solutions d'acide borique connues, ayant des concentrations différentes au plus de 20% et choisies de telle sorte que la solution inconnue à titrer ait une concentration comprise entre celles des solutions de référence. Pendant la formation du complexe et au cours de mesures, un courant d'azote sec barbotte dans le vase de titrage pour chasser le gaz carbonique dont la présence fausse évidemment la qualité du dosage.

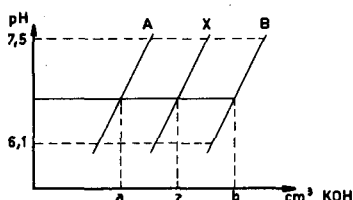
Les courbes de titrages sont linéaires pour :

$$6,7 < \text{pH} < 7,5.$$

On retrouve sur le schéma ci-dessous, le résultat suivant :

$$X = \frac{1}{2} \left[(A + B) + \frac{1}{u} \left\{ \sum_{i=1}^n \left(\frac{x-a}{b-a} \right)_i - \sum_{i=1}^n \left(\frac{b-x}{b-a} \right)_i \right\} (B - A) \right]$$

* Principe Ailloud et Chandanson : Rapport CEA n° 555.



Cette méthode présente un certain nombre d'avantages :

- le titre de la potasse n'intervient pas,
- il n'est pas nécessaire de déterminer le point équivalent.

Le même matériel est utilisé pour effectuer une série de dosages.

Toutes précautions prises, la reproductibilité de dosage d'une solution est meilleure que 0,3%.

La préparation des solutions de référence s'effectue par pesées et fioles jaugées (l'erreur de préparation est inférieure à 0,1%).

Si l'on désire un calage absolu de ces solutions, il convient d'ajouter l'erreur due à l'incertitude qui provient de la qualité du produit utilisé (inférieure à 0,2%).

2.3. Fixation du bore sur les surfaces

Le rapport des surfaces en contact dans le cœur avec la solution est très important : $2 \cdot 10^4 \text{ cm}^2/\text{l}$. Lorsque le réseau est immergé, il s'opère un transfert du bore de la solution vers les structures voisines qui n'affecte pas la réactivité du réseau tant que la circulation n'est pas en fonctionnement. Par contre dès que la circulation est en service, la solution qui s'est appauvrie est remplacée par une solution non altérée (tampon de la pompe de circulation); il en résulte une augmentation de la concentration en bore par litre de cœur et partant, une baisse de réactivité très nette. Le mélange des solutions conduit à une baisse de concentration par litre de solution, indécélable au dosage.

En effet, alors que le réseau est sensible, à l'échelle d'une journée, à une variation de l'ordre de 0,02 à 0,05 mg B/l, soit à un dépôt de $1 \text{ à } 3 \cdot 10^{-9} \text{ g/cm}^2$, les dosages ne permettent pas de déceler de façon significative une variation inférieure à 0,3 mg B/l (solution à 100 mg B/l).

L'état de surface joue un rôle prépondérant. Des plaques d'aluminium recouvertes d'une couche de 5 à 7 g de bayerite ont absorbé en 12 jours dans une solution à 160 mg B/l à 20°C, $1080 \cdot 10^{-9} \text{ g/cm}^2$.

Le lavage permet de récupérer environ 50% du bore absorbé.

Les essais entrepris en pile et en laboratoire ont montré qu'il convient d'envisager une modification de l'état de surface des plaques de structure si l'on désire travailler de façon satisfaisante avec un modérateur empoisonné à froid et surtout jusqu'à 95°C.

Deux solutions ont été essayées :

Le silicatage (dépôt au trempé à chaud dans un bain de silicate de sodium sur des surfaces parfaitement décapées) : Acceptable à froid, inefficace à chaud. L'absorption de $15 \cdot 10^{-9} \text{ g/cm}^2$ à 30°C passe à 90°C à $900 \cdot 10^{-9} \text{ g/cm}^2$. Cette solution a été abandonnée.

Le vernissage (dépôt au trempé d'un vernis phénolique sur des surfaces oxydées anodiquement et polymérisation à chaud) : Aucune absorption n'a

pu être relevée aux essais. Après 4 mois de fonctionnement, avec plusieurs essais à 95°C, la fixation de bore est évaluée à $0,3 \pm 0,2 \cdot 10^{-9}$ g/cm².

2.4. Stabilité des solutions

Une certaine carbonatation des solutions au contact de l'air était à craindre. Des essais en pile limités (durée 4 jours) n'ont rien révélé par les dosages. En laboratoire un essai de 12 jours, réalisé de façon à augmenter le phénomène d'aération, nous a donné l'assurance qu'aucune dérive « apparente au dosage » de la teneur en bore des solutions n'était à craindre.

Une dérive vraie de titre intervient à chaud. Des prélèvements faits fréquemment dans le temps permettent de suivre son évolution et d'introduire d'éventuelles corrections aux mesures du coefficient de réactivité en fonction de la température.

2.5. Variation de titre - Homogénéisation

Les circuits ont été conçus de telle sorte que les modifications de titre puissent se faire aisément. Les fortes variations (dilutions supérieures à 10%) ou reconcentrations ont lieu directement dans la grande cuve. Les faibles variations (de l'ordre de 1%) sont obtenues par utilisation du ballon doseur (dilution) ou envoi de concentrés par la prise de titrage, réseau noyé. L'homogénéisation s'effectue en 15 minutes par le circuit de circulation (renouvellement du volume actif : 1 fois $\frac{1}{2}$). Dans ces conditions, la stabilité est obtenue à $\pm 1 \cdot 10^{-5}$.

2.6. Lavages - Dosage des traces de bore dans l'eau

Après une étude en solution borée, le réseau est lavé à l'eau de ville une première fois. Un rideau de Cd fixe est amené dans la lame centrale, une approche sous critique est ensuite effectuée dans les conditions normales de sécurité. Le premier lavage dure 90 minutes, puis l'eau de ville est remplacée par une charge d'eau déminéralisée qui assure le rinçage de l'installation (temps de rinçage identique au temps de lavage).

Après le rinçage la teneur en bore de l'eau déminéralisée comprise initialement entre 0,05 - 0,10 mg B/l atteint 0,20 - 0,30 mg B/l. L'eau de rinçage est évacuée et remplacée par une nouvelle charge d'eau déminéralisée. Un contrôle effectué sur cette eau a donné une teneur résiduelle de 0,10 mg B/l. Le dosage des traces s'effectue par la méthode d'extraction du bluoborate de bleu de méthylène dans le dichloréthane 1-2 et colorimétrie.

Cette méthode est assez longue par suite de la lente formation de fluoroborate en présence de bifluorure d'ammonium, la réaction réversible qui pour un rendement à 95°C demande 18 h.

20 μ g B/l peuvent être dosés à 25% près, la reproductibilité pour des solutions à 50 μ g B/l atteint 10%.

3. Reproductibilité des mesures - Précisions

3.1. Reproductibilité d'un état de pile, modérateur propre

Les mesures effectuées en réseau propre, avant que ne soient résolus les problèmes posés par la fixation du bore sur les structures, permettent

de chiffrer la reproductibilité d'un état de pile, dans le temps, à la suite d'une intervention locale sur le chargement ou après un démontage complet du réseau.

Une série de mesures effectuées pendant quelques heures montre que l'écart maximum à la moyenne pour une réactivité de $200 \cdot 10^{-5}$ est de l'ordre de $\pm 0,5 \cdot 10^{-5}$.

A l'échelle de la journée, compte tenu d'interventions locales sur le chargement, une réactivité de $200 \cdot 10^{-5}$ est affectée d'un écart maximum de $\pm 0,5 \cdot 10^{-5}$.

D'un jour à l'autre, sans intervention sur le chargement, mais avec vidange du cœur, pour une réactivité de $250 \cdot 10^{-5}$, écart de $2 \cdot 10^{-5}$ mais défini à $\pm 1 \cdot 10^{-5}$ près.

Des manœuvres locales de boîtes combustibles mettent en évidence des effets de $5 \cdot 10^{-5}$ pour un état de $150 \cdot 10^{-5}$.

Après des études en solution borée, à froid comme à chaud, la reproductibilité du cœur propre à $200 \cdot 10^{-5}$ s'obtient à une précision supérieure à $5 \cdot 10^{-5}$.

3.2. Reproductibilité d'un état, modérateur empoisonné

Pendant quelques heures, sans modification du titre de la solution et en l'absence de circulation, un état critique de $170 \cdot 10^{-5}$ se reproduit à une précision supérieure à $1,5 \cdot 10^{-5}$.

Dans les mêmes conditions de temps, après les cycles de circulation pour homogénéisation, consécutifs à une modification du titre, la précision de la reproductibilité est meilleure que $\pm 1 \cdot 10^{-5}$.

Entre deux mesures effectuées sur un chargement identique, après démontage complet et remontage (intervalle entre les mesures : 1 mois), compte tenu des erreurs de détermination des paramètres critiques, l'écart n'est pas significatif.

Des déterminations systématiques faites sur un chargement donné, en utilisant pour plusieurs mesures une même solution ou une solution fraîchement préparée, permettent de chiffrer la reproductibilité dans de telles conditions.

Titre critique après deux ajustements (voir paragraphe suivant):

309,3

309,3

309,8

308,7

308,3

Valeur moyenne pondérée : 309,27 mg B/l.

Ecart maximum d'un point à la moyenne : 1 mg B/l, soit pour la quantité de réactivité contrôlée en l'occurrence, 12‰, une incertitude maximum de $40 \cdot 10^{-5}$.

3.3. Précision à espérer de la mesure du titre critique

Le titre critique expérimental est la donnée directement comparable aux calculs de réseaux effectués en tenant compte de la présence de bore

soluble dans le modérateur. L'équivalent en réactivité du mg B/l n'est qu'un intermédiaire qui sert à la détermination par extrapolation du titre critique C_0 .

Corrections de température effectuées, une mesure sur un chargement donné est caractérisée par les grandeurs suivantes :

- réactivité ρ ($\delta\rho$),
- volume total de solution V (δV),
- concentration de la solution C (δC),
- quantité de bore en solution Q (δQ),
- deux mesures correspondant à deux états voisins fournissent deux estimations de l'efficacité du mg B/l.
- par dosage de la variation de concentration $\gamma_1 = \frac{\rho_1 - \rho_2}{C_1 - C_2}$
- du fait de la variation de volume. Addition, pour une dilution de n volumes v (δv).

$$\gamma_2 = \frac{\rho_1 - \rho_2}{C_1} \cdot \frac{V_1 + u v}{u v}$$

Les erreurs à craindre qui affectent ces deux déterminations sont très différentes.

$$\frac{\delta\gamma_1}{\gamma_1} = \frac{2 \delta\rho}{\Delta\rho} + \frac{2 \delta c}{c} \cdot \frac{c}{\Delta c}$$

$$\frac{\delta\gamma_2}{\gamma_2} = \frac{2 \delta\rho}{\Delta\rho} + \frac{\delta c}{c} + \frac{\delta V_1}{V_1} + u \frac{\delta v}{v}$$

Ordre de grandeur :

$$\delta\rho = 1 \cdot 10^{-5}; \quad \Delta\rho = 50 \text{ à } 200 \cdot 10^{-5}; \quad \frac{\delta v}{v} = 0,02\%;$$

$$\frac{\delta c}{c} = 0,3\%; \quad \frac{\delta V_1}{V_1} = 0,9\%; \quad u = 1 \text{ à } 20; \quad \frac{c}{\Delta c} = 20 \text{ à } 100.$$

Par exemple pour :

$$\Delta\rho = 200 \cdot 10^{-5}, \quad c = 200 \text{ mg B/l et } \gamma \cong 40 \cdot 10^{-5}/\text{mg B/l} (\Delta c = 5 \text{ mg B/l})$$

$$\text{on a :} \quad \frac{\delta\gamma_1}{\gamma_1} = 2,5\% \text{ et } \frac{\delta\gamma_2}{\gamma_2} \cong 2,5\%.$$

Pratiquement on opère de la façon suivante :

Un premier ajustement de $\rho = \gamma_1 (c - C_0)$ est effectué à partir des résultats de dosage; il permet de situer le centre de gravité des mesures [c_1 ($\Delta\delta c_1$), V_1 ($\Delta\delta V_1$)] ensuite utilisé pour un second ajustement en injectant les variations de volume effectuées en vue d'obtenir les différents points critiques. L'incertitude sur C_0 due à l'erreur combinée sur le centre de gravité C_1 et sur γ est à peu près $\Delta\delta C_1$, soit environ 0,3%. Pour une quantité de réactivité contrôlée de 10%, l'imprécision est ainsi de 30 pcm.

3.4. Précision à attendre des mesures du coefficient de température jusqu'à 95°C

Quelques points rendent cette mesure particulièrement délicate :

- Le dégazage de la solution est impératif. Il s'effectue par préchauffage à 95°C pendant quelques heures.
- L'homogénéisation en température à 95°C peut être réalisée à $\pm 0,2^\circ\text{C}$ entre 2 points du cœur ou du réflecteur moyennant un certain soin opératoire.
- La connaissance du titre de la solution et du volume de travail nécessitent des dosages fréquents du fait de l'évaporation ou de la condensation liée au processus opératoire (fluctuations de concentration de 1 à 2%).

Chaque point de mesure est caractérisé par 3 grandeurs :

$$\begin{array}{ll} \text{réactivité } \rho (\delta\rho) & \delta\rho = 1 \cdot 10^{-5} \\ \text{température } \theta (\delta\theta, \Delta\theta) & \delta\theta = 0,3^\circ\text{C} \quad (\text{erreur absolue}) \\ & \Delta\theta = 0,1 - 0,2^\circ\text{C} \quad (\text{homogénéité}) \end{array}$$

L'erreur relative d'un point de température à l'autre est inférieure à 0,1°C.

- Concentration : la précision des mesures est optimum pour une concentration de l'ordre de 50 à 100 mg B/l et varie comme l'indique le tableau ci-dessous :

C mg B/l	Précision 10^{-5}
30	13,5
50	8
100	9
200	16

ANALYSE DES RÉSEAUX A EAU LÉGÈRE ET A URANIUM HAUTEMENT ENRICHI, AVEC ET SANS HAFNIUM, EXPÉRIMENTÉS DANS L'INSTALLATION ALIZÉ II

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Abstract — Résumé — Аннотация — Resumen

ANALYSIS OF LIGHT-WATER, HIGHLY-ENRICHED URANIUM LATTICES, WITH AND WITHOUT HAFNIUM, TESTED IN THE ALIZÉ II INSTALLATION. The author's purpose is to appraise, on the basis of a knowledge of cross-sections and resonance parameters, the extent to which the reactivity of cores uniformly loaded with natural hafnium may be calculated. Two series of critical lattices are used, the first including aluminium alone and the second zircaloy plates instead of aluminium.

ANALYSE DES RÉSEAUX A EAU LÉGÈRE ET A URANIUM HAUTEMENT ENRICHI, AVEC ET SANS HAFNIUM, EXPÉRIMENTÉS DANS L'INSTALLATION ALIZÉ II. Les auteurs se proposent d'apprécier dans quelle mesure on peut calculer la réactivité de cœurs chargés uniformément d'hafnium naturel en se fondant sur la connaissance des sections efficaces et des paramètres de résonance. A cette fin, ils utilisent deux séries de réseaux critiques; la première ne comporte que de l'aluminium tandis que dans la seconde les plaques d'aluminium sont remplacées par des plaques de zircaloy.

АНАЛИЗ ВЫСОКОБОГАТЕННЫХ УРАНОМ ЛЕГКОВОДНЫХ РЕШЕТОК С ГАФНИЕМ И БЕЗ НЕГО В ОПЫТЕ НА УСТАНОВКЕ АЛИЗЕ II. Авторы ставят своей целью установить, в какой степени можно рассчитывать реактивность активной зоны, полностью заполненной природным гафнием, основываясь при этом на знании эффективных сечений и параметров резонанса. С этой целью они применяют две серии критических решеток: первая содержит лишь алюминий, в то время как во второй алюминиевые пластинки заменены пластинками циркаллой.

ANÁLISIS DE LOS RETICULADOS DE AGUA LIGERA-URANIO MUY ENRIQUECIDO, CON Y SIN HAFNIO, EXPERIMENTADOS EN LA INSTALACIÓN ALIZÉ II. Los autores se proponen evaluar la medida en que puede calcularse la reactividad de los cuerpos con carga uniforme de hafnio natural basándose en el conocimiento de las secciones eficaces y de los parámetros de resonancia. Para ello utilizan dos series de reticulados críticos: el primero sólo tiene aluminio, mientras que en el segundo las placas de aluminio han sido sustituidas por placas de Zircaloy.

1. INTRODUCTION

Le but de ces expériences était d'apprécier dans quelle mesure on pouvait calculer, avec une méthode classique, la réactivité de cœurs chargés uniformément d'hafnium naturel à partir des sections efficaces et des paramètres de résonance de ses divers isotopes publiés dans la littérature, et par là même contribuer à améliorer la connaissance de la capture épithermique dans ces cœurs et en particulier de leur intégrale de résonance.

Une première série de réseaux critiques fut réalisée: ils ne contenaient que de l'aluminium. Le but était de tester le modèle utilisé pour le calcul

de la réactivité dans un domaine assez large de variation des paramètres (rapport métal/eau, concentration en ^{235}U , dimension).

Les réseaux de la seconde série étaient chargés uniformément avec un certain nombre de plaques de zircaloy en remplacement de plaques d'aluminium, la proportion des plaques de zircaloy et de zircaloy hafnié à la teneur naturelle étant variable.

2. DESCRIPTION DES RÉSEAUX EXPÉRIMENTÉS

Les cœurs sont construits dans l'installation critique ALIZÉ II (voir la description en [1]). Ils sont formés d'un nombre variable d'éléments de combustible fixés sur une grille selon un réseau rectangulaire. L'installation a été conçue de façon à travailler en « pile noyée » avec un réflecteur minimum de 10 cm d'eau sur toutes les faces. La criticalité est ajustée en réglant la taille.

Les éléments combustibles sont constitués de boîtes d'aluminium de section carrée de 8,5 cm de côté, chargées de plaques de métal amovibles (aluminium uranié, aluminium, aluminium boré, zircaloy ou zircaloy hafnié) dont l'écartement est maintenu par des crémaillères. On dispose de crémaillères correspondant à 40 ou 28 plaques et permettant de réaliser respectivement les rapports de volume métal/eau voisins de 1 et de $\frac{2}{3}$.

Les plaques ont 65 cm de hauteur, 7,9 cm de largeur et 0,1 cm d'épaisseur. Elles créent à l'intérieur de la boîte des lames d'eau d'épaisseur constante. Dans la direction perpendiculaire aux plaques les boîtes ne sont espacées que de 0,1 cm. Cela permet de réaliser une distribution du flux sensiblement périodique dans cette direction grâce à une répartition convenable des plaques de combustible. Dans l'autre direction, une lame d'eau de 1 cm est maintenue entre les boîtes pour permettre le passage des rideaux de sécurité (voir figures 1 et 2).

La teneur en ^{235}U dans les plaques d'aluminium uranié est de 25 mg/cm² (l'enrichissement de l'uranium utilisé est de 90%); la teneur en bore naturel des plaques d'aluminium boré est de 1,22 mg/cm² et la teneur en hafnium des plaques de zircaloy hafnié est de 14,6 mg/cm². Les autres données géométriques de l'élément combustible sont indiquées à la figure 1.

Dans la plupart des cas les réactivités expérimentales ont été évaluées pour des cœurs rectangulaires obtenus, en remplissant incomplètement les boîtes de la dernière rangée parallèle à la direction des plaques.

3. MÉTHODE DE CALCUL DE LA RÉACTIVITÉ

Dans ce qui suit, on entendra par cellule l'ensemble constitué par un élément de combustible et le volume d'eau associé que l'on retrouve périodiquement dans les deux directions du réseau. Nous ferons l'analyse des cœurs expérimentaux dans un modèle semi-homogène qui consiste à remplacer la cellule par un réseau homogène équivalent. Le calcul est fait en théorie de diffusion à quatre groupes avec les coupures usuelles pour les groupes (10 MeV, 0,821 MeV, 5,53 keV, 0,625 eV).

Les constantes rapides sont obtenues pour la pile nue équivalente par le code MUFT IV [2]. On a supposé le réseau homogénéisé géométrique-

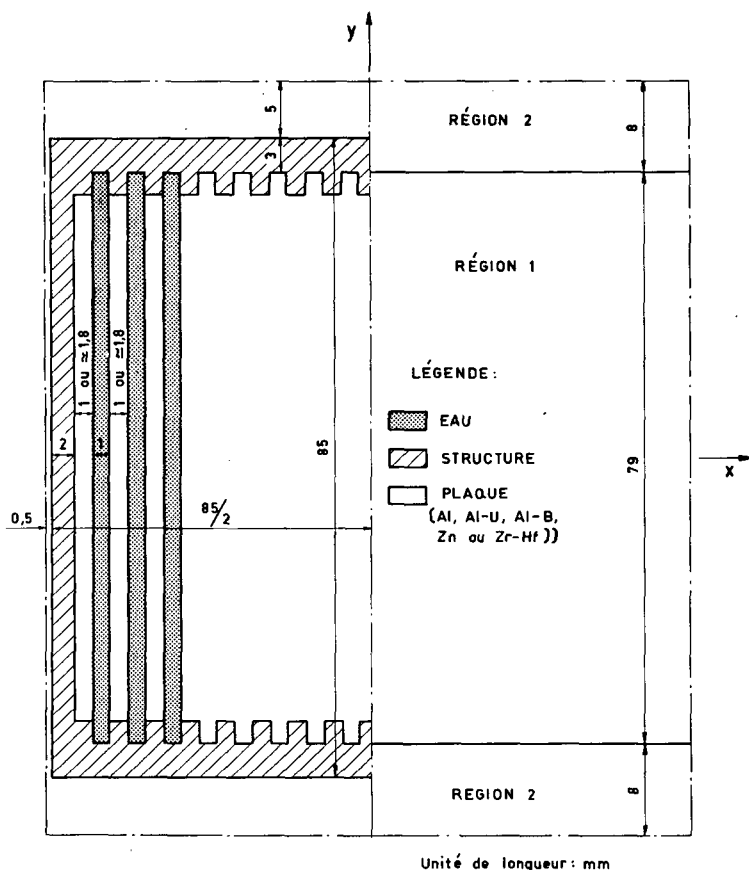


Figure 1

Cellule. Le volume des dents (1,068 ou 1,532 cm³ suivant que la crémaillère est à 41 ou 29 dents) localisé dans la région 1 a été reportée dans la région 2 pour le calcul

ment, le libre parcours des neutrons rapides étant grand vis-à-vis des dimensions des milieux constituant la cellule. Les sections efficaces utilisées sont les valeurs les plus récentes issues de la référence [3] (élément 72 pour l'oxygène, 73 pour le zirconium, 85 et 88 pour les isotopes 235 et 238 de l'uranium). Les deux approximations B1 et P1 sont utilisées concurremment.

Dans le cadre de ces approximations le ralentissement par l'hydrogène est traité exactement et celui des noyaux lourds par la théorie de l'âge.

Nous négligerons la variation spatiale du spectre des neutrons thermiques dans la cellule, faute d'un procédé simple permettant d'en tenir compte. Nous utiliserons un spectre moyen calculé pour un mélange homogène de tous les constituants du réseau en milieu infini, dans le modèle du gaz hydrogène monoatomique, par la méthode SOFOCATE [4]. Les sections efficaces et les coefficients de diffusion du groupe thermique sont déduits

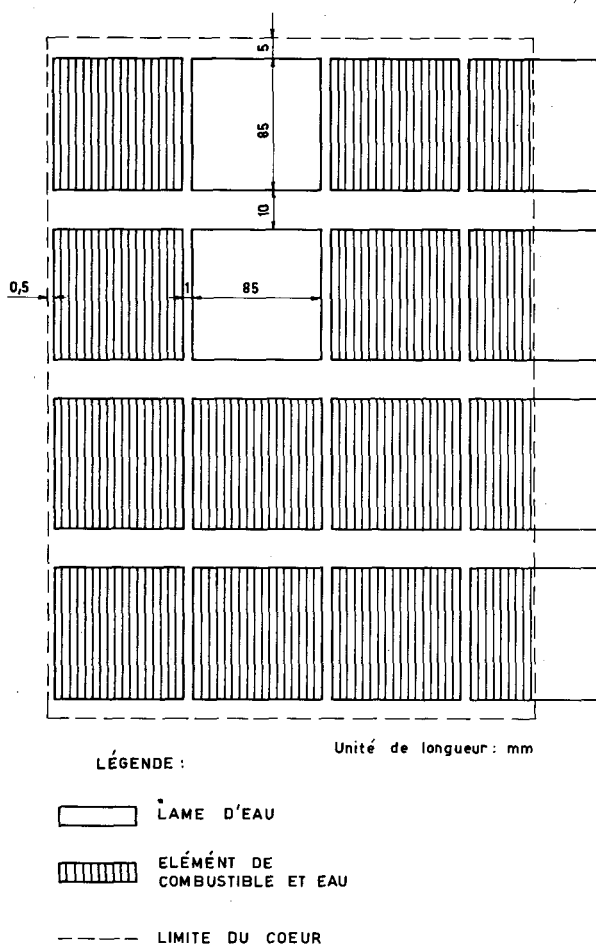


Figure 2

Cœur type d'ALIZÉ II

par pondération sur ce spectre. Les sections efficaces microscopiques adoptées sont tirées de la référence [5].

La géométrie simple de la cellule permet de calculer les facteurs de désavantage en deux temps. La distribution du flux est d'abord évaluée dans la direction perpendiculaire aux plaques sur toute la largeur de la cellule. Par pondération sur cette distribution de flux on obtient des sections efficaces moyennes pour la région 1 ou région combustible (voir figure 1). Le volume restant de la cellule, homogénéisé géométriquement, constitue la région 2.

Le flux est alors calculé dans la seconde direction dans les régions 1 et 2. Une pondération finale sur ces flux donne les sections efficaces d'absorption et de fission et le coefficient de diffusion du réseau homogène équivalent. Les calculs de flux sont faits en approximation 2P3 à l'aide du code FLIP [6].

La réactivité est calculée en deux dimensions (XY), par le code PDQ 02 [7]; on tient compte de la dimension axiale grâce à un laplacien transverse. Ce dernier est calculé comme la différence entre le laplacien matière et le laplacien radial qui rend critique la pile considérée dans la seule direction axiale.

4. ANALYSE DES CŒURS SANS ZIRCONIUM

Les caractéristiques essentielles et les réactivités ρ calculées pour les différents cœurs critiques sont reportées dans le tableau I: ($\rho = (\lambda - 1)/\lambda$ où λ est la valeur propre issue du calcul PDQ). Les valeurs des réactivités sont données dans les deux approximations B1 et P1; elles sont trop élevées dans les deux cas et particulièrement en approximation B1: la moyenne des réactivités pour les huit cœurs est de 0,0238 en approximation B1 et de 0,0191 en P1, les fourchettes étant respectivement de 0,7% et de 1%. L'analyse des résultats a conduit à faire deux corrections qui réduisent ces valeurs de façon appréciable.

D'une part, à la suite des mesures faites à Saclay sur l'aluminium utilisé dans ALIZÉ II, il est apparu préférable d'adopter une section efficace d'absorption à 2200 m/s de 0,250 barn au lieu de 0,230, introduit dans la librairie SOFOCATE, ce qui entraîne une première correction $\Delta\rho_{(Al)}$. D'autre part les constantes de ^{235}U (ν , σ_f , σ_a) qui étaient celles de la librairie SOFOCATE ont été remplacées par le jeu international cohérent issu de la référence [8], ce qui entraîne une seconde correction $\Delta\rho_{(U)}$. Les deux corrections et les valeurs corrigées de la réactivité sont reportées dans le tableau I. Les valeurs moyennes pour les huit cœurs en approximations B1 et P1 sont devenues respectivement 0,0083 et 0,0036; toutes les valeurs se trouvent alors à l'intérieur d'une fourchette de 0,5%.

L'approximation P1 semblerait la meilleure, néanmoins le calcul a été également poursuivi en approximation B1, car les approximations du type B1 convergent plus vite que les approximations P1 [9].

L'excédent de réactivité résiduel pourrait s'expliquer par les fuites de neutrons dans la structure d'aluminium. Egalement, les incertitudes sur le chargement en ^{235}U et en matériaux de structure dans chaque cœur peuvent intervenir.

En conclusion nous disposons d'une méthode de calcul qui nous permet de déterminer la réactivité avec une incertitude de l'ordre de $\pm 0,0025$ dans un domaine assez large de variation de la concentration en ^{235}U et de variation du laplacien qui recouvre celui des expériences réalisées pour l'étude du hafnium.

5. ANALYSE DES CŒURS COMPRENANT DU ZIRCONIUM ET DE L'HAFNIUM (voir tableau II)

L'hafnium dont nous disposons se présentait sous la forme de plaques de zircaloy hafnié à teneur naturelle en hafnium. Les boîtes de tous les cœurs réalisés étaient chargées uniformément de 13 plaques de zircaloy en remplacement du même nombre de plaques d'aluminium, une proportion variable de ces plaques étant hafniée. L'antiréactivité due au hafnium a été com-

TABLEAU II

RÉACTIVITÉS CALCULÉES AVANT ET APRÈS CORRECTION
POUR LES CŒURS CRITIQUES AVEC ZIRCONIUM ET HAFNIUM

Cœur	Chargement complet d'un élément de combustible				V _{métal}	$\frac{V_{Zr}}{V_{eau}}$	N ²³⁵ U	N _{Hf}	B ² _m (en m ⁻²)
	Al	Al-U	Zr	Zr-Hf	V _{eau}	$\frac{N_H}{\times 10^3}$	$\frac{N_H}{\times 10^3}$	$\frac{N_H}{\times 10^3}$	
11/1	18	9	13		1,00	0,252	1,670		54,0
13/6	18	9	12	1	1,00	0,252	1,670	0,143	49,9
23/8	15	12	13		1,00	0,252	2,227		71,9
19/7	15	12	8	5	1,00	0,252	2,227	0,715	53,8
15/10	12	15	2	11	1,00	0,252	2,784	1,573	49,1
21/7	4	12	24		1,00	0,465	2,227		75,2

Cœur	en approximation		$\Delta\rho(Al)$	$\Delta\rho(U)$	ρ corrigé en approximation		$\Delta\rho(Hf)$	ρ corrigé pour l'hétérogénéité en approximation	
	B 1	P 1			B 1	P 1		B 1	P 1
11/1	0,0228	0,0198	-0,0076	-0,0100	0,0052	0,0022		0,0052	0,0022
13/6	0,0221	0,0194	-0,0076	-0,0101	0,0044	0,0017	0,0019	0,0063	0,0036
23/8	0,0195	0,0144	-0,0063	-0,0094	0,0038	-0,0013		0,0038	-0,0013
19/7	0,0182	0,0150	-0,0060	-0,0095	0,0027	-0,0005	0,0055	0,0082	0,0050
15/10	0,0176	0,0150	-0,0049	-0,0091	0,0036	0,0010	0,0075	0,0111	0,0085
21/7	0,0134	0,0084	-0,0045	-0,0094	-0,0005	-0,0055		-0,0005	-0,0055

pensée par une augmentation de la charge en ²³⁵U de façon à maintenir une taille critique sensiblement constante. Cette façon de procéder avait l'avantage de conserver des fuites de neutrons rapides voisines et d'éliminer les difficultés d'interprétation de l'effet parasite du zircaloy, la concentration de celui-ci restant pratiquement constante.

En particulier nous n'avons pas tenu compte dans les calculs de l'anisotropie causée par la transparence de l'aluminium pour les neutrons, effet qui n'existe pas dans le zircaloy. Nous avons ainsi vérifié en réalisant un cœur supplémentaire avec 24 plaques de zircaloy par boîte que la réactivité calculée décroît très nettement avec la concentration en zircaloy; cet effet atteint 0,88% lorsque l'on remplace 46,5% du métal par du zircaloy (voir tableaux I et II).

Nous avons réalisé trois cœurs à 1,5 et 11 plaques de zirconium hafnié par boîte ainsi que deux sans hafnium; nous avons retenu comme référence le cœur sans hafnium dont le laplacien se rapprochait le plus des autres.

Les réactivités ont été calculées en utilisant le schéma de calcul précédant en prenant pour l'hafnium naturel les constantes microscopiques rapides de l'élément 14 de la dernière librairie MUFT publiée [3] et la section efficace d'absorption thermique donnée par la librairie SOFOCATE [5], environ 105 à 2200 m/s; dans le calcul MUFT, le facteur de « self shielding » L fut pris égal à 1. Les corrections ont été faites pour la capture de l'aluminium et les paramètres de ^{235}U comme ci-dessus. Les écarts entre les réactivités calculées sont inférieures à 0,0025.

Dans les calculs nous n'avons pas tenu compte ni de l'effet Doppler, ni de l'hétérogénéité. L'étude a montré que cela était justifié pour le premier effet; quant au second effet, son influence sur la réactivité a été estimée grossièrement comme suit en raisonnant avec un seul groupe rapide. Avec les notations classiques on a pour une variation ΔI de l'intégrale effective due à l'hétérogénéité:

$$\Delta \rho = [1/(1 + \tau B^2)] (\Delta p/p)$$

avec

$$\Delta p/p = (N_{\text{HF}} \Delta I / \xi \Sigma_s).$$

L'intégrale effective I du hafnium naturel en milieu homogène a été calculée à partir des paramètres de résonance de ses différents isotopes dans l'approximation de résonance étroite et par les formules suivantes:

$$I = (\pi/2) (I_j \sigma_0 / E_0) \sqrt{\beta(1 + \beta)}$$

avec

$$\sigma_p = \Sigma_H / N_{\text{HF}} \quad \text{et} \quad \beta = \sigma_p / \sigma_0.$$

(Les notations sont classiques [10]).

L'intégrale effective en milieu hétérogène I^* pour un absorbant de surface S_0 de volume V_0 et ayant N_0 noyaux par cm^3 , peut être réduite simplement à l'expression ci-dessus [11] en remplaçant σ_p , section efficace de modérateur par noyau absorbant, par $S_0(1 - C)/4 N_0 V_0$. C'est-à-dire β par $(1 - C)/(2 + \Sigma_0)$, où C est la correction habituelle de Dancoff. L'expression de C a été donnée par SPINRAD [11] dans le cas de plaques absorbantes d'épaisseur t séparées par des intervalles d'eau d'épaisseur T :

$$C = 2 e^{-\Sigma_1 T} / \left(3 + \Sigma_1 T - e^{-\Sigma_1 T} \right),$$

Σ_0 et Σ_1 étant respectivement les sections efficaces microscopiques de la plaque et du modérateur à l'énergie de résonance.

Ces corrections ainsi que les valeurs de la réactivité correspondante sont reportées dans le tableau II. Il semblerait que la capture épithermique

de l'hafnium soit sous-estimée car les réactivités calculées croissent avec la proportion de celui-ci. Cependant ce résultat ne semble pas significatif compte tenu des diverses incertitudes. Ainsi, considérons cette incertitude lorsqu'elle est la plus faible en valeur relative, ce qui est le cas du réseau le plus chargé en hafnium pour lequel la capture thermique de l'hafnium intervient dans le bilan de réactivité pour 0,063 et la capture épithermique pour 0,095. La réactivité du cœur correspondant est affectée d'une incertitude de $\pm 0,0027$ liée à la précision avec laquelle est connue la section efficace d'absorption du hafnium (± 5 b) et d'une incertitude de $\pm 0,0025$ liée à la précision de la méthode de calcul (voir paragraphe précédent), soit une incertitude totale de $\pm 0,0052$. La réactivité du cœur de référence est affectée de l'incertitude liée à la méthode de calcul soit $\pm 0,0025$. Donc l'écart entre les réactivités calculées, toutes corrections faites pour la pile la plus chargée en hafnium et la pile de référence, soit 0,0060, n'apparaît pas de manière significative. D'un autre côté les conclusions semblent indépendantes du type d'approximation, B1 ou P1.

6. CONCLUSION

Le premier résultat de ces expériences est de montrer qu'il est possible de calculer la réactivité des cœurs à eau légère et uranium enrichi, chargés en hafnium naturel, en utilisant les paramètres de résonance de ce dernier tels qu'ils sont publiés dans la littérature [3], du moins tant que la capture épithermique de l'hafnium ne contribue pas au bilan de réactivité pour plus de 10%.

Le second résultat concerne indirectement l'intégrale de résonance du hafnium naturel. Les paramètres que nous avons utilisés pour le calcul conduisent à une intégrale de résonance de 1968 b qui est à comparer à la valeur de 2850 b donnée par TATTERSTALL *et al.* [12]. Ainsi nos expériences établissent une forte présomption en faveur d'une valeur voisine de 2000 b, résultat qui est confirmé par les mesures récentes de FEINER [13].

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ANALYSIS OF WATER MODERATED UO₂ AND ThO₂ LATTICES

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Abstract — Résumé — Аннотация — Resumen

ANALYSIS OF WATER MODERATED UO₂ AND ThO₂ LATTICES. An extensive series of critical and exponential experiments has been performed with uniform lattices of fuel rods composed of low enriched UO₂ and of fully enriched UO₂ in ThO₂. These lattices were moderated by H₂O and by mixtures of D₂O and H₂O. Lattice spacings ranged from somewhat looser lattices than needed for maximum buckling to very tight lattices in which more than half the fissions were epithermal. Measurements included buckling, ratio of epithermal to thermal captures in U²³⁸ (ρ_{28}) and in Th²³² (ρ_{02}), ratio of epithermal to thermal fissions in U²³⁵ (δ_{25}), ratio of thermal flux in the moderator to that in the fuel (ϕ_M/ϕ_F), and water height reactivity worth leading to $\delta k_{eff}/\delta B^2$.

The primary theoretical analysis of these lattices utilizes a 39-group, Greuling-Goertzel slowing down model in the B₁ approximation for a bare reactor. A comparison analysis is made with the usual one-group, bare reactor model, modified to account for epithermal fissions. The simple model correlates the more thermal lattices with good accuracy, but for the lattices with larger fractions of epithermal fissions the epithermal neutron flux spectrum departs markedly from the usual 1/E shape and the multi-group treatment is required for accurate analysis. The multi-group model using experiment bucklings computes criticality of each of the more than 30 lattices in the series well, for most lattices within $\pm 1\%$ in k_{eff} . Calculated and measured critical bucklings agree well over the entire range $-5M^{-2} < B^2 < 115M^{-2}$, for most lattices within $\pm 2M^{-2}$.

Four types of fuel were used in the series; 3% enriched UO₂, 4% enriched UO₂, ThO₂ with Th to U²³⁵ atom ratio = 25, and ThO₂ with Th to U²³⁵ atom ratio = 15. Although the fuel rod diameters were small, ranging from 0.66 cm to 1.27 cm, calculations indicate 10 to 20% reduction of the effective resonance integral of U²³⁵ due to self-shielding in the resonances. Most of the computed shielding effects occur within the individual fuel rod, with smaller effects coming from the lattice mutual shielding between fuel rods. With this shielding taken into account, calculated values of δ_{25} are in good agreement with measured values over the range $0.05 < \delta_{25} < 1.6$.

Single lump resonance integrals for U²³⁸ calculated from resonance parameters are in agreement with recent values measured by Pettus, using both activation and reactivity determinations. Similar calculations for Th²³² are in appreciably poorer agreement with experiment. Because of this, the resonance cross-sections of Th²³² were adjusted to agree with the measured integral values. Lattice mutual shielding Dancoff effects are very important for the tighter lattices, and there is some uncertainty in determining them. The computed effective lattice resonance integrals lead to calculated values of ρ_{28} and ρ_{02} in good agreement with experiment in the ranges $4 < \rho_{28} < 16$ and $1.3 < \rho_{02} < 8$. Because of the large values of ρ in some of these lattices, a technique was developed to measure ρ directly utilizing intermediate dysprosium foil activations. This technique gave appreciably improved accuracy compared to conventional cadmium ratio measurements.

Calculation of thermal flux ratios in water lattices is complicated by both geometrical and spectral considerations. Two simplified methods are used to calculate ϕ_M/ϕ_F and the results are not in very good agreement with measured values. Because nearly all thermal neutrons in these lattices are absorbed in the fuel, the poor agreement has little effect on the calculated critical buckling or on other lattice parameters. More complex treatments involving both geometrical and spectral dependence are needed for more accurate calculations of this ratio.

Measured and calculated values of $\delta k_{eff}/\delta B^2$ are in fair agreement over the range 30 to 70 cm². The results are consistent with a separable Gaussian non-leakage probability, but the multi-group calculations indicate an appreciable interaction between leakage and resonance absorption. Thus values of the age can not be determined directly from water height measurements, but rather these data give a different extensive parameter for analysis.

ÉTUDE ANALYTIQUE DES RÉSEAUX $\text{UO}_2\text{-H}_2\text{O}$ ET $\text{ThO}_2\text{-H}_2\text{O}$. Une série très complète d'expériences critiques et exponentielles a été exécutée sur des réseaux uniformes comprenant des barres de combustible d' UO_2 faiblement enrichi et d' UO_2 pleinement enrichi en ThO_2 . Comme ralentisseur, on a utilisé H_2O et des mélanges de D_2O et H_2O . Les interstices des réseaux variaient dans une gamme allant de réseaux plus lâches qu'il n'était nécessaire pour obtenir le foisonnement maximum à des réseaux très serrés où plus de la moitié des fissions était d'origine épithermique. Les mesures portaient notamment sur les paramètres ci-après: foisonnement, rapport entre captures épithermiques et thermiques dans ^{238}U (ρ_{28}) et ^{232}Th (ρ_{02}), rapport entre les fissions épithermiques et thermiques dans ^{235}U (δ_{25}), rapport entre le flux thermique dans le ralentisseur et le flux thermique dans le combustible (ϕ_R/ϕ_C) et rapport entre la hauteur du ralentisseur et la réactivité, donnant la relation $\delta k_{\text{eff}}/\delta B^2$.

Pour l'analyse théorique primaire de ces réseaux, on utilise un modèle de ralentissement Greuling-Goertzel à 39 groupes, dans l'approximation B_1 applicable aux réacteurs sans réflecteur. L'auteur procède à une comparaison analytique avec le modèle habituel à un groupe pour réacteurs sans réflecteur, modifié de manière à tenir compte des fissions d'origine épithermique. Ce dernier établit une bonne corrélation pour les réseaux essentiellement thermiques; cependant, lorsqu'il s'agit de réseaux où les fissions d'origine épithermique prédominent, le spectre du flux des neutrons épithermiques s'écarte sensiblement de sa forme habituelle $1/E$ de sorte que le recours à un modèle à plusieurs groupes devient nécessaire pour permettre une analyse suffisamment exacte. Le modèle à plusieurs groupes fondé sur des foisonnements expérimentaux permet de déterminer l'état critique de chacun des réseaux de la série expérimentale - dont le nombre dépasse la trentaine - avec une marge d'erreur qui, pour la plupart des réseaux, est inférieure à $\pm 1\%$ en ce qui concerne le facteur de multiplication effectif (k_{eff}). On obtient une bonne concordance entre les foisonnements critiques calculés et mesurés dans toute la gamme $-5\text{M}^{-2} < B^2 < 115\text{M}^{-2}$, dus à la plupart des cas à $\pm 2\text{M}^{-2}$ près.

Dans la série expérimentale, on a utilisé quatre types de combustibles: UO_2 enrichi à 3%, UO_2 enrichi à 4%, ThO_2 avec un rapport atomique $\text{Th}/^{235}\text{U} = 25$ et ThO_2 avec un rapport atomique $\text{Th}/^{235}\text{U} = 15$. Bien qu'on ait employé des barres de combustible de faible diamètre, allant de 0,66 à 1,27 cm, les calculs indiquent une diminution de 10 à 20% de l'intégrale de résonance effective de ^{235}U , provoquée par un effet d'autoprotection lors des résonances. La plupart des effets d'ombre calculés se produisent à l'intérieur de chaque barre de combustible, alors que des effets moins prononcés sont dus au rôle d'écran que les barres jouent les unes par rapport aux autres dans le réseau. Compte tenu de cet effet d'ombre, les valeurs calculées de δ_{25} concordent bien avec celles qui ont été mesurées dans toute la gamme $0,05 < \delta_{25} < 1,6$.

Les intégrales de résonance dans un seul bloc calculées pour ^{238}U à partir des paramètres de résonance concordent avec les valeurs récemment mesurées par Pettus sur la base de celles qu'il avait obtenues pour l'activation et la réactivité. Des calculs semblables pour ^{232}Th sont moins conformes aux résultats expérimentaux. De ce fait, on a ajusté les sections efficaces de résonance de ^{232}Th pour les mettre en harmonie avec les valeurs des intégrales mesurées. Les effets Dancoff de protection mutuelle dans les réseaux sont très importants pour les réseaux plus serrés, mais leur détermination comporte une certaine incertitude. Les intégrales de résonance effective dans les réseaux obtenues par calcul permettent de déterminer ρ_{28} et ρ_{02} en bonne conformité avec les valeurs expérimentales pour $4 < \rho_{28} < 16$ et $1,3 < \rho_{02} < 8$. Comme dans certains de ces réseaux ρ prend des valeurs élevées, on a élaboré une méthode permettant de déterminer ρ par mesure directe à partir de l'activation de feuilles de dysprosium intercalées. Cette méthode a donné des résultats sensiblement plus précis que les mesures classiques du rapport cadmium.

Le calcul de la valeur relative des flux thermiques dans les réseaux comportant de l'eau est rendu plus complexe par des considérations de géométrie et de spectre. Pour calculer le rapport ϕ_M/ϕ_F on fait appel à deux méthodes simplifiées; les résultats obtenus ne concordent pas très bien avec les valeurs mesurées. Comme, dans ces réseaux, presque tous les neutrons thermiques sont absorbés par le combustible, ce manque de concordance n'a que peu d'influence sur la valeur calculée du foisonnement critique et sur d'autres paramètres du réseau. Des procédés plus complexes qui tiendraient compte à la fois de la géométrie et du spectre seraient nécessaires pour déterminer ce rapport avec plus de précision.

Les valeurs mesurées et calculées de $\delta k_{\text{eff}}/\delta B^2$ concordent relativement bien dans l'intervalle compris entre 30 et 70 cm^2 . Les résultats sont compatibles avec une probabilité gaussienne distincte d'absence de fuite, mais les calculs à plusieurs groupes révèlent une interaction non négligeable des fuites et de l'absorption par résonance. Il est donc impossible de déterminer l'âge directement à partir des mesures du niveau de l'eau, mais ces données fournissent en revanche un paramètre distinct pour les analyses.

АНАЛИЗ РЕШЕТОК ИЗ UO_2 И ThO_2 С ВОДНЫМ ЗАМЕДЛИТЕЛЕМ. Проведена большая серия критических и экспоненциальных опытов с одно-

родными решетками топливных стержней, состоящих из малообогащенного UO₂ и высокообогащенного UO₂ в ThO₂. Эти решетки замедлялись H₂O и смесями D₂O и H₂O. Шаги решеток изменялись в пределе от несколько менее плотных решеток, чем требуется для максимального лапласиана, до очень плотных, в которых больше половины делений были надтепловыми. Измерения включали лапласиан, соотношение между надтепловым захватом и захватом тепловых нейтронов в U²³⁸ (ρ_{28}) и в Th²³² (ρ_{02}), соотношение деления под действием надтепловых нейтронов к делению под действием тепловых нейтронов в U²³⁵ (δ_{25}) отношение потока тепловых нейтронов в замедлителе к потоку тепловых нейтронов в топливе (ϕ_M/ϕ_F) и реактивная способность столба воды, ведущая к $\delta k_{eff}/\delta B^2$.

Для первоначального теоретического анализа этих решеток используются группа-39, модель замедления Грейлинга-Гертцеля в приближении B₁ для реактора без отражателя. Проводится сравнительный анализ с обычной одnogрупповой моделью реактора без отражателя, измененной для потребности счета делений под действием надтепловых нейтронов. Простая модель достаточно точно коррелирует с тепловыми решетками с повышенными тепловыми данными, но у решеток с большими фракциями делений под действием надтепловых нейтронов спектр потока надтепловых нейтронов заметно отличается от обычной формы $1/E$, и требуется многогрупповое приближение для проведения точного анализа. Многогрупповая модель, в которой используются экспериментальные лапласианы, хорошо высчитывает критичность каждой из более чем 30 решеток в серии, для большинства решеток в пределах 1% при k_{eff} . Расчетные лапласианы и измеренные критические лапласианы хорошо согласуются во всем диапазоне $-5M^{-2} < B^2 < 115M^{-2}$, для большинства решеток в пределе $\pm 2M^{-2}$.

В серии были использованы четыре типа топлива: UO₂ 3%-ного обогащения, UO₂ 4%-ного обогащения, ThO₂ с атомным отношением Th к U-235 = 25 и ThO₂ с атомным отношением Th к U²³⁵ = 15. Хотя диаметры топливных стержней были малы, от 0,66 см до 1,27 см, расчеты показывают 10-20%-ное уменьшение эффективного резонансного интеграла U²³⁵ ввиду самоэкранирования при резонансах. Большинство из вычисленных защитных эффектов возникают внутри отдельного топливного стержня, и меньшие эффекты возникают в результате взаимного экранирования решетки между топливными стержнями. С учетом этого экранирования расчетные величины δ_{25} находятся в хорошем соответствии с данными измерений в диапазоне $0,05 < \delta_{25} < 1,6$.

Однократные общие резонансные интегралы для U²³⁸, вычисленные из резонансных параметров, согласуются с последними данными измерений, полученными Петтусом с использованием как определения активации, так и определения реактивности. Аналогичные расчеты для Th²³² значительно хуже согласуются с данными эксперимента. Ввиду этого резонансные поперечные сечения Th²³² приводились в соответствие с измеренными интегральными величинами. Эффекты Данкоффа взаимного экранирования решетки имеют очень важное значение для более плотных решеток, но их определение не совсем точное. Вычисленные эффекты резонансных интегралов решетки приводят к расчетным величинам ρ_{28} и ρ_{02} , которые хорошо согласуются с экспериментальными данными в диапазонах $4 < \rho_{28} < 16$ и $1,3 < \rho_{02} < 8$. Ввиду больших величин ρ в некоторых из этих решеток был разработан метод измерения ρ путем прямого использования активаций промежуточной диспрозиевой фольги. Этот метод

привел к значительно большей точности по сравнению с измерениями условного кадмиевого отношения.

Расчет коэффициентов потока тепловых нейтронов в водных решетках усложняется как по геометрическим, так и по спектральным соображениям. Применяются два упрощенных метода для расчета ϕ_M/ϕ_T , но результаты не очень хорошо согласуются с данными измерений. Так как почти все тепловые нейтроны в этих решетках поглощаются в топливе, плохое согласование данных мало влияет на вычисленный критический лапласиан или на другие параметры решетки. Необходимы более сложные опыты с учетом как диаметральной, так и спектральной зависимости для более точных расчетов этого коэффициента.

Данные измерений и расчета величин $\delta k_{eff}/\delta B^2$ находятся в удовлетворительном соответствии в диапазоне 30–70 см^2 . Результаты согласуются с отдельной гауссовой вероятностью избежания утечки, но многогрупповые расчеты указывают на значительное взаимодействие между утечкой и резонансным поглощением. Таким образом данные возраста не могут быть определены непосредственно из измерений столба воды, но эти данные скорее приводят к другому широкому параметру для анализа.

ANÁLISIS DE RETICULADOS DE UO_2 Y ThO_2 MODERADOS POR AGUA. El autor ha llevado a cabo una extensa serie de experimentos críticos y exponenciales con reticulados uniformes de elementos combustibles de UO_2 ligeramente enriquecido y de UO_2 muy enriquecido mezclado con ThO_2 . Los moderadores usados en estos reticulados eran el H_2O y mezclas de D_2O y H_2O . El espaciado de los reticulados estuvo comprendido entre valores algo superiores a los necesarios para alcanzar un laplaciano máximo y los correspondientes a reticulados muy compactos, en los que más de la mitad de las fisiones son epitérmicas. Entre las mediciones efectuadas, figuran la determinación del laplaciano, de la razón de capturas epitérmicas a capturas térmicas en el uranio-238 (p_{28}) y en el torio-232 (p_{22}), la razón de fisiones epitérmicas a fisiones térmicas en el uranio-235 (δ_{25}), la razón del flujo térmico en el moderador al flujo térmico en el combustible (ϕ_M/ϕ_C), y la influencia del nivel del agua sobre el valor de la reactividad, que conduce a $\delta k_{eff}/\delta B^2$.

El análisis teórico primario de estos reticulados se basa en la utilización de un modelo de moderación de Greuling-Goertzel, de 39 grupos, con la aproximación B_1 para el reactor sin reflector. Se efectúa un análisis comparativo con el modelo usual de un solo grupo, para reactor sin reflector, modificado para tener en cuenta las fisiones epitérmicas. En el modelo simple, la correlación de los reticulados con mayor abundancia de fisiones térmicas ofrece una precisión satisfactoria pero, para los reticulados con una fracción más elevada de fisiones epitérmicas, el espectro del flujo de neutrones epitérmicos se aleja considerablemente de la forma habitual $1/E$ y es preciso recurrir al tratamiento de multi-grupos para que el análisis sea correcto. Mediante el modelo de multi-grupos en que se aplican laplacianos experimentales, se puede calcular satisfactoriamente la criticidad individual de más de 30 reticulados de la serie; la aproximación es $\pm 1\%$ de k_{eff} para la mayoría de esos reticulados. Los laplacianos críticos calculados teóricamente concuerdan bastante bien con los resultados de las mediciones, dentro de todo el intervalo $-5\text{M}^{-2} < B^2 < 115\text{M}^{-2}$, siendo la precisión de $\pm 2\text{M}^{-2}$ para la mayoría de los reticulados.

En esta serie de experimentos se utilizaron cuatro tipos de combustible, a saber: UO_2 enriquecido al 3%, UO_2 enriquecido al 4%, ThO_2 con una razón atómica $\text{Th}/^{235}\text{U} = 25$, y ThO_2 con una razón atómica $\text{Th}/^{235}\text{U} = 15$. Aunque los diámetros de los elementos combustibles eran pequeños (entre 0,66 cm y 1,27 cm), los cálculos indican que se produce una reducción de 10 a 20% de la integral de resonancia efectiva del uranio-235 debido a un efecto de autobloqueo en las resonancias. La mayor parte de los efectos de blindaje calculados se verifican en el interior de las varillas de combustible, reduciéndose esos efectos por la acción de blindaje recíproco que tiene lugar en el reticulado entre las varillas de combustible. Si se tiene en cuenta este efecto de blindaje, los valores calculados de δ_{25} concuerdan satisfactoriamente con los valores medidos en el intervalo $0,05 < \delta_{25} < 1,6$.

Las integrales de resonancia de bloque único para el uranio-238, calculadas a partir de los parámetros de resonancia, coinciden con los valores medidos recientemente por Pettus aplicando tanto determinaciones de la activación como de la reactividad. En cambio, los respectivos cálculos para el torio-232 concuerdan menos con los resultados experimentales. Por esta razón se ajustaron las secciones eficaces de resonancia del

torio-232 de modo que coincidan con los valores de las integrales obtenidos por medición. Los efectos de Dancoff de blindaje recíproco en el reticulado asumen gran importancia en el caso de los reticulados más compactos y en su determinación se presenta cierto grado de incertidumbre. Los valores calculados de ρ_{28} y ρ_{02} deducidos del cómputo de las integrales efectivas de resonancia del reticulado coinciden satisfactoriamente con los resultados experimentales en los intervalos $4 < \rho_{28} < 16$ y $1,3 < \rho_{02} < 8$. Debido a la magnitud de los valores de ρ en algunos de estos reticulados, se ideó un procedimiento para medir ρ directamente, utilizando el método de la activación intermedia de láminas de disprosio. Este procedimiento ofrece una exactitud apreciablemente mayor que el método usual de medición de la razón de cadmio.

El cálculo de las razones de flujo térmico en los reticulados de agua se torna más complicado debido a los factores geométricos y espectrales. Se utilizan dos métodos simplificados para calcular Φ_M/Φ_C y los resultados se alejan bastante de los valores obtenidos por medición. Debido a que en estos reticulados casi todos los neutrones térmicos son absorbidos por el combustible, esa falta de coincidencia ejerce poca influencia en el laplaciano crítico calculado o en los restantes parámetros del reticulado. Para calcular más exactamente esta relación, hay que recurrir a tratamientos de mayor complejidad en que intervienen al mismo tiempo las relaciones de dependencias geométricas y las espectrales.

Los valores medidos y los calculados de $\delta k_e/\delta B^2$ concuerdan satisfactoriamente en el intervalo de 30 a 70 cm². Los resultados son compatibles con una probabilidad gaussiana de no escape separable, pero los resultados del cálculo de multigrupos indican que existe una interacción apreciable entre el escape y la absorción por resonancia. Por tanto, los valores relativos a la edad no pueden determinarse directamente partiendo de las mediciones del nivel del agua, dando más bien estos datos un parámetro extensivo diferente para el análisis.

1. DESCRIPTION OF EXPERIMENTS

1. 1. Background of experiments

Critical experiments performed with uniform lattices of fuel rods in simple geometry form the basis for the major part of our understanding of water moderated reactors. Information from these experiments serves as the basic test of the theory which is used in the design of power reactors. Many of these clean lattice experiments have been performed over the past five years in the Critical Experiment Laboratory of The Babcock & Wilcox Company in Lynchburg, Virginia. These have included work with low enriched uranium oxide (UO₂) and also with fully enriched uranium mixed with thorium oxide (ThO₂). H₂O moderated lattices of each fuel have been formed with pitch spacings ranging from fairly loose lattices through those with maximum buckling, to very closely spaced lattices. A series of experiments with mixtures of H₂O and D₂O as moderator extend the information to equivalent lattice spacings smaller than those normally attainable with H₂O moderator alone. The lattices that are more closely spaced have very large amounts of resonance absorption, and the analysis is very sensitive to the accuracy of the fundamental resonance data.

1. 2. Fuel rods

Seven different types of fuel rods have been used in the experiments. These are described in Table I. The fuel rods were long slender cylinders with metal clad surrounding the oxide fuel. The rods designated NMSR were used in preliminary experiments [1] for the design of the Nuclear Ship Savannah, and those designated CETR were used for the design of the Consolidated Edison Thorium Reactor [2, 3]. The TUPE rods were used in a series of Thorium Uranium Physics Experiments [4] in collaboration with

TABLE I
FUEL ROD DESCRIPTION

Fuel designation	Fertile material	U^{235} Fertile atom ratio	Clad material	Fuel* density g/cm^3	Rod diam cm	Fuel diam cm	Clad thickness cm	Active rod length cm
NMSR-3	U^{238}	0.03128	Stainless steel	9.28	1.270	1.126	0.072	168.9
NMSR-4	U^{238}	0.04242	Stainless steel	9.43	1.270	1.126	0.072	168.9
SSCR-4	U^{238}	0.04242	Stainless steel	9.46	1.208	1.126	0.041	169.4
CETR-4	Th^{232}	0.0388	Stainless steel	8.35	0.792	0.660	0.048	121.9
CETR-7	Th^{232}	0.0667	Stainless steel	8.35	0.792	0.660	0.048	121.9
TUPE-4	Th^{232}	0.0395	Aluminium	8.45	0.785	0.594	0.086	152.4
TUPE-7	Th^{232}	0.0667	Aluminium	8.33	0.782	0.660	0.036	152.4

* Fuel is in the form of either UO_2 or ThO_2 .

TABLE II
LATTICE CONFIGURATIONS

CASE	Fuel designation	Pitch, cm	M/W*	Mole % D O in moderator	Type of experiment
1	NMSR-3	2.196	0.355	0	Critical
2	"	1.684	0.807	0	"
3	"	1.554	1.102	0	"
4	NMSR-4	2.381	0.288	0	"
5	"	2.196	0.355	0	"
6	"	1.684	0.807	0	"
7	"	1.554	1.012	0	"
8	CETR-4	1.023	0.890	0	"
9	"	0.966	1.116	0	"
10	CETR-7	1.933	0.152	0	"
11	"	1.367	0.358	0	"
12	"	1.115	0.656	0	"
13	"	1.023	0.890	0	"
14	"	0.966	1.116	0	"
15	TUPE-4	1.446	0.301	0	"
16	"	1.222	0.480	0	"
17	TUPE-7	1.222	0.480	0	"
18	"	1.023	0.860	0	"
19	"	0.978	1.024	0	"
20	SSCR-4	1.511	1.0064	0	"
21	"	1.511	1.0064	49.66	"
22	"	1.511	1.0064	69.70	"
23	"	1.511	1.0064	73.77	"
24	"	1.511	1.0064	73.77	"
25	"	1.511	1.0064	76.50	"
26	"	1.511	1.0064	80.65	Exponential
27	"	1.511	1.0064	89.14	"
28	TUPE-7	0.983	0.990	55.38	"
29	"	0.983	0.990	60.40	Critical
30	"	0.983	0.990	71.94	Exponential
31	"	0.983	0.990	81.16	Critical**
32	"	0.983	0.990	81.96	Exponential

* Metal-to-Water ratio (M/W) is the volume ratio of fuel rods to water moderator.

** Central zone of a two-zone configuration.

Argonne National Laboratory who performed similar measurements in D₂O moderator [5]. The TUPE-7 and SSCR rods were used in the Spectral Shift Control Reactor programme with mixtures of H₂O and D₂O as moderator [6, 7].

1.3. Lattices

The lattice configurations were obtained by mounting the fuel rods at each end in various metal grid plates which maintained the lattice spacing. All lattices had uniform square pitch. Care was taken to obtain right circular cylindrical configurations, and criticality was achieved with control blades removed by adding fuel rods or by adjusting the moderator height. In several low buckling cases the results of exponential assemblies are reported, since there were not enough fuel rods to obtain criticality. The lattice configurations are listed in Table II.

2. MEASUREMENTS AND MEASURING TECHNIQUES

2.1. Types of measurement

Since the assemblies were measured over an extended period of time and for several different purposes, not all important parameters were measured for all lattices. However, a sufficient number of parameters have been measured that a fairly consistent set of conclusions can be drawn. Values of critical size and buckling have been determined for most cores in the series. Measurements of $\delta k/\delta B^2$ and of δ_{25} (the ratio of epicadmium to subcadmium fissions in U²³⁵) have been made for more than half the lattices. Fewer data have been taken for values of ρ_{28} (ratio of epicadmium to subcadmium captures in U²³⁸), and ρ_{02} (ratio of epicadmium to subcadmium captures in Th²³²), and ϕ_w/ϕ_f (ratio of thermal flux in the water moderator to that in the fuel). The experimental values are shown in Tables III and IV, along with the comparison calculated values.

2.2. Criticality

In the critical experiments, care was taken to obtain nearly perfect cylindrical configurations which were critical with all control blades removed, and with water moderator raised to the full height of the fuel rods. For most cases the control or safety blades were thin enough to slip between rows of fuel rods, so no extra guides were used. Where guides were used, their effect was corrected for by auxiliary experiments so the final critical sizes are for clean, cylindrical lattices. Radial and axial bucklings were measured with gold, indium, or U²³⁵ foils. In a number of lattices, bare and cadmium covered foil activation measurements of the buckling were compared. Generally the cadmium covered activations were found more reliable. Thermal activations were more sensitive to local effects and to location, and it was difficult to place foils at the same position relative to the fuel rods at all points in the lattice. Foils were usually mounted in thin plastic strips or on flexible metal measuring tape. In the exponential as-

TABLE III

EXPERIMENTAL AND CALCULATED LATTICE PARAMETERS

CASE	Critical buckling, (M ²)		Calculated k _{eff} with measured B ²	$\delta k_{eff}/\delta B^2$, cm ²	
	Expt.	Calc.		Expt.	Calc.
1	69	64.7	0.988	27.5	29.4
2	64	60.6	0.989	32.3	33.7
3	51	49.8	0.996	21.4	36.9
4	92	92.2	1.001	---	27.0
5	93	97.3	1.012	23.5	27.2
6	86	83.8	0.993	25.7	31.6
7	69	69.3	1.001	28.5	35.0
8	29.3	27.7	0.994	39.8	38.2
9	26.2	21.5	0.981	---	41.3
10	57	64.8	1.022	---	27.6
11	94	93.7	0.999	---	27.6
12	84	82.7	0.996	---	30.9
13	72	70.6	0.995	---	33.9
14	61	60.0	0.996	34.5	36.9
15	53.55	55.8	1.007	---	32.3
16	64.01	66.0	1.007	33.5	34.1
17	114.2	115.5	1.004	27.7	29.7
18	94.25	92.1	0.993	39.4	35.8
19	85.31	83.1	0.992	43.7	38.5
20	88.0	85.2	0.991	31.9	33.8
21	44.0	41.0	0.985	48.1	51.0
22	18.60	18.4	0.999	60.2	64.8
23	14.33	15.6	1.009	62.1	69.0
24	14.09	15.6	1.011	63.2	68.9
25	10.77	12.6	1.013	65.4	72.0
26	6.53	7.9	1.010	---	77.1
27	-4.68	-2.0	1.025	---	89.5
28	44.8	41.7	0.981	---	61.3
29	39.0	37.1	0.988	59.3	65.1
30	31.4	26.1	0.961	---	76.2
31	---	16.6	---	---	88.8
32	22.7	15.8	0.940	---	89.8

TABLE IV

EXPERIMENTAL AND CALCULATED LATTICE PARAMETERS

CASE	δ_{25}		ρ_{Fertile}		Fertile material	ϕ_W/ϕ_F	
	Expt.	Calc.	Expt.	Calc.		Expt.	Calc.
1	-	0.064	-	1.09	U	-	1.21
2	-	0.137	-	2.26	U	-	1.17
3	-	0.188	-	3.01	U	-	1.15
4	-	0.067	-	1.15	U	-	1.31
5	-	0.080	-	1.38	U	-	1.28
6	-	0.177	-	2.95	U	-	1.22
7	-	0.245	-	3.97	U	-	1.21
8	0.157	0.163	-	1.01	Th	1.14	1.12
9	0.210	0.206	1.28	1.26	Th	-	1.12
10	-	0.047	-	0.31	Th	-	1.27
11	-	0.100	-	0.66	Th	-	1.22
12	-	0.018	-	1.17	Th	-	1.19
13	0.221	0.244	1.49	1.58	Th	1.19	1.18
14	0.292	0.310	2.08	1.99	Th	-	1.18
15	0.053	0.046	-	0.31	Th	1.22	1.14
16	0.085	0.070	-	0.45	Th	1.16	1.13
17	0.130	0.123	-	0.81	Th	1.22	1.21
18	0.181	0.220	-	1.41	Th	1.12	1.19
19	0.266	0.263	-	1.67	Th	1.22	1.18
20	0.266	0.238	4.18	3.88	U	1.27	1.21
21	0.495	0.439	8.3	7.11	U	1.20	1.20
22	0.73	0.668	10.4	11.32	U	1.13	1.21
23	-	0.755	-	12.23	U	-	1.21
24	-	0.755	-	12.23	U	-	1.21
25	0.88	0.823	16.4	13.36	U	1.17	1.21
26	1.02	0.955	-	15.58	U	-	1.21
27	1.56	1.590	-	23.80	U	-	1.22
28	0.56	0.524	-	3.43	Th	-	1.19
29	0.65	0.578	-	3.82	Th	1.18	1.19
30	0.81	0.771	-	5.23	Th	-	1.19
31	1.20	1.045	8	7.37	Th	1.16	1.19
32	1.16	1.083	7	7.65	Th	-	1.20

semblies flux traverses were measured both radially and axially, with the axial exponential attenuation being measured in the central part of the assembly where the radial buckling was constant. Radial measurements were fitted by least squares to $J_0(B_r r)$ over the asymptotic region. Axial measurements in the critical experiments were fitted to $\cos B_z z$, and in the exponential experiments to $e^{-B_z z}$.

2.3. Kinetic measurements

The lattices were assembled with the axis of the cylinder in the vertical direction. Experimental determinations of $\delta\rho/\delta h$ were made by changing the water height, and measuring the reactor period, T . The following form of the inhour equation was used to relate the reactivity ρ to the period:

$$\rho = \frac{\ell}{T k_{\text{eff}}} + \sum_{i=1}^m \frac{\beta_i}{1 + \lambda_i T}$$

where

ℓ = prompt neutron lifetime,

β_i = fraction of delayed neutrons in group i ,

λ_i = decay constant of group i .

Delayed groups included those from U²³⁵ and from U²³⁸ or thorium. Where appropriate, delayed groups were included for the effects of the D (γ , n) reaction, using estimates of the magnitude based on results of GALANIN [8]. Because delayed neutrons are emitted at lower energies than prompt neutrons, they have a larger non-leakage probability. This effect was accounted for by using effective delay fractions corrected by calculated non-leakage probabilities.

The differential worth of water height was measured for several different configurations of the same lattice (different heights and core radii). Since the reactivity is defined by

$$\rho = (k_{\text{eff}}^{-1})/k_{\text{eff}}$$

The differential value is

$$\frac{\delta\rho}{\delta h} = \left(\frac{1}{k_{\text{eff}}}\right)^2 \frac{\delta k_{\text{eff}}}{\delta B^2} \frac{\delta B^2}{\delta h}$$

Assuming constant axial reflector savings λ , and that the radial and axial bucklings are separable,

$$B^2 = B_r^2 + \left(\frac{\pi}{h + \lambda}\right)^2$$

and

$$\delta B^2/\delta h = - \left[2\pi^2/(h + \lambda)^3 \right]$$

Thus for k_{eff} very close to unity,

$$\left(\frac{\delta p}{\delta h} \right)^{-1/3} = - \left(2\pi^2 \frac{\delta k_{\text{eff}}}{\delta B^2} \right)^{-1/3} (h + \lambda),$$

which is a straight line in variables h and $(\delta\rho/\delta h)^{-1/3}$. The line which fitted experimental values of these variables had a slope which determined the experimental value of $\delta k_{\text{eff}}/\delta B^2$ for the lattice.

2.4. Cadmium ratios

The epithermal effects in U^{235} , U^{238} , and Th^{232} were measured by comparing the activation of small samples covered with cadmium to the activation of the same or similar samples without the cover. For U^{235} , thin aluminium- U^{235} alloy foils were inserted in a fuel rod, and the fission product activities for equivalent irradiations were compared by γ -counting. Comparison required the use of a pre-calibrated decay curve for U^{235} fission products. The measurements determined the cadmium ratio of U^{235} fissions, that is, the ratio of the uncovered foil activation to the cadmium covered foil activation. Experimental values of δ_{25} were determined by

$$\delta_{25} = \frac{1}{\text{cadmium ratio} - 1}$$

The cadmium ratios of Th^{232} activation were measured using either thorium metal foils or ThO_2 wafers. These were inserted in a section of a fuel rod for activation. For the cadmium covered runs, a cadmium sleeve replaced a section of the metal clad around the fuel rod. The β -decay of Th^{233} was counted. For a number of cases attempts were made to measure the activity by γ -counting, but the presence of some unknown impurity activity with about the same half-life as Th^{233} obscured the results. Values of ρ_{02} were determined by

$$\rho_{02} = \frac{1}{\text{cadmium ratio} - 1}$$

For U^{238} there are some difficulties in obtaining good results using depleted metal foils in oxide fuel rods. To avoid these difficulties, measurements were made on small sections of the fuel rod itself. After irradiation, these small sections of fuel were dissolved in acid and the fission products were removed by solvent extraction. The uranium was then precipitated, and the

U²³⁹ β-activity was counted. The cadmium ratio was obtained by this method with 2 to 3% accuracy. Values of ρ_{28} were determined by

$$\rho_{28} = \frac{1}{\text{cadmium ratio} - 1}$$

Because the values of ρ_{28} are large (cadmium ratio near unity) small errors in the cadmium ratio led to much larger errors in ρ_{28} . Improved accuracy was obtained by relating the resonance U²³⁸ absorptions to the thermal absorptions through an intermediate thermal activation foil of dysprosium. The dysprosium foil was irradiated inside a fuel rod at the same time as the sections of fuel, and then a comparison between U²³⁸ and dysprosium activations was made by a later irradiation in a thermal flux. The β-activity of Dy¹⁶⁵ was counted directly, and the same dysprosium foil was used in both the reactor and thermal flux irradiations. Careful weighing of the fission-product-free uranium samples was required, and a small correction was applied for the non-1/v cross-section of Dy¹⁶⁴. The value of ρ_{28} determined by the intermediate foil technique has an accuracy of about ± 5%, independent of the absolute value. This is an appreciable improvement for large values of ρ_{28} .

2.5. Thermal flux ratio

The ratio of thermal flux in the water moderator to that in the fuel was measured using foils of dysprosium-aluminum alloy. Round foils were inserted in the fuel rods and sector foils were cut to fit the space between fuel rods. Measured corrections for epicadmium activation were applied. The β-activity of Dy¹⁶⁵ was counted. A number of irradiations were performed for each lattice, but, probably because of the difficulty in aligning the foils accurately in close-packed lattices, the final ratio ϕ_W/ϕ_F was determined to only about 2 or 3%.

3. METHOD OF ANALYSIS

3.1. The basic model and equations

An exact solution of the slowing down problem is possible for hydrogen moderator, and for heavy elements the Fermi age treatment is quite accurate, but slowing down in deuterium moderator requires somewhat more complex treatment. In the GREULING-GOERTZEL approximation[9, 10] deuterium is treated in a hydrogen-like manner, computing the slowing down density, like hydrogen

$$q_H = \int_0^u \Sigma_H(u') e^{-u'} - u \phi(u') du'$$

to be

$$q_D = \frac{\xi_D}{\gamma_D} \int_0^u \Sigma_D(u') e^{\frac{u' - u}{\gamma_D}} \phi(u') du'.$$

Here ξ is the average lethargy gain per collision ($\xi_D = 0.725$ for isotropic scattering), and γ is the Greuling-Goertzel constant, which can also be computed easily from microscopic collision parameters ($\gamma_D = 0.583$ for isotropic scattering). For a finite reactor the spatial variation of the flux can be separated from the energy variation in the equivalent bare reactor approximation. For small systems the use of the B_1 expansion [11] in the Fourier transformed equations leads to more accurate treatment of the leakage than does the usual P_1 approximation. The Greuling-Goertzel equations in the B_1 approximation are used as the basic model in the analysis [12]. These are contained in the BPG Code [13] which has been written for the Burroughs Datatron-205 computer. The code computes the criticality and neutron balance in a bare reactor using up to 80 energy groups (39 were used in the present work). The neutron balance is reformulated by the computer to give 4-group constants which can be used in one- or two-dimensional multiregion calculations. BPG solves the following equations:

$$BJ(u) + [\Sigma^A(u) + \Sigma^{in}(u)] \phi(u) = S(u) - \sum_k \frac{dq_k}{du} - \frac{dq_F}{du} + I(u) \quad (1)$$

$$[h(u) \Sigma^T(u) - \mu(u) \Sigma^S(u) - J(u) - \frac{B}{3} \phi(u) = \sum_m \frac{dp_m}{du} \quad (2)$$

$$\gamma_k(u) \frac{dq_k}{du} + q_k(u) = \xi_k(u) \Sigma_k^S(u) \phi(u) \left[1 - \frac{d\gamma_k}{du} \right] \quad (3)$$

$$q_F(u) = \sum_\ell \xi_\ell(u) \Sigma_\ell^S(u) \phi(u) \quad (4)$$

$$\xi_m(u) \frac{dp_m}{du} + P_m(u) = \eta_m(u) \Sigma_m^S(u) J(u) \left[1 - \frac{d\xi_m}{du} \right] \quad (5)$$

The symbols are defined by:

$$\begin{aligned} u &= \text{lethargy} \\ \phi(u) &= \text{flux} \end{aligned}$$

$J(u)$	=	current
$q(u)$	=	slowing down density from elastic scattering
$p(u)$	=	anisotropic component of slowing down density
$I(u)$	=	source from inelastic scattering
B^2	=	reactor buckling
$\Sigma^A(u)$	=	absorption cross-section
$\Sigma^{in}(u)$	=	inelastic scattering cross-section
$\Sigma^S(u)$	=	elastic scattering cross-section
$\Sigma^T(u)$	=	total cross-section
$S(u)$	=	neutron source (usually from fission)
$\mu(u)$	=	average cosine of the scattering angle (laboratory system)
$\xi(u)$	=	average lethargy gain per collision
$\gamma(u), \xi(u), \eta(u)$	=	slowing down parameters
$h(u)$	=	$\frac{a^2}{3} \left[\tan^{-1} a / (a - \tan^{-1} a) \right]$, where $a = B/\Sigma^T(u)$

In the code it is possible to choose whether the Greuling-Goertzel approximation (subscript k) or the Fermi age approximation (subscript l) is to be used for slowing down from each nuclear species. The model used in the analysis chose all nuclei of mass 27 or less (including Al²⁷) to be treated by Greuling-Goertzel, and all others by Fermi age. Corrections to the slowing down density caused by anisotropy of scattering in the laboratory systems are significant for the lighter elements. Equation (5) accounts for this, and its use gives the consistent form of the B₁ approximation,

If $p(u)$ is zeroed the inconsistent form is obtained. In the analysis, the consistent form was used for slowing down by H and D and the inconsistent form for all other nuclei.

Inelastic scattering is treated by a matrix transfer of neutrons between groups. A combination of evaporation model and discrete level information was used to generate the matrix, following the treatment by AMSTER [14]. Only the matrix for U²³⁸ has been computed, and this is used in the analysis for all inelastic scattering, since U²³⁸ is the major contributor to inelastic scattering in the lattices containing it. It is believed that this matrix also describes well the inelastic process in thorium, because the level scheme of thorium is very similar to U²³⁸.

Anisotropic scattering in the centre of mass system is an important effect at energies in the MeV range. A two-parameter fit has been made to the measured angular distribution [15, 16] for each nuclear species, and from this the variation of $\mu(u)$, $\xi(u)$, $\gamma(u)$, $\xi(u)$, $\eta(u)$ can be computed. Because of the uncertainties in most of the data available, this variation has been computed only for $\mu(u)$ for each element. In the present analysis, ξ , γ , ξ , η are all assumed constant (for each element). The error in the analysis caused by these assumptions should be small, since the major variation, $\mu(u)$, is accounted for. Agreement of calculated ages with measured values substantiates this.

3.2. Resonance absorption

In the multi-group formulation of the BPG Code, the groups are of finite width and generally large compared with the width of a single resonance. In U^{238} and in Th^{232} a large part of the resonance absorption takes place in a few very strongly absorbing lines, and this absorption is found in the code by computing the resonance escape probability for a single resonance and obtaining the product of slowing down density and the absorption probability. If q_i is the slowing down density into group i , and if p is the resonance escape probability for a single peak,

$$\begin{aligned}\text{Absorptions in peak} &= q_i (1-p) \\ &= q_i \left[NR / (\xi \Sigma_s)_i \right]\end{aligned}$$

where NR is the macroscopic resonance integral over the peak and $(\xi \Sigma_s)_i$ is the slowing down power in the group. Resonance integrals of single peaks are obtained from the resonance parameters, using a Doppler broadened Breit-Wigner Shape. The effects of lump size and of lattice mutual shielding are accounted for by escape probabilities computed according to methods of CHERNICK [17], NORDHEIM [18] and DANCOFF and GINSBURG [19]. The measured resonance parameters are related through these models to measured lattice cadmium ratios and criticalities. Measured single pin resonance integrals are used as an intermediate check.

For U^{238} the resonance integral calculated from the resonance parameters fits the relation,

$$R_{28} = 5.8 + 22.0 \sqrt{S/M_{OX}} \text{ barns,}$$

where S/M_{OX} is the surface-to-mass ratio of the oxide fuel lump. HELL-STRAND [20] used an activation technique to measure

$$R_{28} = 4.15 + 26.6 \sqrt{S/M_{OX}} \text{ barns,}$$

and PETTUS et al. [21] have compared activation and reactivity measurements to obtain

$$R_{28} = 0.8 + 28.2 \sqrt{S/M_{OX}} \text{ barns.}$$

These formulas for the resonance integral of U^{238} are not in very good agreement. Because of the agreement between the two different determinations of Pettus, somewhat greater reliance is placed on that formulation. For the analysis, the calculated energy distribution of the resonance integral was used, with the resonance cross-sections renormalized to fit the Pettus value. The lattice effects were computed using a numerical integration of

TABLE V

RESONANCE INTEGRALS

CASE	Fertile material	$\sqrt{S/M_{ox}}$ cm g ^{-1/2}	Single Rod R, barns	Surface fraction	Dancoff factor	Lattice R, barns
1	UO ₂	0.628	18.51	0.576	0.986	17.84
2	UO ₂	0.628	18.51	0.576	0.890	16.25
3	UO ₂	0.628	18.51	0.576	0.833	15.31
4	UO ₂	0.626	18.46	0.572	0.933	17.97
5	UO ₂	0.626	18.46	0.573	0.981	17.76
6	UO ₂	0.626	18.46	0.572	0.886	16.19
7	UO ₂	0.626	18.46	0.572	0.830	15.26
8	ThO ₂	0.867	17.19	0.486	0.585	13.97
9	ThO ₂	0.867	17.19	0.486	0.553	13.31
10	ThO ₂	0.879	17.40	0.486	0.734	17.02
11	ThO ₂	0.879	17.40	0.486	0.694	16.20
12	ThO ₂	0.879	17.40	0.486	0.633	14.95
13	ThO ₂	0.879	17.40	0.486	0.593	14.14
14	ThO ₂	0.879	17.40	0.486	0.561	13.48
15	ThO ₂	0.909	17.95	0.497	0.726	16.86
16	ThO ₂	0.909	17.95	0.497	0.681	15.93
17	ThO ₂	0.880	17.42	0.487	0.667	15.57
18	ThO ₂	0.880	17.42	0.487	0.589	14.06
19	ThO ₂	0.880	17.42	0.487	0.563	13.52
20	UO ₂	0.625	18.44	0.571	0.836	15.36
21	UO ₂	0.625	18.44	0.571	0.753	13.99
22	UO ₂	0.625	18.44	0.571	0.705	13.19
23	UO ₂	0.625	18.44	0.571	0.694	13.00
24	UO ₂	0.625	18.44	0.571	0.694	13.00
25	UO ₂	0.625	18.44	0.571	0.686	12.87
26	UO ₂	0.625	18.44	0.571	0.674	12.67
27	UO ₂	0.625	18.44	0.571	0.647	12.22
28	ThO ₂	0.880	17.42	0.487	0.500	12.24
29	ThO ₂	0.880	17.42	0.487	0.493	12.09
30	ThO ₂	0.880	17.42	0.487	0.476	11.78
31	ThO ₂	0.880	17.42	0.487	0.460	11.41
32	ThO ₂	0.880	17.42	0.487	0.458	11.38

the Dancoff formulation, which was applied to the surface fraction of the resonance integral following the calculations of Nordheim. The effective lattice resonance integral was thus obtained by

$$R_{28} (\text{Lattice}) = R_{28} (\text{Single Lump}) \left[1 - c + c \gamma \right],$$

where c is the surface fraction of the resonance integral, and γ is the Dancoff factor. Values used in the analysis are listed in Table V.

For thorium the data are preliminary and the resonance parameters are more poorly known than for U^{238} . The formula calculated from resonance parameters is

$$R_{02} = 6.25 + 18.65 \sqrt{S/M_{\text{ox}}} \text{ barns.}$$

A recent reevaluation of older data from PETTUS [22] gives

$$R_{02} = 0.2 + 16.4 \sqrt{S/M_{\text{ox}}} \text{ barns,}$$

and preliminary results of HELLSTRAND and WEITMAN [23] give

$$R_{02} = 1.5 + 18.1 \sqrt{S/M_{\text{ox}}} \text{ barns.}$$

For the analysis, the calculated distribution in energy of the resonance integral was renormalized to agree with the Hellstrand and Weitman data, and the lattice integrals were computed in the same manner as for U^{238} . Values are given in Table V.

In these lattices there are important effects of lumping and of lattice shielding for the U^{235} resonance absorption. These effects were determined in a manner similar to that used for the fertile materials. Resonance parameters were used to compute the effective resonance integral of each line assuming a Breit-Wigner shape. The calculations followed work of DRESNER [24] using the effective variable

$$\sqrt{\sigma_p + \gamma S/4NV},$$

according to the recipe of LEVINE [25]. A reduction of 10 to 20% in the U^{235} resonance integral was determined for the lattices, the primary effect being in the fuel rod lumping. To simplify the calculations, the same values of the integrals to 0.4eV were used in all lattices:

$$R_{25} (\text{Absorption}) = 405 \text{ b}$$

$$R_{25} (\text{Fission}) = 270 \text{ b}$$

This assumption has only a small effect on computed criticalities. The effect on computed values of δ_{25} is larger, and is discussed in the detailed analysis.

3.3. Thermal cross-sections

For lattices moderated with H₂O, effective thermal cross-sections were obtained from averages over the appropriate WIGNER-WILKINS spectrum [26]. The use of these spectra rather than a Maxwell-Boltzmann distribution gave about 1% reduction in thermal utilization, primarily caused by the non- $1/v$ cross-section of U²³⁵. For mixtures of H₂O and D₂O moderator, appropriate Wigner-Wilkins spectra were found following a suggestion of AMSTER [26] that deuterium be treated as hydrogen with effective scattering cross-section equal to its slowing down power. Comparison with the heavy moderator spectra of COHEN [27] confirmed that this approximation is in reasonable agreement with better theoretical treatment.

3.4. Spectrum effects

Calculations with the BPG Code indicate that the flux spectrum of water moderated reactors in the slowing down region deviates markedly from the $1/E$ energy dependence that is characteristic of many thermal reactors. Figure 1 shows the calculated spectrum for a typical case (Case 20). At very high energies ($\sim 10^6$ eV) the flux is quite large because the scattering cross-section of hydrogen is low. Going down in energy, the hydrogen cross-section rises rapidly and the flux is reduced rapidly. In the resolved resonance region (1 to 100 eV) heavy resonance absorption reduces the flux appreciably. Because of these major deviations from the $1/E$ dependence usually assumed, it is clear that a type of multi-group representation is needed to analyse water moderated lattices in detail. It is also evident that the values of resonance integrals and of other one-group epithermal cross-sections are strongly dependent on the reactor spectrum in which they are measured or in which they are to be used. The subsequent analysis in this paper demonstrates this.

4. DETAILED ANALYSIS AND COMPARISON WITH EXPERIMENT

4.1. Introduction

Calculations have been performed for a total of 32 clean geometry lattices, using the BPG Code with constants generated as described previously. The lattices are described in Table II and the comparison of experimental and calculated parameters is given in Tables III and IV. Agreement between experimental and calculated values is fairly good over a wide range of lattice configurations, for both U²³⁸ and Th²³² as fertile material. The agreement is not quite as good as might be hoped for some of the lattices, partly due to difficulties in making the measurements, but probably more due to inaccuracies in the data (particularly in the resonance region) that are used in the calculations. There is a definite need for more good measurements of

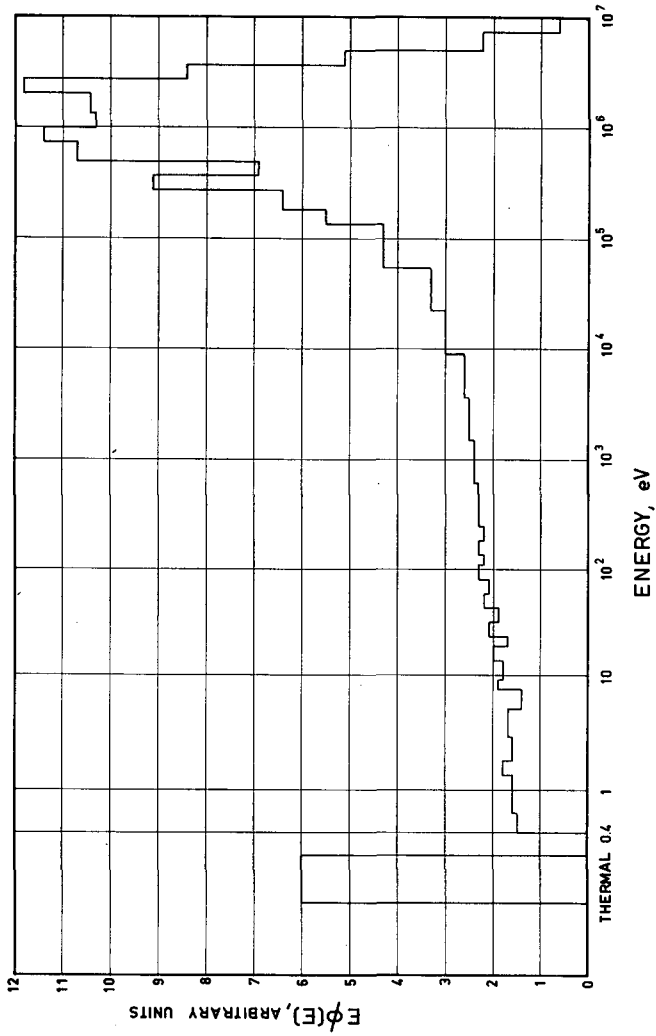


Fig. 1
Flux spectrum in water-moderated reactor
(Case 20)

resonance effects in lattices in order to reduce the disagreement. The comparison of the individual parameters is discussed in the following sections.

4.2. Criticality

The BPG Code is a one region equivalent bare reactor calculation, and determines a value of k_{eff} dependent on the input value of buckling. Thus the value of B^2 required to compute $k_{eff} = 1$ can be determined, or conversely using a measured value of critical buckling, the code determines a value of k_{eff} . Either of these can be compared to the experiment as a measure of the overall accuracy of the calculation, and both are listed in Table III. All calculated bucklings are in fairly good agreement with measured values except for Cases 1, 9, 10, 30, and 32. Cases 1 and 10 are very loose lattices and are sensitive to small errors in thermal utilization. Small errors in calculating thermal flux ratios or spectra, or the presence of unaccounted impurities in the fuel could lead to erroneous results. Cases 9, 30, and 32 are close packed lattices of ThO₂ fuel rods, which are very sensitive to the fairly large uncertainty in the thorium resonance integral, as discussed later. In addition, Cases 30 and 32 are exponential assemblies with large fractions of epithermal fissions, and there is still an unresolved question concerning the validity of the measured buckling. With five exceptions all values of k_{eff} are determined to better than 2%, most within 1%. This agreement over such a wide range of moderation and fuel indicates the general validity of the calculational method, and that it can be applied with accuracy fairly generally.

4.3. Leakage

KOUTS [28] has pointed out that $\delta k_{eff}/\delta B^2$ is an appropriate parameter of the reactor leakage, since it is well defined from a calculational standpoint and can be measured fairly well in water moderated lattices. The agreement between calculated and experimental values shown in Table III is rather poor. Most of the experimental data listed were taken merely as auxiliary aids to the critical experiments and were not intended originally to be basic measurements. In addition the experimental values listed are sensitive to computed values of delayed neutron fraction (β), and the results depend on the axial reflector savings being constant, which was definitely not the case in several experiments where the measurements were made near the end of the active lattice. More careful experimental work in determining this parameter should improve the situation. There are also several minor improvements to the model which could change the calculated values by less than 5%. The effects of anisotropic scattering on the slowing down parameters ξ , γ , ξ , and η in the BPG Code are probably small (the effect on μ was included in the analysis) and can only be determined accurately when more anisotropic scattering data at high energy are available. A transport correction for the absorption and inelastic scattering in the second BPG equation was found [12] to give better agreement with measured ages.

It should be noted that while determination of $\delta k_{eff}/\delta B^2$ gives information about reactor leakage, it is not equivalent to infinite medium age or mi-

gration area. An auxiliary set of calculations used the BPG Code to determine some of the more commonly used leakage representations. In the B_1 approximation, the rigorous mathematical expression for the second moment of the slowing down density [11, 12, 29] is given by

$$\tau(E) = - \lim_{B \rightarrow 0} \left[\frac{1}{q(E, B)} \frac{\delta q(E, B)}{\delta B^2} \right]$$

This expression was evaluated at $E = 0.4$ eV and is listed in Table VI for lattice cases 20, 21, 22, and 24. If the leakage is assumed to be separable from the absorptions in a reactor, the multiplication factor can be written

$$k_{\text{eff}} = k_{\infty} P_{\text{NL}}(B^2)$$

where $P_{\text{NL}}(B^2)$ is the non-leakage probability, which has two generally used forms. A Gaussian kernel has the form

$$P_{\text{NL}} = \exp(-M_G^2 B^2),$$

and differentiation gives

$$M_G^2 = - \frac{1}{P_{\text{NL}}} \frac{\delta P_{\text{NL}}}{\delta B^2}.$$

A Yukawa (or one-group) kernel has the form

$$P_{\text{NL}} = (1 + M_Y^2 B^2)^{-1},$$

which gives, on differentiation,

$$M_Y^2 = \frac{1}{(P_{\text{NL}})^2} \frac{\delta P_{\text{NL}}}{\delta B^2}$$

These two expressions were also evaluated using the BPG Code and measured critical bucklings. The results are compared in Table VI.

TABLE VI

BUCKLING-REACTIVITY CHANGES

CASE	Calculated values, cm ²				Measured values, cm ²
	$\tau(0.4\text{eV})$	M_Y^2	M_G^2	$\delta k_{\text{eff}}/\delta B^2$	$\delta k_{\text{eff}}/\delta B^2$
20	39.8	45.0	33.2	33.8	31.9
21	58.1	60.9	49.4	51.0	48.1
22	74.7	68.9	61.4	64.8	60.2
24	76.2	73.4	66.0	68.9	63.2

It is apparent from the table that $\delta k_{\text{eff}}/\delta B^2$ and age are different quantities. The differences between calculated values of M_G^2 and $\delta k_{\text{eff}}/\delta B^2$ indicate that the leakage is not separable, that is, that

$$\frac{1}{P_{\text{NL}}} \frac{\delta P_{\text{NL}}}{\delta B^2} \neq \frac{1}{k_{\text{eff}}} \frac{\delta k_{\text{eff}}}{\delta B^2}$$

The calculated M_G^2 do agree better with measured $\delta k_{\text{eff}}/\delta B^2$ than do the calculated $\delta k_{\text{eff}}/\delta B^2$, but this is probably because there was appreciable error in the measurements and in the delayed neutron fractions used to interpret them. More measurements made with greater care, and detailed theoretical investigation of the application of the inhour equation and effective delayed neutron fractions to these cores should improve the agreement.

4.4. Epithermal fissions in U²³⁵

The values of δ_{25} shown in Table IV indicate good agreement between experiment and calculations over a very wide range of lattices. Some cases were very thermal systems while others had more than half the U²³⁵ fissions above cadmium cut-off. The agreement over this wide range of δ_{25} values indicates that the fission cross-section of U²³⁵ is represented fairly well in the calculational model. The calculated values used a self shielding of 10% on the resonances and a cadmium cut-off at 0.625 eV, giving the resonance integrals

$$R_{25} \text{ (fission)} = 231 \text{ b}$$

$$R_{25} \text{ (absorption)} = 361 \text{ b}$$

These values were used in all cases. For more accurate calculations, the variation in self shielding from lattice to lattice should be taken into account,

and also the variation of effective energy of the cadmium cut-off. These two effects oppose each other in correcting the resonance integrals, so the use of constant values gives fairly accurate results, as shown in the table. The effects of better thermal flux ratio calculations, and of better thermal spectrum calculations should improve the accuracy of the calculated values of δ_{25} . Experimentally there was some difficulty in locating the U^{235} foils accurately in the fuel rods, and some corrections for the local effects of the cadmium covers could be determined. However, the experimental values are believed to be fairly accurate as they stand.

4.5. Resonance capture in U^{238}

In the series of lattice measurements, four experimental determinations of ρ_{28} were made. Agreement of calculated and experimental values shown in Table IV is fair. Experimental difficulties leading to uncertainties in the reported results include sample location in the fuel rod, effects of cadmium covers, recalibration of samples in other flux spectra, and uniformity of chemical separation. Further experience in these measurements should reduce the uncertainties in future measurements. Computational uncertainty comes from the discrepancy between lump resonance integral measurements and values computed from resonance parameters. In addition, the mutual shielding between fuel rods plays an important role in calculating the lattice values. As mentioned before, the formulation of Dancoff and Ginsburg has been used in the analysis. PETTUS [21, 30] has measured lattice effects and has found them consistent with the simpler approximation of BELL [31].

HARDY et al., [32] have also measured the effects and have found them to agree with the Dancoff and Ginsburg formulation. In close packed water lattices the mutual shielding problem in the resonance region has many similarities to the calculation of thermal flux ratios, and the most correct treatment may turn out to be extremely complex. The direct solution is to resort to Monte-Carlo calculations, but these at present are too lengthy to be used in general reactor design problems. More measurements of ρ_{28} in lattices will aid in the solution of the problem of relating single lump values to effective lattice resonance integrals.

4.6. Resonance capture in Th^{232}

Five measurements of ρ_{02} are available from this series of lattices. Agreement is fairly good between the measured and calculated values, but the measurements are subject to more uncertainty than are the U^{238} measurements. Thorium metal foils were used in Cases 31 and 32, and the ρ values are derived from cadmium ratios rather than from a direct measurement. Thus these cases have considerably more uncertainty than the rest. Cases 9, 13, and 14 were measured with ThO_2 wafers which were slightly different size and density than the fuel. The effects of the cadmium covers were probably larger in these lattices than in the U^{238} cases because the moderator space between fuel rods was smaller. Calculationally the same comments apply as for U^{238} resonance capture. In addition, there is somewhat more uncertainty in the resonance parameter values, and there is some inaccu-

racy in applying the Breit-Wigner single-level formula to the two lowest peaks in thorium at 21.9 and 23.6 eV. In both the ρ_{28} and ρ_{02} calculations the agreement with experiment is good enough over a wide range of lattices to indicate that the calculational representation is nearly correct. More experimental data are needed to reduce the uncertainties.

4.7. Thermal flux ratio

The ratio of thermal flux in the water moderator to that in the fuel was calculated using the approximate method of AMOUYAL *et al.* [33]. This method is based on accurate escape probabilities computed in the fuel, with diffusion theory boundary conditions at the cell boundary in the moderator. Thermal cross-sections were averaged over the Wigner-Wilkins spectrum for a homogenized lattice. Table IV lists the results. The calculated values of ϕ_w/ϕ_f are in fairly poor agreement with the measurements, but are sufficiently close that little error is made in computing the thermal utilization, and subsequently k_{eff} . Although this series of critical and exponential experiments is not sensitive to the discrepancies, they would be important for looser lattices, for lattices with moderator poisoning, or for fuel containing appreciable quantities of plutonium or fission products. A series of calculations using a cylindrical B₃ formulation gave appreciably poorer agreement with experiment. It appears that calculational improvements should include the effects of spectral hardening in the fuel and also the exact geometrical shape. Because all the dimensions in a close packed lattice cell are of the order of a mean free path, it is not clear that higher order Legendre expansions will improve the situation. The most direct method of improvement is through Monte-Carlo calculations. Since these are time consuming and expensive, it would be worthwhile to derive some simple formulae, such as the Amouyal method, with fitting factors based on the more complex treatment. These then would be useful for reactor design application.

4.8 Conclusions from the detailed analysis

The generally good agreement between experiment and calculation for a wide range of water moderated lattices indicates that these systems are fairly well described by the model. More data is needed to improve the accuracy of $\delta k_{eff}/\delta B^2$, ρ_{28} , and ρ_{02} determinations. Minor improvements in the BPG model and in the inhour equation should improve the agreement with $\delta k_{eff}/\delta B^2$ measurements, while more cross-section information in the resonance region is needed to improve agreement with ρ_{28} and ρ_{02} data. Considerably more theoretical and experimental work may be needed to obtain a Dancoff representation that is satisfactory for close packed lattices. Monte-Carlo calculations are the most direct way of obtaining adequate resonance mutual shielding information, and also of obtaining accurate thermal utilization values.

5. ANALYSIS BY ONE-GROUP MODEL AND COMPARISON WITH EXPERIMENT

5.1. The model

Because a large part of previous analysis of water moderated lattices has been in terms of a one-group model, and because there is a natural

tendency to simplify results for discussion, it was felt worthwhile to compare the BPG multi-group analysis with one using a simpler model. Several formulations of a one-group representation have had considerable success in correlating a wide variety of critical experiments on water lattices [2, 28, 34, 35]. The model employed here accounts for epithermal fissions in U^{235} and assumes that these occur at lower energies than do the resonance captures in U^{238} or Th^{232} . Multi-group calculations show that this assumption is good for thorium lattices and not badly in error for uranium lattices. The model is defined by the following equations:

$$k_{\text{eff}} = k_{\infty} P_{\text{NL}}$$

$$k_{\infty} = k_{\text{th}} + k_{\text{res}}$$

$$k_{\text{th}} = \epsilon \eta_{\text{th}}^f p_{25} p_{28} p_{\text{clad}}$$

or

$$k_{\text{th}} = \epsilon \eta_{\text{th}}^f p_{25} p_{02} p_{\text{clad}}$$

$$k_{\text{res}} = \epsilon \eta_{\text{res}} p_{28} (1 - p_{25})$$

or

$$k_{\text{res}} = \epsilon \eta_{\text{res}} p_{02} (1 - p_{25})$$

The symbols are defined as follows

P_{NL} = non-leakage probability

ϵ = fast effect

η_{th} = thermal $\nu(\sigma_f/\sigma_a)$ for U^{235}

η_{res} = average resonance $\nu(\sigma_f/\sigma_a)$ for U^{235}

f = thermal utilization for U^{235}

$p_i = \exp \left[- \frac{N_i R_i}{\xi \bar{\Sigma}_s} \right], \quad i = 25, 28, 02, \text{ clad}$

$\xi \bar{\Sigma}_s$ = slowing down power of the lattice

N_i = atomic concentration in the lattice

R_i = effective resonance integral

Using the equations above, the lattice parameters can be computed:

$$\delta_{25} = k_{\text{res}}/k_{\text{th}}$$

$$\rho_{28} = \left[(1-p_{28}) \Sigma_{25} \right] / \left[p_{28} p_{25} p_{\text{clad}} f \Sigma_{28} \right]$$

$$\rho_{02} = \left[(1-p_{02}) \Sigma_{25} \right] / \left[p_{02} p_{25} p_{\text{clad}} f \Sigma_{02} \right]$$

The thermal utilization is computed with the assumption that the thermal flux in the clad is half way between that in the fuel and that in the water. Thus

$$f = \left[1 + \frac{\Sigma_{28} + \Sigma_{02}}{\Sigma_{25}} + \frac{\Sigma_W \phi_W}{\Sigma_{25} \phi_F} + \frac{1}{2} \frac{\Sigma_{\text{clad}}}{\Sigma_{25}} \left(1 + \frac{\phi_W}{\phi_F} \right) \right]^{-1}$$

In the following analysis the calculated non-leakage probability is taken from the multi-group results.

5.2. Choice of constants

The thermal cross-sections were averaged over a Wigner-Wilkins distribution, and were the same as those used in the BPG multi-group analysis. Values of ϕ_W/ϕ_F were those obtained by the Amouyal approximation as in the previous analysis. The resonance integral for stainless steel clad was 2.85 b; that for aluminium clad was 0.18 b. The U²³⁵ absorption integral to 0.4eV was 405 b, and the fission integral was 270 b. These both included a small correction for self shielding. The U²³⁸ and Th²³² resonance integrals were those listed in Table V, with 1.0 b added to correct for the departure from a 1/E flux at high energies. For U²³⁸, the 1/v part of the integral at low energies, 1.04 b, was also added, and for Th²³² the added integral for this range was 3.24 b. The value of η_{th} for U²³⁵ was 2.07, and an average value over the epithermal region was taken to be $\eta_{\text{res}} = 1.62$. The fast effect was calculated from a fit to data of KLEIN[36], which was used in the form

$$\epsilon = 1.004 + 0.137 \left[(N_H + N_D)/(N_{25} + N_{28}) \right]^{-0.764},$$

for U²³⁸ fuel. For thorium fuel experimental data was not available, but assuming that the fast fission rate in a thorium system is approximately that of a similar uranium system multiplied by the ratio of the fission cross-sections, a similar expression was derived,

$$\epsilon = 1.0007 + .0249 \left[(N_H + N_D)/(N_{25} + N_{02}) \right]^{-0.764}$$

for Th^{232} fuel. N_i in these expressions is the average atomic concentration in the lattice.

5.3. Calculated values

The representative cases, for which most of the parameters had been measured, were chosen for comparison. Table VII lists the cases and compares the calculated criticality and buckling with the BPG multi-group results and with experimental values. It can be seen from the table that the one-group model correlates the light water moderated cores very well, but that the results become more in error as D_2O is added to the moderator and the cores become more epithermal. On the other hand, calculated values of δ_{25} shown in Table VIII are in good agreement for all these cores, partly because of the choice of 0.4 eV for the cadmium cut-off energy for the U^{235} resonance integrals. Calculated values of ρ_{28} and ρ_{02} are in good agreement for light water moderated lattices, becoming more in error for the highly epithermal systems. A somewhat arbitrary choice of fertile material resonance integrals could improve the agreement of the one-group model calculations with the experimental values, but these resonance integrals would not be consistent with measured single lump values and the calculated Dancoff correction. The results of this analysis thus indicate that the one-group model correlates most H_2O moderated lattices well, but the correlation may be appreciably in error for lattices more tightly packed than $M/W = 1$ or for lattices with appreciable amounts of D_2O mixed with H_2O .

TABLE VII

CRITICALITY COMPARISON

CASE	Fuel rods	M/W	% D_2O in moderator	Calculated k_{eff} with measured buckling		Critical buckling, M^{-2}		
				1-group	BPG	1-group	BPG	experiment
8	CETR-4	0.890	0	0.991	0.994	26.9	27.7	29.3
9	CETR-4	1.116	0	0.978	0.981	20.5	21.5	26.2
13	CETR-7	0.890	0	0.995	0.995	70.5	70.6	72
14	CETR-7	1.116	0	0.994	0.996	59.3	60.0	61
19	TUPE-7	1.024	0	0.997	0.992	84.3	83.1	85.31
20	SSCR-4	1.0064	0	0.983	0.991	82.6	85.2	88.0
21	SSCR-4	1.0064	49.66	0.934	0.985	29.9	41.0	44.0
22	SSCR-4	1.0064	69.70	0.905	0.999	2.9	18.4	18.60
25	SSCR-4	1.0064	76.50	0.872	1.013	-7.7	12.6	10.77
29	TUPE-7	0.990	60.40	0.959	0.988	31.9	37.1	39.0

TABLE VIII
COMPARISON OF EPITHERMAL LATTICE PARAMETERS

Case	δ_{25}			ρ_{28} and ρ_{02}		
	1-group	BPG	Experiment	1-group	BPG	Experiment
8	0.178	0.163	0.157	1.02	1.01	-
9	0.225	0.206	0.210	1.28	1.26	1.28
13	0.265	0.244	0.221	1.59	1.58	1.49
14	0.340	0.310	0.292	2.05	1.99	2.08
19	0.272	0.263	0.266	1.62	1.67	-
20	0.242	0.238	0.266	4.25	3.88	4.18
21	0.456	0.439	0.495	8.60	7.11	8.3
22	0.713	0.668	0.73	14.88	11.32	10.4
25	0.881	0.823	0.88	19.58	13.36	16.4
29	0.622	0.578	0.65	4.15	3.82	-

5.4. Values derived from experiments

Using the one-group model it is possible to derive a number of other parameters from the experiments and to obtain derived values of effective resonance integrals that would force the model to fit all the lattices. If the non-leakage probability is assumed to be of the Gaussian form,

$$k_{\text{eff}} = k_{\infty}^G \exp(-M_G^2 B^2),$$

differentiation gives

$$-(1/k_{\text{eff}})(\delta k_{\text{eff}}/\delta B^2) = M_G^2.$$

If the non-leakage probability is assumed to have the one-group form,

$$k_{\text{eff}} = k_{\infty}^Y (1 + M_Y^2 B^2)^{-1},$$

differentiation gives

$$-\left[1/k_{\text{eff}}\right]\left[\delta k_{\text{eff}}/\delta B^2\right] = M_Y^2/k^Y$$

Using these forms, measured values of $\delta k_{\text{eff}}/\delta B^2$ at $k_{\text{eff}} = 1$ can be used to determine effective values of M^2 and k_{∞} . These are compared with results calculated from the BPG multi-group analysis in Table IX. In the BPG analysis

$$M_G^2 = - \left[1/P_{NL} \right] \left[\delta P_{NL}/\delta B^2 \right],$$

$$M_Y^2 = - \left[1/(P_{NL})^2 \right] \left[\delta P_{NL}/\delta B^2 \right],$$

and k_{∞} was calculated by the code with zero leakage. The results of the M^2 comparisons are indicative only of the agreement between BPG calculations and experiment for $\delta k_{\text{eff}}/\delta B^2$ (see Table III).

The derived M_G^2 are the measured $\delta k_{\text{eff}}/\delta B^2$, and the fairly good agreement with the M_G^2 calculated by BPG may be fortuitous because of the uncertainties in the measured values (see section 4.3). The BPG computation of k_{∞} agrees appreciably better with the interpreted values than does the one-group computation for the more epithermal cases. This is primarily because the one-group model does not account properly for the interaction of U^{235} and fertile material resonance absorption. The BPG multi-group model is needed to compute these lattices with accuracy. Comparing the derived k_{∞}^G and k_{∞}^Y with the BPG calculations, there appears to be equal agreement within the uncertainties of the measured $\delta k_{\text{eff}}/\delta B^2$, and thus these

TABLE IX

ONE-GROUP PARAMETERS DERIVED FROM EXPERIMENT

Case	$M_G^2, \text{ cm}^2$		$M_Y^2, \text{ cm}^2$		$k_{\infty}, \text{ Derived}$		$k_{\infty}, \text{ Computed}$	
	Derived	BPG	Derived	BPG	Gaussian	1-group	BPG	1-group
8	39.8	37.4	45.1	41.6	1.124	1.132	1.115	1.110
9	-	40.2	-	44.0	-	-	1.095	1.089
13	-	33.2	-	42.7	-	-	1.295	1.289
14	34.5	35.9	43.7	45.3	1.234	1.267	1.267	1.258
19	43.7	37.6	69.7	53.0	1.452	1.594	1.424	1.418
20	31.9	33.2	44.3	45.0	1.324	1.390	1.364	1.348
21	48.1	49.4	61.0	60.9	1.236	1.268	1.243	1.174
22	60.2	61.4	67.8	68.9	1.118	1.126	1.130	1.020
25	65.4	68.9	70.4	75.2	1.073	1.076	1.096	1.943
29	59.3	61.8	77.1	78.7	1.260	1.301	1.288	1.238

experiments do not distinguish between the Gaussian and one-group forms of the non-leakage probability.

Effective values of the resonance integrals can be derived through the one-group model from measured values of δ_{25} , ρ_{28} , and ρ_{02} . Since

$$\delta_{25} = \frac{k_{\text{res}}}{k_{\text{th}}} = \frac{\eta_{\text{res}} (1 - p_{25})}{\eta_{\text{th}} f p_{25} p_{\text{cl}}},$$

an expression can be found for

$$p_{25} = 1 + \delta_{25} \left[\frac{\eta_{\text{th}} f p_{\text{clad}}}{\eta_{\text{res}}} \right]^{-1}.$$

Using this expression and

$$-\ln p_{25} = \frac{N_{25} R_{25}^A}{\xi \Sigma_s},$$

the derived value of the absorption resonance integral for U²³⁵ is:

$$R_{25}^A = \frac{\xi \Sigma_s}{N_{25}} \ln \left[1 + \delta_{25} \frac{\eta_{\text{th}} f p_{\text{clad}}}{\eta_{\text{res}}} \right].$$

The derived fission resonance integral can also be found, assuming

$$R_{25}^F = \frac{\eta_{\text{res}}}{\nu} R_{25}^A.$$

These are listed in Table X, using calculated numbers for f and p_{clad} , and using experimental values of δ_{25} .

The derived integrals are in fair agreement with the constant values $R_{25}^A = 405$ b and $R_{25}^F = 270$ b used in the one-group criticality calculations. The disagreement is worse for lattices with smaller values of δ_{25} , which is in the opposite direction from that expected from self shielding corrections.

TABLE X

RESONANCE INTEGRALS DERIVED FROM EXPERIMENT

	R_{25}^A, b	R_{25}^F, b	R_{28}, b		R_{02}, b	
Case	Derived *	Derived *	Derived	Calculated	Derived	Calculated
8	363	242	-	-	18.93	18.51
9	379	253	-	-	18.03	17.85
13	348	232	-	-	18.34	18.68
14	354	236	-	-	18.84	18.02
19	399	266	-	-	18.21	18.06
20	444	296	18.23	18.67	-	-
21	438	292	16.45	17.30	-	-
22	416	278	12.35	16.50	-	-
25	407	271	14.20	16.18	-	-
29	422	281	-	-	16.37	16.63

* Calculated U^{255} resonance integrals are $R_{25}^A = 405 b$, $R_{25}^F = 270 b$.

In a similar fashion derived values of R_{28} and R_{02} can be found:

$$\rho_{28} = \frac{(1-p_{28}) \Sigma_{25}}{p_{28} p_{25} p_{\text{clad}} f \Sigma_{28}}$$

$$\rho_{02} = \frac{(1-p_{02}) \Sigma_{25}}{p_{02} p_{25} p_{\text{clad}} f \Sigma_{02}}$$

$$R_{28} = \frac{\xi \Sigma_s}{N_{28}} \ln \left[1 + \rho_{28} f p_{25} p_{\text{clad}} \frac{\Sigma_{28}}{\Sigma_{25}} \right]$$

$$R_{02} = \frac{\xi \Sigma_s}{N_{02}} \ln \left[1 + \rho_{02} f p_{25} p_{\text{clad}} \frac{\Sigma_{02}}{\Sigma_{25}} \right]$$

These derived values of R_{28} and R_{02} were computed using calculated p_{clad} and f , and using p_{25} derived from the δ_{25} measurements, as indicated above. The derived resonance integrals R_{28} and R_{02} are compared in Table X with

those used in the one-group analysis. Comparison with resonance integrals used in the BPG analysis can be made by referring to Table V, remembering that these do not include either the $1/v$ part or the high energy addition needed for the one-group analysis. The agreement between calculated and derived resonance integrals is fairly good except for the cases with very large amounts of resonance absorption.

5.5. Conclusions from the one-group analysis

The one-group analysis of resonance integrals indicates that, in the more epithermal cases, the measured fertile resonance absorption is less than is expected by the model and that the measured U²³⁵ resonance fission is more than is consistent within the model with the less epithermal cases. This points out the interaction and competition in resonance absorption between the fertile and U²³⁵ absorptions, and indicates that a multi-group analysis is required to obtain a good picture of the resonance absorption in the more epithermal systems. Although the one-group model breaks down in these cases, it does correlate the H₂O moderated lattices which have fewer epithermal fissions with good accuracy.

Within the uncertainties of the measured $\delta k_{\text{eff}}/\delta B^2$ either the Gaussian form or the one-group form of separable non-leakage probability used with the one-group criticality model appears to correlate the lattices well. However, BPG multi-group calculations show a definite dependence of k_{∞} on the buckling. More experimental water height data, and minor improvements to the inhour equation are needed to demonstrate experimentally the inaccuracy of a separable non-leakage probability.

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EFFECTIVE U^{238} RESONANCE CAPTURE INTEGRALS IN RODS AND LATTICES

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Abstract — Résumé — Аннотация — Resumen

EFFECTIVE U^{238} RESONANCE CAPTURE INTEGRALS IN RODS AND LATTICES. Recent theoretical calculations [1, 2, 3] of U^{238} resonance absorption in isolated rods and lattices have suggested certain equivalence relations. These equivalence relations are dependent upon the use of the Wigner rational approximation to the escape probability. In this framework lattices are characterized by an effective surface-to-mass ratio which then allows correlation of isolated rod and lattice resonance integrals.

The first part of this paper deals with the measurement of U^{238} resonance neutron capture in isolated uranium metal and UO_2 rods. The second section deals with Monte-Carlo calculations of resonance integrals. This study was made to investigate the range of validity of the equivalence relations under idealized conditions. The third part of this paper infers resonance integrals from measurements of the resonance escape probability, p_{28} , for a number of different lattices. These results are then correlated with the isolated rod measurements by using two of the suggested equivalence relations.

The equivalence relations using either the Dancoff factor or the Bell approximation to it, were tested over a wide range of surface-to-mass ratios by Monte-Carlo calculations. In tight lattices, a breakdown appeared in both formulations, but the Dancoff treatment showed somewhat better agreement than the Bell approximation. Both formulations were also tested experimentally, over a smaller range of surface-to-mass ratios; the Bell approximation is favoured in these comparisons. Within the precision of all the information cited, no clear out preference is indicated. Both formulations work well.

INTÉGRALES EFFECTIVES DE CAPTURE PAR RÉSONANCE DANS ^{238}U , POUR DES BARRES ET POUR DES RÉSEAUX. Des calculs théoriques récents sur les caractéristiques d'absorption par résonance de ^{238}U dans des barres isolées et dans des réseaux donnent à supposer qu'il existe certaines relations d'équivalence. Ces relations sont modifiées si l'on applique l'approximation rationnelle de Wigner à la probabilité de fuite. Dans ce contexte, les réseaux sont caractérisés par un rapport surface/masse qui permet de mettre en corrélation les intégrales de résonance relatives aux barres isolées et aux réseaux.

La première partie du mémoire est consacrée à la mesure de la capture des neutrons par résonance dans ^{238}U en barres isolées d'uranium métal et de UO_2 . La deuxième partie a trait aux calculs des intégrales de résonance par une méthode de Monte-Carlo. Cette étude a été faite pour déterminer dans quel intervalle les relations d'équivalence sont valables, dans des conditions idéales. Dans la troisième partie, les intégrales de résonance sont déterminées par déduction pour plusieurs réseaux différents, à partir de mesures de la probabilité de fuite p_{28} . On établit ensuite une corrélation entre les résultats obtenus et ceux des mesures portant sur des barres isolées, en employant à cet effet deux des relations d'équivalence proposées.

Les relations d'équivalence dans lesquelles sont utilisés soit le facteur Dancoff, soit l'approximation de Bell relative audit facteur, ont été vérifiées pour une gamme très étendue de rapports surface/masse au moyen de calculs par une méthode de Monte-Carlo. Pour les réseaux à interstices étroits, les deux formules ont accusé certains défauts, mais l'emploi du facteur Dancoff a donné des résultats un peu plus proches de la réalité que le recours à l'approximation de Bell. Les deux formules ont également fait l'objet de vérifications expérimentales pour un nombre plus restreint de rapports surface/masse; au cours de ces études comparatives, l'approximation de Bell s'est révélée plus pratique. Compte tenu de la précision des données, les auteurs ne marquent aucune préférence nette pour l'une ou l'autre de ces formules, qui ont toutes deux donné des résultats satisfaisants.

ЭФФЕКТИВНОСТЬ РЕЗОНАНСНЫХ ИНТЕГРАЛОВ ЗАХВАТА U^{238} В СТЕРЖНЯХ И РЕШЕТКАХ. Недавние теоретические расчеты резонансного поглощения U^{238} в изолированных стержнях и решетках приводят к определенным соотношениям эквивалентности. Эти соотношения экви-

валентности зависят от применения рационального приближения Вигнера для вероятности избежания захвата. В рамках этого рассмотрения решетки характеризуются эффективным отношением поверхности к массе, которая затем позволяет сопоставить резонансные интегралы изолированного стержня и решетки.

В первой части этого доклада приводятся данные измерения резонансного захвата нейтронов U^{238} в изолированном металлическом уране и стержнях из UO_2 . Во второй части приводятся расчеты резонансных интегралов методом Монте-Карло. Это исследование проведено в целях изучения предела применимости соотношений эквивалентности при идеальных условиях. В третьей части этого доклада выводятся резонансные интегралы, полученные в результате измерения вероятности избежания резонансного захвата ρ^{28} для ряда различных решеток. Эти результаты затем сравниваются с данными измерений изолированного стержня по двум из предлагаемых соотношений эквивалентности.

Путем использования либо коэффициента Данкоффа, либо приближения Белла соотношения эквивалентности были опробованы в широком диапазоне величин отношения поверхности к массе путем расчетов методом Монте-Карло. В случае плотных решеток наблюдался разрыв в обоих случаях, но метод Данкоффа дал несколько лучшее совпадение данных, чем приближение Белла. Оба положения были также опробованы экспериментально в небольшом диапазоне величин отношения поверхности к массе; приближение Белла дало лучшие результаты в этих сравнениях. С точки зрения точности приведенных данных трудно отдать предпочтение одному из положений. Оба положения могут быть применены.

INTEGRALES EFECTIVAS DE CAPTURA POR RESONANCIA DEL ^{238}U EN BARRAS Y RETICULADOS. A partir de cálculos teóricos recientes de la absorción por resonancia del ^{238}U en barras aisladas y en todo el reticulado, se han deducido ciertas relaciones de equivalencia. Estas relaciones se fundan en el uso de la aproximación racional de Wigner a la probabilidad de escape. En este sistema de cálculo, los reticulados se caracterizan por una razón efectiva superficie: masa que permite luego establecer una correlación entre las integrales de resonancia de cada barra y las correspondientes a los reticulados.

La primera parte de esta memoria se refiere a las mediciones de la captura neutrónica por resonancia en el ^{238}U , en barras aisladas de uranio metálico y de UO_2 . La segunda sección trata de los cálculos de las integrales de resonancia por el método de Montecarlo. El propósito del estudio consistió en determinar el intervalo de validez de las relaciones de equivalencia en condiciones ideales. En la tercera parte, se deducen las integrales de resonancia a partir de las mediciones de la probabilidad de escape por resonancia, ρ^{28} , para una serie de reticulados diferentes. Luego, utilizando dos de las relaciones de equivalencia propuestas, se establece una correlación entre estos resultados y los valores medidos en las barras aisladas.

Mediante el método de Montecarlo, se verificó la exactitud de las relaciones de equivalencia, dentro de un amplio intervalo de valores de la razón superficie: masa, aplicando sea el factor de Dancoff o bien la aproximación de Bell al mismo. En los reticulados compactos, ambos tipos de fórmulas fallan, pero el método de Dancoff da resultados algo más concordantes que la aproximación de Bell. Ambas fórmulas se probaron también experimentalmente, dentro de un intervalo algo más restringido de dicha razón superficie: masa; en este caso, la comparación favorece a la aproximación de Bell. Dentro de los límites de error de todos los datos citados, no se manifiesta una superioridad neta. Ambas series de fórmulas proporcionan resultados satisfactorios.

A. MEASUREMENTS IN ISOLATED RODS

The effective U^{238} resonance neutron capture integrals of uranium metal and UO_2 rods of a variety of diameters have been measured by an activation technique. The experiment employed 10-cm long sections of fuel rod ir-

TEST FUEL LOADING

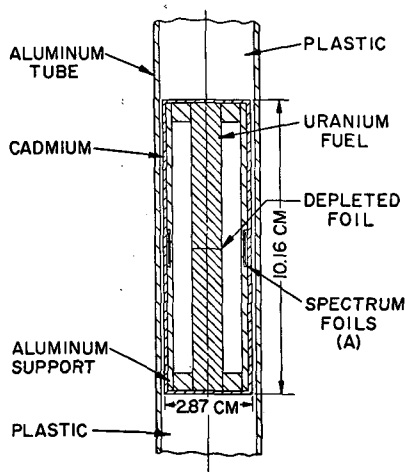


Fig. 1

Test fuel loading

radiated in a cadmium can as shown in Fig. 1. The can was located in a water hole at the centre of the TRX, an H₂O-moderated, slightly enriched, heterogeneous critical facility [4]. In this water hole, the overall spectrum of the epithermal neutron flux is very nearly proportional to $1/E$ up to an energy of approximately 25 keV, above which there appears a flux peak characteristic of H₂O-moderated reactors.

Each measurement of the effective resonance integral for a particular rod consisted of two irradiations. After the first irradiation, called the "fuel rod run", a uniform sample of uranium atoms was extracted from the irradiated fuel rod and made into thin deposits. The U²³⁸ capture activity of these deposits was then measured. In the second irradiation, called the "normalizing run", the deposits were placed in the otherwise empty cadmium can, irradiated and their activities measured exactly as before. With some corrections (for background activity, flux spectrum, etc.) the ratio of activity from the fuel rod run to that from the normalizing run is just the ratio of the effective resonance integral per U²³⁸ atom of the fuel rod to the dilute resonance integral of U²³⁸.

Table I lists the characteristics of the fuel rods for which measurements were done. The cadmium can, containing the 10-cm long section of fuel rod, was mounted between plastic rods in an aluminium tube and positioned at the centre of a 12-cm diam water hole. The cadmium can was 10-cm high, with a wall thickness of 0.076 cm. Its diameter was 2.87 cm for all except the largest metal rod which required a can of 4.10-cm diam. To ensure that no error was caused by this change, measurements on the 2.16-cm diam metal rod were done in both cadmium cans. The results were indistinguishable within the experimental uncertainty.

For each of the five metal rods, a circular uranium metal foil, depleted to 190 ppm U²³⁵ and of the same diameter as the fuel, was carefully posi-

TABLE I

FUEL DESCRIPTION

Fuel	Enrichment	Diameter (cm)	Density (g/cm ³)	$\sqrt{S/M}$ (cm/g ^{1/2})
U Metal	Natural	3.175	18.9	0.259
	Natural	2.159	18.9	0.313
	Natural	1.524	18.9	0.373
	1.3% U ²³⁵	0.983	18.9	0.464
	1.3% U ²³⁵	0.686	18.9	0.558
UO ₂	Natural	2.210	10.5	0.414
	1.3% U ²³⁵	1.524	7.5	0.590
	1.3% U ²³⁵	0.975	10.5	0.623
	1.3% U ²³⁵	0.986	7.5	0.734

tioned at the centre of the fuel section in the cadmium can. After an irradiation of 1 h, the foil was removed and dissolved in dilute HCl. A small amount of H₂O₂ was then added and individual drops of the solution placed on blotter paper. The blotter paper was dried and circular foils punched from the spots made by the drops of solution. The uranium concentration in the deposits appeared to be quite uniform. The effective uranium thickness of the deposits was about 0.3 mg/cm² in every case.

The 74 keV γ -activity resulting from the decay of U²³⁹ (half life 23.5 min) was counted on a NaI scintillation spectrometer from 25 to 70 min after the irradiation.

In addition, there was a depleted uranium metal foil acting as a reactor power monitor. It was placed in a standard reference position where the spectrum was unaffected by the contents of the cadmium can. The 74 keV γ -activity of this foil (undissolved) was counted on the scintillation spectrometer from 70 to 90 min after the irradiation.

The normalizing run was made approximately two days after the fuel rod run. It was an exact duplicate of the first except that the cadmium can now contained only the deposits mounted on a thin aluminium support plate. After irradiation, the activities of the deposits and the monitor foil were counted exactly as before. A small background on the foils was measured immediately before the irradiation and subtracted from the observed counts.

The UO₂ fuel rods were made up of cylindrical pellets with a height-to-diameter ratio of approximately unity. One of these pellets was used as the primary detector of U²³⁸ capture. Because the UO₂ pellets would not dissolve rapidly in acid, it was necessary to develop a somewhat different method of preparing the thin deposits. After irradiation, the central pellet was crushed to a fine powder which was then fused with molten potassium pyrosulphate. After the reaction was complete, approximately one

ml of the molten salt was introduced into ten ml of warm dilute HNO₃ where it dissolved rapidly. Deposits were then prepared as before from this solution. With some practice, the entire procedure from initial crushing to completion of the deposits took less than 10 min. Except for this modification, the UO₂ resonance integral measurements proceeded just as did those for metal rods.

The first significant correction to the measured activities was removal of the fission product γ -counts appearing in the window at 74 keV. This was done by the following method which has been previously reported [5, 6]. One first measures the pulse height spectrum of the fission product γ -activity of a U²³⁵ foil from 50 to 300 keV. One then measures similarly the γ -pulse height spectrum from the U²³⁸ deposits. The two spectrum shapes are normalized in the energy range from 174 to 264 keV where, a few minutes after irradiation, the activity is due entirely to fission products. Assuming that the fission product γ -pulse height spectrum of U²³⁵ is close to that of U²³⁸ from 50 to 300 keV, one can obtain the amount of fission product background appearing in the 74 keV window along with the capture activity.

As a check on this method, in several cases a more direct but more involved technique was also used which consisted of a chemical separation of the uranium from fission products. After the regular deposits were made, the remaining solution was combined with a mixture of ammonium nitrate and hexone (methyl isobutyl ketone) and shaken vigorously for 5 min. It was then allowed to stand for 1 min so that the hexone layer separated out at the top. Deposits were made from drops of the hexone, which contained a large fraction of the uranium and was strongly depleted in fission products. These deposits were subsequently handled exactly as described in the case of the regular deposits except that no fission correction was applied. The resulting resonance integral values obtained by the two methods were in good agreement within the precision with which each could be repeated. It was concluded that uncertainty of the fission correction could not introduce more than a 0.2 b error into the results.

The percent fission correction varies with rod size because epicadmium fission is nearly independent of rod size whereas the U²³⁸ resonance integral is not. Also, the correction was greater for the UO₂ rods than for the metal rods because depleted foils were used in the metal rods whereas the UO₂ pellets were slightly enriched. The fission correction ranged from 2-4% for the metal rods, and from 5-6% for the UO₂ rods. The percent fission correction was nearly constant during the counting time from 30 to 70 min after shut-down. (For a typical metal rod, the correction factor might range from 0.96 to 0.98 during the counting period).

No fission correction was applied to the counts from the normalizing run since the correction was much less than 1% in this case.

The power normalized ratio of U²³⁸ activity from the first run to that from the second is, except for spectrum corrections, just the ratio of the rod effective resonance integral to the infinitely dilute resonance integral. Thus,

$$R_U = \frac{U_A}{U_A^*} \times \frac{U_B}{U_B^*} \times 280 \quad (1)$$

where $U_A^* = U^{238}$ capture activity of the uranium deposit after the fuel rod run.

$U_A = U^{238}$ capture activity of the uranium deposit after the normalizing run.

U_B^* = Activity of the depleted uranium power monitor foil after fuel rod run.

U_B = Activity of the depleted uranium power monitor foil after the normalizing run.

The value 280 b is used for the dilute resonance capture integral of U^{238} [7]. It is in close agreement with the value 282 b measured by a separate experiment at this laboratory.

The choice of a depleted uranium foil as the monitor and the fact that its 74 keV activity was counted on the same counter as the deposits tends to cancel any slight drift in counting electronics or variation in the timing of the irradiations.

R_U as defined above is very nearly the effective resonance integral which is sought. It contains, however, a normalization to the same reactor power level as measured in a remote reference position, whereas it should be normalized to the same resonance neutron flux incident on the cadmium can, as measured by suitable monitors. The detectors chosen to monitor the resonance flux were indium and molybdenum foils placed against the inner surface of the cadmium can as shown in Fig. 1 (labelled Spectrum Foils, Position A). These foils were 1 cm diam and 0.013 cm thick. The indium was alloyed with tin to give 0.1% indium by weight; the molybdenum foil was 99.98% pure molybdenum.

Similar foils were placed with the uranium power monitor in the standard reference position. (Labelled "position B"). After irradiation the 410 keV, 54 min γ -activity from the decay of In^{116} was counted on a NaI scintillation counter. The 140 keV, 66 h activity of Mo^{99} was counted similarly. Since position B was far from the location of the cadmium can, the neutron energy spectrum at position B was constant from one run to another (ie. independent of the contents of the cadmium can). The following ratio was measured for Mo^{98} :

$$R_{Mo} = \left(\frac{Mo_A^*}{Mo_B^*} \right) \div \left(\frac{Mo_A}{Mo_B} \right)$$

where, as for uranium, the starred quantities refer to the fuel rod run and the unstarred quantities refer to the normalizing run. A similar quantity, R_{In} , was obtained for the indium monitors.

It was assumed that the indium activation occurred at 1.41 eV and the molybdenum activation occurred at 480 eV. $R_{Mo} < 1$ or $R_{In} < 1$ indicates a depression of the corresponding flux by the fuel rod placed in the can. Any significant departure of R_{In} , R_{Mo} , or R_{In}/R_{Mo} from unity for a given type of fuel rod would cause serious uncertainty in our knowledge of the flux in-

TABLE II
RESULTS

Fuel	$\sqrt{S/M}^{(a)}$ (cm/g ^{1/2})	R_U (barns)	R_{In}	R_{Mo}	R_{Mon}	$I^{(b)}$ (barns)
U Metal	0.259	11.28 ± 0.17	0.962	0.990	0.976	10.96
	0.313	13.19 ± 0.11	0.974	0.990	0.982	12.83
	0.373	14.70 ± 0.03	0.990	0.994	0.992	14.22
	0.464	17.43 ± 0.16	1.00	1.00	1.00	16.83
	0.558	19.99 ± 0.09	1.00	1.00	1.00	19.39
UO ₂	0.414	16.95 ± 0.15	1.00	1.00	1.00	16.35
	0.590	21.47 ± 0.50	1.00	1.00	1.00	20.87
	0.623	22.10 ± 0.07	1.00	1.00	1.00	21.50
	0.734	24.91 ± 0.05	1.00	1.00	1.00	24.31

$$R_U = \frac{U_A^*}{U_A} \times \frac{U_B}{U_B^*} \times 280 \text{ b}$$

$$R_{In} = \left(\frac{In_A^*}{In_B^*} \right) \div \left(\frac{In_A}{In_B} \right) = \frac{In_A^*}{In_A} \times \frac{In_B}{In_B^*} \text{ etc.}$$

$$R_{Mon} = \frac{1}{2} (R_{In} + R_{Mo})$$

$$I = \frac{R_U}{R_{Mon}} - 0.6 \text{ b}$$

(a) M is mass of uranium metal or of UO₂.

(b) Uncertainty of 0.6 b applies to all the results.

cident on the fuel rod. Only for the largest metal rod was there a distinct effect of this kind and even there it was not serious.

Finally, there was some concern for the possibility of a flux gradient from the edge to the centre of the cadmium can. To check this, activation plots across the empty can were taken with small foils of Mo, Au and U₃O₈. In each case, the activation distribution was flat across the can within about 1%.

The experimental results are shown in Table II. The column labelled R_U lists the average measured uranium activation ratios multiplied by 280 b. These values are already corrected for fission product activity. The columns headed R_{In} and R_{Mo} list the corresponding indium and molybdenum monitor foil activation ratios. Except for the larger diameter metal rods, these ratios. Except for the larger diameter metal rods, these ratios were in-

distinguishable from unity. The presence of a slight flux dip and spectrum shift for the larger pieces of metal fuel is not surprising.

The average of R_{In} and R_{Mo} (called R_{Mon}) was used to correct the measured R_U values. Finally, this was reduced by 0.6 b to account for a departure of the spectrum from $1/E$ (discussed below). Thus the effective resonance integral is

$$I = (R_U/R_{Mon}) - 0.6 \text{ b} \quad (2)$$

The error estimates listed for R_U are standard deviations representing the reproducibility of the measurements. An uncertainty of 1% is assigned to the values of R_{Mon} , which is conservative for the smaller rods. In the case of the largest metal rod ($\sqrt{S/M} = 0.259$), the magnitude of the spectrum shift makes an uncertainty of 2% more realistic. In addition, the fission correction to R_U is uncertain to about 0.2 b. Any error here would systematically affect all rod sizes. The fission correction and the self-shielding correction to the uranium activity observed in the normalizing run were neglected since both were considerably less than 1% and opposed each other.

The remaining source of uncertainty is the flux spectrum in the cadmium can. Calculations of the resonance flux spectrum were done using a 54-group, one-dimensional P_1 programme for the IBM 704, P1MG [8]. The first calculation was for the case of central water hole filled with water (ie. without the cadmium can). The resulting spectrum is labeled (a) in Fig. 2.

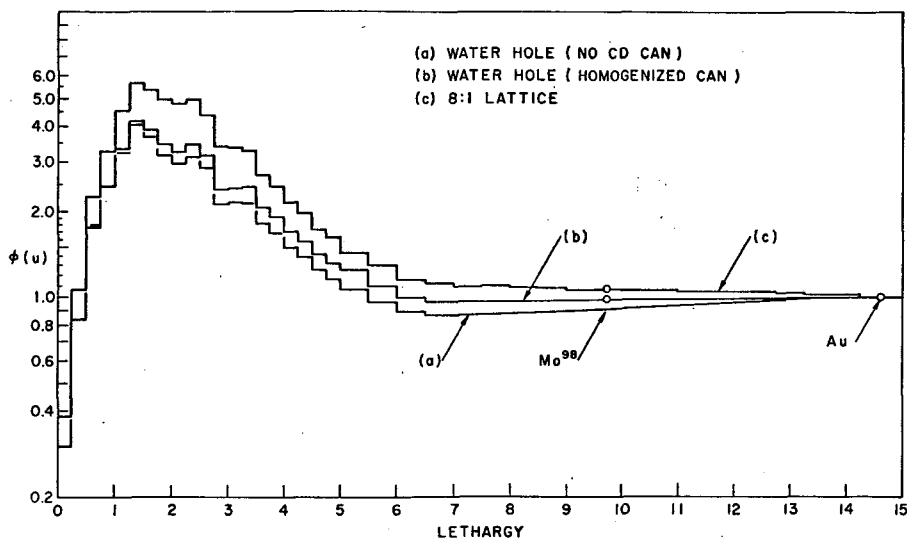


Fig. 2

Calculated flux spectra

In addition to the flux peak at small values of the lethargy* (due mostly to the decrease of the hydrogen cross-section above about 25 keV), $\phi(u)$ rises by about 15% from lethargy 7 to 15. This rise is due to the cumulative effects of the flow of neutrons into the water hole at every lethargy. However, this calculation is not realistic because the cadmium can displace a large portion of the water in the centre of the water hole. The effect of this is to flatten $\phi(u)$ at lethargies greater than seven.

A somewhat more realistic calculation was done in which the can was treated as infinitely long (to preserve the requisite one-dimensional nature of the system). In addition, the can and its uranium fuel rod were homogenized, and resonance capture in the uranium was ignored. The resulting spectrum (b) was indeed much flatter, as was expected.

Because of the uncertainties involved in treating the heterogeneities, a second approach was used, namely to measure the ratio of U²³⁸ capture in the can to that in a reference configuration where the spectrum was known. One could then calculate the ratio U²³⁸ capture in the known reference spectrum to that in a flat spectrum ($\phi(u)$ constant). It would evidently be advantageous for the reference spectrum to be as flat as possible.

The reference configuration chosen was a hexagonal lattice of 0.983-cm diam, 1.3% enriched uranium metal fuel rods at a lattice pitch of 2.88 cm (water-to-uranium volume ratio 8.1:1). The fuel rods were clad with aluminium, having an outer diam of 1.153 cm. The spectrum appropriate to this lattice is shown as (c) in Fig. 2. This spectrum, calculated by PLMG, gave a resonance escape probability, $p = 0.929$ whereas the corresponding measured value was $p = 0.928 \pm 0.001$.

The measurements were made with metal foils of uranium (depleted to 5 ppm U²³⁵), molybdenum and gold as shown in Fig. 3. In case (b), the fuel rod loading was placed in the cadmium can and irradiated in the same manner as a standard resonance integral measurement. The reference measurements (c) employed the same fuel loading and the same foils, with a 0.6-cm high cadmium sleeve. The foils were placed in the central rod of the 8:1 lattice.

Normalized to the same gold activation, the measured ratio of activation in the 8:1 lattice to activation in the cadmium can was 1.084 ± 0.008 for U²³⁸ capture and 1.083 ± 0.008 for Mo⁹⁸ capture. (The equality of these numbers is not significant). The calculated ratio of U²³⁸ capture in a flat flux spectrum ($\phi(u) = 1$) to a capture in spectrum (c) was 0.8938. Thus the ratio of capture in a flat flux spectrum to capture in the cadmium can spectrum was 0.9680 for the 0.983-cm diam uranium metal rod. Since the directly measured value was 17.43 b, the correction amounts to a subtraction of 0.56 b.

From the spectra (b) and (c) of Fig. 2, the Mo⁹⁸ ratio is 1.096, in good agreement with the measured value of 1.083. The calculated U²³⁸ capture ratio in these two spectra was 1.079, also in good agreement with the measured value of 1.084. It is concluded from this agreement that spectrum (b) represents the can spectrum accurately.

* Lethargy $u = 0$ corresponds to 10 MeV.

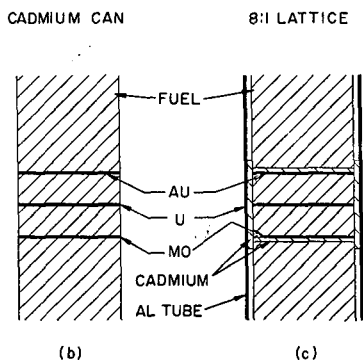


Fig. 3

Foil loadings for spectrum check

To calculate these ratios from the spectra of Fig. 2, it was necessary to assume that the Mo^{98} capture occurs predominantly at an energy of 480 eV, and that depression of the gold capture due to interference by the 6.7 eV resonance of U^{238} was the same in the lattice as in the cadmium can and could thus be ignored.

In order to calculate the U^{238} capture in the several spectra of Fig. 2, it was also necessary to have values for the effective U^{238} resonance capture integral of the rod in small lethargy groups. These values were obtained from NORDHEIM'S recent calculations [9] up to 1000 eV. From 1000 eV to 30 000 eV values were obtained from VERNON [10]. Above 30 000 eV the directly measured cross-section values from BNL-325 [11] were used.

Most of the correction to the measured resonance integral is contributed by the flux peak below lethargy 7. For the 0.983 cm rod, the peak contributes in excess of 0.7 b whereas the flux dip for $u > 7$ causes a deficit of ~ 0.1 b. It was decided to apply a flat 0.6 b correction to the resonance integral for every rod. This should introduce no more error than the uncertainty inherent in the entire spectrum correction procedure. An uncertainty of 0.3 b is assigned to this correction.

The final effective resonance integral values are listed in the last column of Table II. The estimated errors are as follows (average for all rod sizes):

(a) Reproducibility of the raw measurements	1%
(b) Uncertainty of the monitor ratio (due to spectrum shift)	1%
(c) Uncertainty of the fission correction	0.2 b
(d) Uncertainty of the spectrum correction	0.3 b

An overall uncertainty of 0.6 b is felt to be reasonable.

A least squares fit to the measured points yielding the following expressions for the effective resonance integrals (including $1/v$ capture):

$$\text{UO}_2 \quad I = 6.09 + 24.8 \sqrt{S/M_{\text{UO}_2}}, \quad 0.4 < \sqrt{S/M_{\text{UO}_2}} < 0.75 \quad (3)$$

$$\text{U metal} \quad I = 3.91 + 27.8 \sqrt{S/M_{\text{U}}}, \quad 0.2 < \sqrt{S/M_{\text{U}}} < 0.6 \quad (4)$$

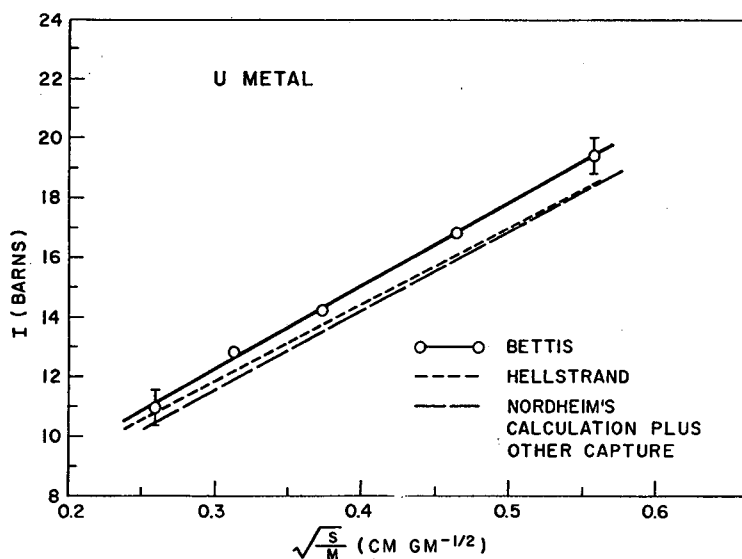


Fig.4

Experimental results for isolated metal rods

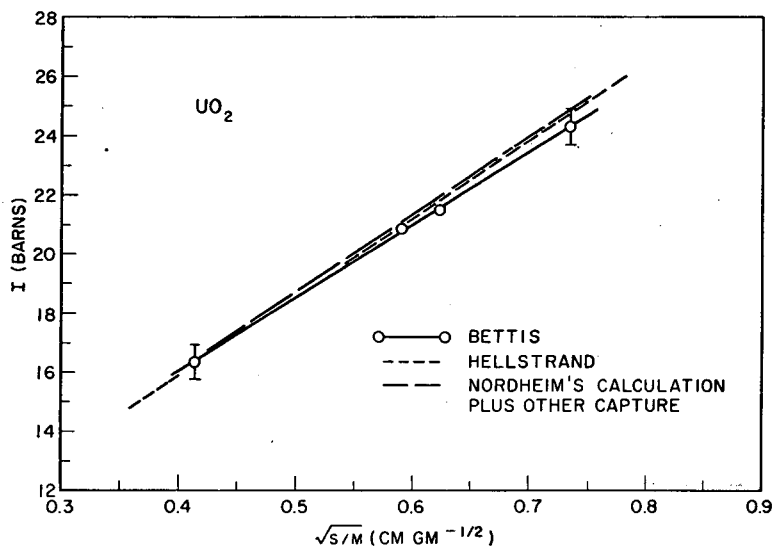


Fig.5

Experimental results for isolated UO₂ rods

Figures 4 and 5 show the measured effective resonance integrals for metal and UO₂ plotted against $\sqrt{S/M}$.

Figures 4 and 5 also compare our results with the most recent calculations of NORDHEIM [9], which have been done for the resolved resonances

up to 1000 eV, and unresolved s-wave resonances above 1000 eV. In addition, the following contributions are added to Nordheim's results:

	$\frac{I}{(b)}$	
(a) $1/v$ capture (all S/M)	+ 1.1	} + 1.7
(b) All capture above 30 keV (all S/M)	+ 0.8	
(c) S-wave capture above 30 keV (all S/M)	- 0.2	
(d) P-wave capture below 30 keV:		

	$\frac{S}{M}$	
Metal	0.259	+ 0.79
	0.558	+ 0.96
UO ₂	0.414	+ 0.90
	0.734	+ 1.10

- (a) is calculated using a U²³⁸ capture cross-section of 2.71 b at E = 0.0253 eV and assuming a cadmium cut-off energy of 0.5 eV.
 (b) is obtained directly from measured capture cross-sections in BNL-325 [8].
 (c) is calculated assuming unshielded capture [1].
 (d) is obtained from calculations reported by CHERNICK and VERNON [3], which assume a p-wave strength function twice that for s-wave resonances. Lack of knowledge of this strength function contributes appreciable uncertainty to these numbers.

It should be pointed out that this direct comparison of resonance integrals (calculated) with resonance integrals (measured) is not entirely unambiguous. The ambiguity is introduced by the fact that the nuclear data used in Nordheim's calculations yield an infinitely dilute resonance integral of only 268 b [9], whereas the measured value is 280 b. The effect on the rod calculations of this 12 ± 4 b discrepancy depends strongly on its origin.

Also shown in Figs. 4 and 5 are HELLSTRAND'S latest results for U metal and UO₂ rods [12], to which he assigns an overall uncertainty of 5%, including uncertainty of the gold resonance integral standard. The UO₂ results are in excellent agreement. Agreement is not as good for the metal results, but is still within the overall uncertainties.

B. MONTE-CARLO STUDY OF EQUIVALENCE RELATIONS

The equivalence relations deduced by several authors [1, 2, 3, 13, 14] are statements connecting lattices with the same resonance integral. Descriptions of the theory are abundant and there is no need to add still another. The result will simply be stated, in the form:

The resonance integrals of lattices of a fixed absorber composition and temperature depend only on the effective surface-to-mass ratio, given by

$$(S/M)^* = (S/M) (1 - C) \quad (5)$$

and do not otherwise depend on geometry and moderator.
 Here, (S/M) is the surface-to-mass ratio of the absorber lump, and $(1 - C)$

is the familiar Dancoff factor, defined as the probability that a neutron leaving an absorber lump collides in the moderator before entering another lump. This equivalence may be derived by use of the Wigner rational approximation to the region-averaged collision probabilities, together with the assumption that each resonance is either broad or narrow with respect to all the scattering materials in the lattice.

The use of the rational approximation to estimate $(1 - C)$ leads to a simpler form for the effective surface-to-mass ratio, due to BELL [13]:

$$(S/M)^*_{\text{Bell}} = (S/M) \left[1 + (S_1/4V_1E_1) \right]^{-1} \quad (6)$$

where S_1/V_1 is the surface-to-volume ratio of the moderator, and E_1 is the scattering cross-section of the moderator. The factor multiplying (S/M) in the above equation will be referred to as the Bell-Dancoff factor, $(1 - C)_{\text{Bell}}$.

A more general equivalence relation has been stated which links lattices in which the absorber contains varying quantities of internal moderator. This statement requires that the resonances all be narrow with respect to the internal moderator, a condition not met for oxygen in U²³⁸. To anticipate later results, the more general equivalence is not as valid as the form given above.

The equivalence relations can be used to predict resonance integrals in a wide variety of configurations on the basis of determinations in a relatively small number of lattices, or perhaps isolated rods. Although the rational approximation is subject to large errors. There is reason to believe that the equivalence relation is better than the approximations leading to it. In particular, for isolated lumps of absorber, the equivalence connects various shapes with the same surface-to-mass ratio. Examination of the actual collision probabilities for slabs, spheres and cylinders [15] reveals differences of only a few percent, whereas the rational approximation predicts no difference, but may be 20% in error. Thus the rational approximation is inadequate for calculation of resonance integrals, but the equivalence relation is still useful in predicting resonance integrals for cylinders, say, from accurate calculations for slabs.

In the hope that the extension to lattices would also be reliable, an investigation of the equivalence relations was undertaken using Monte-Carlo results for resonance escape probabilities. A large number of existing calculations was used, together with some new ones, so that three different sets of resonance parameters are used, corresponding to different stages of parameter measurement. Only resolved resonances were included in the calculations because the major uncertainties associated with the use of the narrow resonance or narrow resonance-infinite absorber approximations occur in these resonances. No corrections for spectrum effects, leakage, or competing absorption are required. Resonance integrals were computed from the expression:

$$I = (-\ln p) \frac{\overline{\xi \Sigma_s} V_{\text{cell}}}{N_u V_o} \quad (7)$$

where $\xi \Sigma_S$ is the slowing down power of the homogenized cell, N_u the U^{238} atom density, V_0 the volume of the fuel, and V_{cell} the cell volume.

The measurements on isolated rods suggest that a plot of resonance integral versus $\sqrt{(S/M)^*}$ should be very nearly linear. Figure 6 shows results for a group of 15 metal-water lattices at 293°K and consisting of hexagonal arrays of cylindrical rods. Resonance parameters used in these

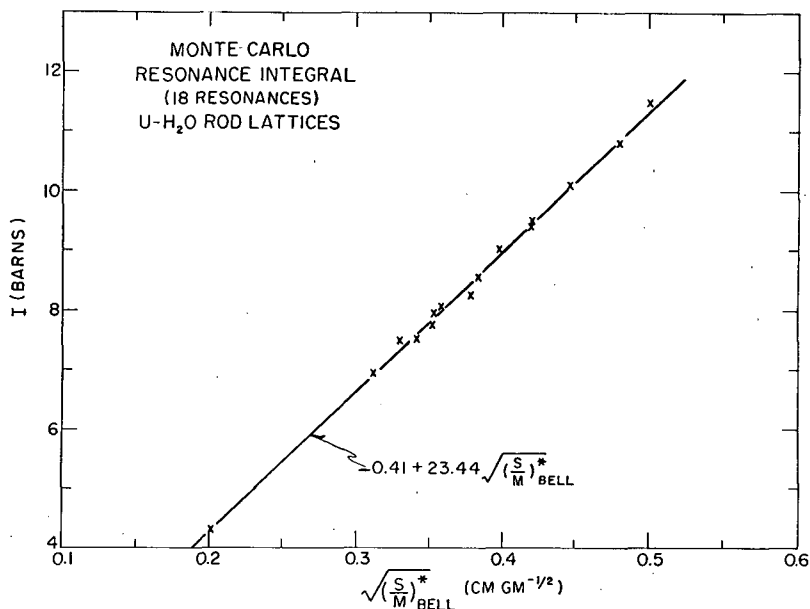


Fig. 6

Monte-Carlo resonance integrals for U metal lattices in terms of the Bell equivalence parameter

calculations were essentially those in the first edition of BNL-325. No $1/v$ absorption was included. Rod diameters ranged from 0.25 in to 2.0 in, and $\sqrt{(S/M)^*}_{\text{Bell}}$ from 0.2 $\text{cm/g}^{1/2}$ to 0.5 $\text{cm/g}^{1/2}$. Within 0.2 b they are represented by the least squares fit

$$I = -0.41 + 23.44 \sqrt{(S/M)^*}_{\text{Bell}} \quad (8)$$

(The negative volume term can be explained by assuming a departure from linearity at small $(S/M)^*$.) The scatter is just about that expected from the Monte-Carlo uncertainties. The tightest lattice considered has a Bell-Dancoff factor of 0.43, compared to 0.94 in the most open lattice.

A group of UO_2 lattices are exhibited in Fig. 7. In most of these cases the geometry was alternating slabs of UO_2 and water. Thirty-nine resolved resonances were included [16]; the temperature was again 293°K. The

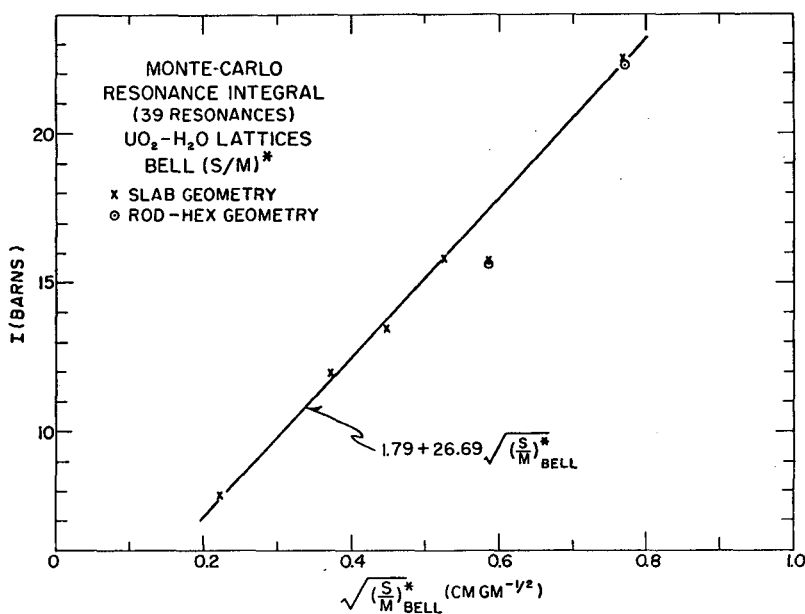


Fig. 7

Monte-Carlo resonance integrals for UO₂ lattices in terms of the Bell equivalence parameter

tightest lattices shown (one a slab, the other a cylinder lattice) depart significantly from the rest of the data; their Bell-Dancoff factor is 0.37. The rest of the calculations are represented, to the accuracy of the Monte Carlo, by

$$I = 1.79 + 26.69 \sqrt{\left(\frac{S}{M}\right)^*_{\text{Bell}}} \quad (9)$$

These range in Bell-Dancoff factor between 0.64 and 0.82. They exhibit a root mean square error of 0.2 b.

The slab lattices used in the straight line fit in Fig. 7 were also calculated at an absorber temperature of 553°K. With the same accuracy, the results were represented by the expression:

$$I = 1.44 + 28.98 \times \sqrt{\left(\frac{S}{M}\right)^*_{\text{Bell}}} \quad (10)$$

The exact Dancoff factors were computed for the cold slab lattices in this group, and the results are shown in Fig. 8. There is improvement in the agreement obtained, but the tightest lattice remains out of line. The use of (S/M)* does extend somewhat the useful range of the equivalence relation; this advantage is to some extent offset by the greater complexity of this calculation for rod lattices.

Two other UO₂ lattices also suggest that the Dancoff factor may be a better approximation in tight lattices. The lattice descriptions are given in Table III, together with the results of the Monte-Carlo calculations which

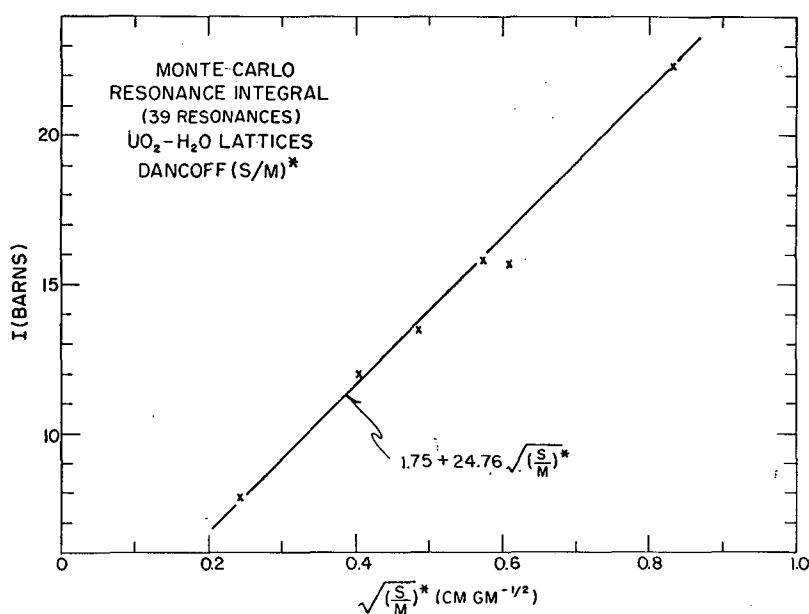


Fig. 8

Monte-Carlo resonance integrals for UO₂ lattices in terms of the Dancoff equivalence parameter

TABLE III

SOME MONTE-CARLO RESULTS FOR TIGHT LATTICES

Geometry	Moderator	S/M	$\sqrt{(S/M)^*}_{\text{Bell}}$	$\sqrt{(S/M)^*}$	I (Barns)
Cylinder, hex. array	H ₂ O	0.381	0.513	0.544	14.9
Slab	H ₂ O	0.762	0.513	0.527	14.3
Cylinder, hex. array	D ₂ O	0.381	0.513	-	14.8

used 27 resonances, essentially those in the second edition of BNL-325. The probable error on the resonance integral is ~ 0.15 b. The lattices have the same $(S/M)^*_{\text{Bell}}$, $0.263 \text{ cm}^2/\text{g}$, and the temperature is 293°K . The resonance integral for the close-packed slab lattice falls somewhat low. For this lattice, $(S/M)^*$ is also somewhat lower than for the other and by an amount consistent with the expected slope of $25\text{-}30 \text{ g}^{1/2}\text{-b/cm}$.

Included in Table III is a deuterium-moderated lattice which shows that the equivalence holds well under a change of moderator. The Dancoff factor, in this case, could not be calculated with the necessary precision.

In conclusion, both formulations give a linear dependence on the effective surface-to-mass ratio over a wide range of lattices. The lines are

distinct, being separated by as much as two barns. In principle, isolated rod calculations could permit a selection since in that limit

$$(S/M)^* = (S/M)_{\text{Bell}}^* = S/M$$

These "wide lattice" calculations would be extremely costly. The experimental study described in the next section makes such a comparison.

C. EXPERIMENTAL RESULTS USING EQUIVALENCE RELATIONS

Experimental tests of the equivalence relations can be extracted from TRX measurements of U²³⁸ resonance escape probability, p^{28} , for lattices and measurements of the effective resonance integrals described in section A. The lattices in question were hexagonal, H₂O-moderated and composed of cylindrical, aluminium clad fuel rods of slightly enriched uranium metal or UO₂.

Table IV lists the measured values of p^{28} from which the lattice effective resonance integrals have been computed. Cases 1 - 6 and 12 - 19, as well as the measurement technique, were reported previously [4]. In case 7, p^{28} is a recently remeasured value; cases 8 - 11 are new. The measured ρ values are also listed in the table.

The resonance integrals were computed from the expression

$$I' = (-\ln p^{28}) \frac{\xi \Sigma_s V_{\text{cell}}}{N_u V_o} \quad (11)$$

A correction to account for the effects on p^{28} of leakage and U²³⁵ absorption was determined as follows. In each case, a 54 energy group P_1 calculation of resonance capture was done for a homogenized lattice by means of the MUFT code* [17]. The resonance shielding factor (L factor) used was such that the calculated value of p^{28} very closely approximated the corresponding measured value. A second calculation was also done which differed from the first only in that the U²³⁵ was omitted and the lattice buckling was reduced by a factor 100. The difference between the resonance integrals inferred from the two calculations, ΔI_1 , was used to correct (I)' to the value appropriate to an infinite lattice without U²³⁵.

In addition, in these H₂O-moderated lattices $\xi \Sigma_s(u)$ decreases rapidly with increasing energy greater than 10 keV. This causes a peaking of the flux in this interval and leads one to infer too large a value for the effective resonance integral from equation (11). The error can be corrected adequately simply removing the excess capture contributed by the flux peak. This was done using the calculated multi-group flux values for groups one to 25, which just span the lethargy region of the flux peak:

$$\Delta I_2 = \sum_{i=1}^{25} (\phi_i - 1) \bar{\sigma}_i^{28} \Delta u_i \quad (12)$$

* For the two-region lattices, a 54 energy group, radially dependent problem was done instead, by means of the PIMG code [8].

TABLE IV
LATTICE RESULTS

Case	Rod diam. (in)	Fuel	Enrichment (Weight %U ²³⁵)	W/U (b)	ρ (c)	P^{238}	$\frac{E_{235}}{V_{cell}} \frac{N_U}{U}$ (barns)	I' (barns)	ΔI_1 (barns)	ΔI_2 (barns)	I (barns)	$\sqrt{\frac{S}{M}} \frac{\Pi_{BeII}}{M}$ * (cm ² g ⁻¹)	$\sqrt{\frac{S}{M}}$ *
1	0.600		1.3	1.5	1.80 $\pm$ 0.18	0.720 $\pm$ 0.014	44.25	14.54	1.10	1.05	12.39 $\pm$ 0.3	0.324	0.350
2	0.600		1.3	2.0	1.24 $\pm$ 0.12	0.794 $\pm$ 0.016	58.89	13.59	1.00	1.05	11.54 $\pm$ 1.2	0.334	0.359
3	0.600		1.3	3.0	0.89 $\pm$ 0.09	0.850 $\pm$ 0.017	87.85	14.28	0.84	1.05	12.39 $\pm$ 1.8	0.346	0.367
4	0.600		1.15	2.0	1.20 $\pm$ 0.03	0.785 $\pm$ 0.005	58.81	14.24	0.79	1.05	12.40 $\pm$ 0.4	0.334	0.359
5	0.600		1.15	3.0	0.93 $\pm$ 0.04	0.833 $\pm$ 0.007	87.72	16.03	0.66	1.07	14.30 $\pm$ 0.8	0.346	0.367
6	0.387	U Metal	1.3	2.0	1.59 $\pm$ 0.03	0.747 $\pm$ 0.005	59.11	17.26	1.05	1.07	15.14 $\pm$ 0.4	0.393	0.424
7	0.387		1.3	3.0	1.066 $\pm$ 0.012	0.8233 $\pm$ 0.0028	88.06	17.15	0.82	1.08	15.25 $\pm$ 0.29	0.412	0.441
8 (a)	0.387		1.3	1.0	3.13 $\pm$ 0.20	0.589 $\pm$ 0.015	29.26	15.49	1.46	1.09	12.94 $\pm$ 0.8	0.346	0.382
9 (a)	0.387		1.3	2.4	1.37 $\pm$ 0.02	0.779 $\pm$ 0.004	68.62	17.14	0.98	1.07	15.08 $\pm$ 0.4	0.402	0.432
10	0.387		1.3	2.4	1.41 $\pm$ 0.06	0.772 $\pm$ 0.008	68.62	17.75	0.98	1.07	15.70 $\pm$ 0.7	0.402	0.432
11 (a)	0.387		1.3	8.1	0.484 $\pm$ 0.02	0.9284 $\pm$ 0.0010	236.08	17.53	0.25	1.10	16.18 $\pm$ 0.23	0.443	0.459
12	0.600		1.3	3.0	1.19 $\pm$ 0.04	0.805 $\pm$ 0.006	90.95	19.73	0.82	1.04	17.87 $\pm$ 0.7	0.489	
13	0.600	UO ₂	1.3	4.0	0.994 $\pm$ 0.013	0.898 $\pm$ 0.003	118.83	21.00	0.73	1.06	19.21 $\pm$ 0.4	0.508	
14	0.600	7.53 g/cm ²	1.3	5.0	0.807 $\pm$ 0.014	0.869 $\pm$ 0.003	148.14	20.80	0.60	1.07	19.13 $\pm$ 0.5	0.522	
15	0.388		1.3	4.0	1.04 $\pm$ 0.05	0.832 $\pm$ 0.007	117.53	21.61	0.71	1.07	19.83 $\pm$ 1.0	0.585	
16	0.388		1.3	5.0	0.901 $\pm$ 0.02	0.853 $\pm$ 0.004	146.61	23.31	0.64	0.08	21.59 $\pm$ 0.7	0.608	
17	0.384	UO ₂	1.3	2.9	1.43 $\pm$ 0.01	0.775 $\pm$ 0.004	86.18	21.97	0.93	1.04	20.00 $\pm$ 0.4	0.498	
18	0.384	10.53 g/cm ²	1.3	3.6	1.15 $\pm$ 0.01	0.815 $\pm$ 0.004	107.16	21.92	0.81	1.06	20.05 $\pm$ 0.5	0.517	
19	0.384		1.3	4.9	0.934 $\pm$ 0.010	0.850 $\pm$ 0.004	143.86	23.38	0.68	1.07	21.63 $\pm$ 0.7	0.540	

(a) Measured in sub-lattice at centre of two region core.

(b) Water-to-uranium volume ratio. For UO₂ rods, "uranium volume" is the volume of a uranium metal rod with the same number of uranium atoms as the UO₂ rod.

(c) ρ = (Epi-cadmium U²³⁸ radiative capture) - (Sub-cadmium U²³⁸ radiative capture)

(d) M is mass of uranium metal or of UO₂.

where $\phi_{25} = 1$. The U²³⁸ capture cross-sections were obtained from experimental values listed in BNL-325 [11]. Errors quoted were calculated from those for the p^{28} values.

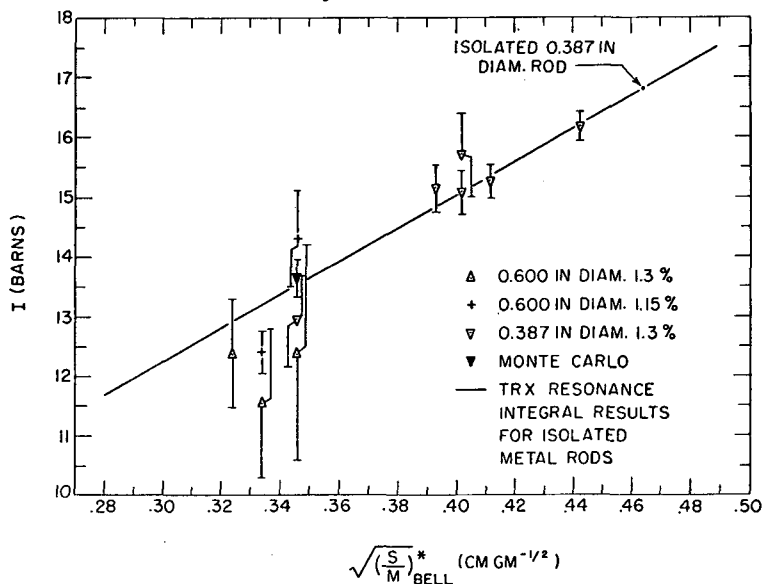


Fig.9

Comparison of measured isolated rod and lattice resonance integrals for metal rods in terms of the Bell equivalence parameter

The equivalence parameter for the Bell formulation, $(S/M)^*_{\text{BELL}}$, was calculated from equation (6) for each lattice. Figures 9 and 10 show the results for uranium metal and UO₂ respectively. Included are the measured isolated rod resonance integrals from section A, to which an overall uncertainty of 0.6 b is assigned. In addition, there is a resonance integral inferred from a Monte-Carlo calculation of p^{28} for case 8 [18]. This calculation was done for an infinite lattice without U²³⁵; 30 000 histories were run, the neutrons being started at 10 keV. The result was p^{28} (10 keV $\rightarrow$ 3 eV) = 0.6925 ± 0.0027 , yielding a resonance integral of 10.75 ± 0.11 b. To this was added 2.9 b to account for the remaining capture.

The metal rod results are perfectly consistent with the assumed equivalence relation. Except for the three high density lattice cases, the UO₂ results show good consistency also. Part of the discrepancy among the UO₂ lattices can probably be attributed to the use of uranium metal foils in the older UO₂ measurements. [5].

The metal results were replotted in terms of the Dancoff equivalence parameter $\sqrt{(S/M)^*}$ (see Fig. 11). Although there is some uncertainty involved in properly calculating $(1 - C)$, particularly in the tighter lattices, our lattice results fall below the isolated rod values when plotted against $\sqrt{(S/M)^*}$. The overall agreement is still considered good.

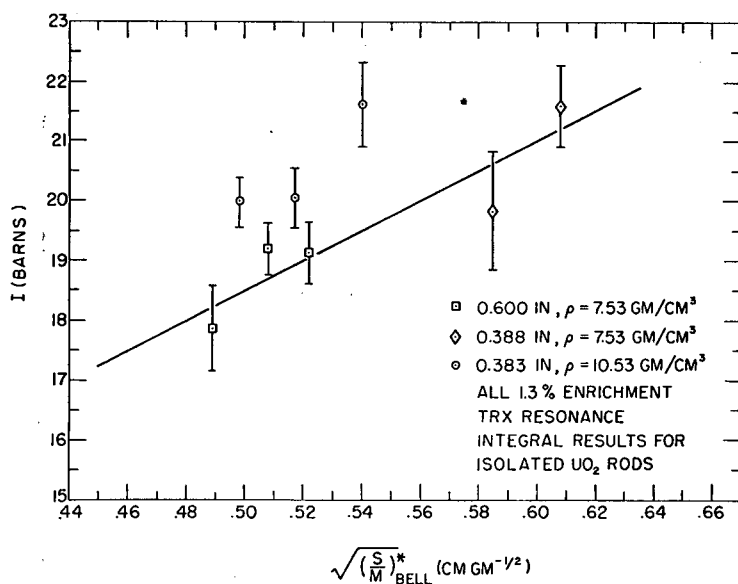


Fig.10

Comparison of measured isolated rod and lattice resonance integrals for UO₂ rods in terms of the Bell equivalence parameter

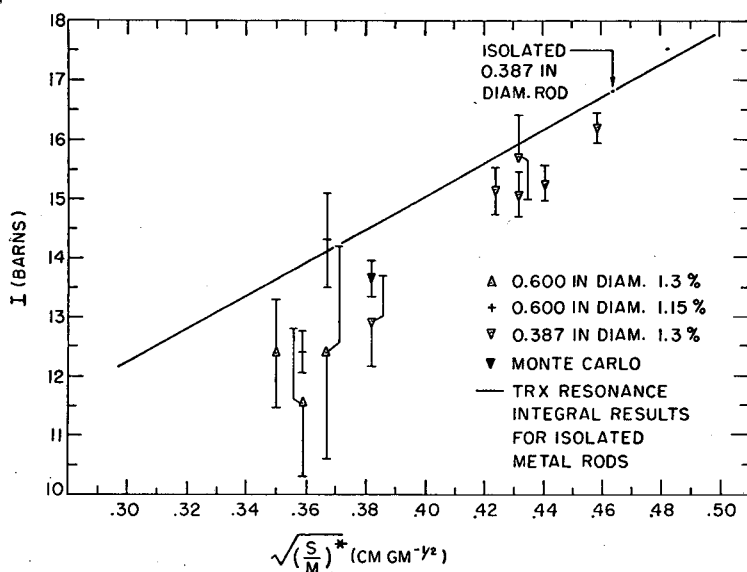


Fig.11

Comparison of measured isolated rod and lattice resonance integrals for metal rods in terms of the Dancoff equivalence parameter

The more general equivalence among lattices of uranium metal and of UO₂ in terms of one parameter does not appear to work as well. The equivalence parameter is, in this case,

$$\lambda_e^{\frac{1}{2}} = \left[\left(\frac{4N}{A^{28}} \right) \sigma_{ps} + \left(\frac{S}{M_U} \right) \right]^{\frac{1}{2}} \quad (13)$$

where

S = surface of fuel (cm²)

M_U = mass of uranium in fuel (g)

N = Avogadro's number

$A^{28} = 238$

σ_{ps} = admixed oxygen potential scattering cross-section per uranium atom. (barns)

On this basis, all of the data should form a single straight line. Referring to Fig. 12, it can be seen that the oxide results are about 2 b lower than the metal results. In order to obtain better agreement it would be necessary to reduce the admixed oxygen moderating cross-section in equation (13). Such a reduction is qualitatively consistent with the fact that the treatment of oxygen in the narrow resonance approximation is not strictly correct for the lowest resonances in U²³⁸.

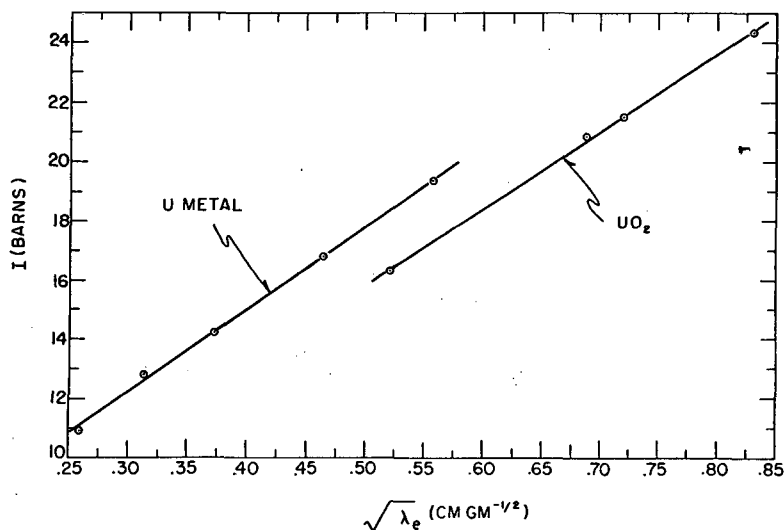


Fig. 12

Comparison of measured resonance integrals for isolated metal and UO₂ rods in terms of the generalized equivalence parameter λ_e

In summary, the equivalence relations using either the Dancoff factor or the Bell approximation to it, were tested over a wide range of surface-to-mass ratios by Monte-Carlo calculations. In tight lattices, a breakdown appeared in both formulations, but the Dancoff treatment showed somewhat

better agreement than the Bell approximation. Both formulations were also tested experimentally, over a smaller range of surface-to-mass ratios; the Bell approximation is favoured in these comparisons. Within the precision of all the information cited, no clear cut preference is indicated. Both formulations work well.

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THE CALCULATION OF THE THERMAL UTILIZATION AND DISADVANTAGE FACTOR IN URANIUM WATER LATTICES

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Abstract — Résumé — Аннотация — Resumen

THE CALCULATION OF THE THERMAL UTILIZATION AND DISADVANTAGE FACTOR IN URANIUM WATER LATTICES. The theoretical methods for computing intracell thermal neutron densities, disadvantage factors, and thermal utilizations which have been developed at the Brookhaven National Laboratory are reviewed. The topics discussed are the computation of the neutron spectra in a lattice, anisotropic scattering by water, the cylindrical cell effect, correlation of spectral moments and average one-group cross-sections, one-group transport theory methods, scattering models of water, and comparison of various theories with experiment. The lattices investigated are water moderated with slightly enriched uranium metal and oxide fuel.

CALCUL DE L'ABSORPTION UTILE DES NEUTRONS THERMIQUES ET DU FACTEUR DE DIMINUTION DE FLUX DANS DES RÉSEAUX URANIUM-EAU. Le mémoire donne un aperçu des méthodes théoriques qui ont été mises au point au Laboratoire national de Brookhaven pour calculer les densités intracellulaires des neutrons thermiques, les facteurs de diminution de flux et les probabilités d'absorption utile des neutrons thermiques. On examine les questions suivantes: calcul des spectres neutroniques dans un réseau; diffusion anisotrope dans l'eau; effet de cellule cylindrique; corrélation des moments de spectre et des sections efficaces moyennes à un groupe; méthodes fondées sur la théorie de transport à un groupe; modèles de dispersion dans l'eau; comparaison de différentes théories avec l'expérience. Les réseaux examinés sont ceux que l'on trouve dans les réacteurs ralentis à l'eau et utilisant comme combustible de l'uranium faiblement enrichi, à l'état de métal et d'oxyde.

РАСЧЕТ ИСПОЛЬЗОВАНИЯ ТЕПЛОВЫХ НЕЙТРОНОВ И КОЭФФИЦИЕНТА ПРОИГРЫША В УРАНОВОДНЫХ РЕШЕТКАХ. Рассматриваются теоретические методы вычисления внутриклеточных плотностей тепловых нейтронов, неблагоприятных факторов и использования тепловых нейтронов, которые были разработаны в Брукхейвенской национальной лаборатории. Обсуждаются следующие темы: вычисление нейтронных спектров в решетке, анизотропное рассеяние с помощью воды, эффект цилиндрической ячейки, корреляция спектральных моментов и усредненные одnogрупповые сечения, теоретические методы одnogрупповых переносов, рассеивающие модели воды и сравнение различных теоретических данных с экспериментальными. Исследуемыми решетками являются решетки с водяным замедлителем и слегка обогащенным металлическим ураном и оксидным топливом.

CÁLCULO DE LA UTILIZACIÓN TÉRMICA Y DEL FACTOR DE DESVENTAJA EN RETICULADOS DE URANIO-AGUA. Se presenta una reseña de los métodos teóricos ideados en el Brookhaven National Laboratory con miras a calcular las densidades de neutrones térmicos en el interior de una célula del reticulado, los factores de desventaja y los coeficientes de utilización térmica. Entre los temas tratados, figuran el cómputo de los espectros neutrónicos en un reticulado, la dispersión anisotrópica por el agua, el efecto de célula cilíndrica, la correlación de los momentos espectrales y de las secciones eficaces medias sobre la base de un solo grupo, los métodos de la teoría de transporte de un solo grupo, los modelos de dispersión en el agua y una comparación de las diversas teorías con los resultados experimentales. En los reticulados estudiados se utiliza agua en calidad de moderador y, como combustible, uranio metálico ligeramente enriquecido y óxido de uranio.

INTRODUCTION.

The measurement of intracell thermal neutron density distributions has been a part of the BNL experimental programme on the physics of ordinary water lattices. These intracell distributions, $N(r)$, are used to compute the neutron density disadvantage factor, δ_N ,

$$\delta_N = \bar{N}_m / \bar{N}_f \quad (1)$$

where $\bar{N}_m$ and $\bar{N}_f$ are the average neutron densities in the moderator and fuel, and the thermal utilization, f

$$f = \frac{\text{absorptions in the fuel}}{\text{total absorptions in the cell}} \quad (2)$$

$$= \frac{\Sigma_{af}^0 g_a \bar{N}_f V_f}{\Sigma_{af}^0 g_a \bar{N}_f V_f + \Sigma_{ac}^0 \bar{N}_c V_c + \Sigma_{am}^0 \bar{N}_m V_m}$$

where superscript zero denotes 2200 m/s values of the absorption cross-section Σ_a , g_a is the non 1/v factor for fuel absorption, V is the volume, and f , c , and m denote fuel, cladding, and moderator. The experimental techniques and the results of many of these measurements have been reported by KOUTS *et al.* [1, 2, 3]. The theoretical prediction of these intracell distributions is complicated by the non-Maxwellian character of the neutron energy spectrum in the lattice and by the two dimensional nature of the lattice (rods in a triangular array). Early calculations using one-group P_3 theory have been reviewed by CHERNICK [4]. These calculations showed the importance of the spectral hardening effect. P_1 methods have been used with cross-sections averaged over a Wigner-Wilkins spectrum for the homogenized lattice [1]. TAKAHASHI [5] and HONECK [6, 7] have computed spectra in the heterogeneous lattice. In this paper we will review the work done at BNL on the theoretical prediction of these intracell distributions.

We have attempted in this work to gain as complete an understanding of the problem as possible and thus be able to evaluate the validity of many simplifications in the theory. Some approximations generate errors which, in some fortuitous cases, tend to cancel and give apparently accurate results. An example is the use of one-group P_3 theory (which underestimates δ) with cross-sections averaged over a Maxwellian spectrum at the material temperature (which overestimates δ). The secondary emphasis is the reduction of our results to a simpler form so that approximate theories can be used with confidence. For example, prescriptions have been obtained for computing average cross-sections. When these cross-sections are used in the method of AMOUYAL *et al.*, [8], surprisingly accurate disadvantage factors are obtained.

The detailed thermal neutron spectrum in a lattice has been measured by CAMPBELL and POOLE [9] and MOSTOVOI [10]. Attempts to reproduce these spectra by HONECK and TAKAHASHI [11] have been unsuccessful, and it was concluded that the measured spectra were not infinite lattice spectra but were distorted by spatial gradients in the finite lattice array.

Infinite medium spectra measured and computed by BEYSTER *et al.*, [12] verify the soundness of the theoretical methods used. We will not discuss detailed spectra here, but we will frequently refer to the first moment of the spectrum, $\bar{v}$,

$$\bar{v} = \int_0^{v^*} dv v N(v) / \int_0^{v^*} dv N(v) \quad (3)$$

where $N(v)$ is the neutron density and v^* is a cut-off for the thermal group. We will use a v^* corresponding to an energy of 0.785 eV, a value large enough to be in the $1/E$ portion of the spectrum.

The foil activities measured in an intracell traverse are proportional to the integrated neutron density since most foils have an absorption cross-section proportional to $1/v$. Older definitions of the disadvantage factor are in terms of the neutron flux.

$$\delta_\phi = \frac{\bar{\phi}_m}{\phi_f} = \frac{\bar{v}_m}{\bar{v}_f} \delta_N \quad (4)$$

We will always use the neutron density disadvantage factor δ_N here, and when appropriate, we will use eq. (4) to convert δ_ϕ from one-group calculations to δ_N . Two other quantities frequently used are the fuel and moderator disadvantage factors.

$$F_f = \frac{\text{neutron density at the fuel surface}}{\text{average neutron density in the fuel}} \quad (5)$$

$$F_m = \frac{\text{average neutron density in the moderator}}{\text{neutron density at the fuel surface}}$$

In those cases where the fuel is clad, these definitions are changed to read "average neutron density in the clad" in place of "neutron density at the fuel surface".

A description of the water moderated lattices investigated at BNL is given in Table I and their dimensions in Table II. The material concentrations and cross-sections are listed in Table III. Except for U^{235} , the cross-sections were taken from [13]. The absorption and fission cross-sections for U^{235} were taken from the "best line" through the data in a review by SAFFORD and HAVENS [14]. The value of η_{25} (2200) of 2.070 is in good agreement with [13]. The value of η for natural uranium of 1.328 agrees well with measurements [13, 15].

We will discuss the following topics in the remaining sections of this paper.

- (1) The multi-group and one-group transport methods to be used.
- (2) Scattering models for water and their effect on the spectra and intracell distributions.

TABLE I
DESCRIPTION OF THE LATTICES

Fuel	Enrichments % U ²³⁵	Rod diam or slab thickness (in)	Cladding thickness (in) and material	Type of lattice	Range of W/U volume Ratios	Fuel density (g/cm ³)
U Metal slabs	1.25	0.122	None	Slab	1.0 → 4.0	18.7
U Metal rods	1.0, 1.15, 1.3	0.25, 0.387, 0.60	0.028 Aluminium ^a	Triangular	1.0 → 4.0	18.9
UO ₂ rods	3.0	0.444	0.028 Stainless Steel	Triangular	1.291 → 4.0	9.3

^a There is an 0.005 in air gap between fuel and cladding.

TABLE II
LATTICE DIMENSIONS

Rod diam (in)	Rod radius (cm)	W/U	Distance between rod centres (cm)	Radius of equiv. cylindrical cell (cm)
0.250	0.3175	1.0	0.9746	0.5117
		1.5	1.0643	0.5588
		2.0	1.1471	0.6023
		3.0	1.2967	0.6808
		4.0	1.4308	0.7512
0.387	0.4915	1.0	1.4412	0.7567
		1.5	1.5860	0.8327
		2.0	1.7188	0.9024
		3.0	1.9573	1.0276
		4.0	2.1697	1.1392
0.600	0.7620	1.0	2.1684	1.1384
		1.5	2.3988	1.2596
		2.0	2.6093	1.3703
		3.0	2.9855	1.5674
		4.0	3.3198	1.7429
0.444	0.5639	1.291	1.7185	0.9022
		1.600	1.8209	0.9560
		2.050	1.9569	1.0274
		2.800	2.1667	1.1375
		4.000	2.4657	1.2945
Slab thickness (in)	Slab thickness (cm)	W/U (nominal)	Water channel thickness (cm)	
0.122	0.3099	1.0	0.3175	
		1.5	0.4763	
		2.0	0.6350	
		3.0	0.9525	
		4.0	1.2700	

- (3) Anisotropic scattering in water and transport corrections.
- (4) Comparison of computed and measured intracell distributions in the slab lattices.
- (5) The cylindrical (or Wigner-Seitz) cell effect.
- (6) Correlation of the spectral moments and the calculation of one-group cross-sections.
- (7) Comparison of computed and measured disadvantage factors and thermal utilizations for the rod lattices.
- (8) Conclusions and future work.

THEORETICAL METHODS

The theoretical methods we use will be briefly described in this section. A more detailed discussion will be found in the references.

TABLE III
MATERIAL CONCENTRATIONS AND CROSS-SECTIONS

Material	Concentration (atoms/b-cm)	σ_a (2200) (b)	σ_f (2200) (b)	Reference
U^{235} in 1.00% U Metal	4.971×10^{-4}	679.2 ^a	578.6 ^a	[14]
1.15% U Metal	5.540×10^{-4}			
1.25% U Metal	5.987×10^{-4}			
1.30% U Metal	6.288×10^{-4}			
3.00% UO_2	6.222×10^{-4}			
U^{238} in 1.00% U Metal	4.732×10^{-2}	2.71 ^b	-	[13]
1.15% U Metal	4.731×10^{-2}			
1.25% U Metal	4.670×10^{-2}			
1.30% U Metal	4.719×10^{-2}			
3.00% UO_2	2.012×10^{-2}			
Oxygen in UO_2	4.148×10^{-2}	0.0	-	[13]
in H_2O	3.344×10^{-2}			
Hydrogen in H_2O	6.689×10^{-2}	0.328	-	[28]
Aluminium ^a	6.031×10^{-2}	0.230	-	[13]
Stainless Steel	8.606×10^{-2}	2.870	-	[13, 43]

^a The aluminium concentration was reduced by 0.86 to account for the air gap.

^b These values along with $\nu_{25} = 2.43$, $g_a^{25} = 0.9720$, and $g_f^{25} = 0.9745$ give $\eta(U^{235}, 2200 \text{ m/s}) = 2.070$ and $\eta(\text{Nat. U.}, \text{Max. Av.}) = 1.328$.

The neutron flux in a lattice is a solution of the transport equation [16].

$$\underline{\Omega} \cdot \nabla \phi(\underline{r}, E, \underline{\Omega}) + \Sigma(\underline{r}, E) \phi(\underline{r}, E, \underline{\Omega}) = H(\underline{r}, E, \underline{\Omega}) \quad (6)$$

$$H(\underline{r}, E, \underline{\Omega}) = S(\underline{r}, E, \underline{\Omega}) + \int d\underline{\Omega}' \int dE' \Sigma_s(\underline{r}, E' \rightarrow E, \mu_0) \phi(\underline{r}, E', \underline{\Omega}') \quad (7)$$

where $(\underline{r}, E, \underline{\Omega})$ are the co-ordinates of a neutron at $\underline{r}$ with energy E moving in direction $\underline{\Omega}$, ϕ is the neutron flux, S is an external source, Σ is the total cross-section, Σ_s is the scattering kernel, and μ_0 is the cosine of the angle between $\underline{\Omega}$ and $\underline{\Omega}'$. The quantity H is called the emission density. An alternate form of eq. (6) is the integral transport equation

$$\phi(\underline{r}, E, \underline{\Omega}) = \int_0^\infty ds H(\underline{r} - s\underline{\Omega}, E, \underline{\Omega}) \exp \left[- \int_0^s ds' \Sigma(\underline{r} - s'\underline{\Omega}, E) \right] \quad (8)$$

which states that the flux is obtained by looking backward along the line $\underline{\Omega}$ and summing the product of the sources along the line and the probability of surviving the flight along s . The THERMOS code [6, 7] was written to solve eq. (8) with certain restrictions on the scattering kernel. There are three versions of this code, each for a different geometry. We will discuss the calculation of the transport kernel for each geometry first.

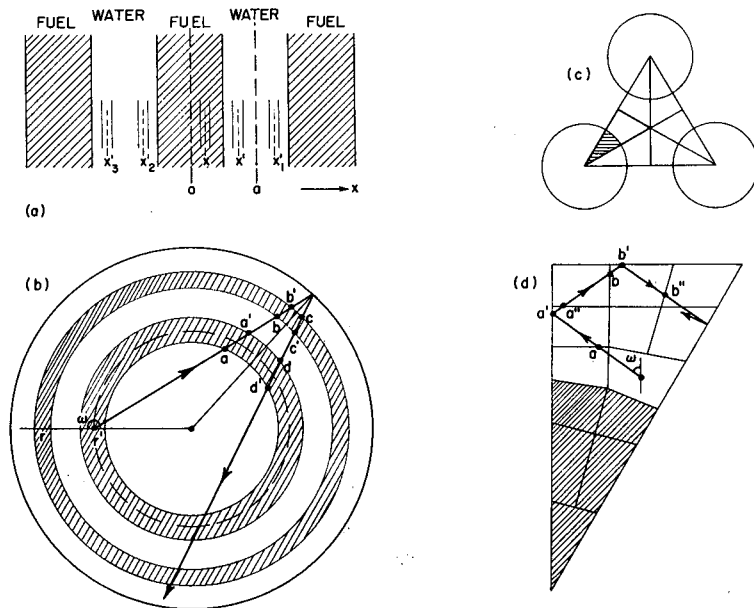


Fig. 1

Lattice geometry

A schematic of a slab lattice is shown in Fig. 1a. We consider only the region between the symmetry lines at $x = 0$ and $x = a$. In this case the variable $\underline{r}$ is x and $\underline{\Omega}$ is μ , the cosine of the angle between the neutron vector and the x axis. Expand ϕ , Σ_s , S , and H in Legendre polynomials. For example,

$$\phi(x, E, \mu) = \sum_{n=0}^{\infty} \frac{2n+1}{2} \phi_n(x, E) P_n(\mu) \quad (9)$$

When these expansions are inserted in (8) and the equation is multiplied by $P_l(\mu)$ and integrated over μ , a set of coupled integral equations is obtained.

$$\phi_l(x, E) = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} dx' T_{ln}(x, x', E) H_n(x', E) \quad (10)$$

where

$$H_n(x', E) = S_n(x', E) + \int_0^\infty dE' \Sigma_{sn}(E' \rightarrow E) \phi_n(x', E') \quad (11)$$

$$T_{ln}(x, x', E) = \frac{2n+1}{2} \int_0^1 \frac{dy}{y} P_n\left(\frac{1}{y}\right) e^{-\left| \int_x^{x'} dz \Sigma(x, E) \right| y} \begin{cases} (-1)^{l+n} & x < x' \\ 1 & x > x' \end{cases} \quad (12)$$

We restrict our attention to the case where the scattering is linearly anisotropic

$$\Sigma_s(E' \rightarrow E, \mu_0) = \frac{1}{2} \Sigma_{s0}(E' \rightarrow E) + \frac{1}{2} \mu_0 \Sigma_{s1}(E' \rightarrow E) \quad (13)$$

in which case the set (10) consists of only two equations. The infinite integration over x' is replaced by an integral from $x' = 0$ to $x' = a$ and a sum over image points such as x'_1 , x'_2 , and x'_3 in Fig. 1a. The final step in computing the transport kernels is to average the kernels over a finite source slab at x' and a receptor slab at x . The transport kernels T_{ln} have then been reduced to a matrix relating the average flux in the slab at x with a uniform source in the slab at x' . Up to 20 slabs are allowed between $x = 0$ and $x = a$.

A schematic of one dimensional cylindrical geometry is shown in Fig. 1b. The outer circle is assumed to be a reflecting boundary. In this geometry the THERMOS code is restricted to isotropic scattering and we need solve only for the scalar flux. Equation (8) becomes

$$\phi_0(r, E) = \int_0^{R_0} dr' T(r, r', E) H_0(r', E) \quad (14)$$

where $T(r, r', E)$ is the average flux at r due to a unit isotropic source at r' . The cylindrical cell is divided into circular shells (up to 20) such as are shown in Fig. 1b. A unit line source is set up at r' in direction ω and the fraction of this source of neutrons which suffer their first collision in each shell is computed such as between points a and a' , b and b' , c and c' , etc. This calculation is repeated at 20 equally spaced values of ω and at all r' . Trapezoidal rule integration over ω yields the desired transport kernel $T(r, r', E)$. The kernel as computed is the average flux in a shell about r due to a unit isotropic source at r' (and not in a shell about r' as we would like). This approximation is valid if the shells are thin in mean free paths. The fact that the transport kernel is symmetric gives a check on the validity of this approximation. For all cells considered here the approximation is found to be excellent.

A schematic of three rods in a triangular lattice is shown in Fig. 1c. The smallest symmetric element is heavily outlined and expanded in Fig. 1d. We again assume isotropic scattering so that eq. (8) becomes

$$\phi_0(\underline{r}, E) = \int d\underline{r}' T(\underline{r}, \underline{r}', E) H_0(\underline{r}', E) \quad (15)$$

The region shown in Fig. 1d is arbitrarily divided into smaller regions (up to 20) bounded by at most five sides. A unit line source is set up at the centre of a region in direction ω , and the fraction of these neutrons which suffer their first collision in each region is computed. The technique is the same as used to compute transport kernels for the cylindrical cell. The symmetry of the kernels is used to check the accuracy of the kernels. Again the assumption is that the regions are small and that the centre is typical of the region. Of course, this is just the case (small rods in a tight lattice) we wish to study. When the regions are large, the assumption of a circular outer boundary is valid and the cylindrical cell can be used.

We now return to the integral eq. (8) and the treatment of the energy variable. We assume that above an energy $E^* = 0.785$ eV the spatial distribution of the flux is known (usually taken as flat), and that there are no chemical binding effects. The integral in (11) from E^* to ∞ can be computed and is the slowing down source into the thermal region. A multi-group method is used in the thermal region $E < E^*$. The range below E^* is divided into 30 velocity groups with spacing selected to give accuracy in regions of high absorption. All quantities are evaluated at the centre of each velocity group. The only averaging over groups is in the case of the kernel $\Sigma_s(E' \rightarrow E)$. Finally the integral equation (10 or 14 or 15), in its matrix form, is solved by power iteration using extrapolation and over relaxation to accelerate convergence. The final result is the average flux in each velocity group and each spatial volume element. A special edit programme computed the required integral and average properties of the flux.

A one-group equation can be obtained by integrating eq. (6) over energy. When average quantities are defined, terms arise like

$$\Sigma(\underline{r}, \underline{\Omega}) = \int_0^{E^*} dE \Sigma(\underline{r}, E) \phi(\underline{r}, E, \underline{\Omega}) / \int_0^{E^*} dE \phi(\underline{r}, E, \underline{\Omega}) \quad (16)$$

which depend on $\underline{\Omega}$. To obtain a tractable one-group equation it is necessary to assume that the cross-sections are independent of the orientation $\underline{\Omega}$, which is true only when the flux at all energies has the same angular distribution. The one-group equation is then

$$\underline{\Omega} \cdot \nabla \phi(\underline{r}, \underline{\Omega}) + \Sigma(\underline{r}) \phi(\underline{r}, \underline{\Omega}) = S(\underline{r}, \underline{\Omega}) + \int d\underline{\Omega}' \Sigma_s(\underline{r}, \underline{\mu}) \phi(\underline{r}, \underline{\Omega}') \quad (17)$$

where

$$\phi(\underline{r}, \underline{\Omega}) = \int_0^{E^*} dE \phi(\underline{r}, E, \underline{\Omega}) \quad (18)$$

$$S(\underline{r}, \underline{\Omega}) = \int_0^{E^*} dE S(\underline{r}, E, \underline{\Omega}) \quad (19)$$

$$\Sigma(\underline{r}) = \int_0^{E^*} dE \Sigma(\underline{r}, E) \int d\Omega \phi(\underline{r}, E, \underline{\Omega}) / \int d\Omega \phi(\underline{r}, \underline{\Omega}) \quad (20)$$

$$\Sigma_S(\underline{r}, \mu_0) = \int_0^{E^*} dE \Sigma_S(\underline{r}, E, \mu_0) \int d\Omega \phi(\underline{r}, E, \underline{\Omega}) / \int d\Omega \phi(\underline{r}, \underline{\Omega}) \quad (21)$$

$$\Sigma_S(\underline{r}, E, \mu_0) = \int_0^{E^*} dE' \Sigma_S(\underline{r}, E \rightarrow E', \mu_0) \quad (22)$$

The scalar flux has been used to average cross-sections. The finite cut-off E^* was chosen so that the integrated flux remains finite. An average velocity $\bar{v}(\underline{r})$ can also be defined.

$$\bar{v}(\underline{r}) = \int_0^{E^*} dE \int d\Omega \phi(\underline{r}, E, \underline{\Omega}) / \int_0^{E^*} \frac{dE}{v} \int d\Omega \phi(\underline{r}, E, \underline{\Omega}) \quad (23)$$

We cannot, however, obtain a one-group equation of the form of (17) for the neutron density, $N(\underline{r}, \underline{\Omega}) = \phi(\underline{r}, \underline{\Omega}) / \bar{v}(\underline{r})$, unless we make the further restriction that $\nabla \bar{v}(\underline{r}) = 0$, that is, the spectrum is the same everywhere in the cell. Thus, if we use average cross-sections in a one-group calculation, we obtain the neutron flux and not the neutron density.

Average cross-sections defined by (20-21) can be obtained from the multi-group THERMOS calculations. A variety of methods are then available for solving the one-group transport equation. The methods that we use here are:

- (1) S_n - The angular distribution of the flux is described by a set of linearly connected points. The particular code used is an S_4 cylinder version with reflecting boundary conditions. The code was obtained from Atomics International [18, 19].
- (2) P_n - The angular distribution of the flux is represented by a set of orthogonal polynomials. The particular code used is a P_3 cylinder version called CEPTRE written at Combustion Engineering [20]. The outer boundary condition is either reflecting (all odd components vanish at the boundary) or cell (zero normal derivative at the boundary and remaining odd components vanish).
- (3) ABH (AMOYAL, BENOIST, and HOROWITZ [8]). The net neutron current at the surface of the rod is computed from collision probabilities in the rod and from diffusion theory in the moderator. The resulting expression for the disadvantage factor is analytic. The same method for slab lattices has been developed by THEYS [21].

We conclude this section by restating the main restrictions of the THERMOS codes.

- (1) Scattering is isotropic in the cylindrical and two dimensional geometries, and linearly anisotropic in slab geometry.
- (2) Up to 20 spatial regions and 30 energy groups can be used.
- (3) The method of computing cylindrical transport kernels is accurate only for systems smaller than 10-20 mfp (mean free path).
- (4) The method of computing two dimensional transport kernels is accurate only for systems smaller than 5-10 mfp.

The water lattices which we will discuss here are small enough so that the last two restrictions are not serious.

SCATTERING MODELS FOR WATER

Four scattering models for water have been studied. They are:

- (1) Hydrogen gas - The motions of the protons in water are like a free gas [22] with no binding between protons or between protons and oxygen.
- (2) Mass 18 gas - The water molecules are treated as rigid structures and replaced by a gas of point particles with mass 18.
- (3) BROWN-ST. JOHN [23, 24]. The water molecules are treated as rigid structures free to rotate. The rotator is then replaced by a free point particle with an "effective rotational" mass of 1.88. A special cross-section is used, $\Sigma_s(v_r)$, where v_r is the relative velocity between the neutron and the particle. This cross-section contains parameters which can be adjusted so that the computed scattering cross-section, $\Sigma_s(v)$, can be fit to the experimental cross-section for water.
- (4) NELKIN water [25]. The dynamics of protons in water are described by three harmonic oscillators, two for the vibrational levels at 0.205 eV and 0.48 eV, and one for the hindered rotational level at 0.06 eV. The motions of the molecule are described by a mass 18 gas.

A fifth model is a modification of the hydrogen gas kernel which corrects the elastic scattering to fit the measured cross-section and uses an elastic anisotropic kernel given by the RADKOWSKY [26] prescription. This model is not used here. In all of the above cases, a free gas kernel with mass 16 was used to approximate the scattering from oxygen.

Each of the four kernels described above was used to compute the flux in a slab lattice. Only isotropic scattering was considered. A free gas kernel with mass 238 was used for the fuel. The disadvantage factor and average moderator speed are shown in Fig. 2. The slopes of the disadvantage factor curves are determined by the energy variation in the total cross-section predicted by each model. Thus the curves for Brown and St. John (BSJ) and Nelkin water are parallel and rise steeply as the average speed increases and the cross-sections increase. The gas law curves are flatter reflecting their smaller change in cross-section with decreasing energy. The level of the disadvantage curves is determined both by cross-section and by moderating ability characterized by the parameter M_2 (see Table IV). The BSJ kernel has the same cross-section variation as the Nelkin kernel but a larger

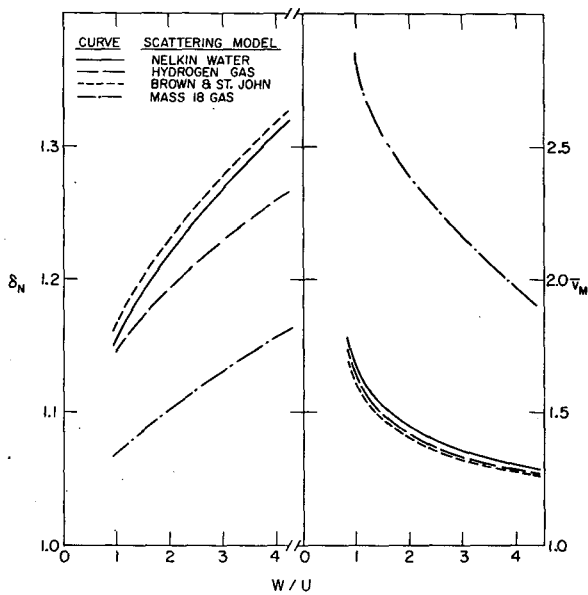


Fig. 2

Cell disadvantage factor and average neutron speed
in the moderator for the slab lattices
using various scattering kernels for water

TABLE IV
SECOND ENERGY MOMENT OF THE ISOTROPIC SCATTERING KERNEL

$M_2 = \frac{1}{T^2} \int_0^\infty dE' M(E') \int_0^\infty dE \Sigma_s(E' \rightarrow E) (E' - E)^2$	
Kernel	$M_2(\text{cm}^{-1})$
H gas	3.85
Mass 18	0.67
BSJ	5.23
Nelkin	3.34

M_2 leading to a softer spectrum and a larger disadvantage factor. The hydrogen gas kernel has nearly the same M_2 as the Nelkin kernel but a lower cross-section leading to a smaller disadvantage factor. The mass 18 kernel has a small M_2 leading to a large average speed and small disadvantage factor. The main conclusion from these results is that the disadvantage

factor, being a spatial effect, is much more sensitive to the total cross-section variation with energy than to the moderating ability of the kernel. BJS and Nelkin water (same cross-section, different M_2) are much closer than Nelkin water and hydrogen gas (same M_2 but different cross-sections).

We will use the Nelkin water kernel throughout the remainder of this paper. This choice is justified by the fact that this kernel predicts infinite medium spectra in good agreement with spectra measured by BEYSTER [12] over a wide range of poison types and concentrations. Secondly, this kernel gives diffusion coefficients and cooling coefficients [27] in good agreement with values recently measured by STARR and KOPPEL [28]. Detailed comparisons of the Nelkin kernel with measurements by EGELSTAFF [29] have been made by GOLDMAN [30]. Goldman finds that the model is in good agreement with the measurements at all but small values of the energy and momentum transfer. Fortunately, this region contributes little to the total cross-section or to the energy transfer moments. Finally, the Nelkin kernel gives a total cross-section which is in excellent agreement with the measured cross-section [13] over the entire thermal energy range.

The ratio of the average absorption cross-section of cadmium to the average absorption cross-section of boron in the moderator has been measured in several lattices [1]. These results plus some later unreported results are given in Table V along with the computed ratios using the Nelkin kernel. The measured and computed ratios are in good agreement but the temperature shifts derived from these ratios are quite different. The ΔT computed from Maxwellian averaged cross-sections is lower by nearly a factor of two than the ΔT computed from the shift of the peak of the computed spectrum. The $1/E$ tail of the spectrum increases the average boron cross-section relative to the average cadmium cross-section leading to a smaller ratio and an apparently smaller temperature shift.

TABLE V
CADMIUM-BORON CROSS SECTION RATIOS

Rod Diam Enrichment (in) %		W/U	T_m (°K)	$\frac{\sigma_{cd}}{\sigma_B} +$ exp.	$\frac{\sigma_{cd}}{\sigma_B} ++$ theory	ΔT^* exp.	ΔT^{**} theory
0.387	1.3	2	290	$4.98 \pm .30$	5.00	35 ± 17	63
		3	290	$5.26 \pm .35$	4.82	57 ± 21	43
		4	290	$4.74 \pm .30$	4.72	14 ± 17	28
	1.15	3	290	$4.63 \pm .50$	4.80	4 ± 30	38
		4	290	$4.62 \pm .50$	4.70	3 ± 30	29
		3	297	$4.73 \pm .50$	4.81	5 ± 30	22

+ An uncertainty of $\pm 10\%$ was assigned to each measurement.

++ Based on 2200 m/s cross-sections of $\sigma_B = 755b$ and $\sigma_{cd} = 2450b$.

** Computed from Maxwellian averaged cross-sections.

** Computed from the most probable speed.

ANISOTROPIC SCATTERING

The THERMOS codes for cylinders and two dimensional geometry are restricted to isotropic scattering. We next investigate the importance of anisotropic scattering and propose an effective isotropic scattering kernel which will partially account for anisotropic scattering. The neutron density was computed on the slab lattices using the Nelkin kernel for the following cases.

- (1) Complete linear anisotropic scattering using both a $\Sigma_{s_0}(E' \rightarrow E)$ and $\Sigma_{s_1}(E' \rightarrow E)$ kernel in eq. (11). This is the reference case and the other cases will be compared with it.
- (2) Isotropic scattering only.
- (3) Transport correction using an isotropic $\Sigma_{tr}(E' \rightarrow E)$ defined by

$$\Sigma_{tr}(E' \rightarrow E) = \Sigma_{s_0}(E' \rightarrow E) - \Sigma_{s_1}(E) \delta(E - E') \quad (24)$$

- (4) Effective isotropic kernel $\Sigma_s^*(E' \rightarrow E)$ defined by

$$\Sigma_s^*(E' \rightarrow E) = \Sigma_{s_0}(E' \rightarrow E) - c \delta(E - E') \quad (25)$$

where c is a constant to be determined. This approximation is an attempt to correct for only the elastic anisotropic scattering which is roughly constant in the thermal energy region.

For each of these cases the moderator disadvantage factor minus one (which should be proportional to Σ_s) and the average neutron speed in the moderator were computed. Define

$$\epsilon_{F_m} = \frac{(F_m - 1) - (F_m - 1)_R}{(F_m - 1)_R} \times 100 \quad (26)$$

$$\epsilon_{\bar{v}_m} = \frac{(\bar{v}_m - 1.128) - (\bar{v}_m - 1.128)_R}{(\bar{v}_m - 1.128)_R} \times 100 \quad (27)$$

where R denotes the reference case 1. The quantities are shown in Fig. 3. We first note that the isotropic scattering approximation is in error by at most 5% in $F_m - 1$ and 0.5% in $\bar{v}_m - 1.128$. The error in the cell disadvantage factor is about 1.5%. The transport approximation overestimates the effect of anisotropic scattering by nearly a factor of two for tight lattices but is quite accurate at large W/U ratios.

Several curves for $\sigma_{s_1}(E' \rightarrow E)$ were drawn and the area under the elastic peak was estimated to be about 5.6b. Thus a value of $c = 5.6 \times 0.0669 = 0.375 \text{ cm}^{-1}$ was tried. The results are also shown in Fig. 3 and it is apparent that in the entire range of W/U ratios the error in F_m and $\bar{v}_m$ are very small. This effective isotropic kernel defined by (25) with $c = 0.375 \text{ cm}^{-1}$ will be used throughout the remainder of the paper for cylinder calculations.

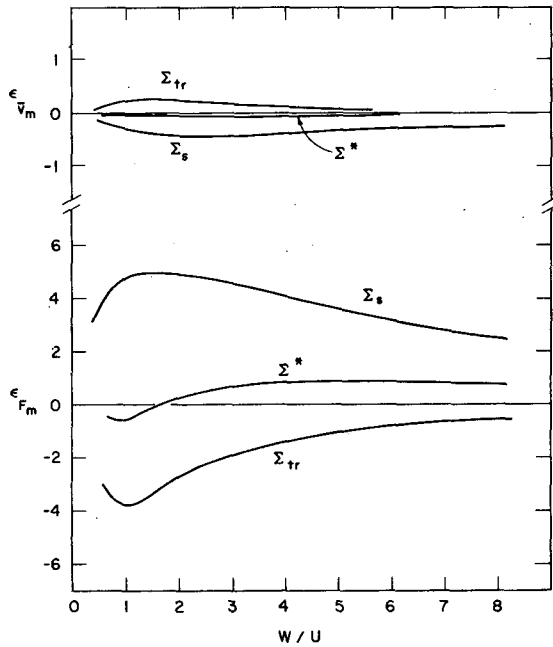


Fig. 3

Percentage errors in the moderator disadvantage factor
and average neutron speed in the moderator
for various transport corrections

The errors in $\bar{v}_m$ are very small for all of the transport approximations because the off-diagonal elements of $\Sigma_{s0}(E' \rightarrow E)$ were not changed. Another type of transport correction

$$\Sigma_{tr}(E' \rightarrow E) = \Sigma_{s0}(E' \rightarrow E) - \Sigma_{s1}(E' \rightarrow E) \quad (28)$$

previously used by this author [31] was found to be very unsatisfactory because of the large errors in $\bar{v}_m$ produced by changing the off-diagonal elements of the scattering matrix.

In summary, the anisotropic scattering from water produces a negligible error in the shape of the spectrum and a small error in the disadvantage factor which can be easily corrected by use of a transport correction.

The reason that these anisotropic scattering effects are small here is that the gradients in the cell are small. In other situations, such as the flux from a point source, leakage into a vacuum, or the flux incident on a control surface, the gradients will be large and anisotropic scattering will be a significant effect. The simple prescriptions given here will not be accurate for these cases.

INTRACELL NEUTRON DENSITY DISTRIBUTIONS IN THE SLAB LATTICES

The neutron density distributions in the slab lattices were computed by the slab version of THERMOS with linear anisotropic scattering. Dysprosium activation (including activation above $E^* = 0.785$ eV) was computed using the dysprosium cross-section measured by SHER and WEINSTOCK [32]. These dysprosium activation distributions differed by less than 0.5% from $1/v$ activation distributions. The calculated and measured intracell distributions are shown in Fig. 4 and the resulting disadvantage factors are shown in Fig. 5. The agreement in the shapes of the curves is good but there is a systematic discrepancy of about 4% in the disadvantage factor. Many attempts have been made to resolve this discrepancy. From the point of view of the theory, the only uncertainty is in the scattering model for water. If we correlate the disadvantage factor with M_2 for models which have the same $\Sigma_S(E)$, we would need an M_2 which is about $\frac{1}{3}$ of the M_2 for Nelkin water to reproduce the experiments. A change in M_2 this large is clearly unreasonable in view of the excellent diffusion cooling coefficient and infinite medium spectra given by the Nelkin kernel. If the $\Sigma_{S1}(E' \rightarrow E)$ is at fault, then from the results of the last section, the $\Sigma_{S1}(E' \rightarrow E)$ would need to be twice as large as given by the Nelkin kernel to reproduce the experiments. Again a change this large is unreasonable since it would lead to $\bar{\mu} > 1$ throughout much of the thermal energy range. Indeed, there is some indication

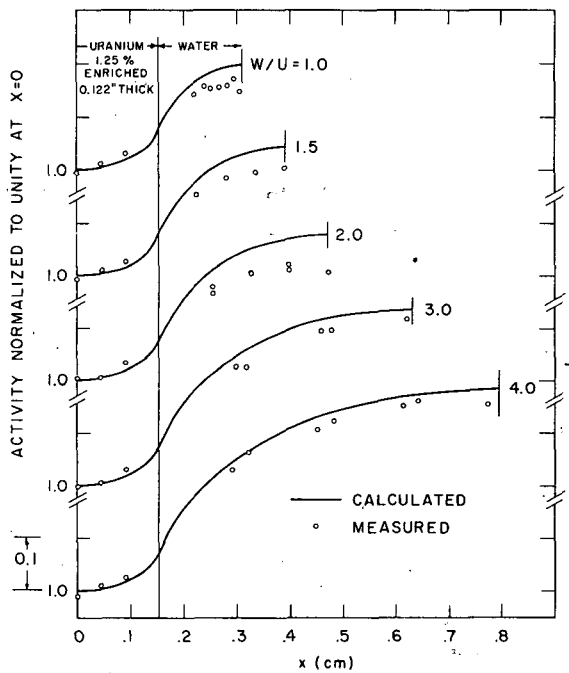


Fig. 4

Dysprosium foil activation traverses in the slab lattices

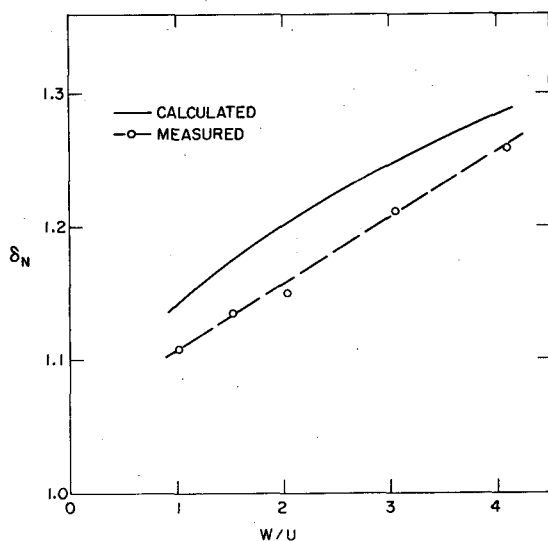


Fig. 5

Neutron density disadvantage factors for the slab lattices

that the $\Sigma_{S_1}(E' \rightarrow E)$ given by the Nelkin kernel is already somewhat too large. We conclude that there is little more that can be done to improve the theory in the case of these slab lattices. Truncation error in the calculation has been checked and found to be insignificant.

The effects of leakage from the finite lattice, contamination of the water, variation in enrichment and lattice dimensions have been investigated theoretically and produce changes in the disadvantage factor at least an order of magnitude smaller than the 4% discrepancy. Experimental studies are now in progress at BNL to determine the perturbation effect of the 7 mil dysprosium aluminium foils and possible self-shielding corrections.

While it may appear that a 4% discrepancy in the disadvantage factor is satisfactory, the discrepancy in $\delta - 1$ is about 20% and this figure is the test of the theory.

THE CYLINDRICAL CELL EFFECT

NEWMARCH [33] has pointed out that the disadvantage factor in a cylindrical cell is always an overestimate of the disadvantage in a square or triangular cell since there are paths in the moderator region where neutrons can never enter the fuel region. These neutrons stay in the moderator, increase the average moderator flux, and increase the disadvantage factor. This effect has also been observed in S_n calculations by THIE [34]. The effect is small when diffusion theory is used [35]. Disadvantage factors for the metal rod lattices were computed using the cylindrical cell version of THERMOS and are shown in Fig. 6. The curves for the small rod sizes exhibit a minimum where the water region is about 0.5 mean free paths thick. The disadvantage factors for the 1.0% enriched, 0.25 in rod are also

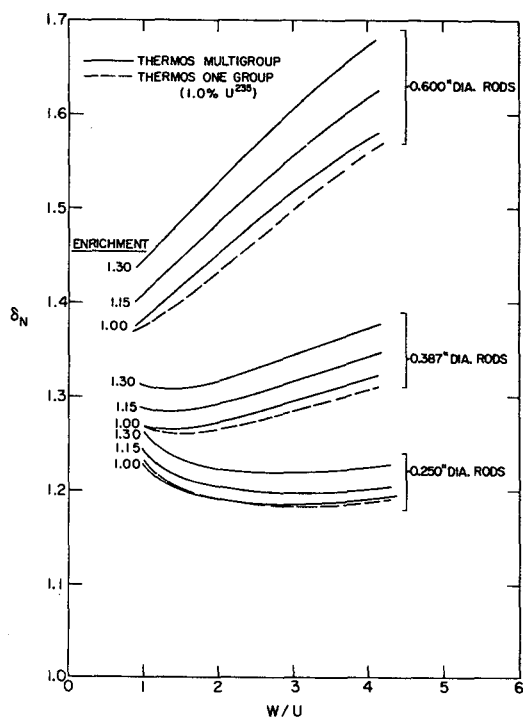


Fig. 6

Neutron density disadvantage factors for the metal rod lattices

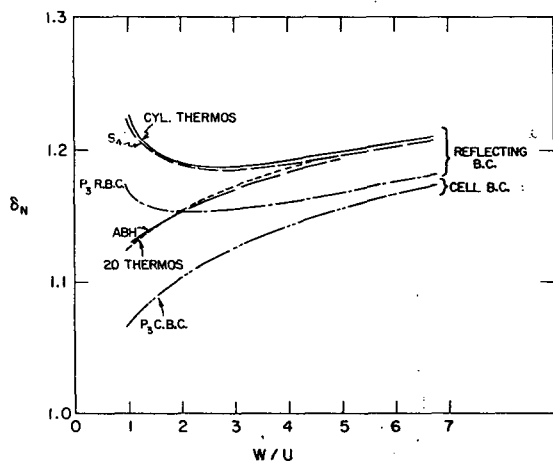


Fig. 7

Neutron density disadvantage factors for the 0.250-in diam
1.00% enriched uranium metal rod lattices.

shown in Fig. 7 for a larger range of W/U ratios. Thus, not only does the cylindrical reflecting boundary condition lead to an overestimate of the disadvantage factor, but when the water thickness becomes very small the disadvantage factor actually increases as the lattice is tightened.

We would like to use some of the one-group transport codes to investigate this effect further. We digress for a moment to observe that when region-wise average cross-sections are computed from the multi-group results and are used in a one-group THERMOS calculation, the disadvantage factors (Fig. 6) are in good agreement with those from the multi-group calculations. The error in the one-group approximation is very small for the 0.25 in rod (where the spectrum is nearly uniform across the cell) and increases to about 1.5% for the 0.600 in rod but the character and shape of curves are preserved.

The disadvantage factors from one-group S_4 , P_3 , and ABH calculations and from the multi-group 2D THERMOS calculations are also shown in Fig. 7. The cylindrical cell effect is again observed in the S_4 and P_3 (reflecting boundary conditions) calculations but not in the ABH and P_3 (cell boundary conditions) calculations. Fuel and moderator disadvantage factors are shown in Fig. 8. We observe that F_m is insensitive to the outer boundary condition and that most of the cylindrical cell effect is due to the increased dip in the fuel rod. Disadvantage factors for the 1.0% enriched, 0.60 in rods are shown

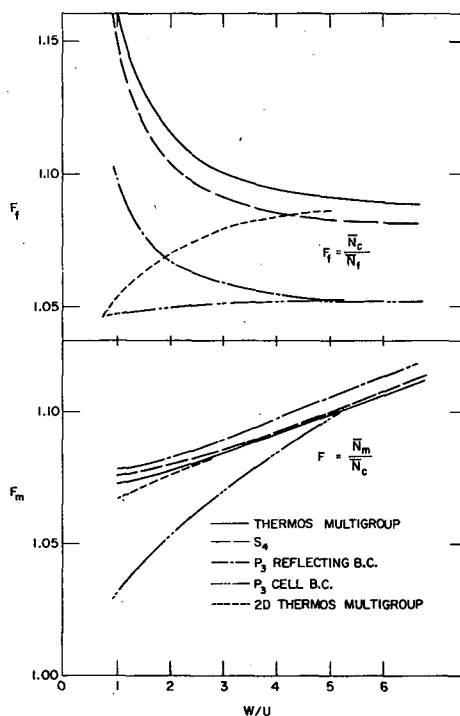


Fig. 8

Fuel and moderator disadvantage factors for the 0.250-in diam.
1.00% enriched uranium metal rod lattices

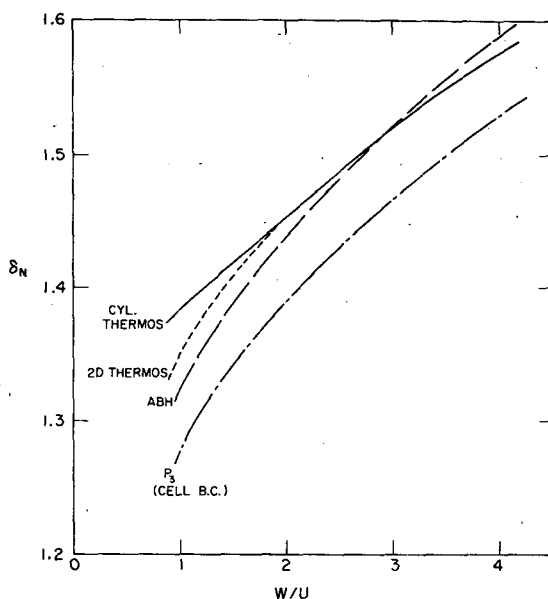


Fig. 9

Neutron density disadvantage factors for the 0.600-in diam.
1.00% enriched uranium metal rod lattices.

in Fig. 9. Here, the water region is thick and the cylindrical cell effect is small. For both rod sizes the error due to the cylindrical cell approximation was found to be proportional to $\exp - 10t_w$, where t_w is the thickness in cm of the cylindrical water region.

At the present time, only a few lattices have been analysed with the 2D THERMOS code. However, the ABH method is seen to give remarkably good agreement with these 2D THERMOS disadvantage factors for the small rods and is in error by less than 1.5% for the large rods. A few calculations at other enrichments also support this agreement. We will then use the ABH method with THERMOS averaged cross-sections to give a complete set of δ and f for all the lattices. The average neutron density in the cladding in the ABH calculations is assumed to be the same as at the rod surface and the cladding is treated as a void in the calculation of the disadvantage factor.

The range of rod sizes, enrichments, and volume ratios investigated is not large enough to indicate how accurate the ABH method is outside this range. Thus we use the ABH method only for interpolation and not extrapolation. Three instances can be cited where the ABH method gives poor results. The surface to average flux in the rod is considerably smaller than predicted by THERMOS, leading to an underestimate of the absorptions in the cladding. This is not serious for aluminium cladding but is important when stainless steel cladding is used. The ABH method does not account for the transmission losses in the cladding. Thus the disadvantage factors are too small for the oxide rods. Finally, when applied to thin slab lattices,

the surface to average flux in the fuel slab is considerably larger than predicted by THERMOS.

CORRELATION OF SPECTRAL MOMENTS

We have seen in the preceding section that a one-group calculation with region-wise averaged cross-sections gives disadvantage factors in good agreement with the corresponding multi-group calculation. Indeed, the ABH method with region-wise averaged cross-sections agrees very well with our most sophisticated calculations, the 2D THERMOS results. We now use the many multi-group calculations to give prescriptions for computing average cross-sections. All of the averages will be taken to a cut-off energy of 0.785 eV.

Most absorption cross-sections are nearly $1/\bar{v}$, so that the most useful single parameter characterizing the spectrum is the average speed in the region, $\bar{v}$. The average cross-section is then $\Sigma_a(2200)/\bar{v}$. The average speed, $\bar{v}$, will increase as the absorption in the cell increases, and will decrease as the thermalizing ability of the moderator increases. Other workers [36, 37, 38] have characterized this relationship by taking the neutron temperature to be proportional to $\Sigma_a/\xi\Sigma_s$, where the 2200 m/s value is used for Σ_a , and the high energy values are used for $\xi\Sigma_s$. Three of the scattering models used (H gas, Nelkin, and BSJ) all have the same high energy $\xi\Sigma_s$, yet behave differently at thermal energies. A parameter which does distinguish between these models is the second energy transfer moment M_2 . We first attempt to correlate the parameter τ_m

$$\tau_m = (\bar{v}_m / 1.128)^2 \quad (29)$$

with a parameter θ' defined by

$$\theta' = [\Sigma_{am}^0 + \Sigma_{ac}^0 (V_c/V_m) + \Sigma_{af}^0 (V_f/V_m)] / M_2 \quad (30)$$

where subscripts m, c, and f denote moderator, clad, and fuel. Superscript zero denotes 2200 m/s values of the absorption cross-section Σ_a . V is the volume, and M_2 refers to the moderator (we ignore M_2 in the fuel and cladding). This correlation is shown in Fig. 10 for the metal rods. The spread of the results can be reduced by a factor of ten if we redefine θ to include both the non $1/v$ factor in the fuel and the average neutron densities.

$$\theta = [\Sigma_{am}^0 + \Sigma_{ac}^0 \frac{V_c}{V_m} \frac{\bar{N}_c}{\bar{N}_m} + \Sigma_{af}^0 g_a \frac{V_f}{V_m} \frac{\bar{N}_f}{\bar{N}_m}] / M_2 \quad (31)$$

This correlation is shown in Fig. 11 for all lattices studied. The values of τ_m for all geometries and all scattering models (except BSJ) are very close to the line

$$\tau_m = 1 + 9.10 \theta \quad (32)$$

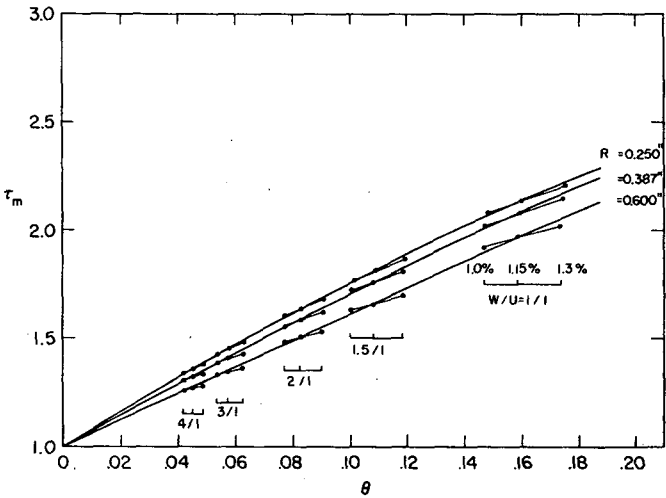


Fig. 10

Moderator neutron spectrum hardness parameter τ_m for the metal rod lattices.
The parameter θ is computed with no hardening of the cross-sections
and a disadvantage factor of unity

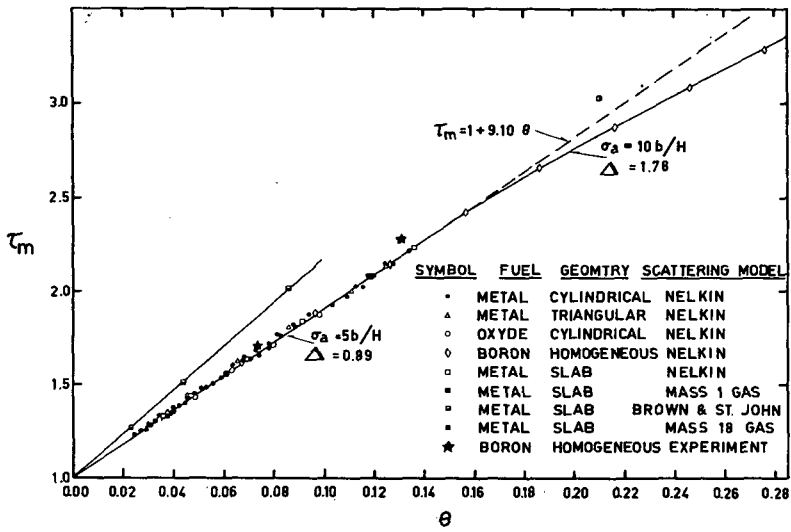


Fig. 11

Moderator neutron spectrum hardness parameter τ_m for all lattices.
The parameter θ is computed from hardened cross-sections
and the computed disadvantage factor

The discrepancy in the case of the BSJ model is not understood. The coefficient 9.10 is much larger than appears in a similar expression using $\theta = \Sigma_a / \xi \Sigma_s$ because of the difference in definition of θ and because a very high energy cut-off was used to define $\bar{v}_m$. Two experimental values of τ_m (computed from the homogeneous boric acid spectra measured by BEYSTER [12]) are included in Fig. 11. The M_2 used in the parameter θ is the value given by the Nelkin model, $M_2 = 3.34 \text{ cm}^{-1}$. These experimental values would lie on the line $\tau_m = 1 + 9.10 \theta$ if the value $M_2 = 3.07 \text{ cm}^{-1}$ is used. The indication is that M_2 given by the Nelkin model is about 8% too large.

It is unfortunate that it is necessary to use a parameter which includes the unknown neutron densities in each region. However, these quantities can be estimated, average cross-sections computed, and the neutron density distribution in the cell calculated. A second iteration using the calculated neutron densities will be quite accurate.

Neutrons entering the fuel from the moderator are absorbed in the fuel and the fuel spectrum is harder than the moderator spectrum. To a first approximation, this increased hardening of the spectrum, $\bar{v}_f - \bar{v}_m$, is proportional to the blackness of the rod, β , the probability that a neutron incident on the rod will be absorbed in the rod. The blackness is a function of the angular distribution of the neutrons incident on the rod. For simplicity we will use the blackness for an isotropic incident flux [39, 40].

$$\beta = \frac{2\Sigma_a^0 R_0 (1 - P_c)}{\Sigma_s \left(1 - \frac{\Sigma_s}{\Sigma} P_c\right)} \quad (33)$$

where R_0 is the radius of the rod and P_c is the collision probability tabulated in [41]. Cross-sections evaluated at 2200 m/s have been used. The values of $(\bar{v}_f - \bar{v}_m)/\beta$ are shown in Fig. 12 as a function of the water thickness, t_w . The spread of values here is much greater than for $\bar{v}_m$, but all curves for the rods appear to be approaching $\bar{v}_f - \bar{v}_m/\beta = 0.30$ as t_w approaches zero and for large t_w . The two dimensional calculations also support this value.

The average speed in the clad can be represented by

$$\bar{v}_c = \frac{1}{2} (\bar{v}_m + \bar{v}_c) \quad (34)$$

an expression which is good to better than 1% for all cases studied.

The remaining parameters needed for one-group calculations are g_a and g_f , the non $1/v$ absorption and fission factors for U^{235} , and the average scattering cross-section of water. These quantities are plotted in Fig. 13 against the $\bar{v}$ of the region. Values of η derived from the g_a and g_f curves are shown in Fig. 14. The scattering cross-sections of the fuel and cladding have been taken as independent of energy.

From the above discussion and results, it is possible to obtain suitable one-group cross-sections for use in transport codes to compute the spatial distribution of the flux. Since most of these cross-sections depend on the unknown spatial distribution, it is necessary to do one iteration.

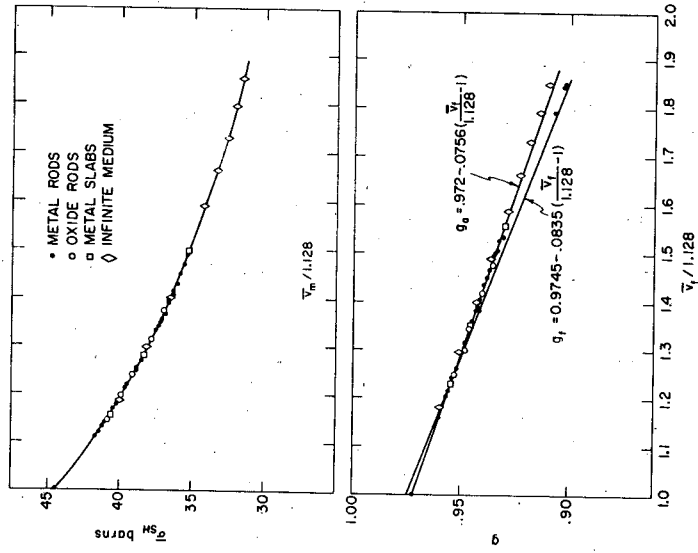


Fig. 13

The average hydrogen scattering cross-section and non 1/v factors for U^{235} as a function of the average neutron speed in the region

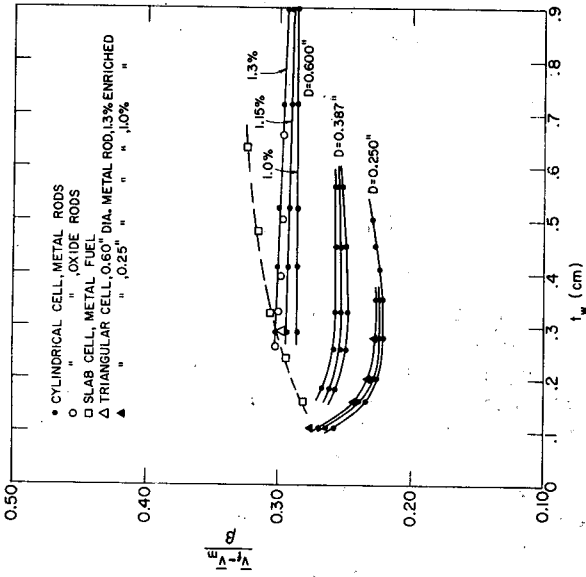


Fig. 12

The parameter $(V_f - V_m)/s$ for all lattices as a function of the water channel thickness

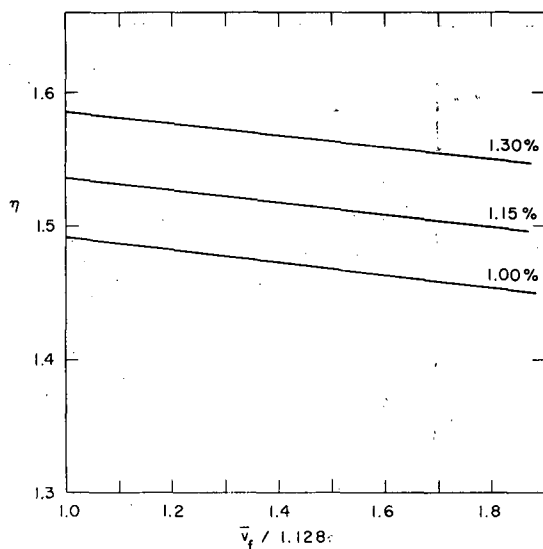


Fig. 14

Average η for the metal rods as a function of the average neutron speed in the fuel for 1.00, 1.15 and 1.30% enriched uranium

COMPARISON OF COMPUTED AND MEASURED DISADVANTAGE FACTORS FOR CYLINDRICAL LATTICES

A tabulation of our results for the neutron density disadvantage factor, δ_N , thermal utilization, f , and η is given in Tables VI-X. BNL experimental values are also included. The calculated non $1/v$ factors for the fuel were used in the calculation of the experimental f .

The best theoretical method is the 2D THERMOS calculation but only a few cases have been run at present. Since the cylindrical cell error decreases rapidly with increasing water channel thickness, the cylinder THERMOS calculations are reliable for large W/U ratios and large rod sizes. Amouyal's method gives δ_N in good agreement with our best THERMOS calculations for all rod sizes and enrichments and is used as an interpolation to cases not investigated with the 2D THERMOS. The disadvantage factors for 1% enriched metal rods are shown in Fig. 15 along with the experimental points. The theory predicts δ_N about 5% larger than given by the experiment. A more extensive comparison is given in Table XI where the ratios of the theoretical δ_N to experimental δ_N are listed. There is no apparent trend with rod size, enrichment, or W/U ratio. The average ratio is 1.058 for the metal rods, 1.062 for the oxide rods, and 1.034 for the metal slabs. Disadvantage factors measured by KLEIN [42] at BAPL are given in Table XII and are in every case larger than the BNL values.

If this systematic discrepancy in δ_N is experimental in origin, two possible causes are the self-shielding of the dysprosium-aluminium and

TABLE VI
0.250 IN DIAM URANIUM METAL RODS

Enrichment % U ²³⁵	w/u	δN				f				η		
		Cylinder Thermos	2D Thermos	P ₃ Cell	ABH	Exp.	Cylinder Thermos	2D Thermos	P ₃ Cell	ABH	Exp.	Cylinder Thermos
1.00	1.0	1.226	1.127	1.078	1.129	-	0.9267	0.9321	0.9342	0.9323	-	1.468
	1.5	1.200	1.141	1.091	1.141	1.105	0.9038	0.9079	0.9111	0.9085	0.9107	1.474
	2.0	1.190	1.149	1.102	1.152	1.091	0.8816	0.8853	0.8891	0.8856	0.8903	1.477
	3.0	1.187	1.175	1.121	1.169	1.114	0.8397	0.8412	0.8466	0.8424	0.8452	1.480
	4.0	1.192	-	1.136	1.183	1.132	0.8005	-	0.8077	0.8025	0.8087	1.482
1.15	1.0	1.242	-	1.083	1.138	-	0.9310	-	0.9385	0.9366	-	1.512
	1.5	1.214	-	1.097	1.151	1.081	0.9095	-	0.9168	0.9142	0.9182	1.518
	2.0	1.204	-	1.109	1.162	1.087	0.8885	-	0.8957	0.8924	0.8979	1.521
	3.0	1.201	-	1.130	1.181	1.128	0.8485	-	0.8559	0.8512	0.8563	1.524
	4.0	1.207	-	1.146	1.196	1.146	0.8109	-	0.8182	0.8129	0.8189	1.526
1.30	1.0	1.262	-	1.090	1.149	-	0.9358	-	0.9430	0.9414	-	1.560
	1.5	1.233	-	1.105	1.164	-	0.9157	-	0.9230	0.9204	-	1.566
	2.0	1.222	-	1.119	1.176	-	0.8960	-	0.9033	0.9000	-	1.570
	3.0	1.219	-	1.142	1.197	-	0.8583	-	0.8655	0.8612	-	1.572
	4.0	1.225	-	1.160	1.214	-	0.8227	-	0.8298	0.8247	-	1.574

TABLE VII
0.387 IN DIAM URANIUM METAL RODS

Enrichment % U ²³⁵	w/u	δ_N					f					
		Cylinder Thermos	2D Thermos	P ₃ Cell	ABH	Exp.	Cylinder Thermos	2D Thermos	P ₃ Cell	ABH	Exp.	Cylinder Thermos
1.00	1.0	1.268	-	1.144	1.197	1.101	0.9309	-	0.9367	0.9368	0.9390	1.468
	1.5	1.265	-	1.176	1.225	1.149	0.9054	-	0.9112	0.9091	0.9130	1.474
	2.0	1.271	-	1.201	1.249	1.183	0.8805	-	0.8861	0.8829	0.8879	1.477
	3.0	1.285	-	1.240	1.287	1.249	0.8329	-	0.8388	0.8343	0.8383	1.480
	4.0	1.319	-	1.270	1.318	1.243	0.7887	-	0.7950	0.7895	0.7988	1.482
1.15	1.0	1.287	-	1.154	1.210	1.142	0.9348	-	0.9406	0.9394	0.9415	1.512
	1.5	1.284	-	1.188	1.241	1.182	0.9106	-	0.9165	0.9138	0.9169	1.518
	2.0	1.291	-	1.216	1.267	1.193	0.8869	-	0.8929	0.8893	0.8946	1.521
	3.0	1.316	-	1.258	1.308	1.246	0.8413	-	0.8471	0.8427	0.8487	1.524
	4.0	1.342	-	1.290	1.341	1.302	0.7986	-	0.8051	0.7994	0.8039	1.526
1.30	1.0	1.311	-	1.167	1.227	1.161	0.9392	-	0.9450	0.9431	0.9456	1.560
	1.5	1.308	-	1.204	1.261	1.167	0.9165	-	0.9224	0.9197	0.9245	1.566
	2.0	1.316	-	1.234	1.290	1.210	0.8941	-	0.8997	0.8966	0.9019	1.570
	3.0	1.345	-	1.281	1.335	1.272	0.8507	-	0.8570	0.8522	0.8577	1.572
	4.0	1.373	-	1.316	1.372	1.307	0.8097	-	0.8164	0.8105	0.8175	1.574

TABLE VIII
0.600 IN DIAM URANIUM METAL RODS

Enrichment % U ²³⁵	w/U	δ_N					f					η	
		Cylinder Thermos	2D Thermos	P ₃ Cell	ABH	Exp.	Cylinder Thermos	2D Thermos	P ₃ Cell	ABH	Exp.	Cylinder Thermos	
1.00	1.0	1.383	1.352	1.283	1.324	1.278	0.9214	0.9309	0.9346	0.9325	0.9345	1.468	
	1.5	1.417	1.409	1.345	1.388	1.321	0.8993	0.9001	0.9041	0.9015	0.9055	1.474	
	2.0	1.452	1.449	1.395	1.439	1.356	0.8696	0.8702	0.8741	0.8712	0.8776	1.477	
	3.0	1.519	1.520	1.473	1.523	1.448	0.8130	0.8132	0.8183	0.8131	0.8205	1.480	
	4.0	1.576	-	1.536	1.590	1.519	0.7605	-	0.7655	0.7594	0.7675	1.482	
1.15	1.0	1.410	1.375	1.302	1.346	1.270	0.9331	0.9346	0.9378	0.9362	0.9393	1.512	
	1.5	1.447	-	1.370	1.415	1.289	0.9043	-	0.9090	0.9066	0.9137	1.518	
	2.0	1.485	-	1.423	1.471	1.383	0.8757	-	0.8804	0.8774	0.8835	1.521	
	3.0	1.558	-	1.508	1.562	1.449	0.8209	-	0.8256	0.8210	0.8315	1.524	
	4.0	1.619	-	1.576	1.634	1.515	0.7697	-	0.7742	0.7685	0.7815	1.526	
1.30	1.0	1.445	-	1.327	1.374	1.285	0.9373	-	0.9421	0.9404	0.9438	1.560	
	1.5	1.485	-	1.401	1.450	1.354	0.9099	-	0.9149	0.9122	0.9172	1.566	
	2.0	1.528	-	1.460	1.511	1.427	0.8926	-	0.8977	0.8943	0.8999	1.570	
	3.0	1.608	-	1.553	1.611	1.482	0.8298	-	0.8345	0.8299	0.8412	1.572	
	4.0	1.675	-	1.628	1.692	1.582	0.7801	-	0.7850	0.7789	0.7900	1.574	

TABLE IX
0.444 IN DIAM URANIUM OXIDE RODS

Enrichment % U ²³⁵	W/UO ₂	δN				f				η		
		Cylinder Thermos	2D Thermos	P ₃ Cell	ABH	Exp.	Cylinder Thermos	2D Thermos	P ₃ Cell	ABH	exp.	Cylinder Thermos
3.00	1.291	1.325	-	1.242	1.258	1.213	0.8001	-	0.8065	0.8074	0.8157	1.814
	1.600	1.339	-	1.268	1.280	1.249	0.7876	-	0.7936	0.7946	0.8017	1.816
	2.050	1.368	-	1.305	1.312	1.278	0.7694	-	0.7751	0.7761	0.7835	1.819
	2.800	1.396	-	1.344	1.347	1.326	0.7398	-	0.7450	0.7464	0.7523	1.820
	4.000	1.442	-	1.398	1.396	1.357	0.6946	-	0.6996	0.7011	0.7100	1.822

TABLE X
0.122 IN THICK URANIUM METAL SLABS

Enrichment % U ²³⁵	W/U	δ_N				
		f		η		
		Thermos	Exp.	Thermos	Exp.	Thermos
1.25	1.025	1.145	1.108	0.9515	0.9545	1.544
	1.537	1.178	1.135	0.9276	0.9318	1.551
	2.049	1.205	1.150	0.9042	0.9100	1.554
	3.074	1.249	1.211	0.8590	0.8649	1.558
	4.098	1.285	1.258	0.8166	0.8221	1.560

TABLE XI
RATIO OF THERMOS δ_N TO EXPERIMENTAL δ_N

Fuel	% U ²³⁵	1.0	1.5	W/U 2.0	3.0	4.0
0.250 in metal rod	1.00	-	1.033	1.053	1.055	1.053
	1.15	-	1.065	1.069	1.047	1.053
	1.30	-	-	-	-	-
0.387 in metal rod	1.00	-	-	-	1.037	1.061
	1.15	-	-	-	1.056	1.031
	1.30	-	-	-	1.057	1.050
0.600 in metal rod	1.00	1.058	1.067	1.069	1.050	1.038
	1.15	1.083	-	1.074	1.075	1.069
	1.30	-	-	1.071	1.085	1.059
0.444 in oxide rod	3.00	-	-	1.070	1.053	1.063
0.122 in metal slab	1.25	1.033	1.038	1.048	1.031	1.021

dysprosium-polyethylene foils used to measure the intracell neutron densities, and the pickup of fission products on the surface of the foils in the fuel region. These effects are currently being investigated at BNL.

CONCLUSIONS

We summarize this work with the following remarks.

- (1) Anisotropic scattering in water can be accurately approximated by an effective isotropic scattering kernel.

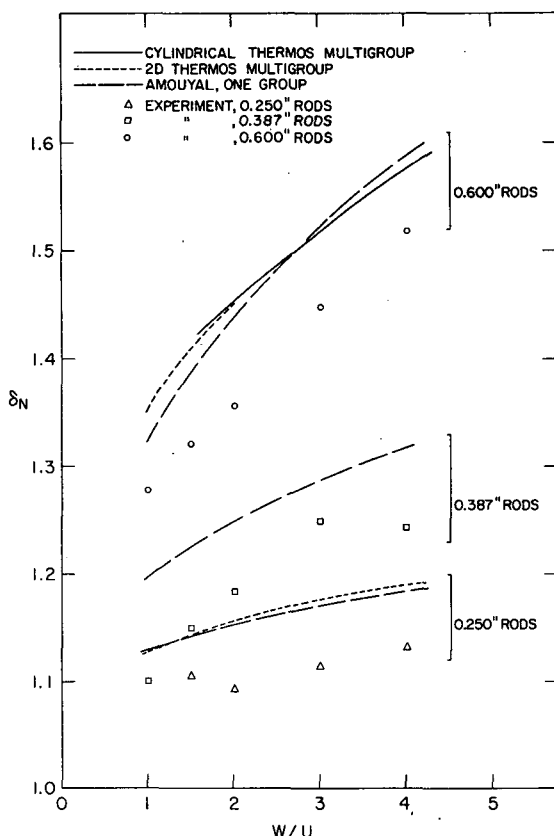


Fig. 15

Neutron density disadvantage factors for the
1% enriched uranium metal rod lattices

- (2) The cylindrical cell approximation can lead to large errors in the disadvantage factor for small rods and small W/U ratios. The error decreases rapidly with increasing thickness of the water region.
- (3) The best computed disadvantage factors are about 6% larger than the measured values for the rod lattices and 3% for the slab lattices. Studies with other scattering models show that the Nelkin water kernel would have to be changed by an unrealistic amount to explain a discrepancy this large.
- (4) It is possible to correlate the speed moments of the spectrum with quantities like the average absorption cross-section, the second energy transfer moment of the scattering kernel, M_2 , and the blackness of the rod. This correlation is insensitive to cell geometry. From these moments one-group cross-sections can be obtained. The resulting one-group disadvantage factors agree well with those obtained from the multi-group calculations.

TABLE XII
COMPARISON OF COMPUTED δ_N WITH VALUES
MEASURED AT BNL AND BAPL

Rod Diam (in)	% U ²³⁵	W/U	δ_N				$\frac{\delta_{\text{Theory}}}{\delta_{\text{BNL Exp.}}}$	$\frac{\delta_{\text{Theory}}}{\delta_{\text{BAPL Exp.}}}$
			Theory	BNL Exp.	BAPL	Exp. ¹		
0.387	1.30	2.0	1.290	1.210		1.21	1.066	1.066
		3.0	1.335	1.272		1.31	1.050	1.019
0.600	1.15	2.0	1.471	1.383		1.45	1.064	1.014
		3.0	1.562	1.449		1.45	1.077	1.077
0.600	1.30	1.5	1.450	1.354		1.39	1.071	1.043
		2.0	1.511	1.427		1.45	1.059	1.042
		3.0	1.611	1.482		1.49	<u>1.087</u>	<u>1.081</u>
						Average	1.067	1.048

¹ These values have an uncertainty of $\pm .04$

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DEVELOPMENTS IN THE REACTOR PHYSICS OF PLUTONIUM IN LIGHT WATER

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Abstract — Résumé — Аннотация — Resumen

DEVELOPMENTS IN THE REACTOR PHYSICS OF PLUTONIUM IN LIGHT WATER. The production of plutonium as a by-product of the operation of power reactors using slightly enriched uranium kindled an interest in the use of such material as a thermal reactor fuel. Because of the chain of plutonium isotopes and their resonance structure, the reactor physics is more complex than is the case with uranium. The present cross-section data are also less satisfactory. Some data on lattices of Pu-Al alloy rods in light water are presented and some comparisons with data on aqueous solutions of plutonium are drawn. Additional recent data on more concentrated solutions are presented and compared with the results of multi-group calculations. Results of a temperature coefficient measurement on Pu-Al fuel rods are also presented.

A burn-up code is described and some results of its application to a light water reactor are given. Efforts to develop improved codes are described with emphasis on the treatment of the spectrum problem with some examples of calculated spectra for water moderated plutonium fuel. Plans for future experimental work on plutonium fuelled light water lattices are presented.

PROGRÈS DE LA PHYSIQUE DES RÉACTEURS POUR LES RÉSEAUX PLUTONIUM-EAU LÉGÈRE. La production accessoire de plutonium lors du fonctionnement des réacteurs de puissance qui utilisent de l'uranium faiblement enrichi a suscité un intérêt pour l'emploi de cette matière comme combustible dans les réacteurs à neutrons thermiques. La chaîne des isotopes du plutonium et leur structure de résonance rendent la physique des réacteurs plus complexe que dans le cas de l'uranium. En outre, les données dont on dispose à l'heure actuelle sur les sections efficaces sont moins satisfaisantes. Le mémoire contient quelques données relatives aux réseaux alliage Pu-Al (sous forme de barres) -eau légère, que l'on compare aux données concernant les solutions aqueuses de plutonium. On y trouve des renseignements complémentaires récents sur des solutions plus concentrées, qui sont comparés aux résultats obtenus par des calculs à plusieurs groupes. Les résultats de la mesure d'un coefficient de température effectuée sur des barres de combustible en Pu-Al sont également présentés.

Le mémoire décrit un code relatif au taux de combustion et indique quelques résultats obtenus lors de son application à un réacteur à eau légère. Il fait état des efforts déployés en vue de mettre au point des codes plus perfectionnés, notamment pour étudier le problème du spectre; à titre d'exemple, on donne des spectres calculés pour un combustible au plutonium utilisé dans un réacteur ralenti à l'eau. Enfin, le mémoire présente des plans pour une étude expérimentale future des réseaux plutonium-eau légère.

ДОСТИЖЕНИЯ В ИЗУЧЕНИИ ФИЗИКИ РЕАКТОРОВ С ПЛУТОНИЕМ В ЛЕГКОЙ ВОДЕ. Производство плутония в качестве побочного продукта во время работы энергетических реакторов, использующих слегка обогащенный уран, вызвало интерес к применению такого материала в качестве топлива для тепловых реакторов. Вследствие наличия цепочки изотопов плутония и их резонансной структуры физика реакторов этого типа является более сложной чем тогда, когда используется один уран. Имеющиеся данные о сечениях также мало удовлетворительны. Приводятся некоторые данные о решетках стержней из сплава Pu-Al в легкой воде и делаются некоторые сравнения с данными о водных растворах плутония. Приводятся также некоторые дополнительные последние данные о более кон-

центрированных растворах, и эти данные сравниваются с результатами многогрупповых вычислений. Указываются также результаты измерения температурного коэффициента топливных стержней из Pu-Al.

Описывается код выгорания и даются некоторые результаты его применения к легководному реактору. Излагаются попытки разработать улучшенные коды, при этом особое внимание уделяется решению проблем спектров с помощью некоторых примеров вычисленных спектров для плутониевого топлива с водным замедлителем. Указываются планы будущей экспериментальной работы по легководным решеткам с топливом из плутония.

PROGRESOS EFECTUADOS EN FÍSICA DE LOS REACTORES PARA LOS RETICULADOS DE PLUTONIO-AGUA LIGERA. La obtención de plutonio como subproducto de la explotación de los reactores de potencia en que se utiliza uranio ligeramente enriquecido ha estimulado el interés por el empleo de dicho material como combustible para reactores térmicos. Debido a las características de la cadena de isótopos del plutonio y a su estructura de resonancia, la física del reactor es en este caso más complicada que en el del uranio. Asimismo, los datos actualmente conocidos sobre las secciones eficaces son menos satisfactorios. En esta memoria se presentan algunos datos relativos a reticulados de elementos de aleación Pu-Al en agua ligera y se establecen algunas comparaciones con los datos referentes a las soluciones acuosas de plutonio. También se presentan datos adicionales obtenidos recientemente con soluciones más concentradas y se comparan con los resultados de los cálculos de multigrupos. Se presentan asimismo los resultados de la medición del coeficiente de temperatura de los elementos combustibles de Pu-Al.

Se describe una clave para el cómputo del grado de combustión y los resultados de su aplicación a un reactor de agua ligera. Se exponen los esfuerzos desplegados para mejorar las claves, especialmente en lo que se refiere al tratamiento del problema del espectro, dando algunos ejemplos de espectros calculados para combustibles de plutonio moderados por agua. Por último, se presentan los programas de futuros trabajos experimentales sobre reticulados de agua ligera con elementos combustibles de plutonio.

1. INTRODUCTION

The prospect that significant quantities of plutonium will be produced by power reactors operating on natural or slightly enriched uranium has led to an increasing interest in plutonium itself as a power reactor fuel. While the basic concept of a fast breeder reactor is attractive, the present economic advantages of that type are not so clear-cut as to exclude other uses of plutonium. In particular, the possible use of plutonium in thermal reactors, possibly in the same reactors in which it was produced, has received considerable attention. Such fuelling schemes have come to be known by the general term of Plutonium Recycle.

The general concept has been dealt with elsewhere, [1, 2] and the purpose of the present paper is confined to the physics of light water lattices using plutonium as fuel. The more general concepts will be involved only as they delineate a range of plutonium concentrations and isotopic compositions that are of interest, and as they indicate the type of physics information required.

Two general types of fuel are of interest: A uniform mixture of plutonium and uranium oxides in which the plutonium plays the same general role as U^{235} in partially enriched uranium; and plutonium alloyed with some metallic diluent (e.g., aluminium, zirconium) to provide fuel for the seed in seed-blanket cores or as a fuel in plutonium burner reactors for special applications.

The isotopic compositions of interest for light water reactors using mixed oxide fuels are indicated by a study by ESCHBACH [3] and others to lie in the following ranges:

Pu ²³⁹	70 - 50%
Pu ²⁴⁰	15 - 22%
Pu ²⁴¹	8 - 16%
Pu ²⁴²	Balance

Concentrations in the fuel will range from a fraction of a percent up to a few percent depending upon the uranium enrichment used for make-up fuel. The lower concentrations would apply for strict recycle type of operation. In alloy fuels the concentration would be similar to U²³⁵ concentrations for the same applications, i. e., a few atom percent, and the isotopic compositions would fall in the same ranges as indicated above.

The physics of plutonium as a thermal reactor fuel is more complex than that of U²³⁵ because of the plutonium isotopic chain:



which provides two fissionable isotopes and one fertile isotope of high cross-section, Pu²⁴⁰, and because of the low lying and prominent resonances in the cross sections of Pu²³⁹ and Pu²⁴¹ and the large resonance in Pu²⁴⁰ at 1 eV.

The resonances in the fissionable isotopes make knowledge of the spectrum much more critical than is the case with U²³⁵. This situation has, in the case of light water lattices, been explored by GREEBLER [4]. The treatment of plutonium fuelled lattices has also been complicated by the circumstance that the nuclear parameters of the plutonium isotopes are less precisely known than those of U²³⁵ or U²³³. A review of the data made by LEONARD [5] indicates that consistency between measurements of the total cross-section, fission cross-section, η , and ν at 2200 m/s is of the order of 5% for Pu²³⁹ whereas it is better than 2% for U²³⁵. A similar analysis could not be conducted for Pu²⁴¹ mainly because of the lack of direct measurements of η .

2. LATTICE MEASUREMENTS

At the present time there are few experimental data available on plutonium-fuelled light water lattices. In the absence of more comprehensive data the results of some experiments conducted at the Hanford Laboratories by V. I. NEELEY *et al.* [6] may be of interest. Since the only published report is in abstract form, some description of the experiments is presented here. In these experiments the critical size of lattices fuelled with Pu-Al alloy rods was determined by means of neutron multiplication and exponential measurements.

The fuel elements were Pu-Al alloy rods with 5 wt. % Pu enrichment. These rods were 24 in long and 0.506-in diam. They were clad in 0.030-in

Zircaloy-2 with 0.020-in and 0.125-in thick end caps. This gave a Zr/Pu atomic ratio of 31.92 and an Al/Pu atomic ratio of 168.20. The average rod contained 11.01 g of Pu. The plutonium contained approximately 5% Pu²⁴⁰.

The experiments were carried out in a 4 ft diam. by 5 ft deep tank of light water. The lattices were hexagonal in shape and placed so as to provide effectively infinite water reflection on all sides. The lattice plates were constructed from lucite. Safety circuits, neutron sources, and BF₃ counters for monitoring the neutron flux were adapted from previous experiments of this type on 3.1% enriched uranium systems [7].

The neutron multiplication measurements were carried to within 96% of the critical mass as determined from the inverse multiplication plots. The critical number of rods, N_c , was found from a least squares fit to the plot of the number of rods divided by the counting rate (N/ϕ) versus the number of rods in the region between 85 - 96% of criticality. It has been shown that a straight line fit to this section of the plot is a very good approximation [8].

The exponential experiments were conducted after the assemblies had been reduced sufficiently in size, since in order to conduct the exponential experiments, it was necessary to reduce the pile size from the near critical condition.

Cylindrical geometry was approximated in the hexagonal lattice using the equation,

$$R_{\text{cylinder}}^2 = \frac{NL^2\sqrt{3}}{2\pi}$$

where

N = number of fuel rods, and

L = centre to centre fuel rod spacing

The extrapolation length was determined by equating the buckling expression for the exponential measurements with the critical buckling as determined from neutron multiplication measurements,

$$B_{\text{critical buckling}}^2 = \left(\frac{2.4048}{R + \lambda_{\text{critical size}}} \right)^2 + \left(\frac{\pi}{h + 2\lambda} \right)^2 = \left(\frac{2.4048}{R + \lambda_{\text{exponential}}} \right)^2 - \left(\frac{1}{b_{11}} \right)^2$$

Table I presents the results of these experiments.

A similar set of experiments is now being performed with rods of a 1.8% Pu-Al alloy [9]. In this case the rod diameter and cladding dimensions were the same as for the 5% alloy but the rod lengths were 44 in.

Preliminary results from these experiments are presented in Table II.

A comparison of these results with measurements on plutonium solutions is instructive. The solution data [10] are available in terms of the spherical critical masses and while this quantity is seldom of direct use in reactor design, the comparisons can be made most directly on this basis since measured extrapolation lengths are available for the lattice experiments but not for solutions. The solution data can be compared meaning-

TABLE I
SUMMARY OF CRITICAL APPROACH AND EXPONENTIAL MEASUREMENTS
WITH Al-5 Wt. % Pu ALLOY RODS IN LIGHT WATER

Experiment	Lattice spacing (in)	H/Pu * (atom ratio)	H ₂ O/V Pu-Al Alloy	Extrapolation length (λ)	Critical No. of 24-in rods (Cyl. geometry)	Critical mass (Spherical geometry)	Buckling (10 ⁻⁶ cm ⁻²)
Exp.	0.75	217.8	1.17	8.10 cm**	356.1 (3.92 Kg Pu*)	3.31 Kg Pu*	9,605
C. A. and Exp.	0.85	354.7	1.86	7.92 cm	230.2 (2.54 Kg Pu*)	2.07 Kg Pu*	10,839
C. A. and Exp.	0.90	426.8	2.23	7.93 cm	192.0 (2.11 Kg Pu*)	1.70 Kg Pu*	11,261
C. A. and Exp.	1.00	582.6	3.06	7.73 cm	170.1 (1.87 Kg Pu*)	1.53 Kg Pu*	10,853
C. A. and Exp.	1.10	755.1	3.96	7.34 cm	166.5 (1.88 Kg Pu*)	1.53 Kg Pu*	10,107
C. A. and Exp.	1.20	944.0	4.95	7.03 cm	181.1 (1.99 Kg Pu*)	1.73 Kg Pu*	8,840
C. A. and Exp.	1.30	1149.3	6.03	6.04 cm	215.5 (2.37 Kg Pu*)	2.12 Kg Pu*	7,681
Exp.	1.50	1609.2	8.44	5.80 cm**	307.8 (3.39 Kg Pu*)	3.10 Kg Pu*	5,332
Exp.	1.80	2422.3	12.70	5.20 cm**			- 197

* Total Pu including all isotopes; the Pu contains approximately 5% Pu²⁴⁰

** Assumed values of λ extrapolated from curve.

For those cases in which the exponential experiment (Exp.) only was conducted the critical number of rods and critical mass were calculated from the measured buckling.

TABLE II (REVISED)

SUMMARY OF CRITICAL APPROACH AND EXPONENTIAL MEASUREMENTS WITH Al-1.8 Wt. % Pu ALLOY RODS IN LIGHT WATER

Lattice spacing (in)	H/Pu (atom ratio)	Critical No. of 44-in rods (Cyl. geometry)	Critical mass Kg Pu	Buckling* (10^{-6}cm^{-2})
0.75	638	580 ± 2	4.13	6335
0.80	819	509	3.62	6339
0.85	1013	494 ± 2	3.52	5954
0.90	1218	516 ± 2	3.67	5340
0.95	1435	578 ± 2	4.12	4582

* The B^2 numbers include $609 \times 10^{-6}\text{cm}^{-2}$ for a vertical buckling and assume an extrapolation length of 7.7 cm since the loadings are totally reflected in all dimensions

fully only with the results of the 5% Pu-Al alloy experiments for reasons which will become apparent later.

The values of the bucklings derived from the experiments were used to compute the volumes of spherical assemblies, and from these the spherical critical masses were derived. Figure 1 shows a plot of these spherical critical masses.

The solution experiments were conducted with varying amounts of nitrate ion present and it has been possible to select from the data a set of values of critical masses in which the parasitic capture in the nitrogen, as measured by b/Pu atom, is the same as the parasitic capture in the aluminium and zirconium in the lattice experiments. The critical masses obtained from this selection are also plotted in Fig. 1. Corresponding points on the two

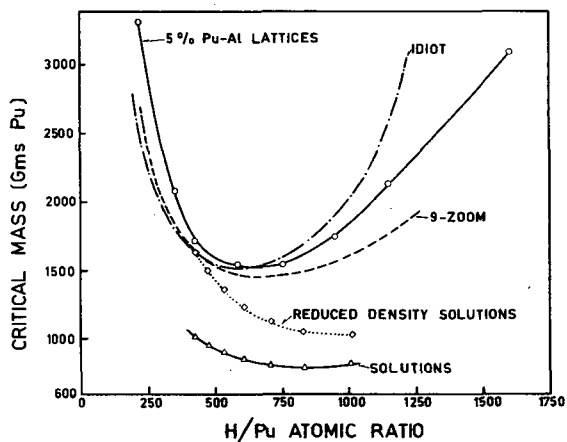


Fig. 1

Plutonium critical masses in light water

curves have the same hydrogen to plutonium atomic ratio and the same parasitic capture as indicated by b/Pu atom. The solution experiments have been corrected to the same Pu^{240} content as that existing in the lattice experiments. This correction was about 40 g in the critical mass and was done in an empirical fashion from data available from the solution experiments. The heavy element scattering cross-section (oxygen, nitrogen, aluminium and zirconium) per plutonium atom is also very nearly the same in the solution and lattice experiments. The difference in the heavy element Σ is hardly significant in the face of the amount of hydrogen present. The two major differences between the experiments are, however, the lower hydrogen density existing in the lattice and the flux disadvantage introduced by incorporating the plutonium in rods.

Since, per plutonium atom, the amounts of hydrogen, parasitic capture and heavy element scattering are the same for corresponding points on the two curves, it is possible to regard the lattice systems as reduced in overall density with respect to the solutions. In both systems the water reflector was, however, at full density for all of the experiments. It is, therefore, not possible to make a simple correction for the density effect by the use of the inverse square rule. However, this rule may be used as a guide for determining the upper limit to the density effect. A curve for solutions, reduced to the density of the corresponding lattices, was obtained in this way and is also shown in Fig. 1. It is evident from the relation of the curves for the more open lattices that the flux disadvantage effects are the major source of difference between the solutions and the lattice experiments. In the case of the more close-packed lattices on the other hand the density effect predominates. To compute a correction including full reflector density in each case would involve adopting some computational model such as a multi-group theory. When this amount of complexity is involved, it then becomes more straightforward to apply the computation methods directly to predict the critical size of the lattices rather than for the purpose of computing corrections. The graph shows two such computations, one of which was done with the IDIOT code [11] and the other of which was done by the process of homogenizing the heterogeneous system and using the 9-ZOOM code [12]. It can be seen that there is fair agreement between both of these computations and the experimental data. For the most open lattices, the critical size is increasingly sensitive to errors in the computations and the increasing divergence of the theories from the experimental results is not unexpected.

IDIOT is a 4-factor formula code devised at Hanford primarily for computations with graphite and heavy water moderated lattices. However, its application in this case to a light water lattice appears to have been relatively successful. In the code, the thermal utilization is computed by a P_3 method and the Westcott cross-section formulation is used. Other factors in the formula are evaluated in the conventional manner. In the 9-ZOOM calculation the lattice was homogenized using the relative fluxes obtained from the P_3 portion of IDIOT. The problem is then treated by an 18-group diffusion calculation in which the thermal neutrons are handled by a modification of the SOFOCATE code. These two approaches bracket the experi-

mental data at the open lattice end and come satisfactorily close to the data for the close-packed lattices.

A comparison of the data from the measurements of the Al-1.8% Pu lattices with solutions, similar to that carried out above, is not possible at present since the higher parasitic capture in the aluminium cannot be matched in existing solution data.

Extension of the comparison into the interesting H/Pu range below 400 is also not possible because data on fully reflected solutions are currently not available. However, the Plutonium Critical Mass Laboratory of the Hanford Laboratories now has in progress a programme of obtaining such data primarily for nuclear safety purposes. Initial measurements have been made with partially reflected and unreflected spherical assemblies [13]. Measurements on fully reflected assemblies are planned in the future. Experiments performed to date have been conducted in a 14-in diam sphere, unreflected and with reflectors of 0.5 in and 1 in of paraffin. Plutonium concentrations have ranged from 68 g/l to 300 g/l and nitrate concentrations have varied correspondingly. An analysis of the errors involved has indicated that the over-all precision of the critical mass values is 3%. For comparison purposes it has been possible to reduce all of the values to equivalent bare spheres and to correct them to the nearest even integer of nitric acid molarity. The results are shown in Fig. 2, along with calculated curves.

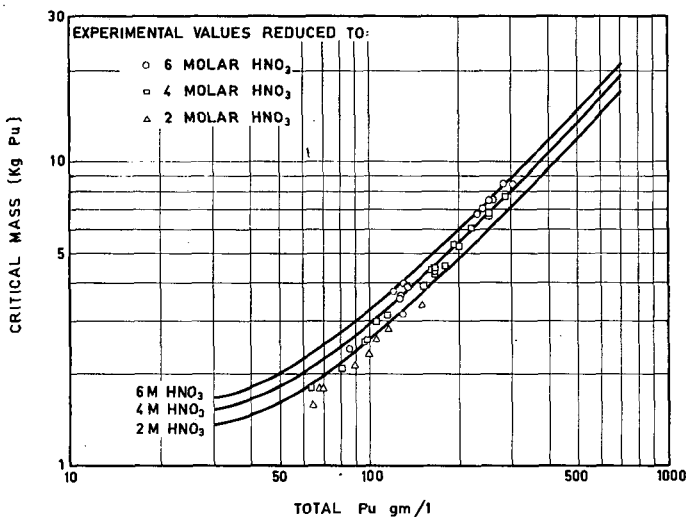


Fig. 2

Critical mass VS Pu density for $\text{Pu}(\text{NO}_3)_4 \cdot \text{H}_2\text{O} + \text{HNO}_3$ bare spheres

Starting with chemical compositions of the solutions, the critical radii were computed [14], using the 9-ZOOM code [12] and an 18-group set of cross-sections [15]. The thermal group cross-sections were computed using the TEMPEST [16] code and substituted for the values on the 9-ZOOM library

tape. This procedure provides greater accuracy and flexibility than the 9-ZOOM library alone. Calculations were carried through for Pu^{240} isotopic concentrations of 0, 2.5, 5, and 7.5%. The agreement is seen to be good from 100 g/l to 300 g/l and high acid concentrations but to leave something to be desired at lower concentrations.

3. OTHER DATA

A measurement of the reactivity temperature coefficient of Pu-Al alloy fuel has been made with plutonium containing 6% Pu^{240} and with plutonium containing 16% Pu^{240} [17]. The plutonium composition of the alloy in the two cases was adjusted to obtain approximately the same Pu^{239} content in both sets of fuel. Although the measurement was made in a graphite-moderated lattice the results are of general interest. The fuel was in the form of 19-rod clusters of Pu-Al alloy containing approximately 2% Pu and of $\frac{1}{2}$ -in rod diam. Measurements were conducted in the Physical Constants Testing Reactor (PCTR) over a temperature range from 20°C to above 400°C. Techniques used were similar to those previously used with uranium fuels [18]. Electrical heating was used. The composition of the two types of fuel was as follows:

Isotope	Atom % (LX)	Atom % (HX)
Pu^{239}	93.434	80.727
Pu^{240}	$6.062 \pm .005$	$15.944 \pm .011$
Pu^{241}	$0.448 \pm .001$	$2.828 \pm .004$
Pu^{242}	$0.0159 \pm .001$	$0.501 \pm .001$
wt. % Pu in Al	1.878	2.078

A measurement was also made with dummy aluminium rods to enable correction for the positive reactivity effect caused by the increase of neutron temperature due to scattering from the heated aluminium. The low Pu^{240} fuel yielded a negative coefficient; the positive effect of the aluminium being overcome by the Doppler effect in plutonium. With the higher Pu^{240} fuel a more negative coefficient was obtained probably due to the Doppler broadening of the Pu^{240} resonance. The temperature coefficients: $1/k_{\infty} (dk_{\infty}/dT)$, are

6% Pu^{240} fuel	-	$0.7 \pm 0.3 \times 10^{-5}/^{\circ}\text{C}$
16% Pu^{240} fuel	-	$1.4 \pm 0.3 \times 10^{-5}/^{\circ}\text{C}$

4. BURN-UP CODES

Fuel compositions of interest for the use of plutonium in light water reactors have been computed [3] by the use of the MELEAGER burn-up code [19]. MELEAGER is a reactor fuel burn-up code in the Fortran language for the IBM 709-7090. Fuel composition, reactivity parameters, and power as a function of exposure are the primary results. The programme has the

following features: (1) Input preparation is minimal; (2) 150 isotopes may be considered, 30 of which may be fertile or fissionable, and radioactive decay may be allowed for; (3) cross-sections are corrected for thermal and epithermal self-shielding as the compositions change; and (4) current running time on the IBM-7090 is of the order of one minute for 20 isotopes, exposed to 10 000 MWd/T.

MELEAGER is primarily a nuclear fuel cycling survey code for well-thermalized reactors and represents a compromise between speed and accuracy.

The typical stages of a calculation are:

- (1) A set of effective cross-sections is formulated from the basic library tape neutron temperature, isotopic concentrations and spectral index. The Westcott cross-sections [20] are used.
- (2) The isotopes under consideration are exposed to the input flux for a time step H , producing a new set of isotopic concentrations.
- (3) If the isotopic composition change is within given error limits, the flow moves to stage (4). If, however, the change is too large, stage (2) is repeated with a time step of $H/2$; if the change is too small, the following time step is doubled.
- (4) The reactor parameters are calculated; they include reactivity, power, and the time averages of these quantities over the interval from zero to the current time.
- (5) If the maximum time has not been reached nor any of the reactivity limits exceeded, the control returns to stage (1) for recalculation of the neutron spectrum and the effective cross-sections.

Any isotopes, real or fictitious, may be included by entering them in the proper places on the library cross-section tape and on the corresponding composition card. Neutron temperature and neutron spectrum either are specified by the input or may be determined from the absorption and slowing down properties. Problems involving either batch irradiation or graded irradiation may be run.

The application of the code to the Advanced Pressurized Water Reactor [21, 22] will furnish an illustrative example. It is a two-zone reactor, with the inner zone having 2.6% enriched uranium oxide fuel and the outer zone having 3.4% enrichment. The chief effect of this is to improve the power distribution. Calculations showed that the zoning had negligible effects upon the burn-up characteristics (reactivity and average fuel composition); for example, the reactivity difference between the two-zone core and an equivalent one-zone core (initial enrichment, 3%) was only 0.002 at 13 000 MWd/T. The uniform core was, therefore, used as the basis of the MELEAGER study. The fuel density was set at 9.256 g uranium per cm^3 . The exposure unit is megawatt days per short ton of uranium.

The fuel element rod is 0.391 in in diameter and clad in 0.22-in Type 347 stainless steel tube. Two hundred 17-elements are formed into a hexagonal assembly 8.40 in on the flats. The thermal utilization of the core, with some allowance for peaking in water gaps, was calculated, using a P_3 code, to be 0.833. Water temperature was 316°C. The neutron spectrum has an effective temperature determined from the moderator temperature of 450°C and the Westcott spectral index "r", which is initially 0.15. The

minimum required reactivity is 1.050, for effects of leakage, temperature coefficients, shut-down margins, samarium, and xenon.

Calculations starting with four enrichments in the neighbourhood of 3% were made which gave the exposures accumulated in each case before the reactivity dropped to 1.050 and the isotopic compositions of the fuel at that time. Since the code strictly calculates for only one point in the reactor, the results are to be understood in the sense of average exposures over the reactor, and average compositions. The results are tabulated below:

	Initial Enrichment	Exposure MWd/T	Plutonium Composition			
			239	240	241	242
1.	2.84%	11953	0.728	0.174	0.084	0.014
2.	3.04%	14070	0.707	0.181	0.094	0.018
3.	3.07%	14353	0.705	0.182	0.095	0.018
4.	3.31%	16853	0.681	0.192	0.103	0.024

Each of these cases could be used as the start of a recycle sequence in which the plutonium from the discharged fuel is mixed with enriched uranium and used to recharge the reactor. At the end of this cycle the process is repeated, etc. The exact values of uranium enrichment are chosen for economic reasons which need not concern us here. Table III shows the exposures and compositions resulting in the sequence which begins with Case 2 above. In this table the plutonium compositions are those at the beginning of each step and consequently those resulting from the end composition of the previous step.

TABLE III

EXPOSURES AND FUEL COMPOSITIONS IN THE BATCH
RECYCLE OF PLUTONIUM IN THE APWR

Step number	Exposure (MWd/T*)	Initial enrichments (Weight fraction)		Weight fraction of total plutonium			
		Total	Uranium	239	240	241	242
1	14070	0.0304	0.0304	0	0	0	0
2	14938	0.0318	0.0276	0.707	0.181	0.094	0.018
3	14947	0.0316	0.0265	0.600	0.209	0.136	0.055
4	15600	0.0325	0.0269	0.549	0.212	0.148	0.091
5	16546	0.0336	0.0277	0.520	0.208	0.151	0.121
6	16722	0.0339	0.0279	0.502	0.201	0.150	0.147

* T equals short tons

The change of spectrum and the effects of self-shielding occurring during this sequence are indicated by the data in Table IV taken at the beginning of the first step and the end of the fourth step in the sequence.

TABLE IV
EFFECTIVE CROSS-SECTIONS AND PARAMETERS IN THE
RECYCLE OF PLUTONIUM IN THE APWR

Parameter	Beginning	End of 4th Step
r	0.15	0.20
U^{235}		
σ_f	500	498
σ_a	610	611
α	0.220	0.227
Pu^{239}		
σ_f	1345	1186
σ_a	2126	1856
α	0.580	0.566
Pu^{240}		
σ_a	2677	1553
Pu^{241}		
σ_f	1403	1370
σ_a	1932	1886
α	0.376	0.376
Pu^{242}		
σ_a	283	262
U^{238}		
σ_a	8.5	10.2

The programme to attempt improvement in the method of doing burn-up calculations is now going on. As a first step in this programme, a study has been made of the spectral characteristics of reactors fuelled with plutonium [23]. To date this work has been directed toward studying the homogeneous infinite medium case to lay the foundations for eventually tackling more complex situations. Calculations of the spectrum have been made for a variety of concentrations of Pu^{239} in water by use of the SPECTRUM code [24].

The spectra obtained in two of these cases, H/Pu ratios of 1000 and of 200, are shown in Fig. 3. The effect of the plutonium resonance and the general hardening of the spectrum is quite marked, especially at the higher plu-

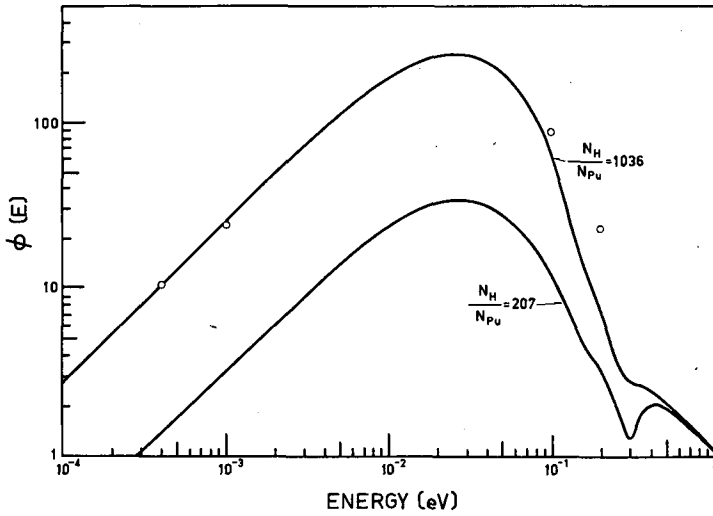


Fig. 3

Calculated spectra in plutonium solutions

tonium concentration. The two curves are normalized for the same total absorption rate in each case.

A neutron temperature was determined for each case by fitting a Maxwellian to the peak of the computed spectrum. Average values of a number of cross-sections of a number of isotopes were obtained for each of the cases. Some of these results are presented in Table V, where the neutron temper-

TABLE V

SPECTRAL HARDENING AND EFFECTIVE CROSS-SECTIONS IN
HOMOGENEOUS PLUTONIUM-HYDROGEN MIXTURES

H/Pu	T_N	$\bar{\sigma}_a(1/v)$	Pu^{239}				
			$\bar{\sigma}_a$	$\bar{\sigma}_f$	$\bar{\sigma}_a(1/v)$	$\bar{\sigma}$	$\bar{\sigma}(1 \text{ eV})$
No absorption	293.7	5.58	982	697	1105	1105	1105
2500	300.1	5.24	998	696	1196	1187	1188
1500	302.3	5.12	1000	695	1226	1215	1217
1000	305.0	4.99	1002	692	1261	1251	1254
500	313.3	4.63	996	679	1351	1354	1362
300	324.0	4.25	975	657	1444	1484	1503
200	337.0	3.87	943	627	1531	1635	1677
100	371.9	3.15	839	548	1672	--	--
67	405.9	2.75	753	488	--	--	--
50	439.0	2.49	685	442	--	--	--

ature average cross-section for a $1/v$ absorber and average cross-sections for the absorption and fission of Pu^{239} are presented. Also presented is an effective $1/v$ cross-section which would produce the same capture rate as the actual plutonium cross-section, and this is compared with the Westcott cross-section and with a Westcott cross-section corrected for the circumstance that the computed spectrum and the average cross-section were carried only to 1 eV. The reasonably good agreement obtained for the more dilute cases begins to disappear as the plutonium concentration increases. The possibility of using a computed neutron temperature in a Westcott formulation of the cross-sections has been investigated. By the method of fitting a Maxwellian to the peak of the computed distribution, it has been shown that the ratio of neutron temperature to moderator temperature can be given by the following equation:

$$T_N = T_M \left[1 + c \frac{\Sigma_a(kT_M)}{\xi \Sigma_s} \right]$$

where:

T_N is the neutron temperature

T_M is the moderator temperature

$\Sigma_a(kT_M)$ is the macroscopic absorption cross-section based on the moderator temperature;

ξ is the average logarithmic energy decrement;

Σ_s is the macroscopic scattering cross-section;

c is a constant.

A constant c has been found to represent the situation quite well with most materials investigated for values $\Sigma_a/\xi \Sigma_s$ up to about one-half, with the value of c being somewhat different for each absorber. A table of these values is presented below:

Absorber	c	Probable error of c
U^{235}	0.68	± 0.01
U^{238}	0.61	± 0.01
Pu^{239}	0.51	± 0.04
Pu^{240}	0.54	± 0.02
$\text{B}^{10} (1/v)$	0.61	± 0.01

In addition, spectra have been computed for mixtures of absorbers, and it has turned out in this case that the temperature has been well represented by using a macroscopic cross-section weighted mean value of c . However, the situation with respect to β in the Westcott formulation is

not nearly so satisfactory. In this case it appears that the values of β computed according to the Westcott formula are too large as compared with those obtained from the computed spectra for any values of $\Sigma_a/\xi\Sigma_s$ greater than about 0.1 and even at this limit some materials, as for example, Pu^{240} , are not well handled.

A new burn-up code is now being formulated which will compute the thermal spectrum and effective cross-sections up to 0.6 eV by use of the TEMPEST code and will handle the higher energies by a 68-group code, GAM. These two codes will produce effective cross-sections for use in a 9-group burn-up calculation, with the 9-group cross-sections being re-computed whenever the compositions change by more than a preset amount. An elaborate, combined Monte-Carlo multi-group diffusion burn-up code, called RBU, is also available for carrying out detailed examination of individual burn-up cases. This code treats the reactor fine structure by Monte-Carlo method, computing group constants, which can then be used in a multi-group diffusion treatment of the entire reactor. The burn-up calculation can proceed, using the multi-group constants with resort to additional Monte-Carlo treatment as specified for any particular problem. Although portions of this code have been used to compute lattice parameters, it is not as yet completely operational [25].

5. PLANS

It is evident from the foregoing that a need exists for more experimental information on light water lattices fuelled with plutonium and especially for lattices with fuels of mixed plutonium and uranium oxides. The Plutonium Recycle Programme includes plans for obtaining some of these measurements. A critical facility is now under construction which will provide the necessary equipment to do the measurements. This facility will be a versatile tool for conducting critical experiments with both light and heavy water moderators and also for making reactivity measurements with samples of irradiated fuel in either environment.

About one year from now, following initial experiments with heavy water the facility will become available for light water work. Its essential parts are a tank, a number of replaceable templates for holding the fuel elements in various lattice spacings, control and safety instrumentation, control and safety rods and a quick dump arrangement for the moderator. Experiments will be limited to atmospheric pressure. The tank is contained in a concrete lined pit below grade level and the shielding is completed by the addition of removable concrete slabs over the top of the pit. It is located adjacent to the canal for cooling of fuel discharged from the Plutonium Recycle Test Reactor (PRTR) and an air-water lock has been provided for bringing such fuel into the critical facility pit. Irradiated fuel rods from reactors other than the PRTR could also be brought in through the same route.

Initial experiments with light water will be designed to confirm and extend the subcritical measurements with Pu-Al alloy fuels by direct determination of critical size, cell flux distribution measurements, and the irradiation of spectral indicators such as paired U^{235} and Pu foils, lutetium, etc. These measurements will be followed by a series with mixed plutonium and uranium oxide fuels. Consideration is being given to the use of both

natural and depleted uranium as a base for these fuels. Plutonium concentrations used will be guided by calculations such as those described above. The isotopic composition will be in the range of 15-20% Pu²⁴⁰. The USAEC is also currently considering the irradiation of such fuels in light water reactors and a programme for determining the changes of isotopic composition and reactivity as a function of exposure. Reactivity measurements on irradiated fuels will be of the substitution type, based upon, and guided by, the results of the light water criticals. It is expected that significant results can be obtained from substitution of single rods or small groups of rods and then compared with isotopic composition measurements on the same, or similar rods.

Such a programme would be similar to that now beginning for the irradiation of both Pu-Al alloy fuels and mixed oxide fuels in the heavy water moderated PRTR. Some fuels irradiated in this programme would also be subsequently available for measurements in light water lattices, thus providing some variety in isotopic compositions.

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MULTI-REGION LATTICE STUDIES: CORRELATION OF THEORY AND EXPERIMENT

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Abstract — Résumé — Аннотация — Resumen

MULTI-REGION LATTICE STUDIES: CORRELATION OF THEORY AND EXPERIMENT. The research and development programmes at Westinghouse Atomic Power Division have emphasized the utilization of machine methods of computation and the evaluation of such methods by means of interpretation of critical experiments. The most recent programme of critical experiments was the programme of Multi-region Lattice Studies carried out under the Fuel Cycle Development Programme of the United States Atomic Energy Commission. This programme involved the measurements of criticality, neutron flux distribution and reaction rates (microscopic parameters) in lattices of light water moderated, low enriched, UO_2 fuel clad in stainless steel. Fuel rods of uranium oxide enriched to 1, 6%, 2, 7% and 3, 7% were utilized in lattices of 2, 5:1 and 4, 5:1 water/uranium ratio. Currently, these experiments are being supplemented by measurements in the same lattices with boric acid dissolved in the moderator. This paper summarizes the present status of a number of experimental and analytical problems and points out where further effort is needed. An attempt is made to point out inter-relationships between different measurements or calculations where significant points of agreement or contradiction can be useful in interpretation of results or in planning future work.

ÉTUDES DES RÉSEAUX A PLUSIEURS ZONES: CORRELATION DE LA THÉORIE ET DE L'EXPÉRIENCE. Les programmes de recherche et de mise au point établis par la Division de l'énergie d'origine nucléaire de la Société Westinghouse ont mis l'accent sur l'utilisation de méthodes de calcul mécanique et sur l'évaluation de ces méthodes par l'interprétation d'expériences critiques. Le dernier ensemble d'expériences critiques était prévu au programme d'étude des réseaux à plusieurs zones qui a été exécuté dans le cadre du programme de mise au point de cycles de combustible établi par la Commission de l'énergie atomique des Etats-Unis. Ces expériences comportaient des mesures de l'état critique, de la distribution du flux de neutrons et des vitesses de réaction (paramètres microscopiques) dans les réseaux de réacteurs ralentis à l'eau légère et utilisant comme combustible de l' UO_2 faiblement enrichi et gainé d'acier inoxydable. Des barres de combustible en oxyde d'uranium enrichi à 1, 6%, 2, 7% et 3, 7% étaient employées dans des réseaux accusant un rapport eau-uranium de 2, 5/1 et de 4, 5/1. Actuellement, ces expériences sont complétées par des mesures portant sur les mêmes réseaux en présence d'acide borique dissous dans le ralentisseur. Le mémoire donne un aperçu de l'état actuel de plusieurs problèmes expérimentaux et analytiques, en soulignant les points dont l'étude exige des efforts plus poussés. L'auteur essaie d'établir des relations entre les différents calculs et mesures qui révèlent une concordance ou un désaccord significatif qui peut être utile pour interpréter des résultats acquis ou préparer les travaux futurs.

ИЗУЧЕНИЕ МНОГОЗОНАЛЬНЫХ РЕШЕТОК: КОРРЕЛЯЦИЯ ТЕОРИИ И ЭКСПЕРИМЕНТА. В программах по исследованиям и развитию при Отделе атомной энергии компании Вестингауз особое внимание уделяется использованию машинных методов расчета и оценки таких методов посредством интерпретации критических экспериментов. Наиболее поздней программой критических экспериментов была программа изучения многозональной решетки, проводившаяся по программе разработки топливного цикла Комиссии по атомной энергии США. Эта программа была связана с измерениями критичности, распределением нейтронного потока и скоростью реакции (микроскопические параметры) в решетках с легководным замедлителем, малообогащенным топливом UO_2 с оболочкой из нержавеющей стали. Топливные стержни из оксидного урана, обогащенного до 1, 6%

2,7% и 3,7%, использовались в решетках при соотношении объемов воды и урана, равном 2,5:1 и 4,5:1. Совсем недавно эти эксперименты были дополнены измерениями, проведенными в тех же самых решетках с борной кислотой, растворенной в замедлителе. Данный доклад кратко излагает современное состояние целого ряда экспериментальных и аналитических проблем и отмечает направления, по которым необходимо прилагать дальнейшие усилия. Делается попытка определить внутреннюю взаимосвязь между различными измерениями или вычислениями, при которых основные точки совпадения или расхождения могут оказаться полезными при толковании результатов или планировании будущей работы.

ESTUDIOS DE RETICULADOS DE ZONAS MÚLTIPLES: CORRELACIÓN ENTRE LOS RESULTADOS TEÓRICOS Y LOS EXPERIMENTALES. En los programas de investigación y desarrollo de la Westinghouse Atomic Power Division, se ha asignado gran importancia a la utilización de los métodos mecánicos de cálculo y a la evaluación de dichos métodos por interpretación de experimentos críticos. La serie más reciente de experimentos críticos fue la de los estudios de reticulados de zonas múltiples, ejecutados como parte del programa de mejoramiento del ciclo de combustible de la Comisión de Energía Atómica de los Estados Unidos. Este programa comprende mediciones de la criticidad, de las distribuciones del flujo neutrónico y de las velocidades de reacción (parámetros microscópicos) en reticulados de elementos combustibles de UO_2 ligeramente enriquecido, revestidos de acero inoxidable y moderados con agua ligera. Se utilizaron barras de óxido de uranio enriquecido al 1,6, al 2,7 y al 3,7%, respectivamente, en reticulados con razones agua/uranio de 2,5: 1 y de 4,5: 1. Como complemento de estos experimentos, se están realizando actualmente mediciones en los mismos reticulados, pero con ácido bórico disuelto en el moderador. En la memoria se resumen los trabajos efectuados hasta el presente con miras a resolver una serie de problemas experimentales y analíticos y se señalan los puntos que conviene seguir estudiando. Se procura poner de manifiesto la relación existente entre los valores medidos y calculados siguiendo diferentes métodos en los casos en que se observan coincidencias o contradicciones que pueden facilitar la interpretación de los resultados o la preparación de la labor futura.

INTRODUCTION

The research and development programmes at Westinghouse Atomic Power Division have emphasized the utilization of machine methods of computation and the evaluation of such methods by means of interpretation of critical experiments. The most recent programme of critical experiments was the programme of Multi-region Lattice Studies carried out under the Fuel Cycle Development Programme of the USAEC. This programme involved the measurements of criticality, neutron flux distributions and reaction rates (microscopic parameters) in lattices of light water moderated, low enriched, UO_2 fuel clad in stainless steel. Fuel rods of uranium oxide enriched to 1,6%, 2,7% and 3,7% were utilized in lattices of 2,5:1 and 4,5:1 water/uranium ratio. Currently, these experiments are being supplemented by measurements in the same lattices with boric acid dissolved in the moderator. This paper summarizes the present status of a number of experimental and analytical problems and points out where further effort is needed. An attempt is made to point out interrelationships between different measurements or calculations where significant points of agreement or contradiction can be useful in interpretation of results or in planning future work.

CORRELATION OF THEORY AND EXPERIMENT

Criticality calculations

Experimentally, the condition of criticality can be determined with greater accuracy than any other condition in the reactor. Results of criticality measurements are relatively unambiguous and free from errors in interpretation. For this reason, a prime requirement for any computational model is that it be capable of predicting criticality. Measurements to an accuracy 0.1% are achieved with reasonable ease. The relationship of reactivity to specific reaction rates to be measured or calculated often supplies a criterion for the evaluation of the experimental technique or computational procedure, i. e. results inconsistent with the accurate prediction of criticality are immediately suspect.

Measurements in the multi-region lattice studies have involved loading to criticality in clean cylindrical configurations with the 2.7% and 3.7% fuel in the 2.5:1 and 4.5:1 lattice plates, and, in addition, numerous configurations involving uniform concentric regions of 1.6%, 2.7% and 3.7% fuel. Also complex rectangular arrays of the three types of fuel rods have been assembled for the purpose of evaluating the ability to predict realistic design configurations. Normally, criticality has been achieved with both an upper and lower water reflector, or sufficient dry core to permit an accurate estimate of reflector savings. In all cases, core height was such as to insure that uncertainty in reflector savings would contribute but little to uncertainty in the criticality calculation.

Analysis of the criticality experiments has been carried out with both few-group (WANDA [1]) and multi-group (PIMG [2]) one dimensional diffusion theory computer programmes and with few-group one dimensional transport theory as embodied in the DSN [3] computer programme. The analysis of the complex rectangular arrays was carried out by means of the PDQ [4] two-dimensional, few-group diffusion theory programme. Few group calculations have been carried out with both conventional diffusion theory and the "mixed number density" [5] model for the thermal group. In "mixed number density" the macroscopic group constants are so defined that neutron density rather than flux is calculated. Group constants other than the diffusion coefficient are determined on the basis of a SOFOCATE [6] calculated Wigner-Wilkins spectrum. The diffusion coefficient is averaged on the basis of a Maxwellian spectrum for the gradient of the thermal neutron flux. This procedure, although semi-empirical in nature, has been shown to be rather well founded on the basis of multi-group thermal calculations carried out with the SLO-PI [7] programme. Thermal group disadvantage factors have been calculated by means of a transport procedure originated by AMOUYAL and BENOIST [8]. Few-group constants for the epithermal and fast neutron groups were calculated by means of the MUFT IV [9] programme. Resonance self-shielding factors ("L factors") required by the MUFT IV programme have been calculated as follows. A semi-empirical "five factor" loop model represented diagrammatically in Fig. 1 has been utilized to calculate the ratio of the epithermal capture rate in U^{238} to the thermalization rate. An "L factor search" is then used to force MUFT IV to give the same ratio. The adjustable parameters of the loop model which

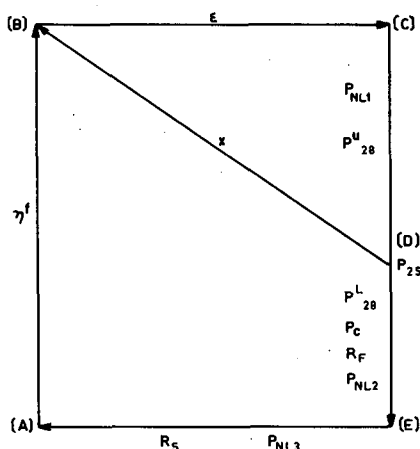


Fig. 1

The neutron life cycle

determine the competition between U^{235} and U^{238} resonances were chosen on the basis of the analysis of Yankee, BR3 and TRX [10] critical experiments to give a best fit between experimental and calculated reactivities. The results of criticality calculations utilizing "loop model" L factors are shown in Table I. Note the high degree of consistency in each lattice and the "bias" of -0.7% in the 2.5:1 core and -0.3% in the 4.5:1.

Current efforts in improving methods for criticality calculations involve an evaluation of methods for removing the "reactivity bias" from the model. Results of specific reaction rate measurements described below undoubtedly provide information relevant to the problem of reactivity bias. Also, there is some room for adjustment in basic physical data which is contained in the libraries of the digital computer programmes. Adoption of the "world consistent" value 2.43 for ν_{235} , for example, would lower the calculated reactivity in our spectrum by about 0.5%. Introduction of procedures for the heterogeneous calculations of fast fission would increase

TABLE Ia

SIGNIFICANT FOUR-GROUP DIFFUSION EIGENVALUES
CONVENTIONAL THERMAL CONSTANTS

Core description	W/U = 2.5:1	W/U = 4.5:1
2.7% cylinder	0.9909 (1-A)	0.9960 (1-B)
2.7% rectilinear	0.9914 (2-A)	0.9967 (2-B)
3.7% cylinder	0.9914 (3-A)	0.9939 (3-B)
1.6 - 2.7% cylinder	0.9885 (8-A)	0.9942 (11-B)
1.6 - 3.7% cylinder	0.9899 (9-A)	0.9939 (12-B)
Small 3-region cylinder	0.9910 (6-A)	0.9963 (9-B)
Small 3-region cylinder	0.9912 (7-A)	0.9958 (10-B)
Large 3-region cylinder	0.9933 (4-A)	0.9950 (4-B)
Large 3-region rectilinear	0.9909 (5-A)	0.9956 (5-B)

TABLE Ib

SIGNIFICANT FOUR-GROUP DIFFUSION EIGENVALUES
MND THERMAL CONSTANTS

Core Description	W/U = 2.5:1	W/U = 4.5:1
2. 7% cylinder	0.9933 (1-A)	0.9989 (1-B)
2. 7% rectilinear	0.9937 (2-A)	0.9985 (2-B)
3. 7% cylinder	0.9943 (3-A)	0.9971 (3-B)
1. 6 - 2. 7% cylinder	0.9900 (8-A)	0.9958 (11-B)
1. 6 - 3. 7% cylinder	0.9915 (9-A)	0.9956 (12-B)
Small 3-region cylinder	0.9922 (6-A)	0.9988 (9-B)
Small 3-region cylinder	0.9934 (7-A)	0.9985 (10-B)
Large 3-region cylinder	0.9953 (4-A)	0.9976 (4-B)
Large 3-region rectilinear	0.9934 (5-A)	0.9984 (5-B)

the calculated multiplication factor by about the same amount. A few percent reduction in the epithermal capture to fission ratio for U^{235} (a quantity not very well known), would result in a reactivity increase sufficient to remove the observed bias. The test of these various alterations is in the evaluation of their effects over the full range of water/uranium ratios and enrichments and in multi-region lattices. The motivation for improvement is to obtain a calculational scheme which can be extrapolated with confidence to new situations and which gives specific reaction rates with accuracy sufficient to insure reliable calculation of reactor fuel characteristics during fuel burn-up.

Neutron flux and power distributions

Of special significance in establishing the power producing capability of a nuclear reactor is the ability to predict with accuracy, the spatial distribution of power. Usually the power rating is limited by the conditions at the point of greatest power density or by the conditions in the hot channel, so that considerable emphasis on the development of techniques for calculating local peaking as well as overall power shapes is justified.

Experimentally, power distributions have been obtained in many configurations involving the three enrichments of fuel by measuring the fission product activation of fuel pellets. In addition, capture and fission activation distributions have been measured with U^{238} foils and capture activation with gold foils. Measurements have been made under uniform lattice conditions and also in the neighbourhood of discontinuities such as silver-cadmium plates, steel plates and aluminium plates. Primary emphasis has been placed on the fuel pellet fission product activation because of its direct interpretation in terms of reactor power distribution.

Most of the computer programmes utilized in the calculation of criticality provide neutron flux and power distributions as well. In general, very good agreement has been observed between experimental and analytical power shapes. Some differences have been observed between results obtain-

ed with the mixed number density model and the conventional model for calculating thermal group constants and also between diffusion theory and transport theory in thin regions and near the core reflector interface. In general, there is excellent agreement between the best theoretical method and the experimental data. Figures 2, 3 and 4 illustrate the comparison between theory and experiment for a three-region cylindrical core and show the differences obtained through the use of various analytical methods.

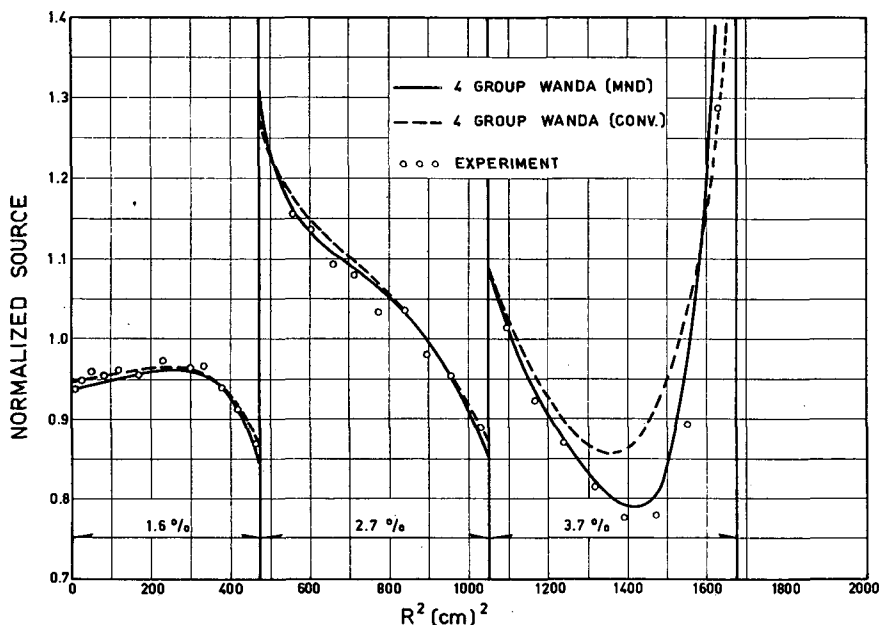


Fig. 2

Radial fuel rod scan - core 6-A - curve 1

Figure 5 shows the results of a comparison between theory and experiment for a large rectangular core requiring two-dimensional analysis. Figures 6 and 7 show results obtained for power distributions in the neighbourhood of a water gap and an absorbing slab. Results indicate that in most instances, the disparity between theory and experiment is less than 5%. On the other hand, there are some systematic effects which warrant future investigations. For example, we have noted a general tendency to underestimate the power density in the 1.6% fuel by a few percent relative to other regions in a multi-region core. It is felt that this effect may contain a clue concerning the source of our reactivity bias. Also, as indicated in Fig. 8, there is a noticeable need for improvement in the representation of "gray" absorbers. Further effort on this problem is under way.

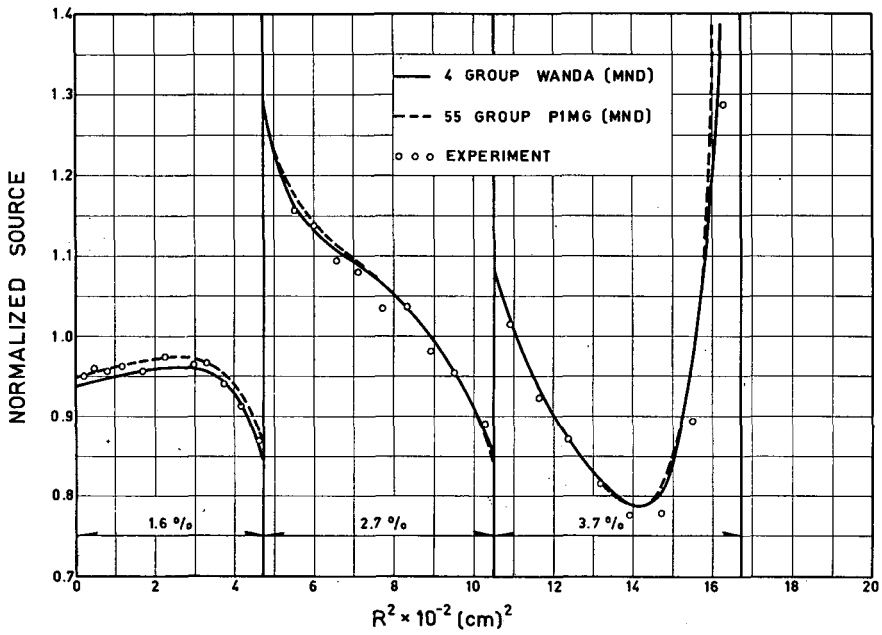


Fig. 3

Radial fuel rod scan - core 6-A - curve 2

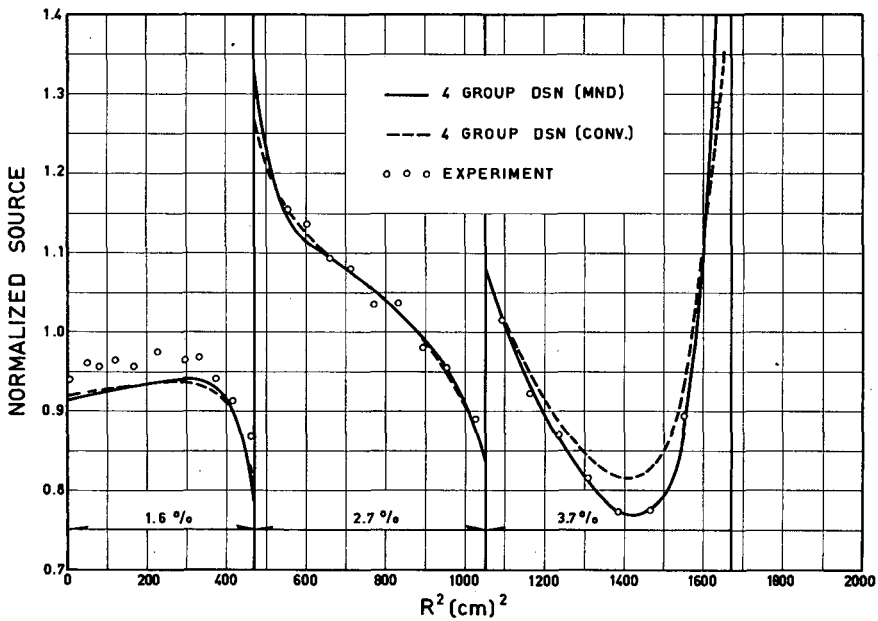


Fig. 4

Radial fuel rod scan - core 6-A - curve 4

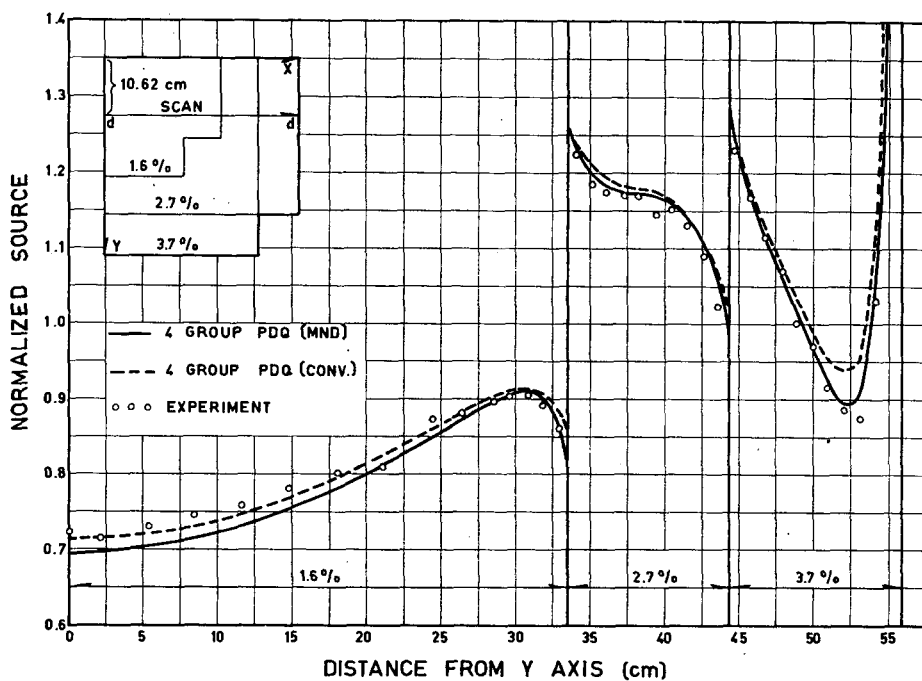


Fig. 5

Fuel rod scan (d, d) - core 5-A - curve 5

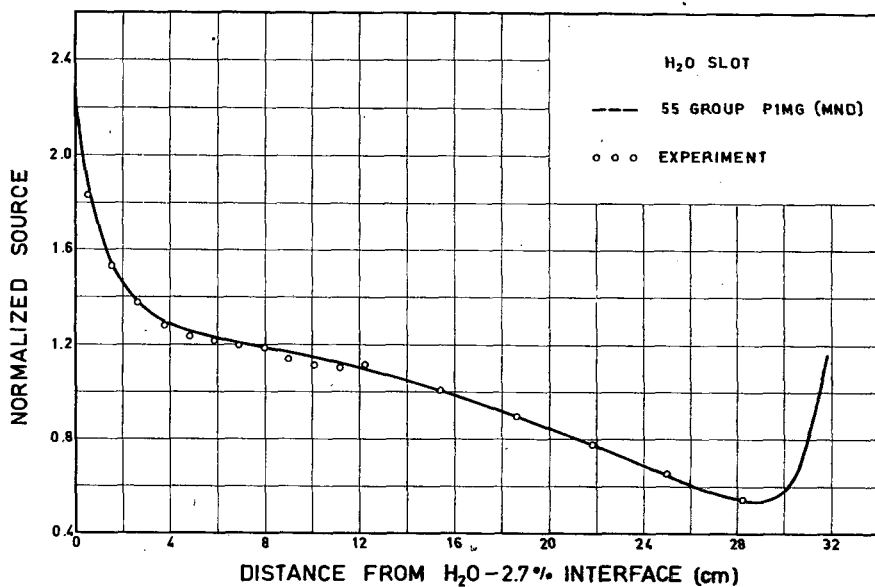


Fig. 6

Fuel rod scan - core I-A - curve 1

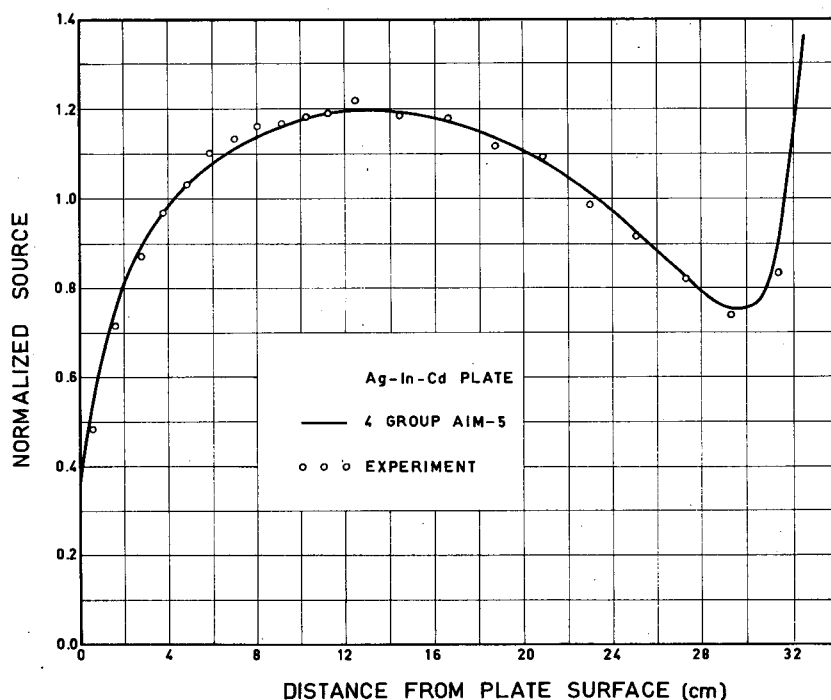


Fig. 7

Fuel rod scan - core VI-A - curve 1

Microscopic parameters

A. General

To supplement the direct measurements of critical size and power distributions, measurements of specific reaction rate ratios, often referred to as microscopic parameters, were also carried out. The parameters selected for study are defined as follows:

- (1) δ^{28} - The ratio of fission rate in U^{238} to the fission rate in U^{235} .
- (2) δ^{25} - The ratio of epicalcium fission rate in U^{235} to the thermal fission rate in U^{235} .
- (3) CR - The ratio of total capture rate in U^{238} to the total fission rate in U^{235} .
- (4) ρ^{28} - The ratio of epicalcium capture rate in U^{238} to thermal capture rate in U^{235} .

Measurements of microscopic parameters provide information useful in the evaluation of methods of calculating the burn-up of the reactor fuel and also make possible the verification of the detailed neutron balance which is assumed in the criticality calculation. The techniques utilized in the measurement of microscopic parameters were based on methods originally

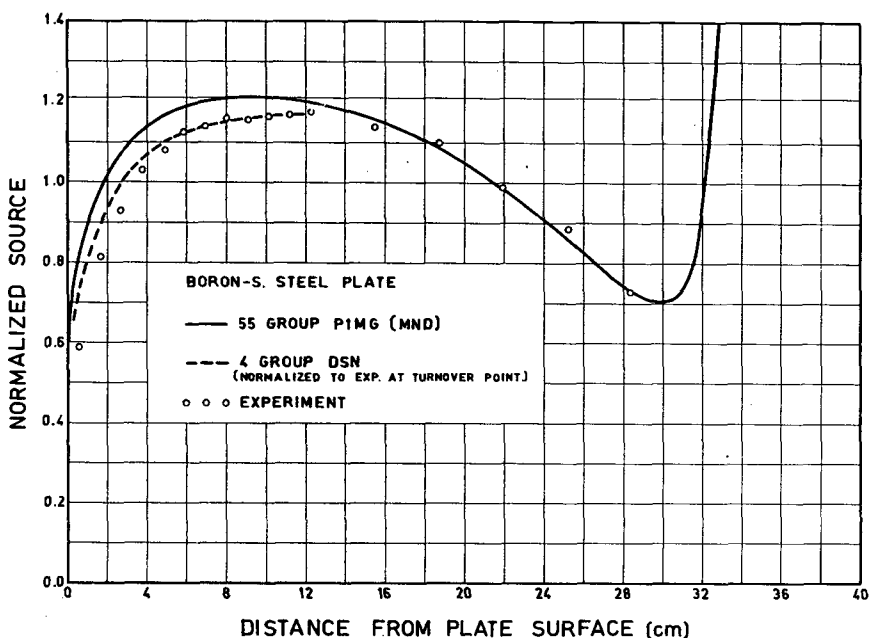


Fig. 8

Fuel rod scan - core V - curve 1

reported by BNL[12] and BETTIS [11] groups and represented modifications of methods employed earlier in the Yankee and BR-3 critical experiments [13]. In general, the measurements of microscopic parameters involve the irradiation of foils inserted between UO_2 pellets in the fuel rods, so that, in the interpretation of the results, assumptions must be made regarding the relationship between processes in the detector foils and processes in the reactor itself. The possibility of perturbation of the reactor by the detector foils must also be considered. Foil activation measurements carried out in the programme of multi-region critical experiments involved the use of 5 mil U^{238} metal foils depleted to 190 ppm in U^{235} and 5 mil highly enriched U^{235} aluminium foils containing sufficient U^{235} to produce the same macroscopic thermal cross-section as that of the 2.7% UO_2 fuel pellets.

B. δ^{28}

The fast fission reaction rate in the lattice is determined by means of the measurement of the parameter δ^{28} . This parameter is determined by the simultaneous irradiation of U^{238} foils and U^{235} foils placed together in the fuel rods of the lattice. Fission product activation is determined and the data are corrected by referring to previous measurements of $p(t)$, the ratio of yield curves for fission product gammas after irradiation of the two uranium isotopes. Values of δ^{28} can be compared with values obtained from the MUFT-SOFOCATE results. To correct for the differences between homogeneous and heterogeneous calculations, the MUFT results

for δ^{26} are increased in accordance with results of monoenergetic analyses carried out independently by Dr. W. Zernik of WAPD and Dr. R. Hellens of Brookhaven National Laboratory. A comparison of the theoretical and experimental values of δ^{28} is shown in Table II.

Table II

THEORETICAL AND EXPERIMENTAL VALUES OF δ^{28}

W/U	Enrichment	MUFT-SOFOCAT	δ^{28} Heterogeneous	Experimental
2.5:1	1.6%	0.089	0.093	0.107 ± 0.006
2.5:1	2.7%	0.080	0.084	0.097 ± 0.007
2.5:1	3.7%	0.075	0.079	0.078 ± 0.005
4.5:1	1.6%	0.058	0.063	0.072 ± 0.001
4.5:1	2.7%	0.051	0.056	0.066 ± 0.002
4.5:1	3.7%	0.048	0.053	0.054 ± 0.002

C. δ^{25}

The parameter δ^{25} is defined as the ratio of episcadmium fission in U^{235} to subcadmium fission in U^{235} . δ^{25} can be obtained from the cadmium ratio of U^{235} by use of the relation $\delta^{25} = 1 / CdR^{25} - 1$. Figure 9 shows the arrangement of foils utilized in this measurement. To eliminate the

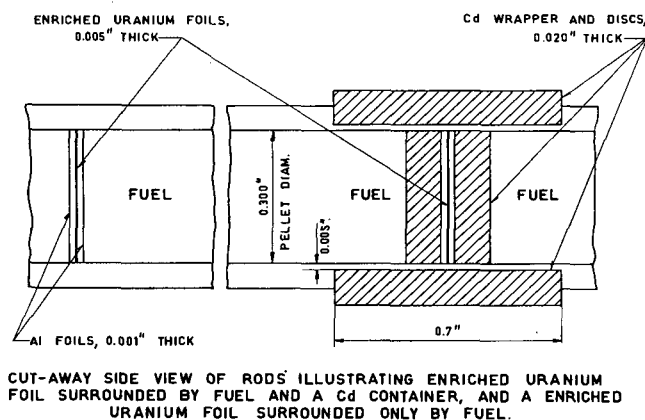


Fig. 9

Exposure geometry for the measurement of the U^{235} Cd ratio for fission in the fuel

uncertainties in the symmetry of the neutron flux in the lattice, two measurements are made, exchanging the positions of the U^{235} and U^{238} foils and determining the cadmium ratio from

$$CdR^{25} = \sqrt{(CdR^{25})_1 \cdot (CdR^{25})_2}$$

The values of δ^{25} were calculated by use of the MUFT-REPETITIOUS-SOFOCATE model. The MUFT-REPETITIOUS-SOFOCATE calculation is one in which the fast neutron reaction rates are determined from the MUFT IV programme. For the resonance group (5530 eV to cadmium cut-off energy), reaction rates are calculated by the REPETITIOUS [14] programme. REPETITIOUS is a modification of the NYU-REP [15] Monte-Carlo programme for resonance escape calculation developed by R. A. Dannels and S. M. Hendley of our laboratory. The modification permits the explicit representation of resonances for U^{235} as well as for U^{238} .

On the basis of a simplified analysis of the cadmium attenuation in slab geometry and after a review of the literature on the subject [16], a cadmium cut-off of 0.380 eV was selected to correspond to 10 mil of cadmium and a cut-off of 0.490 eV to correspond to 20 mil of cadmium. Theoretical values of δ^{25} were calculated for both values of the cut-off to determine the sensitivity of the results of this assumption. Calculated and experimental values are compared in Table III. It is felt that good agreement is indicated between theory and experiment to the accuracy of the measurements, the statistics of Monte Carlo and the determination of the cadmium cut-off.

Table III

THEORETICAL EXPERIMENTAL VALUES OF δ^{25}

W/U	Enrichment	δ^{25} -Theoretical		δ^{25} -Experimental
		$E_c = 0.380 \text{ eV}$	$E_c = 0.490 \text{ eV}$	
2.5:1	1.6%	0.168	0.155	0.144 ± 0.003
2.5:1	2.7%	0.238	0.219	0.207 ± 0.002
2.5:1	3.7%	0.313	0.287	0.267 ± 0.005
4.5:1	1.6%	0.092	0.083	0.080 ± 0.002
4.5:1	2.7%	0.128	0.115	0.114 ± 0.003
4.5:1	3.7%	0.169	0.151	0.156 ± 0.003

D. CR^*

Perhaps the most significant parameter measuring the relationship between processes in the reactor lattice is the conversion ratio. To provide a measure of this quantity we have defined a related parameter CR^* which we refer to as the "modified conversion ratio". CR^* is defined as the

ratio of total capture rate in U^{238} to the total fission rate in U^{235} . This quantity can be measured directly by irradiation of U^{238} and U^{235} foils in the fuel rods of the lattice. To normalize the counting rates of the equipment measuring the neptunium gammas resulting from U^{238} capture and the fission product gammas, a calibration run in a thermal column is required. The modified conversion ratio measurement has the advantage of introducing very little perturbation into the reactor, since no cadmium ratios are required. CR^{*} measurements were carried out in the 4.5:1 lattice utilizing a calibration run in the Brookhaven thermal column. Measurements in the 2.5:1 lattice were also made, but with less accuracy, because the calibration was made in the thermal peak of the multi-region reflector.

The modified conversion ratio was calculated from the results of MUFT-REPETITIOUS-SOFOCATE runs. The comparison between the calculated and experimental results is shown in Table IV. It is apparent that excellent agreement was obtained in the runs involving the thermal column calibration. Further experiments in the 2.5.1 lattice are planned to investigate the disparity between calculation and experiment which, at present, is attributed to the uncertainty of the calibration run carried out in the reflector.

Table IV

MODIFIED CONVERSION RATIOS

W/U	Enrichment	CR [*] Theoretical	CR [*] Experimental
2.5:1	1.6%	0.880	0.854 ± 0.08
2.5:1	2.7%	0.653	0.595 ± 0.06
2.5:1	3.7%	0.530	0.480 ± 0.05
4.5:1	1.6%	0.662	0.668 ± 0.018
4.5:1	2.7%	0.482	0.470 ± 0.010
4.5:1	3.7%	0.407	0.408 ± 0.020

Note: 2.5:1 values are non-asymptotic values obtained in a 3-region core.

E. ρ^{28}

For the determination of ρ^{28} , depleted U^{238} foils are irradiated, one bare and one cadmium covered, in two positions in the reactor where the flux is expected to be the same. The foils are then exchanged, and a second irradiation is made. The cadmium ratio is determined from the expression

$$CdR = \sqrt{(CdR)_1 \cdot (CdR)_2},$$

where CdR refers to the ratio of bare foil activity to cadmium covered foil activity. This procedure results in cancellation of any error due to asymmetry in the flux. The geometry in which the foils are exposed is shown in Fig. 10. Note that both 5 mil foils are separated from the oxide fuel by 1 mil aluminium spacers. To avoid displacement of moderator, a position for the 10 mil cadmium wrapper is provided by milling out a 10

mil depression in the stainless steel clad surrounding the oxide pellets. Twenty mil end covers of cadmium are separated from the U^{238} foil by 100 mil pellets of oxide fuel to avoid the effects of resonance neutron streaming. Foil activations are determined after waiting 48 h for fission product

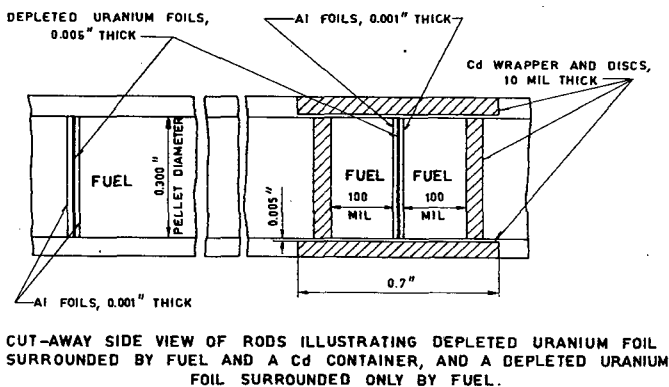


Fig. 10

Exposure geometry for the measurement of the U^{238} Cd ratio for capture in the fuel

decay by counting the 106 keV γ -rays from neptunium decay with a scintillation spectrometer. The parameter ρ^{28} is then obtained by the use of the relation $\rho^{28} = 1/CdR - 1$. The results of the determination of ρ^{28} from the cadmium ratios in the six lattices studies are tabulated in Table V.

Because of a disparity between the measured and calculated values of ρ^{28} obtained from cadmium ratios, an alternate method for determining experimental values utilizing the results of the conversion ratio measurements and the δ^{25} measurements was devised. The relevant equation can be shown to be

$$\rho^{28} = CR^* (\delta^{25} + 1) \frac{\frac{\langle \Sigma_f \rangle_{th}^{25}}{\langle \Sigma_c \rangle_{th}^{28}} - 1}{1}$$

The experimental values of ρ^{28} obtained in this way are also given in Table V.

The analytical values of ρ^{28} can be obtained from the MUFT-SOFOCATE calculations and from the MUFT-REPETITIOUS-SOFOCATE calculations after the selection of a "cadmium cut-off" energy. Results of the analysis are indicated in the final columns of Table V for a cut-off of 0.380 eV. It is interesting to note the agreement between the two analytical values and the considerable disparity between experimental values obtained by two methods. Also, as is apparent, there is considerably better agreement between theory and experiment when the cadmium ratio method is not used.

Table V

CALCULATED AND EXPERIMENTAL VALUES FOR ρ^{28}

W/U	Enrichment	ρ^{28} Theoretical		ρ^{28} Experimental	
		MUFT-SOFOCATE	MUFT-REPETITIOUS SOFOCATE	From Cd R	From CR* & δ^{25}
2.5:1	1.6%	2.501	2.556	2.08 ± 0.09	--
2.5:1	2.7%	3.788	3.893	2.78 ± 0.16	--
2.5:1	3.7%	5.042	5.190	3.45 ± 0.11	--
4.5:1	1.6%	1.426	1.453	1.25 ± 0.11	$1.443 \pm .070$
4.5:1	2.7%	2.136	2.142	1.72 ± 0.07	$2.015 \pm .060$
4.5:1	3.7%	2.759	2.802	1.96 ± 0.08	$2.750 \pm .130$

It should be pointed out that values of cadmium ratios as small as 1.25 are obtained in these experiments, so that errors of the order of 1% in cadmium ratio can lead to errors of 5-6% in ρ^{28} .

Because of the large discrepancy between theory and experiment in the CdR experiments, a number of auxiliary experiments were carried out in an effort to identify the source of any systematic errors which might exist in the experimental method. No single source of error great enough to explain the difficulty was identified. Careful analysis of the data seems to indicate that there is an unaccounted for reduction in the resonance activation of the U^{238} foil of the order of 8% which occurs in the presence of cadmium. This effect could be partially a direct shielding and partially a reduction in slowing down density due to the depression of the fission source. A third effect which could contribute an error of this nature would arise in the event that significant fission γ -activity is counted by the scintillation counter at the energy of the neptunium line. Because the conversion ratio measurement eliminates the use of cadmium in the lattice, it has been adopted as the preferred procedure for evaluating the part of the analytical method pertaining to neutron capture in U^{238} .

To provide an additional check on the validity of the calculation of resonance capture in U^{238} , the REPETITIOUS calculations were utilized to calculate effective resonance integrals for U^{238} in the 2.5:1 and 4.5:1 lattice. The calculation was performed by extrapolating to zero enrichment and no clad from the results for 1.6%, 2.7% and 3.7% enriched fuel in each lattice. The effective resonance integrals were evaluated from the expression

$$-\ln \rho^{28} = [N^{28} / \xi \Sigma_s] (RI)_{\text{eff}}.$$

Values of the calculated resonance integrals for the energy range 5530 eV-0.625 eV are shown in Table VI along with values derived by DRAWBAUGH [17] by application of Bell's equivalence theorem [18] to calculations of ADLER-HINMAN and NORDHEIM [19]. In the interpretation of Drawbaugh's results, three-fourths of the resonance integral of 1.6 b attributed to the range 1000 eV - 30000 eV was assumed to lie below 5530 eV and 1.4 b was

TABLE VI

EFFECTIVE RESONANCE INTEGRALS

W/U	(RI) _{eff} Monte Carlo	(RI) _{eff} Analytical
2, 5:1	19.4	18.7
4, 5:1	20.3	20.4

added to account for $1/v$ capture. Agreement between the Monte Carlo and analytical theory is quite good.

ACKNOWLEDGEMENTS

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PHYSICS CHARACTERISTICS OF SEED-BLANKET LATTICES

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Abstract — Résumé — Аннотация — Resumen

PHYSICS CHARACTERISTICS OF SEED-BLANKET LATTICES. Seed-blanket core arrangements consisting of a natural uranium blanket driven by a highly enriched seed, as exemplified in the Shippingport cores, present a type of light water lattice which is very interesting physicswise and is also of great practical importance as a promising approach to economic nuclear power.

Some of the areas of major physics interest are:

- (1) The phenomena at the boundary between the seed and the blanket in which the neutron flux and spectra are changing rapidly;
- (2) The control characteristics of such an arrangement. The reactor tends to have the control properties of a small highly enriched core;
- (3) The depletion characteristics. By replacing seeds it is possible to drive the blanket to the radiation limit of the materials, estimated to be about 30 000 MWd/T average for uranium oxide fuel. Most of the energy in the blanket at this time will have come from burning plutonium.

The physics characteristics of the Shippingport cores, both initially and throughout life, and the parameter measurements in the critical assemblies will be presented and compared with calculations.

CARACTÉRISTIQUES DES RÉSEAUX GERME-COUCHE FERTILE. Des ensembles germe-couche fertile consistant en une couche d'uranium naturel activée par un germe fortement enrichi, comme c'est le cas des cœurs de la centrale de Shippingport, représentent un type de réseau uranium-eau légère qui offre un intérêt considérable du point de vue de la physique et revêt, en outre, une grande importance du fait qu'il ouvre des perspectives prometteuses pour la production d'énergie d'origine nucléaire dans des conditions de rentabilité.

Parmi les aspects qui présentent un intérêt spécial pour la physique figurent notamment:

1. Les phénomènes qui s'opèrent à la jonction du germe et de la couche fertile, où le flux et les spectres neutroniques subissent des modifications rapides.
2. Les caractéristiques de contrôle d'un tel ensemble. Le réacteur tend à avoir les mêmes propriétés, quant à son contrôle, qu'un cœur de faibles dimensions à uranium fortement enrichi.
3. Les caractéristiques d'appauvrissement. En remplaçant les germes, on parvient à porter le taux de combustion dans la couche fertile à sa valeur maximum, estimée à environ 30 000 MWd/t en moyenne pour le combustible en oxyde d'uranium. La plus grande partie de l'énergie qui est alors produite dans la couche fertile est due à la combustion de plutonium.

L'auteur indique les caractéristiques physiques des cœurs de la centrale de Shippingport, déterminées tant au début que pendant toute la période de leur utilisation, et les paramètres mesurés des ensembles critiques; il compare ces données avec les résultats des calculs.

ХАРАКТЕРИСТИКИ РЕШЕТОК ДВОЙНОЙ АКТИВНОЙ ЗОНЫ. Системы с активной зоной воспроизводства, состоящие из зоны воспроизводства из природного урана, приводящейся в действие высокообогащенной активной зоной, примером чего служат активные зоны реактора Шиппингпорт, представляют собой разновидность легководной решетки, которая весьма интересна с точки зрения физики, а также имеет большое практическое значение как многообещающий путь решения проблемы экономичности ядерной энергетики.

Основными областями, представляющими большой интерес с точки зрения физики, являются:

1. Явления, происходящие на границе между активной зоной и зоной воспроизводства, где поток и спектр нейтронов претерпевают быстрые изменения.

2. Характеристики, регулирующие такую систему. Такого типа реактор может иметь регулирующие качества небольшой высокообогащенной активной зоны.

3. Характеристики обеднения. Путем замены активной зоны возможно довести предельное облучение материала зоны воспроизводства в среднем примерно до 30 000 мгвт-дней/Т для топлива из окиси урана. Большая часть энергии в зоне воспроизводства в это время будет получаться за счет выгорания плутония.

Физические характеристики активных зон реактора Шиппингпорт как в начальной стадии, так и в процессе работы, а также измерения параметров критических сборок, будут представлены в сравнении с расчетами.

CARACTERÍSTICAS DE LOS RETICULADOS DE GERMEN-ENVOLTURA FÉRTIL. Los cuerpos de reactor del tipo "germen-envoltura fértil", que constan de una envoltura de uranio natural activada por un germen muy enriquecido y cuyo ejemplo es el cuerpo del reactor de Shippingport, constituyen un tipo de reticulado de agua ligera muy interesante por sus características físicas, que revisten al mismo tiempo gran importancia práctica debido a las perspectivas que ofrece para la producción económica de energía nucleoelectrica.

Algunos de los aspectos físicos más interesantes son los siguientes:

1. Los fenómenos que se desarrollan en el límite entre el germen y la envoltura fértil, donde el flujo y los espectros neutrónicos varían rápidamente.
2. Las características de control de esta disposición. El reactor tiende a asumir las mismas propiedades, en lo que respecta al control, que un reactor pequeño de combustible altamente enriquecido.
3. Las características de agotamiento. Reemplazando los "gérmenes" es posible activar la envoltura
3. Las características de agotamiento. Reemplazando los "gérmenes" es posible activar la envoltura hasta el límite de resistencia a la radiación de los materiales, que se calcula en unos 30 000 MWd/t en promedio, para el combustible de óxido de uranio. La mayor parte de la energía desarrollada en la envoltura en ese momento habrá procedido de la combustión del plutonio.

En la memoria se describen las características físicas de los núcleos de reactores tipo Shippingport, tanto inicialmente como en el curso de la explotación, así como los resultados de las mediciones de los parámetros en los conjuntos críticos; esos datos se comparan con los resultados de los cálculos.

INTRODUCTION

Seed-blanket core arrangements consisting of a natural uranium blanket driven by a highly enriched seed, as exemplified in the Shippingport cores, present a type of light water lattice which is very interesting to the physicist and is also of great practical importance as a promising approach to economic nuclear power.

The difference between a seed-blanket core arrangement and a spiked core is essentially qualitative. In a seed-blanket core the highly enriched elements are grouped contiguously to concentrate sufficiently large amounts of reactivity in the seed region that the core may be controlled by controlling the seed and the amount of fuel required for criticality is minimized. The entire core then tends to have the dynamic characteristics of the seed, i. e. of a small, highly enriched reactor.

Those general physics characteristics of seed-blanket cores which are of primary interest from a power generation standpoint were presented at the Geneva Conferences on Peaceful Uses of Atomic Energy [1, 2]. This paper emphasizes the properties of the seed-blanket core concept which are of primary interest from the standpoint of the physics of light water lattices.

The work to be covered in this paper consists of the following topics:

1. Measurement and computation of selected parameters in rod-type blanket lattices (typical of first Shippingport core)
2. Measurement and computation of selected parameters in plate-type blanket lattices (typical of second Shippingport core)
3. Results of power sharing measurements in Shippingport Core 1
4. Experiments with homogeneous blanket
5. Post operation evaluation of blanket rods from Shippingport
6. Operation with successive seeds

1. PARAMETERS IN ROD-TYPE BLANKET LATTICES

A. Measurements and computation of the conversion ratio of the Shippingport critical facility blanket

Since one of the important advantages of the seed-blanket core concept is the achievement of extremely high irradiation in the natural uranium blanket, with the major portion of the energy produced by the blanket coming from plutonium, it is of great interest to determine the production rate of fissionable plutonium isotopes in the blanket. The conversion ratio (C. R.) is a measure of this production rate and is defined at the beginning of core life as:

$$\begin{aligned} \text{C. R.} &= \frac{\text{Pu}^{239} \text{ atoms produced in the blanket}}{\text{U}^{235} \text{ atoms destroyed in the blanket}} \quad (1) \\ &= \frac{\text{U}^{238} \text{ radiative captures}}{\text{U}^{235} \text{ fissions} + \text{U}^{235} \text{ radiative captures}} \end{aligned}$$

If the ratio of uranium isotopic concentrations in the blanket is N^{28}/N^{25} , then

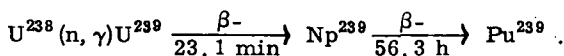
$$\text{C. R.} = \frac{N^{28} \left(\frac{\text{U}^{238} \text{ absorption}}{\text{U}^{238} \text{ atom}} \right)}{N^{25} \left(\frac{\text{U}^{235} \text{ fission}}{\text{U}^{235} \text{ atom}} \right) (1 + \alpha_b^{25})} \quad (2)$$

where $\alpha_b^{25} = \sigma_c^{25}/\sigma_f^{25}$, the cross-sections being averaged over the neutron spectrum in the blanket, and the subscript b refers to the blanket.

The blanket assembly for the first Shippingport core contains fuel rods of 0.357 in natural uranium oxide pellets stacked inside a Zircaloy tube 0.411 in o.d. The oxide pellets have a density equal to 93% of theoretical maximum.

Suppose the conversion ratio is to be determined in a particular blanket position in the critical assembly. At that point a pair of foils is exposed; one foil is made from natural uranium while the other contains uranium enriched to > 90% in the U^{235} isotope. Activation of the natural foil will result from fissions and captures in U^{238} and U^{235} . Ignoring for the moment the

contributions from U^{235} absorptions and U^{238} fissions in the natural uranium foil, the foil is activated by the following reaction:



A measurement of the β -decay of the activated foil for several days following irradiations will give a quantity which is proportional to the number of U^{238} absorptions in the foil. It is therefore possible to write

$$U^{238} \text{ absorptions} = K_1(t) P_b(t). \quad (3)$$

Here $K_1(t)$ is a function reflecting the decay constant of the foil activity, the efficiency of the detector observing the β -decay, and the effective U^{238} absorption cross-section; and $P_b(t)$ is the β -counting rate of the exposed natural uranium foil at time t after irradiation, the subscript b again referring to the blanket.

The activity of the foil containing enriched uranium will be due mainly to fission products from U^{235} fission. If the decay of the fission product γ -radiation is counted as a function of time, a quantity is obtained which is proportional to the U^{235} fission rate, and

$$U^{235} \text{ fissions} = K_2(t) F_b(t). \quad (4)$$

Here $K_2(t)$ is a function reflecting the decay constant of the fission product activity, the efficiency of the detector observing this decay, and the effective U^{235} fission cross-section; and $F_b(t)$ is the fission product γ -ray counting rate of the exposed enriched uranium foil at time t after irradiation.

$$\text{Then} \quad C. R. = \frac{N^{28} K_1(t) P_b(t)}{N^{25} K_2(t) F_b(t) (1 + \alpha_b^{25})}. \quad (5)$$

It is now necessary to determine the functions $K_1(t)$ and $K_2(t)$ to obtain a value for the conversion ratio. Suppose two foils, one natural and one containing enriched uranium, are irradiated in a neutron flux with a large ratio of thermal to epithermal neutrons, so that the epithermal activation of the foils may be neglected. In particular a neutron spectrum similar to that found in a thermal column is desirable, since it is then possible to describe the neutron energy distribution quite precisely.

The activation per atom of the natural uranium foil due to U^{239} radiative captures in the thermal column may be written as

$$(\sigma_a^{28} \varphi)_{th} = K_1(t) P_{tc}(t) \quad (6)$$

and the neutron absorption in the foil containing enriched uranium is

$$(\sigma_a^{25} \varphi)_{th} = K_2(t) F_{tc}(t) (1 + \alpha_{tc}). \quad (7)$$

Here φ is the thermal neutron flux at the irradiation position, and the cross-

sections σ_a^{25} and σ_a^{28} are the absorption cross-sections of U^{235} and U^{238} averaged over the Maxwellian distribution of neutrons in the thermal column. The subscript (tc) refers to the thermal column.

In the last equation it is assumed that the fission products resulting from thermal neutron fission in the thermal column are the same as fission products produced in the reactor spectrum present in the blanket of the critical facility. Experiments at Bettis have a constant cadmium ratio of fission product γ -radiation from highly enriched uranium foils as a function of time after irradiation. The foils were irradiated in a slightly enriched water moderated rod lattice. It is believed that the assumption is therefore valid.

Solving (6) and (7) for $K_1(t)$ and $K_2(t)$ respectively and substituting in (5):

$$C.R. = \left(\frac{N^{28}}{N^{25}} \right) \left[\frac{P_b(t)/P_{tc}(t)}{F_b(t)/F_{tc}(t)} \right] \left(\frac{\sigma_a^{28}}{\sigma_a^{25}} \right)_{th} \left(\frac{1 + \alpha_{tc}^{25}}{1 + \alpha_b^{25}} \right). \quad (8)$$

It is evident that the irradiations in the blanket and the thermal column must have the same duration in order to obtain meaningful values of $K_1(t)$ and $K_2(t)$. In addition, $P_b(t)$ and $P_{tc}(t)$ must be measured at the same time after the end of the irradiation. A similar requirement is made on the experimental values of $F_b(t)$ and $F_{tc}(t)$, but the quantities $F(t)$ and $P(t)$ need not be measured at the same time. In Eq. (8) the only quantity which is not readily obtained is the value of α_b^{25} . However, since the epithermal neutron flux magnitude is much lower than the thermal flux, the weighted value of α_b^{25} is not very much different from α_{tc}^{25} . For practical purposes the ratio $(1 + \alpha_{tc} / 1 + \alpha_b)$ is set equal to unity. The conversion ratio may then be written:

$$C.R. = \left(\frac{\Sigma_a^{28}}{\Sigma_a^{25}} \right)_{th} \left[\frac{P_b(t)/P_{tc}(t)}{F_b(t)/F_{tc}(t)} \right]. \quad (9)$$

In summary, a determination of the conversion ratio by this technique involves measurement of two relative quantities. The Pu^{239} production and U^{235} fission rates in the blanket are compared with the same quantities after irradiation of a similar set of foils in a thermal column. Knowledge of the neutron spectrum in the thermal column makes possible an absolute normalization to obtain a value of the blanket conversion ratio.

U^{235} Fission Measurements

A normalization similar to that described for the Pu^{239} production measurement is made to determine the number of U^{235} fissions by detection of the fission product gamma rays. The foils used to detect the U^{235} fissions are 0.005-in thick, 10 wt. % enriched U^{235} in aluminium. These foils are used to insure that there is no large flux depression caused by the foil when it is placed in the reactor. Calculations indicate a flux depression of $\sim 1.5\%$ with the 10 wt. % uranium in aluminium foils in an isotropic thermal neutron flux.

Results and Comparison with Calculation

Experimental values of the conversion ratios have been obtained thus far in the two outer rows of blanket assemblies. By considering the PWR

to consist of four similar slab reactors which are connected at the corners, data have been taken in the two outer blanket assemblies normal to the centre of the slab which has seven subassemblies. Table I lists the conversion ratio values obtained at the positions specified, and the results are plotted as a function of position in Fig. 1. The Table uses the standard matrix notation for locating the position of a rod in the 11×11 blanket sub-assembly. The blanket assembly closest to the seed is designated as Assembly No. 1.

TABLE I
BLANKET CONVERSION RATIOS

Bundle assembly	Rod position	Conversion ratio
1	1-6	1.06 ± 0.02
	3-6	1.20 ± 0.02
	4-6	1.16 ± 0.03
	6-6	1.21 ± 0.03
	9-6	1.18 ± 0.02
	11-6	1.16 ± 0.03
2	1-6	1.00 ± 0.025
	3-6	1.06 ± 0.025
	6-6	1.14 ± 0.025
	9-6	1.07 ± 0.025
	11-6	0.85 ± 0.025
3	1-1	0.99 ± 0.025
	1-3	1.07 ± 0.03
	1-9	1.05 ± 0.04
	1-11	0.97 ± 0.025

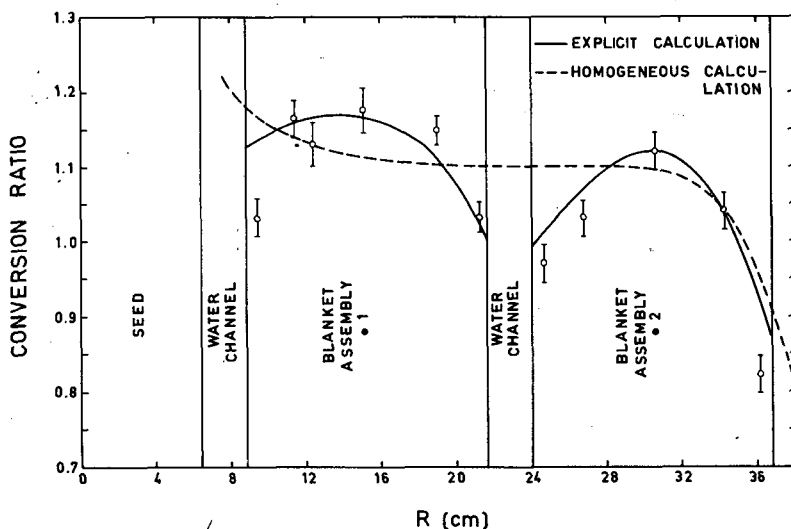


Fig. 1

Conversion ratio as a function of blanket position

The values for conversion ratio obtained by the experimental procedure described above have been compared with the results of analytic calculations. The analysis consisted of several one-dimensional four-energy group diffusion theory calculations for the spatial variation of the energy dependent neutron flux in the region of the reactor where the experimental measurements had been made.

It was stated previously that the PWR core geometry is such that, for the location at which foil measurements were taken, it is reasonable to represent the reactor as a slab arrangement of seed material surrounded by about 1 ft of blanket material and a water reflector. A slab configuration of this type is particularly amenable to a one-dimensional representation. The transverse neutron leakage is treated by assuming that in the transverse coordinate directions the spatial distribution for all neutrons of all energies is the same and follows the fundamental cosine mode shape.

The neutron flux distribution was provided by the IBM-704 WANDA code. The slow-group reactor constants used in the WANDA calculations were provided by the IBM-650 DONATE code; constants for the higher energy groups were obtained by use of the IBM-650 MUFT-III code.

Two geometric representations were examined. In one, the fuel and non-fuel regions were each described as homogeneous mixtures but of different composition. In the second representation the water channels were "smeared out" into the fuel regions so that only the grossly different regions of seed, blanket, and water reflector were differentiated.

The four-group flux distributions that resulted from calculations with these geometric representations were then used in combination with the MUFT-III and DONATE cross-sections to calculate the conversion ratio in the blanket material at various distances from the seed.

The conversion ratio is given by the expression

$$C.R. = \frac{\sum_{g=1}^4 \left[\Sigma_{ag}^{28}(x) \phi_g(x) \right]}{\sum_{g=1}^4 \left[\Sigma_{ag}^{25}(x) \phi_g(x) \right]} \quad (10)$$

where $\phi_g(x)$ represents the neutron flux density in energy group g at position x ,

$\Sigma_{ag}^{28}(x)$ is the macroscopic absorption cross-section for U^{238} in group g at x ,

$\Sigma_{ag}^{25}(x)$ is the corresponding cross-section for U^{235} ,
and $g = 4$ is the thermal group.

The four-energy groups employed in these calculations subdivided the actual spectrum of neutron energies in the following manner:

$$\begin{aligned} g = 1, & \quad 10 \text{ MeV} > E_1 \geq 0.821 \text{ MeV}, \\ g = 2, & \quad 0.821 \text{ MeV} \geq E_2 \geq 5.53 \text{ keV}, \\ g = 3, & \quad 5.53 \text{ keV} \geq E_3 \geq 0.625 \text{ eV}, \\ g = 4, & \quad 0 > E_4 \geq 0.625 \text{ eV}. \end{aligned}$$

In performing the calculations it was convenient to neglect the absorption in group one. This was found to result in values for conversion ratios that are about 2% lower than those obtained with group one absorption included.

The numerical results and their comparison with the experimental data are shown in Fig. 1. The values plotted have been reduced by 3% to account for the variation of the conversion ratio parallel to the interface between seed and blanket. In the calculation, the water channels normal to the seed-blanket interface are not explicitly described. It is therefore necessary to compare the average experimental value obtained with the calculation. If all epithermal fissions in fuel are neglected and epithermal capture in U^{235} is not considered, it is possible to write the conversion ratio as

$$C.R. = \frac{(1 - p^{28}) + p^{28} \left(\Sigma_a^{28} / \Sigma_a \right)_{th}}{p^{28} \left(\Sigma_a^{25} / \Sigma_a \right)_{th}} \quad (11)$$

By using a value of $p^{28} = 0.76$ in Eq. (11), the conversion ratio is found to be 1.17. This value is in reasonable agreement with the average value obtained by the "explicit" or "smeared out" calculation.

Conclusions

From Fig. 1 it can be seen that the calculation which explicitly describes the water channel agrees fairly well with the experimental results. In addition, the "smeared out" calculation appears to describe the average value of the conversion ratio rather well. The increase in the conversion ratio value close to the seed in the "smeared out" case may be ascribed to the failure of the technique to describe the peak of the thermal flux in the water gap between the seed and blanket. The capture rate in U^{235} is therefore greatly reduced relative to capture in U^{238} , and a resultant increase in the calculated conversion ratio is observed. The "explicit" calculation gives these thermal flux peaks and shows the reduction in conversion ratio at the boundaries of the water gaps which is observed experimentally.

B. Measurements and computation of the distribution of U^{238} epithermal capture in the Shippingport critical facility blanket

The purpose of this work was to determine spatial variations in U^{238} resonance capture within a natural uranium blanket cluster. According to WICK [5] a natural uranium fuel plate adjacent to an intercluster water channel should have a resonance neutron absorption about 20% higher than the next nearest fuel plate. A similar effect was therefore looked for in the rod lattice of the blanket of the critical mockup of the first Shippingport core.

Since radiative neutron capture in U^{238} leads to production of Pu^{239} , a highly asymmetric buildup of Pu^{239} near the edge of a fuel cluster could lead to changing thermal power peaks as a function of core lifetime in a core containing significant quantities of U^{238} .

The critical assembly in which these measurements were made is shown in Fig. 2. The cluster in which the measurements were performed is indi-

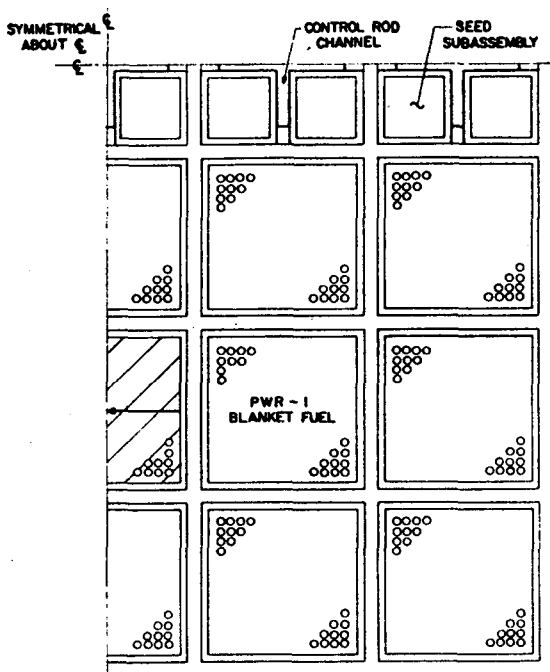


Fig. 2

Critical assembly for measurements of epithermal U^{238} captures.

The arrow indicates the direction in which the epithermal capture distribution was obtained.

cated by cross-hatching, and the direction of the measured distribution is also shown. The blanket assembly consists of an 11×11 array of unit fuel cells, one of which is shown in Fig. 2. This matrix is surrounded by a Zircaloy box 0.275-in thick with an annular water channel 0.50-in thick between Zircaloy boxes. The captures in U^{238} were observed with a γ -ray scintillation spectrometer, which detected the 104 keV x-ray produced in the de-excitation of the excited states of Pu^{239} following radiative capture. The measurements were made using 0.357-in diam $\times$ 0.005-in thick uranium metal foils highly depleted in U^{235} (190 atoms U^{235} per 10^6 atoms U^{238}) covered by 0.020-in thick cadmium. The geometry minimized the effects of fission product contributions to the observed 104 keV activity and eliminated the problems of epicadmium neutrons streaming through the cadmium wrapper without having had a chance to interact with the fuel. No correction to the observed activity was made for contributions from U^{235} or U^{238} fission products. This experiment was designed to measure the relative epithermal captures in U^{238} , and a fission contribution of 10% to the total observed activity would be needed to produce a 1% change in the ratio of maximum to minimum observed activities.

The normalized epithermal neutron activation of the U^{238} foils is shown in Fig. 3, where the relative activations have been normalized to a value of

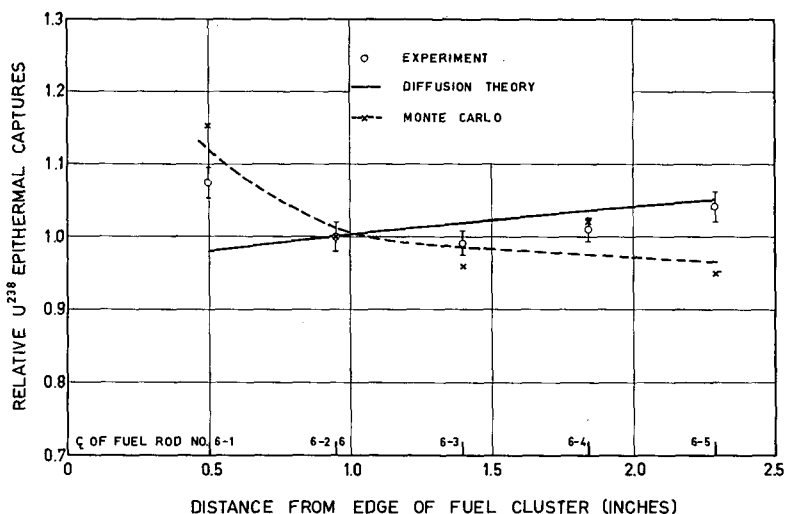


Fig. 3

Experimental and analytic results of epithermal U^{238} captures in PWR-1 blanket cluster
 The dotted curve is drawn only to indicate the trend in the Monte-Carlo results
 and has no quantitative significance.

1.00 at fuel rod position 2. In examining these data, there appears to be a slight rise in the epithermal absorption in U^{238} in the centre of the cluster. To determine the validity of this observation, a four-energy group, two-dimensional diffusion theory calculation using the IBM-704 PDQ code was carried out. A quarter core problem describing the assembly shown in Fig. 2 was performed. Input cross-sections were obtained using the IBM-704 codes MUFT and SOFOCATE. The intercluster H_2O -Zr region around each blanket fuel region was described explicitly in the analysis. The epithermal captures in U^{238} in the blanket fuel regions were computed using the calculated epithermal fluxes and U^{238} cross-sections. The result of this calculation is shown as a solid line in Fig. 3, where the calculation has been normalized to the experiment at rod position 2. The agreement is remarkably good in the central portion of the cluster, but the analysis appears to fail at the interface between the fuel and H_2O -Zr regions, where the slowing down through the resonance region increases.

The results of a Monte-Carlo analysis, NUREP code, are also given in Fig. 3, where the relative captures in each rod have again been normalized to unity at rod position 2. The calculated captures do not agree particularly well with the experiment but the increase of epithermal absorptions in the edge fuel rod is actually overpredicted by NUREP.

It is possible that the diameter of PWR-1 blanket rods is sufficiently large so that the variation in resonance capture predicted by Wick takes place primarily across the diameter of the first fuel rod. To establish the validity of this assumption, the radial distribution of epithermal neutrons in a rod at the edge of the cluster has been measured. The experiment was

performed at rod position 6-1. The observations were made on a rod diameter perpendicular to the intercluster water channel using the "spiral" technique. The spiral used was a natural uranium $\frac{1}{8}$ -in wide metal strip wrapped around a natural uranium metal cylinder to form a 0.357-in diam cylindrical fuel element. The data obtained give the epithermal U^{238} captures as a function of radial position through a metal fuel cylinder which is surrounded by oxide fuel pellets above and below its position in the fuel rod. This arrangement should be considered only as indicative of the general shape of the radial resonance distribution in an oxide fuel pellet, and no quantitative significance should be attached to the magnitudes of the observed distribution.

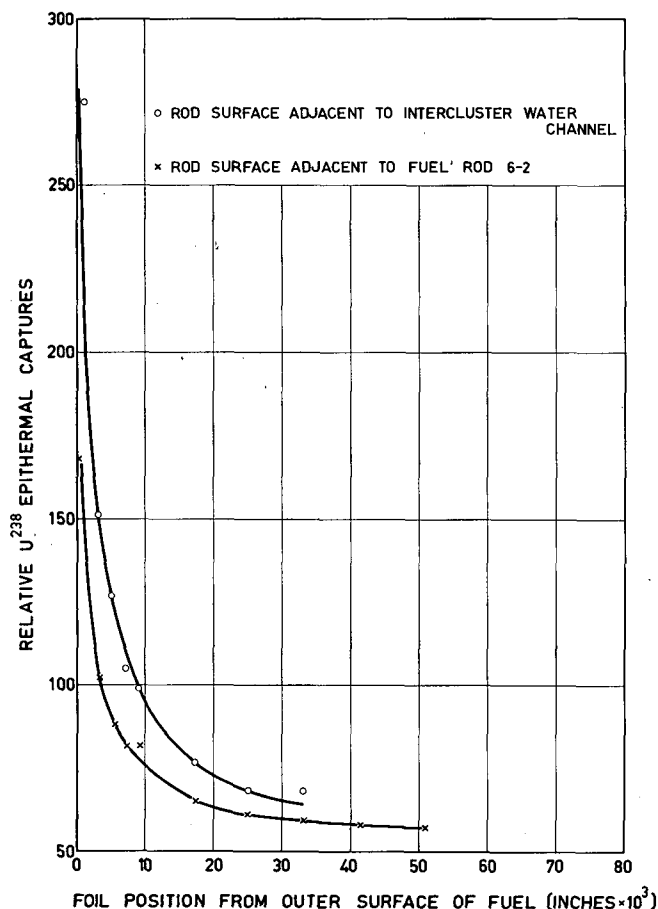


Fig. 4

Radial distribution of epithermal U^{238} captures in blanket fuel rod 6-1.

The results of the radial distribution experiments are given in Fig. 4. The results do show the type of effect predicted by Wick. The U^{238} reson-

ance capture 0.001-in inside the outer surface of the rod adjacent to the wide intercluster water channel is 65% greater than the capture observed at a similar radial position on the other side of the rod. Assuming that rod number 6-2 has a symmetric radial resonance capture with an average magnitude equal to that given by the lower curve of Fig. 3, then the average capture in rod number 6-1 is approximately 6% greater than is calculated for rod number 6-2. This is in qualitative agreement with the data shown in Fig. 3.

2. MEASUREMENT AND COMPUTATION OF SELECTED PARAMETERS IN PLATE TYPE BLANKET LATTICES

Much of this work was similar to that for Shippingport Core 1, except for differences due to the plate geometry. The following work related to the determination of the quantity ρ^{28} , the ratio of epithermal to thermal neutron radiative capture in U^{238} [6].

The design used for the blanket of the second Shippingport core (PWR-2) consists of Zircaloy compartmented and clad natural uranium-oxide platelets, which is quite different from the cylindrical oxide rods used for the first core. A series of experiments was therefore performed to measure the radiative capture in U^{238} in a geometry similar to that of the second Shippingport core. Since the second core is designed to have a blanket burn-up of 20 000 MWd/T average, the ability to predict the distribution of U^{238} production is extremely important.

The thermal neutron absorption in U^{235} and U^{238} can be determined from known cross-sections, since the fuel materials are in the same flux. The resonance region contribution is important only in the Pu^{239} production and not in the U^{235} destruction. In this particular case, where the fuel is natural uranium oxide, only about 6% of the U^{235} absorptions occur epithermally, so calculations are not particularly sensitive to this contribution. In contrast, roughly half of the Pu^{239} production takes place epithermally.

While the resonance escape probability (p^{28}) may be related to ρ^{28} , it is not particularly sensitive to variations in ρ^{28} . For a change of approximately 10% in ρ^{28} , the inferred value of p^{28} changes only 2%. In addition, calculation or measurement of p^{28} requires additional information and theoretical assumptions, while ρ^{28} can be compared directly with analysis.

Figure 5 shows one-quarter of the symmetric double seed five-cluster slab critical facility in which the measurements were made. Two seed regions were used to provide a nearly flat source in the blanket. The seed is fabricated of enriched U^{235} Zirconium alloy fuel, and each seed cluster contains a cross-shaped Cd-Ag alloy control rod. All the blanket clusters except that in the central position are made from natural UO_2 fuel rods originally used to mock up the first PWR core (PWR-1). The central blanket cluster contained a special sub-assembly, approximately 10.5-in high, which was fabricated using Zircaloy compartmented and clad PWR-2 natural uranium oxide platelets. The small PWR-2 sub-assembly should thus allow experiments to give results which are characteristic of the PWR-2 fuel geometry.

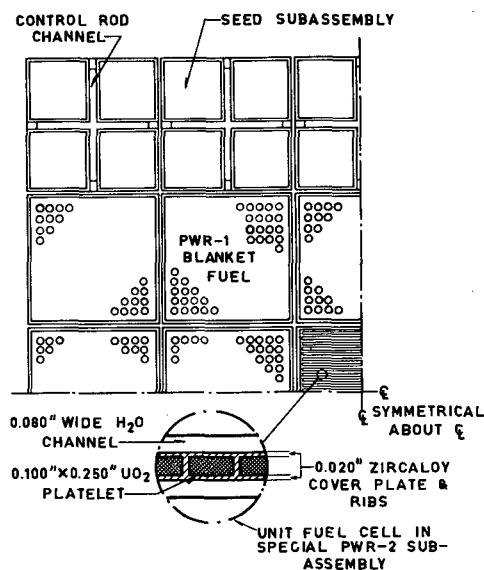


Fig. 5

Cross-section of one-quarter of double seed five-cluster slab in which ρ^{28} measurements were made.

A unit fuel cell of the plate blanket assembly is also shown.

It should be noted that measurements of ρ^{28} were not made adjacent to the seed fuel in the PWR-2 assembly. In blanket subassemblies adjacent to the seed, the neutron spectrum (and ρ^{28}) will vary, depending on the relative positions of the assemblies with respect to the seed. At positions more remote from the seed, the flux falls off asymptotically and is not disturbed by seed spectrum effects. The intent of this investigation was to determine how well diffusion theory predicts Pu production in the PWR-2 blanket fuel geometry. A measurement of ρ^{28} in the PWR-2 core geometry, observing variations in ρ^{28} in blanket assemblies near the seed, will be made at a later date.

Figure 5 shows the positioning of the fuel plates in the core as well as the dimensions of a single platelet unit fuel cell in the special assembly. This sub-assembly is made of 23 10-in high plates, each of which contains 16 such unit fuel cells. A normal fuel plate thus contains 16 axial columns of six 1.5-in long UO_2 platelets, separated by 0.040-in wide Zircaloy ribs. The sub-assembly was made so that the water channels are established by two grids holding the fuel plates as seen in Fig. 6. The sub-assembly is designed so that the single fuel cell disassembly tubes shown in Fig. 6 may be placed in any of the fuel cell positions in the 23 X 16 array.

The measurements in this assembly have determined the quantity ρ^{28} , the ratio of epithermal to thermal neutron radiative capture in U^{238} , in various positions in the assembly. The experiments employed the neptunium separation and counting techniques. In this case, the natural UO_2 fuel itself was used as the detector. An unirradiated fuel pellet was placed in the dis-

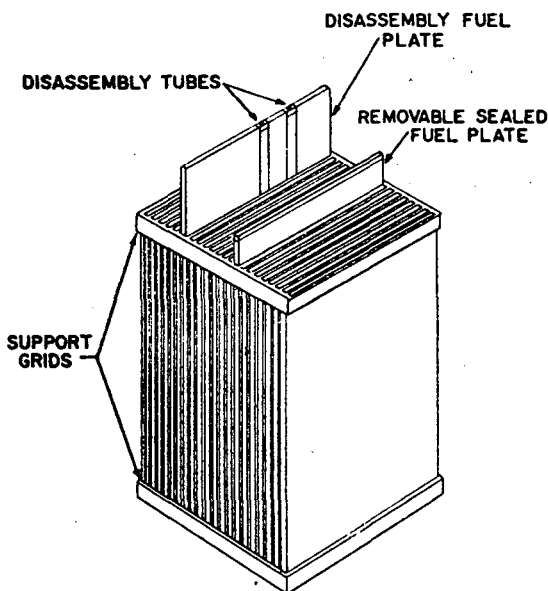


Fig. 6

Special plate blanket assembly showing disassembly fuel plate and disassembly tubes in which ρ^{28} measurements were carried out.

assembly tube as the centre of the axial array of similar fuel pellets, and after sealing the tube and placing it in the proper position in the disassembly fuel plate, it was irradiated in the slab critical assembly shown in Fig. 5. For measurements of the cadmium-covered activation (i. e., epithermal U^{238} captures), the Zircaloy disassembly tube was replaced by an equivalent cadmium tube.

EXPERIMENTAL RESULTS AND ANALYSIS

Values of ρ^{28} are given in Table II, where fuel cell locations in the 23×16 matrix are again identified by matrix notations. The increase in ρ^{28} at the edge of the bundle near the intercluster water channel (i. e., position 1-9) is real, and has been predicted analytically by WICK [5].

The experimental results have been compared with few-group two-dimensional diffusion theory using the IBM-704 PDQ code. Fast neutron constants were obtained using the MUFT or PIMG codes, and the thermal neutron constants were gotten using the IBM-704 SOFOCATE code. The analysis was done by describing the $\frac{1}{4}$ core shown in Fig. 5, with symmetry conditions about the centre lines shown in the figure. The seed fuel regions were homogenized but each cluster of the blanket was described by two regions. The blanket fuel region was made up of a multiplicity of unit fuel cells and was surrounded by an annular H_2O -Zr region which described the Zr box sides and half of each intercluster water channel. The values of ρ^{28}

TABLE II
EXPERIMENTAL VALUES OF ρ^{28} IN DOUBLE SEED AND
BLANKET SLAB

Position	ρ^{28}
1-9	0.86 ± 0.02
2-9	0.80 ± 0.02
3-9	0.86 ± 0.02
8-9	1.00 ± 0.02
12-9	1.10 ± 0.01
12-14	0.88 ± 0.02
12-15	0.86 ± 0.03

were inferred from the spatial few-group flux edit supplied as output by the PDQ code, coupled with U^{238} capture cross-sections obtained from MUFT (or PLMG) and SOFOCATE. The experimental and analytic results have been compared for a traverse perpendicular to the seed blanket interface (i.e., positions 1-9, 2-9, 3-9, etc.). This comparison is given in Fig. 7 where the results of three analytic approaches are shown.

Both MUFT and PLMG require the use of a so-called "self-shielding factor" L to describe the heterogeneous effects on resonance absorption, where L is defined as the ratio of heterogeneous to homogeneous resonance integrals for the fuel cell considered. Stein has developed an approximate method for evaluating the U^{238} heterogeneous resonance integral, which has given results which compare favourably with a Monte-Carlo calculation of the resonance escape probability in a lattice of cylindrical fuel rods. A similar approximation can be made for slab geometry cases. This approximation, however, neglects the effects of Doppler broadening and the two-dimensional geometry of the unit fuel cell under investigation, and there-

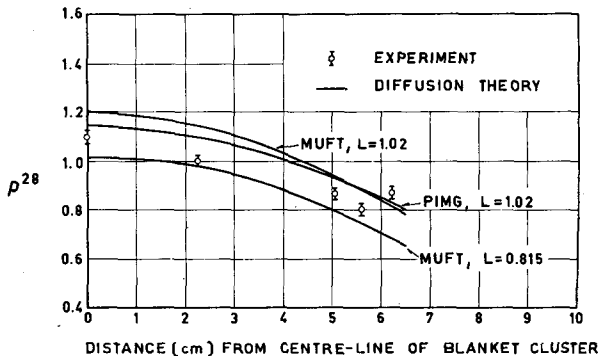


Fig. 7

Experimental values of ρ^{28} as a function of position in the special plate blanket assembly. Diffusion theory calculations using three different sets of fast neutron constants are also given.

fore a series of NUREP Monte-Carlo calculations have been carried out to evaluate these effects. The results indicate that the approximation becomes less satisfactory when Doppler broadening is important, and when a significant volume of the unit fuel cell is occupied by a heavy atom scatterer such as zirconium. Finally, it should be noted that although the L factor is called a "self-shielding factor", it has no physical significance, since the calculations of heterogeneous and homogeneous resonance integrals are usually made with different approximations. For the PWR-2 blanket the L factor inferred if the heterogeneous resonance integral is calculated using NUREP is found to be 1.02 ± 0.01 , which is to be compared with $L = 0.815$ when the Stein approximation is used. The results of two diffusion theory calculations using fast constants obtained by MUFT with these two L factors are shown in Fig. 7. It can be seen that the Stein treatment results in values of ρ^{28} which underestimate the experimental values of ρ^{28} by approximately 10%. Using the NUREP L factor in MUFT to obtain fast neutron constants, the same diffusion theory calculation gives values of ρ^{28} which are overestimated by approximately 7%.

In both of these treatments the fast neutron constants were obtained for the blanket fuel region by assuming that the neutron spectrum in each blanket fuel cluster is the same as that found in an infinite array of unit blanket fuel cells. In the experimental arrangement, this is certainly not the case, since the array of blanket fuel cells is periodically disturbed by zirconium box sides and intercluster water channels. To determine whether improved blanket fast neutron constants could be obtained with a more explicit description of the actual core used, region-wise constants were inferred using the one-dimensional PLMG code. Values of ρ^{28} inferred using PLMG fast constants in PDQ are also shown in Fig. 7 where it can be seen that ρ^{28} is overestimated by approximately 5%.

3. RESULTS OF POWER SHARING MEASUREMENTS IN SHIPPINGPORT CORE 1

Power sharing between the seed (enriched fuel) and blanket (natural uranium) is a parameter of great significance. The reactivity-lifetime of the core depends on the relative depletion of the seed and the blanket. The thermal safety of the core, depending as it does on local power distributions, is based directly on knowledge of the seed-blanket power sharing. The ability to make an accurate prediction of power sharing is necessary for understanding of core operation.

The measurements depend upon the use of thermocouples in the exit water of the various seed and blanket assemblies. Some erratic behaviour of thermocouples was observed during the lifetime of the first seed. Incident to the insertion of the second seed a large proportion of the thermocouples was replaced by an improved type. The results given here pertain to the operation of the core with Seed 2.

Figure 8 shows the fraction of total core power being delivered by the seed as a function of effective full power hours. For comparison the results of a coarse mesh three-dimensional power distribution calculation by the DRACO code are also given.

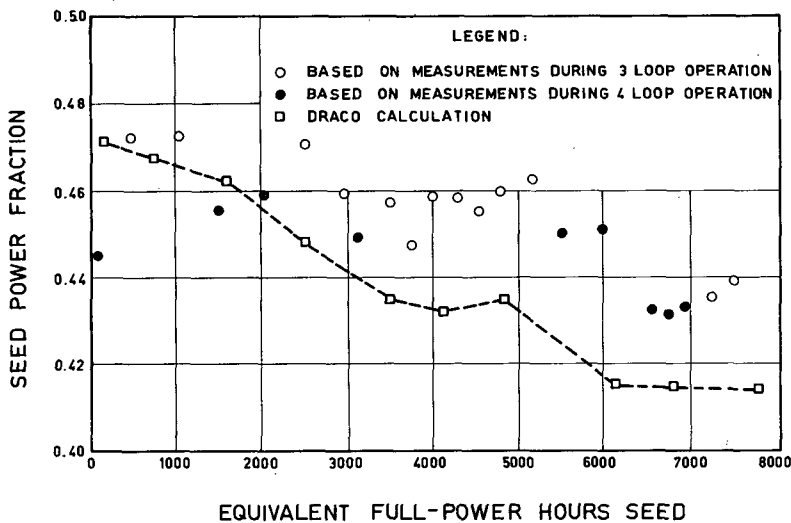


Fig. 8

PWR-I seed 2 power sharing versus EFPH

The following general observations have been made:

1. Calculated and experimental values exhibit similar trends throughout life for all seed assemblies.
2. Absence of any apparent similarity in calculated and measured trends, which may be attributable in some cases to scatter in the measurements, is the prevalent characteristic of the blanket data from beginning to mid-life of Seed 2.
3. A majority of the blanket assemblies show similar trends from mid-life to the end-of-life.
4. Scatter in the seed measurements is less pronounced than for the blanket data.
5. Measurements made during four-loop operation in most cases agree more closely with the calculation than those made during three-loop operation in both seed and blanket.
6. With some exceptions to be discussed, the calculation gives generally lower values than the measurements in the seed.
7. Measurements in several seed clusters exhibit a deviation between calculation and measurement which is roughly linear with EFPH. In all such cases the calculation is high at beginning of life and low at end of life.
8. In the seed, time-wise variations in the measurement tend to lag those in the calculation.
9. In the blanket, the measurements seem to be generally lower than the calculation.

The foregoing statements relate only to the differences between calculated and measured values. In order to derive conclusions concerning the accuracy of the measurements and the calculations, the true values of the

cluster-wise power fractions must be known. However, the approximations in the calculational model and the assumptions discussed subsequently, which are required to convert thermocouple readings to power fractions, make it necessary: (1) to base conclusions regarding the reliability of the calculations on the assumption that the measured values are accurate, and (2) to make conclusions regarding the precision and accuracy of the measurements using the calculated power fractions as the reference.

The following statements can be made regarding the reliability of the DRACO three-dimensional power distribution calculation, assuming that the measurements are accurate:

1. The increases and decreases in measured power fractions with time are reflected in the calculation. Since the time-wise variation is principally a consequence of control rod motion, the three-dimensional DRACO calculation successfully represents this variation, whereas two-dimensional models would be less successful in this respect.

2. In most cases, the calculation tends to exceed the measured seed cluster power fractions early in Seed 2 life and to underestimate them late in life. This tendency is consistent with the behaviour of the overall seed power fraction. This behaviour is probably associated in part with a reactivity behaviour of the irradiated blanket fuel which is more favourable than the calculations have indicated. A second factor contributing to this phenomenon is the necessity of assuming a constant neutron flux level over intervals of around 1000 EFPH in the calculation. Thus, when power fraction is increasing (early in life) the fuel is not sufficiently depleted in the calculation (causing an over-estimation of power output), and when power is decreasing, the fuel is overly depleted (causing an underestimation of power output).

3. From comparisons of two-dimensional detailed and homogenized flux weighted calculations it appears that the flux weighting method, as necessary in DRACO, places too much of the seed power in clusters with rods inserted. This factor was influential in the utilization of a model involving geometrically more detailed calculations in the nuclear design of PWR-2.

4. Although the lesser precision of the blanket cluster power measurement makes comparison more difficult, there appears to be a trend for the calculated values to exceed the measurements by amounts which increase with distance of the given blanket assembly from the seed. This trend is consistent with observations made from previous comparisons of flux-weighted calculations with activation experiments in the PWR-1 mock-up.

By again examining the deviations between calculations and measurement, but now assuming the calculation as the reference, the following statements may be made with regard to reliability of values inferred from thermocouple data:

1. Blanket measurements are generally of lesser quality than seed values as evidenced by the greater scatter. Additional evidence for this is indicated by comparison of symmetrically located measurements which should be identical, but which display marked dissimilarities.

2. A difficulty may exist with measurements made during three-loop operation in that they generally tend to deviate more from prediction than comparable four-loop measurements.

3. The measured power output in seed clusters during that portion of life when rods are inserted tends to be less than predicted. This may be associated with the assumptions concerning local power distribution within the cluster which are used in interpreting thermocouple readings in terms of the total power output per cluster.

An apparent lack of complete mixing in the exit of the seed clusters, along with existing radial gradients in power generation in a rodged seed cluster, makes the cluster power measurement dependent upon the radial position of the thermocouple over the cluster. A worst case study based upon Physics cell calculations and an assumption of no exit water mixing indicates that the measured power for fully rodged clusters would be too low by a factor of 1.7. On the other hand, a study of data obtained from primary and auxiliary thermocouples in such clusters indicated much smaller deviations.

4. EXPERIMENTS WITH HOMOGENEOUS BLANKETS [7]

(a) The blanket regions in Shippingport cores are large (~ 4 migration lengths wide). The flux in much of such a region is in energy equilibrium with the materials present and exhibits the leakage properties which would be seen in an exponential assembly. So as to determine accurately the characteristic flux shape in such a region, a special assembly of PWR-1 blanket materials known as the "homogeneous blanket" has been constructed. The design is such that there is a region of blanket fuel which is (1) far from all boundaries, (2) driven by an approximately one-dimensional source region, and (3) of a sufficient homogeneous extent to allow an adequate measure of the flux shape.

(b) Analytic background

In a homogeneous asymptotic region, i. e., a region which is sufficiently far from all boundaries that only the magnitude of the flux is dependent on position, the solution of the diffusion equations becomes simple. In two energy groups, the diffusion equations are

$$D_f \nabla^2 \phi_f - \left[\Sigma_f^r + \Sigma_f^a \right] \phi_f + \frac{\nu \Sigma_s^f}{\lambda} \phi_f + \frac{\nu \Sigma_s^f}{\lambda} \phi_s = 0 \quad (12)$$

and

$$D_s \nabla^2 \phi_s - \Sigma_s^a \phi_s + \Sigma_f^r \phi_f = 0, \quad (13)$$

where subscripts distinguish fast and slow. Equation (13) immediately becomes

$$\frac{\nabla^2 \phi_s}{\phi_s} = \frac{\Sigma_s^a}{D_s} - \frac{\Sigma_f^r}{D_s} \frac{\phi_f}{\phi_s}. \quad (14)$$

Since ϕ_f and ϕ_s must have the same spatial independence, $\frac{\nabla^2 \phi}{\phi}$ must be constant.

From the identification of terms, where

$$D_s \nabla^2 \phi_s = \text{leakage rate into unit volume} = \text{minus } D_s B^2 \phi_s,$$

$$\Sigma_s^a \phi_s = \text{absorption rate for thermal neutrons, and}$$

$$\Sigma_f^r \phi_f = \text{source to thermal,}$$

we see that a measure of B^2 is a direct measure of the leakage rate.

We may also relate the buckling, B^2 , directly to criticality parameters by eliminating the fluxes from Eqs. (12) and (13). Replacing $\nabla^2 \phi$ and defining the following terms:

$$1/\tau = [\Sigma_f^r + \Sigma_f^a - \nu \Sigma_f^f/\lambda] / D_f, \quad (15)$$

$$L^2 = D_s / \Sigma_s^a, \quad (16)$$

and

$$k_\infty = [\nu \Sigma_s^f \Sigma_f^r / \lambda] / [\Sigma_s^a (\Sigma_f^r + \Sigma_f^a - \nu \Sigma_f^f / \lambda)], \quad (17)$$

From Eqs. (12) and (13)

$$2B^2 = -\left(\frac{1}{\tau} + \frac{1}{L^2}\right) \left[1 - \left\{ 1 + \frac{4[k_\infty - 1] \tau L^2}{(\tau + L^2)^2} \right\}^{\frac{1}{2}} \right]. \quad (18)$$

The second term of the radical is small (~ 0.03 for PWR blanket fuel), so that this expression may be expanded, giving

$$B^2 = \frac{(k_\infty - 1)}{M^2} \left\{ 1 - \left[\frac{(k_\infty - 1) L^2}{(M^2)^2} \right] + \dots \right\}, \quad (19)$$

where $M^2 = \tau + L^2$. For blanket fuel, the second term is a small correction ($\sim 1\%$).

(c) Experiment

A diagram of the experimental arrangement for the homogeneous blanket is shown in Figs. 9 and 10. The homogeneous blanket itself is a 15 in X 15 in array of Shippingport Core 1 mock-up blanket fuel rods (Fig. 11) spaced by aluminium grids which are at about 10 in, 20 in and 70 in from the bottom of the core. The surrounding blanket regions are also Core 1 blanket assemblies with the normal Zircaloy-2 box enclosing the fuel without the usual peripheral water channels. The driving region is an inline slab 30-in long and 6-in wide. The loading of the seed is such as to cause the entire arrangement to be critical with moderator at about 40 in. On the side of the five-cluster slab away from the homogeneous blanket, there is an array of normal PWR-1 blanket clusters, five bundles wide and six bundles deep.

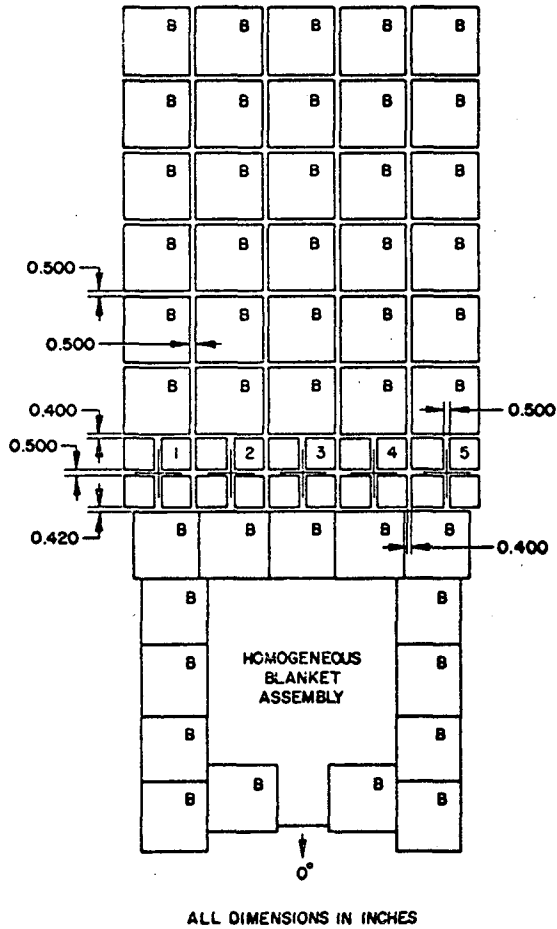


Fig. 9

Five cluster slab seed arrangement with homogeneous blanket II
 Note: "B" indicates PWR blanket bundles;
 the seed clusters are numbered 1-5;
 for detail of homogeneous blanket assembly, see Fig. 2.

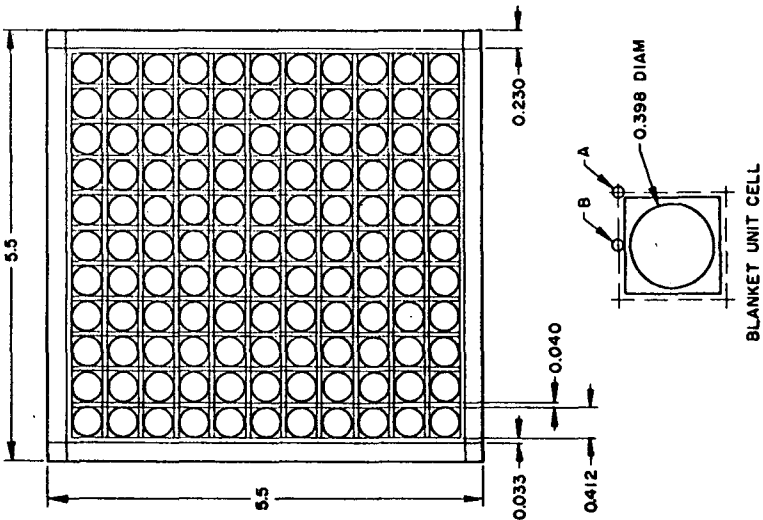


Fig. 11

Cross-section of a blanket bundle

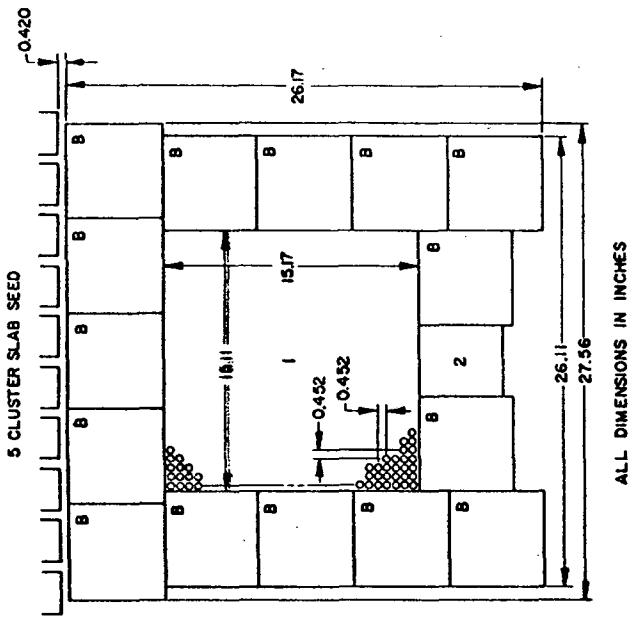


Fig. 10

Detail of homogeneous blanket II

Note: "L" is a 33 x 33 array of blanket tube elements;

"2" is a 9 x 11 fill-in of PWR blanket tube elements;

"B" indicates PWR blanket bundles.

The activation shapes have been measured along a series of traverses. For each of these traverses, a wand containing bare Mn wires is irradiated, and the activation distribution is taken as being equivalent to the flux distribution along that traverse.

The flux shape in the x-direction (horizontal and normal to the seed face), being driven by the seed, becomes exponential in the asymptotic limit.

The experimental flux shapes have been determined for two different compositions of blanket fuel. In one case, the UO_2 in the fuel rod is continuous, while in the other the UO_2 is interrupted each 10 in of length by 1-in long Zircaloy-2 spacers. The latter arrangement corresponds to that used in normal blanket units.

As all flux shapes have been fitted to a cosine or to an exponential, we may describe the fitted curves by

$$\phi = \phi_0 \exp (+ i B_k k),$$

where $k = x, y, z$ and where B_k^2 is real, either positive or negative (the real part and phase factor, of course, being understood). In Table III the experimental measurements are summarized.

TABLE III
SUMMARY OF EXPERIMENTAL B^2 ($\times 10^4/\text{cm}^2$)

	Homogeneous blanket without Zr spacers	Homogeneous blanket with Zr spacers	Normal blanket bundles
B_x^2	-44.2 ± 0.7	-42.0 ± 1.2	-44.0 ± 0.6
B_y^2	12.0 ± 1.4	11.8 ± 1.0	11.4 ± 0.3
B_z^2	6.9 ± 0.2	7.0 ± 0.1	6.3 ± 0.3
$B_x^2 + B_y^2 + B_z^2$	-25.3 ± 1.6	-23.2 ± 1.6	-25.9 ± 0.7

Since these measurements represent the total net leakage from a unit cell of blanket materials, this leakage rate is a function of the materials alone.

Several calculational checks have been made as to the applicability of these numbers to these experiments. For the homogeneous blanket there are several factors given by the available diffusion theory codes that indicate the range which can be treated as asymptotic. In an asymptotic region, the flux shape must be exponential with the same exponent in all energy groups; the flux ratio between any two groups must be independent of position; and, in two-dimensional calculations, the flux shape in a given position must be invariant under lateral displacement. A series of one-dimensional calculations representing an x traverse from the seed through homogeneous blanket, buttressing, and water reflector indicates that there is a region of about 6 to $6\frac{1}{2}$ in in the centre of the blanket which has constant B^2 and flux ratios. Over this range there is negligible change in the flux ratios, but beyond this range the boundary effects have a strong influence, with $\nabla^2 \phi_{\text{TH}} / \phi_{\text{TH}}$ changing by orders of magnitude within ~ 1 in. Artificial changes to the external media can change the magnitude or even the direction of the change

in $\nabla^2\phi/\phi$, but such changes have no apparent effect on the value in the range considered to be asymptotic. The range over which the pointwise $\nabla^2\phi_{TH}/\phi_{TH}$ varies by less than $1 \times 10^{-4}/\text{cm}^2$ in these problems is selected as the range over which the experimental points are fitted.

A two-dimensional diffusion theory calculation has been done for the homogeneous blanket in which water channels, buttressing, and seed have been explicitly represented. In two dimensions, the boundary of the region which may be considered asymptotic is not as clearly defined as in one dimension. The deviation from the asymptotic B^2 which is expected on the basis of this calculation is in the direction of measuring a more negative buckling than the asymptotic value. Assuming the fitting to have been done over a range somewhat greater than the asymptotic range in both x and y , the expected value of the total buckling is not more negative than $-19.3 \times 10^{-4}/\text{cm}^2$, with a "best" estimate (last square fit to apparent extent) being $-18.5 \pm 0.5 \times 10^{-4}/\text{cm}^2$.

In the array of normal bundles, a slightly different criterion is used for establishing the asymptotic range. Because of the heterogeneities in the calculation, the flux ratio is not expected to be constant but should repeat at corresponding locations in successive cells. Also, the ratio of fluxes at corresponding locations in successive cells in an asymptotic limit will be independent of the location. Both those criteria are met over a wider range in the normal blanket array than is practicable to measure. Thus, the buckling measured is presumed to be asymptotic buckling.

In Table IV the best values for calculated and experimental total B^2 's are summarized.

TABLE IV
HOMOGENEOUS BLANKET B^2 ($\times 10^{-4}/\text{cm}^2$)

	Without Zr spacers	With Zr spacers	Normal blanket
Experiment	-25.3 ± 1.6	-23.2 ± 1.6	-25.9 ± 0.7
Asymptotic calculation	-17.2	-16.9	-16.8*
Calculation corresponding to experiment	-18.5	--	-18.7**

* Using volume averaged diffusion constants to describe the blanket cell.

** Using a weighted average of constants to describe the blanket cell, the weighting factor being the relative thermal flux as calculated for a detailed blanket cell.

The difference between calculation and experiment is approximately the same for the cases with and without Zr. It is reasonable to assume, then, that the Zr spacers do not cause a significant discrepancy between calculation and experiment. The discrepancy in all of the cases, however, is well beyond experimental uncertainty.

We may express this discrepancy in several ways. Directly in terms of the buckling, the error is about $7 \pm 4 \times 10^{-4}/\text{cm}^2$, or a 25% (approx) overestimate of the blanket fuel leakage. One method of compensating for errors in representation is by using an eigenvalue other than unity to describe a steady-state situation. If a blanket unit cell were described as having the

experimentally observed leakage, it would be necessary to use an eigenvalue of 1.035 to obtain a steady-state solution.

For the normal blanket bundles, the flux-weighted constants give somewhat better agreement than the volume-weighted constants. The discrepancy when flux-weighted blanket constants are used is of the same magnitude as that for the homogeneous blanket. This does not necessarily imply that the blanket fuel alone is responsible for the blanket cell discrepancy, but it does suggest that a more rigorous set of equivalent average blanket constants would be wasted without improvement in the treatment of the blanket fuel.

The resultant disagreement between calculation and experiment can be compared with previous measurements in similar lattices. An extensive survey of criticality predictions for slightly enriched uranium lattices with various metal-to-water ratios, rod diameters, and enrichments indicated a calculated critical eigenvalue of about 1.01 to 1.02, even when all measurable factors (other than criticality, as such) were accounted for in the calculations.

(d) Computational uncertainties

Thermal utilization

One of the anomalies noted in the homogeneous blanket measurements is that the thermal flux maximum on the diagonal between blanket rods is greater than that midway between adjacent rods (i.e., the flux at point A in Fig. 11 is greater than at B) by some 7%. Diffusion theory predicts this ripple to be of the order of only about 2%. The ripple factor has been measured several times in the homogeneous blanket. The first measurement was done by irradiating a wand containing Mn wires, spacing these in the assembly at 0.50-in intervals. Since the spacing of the rod centres is 0.451-in this places detectors at successive locations between A and B (Fig. 11) (as a vernier). The gross shape, being exponential, can easily be removed, leaving several cycles of the ripple. A second measure has been made over a portion of the homogeneous blanket with 0.030-in spacing of detector wires. Again the measured ripple factor is a consistent $7 \pm 1\%$.

The ripple factor and thermal utilization of the homogeneous blanket have been calculated by several different methods. The most exact mode of calculation (TUT Code) gives a thermal utilization of about 1% less than diffusion theory. The measured ripple factor is more consistent with the TUT calculation and, thus, with the lower thermal utilization. The calculations are done for the blanket unit cell UO_2 cylindrical pellets enclosed in an Al tube 0.908-cm diam in a square water hole 1.153 cm on a side. As this geometry is not easily represented in two dimensions in either cylindrical or cartesian coordinates, an approximation to the geometry is required. All of the calculations are cell geometry (zero gradient at all boundaries) with an assumed zero transverse leakage. Three general types of geometry have been used:

- (1) Approximate two-dimensional representation [TUT, PDQ];
- (2) Slab geometry [WANDA, FLIP]; and
- (3) Cylindrical geometry [WANDA, CLIP].

The latter two geometries have been used only to estimate the validity of several transport approximations and cannot conveniently be used to infer two-dimensional flux distributions.

The diffusion theory (PDQ) geometry is a detailed step approximation to the cylinder and explicit description of the square cell. The Monte-Carlo (TUT) geometry is similar with a less detailed approximation to the cylinder. The volumes of water and UO_2 are maintained, as are the relative water volumes in the corner and along the sides of the cell. The thermal utilizations given by these calculations, using standard design methods materials constants, are

$$f(\text{PDQ}) = 0.901$$

and

$$f(\text{TUT}) = 0.888$$

The various flux distributions obtained from these calculations are summarized in Tables V and VI.

TABLE V
TWO-DIMENSIONAL CALCULATIONS

Code	$\bar{\phi}(\text{water})/\bar{\phi}(\text{fuel})$	$\bar{\phi}(\text{corner water})/\bar{\phi}(\text{side water})$
PDQ	1.005	1.004
TUT	1.07	1.033

TABLE VI
ONE-DIMENSIONAL CALCULATIONS

Approximation	Water channel thickness (cm)	$\bar{\phi}(\text{peak water})$	$\bar{\phi}(\text{fuel})$
		Slab	Cylinder
P-1	0.138	1.041	1.017
	0.214		1.025
	0.383	1.095	1.042
P-3	0.138	1.084	1.054
	0.214	1.122	1.062
	0.383	1.173	1.074
DP-1	0.138	1.146	
	0.214	1.223	
	0.383	1.267	
DP-3	0.138	1.173	
	0.214	1.182	
	0.383	1.215	

If the measured ripple is assumed to be the maximum and minimum of a sinusoidal shape, the expected value of $\phi(\text{corner water})/\phi(\text{side water})$ is about 1.045. This is seen to be reasonably represented by the TUT flux shape.

From Table VI we see, as would be expected, that going to a double-P expansion results in a greater change in flux shape in this case than does a change in either geometry or degree. More thorough intercomparisons of the one-dimensional geometries indicate primarily the inadequacy of representing a "round peg in a square hole" in a one-dimensional geometry.

Resonance escape probability

The experimental measurements relating to resonance escape probability (p) are in agreement with a calculation based on customary MUFT and SOFOCATE constants and measured material compositions.

The experimentally measured quantity is ρ , the ratio of epicadmium U^{238} captures to captures below the Cd cut-off:

$$\rho = (\Sigma_1^3 \Sigma_{ci}^{28} \phi_i) / (\Sigma_{c4}^{28} \phi_4) . \quad (22)$$

In the homogeneous blanket, ρ has been measured. The direct comparison of the experimental ρ with that calculated with the standard constants is:

$$\rho_{\text{expt}} = 1.43 \pm 0.08; \quad \rho_{\text{calc}} = 1.40 .$$

The agreement between calculation and experiment is seen to be well within experimental error.

Some caution is advisable in defining p , the resonance escape probability, and in inferring p from the measured ρ .

The usual definition of p is: (a) the number of neutrons slowing down past the resonance region per neutron entering the resonance region. A second definition is: (b) the number of neutrons not captured in resonance absorption per neutron entering the resonance region. These definitions are seen to differ in the treatment of fast fissions, thus requiring a congruent definition of the fast fission factor in the calculation of criticality. The quantity which is conveniently inferred from measurement is not this p , however, but is p^{28} , in which only resonance capture by U^{238} is included. It is convenient to use a definition of p^{28} which is not analogous to the above definitions, but is the number of neutrons slowing down past the resonance region per neutron which is either slowed down or captured in U^{238} in the resonance region. This definition has the advantage of not containing reference to resonance capture in other than U^{238} . This last definition can be inferred from ρ in the following manner:

$$\frac{1 - p^{28}}{p^{28}} = \frac{\Sigma_1^3 \Sigma_{ci}^{28} \phi_i}{\Sigma_{R3} \phi_3} = \rho \left(\frac{\Sigma_{c4}^{28} \phi_4}{\Sigma_{R3} \phi_3} \right) . \quad (23)$$

Neglecting thermal leakage, this may be written:

$$p_{\text{expt}}^{28} = 1 / \left[1 + \rho \left(\frac{\Sigma_{c4}^{28} \phi_4}{\Sigma_{c4} \phi_4} \right) \right] . \quad (24)$$

The various methods of defining p and p^{28} leads to:

$$p^{(a)} = 0.618$$

$$p^{(b)} = 0.688$$

and

$$p_{\text{expt}}^{28} = 0.685.$$

Considering the agreement between p^{28} and p_{expt}^{28} , which is without allowance for such factors as Doppler broadening, there would appear to be a fortuitous cancellation of errors in the treatment of resonance escape.

Other uncertainties

Another factor which will directly affect criticality is uncertainties in cross-sections. However, making the extreme adjustment of U^{235} cross-sections consistent with the values reported at the 1955 Geneva Conference would account for only slightly more than 1% in blanket criticality.

There is also a noticeable uncertainty in the inventory of blanket materials. This alone can account for approximately 2% in eigenvalue.

Summary of calculational uncertainties

The divergence between calculation and experiment for the homogeneous blanket and estimates of the known uncertainties or errors in the calculation are summarized in Table VII.

TABLE VII
UNCERTAINTIES IN BLANKET CRITICALITY CALCULATIONS

	Eigenvalue	Critical buckling
Experiment	1.000	$-18.5 \times 10^{-4}/\text{cm}^2$
Calculation	1.035	or -25.3
Transport correction for thermal utilization	-0.01	or 2.0
Resonance escape probability	+0.01	- 2.0
	-0.025	+ 5.0
Inventory	± 0.02	± 4.0
U^{235} cross-sections	± 0.01	± 2.0
Corrected calculation	1.018 ± 0.03	or $-21.8 \pm 6 \times 10^{-4}/\text{cm}^2$

The following further observations may be made:

1. To obtain a reasonable estimate of the detailed flux shape associated with PWR-1 blanket rods in terms of a Legendre expansion, a double-P expansion is required.
2. There is agreement between the calculated and measured ρ in spite of apparent inconsistencies in the method of calculation. Therefore,

there appears to be a coincidental adequacy of the resonance escape probability as calculated.

3. A more accurate calculation of p or p^{28} should allow for Doppler broadening and for non- U^{238} resonance capture. Leakage may be neglected.

(e) Application to power distributions

This series of experiments indicates that there is a significant misrepresentation of the blanket in terms of the diffusion theory constants describing the materials. It has been pointed out that this can cause a significant error in the leakage properties ascribed to the blanket, giving an optimistic evaluation of power sharing. Several calculations have been done with modified blanket descriptions to evaluate the error introduced in calculated power distributions by misrepresentation of the asymptotic blanket leakage properties.

The comparisons to be described are based on the measurement of a detailed activation distribution in a PWR-1 Seed 2 full-scale mock-up with a seed loading of 96 kg U^{235} and 256 g of uniformly distributed boron. The core geometry is shown in Fig. 12. The number 4 and number 8 rods were fully inserted, the remainder being in a bank.

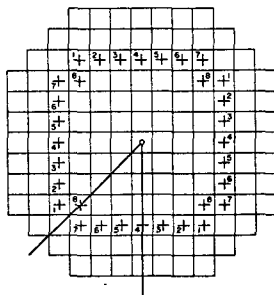


Fig. 12

PWR flexible assembly, 32 rod mock-up
Numbers represent seed cluster and
control rod reference numbers

The "standard" calculation is a four-energy group PDQ core calculation using flux-weighted cell constants. Control rods were explicitly represented calculations by empirical constants. Activations were calculated from the four-group calculated fluxes using activation cross-sections of the detector wires (U^{235} in Zr) in a characteristic spectrum for either seed or blanket. No detailed adjustment of the calculated activations have been made. The average calculated activations quoted below are based on the gross core averages, with the implicit assumption that this is equivalent to a detailed average.

Two additional calculations have been done. For these, the cell flux weighting is unchanged, and only the criticality of seed and/or blanket is

adjusted so as to change the effective leakage properties of the blanket. This is accomplished by the arbitrary adjustments of ν , the number of neutrons per fission, for the seed and the blanket independently. Since a change in ν is equivalent to a change in eigenvalue, we may characterize each calculation by an effective λ_{blanket} , which is the product of the core eigenvalue and the adjustment factor applied to the blanket ν . The calculations which have been done are listed in Table VIII.

TABLE VIII
ADDITIONAL PDQ CORE CALCULATIONS

$2.46/\nu$ (blanket)	$2.46/\nu$ (seed)	λ_{core}	λ_{blanket}
1.000	1.000	0.989	0.989
1.039	1.000	0.980	1.019
1.039	1.025	1.002	1.041

Table IX, summarizes the average activation per orifice region from the experiment and for the various calculations. The calculations are normalized to experiment in the non-rodded seed. If we assume the same error in the power distribution as in the activation distribution, we may infer an experimental power distribution from each of the calculations. The agreement among the various inferred experimental power distributions indicates the adequacy of inferring this from calculated activation distributions. Table IX, therefore, also indicates the calculated misrepresentation in power distribution calculations.

TABLE IX
CALCULATED AND EXPERIMENTAL ACTIVATION AND
POWER DISTRIBUTIONS

Average activation				
		Seed (arbit units)		Blanket (arbit units)
Experiment		9.52		13.90
Calculation				
$\lambda_B = 0.989$		9.52		15.64
$\lambda_B = 1.018$		9.52		15.00
$\lambda_B = 1.041$		9.52		14.37
Power distribution				
	Calculated		Inferred expt	
	% Power in seed	% Power in blanket	% Power in seed	% Power in blanket
$\lambda_B = 0.989$	58.3	41.7	61.3	38.7
$\lambda_B = 1.018$	59.3	40.6	61.5	38.7
$\lambda_B = 1.041$	60.3	39.7	61.3	38.7

From Table IX, it is clear that some improvement in calculated power distribution is obtained using the blanket eigenvalue suggested by the homogeneous blanket experiments (1.035) in preference to unity. It is also clear that a satisfactory comparison of calculation and experiment will not be obtained from adjustment of over-all blanket criticality alone.

(f) Summary

Using normally derived diffusion constants (energy averaging of cross-sections over a consistent neutron spectrum) does not necessarily give a satisfactory agreement in calculated and measured power distribution because of calculational uncertainties. However, a direct clean measurement of asymptotic properties of PWR-1 blanket fuel properties has been made which directly determines the effective leakage properties of that material. Significant improvement in calculated power distributions is obtained by making artificial changes in the blanket description to give agreement with asymptotic measurements.

5. POST-OPERATION EVALUATION OF BLANKET RODS FROM SHIPPINGPORT

(a) During the first seed refueling of Shippingport, after 5806 equivalent full power hours of operation, selected natural uranium dioxide fuel elements were removed from the blanket region of the core. The destructive analyses of these fuel elements for their uranium and plutonium isotopic content afford the first experimental data for natural uranium dioxide irradiated under actual power reactor operating conditions. As part of a programme to utilize data provided by operations at Shippingport to test nuclear design methods, the experimental data thus obtained have been compared with calculated quantities.

A total of ten individual fuel elements were chosen for analysis. This paper gives the measured compositions of the fuel rods; describes the analytical techniques used; and compares the measured and calculated values with regard to the validity of the theoretical methods.

(b) Selection of fuel rods for analysis

Five fuel rods were removed from each of two blanket assemblies at the time of the first refueling. Figure 13, a cross-sectional view of PWR Core 1, indicates the physical locations of these blanket assemblies. The fuel rods selected from these assemblies were among the most highly irradiated in the core due to their proximity to the seed. Due to insufficient depletion, rods located farther from the seed portion of the core were not considered.

Each blanket assembly is composed of a stack of seven fuel rod bundles; one of which is shown in Fig. 14. A bundle consists of an 11×11 array of rigidly assembled fuel rods. The particular rods studied were removed from the third bundle from the bottom of the 6-ft high core as this is the axial region of highest depletion. Figures 15 and 16 identify the five fuel

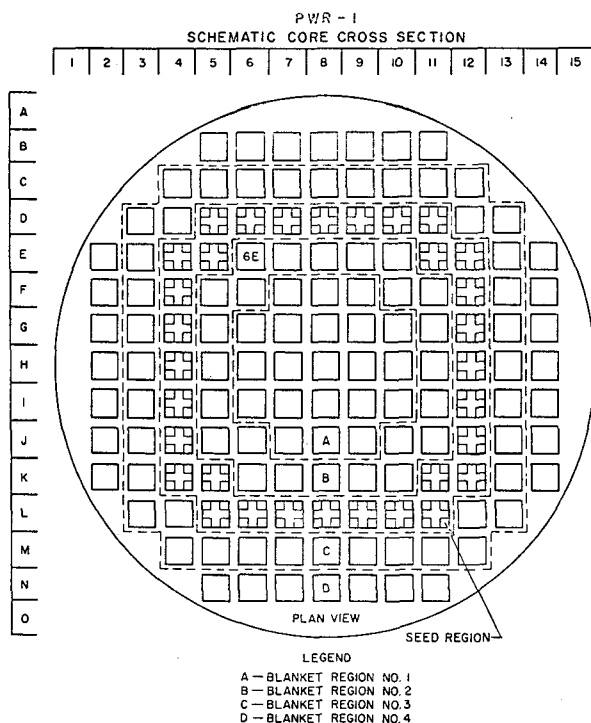


Fig. 13

PWR-I schematic core cross-section

rods taken from each of the bundles. These rods were chosen to reflect the variation of depletion within each bundle. Figure 17 depicts the geometrical details of an individual fuel rod. A rod consists of 26 uranium dioxide pellets contained within a zirconium tube sealed at both ends by zirconium end caps.

(c) Experimental techniques and results

The ten rods chosen were analysed individually for their post-irradiation isotopic composition. Specifically, determinations of total uranium and plutonium content were made and concentrations of the isotopes U^{234} , U^{235} , U^{236} , U^{238} , Pu^{239} , Pu^{240} , Pu^{241} and Pu^{242} were measured.

In addition, the total content of the radioactive fission product Cs^{137} was determined. Due to the rather long half life (28.6 yr) of the γ -activity of Cs^{137} relative to the lifetime of PWR Seed 1, its presence can be used to infer the approximate number of fissions which occurred in the rod provided that the fission yields of Cs^{137} from uranium and plutonium fission are known. With this information the energy release in the rod can be estimated.

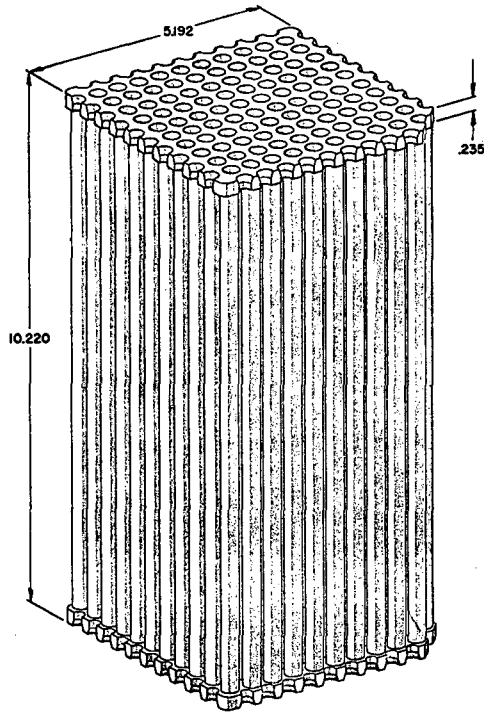


Fig. 14

PWR core I blanket bundle

ASSEMBLY 6 E

SEED

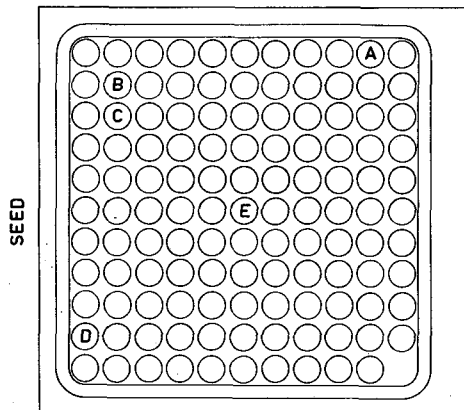


Fig. 15

Blanket bundle - fuel rod pattern

ASSEMBLY 3H

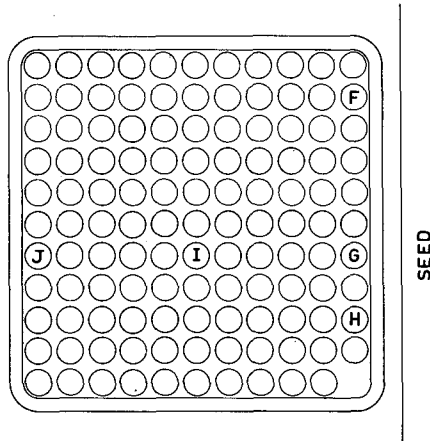


Fig. 16

Blanket bundle - fuel rod pattern.

The total uranium content and the uranium isotopic concentrations were measured mass spectrometrically. The total plutonium content and isotopic concentrations were determined by α -counting and mass analysis. The mass of Cs^{137} present was determined by radiochemical separation and γ -spectrometry.

(d) Analytical techniques

The calculation of fuel rod composition requires neutron flux and cross-section data as a function of core power history. Using this information, the change in isotopic densities due to depletion is computed according to the depletion equations common to Bettis depletion codes.

Flux histories for each rod were taken from a two-energy group, three-dimensional, depletion calculation of Core 1. Because of the large physical size of PWR, this calculation of necessity was performed with a coarse mesh point lattice ($3 \times 3 \times 3$ in) and consequently required homogenization over entire blanket bundles. In general, mesh points were not coincident with blanket rod locations. Estimates of the fluxes at the rod locations were obtained by two-dimensional interpolation in the horizontal plane. The rods of interest occupied the axial region between approximately 20 and 30 in from the core bottom. Mesh planes in the three-dimensional calculation were located at 21, 24, 27 and 30 in. After interpolation in each of these four planes, the axial values were averaged to provide fast and thermal neutron fluxes for each rod.

Due to the homogenization in the three-dimensional depletion calculation, the fluxes obtained reflect only the gross variation in the neutron flux distributions. Corrections to the thermal flux to account for local water channel peaking effects were provided by a two-dimensional blanket bundle

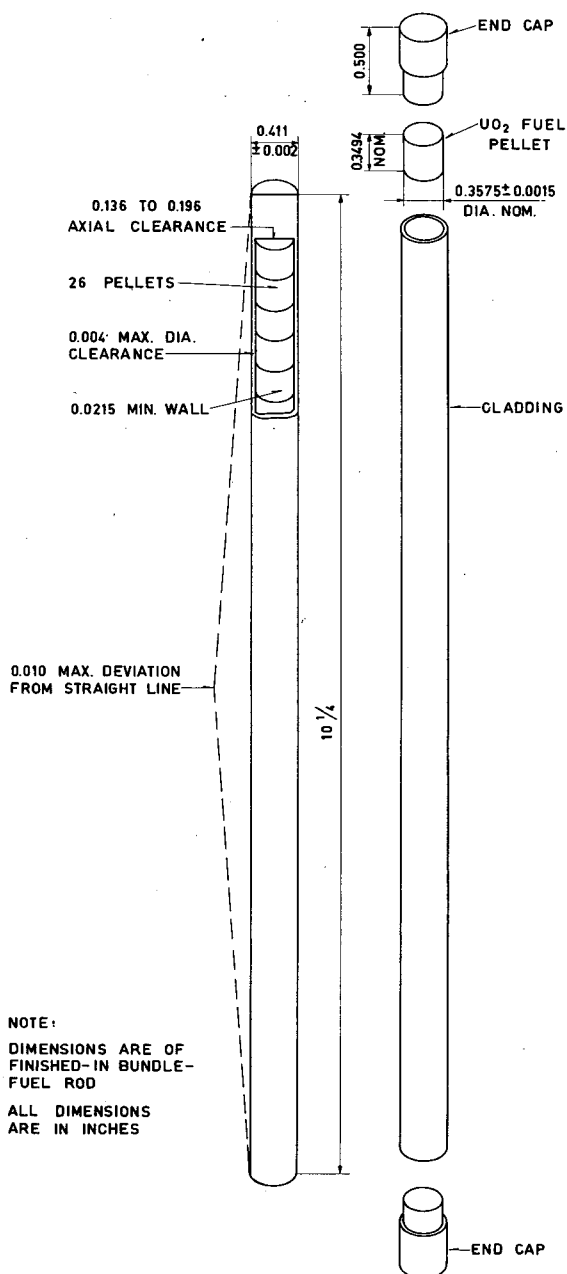


Fig. 17

PWR core I UO_2 fuel rod

cell calculation which described the geometric detail of the bundle. The resulting detailed flux distribution was superimposed on the gross flux shape by multiplying the flux at a rod position by a local factor defined as the ratio of the flux at the rod location to the average flux in the blanket cell.

The three-dimensional depletion study of Core 1 was performed for the most part in time steps of 1000 h during each of which the fluxes were assumed to remain constant. (The initial 1000 h of core life were described by several shorter time steps to account for the effects of the initial reactivity transient due to Xe^{135}). Thus the flux information consists of constant values which apply over successive intervals of 1000 h or less.

Fast and thermal neutron cross-section data were derived from the MUFT and SOFOCATE codes, respectively. Fast and thermal neutron spectra appropriate to a homogenized mixture of the materials in the blanket bundle were assumed. To take into account the effect on neutron cross-sections of the changing material compositions over core lifetime due to depletion, several MUFT and SOFOCATE calculations were performed to obtain cross-sections appropriate to best estimates of the average blanket inventory at various stages of core depletion. Averaging of the several cross-section sets thus generated provided the values used in the calculations. The U^{238} resonance self-shielding factor was computed for the homogenized blanket bundle using the prescription of Stein, and was adjusted to a fuel temperature of 1000°F.

(e) Calculated results - comparison with experiment

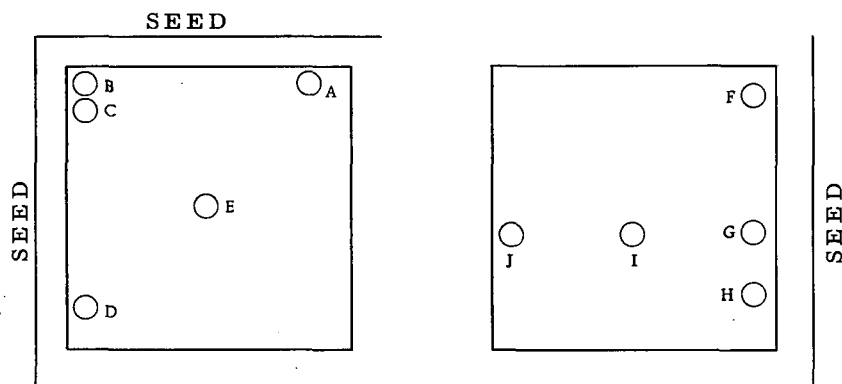
Table X compares experimental and calculated values of the amount of U^{235} remaining in each rod. For ease of reference the physical locations of each rod within the bundle is also depicted.

Comparison of calculated and measured U^{235} contents at 5800 h indicates generally good agreement. The post irradiation U^{235} contents agree with that calculated to within 10% with the exception of Rod B where the difference is 14%. In fact for seven of the rods the difference between calculation and experiment is less than 5%. Closer scrutiny of the results reveals a relationship between the agreement of the calculated value and the location of the rod within the blanket cluster. For rods located on or near the periphery of the array of blanket rods next to the intercluster water channel, the calculations evidence underestimation of the amount of depletion. The calculations for Rods E and I, on the other hand, both of which are at the centre of their respective blanket bundles, indicate if anything a slight over depletion relative to experiment.

The observed discrepancies are (barring experimental errors) presumably due to inaccurate specification of neutron cross-sections and/or flux exposure history. The cross-sections employed in the calculations were values appropriate to a homogeneous mixture of blanket materials. Thus the spatial variations in cross-sections resulting from variation in neutron energy spectrum were ignored. It might be expected that fuel rods on the periphery of the bundle would have been subjected to neutrons with a more moderate energy spectrum than the average spectrum assumed because of their proximity to the water channel. Conversely rods interior to

TABLE X
POST IRRADIATION URANIUM-235 CONTENT
Pre irradiation content - 0.96g

Rod	Measured	Calculated	% Difference
A	0.409 ± 0.001 g	0.419 g	+ 2.4%
B	0.398 ± 0.002 g	0.456 g	+14.6%
C	0.473 ± 0.005 g	0.488 g	+ 3.2%
D	0.475 ± 0.003 g	0.516 g	+ 8.6%
E	0.582 ± 0.001 g	0.572 g	- 1.7%
F	0.488 ± 0.006 g	0.516 g	+ 5.7%
G	0.520 ± 0.002 g	0.545 g	+ 4.8%
H	0.503 ± 0.012 g	0.517 g	+ 2.8%
I	0.654 ± 0.007 g	0.654 g	0%
J	0.648 ± 0.005 g	0.664 g	+ 2.5%



the bundle might be subjected to a somewhat "harder" spectrum than that assumed. Such an effect would tend to account for the observed discrepancies.

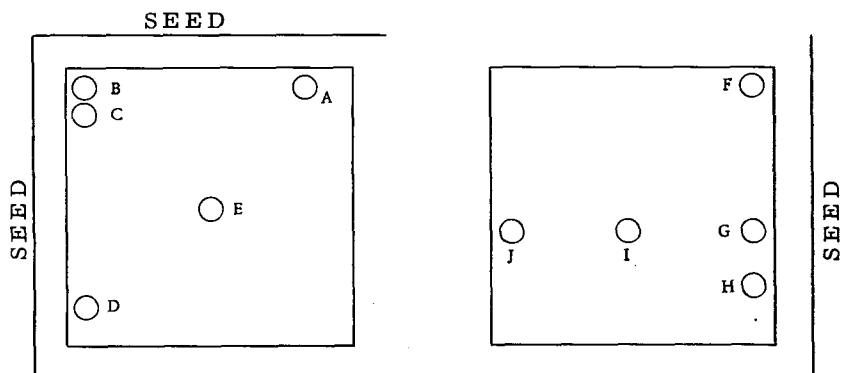
An estimate of the spatial variation of the neutron energy spectrum within a blanket cluster adjacent to the seed portion of the core has been obtained by means of a diffusion theory calculation code (SLOP-1). Comparison of these results with the average spectrum assumed in the fuel rod depletion calculations indicates that the failure to describe the variation of neutron energy spectrum does not result in errors of large enough magnitude to be responsible for the discrepancies involved. Although it is true that the intercluster water channel causes a local variation in the energy spectrum, the interaction between seed and blanket has a compensatory effect. The heavily absorbing seed "hardens" the spectrum for immediately adjacent fuel rods, thus making the average blanket spectrum approximately correct. It is more likely that the flux history assumed for each rod is primarily responsible for the observed differences.

The flux history at each rod location was derived from flux weighted calculations and superposition theory as previously discussed. Previous experience has indicated that this technique of calculating local variations in flux distributions tends to underestimate the extent of such variations. In other words local flux peaks are underestimated and local flux dips are too shallow. Furthermore, it has previously been observed that calculations in which the flux weighting technique tend to underestimate the magnitude of the flux in blanket regions adjacent to the seed and to overestimate the flux farther from the seed. A combination of these effects could account for the differences observed.

It is of interest to note the comparison of Rod J, which is on the periphery of the fuel bundle but distinct from the other such rods in that it is not adjacent to the seed. The fact that the U^{235} depletion is underestimated as it is for the remaining periphery rods suggests that the calculational differences are indeed partially due to the failure of the model to describe local variations within the blanket cell and not primarily due to some more basic error such as an incorrect fall-off of the gross flux distributions with increasing distance from the seed.

TABLE XI
POST IRRADIATION URANIUM-236 CONTENT

Rod	Measured	Calculated	% Difference
A	0.089 ± 0.0010 g	0.092 g	+ 3%
B	0.088 ± 0.0011 g	0.085 g	- 3%
C	0.076 ± 0.0010 g	0.081 g	+ 7%
D	0.078 ± 0.0013 g	0.075 g	- 4%
E	0.063 ± 0.0005 g	0.065 g	+ 3%
F	0.077 ± 0.0013 g	0.074 g	- 4%
G	0.069 ± 0.0013 g	0.069 g	0%
H	0.075 ± 0.0013 g	0.074 g	- 1%
I	0.050 ± 0.0011 g	0.051 g	+ 2%
J	0.048 ± 0.0006 g	0.048 g	0%

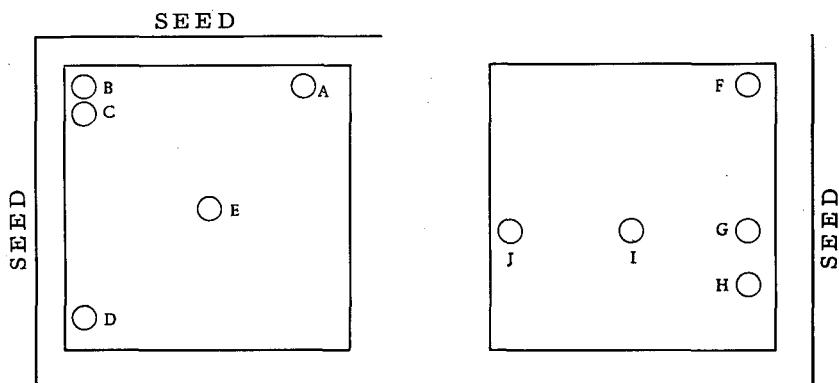


The calculated and experimental U^{236} contents are compared on Table XI. The calculated values are in general slightly higher than would be expected on the basis of the U^{235} depletion results. This is presumably due to the fact that epithermal resonance absorption in U^{236} was neglected in the calculations. In cases where the U^{235} content is in agreement so that the U^{236} production is presumably correct, the inclusion of an estimate of the U^{236} epithermal resonance absorption essentially removes the discrepancy.

Comparison of calculated and experimental total plutonium contents given in Table XII indicates general overestimation of the amount of plutonium present. Such an effect, especially for fuel rods in which the uranium depletion is underestimated, suggests misrepresentation of the resonance capture in U^{238} . Furthermore, as before, a relationship between degree of agreement of calculated values and physical location of the rod within the blanket bundle is observable. Those rods interior to the bundle evidence greater overestimation of total plutonium than do those located on the periphery of the fuel rod array, the greatest discrepancy being $\sim 15\%$ for rod I.

TABLE XII
POST IRRADIATION TOTAL PLUTONIUM CONTENT

Rod	Measured	Calculated	% Difference
A	0.617 ± 0.012 g	0.618 g	+ 0.2%
B	0.574 ± 0.007 g	0.609 g	+ 6.1%
C	0.532 ± 0.007 g	0.604 g	+13.5%
D	0.517 ± 0.007 g	0.549 g	+ 6.2%
E	0.506 ± 0.010 g	0.552 g	+ 9.1%
F	0.522 ± 0.005 g	0.525 g	+ 0.6%
G	0.521 ± 0.003 g	0.514 g	- 1.3%
H	0.503 ± 0.003 g	0.520 g	+ 3.4%
I	0.372 ± 0.007 g	0.428 g	-15.1%
J	0.341 ± 0.007 g	0.361 g	+ 5.9%



The resonance capture in U^{238} was treated by Stein's method. A self-shielding factor defined as the ratio of the effective U^{238} absorption cross-section in a representative cell to the absorption cross-section in the homogenized region was computed. This L factor was included in the MUFT calculation of fast energy group cross-sections, and the resulting energy averaged epithermal U^{238} resonance absorption cross-section was used in the depletion calculations. Due to the geometry of the blanket in which the 11×11 array of blanket rods is surrounded by an intercluster water channel, there is some ambiguity in selecting the representative cell for the L factor calculation. Interior fuel rods have less associated water than do rods on the periphery of the bundle. The L factor used in the calculations was derived on the basis of a representative cell which assumed a water to UO_2 volume ratio appropriate to that in the homogenized blanket. In effect this assumes that the intercluster water channel influences each rod equally regardless of its location relative to the water channel. Such a treatment would be expected to be more appropriate for some rods than for others which could account for the variation in agreement between calculations and experiment.

In order to assess the effect of the description of resonance capture in U^{238} , the calculations were re-run using an L factor which ignored the influence of the intercluster water entirely by assuming the water to UO_2 volume ratio to be that appropriate to a fuel rod and the water immediately surrounding it. These calculations resulted in a general 8% reduction in the amount of plutonium formed. In other words the range of discrepancies was reduced from (+ 15% to -1%) to (+ 7% to -8%).

Due to a fortitious cancellation of errors Pu^{239} contents are in somewhat better agreement than total plutonium as shown on the Table XIII. This is due to the fact that the calculated distribution of plutonium by isotope is shifted to the isotopes of higher mass number relative to experiment. This effect, which partially cancels the overestimation of Pu^{239} due to inaccurate description of resonance escape, is thought to be due at least partially to the fact that the analytical model does not include spatial self-shielding effects in plutonium. The inclusion of spatial self-shielding would have the effect of shifting the calculated distribution toward the lower mass numbers without appreciably altering the total amount of plutonium.

In order to determine the sensitivity of the plutonium isotopic concentration to self-shielding effects, the calculation for Rod A was redone with the fast cross-sections for all plutonium isotopes arbitrarily reduced by 10%. In addition, the thermal absorption of Pu^{239} was reduced by 10% since the resonance structure in the thermal energy range (0 - 0.625 eV) suggests the existence of some thermal self-shielding. In Table XIV the results of this calculation are compared with those which included no shielding and with experiment results. The information in Table XIV indicates that a realistic description of spatial shielding in plutonium could bring the calculated isotopic distribution of plutonium into better agreement.

Another quantity which indicates how successfully blanket fuel depletion is predicted and how well core lifetime behaviour is given is the energy release in the blanket fuel. This quantity, characterized by the number of

TABLE XIII
POST IRRADIATION PLUTONIUM-239 CONTENT

Rod	Measured	Calculated	% Difference
A	0.430 ± 0.009	0.400	- 7.5%
B	0.388 ± 0.005	0.409	+ 5.4%
C	0.387 ± 0.006	0.418	+ 8.0%
D	0.378 ± 0.005	0.394	+ 4.2%
E	0.404 ± 0.008	0.425	+ 5.2%
F	0.382 ± 0.004	0.374	- 2.1%
G	0.393 ± 0.003	0.377	- 4.1%
H	0.372 ± 0.002	0.371	- 0.3%
I	0.312 ± 0.006	0.352	+12.8%
J	0.287 ± 0.006	0.298	+ 3.8%

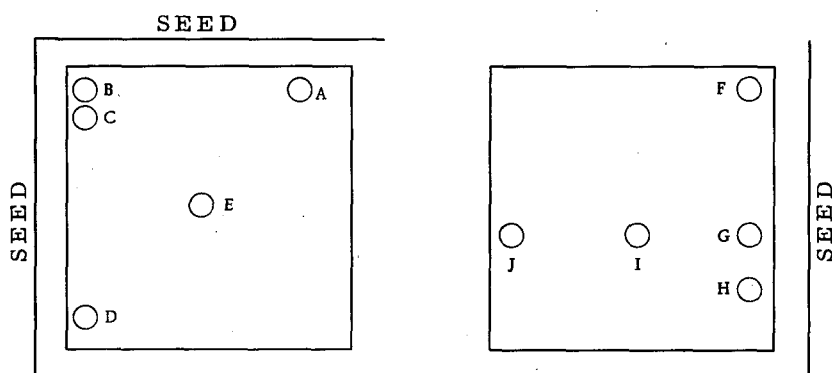


TABLE XIV
ISOTOPIC DISTRIBUTION OF PLUTONIUM IN ROD A

	Measured	Wt. % of Isotopes Calculated	
		No self-shielding	Self-shielding
Pu ²³⁹	69.71 ± 0.04	64.6	71.1
Pu ²⁴⁰	22.20 ± 0.02	23.9	20.2
Pu ²⁴¹	6.68 ± 0.02	9.3	7.0
Pu ²⁴²	1.41 ± 0.02	2.3	1.7

fissions which have occurred, can be inferred from the measured Cs¹³⁷ content of each rod. Table XV compares the total fissions as inferred from the experiment with calculated values.

TABLE XV
 TOTAL FISSIONS

Rod No.	Inferred from exp.	Calculated	Difference calc-exp.
A	2.31×10^{21}	2.50×10^{21}	+ 8.4%
B	2.42×10^{21}	2.29×10^{21}	- 5.4%
C	1.93×10^{21}	2.13×10^{21}	+10.4%
D	1.87×10^{21}	1.87×10^{21}	0%
E	1.38×10^{21}	1.58×10^{21}	+14.5%
F	1.89×10^{21}	1.82×10^{21}	- 4.0%
G	1.82×10^{21}	1.68×10^{21}	- 7.7%
H	1.80×10^{21}	1.81×10^{21}	0%
I	1.01×10^{21}	1.09×10^{21}	+ 7.9%
J	0.95×10^{21}	0.99×10^{21}	+ 4.2%

Due to uncertainties in the experimentally inferred values, it is not clear how much significance can be attached to the comparisons given in Table XV. Inference of the total fissions occurring in a rod requires knowledge of the half life as well as the fission yields of Cs^{137} for the various fissile isotopes. Based on the quoted uncertainties of published values of these quantities the product of yield and decay constant could be in error by 3 or 4%. In addition to these possible consistent uncertainties, the uncertainties associated with measuring the caesium activity in each fuel rod further obscures the comparison. For these reasons no speculation as to possible causes of the discrepancies indicated in Table XV is given, and no attempts at correlating these discrepancies with those previously observed for uranium depletion and plutonium build-up are made. It is observed, however, that the comparison given in Table XV indicates no consistent deviation in the calculations and the discrepancies in general are not extremely large. Assuming that the experimental values on the whole are reasonably indicative of the energy release it appears that there are no serious errors in the calculation of energy release in the blanket.

Continuing study of blanket fuel rod depletion is currently in progress. Isotopic analysis of a number of fuel rods removed from the core at the time of the second refueling, after approximately 13 600 h of fuel power operation, has begun. Preliminary results for the most highly depleted of these rods have been obtained. Table XVI gives the results of the analysis of this rod and compares calculated values obtained using the analytical techniques previously described.

The depletion of U^{235} is seen to be somewhat overestimated by the calculation. Since this rod was located on the periphery of the fuel rod bundle the trend observed in the analyses of the first group of rods appears to be reserved. Data on the remaining rods is necessary however before a conclusion can be drawn.

TABLE XVI

COMPARISON OF PRELIMINARY RESULTS OF FUEL ROD
REMOVED AT TIME OF SECOND REFUELING

	Measured	Calculated
U ²³⁵ content (original content 0.96 g)	0.23 g	0.18 g
U ²³⁶ Depletion	76%	81%
Total Pu content	0.95 g	0.93 g
Pu ²³⁹ content	0.54 g	0.46 g
Pu ²⁴⁰ content	0.26 g	0.23 g
Pu ²⁴¹ content	0.11 g	0.14 g
Pu ²⁴² content	0.04 g	0.09 g

The calculated total plutonium content agrees very well with the experimental value. The Pu²³⁹ content however is underestimated by 15%. It appears that the shifting of calculated plutonium isotopic distribution to isotope of higher mass number observed in the analysis of the first group of rods has become more significant at this stage of blanket depletion. An underestimation of the plutonium content of the blanket of this magnitude could have a significant effect on core reactivity.

(f) Conclusions

From the foregoing discussion it is concluded that no gross misrepresentation in the fuel depletion techniques employed is in evidence. Calculated changes in isotopic content of individual fuel rods due to depletion are seen to be different from measured values by moderate amounts depending on the location of the rod within the 11 × 11 lattice of rods making up a bundle. These differences are for the most part explainable in terms of the failure of the model to account for local variations in neutron flux exposure and resonance escape probability. Over a fuel rod bundle as a whole these errors will tend in part to cancel giving a fair prediction of average depletion.

Improvements in the prediction of the concentrations of individual isotopes are possible through inclusion of U²³⁶ cross-section data and plutonium self-shielding factors.

Preliminary results of analysis of one of the rods removed at the time of the second refueling indicate that the calculated build-up of Pu²³⁹ is considerably underestimated presumably due to the assumption of no self-shielding in plutonium.

6. OPERATION WITH SUCCESSIVE SEEDS

The natural uranium blanket of the Shippingport core was designed to operate for a total of two seeds, the first seed to have a design life of 3000 full power hours and the second seed a design life of 5000 full power hours;

thus the natural uranium blanket of the core was expected to last only 8000 full power hours. Actually the first seed lasted over 5800 full power hours and the second seed over 7900 full power hours, making a total of nearly 14 000 full power hours. The blanket was examined at the end of both the first seed and second seed and found to be in excellent condition. Therefore, a third seed was installed and is operating.

On the basis of metallurgical and physics information obtained from Shippingport and irradiation loops in test reactors, the Bettis Laboratory concludes that it would be feasible to continue operating the present Shippingport core with 10 or more seeds, leaving the original blanket in place. It appears possible to obtain an average burn-up of 30 000 MWd/T for the blanket, this limitation being imposed by metallurgical rather than physics considerations. Of this amount at least 25 000 MWd/T would be furnished by fissioning of U^{238} (either by the fast fission effect or by fissioning of the plutonium isotopes created from the U^{238}). The other 5 000 MWd/T would come from the natural U^{235} content of the blanket. This illustrates one of

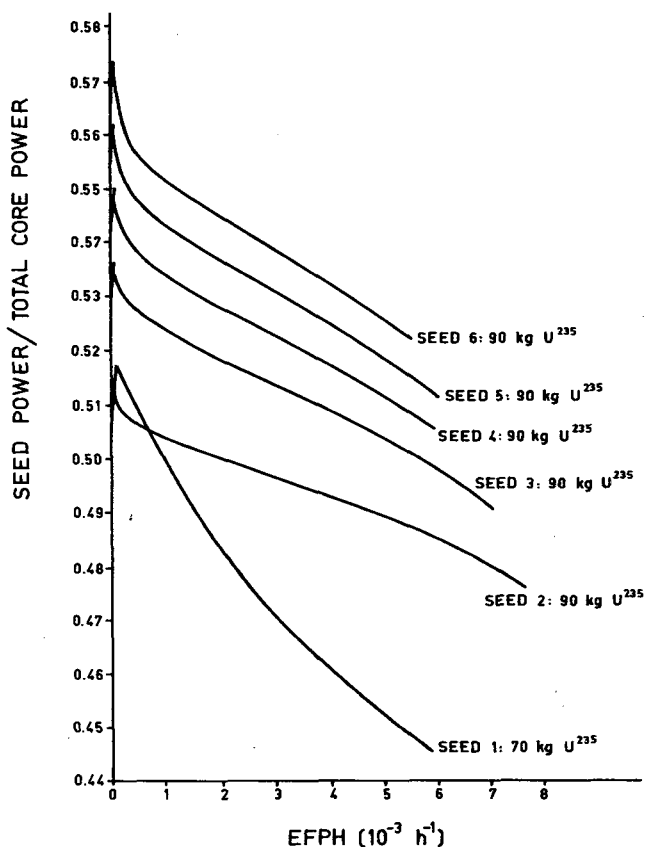


Fig. 18

Seed power fraction versus seed EFPH.

the most important advantages of the seed-blanket concept, obtaining large amounts of energy from U^{238} by burning plutonium in situ without the necessity of extracting the plutonium and fabricating it into the fuel elements.

Figures 18 and 19 give the latest calculated results for the fraction of total core power supplied by the blanket for six successive seeds, and the percentages of blanket power which are obtained from various fissionable isotopes.

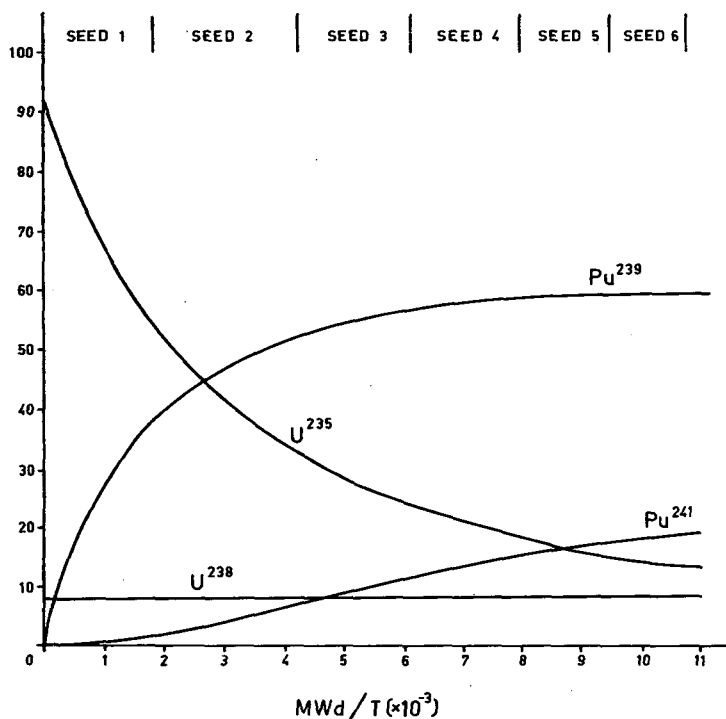


Fig. 19

Percent of blanket power from U^{235} , U^{238} , Pu^{239} , and Pu^{241} versus blanket MWd/T

Because of the size and heterogeneous nature of the Shippingport cores it has not yet proved possible to perform a detailed 3-dimensional life history study of these cores. Considerable work is now underway to develop means of doing such studies with a reasonable expenditure of computing time.

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BOILING LIGHT WATER REACTOR PHYSICS

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Abstract — Résumé — Аннотация — Resumen

BOILING LIGHT WATER REACTOR PHYSICS. The boiling water reactor physics methods frequently used at the Atomic Power Equipment Department, experiments which have verified, or led to, some of the method concepts, and some of the applications to the Dresden Power Station of the Commonwealth Edison Company are described. Material covered includes the basic lattice physics methods used, the treatment of clumped fuel rods, experimental justification of the clumped treatment, treatment of axial variations in lattice composition and, finally, treatment of boiling reactors.

PHYSIQUE DES RÉACTEURS A EAU LÉGÈRE BOUILLANTE. L'auteur décrit les méthodes fréquemment appliquées à l'étude de la physique des réacteurs à eau bouillante au Département du matériel pour la production d'énergie d'origine nucléaire de General Electric, les expériences qui ont permis d'en vérifier ou d'en concevoir certains principes et quelques-unes de leurs applications à la centrale de Dresden de la Commonwealth Edison Company. Le mémoire traite notamment les questions suivantes: méthodes fondamentales pour l'étude de la physique des réseaux, étude des barres de combustible en blocs, justification expérimentale des résultats théoriques ainsi obtenus, étude des variations axiales dans la composition des réseaux et, finalement, étude des réacteurs à eau bouillante.

ФИЗИКА КИПЯЩЕГО ЛЕГКОВОДНОГО РЕАКТОРА. Описываются методы, которые часто применялись в Департаменте оборудования для атомных электростанций США в целях определения физики кипящего легководного реактора. Эти эксперименты подтверждали прежние или давали новые конструктивные методы. Описываются также некоторые случаи их применения к дрезденской атомной электростанции компании "Коммонуэлс Эдисон компани". Имеющийся материал включает в себя основные методы, которые использовались для определения физики решеток, обработку топлива в блоках стержней, экспериментальное подтверждение блоковой обработки, обработку осевых вариаций в составе решетки и, наконец, обработку кипящих реакторов.

FÍSICA DEL REACTOR DE AGUA LIGERA HIRVIENTE. Se describen los métodos corrientemente utilizados para el estudio de la física de los reactores de agua hirviente en el Atomic Power Equipment Department de la empresa General Eléctrica, los experimentos efectuados con objeto de establecer o verificar los conceptos respectivos y algunas de sus aplicaciones en la central de Dresden, perteneciente a la Commonwealth Edison Company. Entre los puntos tratados, figuran los métodos básicos de física de los reticulados, el estudio de los elementos combustibles dispuestos en haces, la justificación experimental de ese procedimiento, el tratamiento de las variaciones axiales en la composición del reticulado, y, por fin, el análisis matemático de los reactores de agua hirviente.

INTRODUCTION

The physics of boiling, as distinct from non-boiling, light water reactors is characterized by the presence of the steam bubbles in the reactor water. These bubbles which from the nuclear standpoint behave like voids lead to a gross change in moderator density in going from the bottom to the top of a reactor core. They emphasize the importance of the "void coeffi-

cient" of reactivity in reactor design. At the top, or effluent end of the reactor, they bring about fuel-to-moderator ratios that lie outside of usual experience in pressurized water lattices. The calculation of BWR criticality requires simultaneous calculation of neutron flux and steam void patterns, and ultimately must assure self-consistency of the two. Determination of control rod "worth" in a boiling reactor requires determination of the change in void pattern as well as flux pattern associated with rod motion. In a reactor operating at power, the concept of worth will be related to reactor power rather than to reactor period.

The goals of a set of reactor physics analytical methods, which are to be applied to actual reactor design and operation, are perhaps somewhat different in emphasis than those developed specifically to achieve an exact solution, or those developed to accentuate conceptual simplicity in understanding the basic processes, with instruction as the primary motive. The so-called practical or engineering methods aim toward a combination of facility in use and precision adequate to achieve optimal and reliable design and efficient performance. This means that any analytical errors which are small compared to tolerances which can be achieved in manufacture or to reproducibility of operation are of no significance, and that, as method precision approaches this point, precision is increasingly compromised in the interest of solution speed and simplicity.

The paper will describe the boiling reactor physics methods frequently used at the Atomic Power Equipment Department of the General Electric Company, experiments which have verified, or led to, some of the method concepts, and some of the applications to the Dresden Power Station of the Commonwealth Edison Company, located southeast of Joliet in the State of Illinois. The development and testing of these methods has been a major function of the Engineering and Vallecitos Laboratory Physics groups. The paucity of this work which has been reported in the literature to date hardly reflects the work that has been done, and in no way indicates the many names which deserve mention in a summary of this kind. The material of this paper will cover the basic lattice physics methods used, the treatment of clumped fuel rods, experimental justification of the clumped treatment, treatment of axial variations in lattice composition and, finally, treatment of boiling reactors. Although all sections of the paper but the last apply equally well to boiling and non-boiling situations, these sections are included to define and specify the methods being discussed. Many of the methods are similar to those used by other groups, but, since this paper will not present mathematical detail, no attempt has been made to exclude these from the descriptions.

THE BASIC LATTICE METHODS

The basic lattice method selected is a three-group diffusion theory method with the group boundaries chosen so that all fission neutrons are emitted in the fast group, and so that all thermal neutrons are included in the lowest energy or thermal group. Resonance capture occurs primarily in the second or intermediate group. The group boundaries are at 5.5 keV and 0.625 eV.

The fast and intermediate group constants and neutron interaction rates are calculated using a 54-group computation, which is a modified form of MUFT-IV [1]. MUFT calculates the group constants using a Fourier transformation spectrum analysis. The fast group diffusion coefficient and the cross-sections for removal and scattering into the intermediate group are obtained as flux-weighted averages over the energy range of the fast group. The fast-group constants obtained from this computation are modified to obtain the final constants by replacing the MUFT result for fast fissions in U^{238} with a separate, more accurate calculation.

The intermediate group is calculated using the following procedure. The ratio of the heterogeneous to homogeneous resonance integral for U^{238} , the "L-Factor", is calculated for a rod equivalent to the actual lattice rod. Equivalence means that the rod has a surface equal to the true rod surface reduced by a DANCOFF-GINSBERG [2] efficiency factor. The equivalent rod "L-Factor" is determined by STEIN'S [3] technique. The average atomic concentrations of the various atomic species are determined for the fuel cell, and these together with the "L-Factor" are used in the multi-group computation of group constants, MUFT-IV.

An independent calculation of the resonance capture probability is made using HELLSTRAND'S [4] correlation of effective surface-to-mass ratio and effective resonance integral. This is reduced using the appropriate Dancoff correction. This independent calculation of the resonance capture is then used to replace the approximately correct U^{238} capture computed by MUFT, and thereby modify the MUFT group constants for the intermediate group. It has been pointed out by R. L. CROWTHER [5] that this method applies accurately in a well moderated $1/E$ spectrum such as was used in Hellstrand's integral measurements. In the case of tightly packed lattices with substantial depletion of the slowing-down density at lower energies by strong absorption in the resonance region, it is more accurate to use a more careful multi-group treatment which uses U^{238} resonance capture based on first order narrow resonance or infinite mass approximations which are applied selectively to individual resonances. The resonance capture is corrected for Doppler broadening by a Nordheim type of calculation. This treatment is found to give very good agreement with Hellstrand's results for well-moderated lattices. This refinement is beginning to be used routinely for analysis of high void content lattices.

The thermal group is treated using a hardened thermal spectrum in a cylindrical P-3 calculation for the individual fuel rods. The thermal spectrum is obtained using the WILKINS heavy gas thermalization model [6]. The P-3 calculation considers the fuel, clad and moderator as separate regions and treats a cylindrical annulus of moderator chosen to have the same area as the moderator in the actual non-cylindrical fuel rod cell. The resulting nuclear reaction rates are used to determine thermal group parameters for an equivalent homogenized cell. Although this treatment has been found satisfactory in the relatively close-packed boiling water reactor lattices, the assumption of a uniform spectrum throughout the cell has been found inadequate in lattices with very large spacings. Such lattices were studied in connection with the United States Atomic Energy Commission Superheat Programme [7].

These investigations dealt with uniform lattices of hollow annular fuel elements. The fuel elements were 1.50-in diam and consisted of stainless-clad, UO_2 , hollow cylinders with inner and outer flow passages. The lattices were arranged with fuel pitches of 1.8 to 2.0 in. Serious discrepancies between measurement and calculation were observed in reactivity, void coefficients and temperature coefficients. These discrepancies were almost entirely eliminated by division of the cell into two zones, each characterized by a different thermal spectrum. The spectrum of the inner zone, defined to contain one mean free path of water outside of the fuel, is hardened using the Wilkins procedure as before. The spectrum of the outer zone is constructed to be a function of its thickness. With this construction, the parameters of the outer zone approach those of the hardened inner zone if the outer zone is very thin, but approach those for water at ambient temperature if the zone is thick compared to a mean free path. An example of the improvement obtained by this method is shown in Fig. 1. In this Figure, the measured temperature coefficient is plotted as a function of temperature along with analytical results based on the design model with and without this modification.

The conversion ratio in these uniform superheat lattices was carefully measured by a new technique [8] in which gammas characteristic of Np^{239} decay were separated from fission product gammas and compared, using a coincidence method with a γ -spectrometer. These conversion ratios were calculated [9] using the standard model without the modified thermal group treatment. The resulting conversion ratios are tabulated in comparison with the measured values.

<u>Lattice</u>		<u>CR</u>	
<u>Pitch</u>	<u>Flooding</u> (of both flow channels)	<u>Calculated</u>	<u>Measured</u>
1.8	No	0.341	0.329 ± 0.011
1.8	Yes	0.303	0.300 ± 0.006
2.0	No	0.269	0.267 ± 0.004
2.0	Yes	0.259	0.257 ± 0.004

The agreement is seen to be quite good. It is important to notice that the conversion in this 3.4% enriched lattice is almost entirely a resonance group process and, hence, little affected by deficiencies in the thermal group treatment.

NUCLEAR DATA

The nuclear data used in these analyses are considered the best available at the present time. Thermal cross-sections from all isotopes are taken from standard source material [10, 11, 12]. The epithermal cross-sections are derived from standard multi-group files and experimentally determined resonance integrals [13, 14, 15].

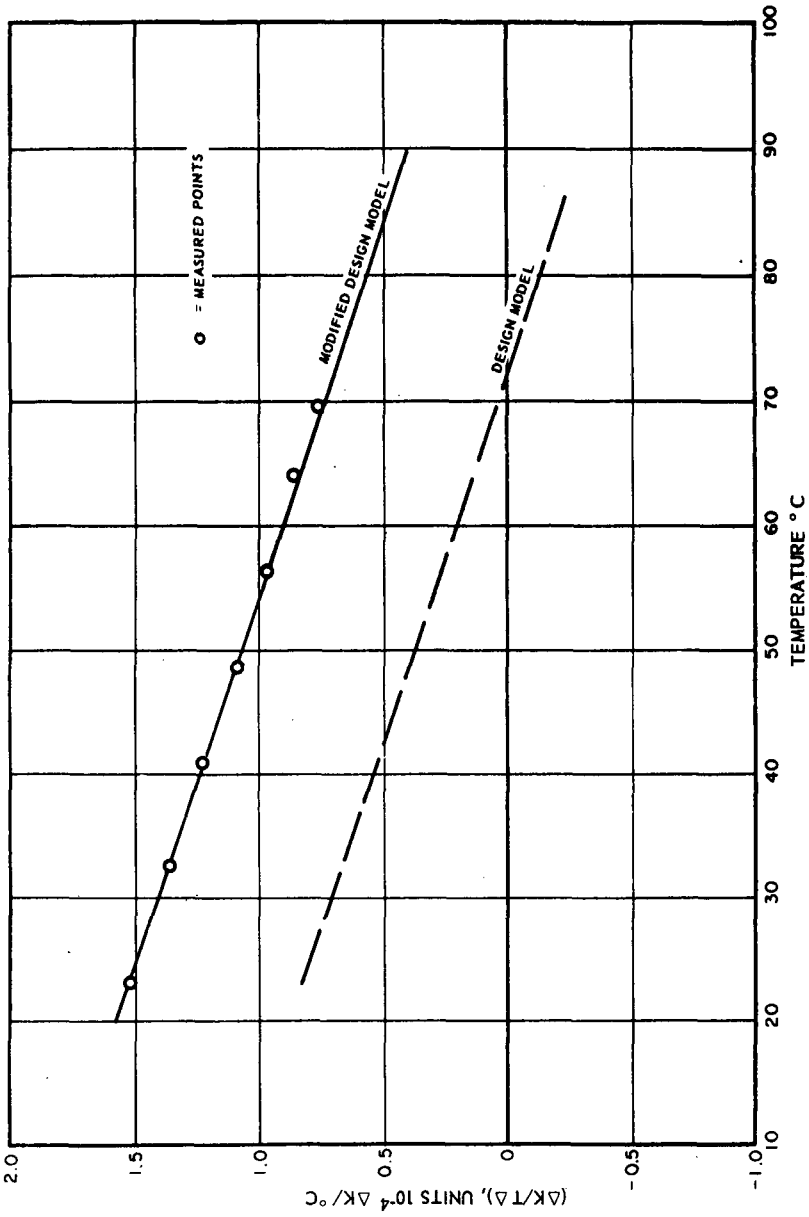


Fig. 1

AEC Superheat criticals - Temperature coefficient - 1.9-in Pitch F-F

METHODS FOR CLUMPED LATTICES

In a "clumped" lattice, the unit lattice cell is formed by a clump (or bundle) of fuel rods, the surrounding associated water gaps, the flow channel partitions, and the control elements or portions thereof. The Dresden lattice cell, for example, consists of a 6×6 array of half-inch rods. The fuel bundle is encased in a square, box-shaped Zircaloy "channel." The channel is surrounded by water gaps; thin on two of the four sides, thicker on the other two. One-half of a cruciform cross-section control element spans the greater part of each of the two thicker gaps when it is inserted. The cell is about 5 in across.

For analysis, the clumped lattice unit cell is usually subdivided into a number of regions within each of which the composition and environment is uniform, or at least approximately so. For example, in treating the Dresden cell, all of the inner sixteen rods are treated as one region, the corner rods as another, etc. The cell is represented by twelve different regions in all.

Three-group cell constants are determined for an equivalent homogeneous unit cell as follows: Group constants are determined for each of the regions within the unit cell as described in the preceding section on basic lattice methods. The unit cell is then treated as an eigenvalue problem, using the two-dimensional three-group diffusion code, PDQ-4, and assuming zero current at the edges of the cell. The boundary conditions at the blade surface are taken as black with an extrapolated end point in the thermal group and grey with the current related to the computed number of blade captures in the intermediate group.

In computing the group constants at the edges of the fuel bundle, the following conventions are adopted. The effect of clumping on the fast fission of U^{238} is generally treated by homogenizing the entire unit cell in calculating the homogeneous portion of the fast effect. In defining the cylindrical cell for calculation of resonance capture and thermal utilization by the rods at the outer edge of a clump, only water within the fuel channel is included. The fuel region of the Dresden element is divided into six different zones, each with a different Dancoff correction. The rods on the outside of the fuel element bundles are exposed to more water, and therefore have a much smaller Dancoff correction than the rods in the centre of the bundle which are surrounded by a number of rows of other rods. In considering the thermal group, water in the gaps between channels is assumed to contain neutrons with a thermal spectrum at the ambient or unhardened temperature. The eigenvalue for the problem, ν_{critical} , is used to determine an equivalent $(k_{\infty}-1)$ for the cell. The homogenized group constants for the cell are then computed as flux-weighted cell averages of the diffusion coefficients and various neutron interaction rates.

Note that the procedure assumes that the effect of a control cross can be replaced by a unique addition of homogenized poison to the cell, regardless of the array of control rods in the total reactor lattice. The validity of this assumption was required to be established by test.

EXPERIMENTAL JUSTIFICATION OF CLUMPED LATTICE TREATMENT

Experimental justification of the basic lattice methods used has included extensive checking of calculations with several sets of uniform lattice critical experiment data. Calculations have been checked against measurements of reactivity thermal utilization, resonance escape probability fast fission effect, power and flux distributions, spectrum measurements, cadmium ratios and other experimental conditions [16]. The agreement is within the experimental error for the various quantities in most cases and appears to be quite good.

Using the Critical Experiment Facility at the Vallecitos Atomic Laboratory, the following results were obtained: the measured k_{eff} of a uniform lattice of $\frac{1}{2}$ -in 1.3% enriched, UO_2 -filled, aluminium-clad rods differed from calculation by a negligible amount. The measured k_{eff} of a simple clumped lattice of an approximate Dresden array differed by a fraction of a percent from the calculated value. The thermal utilization of the clumped lattice was measured in terms of comparison of the activity induced in a circular manganese foil sandwiched between oxide fuel pellets in a fuel rod with the activity induced in manganese dissolved in the lattice water. Great care was taken in an attempt to include all factors, and it was concluded from this experiment that treatment of the water in the gaps around the fuel bundles as "unhardened" was preferable, although the difference in the two calculated results was only 1%. Specifically, the results were

	<u>f</u>	<u>Δf</u>
Measured	0.7639 $\pm$ 0.0042	
Calculated "unhardened"	0.7699 $\pm$ 0.0060	
Calculated "hardened"	0.7771 $\pm$ 0.0132	

A set of experiments was conducted to determine the validity of homogenizing control rods throughout a fuel element cell. One experiment using a critical lattice explored the flux perturbation in cells surrounding a control cross when it was half way inserted. Vertical wire traverses in various locations showed strong flux perturbations near the fuel rods adjacent to the cross, which were pretty well damped out outside of a cylindrical surface which just contained the cross. A more crucial test was obtained by an experiment in which the water in the four elements about a control cross position was partitioned off from the remainder of the water in the lattice by a plastic container. The water in this container was then replaced by dissolved poison. With this apparatus, a poison concentration was determined which produced a reactivity change identical with that produced by a fully inserted poison cross. In a second experiment, three control crosses were replaced, one at a time, with this equivalent amount of poison solution. To within $\pm 0.5\%$ total worth, the homogenized cross was found equivalent to the discrete cross in all situations.

An analytical test of homogenization was performed using CURE to calculate a 5×6 element array with two different control rod configurations. One had a single cross inserted near the centre of the array; the other had

two crosses inserted in corner locations. Both configurations were calculated with both discrete and homogenized representations, with the following results:

Representation	ρ	
	Single cross	Two crosses
Discrete	0.0364	0.0119
Homogenized	0.0344	0.0144
Δk	0.0020	0.0025

It is to be noticed that these experimental and analytical tests of homogenization are both for cold lattices. It has been shown that it should be valid when the diffusion length is small compared to the control rod width [17]. For this reason, further tests of the assumption in hot and boiling lattices will be made to determine the limits of applicability.

TREATMENT OF AXIAL VARIATIONS IN LATTICE CONFIGURATION

So far we have discussed effects of clumping and other non-uniformities in the horizontal plane; i. e. the plane perpendicular to the fuel rods. In this section we will discuss the subject of non-uniformities along the vertical axis of the core, but confine the discussion to ones that are horizontally constant. In these cases, separability of the vertical and horizontal variables has been found to be a very satisfactory approximation.

One set of experiments measured the reactivity effect and flux shape produced by separating the fuel rods into an upper and a lower portion by a dividing plane of metal spacers several inches long. It was found that the vertical flux data could be used to obtain both the vertical buckling and the approximate vertical buckling that would have been obtained in the absence of end spacers. The change in core buckling corresponded to the observed decrease in k_{eff} . This vertical buckling also could be calculated satisfactorily using a one-dimensional, three-group model.

Another set of experiments lending credence to the separable variable approach was the so-called "Forest Experiment." In these experiments a "forest" of control rods was uniformly inserted part way into a critical assembly core as a bank. Thus the lattice consisted of one layer of "rodded elements," i. e. elements with inserted control rods, and another layer which was unrodded. The characteristics of the rodded lattice were assumed determined by the equivalent dissolved poison experiment described earlier in this paper. The reactivity of the resulting lattice corresponded closely with these assumptions.

Overall test of these methods was provided during the initial loading of the Dresden reactor. During the loading a very large number of critical arrays were constructed with a corresponding number of inserted control rod patterns. Whenever two-dimensional calculations were available to compare with experiment, the agreement in k_{eff} was to within a fraction of one percent.

TREATMENT OF BOILING REACTORS

The methods for treatment of an actual reactor situation are more complex than the preceding methods. The actual reactor is usually a three-dimensional problem. In addition, when the reactor is boiling, the distribution of steam voids in the reactor depends directly on the distribution of power through the volume of reactor core and on the channel inlet flow distribution across the bottom of the core. The power distribution, in turn, depends on the steam void distribution, requiring the two distributions, power and steam voids, to form a self-consistent set. Present calculational techniques are restricted to two-dimensional analyses because of practical limits on computing time. Hence the three-dimensional self-consistent void-power distribution must be synthesized from one- and two-dimensional power distributions. The effect of moderator voids is included explicitly in the synthesis, using an experimental correlation between voids and steam quality.

To date, several techniques have been used for boiling reactor simulation [18, 19, 20, 21]. In design work, the first rough scoping calculations of the influence and generation of steam voids are done using a single axial dimensional computation with self-consistent steam void calculation. The bulk of the design analysis is done using the BORE two-dimensional (r, z) code which represents the horizontal plane by a single radial dimension. For this code the basic input consists of the homogenized three-group diffusion equation constants which we have defined in earlier sections of this paper. The experimental steam void-quality correlation is used in a built-in computation of a self-consistent steam void pattern. The horizontal extent of the reactor is divided into annular cylindrical regions. In general the region boundaries will cut through fuel clump and control cross boundaries so that a cross-sectional view of this annulus taken at any particular elevation will show that it is composed of a fraction, f_r , of rodged elements, plus a fraction, $1-f_r$, of unrodged elements. In computing the appropriate three-group constants for the region of this annulus at this elevation, the effective fraction of rodged elements must be determined. For this purpose an auxiliary calculation has been performed. Two-dimensional studies of the flux peaking and rodged cell worth for uniform rod arrays with various spacings have been carried out. These calculations give the relative weighting of the rodged and unrodged elements as a function of rod density. This relationship is used to determine the proper effective rodged fraction for use in the annular calculation.

One especially useful method of using the BORE code has been the so-called backwards method in which the power distribution and corresponding steam void distribution are assumed and used to calculate the distribution of thermal group absorption which regenerates the input power distribution. The distribution of absorption is used to determine the approximately equivalent array of inserted control rods.

A second technique has proved very useful in guiding reactor operation. In this technique the reactor is divided into a small number of horizontal layers. The dividing planes are chosen so that, throughout the height of each layer, the pattern of inserted control rods is uniform. In the case of the Dresden reactor, for example, with twelve notch positions for its rods,

the maximum number of such layers (so far, a smaller number has always been used) would be thirteen, not including the top and bottom reflecting layers. These layers are connected axially by an appropriate one-dimensional technique in which each actual layer is replaced by an equivalent homogenized layer. The individual layer calculation is carried out using PDQ-4 with the usual fuel cell homogenized group constants. In the layer type of calculation, the steam void distribution is approximated to correspond to the proper degree of inlet subcooling and total steam flow. Inlet orificing provides essentially uniform void concentration at each elevation under practical conditions of optimized power flattening. The difference between inlet and exit enthalpies is approximately proportional to the flow, and this in turn to channel power. The power input in each layer of an individual channel is also approximately proportional to channel flow and channel power.

In the Dresden reactor, running at full power, the situation is somewhat simpler. The majority of the control rods are either fully inserted or fully withdrawn. Thus the reactor is divided into an upper, boiling zone and a lower, non-boiling zone. Partly inserted rods, to the extent that they are inserted to a position near to the boiling boundary, do not increase the number of layers required. Normal operation corresponds to a void concentration in the upper region of 25%, with equal amounts of power being generated in the two layers. With these assumptions, the flux distributions in the two halves were calculated and averaged to give the "channel" power distribution over the horizontal plane, (Fig. 2). An experimental determination of the power distribution was made, using the " γ -probe" method. In this method, a small ion chamber is used to scan the fission product γ -flux along a sample number of flow channels between fuel rods and in a large number of fuel elements. The scanning was done during shut-down after an extended run at steady power. Integration of the power along a single scan and over the element gives the channel power. The experimental results are shown in Fig. 3. It should be noticed that Fig. 2 is for a mirror image of the arrangement in Fig. 3. With this in mind, it is seen that the agreement between computed and observed channel power is quite good.

CALCULATION OF FUEL DEPLETION AND CONVERSION

In evaluating fuel depletion and conversion effects, the reactor is divided into several zones. The degree of fineness in choosing these zones will depend upon the requirements of any particular problem. Generally, each fuel bundle is considered discretely in the horizontal plane, and the axial dimension of the reactor is divided into a few layers. For isotopic composition calculations, the procedure is as follows: Given a set of nuclear parameters for a three-group model, an analysis is performed to determine the power and void distribution in each of the zones. This set of calculations is performed at a given exposure condition for the reactor. The resulting power distribution is used to project the exposure increment for each zone to the next time interval. The new exposure distribution, void distribution and control rod distribution are used to obtain a new set of nuclear parameters for each zone, and the process is repeated again. In design, this

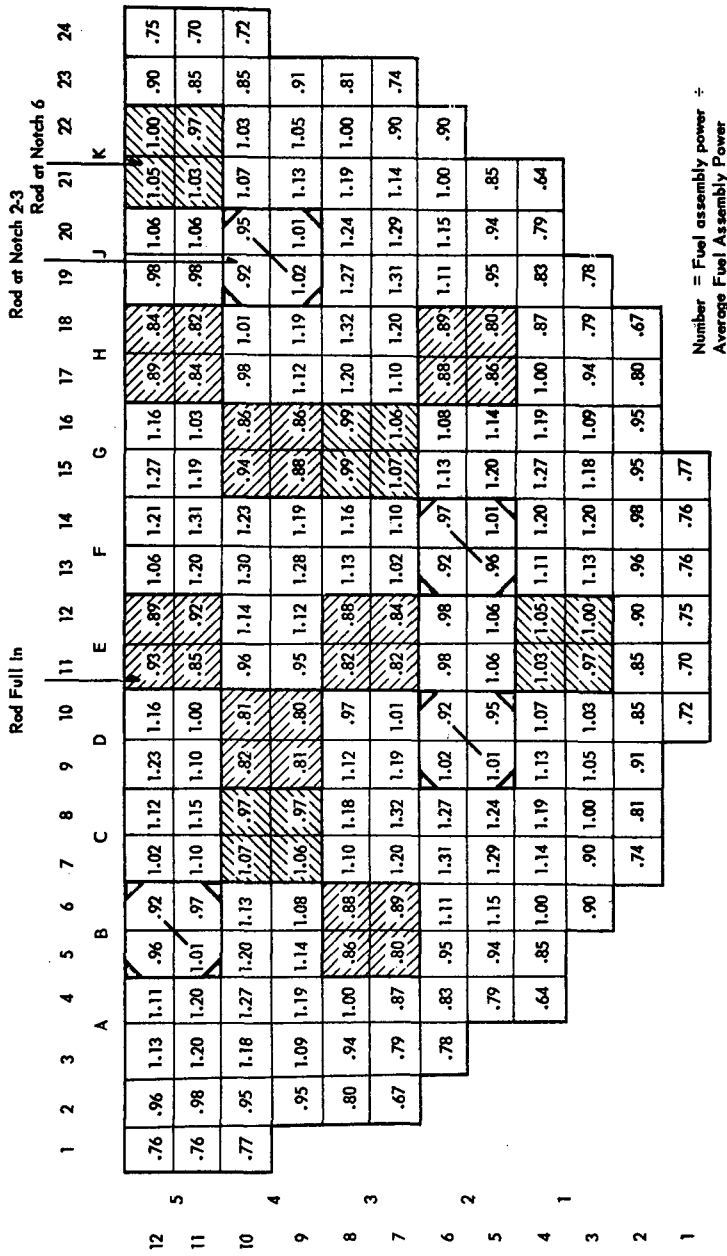


Fig. 2

Full power radial power distribution synthesized from two full core PDQ-4 computations

is continued until an end-of-life condition is reached for the reactor. In order to make the process practical, the dependence of the three-group nuclear parameters on voids, exposure and control rod density are predicted in advance. This facilitates the performance of these stepwise calculations. In as much as the void distribution varies somewhat over the lifetime of the fuel, there is a certain degree of approximation involved in this approach. However, since gross changes in the void distribution occur rather infrequently, and since a rather coarse axial zoning of the reactor is used, the average void fraction in each of the zones is either sensibly constant or a slowly varying function of exposure. In performing these analyses, the effect of control rods on reactivity and hence power distribution is not treated explicitly, but is included in the homogenization of the lattice fine structure to determine the three-group nuclear parameters for these conditions to be included in the horizontal calculation. This averaging out of the control rod effect to obtain parameters for any average control rod density condition is based on the fact that the control rod pattern in the Dresden reactor is rotated quite frequently during the life of the fuel. Hence there are very few bundles which undergo extended periods of exposure with a fixed control pattern and bundles in symmetrically equivalent positions experience the same exposure history in terms of average control rod environment.

The actual calculation of depletion effects is done by numerical integration of the depletion equations for the various isotopes using the appropriate three-group microscopic cross-sections which are developed in the course of normal cell calculations for the fuel bundle. A number of depletion calculations is required to obtain representative isotopic composition data for various conditions of interest in the reactor; thus, several calculations must be made for various void fractions, the spatial variation of burn-up within

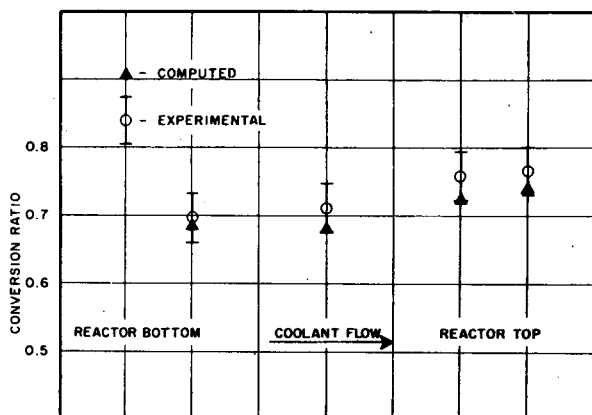


Fig. 4

Comparison of measured and computed conversion ratio in the Dresden reactor -
Comparison given for axial positions along a centre-of-bundle rod.

the fuel bundle is considered explicitly in the cell calculation. Properly weighted flux ratios characteristic of the average condition (control rod in versus control rod out) are used in these calculations. The results of these calculations are isotopic content, reactivity and power distributions for the fuel bundle as a function of exposure for various void fractions. These results are used as a basis for the determination of isotopic composition in the various zones for the three-dimensional mock-up of the reactor.

Fuel irradiated for a relatively short period of time (60 to 180 MWd/t) during the initial phases of Dresden power operation has been analysed for change in isotopic content, and the expected changes computed according to the analytical methods described [22]. Samples were taken from twelve locations in a fuel bundle removed from a central location. Conversion ratios were determined by measuring the Pu^{239} to U^{238} ratio and by measuring the yields of Cs^{137} , Ce^{144} and Zr^{45} . The agreement was found to be reasonably good; see Fig. 4 where measured and calculated conversion ratios are plotted as a function vertical position in the fuel bundle.

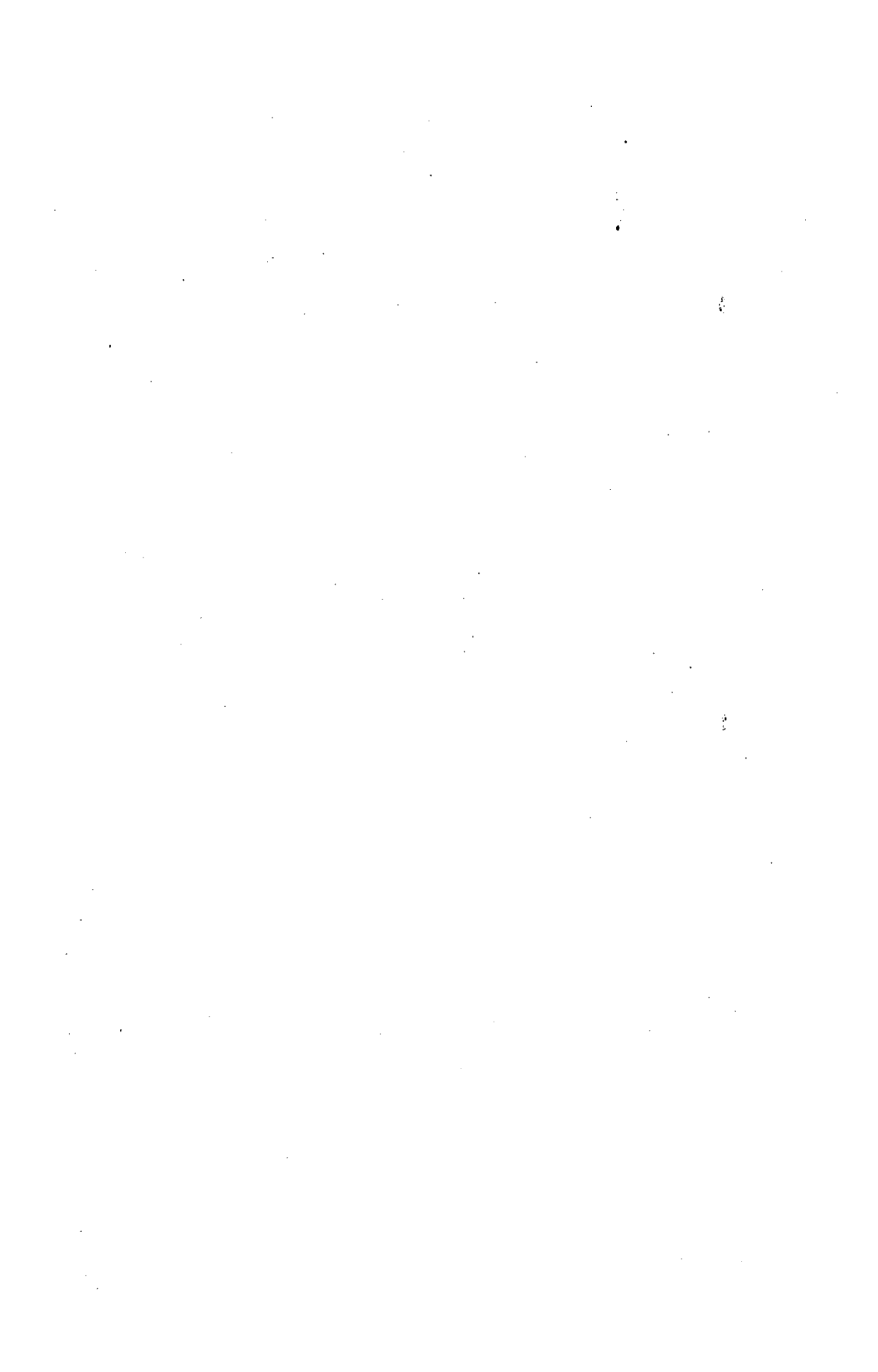
CONCLUSION

The methods used for predicting performance of boiling water reactors have been in a continual state of evolution. The methods now in use perform admirably as far as they have been tested to date. However, extremes of conditions for which they have not been tested may emphasize the need for inclusion of additional precision. Study of superheat lattices with a thick water annulus has already demonstrated the limitations of assuming a single neutron temperature in the neighbourhood of a fuel rod. Resonance absorption effects with high void content conditions are now being described in greater detail. A three-dimensional code which had at least one point in the horizontal plane for each control rod or equivalent position and an axial point for each control rod notch position would provide a simple one-to-one correspondence between the analytical model and the operating reactor configuration which is not now available. The correct description of extended burn-up depends on adequate knowledge of fissile nuclide and fission product cross-sections, possible need for attention to radial location of isotopes in rods and to possible isotope migration along fuel rods.

There are a number of factors under development which may aid in simplifying and speeding the calculations, and ultimately aid practical methods in incorporating the important additional complicating effects. One of these is the ever-increasing speed and capacity of the digital computing machines; and another the development of more powerful and efficient solution techniques. However, the important new factor is going to be the strong impact of reactor operating experience. This will emphasize calculation of only quantities which are significant from the operational point of view. Certain shortcuts to understanding and predicting reactor performance have already developed in operating the Dresden reactor. More may be expected. Meanwhile, today's intensive study of reactor physics should rapidly speed improvement and refinement of all of our physics methods.

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