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4 - 8 February 2008

Detailed description of the Intra Nuclear Cascade from Liege: INCL4

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INCL4: Intra Nuclear Cascade of Liège

Detailed description of the code
(physics and parameterizations)

Alain BOUDARD (CEA-SPhN)
for « Advanced workshop on Spallation Model Codes », Trieste February 2008

Content

1) Historical aspect
2) Basis of the model
3) New ingredients
4) Conclusions (future)

The true father is….. J. Cugnon
Univ Liège, Institut de Physique B5

with

J. Vandermeulen and T. Mizutani

at the conception of the code

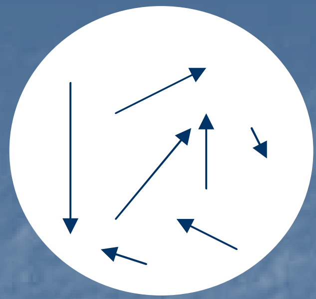
… and contributions from many others:

D. Kinet, M.C. Lemaire, D. L'Hote,
L. Pienkowski….

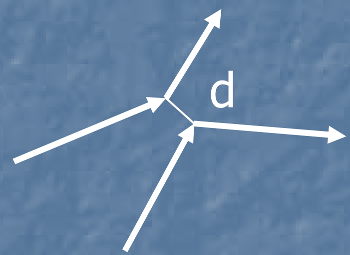
More recently (INCL4):

S. Vuillier, C. Volant, S.Leray, P. Henrotte,
Th. Aoust… and myself.

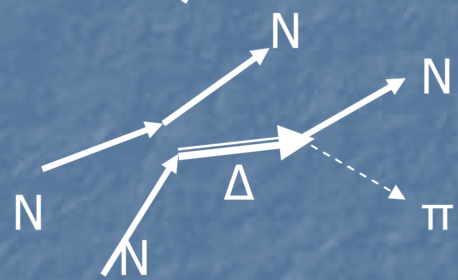
Main characteristics of INCL:



Nucleus are made of nucleons explicitly treated as classical particles randomly distributed in a realistic r-space region and all moving according to their momentum randomly distributed in a Fermi sphere.



Interaction takes place when the minimal distance of approach is smaller than the cross section (actually $\sqrt{\sigma/\pi}$ from $\sigma = \pi \cdot d^2$)



Inelastic NN cross section is treated through a Δ (3/2,3/2:1232 MeV) formation with the Δ as a resonance decaying after some time in πN.

The following interactions are considered and parameterized according to the free interaction (including angular dependence):

NN->NN NN<->NΔ NΔ->NΔ ΔΔ->ΔΔ Δ<->πN

All particles (N, Δ and π) are **explicitly followed in time**. They are moving within straight lines at constant speed between two interactions.

Historical:

~1980 **First version for Heavy Ion collisions around 1 GeV per nucleon**

J. Cugnon, T. Mizutani, J. Vandermeulen N.P. A352 (1981) 505
J.Cugnon, D. Kinet, J.Vandermeulen N.P. A379 (1982) 553

No nuclear potential, no difference np versus pp,
no Pauli blocking but no soft NN collisions.

Nuclear compression, π and nucleon high energy spectra
and multiplicity are studied.

1987 **A version for N-A is built** J. Cugnon N.P. A462 (1987) 751

A nuclear potential is introduced with a transmission probability
at the surface. The NN interaction and the Δ treatment has been refined.

Nuclear stopping power computed, dynamics of the reaction
in the range 100 MeV-20 GeV and high energy nucleon spectra studied.

1989 **A version for antiproton-A** J.Cugnon, P.Deneye, J. Vandermeulen N.P. A500 (1989) 701

Curvature of the p-bar track in the nuclear field, annihilation as a multipion
production following the phase-space.

π spectra and multiplicity, p spectra studied.

Historical:

~1985-1995 **Interactions (parameterisation) continuously improved in series of papers.**

A step for the Δ: J. Cugnon, M.C. Lemaire N.P. A489 (1988) 781

Detailed in: J. Cugnon, D. L'Hote, J. Vandermeulen NIM B111 (1996) 215

1997 **First version of what will be INCL4** J. Cugnon, C. Volant, S. Vuillier N.P. A620 (1997) 475

Additional improvements of NN (dσ/dΩ) above 300 MeV

…and of NN->NΔ following LNS experimental results

J.Cugnon, S.Leray, E.Martinez, Y.Patin, S.Vuillier, P.R. C56 (1997) 2431

Coupling with the Dresner-Atchison de-excitation code

Study of n spectra and of many physical ingredients on the Observables: (cascade stopping time, Pauli treatment, nuclear diffuseness, refraction….)

Historical:

2002 **Official paper on INCL4** A.Boudard,J.Cugnon,S.Leray,C.Volant P.R. C66 (2002) 44615

Diffuse nuclear surface (realistic densities), Pauli complemented by long range correlations (CDPP), light ions as projectiles (<4He), angular momentum of the remnant, stopping time fixed….

Coupling with ABLA and comparison with many observables (n, p, π spectra and multiplicities, residual nuclei etc.)

ABLA: De-excitation part of the A-A code developed at GSI
See presentation in this school.

INCL4.2-ABLA in the transport code LAHET

(J.C. David work, private version from D. Prael)

2003-2005 **Same version in MCNPX** (β version and now public)

(J. Hendrix work)

Historical:

2004 **INCL4.3: Cluster emission added** (d,t,3He,4He)

A.Boudard, J.Cugnon, S.Leray, C.Volant N.P. A740 (2004) 195

Not yet public:

Study of INCL at low energy (<100MeV)

P. Henrotte, J. Cugnon Eur. Phys. J. A16 (2003) 393

Nuclear potential different for p and n and energy dependent $V(\tau, E)$

Th Aoust, J.Cugnon Eur. Phys. J. A21 (2004) 79

Potential for π introduced V_π

Th Aoust, J.Cugnon P.R. C74 (2006) 64607

2007 **INCL4.4 code with all the above improvements: first results**

A.Boudard (et al.) Nice Nuclear Data 2007 Conference

We will describe here the version 4.4, mentioning the still open choices.

2008 **INCL4.2 translated in C++ in GEANT4** (not fully tested)

P. Kaitaniemi-A.Heikkinen work

Deliberate choice of authors:

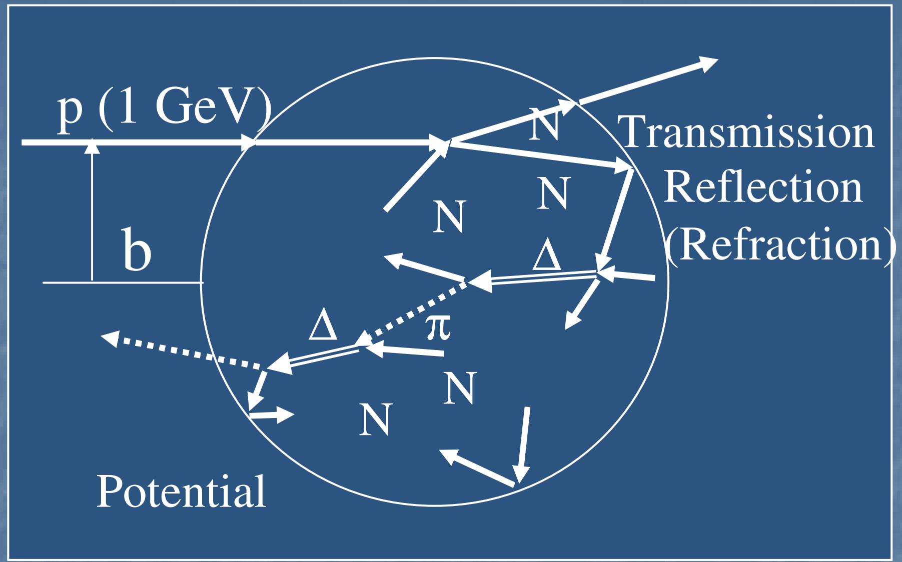
- A model with physical (justified?) ingredients
- Reduced phenomenology (or fitting processes)

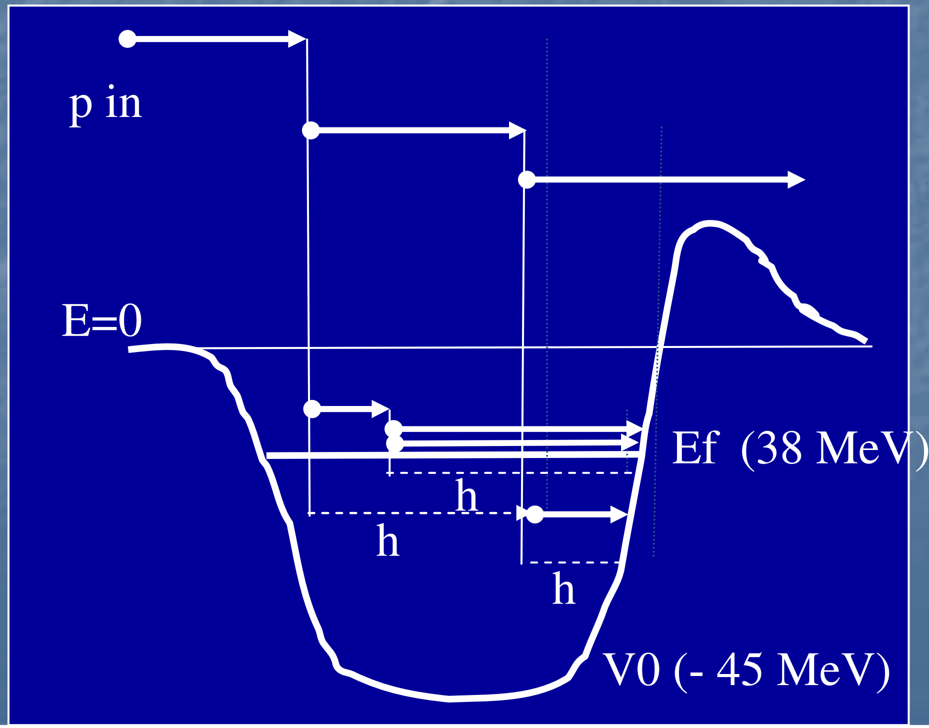
> For a better understanding of the reaction mechanism
>
> … and the hope to be more predictive in extrapolations

- Even if possibly less precise as event generator

Ingredients of the model

1) Target preparation
2) Entering particles
3) Propagation (t dependence)
4) Interactions
5) Escaping particles
6) End of the cascade





1)Target preparation

Z,N nucleons randomly distributed in r and p space

p-space distribution   Spherical distribution up to pF = 270 MeV/c (TF = 38.17 MeV)

r-space distribution   Realistic densities taken from electron scattering

H. De Vries, C. De Jager, C. De Vries Atom. Dat. And Nuc. Dat. Tables 36 (1987) 495

27<A

$$\rho(r) = \frac{1}{1+\exp(\frac{r-R_0}{a})}$$
$$R_0 = (1.063 + 2.745E-4\cdot A)\cdot A^{1/3}$$
$$a = 0.510 + 1.63\cdot E-4\cdot A$$

Woods-Saxon

5<A<28

$$\rho(r) = (1.+R_0\cdot\left(\frac{r}{a}\right)^2)\cdot\exp(-\left(\frac{r}{a}\right)^2)$$

Modified Harmonic Oscillator

R0 and a explicitly tabulated

A<6

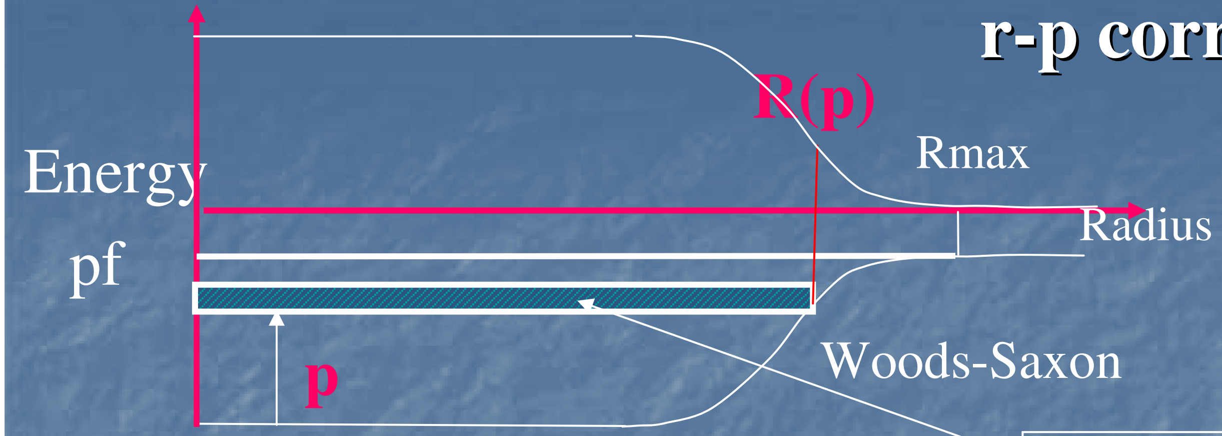
Gaussian with r.m.s. explicitely tabulated

Note: Same shape density for p and n

Any reasonable r shape can be given but only spherical r and p distributions.

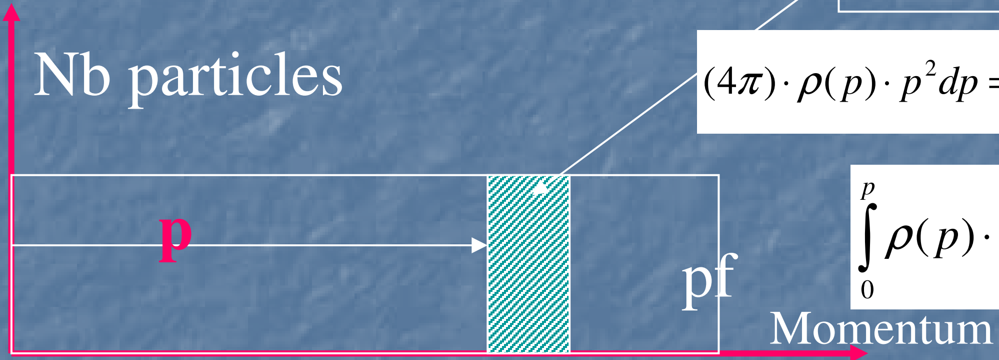
No constrain $\sum \vec{r} = \sum \vec{p} = \sum \vec{l} = 0$ event by event but good on mean

r-p correlated densities



V(r) related to ρ(r)

Correlation



$$(4\pi)\cdot\rho(p)\cdot p^2 dp = \frac{(4\pi)}{3}\cdot r^3 d(-\rho(r)) = -\frac{(4\pi)}{3}\cdot r^3\,\frac{d(\rho(r))}{dr}\cdot dr$$

$$\int_0^p \rho(p)\cdot p^2 dp = \int_0^{R(p)} -\frac{d(\rho(r))}{dr}\cdot\frac{r^3}{3}\,dr$$

$$\left(\frac{p}{pf}\right)^3 = F(R(p))$$

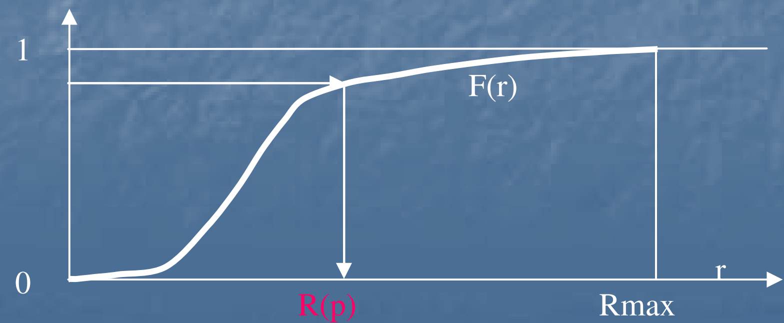
Calculation:

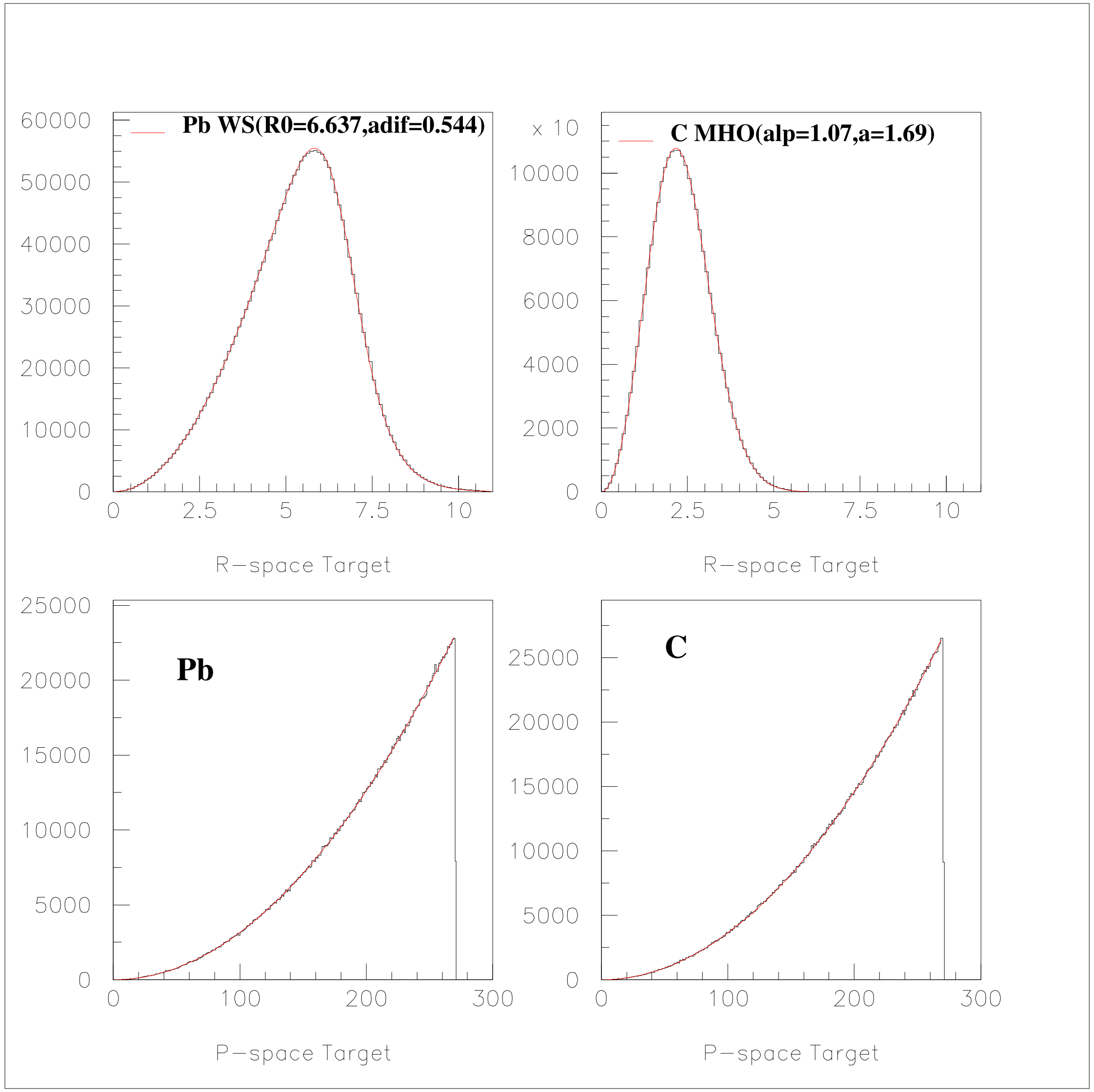
p random in the pf sphere

r random in the spere R(p)

(Note that V(r) is also defined)

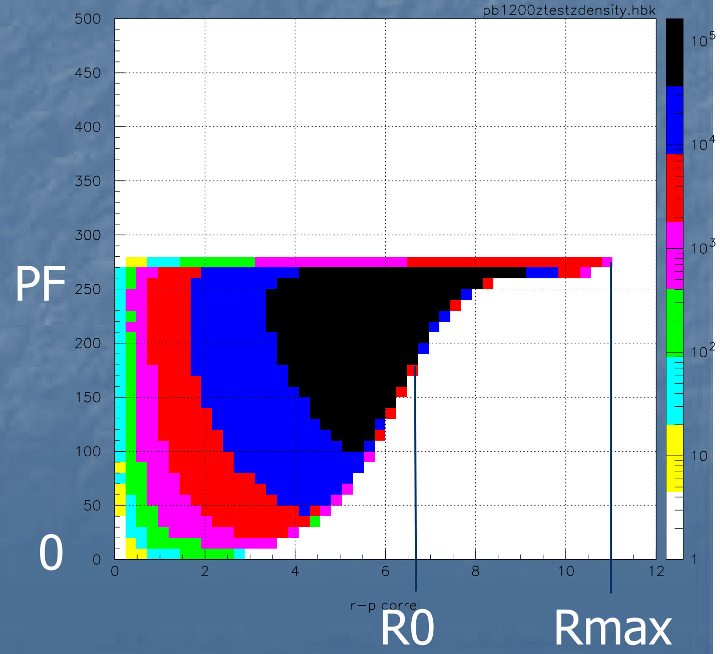
$$\left(\frac{p}{pf}\right)^3$$





Monte-Carlo (black histo) compared to functional densities (red lines) and r-p correlation.

Property:
With moving nucleons (without interaction): STABLE in time.



Isospin dependent potentials

Simple case: same potential

4 nucleons in a cell h^3

$$\frac{4}{3}\pi p_F^3 \cdot \frac{4}{3}\pi R_0^3 = \frac{A}{4}h^3$$

Nuclear density

$$\rho_0 = A \Big/ \tfrac{4}{3}\pi R_0^3$$

$\rho_0 = 0.17$ nucl/fm^3
from R_0=6.637 for lead

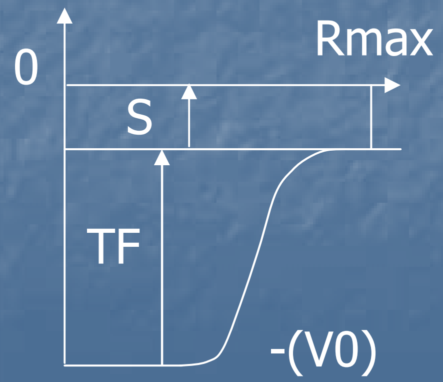
So that

$$p_F = \left(\frac{3\pi^2}{2}\rho_0\right)^{1/3}\frac{h}{2\pi}$$

pF = 270 MeV/c

p-n two separate populations in the same volume

$$\frac{Z}{p_{Fp}^3} = \frac{N}{p_{Fn}^3} = \frac{A}{2\cdot p_F^3} \rightarrow \quad p_{Fp} = p_F\left(\frac{2Z}{A}\right)^{1/3} \quad p_{Fn} = p_F\left(\frac{2N}{A}\right)^{1/3}$$



On mass-shell particles: $(T+m)^2 - p^2 = m^2$

V0 = TF + S We take Sp=Sn= 6.83 MeV; V0= 45 MeV

Could be different and adapted to the target mass region.

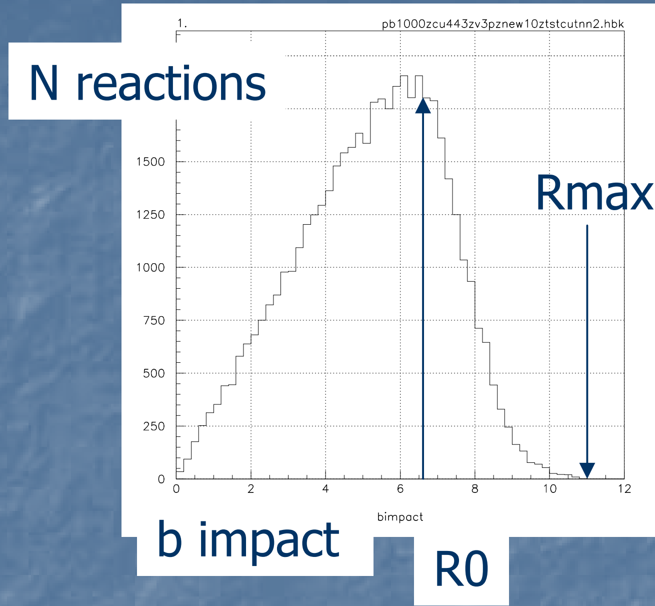
For Lead: V0p= 39.5 MeV; V0n= 50 MeV

2) Entering particles (N or π)

➢A volume of calculation: sphere Rmax=R0+8*a

(less than 10^{-4} reactions missing)

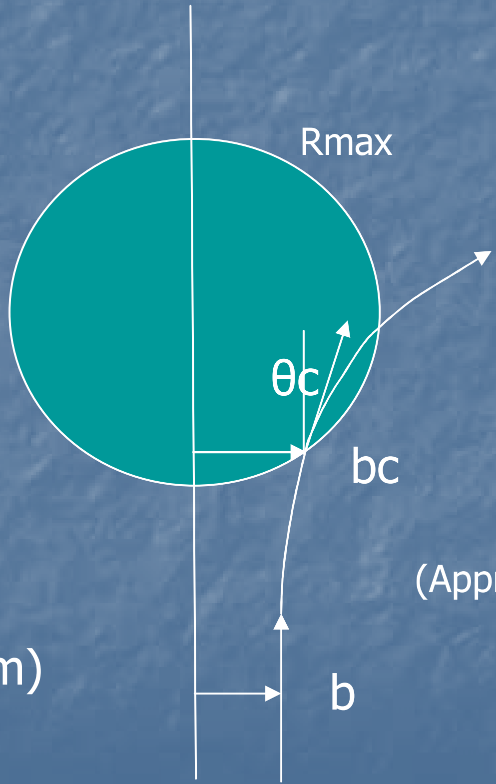
➢Impact parameter b random [0-Rmax]



➢Coulomb distortion:

Rutherford hyperbolic track

(b,0)->(bc,θc)



$$\sigma geom = \pi \cdot R\max^2$$

$$\sigma reac = \sigma geom \cdot \frac{Nreactions}{Nprojectiles}$$

(Approximate global Coulomb correction)

$$\sigma = \sigma reac\left(1 - \frac{1.44 zZ}{T\sqrt{\sigma reac/\pi}}\right)$$

$$(1.44 = \frac{1}{137} \cdot \frac{h}{2\pi})MeV \cdot fm$$

➢Gain the energy V0
(accordingly the momentum)

➢No nuclear refraction

Composite projectiles (d,t,3He,4He…..12C)

➢Gaussian distributions in r-p space with Σr=Σp=0
(Density from Paris potential for deuterons)

rms	t	3He	4He
R (fm)	1.80	1.80	1.63
P MeV/c	110	110	153

➢Lorentz boosted in the lab. frame

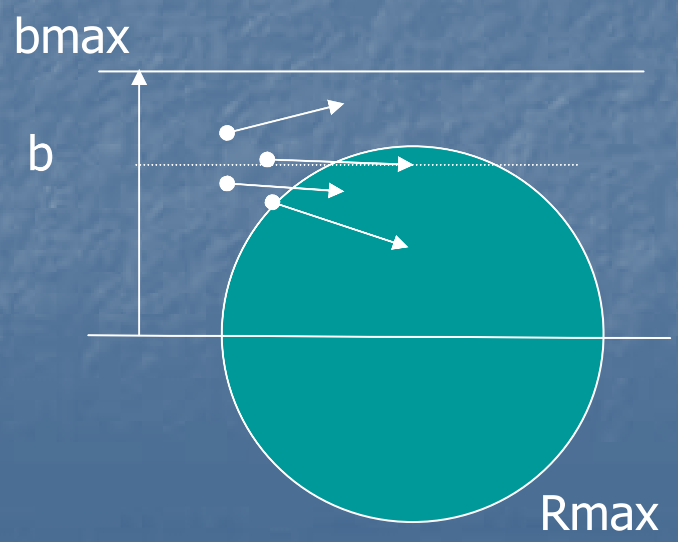
➢Rescaling of nucleon energies to have perfect nomunal incident energy (binding)
(but momentum slightly biased due to on mass shell nucleons)

➢bmax = Rmax + cluster_rms

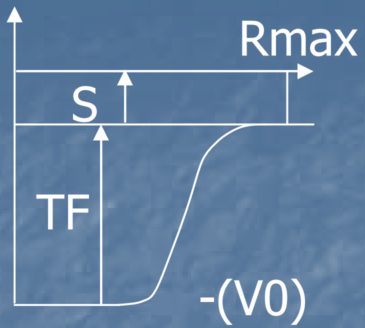
➢Projectile nucleons can miss the target
(or enter later)
No clusterization of projectile spectators

➢Calculation starts when first nucleon enters Rmax sphere

➢Not extensively tested on observables



3) Propagation



Avatars: Interaction NN, or NΔ or πN
Δ decay
Particle at the surface (V(r) or Rmax)

Time list: t=0 when the first nucleon (THE participant) enters at Rmax

Constant velocity and straight trajectories!

Can compute the time of all avatars ti and tij

(No loop with steps in time)

tij list is limited → i or j is a participant
$\sqrt{s}$ is larger than cutnn (parameter)
dij minimal distance smaller than $\sqrt{\sigma tot(s)/\pi}$

The smallest time of the list is executed

Nucleons which experience an avatar with success **becomes participant**
Clear the following times for them and compute the new ones
Propagate at once all particles

Execute the next time if not larger than tfin (parameter)

CUTNN parameter

$$\sqrt{s} \geq \Lambda = 1925 MeV = 2m_N + 48.5 MeV$$

Idea behind: No low energy NN interaction (soft collisions)

-Included in the mean nuclear potential
-Most of them rejected by a Pauli blocking

Tested: Above ~200 MeV a reduction does not change cross sections (but enlarge the computing time)

For low energy calculations ($T<100$ MeV):

No cut but no interaction if both nucleons are below TF

Conclusion:
The cut NN can be suppressed (with NO interaction if both nucleons are below TF)
The Fermi energy for at least one nucleon acts like a more natural cut.

4) Interactions

NN cross sections: σ tot = σ elast + σ inelast

σ inelast is supposed to be the Δ production

pp=nn (mb) (p lab momentum)

$\sigma_{tot} = 34\cdot(p/0.4)^{-2.104}$	$p < 0.44 GeV/c$
$\sigma_{tot} = 23.5 + 1000(p-0.7)^4$	$0.44 < p < 0.8$
$\sigma_{tot} = 23.5 + 24.6/(1+\exp(-10\cdot p+12))$	$0.8 < p < 1.5$
$\sigma_{tot} = 41 + 60(p-0.9)\exp(-1.2p)$	$1.5 < p$

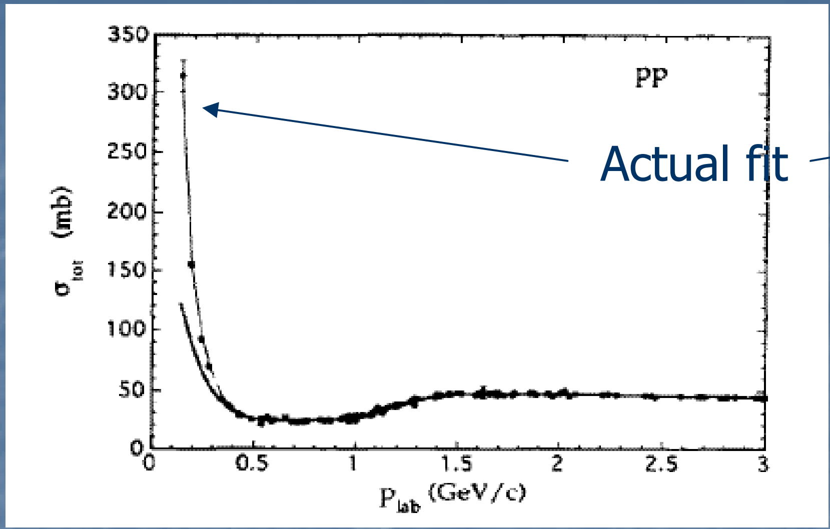
$\sigma_{elast} = 34(p/0.4)^{-2.104}$	$p < 0.44 GeV/c$
$\sigma_{elast} = 23.5 + 1000(p-0.7)^4$	$0.44 < p < 0.8$
$\sigma_{elast} = 1250/(50+p) - 4\cdot(p-1.3)^2$	$0.8 < p < 2.0$
$\sigma_{elast} = 77/(p+1.5)$	$2.0 < p$

np

$\sigma_{Tot} = 6.3555\, p^{-3.2481}\cdot\exp(-0.377(Log(p))^2)$	$p < 0.45 GeV/c$
$\sigma_{Tot} = 33 + 196\sqrt{\lvert p-0.95\rvert^5}$	$0.45 < p < 0.8$
$\sigma_{Tot} = 24.2 + 8.9p$	$0.8 < p < 1.5$
$\sigma_{Tot} = 42$	$1.5 < p < 2.0$
	$2.0 < p$

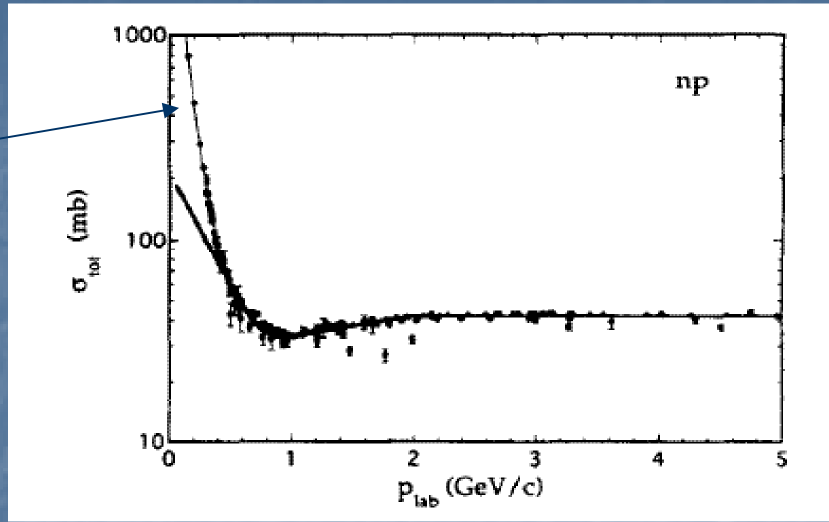
$\sigma_{Elast} = 6.3555\, p^{-3.2481}\exp(-0.377(Log(p))^2)$	$p < 0.45 GeV/c$
$\sigma_{Elast} = 33 + 196\sqrt{\lvert p-0.95\rvert^5}$	$0.45 < p < 0.8$
$\sigma_{Elast} = 31/\sqrt{p}$	$0.8 < p < 1.5$
$\sigma_{Elast} = 77/(p+1.5)$	$1.5 < p < 2.0$
	$2.0 < p$

σtot

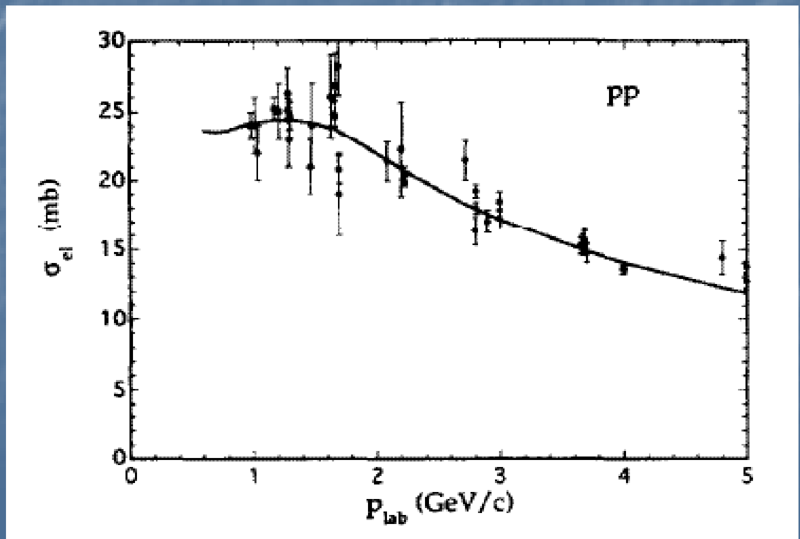


Actual fit

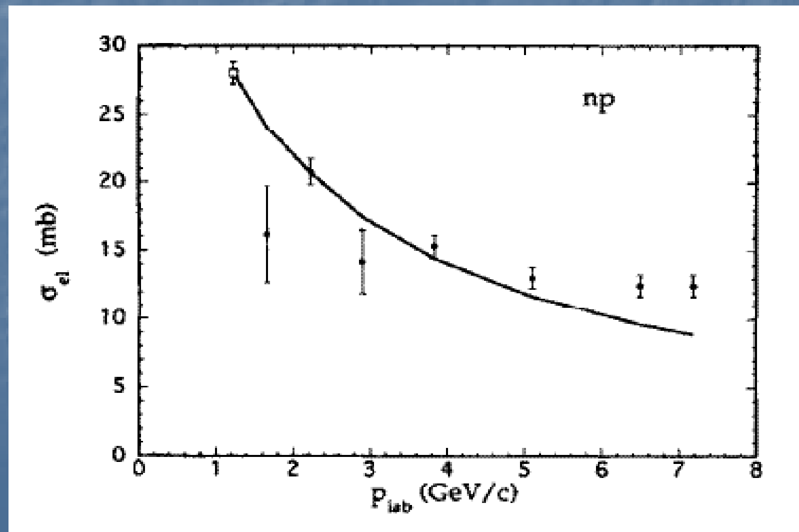
σtot



σelastic



σelastic



Baryon-Baryon cross sections

(assumes only isospin C.G. dependences)

NN->NΔ

$$\sigma(pp \to p\Delta^{+}) = \tfrac{1}{4}\sigma_{inel}(pp)$$
$$\sigma(pp \to n\Delta^{++}) = \tfrac{3}{4}\sigma_{inel}(pp)$$

$$\sigma(nn \to n\Delta^{0}) = \tfrac{1}{4}\sigma_{inel}(nn)$$
$$\sigma(nn \to p\Delta^{-}) = \tfrac{3}{4}\sigma_{inel}(nn)$$

$$\sigma(np \to n\Delta^{+}) = \tfrac{1}{2}\sigma_{Inel}(np)$$
$$\sigma(np \to p\Delta^{0}) = \tfrac{1}{2}\sigma_{Inel}(np)$$

NΔ->NN From NN->NΔ and detailed balance

This cross section has been multiplied by 3 (empirical factor partly justified)
to increase the π absorption.

NΔ->NΔ = ΔΔ->ΔΔ = pp->pp (Same elastic scattering cross section)

Note: All the np inelastic cross section (T=0 and 1) cannot go through a Δ production. (T=0 impossible but is a small channel at the Δ energy)

Angular distributions

pp elastic scattering

(symmetrized around 90°)

$$\frac{d\sigma}{dt} \propto e^{B_{pp}\cdot t}$$

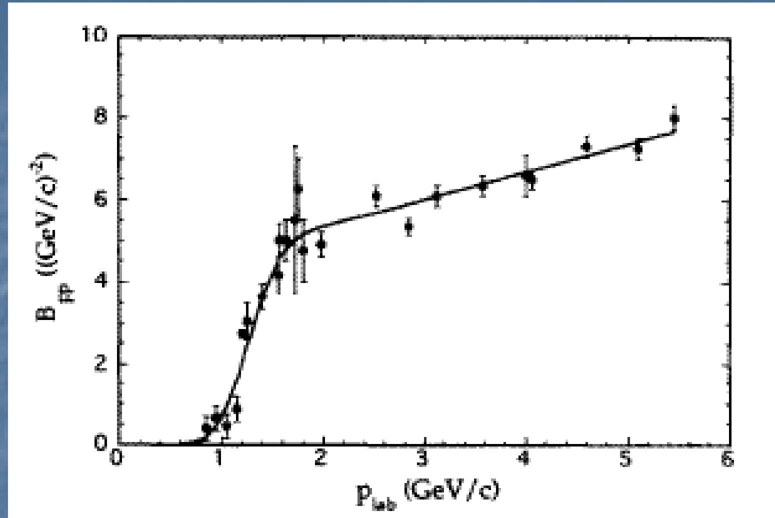
$$t = -2p_{cm}^2(1-\cos\theta_{cm})$$

$$p_{lab} < 2GeV/c$$

$$B_{pp} = \frac{5.5\,p_{lab}^8}{7.7 + p_{lab}^8}$$

$$p_{lab} > 2GeV/c$$

$$B_{pp} = 5.334 + 0.67(p_{lab} - 2)$$



np elastic scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} \propto e^{B_{np}t} + a\cdot e^{B_{np}u} + c\cdot e^{\alpha_c u}$$

$$u = -2.p_{cm}^2(1+\cos\theta_{cm})$$

$$p_{lab} < 0.8GeV/c$$

$$B_{np} = \frac{7.16 - 1.63\,p_{lab}}{1+\exp(-\frac{p_{lab}-0.45}{0.05})}$$

$$a = 1; c = 0$$

$$0.8 < p_{lab} < 1.1$$

$$B_{np} = 9.87 - 4.88\,p_{lab}$$

$$p_{lab} > 1.1$$

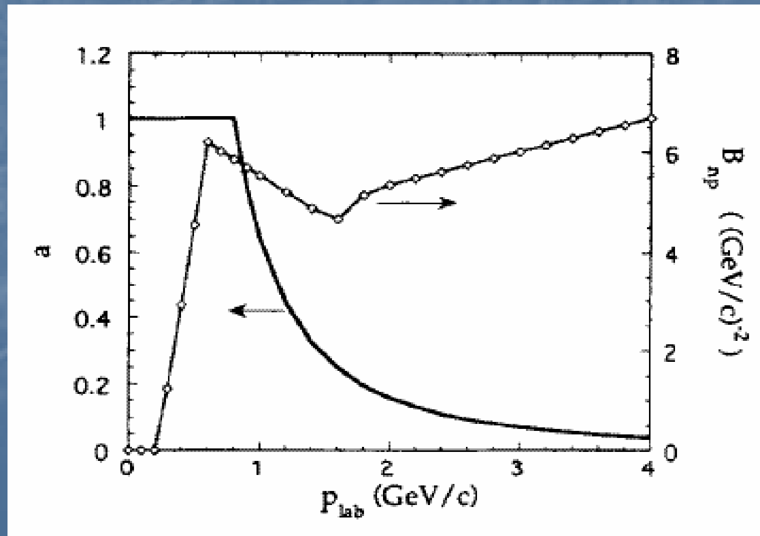
$$B_{np} = 3.68 + 0.76\,p_{lab}$$

$$p_{lab} > 0.8$$

$$a = (0.8/p_{lab})^2$$

$$c = Max\left(6.23e^{-1.79\,p_{lab}}; 0.3\right)$$

$$\alpha_c = 100.$$



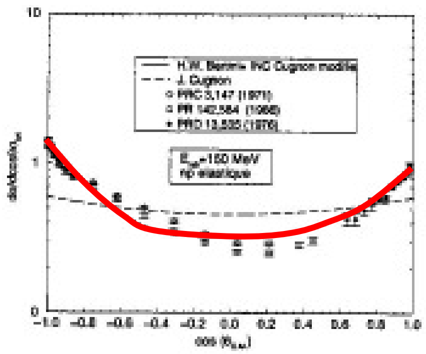


Fig. 53.b : Diffusion np élastique à basse énergie incidente.

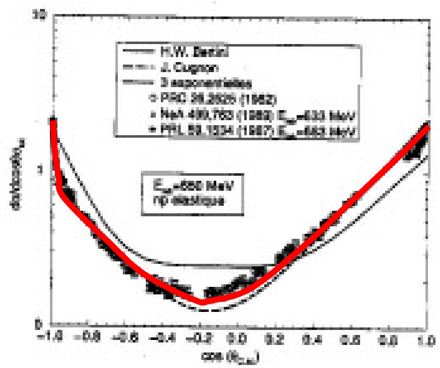


Fig. 54.a : Diffusion np élastique à haute énergie incidente.

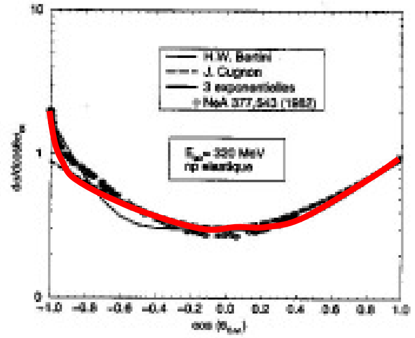


Fig. 54.c : Diffusion np élastique à haute énergie incidente.

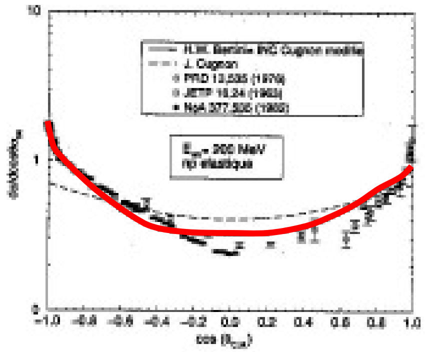


Fig. 53.c : Diffusion np élastique à basse énergie incidente.

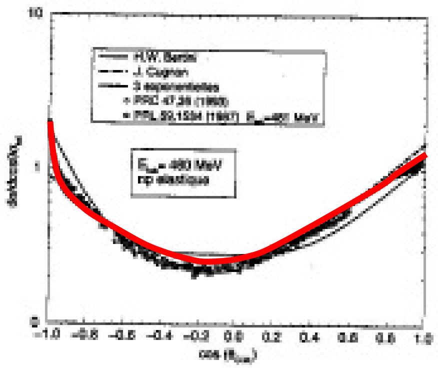


Fig. 54.b : Diffusion np élastique à haute énergie incidente.

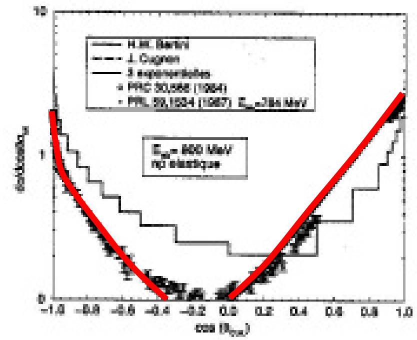


Fig. 54.d : Diffusion np élastique à haute énergie incidente.

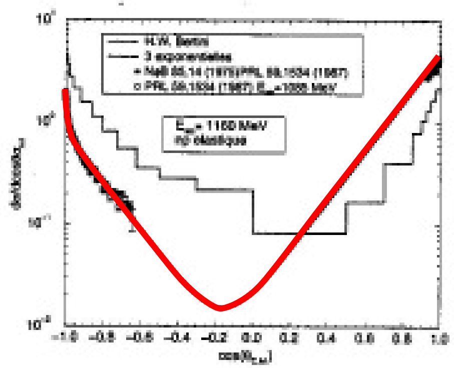


Fig. 54.e : Diffusion np élastique à haute énergie incidente.

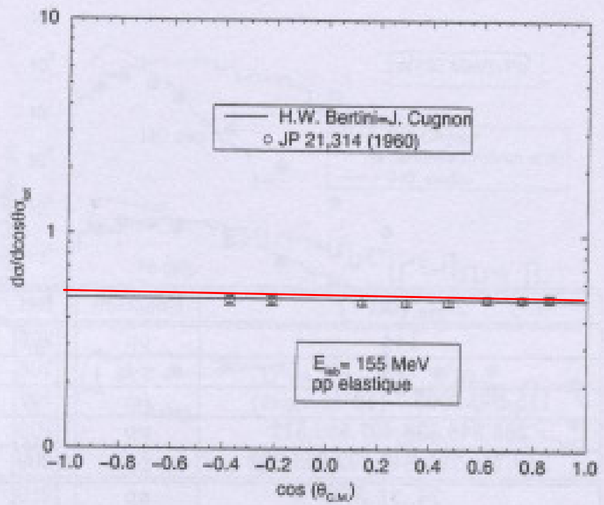


Fig. 52.a : Diffusion pp élastique.

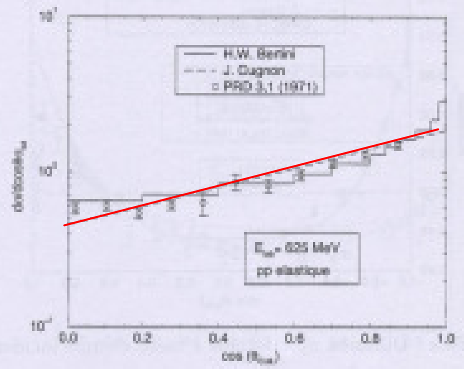


Fig. 52.b : Diffusion pp élastique.

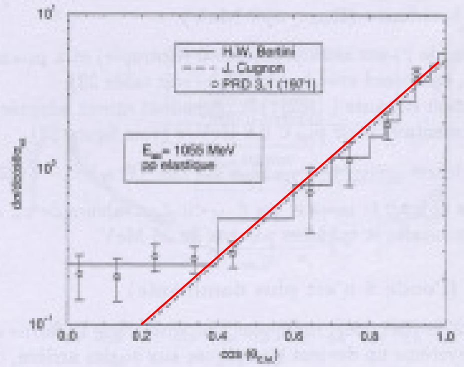


Fig. 52.c : Diffusion pp élastique.

Note:

The φ angle is at random in the CM (maximize the stochasticity)

Δ resonance, πN->Δ cross sections

Δ, spin 3/2, isospin 3/2, Breit Wigner m0=1232 MeV, Γ=118 MeV, P wave

Δ mass distribution:

$$f(M_\Delta) = C \cdot \frac{q^3}{q^3 + 180^3} \cdot \frac{1}{1 + 4\left(\frac{M_\Delta - 1215}{130}\right)^2}$$

$$q = \frac{\sqrt{\left[M_\Delta^{\,2} - (m_N + m_\pi)^2\right] \cdot \left[M_\Delta^{\,2} - (m_N - m_\pi)^2\right]}}{2M_\Delta}$$

$$m_N + m_\pi < M_\Delta < \sqrt{s} - m_N$$

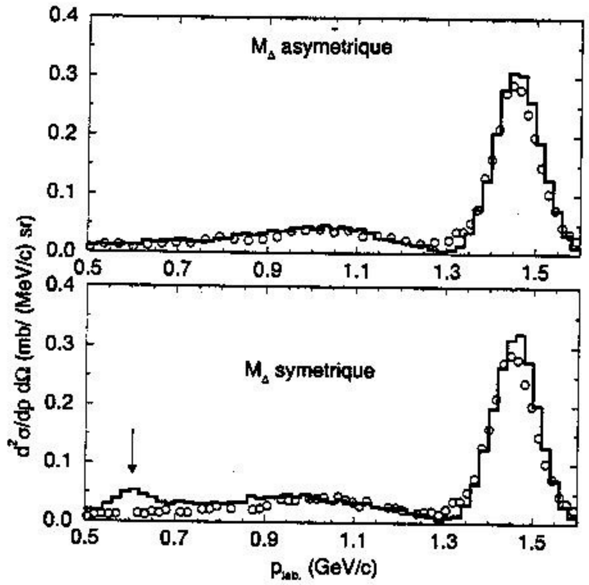
q is the momentum in the CM of the Δ->Nπ decay.

M_Δ is also limited by energy conservation

Note: This distribution with phase space restriction has been shown to be better than the pure Breit-Wigner.
- It gives correctly the π^+p cross section
- It suppress a spurious pic in np->pX cross sections in the forward direction

J.Cugnon et al, P.R. C56 (1997) 2431
S. Vuillier PhD, E. Martinez PhD

np->pX at 800MeV and 0°
E. Martinez data (LNS)



Δ life time:

t_Δ randomly chosen in an exponential law

$$\rho(t) = \exp\left(-\frac{q^3}{q^3+180^3}\cdot\frac{2\pi\cdot 115}{h}\cdot t\right)$$

(t_Δ a new potential avatar-Δ decay- in the list)

NN->NΔ Angular distribution:

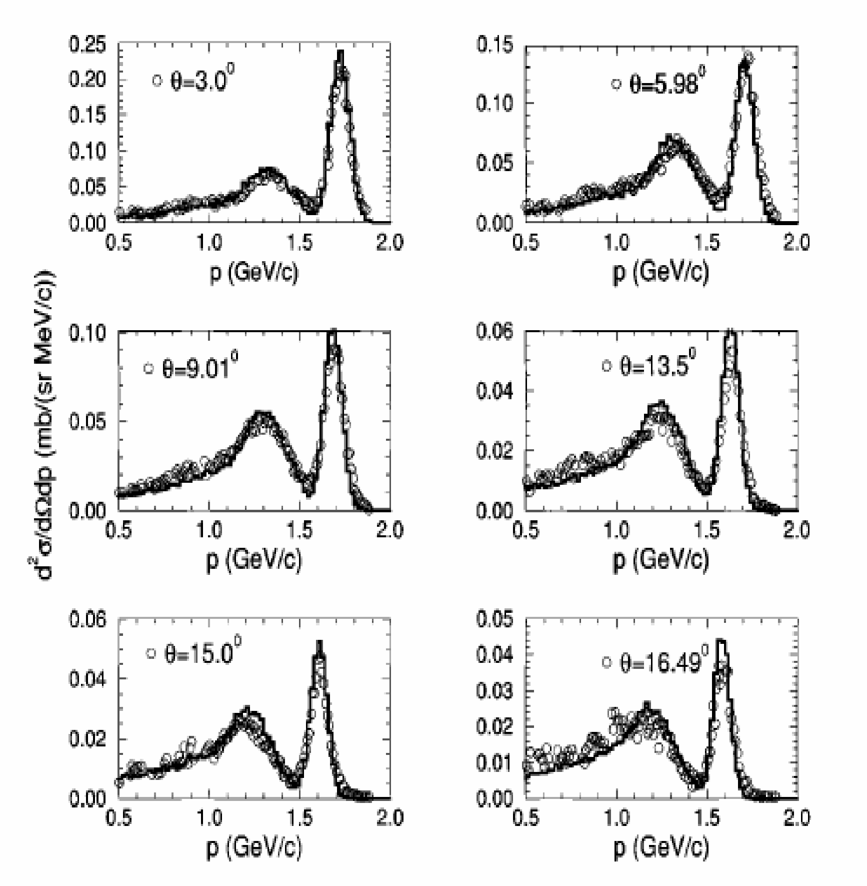
$$\frac{d\sigma}{dt} \propto e^{-B_{in}\cdot t}$$

$$p_{lab} < 1.4 GeV/c$$

$$B_{in} = 5.287/\left(1+\exp((1.3-p_{lab})/0.05)\right)$$

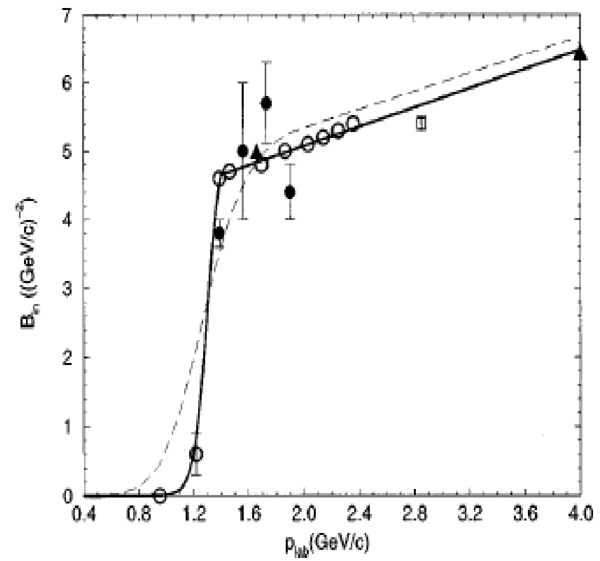
$$p_{lab} > 1.4$$

$$B_{in} = 4.65 + 0.706\cdot(p_{lab} - 1.4)$$



np->pX at 1.03 GeV

G.Bizard et al.
NPB108(1976)189



πN->Δ cross sections:

INCL4.2 (4.3) Only the true Δ resonance region is parameterized

INCL4.4 Fit up to ~5 GeV   Th.Aoust,J.Cugnon PRC74 (2006) 64607

$$\sqrt{s} < 1290 MeV$$

$$\sigma(\pi^+ p) = \frac{326.5}{1+4\left(\frac{\sqrt{s}-1215}{110}\right)^2} \cdot \frac{q^3}{q^3+180^3}$$

$$\sqrt{s} > 1290 MeV$$

$$\sigma(\pi^+ p) = Polynomial(s)$$

$$\sqrt{s} < 1290 MeV$$

$$\sigma(\pi^- p) = \frac{1}{3} \cdot \sigma(\pi^+ p)$$

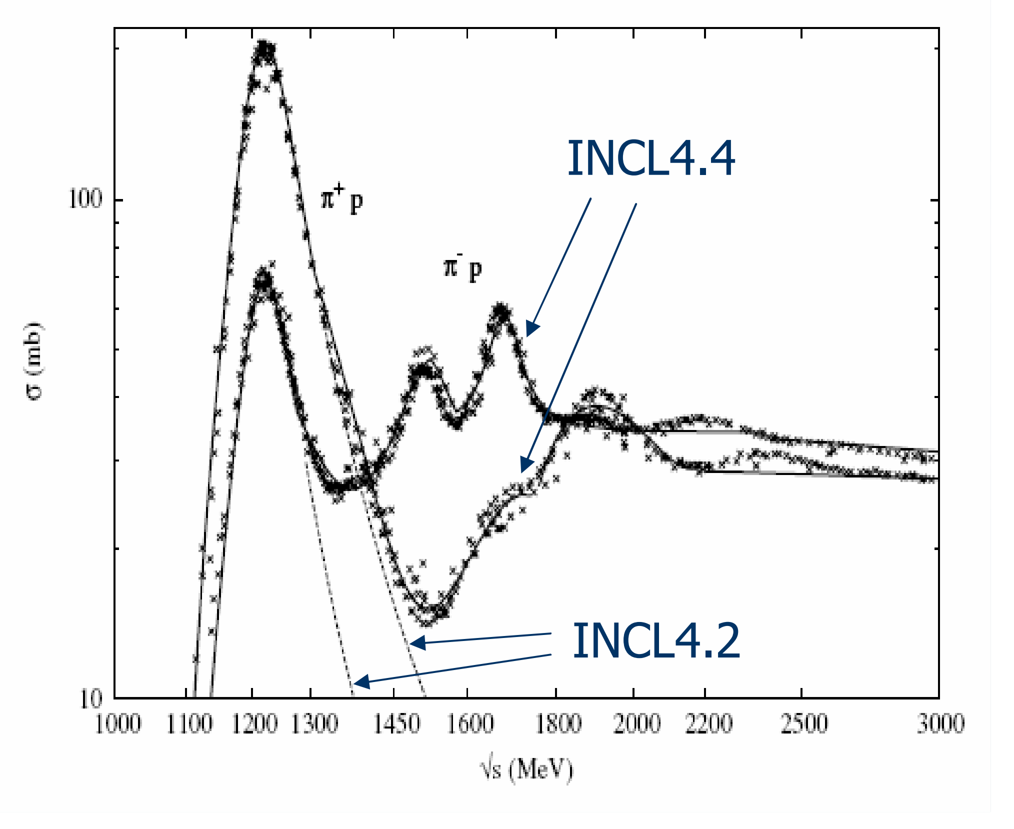
$$\sqrt{s} > 1290 MeV$$

$$\sigma(\pi^- p) = Polynomial(s)$$

$$\sigma(\pi^0 p) = \frac{\sigma(\pi^+ p) + \sigma(\pi^- p)}{2}$$

All channels:5 mb minimal below the Δ

πN total cross sections (PDG)



« Δ »->Nπ

Above the true Δ region the life time of the pseudo resonance is decreased by ~2

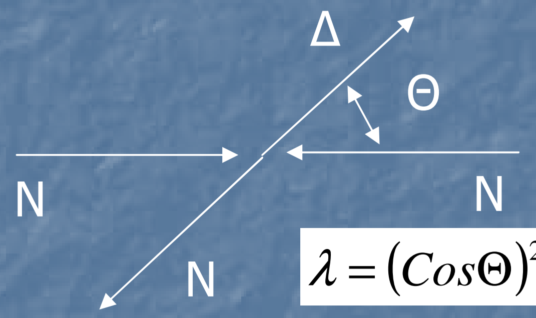
$\sqrt{s} > 1500 MeV$ $\Gamma = 115 MeV \rightarrow 200 MeV$

Angular distribution (Justified for the Δ as a P wave)

$$\left.\frac{d\sigma}{d\Omega}\right|_{CM} = 1 + 3\lambda \cdot (Cos\,\vartheta_\pi)^2$$

λ is the helicity:

(λ is 0 in πN->Δ)



$\lambda = (Cos\Theta)^2$

Pauli blocking

This is the main quantum ingredient of this approach

Fermions-> 4 nucleons (2p-2n) in a cell $h^3=(197\text{MeV.fm})^3$

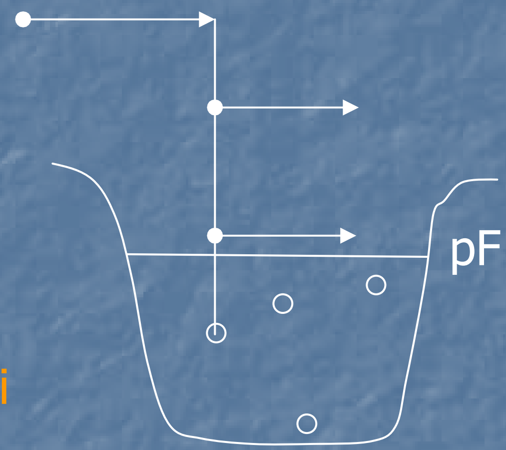
NN->NN (or NΔ); Δ->Nπ possible in free space **can be forbidden in the nucleus**

Macroscopic point of view

Target nucleus is a ground state->No vacancy below the Fermi level

First interaction: pi (pj) > pF

Keep it for ever-> Strict Pauli blocking



After some collisions X holes are created below pF

pi<pF accepted with a proba X/A

Carefully applied to p,n population and depleted nucleus -> Global statistical Pauli

Microscopic point of view

Count effectively the particles in a cell around $\vec{p}_i, \vec{r}_i$ -> **Local statistical Pauli**

(Correlation of holes, shadowing…but spurious statistical holes)

Adopted in INCL; other ones for test.

Pauli in INCL

➢Compute a local occupation factor f   in a cell (3.18fm; 200MeV/c) =2.38h^3 for less fluctuations

•Block the nucleon with the probability f.

•If it is a Δ blocked decay, give a next chance of decay at a time
(Physics behind is the **reduced width of the Δ inside the nucleus**)

$$t' = t \cdot \frac{f}{1-f}$$

➢Coherent Dynamical Pauli Principle (CDPP… a name!)

At any time after a collision or a decay:

$$\sum_{p_i \le pF} \overline{T}_i > \left[\sum_{j \in A_T} T_j^0 - (A_T - A_{rem}^F) T_F \right]$$

Current sum of kinetic energies below Fermi level (including mΔ-mN)

Target ground state

Number of nucleons extracted below Fermi level

[Minimal energy below Fermi level]

Corrects bad statistical choice.
Based on minimal energy or g.s. evolution.

Otherwise blocking.

5) Escaping particles

- **A nucleon at Rmax with T>V can escape**

With a transmission probability (even for n)
a transparence of the coulomb barrier (for charged particles)
and a coulomb barrier computed at Rc=R0 + RMS (0.88fm for p)

$$T = \frac{4pp'}{(p+p')^2} \cdot G$$

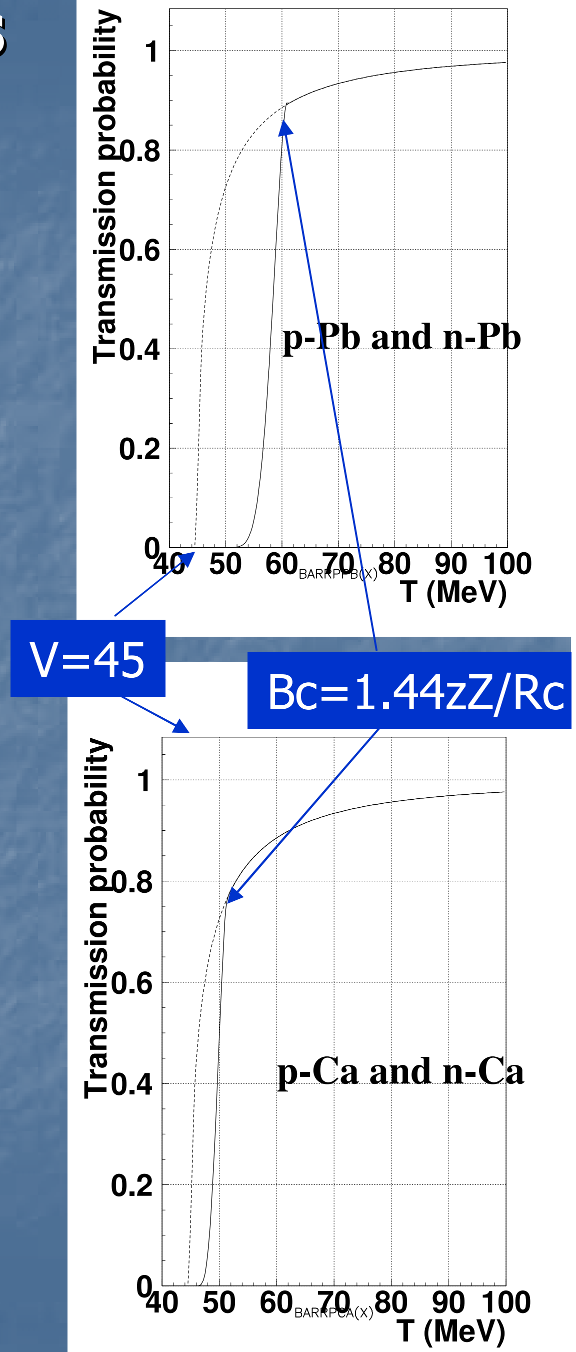
p,p' momentum in and out,
G Gamow factor

(Otherwise it is reflected)

- **Charged particles have a coulomb deflection**

(as for projectiles, following asymptotic Rutherford tracks)

- **A nuclear diffraction is possible but not used**



Cluster production (d,t,3He,4He)

Outside of the INC hypothesis (independent nucleons)…. But observed

⟶ **Phenomenological approach** based on:

- Fluctuation of nucleon positions producing clusters
- Clusters possibly pushed outside if at the surface of the nucleus

When a nucleon i has enough energy to escape:

When it is at $R_0 + h(=2\,fm)$ (h empirical parameter to define "at the surface")

Search for nucleons in it's vicinity so that $\Delta r_{ij} \cdot \Delta p_{ij} \leq 387\, fm \cdot MeV / c$

Phase space (387) second empirical parameter

with the correct isospin, should have enough energy to escape

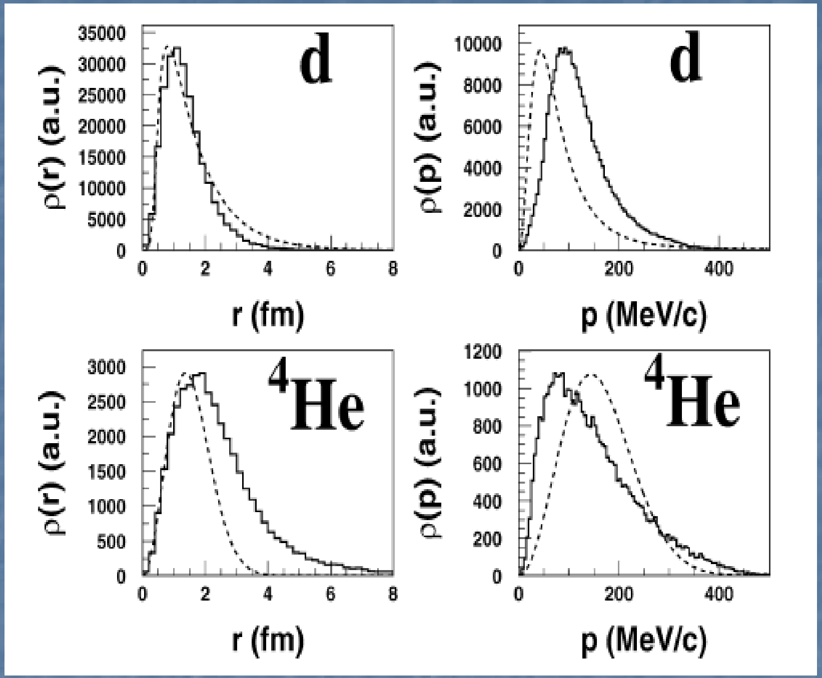
among all possible escaping clusters, give priority to the heaviest (50/50 for t/3He)

Parameters readjusted after A.Boudard et al. N.P. A740(2004)195

Gives reasonable r and p space densities
for the selected clusters:

Dashed line: d density from Paris potential or gaussian model for 4He.

Histogram: selected by the INCL procedure (independent from the target nature and the beam energy)



With only 2 parameters gives reasonable cluster production:

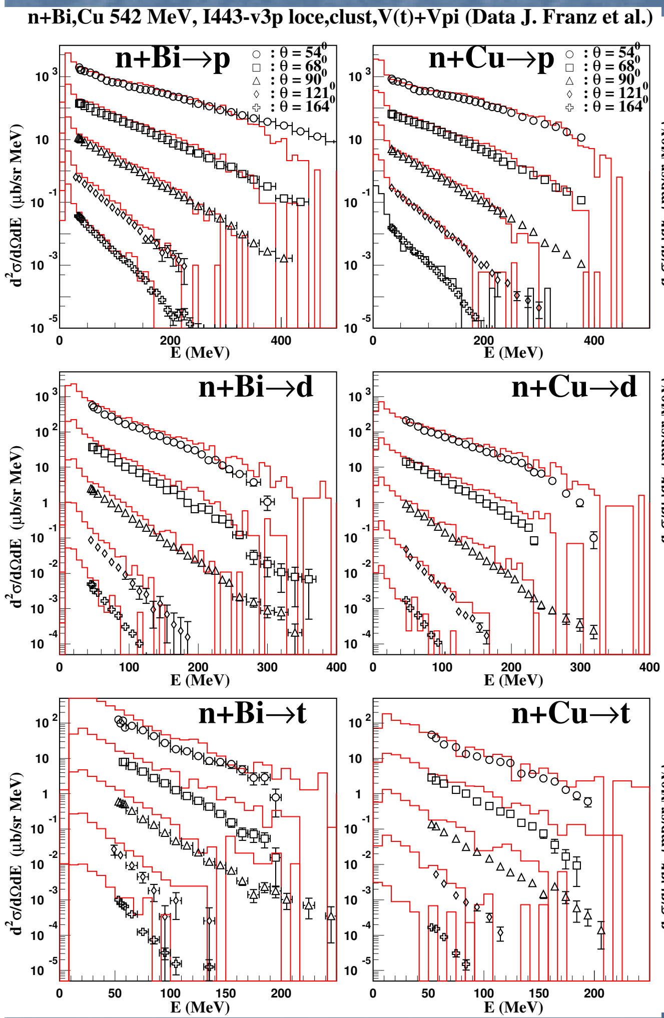
(But strong over production not yet understood at low beam energy <50 MeV)

INCL4.4 coupled with ABLA (only n,p and 4He evaporation)

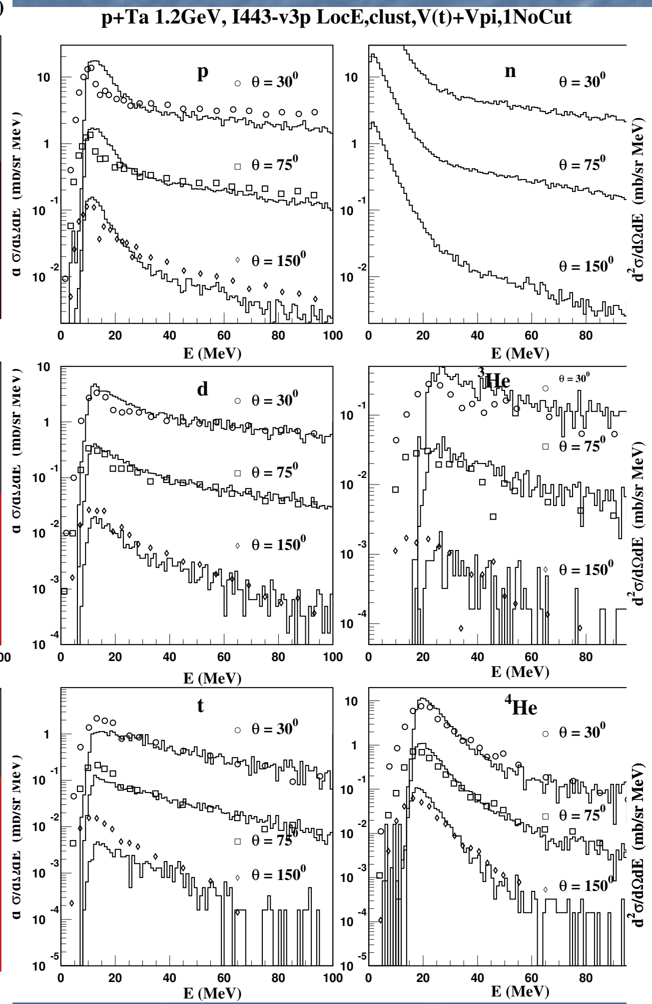
n(542MeV) + Bi,Cu

p(1.2GeV) + Ta

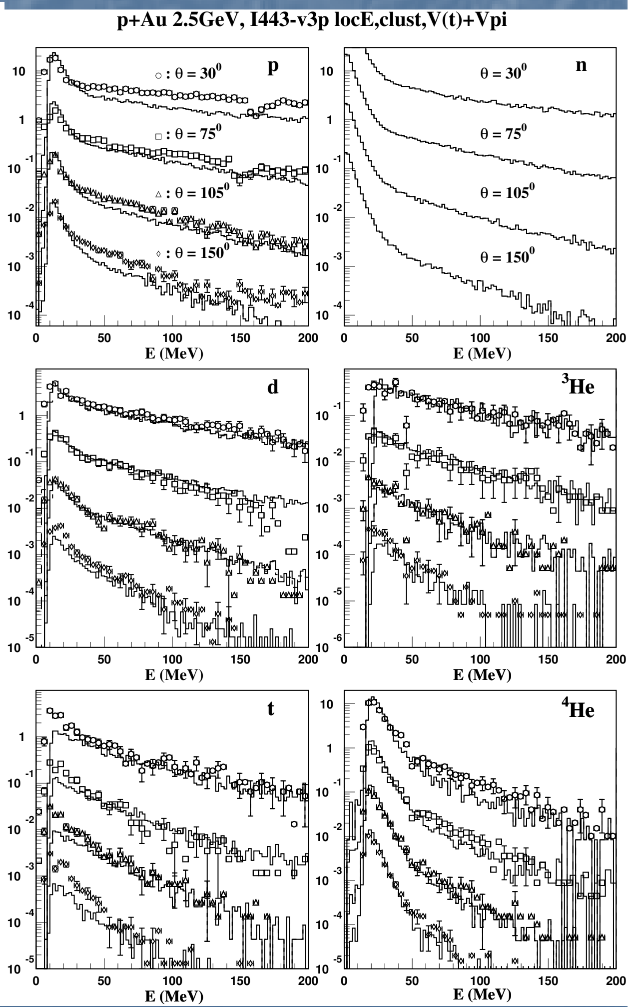
p(2.5 GeV) + Au



Data J. Franz et al. Nucl. Phys. A510 (1990) 774



Data: C. Herbach et al. Nucl. Phys. A765 (2006) 426



Data: A. Letourneau et al. Nucl. Phys. A712 (2002) 133

6) End of the cascade

At some time, the cascade must be stopped:

- Energy of nucleons inside the nucleus too low to be treated by INC
- The beam kinetic energy has been randomized on many nucleons
- Energy spectra of ejected nucleons (evaporation like) are may be too hot.
- The nucleus is a ~thermalized source better treated by evaporation models

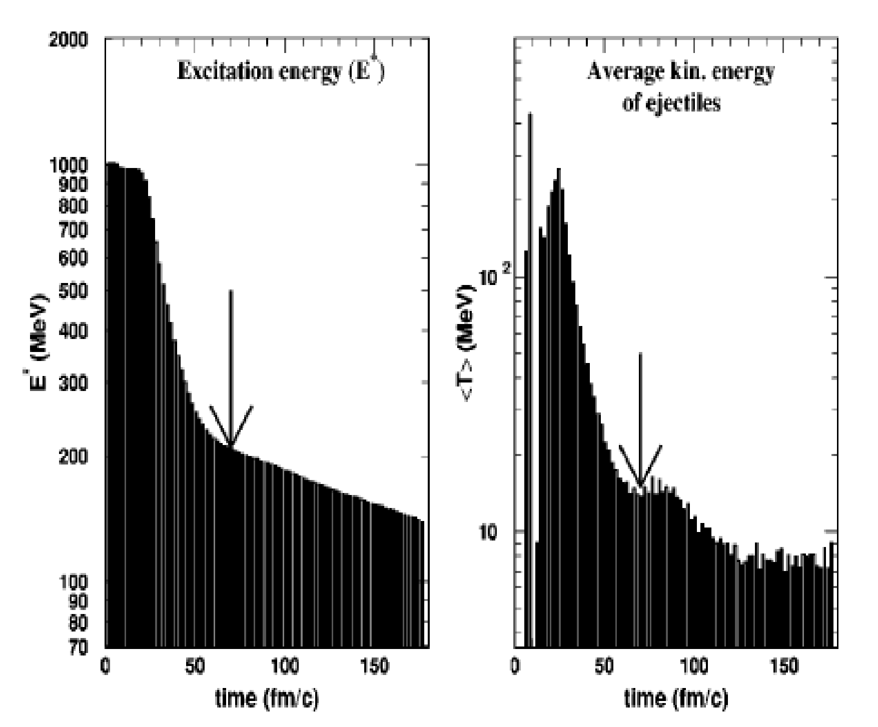
Stopping time value Tfin is determined one time for ever from the time evolution of:

- Excitation energy (a break),
- Average energy of ejected nucleons (between 10 and 20 MeV)
- Randomization (reached) of participant momenta

(beam energy 0.2 to 2GeV, target (Al-U) Impact parameter from central to surface)

$$Tfin = 70\,fm/c \cdot \left(\frac{A_T}{208}\right)^{0.16}$$

(Or could be also a function of R0)



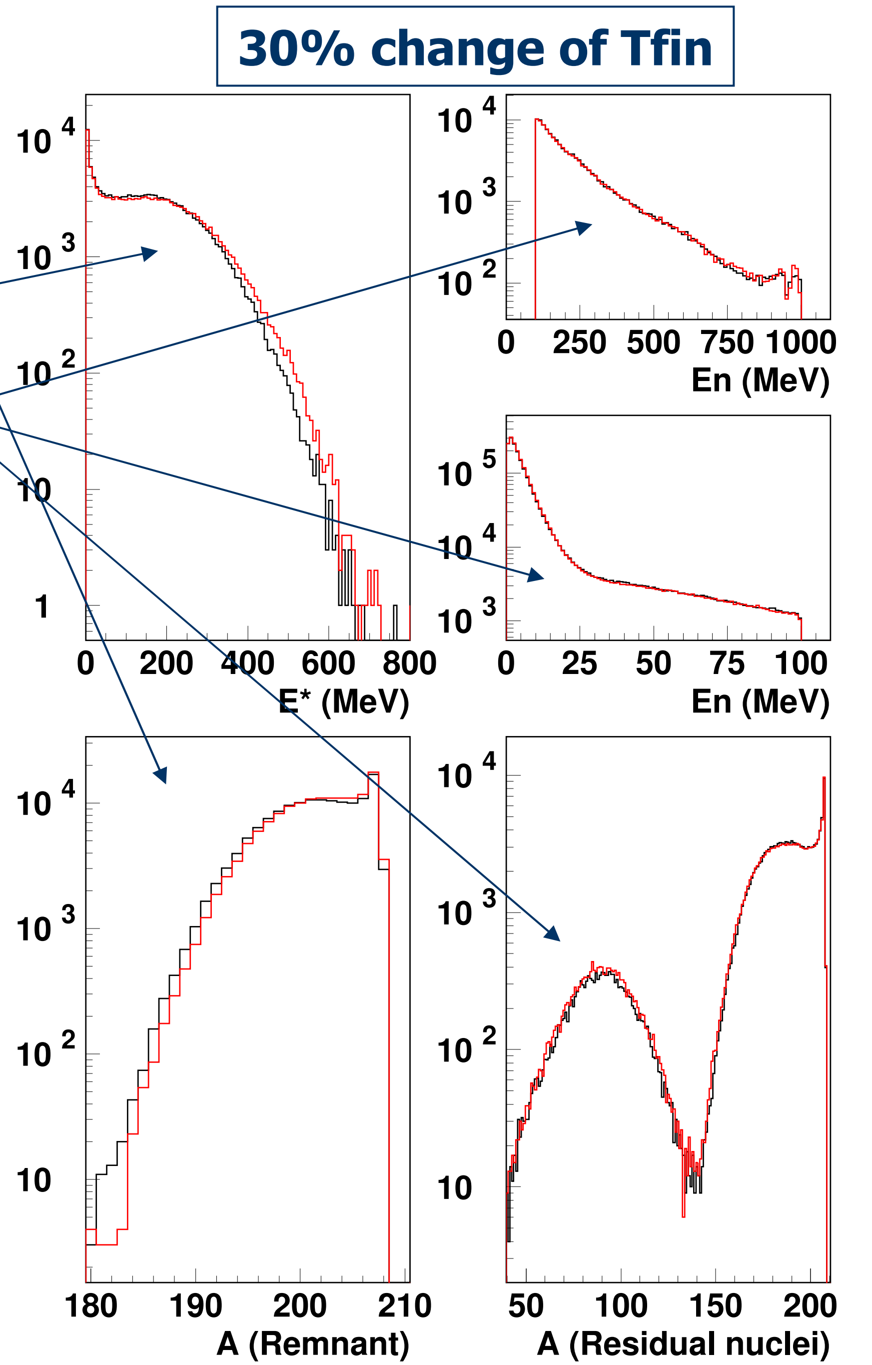
Observables are not very sensitive to Tfin value:

Intermediate quantities: E*, A (Remnant)

Observables: n spectra, residual nuclei A

To save computation time on quasielastic events:
Stop cascade also if no pions inside and all nucleons inside have:

$$T \le T_F + 10 MeV$$



Characteristics of the remnant nuclei

Evaporation needs the nature A,Z of the remnant, its excitation energy E* and spin J

At the end of the cascade the remaining Δ are forced to decay and pions emitted or added to the excitation energy.

$$E^* = \sum_{i \in A_{rem}} T_i - \left[\sum_{j \in A_T} T_j^0 - (A_T - A_{rem}) T_F \right]$$

(Compare to CDPP…)

All kinetic energies inside

Target ground state

Maximal energy for the emission of nucleons

[Running GS of the remnant]

Characteristics of the remnant nuclei (2)

During all the cascade, the nucleus was centered at r=0 and at rest.

(Energy was conserved at all levels but not momentum)

The proper momentum conservation will give now the remnant recoil

(iterative process, Erec is small <10MeV)

$$\vec{p}_{rem} = \vec{p}_{Beam} - \sum \vec{p}_{out}$$

$$E_{rec\_rem} = \sqrt{p_{rem}^2 + M_{rem}^2} - M_{rem}$$

The sum of energies was correct.
Rescale all of them so that we have energy AND momentum conservation:

$$\sum e_{old}^{out} = \sum e_{new}^{out} + E_{rec\_rem}$$

$$p_{new}^{out} = \sqrt{e_{new}^{out}\left(e_{new}^{out} + 2m\right)}$$

$$\vec{p}_{rem}^{new} = \vec{p}_{beam} - \sum \vec{p}_{new}^{out}$$

$$E_{rec\_rem}^{new} = \sqrt{p_{rem}^2 + M_{rem}^2} - M_{rem}$$

Angular momentum

$$\vec{L} = \vec{b} \wedge \vec{p}_{beam} - \sum \vec{l}_{out} - (\vec{r}_{rem} - \vec{r}_{target}) \wedge \vec{p}_{rem}$$

$$J = L \cdot \frac{2\pi}{h}$$

New physical ingredients (INCL4.4)

V(τ,E) p and n population dissociated

$$V_{0n} - V_{0p} = \Delta V(\Delta\tau = 2)$$

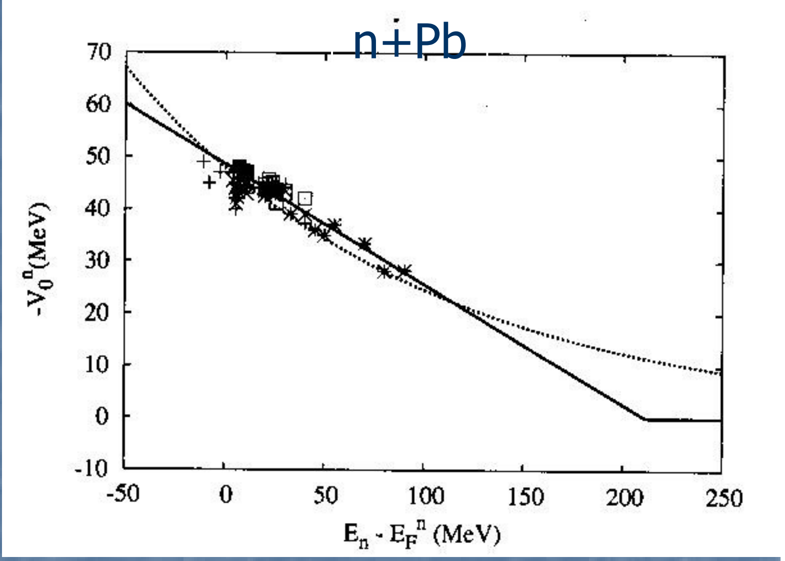
From optical potentials, deepness is a function of beam energy

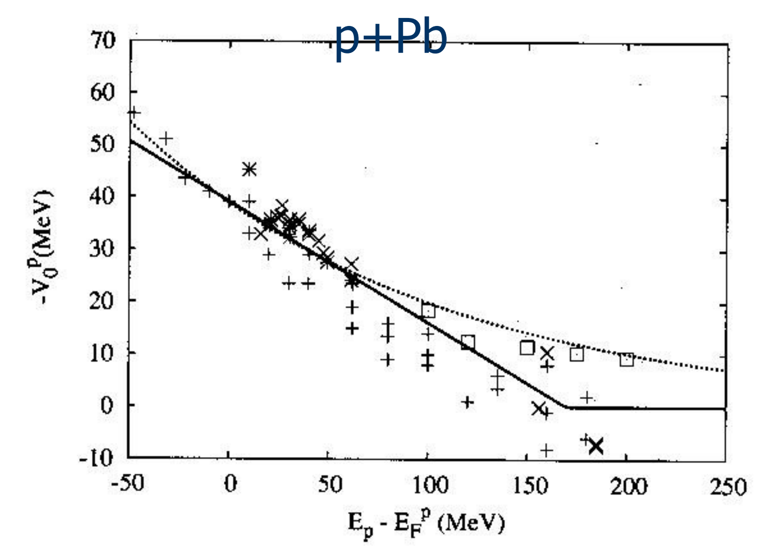
E<E0 $V(E) = V_0(E) - \alpha(E - E_F)$

E>E0 $V(E) = 0$

$\alpha = 0.223$ parameter

(E0~200MeV comes from Fermi energy and separation energy in a coherent way)





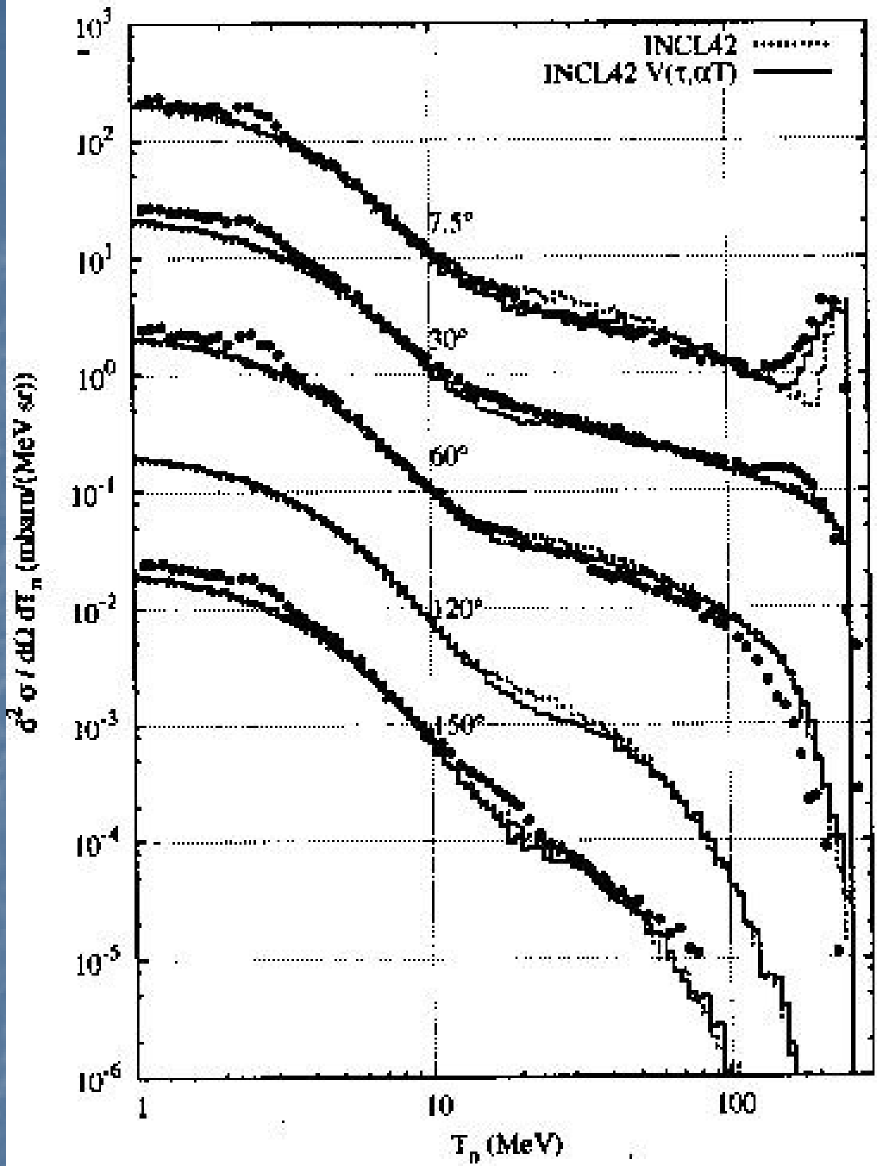
➢Technically, the 2 body kinematics have to be solved iteratively

Details in: Th Aoust, J.Cugnon Eur. Phys. J. A21 (2004) 79
J.P.Jenkenne, Mahaux, Startor P.R. C43(1991)2211

➢Enlarge the n quasi elastic peak (and not the p one)

➢Lowers the n multiplicity

p(1.2GeV)+Pb n above 20 MeV:
3.17->2.90
experimental value: 2.7+/-0.3



p(256MeV)+Pb

M.Meier et al Nuc.Sc.Ener.110(1992)289

Real attractive potential for pions

Th.Aoust, J.Cugnon P.R. C74(2006)64607

An adjusted potential for r<Rc

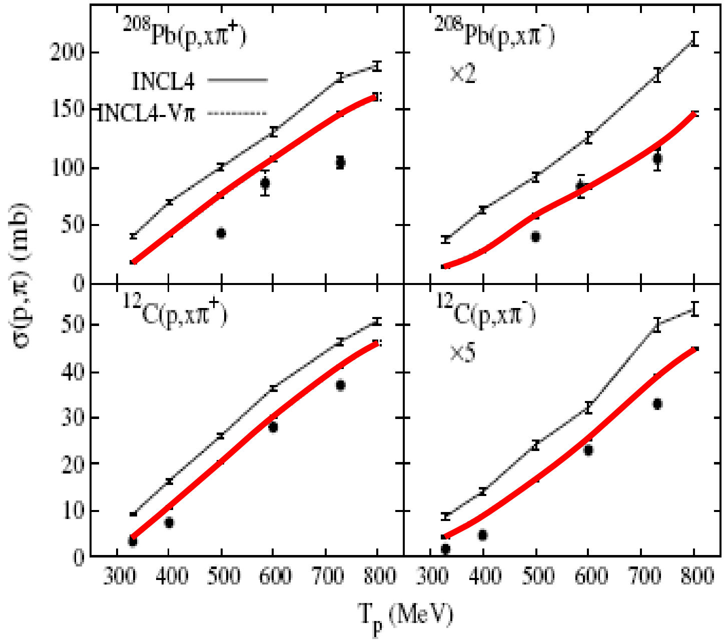
$$V(r,\tau_3) = V_0 + V_1\tau_3\frac{(N-Z)}{A} + \tau_3\frac{1.25Ze^2}{R_0}$$

$$V_0 = -30.6MeV$$

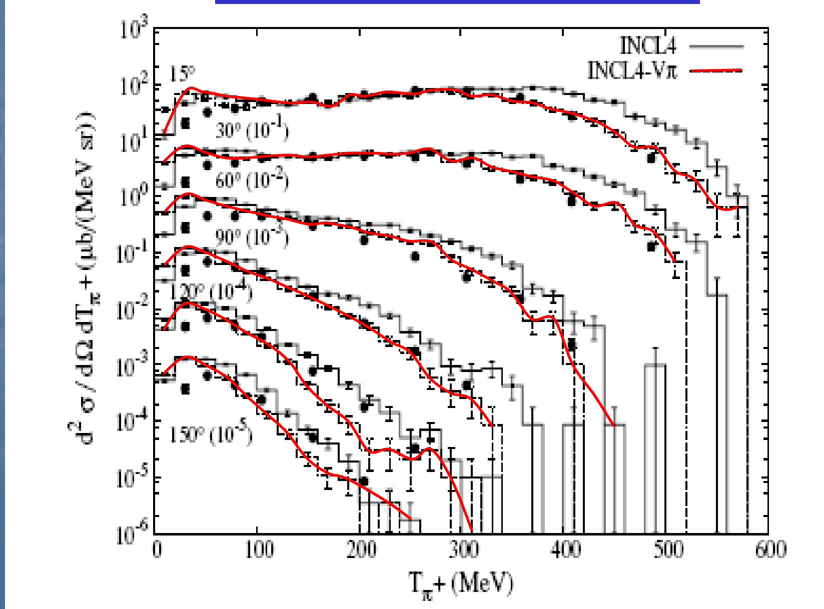
$$V_1 = -71.0MeV$$

Coulomb barrier evaluated at Rc

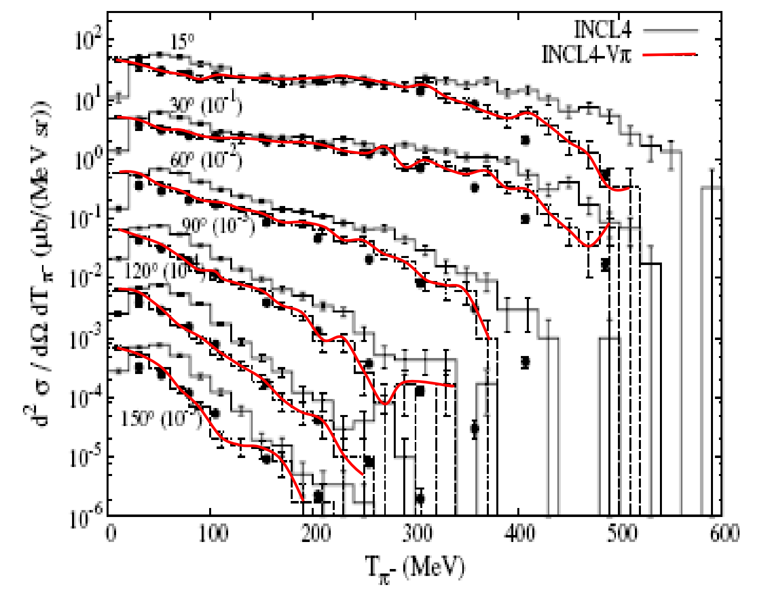
$$R_c = R_0 + 2fm$$



p(730MeV)+Pb-> π+



p(730MeV)+Pb-> π-



D.Cochran et al,P.R. D6(1972)3085

Local energy corrections

Basic in INCL: “motion of nucleons on straight lines at constant speed”

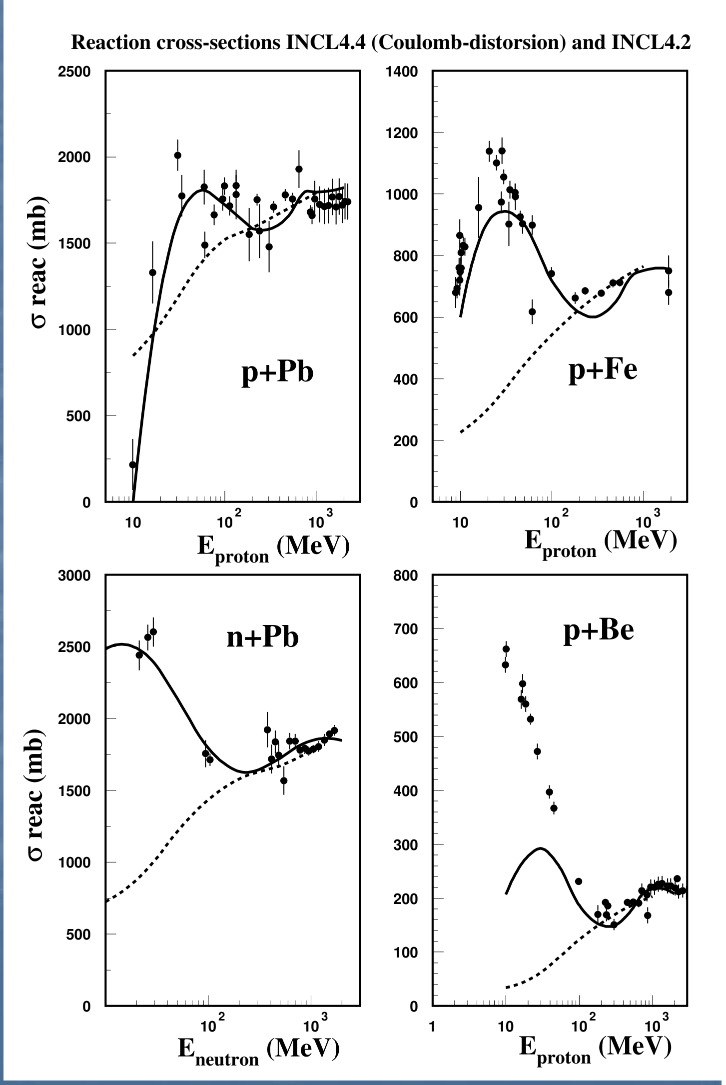
Correction: use of energy ABOVE the potential to evaluate BB cross sections and kinematics

Without low energy cut on NN for the first interaction and Coulomb distortions:

Get a correct reaction cross section down to very low energies

(very nice if used as event generator)

But E* too high without NN energy cut Λ for next interactions….
Increasing interactions (boiling) at the surface!



Getting observables

For all computed reactions record of E,p,θ,φ for all particles and nuclei at the end with index (emitted by the cascade or the deexcitation, fission) and global characteristics (b_impact, Remant characteristics, fissioning nuclei…)

Treated by PAW (as NTUPLE) or ROOT (as TREE)

Any correlation with any binning or conditions can then be used to produce histos (to compare with experiments or to understand the model)

The normalization for N counts in a bin comes from the model:

$$\sigma geom = \pi \cdot R\max^2$$

$$\sigma = \sigma geom \cdot \frac{N}{Nprojectiles}$$

For the cascade part -> can record any characteristics at time steps (avatars)
Could produce a "movie" of reactions (in the semi-classical description)

Computation time: (laptop-Intel Core Duo 1.66GHz)
p+Pb 1 GeV, 300 000 shots, 156 000 reactions: 2500 seconds, Ntuple 215 Mbytes

Conclusions

INCL4.2 Pure cascade, no parameters (We don't consider Tfin as a parameter)

INCL4.3 Production of clusters 2 parameters (with physical meaning)

INCL4.4 Energy dependent potential: A slope parameter (from optical pot)

Pion potential: 2~3 parameters (2 pot., a radius; ~optical potential)

Local energy (related to ΛNN cut) To be used or not ?

Difficulties to go down in energy (~50 MeV)
(Limits of the model? Should accept NN effective? Preequilibrium?)

(Needs of the coupling with ABLA07 in progress to make final decisions)

Future: Asymmetric A-A reactions

A.Boudard for the collaboration (J. Cugnon, S. Leray, J.C. David, Th Aoust…..)
Many fruitful discussions and tests with Y. Yariv