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Joint ICTP-IAEA Advanced Workshop on Model Codes for Spallation Reactions

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ISABEL-INC Model for High-Energy Hadron-Nucleus Reactions

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Outline

- Introduction
  - Why INC?
  - What is INC?
  - Basic Assumptions ➡ Requirements
  - Expected limitations
- Isabel (Etgar…)
  - History
  - Basic Ideas, Justification
  - Implementation
  - Output
  - Nuclear Model
  - Cross sections
  - Additional details: Fermi sea depletion, Pauli blocking
  - Examples

Why INC?

- Transport codes for projectile in energy range up to few GeV important for many applications (e.g. RIB, Spallation Sources)

- Existing cross-section libraries limited to 150MeV or, for some isotopes to 20 MeV (for radioactive "residua" – 20 MeV)

➡ **Need for fast "event generator" code to fill the 20 MeV – 3 GeV gap**

What is INC?

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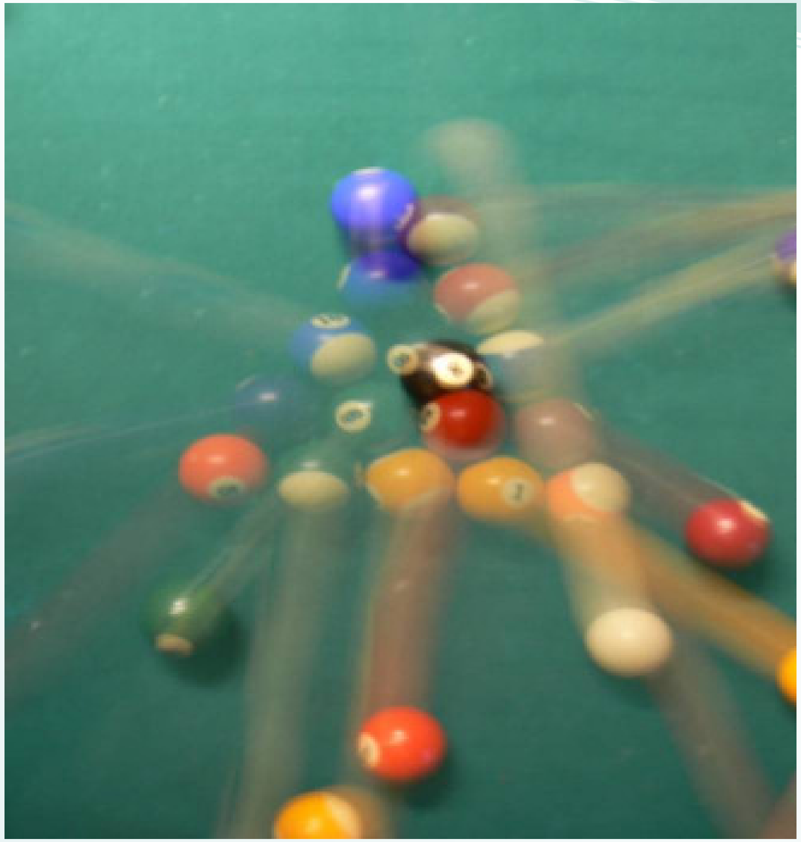
intranuclear cascade model
(¦in·trə′nü·klē·ər kas′kād ′mäd·əl)

(*nuclear physics*) A model of nuclear collisions that assumes a series of independent nucleon-nucleon collisions between particles that act like billiard balls.



INC Models (seriously)

R.Serber, Phys. Rev. 72, 1114 (1947)

- Particle on Nucleus reaction treated as series of two-body scatterings
- “Realistic” target density and momentum distributions (Fermi sea)
- Approximated Pauli principle
- “Fast Phase” followed by “slow” target de-excitation
- No “fitting parameters”

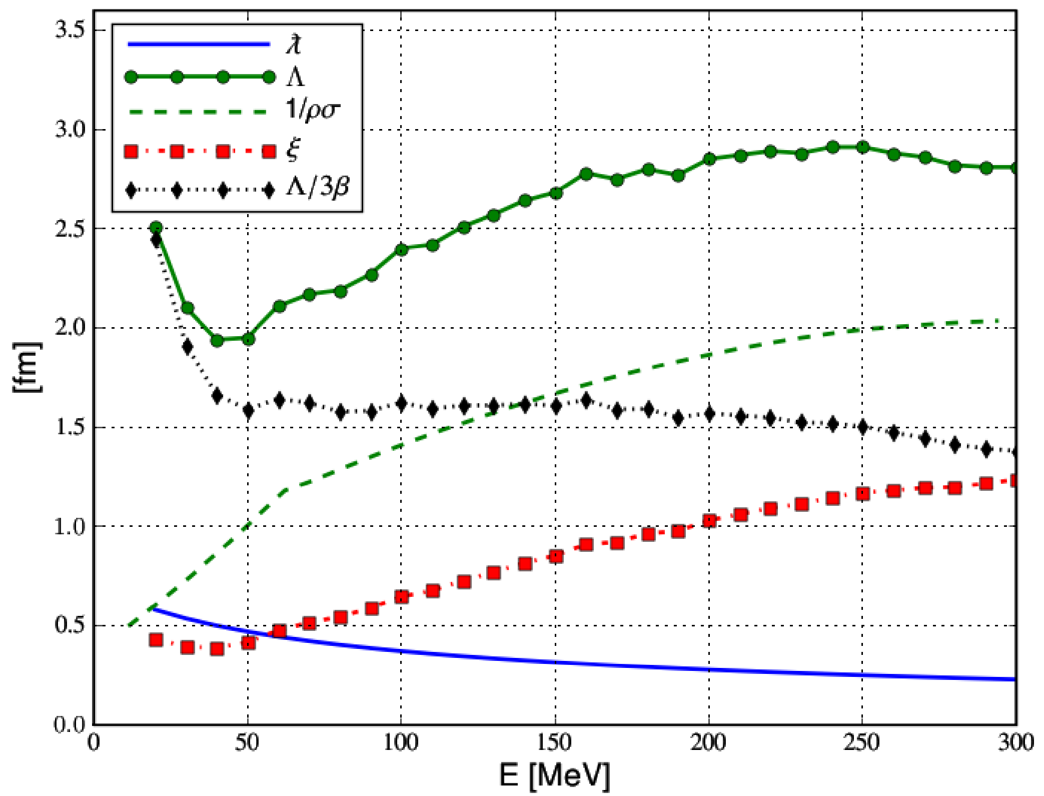
Assumptions ➡ Requirements (1)

- Many-body scattering in terms of on shell single particle cross sections
  - **"Deep Inelastic" collisions, "Energetic" collisions**
- Interacting particles followed on classical trajectories
  - **de Broglie wave-length shorter than inter-nucleon distance**
    $\lambda\!\!\!^{-} << d$
- Asymptotic value of scattered wave before next collision
  - **de Broglie wave-length shorter than m.f.p.**
    $\lambda\!\!\!^{-} < \Lambda$

Assumptions ➡ Requirements (2)

- Interference terms between collisions cancel out
  - **m.f.p. shorter than target radius**
    $\Lambda < R$
- Independent scattering from different nucleons in the target
  - **m.f.p. (Λ) larger than inter-nucleon distance (d); Time between interactions ($\Lambda/\beta c$) shorter than interaction time (10^{-23}sec.)**
    $\Lambda > d$
    $\Lambda/\beta c >\approx 10^{-23}\text{sec.} \Rightarrow \Lambda/3\beta >\approx 1$

Central collision p+^{208}Pb



$ƛ << d < \Lambda < R$

$\Lambda / 3\beta > \approx 1 fm$

$\xi \equiv \Lambda / ƛ / 10$

$\xi > 1.0 \Rightarrow E > \approx 200 MeV$

Expected limitations

- $E_{inc} > \approx 50$ MeV for:
  - Total nucleon yields
  - Peripheral collisions, e.g. “quasi-elastic”,(p,2p)
- $E_{inc} > \approx 200$ MeV for:
  - “Violent reactions” (high multiplicity, high excitation energy)

Significant discrepancies expected for outgoing particles for E_{inc} lower than few tens MeV

איזבל ISABEL

אשד תוך גרעיני
Eshed Toch Gar'ini
→ ETGAR אתגר
Etgar = Challenge

History

- R.Serber, Phys. Rev. **72**, 1114 (1947)
- M.L.Goldberger, Phys. Rev. **74**, 1269 (1948)
- N.Metropolis et al., Phys. Rev. **110**, 185 (1958); Phys. Rev. **110**, 204 (1958)
- **VEGAS**: K.Chen et al., Phys. Rev. **166**, 949 (1968)
- **ISOBAR**: G.D.Harp et al., Phys. Rev. **C8**, 581 (1973); **C10** 2387 (1974)
- **ISABEL**: Y.Yariv and Z.Fraenkel, Phys. Rev. **C20**, 2227 (1979); Phys. Rev. **C24**, 488 (1981)
- **ETGAR...**

Basic Ideas of Time-Like MC

- **Target nucleus represented by continuous density distribution in a potential well according to degenerate "local density" Fermi gas momentum distribution ("Fermi Sea")**
- **Cascade evolution divided into small "time intervals". The probability of interaction of a participant in a time interval $\delta\tau$ is $P(\delta\tau) \approx \rho\sigma\delta\tau$. After each interaction the target Fermi Sea is "depleted".**
- **Only "Pauli allowed" interactions are permitted. Fermi Sea "depletion" is taken into account.**
- **Participants below "energy cutoff" (EC) are "absorbed" EC ≈BE for neutrons, EC ≈BE +CB for protons**
- **Event ends when all participants escape or are absorbed**

Justification (1)

K.Chen et al., Phys. Rev. 166, 949 (1968)

- **Probability per unit time of a particle to interact with the nucleons of the nucleus**

$$Q = \int \sigma_{12} v_{12} \frac{\partial \rho_2}{\partial \vec{p}_2} d\vec{p}_2 \approx \sum \sigma_{12} v_{12} \frac{\partial \rho_2}{\partial \vec{p}_2} \Delta \vec{p}_2$$

- **For degenerate Fermi Gas**

$$\frac{\partial \rho_2}{\partial \vec{p}_2} \Delta \vec{p}_2 = \frac{\rho}{n} \qquad Q = \frac{\rho}{n} \sum_{i=1}^{n} v_{12_i} \sigma_{12_i} \equiv \frac{\rho}{n} \sum_{1}^{n} \sigma'_i$$

Justification (2)

K.Chen et al., Phys. Rev. 166, 949 (1968)

- **Probability that a collision takes place in time τ is**

$$N(\tau) = 1 - \exp(-Q\tau) = 1 - \prod_{i=1}^{n} \exp(-\rho\sigma_i' \frac{\tau}{n})$$

- **Probability of collision of a cascade particle in $\delta\tau = \tau/n \approx$ probability of its collision with hypothetical nucleon gas of density ρ and momentum p_i**

$$N(\delta\tau) = 1 - \exp(-\rho\sigma_i'\delta\tau) \approx \rho\sigma_i'\delta\tau$$

Implementation (1)
General Initialization

- Read reaction details & “options”:
  - Projectile type and energy, Target A,Z
  - Nuclear model (shape: WS, Folded Yukawa...) number of regions and their radia
  - Fermi sea & Pauli principle depletion methods
  - Energy cutoffs, refraction etc.
- Calculate densities, Fermi momenta & energies in different regions
- Calculate first (small) time interval $\tau=\Lambda/n$

Implementation (2)

Event Initialization

- Chose impact parameter from b^2 distribution
- Push projectile over target boundary. Change its kinetic energy and possibly refract.
  It is the first “active participant”

Implementation (3)
Participant motion

- In small time interval, τ, participant is pushed distance $\delta a = \tau\beta$
- If it crosses region boundary its kinetic energy is corrected and it is (possibly) refracted/reflected
- If it leaves the target it is recorded and "ceases" to be a "active participant" – it is not followed any more

Implementation (4)
Interaction

- Possible interaction partner (*l, p*) is chosen from the Fermi sea. Probability of interaction $P(\delta a) \approx \sigma\rho\delta a$ is calculated
- If $\xi \leq P(\delta a)$ for random $0 \leq \xi < 1$ - interaction occurs
  - Identity of outgoing particles is determined from branching ratios
  - Reaction kinematics is performed. Pauli principle is checked. If it is violated – no interaction occurs
  - New participant is created. Outgoing particles above “energy cutoff” become “active participants”. Those below “energy cutoff” are “deactivated” and are not followed any more – they contribute to residual target excitation energy and momenta
  - Fermi sea is “depleted”
  - If needed, the next time interval is decremented, $\tau_i = \min(\Lambda_i / n)$

Hadron-Hadron Interactions in ISABEL

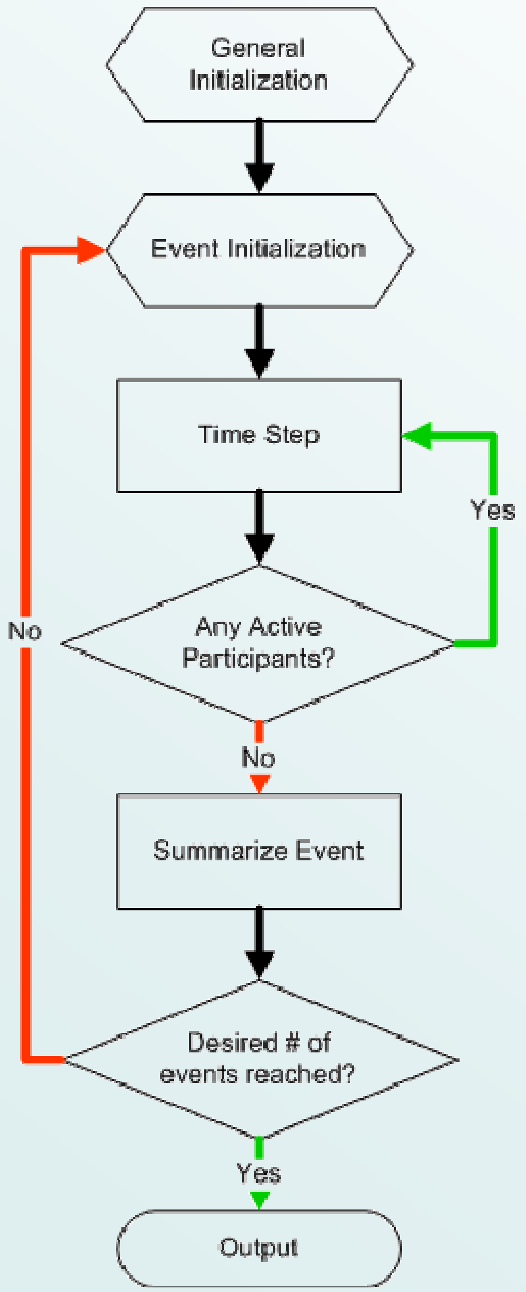
- On-mass-shell, free cross sections
  - **El**astic

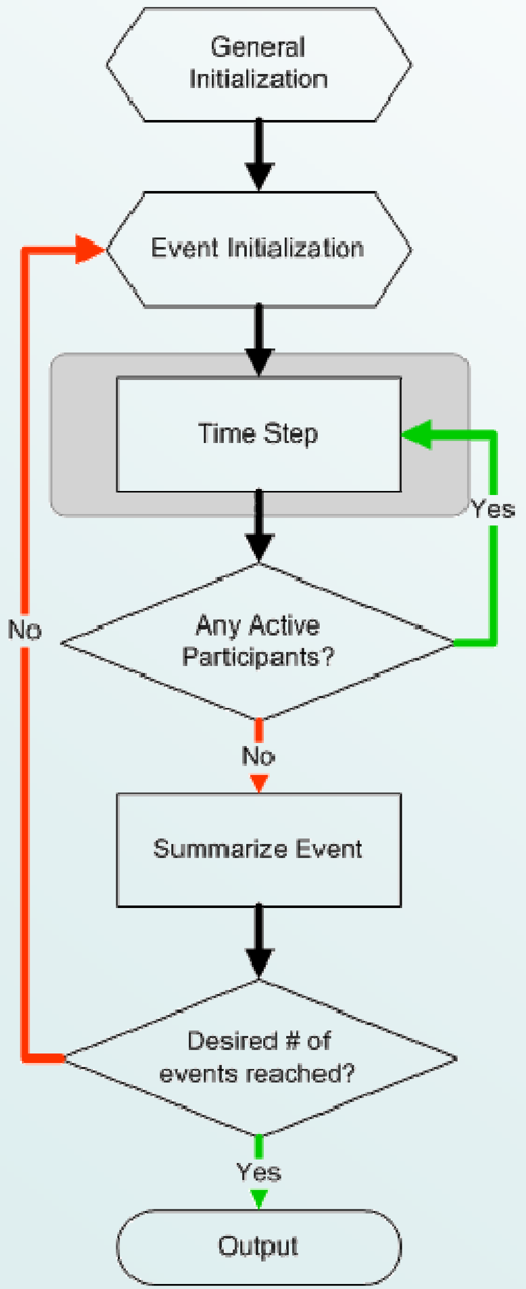
$$N + N \Rightarrow N + N$$

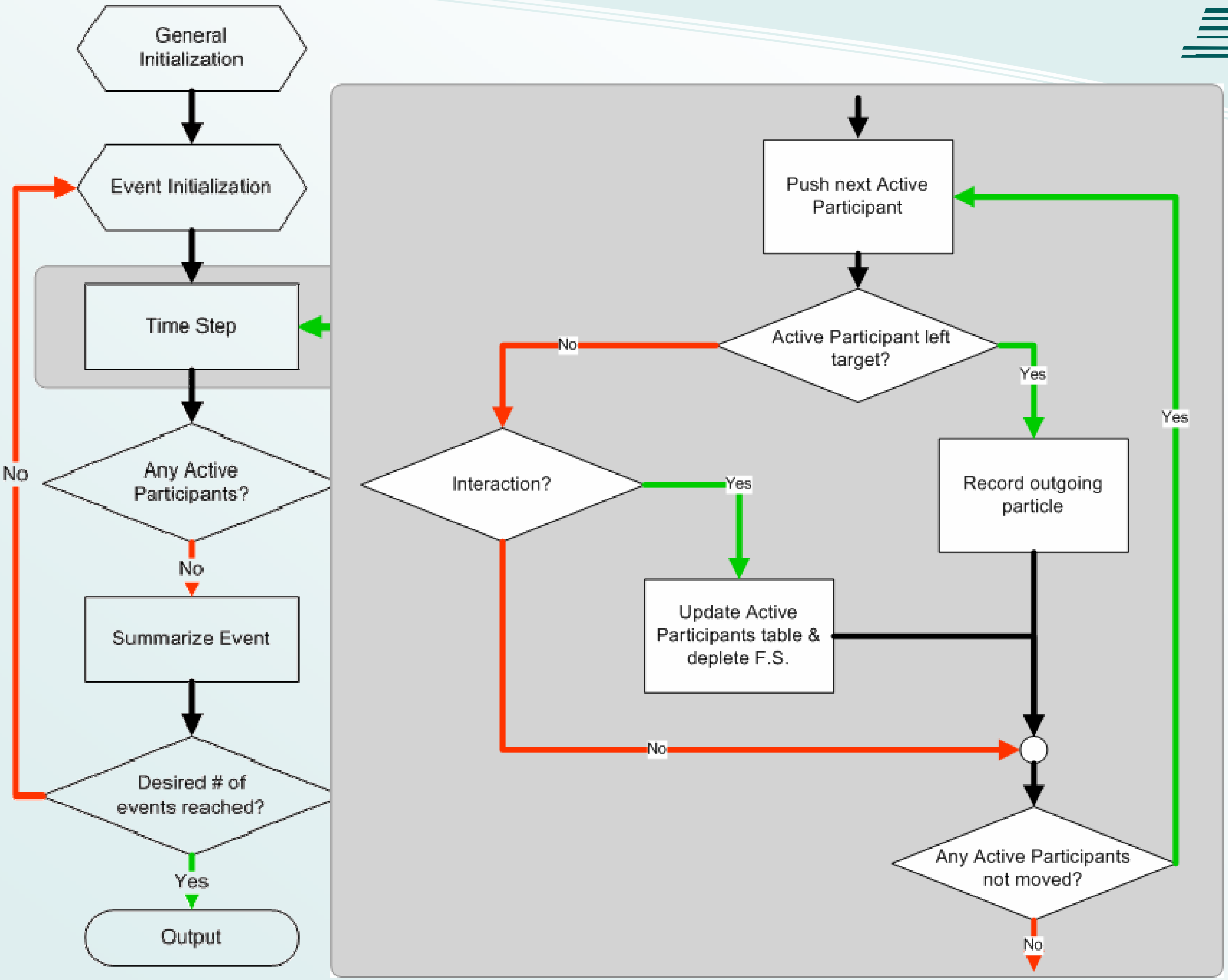
  - **Inel**astic (1π production & absorption)

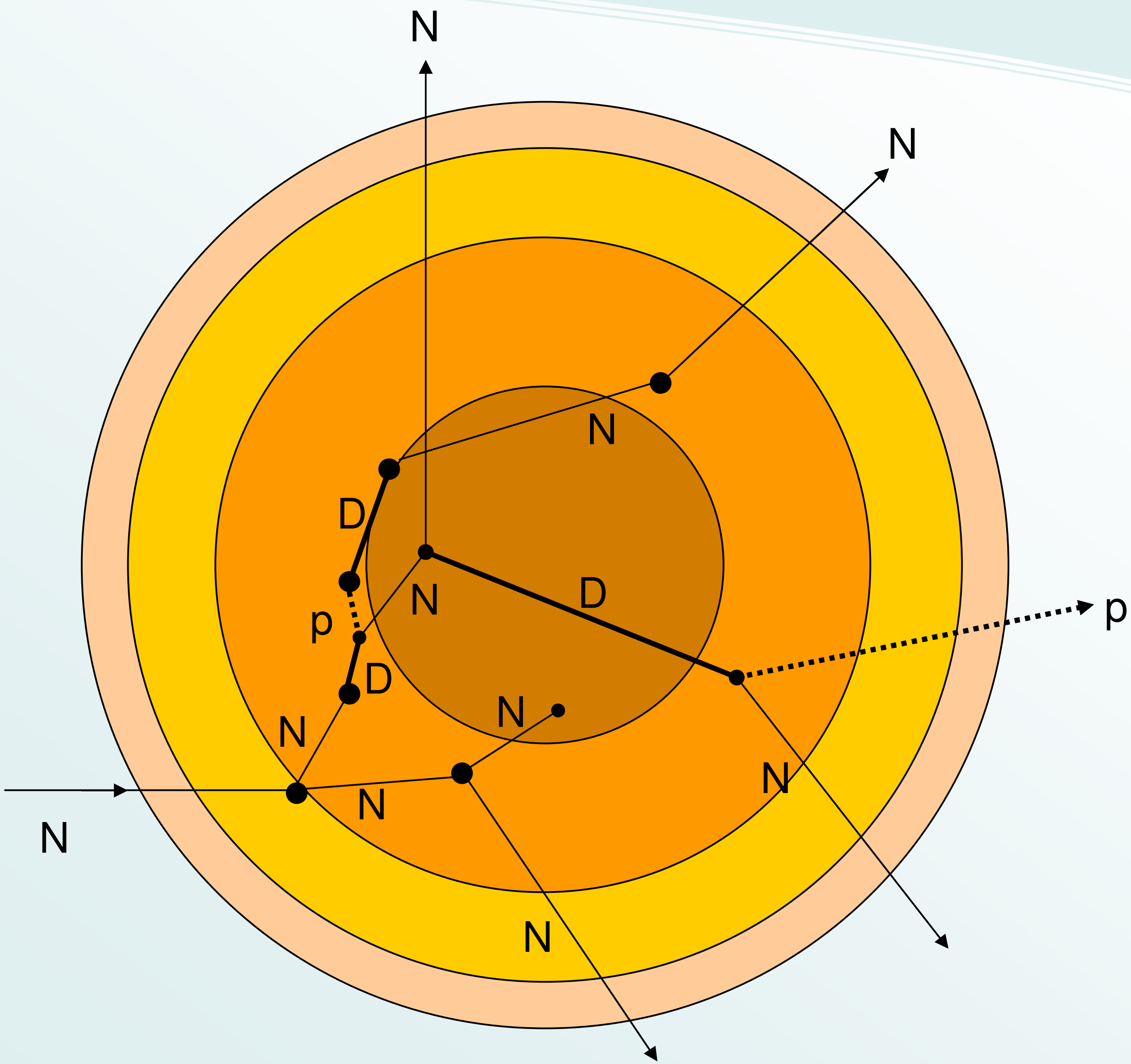
$$N + N \Leftrightarrow \Delta_{33} + N$$

$$\Delta_{33} \Leftrightarrow \pi + N$$


General Initialization
Event Initialization
Time Step
Yes
Any Active Participants?
No
No
Summarize Event
Desired # of events reached?
Yes
Output


General Initialization
Event Initialization
Time Step
Yes
No
Any Active Participants?
No
Summarize Event
Desired # of events reached?
Yes
Output


General Initialization
Event Initialization
Time Step
Any Active Participants?
No
Summarize Event
Desired # of events reached?
Yes
No
Output
Push next Active Participant
Active Participant left target?
No
Yes
Interaction?
Yes
Record outgoing particle
Update Active Participants table & deplete F.S.
No
Any Active Participants not moved?
Yes
No


N
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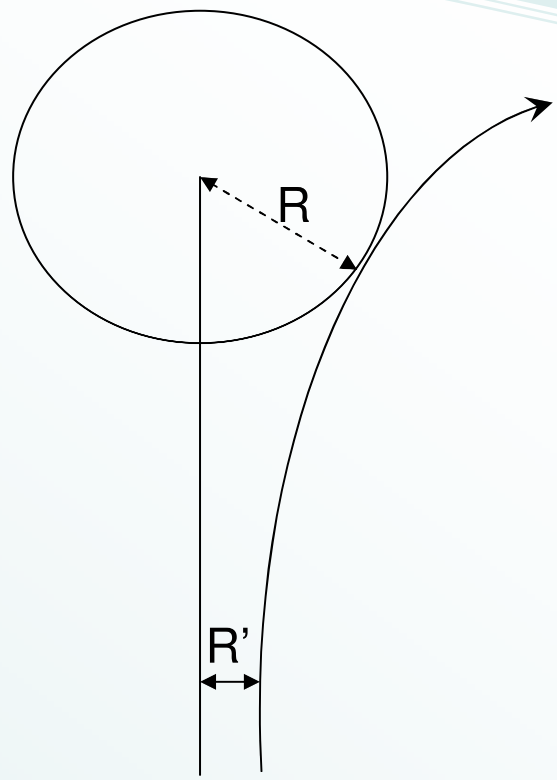
N

Output

- Total reaction cross-section

$$\sigma_R = \pi R^2 * \frac{N_{tot} - N_{Transp.}}{N_{tot}} * (1 - \frac{V_{Coul}(R)}{E_K^{\mathrm{Proj.}}})$$

- Outgoing particle statistics → "Fast" particle spectra
- Residual target momenta and excitation energy from:
  - In-Out balance
    or
  - Particle-Hole considerations



Nuclear Model

- Continuous charge distribution, e.g.

$$\rho(r) = \rho_0 / [1 + \exp(r - c)/a]$$

$$c = 1.07 A^{1/3} Fm; a = 0.55 Fm$$

Nucleus divided into several regions of constant density. Ratio of proton to neutron density Z/(A-Z)

- Momentum distribution - degenerate Fermi Gas

$$E_{F_i} = (\hbar^2 / 2m_i)(3\pi^2 \rho_i)^{2/3}$$

$$i = proton, neutron; m_i = nucleon\_mass; \rho_i = density$$

- Potential depth (J.N. Ginocchio, Phys. Rev. **C17**, 195 (1978))

$$V_i = E_{F_i} + (Separation\_Energy)_i$$

$$V_{\Delta^{++}} = V_p; V_{\Delta^+} = V_p + \frac{(V_p + V_n)}{3}; V_{\Delta^0} + \frac{(V_p + V_n)}{3} = V_n; V_{\Delta^-} = V_n$$

Hadron-Hadron Cross Sections (1)

- N+N
  - σ_{tot}, σ_{inel} , σ_{el}
    G.D.Harp, Phys. Rev. **C10**, 2387 (1974)
    Arndt phase shift analysis
  - $d\sigma_{el}/d\omega$
    P.C.Clements, L.Winsberg, UCRL 9043 (1960), unpublished

Hadron-Hadron Cross Sections (2)

- N+N → N+Δ
  - Type of outgoing N, Δ determined by Isotopic Spin consideration Z.Fraenkel, Phys. Rev. **130**, 2407 (1963)
  - Mass of Δ is chosen from distribution:

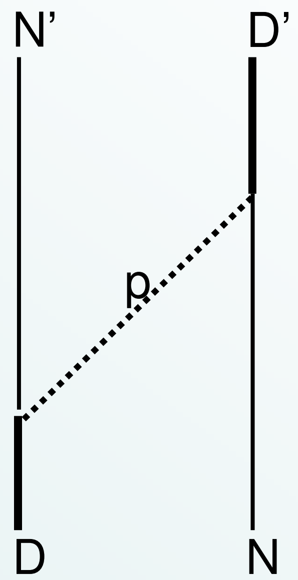
    $$P(m_\Delta, E_{cm}^{N+N}) = const. * \sigma_{tot}^{\pi^+ + p}(E_{cm}^{N+N}) * F(m_\Delta, E_{cm}^{N+N})$$

    $$m_\pi + m_N < m_\Delta < m_\pi + m_N + 500MeV$$

    F = two body phase factor for the produced N+Δ
    S.Lindenbaum and R. Sternheimer, Phys. Rev. **105**, 1874 (1957); **109**, 1723 (1958); **123**, 333 (1961)
  - $P(\cos_{cm})=.25+.75*(\cos_{cm})^2$

Hadron-Hadron Cross Sections (3)

- Δ+N → N+N (π capture)
  - Type of outgoing N, Δ determined by Isotopic Spin consideration
  - σ, dσ/dω calculated from inverse process (Δ production) using the principle of "detailed balance"
  - Δ production calculated using theoretical model (OPE)
- Δ+N → Δ'+N' ("exchange")
  - Naively two step process:
    - Decay of initial Isobar, Δ→π+N'
    - Interaction of decay π with another Nucleon, π+N→Δ'



G.D.Harp et al., Phys. Rev. **C6**, 581 (1973),

Z.Fraenkel, Phys. Rev. **130**, 2407 (1963)

Z.Fraenkel, Nuovo Cimento **30**, 512 (1963)

Hadron-Hadron Cross Sections (4)

- $\pi+N \rightarrow \Delta \rightarrow \pi'+N'$
  (elastic & charge exchange)
  - Experimental $d\sigma/d\omega$ + isospin considerations G.Giacomelli et al., CERN/HERA 69-1 (1969)
  - For Δ decaying without interaction proper $\pi+N$ differential cross section
  - Isotropic Δ decay after scattering or exchange

Hadron-Hadron Cross Sections (5)

- $\Delta \rightarrow \pi + N$
  - Energy dependant Δ width
    J.N. Ginocchio, Phys. Rev. **C17**, 195 (1978)

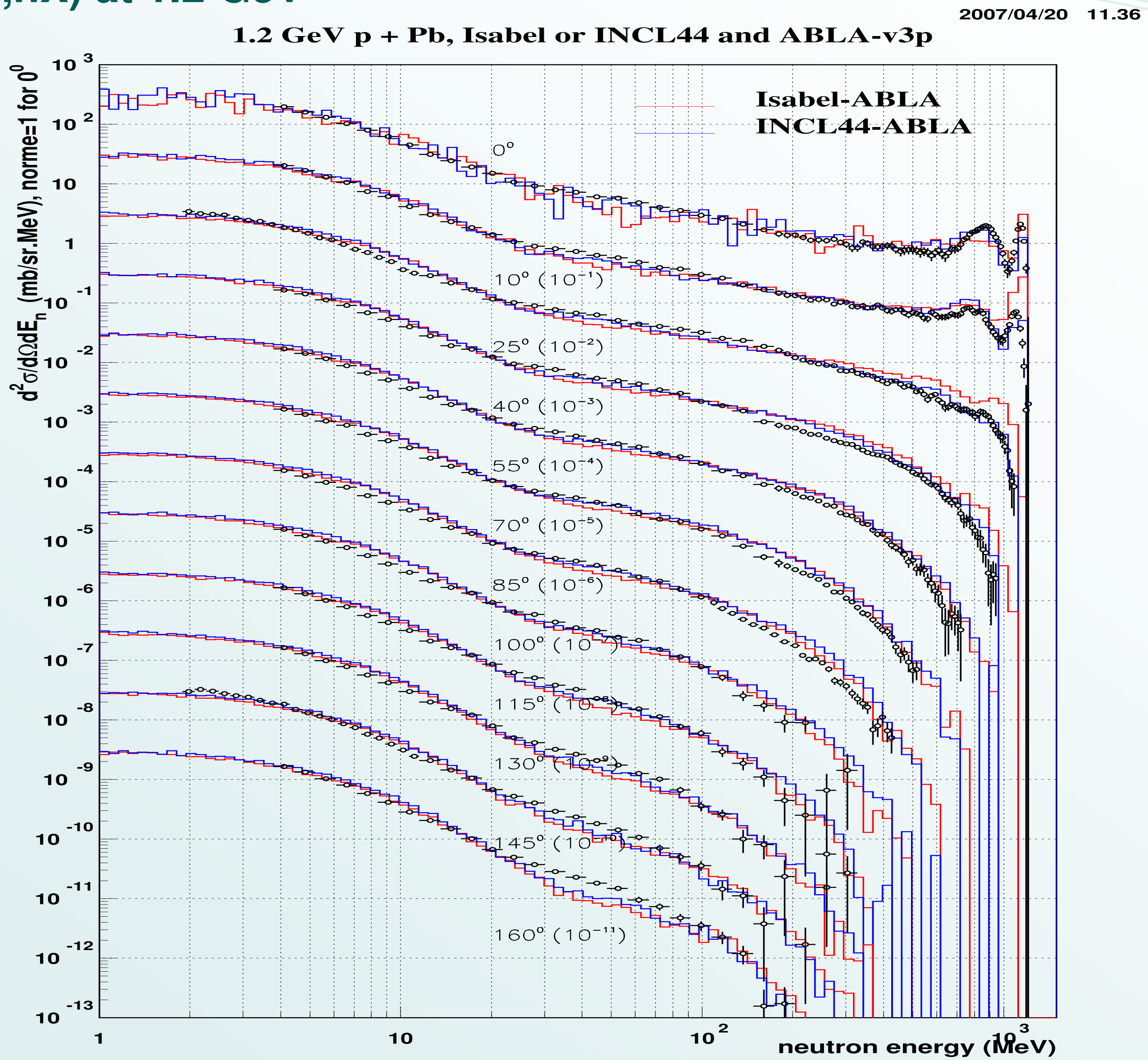
Density depletion

- After each interaction Fermi sea density, ρ_i, is depleted
  - **Fast rearrangement**: ρ_i of the "partner type" Fermi sea is uniformly reduced for the whole nucleus
  - **Slow rearangement**: "partner type" hole of radius *r* is punched in the position of the interaction. No interactions are allowed in the hole with particles of "partner type" .

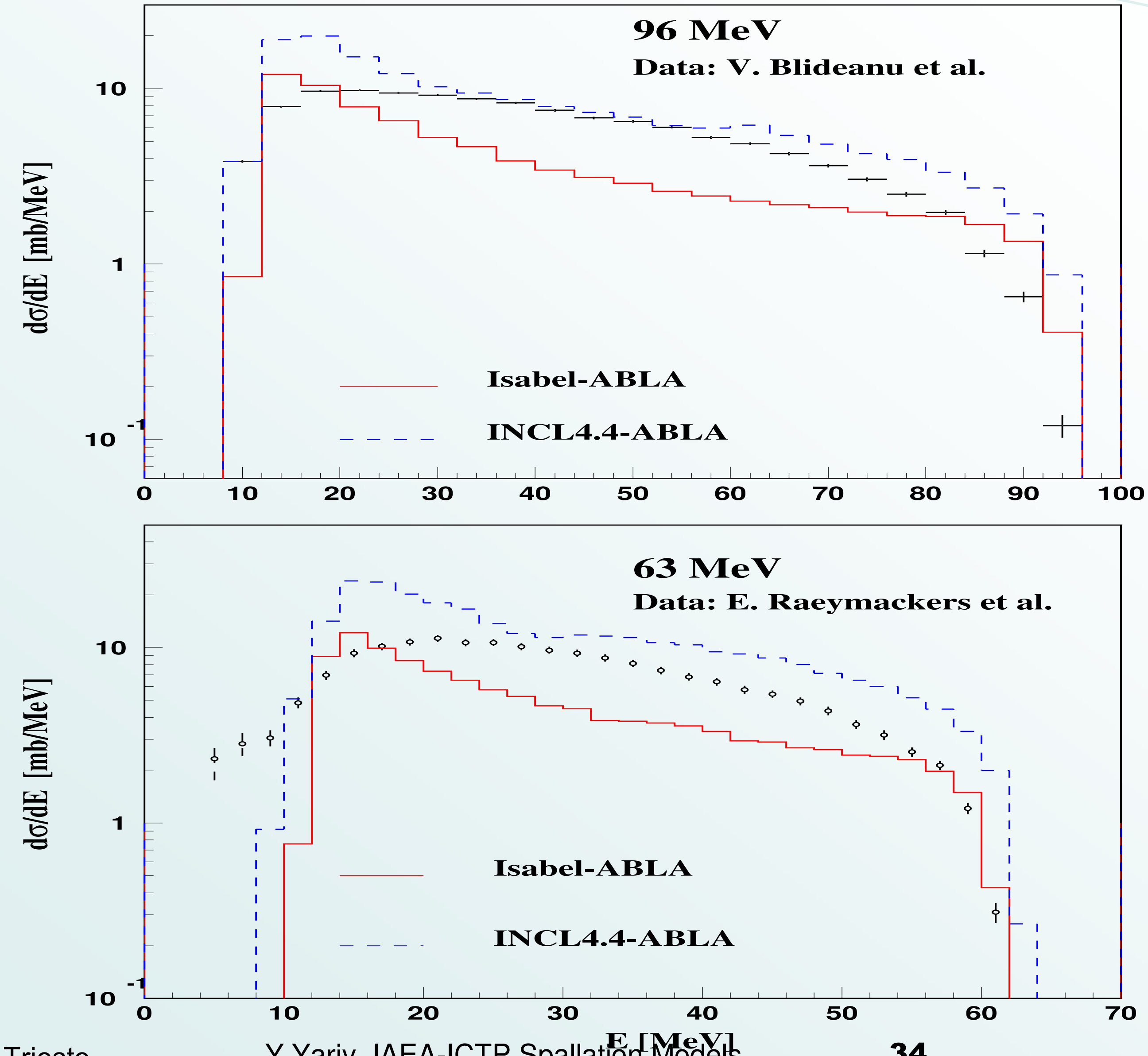
Pauli Blocking

- Options:
  - **Full Pauli Blocking**: Interaction resulting in nucleon falling below Fermi sea is forbidden
  - **"Depleted" Pauli Blocking:** Reaction resulting in nucleon falling below Fermi sea is allowed with probability of the relative depletion of the Fermi sea

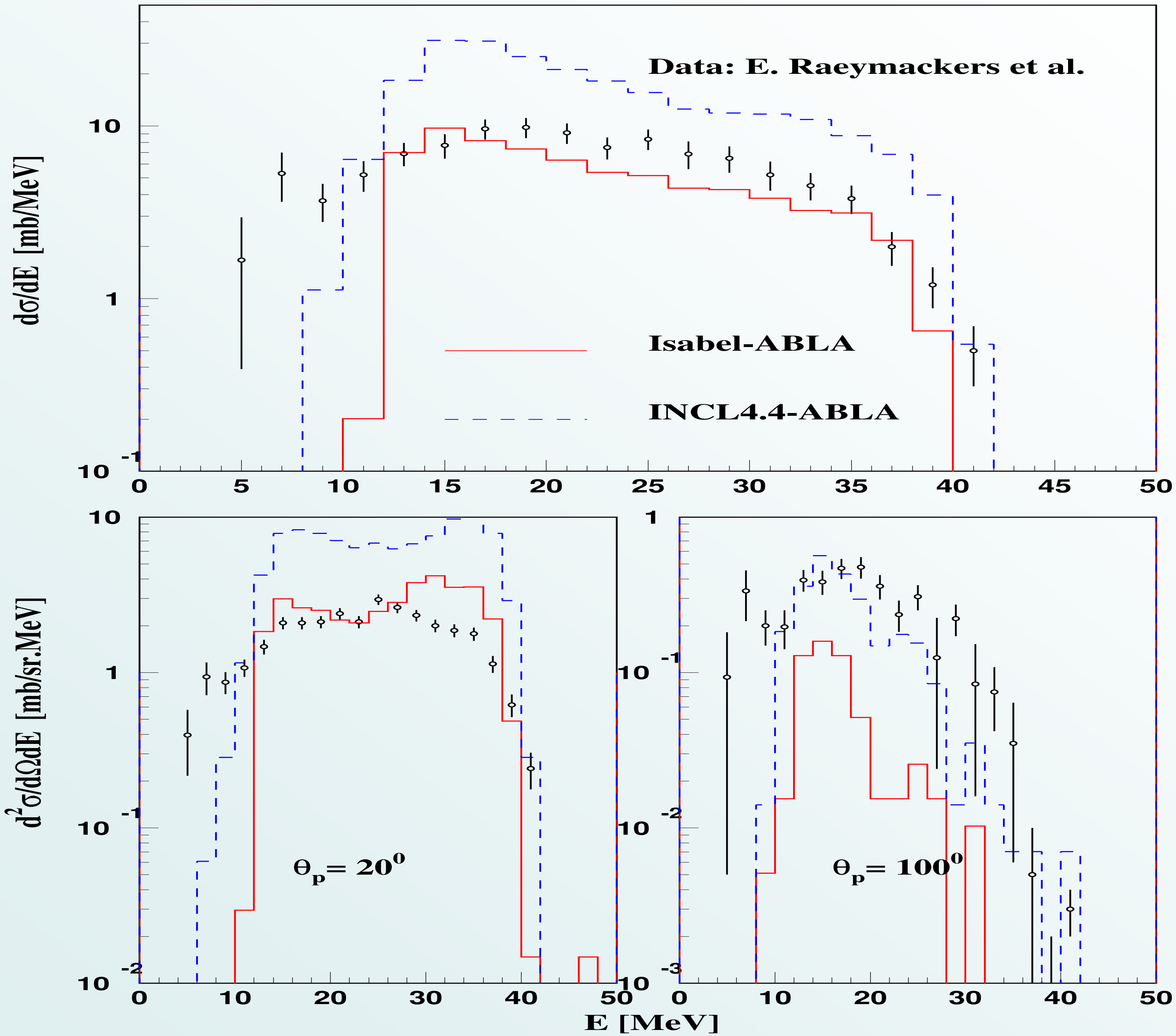
p(^{208}Pb,nX) at 1.2 GeV



n(^{208}Pb,pX) at 96 MeV, n(^{209}Bi,pX) at 63 MeV



n(^{209}Bi,pX) at 41 MeV



Thank You!

Questions, Remarks?